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FINANCIAL INTEGRATION, SPECIALIZATION, AND SYSTEMIC RISK

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ABSTRACT

Financial Integration, Specialization, and Systemic Risk*

This paper studies the implications of cross-border financial integration for financial stability when banks' loan portfolios adjust endogenously. Banks can be subject to sectoral and aggregate domestic shocks. After integration they can share these risks in a complete interbank market. When banks have a comparative advantage in providing credit to certain industries, financial integration may induce banks to specialize in lending. An enhanced concentration in lending does not necessarily increase risk, because a well-functioning interbank market allows to achieve the necessary diversification. This greater need for risk sharing, though, increases the risk of cross-border contagion and the likelihood of widespread banking crises. However, even though integration increases the risk of contagion it improves welfare if it permits banks to realize specialization benefits.

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Keywords: financial contagion, financial integration, interbank market, risk sharing and specialization

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1 Introduction

Large and complex financial institutions increasingly dominate the financial systems of industrial countries. Partly to further enhance scale, partly for domestic competition policy and partly for diversifying revenue streams and risks, these financial institutions transact more and more across borders. They link the financial systems of different countries and foster international financial integration. By diversifying their risks more they improve the resilience of the international financial system against idiosyncratic shocks. At the same time, however, the risk of financial contagion is extended from the national level to the international arena. Due to international integration a default of one such institution can now have more severe negative externalities on financial intermediaries abroad. As the recent turbulence in the global financial system following the failure of Lehman Brothers in September 2008 vividly showed, these externalities may arise from direct exposures, from asymmetric information about them or from large failures causing liquidity dry-ups in key markets.[1] The increasing cross-border activities and risk exposures of major financial intermediaries are particularly challenging, as the main regulatory and supervisory setups in banking, securities and insurance business remain predominantly at the national level, and therefore may not be able to effectively address cross-border contagion risk.

Theoretical studies that deal with this trade-off between the benefits from diversification and the expected costs from financial contagion focus on the integration through the interbank market, because banks remain at the core of financial systems and tend to be particularly linked among each other. For a number of reasons (large and complex financial conglomerates, trading links between different types of financial institutions, e.g., through new credit risk transfer markets, or banks' prime

[1] An early case of international financial contagion due to direct exposures was the Herstatt crisis in 1974. A more recent example of international systemic risk related to market illiquidities was the Long Term Capital Management (LTCM) crisis in 1998. For a discussion of these and many other cases, see Basel Committee on Banking Supervision (2004).

broker activities for hedge funds), however, the analysis carries over to other large financial intermediaries. Moreover, the one and a half decades prior to the recent financial crisis have witnessed exponential growth of cross-border bank activities (see figure 1). The overwhelming part of this is constituted of interbank assets and liabilities.

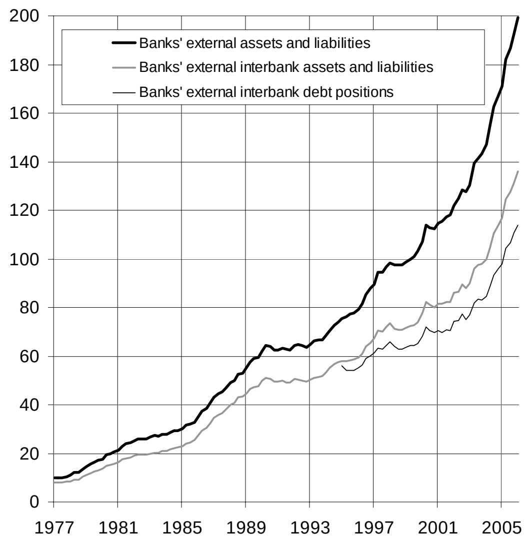
Previous studies of the welfare implications of integrated interbank markets, however, took the corporate lending behavior of banks as given. This implies that the distribution of idiosyncratic shocks across regions is not affected by financial integration.[2] This assumption is problematic because one should expect that the portfolios of financial institutions react to the openness of financial markets. In order to fully evaluate the allocative effects of financial integration one needs to endogenize the loan portfolios of domestic or international banks.

In this paper we follow this idea. We analyze the welfare effects of financial integration taking into account that the improved scope for risk sharing through integrated financial markets affects banks' specialization which in turn influences the cross-country distribution of bank specific shocks. More precisely, we develop a model in which each local bank has a comparative advantage in lending to a specific sector, because this sector is most productive in the respective bank's country.[3]

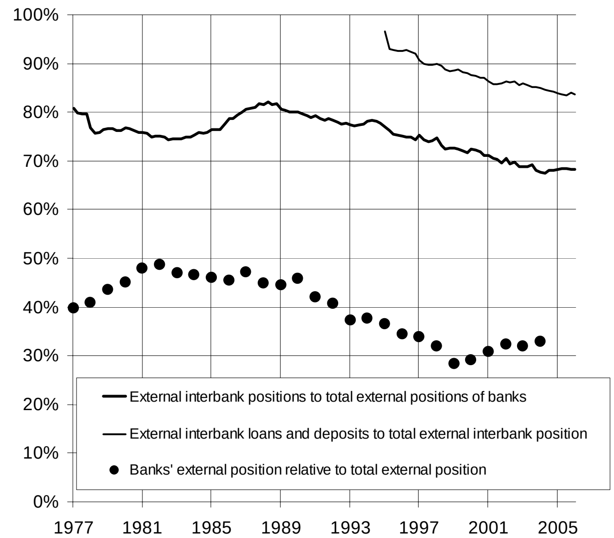
[2]While Allen and Gale (2004a,b) and Fecht (2004) focus on interrelations between banks through the general asset market, Allen and Gale (2000), Freixas, Parigi, and Rochet (2000), Fecht and Grüner (2006), as well as Fecht, Grüner, and Hartmann (2007) focus on the interbank deposit market. All of these studies assume a given distribution of the idiosyncratic shocks.
In contrast, two papers analyze the impact of interbank markets on banks' investment choices, focusing on moral hazard problems and the incentives for peer monitoring. Rochet and Tirole (1996) assess the incentives for peer monitoring in order to draw conclusions about the scope for a system-wide banking crisis in this context. Freixas and Holthausen (2004) discuss the implications of greater asymmetric information about foreign compared to domestic banks for the structure and integration of an interbank market. None of these two papers, however, focus on the relationship between interbank market integration and cross-border contagion.

[3]See Acharya, Hasan, and Saunders (2006) for empirical evidence of these specialization benefits in banking.

Figure 1: Development and composition of banks' external assets and liabilities



(a)



(b)

Panel (a) reports the development of 1) banks' cross-border asset and liability holdings, 2) cross-border interbank assets and liabilities, and 3) cross-border interbank debt positions according to an index constructed based on the BIS Locational Banking Statistics, whereby cross-border asset liabilities held in 2000Q1 are set to 100.

Panel (b) reports 1) banks' cross-border assets and liabilities according to the BIS Locational Banking Statistics relative to the sum of foreign assets and liabilities following Lane and Milesi-Ferretti (2007), 2) the share of cross-border interbank assets and liabilities in cross-border assets and liabilities of banks according to the BIS Locational Banking Statistics, and 3) the share of cross-border interbank debt claims in cross-border interbank assets and liabilities according to the BIS Locational Banking Statistics.

Source: BIS Locational Banking Statistics predominatingly covers OECD countries' banking sectors.

Source: BIS "Locational banking statistics", http://www.bis.org/statistics/bankstats.htm.

Since the timing of loan repayments is uncertain across sectors a trade-off between specialization in lending and diversifying liquidity risks arises.

Integration through an interbank market allows banks to reallocate funds across borders and share their liquidity risks. As the scope for diversification through an interbank market improves, banks may choose to increase their lending to the most profitable sector in their region, because the need to diversify through their loan portfolio diminishes. This endogenously raises banks' exposure to specific sectoral shocks and further increases the need for diversification through the interbank market. Thus, the more pronounced is the specialization in the loan book the greater is the need for risk sharing and the more reliant are regional financial institutions on a well-functioning integrated interbank market. But if banks rely to a larger extent on the interbank market to buffer liquidity shocks the risk of contagion grows. If the sector in which one bank is specialized suffers from an adverse liquidity shock, this bank might not be able to raise the needed liquidity in the integrated interbank market, if the foreign bank is at the same time hit by a domestic shock, for instance, due to an operational problem. In that way the failure of one bank as a consequence of a severe domestic shock is transmitted over an integrated interbank market to banks across borders and might ultimately destabilize banks that were initially not affected by the shock.[4]

These results match very well recent empirical evidence by Bonfiglioli (2008) on the role of financial integration for national productivity. According to her analysis, financial integration raises total factor productivity. Moreover, she finds that financial integration slightly raises the risk of financial contagion. Both observations are in line with the present theory.

We also analyze how financial integration affects overall financial stability and welfare. If banks already reap the benefits of specialization without the risk sharing opportunities of an integrated interbank market, then financial integration does not

[4]It is interesting to note that this channel of interbank contagion is not based on the loss of interbank deposits as in Allen and Gale (2000) or Freixas, Parigi, and Rochet (2000).

change the portfolio composition and bank specific liquidity shocks. However, an interbank market allows banks to pool these risks and might thus be welfare enhancing. If it is only financial integration that induces banks to specialize in their lending portfolio then the severity of idiosyncratic risk exposure increases. But the enhanced risk sharing through the interbank market compensates this. However, it also makes banks dependent on the liquidity provision from the cross-border banking market. This channel for cross-border contagion further enhances banks' default risk. However, in our model the higher systemic risk is exactly offset by the lower exposure to domestic shocks. Thus while individual banks' default probability remains unaffected, the risk of a joint banking crisis increases. As long as wide-spread banking crises are not more costly than national banking crises the economic welfare overall improves because of the benefits from specialization. In sum, the changes that financial integration might induce on banks lending behavior have important implications for the relationship between integration and stability and for welfare.

There is a developing, primarily empirical literature about the benefits and costs of financial globalization and capital account liberalization. One part of this literature suggests that countries with sound macroeconomic policies, good economic institutions, advanced financial development and openness as well as good human capital (i.e. industrial countries and, perhaps, a few advanced emerging market countries) are able to reap the risk sharing benefits of international financial integration, whereas countries that are below certain levels for these variables (i.e. most developing and emerging market countries) are not able to benefit.[5] The small part

[5]See, for example, the two recent surveys by Henry (2006) and Kose, Prasad, Rogoff, and Wei (2009). Stulz (2005) discusses the agency problems that hinder less developed countries from reaping the benefits of financial integration. Bekaert, Harvey and Lundblad (2001, 2005 and 2006) and Bekaert, Harvey, Lundblad, and Siegel (2006) find even more generally valid positive effects of equity market liberalizations. Morgan, Rime, and Strahan (2004) estimate that banking integration through the removal of branching restrictions in the United States reduced and aligned state-level business cycles, as measured by gross state product, employment and personal income growth. Matsuyama (2007) presents a broad theoretical framework.

of this literature most closely related to our work asks how financial openness or the presence of capital controls affects the likelihood of financial crises. Despite concerns sometimes raised in policy circles, there does not seem to be systematic evidence suggesting that greater financial integration increases the likelihood of crises, quite the contrary.[6] Still, particularly in developing countries weak financial supervision, contract enforcement problems and unsound macroeconomic policies may sometimes adversely interact with too fast financial liberalization and thereby contribute to financial instability.[7] There is also some evidence that cross-border contagion risks among industrial countries are increasing in conjunction with the financial integration process.[8] Hence, also the available empirical research suggests that the welfare analysis of international financial integration needs to consider both efficiency and stability implications.[9]

The relationship between efficiency and stability implications of financial integration emphasized in our paper is strongly related to the one put forward in Allen and Gale (2000) and Freixas, Parigi, and Rochet (2000). They also show that finan-

[6]Controlling for selection bias, Glick, Guo, and Hutchison (2006) estimate that countries with fewer restrictions on capital flows experience a smaller probability of currency crises than countries that restrict capital flows more. Bonfiglioli and Mendicino (2004) find that the frequency of banking crises is about the same in countries with capital controls and restrictions on equity transactions as it is in countries without such controls and restrictions. Moreover, the adverse effects of banking crises on economic growth turn out to be less severe in countries with less restricted capital accounts. Demirguc-Kunt and Detragiache (2001) find that financial liberalizations increase the likelihood of banking crises, but they only consider domestic interest rate liberalizations and they do not look at the removal of restrictions on foreign capital. See Ferguson, Hartmann, Panetta, and Portes (2007) for a review and similar results from estimations using de facto measures of integration rather than de jure measures of capital controls.

[7]See for example Eichengreen, Mussa, DellArriccia, Detragiache, Milesi-Ferretti, and Tweedie (1998), Williamson and Mahar (1998) or Ishii and Habermeier (2002) for broad overviews and policy discussions.

[8]See Hartmann, Straetmans, and de Vries (2006), Degryse and Nguyen (2007) and van Lelyveld and Liedorp (2006).

[9]See also Tirole (2002) and Eichengreen (2003).

cial integration through the interbank market allows to diversify regional liquidity shocks efficiently while entailing the risk of financial contagion between banks from different regions. But they do not allow for the important endogenous response of bank balance sheets, in particular specialization in lending. Moreover, while in their model liquidity shocks result from stochastic withdrawals of depositors, in our model liquidity shocks stem from uncertainty in the timing of loan repayments (similar to the assumptions underlying Diamond and Rajan (2005)). Non-performing loans are often not defaulting loans but are repaid later than expected, thereby constituting an important liquidity risk. Furthermore, while in Allen and Gale (2000) and Freixas, Parigi, and Rochet (2000) it is the larger credit exposure that leads to cross regional contagion among banks, in our paper it is the greater dependency on liquidity from the interbank market that makes banks more susceptible to contagious market dry-ups.

Our paper is also related to a literature on the relative benefits of bank diversification. Hanson, Pesaran, and Schuermann (2005) suggest that the scope for the international diversification of credit risk is substantial. Winton (1999), however, warns on theoretical grounds that reduced incentives for monitoring borrowers may offset prima facie asset diversification benefits. DeLong (2001) finds that the announcement effects of bank mergers that are focused in both activity and geography suggest more creation of stockholder value than other types of mergers. These results are also consistent with our result that greater specialization through cross-border integration and diversification through the interbank market may be welfare improving.

Last, the paper is related to an earlier debate about optimum currency areas. In this debate it was a widely held argument that the criteria of what constitutes an optimum currency area is endogenous. According to the main proponents of that view–Frankel and Rose (1998)–the deeper economic integration that goes along with a greater monetary integration affects the correlation of business cycles across member countries which in turn affects the costs of a common monetary policy. One important effect that these authors stress is that by reducing obstacles to international trade a monetary union 1) enables countries to capture benefits from comparative advantages whether they are due to technological differences, differences in factor endowments or whether they result from economies of scale, 2) fosters national specialization and 3) ultimately leads to less correlated business cycles.

Similarly, in a recent study Heathcote and Perri (2004) showed that in the course of financial globalization the correlation of the U.S. business cycle with the rest of the world has declined. However, they argue that financial globalization amplified an exogenous reduction in the correlation of productivity shocks by enlarging cross-border capital flows. More related to our view, Kalemli-Ozcan, Sorensen, and Yosha (2003) provide evidence that indeed a deeper integration of international asset markets improves cross-regional risk sharing and leads to greater specialization in production, as first supposed by Helpman and Razin (1978).

2 Assumptions

Consider a three period economy $t = 0, 1, 2$ consisting of regions $j \in \{A; B\}$. In each region there is a continuum of households with the same utility function:

$$U(c_1; c_2) = c_1 + c_2.$$

Thus households are assumed to be risk-neutral.

In $t = 1$ a fraction $q > 1/2$ of households receives the blueprint of a production technology which produces a return $X > 1$ in $t = 2$. This investment opportunity is not publicly observable and is only available to the respective household.[10]

[10]Introducing this private investment opportunity we basically have a linearized version of the Diamond/Dybvig utility function (see Diamond and Dybvig (1983)). Impatient household are those with a private investment opportunity. They have a higher pay-off from goods available in $t = 1$ while patient households, i.e. those with no private investment opportunity, are indifferent between consumption goods in $t = 1$ and $t = 2$. Note that because of this linear pay-off function welfare considerations differ from standard Diamond/Dybvig based models because the marginal

Apart from a storage technology that allows to transfer funds from one period to the next without paying any interest, there are two investment technologies available, that differ in their regional return. Technology S produces a region specific return S_j for each unit invested in $t = 0$ and technology R produces a return R_j, with $X > R_j, S_j > 1$. We assume that region A has an advantage in technology S while region B has the same advantage in using technology R:

$$S_B = R_A < R_B = S_A$$

These regional advantages in the return from the two investment technology can be explained, for instance, by differences in the resources available in the two regions.

A liquidity risk for banks and thus a reason to invest in the storage technology emerges in our model because the timing of the cash-flow realized from investments in technology S and R is uncertain. With probability e sector R is hit by a shock and the investments in this technology cannot be realized before $t = 2$ while the returns from technology S are realized in $t = 1$. With the same probability a sectoral shock hits sector S and technology S produces late while technology R is early.[11] When liquidated before maturity the return of both technologies is $\epsilon \approx 0$.

In addition to sectoral shocks, with probability f a regional shock hits either region and both technologies in the respective region produce late, while only one technology is late in the other region. We assume that the probability for such a regional shock is close to zero. For simplicity we fix the probability that both technologies produce an early return at zero.[12] The joint probability distribution of

utility of impatient households is not decreasing.

[11]Note that our assumptions ensure for simplicity that banks can fully diversify sectoral liquidity shocks. With a portfolio that fully diversifies these shocks the cash-flows generated in $t = 1$ and in $t = 2$ are identical. To ensure that banks still have an incentive to hold liquidity we need to assume $q > 1/2$. Alternatively we could also assume that part of the returns on technology S and R is always late, i.e. only realized in $t = 2$. This would clearly not affect our results but make the notation messier.

[12]A positive probability of early returns in both sectors would not affect any of our results unless this probability is too large.

the cash flows $(C_1; C_2)$ in $t = 1$ and $t = 2$ of the two technologies in the two regions is summarized in the following table.

			Region A	
		$(R_A; S_A)$	$(S_A; R_A)$	$(0; S_A + R_A)$
	$(R_B; S_B)$	e	0	f
Region B	$(S_B; R_B)$	0	e	f
	$(0; S_B + R_B)$	f	f	0

Obviously,

$$2e + 4f = 1.$$

Households cannot invest directly in those technologies. They can only invest their funds with their local bank. Banks can only raise funds from households in their respective region and they can only invest in the two technologies in their home region. Cross-border retail business and cross-border lending is not feasible. We also assume that there is only one bank operating in each region. But we assume that the regional banking market is a contestable market. Thus banks are forced to offer to households the deposit contract that maximizes their $t = 0$-expected utility.[13] A deposit contract promises a repayment d_1 to all depositors that withdraw in $t = 1$. The banks' cash-flow in $t = 1$ is not verifiable and thus not contractible. Deposit contracts with a repayment d_1 contingent on the cash-flow realized in $t = 1$ are not feasible. However, depositors can observe the banks' $t = 1$ cash-flow. Consequently, if the remaining assets after repaying d_1 to impatient depositors are more than sufficient to repay the patient depositors $d_2 = d_1$ in $t = 2$ then the bank's remaining funds are distributed to the patient depositors in $t = 2$. If the bank's assets are insufficient to repay the impatient depositors d_1 and patient ones $d_2 \geq d_1$ in $t = 2$, late depositors run to be first in line to withdraw in $t = 1$.[14] We assume that patient

[13]Following the reasoning of Diamond and Dybvig (1983) it is easy to see that also in our environment a deposit contract is an optimal contractual arrangement insuring households against the risk of being impatient given that these shocks are unobservable.

[14]Here we simply assume that banks can only use deposit contracts that do not allow for a

and impatient depositors have the same chance of getting a certain position in the line.

In sum, the timing of the model is as follows:

t=0: Banks offer $\{d_1; d_2\}$.
Households invest in deposits at local bank.
Banks invest in liquidity and in technologies S *and* R.

t=1: Households receive private investment opportunity.
Households observe liquidity available to their bank.
If a bank has sufficient funds to pay its impatient HH d_1 *and patient HH* d_2,
then only impatient depositors withdraw and bank repays
If a bank has insufficient funds to repay d_1 *and* d_2,
then depositors run and bank is liquidated and
liquidation proceeds repaid on first come first served basis.

t=2: Cash flow from late projects realized.
Banks pay d_2 *on not yet withdrawn deposits from households.*

3 Optimal allocation with separate banks

In this section we study the optimal allocation given that banks do not dispose of any means to share risks across regions.

3.1 Diversified banks

First, we analyze the optimal investment portfolio and deposit contract of a bank that runs the risk of becoming illiquid if its is hit by a regional shocks, but that plans to honor the deposit contract in any other case. Without loss of generality we focus on a bank operating in region A.

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suspension of convertibility. However, it is straightforward that a commitment problem of the bank manager à la Diamond and Rajan (2001) could be easily integrated in this setting and would endogenously derive a deposit contract including a sequential service constraint without a suspension of convertibility as the optimal contractual arrangement.

Define l_0 as the fraction invested in $t = 0$ in liquidity holdings, $k = 1 - l_0$ as the fraction invested into the two production technologies, and x_A the fraction of k invested in the inferior production technology R.

Unless it is hit by a regional shock bank A can realize from each unit k of capital investment a minimum t_1-cash-flow given by

$$\Phi_1 = Min\left[R_A x_A; S_A\left(1 - x_A\right)\right]. \tag{1}$$

Given that bank A disregards the risks of a regional shock, the expression $\Phi_1 k_A$ gives the liquidity inflow from investments in the production technologies that the bank can rely on in $t = 1$ when deciding about the optimal short-term repayment on the deposit contract. Any additional liquidity inflow is only available in certain favorable states. It is not available with certainty to refinance short-term repayments. Thus if the bank wants to avoid ending up in a liquidity crisis due to sectoral shocks it will not rely on those additional funds for the anticipated short-term withdrawals. Instead it will store this extra liquidity for additional long-term repayments of deposits. Thus returns from production technologies available to refinance d_2 are given by $\Phi_2 k_A$ with

$$\Phi_2 = Max\left[R_A x_A; S_A\left(1 - x_A\right)\right]. \tag{2}$$

Consequently, a safe optimal deposit contract that an autarkic bank can always meet except if it is hit by a regional liquidity shock solves $(P1)$.

$$(P1)\begin{cases} \max\limits_{d_1;d_2;l_0} & 2f\left(qX + (1-q)\right)l_0 + (2e+2f)\left(qXd_1 + (1-q)\,d_2\right) \\ \text{s.t.} & qd_1 = \Phi_1\left(1 - l_0\right) + l_0 \qquad (BC1) \\ & (1-q)\,d_2 = \Phi_2\left(1 - l_0\right) \qquad (BC2) \\ & d_1 \leq d_2 \qquad (IC) \end{cases}$$

The bank maximizes depositors' expected utility whereby it runs the risk that with probability $2f$ it will be hit by a regional shock. In that case the bank anticipates to have insufficient cash in $t = 1$ to repay d_1 to impatient depositors. Thus it expects to be liquidated in which case it will be only able to repay on average the per capita liquidity holding l_0 to its depositors. Since in a run patient and impatient households have the same chance of receiving a repayment on their deposits the expected utility from receiving a unit repaid in that state is given by $qX - (1-q)$. In those states in which there is only a sectoral shock (happening with probability $2e$) or in which the other region is hit be a regional shock (probability $2f$) the bank plans to repay the promised amount d_1 to impatient and d_2 to patient depositors. Impatient depositors can use the proceeds received in $t = 1$ to apply their private technology generating a return $X > 1$ in $t = 2$ on each unit invested, while patient depositors consume the repayment d_2 in $t = 2$.

The budget constraint $(BC1)$ ensures that the funds supposed to be repaid to impatient depositors do not exceed the liquidity holding plus the t_1-cash-flow from capital investment that is realized given no regional shock in region A. $(BC2)$ provides that the cash-flow available in $t = 2$ from late investment projects is sufficient to repay patient depositors. The incentive compatibility constraint (IC) ensures that patient depositors do not have an incentive to withdraw early and consume the proceeds immediately.

Since $X > 1$ the bank maximizes depositors' expected utility by increasing as much as possible the short-term repayment on deposits. Thus for the optimal deposit contract (IC) holds with equality. Taking that into account it follows from $(BC1)$ and $(BC2)$ that

$$(1-q)\,\Phi_1\,(1-l_0) + (1-q)\,l_0 = q\Phi_2\,(1-l_0)\,.$$

Consequently, the optimal liquidity holding is

$$l_0^D = \frac{q\Phi_2 - (1-q)\,\Phi_1}{q\Phi_2 - (1-q)\,\Phi_1 + (1-q)}.$$

Reinserting in $(BC1)$ and $(BC2)$ yields

$$d_D = d_1 = d_2 = \frac{\Phi_2}{q\Phi_2 - (1-q)\,\Phi_1 + (1-q)}. \tag{3}$$

From (1), (2), and (3) it is easy to see that for $x_A > S_A/\left(R_A + S_A\right)$

$$d_D = \frac{R_A x_A}{qR_A x_A - (1-q) S_A (1-x_A) + (1-q)}$$

and

$$\frac{\partial d_D}{\partial x_A} = \frac{-(1-q)(S_A - 1) R_A}{(qR_A x_A - (1-q) S_A (1-x_A) + (1-q))^2} < 0.$$

It is also easy to see from (1), (2), and (3) that for $x_A < S_A/(R_A + S_A)$

$$d_D = \frac{S_A (1-x_A)}{qS_A (1-x_A) - (1-q) R_A x_A + (1-q)}.$$

and

$$\frac{\partial d_D}{\partial x_A} = \frac{(R_A - 1) S_A (1-q)}{qS_A (1-x_A) - (1-q) R_A x_A + (1-q)} > 0.$$

So obviously d_D is maximized for $\hat{x}_A = S_A/(R_A + S_A)$. For $x_A = \hat{x}_A$ the bank fully diversifies sectoral liquidity shocks and receives the same deterministic cash flow Φ in $t = 1$ and $t = 2$ given no regional shocks in region A:

$$\Phi = \Phi_1 (\hat{x}_A) = \Phi_2 (\hat{x}_A) = \frac{R_A S_A}{R_A + S_A}.$$

Thus investing in the portfolio $(l_0^*; \hat{x}_A)$ with

$$l_0^* = \frac{(2q-1)}{(2q-1) + (1-q)\Phi^{-1}}$$

the bank can offer an optimal deposit contract

$$d_D^* = \frac{1}{(2q-1) + (1-q)\Phi^{-1}}$$

Since $\partial\Phi/\partial(S_A/R_A) < 0$, it is easy to see that increasing benefits from specialization, i.e. a higher ratio S_A/R_A lead to lower repayments of a diversified bank:

$$\frac{\partial d_D^*}{\partial \Phi}\frac{\partial \Phi}{\partial S/R} < 0$$

Note that $\hat{x}_A > 1/2$. Thus a portfolio with fully diversified sectoral cash flow shocks implies that bank A has to invest a larger fraction of its capital in the inferior technology R_A in order to maximize the minimum period 1 return. Obviously, the bigger the benefits from specialization, i.e. the bigger S_A/R_A, the smaller is this cash flow of a portfolio that fully diversifies sectoral shocks.

Lemma 1 *The optimal deposit contract of a bank that wants to avoid a liquidity shortage in all but those states in which it suffers from a regional shock is characterized by* $d_1 = d_2 = d_D^*$. *The repayments on this optimal deposit contract decline with increasing benefits from specialization.*

Given this maximum repayment that the bank can promise in $t = 1$ the expected utility of households in the respective regions is

$$EU^D = 2f\,(qX + (1-q))\,l_0^* + (2e + 2f)\,(qX + (1-q))\,d_D^* \tag{4}$$

It is easy to see that bank B will offer the same deposit contract and will hold the same amount of liquidity as bank A. The only difference is that bank B will invest more of its capital into technology S: $\hat{x}_B = 1 - \hat{x}_A$. Thus following this diversified strategy both banks are forced to invest the larger fraction of their capital into the technology in which they have a *dis*advantage.

3.2 Undiversified banks

Assume now that bank A follows a more risky strategy and offers a deposit contract that it can only honor if the regionally more productive technology S generates the cash-flow already in $t = 1$. This means that the bank anticipates to be liquidated not only if a regional shock hits region A but also if technology S is affected by a sectoral shock. Since the liquidation value is zero for both production technologies the portfolio decision x_A does not matter for bankruptcy returns. The portfolio decision only affects the repayment on deposits in those states in which technology S produces early returns. Since the bank can always shift resources between $t = 1$ and $t = 2$ using the storage technology it is obviously optimal for the bank to invest only in liquidity and technology S. Consequently, the optimal deposit contract here simply solves

$$
(P1')\begin{cases}
\max\limits_{d_1;d_2;l_0} & (e+3f)\left(qX+(1-q)\right)l_0+(e+f)\left(qXd_1+(1-q)\,d_2\right) & \\
\text{s.t.} & qd_1=S_A\left(1-l_1\right)\left(1-l_0\right)+l_0 & (BC1)\\
 & (1-q)\,d_2=S_Al_1\left(1-l_0\right) & (BC2)\\
 & d_1\leq d_2 & (IC)
\end{cases}
$$

The optimal deposit contract maximizes depositors expected utility given that it can only repay the liquidation value l_0 if sector S is hit by a sectoral shock (which happens with probability $e+f$) or region A is affected by a regional shock (which happens with probability $2f$). In the run that leads to the liquidation, patient and impatient depositors have the same chance of receiving their repayment. Thus the expected utility in this case is given by the weighted average of patient and impatient depositors. Only if sector S generates an early cash-flow and region A is not hit by a regional shock then the bank will provide the promised repayments d_1 and d_2 on deposits, whereby impatient depositors receiving d_1 have a marginal benefit of $X>1$ from repayments, while patient depositors who receive d_2 have a marginal utility of 1.

The budget constraint $(BC1)$ in $(P1')$ states that the repayments to impatient depositors must not exceed the liquidity holdings l_0 of bank A plus a fraction $1-l_1$ of the cash-flow generated from the investment in technology S. l_1 measures the fraction of the cash-flow from capital investment that is not needed to repay impatient depositors. It is stored in reserves for one period to refinance the payment to patient depositors. Thus $(BC2)$ requires that this stored cash-flow is sufficient for the required repayments to the patient depositors. (IC) again ensures that patient depositors do not withdraw in $t=1$.

The bank maximizes depositors utility in those states in which it remains solvent, by repaying as much as possible to impatient depositors. Thus (IC) will hold with equality and it follows from $(BC1)$ and $(BC2)$ that

$$(1-q)\,S_A\left(1-l_1\right)\left(1-l_0\right)+(1-q)\,l_0=qS_Al_1\left(1-l_0\right).$$

Thus the optimal risky deposit contract is determined by

$$l_1 = (1-q)\frac{S_A(1-l_0)+l_0}{S_A(1-l_0)}$$

and

$$d_U = S_A(1-l_0)+l_0.$$

This risky strategy provides depositors with an expected utility given by

$$EU^R(l_0) = (e+3f)(qX+(1-q))\,l_0 + (e+f)(qX+(1-q))(S_A-(S_A-1)\,l_0)\,. \tag{5}$$

Hence

$$\frac{\partial EU^R}{\partial l_0} = [(e+3f)-(e+f)(S_A-1)](qX+(1-q))\,.$$

Consequently, the optimal risky strategy of an autarkic bank involves $l_0 = 0$ if

$$2f-(e+f)(S_A-2)<0$$

$$\Leftrightarrow S_A > 2+\frac{2f}{(e+f)}. \tag{6}$$

Thus assuming that (6) holds[15] then the expected utility that can be achieved by the risky deposit contract $d_U^* = S_A$ is

$$EU^U = (e+f)(qX+(1-q))\,S_A. \tag{7}$$

3.3 Safe banks

Alternatively the bank could also offer a deposit contract that it could honor even if it is hit by a regional shock. Obviously, in order to follow that strategy the bank

[15]Note that if (6) does not hold, then the bank would prefer to invest only in liquidity $l_0 = 1$, which implies $d = 1$ and would make the bank redundant. The expected utility in that case is

$$EU^A = (2e+4f)(qX+(1-q))\,.$$

has to hold sufficient liquidity to repay early withdrawals even if both technologies provide a late return. But given that it holds sufficient liquidity there is no need for the bank to invest in a diversified portfolio. Thus following this strategy bank A will choose $x_A = 0$ and offer the deposit contract that solves

$$(P1'')\begin{cases} \max\limits_{d_1;d_2;l_0} & (2e+4f)\left(qXd_1+(1-q)\,d_2\right) & \\ \text{s.t.} & qd_1 = l_0 & (BC1) \\ & (1-q)\,d_2 = S_A\,(1-l_0) & (BC2) \\ & d_1 \leq d_2 & (IC) \end{cases}$$

A safe bank will always $((2e+4f)=1)$ repay d_1 and d_2 to its impatient and patient depositors, respectively, whereby again the impatient ones have a marginal utility of $X > 1$ from each unit repaid, while patient depositors have only a marginal utility of 1. To be able to always repay d_1 the bank has to hold liquidity against the early repayments, because only these funds are available with certainty in $t = 1$. Thus $(BC1)$ in $(P1'')$ ensures that the bank holds sufficient liquidity to refinance the repayment to impatient depositors. Since the short-term repayments are always met by the liquidity holdings the bank invests all the funds that are used to refinance the repayment to patient depositors in the most productive technology S. If this technology is late it does not matter since the bank needs the funds only in $t = 2$ to repay the patient depositors. If the technology generates an early cash-flow the bank will store the funds until $t = 2$. Obviously, any investment in technology R would only reduce the possible payment to patient depositors. (IC) again ensures that patient depositors keep their deposits until $t = 2$.

Taking again into account that (IC) will hold with equality it follows from $(BC1)$ and $(BC2)$ that

$$l_0^S = \frac{qS_A}{(1-q)+qS_A}$$

and

$$d_S^* = d_1 = d_2 = \frac{S_A}{(1-q)+qS_A}.$$

The expected utility from such a deposit contract is

$$EU^S = (2e + 4f)(qX + (1 - q)) d_S^*.$$

3.4 Optimal deposit contract

Now we turn to the question under which parameter setting the different strategies are optimal for the bank. First we focus on parameter settings in which banks choose a diversified portfolio and offer d_D^*. Thus we consider cases in which

$$EU^D > EU^U \tag{8}$$

and

$$EU^D > EU^S. \tag{9}$$

Condition (8) requires that

$$(2e + 4qf) d_D^* > (e + f) S_A$$

Reinserting the optimal deposit contract d_D^* we can derive for a given R_A an upper threshold $\bar{S}$ for the returns of the more productive technology S in region A:

$$\frac{e(1 + q) + (5q - 1) f}{(e + f)\left((2q - 1) + \frac{(1-q)}{R_A}\right)} =: \bar{S} > S_A. \tag{10}$$

Thus as long as the returns on technology S_A are lower than threshold $\bar{S}$ bank A prefers a diversified over a fully specialized portfolio. Intuitively, for a given R_A a higher return on technology S_A increases specialization benefits. In order to ensure that full specialization is not preferable over diversification the benefits from specialization must not be too large.

We also need to ensure that (10) is consistent with our initial assumption, i.e. $\bar{S} > S_A > R_A > 1$. Reinserting $\bar{S}$ in this inequality allows us to derive a threshold for R_A:[16]

[16]See appendix for details. Note that $\bar{R} > 1$ for $q \geq 1/2$.

$$\frac{2qe + (6q-2)\,f}{(2q-1)\,(e+f)} =: \bar{R} > R_A \tag{11}$$

Hence, if this condition holds, there are values $S_A > R_A > 1$ such that $EU^D > EU^U$.

Condition (9) holds if

$$(2e + 4qf)\,d_D^* > (2e + 4f)\,d_S^*.$$

Reinserting d_D^* and d_S^* allows us to derive for a given R_A a lower threshold $\underline{S}$ for the return of the more productive technology S in region A.

$$\frac{2f(1-q)^2}{[q\,(e+2qf) - (e+2f)\,(2q-1)] - (e+2f)\,\frac{(1-q)}{R_A}} := \underline{S} < S_A. \tag{12}$$

As long as the returns on technology S in region A are higher than the threshold a diversified bank is providing a higher expected utility to depositors than a safe bank. Intuitively, since safe banks can avoid liquidation in case of regional liquidity shocks the more likely regional liquidity shocks are (the higher f) the more preferable are safe banks and the larger is the threshold in (12). Furthermore, the larger the opportunity costs of holdings liquidity, i.e. the higher the returns on the two investment opportunities R and S, the less preferable is the safe strategy. However, because the few assets that a safe bank has are investments only in the more productive technology S, larger benefits from specialization, i.e. a lower R_A for a given S_A in (12), make safe banks more preferable compared to diversified banks.

The deposit contract offered by a diversified bank is the optimal contract under autarky if both conditions (12) and (10) are simultaneously satisfied. To see that this can be the case consider the two thresholds $\bar{S}$ and $\underline{S}$ at $q = 1/2$:

$$\bar{S}(q = 1/2) = \frac{e+3f}{e+f} R_A > 0$$

and

$$\underline{S}(q = 1/2) = \frac{fR_A}{(e+f)\,R_A - (e+2f)} < 0 \quad \forall R_A > \frac{(e+2f)}{(e+f)}.$$

Since we know that for $q \to 1/2$ the upper threshold $\bar{R}$ for R_A goes to infinity we can conclude that at $q = 1/2$, for all

$$\frac{(e+2f)}{(e+f)} < R_A \tag{13}$$

conditions (12), (10), and the condition that $S_A > R_A$, can be simultaneously satisfied. Moreover, for q sufficiently close to $1/2$ continuity of utilities in q guarantees that the conditions can also be satisfied.

Thus we can summarize the findings in the following proposition:

Proposition 2 *For q sufficiently close to $1/2$ conditions (12), (10), and $S_A > R_A$ can be simultaneously fulfilled and autarkic banks invest into a diversified portfolio of technologies S and R. They invest the larger fraction into the inferior technology.*

One can easily prove that there are parameters such that undiversified banks are better than safe or diversified banks. They are better than safe banks if

$$EU^U > EU^S \Leftrightarrow S_A > \hat{S} := \frac{2e+4f}{q(e+f)} - \frac{(1-q)}{q}. \tag{14}$$

Hence, if

$$S_A > \max\left\{\frac{2e+4f}{q(e+f)} - \frac{(1-q)}{q}, \frac{e(1+q)+(5q-1)f}{(e+f)\left((2q-1)+\frac{(1-q)}{R_A}\right)}\right\} \tag{15}$$

undiversified banks are optimal.

4 Optimal allocation with integrated banks

In this section we first derive the constraint efficient allocation and then show to what extent this constraint efficient allocation can be implemented by an interbank market.

4.1 The constrained efficient allocation

Consider the allocation that a social planner would implement given that he also cannot observe whether a specific household has a private investment opportunity or not. Thus we look for the efficient allocation under the constraint that it has to be incentive compatible for patient households not to claim to be impatient. However, the social planner can shift resources freely between regions. Thus he will obviously not invest in technologies R_A and S_B; he will only make use of the most productive technologies S_A and R_B, whereby $S_A = R_B$. Given that f is sufficiently low the social planner will only diversify sector-specific shocks. Thus the constraint efficient consumption allocation that a social planner will offer solves $(P2)$:

$$(P2)\begin{cases} \max\limits_{d_1;d_2;l_0} & 2f\left(qXl_0 + S_A\left(1-l_0\right)\right) + \left(2e+2f\right)\left(qXd_1 + \left(1-q\right)d_2\right) \\ \text{s.t.} & qd_1 = S_A\left(1-l_0\right)/2 + l_0 \qquad (BC1) \\ & \left(1-q\right)d_2 = S_A\left(1-l_0\right)/2 \qquad (BC2) \\ & d_1 \leq d_2 \qquad (IC) \end{cases}$$

Since it is optimal for the social planner to fully smooth sectoral cash-flow shocks, he invests half of the capital investments $1 - l_0$ in technology S_A and the other half in technology R_B. $(BC1)$ requires that in both regions the repayments to impatient depositors do not exceed the liquidity held by the planner per region plus half of the early cash-flow available in the economy. Given that sector S is early all cash-flow generated in the economy is produced by technology S in region A and half of these returns are transferred by the social planner to the other region. In contrast half of the late produced cash-flow from technology R in region B is transferred to region A to be paid to the patient household in this region. Given that $S_A = R_B$ this is reflected in $(BC2)$. In case of the opposite sectoral cash-flow shock the cross-regional transfers are simply reversed. Since we are assuming that also the social planner cannot observe households' idiosyncratic liquidity shocks (IC) has again to be taken into account.

The social planner maximizes the expected utility of households in both regions.

Thereby he has to take into account that he will only be able to repay the planned amounts d_1 and d_2 if the region in which the sector is located that is supposed to produce early returns is not hit by a regional shock. With probability $2e$ there is no regional shock and with probability $2f$ there is only a regional shock in the region with the production technology that is late anyway. Thus with probability $2e + 2f$ the planner can pay d_1 and d_2 to the impatient and patient households, respectively. With probability $2f$, however, the region A is hit by a regional shock when technology S was producing early or region B has a shock when technology R should be early. In these cases the social planner can only repay the liquidity holdings to the impatient households, while he can divide the entire return on capital investment $S_A(1-l_0)$ by the $1-q$ patient households.[17]

Since f is assumed to be sufficiently small the planner maximizes also the short-term repayment to impatient households d_1 because it generates the maximum expected marginal utility. Thus (IC) holds again with equality at the optimal deposit contract. As a consequence it follows from $(BC1)$ and $(BC2)$ that

$$qS_A(1-l_0) = (1-q)S_A(1-l_0) + (1-q)2l_0.$$

Thus the optimal liquidity holding per region is

$$l_0^I = \frac{(2q-1)S_A}{2(1-q)+(2q-1)S_A},$$

and the optimal payment to patient and impatient households is

$$d_I = d_1 = d_2 = \frac{S_A}{2(1-q)+(2q-1)S_A}.$$

It is easy to see that from $S_A = R_B$, $S_A > R_A$ and $S_B < R_B$ follows that $d_I > d_D^*$ and $l_0^I > l_0^D$. Consequently, the social planner improves households' welfare compared to autarkic banks. He does not only avoid inefficient liquidation but he also fully reaps the benefits of specialization.

[17]Thus we implicitly assume that the social planner is not forced to liquidate assets when he cannot meet the planned payment to impatient depositors. We rather assume that he suspends payments when liquidity is insufficient.

4.2 Implementation through an interbank market

Now assume that there is an interbank market open in $t = 1$. In this interbank market banks can trade liquidity against future cash-flow from some capital investment at an equilibrium interest rate. Since there is no investment alternative to the storage technology for excess liquidity in $t = 1$ (cash that is already available in $t = 1$ but is only needed in $t = 2$ to refinance the repayment to patient depositors) banks will offer any excess cash holdings in the interbank market at a riskless interest rate $i \geq 0$.

We assume that the initial liquidity holding (l_0) are publicly observable and verifiable and thus contractible in $t = 0$. This assumption can essentially be viewed as reflecting regulatory liquidity requirements.[18] However, the investment portfolio x_j, the deposit contract that banks offer their respective regional depositors and the realization of regional and sectoral liquidity shocks are not verifiable and thus not contractible.

Thus the interbank market is a Bayesian game with the following stages: In $t = 0$ 1) banks mutually sign a contract about their liquidity holdings, 2) individually design a deposit contract that they offer households in their region and 3) collect deposits and invest them in a portfolio of technology S and R in their region. Then in $t = 1$ liquidity shocks materialize and dependent on their private liquidity shock banks offer or demand liquidity in the interbank market against repayment in $t = 2$.

Taking the interbank market into account overall timing of our model is now given as follows:

[18] Note also that we take this assumption to abstract from the usual underinvestment in liquidity known from Bhattacharya and Gale (1987) and Bhattacharya and Fulghieri (1994). It is easy to see that if banks could not verify each other's initial liquidity holding also in this setting banks would underinvest in liquidity and free ride on the liquidity provision of their counterparty. As our focus is to show that contagion also occurs if the interbank market is most efficient we abstract from these market inefficiencies. Fecht and Grüner (2006) show that unsecured interbank deposits are a way to eliminate this underinvestment problem.

t=0: Banks commit to hold a certain fraction of their funds in liquidity.

Based on this commitment banks form expectations about $t = 1$ *excess liquidity.*

Banks offer $\{d_1; d_2\}$.

Households invest in deposits at local bank.

Banks invest in liquidity and in technologies S *and* R.

t=1: Households learn about private investment opportunity.

Regional and sectoral cash-flow shocks materialize.

Banks might borrow/lend IB *from/to other banks.*

Households observe liquidity available to their bank.

If bank has sufficient funds to pay impatient HH d_1 *and patient HH* d_2,

then only impatient depositors withdraw and bank repays.

If bank has insufficient funds to repay d_1 *and* d_2,

then depositors run and bank is liquidated and

liquidation proceeds repaid on first come first served basis.

t=2: Cash flow from late projects realized.

Banks repay/receive IB *to/from other banks.*

Banks pay d_2 *on deposits not yet withdrawn from households.*

It is easy to see that the cross-border risk-sharing together with a fully specialized portfolio, as derived in the previous subsection, can be achieved in an equilibrium of this Bayesian game. To prove this assume first that banks offer the optimal deposit contract d_I and agree to hold l_0^I liquid reserves. Furthermore assume that both banks are fully specialized in their respective most efficient technology. In that case if bank A (B) suffers from a liquidity shortage–either due to a sectoral shock to technology S (R) or a regional shock–it will always demand liquidity $IB_D = \left(1 - l_0^I\right) S_A/2$ ($IB_D = \left(1 - l_0^I\right) R_B/2$) in the $t = 1$ interbank market and can promise to repay this amount at $t = 2$. If bank A (B) has excess liquidity because the technology S (R) produces cash flow already in $t = 1$ it can exactly offer $IB_S = \left(1 - l_0^I\right) S_A/2$ ($IB_S = \left(1 - l_0^I\right) R_B/2$). And banks will be willing to offer their entire excess liquidity in the market as long as they receive the same amount back in $t = 2$ since their alternative would be to store the excess liquidity. Thus given that banks are fully specialized the interbank market is a self revealing mechanism and ensures that banks can sustain sectoral liquidity shocks.

The questions remains whether banks have an incentive to fully specialize or not. Assume that bank A is less than fully specialized and holds a fraction $\overline{x_A} > 0$ in technology R, while bank B is fully specialized. It is easy to see that bank A cannot repay a deposit contract d_I if technology S is hit by a liquidity shock because it can only borrow $IB_D = \left(1 - l_0^I\right)(1 - x_A)S_A - d_I/2$ in the interbank market. Together with the early cash flow from technology R $\left(1 - l_0^I\right) x_A R_A$ this is insufficient to repay d_I to the impatient depositors since $R_A < S_A$. Similarly, if technology S is early and R late, the cash flow available in $t = 2$ is lower than under full specialization and insufficient to repay the patient depositors d_I. Thus a bank that is less than fully specialized can only offer a lower deposit contract than d_I. Hence, given the described interbank market banks always have an incentive to self reveal their regional liquidity shocks (offer excess liquidity in the interbank market), will fully specialize and will offer the second best deposit contract.

Intuitively, with an interbank market the diversification of liquidity risks is decoupled from banks' investment decisions. Since bank A only invests in technology S and bank B only in technology R while sectoral cash-flow risks are diversified with the respective interbank payments, banks in this case also offer the same deposit contract as the social planner does. Since $d_I > d_D^*$ and $l_0^I > l_0^D$ both banks therefore also provide depositors with a higher expected repayment than autarkic diversified banks.

However, it is easy to see that banks following this strategy rely on the liquidity provision through the interbank market in case the technology that they are specialized in generates returns not before $t = 2$. If, for instance, bank A does not receive IB funds in the interbank market in $t = 1$ when technology S is delayed it has insufficient funds available to repay d_I to the impatient depositors. Since banks, in contrast to the social planner, cannot suspend convertibility, a run on bank A

is unavoidable and the bank is liquidated.[19] Consequently, with a specialization in lending banks expose themselves to a liquidity risk in the interbank market. This generates the risk of spill-overs of regional liquidity shocks and cross-border contagion. If region B is hit by a regional shock and all investments in that region repay late while also technology S is delayed in region A, bank A will collapse simply because it relies on a liquidity inflow from the interbank market due to its specialization.

These findings are summarized in the following proposition:

Proposition 3 *Financial integration through an interbank market enables banks to specialize ($x_A = 0$; $x_B = 1$) without being destabilized by sectoral shocks. However, specialization brings about the risk of contagion.*

In the instances of financial contagion that occurs with probability $2f$ banks have insufficient liquidity in $t = 1$ to repay d_1 to impatient depositors. Both banks will be liquidated and can only repay on average the per capital liquidity holding l_0. Thus depositors' expected utility under integration and specialization is given by

$$EU^I = 2f\left(qX + (1-q)\right) l_0^I + (2e + 2f)\left(qX + (1-q)\right) d_I. \tag{16}$$

Note that because banks cannot suspend convertibility an inefficient liquidation of both banks is unavoidable in case of an aggregate liquidity shortage. Thus an integrated interbank market cannot implement the constraint efficient allocation that a social planner would achieve. Consequently, the utility that an integrated interbank market and fully specialized banks can provide is lower than the welfare that a social planner achieves because consumption is lower in case of aggregate liquidity shortages.

[19] For a detailed explanation of this assumption see footnote 14.

5 Stability and welfare implications of integration

Both the stability and welfare implications of financial integration through an interbank market depend on the optimal bank behavior under autarky. We now analyze all three cases from the previous section.

5.1 Autarkic banks with an undiversified portfolio

Consider first the case in which autarkic banks hold an undiversified portfolio and specialize in the most productive technology in their respective region at the risk of being illiquid if this technology generates a return only with a delay. In this case a bank fails with probability $e+3f$ while the probability for each bank to fail under an integrated interbank market is only $2f$. Thus financial stability is strictly increasing through financial integration in this case, while the probability of a joint banking failure in both regions remains unchanged. The welfare implications of financial integration are ambiguous in this case. The optimal deposit contract of bank A is given by $d_U^* = S_A$ and expected utility of depositors is given by EU^U. Banks integrate through an interbank market if

$$EU^I > EU^U \tag{17}$$

$$\Leftrightarrow \frac{(6q-2)f - 2qe}{(e+f)(2q-1)} := \bar{\bar{S}} > S_A. \tag{18}$$

Intuitively, while an undiversified bank under autarky is more likely to fail and thus implies higher liquidation costs, such a bank does not need to hold any liquidity to repay some of the impatient depositors. It will only honor the deposit contract in the state in which all returns from technology S are realized in $t = 1$. Thus, compared to integrated and specialized banks an autarkic undiversified bank saves on the opportunity costs of holding liquidity. These costs decline as the fraction of impatient households approaches 1/2. Consequently, for a q sufficiently close to 1/2 (18) always holds. The following proposition summarizes our findings on the case of autarkic banks with an undiversified portfolio.

Proposition 4 *Banks choose to specialize under autarky and prefer integration to autarky with specialization if* $S_A > \bar{S}$, $S_A > \hat{S}$, *and* $S_A < \bar{\bar{S}}$ *simultaneously hold. In this case financial integration reduces the probability of an individual bank failure from* $e + 3f$ *to* $2f$, *while the probability of a joint failure of banks in both regions remains unchanged at* $2f$.

Note that all three conditions and the condition that $S_A > R_A > 1$ can be made compatible by an appropriate choice of parameters if q is sufficiently close to $1/2$. The reason is that the two lower bounds for S_A in (15) are finite for $q = 1/2$.

5.2 Autarkic banks with a safe portfolio

Next, consider the case in which autarkic banks optimally follow a safe strategy and hold sufficient liquidity to serve the deposit contract even in case of a delay of both technologies in their respective region due to a regional shock. Banks would in this case prefer to use an interbank market to reallocate liquidity and share liquidity risks if:

$$EU^I > EU^S \tag{19}$$

$$\Leftrightarrow \frac{(1+q)\,e + 2\,(2-q+q^2)\,f}{(1-q)\,e + 2\,(1-2q+q^2)\,f} := \underline{\underline{S}} < S_A. \tag{20}$$

However, one can show that condition (20) cannot be satisfied simultaneously with $S_A < \hat{S}$, which would ensure that $EU^S > EU^R$, and $S_A < \underline{S}$, which is required to have $EU^S > EU^D$ (see appendix). In other words, whenever banks find it optimal under autarky to hold sufficient liquidity buffers to sustain even regional shocks, they will not integrate over an interbank market to share liquidity risks. Intuitively, if opportunity costs of holding sufficient liquidity buffers are low enough and the expected costs from liquiditation due to a regional shock are high enough to keep banks from entering liquidity risks under autarky they will also refrain from assuming liquidity risks when an integrated interbank market becomes available. Access to a cross regional interbank market will not induce banks to reduce their liquidity buffers, insure liquidity risks in the interbank market and become exposed to liquidity shocks and contagion. Thus we have the following proposition:

Proposition 5 *Banks that hold sufficient liquidity buffers to sustain any liquidity shock under autarky will never use a cross-border interbank market to reallocate liquidity, because $S_A < \hat{S}$, $S_A > \underline{\underline{S}}$, and $S_A < \underline{S}$ never simultaneously hold.*

Proof. See appendix. ■

5.3 Autarkic banks with a diversified portfolio

Finally, consider the case in which banks hold a portfolio that diversifies only sectoral liquidity shocks under autarky. Given that financial integration and specialization brings about the risk of contagion it depends on the expected costs of contagion relative to the gains from specialization whether banks prefer an integrated interbank market or not. Banks and depositors benefit from integration if

$$EU^I > EU^D,$$

which can be rewritten as

$$2fl_0^I + (2e + 2f)\, d_I > 2fl_0^D + (2e + 2f)\, d_D^*.$$

From reinserting $\left(d_D^*, l_0^D\right)$ and $\left(d_I; l_0^I\right)$ it is obvious that this always holds since

$$e + f > -(2q - 1)\, f.$$

As the probability f of regional shocks is the same in both regions in our set-up the expected welfare losses due to contagion are always overcompensated. With an integrated interbank market each banks' exposure to its own regional shock is reduced by f: Bank A, for instance, will be able to sustain a regional shock in region A as long as technology R produces early in region B. Thus an integrated interbank market enables banks to sustain some (but not all) liquidity shocks in their home region, which a diversified autarkic bank could not sustain. Consequently, while contagion increases the probability of a joint banking crisis in both regions by f this is completely offset by a reduction in the exposure to regional shocks in the home region. Therefore, the probability of default of a bank is unaffected by financial integration, while the expected repayments on deposits strictly increase.

Proposition 6 *Banks which optimally invest in diversified loan portfolios under autarky ($\bar{S} > S_A$ and $\underline{S} < S_A$) always prefer to pool liquidity risk in an integrated interbank market and specialize in their lending portfolios. Since regional shocks occur with the same probability f the default probability of each bank remains unchanged, while financial contagion increases the probability of joint banking crisis in both regions from* 0 *to* $2f$*.*

In sum, the analysis in this section shows that availability of a cross-border interbank market may reduce the default probability of one individual bank while it may increase the probability of contagion and joint banking crises in both region. If an integrated interbank market allows banks that already run a fully specialized portfolio to pool their liquidity risks, this strictly increases financial resilience. However, the probability of a joint failure of banks in both regions remains unchanged. For banks that hold a diversified asset portfolio under autarky an integrated interbank market along with portfolio specialization does not affect individual banks' default probability. In this case the increase in systemic risk is exactly offset by reduced exposure to domestic shocks. Banks that hold sufficient liquidity to overcome any shock will not use a cross-border interbank market to reallocate liquidity. Thus the availability of a cross-border interbank market has no stability implications in that case.

Welfare implications can summarized as follows. Since banks will maximize depositors' expected utility by not using an integrated interbank market to share liquidity risks if it becomes available, financial integration does not increase welfare. Banks that do not hold liquidity buffers but are fully specialized under autarky are enabled to share liquidity risks across borders which may enhance welfare. For banks that only reap the benefits from specialization if they can diversify liquidity shocks in an integrated interbank market the availability of a cross-regional interbank market clearly increases welfare. Since banks that hold sufficient liquidity to overcome any shock will note share risks in an integrated interbank market, the availability of such a market has no welfare implication in this case.

6 Conclusion

When assessing the benefits from financial integration it has to be taken into account that the greater scope for diversification through financial integration may foster specialization which in turn increases the need for diversification. Thus, sticking to the status quo of cross-country correlations of banks' liquidity does not allow to assess the costs and benefits from financial integration. It underestimates the benefits but it also undervalues the risk of financial contagion. This may have important empirical implications. Approaches like Imbs and Mauro (2007) and Fecht, Grüner, and Hartmann (2007) that try to assess the benefits from financial integration based on the given cross-country correlation of shocks could therefore lead to biased conclusions in this regard. Empirical estimates of the benefits of financial globalization should take the endogenous impact on the correlation structure into account.

Regarding the financial stability implications of financial integration our analysis shows that integration weakly reduces the probability of individual banking crises, while at the same time it may increase the risk of contagion and thus the probability of widespread banking failures. In particular if the improved ability to pool liquidity risks in an integrated interbank market induces banks to specialize in their lending they become more dependent on interbank market liquidity provision and systemic risk strictly rises.

In terms of policy implications, the greater contagion risk still puts pressure on policy makers to adjust supervisory approaches and structures to the geographical scope of banking activities.[20] While supervisory structures should develop to take greater account of cross-border risks our analysis also suggests that financial integration should not be resisted on stability grounds, at least not in industrial countries with relatively well-functioning interbank markets and more limited contract enforcement problems. In fact, greater specialization in lending to the most profitable sectors through better bank risk sharing enhances overall welfare even though the risk of cross border financial contagion rises.

Obviously, our analysis does not take other negative externalities into account that can be associated with banking crises. It is easy to see that in our model particularly externalities that grow with the breadth of banking crises could bring about situations in which banks decide to integrate through an interbank market and specialize in their lending portfolio while this is not welfare enhancing. However, including this into our formal analysis we leave for further research.

Our analysis could in principle also be applied to a single country with different regions and different absolute advantages in production. However, there are reasons why we believe that our model is better suited to address the role of cross border interbank markets. First, in our model the only financial link between regions is the interbank market. In particular, there is no bank lending from one region to another. In practice, informational problems are likely to be more pronounced in cross border than in interregional retail business. Moreover, due to a unified regulatory framework, it is much easier for banks to directly invest in another region of the same country than to invest internationally. As a consequence even in a multi-country region with a common currency and therefore as highly integrated as the euro area, cross-border retail lending and deposit taking by banks is only a small fraction of cross-border interbank lending and borrowing (see e.g. Baele et al., 2004, and ECB, 2011). Hence, the role that an interbank market plays in permitting efficient specialization should be much smaller at the national level than in the case

[20] See DellArricia and Marquez (2006) for a theoretical analysis of the relationship between financial integration and supervisory structures.

of international financial integration. Second, at the national level, a central bank can easily cope with banking crises that are driven by pure liquidity shocks like those emphasized in our model. The central bank can easily provide additional liquidity to the banking sector to overcome those shocks. In an international context, any such intervention requires some explicit or implicit coordination among central banks. Therefore, the trade-off between efficient specialization and contagion should be more important in an international context. In the national context, even without an integrated interbank market, specialization may obtain when regions can rely on central bank intervention. Similarly, contagion effects that arise due to integration would not be as important because the central bank can easily intervene in case of an aggregate liquidity shortage. Third, given the closer similarities of regions within a country, the benefits from integration and specialization should be smaller in the case of interregional integration.

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Appendix

Deriving $\bar{S}$: A diversified bank is preferable over an undiversified banks if

$$(2e+4qf)\,d_D^{*} > (e+f)\,S_A$$

inserting the deposit contract d_D^{*} this can be simplified to

$$\frac{2e+4qf}{e+f} > (2q-1)\,S_A + (1-q)\left(1+\frac{S_A}{R_A}\right)$$

$$\Leftrightarrow \frac{2e+4qf}{e+f} - (1-q) > (2q-1)\,S_A + (1-q)\left(\frac{S_A}{R_A}\right)$$

$$\Leftrightarrow \frac{e\,(1+q)+(5q-1)\,f}{(e+f)\left((2q-1)+\frac{(1-q)}{R_A}\right)} = \bar{S} > S_A.$$

Proof of the existence of an R_A such that $\bar{S} > R_A > 1$:

$$\frac{e\,(1+q)+(5q-1)\,f}{(e+f)\left((2q-1)+\frac{(1-q)}{R_A}\right)} > R_A$$

$$\Leftrightarrow e\,(1+q)+(5q-1)\,f > (e+f)\,((2q-1)\,R_A+1-q)$$

$$\Leftrightarrow 2qe+(6q-2)\,f > (e+f)\,(2q-1)\,R_A$$

$$\Leftrightarrow \frac{2qe+(6q-2)\,f}{(2q-1)\,(e+f)} =: \bar{R} > R_A.$$

The upper bound on $\bar{R}$ exceeds 1 if

$$2qe+(6q-2)\,f > (2q-1)\,(e+f)$$

$$\Leftrightarrow (4q-1)\,f+e > 0$$

$$\Leftrightarrow 4qf+e > f.$$

Deriving $\underline{S}$: A diversified bank is preferable over a safe bank if

$$(2e + 4qf)\, d_D^* > (2e + 4f)\, d_S^*.$$

Reinserting d_D^* and d_S^* yields

$$\frac{(1-q)S_A^{-1} + q}{(2q-1) + (1-q)\Phi^{-1}} > \frac{e+2f}{e+2qf}$$

$$\Leftrightarrow \frac{(1-q)}{S_A} + q > \frac{e+2f}{e+2qf}\left[(2q-1) + (1-q)\left(\frac{R_A + S_A}{R_A S_A}\right)\right]$$

$$\Leftrightarrow (1-q)\,(e+2qf) - (e+2f)\,(1-q)\left(\frac{R_A+S_A}{R_A}\right) > \left[(e+2f)\,(2q-1) - q\,(e+2qf)\right] S_A$$

$$\Leftrightarrow (1-q)\,(e+2qf) - (e+2f)\,(1-q)\left(1 + \frac{S_A}{R_A}\right) > \left[(e+2f)\,(2q-1) - q\,(e+2qf)\right] S_A$$

$$\Leftrightarrow -2f(1-q)^2 - (e+2f)\,(1-q)\frac{S_A}{R_A} > \left[(e+2f)\,(2q-1) - q\,(e+2qf)\right] S_A$$

$$\Leftrightarrow 2f(1-q)^2 + (e+2f)\,(1-q)\frac{S_A}{R_A} < \left[q\,(e+2qf) - (e+2f)\,(2q-1)\right] S_A$$

$$\Leftrightarrow \frac{2f(1-q)^2}{\left[q\,(e+2qf) - (e+2f)\,(2q-1)\right] - (e+2f)\,\frac{(1-q)}{R_A}} := \underline{S} < S_A.$$

Deriving $\underline{\underline{S}}$:

$$EU^I > EU^S$$

$$(2e + 4fq)\, d_I > (2e + 4f)\, d_S.$$

$$\Leftrightarrow (2e+4fq)\,\frac{S_A}{2\,(1-q) + (2q-1)\,S_A} > (2e+4f)\,\frac{S_A}{(1-q) + qS_A}.$$

$$\Leftrightarrow (e+2fq)\left[(1-q) + qS_A\right] > (e+2f)\left[2\,(1-q) + (2q-1)\,S_A\right].$$

$$\Leftrightarrow (e+2fq)\,(1-q) + (e+2fq)\,qS_A > 2\,(e+2f)\,(1-q) + (e+2f)\,(2q-1)\,S_A.$$

$$\Leftrightarrow \left[(e+2fq)\,q-(e+2f)\,(2q-1)\right]S_A > \left[2\,(e+2f)-(e+2fq)\right](1-q)$$

$$\Leftrightarrow \frac{2\,(e+2f)-(e+2fq)\,(1-q)}{(e+2fq)\,q-(e+2f)\,(2q-1)} := \underline{\underline{S}} < S_A$$

$$\Leftrightarrow \frac{(1+q)\,e+2\,(2-q+q^2)\,f}{(1-q)\,e+2\,(1-2q+q^2)\,f} := \underline{\underline{S}} < S_A$$

Proof of proposition 5: In order to prove that $S_A < \hat{S}$, $S_A > \underline{\underline{S}}$, and $S_A < \underline{S}$ cannot simultaniously hold, t is sufficient to prove that $S_A < \hat{S}$ and $S_A > \underline{\underline{S}}$ cannot simultaneously hold. We have

$$\frac{2e+4f}{q\,(e+f)} - \frac{(1-q)}{q} = \hat{S} > S_A$$
$$\frac{(1+q)\,e+2\,(2-q+q^2)\,f}{(1-q)\,e+2\,(1-2q+q^2)\,f} = \underline{\underline{S}} < S_A.$$

Note that for $q = 1/2$ we have $\hat{S} = \underline{\underline{S}}$:

$$\hat{S}\,(q=1/2) = \frac{2e+4f-\frac{(e+f)}{2}}{\frac{(e+f)}{2}} = \frac{3e-7f}{e+f}$$
$$\underline{\underline{S}}\,(q=1/2) = \frac{\frac{3}{2}e+2\left(\frac{3}{2}+\frac{1}{4}\right)f}{\frac{1}{2}e+2\frac{1}{4}f} = \frac{3e-7f}{e+f}$$

Thus in order to prove $S_A < \hat{S}$ and $S_A > \underline{\underline{S}}$ cannot simultaneously hold it is sufficient to show that

$$\frac{\partial \underline{\underline{S}}}{\partial q} > \frac{\partial \hat{S}}{\partial q}$$

To see this note that

$$
\begin{aligned}
\frac{\partial \hat{S}}{\partial q} &= \frac{d\left(\frac{e(1+q)+f(3+q)}{q(e+f)}\right)}{dq} \\
&= \frac{(e+f)^2 q - (e+f)\left(e(1+q)+f(3+q)\right)}{\left[q(e+f)\right]^2} \\
&= \frac{(e+f)^2 q - (e+f)(eq+e+3f+fq)}{\left[q(e+f)\right]^2} \\
&= -\frac{(e+3f)}{q^2(e+f)} < 0.
\end{aligned}
$$

Furthermore, note that

$$
\begin{aligned}
\frac{\partial \underline{\underline{S}}}{\partial q} &= \frac{d\left(\frac{(1+q)e+2\left(2-q+q^2\right)f}{(1-q)e+2(1-2q+q^2)f}\right)}{dq} \\
&= \frac{(e-2(1-q)f)\left[(1-q)e+2(1-q)^2 f\right] + (e+2(2-q)f)\left[(1+q)e+2\left(2-q+q^2\right)f\right]}{\left[(1-q)e+2\left(1-2q+q^2\right)f\right]^2} > 0
\end{aligned}
$$

always holds because

$$
(e-2(1-q)f)\left[(1-q)e+2(1-q)^2 f\right]+(e+2(2-q)f)\left[(1+q)e+2\left(2-q+q^2\right)f\right] > 0
$$

$$
\Leftrightarrow 2e^2+2\left(2-q+q^2\right)fe+2(2-q)(1+q)ef+\left[4(2-q)\left(2-q+q^2\right)-4(1-q)^3\right]f^2 > 0
$$

$$
\Leftrightarrow (2-q)\left(2-q+q^2\right)-(2-q)\left(1-2q+q^2\right)+\left(1-2q+q^2\right) > 0
$$

$$
\Leftrightarrow (2-q)\left(2-q+q^2-1+2q-q^2\right)+\left(1-2q+q^2\right) > 0.
$$