

DISCUSSION PAPER SERIES

No. 8932

THE ROLE OF SOCIAL NETWORKS AND PEER EFFECTS IN EDUCATION TRANSMISSION

Sebastian Bervoets,
Antoni Calvó-Armengol and Yves Zenou

*LABOUR ECONOMICS
and PUBLIC POLICY*



Centre for **E**conomic **P**olicy **R**esearch

www.cepr.org

Available online at:

www.cepr.org/pubs/dps/DP8932.asp

THE ROLE OF SOCIAL NETWORKS AND PEER EFFECTS IN EDUCATION TRANSMISSION

Sebastian Bervoets, Aix-Marseille School of Economics (GREQAM)
Antoni Calvó-Armengol*
Yves Zenou, Stockholm University, IFN and CEPR

Discussion Paper No. 8932
April 2012

Centre for Economic Policy Research
77 Bastwick Street, London EC1V 3PZ, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

This Discussion Paper is issued under the auspices of the Centre's research programme in **LABOUR ECONOMICS and PUBLIC POLICY**. Any opinions expressed here are those of the author(s) and not those of the Centre for Economic Policy Research. Research disseminated by CEPR may include views on policy, but the Centre itself takes no institutional policy positions.

The Centre for Economic Policy Research was established in 1983 as an educational charity, to promote independent analysis and public discussion of open economies and the relations among them. It is pluralist and non-partisan, bringing economic research to bear on the analysis of medium- and long-run policy questions.

These Discussion Papers often represent preliminary or incomplete work, circulated to encourage discussion and comment. Citation and use of such a paper should take account of its provisional character.

Copyright: Sebastian Bervoets, Antoni Calvó-Armengol and Yves Zenou

* Antoni Calvó-Armengol passed away in November of 2007. His friendship, energy and talents are sorely missed.

ABSTRACT

The Role of Social Networks and Peer Effects in Education Transmission

We propose a dynastic model in which individuals are born in an educated or uneducated environment that they inherit from their parents. We study the role of social networks on the correlation in the parent-child educational status independent of any parent-child interaction. We show that the network reduces the intergenerational correlation, promotes social mobility and increases the average education level in the population. We also show that a planner that encourages social mobility also reduces social welfare, hence facing a trade off between these two objectives. When individuals choose the optimal level of social mobility, those born in an uneducated environment always want to leave their environment while the reverse occurs for individuals born in an educated environment.

JEL Classification: I24, J13, Z13

Keywords: education, intergenerational correlation, social mobility, strong and weak ties

Sebastian Bervoets
Aix-Marseille School of Economics
GREQAM
Centre de la Vieille Charité
2, rue de la charité
13002 Marseille

Yves Zenou
Department of Economics
Stockholm University
10691 Stockholm
SWEDEN

Email:
sebastian.bervoets@univmed.fr

Email:
yves.zenou@ne.su.se

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=175313

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=126027

Submitted 30 March 2012

1 Introduction

Explaining the educational outcomes of children is one of the most challenging questions faced by economists. Most studies have found that school quality (e.g., Card and Krueger, 1992, 1996) and family background (e.g., Ermisch and Francesconi, 2001, Sacerdote, 2002, Plug and Vijverberg, 2003) have a significant and positive impact on the level of education of children. Parents obviously influence their children’s school performance by transmitting their genes to the children, but they also influence them directly, via, for example, their parenting practices and the type of schools to which they send their children (Björklund et al., 2006; Björklund and Salvanes, 2011). Neighborhood and peer effects have also an important impact on educational outcomes of children (Sacerdote, 2001; Durlauf, 2004; Ioannides and Topa, 2010; Sacerdote 2010; Patacchini and Zenou, 2011).

We study the intergenerational relationship between parents’ and offspring’s long-run educational outcomes.¹ In every society, people’s educational achievement is positively correlated with their parents’ education or with other indicators of their parents’ socio-economic status (Björklund and Salvanes, 2011). In the present paper, we illustrate how this correlation can result from *peer effects*, abstracting from any direct parental influence.

To be more precise, we develop a dynastic model where, at each period of time, with some probability, a person (the parent or the father) dies and is replaced by a new born (the child or the son). The son thereby never interacts with his father. Instead, individuals are born in an environment (local community) that they inherit from their parents. This environment is modelled as a *dyad* in which the newborn interacts with a partner, called his strong tie. This strong tie is precisely the individual who interacted in the father’s dyad before he died. Using the language of the cultural transmission literature (Bisin and Verdier, 2000, 2001), in our model, there is no vertical transmission (i.e. socialization inside the family) but only horizontal transmission (i.e. socialization outside the family).

We want to show that, even if a parent and his child never live together at the same time (and thus never interact with each other), there is still a significant correlation between the educational achievement of the father and the son because of peer effects, i.e. the son

¹See Björklund and Jäntti (2009), Black and Devereux (2011) and Björklund and Salvanes (2011) for recent reviews of intergenerational transmission of income and education.

is exposed to the same (educational) environment as his father. In our model, individuals meet both weak and strong ties, where weak ties are modelled as members of other dyads.

Individuals choose their optimal educational efforts but we assume that interacting with an uneducated individual (strong or weak tie) lowers the returns from effort. This cost captures the idea of *negative peer effects*, i.e. the fact that uneducated role models can distract individuals from educating themselves by, for example, proposing activities that are not related to education (watching TV, going to the movies, etc.). On the contrary, there is no cost of interacting with educated peers.

In a benchmark model, we first show that, indeed, even though the parent and his child never live together at the same time, there is a positive correlation between their educational status because there is a substantial overlap in the surroundings that have influenced their education decisions. This correlation is shown to be quadratic in the cost of inheriting of an uneducated local community. Indeed, this cost is the only ingredient of the model which differentiates individuals born in favorable environments from those born in unfavorable ones. The quadratic form of this correlation comes from the fact that relations between parents and children, rather than being direct, have to transit by the community.

We then build on this benchmark model to introduce the possibility for individuals to interact with peers outside of their local community, called weak ties. Whether the fraction of time spent with weak ties is exogenously fixed by a social planner, or is endogenously chosen by the individuals, the intergenerational correlation is significantly reduced, thereby promoting social mobility and increasing the average education level in the population. The network, defined here as the interactions with both types of ties, is thus shown to have a great importance, in terms of public policies aiming at reducing the social inertia communities are facing.

When the socializing decisions are exogenously chosen by the social planner, we show that if the planner's aim is to reduce the correlation in education status between generations (or equivalently, to promote social mobility), he will choose to mix individuals by encouraging them to interact with weak ties. By doing so, the social planner will also increase the average level of education in the population. However, we also show that this policy decreases social welfare. This is because the individuals born in a favorable environment are penalized by

such a policy while those born in an unfavorable environment are better off. The net effect turns out to be negative as the loss of the former are not totally compensated by the gains of the latter. This illustrates a common trade-off faced by a social planner between equity (i.e. decreasing intergenerational correlation) and efficiency.

When the socializing decisions are endogenously chosen by the individuals, those with uneducated strong ties always want to meet weak ties while the reverse occurs for individuals with educated strong ties. This is simply because the former will, in the worse case, meet another uneducated person and, at best, meet someone educated, and the reverse will happen for the latter. We show that when individuals can escape their inherited environment, intergenerational correlation decreases while the average education level in the population increases.

The rest of the paper unfolds as follows. In the next section, we relate our model to the relevant theoretical literatures. Section 3 exposes the model with no networks while Section 4 focuses on exogenous networks, i.e. exogenously fixed levels of social mixing. In Section 5, the network is endogenous since individuals choose how much time they spend with weak and strong ties. Finally, Section 6 concludes. All proofs of propositions, lemmas and remarks can be found in the Appendix.

2 Related literature

Our model is related to different literatures. First, it is related to the literature on peer effects in education. De Bartolome (1990) and Benabou (1993) are the standard references for peer and neighborhood effects in education. In this multi-community approach, individuals can acquire high or low skills or be unemployed. The costs of acquiring skills are decreasing in the proportion of the community that is highly skilled but this decrease is larger for those acquiring high skills. This leads to *sorting* although ex ante all individuals are identical. There are other models of sorting (see Fernández, 2003, for a survey) but they are quite different from our approach since they are static and focus on the role of peer effects in sorting. While there is an extensive empirical literature on the intergenerational transmission of income and education that focuses on the correlation of parental and chil-

dren’s permanent income or education (Björklund and Jäntti, 2009; Black and Devereux, 2011; Björklund and Salvanes, 2011), there are very few theoretical models studying this issue. Ioannides (2002, 2003) analyses the intergenerational transmission of human capital² by explicitly developing a dynamic model of human capital formation with a neighborhood selection. The idea here is to study the impact of parental education and the distribution of educational attainment within a relevant neighborhood on child educational attainment. From a theoretical viewpoint, Ioannides obtains a complete characterization of the properties of the intertemporal evolution of human capital. From an empirical viewpoint, he finds that there are strong neighboring effects in the transmission of human capital and that parents’ education and neighbors’ education have non linear effects that are consistent with the theory. Using a cultural transmission model à la Bisin and Verdier (2000, 2001), Patacchini and Zenou (2011) analyze the intergenerational transmission of education focusing on the interplay between family and neighborhood effects. They develop a theoretical model suggesting that both neighborhood quality and parental effort are of importance for the education attained by children. Their model proposes a mechanism explaining why and how they are of importance, distinguishing between high- and low-educated parents. Empirically, they find that the better is the quality of the neighborhood, the higher is the parents’ involvement in their children’s education. Finally, Calvó-Armengol and Jackson (2009) study the intergenerational transmission of education where an individual sees higher returns to adopting a behavior as many neighbors adopt the behavior (strategic complements in actions). They find that the parent-child correlation can exist without direct parental effect. Our paper is however quite different to these approaches since we explicitly model the social network each individual is embedded in and analyze how it affects the transmission of education.

Second, our paper is also related to the social network literature. There is a growing interest in theoretical models of peer effects and social networks (see e.g. Akerlof, 1997; Glaeser et al., 1996; Ballester et al., 2006; Calvó-Armengol et al., 2009; Goyal, 2007; Jackson, 2008; Ioannides, 2012). There is, however, nearly no theoretical model that looks at the impact

²See also Borjas (1992) who shows that the average human capital level of the ethnic group in the parents’ generation plays a crucial role in intergenerational mobility, and slows down the convergence in the average skills of ethnic groups across generations.

of social networks on education and, in particular, on the intergenerational transmission of education. In the present paper, we model the network as the interaction between strong and weak ties. In his seminal contributions, Granovetter (1973, 1974, 1983) defines *weak ties* in terms of lack of overlap in personal networks between any two agents, i.e. weak ties refer to a network of acquaintances who are less likely to be socially involved with one another. Formally, two agents A and B have a weak tie if there is little or no overlap between their respective personal networks. Vice versa, the tie is *strong* if most of A's contacts also appear in B's network. In this context, Granovetter (1973, 1974, 1983) develops the idea that weak ties are superior to strong ties for providing support in getting a job. Indeed, in a close network where everyone knows each other, information is shared and so potential sources of information are quickly shaken down so that the network quickly becomes redundant in terms of access to new information. In contrast Granovetter stresses the *strength of weak ties* involving a secondary ring of acquaintances who have contacts with networks outside ego's network and therefore offer new sources of information on job opportunities.³ In our model, we stress the role of strong ties as an important mean for the transmission of education. In other words, even though there is no direct influence from the parents, their indirect influence through the inheritance of strong ties affects positively the correlation between the parent and the child.

3 The benchmark model without social networks

3.1 Model

There are n individuals in the economy⁴. We assume that individuals belong to mutually exclusive two-person groups, referred to as *dyads*. We say that two individuals belonging to the same dyad hold a *strong tie* to each other. We assume that dyad members do not change over time unless one of them dies. A strong tie is created once and for ever and can

³For surveys on this issue, see Ioannides and Loury (2004) and Topa (2011). For early models on weak and strong ties, see Montgomery (1994) and Calvó-Armengol et al. (2007).

⁴We assume throughout that n is large, and all the propositions in the paper should be understood as limiting propositions

never be broken. Thus, we can think of strong ties as links between members of the same family, or between very close friends. In this section and only here, we assume that different dyads do not interact.

We consider a dynamic model, where, at each period, each individual in the dyad can die with probability $1/n$. When a person dies, he is automatically replaced by a new born who is his son. The son is then matched with the individual who was previously in the same dyad (strong tie) as his father. As stated in the introduction, this is the way we capture the interaction between a father and a son. The only aspect that the son inherits from his father is his father's social environment or local community, here the father's strong tie. There is no other interaction between the father and the son. In particular, the father and the son never live at the same time. This is because we want to analyze the effect of the environment (peer effects) on the child's education outcomes, independent of any parent-child interaction.

Individuals can be of two types: $j = 0$ (non-educated) or $j = 1$ (educated). We now describe how individuals determine their type. Each individual i is born with a λ_i (her *ability to learn education*), which is uniformly distributed on the interval $[0, 1]$. The new born knows the type of his partner and his own λ_i and he must decide his optimal education effort level, given his ability λ_i and the type j of his strong tie. Observe that the education decision is only made once and for all, at the birth of the individual. This implies that no one can change his type during his lifetime.

The utility of individual i (of type λ_i) who meets individual j and exerts effort e_{ij} is given by:

$$U_{i0}(\lambda_i) = \lambda_i e_{i0} - \frac{1}{2} e_{i0}^2 - \alpha e_{i0}$$

if he meets an uneducated person ($j = 0$), and

$$U_{i1}(\lambda_i) = \lambda_i e_{i1} - \frac{1}{2} e_{i1}^2$$

if he meets an educated person ($j = 1$). In this formulation, higher ability individuals obtain higher benefits from education, i.e. $\lambda_i e_{ij}$ is increasing in λ_i . The cost of effort is increasing and convex in effort and is equal to $\frac{1}{2} e_{ij}^2$. We assume that there is an extra cost of exerting effort in education, αe_{i0} with $\alpha \geq 0$, when meeting someone *who is not educated*. This cost captures the idea of *negative peer effects*, i.e. the fact that uneducated role models

can distract individuals from educating themselves by, for example, proposing activities that are not related to education (watching TV, going to the movies, etc.). There is plenty of evidence of positive and negative peer effects in education (see, in particular, Sacerdote, 2001; Ioannides, 2003; Goux and Maurin, 2007; Calvó-Armengol et al., 2009; Patacchini and Zenou, 2011). First-order conditions yield:

$$e_{i0} = \max\{0, \lambda_i - \alpha\} \quad (1)$$

$$e_{i1} = \lambda_i \quad (2)$$

We assume that when someone is not educated, he obtains an exogenous utility level equal to $\bar{U} > 0$ (his outside option, which could be, for example, the minimum wage). We would like now to calculate the threshold value of λ , denoted by $\tilde{\lambda}_0$ (resp. $\tilde{\lambda}_1$) above which an individual i who has an *uneducated* (resp. *educated*) strong tie will get educated. An individual with an uneducated strong tie will be indifferent between education and non-education if and only if:

$$U_{i0}(\tilde{\lambda}_0) = \bar{U}$$

while, for an individual with an educated strong tie, we have:

$$U_{i1}(\tilde{\lambda}_1) = \bar{U}$$

Solving these equations lead to:

$$\tilde{\lambda}_0 = \sqrt{2\bar{U}} + \alpha \quad (3)$$

$$\tilde{\lambda}_1 = \sqrt{2\bar{U}} \quad (4)$$

so we assume (to have interior solutions)

$$0 < \sqrt{2\bar{U}} < \sqrt{2\bar{U}} + \alpha < 1 \quad (5)$$

In equilibrium, we obtain the following utilities:

$$U_{i0}^*(\lambda_i) = \max\left\{\bar{U}, \frac{(\lambda_i - \alpha)^2}{2}\right\}$$

$$U_{i1}^*(\lambda_i) = \max \left\{ \bar{U}, \frac{\lambda_i^2}{2} \right\}$$

Let us now calculate the probability p_0 (resp. p_1) that an individual with an uneducated (resp. educated) strong tie will get educated. We easily obtain:

$$p_0 = 1 - \tilde{\lambda}_0 = 1 - \sqrt{2\bar{U}} - \alpha \quad (6)$$

$$p_1 = 1 - \tilde{\lambda}_1 = 1 - \sqrt{2\bar{U}} \quad (7)$$

The following figure summarizes the probability of getting educated for all individuals.

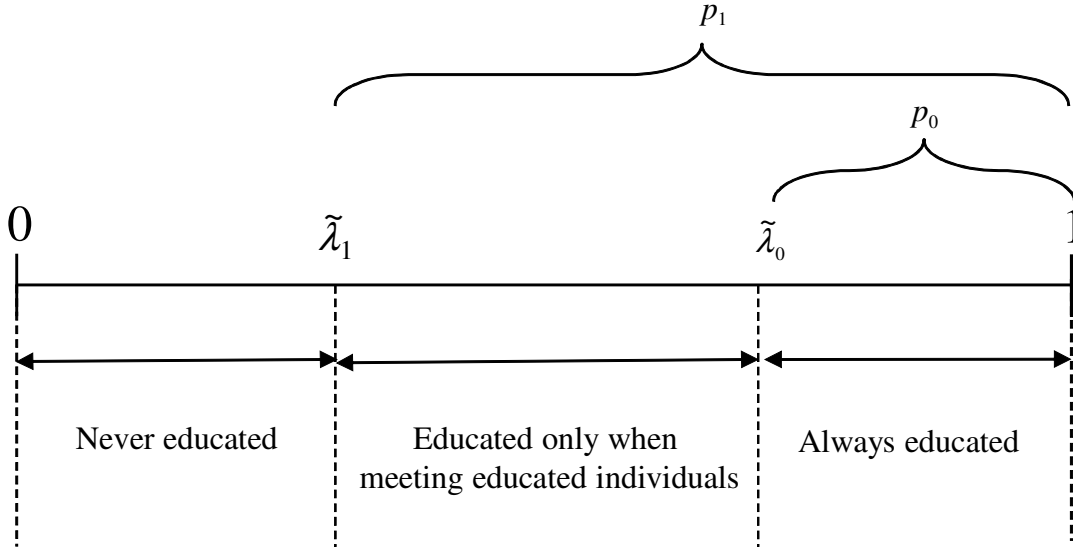


Figure 1: The different probabilities of being educated

3.2 Steady-state equilibrium

So far, we have described what happens within a period. Let us now explain the dynamics of the model and determine the steady-state equilibrium. Each generation of a given dynasty (or family) consists of one individual. At the beginning of each period, one dynasty is randomly chosen and its member replaced by a new individual. This happens with equal

probability across dynasties. We refer to the *new individual* as the *child* and to the *old individual* as the *parent*.

In equilibrium, the share η_1 of educated individuals is given by

$$\eta_1 = 1 - \tilde{\lambda}_0 + (\tilde{\lambda}_0 - \tilde{\lambda}_1)\eta_1 = p_0 + (p_1 - p_0)\eta_1$$

Indeed, the fraction of educated individuals are either those who have an ability between $\tilde{\lambda}_0$ and 1 since they get educated whatever the status of their partner (see Figure 1) or those who have an ability between $\tilde{\lambda}_1$ and $\tilde{\lambda}_0$ and who are matched with an educated partner (this happens with probability η_1). Rearranging this expression, we obtain:

$$\eta_1 = \frac{p_0}{1 + p_0 - p_1}$$

Using (6) and (7), we obtain:

$$\eta_1 = \frac{1 - \sqrt{2\bar{U}} - \alpha}{1 - \alpha} \quad (8)$$

Of course, when $\sqrt{2\bar{U}} = 0$ (i.e. there is no outside option) then everyone will prefer getting educated, while if $\sqrt{2\bar{U}} = 1 - \alpha$ (i.e. $\bar{U} = \frac{(1-\alpha)^2}{2}$ which is the highest possible payoff when getting educated) then no one will get educated.

3.3 The correlation in education between parents and children

We would now like to calculate the intergenerational correlation in education between parents and children. Though they do not interact with each other, there is a correlation which goes through the social network (i.e. strong tie) the parent “transmits” to his child. Let X refer to the education status of the parent and Y to the education status of the child. The *intergenerational correlation* is given by:

$$Cor(X, Y) = \frac{Cov(X, Y)}{\sqrt{Var(X)}\sqrt{Var(Y)}}$$

where $Cov(X, Y)$ is the *covariance* between the educational status of the parent and the child while $Var(X)$ and $Var(Y)$ are the *variances* of the statuses of the parent and the child. We have:

$$Cov(X, Y)_{dyad} = \mathbb{E}[(X = 1)(Y = 1)] - [\mathbb{E}(X = 1)] [\mathbb{E}(Y = 1)] = \eta_{11} - \eta_1^2$$

$$\text{Var}(X)_{dyad} = \mathbb{E}[X = 1] - [\mathbb{E}(X = 1)]^2 = \eta_1 - \eta_1^2$$

where η_1 is the *marginal probability* that an individual chooses state 1, i.e. the probability of being educated in steady state (it is given by (8)), and η_{11} , the *joint probability* that an individual is in state 1 and his father was in state 1.

Individuals can be in either of two different states: educated (state 1) and uneducated (state 0). Dyads, which consist of paired individuals, are, in steady state, in one of three different states:

- (i) both members are educated (11);
- (ii) one member is educated and the other is not (01) or (10);
- (iii) both members are uneducated (00).

The steady state distribution of dyads is given by $\mu = \{\mu_{00}, \mu_{10}, \mu_{01}, \mu_{11}\}$, where μ_{ij} stands for the *fraction of dyads* in state (ij) .⁵ Obviously, by symmetry, $\mu_{10} = \mu_{01}$.

Proposition 1 *Assume (5). When dyads do not interact with each other, the parent-child correlation is equal to:*⁶

$$\text{Cor}_{dyad} = \alpha^2 \tag{9}$$

This is an interesting result because it shows that the correlation between a father's and a son's statuses is *positive*, even though they never interact with each other. The intuition is simple: when the father is educated, his strong tie is likely to be educated (because the father has influenced him) and therefore the son will benefit from a favorable environment (i.e. an educated strong tie) and, as a result, his chances to be educated are higher.

In our setting, the correlation (9) increases with α , the cost of interacting with an uneducated strong tie. Indeed, the difference in individual efforts ($e_{i1} - e_{i0}$) between meeting an educated and an uneducated strong tie is equal to α . If this difference is small, it is almost the same to be matched with an educated or a non-educated partner and newborns decide

⁵Alternatively, μ_{ij} can be interpreted as the *fraction of time* a typical dyad spends in state (ij)

⁶More generally, if λ is distributed between $\underline{\lambda}$ and $\bar{\lambda}$, then the correlation is given by $\left(\frac{\alpha}{\bar{\lambda} - \underline{\lambda}}\right)^2$.

whether to educate or not independently of the status of their strong tie. If this difference is large, individuals' decisions strongly depend on the status of their partner.

The quadratic form of the correlation is due to the pattern of influences between parents and children, as they transit through the community. This “two-step” mechanism explains the square.

We notice that α can also be written as $p_1 - p_0$. Indeed, p_1 is the probability of acting in the *same* way as an educated dyad partner while p_0 is the probability of choosing the *opposite* of an uneducated dyad partner. For instance, if $p_1 = 1$ and $p_0 = 0$, which means that individuals always act according to their strong tie, then the correlation between the education status of parents and children is perfect, i.e. $Cor_{dyad} = 1$. If $p_1 = p_0$, which means that individuals act as frequently the same way and the opposite way as their strong tie, then there is no correlation at all and $Cor_{dyad} = 0$. Finally, if $p_0 = 1$ and $p_1 = 0$, which means that individuals always act exactly in the opposite way as their strong ties, then the correlation is also perfect, i.e. $Cor_{dyad} = 1$.

This benchmark case illustrates in a very simple model how positive correlation can appear as a result of indirect transmission of behavior through peers, as was pointed out by Calvó-Armengol and Jackson (2009). However, individuals usually interact with people outside of their local community and this might have a significant impact on correlation. We explore this in the next section by introducing weak ties.

4 Social networks and exogenous interactions

We introduce social networks by assuming that individuals are exogenously forced to interact with people outside of their own local community. We assume that a newborn will spend a fraction ω of his time with *weak ties* and a fraction $1 - \omega$ of his time with his *strong tie*⁷. In this framework, ω captures the intensity of social interactions of each individual which we interpret as his social network, since it indicates how often an individual meets people outside his dyad.

⁷Observe that strong ties and weak ties are assumed to be substitutes, i.e. the more someone spends time with weak ties, the less he has time to spend with his strong tie.

We consider ω to be set exogenously in this section and interpret it as a public policy of social integration or social mixing since by increasing ω , one raises the chance that educated and uneducated individuals meet. Such policies, like the Moving to Opportunity (MTO) programs (Katz et al., 2001), have been implemented in the United States. By giving housing assistance to low-income families, the MTO programs help them relocate to better and richer neighborhoods. The results of most MTO programs (in particular for Baltimore, Boston, Chicago, Los Angeles and New York) show a clear improvement of the well-being of participants and better labor market outcomes (see, in particular, Ladd and Ludwig, 2001, Katz et al., 2001, Rosenbaum and Harris, 2001). We will endogenize ω in Section 5 below.

The timing is the same as in the previous section, except that a newborn will only spend a fraction $1 - \omega$ of his time in his dyad and a fraction ω with a weak tie, who is chosen at random in the rest of the population.⁸

4.1 Model

The utility of an individual i whose strong tie is uneducated is now given by:

$$U_{i0}(\lambda_i) = \lambda_i e_{i0} - \frac{1}{2} e_{i0}^2 - \omega(1 - \eta_1) \alpha e_{i0} - (1 - \omega) \alpha e_{i0}$$

where η_1 is the time spent with educated weak ties in steady state. Individual i , who has an uneducated strong tie, can either meet an uneducated weak tie (with probability $\omega(1 - \eta_1)$) and, as in the previous section, bears a penalty of α per unit of effort or, with probability $\omega\eta_1$, this individual can meet an educated weak tie and does not suffer any negative peer effect. We can write $U_{i0}(\lambda_i)$ as:

$$U_{i0}(\lambda_i) = \lambda_i e_{i0} - \frac{1}{2} e_{i0}^2 - (1 - \omega\eta_1) \alpha e_{i0}$$

⁸Note that because of this randomness, the utility which follows should be considered as an *expected* utility. However, we can interpret the model as follows: instead of meeting someone at random who will be educated or not, the newborn meets everyone and splits the fraction ω of his time between everyone. In that case we do not deal with expected utility, therefore we have chosen not to write the problem in terms of expected utilities. However in what follows, we sometimes interpret some terms as probabilities instead of as fractions of time, for simplicity.

Similarly, the utility of an individual i who has an educated strong tie is given by:

$$U_{i1}(\lambda_i) = \lambda_i e_{i1} - \frac{1}{2} e_{i1}^2 - \omega(1 - \eta_1) \alpha e_{i1}$$

The first-order conditions give:

$$e_{i0} = \max \{0, \lambda_i - (1 - \omega\eta_1) \alpha\} \quad (10)$$

$$e_{i1} = \max \{0, \lambda_i - (1 - \eta_1) \omega \alpha\} \quad (11)$$

Plugging back e_{ij} in the utility function and accounting for the outside option \bar{U} yields:

$$U_{i0}(\lambda_i) = \max \left\{ \bar{U}, \frac{[\lambda_i - (1 - \omega\eta_1) \alpha]^2}{2} \right\} \quad (12)$$

$$U_{i1}(\lambda_i) = \max \left\{ \bar{U}, \frac{[\lambda_i - (1 - \eta_1) \omega \alpha]^2}{2} \right\} \quad (13)$$

As in the previous section, we can determine the threshold values $\tilde{\lambda}_0$ and $\tilde{\lambda}_1$ as follows:

$$\tilde{\lambda}_0 = \sqrt{2\bar{U}} + (1 - \omega\eta_1) \alpha \quad (14)$$

$$\tilde{\lambda}_1 = \sqrt{2\bar{U}} + (1 - \eta_1) \omega \alpha \quad (15)$$

The probability p_0 that an individual with an uneducated strong tie will get educated and the probability p_1 that an individual with an educated strong tie will get educated are given by:⁹

$$p_0 = 1 - \tilde{\lambda}_0 = 1 - \sqrt{2\bar{U}} - (1 - \omega\eta_1) \alpha \quad (16)$$

$$p_1 = 1 - \tilde{\lambda}_1 = 1 - \sqrt{2\bar{U}} - (1 - \eta_1) \omega \alpha \quad (17)$$

In order to close the model, we determine the value of η_1 as follows:

$$\begin{aligned} \eta_1 = & \eta_1 \{ (1 - \omega)p_1 + \omega [\eta_1 p_1 + (1 - \eta_1)p_0] \} \\ & + (1 - \eta_1) \{ (1 - \omega)p_0 + \omega [\eta_1 p_1 + (1 - \eta_1)p_0] \} \end{aligned}$$

⁹Below we make sure that these probabilities are between 0 and 1.

Indeed, in equilibrium, a newborn gets educated if either (i) he meets an educated strong tie (probability η_1), spends a fraction $1 - \omega$ of his time with this strong tie and gets educated (probability p_1) and spends a fraction ω of his time with a weak tie who can be either educated and the newborn gets educated (probability $\eta_1 p_1$) or who can be uneducated and the newborn gets educated (probability $(1 - \eta_1)p_0$) or (ii) meets an uneducated strong tie (probability $1 - \eta_1$), spends a fraction $1 - \omega$ of his time with this strong tie and gets educated with probability p_0 and spend a fraction ω of his time with a weak tie who can be either educated and the newborn gets educated (probability $\eta_1 p_1$) or who can be uneducated and the newborn gets educated (probability $(1 - \eta_1)p_0$).

This expression can be simplified and we easily obtain:

$$\eta_1 = \frac{p_0}{1 + p_0 - p_1} \quad (18)$$

By replacing p_0 and p_1 by their values in (16) and (17), and solving in η_1 , we obtain:

$$\eta_1 = \frac{1 - \alpha - \sqrt{2\bar{U}}}{1 - \alpha} \quad (19)$$

and $0 \leq \eta_1 \leq 1$ if

$$0 \leq \sqrt{2\bar{U}} \leq 1 - \alpha \quad (20)$$

which also implies that both p_0 and p_1 are between 0 and 1.

Looking at (18) and (19), it is easily verified that the individual probability of being employed, η_1 , is increasing in both p_0 and p_1 and decreasing in α . Furthermore, p_0 is increasing in the time spent with weak ties, ω , while p_1 is decreasing with ω . Finally, η_1 , p_0 and p_1 are all decreasing in \bar{U} . These results are easy to interpret. Take for instance \bar{U} . When the outside option increases, a higher share of individuals will prefer not investing in efforts and the fraction of educated η_1 will decrease. The same happens when the penalty α increases, because the educational effort of individuals decreases (the productivity of education decreases) and hence the utility level is reduced and a larger share of the population will be below \bar{U} , decreasing the share of educated individuals. Finally, when ω increases, each individual spends more time with weak ties. If his strong tie is uneducated, then his probability of being educated p_0 increases because he has more chances to meet an educated

weak tie. If his strong tie is educated, he spends less time with his strong tie who is educated and bears the risk of paying a cost α if matched with an uneducated weak tie.

4.2 Steady-state equilibrium and intergenerational correlation

We are now able to determine the intergenerational correlation. We have:

Proposition 2 *Assume (20). When dyads interact with each other, the parent-child correlation is equal to:*

$$Cor_{exo} = (1 - \omega)^4 \alpha^2 \quad (21)$$

When α increases, which means that the penalty of meeting someone who is uneducated increases, the correlation increases because the influence of strong ties is higher. And as seen in the previous section, this induces an increase in correlation which takes a quadratic form.

On the contrary, when ω increases, the correlation is naturally reduced because individuals are more influenced by their weak ties than their strong ties (i.e. the father’s influence). This is an interesting result, especially from a policy viewpoint. If the planner wants to *encourage social mobility* (Piketty, 1995), he can either decrease α or, more simply and more efficiently, increase ω . This will increase the interaction between low- and high-educated families, which can help the former increase the educational level of their children (peer effects). As a result, any policy promoting “social” integration or “social” mixing (meaning here that low-educated individuals meet more educated families) would also have positive effects on these children’s educational achievement. This model can therefore partly explain the success of the MTO programs, explained earlier. Another illustration of this type of policy (social mixing) is to have school reforms such as the ones implemented in different countries, which aim was to prolong schooling at the lower secondary level. Indeed, such a reform can be interpreted as an early investment in human capital that is complementary to later skills that are acquired (Cunha et al., 2007). If this investment has a stronger effect on pupils with a disadvantaged background, we will expect that it may lead to a higher probability of completion of high school and university, and thus make completion of these degrees less dependent on family background. One may also expect that staying longer in school will lead to a change in preferences for the value of schooling and in time preferences,

leading to more investment in human capital, especially for children with less educated parents. Several recent papers have analyzed the impact of these comprehensive school reforms, particularly in Europe and Scandinavia, on aspects related to the persistence of education across generations. Meghir and Palme (2005) and Aakvik et al. (2010) analyze the effect on earnings and educational attainment of the comprehensive school reforms that took place in Sweden in the 1950s and Norway in the 1960s, respectively, where mandatory schooling was extended by two years and all students had to attend the same track. The reforms used in the studies were implemented as a natural experiment, with the new mandatory schools being adopted at different times in different municipalities. Both studies find support for a weakening of the effect of family background for disadvantaged pupils with parents with low educational attainment. Pekkarinen et al. (2009) more directly assess the effect on the persistence in income across generations using a similar reform in Finland in the 1970s, but the focus of this reform was more explicitly on reduced tracking. They find support for a significant decrease in the intergenerational income elasticity (for fathers and sons) in Finland from about 0.29 to about 0.23. This is a quite strong effect, given that the standard result is that intergenerational income elasticity in the U.S. and U.K. is about 0.40 and about 0.20 in other Nordic countries and Canada (Björklund and Jäntti, 2009). In our model, if the planner wants to increase social mobility and thus to reduce the role of family background on children's educational outcomes, then prolonging schooling is a way to make children from different backgrounds to interact with each other and to increase the chance for the disadvantaged students to be educated. In our model, the planner can perform such a policy by decreasing the intergenerational correlation between parents and children. This will be analyzed in the next section.

Observe that, in our model, children inherit from their parents a local community with a certain education status. If we fix α , then increasing ω from 0.1 to 0.5 reduces the correlation Cor_{exo} by more than 90 percent and thus increase social mobility. Of course, while individuals with uneducated strong ties are favored by such policies, those with educated strong ties are reluctant to go outside their dyad and take the risk of meeting uneducated weak ties. This is related to the standard policy debate on desegregation (Guryan, 2004; Rivkin and Welch, 2006) where mixing students of different backgrounds favors the less educated ones but have

a negative effect on the more educated students.¹⁰

4.3 Welfare analysis

As stated above, an increase in ω has a positive effect on low-educated individuals but can be harmful for high-educated individuals. Because of this trade-off, we would like now to study the welfare consequences of this issue. The total welfare is given by:

$$\mathcal{W} = \int_0^{\tilde{\lambda}_1} \bar{U} d\lambda + \int_{\tilde{\lambda}_1}^{\tilde{\lambda}_0} (1 - \eta_1) \bar{U} d\lambda + \int_{\tilde{\lambda}_1}^1 \eta_1 U_{i1}(\lambda) d\lambda + \int_{\tilde{\lambda}_0}^1 (1 - \eta_1) U_{i0}(\lambda) d\lambda \quad (22)$$

The social planner can have two objectives. First, he might want to maximize the sum of utilities (22) of all agents. Second, he might want to minimize the intergenerational correlation in educational status between the father and the son. Minimizing this correlation is equivalent to reducing the impact of family background (measured here by the “neighborhood quality or peers” that the child inherits from his parent) on the child’s educational attainment, a policy that has been adopted by most democratic societies (Björklund and Salvanes, 2011). However, these two objectives are contradictory:

Proposition 3

- (i) *If the objective of the planner is to maximize total welfare (22), then it is optimal to set the time spent with weak ties to $\omega^o = 0$.*
- (ii) *If the objective is to minimize the intergenerational correlation, i.e. to favor social mobility, then it is optimal to set the time spent with weak ties to $\omega^o = 1$.*

This proposition shows that, depending on the objective function, the outcome may be very different. Indeed, if the planner maximizes the sum of all utilities, then because both educated and uneducated families matter, he will impose that individuals only stay with their strong ties, i.e. $\omega^o = 0$. This implies that the correlation in employment status between the father and the son will be equal to α^2 . This is because it is always costly for sons whose

¹⁰See Sáez-Martí and Zenou (2012) who obtain a similar result using a model of cultural transmission and statistical discrimination.

strong ties are educated to meet weak ties since, if they meet an educated weak tie, the benefit is zero while, if they meet an uneducated one, the cost is αe_{i1} . This is, however, less true for sons who have an uneducated strong tie since if they don't meet weak ties, they will always pay a cost of αe_{i1} while, if they meet weak ties, there is a probability $\omega \eta_1$ that they will pay no cost by meeting an educated weak tie. Since the latter effect is weaker than the former (because both utilities are quadratic in efforts, which implies that the loss of utility of a person with an educated strong tie meeting weak ties is not sufficiently compensated by the gain of utility of a person with an uneducated strong tie meeting educated weak ties), the planner finds it optimal to set $\omega^o = 0$. On the contrary, when the objective is to promote social mobility by minimizing the intergenerational correlation in educational status between the father and the son, then this trade off does not exist anymore and therefore the planner sets $\omega^o = 1$ so that the correlation is equal to zero, which means that a son who inherits an educated strong tie and the one who inherits an uneducated strong tie have the same chance of becoming educated.

Remark 1 *Both the aggregate welfare and the correlation are decreasing and convex. Thus increasing ω decreases both the correlation and the welfare very fast.*

5 The role of social networks

We will now endogeneize ω so that individuals choose both educational effort and the time spent with their strong (or weak) tie. The timing is now as follows. At each period of time, a person (the father) dies at a random rate and is replaced by a newborn (the son) who takes his place in the dyad. The son then discovers the type of his strong tie (educated or not educated) as well as his λ_i . He then optimally decides ω_{ij} , the time spent with weak and strong ties and then e_{ij} the optimal education effort level.

5.1 Model

Individual ij (i.e. individual who is of type λ_i and who has a strong tie $j = 0, 1$) first decides ω_{ij} and then chooses e_{ij} . The utility of each individual ij who puts efforts ω_{ij} and e_{ij} is now

given by:¹¹

$$U_{i0}(\lambda_i) = \lambda_i e_{i0} - \frac{1}{2} e_{i0}^2 - (1 - \omega_{i0} \eta_1) \alpha e_{i0}$$

$$U_{i1}(\lambda_i) = \lambda_i e_{i1} - \frac{1}{2} e_{i1}^2 - \omega_{i1} (1 - \eta_1) \alpha e_{i1}$$

These utility functions are exactly the same as in the previous section. As usual, we solve the model backward. First-order conditions on efforts yield:

$$e_{i0}^* = \max \{0, \lambda_i - (1 - \omega_{i0} \eta_1) \alpha\}$$

$$e_{i1}^* = \max \{0, \lambda_i - (1 - \eta_1) \omega_{i1} \alpha\}$$

which imply:

$$U_{i0}^*(\lambda_i) = \frac{1}{2} (e_{i0}^*)^2 = \max \left\{ \bar{U}, \frac{[\lambda_i - (1 - \omega_{i0} \eta_1) \alpha]^2}{2} \right\} \quad (23)$$

$$U_{i1}^*(\lambda_i) = \frac{1}{2} (e_{i1}^*)^2 = \max \left\{ \bar{U}, \frac{[\lambda_i - \omega_{i1} (1 - \eta_1) \alpha]^2}{2} \right\} \quad (24)$$

Let us now solve the first stage for ω_{ij} . Since both $(1 - \omega_{i0} \eta_1) > 0$ and $\omega_{i1} (1 - \eta_1) > 0$, it should be clear that:

$$\omega_{i0}^* = 1 \quad \text{and} \quad \omega_{i1}^* = 0 \quad (25)$$

This is quite intuitive. Individuals who inherited an *uneducated strong tie* from their father always want to meet weak ties because this gives the possibility of meeting an educated person and avoid the penalty α . On the other hand, those whose *strong ties are educated* never want to meet weak ties because of the risk of the penalty α .

What is interesting here is to compare the result when individuals choose how much time they spend with their weak (or strong) ties (see (25)) and the one when it is the planner who makes this choice (Proposition 3). We have seen that the planner will choose $\omega_{i0}^* = \omega_{i1}^* = 0$ if it maximizes the sum of all utilities while it will choose $\omega_{i0}^* = \omega_{i1}^* = 1$ if it wants to increase

¹¹Observe that we do not introduce a cost of socialization, $-\frac{1}{2}\omega^2$, because we want to stay in line with the model of the previous section. We also believe that educational costs are much higher than socialization costs so that the latter can be ignored. Moreover it sets aside the problems of multiplicity of equilibria, which are not the focus of this paper

social mobility (or equivalently reduces the intergenerational correlation). This result is in sharp contrast with (25) where individuals with uneducated strong ties decide to spend all their time with weak ties, i.e., $\omega_{i0}^* = 1$ while individuals with educated strong ties prefer to spend all their time with strong ties, i.e. $\omega_{i1}^* = 0$. This is due to the fact that the planner internalizes the externalities generated by these choices while individuals don't. For individuals with uneducated strong ties, it is always beneficial to meet weak ties in order to avoid to pay the penalty α while, the planner will make such a choice only if it wants to increase social mobility.

Using (25), we easily obtain:

$$\tilde{\lambda}_0 = \sqrt{2\bar{U}} + (1 - \eta_1) \alpha \quad \text{and} \quad \tilde{\lambda}_1 = \sqrt{2\bar{U}} \quad (26)$$

Furthermore,

$$p_0 = 1 - \sqrt{2\bar{U}} - (1 - \eta_1) \alpha \quad \text{and} \quad p_1 = 1 - \sqrt{2\bar{U}} \quad (27)$$

and

$$U_{i0}(\lambda_i) = \max \left\{ \bar{U}, \frac{[\lambda_i - (1 - \eta_1) \alpha]^2}{2} \right\}$$

$$U_{i1}(\lambda_i) = \max \left\{ \bar{U}, \frac{\lambda_i^2}{2} \right\}$$

Let us compute the value of η_1 . Again, individuals whose ability exceeds $\tilde{\lambda}_0$ will get educated whatever the status of their strong tie. They represent a mass of size p_0 . Those whose ability is lower than $\tilde{\lambda}_1$ will never get educated while those such that $\tilde{\lambda}_0 > \lambda_i > \tilde{\lambda}_1$ will get educated only if they meet an educated (strong or weak) tie. There is a mass $p_1 - p_0$ of these individuals. As a result,

$$\eta_1 = p_0 + (p_1 - p_0)[\eta_1 + (1 - \eta_1)\eta_1]$$

Indeed, in order to meet an educated (strong or weak) tie, either the strong tie is educated (with probability η_1 and, in that case, the newborn i always stays with him as he will choose $\omega_{i1} = 0$) or the strong tie is uneducated (with probability $1 - \eta_1$) and then the newborn i chooses $\omega_{i0} = 1$ so that he needs to meet an educated weak tie (with probability η_1). Rearranging these terms, we obtain:

$$\eta_1 = p_0 + (p_1 - p_0)[\eta_1(2 - \eta_1)]$$

Using the fact that $p_1 - p_0 = (1 - \eta_1)\alpha$, we get η_1 as a solution of:

$$F(\eta_1) \equiv \alpha\eta_1^3 - 3\alpha\eta_1^2 + (3\alpha - 1)\eta_1 + 1 - \sqrt{2\bar{U}} - \alpha = 0 \quad (28)$$

This is equivalent to:

$$F(\eta_1) \equiv \alpha(\eta_1 - 1)^3 - \eta_1 + 1 - \sqrt{2\bar{U}} = 0 \quad (29)$$

We have the following lemma which guarantees the existence of a unique equilibrium and gives some comparative-statics results.

Lemma 1 *If $\sqrt{2\bar{U}} < 1 - \alpha$, there exists a unique solution $\eta_1^* \in [0, 1]$ to (29). Furthermore,*

$$\frac{\partial \eta_1^*}{\partial \bar{U}} < 0 \quad \text{and} \quad \frac{\partial \eta_1^*}{\partial \alpha} < 0$$

5.2 Steady-state equilibrium and intergenerational correlation

When the socialization decisions are endogenous, there is still a positive intergenerational correlation.

Proposition 4 *If $\sqrt{2\bar{U}} < 1 - \alpha$, there exists a unique equilibrium for which $\omega_{i0}^* = 1$, $\omega_{i1}^* = 0$ and η_1 is given by the solution to (29). In that equilibrium, the correlation between the educational status of the father and the son is equal to:*

$$Cor_{net} = \left(\frac{p_1 - \eta_1}{1 - \eta_1} \right)^2 = \left(\frac{1 - \sqrt{2\bar{U}} - \eta_1}{1 - \eta_1} \right)^2 \quad (30)$$

In the model without interactions (Section 3) or in the model with exogenous level of interactions (Section 4), the correlation could be expressed as a function of the quantity¹² $(p_1 - p_0)$, which can be written as $(p_1 - \eta_1) + [(1 - p_0) - (1 - \eta_1)]$. The first term measures the bias in the probability of getting educated induced by the chance of having an educated strong tie. Said differently, it is the difference between the conditional probability and the overall probability of getting educated. Similarly, the second term measures how much having an uneducated strong tie impacts on the probability of not getting educated.

¹²In Proposition 2 the correlation can be written as $(1 - \omega)^2(p_1 - p_0)^2$.

In Proposition 4, the correlation can be explained as follows. When a newborn meets an educated strong tie, his extra chance of being educated can again be written as $p_1 - \eta_1$. This should be normalized by the probability of meeting an uneducated individual in the population, $1 - \eta_1$. Now the other term disappears because when meeting an uneducated strong tie, newborns spend all their time with weak ties and thus incur no bias vis à vis the average. The quadratic form appears for the same reasons as before.

Remark 2 *Contrary to the previous sections, the correlation Cor_{net} (positively) depends on \bar{U} . This is due to the fact that a change in \bar{U} affects both p_0 and p_1 . In Sections 3 and 4 both probabilities were affected in the same way and cancelled out in the quantity $p_1 - p_0$. Here, it is no longer the case because of the asymmetry in behavior of individuals, depending on the status of their strong tie.*

We would now like to compare the results between the different models and to highlight the role of social interactions and networks on the intergenerational correlation and the equilibrium share of educated individuals.

Proposition 5 *The correlation in education status between father and son is always lower when there are social interactions than when there are not, i.e. $Cor_{net} < Cor_{dyad}$. Moreover, the equilibrium share of educated individuals is always higher when there are social interactions than when there are not, i.e. $\eta_{1,net} > \eta_{1,dyad}$.*

This proposition shows that the network reduces the correlation and increases the average education level in the population. As a result, taking into account weak ties in the network changes dramatically both the share of educated individuals and the intergenerational correlation between the father and the son.

6 Conclusion

In this paper, we developed a dynastic model where, at each period of time, with some probability, a person (the parent) dies and is replaced by a new born (the child). The new born takes exactly the same position as the father in the dyad and thus interacts with the

same person (strong tie), i.e. the local community of his father. There is therefore no vertical transmission but only horizontal transmission via peer and neighborhood effects. We assume that meeting an uneducated individual (strong or weak tie) has a cost when providing educational effort. This cost captures the idea of negative peer effects, i.e. the fact that uneducated role models can distract individuals from educating themselves by, for example, proposing activities that are not related to education (watching TV, going to the movies, etc.). On the contrary, there is no cost of interacting with educated peers.

We first show that, even though the parent and his child never live together at the same time (and thus never interact with each other), there is always a positive correlation between their educational status because there is a substantial overlap in the surroundings that have influenced their education decisions. We then study the role of networks on the correlation in parent-child educational status independent of any parent-child interaction. Here networks are captured by weak and strong ties and the individual has to decide how much time he wants to spend with each type of ties. We show that the network reduces the intergenerational correlation, promotes social mobility and increases the average education level in the population. We also show that a planner that maximizes social welfare does not want individuals with a good and bad educational environment to interact with each other. We also show that if the planner's aim is to reduce the intergenerational correlation in education status between the parent and the child (or equivalently promote social mobility), he prefers that plenty of interactions take place between these two groups. When the individuals make these choices themselves, those with uneducated strong ties always want to meet weak ties while the reverse occurs for individuals with educated strong ties.

We believe that this paper sheds some light on the effect of the inherited neighborhood and peers on children's education outcomes. It is indeed difficult to empirically distinguish between direct parental and social influences on education. But only focussing on the latter and providing a mechanism by which this effect takes place, our model provides some predictions that allow one to distinguish social effects from direct effects.

References

- [1] Aakvik, A., Salvanes, K.G. and K. Vaage (2010), “Measuring heterogeneity in the returns to education using an education reform,” *European Economic Review* 54, 483-500.
- [2] Akerlof, G. (1997), “Social distance and social decisions,” *Econometrica* 65, 1005-1027.
- [3] Ballester, C., Calvó-Armengol, A. and Y. Zenou (2006), “Who’s who in networks. Wanted: the key player,” *Econometrica* 74, 1403-1417.
- [4] Benabou, R. (1993), “Workings of a city: Location, education, and production,” *Quarterly Journal of Economics* 108, 619-652.
- [5] Bisin, A. and T. Verdier (2000), “Beyond the Melting Pot: Cultural Transmission, Marriage, and the Evolution of Ethnic and Religious Traits,” *Quarterly Journal of Economics* 115, 955-988.
- [6] Bisin, A. and T. Verdier (2001), “The Economics of Cultural Transmission and the Dynamics of Preferences,” *Journal of Economic Theory* 97, 298-319.
- [7] Björklund, A. and M. Jäntti (2009), “Intergenerational income mobility and the role of family background,” in W. Salverda, B. Nolan and T. Smeeding (Eds.), *Oxford Handbook of Economic Inequality*, New York: Oxford University Press.
- [8] Björklund, A., Lindahl, M. and E. Plug (2006), “The origins of intergenerational associations: Lessons from Swedish adoption data,” *Quarterly Journal of Economics* 121, 999-1028.
- [9] Björklund, A. and K.J. Salvanes (2011), “Education and family background: Mechanisms and policies,” In: E.A. Hanushek, S. Machin and L. Woessmann (Eds.), *Handbook of the Economics of Education*, vol. 3, Amsterdam: Elsevier Science, pp. 201-247.
- [10] Black, S.E. and P.J. Devereux (2011), “Recent developments in intergenerational mobility,” In: O. Ashenfelter and D. Card (Eds.), *Handbook of Labor Economics*, vol. 4B, Amsterdam: Elsevier Science, pp. 1487-1541.

- [11] Borjas, G.J. (1992), "Ethnic capital and intergenerational mobility," *Quarterly Journal of Economics* 107, 123-150.
- [12] Calvó-Armengol, A. and M. Jackson (2009), "Like father, like son: Network externalities, parent-child correlation in behavior, and social mobility," *American Economic Journal: Microeconomics* 1, 124-150.
- [13] Calvó-Armengol, A., Patacchini, E., and Y. Zenou (2009), "Peer effects and social networks in education," *Review of Economic Studies* 76, 1239-1267.
- [14] Calvó-Armengol, A., Verdier, T. and Y. Zenou (2007), "Strong and weak ties in employment and crime," *Journal of Public Economics* 91, 203-233.
- [15] Card, D. and A. Krueger (1992), "Does school quality matter? Returns to education and the characteristics of public schools in the United States," *Journal of Political Economy* 100, 1-40.
- [16] Card, D. and A. Krueger (1996), "Labor market effects of school quality: Theory and evidence," In: G. Burtless (Ed.), *Does Money Matter? The Link between Schools, Student Achievement and Adult Success*, Washington D.C: The Brookings Institution.
- [17] Cunha, F., Heckman, J.J., Lochner, L.L. and D. Masterov (2007), "Interpreting the evidence of life cycle skill formation," In: E. Hanushek and F. Welch (Eds.), *Handbook of the Economics of Education*, vol. 1, Amsterdam: North-Holland.
- [18] De Bartolome, C. (1990), "Equilibrium and inefficiency in a community model with peer group effects," *Journal of Political Economy* 98, 110-133.
- [19] Durlauf, S.E. (2004), "Neighborhood effects," In: J.V. Henderson and J-F. Thisse (Eds.), *Handbook of Regional and Urban Economics Vol. 4*, Amsterdam: Elsevier Science, pp. 2173-2242.
- [20] Ermisch, J. and M. Francesconi (2001), "Family matters: Impacts of family background on educational attainments," *Economica* 68, 137-156.

- [21] Fernández, R. (2003), “Sorting, education and inequality,” In: M. Dewatripont, L. Hansen, and S. Turnovsky (Eds.), *Advances in Economics and Econometrics*, Cambridge: Cambridge University Press.
- [22] Glaeser, E.L., Sacerdote, B., and J.A. Scheinkman (1996), “Crime and social interactions,” *Quarterly Journal of Economics* 111, 508-548.
- [23] Goux, D. and E. Maurin (2007), “Close neighbours matter: Neighbourhood effects on early performance at school,” *Economic Journal* 117, 1193-1215.
- [24] Goyal, S. (2007), *Connections: An Introduction to the Economics of Networks*, Princeton: Princeton University Press.
- [25] Granovetter, M.S. (1973), “The strength of weak ties,” *American Journal of Sociology* 78, 1360-1380.
- [26] Granovetter, M.S. (1974), *Getting a Job: A Study of Contacts and Careers*, Cambridge, MA: Harvard University Press.
- [27] Granovetter, M.S. (1983), “The strength of weak ties: A network theory revisited,” *Sociological Theory* 1, 201-233.
- [28] Guryan, J. (2004), “Desegregation and black dropout rates”, *American Economic Review* 94, 919-943.
- [29] Ioannides, Y.M. (2002), “Nonlinear neighbourhood interactions and intergenerational transmission of human capital,” In: G. Bitros and Y. Katsoulakos (Eds.), *Essays in Economic Theory, Growth and Labour Markets: A Festschrift in Honour of Emmanouel Drandakis*, Gloucester: Edward Elgar, pp. 75-112.
- [30] Ioannides, Y.M. (2003), “Empirical nonlinearities and neighbourhood effects in the intergenerational transmission of human capital,” *Applied Economics Letters* 10, 535-539.
- [31] Ioannides, Y.M. (2012), *From Neighborhoods to Nations: The Economics of Social Interactions*, Princeton: Princeton University Press.

- [32] Ioannides, Y.M. and D.L. Loury (2004), “Job information networks, neighborhood effects, and inequality,” *Journal of Economic Literature* 42, 1056-1093.
- [33] Ioannides, Y.M. and G. Topa (2010), “Neighborhood effects: Accomplishments and looking beyond them,” *Journal of Regional Science* 50, 343-362.
- [34] Jackson, M.O. (2008), *Social and Economic Networks*, Princeton: Princeton University Press.
- [35] Katz, L.F., Kling, J.R. and J.B. Liebman (2001), “Moving to opportunity in Boston: Early results of a randomized mobility experiment,” *Quarterly Journal of Economics* 116, 607-654.
- [36] Ladd, H.F. and J. Ludwig (1997), “Federal housing assistance, residential relocation, and educational opportunities: Evidence from Baltimore,” *American Economic Review* 87, 272-277.
- [37] Meghir, C. and M. Palme (2005), “Education reform, ability and family background,” *American Economic Review* 95, 414-424.
- [38] Montgomery, J.D. (1994), “Weak ties, employment, and inequality: An equilibrium analysis,” *American Journal of Sociology* 99, 1212-1236.
- [39] Patacchini, E. and Y. Zenou (2011), “Neighborhood effects and parental involvement in the intergenerational transmission of education,” *Journal of Regional Science* 51, 987-1013.
- [40] Pekkarinen, T., Uusitalo, R. and S. Pakkala (2009), “School tracking and intergenerational income mobility: Evidence from the Finnish comprehensive school reform,” *Journal of Public Economics* 93, 965-973.
- [41] Piketty, T. (1995), “Social mobility and redistributive politics,” *Quarterly Journal of Economics* 110, 551-584.
- [42] Plug, E. and W. Vijverberg (2003), “Schooling, family background, and adoption: Is it nature or is it nurture?” *Journal of Political Economy* 111, 611-641.

- [43] Rivkin, S. and F. Welch (2006), “Has school desegregation improved academic and economic outcomes for blacks?” In: E.A. Hanushek and F. Welch (Eds.), *Handbook of the Economics of Education*, vol. 2, Amsterdam: Elsevier Science, pp. 1019-1049.
- [44] Rosenbaum, E. and L.E. Harris (2001), “Residential mobility and opportunities: Early impacts of the Moving to Opportunity demonstration program in Chicago,” *Housing Policy Debate* 12, 321-346.
- [45] Sacerdote, B. (2001), “Peer effects with random assignment: Results for Dartmouth roommates,” *Quarterly Journal of Economics* 116, 681-704.
- [46] Sacerdote, B. (2002), “The nature and nurture of economic outcomes,” *American Economic Review Papers and Proceedings* 92, 344-348.
- [47] Sacerdote, B. (2010), “Peer effects in education: How might they work, how big are they and how much do we know thus far?” In: E.A. Hanushek, S. Machin and L. Woessmann (Eds.), *Handbook of the Economics of Education*, Vol. 3, Amsterdam: Elsevier Science, pp. 249-277.
- [48] Sáez-Martí, M. and Y. Zenou (2012), “Cultural transmission and discrimination,” Unpublished manuscript, Stockholm University.
- [49] Topa, G. (2011), “Labor markets and referrals,” In: J. Benhabib, A. Bisin and M.O. Jackson (Eds.), *Handbook of Social Economics*, Amsterdam: Elsevier Science, pp. 1193-1221.

APPENDIX

Proof of Proposition 1: Since η_1 is determined by (8), we need to derive the joint probability η_{11}

$$\eta_{11} = p_1\mu_{11} + \frac{1}{2}p_0\mu_{01} + \frac{1}{2}p_0\mu_{10}$$

Observe first that

$$\mu_{11} = p_1\eta_1$$

Indeed, for a dyad to be in state 11 (an individual and his strong tie are both educated), it has to be that a newborn is born in a dyad with an educated strong tie (with probability η_1) and gets educated (with probability p_1). Using a similar argument, we also have:

$$\mu_{10} = \mu_{01} = \frac{1}{2}(1 - p_1)\eta_1 + \frac{1}{2}p_0(1 - \eta_1) = (1 - p_1)\eta_1$$

and

$$\mu_{00} = (1 - p_0)(1 - \eta_1)$$

From these three expressions, we obtain:

$$\begin{aligned} \eta_{11} &= p_1\mu_{11} + p_0\mu_{01} \\ &= \eta_1 [p_1^2 + p_0(1 - p_1)] \end{aligned}$$

Finally, we have:

$$Cor(X, Y)_{dyad} = \frac{\eta_{11} - \eta_1^2}{\eta_1(1 - \eta_1)} = \frac{p_1^2 + p_0(1 - p_1) - \eta_1}{1 - \eta_1}$$

which, after some manipulations, leads to (9). ■

Proof of Proposition 2: From equation (19), we have:

$$\eta_1 = \frac{1 - \alpha - \sqrt{2\bar{U}}}{1 - \alpha - \omega\delta} = \frac{p_0}{1 + p_0 - p_1}$$

Now, the joint probability to have both a newborn and his father educated, η_{11} , is given by

$$\eta_{11} = (1 - \omega)(\mu_{11}p_1 + \mu_{10}p_0) + \omega(\mu_{11} + \mu_{10})[\eta_1p_1 + (1 - \eta_1)p_0]$$

Indeed, in order to have both a newborn and his father in state 1, there are two possibilities:

(i) either the son interacts within his dyad (probability $(1 - \omega)$). In that case, the father has to be in state 1, which is the case with probability 1 if it is a 11 dyad (μ_{11}) and with probability $1/2$ if it is a 10 or 01 dyad (μ_{10} or μ_{01}). The son will get educated with probability p_1 if the father was in the dyad 11 and with probability p_0 if the father was in a dyad 10 or 01.

(ii) or the son interacts with a weak tie (with probability ω). In that case, the father has to be educated (with probability $\mu_{11} + \mu_{10}$) and then the son gets educated with probability p_1 if he meets an educated individual (with probability η_1) and with probability p_0 if he meets an uneducated individual (with probability $(1 - \eta_1)$).

In this framework, μ_{11} is given by

$$\mu_{11} = \eta_1[(1 - \omega)p_1 + \omega(\eta_1 p_1 + (1 - \eta_1)p_0)]$$

and μ_{10} is given by

$$\begin{aligned} \mu_{10} &= \frac{1}{2}(1 - \eta_1) \{(1 - \omega)p_0 + \omega[\eta_1 p_1 + (1 - \eta_1)p_0]\} \\ &\quad + \frac{1}{2}\eta_1 \{(1 - \omega)(1 - p_1) + \omega[\eta_1(1 - p_1) + (1 - \eta_1)(1 - p_0)]\} \end{aligned}$$

Indeed, for a 10 dyad to form, either an individual meets a type-0 individual (with probability $(1 - \eta_1)$) and gets educated (either by staying within the dyad $((1 - \omega)p_0)$ or outside the dyad $(\omega(\eta_1 p_1 + (1 - \eta_1)p_0))$), or an individual meets a type-1 individual (with probability η_1) and decides not to educate (either by staying within the dyad $((1 - \omega)(1 - p_1))$ or outside the dyad $(\omega(\eta_1(1 - p_1) + (1 - \eta_1)(1 - p_0)))$.

Observing that $\eta_1 p_1 + (1 - \eta_1)p_0 = \eta_1$, that $\eta_1(1 - p_1) = (1 - \eta_1)p_0$ and that $\eta_1(1 - p_1) + (1 - \eta_1)(1 - p_0) = 1 - \eta_1$, we have:

$$\begin{aligned} \mu_{11} &= \eta_1[(1 - \omega)p_1 + \omega\eta_1] \\ \mu_{10} &= \frac{1}{2}(1 - \eta_1)[(1 - \omega)p_0 + \omega\eta_1] + \frac{1}{2}\eta_1[(1 - \omega)(1 - p_1) + \omega(1 - \eta_1)] \\ \mu_{10} &= \frac{1}{2}(1 - \eta_1)(2\omega\eta_1) + \frac{1}{2}(1 - \omega)[\eta_1(1 - p_1) + (1 - \eta_1)p_0] \\ \mu_{10} &= \omega(1 - \eta_1)\eta_1 + (1 - \omega)\eta_1(1 - p_1) \end{aligned}$$

Furthermore, we have:

$$\mu_{11}p_1 + \mu_{10}p_0 = \eta_1[(1 - \omega)(p_1^2 + p_0(1 - p_1)) + \omega(p_1\eta_1 + p_0(1 - \eta_1))]$$

$$\mu_{11}p_1 + \mu_{10}p_0 = \eta_1[(1 - \omega)(p_1^2 + p_0(1 - p_1)) + \omega\eta_1]$$

This implies that

$$\eta_{11} = (1 - \omega)^2\eta_1(p_1^2 + p_0(1 - p_1)) + \omega(1 - \omega)\eta_1^2 + \omega\eta_1^2$$

and

$$\begin{aligned} \frac{\eta_{11} - \eta_1^2}{\eta_1} &= (1 - \omega)^2 [p_1^2 + p_0(1 - p_1)] + (2\omega - \omega^2 - 1)\eta_1 \\ &= (1 - \omega)^2 [p_1^2 + p_0(1 - p_1) - \eta_1] \end{aligned}$$

Finally

$$\begin{aligned} Cor_{exo} &= \frac{\eta_{11} - \eta_1^2}{\eta_1(1 - \eta_1)} \\ &= (1 - \omega)^2 \frac{p_1^2 + p_0(1 - p_1) - \eta_1}{1 - \eta_1} \\ &= (1 - \omega)^2 (p_1 - p_0)^2 \\ &= (1 - \omega)^4 \alpha^2 \end{aligned}$$

which is (21). ■

Proof of Proposition 3: Let us first analyze (i). The total welfare is given by (22), which is

$$\mathcal{W} = \int_0^{\tilde{\lambda}_1} \bar{U} d\lambda + \int_{\tilde{\lambda}_1}^{\tilde{\lambda}_0} (1 - \eta_1) \bar{U} d\lambda + \int_{\tilde{\lambda}_1}^1 \eta_1 U_{i1}(\lambda) d\lambda + \int_{\tilde{\lambda}_0}^1 (1 - \eta_1) U_{i0}(\lambda) d\lambda$$

The first two terms can be calculated and it is easily shown that:

$$\int_0^{\tilde{\lambda}_1} \bar{U} d\lambda + \int_{\tilde{\lambda}_1}^{\tilde{\lambda}_0} (1 - \eta_1) \bar{U} d\lambda = \tilde{\lambda}_0 \bar{U} - \eta_1 \bar{U} (\tilde{\lambda}_0 - \tilde{\lambda}_1)$$

which using (14) and (15) gives

$$K \equiv \int_0^{\tilde{\lambda}_1} \bar{U} d\lambda + \int_{\tilde{\lambda}_1}^{\tilde{\lambda}_0} (1 - \eta_1) \bar{U} d\lambda = \bar{U} \left[\sqrt{2\bar{U}} + (1 - \eta_1) \alpha \right]$$

which is independent of ω (see (19)) and thus we can ignore these first two terms. So the planner maximizes

$$\int_{\tilde{\lambda}_1}^1 \eta_1 U_{i1}(\lambda) d\lambda + \int_{\tilde{\lambda}_0}^1 (1 - \eta_1) U_{i0}(\lambda) d\lambda$$

Using (12) and (13), we have:

$$\begin{aligned} & \int_{\tilde{\lambda}_1}^1 \eta_1 U_{i1}(\lambda) d\lambda + \int_{\tilde{\lambda}_0}^1 (1 - \eta_1) U_{i0}(\lambda) d\lambda \\ &= \frac{1}{6} \left\{ \eta_1 \left[(\lambda_i - \omega\alpha + \omega\eta_1\alpha)^3 \right]_{\tilde{\lambda}_1}^1 + (1 - \eta_1) \left[(\lambda_i - \alpha + \omega\eta_1\alpha)^3 \right]_{\tilde{\lambda}_0}^1 \right\} \end{aligned}$$

Using (14) and (15), we see that

$$\tilde{\lambda}_1 - \omega\alpha + \omega\eta_1\alpha = \sqrt{2\bar{U}} = \tilde{\lambda}_0 - \alpha + \omega\eta_1\alpha$$

As a result, we obtain:

$$\begin{aligned} \mathcal{W} &= K - \frac{1}{6} \left(\sqrt{2\bar{U}} \right)^3 + \frac{1}{6} \left[\eta_1 (1 - \omega\alpha + \omega\eta_1\alpha)^3 + (1 - \eta_1) (1 - \alpha + \omega\eta_1\alpha)^3 \right] \\ &= K' + \frac{1}{6} \left[\eta_1 (1 - \omega\alpha + \omega\eta_1\alpha)^3 + (1 - \eta_1) (1 - \alpha + \omega\eta_1\alpha)^3 \right] \end{aligned}$$

We are looking for a ω^* such that $\frac{\partial \mathcal{W}}{\partial \omega} = 0$. We have:

$$\frac{\partial \mathcal{W}}{\partial \omega} = \frac{\eta_1 \alpha (\eta_1 - 1)}{2} \left[(1 - \omega\alpha + \omega\eta_1\alpha)^2 - (1 - \alpha + \omega\eta_1\alpha)^2 \right]$$

There are two solutions to $\frac{\partial \mathcal{W}}{\partial \omega} = 0$, which are either

$$\omega^* = 1$$

or

$$\omega^* = \frac{2 - \alpha}{\alpha(1 - 2\eta_1)}$$

Since the latter is either strictly greater than 1 or negative, the unique solution is $\omega^* = 1$.

Let us now analyze (ii). The correlation is given by (21), that is

$$Cor_{exo} = (1 - \omega)^2 (p_1 - p_0)^2 = (1 - \omega)^4 \alpha^2$$

Since $\frac{\partial Cor_{exo}}{\partial \omega} < 0$, it is should be clear that the solution to this program is $\omega^* = 0$. ■

Proof of Remark 1:

$$\frac{\partial^2 \mathcal{W}}{\partial \omega^2} = \frac{\eta_1 \alpha^2 (\eta_1 - 1)}{2} [-1 + \eta_1 \alpha - 2\omega \eta_1 \alpha + \omega \alpha]$$

which has a constant sign over $[0, 1]$, and

$$\frac{\partial^2 \mathcal{W}}{\partial \omega^2} \Big|_{\omega=0} > 0$$

■

Proof of Lemma 1: Assume

$$\sqrt{2\bar{U}} < 1 - \alpha \tag{31}$$

and we have (see (29))

$$F(\eta_1) \equiv \alpha (\eta_1 - 1)^3 + \left(1 - \sqrt{2\bar{U}} - \eta_1\right)$$

Since $\lim_{\eta_1 \rightarrow -\infty} F(\eta_1) = -\infty$ and $\lim_{\eta_1 \rightarrow +\infty} F(\eta_1) = +\infty$ and since $F(0) = 1 - \sqrt{2\bar{U}} - \alpha > 0$ (which is (31)) and $F(1) = -\sqrt{2\bar{U}} < 0$, then the cubic equation $F(\eta_1)$ has necessarily a unique solution $\eta_1^* \in [0, 1]$.

Let us now totally differentiate (29). Observe that $\partial F(\eta_1)/\partial \eta_1 < 0$ since it is the slope of (29) (the cubic equation in η_1) taken at the point η_1^* , where the polynomial goes from positive to negative values. Thus,

$$\text{sign} \frac{\partial \eta_1^*}{\partial x} = \text{sign} \frac{\partial F(\eta_1)}{\partial x}$$

where $x = \alpha, \bar{U}$. We easily obtain:

$$\begin{aligned} \frac{\partial F(\eta_1)}{\partial \alpha} &= (\eta_1 - 1)^3 < 0 \\ \frac{\partial F(\eta_1)}{\partial \bar{U}} &= -\frac{1}{\sqrt{2\bar{U}}} < 0 \end{aligned}$$

■

Proof of Proposition 4: Let us calculate the correlation between the father and son.

This correlation is given by:

$$Cor_{net} = \frac{\eta_{11} - \eta_1^2}{\eta_1(1 - \eta_1)}$$

We have

$$\eta_1 = \eta_1 p_1 + (1 - \eta_1)[\eta_1 p_1 + (1 - \eta_1)p_0]$$

from which we derive

$$\eta_1 p_1 + (1 - \eta_1)p_0 = \frac{\eta_1(1 - p_1)}{(1 - \eta_1)} \quad (32)$$

The steady-state distributions are given by

$$\begin{aligned} \mu_{11} &= \eta_1 p_1 \\ \mu_{10} &= \frac{1}{2}\eta_1(1 - p_1) + \frac{1}{2}(1 - \eta_1)[\eta_1 p_1 + (1 - \eta_1)p_0] \end{aligned}$$

Indeed, for a μ_{11} dyad to be formed, it must be that a newborn meets an educated strong tie (probability η_1) and that he gets educated. But since $\omega_{i1}^* = 0$ in equilibrium, he only interacts with his strong tie and gets educated with probability p_1 .

Accordingly, for a μ_{10} (or a μ_{01}) dyad to be formed, it must be that either a newborn meets an educated strong tie (he then sets $\omega_{i1} = 0$) and does not get educated, with probability $(1 - p_1)$ or the newborn meets a non educated strong tie (probability $1 - \eta_1$, he then sets $\omega_{i0} = 1$) and needs to get educated whatever the status of the weak tie ($\eta_1 p_1$ if he meets an educated weak tie or $(1 - \eta_1)p_0$ if he meets a non educated weak tie). Rearranging and using (32), we obtain:

$$\begin{aligned} \mu_{11} &= \eta_1 p_1 \\ \mu_{10} &= \eta_1(1 - p_1) \end{aligned}$$

In turn η_{11} is given by

$$\eta_{11} = \mu_{11} p_1 + \mu_{10} [\eta_1 p_1 + (1 - \eta_1)p_0]$$

Indeed, for the father and the son to be both educated, it must be the case that the father was educated and that the son gets educated. Either the father was part of a μ_{11} dyad and then the son meets an educated strong tie, in which case he gets educated with probability p_1 , or the father was in a μ_{10} dyad and then the son meets an uneducated strong tie, in which case he only interacts with weak ties and gets educated with probability $\eta_1 p_1 + (1 - \eta_1)p_0$.

Replacing for μ_{11} and μ_{10} , we get

$$\begin{aligned} \eta_{11} &= \eta_1 p_1^2 + \eta_1(1 - p_1)[\eta_1 p_1 + (1 - \eta_1)p_0] \\ &= \eta_1 \left[p_1^2 + (1 - p_1)^2 \left(\frac{\eta_1}{1 - \eta_1} \right) \right] \end{aligned}$$

Hence

$$\begin{aligned}
\frac{\eta_{11} - \eta_1^2}{\eta_1} &= p_1^2 + (1 - p_1)^2 \frac{\eta_1}{1 - \eta_1} - \eta_1 \\
&= \frac{p_1^2 - 2\eta_1 p_1 + \eta_1^2}{1 - \eta_1} \\
&= \frac{(p_1 - \eta_1)^2}{1 - \eta_1}
\end{aligned}$$

and finally

$$Cor = \left(\frac{p_1 - \eta_1}{1 - \eta_1} \right)^2$$

which is (30). ■

Proof of Remark 2: Differentiating Cor_{net} gives

$$\frac{\partial Cor_{net}}{\partial \bar{U}} = -2 \left(\frac{p_1^* - \eta_1^*}{1 - \eta_1^*} \right) \frac{\frac{\partial \eta_1^*}{\partial \bar{U}} (1 - p_1^*) + \frac{(1 - \eta_1^*)}{\sqrt{2\bar{U}}}}{(1 - \eta_1^*)^2}$$

But $1 - p_1^* = \sqrt{2\bar{U}}$ and

$$\frac{\partial \eta_1^*}{\partial \bar{U}} = -\frac{\partial F(\eta_1)/\partial \bar{U}}{F'(\eta_1)}$$

so

$$Sgn \left(\frac{\partial Cor_{net}}{\partial \bar{U}} \right) = Sgn \left(2\bar{U} \frac{\partial F(\eta_1)/\partial \bar{U}}{F'(\eta_1)} + (1 - \eta_1^*) \right)$$

Thus,

$$\frac{\partial Cor_{net}}{\partial \bar{U}} > 0 \iff \sqrt{2\bar{U}} > (\eta_1 - 1)F'(\eta_1)$$

Using (29) one can write

$$\begin{aligned}
\sqrt{2\bar{U}} &= \alpha\eta_1^3 - 3\alpha\eta_1^2 + (3\alpha - 1)\eta_1 + 1 - \alpha \\
&= (\eta_1 - 1)[\alpha\eta_1^2 - 2\alpha\eta_1 + \alpha - 1]
\end{aligned}$$

and check that the above inequality is always true. ■

Proof of Proposition 5: Let us first show that $Cor_{net} < Cor_{dyad}$. This amounts to show that

$$Cor_{net} = \left(\frac{p_1 - \eta_1}{1 - \eta_1} \right)^2 < \alpha^2$$

This is equivalent to:

$$\frac{p_1 - \eta_1}{1 - \eta_1} < \alpha \Leftrightarrow \frac{p_1 - \alpha}{1 - \alpha} < \eta_1$$

Because η_1 is the solution of $F(\eta_1) = 0$ (see 29)), then

$$\frac{p_1 - \alpha}{1 - \alpha} < \eta_1$$

is equivalent to $F\left(\frac{p_1 - \alpha}{1 - \alpha}\right) > 0$ because $\frac{p_1 - \alpha}{1 - \alpha} > 0$. We have:

$$\begin{aligned} & F\left(\frac{p_1 - \alpha}{1 - \alpha}\right) \\ = & \frac{(p_1 - \alpha)}{(1 - \alpha)^3} [\alpha(p_1 - \alpha)^2 - 3\alpha(1 - \alpha)(p_1 - \alpha) + (3\alpha - 1)(1 - \alpha)^2 + (1 - \alpha)^3] \end{aligned}$$

Thus

$$\text{sgn} \left\{ F\left(\frac{p_1 - \alpha}{1 - \alpha}\right) \right\} = \text{sgn} \{ p_1^2 + p_1(\alpha - 3) - \alpha + 2 \}$$

But the term in brackets is always positive when $p_1 \leq 1$. Hence $F\left(\frac{p_1 - \alpha}{1 - \alpha}\right) > 0$ and therefore

$$Cor_{net} = \left(\frac{p_1 - \eta_1}{1 - \eta_1} \right)^2 < \alpha^2$$

Let us now show that $\eta_{1,net} > \eta_{1,dyad}$. Notice that the equilibrium share of educated individuals in Section 3 is given by

$$\eta_1 = \frac{1 - \sqrt{2\bar{U}} - \alpha}{1 - \alpha} = \frac{p_1 - \alpha}{1 - \alpha}$$

As we have just shown, $F\left(\frac{p_1 - \alpha}{1 - \alpha}\right) > 0$ and therefore $\eta_{1,net} > \eta_{1,dyad}$. ■