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A DARWINIAN PERSPECTIVE ON "EXCHANGE RATE UNDERVALUATION"

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ABSTRACT

A Darwinian Perspective on "Exchange Rate Undervaluation"*

Though the real exchange rate is a key price for most economies, our understanding of its determinants is still incomplete. This paper studies the implications of status competition in the marriage market for the real exchange rate. In theory, a rise in the sex ratio (increasing relative surplus of men) can generate a decline in the real exchange rate (RER) through both a savings channel and an effective labor supply channel. The effects can be quantitatively large if the biological desire for a marriage partner is strong. Empirically, we show that within China, those regions with a faster increase in the sex ratio also exhibit a faster decline in the RER (the relative price of nontradables). Furthermore, across countries, those with a high sex ratio tend to have a low real exchange rate, beyond what can be explained by the Balassa-Samuelson effect, financial underdevelopment, dependence ratio, and exchange rate regime classifications. As an application, the estimation suggests that these structural factors can account for the Chinese exchange rate almost completely.

JEL Classification: F3, F4, J1, J7
Keywords: currency manipulation, equilibrium real exchange rate, surplus men

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1 Introduction

The real exchange rate (RER) is one of the most important relative prices for most economies. It is also often a source of international tension - witness the intense controversy over the Chinese exchange rate policy. Yet, our understanding of its determinants is still incomplete. This paper proposes a Darwinian determinant of the real exchange rate that we argue should play an important (but still neglected) role in understanding global exchange rate patterns.

For concreteness, let us reflect a bit more on the international controversy about the Chinese exchange rate, although the mechanism proposed in the paper should be relevant for many other economies that share some common features with China. The Chinese real exchange rate is widely believed to be substantially undervalued. Relative to the purchasing power parity (PPP), the exchange rate appears to be undervalued by 40% or more. The standard narrative attributes this pattern to government interventions in the currency market.

However, this narrative is only correct if one has already taken into account all the important structural determinants of the real exchange rate. In this paper, we investigate one such determinant that is missing from the standard approach to assess the equilibrium exchange rate. In the alternative narrative, the appearance of the real exchange rate undervaluation is an outcome of an imbalance in the sex ratio in the pre-marital age cohort that began around 2003 and has become progressively worse since then. The Chinese exchange rate policy only became a source of international tension since 2003, and we will argue that the timing is not coincidental.

The sex ratio imbalance itself starts from some technology and policy shocks, unrelated to the nominal exchange rate policy. The initial technology shock in the new narrative was the spread of ultrasound B machines in China since the 1980s that allowed expectant parents to easily detect the gender of the fetus and abort the child they did not want. 1985 was the first year in which half of the local (county level) hospitals acquired at least one such machine (Li and Zheng, 2009). The initial policy shock was the implementation of a strict version of the family planning policy (popularly known as the "one-child policy") that severely restricts the number of children a couple can have. By interacting with a long-existing parental preference for sons, the combination of the two shocks started to produce an unnaturally high ratio of boys to girls at birth from the early 1980s, and the sex ratio at birth became progressively worse as the use of ultrasound machines became more widespread, and the enforcement of the family planning policy tightened over time. Around 2003, the first cohort born with an excess number of males was entering the marriage market. The competition for a marriage partner by young men became progressively more fierce since then. In 2007, the sex ratio for the pre-marital age cohort (5-20) was about 115 young men per 100 young women. This implies that about one out of every nine young men cannot get married, mathematically speaking.

Our theory predicts that a rise in the sex ratio in the pre-marital age cohort can lead to an appearance of an undervalued exchange rate relative to the PPP. This happens through both a savings channel and a labor supply channel.

How would a rise in the sex ratio imbalance trigger a significant increase in the savings rate starting from about 2003? The key is to recognize that family wealth is an important status variable in the marriage market (other things equal). As the competition for brides intensifies, young men and their parents raise their savings rate in order to improve their relative standing in the marriage market. (Of course, any complete story has to investigate why the behavior by women or their parents does not undo the competitive savings story. This we will do in the model. In addition, we will argue that the corporate savings will also go up in response to a higher sex ratio.)

When the economy-wide savings rate rises, the real exchange rate often falls. To see this, we recognize that a rise in the savings rate implies a reduction in the demand for both tradable and non-tradable goods. Since the price of the tradable goods is tied down by the world market, this translates into a reduction in the relative price of the non-tradable goods, and hence a decline in the value of the real exchange rate (a departure from the PPP). The effect would be persistent if there are frictions that impede the reallocation of factors between the tradable and non-tradable sectors. The savings channel can be economically and quantitatively significant if the biological desire for a marriage partner is sufficiently strong.

The second channel for the sex ratio imbalance to affect the real exchange rate works through effective labor supply. A rise in the sex ratio can also motivate men to cut down leisure and increase labor supply. This leads to an increase in the economy-wide effective labor supply. If the non-tradable sector is more labor intensive than the tradable sector, this generates a Rybzinsky-like effect, leading to an expansion of the non-tradable sector at the expense of the tradable sector. The increase in the supply of non-tradable goods leads to an additional decline in the relative price of non-tradables and a further decline in the value of the RER. Again, the labor supply channel can be economically powerful if the biological desire to avoid involuntary bachelorhood is strong.

Putting the two channels together, a rise in the sex ratio generates a real exchange rate that appears too low relative to the purchasing power parity (or relative to the standard approach used by the IMF to assess equilibrium exchange rates that includes additional terms beyond PPP but does not include the sex ratio, savings rate, and effective labor supply). Because the effect of a skewed sex ratio on the real exchange rate comes from competition for sex partners, this is fundamentally a Darwinian perspective on the exchange rate.

Of course, other structural factors may also have contributed to an increase in the aggregate savings rate (e.g., an increase in government savings or an increase in private-sector precautionary savings) or an increase in the effective labor supply (e.g., gradual relaxation of restrictions on rural-urban migration). These other factors would reinforce the Darwinian mechanism discussed in this paper, causing the real exchange rate to fall further.

A desire to enhance one's prospects in the marriage market through a higher level of wealth could be a motive for savings or labor supply even in countries with a balanced sex ratio. But such a motive is not as easy to detect when the competition is modest. When the sex ratio gets out of balance, obtaining a marriage partner becomes much less assured. A host of behaviors that are motivated by a desire to succeed in the marriage market may become magnified. But sex ratio imbalances so far have not been investigated by macroeconomists. This may be a serious omission.

A sex ratio imbalance is a common demographic feature in many economies, especially in East, South, and Southeast Asia, such as Korea, India, Vietnam, Singapore, Taiwan and Hong Kong, in addition to China. It is quite possible that the sex ratio effect plays an important role in the real exchange rate of these economies. To be clear, most countries in the world do not have a severe sex ratio imbalance. Correspondingly, there are not enough variations in the sex ratio among these countries to detect its role in determining the level of the exchange rate. However, if one only considers the standard determinants of the real exchange rate and ignores the sex ratio effect, one could mistakenly conclude that countries with a severe sex ratio imbalance have a severely undervalued currency. This set of countries happens to include China - the world's second largest economy and the largest exporter. Given the enormous effort by international financial institutions and many national governments to pass judgment on its exchange rate, getting it right has global importance.

The empirical evidence on the savings channel is provided by Wei and Zhang (2011a). First, across rural households with a son, they document that the savings rate tends to be higher in regions with a higher sex ratio imbalance (holding constant family income, age, gender, and educational level of the household head and other household characteristics). In comparison, for rural households with a daughter, their savings rate appears to be uncorrelated with the local sex ratio. Across cities, both households with a son and households with a daughter tend to have a higher savings rate in regions with a more skewed sex ratio, although the elasticity of the savings rate with respect to the sex ratio tends to be bigger for son families. Second, across Chinese provinces, they find a strong positive correlation between the local savings rate and the local sex ratio, after controlling for the age structure of the local population, income level, inequality, recent growth rate, local birth rate, local enrollment rate in the social safety net, and other factors. Third, to go from correlation to causality, they explore regional variations in the enforcement of the family planning policy as instruments for the local sex ratio, and confirm the findings in the OLS regressions. The sex ratio effect is both economically and statistically significant. While the Chinese household savings rate approximately doubled from 16% (of disposable income) in 1990 to 31% in 2007, Wei and Zhang (2011) estimate that the rise in the sex ratio could explain about half the increase in the household savings rate.

Besides the papers cited above, there are four bodies of work that are related to the current paper. First, the theoretical and empirical literature on the real exchange rate is too voluminous to summarize comprehensively here. Sarno and Taylor (2002) and Chinn (2012) provide recent surveys. Second, the literature on status goods, positional goods, and social norms (e.g., Cole, Mailath and Postlewaite, 1992, Corneo and Jeanne, 1999, Hopkins and Kornienko, 2004 and 2009) has offered many useful insights. One key point is that when wealth can improve one's social status (including improving one's standing in the marriage market), in addition to affording a greater amount of consumption goods, there is an extra incentive to save. This element is in our model as well. However, all existing theories on status goods feature a balanced sex ratio. Yet, an unbalanced sex ratio presents some non-trivial challenges. In particular, while a rise in the sex ratio is an unfavorable shock to men, it is a favorable shock to women. Could the women strategically reduce their savings so as to completely offset whatever increments in savings men may have? In other words, the impact on aggregate savings from a rise in the sex ratio appears ambiguous. Our model will address this question. In any case, the literature on status goods has no discernible impact in macroeconomic policy circles. For example, while there are voluminous documents produced by the International Monetary Fund or speeches by US officials on China's high savings rate, no single paper or speech thus far has pointed to a possible connection with its high sex ratio imbalance.

A third related literature is the economics of family, which is also too vast to be summarized here comprehensively. One interesting insight from this literature is that a married couple's consumption has a partial public goods feature (Browning, Bourguignon and Chiappori, 1994; Donni, 2006). We make use of this feature in our model as well. An insightful paper by Bhaskar and Hopkins (2011) studies parental investment in their children before they go to the marriage market. When there is a surplus of boys, parents overinvest in boys and underinvest in girls but the total investment in children is excessive. Du and Wei (2010) examine the effect of higher sex ratios for aggregate savings and current account balances. None of the papers in this literature explores the general equilibrium implications for exchange rates from a change in the sex ratio.

The fourth literature examines empirically the causes of a rise in the sex ratio. The key insight is that the proximate cause for the recent rise in the sex ratio imbalance is sex-selective abortions, which have been made increasingly possible by the spread of Ultrasound B machines. There are two deeper causes for the parental willingness to disproportionately abort female fetuses. The first is the parental preference for sons, which in part has to do with the relatively inferior economic status of women. When the economic status of women improves, sex-selective abortions appear to decline (Qian, 2008). The second is either something that leads parents to voluntarily have a lower fertility rate than earlier generations, or a government policy that limits the number of children a couple can have. In regions of China where the family planning policy is less strictly enforced, there is also less sex ratio imbalance (Wei and Zhang, 2009). Bhaskar (2011) examines parental sex selections and their welfare consequences.

The rest of the paper is organized as follows. In Section 2, we construct a simple overlapping generations (OLG) model with only one gender, and show that structural shocks can produce a real exchange rate depreciation. In Section 3, we present an OLG model with two genders, and demonstrate that a rise in the sex ratio could lead to a fall in the value of the real exchange rate. In Section 4, we calibrate the model to see if and when the sex ratio imbalance can produce changes in the real exchange rate whose magnitude is economically significant. In section 5, we provide some empirical evidence on the connection between the sex ratio and the real exchange rate. Section 6 offers concluding remarks and discusses possible future research.

2 A benchmark model with one gender

We start with a simple benchmark model with one gender. This allows us to see both the savings channel and the labor supply channel in a transparent way. The setup is standard, and the discussion will pave the way for a model in the next section that features two genders and an unbalanced sex ratio.

2.1 Consumers

There are two types of agents: consumers and producers. Consumers live for two periods: young and old. In the first period (young), a representative consumer supplies labor in exchange for labor income, consumes a part of the income and saves the rest. In the second period (old), she does not work and consumes her savings with interest.

The final good C_t consumed by the representative consumer consists of two parts: a tradable good C_{Tt} and a non-tradable good C_{Nt}.

$$C_t = \frac{C_{Nt}^{\gamma} C_{Tt}^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}$$

We normalize the price of the tradable good to be one, and let P_{Nt} denote the relative price of the non-tradable good. The consumer price index is $P_t = P_{Nt}^{\gamma}$.

The optimization problem for the representative consumer is

$$\max \; u(C_{1t}) + \beta u(C_{2,t+1})$$

with the intertemporal budget constraint

$$P_t C_{1t} = (1 - s_t) w_t \text{ and } P_{t+1} C_{2,t+1} = R s_t w_t$$

where w_t is the wage rate. We assume that everyone supplies one unit of labor inelastically. Then w_t is also the total first period income for a young consumer. s_t is the savings rate of the young cohort. R is the gross interest rate in units of the tradable good.

The optimal conditions is

$$\frac{u'_{1t}}{P_t} = \beta R \frac{u'_{2,t+1}}{P_{t+1}} \tag{2.1}$$

We start with the case of a small open economy, and assume that the law of one price for the tradable good holds. The price of the tradable good is determined by the world market, and is set to be one in each period. The interest rate R in units of the tradable good is also a constant. For simplicity, we assume $\beta R = 1$.

2.2 Producers

There are two sectors in the economy: a tradable good sector and a non-tradable good sector. Both markets are perfectly competitive. For simplicity, we make the same assumption as in Obstfeld and Rogoff (1996) that only the tradable good can be transformed into capital used in production.[1]

2.2.1 Tradable good producers

For simplicity, we assume a complete depreciation of capital at the end of every period. Tradable producers maximize

$$\max \; E_t \sum_{\tau=0}^{\infty} (R)^{-\tau} \left[Q_{T,t+\tau} - w_{t+\tau} L_{T,t+\tau} - K_{T,t+\tau+1} \right]$$

where the production function is

$$Q_{Tt} = \frac{A_{Tt} K_{Tt}^{\alpha_T} L_{Tt}^{1-\alpha_T}}{\alpha_T^{\alpha_T} (1-\alpha_T)^{1-\alpha_T}}$$

Without any unanticipated shocks, the factor demand functions are, respectively,

$$R = \frac{1}{\alpha_T^{\alpha_T} (1-\alpha_T)^{1-\alpha_T}} \alpha_T A_{Tt} \left(\frac{L_{Tt}}{K_{Tt}} \right)^{1-\alpha_T} \tag{2.2}$$

$$w_t = \frac{1}{\alpha_T^{\alpha_T} (1-\alpha_T)^{1-\alpha_T}} (1-\alpha_T) A_{Tt} \left(\frac{K_{Tt}}{L_{Tt}} \right)^{\alpha_T} \tag{2.3}$$

It is useful to note that when there is an unanticipated shock in period t, (2.2) does not hold since K_{Tt} is a predetermined variable.

2.2.2 Non-tradable good producers

non-tradable good producers maximize the following objective function:

$$\max \; E_t \sum_{\tau=0}^{\infty} (R)^{-\tau} \left[P_{N,t+\tau} Q_{N,t+\tau} - w_{t+\tau} L_{N,t+\tau} - K_{N,t+\tau+1} \right]$$

with the production function given by

$$Q_{Nt} = \frac{A_{Nt} K_{Nt}^{\alpha_N} L_{Nt}^{1-\alpha_N}}{\alpha_N^{\alpha_N} (1-\alpha_N)^{1-\alpha_N}}$$

[1] Relaxing this assumption will not change any of our results qualitatively.

Without unanticipated shocks, we have

$$R = \frac{1}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}} P_{Nt}\alpha_N A_{Nt}\left(\frac{L_{Nt}}{K_{Nt}}\right)^{1-\alpha_N} \tag{2.4}$$

$$w_t = \frac{1}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}} P_{Nt}(1-\alpha_N) A_{Nt}\left(\frac{K_{Nt}}{L_{Nt}}\right)^{\alpha_N} \tag{2.5}$$

If there is an unanticipated shock in period t, (2.4) does not hold.

In equilibrium, the market clearing condition for the non-tradable good pins down the price of the non-tradable good,

$$Q_{Nt} = \frac{\gamma P_t\left(C_{2t} + C_{1t}\right)}{P_{Nt}} \tag{2.6}$$

Let x denote the total number of young people in the economy, then the labor market clearing condition is given by

$$L_{Tt} + L_{Nt} = x \tag{2.7}$$

Definition 1 *An equilibrium in the small open economy is a set* $\{s_t, K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$ *that satisfies the following conditions:*

(i) The households' savings rates, $s_t = \{s_{it}, s_{-i,t}\}$*, maximize the household's welfare*

$$s_t = \arg\max\left\{ V_t \middle| \, s_{-i,t}, K_{Tt+1}, K_{Nt+t}, L_{Tt}, L_{Nt}, P_{Nt}\right\}$$

(ii) The allocations of capital stock and labor, and the output of the non-tradable good clear the factor and the output markets, and maximize the firms' profit. In other words, $\{K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$ *solves (2.2), (2.3), (2.4), (2.5), (2.6) and (2.7).*

2.3 From the savings rate or labor supply to the exchange rate

To discuss the savings channel, we consider an unanticipated increase in the discount factor β that makes the young cohort more patient. To discuss the labor supply channel, we consider an unanticipated increase in the number of young people L that enlarges the labor supply in the economy. As a result of either of these shocks, in period t, (2.3) and (2.5) hold, but (2.2) and (2.4) fail.

The market clearing condition for the non-tradable good can be re-written as

$$\frac{P_{Nt}A_{Nt}K_{Nt}^{\alpha_N}L_{Nt}^{1-\alpha_N}}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}} = \gamma\left(Rs_{t-1}^{young}w_{t-1} + (1-s_t^{young})w_t\right)$$

We can solve (2.1), (2.5), (2.3) and (2.6) to obtain the equilibrium in period t. To simplify, we assume that the per period utility function is of the log form, i.e., $u(C) = \ln(C)$. Following Obstfeld and Rogoff (1996) and assuming that the non-tradable good sector is relatively more labor-intensive, i.e., $\alpha_N < \alpha_T$, we can obtain the following proposition.

Proposition 1 *(i) With an increase in the discount factor β of the young cohort, the young raise their savings rate, and the price of the non-tradable good falls. As a result, the real exchange rate depreciates.*

(ii) With an increase in the total number of young people x, the price of the non-tradable good falls. As a result, the real exchange rate depreciates.

Proof. See Appendix A. ■

While a formal proof is relegated to Appendix A, we provide some intuition here. In the period in which the shock to the discount factor occurs, as a representative consumer becomes more patient, he would save more and consume less. The reduction in aggregate consumption implies a reduction in the demand for both tradable and non-tradable goods. As the price of the tradable good is tied down by the world market, this leads to a decrease in the relative price of the non-tradable good (and a depreciation of the real exchange rate).

Now consider the intuition behind the labor supply channel. Under the assumption that the non-tradable sector is more labor intensive than the tradable sector, an increase in the number of labor generates a Rybzinsky-like effect, leading to an expansion of the non-tradable sector relative to the tradable sector. The increase in the supply of non-tradable good puts downward pressure on the relative price of non-tradable and produces a decline in the value of the RER.

In summary, without currency manipulations, real factors that lead to a rise in either a country's savings rate or its labor supply can simultaneously produce a fall in the real exchange rate.

Note that the effect on the RER lasts for one period. In period $t+1$, since the shock has been observed and taken into account by consumers and firms, (2.2) and (2.4) hold in equilibrium. By solving (2.2), (2.3), (2.4) and (2.5), we have

$$P_{Nt} = R^{\frac{\alpha_N - \alpha_T}{1-\alpha_T}} \text{ and } P_{t+1} = R^{\frac{\gamma(\alpha_N - \alpha_T)}{1-\alpha_T}}$$

In other words, the price of the non-tradable good and the consumer price index go back to their initial levels. Later in the paper, we will demonstrate how frictions in the factor market can produce longer-lasting effects on the real exchange rate.

3 A Model with Mating Competition

We now consider a model with two genders and a desire for marriage (or for sexual partner). Within each cohort, there are both men and women. A marriage can take place at the beginning of a cohort's second period, but only between a man and a women in the same cohort. Once married, the husband and wife pool their first-period savings together and consume an identical amount in the second period. The second period consumption within a marriage has a partial public good feature. In other words, the husband and wife can each consume more than half of their combined second period income. Everyone is endowed with an ability to give his/her spouse some additional emotional utility (or "love"). This emotional utility is a random variable in the first period with a common and known distribution across all members of the same sex, and its value is realized and becomes public information when an individual enters the marriage market. There are no divorces.

Each generation is characterized by an exogenous ratio of men to women $\phi(\geq 1)$. All men are identical ex ante, and all women are identical ex ante. Men and women are symmetric in all aspects - in particular, men do not have an intrinsic tendency to save more or to work more - except that the sex ratio may be unbalanced.

Throughout the model, we maintain the assumption of an exogenous sex ratio. While it is surely endogenous in the long run as parental preference should evolve, the assumption of an exogenous sex ratio can be defended on two grounds. First, the technology that enables the rapid rise in the sex ratio has only become inexpensive and widely accessible in developing countries within the last 25 years or so. As a result, it is reasonable to think that the rising sex ratio has a major effect only on the relatively young cohort's savings decisions, but not those who have passed half of their working careers. Second, in terms of cross country experience, most countries with a skewed sex ratio have not shown a sign of reversal. This suggests that, if the sex ratio follows a mean reversion process, the speed of reversion is likely to be very low.

3.1 A small open economy

For ease of discussion, we start with a small open economy with an exogenous labor supply. As in the benchmark model, the price of the tradable good is always one and the interest rate in units of the tradable good is a constant R.

A Representative Woman's Optimization Problem

A representative woman makes her consumption/saving decisions in her first period, taking into account the choices by men and all other women, and the likelihood that she will be married. If she fails to get married, her second-period consumption is given by $P_{t+1}C_{2,t+1}^{w,n} = Rs_t^w y_t^w$, where R, y_t^w and s_t^w are the gross interest rate of an international bond, her first period income, and her savings rate, respectively, all in units of the tradable good. If she is married, her second-period consumption is given by $P_{t+1}C_{2,t+1}^{w} = \kappa\left(Rs_t^w y_t^w + Rs_t^m y_t^m\right)$, where y_t^m and s_t^m are her husband's first period endowment and savings rate, respectively. κ $\left(\frac{1}{2} \leq \kappa \leq 1\right)$ represents the notion that consumption within a marriage is a public good with congestion. As an example, if two spouses buy a car, both can use it. In contrast, were they single, they would have to buy two cars. When $\kappa = \frac{1}{2}$, the husband and the wife only consume private goods. When $\kappa = 1$, then all the consumption is a public good with no congestion[2].

The optimal savings rate is chosen to maximize the following objective function:

$$V_t^w = \max_{s_t^w} u(C_{1t}^w) + \beta E_t \left[u(C_{2,t+1}^w) + \eta^m \right]$$

subject to the budget constraints that

$$P_t C_{1t}^w = (1 - s_t^w) y_t^w \tag{3.1}$$

$$P_{t+1} C_{2,t+1}^w = \begin{cases} \kappa \left(R s_t^w y_t^w + R s_t^m y_t^m \right) & \text{if married} \\ R s_t^w y_t^w & \text{otherwise} \end{cases} \tag{3.2}$$

where E_t is the conditional expectation operator. η^m is the emotional utility (or "love") she obtains from her husband, which is a random variable with a distribution function F^m. Bhaskar (2011) also introduces a similar "love" variable.

A Representative Man's Optimization Problem

A representative man's problem is symmetric to a women's problem. In particular, if he fails to get married, his second period consumption is given by $P_{t+1} C_{2,t+1}^{m,n} = R s_t^m y^m$. If he is married, his second period consumption is given by $P_{t+1} C_{2,t+1}^m = \kappa \left(R s_t^w y_t^w + R s_t^m y_t^m \right)$. He will choose his savings rate to maximize the following value function

$$V_t^m = \max_{s_t^m} u(C_{1t}^m) + \beta E_t \left[u(C_{2,t+1}^m) + \eta^w \right]$$

subject to the budget constraints that

$$P_t C_{1t}^m = (1 - s_t^m) y_t^m \tag{3.3}$$

$$P_{t+1} C_{2,t+1}^m = \begin{cases} \kappa \left(R s_t^w y_t^w + R s_t^m y_t^m \right) & \text{if married} \\ R s_t^m y_t^m & \text{otherwise} \end{cases} \tag{3.4}$$

where V^m is his value function. η^w is the emotional utility he obtains from his wife, which is drawn from a distribution function F^w.

The Marriage Market[3]

In the marriage market, every woman (or man) ranks all members of the opposite sex by a combination of two criteria: (1) the level of wealth (which is determined solely by the first-period savings), and (2) the size of "love" she/he can obtain from her/his spouse. The weights on the two criteria are

[2] By assuming the same κ for the wife and the husband, we abstract from a discussion of bargaining within a household. In an extension later in the paper, we allow κ to be gender specifc, and to be a function of both the sex ratio and the relative wealth levels of the two spouses, along the lines of Chiappori (1988 and 1992) and Browning and Chiappori (1998). This tends to make the response of the aggregate savings stronger to a given rise in the sex ratio.

[3] We use the word "market" informally here. The pairing of husbands and wives is not done through prices.

implied by the utility functions specified earlier. More precisely, woman i prefers a higher ranked man to a lower ranked one, where the rank on man j is given by $u(c_{2w,i,j}) + \eta_j^m$. Symmetrically, man j assigns a rank to woman i based on the utility he can obtain from her $u(c_{2m,j,i}) + \eta_i^w$. To ensure that the preference is strict for both men and women, whenever there is a tie in terms of the above criteria, we break the tie by assuming that a woman prefers j if $j < j'$ and a man does the same. Note that "love" is not in the eyes of a beholder in the sense that every woman (man) has the same ranking over men (women).

The marriage market is assumed to follow the Gale-Shapley algorithm, which produces a unique and stable equilibrium of matching (Gale and Shapley, 1962; and Roth and Sotomayor, 1990). The algorithm specifies the following: (1) Each man proposes in the first round to his most preferred choice of woman. Each woman holds the proposal from her most preferred suitor and rejects the rest. (2) Any man who is rejected in round k-1 makes a new proposal in round k to his most preferred woman among those who have not have rejected him. Each available woman in round k holds the proposal from her most preferred man and rejects the rest. (3) The procedure repeats itself until no further proposals are made, and the women accept the most attractive proposals.[4]

With many women and men in the marriage market, all women (and all men) approximately form a continuum and each individual has a measure close to zero. Let I^w and I^m denote the continuum formed by women and men respectively. We normalize I^w and let $I^w = (0, 1)$. Since the sex ratio is ϕ, the set of men $I^m = (0, \phi)$. Men and women are ordered in such a way that a higher value in the set means a higher ranking by members of the opposite sex.

In equilibrium, there exists a unique mapping (π^w) for women in the marriage market, $\pi^w : I^w \to I^m$. That is, woman i ($i \in I^w$) is mapped to man j ($j \in I^m$), given all the savings rates and emotional utility draws. This implies a mapping from a combination (s_i^w, η_i^w) to another combination (s_j^m, η_j^m). Before she enters the marriage market, she knows only the distribution of her own type but not the exact value. As a result, the type of her future husband (s_j^m, η_j^m) is also a random variable. Woman i's second period expected utility is

$$\begin{aligned}
& \int \max\left[\ u\left(c_{2w,i,j}\right) + \eta^m_{\pi^w\left(\ i|s_i^w,\eta_i^w,s_{-i}^w,\eta_{-i}^w,s^m,\eta^m\right)},\quad u\left(Rs_i^w y_i^w\right)\ \right] dF^w\left(\eta_i^w\right) \\
= & \int_{\bar{\pi}_i^w} \left[u\left(c_{2w,i,j}\right) + \eta^m_{\pi^w\left(\ i|s_i^w,\eta_i^w,s_{-i}^w,\eta_{-i}^w,s^m,\eta^m\right)}\right] dF^w\left(\eta_i^w\right) + \int^{\bar{\pi}_i^w} u\left(Rs_i^w y_i^w\right) dF^w\left(\eta_i^w\right)
\end{aligned}$$

where $\bar{\pi}_i^w$ is her threshold ranking on men such that she is indifferent between marriage or not. Any lower-ranked man, or any man with $\pi_i^w < \bar{\pi}_i^w$, won't be chosen by her.

Since we assume there are (weakly) fewer women than men, we expand the set I^w to $\widetilde{I}^w$ so that $\widetilde{I}^w = (0, \phi)$. In the expanded set, women in the marriage market start from value $\phi - 1$ to ϕ. The

[4] If only women can propose and men respond with deferred acceptance, the same matching outcomes will emerge. What we have to rule out is that both men and women can propose, in which case, one cannot prove that the matching is unique.

measure for women in the marriage market remains one. In equilibrium, there exists a unique mapping for men in the marriage market: $\pi^m : I^m \to \widetilde{I}^w$, where π^m maps man j ($j \in I^m$) to woman i ($i \in I^w$). Those men with a low value $i < \phi - 1$ in set $\widetilde{I}^w$ will not be married. In that case, $\eta^w_{\pi^m(j)} = 0$ and $c_{2m,j,i} = Rs^m_j y^m_j$. In general, man j's second period expected utility is

$$\begin{aligned} & \int \max\left[\ u\left(c_{2m,j,i}\right) + \eta^w_{\pi^m\left(j|s^m_j,\eta^m_j,s^m_{-j},\eta^m_{-j},s^w,\eta^w\right)},\quad u\left(Rs^m_j y^m_j\right)\ \right] dF^m\left(\eta^m_j\right) \\ = & \int_{\bar{\pi}^m_j} \left[u\left(c_{2m,j,i}\right) + \eta^w_{\pi^m\left(j|s^m_j,\eta^m_j,s^m_{-j},\eta^m_{-j},s^w,\eta^w\right)}\right] dF^m\left(\eta^m_j\right) + \int^{\bar{\pi}^m_j} u\left(Rs^m_j y^m_j\right) dF^m\left(\eta^m_j\right) \end{aligned}$$

where $\bar{\pi}^m_j$ is his threshold ranking on all women. Any woman with a poorer rank, $\pi^m_j < \bar{\pi}^m_j$, will not be chosen by him.

We assume that the density functions of η^m and η^w are continuously differentiable. Since all men (women) in the marriage market have identical problems, they make the same savings decisions. In equilibrium, a positive assortative matching emerges for those men and women who are married. In other words, there exists a mapping M from η^w to η^m such that

$$\begin{aligned} 1 - F^w(\eta^w) &= \phi\left(1 - F^m\left(M(\eta^w)\right)\right) \\ &\Leftrightarrow \\ M(\eta^w) &= \left(F^m\right)^{-1}\left(\frac{F^w(\eta^w)}{\phi} + \frac{\phi - 1}{\phi}\right) \end{aligned}$$

For simplicity, we assume that η^w and η^m are drawn from the same distribution, $F^w = F^m = F$. The lowest possible value of emotional utility $\eta^{\min}$ is sufficiently small (which can be negative) so that some women and some men may not get married. Let $\bar{\eta}^w$ and $\bar{\eta}^m$ denote the threshold values for women's and men's emotional utilities in equilibrium, respectively. Only women (men) with emotional utilities higher than the threshold value $\bar{\eta}^w$ ($\bar{\eta}^m$) will get married. In other words,

$$\bar{\eta}^w = \max\left\{u_{2m,n} - u_{2m}, M^{-1}\left(\bar{\eta}^m\right)\right\} \text{ and } \bar{\eta}^m = \max\left\{u_{2w,n} - u_{2w}, M(\bar{\eta}^w)\right\} \tag{3.5}$$

For woman i, given all her rivals' and men's savings decisions and η^w, her second period utility is

$$\delta^w_i u\left(\frac{\kappa\left(Rs^w_i y^w + Rs^m y^m\right)}{P_{t+1}}\right) + \left(1 - \delta^w_i\right) u\left(\frac{Rs^w y^w}{P_{t+1}}\right) + \int_{\tilde{\eta}^w_i \geq \bar{\eta}^w} M(\tilde{\eta}^w_i) dF(\eta^w_i)$$

where $\tilde{\eta}^w_i = u\left(\frac{\kappa(Rs^w_i y^w + Rs^m y^m)}{P_{t+1}}\right) - u\left(\frac{\kappa(Rs^w y^w + Rs^m y^m)}{P_{t+1}}\right) + \eta^w_i$. δ^w_i is the probability that woman i will get married,

$$\begin{aligned} \delta^w_i &= \Pr\left(u\left(\frac{\kappa\left(Rs^w_i y^w + Rs^m y^m\right)}{P_{t+1}}\right) - u\left(\frac{\kappa\left(Rs^w y^w + Rs^m y^m\right)}{P_{t+1}}\right) + \eta^w_i \geq \bar{\eta}^w \,\middle|\, Rs^w y^w, Rs^m y^m\right) \\ &= 1 - F\left(\bar{\eta}^w - u\left(\frac{\kappa\left(Rs^w_i y^w + Rs^m y^m\right)}{P_{t+1}}\right) + u\left(\frac{\kappa\left(Rs^w y^w + Rs^m y^m\right)}{P_{t+1}}\right)\right) \end{aligned} \tag{3.6}$$

Due to symmetry, we drop the sub-index i for women. Given men's savings decisions, the first order condition for her optimization problem is

$$-u_{1w}'\frac{y^w}{P_t}+\beta\left[\begin{array}{c}\delta^w u_{2w}'\frac{\partial c_{2w}}{\partial s^w}+(1-\delta^w)\,u_{2w,n}'\frac{R}{P_{t+1}}y^w+\frac{\partial\int_{\tilde{\eta}^w\geq\bar{\eta}^w}M(\tilde{\eta}^w)dF(\eta^w)}{\partial s^w}\\ +\frac{\partial\delta^w}{\partial s^w}\left(u_{2w}-u_{2w,n}\right)\end{array}\right]=0 \tag{3.7}$$

where

$$\begin{aligned}\frac{\partial\int_{\tilde{\eta}^w\geq\bar{\eta}^w}M(\tilde{\eta}^w)dF(\eta^w)}{\partial s^w} &= \kappa u_{2w}'\frac{Ry^w}{P_{t+1}}\left[\int_{\bar{\eta}^w}M'(\eta^w)dF(\eta^w)+M(\bar{\eta}^w)f(\bar{\eta}^w)\right]\\ \frac{\partial\delta^w}{\partial s^w} &= f\left(\bar{\eta}^w\right)\kappa u_{2w}'\frac{Ry^w}{P_{t+1}}\end{aligned}$$

Similarly, a representative man's second-period utility, given his rivals' and all women's savings decisions, is

$$\delta_j^m u\left(\frac{\kappa\left(Rs^w y^w+Rs_j^m y^m\right)}{P_{t+1}}\right)+(1-\delta_j^m)u\left(\frac{Rs_j^m y^m}{P_{t+1}}\right)+\int_{\tilde{\eta}_j^m\geq\bar{\eta}^m}M^{-1}\left(\tilde{\eta}_j^m\right)dF(\eta_j^m)$$

where $\tilde{\eta}_j^m = u\left(\frac{\kappa\left(Rs^w y^w+Rs_j^m y^m\right)}{P_{t+1}}\right)-u\left(\frac{\kappa\left(Rs^w y^w+Rs^m y^m\right)}{P_{t+1}}\right)+\eta_j^m$ and δ_j^m is the probability he gets married

$$\begin{aligned}\delta_j^m &= \Pr\left(u\left(\frac{\kappa\left(Rs^w y^w+Rs_j^m y^m\right)}{P_{t+1}}\right)-u\left(\frac{\kappa\left(Rs^w y^w+Rs^m y^m\right)}{P_{t+1}}\right)+\eta_j^m\geq\bar{\eta}^m\,\middle|\,Rs^w y^w,Rs^m y^m\right)\\ &= 1-F\left(\bar{\eta}^m-u\left(\frac{\kappa\left(Rs^w y^w+Rs_j^m y^m\right)}{P_{t+1}}\right)+u\left(\frac{\kappa\left(Rs^w y^w+Rs^m y^m\right)}{P_{t+1}}\right)\right)\end{aligned} \tag{3.8}$$

The first order condition for a representative man's optimization problem is

$$-u_{1m}'y^m+\beta\left[\begin{array}{c}\delta^m u_{2m}'\frac{\partial c_{2m}}{\partial s^m}+\frac{\partial\int_{\tilde{\eta}^m\geq\bar{\eta}^m}M^{-1}(\tilde{\eta}^m)dF(\eta^m)}{\partial s^m}+(1-\delta^m)\,u_{2m,n}'\frac{RP_t}{P_{t+1}}y^m\\ +\frac{\partial\delta^m}{\partial s^m}\left(u_{2m}-u_{2m,n}\right)\end{array}\right]=0 \tag{3.9}$$

where

$$\begin{aligned}\frac{\partial\int_{\tilde{\eta}^m\geq\bar{\eta}^w}M^{-1}(\tilde{\eta}^m)dF(\eta^m)}{\partial s^m} &= \kappa u_{2m}'\frac{RP_t}{P_{t+1}}y^m\left[\int_{\bar{\eta}^m}\frac{\partial M^{-1}(\eta^m)}{\partial\eta^m}dF(\eta^m)+M^{-1}(\bar{\eta}^m)f(\bar{\eta}^m)\right]\\ \frac{\partial\delta^m}{\partial s^m} &= f\left(\bar{\eta}^m\right)\kappa u_{2m}'\frac{RP_t}{P_{t+1}}y^m\end{aligned}$$

For simplicity, we assume that women and men will earn the same first period labor income and that there is no tax, i.e., $y_t^w = y_t^m = w_t$. For this section, we normalize the measure of young people to one; the labor market clearing condition then becomes

$$L_{Tt} + L_{Nt} = 1 \tag{3.10}$$

We now define an equilibrium in this economy.

Definition 2 *An equilibrium is a set of savings rates, capital and labor allocation by sector, and the relative price of non-tradable good* $\{s_t^w, s_t^m, K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$ *that satisfies the following conditions:*

(i) The savings rates by the representative woman and the representative man, conditional on other women and men's savings rates, $s_t^w = \left\{s_{it}^w, s_{-i,t}^w\right\}$ *and* $s_t^m = \left\{s_{jt}^w, s_{-j,t}^m\right\}$*, maximize their respective utilities*

$$\begin{aligned} s_{it}^w &= \arg\max\left\{ V_t^w |\, s_{-i,t}^w, s_t^m, K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\right\} \\ s_{jt}^w &= \arg\max\left\{ V_t^m |\, s_t^w, s_{-j,t}^m, K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\right\} \end{aligned}$$

(ii) The markets for capital, labor, and tradable and non-tradable goods clear, and firms maximize their profits. In other words, $\{K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$ *solves (2.2), (2.3), (2.4), (2.5), (2.6) and (3.10).*

We now consider an unanticipated shock to the sex ratio, i.e., the sex ratio for the young cohort rises from one to $\phi(> 1)$ from period t onwards. The nature of the shock is motivated by facts about the sex ratio in China. Since a severe sex ratio imbalance for the pre-marital age cohort is a relatively recent phenomenon, the older generations' savings decisions were largely made when there was no severe sex ratio imbalance. As the shock is unanticipated, (2.2) and (2.4) do not hold in period t.

As in the benchmark model, the market clearing condition for the non-tradable good can be re-written as

$$\frac{P_{Nt}A_{Nt}K_{Nt}^{\alpha_N}L_{Nt}^{1-\alpha_N}}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}} = \gamma\left(Rs_{t-1}w_{t-1} + (1-s_t)w_t\right) \tag{3.11}$$

where $s_t = \frac{\phi}{1+\phi}s_t^m + \frac{1}{1+\phi}s_t^w$ is the aggregate savings rate by the young cohort in period t.

By (2.3) and (2.5), we have

$$\frac{1}{\alpha_T^{\alpha_T}(1-\alpha_T)^{1-\alpha_T}}(1-\alpha_T)A_{Tt}\left(\frac{K_{Tt}}{1-L_{Nt}}\right)^{\alpha_T} = \frac{1}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}}P_{Nt}(1-\alpha_N)A_{Nt}\left(\frac{K_{Nt}}{L_{Nt}}\right)^{\alpha_N} \tag{3.12}$$

We can solve (3.7), (3.9), (3.11), (2.3) and (2.5) to obtain the equilibrium in period t. With some restrictions on the utility function and the distribution of emotional utility, we have the following proposition.

Proposition 2 *Assume that the per period utility function is of log form, $u(C) = \ln C$, for everyone, and that η is drawn from a uniform distribution, then, as the sex ratio in the young cohort rises, a representative man increases his savings rate while a representative woman reduces her savings. However, the aggregate savings rate in the young cohort rises unambiguously. The real exchange rate depreciates.*

Proof. See Appendix B. ■

A few remarks are in order. First, it is perhaps not surprising that the representative man raises his savings rate in response to a rise in the sex ratio because the need to compete in the marriage market becomes greater. How does the representative woman respond? Because she anticipates being able to free-ride on her future husband's increased savings, she does not need to sacrifice her first-period consumption as much as she otherwise would have to, and therefore can reduce her own savings rate.

Second, why does the aggregate savings rate rise unambiguously in response to a rise in the sex ratio? The answer comes from both an intensive margin and an extensive margin. On the intensive margin, the increment in the representative man's savings can be shown to be greater than the reduction in the representative woman's savings. Heuristically, the representative man raises his savings rate for two separate reasons: in addition to improving his relative standing in the marriage market, he wants to smooth his consumption over the two periods and would raise his savings rate to make up for the lower savings rate by his future wife. The more his future wife is expected to cut down her savings, the more he would have to raise his own savings to compensate. This ensures that his incremental savings is more than enough to offset any reduction in his future wife's savings. On the extensive margin, a rise in the sex ratio implies a change in the mix of the population with relatively more higher-saving men and relatively fewer lower-saving women. While both margins contribute to a rise in the aggregate savings rate, we verify in our calibrations that the intensive margin is quantitatively more important.

Third, once we obtain an increase in the aggregate savings rate, the logic from the previous one-gender benchmark model applies. In particular, the relative price of the non-tradable good declines and hence the real exchange rate depreciates.

Similar to the benchmark model with a single gender, once the shock is observed and taken into account in period $t+1$, (2.2) and (2.4) hold in equilibrium. By solving (2.2), (2.3), (2.4) and (2.5), we have

$$P_{Nt} = R^{\frac{\alpha_N - \alpha_T}{1-\alpha_T}} \text{ and } P_{t+1} = R^{\frac{\gamma(\alpha_N - \alpha_T)}{1-\alpha_T}}$$

This means that the real exchange rate will return to their previous values after one period.

3.2 Mixed-strategy equilibrium

In this section, we extend our benchmark model by considering an endogenous choice of entering/exiting the marriage market. Formally, we consider a mixed-strategy game in which (a) a representative woman will choose the probability of entering the marriage market ρ^w, a savings rate if she decides to enter, and a separate savings rate if she decides to abstain from the marriage market; and (b) a representative man has similar choices.

The representative woman will have the same optimization problem as in the previous section if she enters the marriage market. She can also choose to be single, and if she does so, her life-time utility is

$$V_n^w = \max_{s_n^w} u(c_{1w,n}) + \beta u(c_{2w,n})$$

where V_n^w denotes the value function of a woman who is single throughout her life.

Her overall optimization problem in the mixed-strategy game is

$$\max_{\rho^w, s^w, s_n^w} \rho^w V^w + (1 - \rho^w) V_n^w$$

Obviously, she will choose $\rho^w = 1$ if and only if $V^w > V_n^w$.

Similarly, a representative man's overall optimization problem is

$$\max_{\rho^m, s^m, s_n^m} \rho^m V^m + (1 - \rho^m) V_n^m$$

where V_n^m denotes the value function of a representative man who is single throughout his life, and ρ^m is his probability of entering the marriage market. He would decide to enter the marriage market with probability one if and only if the expected utility of doing so is greater than otherwise, or $V^m > V_n^m$.

Now we can show a more general proposition:

Proposition 3 *Assume that the utility function is of log form and that emotional utility is drawn from an independent and identical uniform distribution, then there exists a threshold value $\phi_1 > 1$ that satisfies $V^m = V_n^m$.*

(i) For $\phi < \phi_1$, both women and men choose to enter the marriage market with probability one. In addition, as the sex ratio rises, the savings rate of a representative man increases while the savings rate of a representative woman declines. However, the aggregate savings rate in the young cohort increases unambiguously. Importantly, the real exchange rate declines.

(ii) For $\phi \geq \phi_1$, as the sex ratio rises, a representative man chooses a positive probability of being single while a representative woman still chooses to enter the marriage market with probability one. The change in the real exchange rate is ambiguous.

Proof. See Appendix C. ■

A few remarks are in order. First, why is there a threshold? We can see the intuition by considering evolving incentives at different initial values of the sex ratio. When the sex ratio rises from a mildly unbalanced level, the intuition behind Proposition 2 applies. That is, the representative man responds to a higher sex ratio by raising his savings rate out of a desire to outcompete other men (and also to compensate for an anticipated decline in his future wife's savings rate). However, when the initial sex ratio is already sufficiently high, his calculations can be different. On the one hand, his savings rate is already very high, so that any additional increase is quite painful. On the other hand, his probability of getting married is objectively low regardless of his extra effort. In this case, in response to an additional increase in the sex ratio, the representative man may decide that the extra sacrifice associated with a further rise in the savings rate is not justified by the vanishingly small probability of getting married. This is why he decides to switch to a mixed strategy with some probability of not entering the marriage market.

Second, when we calibrate the models, we discover that for reasonable parameters and for a reasonable range of the sex ratio (i.e., for the sex ratio from 1 to 1.5), we do not see the threshold effect. In other words, while the threshold ϕ_1 is a theoretical possibility, it probably does not arise for the realistic levels of the sex ratio imbalance observed in the data. This means that, in the real world, only the first part of Proposition 3 is relevant. Therefore, a rise in the sex ratio in the actual data is likely to generate a decline in the real exchange rate.

Third, a rise in the sex ratio has different welfare effects on men versus women. A higher sex ratio produces two sources of welfare loss for men. A direct reduction in welfare comes from a reduction in the objective probability for men to get married. An indirect reduction in welfare comes from the increase in the competitive savings that causes them to deviate from optimal intertemporal consumption smoothing. Sadly for men as a group, the increased competitive savings do not fundamentally reduce the total number of men who cannot get married in general equilibrium. Interestingly, the welfare effect for women is ambiguous. On the one hand, women could gain both from their future husbands' higher savings rates and from the improved probability to be matched with men with a higher level of emotional utility. On the other hand, precisely because men have raised their savings rate, they become more reluctant to share their high savings rate with a low-type woman. As a result, a representative woman's ex ante chance of getting married can also decline. These two opposing forces produce an ambiguous net effect for women. It is useful to note that, while a representative woman could lose from a higher sex ratio, her ex ante utility level is always higher than that of the representative man.

3.3 Capital adjustment costs and a prolonged effect on the exchange rate

Without additional frictions, a shock to the sex ratio can only affect the real exchange rate for one period. If there are capital adjustment costs in each sector, the effect on the real exchange rate can be prolonged. As is standard in the literature, we assume a quadratic capital adjustment cost, then the optimization problems for firms in the tradable good sector and the non-tradable good sector become, respectively,

$$\max \ E_t \sum_{\tau=0}^{\infty} (R)^{-\tau} \left[Q_{T,t+\tau} - w_{t+\tau} L_{T,t+\tau} - I_{T,t+\tau} \right]$$

and

$$\max\ E_t \sum_{\tau=0}^{\infty} (R)^{-\tau} \left[P_{N,t+\tau}Q_{N,t+\tau} - w_{t+\tau}L_{N,t+\tau} - I_{N,t+\tau}\right]$$

Raising the capital to $K_{i,t+1}$ in period $t+1$ requires time t investment I_{it} to satisfy

$$I_{it} = K_{i,t+1} - (1-\delta)K_{it} + \frac{b}{2}\left(\frac{K_{i,t+1}}{K_{it}} - 1\right)^2 K_{it},\ i = T, N$$

Then (2.2) and (2.4) become, respectively,

$$\begin{aligned} R &= 1-\delta+\frac{1}{\alpha_T^{\alpha_T}(1-\alpha_T)^{1-\alpha_T}}\alpha_T A_{T,t+1}\left(\frac{L_{T,t+1}}{K_{T,t+1}}\right)^{1-\alpha_T} \\ &\quad -bR\left(\frac{K_{T,t+1}}{K_{Tt}}-1\right)+\frac{b}{2}\left(\left(\frac{K_{T,t+2}}{K_{T,t+1}}\right)^2-1\right) \qquad (3.13) \\ R &= 1-\delta+\frac{1}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}}P_{N,t+1}\alpha_N A_{N,t+1}\left(\frac{L_{N,t+1}}{K_{N,t+1}}\right)^{1-\alpha_N} \\ &\quad -bR\left(\frac{K_{N,t+1}}{K_{Nt}}-1\right)+\frac{b}{2}\left(\left(\frac{K_{N,t+2}}{K_{N,t+1}}\right)^2-1\right) \qquad (3.14) \end{aligned}$$

Without capital adjustment costs, i.e., $b = 0$, the price of the non-tradable good will go back to its equilibrium level in period $t+1$. With capital adjustment costs, i.e., $b > 0$, then

$$P_{N,t+1} = \frac{\frac{1}{\alpha_T^{\alpha_T}(1-\alpha_T)^{1-\alpha_T}}\alpha_T A_{Tt+1}\left(\frac{L_{Tt+1}}{K_{Tt+1}}\right)^{1-\alpha_T} - bR\left(\frac{K_{T,t+1}}{K_{Tt}}-\frac{K_{N,t+1}}{K_{Nt}}\right)-\frac{b}{2}\left(\left(\frac{K_{T,t+2}}{K_{T,t+1}}\right)^2-\left(\frac{K_{N,t+2}}{K_{N,t+1}}\right)^2\right)}{\frac{1}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}}\alpha_N A_{Nt+1}\left(\frac{L_{Nt+1}}{K_{Nt+1}}\right)^{1-\alpha_T}}$$

$P_{N,t+1}$ is now a function of $\frac{K_{T,t+1}}{K_{Tt}}$, $\frac{K_{N,t+1}}{K_{Nt}}$, $\frac{K_{T,t+2}}{K_{T,t+1}}$ and $\frac{K_{N,t+2}}{K_{N,t+1}}$. If $\frac{K_{T,t+1}}{K_{Tt}} \neq \frac{K_{N,t+1}}{K_{Nt}}$ and $\frac{K_{T,t+2}}{K_{T,t+1}} \neq \frac{K_{N,t+2}}{K_{N,t+1}}$, $P_{N,t+1}$ is not a constant. This means that, with capital adjustment costs, the price of the non-tradable good does not return immediately to its long-run equilibrium level. As a result, a rise in the sex ratio can have a long-lasting and depressing effect on the real exchange rate.

3.4 Two large countries

We now turn to a world with two large countries: Home and Foreign. Assume that they are identical in every respect except for their sex ratios. Specifically, in period t, the sex ratio of the young cohort in Home rises from one to ϕ ($\phi > 1$), while Foreign always has a balanced sex ratio. Households in each country consume a tradable good and a non-tradable good.

$$C_t = \frac{C_{Nt}^{\gamma}C_{Tt}^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \text{ and } C_t^* = \frac{(C_{Nt}^*)^{\gamma}(C_{Tt}^*)^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}$$

where C_t and C_t^* represent home and foreign consumption indexes, respectively. Since we choose the tradable good as the numeraire, the consumer price index is $P_t = P_{Nt}^{\gamma}$, where P_{Nt} is the price of the home produced non-tradable good. Similarly, the consumer price index in Foreign is $P_t^* = (P_{Nt}^*)^{\gamma}$.

The rise in Home's sex ratio in period t is assumed to be unanticipated. As a result, (2.2) and (2.4) fail in both Home and Foreign. Since two countries are identical except the sex ratios, by Proposition 2, Home will have a lower relative non-tradable good price than Foreign. This means that Home will experience a real exchange rate depreciation relative to Foreign in period t. Similar to previous analysis, Home will also experience a real appreciation in period $t+1$.

3.5 Endogenous labor supply

We turn to the case of endogenous labor supply. Just as a male raises his savings rate to gain a competitive advantage in the marriage market, he may choose to increase his supply of labor for the same reason in response to a rise in the sex ratio. This can translate into an increase in the effective aggregate labor supply if women do not decrease their labor supply too much. If the production of the non-tradable good is more labor-intensive[5], the increase in the effective labor supply can further reduce the relative price of the non-tradable good (and the value of the real exchange rate). Therefore, endogenous labor supply could reinforce the savings channel from the sex ratio shock, leading to an additional reduction in the real exchange rate.

We allow each person to endogenously choose the first period labor supply and the utility function of the first period is $u(C) + v(1-L)$, where L is the labor supply and $v(1-L)$ is the utility function of leisure as in the benchmark model. Again, for simplicity, we assume no tax on the labor income. The utility function governing the leisure-labor choice is the same for men and women. In other words, by assumption, men and women are intrinsically symmetric except for their ratio in the society.

We can rewrite the optimization problem for a representative woman as following:

$$\max \; u(C_{1t}^w) + v(1 - L_t^w) + \beta E_t \left[u(C_{2,t+1}^w) + \eta^m \right]$$

with the budget constraint

$$\begin{aligned} P_t C_{1t}^w &= (1 - s_t^w) w_t L_t^w \\ P_{t+1} C_{2,t+1}^w &= \begin{cases} \kappa \left(R s_t^w L_t^w + R s_t^m L_t^m \right) w_t & \text{if married} \\ R s_t^w w_t L_t^w & \text{otherwise} \end{cases} \end{aligned}$$

[5] This is confirmed by the data from the Chinese Input-Output Tables of 2002 and 2007 (the most recent two tables released).

The first order conditions with respect to her savings rate and labor supply are

$$-u'_{1w}\frac{w_t L_t^w}{P_t}+\beta\left[\begin{array}{c}\delta^w \kappa u'_{2w}\frac{R w_t L_t^w}{P_{t+1}}+(1-\delta^w)\,u'_{2w,n}\frac{R w_t L_t^w}{P_{t+1}}+\frac{\partial\int_{\tilde{\eta}^w\geq\bar{\eta}^w}M(\tilde{\eta}^w)dF(\eta^w)}{\partial s^w}\\ +\frac{\partial\delta^w}{\partial s^w}\left(u_{2w}-u_{2w,n}\right)\end{array}\right]=0 \tag{3.15}$$

and

$$u'_{1w}\frac{(1-s_t^w)w_t}{P_t}+\beta\left[\begin{array}{c}\delta^w \kappa u'_{2w}\frac{R s_t w_t}{P_{t+1}}+(1-\delta^w)\,u'_{2w,n}\frac{R s_t w_t}{P_{t+1}}+\frac{\partial\int_{\tilde{\eta}^w\geq\bar{\eta}^w}M(\tilde{\eta}^w)dF(\eta^w)}{\partial L_t^w}\\ +\frac{\partial\delta^w}{\partial L_t^w}\left(u_{2w}-u_{2w,n}\right)\end{array}\right]-v'_w=0 \tag{3.16}$$

respectively. Notice that $\frac{\partial C_{2,t+1}^w}{\partial L_t^w}=\frac{\partial C_{2,t+1}^w}{\partial s_t^w}\frac{s_t^w}{L_t^w}$ and $\frac{\partial\int M(\tilde{\eta}^w)d\tilde{F}^w(\tilde{\eta}^w)}{\partial L_t^w}=\frac{\partial\int M(\tilde{\eta}^w)d\tilde{F}^w(\tilde{\eta}^w)}{\partial s_t^w}\frac{s_t^w}{L_t^w}$. By (3.15) and (3.16), we have

$$\frac{w_t}{P_t}=\frac{v'_w}{u'_{1w}} \tag{3.17}$$

The optimization problem for a representative man is similar:

$$\max\ u(C_{1t}^m)+v(1-L_t^m)+\beta E_t\left[u(C_{2,t+1}^m)+\eta^w\right]$$

with the budget constraint

$$\begin{aligned}P_t C_{1t}^m &= (1-s_t^m)w_t L_t^m\\ P_{t+1}C_{2,t+1}^m &= \begin{cases}\kappa\left(Rs_t^m L_t^m+Rs_t^m L_t^m\right)w_t & \text{if married}\\ Rs_t^m w_t L_t^m & \text{otherwise}\end{cases}\end{aligned}$$

The optimization conditions for his savings rate and labor supply are

$$-u'_{1m}\frac{w_t L_t^m}{P_t}+\beta\left[\begin{array}{c}\delta^m \kappa u'_{2m}\frac{R w_t L_t^m}{P_{t+1}}+(1-\delta^m)\,u'_{2m,n}\frac{R w_t L_t^m}{P_{t+1}}+\frac{\partial\int_{\tilde{\eta}^m\geq\bar{\eta}^m}M(\tilde{\eta}^m)dF(\eta^m)}{\partial s^m}\\ +\frac{\partial\delta^m}{\partial s^m}\left(u_{2m}-u_{2m,n}\right)\end{array}\right]=0 \tag{3.18}$$

and

$$\frac{w_t}{P_t}=\frac{v'_m}{u'_{1m}} \tag{3.19}$$

respectively.

On the supply side, all equilibrium conditions other than the labor market clearing condition remain the same. If we normalize the measure of the young cohort to be one, then the labor market clearing condition becomes

$$L_{Tt}+L_{Nt}=\frac{1}{1+\phi}L_t^w+\frac{\phi}{1+\phi}L_t^m \tag{3.20}$$

We now define an equilibrium for such an economy.

Definition 3 *An equilibrium is a set* $\{(s_t^w,L_t^w),(s_t^m,L_t^m),K_{T,t+1},K_{N,t+1},L_{Tt},L_{Nt},P_{Nt}\}$ *that satis-*

fies the following conditions:

(i) The savings and labor supply decisions by women and men, $(s_t^w, L_t^w) = \left\{s_{it}^w, s_{-i,t}^w, L_{it}^w, L_{-i,t}^w\right\}$ *and* $(s_t^m, L_t^m) = \left\{s_{it}^m, s_{-i,t}^m, L_{it}^m, L_{-i,t}^m\right\}$*, maximize their utilities, respectively,*

$$\begin{aligned}
(s_{it}^w, L_{it}^w) &= \arg\max\left\{V_t^w | \left(s_{-i,t}^w, L_{-i,t}^w\right), (s_t^m, L_t^m), K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\right\} \\
\left(s_{jt}^m, L_{jt}^m\right) &= \arg\max\left\{V_t^m | (s_t^w, L_t^w), \left(s_{-j,t}^m, L_{-j,t}^m\right), K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\right\}
\end{aligned}$$

(ii) The markets for both goods and factors clear, and firms' profits are maximized. In other words, $\{K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$ *solves (2.2), (2.3), (2.4), (2.5), (2.6) and (3.20).*

As before, we assume that $u(C) = \ln C$. We let L_t denote the aggregate labor supply in period t, and assume that the utility function for leisure satisfies the feature[6]

$$\frac{2\left(v''\right)^2}{v'} + \frac{v'}{L^2} - \frac{v''}{L} - v''' > 0 \tag{3.21}$$

If the country stays at the initial equilibrium (before period t) with non-negative net foreign assets, then we can show the following proposition:

Proposition 4 *Under the same assumptions as in Proposition 2, as the sex ratio (in the young cohort) rises in period t, a representative man increases both his labor supply and his savings rate, while a representative woman's savings rate and labor supply decline. The real exchange rate depreciates.*

Proof. See Appendix D. ■

In response to a rise in the sex ratio, for the same reason that men reduce their consumption and increase their savings rate, they cut down their leisure and increase their labor supply. Similarly, for women, for the same reason that induces them to reduce their savings, they may reduce their labor supply (and increase leisure). In the aggregate, for the same reason that the increase in savings by men is more than enough to offset the decrease in savings by women, the increase in labor supply by men is also larger than the decrease in labor supply by women. Therefore, the aggregate labor supply rises in response to a rise in the sex ratio.

With a fixed labor supply (in the previous subsections), it is worth remembering that the non-tradable sector shrinks after a rise in the sex ratio. Specifically, a decline in the relative price of the non-tradable good (due to the savings channel) makes it less attractive for labor and capital to stay in the non-tradable sector. Now, with an endogenous labor supply, the total effective labor supply increases after a rise in the sex ratio according to Proposition 3. By a logic similar to the Rybczynski theorem, this by itself has a tendency to induce an expansion of the non-tradable sector if the production of the non-tradable good is more labor intensive. Relative to the case of a fixed labor

[6] It is easy to show that utility function $v(1-L) = \frac{B(1-L)^{1-\theta}}{1-\theta}$ $(\theta \geq 1)$ satisfies the assumption.

supply, adding the effect of endogenous labor supply leads to either an expansion of the non-tradable sector, or at least a smaller reduction in the size of the non-tradable sector. The exact scenario depends on parameter values. However, regardless of what happens to the size of the non-tradable sector, the price of the non-tradable good (and the value of the real exchange rate) must fall by a greater amount when the endogenous labor supply effect is added to the savings effect.

4 Some Numerical Examples

Before we go to our empirical results, we provide some numerical examples, both to illustrate the possible quantitative effects of a rise in the sex ratio, and to explore robustness of the main results to different values of the key parameters. To build in more realism relative to the benchmark model, we introduce three modifications. First, we assume that each cohort lives 50 periods, working in the first 30 periods, and retiring in the remaining 20 periods. Second, we follow the literature and introduce a cost for distributing tradable goods. Third, we also introduce capital adjustment costs.

4.1 An OLG model in which a cohort lives 50 periods

For a representative man or woman, if he or she gets married, the marriage takes place in the (exogenously predetermined) τth period, which is common for both men and women. To determine the value of τ, we have to balance two considerations. On the one hand, if it is the representative agent's own marriage, it would be reasonable to set a relatively low value for τ, perhaps around 5. On the other hand, data suggests that a big part of the savings and work effort responses to higher sex ratios come from actions taken by parents for their children (Wei and Zhang, 2011a and 2011b). While our model does not formally feature parental savings or parental income transfers to children, we do not want the simulations to ignore completely this important data feature. If τ is to represent the number of working years a parent has when his/her child gets married, we may set a relatively high value, perhaps around 25. As a compromise, we set $\tau = 15$ as the benchmark value in our simulations.

For a robustness check, we will also report results when $\tau = 10$. One may also think that τ should take a value greater than 15. Generally speaking, the greater the value of τ, the stronger is the real exchange rate response to a given rise in the sex ratio.

We now describe the representative woman's optimization problem. Her objective function is

$$\max \sum_{t=1}^{\tau-1} \beta^{t-1} U_t^w + E_1 \left[\sum_{t=\tau}^{50} \beta^{t-1} \left(U_t^w + \eta^m \right) \right]$$

where

$$U_t^w = \begin{cases} u(c_t^w) + v(1 - L_t^w) & \text{if } t \leq 30 \\ u(c_t^w) + v(1) & \text{if } t > 30 \end{cases}$$

For $t < \tau$, when she is still single, the intertemporal budget constraint is

$$A_{t+1} = R\left(A_t + y_t^w - P_t c_t^w\right)$$

where A_t is the wealth held by her at the beginning of period t. $y_t^w = w_t L_t^w$ is her labor income at the age t. After marriage ($t \geq \tau$), her family budget constraint becomes

$$A_{t+1}^H = \begin{cases} R\left(A_t^H + w_t\left(L_t^w + L_t^m\right) - \frac{P_t c_t^H}{\kappa}\right) & \text{if } t \leq 30 \\ R\left(A_t^H - \frac{P_t c_t^H}{\kappa}\right) & \text{if } t > 30 \end{cases}$$

where A_t^H is the level of family wealth at the beginning of period t. c_t^H is the (common) consumption good consumed by the wife and the husband, which takes the same form as in the two-period OLG model. If she remains single, her budget constraint after period τ is

$$A_{t+1}^{w,n} = \begin{cases} R\left(A_t^{w,n} + w_t L_t^w - P_t c_t^{w,n}\right) & \text{if } t \leq 30 \\ R\left(A_t^{w,n} - P_t c_t^{w,n}\right) & \text{if } t > 30 \end{cases}$$

The representative man's optimization problem is similar.

4.2 Parameters used in the examples

We assume the same sub-utility function on leisure as in the two-period model, $v(1 - L) = B\ln(1 - L)$. Parameter B is set to match the fact that the equilibrium labor supply (under a balanced sex ratio) is 1/3 (corresponding to 8 working hours per day). We assume that there is a lower bound for labor supply $\bar{L}$, $L_t^i \geq \bar{L}$ $(i = w, m)$. This is to prevent women from reducing their labor supply too much and generating an unrealistic reduction in their labor supply when the sex ratio rises. A possible justification for the lower bound is this: It may be unrealistic for most people to find a job that allows for downward adjustment of working hours in a flexible manner. Part-time jobs such as babysitting are not generally available in all industries. On the other hand, one can often do overtime or moonlighting for a second job. In other words, it is relatively easier to adjust labor supply upward in a fractional manner than to adjust it downward. We set $\bar{L} = 1/3$. This value is chosen so that both men and women would supply labor around 1/3 in the equilibrium with a balanced sex ratio. [In unreported simulations, we verify that relaxing this assumption does not dramatically change the quantitative results on the overall RER response to a higher sex ratio (although the responses by some other variables could change more noticeably)].

Following Song, Storesletten, and Zilibotti (2011), we take 1.0175 as the annual gross interest rate in China. The subjective discount factor is set at $\beta = 1/R$. Using China's Input-Output Table in 2007, we calibrate the labor-intensities in the tradable good sector and the non-tradable good sector, respectively. To be more precise, we first define a sector as a non-tradable good sector if the ratio of its sum of exports and imports to the gross sectoral output is below the median. By this criterion, most service sectors are non-tradables, whereas most manufacturing sectors are classified as tradable sectors. We then compute the shares of labor costs in total production costs for the two aggregated sectors by matching the mean and the variance of the two aggregated sectors to the same moments for the labor shares in all sectors in the input-output table.[7] This procedure gives us the results that $\alpha_T = 0.65$ and $\alpha_N = 0.23$.

Burstein, Neves, and Rebelo (2003) find that the costs of distributing tradable goods (transportation, wholesaling and retailing) are important for understanding the movements in the RER. For this reason, we assume that consuming the tradable good requires the use of distribution services (represented by the use of the non-tradable good)[8]. Since the total available non-tradable good can be either consumed or used to supply distribution services, we have, in equilibrium,

$$Q_{Nt} = C_{Nt} + \zeta C_{Tt}$$

This will only change the market clearing condition for the non-tradable good and the final price of the tradable good

$$Q_{Nt} = \frac{\gamma P_t \left(C_{2t} + C_{1t}\right)}{P_{Nt}} + \zeta \frac{\left(1-\gamma\right) P_t \left(C_{2t} + C_{1t}\right)}{P_{Tt}}$$

where

$$P_{Tt} = 1 + \zeta P_{Nt}$$

The distribution margin (the fraction of distribution cost in the final tradable good price) is

$$\mu = \frac{\zeta P_{Nt}}{1 + \zeta P_{Nt}}$$

We set $\gamma = 0.44$ to match the Chinese data - the share of all non-tradable sectors collectively accounts for 44% of the final consumption good basket. Following Burstein, Neves, and Rebelo (2003), we choose parameter ζ so that the distribution margin μ is 50%. We also consider $\mu = 0.25$ as a robustness check.

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[7] Suppose there are two sectors, tradable and non-tradable, in the economy with weight ω_T and ω_N. We compute the labor shares in the two sectors $1 - \alpha_T$ and $1 - \alpha_N$ by the following methodology:

$$\begin{aligned} 1 - \bar{\alpha} &= \omega_T \left(1 - \alpha_T\right) + \omega_N \left(1 - \alpha_N\right) \\ \sigma_\alpha^2 &= \omega_T \left(\left(1 - \alpha_T\right) - \left(1 - \bar{\alpha}\right)\right)^2 + \omega_N \left(\left(1 - \alpha_N\right) - \left(1 - \bar{\alpha}\right)\right)^2 \end{aligned}$$

where $1 - \bar{\alpha}$ and σ_α^2 are the mean and variance of labor shares in all sectors.

[8] No qualitative results will change if we add the distribution cost to our benchmark model. Mathematically, we can show that for any $0 \leq \zeta < 1$, under the assumption in Propositions 2 and 4, the aggregate demand for the non-tradable good, $C_{Nt} + \zeta C_{Tt}$, is a decreasing function of the sex ratio, which ensures the same results in Propositions 2 and 4. Intuitively, the aggregate demand for the non-tradable good consists of (i) the nontradable consumption good directly demanded by consumers and (ii) the non-tradable services used in distributing the tradable consumption good (which is positively correlated with the demand for the tradable good). As the sex ratio rises, the aggregate savings rate rises. Then the demand for both the tradable and the non-tradable consumption goods falls which in turn leads to a fall in the aggregate demand for the non-tradable goods. Based on the same logic, the relative price of the non-tradable good falls which in turn leads to a real exchange rate depreciation.

For the congestion index in the within-marriage consumption allocation, we set $\kappa = 0.8$. We also choose $\kappa = 0.7$ and 0.9 for robustness checks. As in Song et al. (2011), we set the annual capital depreciation rate δ to be 0.1.

For the quadratic capital adjustment costs in production, $\frac{b}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t$, there is no consensus on the adjustment parameter b. Its value ranges from 0.5 to 20 in the literature. Following Gali et al. (2004), we choose the value for parameter b such that the elasticity of the investment-capital ratio with respect to Tobin's Q to be one in the benchmark[9]. This implies that $b = 10$. Based on data on a panel of Chinese manufacturing plants, Wu (2011) estimates a capital adjustment function (embedded in a structural model) that is more complex than ours. Her estimates suggest that the coefficient for the quadratic term in the capital adjustment cost in China can be almost 10 times larger than the corresponding coefficient estimated for the United States by Cooper and Haltiwanger (2006). While we use $b = 10$ as our baseline case, we will also consider $b = 5$ and 15 for robustness checks.

The Chinese data suggests an interesting (and maybe peculiar) feature about a typical worker's life-time earnings profile. Using data from urban household surveys, Yang and Song (2011) document that a typical worker in China faces a fairly flat life-time (real) earnings profile (although the starting salary of each successive cohort tends to rise fast). Within a given cohort, we also assume a flat earnings profile over time. Since we do not consider an exogenous growth in productivity, we do not feature a steady rise in income from one cohort to the next.

The emotional utility η needs to follow a continuously differentiable distribution. We assume a normal distribution which might be more realistic than the uniform distribution used in the analytical model. To find the appropriate values for the mean and the standard deviation for this distribution is a bit challenging. We use the empirical literature on the effects of marital status and income on happiness as our guide.

To be more specific, to choose the mean value for emotional utility, we perform the following

[9] Tobin's Q for firm i is defined as

$$q_{it} = \frac{P_{K_i't}}{P_{It}}$$

where $P_{K_i't}$ denotes the marginal value of capital at firm i installed at the beginning of time $t+1$ and P_{It} is the price of investment goods, which is unity. Profit maximization implies that the value of a marginal unit of installed capital is equal to its cost. That is

$$P_{K_i't} = \frac{P_{It}}{MPI_{it}} = \frac{1}{MPI_{it}}$$

where

$$MPI_{it} = \frac{dK_{it+1}}{dI_{it}}$$

Since K_{it} is predetermined, under the quadratic capital adjustment cost function,

$$q_{it} = 1 + b\left(\frac{K_{it+1}}{K_{it}} - 1\right)$$

Then, the elasticity of investment-capital ratio with respect to Tobin's Q in the steady state is

$$\frac{d\left(I_{it}/K_{it}\right)}{dq_{it}} \frac{q_{it}}{I_{it}/K_{it}} = \frac{1}{b\delta}$$

thought experiment. Holding all other factors constant, we can compute the annual income compensation needed for a representative person to be indifferent between being a life-time bachelor and getting married. Let $CY_\tau = \{y_\tau + compensation_\tau, y_{\tau+1} + compensation_{\tau+1}, \cdots, y_{50} + compensation_{50}\}$ be a vector of all his future incomes after period τ. Then

$$u\left(\kappa\left(c_t^w + c_t^m\right)\right) + v\left(1 - L_t^m\right) + E\left(\eta\right) = u\left(c_{n,t}^m\left(CY_\tau\right)\right) + v\left(1 - L_{n,t}^m\right)$$

where $c_{n,t}^m\left(CY_\tau\right)$ is his consumption function in period t. L_t^m and $L_{n,t}^m$ are period t's labor supplies by a married man and a life-time bachelor, respectively.

Under a balanced sex ratio, we can re-write a representative man's optimization problem as the following

$$\max_{c_{1t}^m, c_{2t}^m} \left(\sum_{t=1}^{\tau-1} \beta^{t-1}\right) \left[u(c_{1t}^m) + v(1 - L_{1t}^m)\right] + \left(\sum_{t=\tau}^{50} \beta^{t-1}\right) E_1\left[\left(u(c_{2t}^m) + v(1 - L_{2t}^m) + \eta^m\right)\right]$$

where c_{1t}^m and c_{2t}^m are consumptions before and after period τ, respectively; and L_{1t}^m and L_{2t}^m ($L_{2t}^m = 0$ for $t > 30$) are labor supplies before and after period τ, respectively. In equilibrium, since wage is a constant, c_{1t}^m, c_{2t}^m, L_{1t}^m and L_{2t}^m are also constants. Since we choose parameter B - the parameter for the disutility of work - to let the equilibrium labor supply be 1/3, we further re-write the optimization for the representative man as

$$\max_{c_1^m, c_2^m} \left(\sum_{t=1}^{\tau-1} \beta^{t-1}\right) \left[u(c_t^m) + \widehat{\beta} E_1\left[\left(u(c_2^m) + \eta^m\right)\right] + t.i.p\right]$$

where we use $t.i.p$ to denote those terms that are independent of the optimization problem and $\widehat{\beta} = \left(\sum_{t=\tau}^{50} \beta^{t-1}\right) / \left(\sum_{t=1}^{\tau-1} \beta^{t-1}\right)$. The budget constraint in this case is

$$A_\tau = \frac{R - R^\tau}{1 - R}\left(w - c_1^m\right) \tag{4.1}$$

$$\left(\sum_{t=\tau}^{50} R^{-(t-\tau)}\right) c_2^m = A_\tau + \frac{1}{3}\left(\sum_{t=\tau}^{30} R^{-(t-\tau)}\right) w \tag{4.2}$$

Similar to our two-period OLG model, under a balanced sex ratio, the first order condition with respect to $\frac{A_\tau}{\frac{R-R^\tau}{1-R}}$ is

$$-u'_{1m} + \widehat{\beta}\frac{R - R^\tau}{1 - R}\left[\kappa u'_{2m}\left(\delta^m + \left(1 - F\left(\overline{\eta}^m\right)\right)\right) + \left(1 - \delta^m\right) u'_{2m,n}\right] = 0$$

where $\delta^m = 1 - F\left(\overline{\eta}^m\right)$. Under the log utility assumption, and due to the symmetry between men and women, the first order condition becomes

$$-\frac{1}{c_1^m} + \widehat{\beta}\frac{R - R^\tau}{1 - R}\frac{1}{c_2^m} = 0 \tag{4.3}$$

Combining (4.1), (4.2) with (4.3), we can solve the model under a balanced sex ratio in which the consumption does not depend on any emotional utility assumptions.

Given CY_τ and A_τ obtained from the previous solution, we can also compute a life-time bachelor's consumption in each period after age τ. Blanchflower and Oswald (2004), by regressing self-reported well-being scores on income, marriage status, and other determinants, estimate that a lasting marriage is, on average, worth \$100,000 (in 1990 dollars) per year (every year) in the United States during 1972-1998. Since GDP per person employed is about \$48,000 during the same period, this implies that a lasting marriage per year is worth more than twice the average working income for employed people. We take the ratio of income compensation to the average wage income $m = 100,000/48,000 \simeq 2.08$ as the benchmark. This implies that

$$CY_\tau = \left\{ \underbrace{(1+m)wL_{n,2}^m, \cdots, (1+m)wL_{n,2}^m}_{t \leq 30}, \underbrace{mwL_{n,2}^m \cdots, mwL_{n,2}^m}_{t>30} \right\}$$

Since the United States has a balanced sex ratio over that period, we calibrate the mean value of the emotional utility/love by using the previous model results:[10]

$$E(\eta) = u\left(c_{n,t}^m\left(CY_\tau\right)\right) + v\left(1 - L_{n,t}^m\right) - \left[u\left(\kappa\left(c_t^w + c_t^m\right)\right) + v\left(1 - L_t^m\right)\right]\Big|_{\phi_t = 1}$$

As a robustness check, we will also consider $m = 1$.

As for the standard deviation of emotional utility, we set $\sigma = 0.1$ in the benchmark and $\sigma = 0.2$ as a robustness check. This is consistent with the empirical results in Blanchflower and Oswald (2004) that the absolute values of the t statistics, $|t.stat| = \frac{E(\eta)}{\sigma}$, for the coefficient estimates on the dummy variable "never married" (the United States) or the dummy variable "married" (the United Kingdom) for men and women are between 3 and 16.

For technical reasons, we also assume that there exists an exogenous marriage market friction that everybody in the marriage market faces a small possibility of not getting matched that is independent of the sex ratio. We choose that probability to be 0.001.[11]

We summarize our choices of the parameter values in the following table.

[10] Under the log utility assumption, w will disappear when we calculate the mean of the emotional utility $E(\eta)$.

[11] This assumption plays no role in the theoretical analysis but ensures that we obtain real roots in the numerical computations. Small departures from this value have little effect on the quantitative results.

Choice of Parameter Values

Parameters	*Benchmark*	*Source and robustness checks*
Annual gross interest rate	$R = 1.0175$	Song et al. (2010)
Discount factor	$\beta = 1/R$	
Non-tradable sector capital-intensity	$\alpha_N = 0.23$	Authors' calculation based on 2007 Chinese input-output table
Tradable sector capital-intensity	$\alpha_T = 0.65$	Authors' calculation based on 2007 Chinese input-output table
Share of non-tradable good in the consumption basket	$\gamma = 0.44$	Authors' calculation based on 2007 Chinese input-output table
Congestion index	$\kappa = 0.8$	$\kappa = 0.7, 0.9$ as robustness checks.
Capital adjustment cost	$b = 10$	$b = 5, 15$ as robustness checks
Love, mean	$m = 2.08$	$m = 1$ as a robustness check
Love, standard deviation	$\sigma = 0.1$	$\sigma = 0.2$ as a robustness check
Marriage age	$\tau = 15$	$\tau = 10$ as a robustness check
Distribution margin	$\mu = 0.5$	$\mu = 0.25$ as a robustness check

4.3 Numerical results

We solve the multi-period model by iterations. Given a guess for a sequence of the wage rates and future capital accumulation path, we solve the equilibrium $\{(s_t^w, L_t^w), (s_t^m, L_t^m), K_{T,t+1}, K_{N,t+1}, L_{Tt}, L_{Nt}, P_{Nt}\}$ from equations (3.15), (3.17), (3.18), (3.19), (3.12), (3.13), (3.14), (2.6) and (3.20). By (2.3) or (2.5), we can generate new sequences of the wage rates and capital stocks. If the implied sequences are consistent with our guess, then an equilibrium is considered found. Otherwise, we update the guess for the sequences and iterate until we achieve a convergence.

To calibrate the size of the sex ratio shock, we use Chinese demographic data as a guide (see Wei and Zhang, 2011a). We let the sex ratio in the model rise continuously and smoothly from 1 to 1.15 in period 6, and continue to rise until it reaches 1.20 in period 8 and stay at that level in all subsequent periods[12]

The simulation results for the baseline case are shown in Figure 1. As the sex ratio rises (by following a pre-specified path), the real exchange rate depreciates by more than 4 percent by year 6, and continues to depreciate to a cumulative 9.1 percent by year 15. In the figure, we also consider two robustness checks by varying the value of κ. As it turns out, the real exchange rate response is not sensitive to these changes in κ.

In Figure 2, we present how the response pattern changes when we vary the capital adjustment cost. With a higher cost of capital adjustment, $b = 15$, the real exchange rate depreciation become somewhat stronger. The converse is true when the adjustment cost is lower to $b = 5$. Overall, these

[12] The sex ratio for the pre-marital age cohort in China was close to normal before 2000 but rose to 1.15 by 2007 and is projected to be 1.20 around 2025 (Wei and Zhang, 2011a).

responses are all economically significant even though the magnitude depends on the exact adjustment cost.

In Figure 3, we report a robustness check when we vary the mean value of emotional utility/love. While at a lower value, $m = 1$, which is substantially lower than the baseline value inferred from Blanchflower and Oswald (2004), the response of the real exchange rate is weaker (but still significant economically). Naturally, if we experiment with a higher value of m, we would have obtained a stronger RER response than the baseline simulation.

In Figure 4, we vary the standard deviation of the emotional utility. With a larger standard deviation, $\sigma = 0.2$, men are more likely to be matched with low-type women and hence they are less desperate to avoid bachelorhood. This causes them to exert less effort to succeed in the marriage market. As a result, the RER response becomes weaker, but is still economically significant.

In Figure 5, we conduct a sensitivity check on the timing of marriage by assuming $\tau = 10$ (instead of the baseline value of $\tau = 15$). The real exchange rate depreciation in the first ten periods is comparable to the baseline case (or even stronger in some periods), but the reversal to the long-run steady state occurs earlier (at the 10th period instead of the 15th period).

Finally, in Figure 6, we perform a robustness check on the value of the distribution margin by assuming a very low margin, $\mu = 0.25$, which is only half of what is estimated by Burstein, Neves, and Rebelo (2003). In that case, a rise in the sex ratio generates a smaller but still sizable response in the real exchange rate.

To summarize, these numerical examples suggest that a rise in the sex ratio can produce an economically meaningful reduction in the real exchange rate. Relative to the standard approach to assess the exchange rate, for example, of the kind used by the International Monetary Fund, the exchange rate of a country with a sex ratio imbalance may appear "undervalued." Because the existing literature does not provide a tight guidance on the values of all the parameters, the numerical examples are only suggestive.

5 Some empirics

Since the effect of an unbalanced sex ratio on the exchange rate is relatively novel, it is useful to present and discuss some empirical evidence. We proceed in three steps. First, on the effects of the sex ratio on savings rate and effective labor supply, we do not have to re-invent the wheel as we can point to relevant evidence from the recent empirical literature. Second, we provide some new evidence on the connection between the sex ratio and the real exchange rate (or the relative price of non-tradable goods) across regions within a country. Third, we present some new cross-country evidence on the relationship between the sex ratio and the real exchange rate.

5.1 Micro-level evidence on savings

We first review the evidence in Wei and Zhang (2011a) that a higher sex ratio has led to a rise in the household savings rate in China. At the household level, families with a son tend to save more than families with a daughter, the more interesting pattern is that families with a son in regions with a higher sex ratio tend to save more than their counterparts in regions with a lower sex ratio. This is true after taking into account the effects of other family and regional characteristics on the savings rate. In other words, it takes a combination of having a son at home and living in a region with an unbalanced sex ratio for parents to raise their savings rate.

At the province level, where one can compute the local aggregate savings rate in a way that is parallel to the national savings rate, the finding is that regions with a higher sex ratio tend to have a higher aggregate savings rate. This means that the higher savings rates by families with a son are not offset by dis-savings of other households. In other words, in general equilibrium, a higher sex ratio is associated with a higher savings rate.

To go from correlation to causality, Wei and Zhang (2011a) use regional variations in the enforcement of the family planning policy as instruments for the local sex ratio. The idea is that a harsher penalty for violating birth quotas is likely to induce families to engage in more aggressive sex selective abortions, leading to a more unbalanced sex ratio. Their first stage regression confirms that this is indeed the case. The instrumental variable regressions confirm a positive effect of a higher sex ratio on the savings rate. Based on the IV regressions, they estimate that the observed rise in the sex ratio may explain about 50-60% of the observed rise in the household savings rate in China.

Interestingly, there is some evidence that a substantial part of the increase in Chinese corporate savings may also come from the sex ratio imbalance. First, Bayoumi, Tong, and Wei (2011) present evidence that private-owned firms tend to have a higher savings rate than state-owned firms (since the private firms cannot rely on banks or the capital market to raise funds). Second, Wei and Zhang (2011b) show that a higher sex ratio has substantially stimulated private entrepreneurship - entrepreneurship is viewed as another way to raise the family wealth and hence to enhance one's status in the marriage market. The increase in the sex ratio during 1995-2004 is estimated to explain about 20% of the increase in the number of private firms in urban areas, and about 40% of the increase in the number of private firms in rural areas. Since private firms outnumber state-owned firms by a large margin, savings by private firms are an important part of the overall corporate savings. Therefore, a rise in the sex ratio may have raised both the corporate savings rate and the household savings rate.

5.2 Micro-level evidence on effective labor supply

Some evidence that a higher sex ratio has increased effective labor supply is provided in Wei and Zhang (2011b). In particular, based on a survey of rural households, they examine whether/how a household's supply of labor as a migrant worker is affected by the local sex ratio. (Because of the government restrictions on formal migration from a rural area to a city, most migrant workers' time in a city are temporary. Their children typically cannot go to local schools or enjoy local health benefits. As a result, being a migrant worker involves a lot of sacrifice associated with being away from the family on an extended period of time.) Our theory would predict that a combination of having a son at home and living in a region with a higher male/female ratio would raise the willingness for a household to supply more labor. The evidence (using a Tobit specification) is consistent with this prediction: the number of days a rural migrant worker chooses to work away from home tends to rise with the local sex ratio, especially if the migrant worker has an unmarried son at home.

Wei and Zhang (2011b) also examine another dimension of effective labor supply: willingness to accept intrinsically unpleasant or dangerous jobs. Such jobs are defined as those in mining or construction, or with exposure to extreme heat, cold or hazardous material. People accept such a job presumably for a better wage. Our theory would again suggest that a combination of having an unmarried son at home and living in a region with a higher male/female ratio would motive people to be more willing to accept such jobs. The empirical patterns (from a Probit regression) are indeed consistent with this prediction.

Interestingly, while some parents with a daughter also work as migrant workers and/or accept jobs that are intrinsically unpleasant or dangerous, their supply of work effort is uncorrelated with the local sex ratio. In other words, if a higher sex ratio produces incentives for parents with a daughter to reduce their work effort, it is likely to have been offset by opposing incentives (such as a desire to protect their daughter's bargaining power within a marriage). Whatever the exact mechanism, Wei and Zhang (2011b) do not find evidence that the labor supply by parents with a daughter declines with a higher sex ratio (while the parents with a son do supply more labor). Overall, a higher sex ratio appears to lead to a net increase in the labor supply.

5.3 Some within-country evidence on the sex ratio and the real exchange rate

One place to check the validity of our hypothesis is to examine regional variations in the real exchange rate (or the relative price of non-tradables) within a geographically large country. In this sub-section, we explore regional variations in China. An important advantage of a within-country study is that cultural norms, legal institutions, social security systems, and other factors can more plausibly be held constant across regions within a country than across countries. A potential disadvantage is that the regional sex ratio might not adequately capture the relative tightness of the local marriage market. Fortunately, for China, Wei and Zhang (2011) provide evidence that the marriage market is still very local - over 90% of marriages take place between a man and a woman from the same rural county or the same city. Mobility for marriage purposes is relatively low. (Migrant workers tend to go back to their home county to get married, or go out to work after getting married.)

We run regressions of the following type:

$$\Delta\left(\frac{P_{Nt}}{P_{Tt}}\right)_i = \alpha + \beta \cdot \Delta\text{sex ratio}_i + \gamma Z_i + error_i$$

where the dependent variable is the cumulative change in the log price of the non-tradable sector relative to that of the tradable for province i from 2001 to 2005, the first regressor is the cumulative change in the sex ratio for the same province over the same period, and Z_i is a set of control variables. α, β and γ are parameters to be estimated.

We obtain data on price indices (and their sub-indices) for 31 provinces over 2001-2005 from the China Statistics Yearbook database. The broad sectors are Food, Tobacco & Liquor, Clothing, Household Facilities & Services, Health, Transportation & Communication, Recreation & Education & Culture, and Residence. We construct two measures of the changes in the relative price of non-tradable goods. For the first measure, we define five broad sectors (Household Facilities and Services, Health, Transportation & Communication, Recreation & Education & Culture and Residence) as non-tradables, and the rest as tradables.

However, some of the broad non-tradable sectors may have sizable tradable components. As a refinement, we also construct a second measure of the relative price of the non-tradable by taking a more conservative approach that excludes plausibly tradable sub-components from otherwise non-tradable broad sectors. More precisely, for the non-tradable basket, we only include the subsectors of Health, Transportation & Communication, Household Services under the broad sector of Household Facilities and Services, the subsector of Recreation & Culture Services under the broad sector of Recreation & Education & Culture, and the subsector of House Renting & Utility under the broad sector of Residence. While some of the choices are judgment calls, the goal is to check if the basic relationship between the local relative price and the local sex ratio is robust to some perturbations of the definition of the relative price.

Somewhat inconveniently, instead of reporting the raw values of the price indices, the data source (China Statistics Yearbook) reports only annual changes of the indices with a potentially changing base from year to year. We accumulate the changes in the price indexes from 2001 to 2005. Because the data source does not report the weights assigned to various sub-indices in the CPI basket, we back out the weights by running an OLS regression of the changes in the overall CPI index on the changes in the sub-indices without an intercept.

The summary statistics on province-level variables - changes in the relative price of non-tradables from 2001 to 2005, measured in two ways as described above, and changes in the local sex ratio (which come from Wei and Zhang, 2011a), and some other variables - are reported in Table 1. During the period, the relative price of the non-tradable good by the first measure dropped by 5.9% on average. With a standard deviation of 4.3%, there are substantial variations across different provinces. By the second measure, the relative price of non-tradable goods declined by 1.8% on average. With a standard deviation of 6.2%, the coefficient of variations (or the ratio of standard deviation to mean) is even bigger for the second measure.

The sex ratio for the pre-marital age cohort during the same period rose by 2.1 basis points (or an increase in the sex ratio from 1079 young men per 1000 young women to 1100 young men per 1000 young women). With a standard deviation of 3.3%, there are substantial variations across provinces in terms of the changes in the local sex ratio. These variations are useful for econometric identifications.

Table 2 reports the regression results. In Column 1, we regress the cumulative change in the relative price of non-tradables (first measure) on the cumulative change in the sex ratio (and an intercept). The coefficient on the sex ratio is negative and statistically significant. This implies that within China, regions with a faster relative increase in the sex ratio tend to exhibit a faster relative decline in the real exchange rate, which is consistent with our theory.

A well-known result in empirical international finance is that the real exchange rate tends to be systematically lower in poorer countries (or regions). A common explanation for this pattern is the Balassa-Samuelson theory. Separately, another demographic feature - the age structure of the population - is often used to capture the life cycle hypothesis on a country's savings behavior. In Column 2, we add both the change in local per capita income and the change in the share of working-age people in the local population as controls. These two variables turn out to be insignificant. In any case, the coefficient on the sex ratio remains negative and significant.

In Columns 3 and 4 of Table 2, we perform a simple robustness check by using the second measure of the relative price of non-tradables where the definition of a "non-tradable" is somewhat stricter. The results are qualitatively the same. In particular, the coefficients on the sex ratio are negative and statistically significant. In other words, regions with higher sex ratios tend to have lower values of the real exchange rate.

Because we are not aware of any other theory that predicts a negative association between the sex ratio and the real exchange rate, we find the patterns in Table 2 interesting. Nonetheless, there are other control variables one could think of such as regional financial development. Unfortunately, we do not have reliable data on these variables. We will next turn to some international evidence for which we could consider additional control variables.

5.4 Some cross-country evidence on the exchange rate

We now provide some suggestive cross-country evidence on how the sex ratio imbalance may affect the real exchange rate. We first run regressions based on the following specification:

$$\ln RER_i = \alpha + \beta \cdot \text{sex ratio}_i + \gamma \cdot Z_i + \varepsilon_i$$

where RER_i is the real exchange rate for country i. Z_i is the set of control variables. We consider a sequentially expanding list of control variables including log GDP per capita, financial development index, government fiscal deficit, dependence ratio, and de facto exchange rate regime classifications.

The data for the real exchange rate and real GDP per capita are obtained from Penn World Table 6.3. The "price level of GDP" in the Penn World Table is equivalent to the inverse of the real exchange rate in the model: A higher value of the "price level of GDP" means a lower value of the real exchange rate. The sex ratio data is obtained from the World Factbook. As we are not able to find the sex ratio for the age cohort 10-25 for a large number of countries, we use age group 0-15 instead to maximize the country coverage.

We use two proxies for financial development. The first is the ratio of private credit to GDP, from the World Bank's WDI dataset. This is perhaps the most commonly used proxy in the standard literature. There is a clear outlier with this proxy: China has a very high level of bank credit, exceeding 100% of GDP. However, 80% of the bank loans go to state-owned firms, which are potentially less efficient than private firms (see Allen, Qian, and Qian, 2004). To deal with this problem, we modify the index by multiplying the credit to GDP ratio for China by 0.2. Because this measure is far from being perfect, we also use a second measure, which is the level of financial system sophistication as perceived by a survey of business executives reported in the Global Competitiveness Report (GCR).

For exchange rate regimes, we use two de facto classifications. The first comes from Reinhart and Rogoff (2004), who classify all regimes into four groups: peg, crawling peg, managed floating and free floating. The second classification comes from Levy-Yeyati and Sturzenegger (2005), who use three groups: fix, intermediate and free float.

For the dependent variable, log RER, and most regressors where appropriate, we use their average values over the period 2004-2008. The averaging process is meant to smooth out business cycle fluctuations and other noises. The period 2004-2008 is chosen because it is relatively recent, and the data are available for a large number of countries. (We have also examined a single year, 2006, and obtained similar results).

Table 3 provides summary statistics for the key variables. The log RER ranges from -2.22 to 0.41 in the sample, with a mean of -0.74 and a standard deviation of 0.59. The value of log RER for China indicates a substantial undervaluation on the order of 45% when compared to the simple criterion of purchasing power parity.

For the sex ratio for the age cohort 0-15, both the mean and the median across countries are 1.04, and the standard deviation is 0.02. For this age cohort, all countries in the sample have a sex ratio that is at least 1. The sex ratio for most of the countries is between 1 and 1.07. The following economies have a sex ratio that is 1.07 or higher: China (1.13), Macao (1.11), Korea (1.11), Singapore (1.09), Switzerland (1.08), Hong Kong (1.08), Vietnam (1.08), Jordan (1.07), Portugal (1.07) and India (1.07). They represent the most skewed sex ratios in the sample. China, by far, has the most unbalanced sex ratio in the world. If the same sex ratio persists into the marriage market, then at least one out of every eight young men cannot get married. As wives are typically a few years younger than their husbands, the actual probability of not being able to marry is likely to be modestly better in a country with a growing population (for which later cohorts are slightly larger). Nonetheless, the relative tightness of the marriage market for men across countries should still be highly correlated with this sex ratio measure. In addition, unlike most other countries, China exhibits a progressively smaller age cohort over time (for the population younger than 40) as a result of its strict family planning policy. As a result, the relative tightness of the marriage market for Chinese men when compared to their counterparts in other countries is likely to be worse than what is represented by this sex ratio. Furthermore, the Chinese sex ratios at birth in 1990 and 2005 are estimated to be 1.15 and 1.20, respectively (see Wei and Zhang, 2009). This implies that the sex ratio for the pre-marital age cohort will likely worsen in the foreseeable future.

We present a series of regressions in Table 4. The first column shows that the real exchange rate tends to be lower in poorer countries. This is commonly interpreted as confirmation of the Balassa-Samuelson effect. In Column 2, we add a proxy for financial development by the ratio of private sector credit to GDP. The positive coefficient on the new regressor indicates that countries with a poorer financial system tend to have a lower RER. In Column 3, we add the sex ratio. The coefficient on the sex ratio is negative and statistically significant, indicating that countries with a higher sex ratio tend to have a lower RER. Since oil exporting countries have a current income that is likely to be substantially higher than their permanent income (until they run out of the oil reserve), their RER patterns may be different from other economies. In Column 4, we exclude major oil exporters and re-do the regression. This turns out to have little effect on the result. In particular, countries with a higher sex ratio continue to exhibit a lower RER.

In Column 5 of Table 4, we add several additional control variables: government fiscal deficit, terms of trade, capital account openness, and dependency ratio. Due to missing values for some of these variables, the sample size is dramatically smaller (a decline from 123 in Column 4 to 75 in Column 5). Of these variables, the dependency ratio is the only significant variable. The positive coefficient on the dependency ratio (0.009) means that countries with a low dependency ratio (fewer children and retirees as a share of the population) tend to have a low RER. By the logic of the life-cycle hypothesis, a lower dependency ratio produces a higher savings rate. By the model in Section 2, this could lead to a reduction in the value of the real exchange rate. It is noteworthy, however, that even with these additional controls and in a smaller sample, the sex ratio effect is still statistically significant, although its point estimate is slightly smaller.

In Column 6 of Table 4, we take into account exchange rate regimes using the Reinhart-Rogoff (2004) de facto regime classifications. Relative to the countries on a fixed exchange rate regime (the omitted group), those on a crawling peg appear to have a lower RER. Countries on other currency regimes do not appear to have a systematically different RER. With these controls, the negative effect of the sex ratio on the RER is still robust. In Column 7, we measure exchange rate regimes by the de facto classifications proposed by Levy Yeyati and Sturzenegger (2003). It turns out that this does not affect the relationship between the sex ratio and the real exchange rate.

In Table 5, we re-do the regressions in Table 4 except that we now measure a country's financial development by the financial system sophistication index from the Global Competitiveness Report. The results are broadly similar to Table 4. In particular, the coefficients on the sex ratio are negative in all five cases, and are significant in four of the five cases. The sex ratio coefficient is (marginally) not significant in Column 6 of Table 5, where the Reinhart-Rogoff exchange rate classifications are used as controls. We note, however, that this regression also has far fewer observations (35 only), which also reduces the power of the test. In any case, when the LYS exchange rate classifications are used instead (reported in Column 7), the sex ratio coefficient becomes significant again.

In sum, we find that the sex ratio has an impact on the real exchange rate in a way consistent with our theory: as the sex ratio rises, a country tends to have a real exchange rate depreciation.

To be clear, as the sex ratio imbalance is a severe problem only in a subset of countries, it is not a key fundamental for the real exchange rate in most countries. Nonetheless, for those countries with a severe sex ratio imbalance, including China, one might not have an accurate view on the equilibrium exchange rate unless one takes it into account. To illustrate the quantitative significance of the empirical relations, we compute the extent of the Chinese real exchange rate undervaluation (or the value of the RER relative to what can be predicted based on the fundamentals) by taking the point estimates in Columns 1-2 and 5 of Tables 4 and 5, respectively, at their face value. The results are tabulated in Table 6. As noted earlier, relative to the simple-minded PPP, the Chinese exchange rate is undervalued by about 45%. Once we adjust for the Balassa-Samuelson effect, the extent of the undervaluation becomes 55% (column 1 of Table 6) - apparently the Chinese RER is even lower than other countries at the comparable income level. If we additionally consider financial underdevelopment (proxied by the ratio of private sector loans to GDP), the Chinese RER undervaluation is reduced to 43% (column 2, row 1 of Table 6), which is still economically significant. If we also take into account government deficit, terms of trade, and capital account openness, the extent of the RER undervaluation is 35% (column 3, row 1). If we further take into account the dependency ratio, the extent of undervaluation drops to 18% (column 4, row 1). Finally, if we add the sex ratio effect, not only can we eliminate the appearance of undervaluation, the Chinese real exchange rate would appear to be overvalued by 8% (column 5, row 1 of Table 6).

If we proxy financial development by the rating of financial system sophistication, and also take into account the sex ratio effect and other structural variables, the extent of the Chinese RER undervaluation becomes 2% (column 5, row 2 of Table 6), which is a very small amount as the extent of exchange rate fluctuation for major exchange rates such as the euro/dollar rate can easily exceed 2% a year. These calculations illustrate that the sex ratio is an economically important determinant of the real exchange rate, though not the only one. One could exaggerate the role of currency manipulation in determining the RER if not taking into the relevant structural factors.

6 Conclusion

Standard models used to assess equilibrium exchange rates do not take into account sex ratio imbalance. We show that a dramatic rise in the sex ratio for the premarital age cohort in China since 2003 could logically generate a depreciation of the real exchange rate. If other factors have also contributed to a rise in the Chinese savings rate, such as a reduction in the dependency ratio, or a rise in the corporate and government savings rates, they can complement the sex ratio effect and reinforce an appearance of an undervalued currency even when there is no manipulation. To be clear, this paper does not make a judgement on whether a policy-induced undervaluation occurs in any particular country. Instead, it illustrates potential pitfalls in assessing the equilibrium exchange rate when important structural factors are not accounted for.

Empirically, countries with a high sex ratio do appear to have a low value of the real exchange rate. If we take the econometric point estimates at the face value, one can account for virtually all of the departure of the Chinese real exchange rate from the purchasing power parity with the sex ratio effect and other structural factors.

In future research, the model could be extended to allow for endogenous adjustment of the sex ratio. (Data suggests that such an adjustment is slow, as most countries that have a sex ratio imbalance continue to exhibit a deterioration over time.) This is not easy to do technically, but will help us to assess the speed of the reversal of the sex ratio and the unwinding of the currency "undervaluation."

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A Proof of Proposition 1

Proof. (i) By totally differentiating the equations (2.1), (2.6), (2.3) and (2.5), we have

$$\Omega \cdot \begin{bmatrix} ds_t & dw_t & dP_{Nt} & dL_{Nt} \end{bmatrix}^T = \begin{bmatrix} z_1 & z_2 & z_3 & z_4 \end{bmatrix}^T d\beta$$

where

$$\begin{aligned}
\Omega_{11} &= -\frac{1}{(1-s_t)^2} - \frac{\beta}{s_t^2},\ \Omega_{12} = \Omega_{13} = \Omega_{14} = 0 \\
\Omega_{21} &= \gamma w_t x,\ \Omega_{22} = -\gamma(1-s_t)x,\ \Omega_{23} = Q_{Nt},\ \Omega_{24} = w_t \\
\Omega_{31} &= 0,\ \Omega_{32} = 1,\ \Omega_{33} = 0,\ \Omega_{34} = -\frac{\alpha_T w_t}{x - L_{Nt}} \\
\Omega_{41} &= 0,\ \Omega_{42} = 1,\ \Omega_{43} = -\frac{w_t}{P_{Nt}},\ \Omega_{44} = \frac{\alpha_N w_t}{L_{Nt}}
\end{aligned}$$

and

$$z_1 = -\frac{1}{s_t},\ z_2 = z_3 = z_4 = 0$$

The determinant of matrix Ω is

$$\det(\Omega) = \Omega_{11} \cdot \det \begin{pmatrix} \Omega_{22} & \Omega_{23} & \Omega_{24} \\ \Omega_{32} & \Omega_{33} & \Omega_{34} \\ \Omega_{42} & \Omega_{43} & \Omega_{44} \end{pmatrix}$$

and

$$\det \begin{pmatrix} \Omega_{22} & \Omega_{23} & \Omega_{24} \\ \Omega_{32} & \Omega_{33} & \Omega_{34} \\ \Omega_{42} & \Omega_{43} & \Omega_{44} \end{pmatrix} = -\frac{w_t^2}{P_{Nt}} - \frac{\alpha_N w_t}{L_{Nt}} Q_{Nt} + \frac{\alpha_T w_t}{x - L_{Nt}} \frac{1}{P_{Nt}} \left(\gamma(1-s_t)x w_t - P_{Nt} Q_{Nt}\right)$$

Since the consumption on the non-tradable goods by the young cohort must be less than the aggregate non-tradable good consumption, it follows that $\gamma(1-s_t)x w_t < P_{Nt} C_{Nt}$. Therefore,

$$\det \begin{pmatrix} \Omega_{22} & \Omega_{23} & \Omega_{24} \\ \Omega_{32} & \Omega_{33} & \Omega_{34} \\ \Omega_{42} & \Omega_{43} & \Omega_{44} \end{pmatrix} < 0$$

and $\det(\Omega) > 0$

By Cramer's rule, we can show that

$$\frac{ds_t}{d\beta} = \frac{\det \begin{pmatrix} z_1 & \Omega_{12} & \Omega_{13} & \Omega_{14} \\ z_2 & \Omega_{22} & \Omega_{23} & \Omega_{24} \\ z_3 & \Omega_{32} & \Omega_{33} & \Omega_{34} \\ z_4 & \Omega_{42} & \Omega_{43} & \Omega_{44} \end{pmatrix}}{\det(\Omega)} = \frac{z_1}{\Omega_{11}} > 0 \tag{A.1}$$

Since $z_2 = z_3 = z_4 = 0$, the change in β does not directly affect w_t, P_{Nt} and L_{Nt}. We can rewrite the differential system regarding dw_t, dP_{Nt} and dL_{Nt} as

$$\begin{pmatrix} \Omega_{22} & \Omega_{23} & \Omega_{24} \\ \Omega_{32} & \Omega_{33} & \Omega_{34} \\ \Omega_{42} & \Omega_{43} & \Omega_{44} \end{pmatrix} \cdot \begin{pmatrix} dw_t \\ dP_{Nt} \\ dL_{Nt} \end{pmatrix} = \begin{pmatrix} -\gamma w_t \frac{ds_t}{d\beta} \\ 0 \\ 0 \end{pmatrix} d\beta$$

We define Δ as in the following

$$\Delta = \det \begin{pmatrix} \Omega_{22} & \Omega_{23} & \Omega_{24} \\ \Omega_{32} & \Omega_{33} & \Omega_{34} \\ \Omega_{42} & \Omega_{43} & \Omega_{44} \end{pmatrix} < 0$$

The price of the non-tradable good

$$\frac{dP_{Nt}}{d\beta} = \frac{\partial P_{Nt}}{\partial s_t}\frac{ds_t}{d\beta} = \frac{\gamma w_t}{\Delta}\frac{ds_t}{d\beta}\left(\frac{\alpha_N w_t}{L_{Nt}} + \frac{\alpha_T w_t}{x - L_{Nt}}\right) < 0 \tag{A.2}$$

The price of the non-tradable good falls as β increases, which in turn leads to a real exchange rate depreciation.

(ii) Consider the case of an unanticipate increase in the number of young people. Let l_{Nt} and l_{Tt} denote the share of labor input in non-tradable good sector and tradable good sector, respectively.

$$l_{Nt} = \frac{L_{Nt}}{x} \text{ and } l_{Tt} = 1 - l_{Nt}$$

Similar to (i), by totally differentiating equations (2.1), (2.6), (2.3) and (2.5), we have

$$\Omega \cdot \begin{bmatrix} ds_t & dw_t & dP_{Nt} & dl_{Nt} \end{bmatrix}^T = \begin{bmatrix} z_1' & z_2' & z_3' & z_4' \end{bmatrix}^T dx_t$$

where Ω is the same matrix as in (i) and

$$z_1' = 0,\ z_2' = \gamma(1 - s_t) w_t - w_t l_{Nt},\ z_3' = -\frac{\alpha_T w_t l_{Tt}}{L_{Tt}},\ z_4' = -\frac{\alpha_N w_t l_{Nt}}{L_{Nt}}$$

Under the assumption of log utility, the optimal savings rate choice is independent of the wage rate. The differential system can be re-written as

$$\begin{pmatrix} \Omega_{22} & \Omega_{23} & x\Omega_{24} \\ \Omega_{32} & \Omega_{33} & x\Omega_{34} \\ \Omega_{42} & \Omega_{43} & x\Omega_{44} \end{pmatrix} \cdot \begin{pmatrix} dw_t \\ dP_{Nt} \\ dL_{Nt} \end{pmatrix} = \begin{pmatrix} z_2' \\ z_3' \\ z_4' \end{pmatrix}$$

By (2.5) and (2.6), we have

$$w_t L_{Nt} = (1 - \alpha_N)\gamma\left((1 - s_t) w_t x_t + R s_{t-1} w_{t-1} x_{t-1}\right)$$

then

$$z_2' = \gamma\,(1-s_t)\,w_t - w_t l_{Nt} = \alpha_N \gamma\,(1-s_t)\,w_t - (1-\alpha_N)\gamma R s_{t-1} w_{t-1} \frac{x_{t-1}}{x_t}$$

The price of the non-tradable good

$$\begin{aligned} \frac{dP_{Nt}}{dx_t} &= \frac{1}{\Delta} \left[\begin{array}{c} \frac{\alpha_T w_t}{x_t} \left(\gamma\,(1-s_t)\, x_t \frac{\alpha_N w_t}{L_{Nt}} + w_t \right) + \frac{\alpha_N w_t}{x_t} \left(\gamma\,(1-s_t)\, x_t \frac{\alpha_T w_t}{L_{Tt}} - w_t \right) \\ + \left((1-\alpha_N)\gamma R s_{t-1} w_{t-1} \frac{x_{t-1}}{x_t} - \alpha_N \gamma\,(1-s_t)\, w_t \right) \left(\frac{\alpha_T w_t}{L_{Tt}} + \frac{\alpha_N w_t}{L_{Nt}} \right) \end{array} \right] \\ &= \frac{ w_t^2 \frac{\alpha_T - \alpha_N}{x_t} + (1-\alpha_N)\gamma R s_{t-1} w_{t-1} \frac{x_{t-1}}{x_t} \left(\frac{\alpha_T w_t}{L_{Tt}} + \frac{\alpha_N w_t}{L_{Nt}} \right) + \gamma\,(1-s_t)\, \frac{\alpha_N (\alpha_T - \alpha_N) w_t}{L_{Nt}}^2 }{\Delta} < 0 \end{aligned}$$

As the number of young people increases, the price of the non-tradable good falls, which in turn leads to a depreciation in the real exchange rate. ■

B Proof of Proposition 2

Proof. The first order conditions for a woman and a man, respectively, are:

$$-u_{1w}' + \beta R \frac{P_t}{P_{t+1}} \left[\begin{array}{c} \kappa u_{2w}' \left(\delta^w + \left[\frac{1}{\phi}\,(1 - F\,(\bar{\eta}^w)) + M(\bar{\eta}^w) f(\bar{\eta}^w) \right] \right) + (1-\delta^w)\, u_{2w,n}' \\ + f\,(\bar{\eta}^w)\, \kappa u_{2w}' \,(u_{2w} - u_{2w,n}) \end{array} \right] = 0 \quad \text{(B.1)}$$

$$-u_{1m}' + \beta R \frac{P_t}{P_{t+1}} \left[\begin{array}{c} \kappa u_{2m}' \left(\delta^m + \left[\phi\,(1 - F\,(\bar{\eta}^m)) + M^{-1}(\bar{\eta}^m) f(\bar{\eta}^m) \right] \right) + (1-\delta^m)\, u_{2m,n}' \\ + f\,(\bar{\eta}^m)\, \kappa u_{2m}' \,(u_{2m} - u_{2m,n}) \end{array} \right] = 0 \quad \text{(B.2)}$$

We show by contradiction that $\bar{\eta}^w = u_{2m,n} - u_{2m}$ and $\bar{\eta}^m = M(\bar{\eta}^w)$ hold for $\phi \geq 1$. Suppose not, then

$$\bar{\eta}^m > M(\bar{\eta}^w) \geq \bar{\eta}^w$$

where the second inequality holds because $\phi \geq 1$. Then we have

$$u\left(\frac{R s_t^w w_t}{P_{t+1}} \right) - u\left(\frac{\kappa (R s_t^w w_t + R s_t^m w_t)}{P_{t+1}} \right) > u\left(\frac{R s_t^m w_t}{P_{t+1}} \right) - u\left(\frac{\kappa (R s_t^w w_t + R s_t^m w_t)}{P_{t+1}} \right)$$

and hence, $s_t^w > s_t^m$.

Then

$$\begin{aligned} u_{1w}' &= \delta^w \kappa u_{2w}' \left(1 + \left[\frac{1}{\phi}\,(1 - F\,(\bar{\eta}^w)) + M(\bar{\eta}^w) f(\bar{\eta}^w) \right] \right) + (1-\delta^w)\, u_{2w,n}' + f\,(\bar{\eta}^w)\, \kappa u_{2w}' \,(u_{2w} - u_{2w,n}) \\ &< \delta^m \kappa u_{2m}' \left(1 + \left[\phi\,(1 - F\,(\bar{\eta}^m)) + M^{-1}(\bar{\eta}^m) f(\bar{\eta}^m) \right] \right) + (1-\delta^m)\, u_{2m,n}' + f\,(\bar{\eta}^m)\, \kappa u_{2m}' \,(u_{2m} - u_{2m,n}) = u_{1m}' \end{aligned}$$

which means that[13]

$$s_t^m > s_t^w$$

Contradiction! Therefore, we have $\bar{\eta}^m = M(\bar{\eta}^w)$ and $s^m \geq s^w$ for $\phi \geq 1$.

Totally differentiating the equations (B.1), (B.2), (3.11), (2.3) and (2.5), we can obtain

$$\Omega \cdot \left[\begin{array}{ccccc} ds_t^w & ds_t^m & dw_t & dP_{Nt} & dL_{Nt} \end{array} \right]^T = \left[\begin{array}{ccccc} z_1 & z_2 & z_3 & z_4 & z_5 \end{array} \right]^T d\phi$$

where Ω is a 5×5 matrix with elements

$$\begin{aligned}
\Omega_{11} &= u''_{1w} w_t + \beta \left(R \frac{P_t}{P_{t+1}} \right)^2 w_t \left[\begin{array}{c} \kappa^2 u''_{2w} \left(\left(1 + \frac{1}{\phi}\right) \left(1 - F\left(\bar{\eta}^w\right)\right) \right) + \left(1 - \delta^w\right) u''_{2w,n} \\ + f\left(\bar{\eta}^w\right) \kappa^2 u''_{2w} \left(u_{2w} + M(\bar{\eta}^w) - u_{2w,n}\right) + 2 f\left(\bar{\eta}^w\right) \kappa u'_{2w} \left(\kappa u'_{2w} - u'_{2w,n}\right) \end{array} \right] \\
\Omega_{12} &= \beta \left(R \frac{P_t}{P_{t+1}} \right)^2 w_t \left[\begin{array}{c} \kappa^2 u''_{2w} \left(\left(1 + \frac{1}{\phi}\right) \left(1 - F\left(\bar{\eta}^w\right)\right) \right) + f\left(\bar{\eta}^w\right) \kappa^2 u''_{2w} \left(u_{2w} + M(\bar{\eta}^w) - u_{2w,n}\right) \\ + f\left(\bar{\eta}^w\right) \kappa^2 u'^2_{2w} + f(\bar{\eta}^w) \left(u'_{2m,n} - \kappa u'_{2m}\right) \left(u'_{2w,n} - \kappa u'_{2w}\right) \end{array} \right] \\
\Omega_{13} &= \Omega_{14} = \Omega_{15} = 0 \\
\Omega_{21} &= \beta \left(R \frac{P_t}{P_{t+1}} \right)^2 w_t \left[\kappa^2 u''_{2m} \left((1 + \phi) \left(1 - F\left(M\left(\bar{\eta}^w\right)\right)\right) \right) + f\left(\bar{\eta}^w\right) \kappa u'_{2m} \left(\left(1 + \frac{1}{\phi}\right) \kappa u'_{2m} - \frac{1}{\phi} u'_{2m,n} \right) \right] \\
\Omega_{22} &= u''_{1m} w_t + \beta \left(R \frac{P_t}{P_{t+1}} \right)^2 w_t \left[\begin{array}{c} \kappa^2 u''_{2m} \left((1 + \phi) \left(1 - F\left(M\left(\bar{\eta}^w\right)\right)\right) \right) + \left(1 - \delta^m\right) u''_{2m,n} \\ + f\left(\bar{\eta}^w\right) \left(\kappa u'_{2m} - u'_{2m,n}\right) \left(\left(1 + \frac{1}{\phi}\right) \kappa u'_{2m} - \frac{1}{\phi} u'_{2m,n} \right) \end{array} \right] \\
\Omega_{23} &= \Omega_{24} = \Omega_{25} = 0
\end{aligned}$$

$$\begin{aligned}
\Omega_{31} &= \frac{\gamma w_t}{1 + \phi}, \, \Omega_{32} = \frac{\gamma \phi w_t}{1 + \phi}, \, \Omega_{33} = -\gamma(1 - s_t), \, \Omega_{34} = Q_{Nt}, \Omega_{35} = w_t \\
\Omega_{41} &= \Omega_{42} = 0, \, \Omega_{43} = 1, \, \Omega_{44} = 0, \, \Omega_{45} = -\frac{\alpha_T w_t}{L_{Tt}} \\
\Omega_{51} &= \Omega_{52} = 0, \, \Omega_{53} = 1, \, \Omega_{54} = -\frac{w_t}{P_{Nt}}, \, \Omega_{55} = \frac{\alpha_N w_t}{L_{Nt}}
\end{aligned}$$

and

$$z_1 = 0, \, z_2 = \frac{1}{\phi^2} \left[1 - F(\bar{\eta}^w)\right] \left(\kappa u'_{2m} - u'_{2m,n}\right), \, z_3 = -\frac{\gamma w_t \left(s_t^m - s_t^w\right)}{1 + \phi}, \, z_4 = z_5 = 0$$

Under the assumption that $u(c) = \ln c$, we have

$$\kappa u'_{2m} = \frac{1}{R\left(s_t^m + s_t^w\right) w_t / P_{t+1}} < \frac{1}{R s_t^m w_t / P_{t+1}} = u'_{2m,n}$$

[13] **The inequality holds because (i)**

$$\frac{1}{\phi} \left(1 - F\left(\bar{\eta}^w\right)\right) + M(\bar{\eta}^w) f(\bar{\eta}^w) = \phi \left(1 - F(\bar{\eta}^m)\right) + M^{-1}(\bar{\eta}^m) f(\bar{\eta}^m)$$

by using the uniform distribution assumption; and (ii),

$$u_{2m} - u_{2m,n} > u_{2w} - u_{2w,n}$$

then $z_2 < 0$.

The determinant of matrix Ω is

$$\det(\Omega) = \det\begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix} \cdot \det\begin{pmatrix} \Omega_{33} & \Omega_{34} & \Omega_{35} \\ \Omega_{43} & \Omega_{44} & \Omega_{45} \\ \Omega_{53} & \Omega_{54} & \Omega_{55} \end{pmatrix}$$

We can show that

$$\begin{aligned} \det\begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix} &= positve.terms - \frac{Ryf\left(\bar{\eta}^{w}\right)\left(\kappa u'_{2m} - u'_{2m,n}\right)^{2}\Omega_{11}}{\phi} + u''_{1m}u''_{1w}y^{2} \\ &+ \frac{\left(\kappa u'_{2}\right)^{2}R^{2}y^{2}\left[u'_{2w,n}u'_{2m,n} - \kappa u'_{2}\left(u'_{2w,n} + u'_{2m,n}\right)\right]}{\left(\eta^{\max} - \eta^{\min}\right)^{2}} \end{aligned}$$

Under the assumption that $u(c) = \ln c$, the last term on the right hand side is zero. Then

$$\det\begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix} > 0$$

and

$$\det\begin{pmatrix} \Omega_{33} & \Omega_{34} & \Omega_{35} \\ \Omega_{43} & \Omega_{44} & \Omega_{45} \\ \Omega_{53} & \Omega_{54} & \Omega_{55} \end{pmatrix} = -\frac{w_t^2}{P_{Nt}} - \frac{\alpha_N w_t}{L_{Nt}}Q_{Nt} + \frac{\alpha_T w_t}{1 - L_{Nt}}\frac{1}{P_{Nt}}\left(\gamma(1 - s_t)w_t - P_{Nt}Q_{Nt}\right)$$

Notice that the consumption of the non-tradable good by the young cohort must be less than the aggregate non-tradable good consumption, then $\gamma(1 - s_t)w_t < P_{Nt}Q_{Nt}$. Therefore,

$$\det\begin{pmatrix} \Omega_{33} & \Omega_{34} & \Omega_{35} \\ \Omega_{43} & \Omega_{44} & \Omega_{45} \\ \Omega_{53} & \Omega_{54} & \Omega_{55} \end{pmatrix} < 0$$

and $\det(\Omega) < 0$.

Then

$$\frac{ds_t^m}{d\phi} = \frac{z_2\Omega_{11}}{\det\begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix}} > 0$$

and

$$
\begin{aligned}
\frac{ds_t^w}{d\phi} &= -\frac{z_2\beta\left(\frac{RP_t}{P_{t+1}}\right)^2 \frac{w_t}{P_t}\left(\left(1+\frac{1}{\phi}\right)(1-F(\bar{\eta}^w)) + f(\bar{\eta}^w)\left(M(\bar{\eta}^w) - \bar{\eta}^w + \log\left(\frac{s_t^m}{s_t^w}\right)\right) - 2\right)}{(s_t^m + s_t^w)^2 \det\begin{pmatrix}\Omega_{11} & \Omega_{12}\\ \Omega_{21} & \Omega_{22}\end{pmatrix}} \\
&= \frac{z_2\beta\left(\frac{RP_t}{P_{t+1}}\right)^2 \frac{w_t}{P_t}\left(2\log\kappa + \log\left(2 + \frac{s_t^m}{s_t^w} + \frac{s_t^w}{s_t^m}\right)\right)}{(\eta^{\max} - \eta^{\min})(s_t^m + s_t^w)^2 \det\begin{pmatrix}\Omega_{11} & \Omega_{12}\\ \Omega_{21} & \Omega_{22}\end{pmatrix}} \\
&< \frac{2z_2\beta\left(\frac{RP_t}{P_{t+1}}\right)^2 \frac{w_t}{P_t}\log(2\kappa)}{(\eta^{\max} - \eta^{\min})(s_t^m + s_t^w)^2 \det\begin{pmatrix}\Omega_{11} & \Omega_{12}\\ \Omega_{21} & \Omega_{22}\end{pmatrix}} < 0
\end{aligned}
$$

However, the aggregate savings rate by the young cohort, $s_t^{young} = \frac{\phi}{1+\phi}s_t^m + \frac{1}{1+\phi}s_t^w$, rises as the sex ratio becoms more unbalanced.

$$
\begin{aligned}
\frac{ds_t^{young}}{d\phi} &= \frac{s_t^m - s_t^w}{(1+\phi)^2} + \frac{\phi-1}{1+\phi}\frac{ds_t^m}{d\phi} + \frac{1}{1+\phi}\left(\frac{ds_t^m}{d\phi} + \frac{ds_t^w}{d\phi}\right) \\
&= \frac{s_t^m - s_t^w}{(1+\phi)^2} + \frac{\phi-1}{1+\phi}\frac{ds_t^m}{d\phi} + \frac{z_2\left(u''_{1w}y + Ry\begin{bmatrix}(1-\delta^w)\,u''_{2w,n} \\ -f(\bar{\eta}^w)\left(\kappa u'_2\left(u'_{2w,n} - u'_{2m,n}\right) + u'_{2w,n}u'_{2m,n}\right)\end{bmatrix}\right)}{(1+\phi)\det\begin{pmatrix}\Omega_{11} & \Omega_{12}\\ \Omega_{21} & \Omega_{22}\end{pmatrix}}
\end{aligned}
$$

where all the terms on the right hand side are positive and hence, $\frac{ds_t^{young}}{d\phi} > 0$.

As in the proof of Proposition 1, we rewrite the system regarding the changes in P_{Nt}, w_t and L_{Nt} as following

$$
\begin{pmatrix}\Omega_{33} & \Omega_{34} & \Omega_{35}\\ \Omega_{43} & \Omega_{44} & \Omega_{45}\\ \Omega_{53} & \Omega_{54} & \Omega_{55}\end{pmatrix}\cdot\begin{pmatrix}dw_t\\ dP_{Nt}\\ dL_{Nt}\end{pmatrix} = \begin{pmatrix}-\gamma w_t\frac{ds_t^{young}}{d\phi}\\ 0\\ 0\end{pmatrix}d\phi
$$

Following the same steps as in the proof of Proposition 1, we can show that

$$
\frac{dP_{Nt}}{d\phi} = \frac{\partial P_{Nt}}{\partial s_t^{young}}\frac{ds_t^{young}}{d\phi} = \frac{\gamma w_t}{\Lambda}\frac{ds_t^{young}}{d\phi}\left(\frac{\alpha_N w_t}{L_{Nt}} + \frac{\alpha_T w_t}{1-L_{Nt}}\right)
$$

where Λ is defined as

$$
\Lambda = \det\begin{pmatrix}\Omega_{33} & \Omega_{34} & \Omega_{35}\\ \Omega_{43} & \Omega_{44} & \Omega_{45}\\ \Omega_{53} & \Omega_{54} & \Omega_{55}\end{pmatrix} < 0
$$

Since $\frac{ds_t^{young}}{d\phi} > 0$, $\frac{dP_{Nt}}{d\phi} < 0$, which results in a fall in the consumption price index and therefore a real exchange rate depreciation in period t. ■

C Proof of Proposition 3

Proof. If $u(c) = \ln c$, solving the first order condition under a balanced sex ratio for both men and women in the marriage market, we can obtain

$$-\frac{1}{1-s_t} + \beta R \frac{P_t}{P_{t+1}} \frac{1}{s_t} = 0$$

which is the same optimal condition when a man or a woman chooses to be single. For a representative woman, at the balanced sex ratio, if she chooses to enter the marriage market, with probability $F(\bar{\eta})$ she can get married and receive welfare

$$\begin{aligned} V_t^w &= \ln\left(\frac{(1-s_t)w_t}{P_t}\right) + \beta F(\bar{\eta}) \ln\left(\frac{\kappa R(2s_t) w_t}{P_{t+1}}\right) + \beta(1-F(\bar{\eta}))\ln\left(\frac{Rs_t w_t}{P_{t+1}}\right) + E[\eta | \eta^w \geq \bar{\eta}] \\ &> \ln\left(\frac{(1-s_t)w_t}{P_t}\right) + \beta \ln\left(\frac{Rs_t w_t}{P_{t+1}}\right) = V_{n,t}^w \end{aligned}$$

where the inequality holds because $\kappa > 1/2$ and $E[\eta | \eta^w \geq \bar{\eta}] > 0$. Therefore, entering the marriage market is a dominant strategy for all women. Since men and women are symmetric when $\phi = 1$, all men and all women will enter the marriage market with probability one at the balanced sex ratio.

As we have shown in Proposition 2,

$$\frac{ds_t^m}{d\phi} > 0 \text{ and } \frac{ds_t^m}{d\phi} + \frac{ds_t^w}{d\phi} > 0$$

we can show that

$$\begin{aligned} \frac{\partial V_t^m}{\partial \phi} &= y\left(-u'_{1m} + \beta R \frac{P_t}{P_{t+1}}\left(\kappa \delta^m u'_{2m} + (1-\delta^m) u'_{2m,n}\right)\right)\frac{ds_t^m}{d\phi} \\ &\quad + \beta R \frac{P_t}{P_{t+1}} \delta^m y \kappa u'_{2w} \frac{ds_t^w}{d\phi} - \beta \int_{M(\bar{\eta}^w)} [1 - F(\eta)]\, d\eta \\ &< -\beta \int_{M(\bar{\eta}^w)} [1-F(\eta)]\, d\eta - \beta R \frac{P_t}{P_{t+1}} (\phi - 1)\, \delta^m y \kappa u'_{2w} \frac{ds_t^m}{d\phi} < 0 \end{aligned} \tag{C.1}$$

where the first equality in (C.1) holds because

$$
\begin{aligned}
\frac{\partial \delta^m}{\partial \phi} &= -\frac{1-F\left(\bar{\eta}^w\right)}{\phi^2}\left(u_{2m}-u_{2m,n}\right) \\
&\quad -\frac{Ryf(\bar{\eta}^w)}{\phi}\left[u'_{2m,n}\frac{ds_t^m}{d\phi}-\kappa u'_{2m}\left(\frac{ds_t^m}{d\phi}+\frac{ds_t^w}{d\phi}\right)\right]\left(u_{2m}-u_{2m,n}\right) \\
\frac{\partial\left(\int_{M(\bar{\eta}^w)} M^{-1}(\eta^m)dF(\eta^m)\right)}{\partial \phi} &= -\int_{M(\bar{\eta}^w)}\left[1-F\left(\eta\right)\right]d\eta-\frac{\bar{\eta}^w\left(1-F(\bar{\eta}^w)\right)}{\phi^2} \\
&\quad -\frac{\bar{\eta}^w f\left(\bar{\eta}^w\right)}{\phi}\left[u'_{2m,n}\frac{ds_t^m}{d\phi}-\kappa u'_{2m}\left(\frac{ds_t^m}{d\phi}+\frac{ds_t^w}{d\phi}\right)\right]
\end{aligned}
$$

and

$$
\delta^m < 1 < \phi\left(1-F\left(\bar{\eta}^m\right)\right)+\bar{\eta}^m f(\bar{\eta}^m) \text{ and } \frac{ds_t^w}{d\phi} \leq \frac{ds_t^m}{d\phi}
$$

Men lose as the sex ratio rises while the effect on women's welfare is ambiguous.

Now consider women's welfare. Given the equilibrium s_t^m and s_t^w under a sex ratio ϕ, if one woman deviates from the equilibrium choice s_t^w, for instance, by choosing a savings rate $s_t^{w\prime} = s_t^m$, she would receive a lower life-time utility $V_t^{w\prime}$ $(\leq V_t^w)$. Since $s_t^{w\prime} = s_t^m \geq s_t^w$, this woman will have a better situation than all other women in the marriage market, i.e., she is more likely to get married and also more likely to marry a better man. Then

$$
\begin{aligned}
V_t^{w\prime} &= u_{1w'}+\beta\left[\delta' u_{2w'}+\left(1-\delta'\right)u_{2w',n}+\int_{\bar{\eta}^w} M\left(\eta^w+u_{2w'}-u_{2w}\right)dF\left(\eta^w\right)\right] \\
&\geq u_{1w'}+\beta\left[\left(1-F(\bar{\eta}^w)\right)u_{2w'}+F\left(\bar{\eta}^w\right)u_{2w',n}+\int_{\bar{\eta}^w} M\left(\eta^w\right)dF\left(\eta^w\right)\right] \\
&= u_{1m}+\beta\left[\left(1-F(\bar{\eta}^w)\right)u_{2m}+F\left(\bar{\eta}^w\right)u_{2m,n}+\int_{\bar{\eta}^w} M\left(\eta^w\right)dF\left(\eta^w\right)\right] \\
&\geq u_{1m}+\beta\left[\left(1-F(M\left(\bar{\eta}^w\right))\right)u_{2m}+F\left(M\left(\bar{\eta}^w\right)\right)u_{2m,n}+\int_{M(\bar{\eta}^w)} M^{-1}\left(\eta^m\right)dF\left(\eta^m\right)\right] = V_t^m
\end{aligned}
$$

where $u_{1w'}$, $u_{2w'}$ and $u_{2w',n}$ denote the first period consumption-led utility, the second period consumption-led utility when she gets married, and the second period utility when she fails to get matched with any man, respectively. u_{2w} is the second period consumption-led utility for all other women who get married. The first inequality holds because the woman faces a greater possibility of getting married and also she will receive a higher expected emotional utility from her husband. The second inequality holds both because she is more likely than the representative man to get married and because she is expecting to receive higher emotional utility from her spouse than the representative man.

Therefore, for $\phi \geq 1$, we can show that $V_t^w \geq V_t^{w\prime} \geq V_t^m$, the representative woman always achieves higher welfare than the representative man.

For the representative man in the marriage market, given his rivals' choices, if he choose to stay in the marriage market, he will follow the first order condition (B.2) and achieves an approximate life time utility $u_{1m} + \beta u_{2m,n}$. If he chooses to be single, he maximizes the life time utility $u_1 + \beta u_2$. The first order condition in this case is

$$-u'_{1m} + u'_{2m} = 0 \tag{C.2}$$

The two savings decisions, in the marriage market and being single, will be different since the man will follow different first order conditions. Then

$$V_n^m = \max u_1 + \beta u_2 > u_{1m} + \beta u_{2m,n} \to V^m$$

when $\phi \to \infty$. The representative man will then choose to be single which violates the assumption that, for all ϕs, entering the marriage market is the dominant strategy for all men. Therefore, a threshold as ϕ_1 exists and at $\phi \geq \phi_1$, $V_n^m = V^m$.

For $\phi \geq \phi_1$, with probability $\frac{\phi_1}{\phi}$, the representative man will choose to enter the marriage market, and with probability $1 - \frac{\phi_1}{\phi}$, he remains single. For the representative woman, since she earns the same first period income as he, we can show that

$$V_n^w = V_n^m = V^m < V^w$$

the representative woman will enter the marriage market with probability one.

As for the aggregate savings rate in the young cohort, we have shown in Proposition 1 that for $\phi < \phi_1$, as the sex ratio rises, the aggregate savings rate in the young cohort will rise. For $\phi \geq \phi_1$, as the sex ratio rises, some men begin quitting the marriage market and choose a different savings rate according to (C.2). Compare (B.2) with (C.2), it is ambiguous whether $s^m > s_n^m$ or not, then the effect on the aggregate savings rate is ambiguous. ■

D Proof of Proposition 4

Proof. If $u(C) = \ln C$, for $\phi < \phi_1$, by the optimal labor supply condition, we have

$$0 < \frac{dL_t^i}{ds_t^i} = \frac{\left(v'_i L_t^i\right)^2}{v'_i - v''_i L_t^i} \tag{D.1}$$

where $i = w, m$.

Similar to the proof of Proposition 2, we can show that $\bar{\eta}^m = M(\bar{\eta}^w)$ and $s^m L^m \geq s^w L^w$ for $\phi \geq 1$. Since at $\phi = 1$, women and men are symmetric, and hence $s^m = s^w$ and $L^m = L^w$. For $\phi \geq 1$, by (D.1), $s^m L^m \geq s^w L^w$ means $s^m \geq s^w$ and $L^m \geq L^w$.

Similar to the proof of Proposition 2, we can rewrite the first order conditions with respect to women and men's savings rate as

$$-u'_{1w}+\beta R\frac{P_t}{P_{t+1}}\left[\begin{array}{c}\kappa u'_{2w}\left(1+\frac{1}{\phi}\right)\left(1-F\left(\bar{\eta}^w\right)\right)+\left(1-\delta^w\right)u'_{2w,n}\\ +f\left(\bar{\eta}^w\right)\kappa u'_{2w}\left(u_{2w}+M(\bar{\eta}^w)-u_{2w,n}\right)\end{array}\right]=0 \quad \text{(D.2)}$$

$$-u'_{1m}+\beta R\frac{P_t}{P_{t+1}}\left[\kappa u'_{2m}\left(1+\phi\right)\left(1-F\left(\bar{\eta}^m\right)\right)+\left(1-\delta^m\right)u'_{2m,n}\right]=0 \quad \text{(D.3)}$$

Totally differentiating equations (D.2), (D.3), (3.11), and (3.12), and plugging the expression of $\frac{dL_t^i}{ds_t^i}$ into women and men's first order conditions with respect to savings rate, we can obtain

$$\Omega\cdot\left[\begin{array}{ccccc} ds_t^w & ds_t^m & dw_t & dP_{Nt} & dL_{Nt}\end{array}\right]^T=\left[\begin{array}{ccccc} z_1 & z_2 & z_3 & z_4 & z_5\end{array}\right]^T d\phi$$

where

$$\Omega_{11} = \frac{w_t L_t^w}{P_t}\left\{u''_{1w,t}+\beta\left(\frac{RP_t}{P_{t+1}}\right)^2\left[\begin{array}{c}\kappa^2 u''_{2w}\left(\left(1+\frac{1}{\phi}\right)\left(1-F\left(\bar{\eta}^w\right)\right)\right)+\left(1-\delta^w\right)u''_{2w,n}\\ +f\left(\bar{\eta}^w\right)\kappa^2 u''_{2w}\left(u_{2w}+M(\bar{\eta}^w)-u_{2w,n}\right)\\ +2f\left(\bar{\eta}^w\right)\kappa u'_{2w}\left(\kappa u'_{2w}-u'_{2w,n}\right)\end{array}\right]\left(1+\frac{s_t^w}{L_t^w}\frac{dL_t^w}{ds_t^w}\right)\right\}$$

$$\Omega_{12} = \beta\left(\frac{RP_t}{P_{t+1}}\right)^2\left[\begin{array}{c}\kappa^2 u''_{2w}\left(\left(1+\frac{1}{\phi}\right)\left(1-F\left(\bar{\eta}^w\right)\right)\right)\\ +f\left(\bar{\eta}^w\right)\kappa^2 u''_{2w}\left(u_{2w}+M(\bar{\eta}^w)-u_{2w,n}\right)\\ +f\left(\bar{\eta}^w\right)\kappa^2 u'^2_{2w}+f(\bar{\eta}^w)\left(u'_{2m,n}-\kappa u'_{2m}\right)\left(u'_{2w,n}-\kappa u'_{2w}\right)\end{array}\right]\left(1+\frac{s_t^m}{L_t^m}\frac{dL_t^m}{ds_t^m}\right)\frac{w_t L_t^m}{P_t}$$

$$\Omega_{13} = \Omega_{14}=\Omega_{15}=0$$

$$\Omega_{21} = \beta\left(\frac{RP_t}{P_{t+1}}\right)^2\left[\begin{array}{c}\kappa^2 u''_{2m}\left(\left(1+\phi\right)\left(1-F\left(M\left(\bar{\eta}^w\right)\right)\right)\right)\\ +f\left(\bar{\eta}^w\right)\kappa u'_{2m}\left(\left(1+\frac{1}{\phi}\right)\kappa u'_{2m}-\frac{1}{\phi}u'_{2m,n}\right)\end{array}\right]\left(1+\frac{s_t^w}{L_t^w}\frac{dL_t^w}{ds_t^w}\right)\frac{w_t L_t^w}{P_t}$$

$$\Omega_{22} = \frac{w_t L_t^m}{P_t}\left\{u''_{1m}+\beta\left(\frac{RP_t}{P_{t+1}}\right)^2\left[\begin{array}{c}\kappa^2 u''_{2m}\left(\left(1+\phi\right)\left(1-F\left(M\left(\bar{\eta}^w\right)\right)\right)\right)+\left(1-\delta^m\right)u''_{2m,n}\\ +f\left(\bar{\eta}^w\right)\left(\kappa u'_{2m}-u'_{2m,n}\right)\left(\left(1+\frac{1}{\phi}\right)\kappa u'_{2m}-\frac{1}{\phi}u'_{2m,n}\right)\end{array}\right]\left(1+\frac{s_t^m}{L_t^m}\frac{dL_t^m}{ds_t^m}\right)\right\}$$

$$\Omega_{23} = \Omega_{24}=\Omega_{25}=0$$

$$\Omega_{31} = \frac{\gamma w_t}{1+\phi}\left(L_t^w+s_t^w\frac{dL_t^w}{ds_t^w}\right),\ \Omega_{32}=\frac{\gamma\phi w_t}{1+\phi}\left(L_t^m+s_t^m\frac{dL_t^m}{ds_t^m}\right)$$

$$\Omega_{33} = -\gamma\left[\frac{(1-s_t^w)L_t^w}{1+\phi}+\frac{\phi(1-s_t^m)L_t^m}{1+\phi}\right],\ \Omega_{34}=\frac{A_{Nt}K_{Nt}^{\alpha_N}L_{Nt}^{1-\alpha_N}}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}},\ \Omega_{35}=\frac{P_{Nt}(1-\alpha_N)A_{Nt}K_{Nt}^{\alpha_N}L_{Nt}^{-\alpha_N}}{\alpha_N^{\alpha_N}(1-\alpha_N)^{1-\alpha_N}}$$

$$\Omega_{41} = -\frac{\alpha_T w_t}{\frac{1}{1+\phi}L_t^w+\frac{\phi}{1+\phi}L_t^m-L_{Nt}}\frac{1}{1+\phi}\frac{dL_t^w}{ds_t^w},\ \Omega_{42}=-\frac{\alpha_T w_t}{\frac{1}{1+\phi}L_t^w+\frac{\phi}{1+\phi}L_t^m-L_{Nt}}\frac{\phi}{1+\phi}\frac{dL_t^m}{ds_t^m}$$

$$\Omega_{43} = -1,\ \Omega_{44}=0,\ \Omega_{45}=\left(\frac{\alpha_T}{1-\alpha_T}\right)^{1-\alpha_T}(1-\alpha_T)A_{Tt}K_{Tt}^{\alpha_T}\left(\frac{1}{1+\phi}L_t^w+\frac{\phi}{1+\phi}L_t^m-L_{Nt}\right)^{-\alpha_T-1}$$

$$\Omega_{51} = \Omega_{52}=0,\ \Omega_{53}=-1,\ \Omega_{54}=\frac{w_t}{P_{Nt}},\ \Omega_{55}=-\left(\frac{\alpha_N}{1-\alpha_N}\right)^{1-\alpha_N}(1-\alpha_N)A_{Nt}K_{Nt}^{\alpha_N}{L_{Nt}}^{-\alpha_N-1}$$

and

$$z_1 = 0,\ z_2 = \frac{[1 - F(\bar{\eta}^w)]\left(\kappa u'_{2m} - u'_{2m,n}\right)}{\phi^2},\ z_3 = -\frac{\gamma w_t\left(s_t^m L_t^m - s_t^w L_t^w\right)}{1+\phi},\ z_4 = \frac{\alpha_T w_t \frac{L_t^m - L_t^w}{(1+\phi)^2}}{\frac{1}{1+\phi}L_t^w + \frac{\phi}{1+\phi}L_t^m - L_{Nt}},\ z_5 = 0$$

The determinant of matrix Ω is

$$\det(\Omega) = \det\begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix} \cdot \det\begin{pmatrix} \Omega_{33} & \Omega_{34} & \Omega_{35} \\ \Omega_{43} & \Omega_{44} & \Omega_{45} \\ \Omega_{53} & \Omega_{54} & \Omega_{55} \end{pmatrix}$$

Under the assumption that $E\eta$ is sufficiently large, it is easy to show that

$$\det\begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix} > 0$$

and

$$\begin{aligned} \det\begin{pmatrix} \Omega_{33} & \Omega_{34} & \Omega_{35} \\ \Omega_{43} & \Omega_{44} & \Omega_{45} \\ \Omega_{53} & \Omega_{54} & \Omega_{55} \end{pmatrix} &= \frac{\alpha_T w_t}{L_t - L_{Nt}} \frac{1}{P_{Nt}}\left(\gamma\left(\frac{(1-s_t^w)L_t^w}{1+\phi} + \frac{\phi(1-s_t^m)L_t^m}{1+\phi}\right) w_t - P_{Nt}Q_{Nt}\right) \\ &\quad - \frac{w_t^2}{P_{Nt}} - \frac{\alpha_N w_t}{L_{Nt}} Q_{Nt} \end{aligned}$$

Notice that the consumption of the non-tradable good by the young cohort must be less than the aggregate non-tradable good consumption, then $\gamma\left[\frac{(1-s_t^w)L_t^w}{1+\phi} + \frac{\phi(1-s_t^m)L_t^m}{1+\phi}\right] w_t < P_{Nt}C_{Nt}$. Therefore,

$$\det\begin{pmatrix} \Omega_{33} & \Omega_{34} & \Omega_{35} \\ \Omega_{43} & \Omega_{44} & \Omega_{45} \\ \Omega_{53} & \Omega_{54} & \Omega_{55} \end{pmatrix} < 0$$

and $\det(\Omega) < 0$

Then
$$\frac{ds_t^m}{d\phi} = \frac{z_2\Omega_{11}}{\det\begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix}} > 0 \text{ and } \frac{ds_t^w}{d\phi} = -\frac{z_2\Omega_{12}}{\det\begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix}}$$

By the definition of $\bar{\eta}^w$ and $M(\bar{\eta}^w)$, it is easy to show that

$$u_{2w} + M(\bar{\eta}^w) - u_{2w,n} = u_{2w} + M(\bar{\eta}^w) - u_{2m,n} + u_{2m,n} - u_{2w,n} = M(\bar{\eta}^w) - \bar{\eta}^w + u_{2m,n} - u_{2w,n}$$

By the definition of $\bar{\eta}^w$,

$$\bar{\eta}^w = \log\left(\frac{Rs_t^m L_t^m}{P_{t+1}}\right) - \log\left(\frac{\kappa R\left(s_t^m L_t^m + s_t^w L_t^w\right)}{P_{t+1}}\right)$$

Similar to the proof of Proposition 2, under the log utilty assumption, we can show that

$$\begin{aligned}\frac{ds_t^w}{d\phi} &= -\frac{z_2\beta\left(\frac{RP_t}{P_{t+1}}\right)^2\left(1+\frac{s_t^m}{L_t^m}\frac{dL_t^m}{ds_t^m}\right)\frac{w_t L_t^m}{P_t}\left(\left(1+\frac{1}{\phi}\right)\left(1-F\left(\bar{\eta}^w\right)\right)+f\left(\bar{\eta}^w\right)\left(M(\bar{\eta}^w)-\bar{\eta}^w+\log\left(\frac{s_t^m L_t^m}{s_t^w L_t^w}\right)\right)-2\right)}{\left(s_t^m L_t^m + s_t^w L_t^w\right)^2 \det\begin{pmatrix}\Omega_{11} & \Omega_{12}\\ \Omega_{21} & \Omega_{22}\end{pmatrix}} \\ &< \frac{2z_2\beta\left(\frac{RP_t}{P_{t+1}}\right)^2\left(1+\frac{s_t^m}{L_t^m}\frac{dL_t^m}{ds_t^m}\right)\frac{w_t L_t^m}{P_t}\log\left(2\kappa\right)}{\left(\eta^{\max}-\eta^{\min}\right)\left(s_t^m L_t^m + s_t^w L_t^w\right)^2 \det\begin{pmatrix}\Omega_{11} & \Omega_{12}\\ \Omega_{21} & \Omega_{22}\end{pmatrix}} < 0\end{aligned}$$

where the last inequality holds because $\kappa > 1/2$. By (D.1), we have

$$\frac{dL_t^m}{d\phi} > 0 \text{ and } \frac{dL_t^w}{d\phi} < 0$$

The aggregate savings rate in the young cohort is

$$s_t^{young} = \frac{\frac{\phi}{1+\phi}s_t^m L_t^m + \frac{1}{1+\phi}s_t^w L_t^w}{\frac{\phi}{1+\phi}L_t^m + \frac{1}{1+\phi}L_t^w} = \frac{\frac{\phi}{1+\phi}L_t^m}{\frac{\phi}{1+\phi}L_t^m + \frac{1}{1+\phi}L_t^w}s_t^m + \frac{\frac{1}{1+\phi}L_t^w}{\frac{\phi}{1+\phi}L_t^m + \frac{1}{1+\phi}L_t^w}s_t^w$$

then

$$\frac{ds_t^{young}}{d\phi} = \frac{\frac{\phi}{1+\phi}L_t^m}{\frac{\phi}{1+\phi}L_t^m + \frac{1}{1+\phi}L_t^w}\frac{ds_t^m}{d\phi} + \frac{\frac{1}{1+\phi}L_t^w}{\frac{\phi}{1+\phi}L_t^m + \frac{1}{1+\phi}L_t^w}\frac{ds_t^w}{d\phi} + \frac{d\left(\frac{\frac{\phi}{1+\phi}L_t^m}{\frac{\phi}{1+\phi}L_t^m + \frac{1}{1+\phi}L_t^w}\right)}{d\phi}\left(s_t^m - s_t^w\right)$$

The sum of the first two terms on the right hand side

$$
\begin{aligned}
& \frac{\frac{\phi}{1+\phi}L_t^m}{\frac{\phi}{1+\phi}L_t^m+\frac{1}{1+\phi}L_t^w}\frac{ds_t^m}{d\phi}+\frac{\frac{1}{1+\phi}L_t^w}{\frac{\phi}{1+\phi}L_t^m+\frac{1}{1+\phi}L_t^w}\frac{ds_t^w}{d\phi}>\frac{\frac{1}{1+\phi}}{\frac{\phi}{1+\phi}L_t^m+\frac{1}{1+\phi}L_t^w}\left(L_t^m\frac{ds_t^m}{d\phi}+L_t^w\frac{ds_t^w}{d\phi}\right) \\
= \quad & \frac{\frac{1}{1+\phi}}{\frac{\phi}{1+\phi}L_t^m+\frac{1}{1+\phi}L_t^w}\frac{z_2}{\det\begin{pmatrix}\Omega_{11} & \Omega_{12}\\ \Omega_{21} & \Omega_{22}\end{pmatrix}}\left(L_t^m\Omega_{11}-L_t^w\Omega_{12}\right) \\
> \quad & \frac{\frac{1}{1+\phi}\frac{w_tL_t^mL_t^w}{P_t}}{\frac{\phi}{1+\phi}L_t^m+\frac{1}{1+\phi}L_t^w}\frac{z_2}{\det\begin{pmatrix}\Omega_{11} & \Omega_{12}\\ \Omega_{21} & \Omega_{22}\end{pmatrix}}\begin{pmatrix} u''_{1w,t}+\beta\left(\frac{RP_t}{P_{t+1}}\right)^2\kappa^2u''_{2w}\left(\left(1+\frac{1}{\phi}\right)\left(1-F\left(\bar{\eta}^w\right)\right)\right)\frac{s_t^w}{L_t^w}\frac{dL_t^w}{ds_t^w} \\ -\beta\left(\frac{RP_t}{P_{t+1}}\right)^2\left[\kappa^2u''_{2w}\left(\left(1+\frac{1}{\phi}\right)\left(1-F\left(\bar{\eta}^w\right)\right)\right)\right]\frac{s_t^m}{L_t^m}\frac{dL_t^m}{ds_t^m}\end{pmatrix} \\
> \quad & \frac{\frac{1}{1+\phi}\frac{w_tL_t^mL_t^w}{P_t}}{\frac{\phi}{1+\phi}L_t^m+\frac{1}{1+\phi}L_t^w}\frac{z_2}{\det\begin{pmatrix}\Omega_{11} & \Omega_{12}\\ \Omega_{21} & \Omega_{22}\end{pmatrix}}\left(-\frac{u'_{1w,t}}{(1-s_t^w)L_t^w}+\frac{\beta\left(\frac{RP_t}{P_{t+1}}\right)\kappa u'_{2w}\left(\left(1+\frac{1}{\phi}\right)\left(1-F\left(\bar{\eta}^w\right)\right)\right)}{s_t^mL_t^m+s_t^wL_t^w}\frac{s_t^m}{L_t^m}\frac{dL_t^m}{ds_t^m}\right) \\
> \quad & \frac{\frac{1}{1-s_t^m}\frac{1}{1+\phi}\frac{w_tL_t^mL_t^w}{P_t}}{\frac{\phi}{1+\phi}L_t^m+\frac{1}{1+\phi}L_t^w}\frac{z_2}{\det\begin{pmatrix}\Omega_{11} & \Omega_{12}\\ \Omega_{21} & \Omega_{22}\end{pmatrix}}\begin{pmatrix} -\frac{(1-s_t^m)L_t^m}{(1-s_t^w)L_t^w}\frac{s_t^mL_t^m+s_t^wL_t^w}{s_t^mL_t^m}u'_{1w,t} \\ +\beta\left(\frac{RP_t}{P_{t+1}}\right)\kappa u'_{2w}\left(\left(1+\frac{1}{\phi}\right)\left(1-F\left(\bar{\eta}^w\right)\right)\right)\end{pmatrix}
\end{aligned}
$$

where the last inequality holds because

$$
\frac{s_t^m}{L_t^m}\frac{dL_t^m}{ds_t^m}=\frac{s_t^m}{1-s_t^m}\frac{1}{1-\frac{v''_mL_t^m}{v'_m}}<\frac{s_t^m}{1-s_t^m}
$$

We can show that by (D.2) and (D.3)

$$
\begin{aligned}
\frac{(1-s_t^m)L_t^m}{(1-s_t^w)L_t^w}\frac{s_t^mL_t^m+s_t^wL_t^w}{s_t^mL_t^m} \quad & = \quad \frac{\begin{pmatrix}\kappa u'_{2w}\left(1+\frac{1}{\phi}\right)\left(1-F\left(\bar{\eta}^w\right)\right)+\left(1-\delta^w\right)u'_{2w,n} \\ +f\left(\bar{\eta}^w\right)\kappa u'_{2w}\left(u_{2w}+M(\bar{\eta}^w)-u_{2w,n}\right)\end{pmatrix}}{\kappa u'_{2m}\left(1+\phi\right)\left(1-F\left(\bar{\eta}^m\right)\right)+\left(1-\delta^m\right)u'_{2m,n}}\frac{s_t^mL_t^m+s_t^wL_t^w}{s_t^mL_t^m} \\
& > \quad \frac{\left(1+\frac{1}{\phi}\right)\left(1-F\left(\bar{\eta}^w\right)\right)+F\left(\bar{\eta}^w\right)\frac{s_t^mL_t^m+s_t^wL_t^w}{s_t^wL_t^w}+f\left(\bar{\eta}^w\right)\left(M(\bar{\eta}^w)-\bar{\eta}^w\right)}{\frac{s_t^mL_t^m}{s_t^mL_t^m+s_t^wL_t^w}\left(1+\phi\right)\left(1-F\left(\bar{\eta}^m\right)\right)+\left(1-\delta^m\right)} \\
& > \quad \frac{1+\frac{1}{\phi}\left(1-F\left(\bar{\eta}^w\right)\right)+F\left(\bar{\eta}^m\right)-F\left(\bar{\eta}^w\right)}{\left(1+\frac{1}{\phi}\right)\left(1-F\left(\bar{\eta}^w\right)\right)+F\left(\bar{\eta}^m\right)}=1
\end{aligned}
$$

and

$$
-u'_{1w,t}+\beta\left(\frac{RP_t}{P_{t+1}}\right)\kappa u'_{2w}\left(\left(1+\frac{1}{\phi}\right)\left(1-F\left(\bar{\eta}^w\right)\right)\right)<0
$$

then

$$
\frac{\frac{\phi}{1+\phi}L_t^m}{\frac{\phi}{1+\phi}L_t^m+\frac{1}{1+\phi}L_t^w}\frac{ds_t^m}{d\phi}+\frac{\frac{1}{1+\phi}L_t^w}{\frac{\phi}{1+\phi}L_t^m+\frac{1}{1+\phi}L_t^w}\frac{ds_t^w}{d\phi}>0
$$

Since $\frac{dL_t^m}{d\phi} > 0$ and $\frac{dL_t^w}{d\phi} < 0$, we can show that

$$\frac{d\left(\frac{\frac{\phi}{1+\phi}L_t^m}{\frac{\phi}{1+\phi}L_t^m+\frac{1}{1+\phi}L_t^w}\right)}{d\phi} = \frac{L_t^m L_t^w + \phi\left(L_t^w\frac{dL_t^m}{d\phi} - L_t^m\frac{dL_t^w}{d\phi}\right)}{\left(\phi L_t^m + L_t^w\right)^2} > 0$$

Then

$$\frac{ds_t^{young}}{d\phi} > 0$$

The aggregate labor supply in period t

$$\begin{aligned}
\frac{dL_t}{d\phi} &= \frac{\phi}{1+\phi}\frac{dL_t^m}{ds_t^m}\frac{ds_t^m}{d\phi} + \frac{1}{1+\phi}\frac{dL_t^w}{ds_t^w}\frac{ds_t^w}{d\phi} + \frac{L_t^m - L_t^w}{(1+\phi)^2} \\
&> \frac{1}{1+\phi}\frac{z_2}{\det\begin{pmatrix}\Omega_{11} & \Omega_{12}\\ \Omega_{21} & \Omega_{22}\end{pmatrix}}\left(\Omega_{11}\frac{dL_t^m}{ds_t^m} - \Omega_{12}\frac{dL_t^w}{ds_t^w}\right) \\
&> \frac{1}{1+\phi}\frac{w_t}{P_t}\frac{z_2}{\det\begin{pmatrix}\Omega_{11} & \Omega_{12}\\ \Omega_{21} & \Omega_{22}\end{pmatrix}}\left(\begin{array}{c} u''_{1w,t}L_t^w\frac{dL_t^m}{ds_t^m} + \beta\left(\frac{RP_t}{P_{t+1}}\right)^2\kappa^2 u''_{2w}\left(\left(1+\frac{1}{\phi}\right)\left(1-F\left(\bar{\eta}^w\right)\right)\right)\left(L_t^w + s_t^w\frac{dL_t^w}{ds_t^w}\right)\frac{dL_t^m}{ds_t^m} \\ -\beta\left(\frac{RP_t}{P_{t+1}}\right)^2\left[\kappa^2 u''_{2w}\left(\left(1+\frac{1}{\phi}\right)\left(1-F\left(\bar{\eta}^w\right)\right)\right]\left(L_t^m + s_t^m\frac{dL_t^m}{ds_t^m}\right)\frac{dL_t^w}{ds_t^w}\end{array}\right) \\
&> \frac{1}{1-s_t^w}\frac{dL_t^m}{ds_t^m}\frac{1}{1+\phi}\frac{z_2}{\det\begin{pmatrix}\Omega_{11} & \Omega_{12}\\ \Omega_{21} & \Omega_{22}\end{pmatrix}}\left(-u'_{1w,t} + \frac{\beta\left(\frac{RP_t}{P_{t+1}}\right)\kappa u'_{2w}\left(\left(1+\frac{1}{\phi}\right)\left(1-F\left(\bar{\eta}^w\right)\right)\right)s_t^m L_t^m}{s_t^m L_t^m + s_t^w L_t^w}\right) \\
&> \frac{1}{1-s_t^w}\frac{dL_t^m}{ds_t^m}\frac{1}{1+\phi}\frac{z_2}{\det\begin{pmatrix}\Omega_{11} & \Omega_{12}\\ \Omega_{21} & \Omega_{22}\end{pmatrix}}\left(-u'_{1w,t} + \beta\left(\frac{RP_t}{P_{t+1}}\right)\kappa u'_{2w}\left(\left(1+\frac{1}{\phi}\right)\left(1-F\left(\bar{\eta}^w\right)\right)\right)\right) > 0
\end{aligned}$$

where the third inequality holds because, under the assumption

$$\frac{2\left(v''\right)^2}{v'} + \frac{v'}{L^2} - \frac{v''}{L} - v''' > 0$$

we can show that

$$\frac{d\left(\frac{1}{L}\frac{dL}{ds}\right)}{ds} = \left(\frac{v'}{v'/L - v''}\right)^2\left(\frac{2\left(v''\right)^2}{v'} + \frac{v'}{L^2} - \frac{v''}{L} - v'''\right)\frac{dL}{ds} > 0$$

and hence $\frac{1}{L_t^m}\frac{dL_t^m}{ds_t^m} > \frac{1}{L_t^w}\frac{dL_t^w}{ds_t^w}$. The fourth inequality holds because

$$s_t^m\frac{dL_t^w}{ds_t^w} = \frac{s_t^m}{1-s_t^w}\frac{L_t^w}{1-\frac{v''_w L_t^w}{v'_w}} < \frac{s_t^m L_t^m}{1-s_t^w}$$

and the last inequality holds because of (D.2).

Similar to the proof of Proposition 1, we can rewrite the system as

$$\begin{pmatrix} \Omega_{33} & \Omega_{34} & L_t\Omega_{35} \\ \Omega_{43} & \Omega_{44} & L_t\Omega_{45} \\ \Omega_{53} & \Omega_{54} & L_t\Omega_{55} \end{pmatrix} \cdot \begin{pmatrix} dw_t \\ dP_{Nt} \\ dl_{Nt} \end{pmatrix} = \begin{pmatrix} z_3'' \\ -\frac{\alpha_T w_t}{L_t}\frac{dL_t}{d\phi} \\ -\frac{\alpha_N w_t}{L_t}\frac{dL_t}{d\phi} \end{pmatrix} d\phi$$

where

$$z_3'' = \left(\gamma\left(1 - s_t^{young}\right) w_t - w_t l_{Nt}\right) \frac{dL_t}{d\phi} - \gamma w_t \frac{ds_t^{young}}{d\phi}$$

Similar to the proof of Proposition 1, the sex ratio will affect the price of the non-tradable good via two channels: the savings channel and the labor supply channel. We can write $\frac{dP_{Nt}}{d\phi}$ in two parts

$$\begin{aligned} \frac{dP_{Nt}}{d\phi} &= \frac{\gamma w_t}{\Delta}\frac{ds_t^{young}}{d\phi}\left(\frac{\alpha_N w_t}{L_{Nt}} + \frac{\alpha_T w_t}{x - L_{Nt}}\right) \\ &+ \frac{1}{\Delta}\left[\begin{array}{c} \frac{\alpha_T w_t}{L_t}\left(\gamma\left(1 - s_t^{young}\right) L_t \frac{\alpha_N w_t}{L_{Nt}} + w_t\right) + \frac{\alpha_N w_t}{L_t}\left(\gamma\left(1 - s_t^{young}\right) L_t \frac{\alpha_T w_t}{L_{Tt}} - w_t\right) \\ + \left((1-\alpha_N)\gamma R s_{t-1} w_{t-1}\frac{L_{t-1}}{L_t} - \alpha_N \gamma\left(1 - s_t^{young}\right) w_t\right)\left(\frac{\alpha_T w_t}{L_{Tt}} + \frac{\alpha_N w_t}{L_{Nt}}\right) \end{array}\right] \frac{dL_t}{d\phi} \end{aligned}$$

where Δ is defined as in the proof of Proposition 1. Since $\frac{ds_t^{young}}{d\phi} > 0$ and $\frac{dL_t}{d\phi} > 0$, similar to the proof of Proposition 1

$$\frac{dP_{Nt}}{d\phi} < 0$$

That is, the price of the non-tradable good falls, leading to a depreciation of the real exchange rate. ■

Figure 1: Impulse responses of RER, aggregate savings rate and CA/GDP
tao=15, b=1, m=2.08, std dev=0.1, distribution margin=0.5

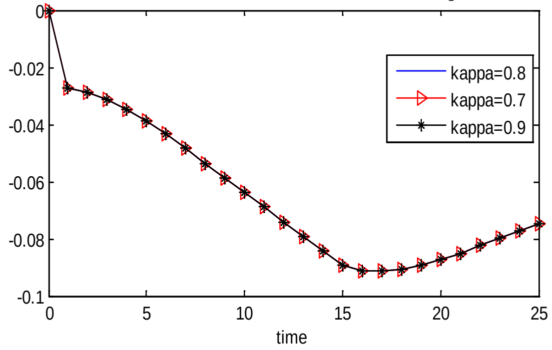

Figure 2: Robustness Checks by Varying the Capital Adjustment Cost Parameter b
(tao=15, kappa=0.8, m=2.08, std dev=0.1, distribution margin=0.5)

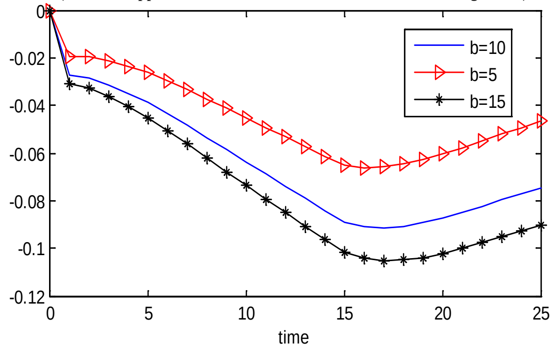

Figure 3: Robustness Checks by Varying the Mean Value of Emotional Utility (Love) m
(tao=15, kappa=0.8, b=1, std dev=0.1, distribution margin=0.5)

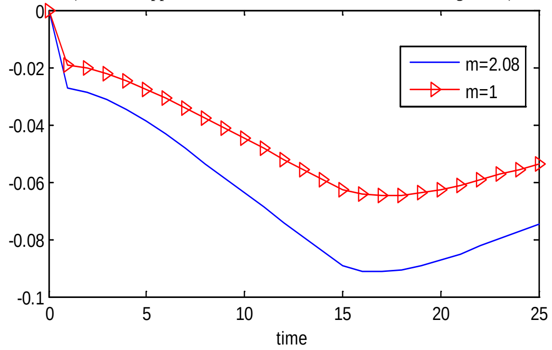

Figure 4: Robustness Checks by Varying the Standard Deviation of Emotional Utility (Love)
(tao=15, kappa=0.8, b=1, m=2.08, distribution margin=0.5)

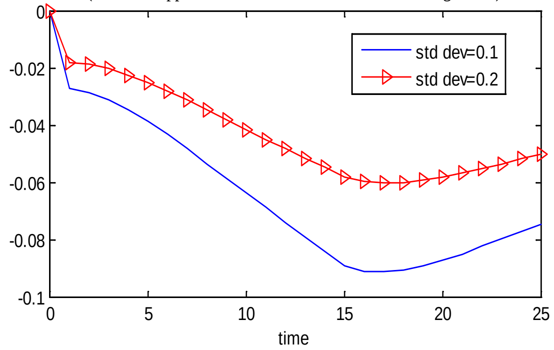

Figure 5: Robustness Checks by Varying the Timing of Marriage
(kappa=0.8, b=1, m=2.08, std dev=0.1, distribution margin=0.5)

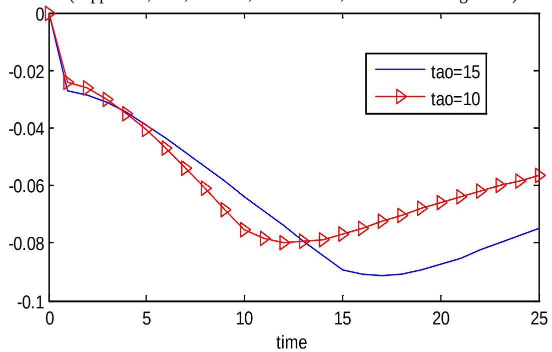

Figure 6: Robustness Checks by Varying the Distribution Margin Parameter
(tao=15, kappa=0.8, b=1, m=2.08, std dev=0.1)

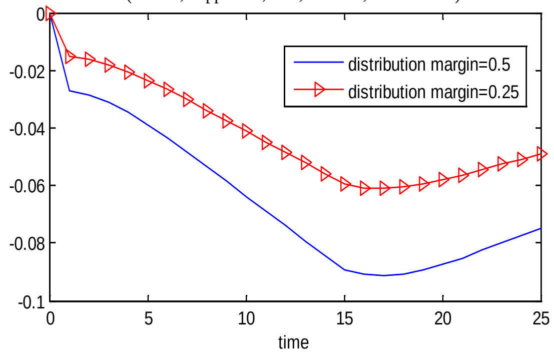

Table 1: Summary Statistics on Regional Variations in China, Cumulative over 2001-2005

Variable	Mean	Median	Standard deviation	Min value	Max value
Full sample					
Cumulative change in log(Pn/Pt), measure 1	-0.059	-0.057	0.043	-0.141	0.024
Cumulative change in log(Pn/Pt), measure 2	-0.018	-0.019	0.062	-0.140	0.128
Δ(sex ratio)	0.021	0.016	0.033	-0.037	0.155
Δ(log per capita income)	0.351	0.361	0.072	0.188	0.489
Δ(Share of working age population)	0.038	0.034	0.011	0.026	0.062

Notes: The price of non-tradable relative to tradable is measured by two ways as described in the text.

Table 2: Local Real Exchange Rate and Sex Ratio across Chinese Provinces during 2001-2005

	(1)		(2)	
Variables	Δlog(Pn/Pt), measure 1		Δlog(Pn/Pt), measure 2	
Δ(sex ratio)	-0.454*	-0.478*	-0.806**	-0.857**
	(0.223)	(0.241)	(0.310)	(0.321)
Δ(log per capita income)		-0.012		-0.081
		(0.109)		(0.145)
Δ(Share of working age population)		0.486		1.45
		(0.723)		(0.964)
Observations	31	31	31	31
R-squared	0.13	0.14	0.19	0.26

Notes: All regressions have an intercept which is not reported. Standard errors are in parentheses, ** and * indicate statistically significant at the 5% and 1% levels, respectively.

Table 3: Summary Statistics on International Variables, Averaged over 2004-2008

Variable	Mean	Median	Standard deviation	Min value	Max value
Ln(RER)	-0.74	-0.80	0.59	-2.22	0.41
Real GDP per capita (US$)	12986	8032	12991	367	72937
Private credit (% of GDP)	57.11	35.07	56.41	2.08	319.7
Financial system sophistication	3.78	3.66	0.79	2.52	5.28
Sex ratio	1.04	1.04	0.02	1.00	1.13
Fiscal deficit (% of GDP)	-1.11	-0.19	5.16	-21.92	13.23
Terms of trade	110	102	30.8	25.5	211
Capital account openness	0.44	0.12	1.64	-1.83	2.50
Dependency ratio	60.78	54.86	17.62	28.80	107.55

- The real exchange rate data is obtained from Penn World Tables 6.3. The variable "*p*" (called "price level of GDP") in the Penn World Tables is equivalent to the real exchange rate relative to the US dollar: A lower value of *p* means a depreciation in the real exchange rate.
- For the ratio of private credit (% of GDP), we follow Allen, Qian and Qian (2004) and modify the measure for China by multiplying 0.2 to the credit to GDP ratio. This is to correct for the fact that only 20% of the bank loans go to private firms. Financial system sophistication from the Global Competitiveness Report is another measure for the financial development.
- Fiscal deficit data is obtained from IFS database. Terms of trade index is defined as the ratio of export price index to the import price index, which is from Worldbank database. We use the capital account openness index in Chinn and Ito (2008) to measure the degree of capital controls. A higher value means less capital control. Dependency ratio data can be obtained from Worldbank database.

Table 4: Ln(real exchange rate) and the sex ratio, using private credit to GDP ratio as the measure of financial development

	(1) All countries	(2) All countries	(3) All countries	(4) Excluding major oil exporters	(5) Excluding major oil exporters	(6) Excluding major oil exporters	(7) Excluding major oil exporters
Sex ratio			-4.293**	-4.010**	-3.461*	-3.357**	-3.966**
			(1.663)	(1.707)	(1.760)	(1.538)	(1.687)
Ln(GDP per capita)	0.320**	0.192**	0.238**	0.236***	0.413**	0.416**	0.409**
	(0.030)	(0.038)	(0.041)	(0.044)	(0.075)	(0.064)	(0.071)
Private credit (% of GDP)		0.004**	0.004**	0.004**	0.003**	0.003**	0.002**
		(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Fiscal deficit					-0.005	0.002	-0.003
					(0.009)	(0.008)	(0.009)
Terms of trade					-0.001	-0.002	-0.001
					(0.001)	(0.001)	(0.001)
Capital account openness					0.0334	0.023	0.0283
					(0.029)	(0.025)	(0.028)
Dependency ratio					0.009**	0.009**	0.007*
					(0.004)	(0.003)	(0.004)
Crawling peg (RR)						-0.412**	
						(0.076)	
Managed floating (RR)						-0.122	
						(0.080)	
Free floating (RR)						-0.092	
						(0.114)	
Intermediate (LYS)							-0.158*
							(0.088)
Float (LYS)							-0.233**
							(0.080)
Observations	142	132	132	123	75	73	75
R-squared	0.447	0.543	0.566	0.581	0.771	0.842	0.801

Dependent variable = ln(RER). Standard errors are in parentheses, ** p<0.05, * p<0.1

Table 5: Ln(real exchange rate) and the sex ratio, using financial system sophistication as the measure of financial development

	(1) All countries	(2) All countries	(3) All countries	(4) Excluding major oil exporters	(5) Excluding major oil exporters	(6) Excluding major oil exporters	(7) Excluding major oil exporters
Sex ratio			-6.125**	-6.187**	-4.601*	-4.385	-6.217**
			(1.963)	(1.995)	(2.285)	(2.760)	(2.757)
Ln(GDP per capita)	0.320**	0.483**	0.445**	0.449**	0.556**	0.559**	0.541**
	(0.030)	(0.082)	(0.077)	(0.088)	(0.119)	(0.123)	(0.121)
Financial system sophistication		0.165*	0.247**	0.240**	0.117	0.0589	0.151
		(0.089)	(0.086)	(0.100)	(0.104)	(0.122)	(0.110)
Fiscal deficit					0.01	0.0104	0.015
					(0.013)	(0.014)	(0.014)
Terms of trade					-0.004*	-0.005*	-0.004
					(0.002)	(0.003)	(0.003)
Capital account openness					0.045	0.0417	0.022
					(0.042)	(0.051)	(0.048)
Dependency ratio					0.002	0.005	-0.002
					(0.008)	(0.010)	(0.009)
Crawling peg (RR)						-0.226	
						(0.154)	
Managed floating (RR)						-0.058	
						(0.108)	
Free floating (RR)						-0.000	
						(0.174)	
Intermediate (LYS)							0.041
							(0.143)
Float (LYS)							-0.118
							(0.123)
Observations	142	54	54	49	36	35	36
R-squared	0.447	0.75	0.791	0.798	0.882	0.885	0.887

- Dependent variable = log(RER). Standard errors are in parentheses, ** $p<0.05$, * $p<0.1$

Table 6: Real exchange rate undervaluation: The Case of China

	% of RER undervaluation				
	(1) Only BS	(2) FD + BS	(3) Add GD+TT+KA	(4) Add DR	(5) Add SR
Financial development index					
Private credit (% of GDP)	55.26	43.45	35.44	17.90	-7.86
Financial system sophistication	55.26	46.38	31.31	16.78	2.24

Notes:

A. Excess RER undervaluation = model prediction – actual log RER. (A positive number describes % undervaluation).

B. The five columns include progressively more regressors:

- (1) The only regressor (other than the intercept) is log income, a proxy for the Balassa-Samuelson (BS) effect;
- (2) Add financial development (FD) to the list of regressors;
- (3) Add government fiscal deficit (GD), terms of trade (TT), and capital account openness (KA);
- (4) Add the dependence ratio (DR);
- (5) Add the sex ratio (SR)

C. The last two rows correspond to estimates when two different proxies for financial development are used. The first row uses the ratio of credit to the private sector to GDP, and the second row uses an index of local financial system sophistication from the Global Competitiveness Report.