DISCUSSION PAPER SERIES

No. 8849

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INDUSTRIAL ORGANIZATION

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ISSN 0265-8003

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Discussion Paper No. 8849
February 2012

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CEPR Discussion Paper No. 8849

February 2012

ABSTRACT

Dynamic Price Competition with Switching Costs*

We develop a continuous-time dynamic model with switching costs. In a relatively simple Markov Perfect equilibrium, the dominant firm concedes market share by charging higher prices than the smaller firm. In the short-run, switching costs might have two types of anti-competitive effects: first, higher switching costs imply a slower transition to a symmetric market structure and a slower rate of decline for average prices; and second, if firms are sufficiently asymmetric, an increase in switching costs also leads to higher current prices. However, as market structure becomes more symmetric, price competition turns fiercer and in the long-run, switching costs have a pro-competitive effect. From a policy perspective, we conclude that switching costs should only raise concerns in concentrated markets.

JEL Classification: C61, L13 and L41
Keywords: continuous-time model, firms' asymmetries, Markov-perfect equilibrium and switching costs

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* We thank Juan Pablo Rincón (Universidad Carlos III de Madrid) for many helpful suggestions. Natalia Fabra acknowledges financial support awarded by the Spanish Ministry of Science and Innovation, Project 2010/02481/001.

Submitted 8 Feburary 2012

1 Introduction

Many products and technologies exhibit switching costs (i.e., costs that customers must bear when they adopt a new product or technology). For example, switching costs arise when there is limited compatibility between an old product (or technology) and a newly adopted one. In this case, the specific investments that a customer may have incurred in relation to the utilization of the old product (or technology) may fully or partially depreciate. When compatibility is highly valued by customers, they are discouraged from changing products. Customers face a potential lock-in effect which confers market power to the firm. As the history of past choices affects future product choice or technology adoption decisions, market share is a valuable asset. The incentives to exploit current customers (by charging high prices) and to increase market share (by offering low prices) are countervailing. A priori, it seems that the net effect of switching costs on the nature of dynamic price competition is unclear. Nonetheless, the conventional wisdom distilled from the literature seems to suggest that switching costs are anti-competitive (see Klemperer (1995) and Farrell and Klemperer (2007) for a survey of this literature). In this paper, we show that when switching costs are not too high so that switching takes place in equilibrium, this conventional wisdom is partially incorrect- at least, in steady state or when firms are sufficiently symmetric.

We develop a continuous-time dynamic equilibrium model with switching costs and show that, in a relatively simple Markov Perfect equilibrium, the dominant firm concedes market share by charging higher prices to current customers. In the short-run, switching costs might have two types of anti-competitive effects: first, higher switching costs imply a slower transition to a symmetric market structure and a slower rate of decline for average prices; and second, if firms are sufficiently asymmetric, an increase in switching costs also leads to higher prices. However, as market structure becomes more symmetric, price competition turns fiercer and in the long-run, switching costs have a pro-competitive effect. However, if there are obstacles that stop firms from becoming sufficiently symmetric, the anti-competitive effects of switching costs might prevail.

This paper is organized as follows. In Section 2 we review the relevant literature on the subject and relate it to our results. In Sections 3 and 4 we introduce and analyze a dynamic pricing model with switching costs in order to characterize the evolution of market structure and prices. In Section 5, we extend the basic model by allowing for asymmetric switching costs across firms. In Section 6 we discuss the limitations of the model, and we conclude in Section 7.

2 Switching costs: pro-competitive or anti-competitive?

Klemperer (1987a) and (1987b) are the first published works that aimed to analyze the impact of switching costs on the nature of price competition. In these papers, the author developed a two-period model in which, in the first period, consumers choose a product (or technology) for the first time. In the second period, consumers may choose a different product in which case they face switching costs. In equilibrium, prices follow a pattern of “bargains” followed by “rip-offs”. To circumvent the potential “end of horizon” effect in these two-period models, infinite-horizon models in which firms face overlapping generations of consumers have been analyzed (Farrell and Shapiro (1988), Padilla (1995) and To (1995)).

In all of these papers, firms face a trade-off between maximizing current versus future profits. Maximizing current profits calls firms to exploit their loyal consumers (“harvesting” effect), whereas maximizing future profits calls firms to decrease current prices in order to attract new customers (“investing” effect). The previous papers concluded that the former effect dominates, so that switching costs have anti-competitive effects.

However, these analyses omitted an equally important effect: the fact that switching costs affect current market competition even in a static setting. This effect was hidden in these models by the lack of switching in equilibrium (either by assumption, or because switching costs were assumed very large).

Instead, if switching takes place, firms want to attract new consumers, not just to exploit them in the future, but also as a source of current profits. This effect, which Arie and Grieco (2011) refer to as the “compensating” effect, mitigates the “harvesting” effect, and fosters more competitive outcomes.

To understand firms’ pricing incentives, it is useful to interpret switching costs as a firm-subsidy when consumers are loyal to the firm (i.e., the firm can afford raising its price by the value of the switching cost without loosing consumers). Similarly, switching costs are analogous to a firm-tax when consumers are switching to the firm’s product (i.e., the firm has to reduce its price by the amount of the switching cost in order to attract new consumers). Hence, for a large firm (which has more loyal consumers than consumers willing to switch into its product), the net effect of switching costs is that of a subsidy, whereas for a small firm the net effect of switching costs is that of a tax. Therefore, if we just focus on the static effects, an increase in switching costs implies that the large firm becomes less aggressive while the small firm becomes more so. In the short-run, this translates into an increase in prices.

However, in a dynamic setting, the investing effect induces firms to reduce current prices, thus suggesting that the net effect of switching costs on equilibrium prices might be ambiguous. In this paper we show that the issue of which of the three effects –whether harvesting, compensating or investing – dominates critically depends on the degree of market share asymmetries. If firms are symmetric, the compensating and harvesting effects cancel out. As the investing effect is the only effect that remains, an increase in switching costs reduces prices. In contrast, if firms are very asymmetric, the compensating effect barely mitigates the harvesting effect, so that an increase in switching costs leads to higher prices.

It turns out that, in steady state, firms become symmetric precisely because in previous periods the dominant firm priced less aggressively than the smaller one. This implies that long-run equilibrium prices decrease with switching costs (as long as this value is small enough so as to allow for switching in equilibrium).

There is recent a string of papers showing that switching costs can be pro-competitive as they lead to lower prices (Cabral (2008) and (2010), Dubé *et al.* (2009), Doganoglu (2010), Arie and Grieco (2011), and Rhodes (2011)).[1] While most of the theoretical literature appeals to price discrimination to show that switching costs can lower prices, Arie and Grieco (2011) identify another channel which is common to our model: if consumers switch in equilibrium, then a firm may lower price to partially offset the costs of consumers that are switching to the firm. Rhodes (2011) also arrives at similar conclusions using an overlapping generations model. Our paper, which was developed independently from Arie and Grieco (2011) and Rhodes (2011), differs from theirs in its modelling assumptions. Notably, they develop a discrete time model with very general functional assumptions whereas we derive our conclusions from a simpler continuous time model. This makes our model more tractable, allowing to shed light on the pricing dynamics and the properties of the equilibria both at and before steady state.

We close this section by referring to the literature addressing the interplay between switching costs and price discrimination (in our model, firms cannot price discriminate). With price discrimination, firms may charge low prices to new customers (in order to steal customers from competitors) and high prices to existing customers (to exploit switching costs). Chen (1997) shows that proportional increases in switching costs increase equilibrium profits, which is consistent with the view that switching costs are anti-competitive. However, Cabral (2008) shows that a small increase in the switching cost (from a cost of zero) reduces average price, a result that is in line with our characterization of the dynamic pricing equilibrium (Proposition 1).

3 The Model

We consider a market in which two firms, say 1 and 2, compete to provide a service which is demanded continuously over time. Firms have identical marginal costs normalized to zero. We assume a unit mass of infinitely lived consumers. In each period, each consumer chooses the service provided by one of the two firms. A consumer's maximum willingness to pay for firm i's service in period t is $v + \epsilon_{i,t}$, where $\epsilon_{i,t}$ is a random term for unobserved factors, which we assume *i.i.d.* across sellers and periods. This idiosyncratic shock implies that, even though consumers are identical ex-ante, they become heterogenous ex-post, thus introducing

[1] There are also numerical and empirical papers. For instance, by means of a numerical testbed, Dubé *et al.* (2009) show that depending upon the magnitude of switching costs, switching may indeed occur in equilibrium and that the net effect on prices is ambiguous. In an empirical paper, Viard (2007) finds that lower switching costs (i.e. number portability) led to lower prices for toll-free services.

horizontal differentiation across firms.[2] The utility for a consumer currently served by firm $i \in \{1,2\}$ at time $t > 0$ is given by

$$u_{i,t} = v + \epsilon_{i,t} - p_i,$$

where p_i is the price charged by firm $i \in \{1,2\}$. We assume that v is sufficiently large so that consumers always demand the service from one of the two firms, i.e., the market is covered.

Given current prices, consumers may switch incurring a cost $\frac{s}{2}$. Throughout the paper, we will assume that s is sufficiently small so as to guarantee that there will switching in equilibrium. In particular, we assume $s \in \left(0, \frac{3}{5}\right)$. Consumers decide myopically so that a consumer that is currently served by firm j would opt for firm i provided that

$$u_{i,t} - \frac{s}{2} = v + \epsilon_{i,t} - p_i - \frac{s}{2} > u_{j,t} = v + \epsilon_{j,t} - p_j. \tag{1}$$

Assuming that $\epsilon_{i,t} - \epsilon_{j,t}$ is uniformly distributed in $[-\frac{1}{2}, \frac{1}{2}]$,[3] the probability that a randomly chosen customer served by firm j switches to firm i, q_{ji}, is given by

$$q_{ji} = \Pr\left(\epsilon_{j,t} - \epsilon_{i,t} < -\frac{s}{2} - p_i + p_j\right).$$

A consumer that is currently served by firm i maintains this relationship if

$$u_{i,t} = v + \epsilon_{i,t} - p_i > u_{j,t} - \frac{s}{2} = v + \epsilon_{j,t} - p_j - \frac{s}{2}.$$

Hence, the probability q_{ii} that a randomly chosen customer already served by firm i maintains this relationship is:

$$q_{ii} = \Pr\left(\epsilon_{i,t} - \epsilon_{j,t} > -\frac{s}{2} + p_i - p_j\right).$$

Assuming that $p_i - p_j \in \left[-\frac{1}{2}(1-s), \frac{1}{2}(1+s)\right]$ so that q_{ji} and q_{ii} belong to $(0,1)$ (i.e., there is switching in *both* directions) we have:

$$\begin{aligned} q_{ji} &= \frac{1}{2}(1-s) - p_i + p_j \\ q_{ii} &= \frac{1}{2}(1+s) - p_i + p_j. \end{aligned}$$

Note that $q_{ii} \geq q_{ji}$ reflects the fact that for given prices it is more likely to retain customers than to steal them from the rival.

Let $x_i(t)$ denote firm i's market share at time $t > 0$. Since the collection $\{\epsilon_{i,t} - \epsilon_{j,t} : t > 0\}$ is *i.i.d.*, firm i's expected market share at time $t+1$ can be expressed as:

$$\begin{aligned} x_i(t+1) &= q_{ii} x_i(t) + q_{ji}(1 - x_i(t)) \\ &= x_i(t)\left[q_{ii} - q_{ji}\right] + q_{ji}. \end{aligned}$$

[2]A similar specification is used in Cabral (2008) and Arie and Grieco (2011). Biglaiser *et al.* (2010) introduce heterogeneity across consumers by allowing for heterogenous switching costs.

[3]Results are robust to perturbing this distributional assumption. Details available upon request.

Or equivalently,

$$x_i(t+1) - x_i(t) = x_j(t)q_{ji} - x_i(t)(1 - q_{ii}),$$

i.e., the net change in market share is equal to the expected number of customers that firm i steals from firm j, minus the customers that firm j steals from firm i. To develop a continuous time model, q_{ji} and $1-q_{ii}$ are interpreted as the rate at which switching between firms occur. Hence, in an infinitesimal time interval $dt > 0$:

$$x_i(t+dt) - x_i(t) = x_j(t)q_{ji}dt - x_i(t)(1 - q_{ii})dt,$$

where the first and second terms respectively represent the gain and the loss of consumers, i.e., those switching from firm j to firm i, and vice-versa. The profits accrued by firm i in the time interval $[t, t+dt)$ are $\pi_i = p_i x_i(t+dt)dt$.

Under the assumption that both q_{21} and q_{11} belong to $(0,1)$, we have:

$$\frac{x_1(t+dt) - x_1(t)}{dt} = -x_1(t)(1-s) + \frac{1-s}{2} - p_1 + p_2. \tag{2}$$

In the limit, as $dt \to 0$, we obtain:

$$\dot{x}_1(t) = -x_1(t)(1-s) + \frac{1-s}{2} - p_1 + p_2,$$

and the rates at which profits are accrued can be written as:

$$\begin{aligned} \pi_1 &= p_1 \left(x_1(t)s + \frac{1-s}{2} - p_1 + p_2 \right) \\ \pi_2 &= p_2 \left(-x_1(t)s + \frac{1+s}{2} + p_1 - p_2 \right). \end{aligned}$$

3.1 Dynamic Equilibrium

Given the assumption of full market coverage, payoff relevant histories are subsumed in the state variable $x_1 \in [0,1]$. Assume a discount rate $\rho > 0$. A stationary Markovian pricing policy is a map $p_i : [0,1] \to [0, \frac{1}{2}]$. We restrict our attention to the set of continuous and bounded Markovian pricing policies, say $\mathcal{P}$. For a given strategy combination $(p_i, p_j) \in \mathcal{P} \times \mathcal{P}$ and initial condition, $x_1(\tau) \in [0,1]$ and $\tau < \infty$, the value function is defined as

$$V_i^{(p_i,p_j)}(x_1(\tau)) = \int_\tau^\infty e^{-\rho t} \pi_i(p_i(x_1(t)), p_j(x_1(t)), x_1(t))dt.$$

A stationary Markovian strategy combination $(p_i^*, p_j^*) \in \mathcal{P} \times \mathcal{P}$ is a Markov Perfect equilibrium (MPE) iff

$$V_i^{(p_i^*,p_j^*)}(x_1(\tau)) \geq V_i^{(p_i,p_j^*)}(x_1(\tau)),$$

for all $p_i \in \mathcal{P}, i \in \{1,2\}$ and $x_1(\tau) \in [0,1]$ and $\tau < \infty$.

4 Analysis

As we shall show, when switching costs are not too high, the tension between the "harvesting" effect (exploit loyal customers via high prices), the "compensating" effect (attract new customers via low prices to increase *current* sales) and the "investing" effect (attract new customers via low prices to increase *future* sales) leads to more competitive prices in the long-run. However, in the short run, the degree of market share asymmetries will determine which of these effects dominates.

4.1 Static Setting

To understand pricing incentives in the long run, let us first characterize equilibrium pricing in the static setting. This allows to isolating the harvesting and compensating effects from the investing effect, as the latter only arises in the dynamic setting. The first order conditions in the static setting are:

$$\begin{aligned} \frac{\partial \pi_1}{\partial p_1} &= x_1 s + \frac{1-s}{2} - 2p_1 + p_2 = 0 \\ \frac{\partial \pi_2}{\partial p_2} &= -x_1 s + \frac{1+s}{2} + p_1 - 2p_2 = 0, \end{aligned}$$

from which we derive the best reply functions:

$$\begin{aligned} R_1^S(p_2) &= \tfrac{1}{2}\left(p_2 + s\left(x_1 - \tfrac{1}{2}\right) + \tfrac{1}{2}\right) \\ R_2^S(p_1) &= \tfrac{1}{2}\left(p_1 - s\left(x_1 - \tfrac{1}{2}\right) + \tfrac{1}{2}\right). \end{aligned}$$

Adding switching costs does not alter the fact that prices are strategic complements; that is, a firm optimally responds to a rival's price increase by increasing its own. Figure 1 below plots firms' best reply functions for two values of switching costs.

The large firm (e.g., firm 1) has more to gain by increasing the price and exploit its loyal consumers (more than half) than it has to lose by reducing the price to attract its rival's loyal consumers (less than half). Hence, the large firm behaves less aggressively than the small one, i.e., $R_1^S(p) > R_2^S(p)$.

Solving for equilibrium prices we obtain

$$\begin{aligned} p_1^S(x_1) &= \tfrac{s}{3}\left(x_1 - \tfrac{1}{2}\right) + \tfrac{1}{2} \\ p_2^S(x_1) &= -\tfrac{s}{3}\left(x_1 - \tfrac{1}{2}\right) + \tfrac{1}{2}. \end{aligned}$$

Suppose $x_1 > \frac{1}{2}$. The equilibrium price of the large firm exceeds that of its smaller competitor,

$$p_1^S - p_2^S = \frac{2}{3} s \left(x_1 - \frac{1}{2}\right) > 0.$$

As a consequence, the large firm loses customers in favour of its smaller competitor, but it still remains large.

The following lemma summarizes the comparative statics of equilibrium outcomes in the static setting as switching costs s increase:

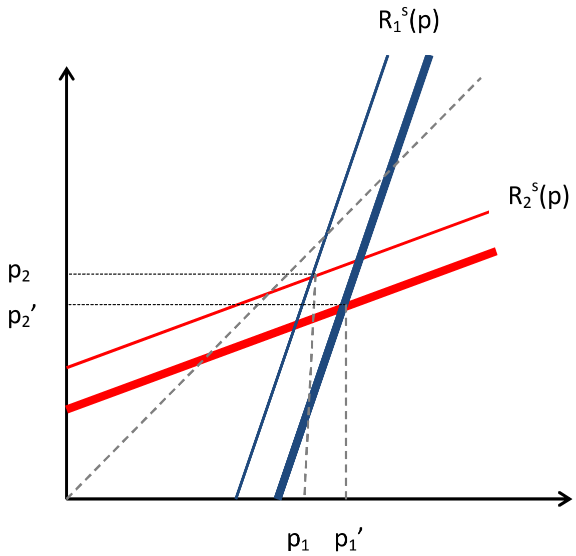

Figure 1: Firms' best replies in the static setting for low (thin lines) and high (thick lines) swicthing costs s

Lemma 1 *In a static setting:*

(i) If market shares are asymmetric, an increase in switching costs s raises the price charged by the large firm, reduces the price charged by the small firm, increases the average market price, and makes market shares even more asymmetric.

(ii) If market shares are symmetric, switching costs have no effect on equilibrium outcomes.

When firms' market shares are asymmetric, i.e., $x_1 > \frac{1}{2}$, an increase in switching costs s implies an outward shift in the large firm's best reply function and an inward shift in the small firm's best reply function (see Figure 1). In other words, switching costs make the large firm less aggressive and the small firm more so. As already noted, this is consistent with the view that switching costs can be interpreted as a subsidy for the large firm and a tax for the small one.

Since, as s increases, the shifts in firms' best replies are of the same magnitude, the equilibrium point moves down to the right, i.e., the price of the large firm goes up while the price of the small firm goes down. Given that the price charged by the large firm has a stronger impact on the average market price, an increase in s implies that the average price in the market also goes up. This corresponds to the conventional wisdom according to which prices are increasing in switching costs. It also follows that an increase in s enlarges the price differential, so that market shares become even more asymmetric.

Last note that if firms were symmetric, i.e., $x_1 = \frac{1}{2}$, switching costs would have no impact on equilibrium outcomes in a static setting.

4.2 Dynamic Setting

We are now ready to characterize equilibrium pricing in the dynamic setting.

Proposition 1 *The unique Markov Perfect Equilibrium in affine pricing strategies is:*[4]

$$\begin{aligned} p_1^D(x_1) &= p_1^S(x_1) + \tfrac{1}{3}(\lambda_2 - 2\lambda_1) \\ p_2^D(x_1) &= p_1^S(x_1) + \tfrac{1}{3}(2\lambda_2 - \lambda_1) \end{aligned}$$

with $\lambda_1 = ax_1 + b > -\lambda_2 = -ax_1 + b > 0$, *where* $a \in (0, \frac{s}{2})$ *is the smallest root of the quadratic equation*

$$2a^2 - 3\left(2 + \rho - \frac{7}{9}s\right)a + \frac{2}{3}s^2 = 0, \tag{1}$$

and

$$b = \frac{1}{1+\rho}\frac{3-s}{3}\left(\frac{s}{3} + \frac{a}{2}\right).$$

Proof. See the appendix. ■

In the proof we make use of the notion of a Hamiltonian (see Dockner *et al.* (2000)), that is:

$$\mathcal{H}_i = e^{-\rho t}[\pi_i + \lambda_i \dot{x}_1],$$

for $i \in \{1, 2\}$, where $\lambda_i = \frac{\partial V_i}{\partial x_1}$ is the co-state variable. A necessary condition for optimality is:

$$\frac{\partial \pi_i}{\partial p_i} = -\lambda_i \frac{\partial \dot{x}_1}{\partial p_i},$$

which captures the inter-temporal trade-offs inherent in equilibrium pricing: i.e., marginal revenue equals the (marginal) opportunity cost (value loss) associated with market share reduction. This condition gives rise to a sort of "instantaneous" best reply functions:

$$\begin{aligned} R_1^D(p_2) &= \tfrac{1}{2}\left(p_2 + s\left(x_1 - \tfrac{1}{2}\right) + \tfrac{1}{2}\right) - \tfrac{\lambda_1}{2} \\ R_2^D(p_1) &= \tfrac{1}{2}\left(p_1 - s\left(x_1 - \tfrac{1}{2}\right) + \tfrac{1}{2}\right) + \tfrac{\lambda_2}{2}. \end{aligned}$$

These can also be expressed as

$$\begin{aligned} R_1^D(p_2) &= R_1^S(p_2) - \tfrac{\lambda_1}{2} \\ R_2^D(p_1) &= R_2^S(p_1) + \tfrac{\lambda_2}{2}. \end{aligned}$$

Therefore, as compared to the static setting, firms' best reply functions in the dynamic setting shift in, thus implying that equilibrium prices are lower. This is a direct consequence of the "investing effect": firms compete more aggressively to attract new customers as these will become loyal, and thus valuable, in the future.

[4] Other MPE in non-linear strategies may exist. However, a complete characterization of MPE is beyond the scope of this paper.

In the dynamic setting, it is still true that the large firm behaves less aggressively than the small one, i.e., $R_1^D(p) > R_2^D(p)$, thus implying that the large firm's equilibrium price is higher than that of the small firm,

$$p_1^D - p_2^D = \frac{2}{3}(s-a)\left(x_1 - \frac{1}{2}\right) > 0.$$

As compared to the static setting, the large firm's best reply function has shifted in by a larger amount, $\frac{\lambda_1}{2}$, than that of the small one, $-\frac{\lambda_2}{2}$ (recall that $\lambda_1 > -\lambda_2$). This derives from the fact that the investing effect is stronger for the large firm than for the small one: attracting new customers today is more valuable for the large firm, given that the price it charges to its loyal consumers is higher. It follows that the price differential, while still positive, is now smaller than in the static setting, i.e., $p_1^D - p_2^D < p_1^S - p_2^S$.

Concerning dynamics, the fact that the large firm has the higher price implies that the large firm concedes market share in favour of the smaller one. Therefore, firms' asymmetries fade away over time. In particular, the equilibrium state dynamics are described by:

$$\begin{aligned}\dot{x}_1(t) &= -x_1(t)(1-s) + \frac{1-s}{2} - p_1^D + p_2^D \\ &= -\left(x_1(t) - \frac{1}{2}\right)\left(1 - \frac{s+2a}{3}\right) < 0,\end{aligned}$$

whose solution is:

$$x_1(t) = x_1(0)e^{-\left(1-\frac{s+2a}{3}\right)t} + \frac{1}{2}.$$

Furthermore, as the large firm loses market share, its incentives to price high diminish, so that competition becomes more intense. Hence, the average price in the market is *decreasing* over time. In detail, let $p(t) = p_1(x_1(t))x_1(t) + p_2(x_1(t))x_2(t)$ denote the average price charged in the market. It follows that:

$$\begin{aligned}\dot{p}(t) &= \left[\left(\frac{\partial p_1}{\partial x_1} - \frac{\partial p_2}{\partial x_1}\right)x_1 + (p_1 - p_2)\right]\dot{x}_1 + \frac{\partial p_2}{\partial x_1}\dot{x}_1 \\ &= \left[\frac{\partial p_1}{\partial x_1}x_1 + (p_1 - p_2) + \frac{\partial p_2}{\partial x_1}(1 - x_1)\right]\dot{x}_1 \\ &= \left[\frac{4}{3}(s-a)\left(x_1 - \frac{1}{2}\right)\right]\dot{x}_1 < 0.\end{aligned}$$

Note that in steady state firms become symmetric, as $\lim_{t\to\infty} x_1(t) = \frac{1}{2}$, and that both firms' equilibrium prices converge to their lowest level,

$$\lim_{t\to\infty} p_i(t) = \frac{1}{2} - \frac{a}{3} - \frac{1}{9}\frac{3-s}{1+\rho}\left(\frac{s}{3} + \frac{a}{2}\right).$$

These results are summarized next:

Lemma 2 *In a dynamic setting:*

(i) Firms' market shares become more symmetric over time, and they become fully symmetric in steady state.

(ii) The average market price is decreasing *over time, and it is thus lowest in steady state.*

4.3 What is the effect of switching costs?

We end this section by performing comparative statics of equilibrium outcomes as switching costs s increase. We start by focusing on equilibrium prices charged by the two firms at a given point in time before reaching the steady state:

Lemma 3 *Out of steady state:*

(i) An increase in s reduces the price charged by the small firm.

(ii) There exists $\widehat{x}_1 > 1/2$ such that an increase in s reduces the price charged by the large firm if and only if $x_1 < \widehat{x}_1$.

(iii) An increase in s enlarges the price differential.

(iv) There exists $\widetilde{x}_1 > \widehat{x}_1$ such that an increase in s reduces the average market price if and only if $x_1 < \widetilde{x}_1$.

Proof. See the Appendix. ■

When switching costs increase, price choices reflect two countervailing incentives. Just as we described in the static setting, an increase in s changes the harvesting and compensating effects, inducing the large (small) firm to price less (more) aggressively. However, in a dynamic setting, a higher s also implies a greater value of attracting customers so as to increase future profits (investing effect).

For the small firm, all three effects point to the same direction. Accordingly, the price charged by the small firm unambiguously decreases in the switching cost s. In contrast, the large firm faces countervailing incentives as s increases. Since the incentives to charge higher prices today are greater the larger the firm's market share, there exists a critical market share $\widehat{x}_1$ below (above) which the investing (harvesting) effect dominates, so that the price charged by the large firm decreases (increases) in s.

As an illustration, Figure 2 depicts the price charged by the two firms as a function of the switching cost s for different values of the large firm's market share. As it can be seen, the price charged by the large firm decreases in s for low values of x_1 but increase in s for high values of x_1. In contrast, the price charged by the small firm is always decreasing in s. In all cases, the vertical distance between the prices charged by the two firms widens up as s goes up.

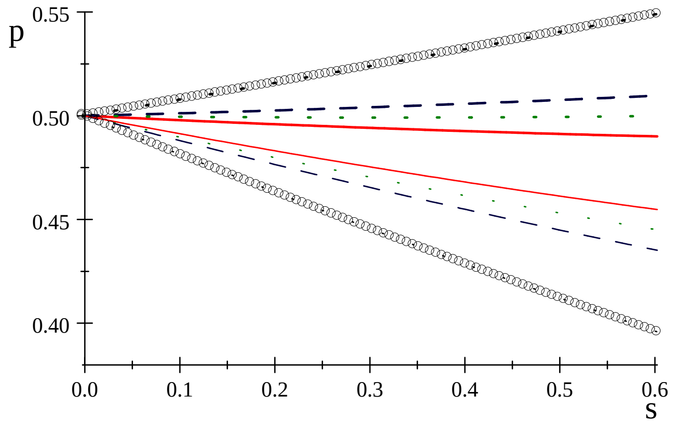

Figure 2. Prices charged by the large firm (thick lines) and small firm (thin lines) as a function of the switching cost s, assuming $\rho = 5$ and $x_1 = 0.6$ (solid), $x_1 = 0.65$ (dots), $x_1 = 0.8$ (dash), and $x_1 = 0.9$ (circles)

As s increases, the average price changes as follows:

$$\frac{\partial p\left(t\right)}{\partial s} = \frac{\partial\left(p_1 - p_2\right)}{\partial s}x_1 + \left(p_1 - p_2\right)\frac{\partial x_1}{\partial s} + \frac{\partial p_2}{\partial s}.$$

The first term is positive given that an increase in s enlarges the price differential. However, the second and third terms are negative. Hence, the sign of the effect of s on average prices is ambiguous. In particular, an increase in s leads to a reduction in the average price only when firms are sufficiently symmetric, i.e., if $x_1 < \widetilde{x}_1$. Note that $\widetilde{x}_1 > \widehat{x}_1$ as $x_1 < \widehat{x}_1$ is a sufficient condition for the average price to go down in s, as both firms' prices are decreasing in s (part (ii) of the Lemma). In sum, an increase in switching costs might be pro-competitive or anticompetitive depending on whether firms are more or less symmetric. Figure 3 provides numerical support to this claim.

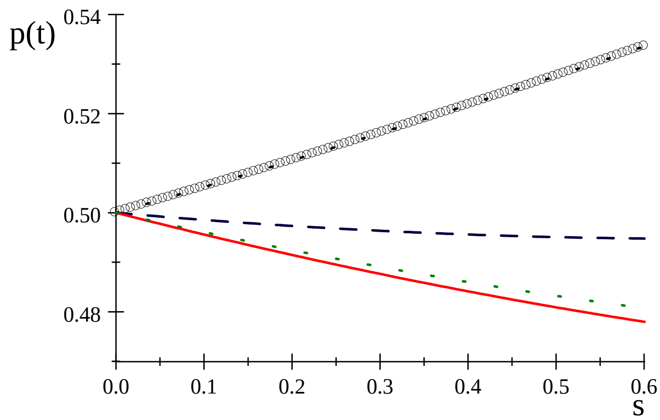

Figure 3. Average price as a function of the switching cost s, assuming $\rho = 5$ and $x_1 = 0.6$ (solid), $x_1 = 0.65$ (dots), $x_1 = 0.8$ (dash), and $x_1 = 0.9$ (circles)

The fact that the price differential across firms goes up in s (part (iii) of the Lemma) implies that higher switching costs also slow down the transition to a symmetric market structure, and hence lead to a lower rate of decline in average prices. This can be seen as an anti-competitive effect of switching costs in the short-run, which arises regardless of the degree of firms' asymmetries. The following Lemma summarizes the effect of switching costs on the equilibrium dynamics.

Lemma 4 *An increase in switching costs s :*
(i) reduces the rate of decline of average prices and
(ii) delays the transition to the steady state.

Proof. See the appendix. ■

Nonetheless, in the long-run, switching costs are pro-competitive: the *higher* the switching cost, the *lower* the equilibrium price in steady state. Indeed, in steady state, once firms have become fully symmetric, only the investing effect plays a role. Hence, an increase in s, which increases the future value of current sales, makes competition fiercer and thus lowers equilibrium prices.

Lemma 5 *In steady state, increasing switching costs reduce prices.*

5 Asymmetric Switching Costs

In this section we consider the case in which switching costs are asymmetric. More specifically, customers switching from firm 1 to firm 2 incur a cost $\frac{s}{2}$, $s \in (0, \frac{3}{5})$, while customers switching in the other direction bear no cost. As before we assume hat $\epsilon_{i,t} - \epsilon_{j,t}$ is uniformly distributed in $[-\frac{1}{2}, \frac{1}{2}]$. The probability that a randomly chosen customer served by firm 2 switches to firm 1, q_{21}, and the probability that a randomly chosen customer already served by firm 1 maintains this relationships, q_{11}, are now given by

$$\begin{aligned} q_{21} &= \Pr\left(\epsilon_{2,t} - \epsilon_{1,t} < -p_1 + p_2\right) \\ q_{11} &= \Pr\left(\epsilon_{1,t} - \epsilon_{2,t} > -\frac{s}{2} + p_1 - p_2\right). \end{aligned}$$

Assuming that $p_1 - p_2 \in \left[-\frac{1}{2}(1-s), \frac{1}{2}(1+s)\right]$ so that q_{21} and q_{11} belong to $(0,1)$ (i.e., there is switching in *both* directions) we have:

$$\begin{aligned} q_{21} &= \frac{1}{2} - p_1 + p_2 \\ q_{11} &= \frac{1}{2}(1+s) - p_1 + p_2. \end{aligned}$$

We revisit our discrete choice model so that the difference equation (2) is now

$$\begin{aligned} \frac{x_1(t+dt) - x_1(t)}{dt} &= q_{21} - x_1(t)(1 - q_{11} + q_{21}) \\ &= \frac{1}{2} - p_1 + p_2 - x_1(t)\left(1 - \frac{s}{2}\right). \end{aligned}$$

In the limit as $dt \to 0$ we obtain:

$$\dot{x}_1 = -x_1(t)\left(1 - \frac{s}{2}\right) + \frac{1}{2} - p_1 + p_2.$$

The instantaneous rate at which revenue accrues for firms 1 and 2 can be expressed as:

$$\begin{aligned} \pi_1 &= p_1 \left(x_1(t)\frac{s}{2} + \frac{1}{2} - p_1 + p_2 \right) \\ \pi_2 &= p_2 \left(-x_1(t)\frac{s}{2} + \frac{1}{2} - p_2 + p_1 \right). \end{aligned}$$

5.1 Analysis

In the static setting, firms' best reply functions are:

$$\begin{aligned} R_1^S(p_2) &= \tfrac{1}{2}\left(p_2 + \tfrac{s}{2}x_1 + \tfrac{1}{2}\right) \\ R_2^S(p_1) &= \tfrac{1}{2}\left(p_1 - \tfrac{s}{2}x_1 + \tfrac{1}{2}\right). \end{aligned}$$

Therefore, equilibrium prices are,

$$\begin{aligned} p_1^S(x_1) &= \frac{s}{6}x_1 + \frac{1}{2} \\ p_2^S(x_1) &= -\frac{s}{6}x_1 + \frac{1}{2}. \end{aligned}$$

Note that firm 1, which is protected by switching costs, prices less aggressively regardless of whether it is large or not, i.e., regardless of whether $x_1 > \frac{1}{2}$ or $x_1 < \frac{1}{2}$.

In the following result we revisit the structure of dynamic equilibrium pricing policies.

Proposition 2 *The affine pricing strategies*[5]

$$\begin{aligned} p_1^D(x_1) &= p_1^S(x_1) + \tfrac{1}{3}(\lambda_2 - 2\lambda_1) \\ p_2^D(x_1) &= p_1^S(x_1) + \tfrac{1}{3}(2\lambda_2 - \lambda_1), \end{aligned}$$

with $\lambda_1 = ax_1 + b > -\lambda_2 = -ax_1 + b > 0$, *where* $a \in (0, \frac{s}{2})$ *is the smallest root of the quadratic equation*

$$2a^2 - 3\left(2 + \rho - \frac{11}{18}s\right)a + \frac{a^2}{3} = 0,$$

and

$$b = \frac{1}{1+\rho}\left(\frac{s}{3} + \frac{a}{2}\right),$$

are a Markov Perfect Equilibrium.

[5] Other MPE in non-linear strategies may exist. However, a complete characterization of MPE is beyond the scope of this paper.

Proof. See the appendix. ■

In this case, the equilibrium dynamics are:

$$\begin{aligned}\dot{x}_1 &= \left(\frac{s}{2}-1\right)x_1+\frac{1}{2}-\frac{2}{3}\left(\frac{s}{2}-a\right)x_1\\ &= -(1-\frac{1}{3}(\frac{s}{2}+2a))x_1+\frac{1}{2}.\end{aligned}$$

The solution is

$$x_1(t)=x_1(0)e^{-(1-\frac{1}{3}(\frac{s}{2}+2a))t}+\frac{1}{2-\frac{1}{3}(s+4a)}.$$

Since $a<\frac{s}{2}$ we have $0<\frac{s+4a}{3}<s<\frac{3}{5}$ and the long-run market share is:

$$\lim_{t\to\infty}x_1(t)=\frac{1}{2-\frac{1}{3}(s+4a)}\in\left(\frac{1}{2},\frac{5}{7}\right).$$

In the long-run, the asymmetric structure of the switching costs allows the firm from which it is costly to switch (i.e., firm 1) to maintain (or attain) a degree of market dominance while charging higher prices. To see why this is the case, recall that switching in both directions *always* takes place (some customers switch to firm 1, the higher priced firm, due to unobservable factors). Hence, once customers switch to firm 1 they are in a certain sense "locked-in". The level of asymmetry in the long-run structure of market shares is increasing in the magnitude of the switching cost, and the more asymmetric firms are the higher the level of steady state prices. This conclusion, which contrasts with our previous result, shows that asymmetric switching costs might have anticompetitive effects by creating or reinforcing firm dominance.

6 Discussion: Limitations of the Model

In our model customers are assumed to behave myopically. However, more sophisticated customers, who aim to maximize their total discounted consumption surplus over the infinite horizon, may abstain from switching when they correctly anticipate price increases in the future. These sophisticated customers would likely be more demanding than myopic customers when switching to the large firm, knowing that its future prices will be higher than those of the smaller firm. For the opposite reason, they would require a smaller instantaneous surplus gain when switching to the smaller firm. It is true that the smaller firm will not need to price as aggressively in the short-run in order to steadily gain market share. However, the former effect is likely to dominate, thus implying that the speed of convergence to the steady state is faster with sophisticated than with myopic consumers.

In the long-run however, when firms have become fully symmetric, both firms charge equal prices. Hence, maximizing instantaneous surplus is equivalent to maximizing total discounted surplus over the infinite horizon. In other words, the presence of sophisticated consumers will speed up convergence but will have no impact on the long-run equilibrium. Thus, the qualitative features of Proposition 1 are likely to be preserved.

On the contrary, the results of Proposition 2 are likely to be significantly affected by the incorporation of more sophisticated customers. The qualitative nature of the equilibrium in that proposition indicates that in the long-run, the firm from which it is costly to switch attains market dominance while charging higher prices. Evidently, sophisticated customers would be less likely to switch in anticipation of such "rip-off".

A second limitation of this model pertains to our assumption of no-growth in demand. In a growing market, the qualitative implications of switching costs are likely to differ from those presented in this paper. With a relatively low discount factor and significant market growth, the "investing" effect dominates the "harvesting" and "compensating" effects in the short-run, so that switching costs may end up inducing aggressive pricing in the short-run and less competitive outcomes in the long-run. A formalization of this intuition is the subject of future work.

7 Conclusions

Many information technology products and technologies exhibit switching costs (i.e., costs that customers must bear when they adopt a new product or technology). Typically, switching costs arise when there is limited compatibility between an old product (or technology) and a newly adopted one. In this paper, we have analyzed the effect of switching costs on the nature of dynamic price competition.

We have shown that switching costs can be pro-competitive when the magnitude of switching costs is not too high and firms are not too asymmetric. In a Markov Perfect equilibrium, the dominant firm concedes market share by charging higher prices to current customers in the short-run. As market structure becomes more symmetric, price competition becomes fiercer. The average price charged in the market is decreasing over time and in the long-run equilibrium prices are decreasing in the magnitude of switching costs.

However, before steady state is reached, switching costs have an ambiguous effect on market prices. When firms are sufficiently asymmetric, an increase in switching costs implies that the large firm behaves less aggressively in order to exploit its customer base. It is only when firms become sufficiently symmetric over time that the investing effect dominates, thus leading to lower prices in markets with higher switching costs. Therefore, from a policy perspective, the presence of switching costs should raise more concerns in concentrated industries, or in those markets in which asymmetries in switching costs constitute an obstacle for the convergence to a symmetric market structure.

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Appendix: Proofs

Proof of Proposition 1

The Hamiltonians are

$$\mathcal{H}_i = e^{-\rho t}[\pi_i + \lambda_i \dot{x}_1]$$

for $i \in \{1, 2\}$. The Hamiltonians are strictly concave so that first order conditions for MPE are also sufficient (see Dockner et al. (2000)),

$$\frac{\partial \mathcal{H}_i}{\partial p_i} = 0$$

$$-\frac{\partial \mathcal{H}_i}{\partial x_1} - \frac{\partial \mathcal{H}_i}{\partial p_j}\frac{\partial p_j}{\partial x_1} = \dot{\lambda}_i - \rho \lambda_i$$

for $i \in \{1, 2\}$. These respectively lead to:

$$p_1 = \frac{1}{2}\left(p_2 + s\left(x_1 - \frac{1}{2}\right) + \frac{1}{2} - \lambda_1\right) \tag{A.1}$$

$$-sp_1 + (1-s)\lambda_1 - (p_1 + \lambda_1)\frac{\partial p_2}{\partial x_1} = \dot{\lambda}_1 - \rho\lambda_1 \tag{A.2}$$

$$p_2 = \frac{1}{2}\left(p_1 - s\left(x_1 - \frac{1}{2}\right) + \frac{1}{2} + \lambda_2\right) \tag{A.3}$$

$$sp_2 + (1-s)\lambda_2 - (p_2 - \lambda_2)\frac{\partial p_1}{\partial x_1} = \dot{\lambda}_2 - \rho\lambda_2. \tag{A.4}$$

Equations (A.1) and (A.3) are firms' best reply functions. Using them we can obtain equilibrium prices,

$$\begin{aligned} p_1 &= \frac{s}{3}\left(x_1 - \frac{1}{2}\right) + \frac{1}{2} + \frac{1}{3}(\lambda_2 - 2\lambda_1) \\ p_2 &= -\frac{s}{3}\left(x_1 - \frac{1}{2}\right) + \frac{1}{2} + \frac{1}{3}(2\lambda_2 - \lambda_1). \end{aligned}$$

Thus,

$$\frac{\partial p_2}{\partial x_1} = -\frac{\partial p_1}{\partial x_1} = -\frac{s}{3}\cdot$$

Substituting into (A.2) and (A.4) we obtain

$$\begin{aligned} \frac{2s}{3}p_2 + (1-\frac{2s}{3})\lambda_2 &= \dot{\lambda}_2 - \rho\lambda_2 \\ -\frac{2s}{3}p_1 + (1-\frac{2s}{3})\lambda_1 &= \dot{\lambda}_1 - \rho\lambda_1\cdot \end{aligned}$$

We solve this system of differential equations by the method of undetermined coefficients. Assume $\lambda_i = a_i x_1 + b_i$ for $i \in \{1,2\}$. Substitution into the last equation yields

$$\begin{gathered} -\tfrac{2s}{3}\left(\tfrac{s}{3}(x_1-\tfrac{1}{2}) + \tfrac{1}{2} + \tfrac{1}{3}(a_2x_1 + b_2 - 2a_1x_1 - 2b_1)\right) + (1-\tfrac{2s}{3})(a_1x_1+b_1) \\ = \\ a_1\dot{x}_1 - \rho(a_1x_1+b_1) \\ = \\ a_1\left(-x_1(1-s) + \tfrac{1-s}{2} - p_1 + p_2\right) - \rho a_1x_1 - \rho b_1 \\ = \\ a_1\left(-x_1(1-s) + \tfrac{1-s}{2} - \tfrac{2s}{3}(x_1-\tfrac{1}{2}) + \tfrac{\lambda_1+\lambda_2}{3}\right) - \rho a_1x_1 - \rho b_1 \\ = \\ a_1\left(-x_1(1-s) + \tfrac{1-s}{2} - \tfrac{2s}{3}(x_1-\tfrac{1}{2}) + \tfrac{a_1x_1+b_1+a_2x_1+b_2}{3}\right) - \rho a_1x_1 - \rho b_1\cdot \end{gathered}$$

This results in the following two equations:

$$-\frac{2}{9}s^2 + \frac{2}{9}s(2a_1 - a_2) + (1-\frac{2s}{3})a_1 = -(1-\frac{s}{3})a_1 + \frac{1}{3}(a_1+a_2)a_1 - \rho a_1 \tag{A.5}$$

$$-\frac{s}{3}(1-\frac{s}{3}) - \frac{2s}{9}(b_2 - 2b_1) + b_1(1-\frac{2s}{3}) = \frac{1}{3}(b_1+b_2)a_1 + \frac{a_1}{2}(1-\frac{s}{3}) - \rho b_1 \tag{A.6}$$

In a similar fashion, we obtain two additional equations:

$$-\frac{2}{9}s^2 + \frac{2}{9}s(2a_2 - a_1) + (1-\frac{2s}{3})a_2 = -(1-\frac{s}{3})a_2 + \frac{1}{3}(a_1+a_2)a_2 - \rho a_2 \tag{A.7}$$

$$\frac{s}{3}(1-\frac{s}{3}) + \frac{2s}{9}(2b_2 - b_1) + b_2(1-\frac{2s}{3}) = \frac{1}{3}(b_1+b_2)a_2 + \frac{a_2}{2}(1-\frac{s}{3}) - \rho b_2 \tag{A.8}$$

Thus, substracting (A.5) from (A.7) we get:

$$\left[1 - \frac{s}{3} - \frac{1}{3}(a_1+a_2) + \frac{2}{9}s + 1 - \frac{2s}{3} + \rho\right](a_1 - a_2) = 0\cdot$$

Hence, $a_1 - a_2 = 0$. Let $a_1 = a_2 = z$, we solve the quadratic equation implicit in (A.5):

$$g(z) = \frac{2}{3}z^2 - \left(2 + \rho - \frac{7}{9}s\right)z + \frac{2}{9}s^2 = 0\cdot$$

Note that since $g(0) > 0$ and $g(\frac{s}{2}) < 0$, a solution $a \in (0, \frac{s}{2})$ exists. Then (A.6) and (A.8) imply $b_2 = -b_1$ and from (A.8):

$$b_1 = \frac{1}{3}\frac{3-s}{1+\rho}\left(\frac{s}{3} + \frac{a}{2}\right).$$

Last, we note that given the assumption $s < \frac{3}{5}$ the pricing policies satisfy:

$$p_1(x_1) - p_2(x_1) = \frac{2}{3}(s-a)\left(x_1 - \frac{1}{2}\right) \in \left(-\frac{1-s}{2}, \frac{1-s}{2}\right),$$

so that $q_0, q_1 \in (0,1)$.

Proof of Lemma 2

We first note that implicit differentiation in (1) yields:

$$\frac{\partial a}{\partial s} = \frac{4s + 7a}{9(2+\rho) - 7s - 12a} \in (0,1)\cdot$$

(i) Using this result, it is straightforward to see that $\frac{\partial p_2}{\partial s} < 0$. Taking derivatives,

$$\begin{aligned}\frac{\partial p_2}{\partial s} &= -\frac{1}{3}\left(\left(1 - \frac{\partial a}{\partial s}\right)\left(x_1 - \frac{1}{2}\right) + \frac{\partial a}{\partial s}\right) \\ &\quad -\frac{1}{3}\frac{1}{1+\rho}\left(\frac{2}{3}\left(\frac{s}{3} + \frac{a}{2}\right) + \left(1 - \frac{s}{3}\right)\left(\frac{1}{3} + \frac{1}{2}\frac{\partial a}{\partial s}\right)\right).\end{aligned}$$

(ii) Taking derivatives,

$$\begin{aligned}\frac{\partial p_1}{\partial s} &= \frac{1}{3}\left(\left(1 - \frac{\partial a}{\partial s}\right)x_1 - \frac{1}{2}\left(1 + \frac{\partial a}{\partial s}\right)\right) \\ &\quad -\frac{1}{3}\frac{1}{1+\rho}\left(\frac{2}{3}\left(\frac{s}{3} + \frac{a}{2}\right) + \left(1 - \frac{s}{3}\right)\left(\frac{1}{3} + \frac{1}{2}\frac{\partial a}{\partial s}\right)\right).\end{aligned}$$

The second term is negative, while the sign of the first term cannot be determined in general. Solving for x_1, expression above is positive if and only if

$$x_1 > \widehat{x}_1 = \frac{1}{\left(1 - \frac{\partial a}{\partial s}\right)}\left(\frac{1}{1+\rho}\left(\frac{2}{3}\left(\frac{s}{3} + \frac{a}{2}\right) + \left(1 - \frac{s}{3}\right)\left(\frac{1}{3} + \frac{1}{2}\frac{\partial a}{\partial s}\right)\right) + \frac{1}{2}\left(1 + \frac{\partial a}{\partial s}\right)\right).$$

The fact that $\widehat{x}_1 > \frac{1}{2}$ follows since $\frac{\partial p_1}{\partial s}$ is weakly increasing in x_1 and $\frac{\partial p_1}{\partial s} < 0$ for $x_1 = \frac{1}{2}$, as the first term becomes $-\frac{\partial a}{\partial s} < 0$.

(iii) It follows from the fact that the price differential $p_1^D - p_2^D$ is directly proportional to $s - a$ and, as shown above, $\frac{\partial a}{\partial s} < 1$.

(iv) The proof is provided in the main text.

Proof of Lemma 3

(i) It follows from the fact that $\dot{p}(t)$ is inversely proportional to $s - a$ and, as shown above, $\frac{\partial a}{\partial s} < 1$. (ii) The transition to the steady state occurs at a rate which is inversely proportional to $\frac{s+2a}{3}$, and as shown above, $\frac{\partial a}{\partial s} > 0\cdot$

Proof of Lemma 4

Steady-state prices are

$$\lim_{t\to\infty} p_i(t) = \frac{1}{2} - \frac{a}{3} - \frac{1}{9}\frac{3-s}{1+\rho}\left(\frac{s}{3}+\frac{a}{2}\right).$$

Taking derivatives w.r.t. s,

$$-\frac{1}{3}\frac{\partial a}{\partial s} + \frac{1}{9}\frac{1}{1+\rho}\left(\frac{1}{6}\left(3a+4s-6\right) - \left(3-s\right)\frac{1}{2}\frac{\partial a}{\partial s}\right).$$

Note $3a+4s-6<0$ if $a<-\frac{4}{3}s+2$. This condition is satisfied since $a<\frac{s}{2}$ and $s<\frac{3}{5}$. It thus follows that expression above is negative.

Proof of Proposition 2

As in the proof of proposition 1, the Hamiltonians are

$$\mathcal{H}_i = e^{-\rho t}[\pi_i + \lambda_i \dot{x}_1],$$

for $i\in\{1,2\}$. First order conditions (which in this case due to concavity are also sufficient) are:

$$\frac{\partial \mathcal{H}_i}{\partial p_i} = 0$$
$$-\frac{\partial \mathcal{H}_i}{\partial x_1} - \frac{\partial \mathcal{H}_i}{\partial p_j}\frac{\partial p_j}{\partial x_1} = \dot{\lambda}_i - \rho\lambda_i.$$

The first order conditions lead to:

$$p_1 = \frac{1}{2}\left(p_2 + \frac{s}{2}x_1 + \frac{1}{2} - \lambda_1\right) \tag{B.1}$$

$$-\frac{s}{2}p_1 + (1-\frac{s}{2})\lambda_1 - (p_1+\lambda_1)\frac{\partial p_1}{\partial x_1} = \dot{\lambda}_1 - \rho\lambda_1 \tag{B.2}$$

$$p_2 = \frac{1}{2}\left(p_1 - \frac{s}{2}x_1 + \frac{1}{2} + \lambda_2\right) \tag{B.3}$$

$$\frac{s}{2}p_2 + (1-\frac{s}{2})\lambda_2 - (p_2-\lambda_2)\frac{\partial p_2}{\partial x_1} = \dot{\lambda}_2 - \rho\lambda_2. \tag{B.4}$$

Here, (B.1) and (B.3) imply, the equilibrium prices are of the form:

$$\begin{aligned} p_1 &= \frac{s}{6}x_1 + \frac{1}{2} + \frac{\lambda_2 - 2\lambda_1}{3} \\ p_2 &= -\frac{s}{6}x_1 + \frac{1}{2} + \frac{2\lambda_2 - \lambda_1}{3}. \end{aligned}$$

Substituting into (B.2) and (B.4) we obtain

$$\begin{aligned} \frac{2s}{3}p_2 + (1-\frac{2s}{3})\lambda_2 &= \dot{\lambda}_2 - \rho\lambda_2 \\ -\frac{2s}{3}p_1 + (1-\frac{2s}{3})\lambda_1 &= \dot{\lambda}_1 - \rho\lambda_1. \end{aligned}$$

We solve using method of undetermined coefficients. Assume $\lambda_i = a_i x_1 + b_i$ for $i = 1, 2$. Substitution into the last equation yields

$$-\tfrac{2s}{3}\left(\tfrac{s}{6}x_1 + \tfrac{1}{2} + \tfrac{1}{3}(a_2x_1 + b_2 - 2a_1x_1 - 2b_1)\right) + (1 - \tfrac{2s}{3})(a_1x_1 + b_1)$$
$$=$$
$$a_1\dot{x}_1 - \rho(a_1x_1 + b_1)$$
$$=$$
$$a_1\left(-x_1(1 - \tfrac{s}{2}) + \tfrac{1}{2} - p_1 + p_2\right) - \rho a_1 x_1 - \rho b_1$$
$$=$$
$$a_1\left(-x_1(1 - \tfrac{s}{2}) + \tfrac{1}{2} - \tfrac{2s}{6}x_1 + \tfrac{\lambda_1+\lambda_2}{3}\right) - \rho a_1 x_1 - \rho b_1$$
$$=$$
$$a_1\left(-x_1(1 - \tfrac{s}{2}) + \tfrac{1}{2} - \tfrac{2s}{6}x_1 + \tfrac{a_1x_1+b_1+a_2x_1+b_2}{3}\right) - \rho a_1 x_1 - \rho b_1.$$

This results in the following two equations:

$$-\frac{s^2}{9} + \frac{2}{9}s(2a_1 - a_2) + (1 - \frac{2s}{3})a_1 = -(1 - \frac{s}{6})a_1 + \frac{1}{3}(a_1 + a_2)a_1 - \rho a_1 \tag{B.5}$$

$$-\frac{s}{3} - \frac{2s}{9}(b_2 - 2b_1) + b_1(1 - \frac{2s}{3}) = \frac{1}{3}(b_1 + b_2)a_1 + \frac{a_1}{2} - \rho b_1 \tag{B.6}$$

In a similar fashion, we obtain two additional equations:

$$-\frac{s^2}{9} + \frac{2}{9}s(2a_2 - a_1) + (1 - \frac{2s}{3})a_2 = -(1 - \frac{s}{6})a_2 + \frac{1}{3}(a_1 + a_2)a_2 - \rho a_2 \tag{B.7}$$

$$\frac{s}{3} + \frac{2s}{9}(2b_2 - b_1) + b_2(1 - \frac{2s}{3}) = \frac{1}{3}(b_1 + b_2)a_2 + \frac{a_2}{2} - \rho b_2. \tag{B.8}$$

As in the proof of proposition 1, we can show that $a_1 = a_2$ and $b_1 = -b_2$. Let $a_1 = a_2 = z$, we solve the quadratic equation implicit in (B.5):

$$g(z) = \frac{2}{3}z^2 - \left(2 + \rho - \frac{11}{18}s\right)z + \frac{s^2}{9} = 0,$$

and from (B.6) we obtain

$$b_1 = \frac{1}{1+\rho}\left(\frac{s}{3} + \frac{a}{2}\right).$$

It follows that

$$\begin{aligned} p_1 &= \frac{1}{3}\left(\frac{s}{2} - a\right)x_1 + \frac{1}{2} - \frac{1}{1+\rho}\left(\frac{s}{3} + \frac{a}{2}\right) \\ p_2 &= -\frac{1}{3}\left(\frac{s}{2} - a\right)x_1 + \frac{1}{2} - \frac{1}{1+\rho}\left(\frac{s}{3} + \frac{a}{2}\right). \end{aligned}$$

Note that

$$p_1(x_1) - p_2(x_1) = \frac{2}{3}\left(\frac{s}{2} - a\right)x_1 \in \left(-\frac{1}{2}(1-s), \frac{1}{2}(1-s)\right)$$

provided $s < \frac{3}{5}$.