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PUBLIC INVESTMENT AND WELFARE: THEORY AND EMPIRICAL IMPLICATIONS

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ABSTRACT

Public Investment and Welfare: Theory and Empirical Implications*

This paper analyses the determinants of public expenditures allocated to investment. We perform welfare analysis in an overlapping generations model with public consumption, public investment, debt and taxes. The optimal public investment share depends positively on the productive contribution of public investment, and negatively on the preference for public consumption relative to private consumption and on the weight that the government assigns to the older generation. We then provide empirical evidence that the old age dependency ratio is the main determinant of the international variability of the share of public resources allocated to investment.

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NON-TECHNICAL SUMMARY

Public investment has a different impact on economic activity than does public consumption expenditure. Since public investment in the form of infrastructure raises the stock of physical capital, and public investment in education increases human capital, both enhance future rather than current consumption. Furthermore, if public investment affects positively the marginal productivity of private capital, private investment, private output and private consumption will increase. Recent research has shown that if the externality in production of public capital is very strong, an increase in public investment may even affect the economic growth rate permanently.

Despite the renewed interest in the macroeconomic effects of public investment, the recent theoretical and empirical literature has left unanswered the question of what determines public investment. This paper is an attempt to characterize the determinants of the share of public expenditure that is allocated to investment. We analyse a simple overlapping generations model with households, firms and a government sector. We assume that the government provides public consumption in the form of a pure public good and also spends on public investment. The latter increases the stock of public capital; due to externalities in production, public investment raises the marginal productivity of private capital and private output. The government can finance its expenditures by raising taxes and issuing debt.

If the government wishes to maximize welfare, it faces an intergenerational trade-off. Old households care about public investment only in so far as it increases current output, allowing the government to raise more taxes and spend more on the public good. Thus the elderly would like the government to set public investment at a relatively low level. Young households, instead, will enjoy the future benefits of public investment, and would like to see the government spend relatively little on public consumption.

The model implies that the welfare-maximizing share of public investment on total government outlays depends positively on the productive contribution of public investment and negatively on the preference for public consumption relative to private consumption. The model also implies that the optimal share depends negatively on the weight that the government assigns to old households. If the productive contribution of public investment is high enough to induce self-sustained economic growth, in the long run it is optimal for the government to spend only on public investment.

In the second part of the paper we compare the actual values of the public investment share with the theoretical values predicted by the theory and test the main predictions of the model. We use average data on a cross-section of 70 countries over the period 1970-85. In the sample the average ratio of public investment to total government expenditures is 24%. According to our model, this ratio is consistent with a relatively low elasticity of private output with respect to public capital, a relatively strong preference for public consumption relative to private consumption or a relatively strong weight given by the government to the older generation.

In the econometric analysis we regress the public investment share on the overall expenditure-GDP ratio, per capita GDP, primary and secondary school enrollment rates, a proxy for political instability and the old-age dependency ratio (a proxy for the weight that governments assign to the older generation). Overall, the regression explains only 14% of the international variability in the public investment share. Except for the old-age dependency ratio, none of the variables helps explain the international variability of the public investment share. The structure of the population has a considerable impact on the composition of the government budget: an increase in the old-age dependency ratio from 5–16% (the average values corresponding, respectively, to Africa and the OECD) is associated with a reduction in the public investment share of 7.4%. The negative impact of the dependency ratio on the composition of government expenditure suggests that population ageing provides one possible explanation for the decline in public investment observed in virtually all OECD countries in the last two decades.

1. Introduction

It has long been recognized that the impact of public investment expenditure on economic activity differs from that of public consumption.[1] More recently, the resurgence of interest in the determinants of economic development has spurred a series of theoretical studies on the connection between public investment and growth (Barro, 1990; Glomm and Ravikumar, 1992; Barro and Sala-i-Martin, 1992) and of empirical studies on the macroeconomic impact of public investment (e.g. Aschauer, 1989).

The natural question - which this recent research has left largely unanswered, however - is what determines public investment. Since public investment is widely acknowledged to stimulate output, it becomes important to understand how and why governments allocate resources between investment and consumption. The determinants of the share of total government outlays allocated to investment lend themselves to both theoretical welfare analysis and empirical scrutiny. In this paper we try to make progress on both counts.

We start in section 2 with a descriptive analysis of the international variability in the share of public investment in government budgets, using the data set assembled by Barro and Wolf (1989), and uncovering two stylized facts. First, the public investment share exhibits considerable variability between countries, but relatively little variability between geographical areas. Second, the only variable that appears to have some explanatory power vis-à-vis the composition of government outlays is the age structure

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[1] Arrow and Kurz (1970) provide one of the first studies on the interaction between private output and public capital.

of the population: where the share of the elderly is high, governments allocate a smaller fraction of budgetary expenses to investment.

In section 3 we propose a theoretical framework consistent with these features of the data. We analyze a simple overlapping generations model in which public investment has a direct positive effect on private production, and in which the volume of such investment has to be balanced against that of public consumption. We assume that the latter is in the form of a pure public good that directly enters households' utility functions.

When public investment stimulates private output, government revenues and saving increase. The higher revenues allow the government to spend more on public consumption; and higher saving enhances the private consumption of future generations. The former benefit of public investment accrues to all generations currently living; the latter, however, is enjoyed only by those who will be alive in the future. Thus the young should want to set public investment at a higher rate than the old.

The model implies that the public investment share that maximizes social welfare depends: (i) positively on the productive contribution of public investment; (ii) negatively on the preference for public consumption as against private consumption; (iii) negatively on the weight that the government assigns to old households (this effect disappears if the government is following the golden rule). A significant and realistic feature of our model is that we allow the issue of public debt, not constraining the public sector to balance its budget in each period. We also show that if the model features endogenous growth, in the long run it is optimal to spend only on public investment.

In section 4 we take the old age dependency ratio (the share of the elderly in total population) as a proxy for the weight assigned by fiscal authorities to the current retired generation. Formal econometric evidence that this ratio is indeed the main determinant of the international variability of the share of public resources allocated to investment is presented. Section 5 summarizes the paper.

2. International figures on public investment

The main international reference for fiscal variables is the data set assembled by Barro and Wolf (1989), which makes available averages of public investment, government consumption and other budget items in 76 countries over the period 1970-1985; since for six of them the dependency ratio is not available, our sample comprises 70 countries (see Table 1). In order to to separate the budget items that contribute to current consumption from those that enhance future consumption, we take a broad definition of public investment. In addition to infrastructural spending, which increases the stock of real capital, we include public expenditure on education, which raise the stock of human capital.

Table 1 shows the GDP ratio of public investment and of total government outlays (investment, government purchases of goods and services and transfer payments) over the period 1970-1985 by main geographical and economic areas. The average ratio of public investment to GDP is 0.07; that of total government outlays to GDP is 0.30; and the average public investment share, i.e. the ratio of investment to total government outlays, is 0.24. While this share exhibits considerable variability from country to country, it varies relatively little across main geographical and economic areas, ranging only from 0.26 in Latin America to 0.22 in the OECD.

In Table 1 we also report a number of other variables that might well be expected to correlate with the public investment share. Countries with comparatively lower initial per capita GDP (measured as the ratio to the sample mean) may need to invest more in order to catch up with richer countries. Where the initial level of human capital (measured by the 1970 primary and secondary enrollment rate) is lower, governments may choose to invest more in education to close the gap with countries that are better endowed.[2] Political instability (proxied by the number of revolutions and coups per year in 1960-1985) may discourage or even prevent long-term government planning in the form of public investment. Finally, the old age dependency ratio, i.e. the share of the population over 60 years old in the total population, may reflect the weight that the government assigns to the elderly in framing fiscal policy.[3]

All these variables exhibit substantial variability across geographical areas, but the public investment share is only weakly correlated with most of them. The highest correlation coefficient (-0.26) is with the old age dependency ratio; proxies for initial resources and for the initial level of human capital are negatively but weakly correlated with the public investment share; contrary to our expectations, the proxy for political instability correlates positively.

These patterns are confirmed most graphically by Figures 1A, 1B and 1C, which plot the public investment share against total government outlays, initial per

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[2]As is explained by Barro (1991) "These variables [...] measure number of students enrolled in the designated grade levels relative to the total population of the corresponding age group. Because of this definition it is possible for the values to exceed 1.0" (p. 414).

[3] Demographic structure was brought to bear on the classic public finance problem of the link between the size of the public sector and economic development in a series of empirical papers in the seventies. These papers relate the expenditure-GDP ratio (Kelley, 1976) or the tax-GDP ratio (Bolnick, 1978) to the old age dependency ratio and to other demographic variables, with mixed results. More recently, Engen and Skinner (1991) use demographic variables as instruments for government consumption in a reduced-form growth regression. The present paper, however, focuses on the structure of the government budget rather than on its size.

capita GDP and the old age dependency ratio. The regression line reported in each figure is obtained by plotting the fitted values of a bivariate OLS regression of the public investment share against the variable whose values are given on the horizontal axis.

Figures 2A, 2B and 2C specialize the descriptive analysis of the previous figures to the 23 OECD countries in the sample. Even within this relatively homogeneous set, the dispersion of the public investment share is substantial. The share is negatively correlated with total outlays and the dependency ratio and uncorrelated with per capita GDP.

To summarize, our descriptive analysis uncovers two stylized facts: (i) the public investment share displays considerable dispersion between countries; (ii) the demographic structure is the only variable possessing some potential power to explain the composition of the government budget. In the next section we present a theoretical model that is consistent with these patterns.

3. The model

To illustrate the problem of a government choosing how to allocate budgetary resources between consumption and investment, we use an overlapping generations model in which households live for two periods. The government finances public consumption and public investment by raising taxes and issuing debt. Public consumption directly enters households' utility function; public investment raises the productivity of private capital and contributes directly to private production.

After stating the specific assumptions that we use to describe each sector of the economy, we characterize the equilibrium and perform welfare analysis. We then analytically derive the welfare-maximizing expression for the share of public investment in total expenditure.

3.1 The economy

We assume that households earn labor income only in the first period of their lives: when young they consume and save for retirement, and when old they consume the savings accumulated in the first period. Households derive utility from private consumption c and from a pure public good c_g; the young also pay a proportional income tax on labor income. We assume that utility is separable within and across periods and that preferences are given by

$$U(c_t^y, c_{gt}, c_{t+1}^o, c_{gt+1}) = \gamma \ln c_t^y + (1-\gamma) \ln c_{gt} + \delta \gamma \ln c_{t+1}^o + \delta (1-\gamma) \ln c_{gt+1}\,, \quad (1)$$

where the superscripts 'y' and 'o' indicate the younger and the older generations, the time subscript refers to the timing of consumption, δ is the discount factor, and γ the preference for private consumption relative to public consumption. Households maximize utility subject to the intertemporal budget constraint

$$c_t^y + \frac{c_{t+1}^o}{R_{t+1}} = w_t - t_t\,, \quad (2)$$

where w_t, R_{t+1} and t_t are, respectively, the wage rate, the interest rate factor and total taxes levied on labor income. Utility maximization yields the household saving function

$$s_t^y = \theta\,(w_t - t_t)\,, \quad (3)$$

where $\theta = \delta/(1+\delta)$.

Private output y is produced by combining private capital k, public capital g and labor according to the Cobb-Douglas production function

$$y_t = A\, g_t^{\phi}\, k_t^{\alpha}\, , \tag{4}$$

where the labor force has been normalized to unity and assumed to be constant. Thus, as in Barro (1990), we assume that public capital raises the productivity of private capital to an extent that depends on the parameter ϕ.

Initially we assume $\alpha+\phi<1$, so that production exhibits diminishing returns to the inputs that can be accumulated. Since the parameter A is also initially assumed to be constant, the system does not display long-run growth. The implications of our model in the presence of exogenous and endogenous growth are analyzed in section 3.5. We assume that there is full depreciation of public and private capital, implying that the flow of investment equals the stock of capital in the public and private sector (this assumption is relaxed in section 3.6).

Firms maximize profits in a competitive environment, hiring labor and purchasing capital according to the first order conditions

$$w_t = (1-\alpha)\, A\, g_t^{\phi}\, k_t^{\alpha} = (1-\alpha)\, y_t \tag{5}$$

$$R_t = \alpha\, A\, g_t^{\phi}\, k_t^{\alpha-1} = \alpha(y_t/k_t)\, . \tag{6}$$

Each period the government finances current expenditures, public investment and interest payments on public debt through tax revenues and new debt issues

$$b_{t+1} - b_t = (R_t-1)b_t + g_t + c_{gt} - t_t\, , \tag{7}$$

where b is the stock of government debt and g is public investment, equal to the public capital stock under full depreciation (we introduce depreciation in section 3.6).

A range of fiscal policies may be compatible with the intertemporal budget constraint. Since we are interested in studying the steady-state composition of the budget, we assume that fiscal policy is defined by a set of three exogenous instruments, the tax rate $\tau = t/y$, the share of public investment in government expenditures $\beta = g/(g+c_g)$, and the debt-output ratio $\lambda = b/y$. Note that since $w_t = (1-\alpha)y_t$, the first assumption implies that taxes are levied on the young in proportion to their earnings. The instrument β will be the main object of the welfare analysis in sections 3.3 and 3.4.

3.2 Equilibrium

In equilibrium, total saving by the young must equal the sum of private capital plus government debt. The latter evolves according to equation (7). Thus the model can be reduced to a two-equation system, the capital market equilibrium condition (CME) and the government budget constraint (GBC),

$$k + b = s^y = \theta\,(w - t) = \theta\,(1 - \alpha - \tau)\,y \qquad (8)$$

$$(R - 1)\,b = \tau\,y - g - c_g\,. \qquad (9)$$

Since we confine ourselves to a steady-state analysis, the time subscripts have been dropped. Note from equation (9) that in the steady-state of a non-growing economy the stock of debt is constant, so that the budget deficit must be equal to zero. This implies that, in steady-state, interest payments must equal the primary surplus. Denoting by $\lambda = (b/y)$ the debt-output ratio, dividing equations (8) and (9) by y, using the equilibrium condition $R=\alpha y/k$, and the definition of the fiscal instruments τ, β, and λ, we can transform equations (8) and (9) as follows

$$(k/y) = \theta\,(1-\alpha-\tau) - \lambda \tag{10}$$

$$(g + c_g)/y - \tau = H(\lambda)\,, \tag{11}$$

where $H(\lambda)$ is the sustainable primary deficit, and it is equal to

$$H(\lambda) = \lambda\left[1 - \frac{\alpha}{\theta\,(1-\alpha-\tau) - \lambda}\right] = \lambda[1-R(\lambda)].$$

Since public debt crowds out private capital, the capital-output ratio in the CME equation (10) is a monotonically decreasing function of λ; it is this relation between λ and k/y which implies $R'(\lambda)>0$. On the other hand, the relation between the debt-output ratio λ and interest payments is non-linear, as shown by the GBC condition (11). Figure 3 plots the $H(\lambda)$ function. In the dynamically efficient region ($R>1$), raising debt unambiguously raises interest payments, thus reducing the sustainable primary deficit. If instead the economy is dynamically inefficient ($R<1$), the government is effectively raising interest revenues from the sale of public debt at a rate equal to (1-R). In this case, an increase in debt has two opposite effects: it increases the tax base λ, but lowers the tax rate (1-R). Finally, at the golden rule level of the interest rate ($R=1$), not only the total budget, but also the primary budget must be balanced.

Plugging the production function (4) and the GBC (11) into the definition of the share β of public investment on total public expenditure, we derive an expression for the steady-state level of public investment in terms of private capital and fiscal policy instruments

$$g = \{\beta[\tau+H(\lambda)]\,Ak^{\alpha}\}^{\frac{1}{1-\phi}}\,. \tag{12}$$

Substituting the expression for g in the CME, we obtain the steady-state solution of private capital and private output

$$y = \{[\theta\,(1-\alpha-\tau)-\lambda]^{\alpha}\,A\,[\beta(\tau+H(\lambda))]^{\phi}\}^{\frac{1}{1-\alpha-\phi}}\,. \tag{13}$$

The equation above indicates that the steady-state level of y varies directly with the fraction of total spending devoted to public investment, β, and with the productive contribution of public capital to private output, ϕ. An increase in the tax rate, however, will have two opposite effects on steady-state output: contractionary, due to the effect of taxation on household saving, and expansionary deriving from the fact that the government has more resources to channel towards productive public investment (to an extent that depends on β). This trade-off is similar to that highlighted by Barro (1990) in a model with infinite horizon and balanced budget.

If the economy is dynamically efficient, an increase in the debt-output ratio (an increase in λ), is associated with lower steady-state output, because of the direct crowding-out effect on private capital (the term in the first bracket), and because of the reduction in public investment due to the lower equilibrium primary deficit (the $H(\lambda)$ term). If the economy is dynamically inefficient, this latter effect may be positive and could outweigh the former.[4]

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[4] Note that in this economy a competitive equilibrium may be also subject to static inefficiencies. For instance, this occurs if the government sets public investment at a level that violates the equality between the marginal rate of transformation between public and private capital.

3.3 Welfare analysis in the command economy

Before performing the second best welfare analysis of a government that controls only the three policy instruments posited, we solve the problem of a centralized economy in which a social planner with full control of the economy maximizes the utility function of a representative generation. In golden rule the only constraint in this economy is that total consumption and investment cannot exceed total production

$$c^y + c^o + c_g + k + g \leq A\, g^{\phi} k^{\alpha} . \tag{14}$$

From the first order conditions, $\frac{\partial U}{\partial c^y} = \frac{\partial U}{\partial c^o} = \frac{\partial U}{\partial c_g}$ and $\frac{\partial y}{\partial k} = \frac{\partial y}{\partial g} = 1$, one immediately derives an expression for private output

$$y = (A\, \phi^{\phi}\, \alpha^{\alpha})^{\frac{1}{1-\alpha-\phi}} , \tag{15}$$

and for the first best values of public consumption and investment,

$$c_g = (1-\gamma)\,(1-\alpha-\phi)\, y \qquad\qquad g = \phi\, y . \tag{16}$$

Thus, the golden rule value of the share of investment in total government expenditure is equal to

$$\beta = \frac{g}{g+c_g} = \frac{\phi}{\phi+(1-\gamma)(1-\alpha-\phi)} . \tag{17}$$

The last expression indicates that β depends positively on the strength of the productive contribution of public capital (ϕ), and negatively on people's preference for public consumption ($1-\gamma$).[5]

3.4 Welfare analysis in the decentralized economy

We now illustrate the solution to the problem of a government that sets the fiscal policy instruments with the objective of maximizing the welfare of the population. In golden rule this amounts to considering the utility of a representative generation, as was done in the case of the centralized economy. In a decentralized economy, instead, the government aims at maximizing social welfare within the set of competitive equilibria and takes into account also the utility of the present older generation.

In steady-state the utility of the younger generation depends on the equilibrium values of current and future private and public consumption, and thus on the equilibrium wage and interest rates corresponding to the steady-state level of output in equation (13). An increase in β inflicts a utility loss on the young that is proportional to their preference for public consumption. On the other hand, there are two sources of utility gain from an increase in the public investment share: (i) it raises the wage rate of the young and therefore their level of private consumption; (ii) it also raises the saving of the young, allowing higher future consumption.

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[5] It can be shown that the golden rule value of β in (17) is an upper bound of the set of Pareto optimal public investment shares. In fact, imposing the additional constraint $c^o \geq \bar{c}$ to the maximization problem of the social planner implies $\frac{\partial U}{\partial c^y} \leq \frac{\partial U}{\partial c^o}$ and $c_g \geq (1-\gamma)(1-\alpha-\phi)$. Since at the optimum $g = \phi y$, the set of Pareto optimal β is bounded above by (17). A lower bound for β can be found by setting public consumption at its highest level relative to private output. This occurs when $c^y+c^o=0$, implying $c_g = (1-\gamma)(1-\alpha-\phi)$. Optimality still requires $g = \phi y$, so the lower bound is $\beta = \phi/(1-\alpha)$.

The utility of the elderly, by contrast, depends on private consumption, which is predetermined by the values of factor prices and output (denoted by w_o, R_o and y_o), and on the consumption of the pure public good, which is consumed at the same level as that enjoyed by the young. Since the elderly will not enjoy future consumption, they gain less than the young from an increase in β. In fact, if public investment did not contribute to current production, and therefore did not allow the government to collect taxes and finance the supply of the public good, the older generation would want to set $\beta=0$.

The foregoing considerations imply that the welfare function depends on the following equilibrium values of private and public consumption

$$c^y = (1-\theta)(1-\alpha-\tau)y \,, \tag{18}$$

$$c^o = R\theta\,(1-\alpha-\tau)y \,, \tag{19}$$

$$c_g = (1-\beta)[\tau + H(\lambda)]\,y \,. \tag{20}$$

Denoting by π and $(1-\pi)$ the weights assigned by the government to the older and younger generations, and substituting the above expressions into the utility function, social welfare L is given by

$$L = (1-\pi)\{\gamma \ln (1-\theta)(1-\alpha-\tau)y + (1-\gamma) \ln R\theta(1-\alpha-\tau)y + (1+\delta)(1-\gamma) \ln (1-\beta)[\tau + H(\lambda)]\,y\}$$
$$+ \pi\,\{\gamma \ln R_o\,\theta(1-\alpha-\tau)y_o + (1-\gamma) \ln (1-\beta)[\tau + H(\lambda)]\,y\}\,. \tag{21}$$

Substituting the equilibrium value of output (13) into this welfare function and maximizing with respect to β, we obtain

$$\beta = \frac{\phi}{\phi+(1-\gamma)(1-\alpha-\phi)\frac{1+\delta(1-\pi)}{1+\delta-\pi(\delta+\gamma)}}. \tag{22}$$

The expression for β is the same as in the command economy, except for the presence of an additional term in the denominator. The optimal β is still a positive function of ϕ and a negative function of $(1-\gamma)$. Furthermore, if $\pi=0$, the expression reduces to equation (17), the golden rule level of β. Since the derivative of β with respect to π is unambiguously negative, the larger the weight assigned to the older generation, the lower the welfare-maximizing share of public investment. When $\pi=1$, the optimal β reaches a minimum: $\beta=\phi/(1-\alpha)$.[6] This shows that in our model it is not optimal to set $\beta=0$ even if $\pi=1$: public capital enhances private production, which in turn allows the government to raise revenues and to provide public consumption for the older generation as well.

Our welfare analysis indicates that the welfare-maximizing public investment share depends only on preferences and technology, and not at all on the level of public debt, its ratio to output, the tax rate or the overall level of spending. This is the main empirical prediction of the model that we will test in the following section. But first we explore whether this conclusion holds if one relaxes two important assumptions, namely the absence of growth and the full depreciation of the stock of public capital.

3.5 Growth

Let us examine how the model is modified when provision is made for technical progress or constant returns to scale to the inputs that can be accumulated ($\alpha+\phi=1$).

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6 Thus the set of set of optimal values of β in the market economy is the same set of Pareto optimal β in the command economy (see footnote 5).

The former case corresponds to the case of exogenous growth, the latter to endogenous growth.

Suppose first that the parameter A, rather than being a constant, grows at the exogenous rate μ. This modifies the production function as follows

$$y_t = A(1+\mu)^t g_t^{\phi} k_t^{\alpha} \tag{23}$$

In this growing economy the steady-state growth rate of public capital, private capital, output and wages is equal to[7]

$$(1+\rho) = (1+\mu)^{\frac{1}{1-\alpha-\phi}} . \tag{24}$$

In the presence of growth, the CME and GBC conditions (equations 10 and 11) are modified in a straightforward way,

$$(k/y) = \frac{\theta\,(1-\alpha-\tau)}{1+\rho} - \lambda \tag{25}$$

$$(g+c_g)/y - \tau = H(\rho,\lambda) , \tag{26}$$

where $H(\rho,\lambda) = \lambda\left[(1+\rho) - \frac{\alpha(1+\rho)}{\theta(1-\alpha-\tau) - \lambda(1+\rho)}\right]$.

Since the functional forms of equations (25) and (26) are identical to (10) and (11), the effect of public debt is the same as in the non-growing economy, the only difference being that the condition for dynamic efficiency in Figure 3 is replaced by

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[7] Expression (24) is obtained by taking the first differences of equation (23) and considering that, in steady-19

state, output, private capital and public capital grow at the common rate ρ.

$R=(1+\rho)$. Following the same steps as in the no-growth economy, we obtain the expression for output along the steady-state growth path

$$y_t = \{[\theta(1-\alpha-\tau) - \lambda]^{\alpha} A \{\beta[\tau+H(\rho,\lambda)]\}^{\phi}\}^{\frac{1}{1-\alpha-\phi}} (1+\mu)^{\frac{t}{1-\alpha-\phi}} . \qquad (27)$$

It can be immediately shown that the welfare-maximizing level of β is unchanged with respect to the non-growth economy, in both the command and the market economy. Thus even in a growing economy the optimal ratio of public investment to total government outlays depends on preferences and technology, and on the weight that the government assigns to the utility of the older generation.

Suppose now that $\alpha+\phi = 1$, so that production displays constant returns with respect to the inputs that can be accumulated. Thus, the productive contribution of public capital is high enough to induce self-sustained economic growth. The CME and GBC relations are still given by equations (25) and (26); using the definition of public investment in the GBC, $g=\beta[\tau+H(\rho,\lambda)]y$, it is readily demonstrated that in this case the interest factor R is constant and equal to

$$R^* = \alpha y/k = \alpha A(g/k)^{1-\alpha} = \alpha\, A^{\frac{1}{\alpha}} [\beta(\tau+H(\rho,\lambda)]^{\frac{1-\alpha}{\alpha}} , \qquad (28)$$

where $H(\rho,\lambda)$ is defined following equation (26). Substituting the expression for (α/R) into the term (k/y) in the CME, we obtain an implicit expression for the steady-state growth rate

$$(1+\rho) = \frac{\theta(1-\alpha-\tau)}{A^{-\frac{1}{\alpha}} \{\beta[\tau+H(\rho,\lambda)]\}^{-\frac{1-\alpha}{\alpha}} + \lambda} . \qquad (29)$$

The noteworthy feature of equation (29) is that due to the non-monotonic relation between the debt-output ratio λ, the growth rate $(1+\rho)$ and the sustainable primary deficit $H(\rho,\lambda)$, the growth rate does not fall unambiguously with λ over its whole feasible range, as in the Saint-Paul (1992) version of the Blanchard model of perpetual youth and in the Grossman and Yanagawa (1993) model with debt as a pure bubble. In our model, instead, an increase in λ from sufficiently low values allows the government to spend more on public investment and therefore to raise the growth rate (see Appendix). As a result of the non-linearity of the GBC, the relation between the public investment share and the growth rate is also non-monotonic, so that there is a range of parameter values over which an increase in β reduces $(1+\rho)$.

Equation (29) simplifies considerably if we confine ourselves to the case of no public debt ($\lambda=0$). Then the steady-state growth rate becomes

$$(1+\rho) = A^{\frac{1}{\alpha}} (\beta\tau)^{\frac{1-\alpha}{\alpha}} \theta(1-\alpha-\tau). \tag{30}$$

As is shown in the Appendix, the welfare analysis indicates that the benefit of public investment increases over time, so that, eventually, it becomes so large as to swamp any cost of foregone private consumption. It is therefore optimal for the government to spend nothing on public consumption and to set $\beta=1$ as t tends to infinity.

Further specializing this expression by setting $\beta=1$, one obtains a solution for steady-state growth that is comparable to that of Barro (1990). In this particular case, in our model growth is maximized by setting $\tau=(1-\alpha)^2$. Note that this implies, as in Barro, that the optimal tax rate on wages is equal to $(1-\alpha)$, the share of public capital in production.

3.6 Depreciation

Suppose that one wants to distinguish between the stock of public capital, g, and the flow of public investment i_g, and introduce depreciation into the model. Stocks and flows are linked by the following equation:

$$i_{gt} = g_t - (1-\eta)g_{t-1} = g_t \frac{\eta+j_t}{1+j_t}, \tag{31}$$

where η is the constant depreciation rate of the stock of public capital and j_t the rate of growth of public capital, i.e. $j_t = (g_t/g_{t-1}) - 1$. In steady-state, in the absence of growth, $j=0$, and the stock of public capital is equal to public investment divided by the depreciation rate,

$$g = \frac{i_g}{\eta} = \frac{\beta[\tau+H(\lambda)]y}{\eta}. \tag{32}$$

The second equality follows from the definition of the policy instruments.

The model then has the same solution as in the case without depreciation, the only difference being that the parameter β is replaced by β/η. But the main result of our welfare analysis is unchanged: the public investment share is independent of the rate of depreciation of public capital.

4. Empirical evidence

The main empirical implication of the model is that the optimal share of public investment over total government spending depends on the productivity of public and private capital (ϕ and α), on the preference for public consumption (1-γ) and on the weight assigned by the government to the older generation (π). In this section we compare the theoretical values of the model with the data and test its main predictions using the Barro and Wolf (1989) data set.

Table 2 summarizes the implied values of β for plausible ranges of the relevant parameters. In all cases we set the discount factor δ equal to 0.9. We allow the elasticity of output with respect to capital to vary between 0.1 and 0.5, which encompasses all realistic estimates of the share of capital income in total national income. The evidence about the substitutability between public and private consumption is scant. Using Euler equations, Aschauer (1985) estimates it to be 0.23, implying a value of (1-γ) of 0.19. The weight π varies from 0 (the golden rule case) to 1 (the government sets its policy instruments taking only the current older generation into account).

As Hulten and Schwab (1992) note, there are conflicting estimates of the productive contribution of public investment. Holtz-Eakin (1988), using aggregate United States data, finds that the elasticity of output with respect to public capital is 0.2, and Aschauer (1989) finds even higher values; similar conclusions are reached by Berndt and Hansson (1991) for Sweden. Tatom (1991), after taking into account the non-stationarity of output and capital, finds a small and insignificant elasticity. Holtz-Eakin (1992), using state level variables, also finds a value of ϕ close to zero.

Table 2 shows that the β range that is consistent with reasonable parameter values varies between 0.15 and 0.50. It is of interest to compare these theoretical values of β with their empirical counterparts. The average ratio of public investment to total government spending reported in Table 1 is 0.24. There are several parameter values that are consistent with this figure: a relatively low elasticity of private output with respect to public capital (ϕ around 0.1 or less), a relatively strong preference for public consumption with respect to private consumption ($1-\gamma$ around 0.3 or higher), and a relatively strong weight given to the older generation (π around 0.5 or higher).

To assess the main factors determining the public investment share empirically, we regress β on the variables listed in Table 1, i.e. total government expenditure, the old age dependency ratio, the ratio of per capita GDP in 1970 to the sample mean, the primary and secondary school enrollment rates, a proxy for political instability, and continental dummies for Africa, Asia and Latin America.

We take the share of the elderly in the population as a proxy for π, the weight that the government assigns to the older generation. The welfare analysis suggests that its coefficient should be zero if governments follow the golden rule of capital accumulation, and negative if $\pi>0$.

Table 2 presents the coefficient estimates. Overall, the regression explains only 14 percent of the international variability in the public investment share. None of the variables is significantly different from zero, except for the old age dependency ratio, which is negative and significant at the 10 percent level. The magnitude of this coefficient indicates that an increase in the dependency ratio from 5 to 16 percent (the average values corresponding, respectively, to Africa and the OECD) is associated with a reduction in the share of public investment of 7.4 percentage points. Thus, the structure of the population has a considerable impact on the composition of government budgets.

Since our results may be sensitive to the presence of influential values, we repeat the estimation using a robust estimation method[8] and least absolute deviations (LAD). The results are qualitatively unaffected: the magnitude of the coefficient of the dependency ratio rises and in the LAD estimation the coefficient is significantly different from zero at the 1 percent level.

Table 3 repeats the estimation for the sub-sample of 23 OECD countries. The coefficient of the dependency ratio is of a similar order of magnitude as in Table 2. The most notable difference with respect to the larger sample is that the coefficient of the variable 'primary enrollment rate' is negative and significantly different from zero.

5. Conclusions

We have analyzed an overlapping generations model in which public investment has a direct positive effect on private production. The model implies that the share of total government outlays allocated to investment which maximizes social welfare depends positively on the productive contribution of public investment and negatively on the preference for public consumption as against private consumption. If the government does not follow the golden rule of capital accumulation, the welfare-maximizing public investment share also depends negatively on the weight that the government assigns to old households.

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8 The robust estimation method performs an initial OLS regression, calculates Cook's distance, eliminates the gross outliers for which Cook's distance exceeds 1, and then performs iterations based on Huber weights followed by iterations based on a biweight function. This routine is programmed in the STATA econometric software.

In the empirical analysis we posit the old age dependency ratio as a valid proxy for the weight that governments attach to the elderly in fiscal policy design, and for a sample of 70 countries that ratio proves to be the only variable with some explanatory power with respect to the international variability of the public investment share. A 10-point increase in the old age dependency ratio explains a reduction in the investment share of between 6.7 and 9.7 percentage points. A smaller sample of 23 OECD countries yields similar results.

If confirmed by individual country studies, our cross-country results suggest that the increase in the elderly population in the last two decades provides one rationale for the parallel decline in public investment observed in virtually all OECD countries.

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Appendix

1. The relation between (1+ρ), λ and β when growth is endogenous.

Let $\sigma=\theta\,(1-\alpha-\tau)$, $\Psi = A^{\frac{1}{\alpha}}\{\beta[\tau+H(\rho,\lambda)]\}^{-\frac{1-\alpha}{\alpha}}$ and H_ρ and H_λ the derivatives of $H(\rho,\lambda)$ with respect to ρ and λ. Equation (29) in the text can then be written as $(1+\rho)=\sigma/(\Psi+\lambda)$. Note also that the two partial derivatives of $H(\rho,\lambda)$ are linked by the relation $H_\lambda=(1+\rho)H_\rho/\lambda$ and that $\Psi_H=-(1-\alpha)\Psi/\alpha(\tau+H)$. Taking the derivative of ρ with respect to λ and multiplying both terms by $-\sigma/(\Psi+\lambda)^2=-(1+\rho)^2/\sigma$, we obtain

$$\frac{d\rho}{d\lambda}=\frac{1-\dfrac{(1-\alpha)\Psi}{\alpha(\tau+H)}H_\lambda}{\dfrac{-\sigma}{(1+\rho)^2}+\dfrac{\lambda(1-\alpha)\Psi}{(1+\rho)\alpha(\tau+H)}H_\lambda}.$$

The growth rate and the debt-output ratio are related non-monotonically. In fact, the derivative is not unambiguously negative (a sufficient condition for $d\rho/d\lambda<0$ is $H_\lambda<0$). The derivative is positive if H_λ belongs to the interval

$$\frac{\alpha(\tau+H)}{(1-\alpha)\Psi}<H_\lambda<\frac{\alpha(\tau+H)}{(1-\alpha)\Psi}\,\frac{\sigma}{\lambda(1+\rho)}.$$

Note that also the reverse inequality of the expression above guarantees that $d\rho/d\lambda<0$. However, we rule this case out because it implies $[\sigma-\lambda(1+\rho)]<0$, which from equation (25) entails a negative capital-output ratio.

Noting further that $\Psi_\beta=-(1-\alpha)\Psi/\alpha\beta$, the derivative of the growth rate with respect to the public investment share is

$$\frac{d\rho}{d\beta} = \frac{-\frac{(1-\alpha)\Psi}{\alpha\beta}}{\frac{-\sigma}{(1+\rho)^2} + \frac{\lambda(1-\alpha)\Psi}{(1+\rho)\alpha(\tau+H)} H_\lambda} .$$

This derivative is positive if $H_\lambda \leq 0$ and negative if $H_\lambda > \frac{\sigma\alpha(\tau+H)}{\lambda(1+\rho)(1-\alpha)\Psi}$. Thus an increase in the public investment share does not unambiguously raise the growth rate.

2. Welfare analysis with endogenous growth

When growth is endogenous and there is no public debt (λ=0), the welfare function is

$$L = (1-\pi)[\gamma \ln (1-\theta)(1-\alpha-\tau)y_o(1+\rho)^t + \delta\gamma \ln R\theta(1-\alpha-\tau)y_o(1+\rho)^t +$$
$$+ (1-\gamma) \ln (1-\beta)\tau y_o(1+\rho)^t + \delta(1-\gamma) \ln (1-\beta)\tau y_o(1+\rho)^{t+1}] +$$
$$+ \pi [\gamma \ln R\, \theta(1-\alpha-\tau)y_o(1+\rho)^{t-1} + (1-\gamma) \ln (1-\beta)\tau\, y_o(1+\rho)^{t-1}].$$

Substituting equation (30) for the steady-state growth rate (1+ρ), equation (28) with λ=0 for the interest factor R, and maximizing with respect to β we obtain

$$\beta = \frac{[(1-\alpha)/\alpha]\{(1-\pi)\delta+[1+\delta(1-\pi)t\}}{[(1-\alpha)/\alpha]\{(1-\pi)\delta+[1+\delta(1-\pi)t] + (1-\gamma)[1+\delta(1-\pi)]} .$$

As t tends to infinity, the welfare maximizing level of β tends to 1.

Figure 1A: Entire sample

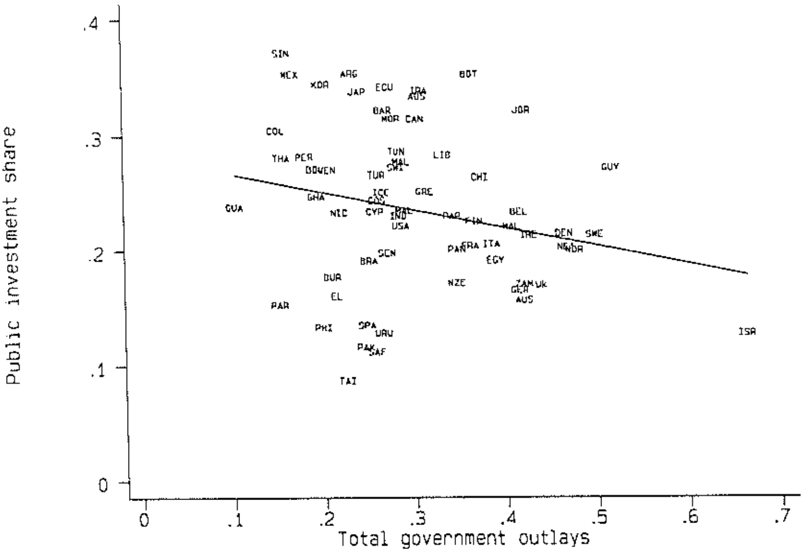

Figure 1B: Entire sample

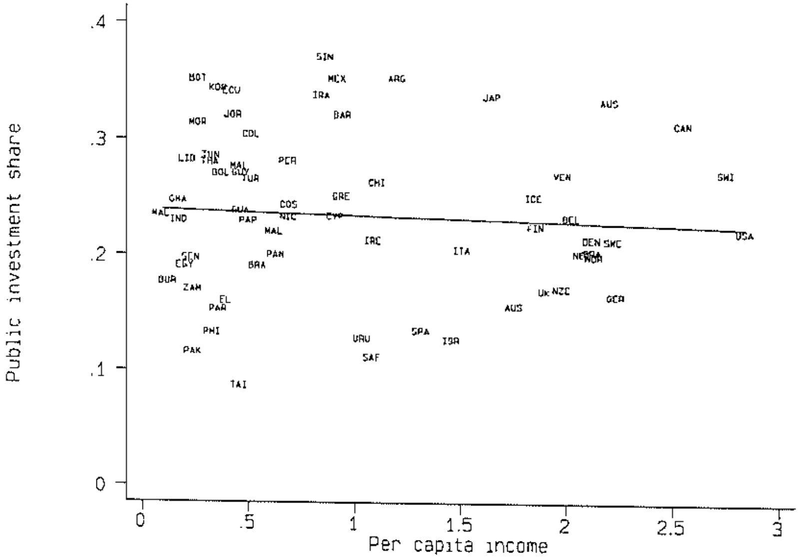

Figure 1C: Entire sample

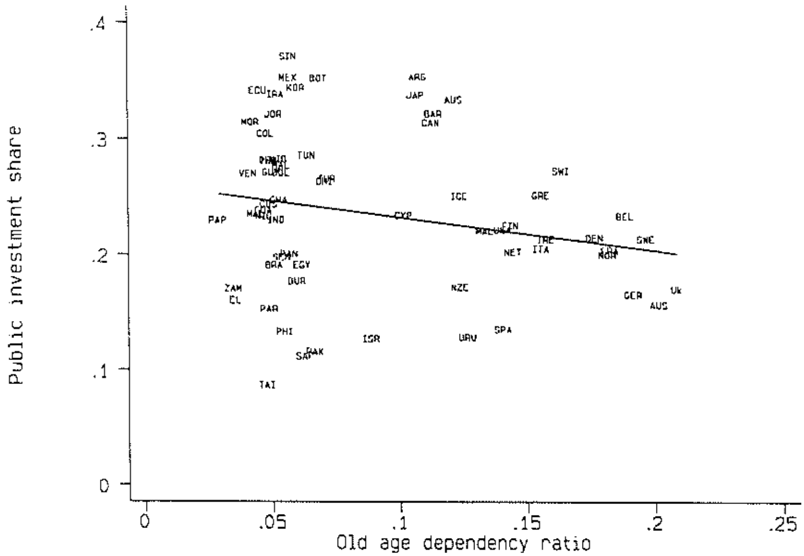

Figure 2A: OECD sub-sample

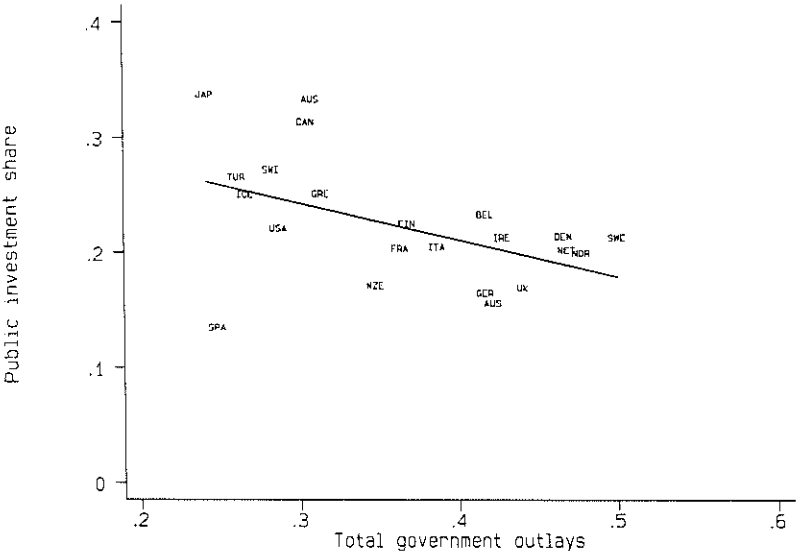

Figure 2B: OECD sub-sample

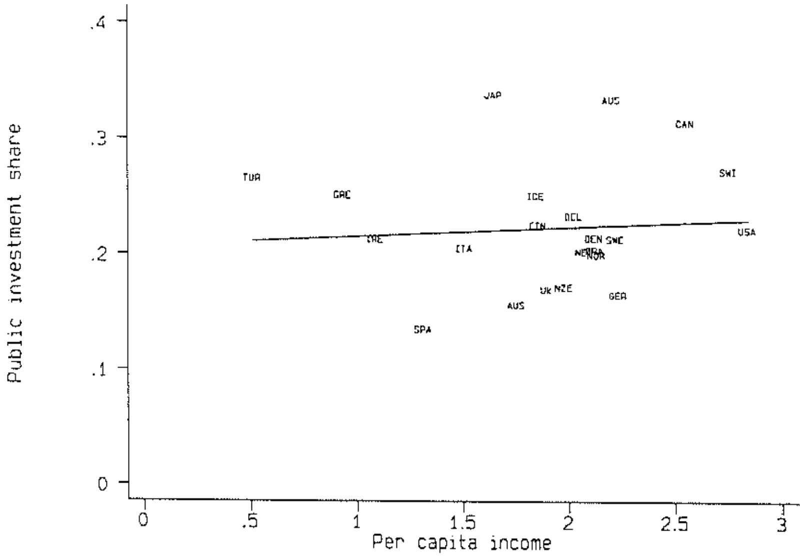

Figure 2C: OECD sub-sample

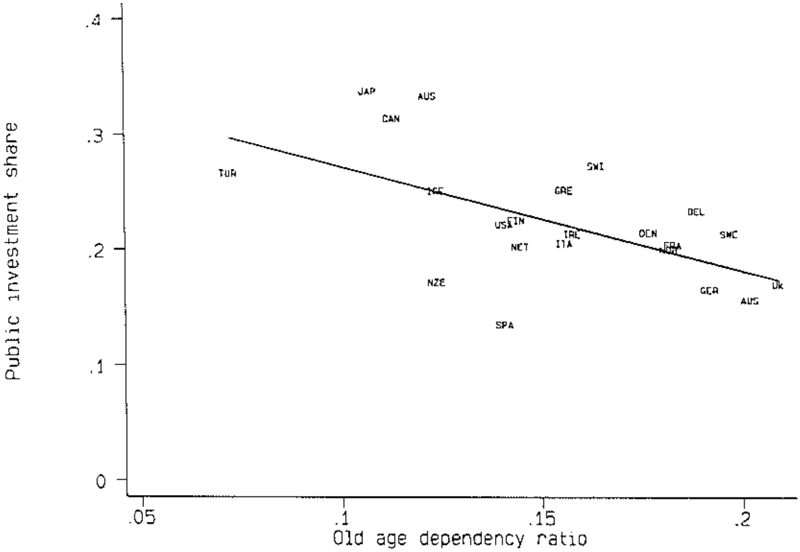

FIGURE 3

The relation between the debt-output ratio and the sustainable primary deficit

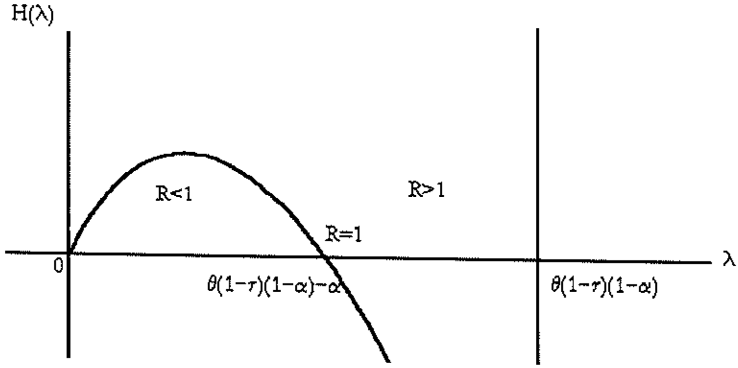

Table 1

Public investment and total government outlays:

an international comparison [a]

	Africa (1)	Asia (2)	Latin America (3)	OECD (4)	Total sample (5)	Correlation with the public investment share (6)
Public investment as a ratio to GDP, 1970-1985	0.07	0.06	0.06	0.08	0.07	0.49
Total government outlays as a ratio to GDP, 1970-1985	0.30	0.28	0.24	0.32	0.30	-0.28
Public investment share, 1970-1985	0.25	0.23	0.26	0.22	0.24	1.00
standard deviation	0.07	0.10	0.07	0.05	0.07	
minimum	0.11	0.08	0.13	0.13	0.08	
maximum	0.35	0.37	0.35	0.33	0.37	
Per capita GDP in 1970 as a ratio to the sample mean	0.29	0.50	0.73	1.91	1.00	-0.11
Primary school enrollment rate in 1970	0.67	0.91	0.95	1.07	0.93	-0.11
Secondary school enrollment rate in 1970	0.14	0.33	0.31	0.71	0.42	-0.16
Number of revolutions and coups per year, 1960-1985	0.10	0.22	0.29	0.04	0.15	0.13
Old age dependency ratio, 1970-1985	0.05	0.06	0.06	0.16	0.09	-0.26
Number of countries	12	12	19	23	70	

a. Except for the old age dependency ratio, all data are taken from Barro and Wolf (1989). The dependency ratio is the ratio of the population over 60 to total population (1970-85 average or available sub-periods) and is drawn from the United Nations, *Demographic Yearbook*, various years. We list here the 70 countries included in the sample. Africa: Botswana, Egypt, Ghana, Kenya, Liberia, Malawi, Morocco, Senegal, South Africa, Swaziland, Tunisia, Zambia; Asia: Burma, India, Iraq, Israel, Jordan, Korea, Malaysia, Pakistan, Philippines, Singapore, Taiwan, Thailand; Latin America: Barbados, Costarica, Dominican Republic, El Salvador, Guatemala, Mexico, Nicaragua, Panama, Argentina, Bolivia, Brazil, Chile, Colombia, Ecuador, Guyana, Paraguay, Peru, Uruguay, Venezuela; OECD: Japan, Austria, Belgium, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Luxembourg, Netherlands, Norway, Spain, Sweden, Switzerland, Turkey, United Kingdom, Canada, United States, Australia, New Zealand; the sample also includes: Cyprus, Malta, Fiji and Papua.

Table 2

Public investment share for various parameter values [a]

ϕ (1)	β (2)	α (3)	β (4)	$(1-\gamma)$ (5)	β (6)	π (7)	β (8)
0.01	0.04	0.10	0.24	0.10	0.53	0.00	0.36
0.05	0.16	0.20	0.26	0.20	0.38	0.25	0.33
0.10	0.30	0.30	0.30	0.30	0.30	0.50	0.30
0.15	0.41	0.40	0.34	0.40	0.25	0.75	0.24
0.20	0.50	0.50	0.39	0.50	0.22	1.00	0.14

a. In column 2 we report the values of β obtained by holding constant all parameter values except ϕ; in column 4, all except α; in column 6, all except $(1-\gamma)$; in column 8, all except π. The baseline parameter values, reported in the third row of the table, are: $\phi = 0.1$, $\alpha = 0.3$, $(1-\gamma) = 0.3$, $\pi = 0.5$ and $\delta = 0.9$.

Table 3

Dependent variable: public investment share in total government outlays
(sample of 70 countries)

	Ordinary Least Squares (1)	Robust Regression (2)	Least Absolute Deviations (3)
Total government outlays as a ratio to GDP, 1970-1985	-0.109 (-1.06)	-0.165 (-1.46)	-0.040 (-0.38)
Per capita GDP in 1970 as a ratio to the sample mean	0.020 (0.95)	0.019 (0.80)	0.007 (0.02)
Primary school enrollment rate in 1970	-0.006 (-0.10)	0.025 (0.40)	0.075 (1.41)
Secondary school enrollment rate in 1970	0.030 (0.41)	0.054 (0.67)	0.093 (1.27)
Number of revolutions and coups per year, 1960-1985	0.021 (0.44)	0.016 (0.31)	0.039 (0.97)
Old age dependency ratio, 1970-1985	-0.674 (-1.84)	-0.749 (-1.87)	-0.957 (-2.64)
Africa	-0.007 (-0.16)	0.009 (0.21)	0.035 (1.06)
Asia	-0.032 (-0.91)	-0.018 (-0.45)	-0.017 (-0.55)
Latin America	-0.014 (-0.42)	-0.017 (-0.46)	-0.009 (-0.32)
Constant	0.313 (4.92)	0.301 (4.31)	0.219 (4.13)
R^2	0.139	-.-	0.121

a. See Table 1 for data sources and country list.

Table 4

Dependent variable: public investment share in total government outlays
(sample of 23 OECD countries)

	Ordinary Least Squares	Robust Regression	Least Absolute Deviations
Total government outlays as a ratio to GDP, 1970-85	-0.120 (-0.75)	-0.127 (-0.73)	-0.234 (-0.96)
Per capita GDP in 1970 as a ratio to the sample mean	0.015 (0.85)	0.015 (0.79)	0.024 (0.86)
Primary school enrollment rate in 1970	-0.219 (-2.48)	-0.199 (-2.08)	-0.114 (1.08)
Secondary school enrollment rate in 1970	0.004 (0.06)	0.003 (0.04)	0.076 (0.91
Old age dependency ratio, 1970-1985	-0.712 (-1.87)	-0.718 (-1.75)	-0.925 (-1.72)
Constant	0.560 (5.16)	0.563 (4.64)	0.482 (4.06)
R^2	0.567	-.-	0.330

a. See Table 1 for data sources and country list.