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Claudia Foroni, Massimiliano Marcellino and Christian Schumacher

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Claudia Foroni, European University institute
Massimiliano Marcellino, European University Institute, Bocconi University and CEPR
Christian Schumacher, Deutsche Bundesbank

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Centre for Economic Policy Research
77 Bastwick Street, London EC1V 3PZ, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

U-MIDAS: MIDAS regressions with unrestricted lag polynomials*

Mixed-data sampling (MIDAS) regressions allow to estimate dynamic equations that explain a low-frequency variable by high-frequency variables and their lags. When the difference in sampling frequencies between the regressand and the regressors is large, distributed lag functions are typically employed to model dynamics avoiding parameter proliferation. In macroeconomic applications, however, differences in sampling frequencies are often small. In such a case, it might not be necessary to employ distributed lag functions. In this paper, we discuss the pros and cons of unrestricted lag polynomials in MIDAS regressions. We derive unrestricted MIDAS regressions (U-MIDAS) from linear high-frequency models, discuss identification issues, and show that their parameters can be estimated by OLS. In Monte Carlo experiments, we compare U-MIDAS to MIDAS with functional distributed lags estimated by NLS. We show that U-MIDAS performs better than MIDAS for small differences in sampling frequencies. On the other hand, with large differing sampling frequencies, distributed lag-functions outperform unrestricted polynomials. The good performance of U-MIDAS for small differences in frequency is confirmed in an empirical application on nowcasting Euro area and US GDP using monthly indicators.

JEL Classification: C53 and E37
Keywords: distributed lag polynomals, mixed data sampling, nowcasting and time aggregation

Claudia Foroni
Department of Economics
European University Institute
Via della Piazzuola 43
50133 Florence
ITALY

Email:
Claudia.Foroni@EUI.eu

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Massimiliano Marcellino
Department of Economics
European University Institute
Via della Piazzuola 43
50133 Florence
ITALY

Email:
massimiliano.marcellino@eui.eu

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Christian Schumacher
Economic Research Centre
Deutsche Bundesbank
Wilhelm-Epstein-Straße 14
60431 Frankfurt/Main
GERMANY

Email: christian.schumacher@bundesbank.de

For further Discussion Papers by this author see:
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1 Introduction

Economic time series differ substantially with respect to their sampling frequency. For example, financial variables are observable daily or even intra-daily, whereas national accounts data such as GDP is available at quarterly frequency depending on the rules applied in statistical agencies. This raises the problem of how to conduct empirical analyses on the relationships between variables sampled at different frequencies.

The simplest solution is to work at the lowest frequency in the data, e.g. quarterly when some variables are available on a monthly basis and others on a quarterly basis. This requires time aggregation of high-frequency variables with a loss of potentially relevant high-frequency information, and a convolution of the dynamic relationships among the variables (see e.g. Marcellino (1999)).

As an alternative, mixed data-sampling (MIDAS) regressions as proposed by Ghysels, Santa-Clara, and Valkanov (2005, 2006), Ghysels, Sinko, and Valkanov (2007) and Andreou, Ghysels and Kourtellos (2010a, 2010b), amongst others, directly relate variables sampled at different frequencies without loosing high-frequency information. To allow for dynamics, MIDAS regressions are typically based on distributed lag polynomials such as the exponential Almon lag to ensure a parsimonious specification (Ghysels, Sinko, and Valkanov, 2007). Due to the non-linearity of the lag polynomials, MIDAS regressions are typically estimated by non-linear least squares (NLS) following the literature on distributed lag models (Lütkepohl, 1981; Judge et al., 1985).

MIDAS regressions have been applied in the financial literature, see for example Ghysels, Rubia, and Valkanov (2009) in the context of volatility forecasting. In the macroeconomic literature, applications are often related to nowcasting and forecasting. For example, Clements and Galvao (2008, 2009) proposed to use MIDAS for forecasting quarterly GDP growth using monthly business cycle indicators, see also Kuzin, Marcellino, and Schumacher (2011), Bai, Ghysels, and Wright (2010), Marcellino and Schumacher (2010), amongst others. The recent application by Andreou, Ghysels and Kourtellos (2010b) proposes MIDAS regressions when daily financial data is used to forecast quarterly GDP.

An alternative way to handle mixed-frequency data requires to write the model in state space form with time-aggregation schemes, see e.g. Mariano and Murasawa (2003). Kuzin et al. (2011) compare mixed frequency VARs estimated with the Kalman filter with MIDAS regressions, finding an unclear ranking but confirming the good performance of MIDAS. Bai et al. (2010) compare MIDAS regressions to state space models and discuss the approximating properties of MIDAS.

In this paper, we study the performance of a variant of MIDAS which does not resort to functional distributed lag polynomials. In particular, we discuss the pros and cons of MIDAS regressions with unrestricted linear lag polynomials, which do not require NLS, but can be estimated by OLS. We will call this approach from now on unrestricted MIDAS, or U-MIDAS in brief, and compare it to the standard MIDAS approach based on the exponential Almon lag following Ghysels et al. (2005, 2006), which we simply denote as MIDAS. One reason that motivates the use of U-MIDAS in macroeconomic applications is that the difference between sampling frequencies is in many applications not so high. For example, many of the cited papers use monthly data, such as survey outcomes or industrial production, to predict quarterly GDP growth. In that case, the number of monthly lags necessary to estimate the lag polynomials might not be too large, implying that a curse of dimensionality might not be relevant. However, when financial data come into play as in Andreou, Ghysels, and Kourtellos (2010b) and Monteforte and Moretti (2010), we face more severe limits in the degrees of freedom and functional lag polynomials may be preferable.

Koenig, Dolmas, and Piger (2003) already proposed U-MIDAS in the context of real-time estimation. Clements and Galvao (2008, 2009) also considered U-MIDAS to forecast quarterly GDP, and Marcellino and Schumacher (2010) provide an application in a factor model framework. Rodriguez and Puggioni (2010) discuss Bayesian estimation of unrestricted MIDAS equations. However, none of these papers systematically studies the role of the functional form of the lag polynomial.

We expand on the existing literature in the following respects. We discuss how U-MIDAS regressions can be derived in a general linear dynamic framework, and under which conditions the parameters of the underlying high-frequency model can be identified.

Next, we provide Monte Carlo simulations that help highlighting the advantages and disadvantages of U-MIDAS versus MIDAS. The basic design of the exercise is similar to that of Ghysels and Valkanov (2006), where a high-frequency VAR(1) is specified. We look both at the in-sample and out-of-sample nowcasting performance. We find that if the frequency mismatch is small, i.e. when mixing monthly and quarterly data, U-MIDAS is indeed better than MIDAS. With larger differences in sampling frequencies, MIDAS with exponential Almon lag polynomials is instead preferable. We also consider the case in which the restricted MIDAS model is the true DGP. Even in this favorable set up for functional lag polynomials, it turns out that U-MIDAS is still preferable when the frequency mismatch is small.

Finally, we carry out an empirical exercise, where GDP growth in the US and euro area are related to different monthly indicators. In the comparison, we clearly find a better in-sample performance of the U-MIDAS model, confirming the results of the Monte Carlo experiments. The evidence is more mixed when looking at the out-of-sample nowcasting performance. However, for a number of indicators, U-MIDAS can outperform MIDAS also out-of-sample. We conclude that U-MIDAS can be a strong competitor for MIDAS. This generally holds if the differences in sampling frequency in the data are small, in particular, when mixing quarterly and monthly data.

The paper proceeds as follows. In Section 2 we provide a theoretical motivation for U-MIDAS in a linear dynamic framework and discuss its use for nowcasting and forecasting. In Section 3 we present the results of the Monte Carlo experiments. In Section 4 we discuss the empirical nowcast exercises for the US and in Section 5 for the euro area. In

Section 6 we summarize the main results and conclude.

2 The rationale behind U-MIDAS and its use in forecasting

In this section we derive the Unrestricted MIDAS (U-MIDAS) regression approach from a general dynamic linear model, consider its use as a forecasting device, and compare it with the original MIDAS specification of Ghysels et al. (2005, 2006).

2.1 U-MIDAS regressions in dynamic linear models

We assume that y and the N variables x are generated by the $VAR(p)$ process

$$\begin{pmatrix} \underset{1\times1}{a(L)} & \underset{1\times N}{-b(L)} \\ \underset{N\times1}{-d(L)} & \underset{N\times N}{C(L)} \end{pmatrix} \begin{pmatrix} \underset{1\times1}{y_t} \\ \underset{N\times1}{x_t} \end{pmatrix} = \begin{pmatrix} \underset{1\times1}{e_{yt}} \\ \underset{N\times1}{e_{xt}} \end{pmatrix}, \tag{1}$$

or

$$a(L)y_t = b_1(L)x_{1t} + ... + b_N(L)x_{Nt} + e_{yt} \tag{2}$$

$$C(L)x_t = d(L)y_t + e_{xt} \tag{3}$$

where $a(L) = 1 - a_1L - ... - a_pL$, $b(L) = (b_1(L), ..., b_N(L))$, $b_j(L) = b_{j1}L + ... + b_{jp}L^p$, $j = 1, ..., N$, $d(L) = (d_1(L), ..., d_N(L))'$, $d_j(L) = d_{j1}L + ... + d_{jp}L^p$, $C(L) = I - C_1L - ... - C_pL^p$, and the errors are jointly white noise. For simplicity, we suppose that the starting values $y_{-p}, ..., y_0$ and $x_{-p}, ..., x_0$ are all fixed and equal to zero, which coincides with the unconditional expected value of y and x. Different lag lengths of the polynomials in (2) and (3) can be easily handled, but at the cost of an additional complication in the notation.

We then assume that x can be observed for each t, while y can be only observed every k periods. For example, $k = 3$ when t is measured in months and y is observed quarterly (e.g., x could contain industrial production and y GDP growth), while $k = 4$ when t is measured in quarters and y is observed annually (e.g., x could contain GDP growth and y fiscal variables that are typically only available on an annual basis). Let us indicate the aggregate (low) frequency by τ, while Z is the lag operator at τ frequency, with $Z = L^k$ and $Zy_\tau = y_{\tau-1}$. In the sequel, HF indicates high frequency (t) and LF low frequency (τ).

Let us then introduce the operator

$$\omega(L) = \omega_0 + \omega_1L + ... + \omega_{k-1}L^{k-1}, \tag{4}$$

which characterizes the temporal aggregation scheme. For example, $\omega(L) = 1 + L + ... + L^{k-1}$ in the case of flow variables and $\omega(L) = 1$ for stock variables.

While general, this framework still imposes a few restrictions. In particular, y is univariate and there are no MA components in the generating mechanism of y and x. These restrictions simplify substantially the notation, and are helpful for the identification of the parameters of the HF model for y given the LF model. The framework remains general enough to handle the majority of empirical applications, and the extensions are theoretically simple but notationally cumbersome.

The method we adopt to derive the generating mechanism for y at LF is similar to that introduced by Brewer (1973), refined by Wei (1981) and Weiss (1984), and further extended by Marcellino (1999) to deal with general aggregation schemes and multivariate processes.

Let us introduce a polynomial in the lag operator, $\beta(L)$, whose degree in L is at most equal to $pk-p$ and which is such that the product $h(L)=\beta(L)a(L)$ only contains powers of $L^k=Z$, so that $h(L)=h(L^k)=h(Z)$. It can be shown that such a polynomial always exists, and its coefficients depend on those of $a(L)$, see the above references for details.

In order to determine the AR component of the LF process, we then multiply both sides of (2) by $\omega(L)$ and $\beta(L)$ to get

$$h(L^k)\omega(L)y_t=\beta(L)b_1(L)\omega(L)x_{1t}+...+\beta(L)b_N(L)\omega(L)x_{Nt}+\beta(L)\omega(L)e_{yt}. \qquad (5)$$

Thus, the order of the LF AR component, $h(Z)$, is at most equal to p. In addition, the polynomial $h(L^k)$ can be decomposed into

$$\prod_{s=1}^{h}\prod_{i=1}^{k}(1-\frac{1}{h_{si}}L), \qquad (6)$$

where $h<p$ is more precisely defined in Appendix A.1, and at least one h_{si} for each s has to be such that $a(h_{si})=0$.

It can be shown that, in general, there is an MA component in the LF model, $q(Z)u_{yt}$. Its order, q, coincides with the highest multiple of k non zero lag in the autocovariance function of $\beta(L)\omega(L)e_{yt}$. The coefficients of the MA component have to be such that the implied autocovariances of $q(Z)u_{yt}$ coincide with those of $\beta(L)\omega(L)e_{yt}$ evaluated at all multiples of k.

Let us consider now the x variables, which are observable at frequency t. The polynomials $\beta(L)b_j(L)\omega(L)=b_j(L)\beta(L)\omega(L)$, $j=1,...,N$, are at most of order $pk+k-1$. Each term $\beta(L)\omega(L)x_{jt}$ is a particular combination of HF values of x_j that affects the LF values of y.

In Appendix A.1 we show that, under certain rather strict conditions, it is possible to recover the polynomials $a(L)$ and $b_j(L)$ that appear in the HF model for y from the LF model, and therefore also $\beta(L)$ can be identified. In this case we can use the *exact*

MIDAS model

$$
\begin{aligned}
h(L^k)\omega(L)y_t &= b_1(L)z_{1t} + ... + b_N(L)z_{Nt} + q(L^k)u_{yt}, \qquad (7)\\
z_{jt} &= \beta(L)\omega(L)x_{it}, \qquad j = 1, ..., N,\\
t &= k, 2k, 3k, ...
\end{aligned}
$$

The left-hand side of this is equation contains the LF variable y, obtained from time aggregation $\omega(L)y_t = y_\tau$. The LF variable is regressed on its own LF lags and on lags of x_{jt} for $j = 1, ..., N$. As the polynomials $a(L)$ and $b_j(L)$ are identified, there is no need for a polynomial approximation.

When $\beta(L)$ cannot be identified, we can use an *approximate unrestricted MIDAS* model based on a linear lag polynomial such as

$$
\begin{aligned}
c(L^k)\omega(L)y_t &= \delta_1(L)x_{1t-1} + ... + \delta_N(L)x_{Nt-1} + \epsilon_t, \qquad (8)\\
t &= k, 2k, 3k, ...
\end{aligned}
$$

where $c(L^k) = (1 - c_1L^k - ... - c_cL^{kc})$, $\delta_j(L) = (\delta_{j,0} + \delta_{j,1}L + ... + \delta_{j,v}L^v)$, $j = 1, ..., N$. We label this approach hereafter *unrestricted MIDAS* or simply *U-MIDAS*. [1] [2]

Notice that, since the polynomials $\delta_i(L)$ operate at HF while $c(L^k)$ in LF, the matrix of regressors in (8) is of the type

$$
\begin{matrix}
y_0 & \cdots & y_{-kc} & \delta_{1,0}x_{1,k-1} & \cdots & \delta_{1,v}x_{1,k-v-1} & \cdots & \delta_{N,0}x_{N,k-1} & \cdots & \delta_{N,v}x_{N,k-v-1}\\
y_k & \cdots & y_{-(k-1)c} & \delta_{1,0}x_{1,2k-1} & \cdots & \delta_{1,v}x_{1,2k-v-1} & \cdots & \delta_{N,0}x_{N,2k-1} & \cdots & \delta_{N,v}x_{N,2k-v-1}\\
\vdots & & \vdots & \vdots & & \vdots & & \vdots & & \vdots\\
y_{Tk-k} & \cdots & y_{Tk-kc} & \delta_{1,0}x_{1,Tk-1} & \cdots & \delta_{1,v}x_{1,Tk-v-1} & \cdots & \delta_{N,0}x_{N,Tk-1} & \cdots & \delta_{N,v}x_{N,Tk-v-1}
\end{matrix}
$$

As an example, if $\omega(L) = 1$, i.e. y is a stock variable, and $k = 3$ (i.e., t is monthly and τ is quarterly), the matrix of regressors becomes

$$
\begin{matrix}
y_0 & \cdots & y_{-3c} & \delta_{1,0}x_{1,3-1} & \cdots & \delta_{1,v}x_{1,3-v-1} & \cdots & \delta_{N,0}x_{N,3-1} & \cdots & \delta_{N,v}x_{N,3-v-1}\\
y_3 & \cdots & y_{-2c} & \delta_{1,0}x_{1,6-1} & \cdots & \delta_{1,v}x_{1,6-v-1} & \cdots & \delta_{N,0}x_{N,6-1} & \cdots & \delta_{N,v}x_{N,6-v-1}\\
\vdots & & \vdots & \vdots & & \vdots & & \vdots & & \vdots\\
y_{3T-3} & \cdots & y_{3T-3c} & \delta_{1,0}x_{1,3T-1} & \cdots & \delta_{1,v}x_{1,3T-v-1} & \cdots & \delta_{N,0}x_{N,3T-1} & \cdots & \delta_{N,v}x_{N,3T-v-1}
\end{matrix}
$$

Note that if we assume that the lag orders c and v are large enough to make the error term ϵ_t uncorrelated, then, all the parameters in the U-MIDAS model (8) can be estimated by simple OLS (while the aggregation scheme $\omega(L)$ is supposed known). From

[1] The static version of U-MIDAS corresponds to the direct mixed frequency regression model of Kvedaras and Rackauskas (2010). They only consider static regressions, but allow for a larger set of aggregation schemes.

[2] In general, the error term ϵ_t has an MA structure. However, for the sake of simplicity, we will work with an AR approximation throughout, since this does not affect the main points we want to make and simplifies both the notation and the estimation

a practical point of view, the lag order v could differ across variables, and v_i and c could be selected by an information criterion such as BIC. We will follow this approach in the Monte Carlo experiments and in the empirical applications, combining it with the use of information criterion for lag length selection.

2.2 Forecasting with U-MIDAS

To start with, let us consider the case where the forecast origin is in period $t = Tk$ and the forecast horizon measured in t time is $h = k$, namely, one LF period ahead. Using standard formulae, the optimal forecast (in the MSE sense and assuming that ϵ_t is uncorrelated) can be expressed as

$$\widehat{y}_{Tk+k|Tk} = (c_1 L^k + ... + c_c L^{kc}) y_{Tk+k} + \delta_1(L)\widehat{x}_{1Tk+k-1|Tk} + ... + \delta_N(L)\widehat{x}_{NTk+k-1|Tk}, \quad (9)$$

where $\widehat{x}_{iTk+j|Tk} = x_{iTk+j|Tk}$ for $j \leq T$.

A problem with the expression in (9) is that forecasts of future values of the HF variables x are also required. Following e.g. Marcellino et al. (2006), a simpler approach is to use a form of direct estimation and construct the forecast as

$$\widetilde{y}_{Tk+k|Tk} = \widetilde{c}(L^k) y_{Tk} + \widetilde{\delta}_1(L) x_{1Tk} + ... + \widetilde{\delta}_N(L) x_{NTk}, \quad (10)$$

where the polynomials $\widetilde{c}(Z) = \widetilde{c}_1 L^k + ... + \widetilde{c}_c L^{kc}$ and $\widetilde{\delta}_i(L)$ are obtained by projecting y_t on information dated $t - k$ or earlier, for $t = k, 2k, ..., Tk$. We will use this approach in the Monte Carlo simulations and empirical applications. In general, the direct approach of (10) can also be extended to construct hk-step ahead forecasts given information in Tk:

$$\overline{y}_{Tk+hk|Tk} = \overline{c}(L^k) y_{Tk} + \overline{\delta}_1(L) x_{1Tk} + ... + \overline{\delta}_N(L) x_{NTk}, \quad (11)$$

where the polynomials $\overline{c}(Z)$ and $\overline{\delta}_i(L)$ are obtained by projecting y_t on information dated $t - hk$ or earlier, for $t = k, 2k, ..., Tk$.[3]

The conditioning information set for forecasting in (10) contains HF information up to the end of the sample of the LF variable, namely period Tk. An advantage of the MIDAS approach is that it also allows for incorporating leads of the HF variable x_t for the projections. This is due to the fact that observations of HF indicators are much earlier available than the observations of the LF models, for example, surveys or industrial production. MIDAS with leads can exploit this early information and thus is in particular helpful for nowcasting, namely, computing and updating projections of the LF variable for the current period given all potential HF information which is available (Giannone et al., 2008; Marcellino and Schumacher, 2010; Andreou et al., 2010b; Kuzin et al., 2011).

[3] Marcellino and Schumacher (2010) present the details of the derivation of a direct forecasting equation for the case where the regressors are factors extracted from a large set of high frequency indicators. A similar approach can be used in this context to derive (10) from a given HF VAR DGP.

Nowcasting with HF indicators becomes politically relevant, because the publication lags for many LF variables are quite substantial. For example, quarterly GDP in US is typically published after about four weeks in the subsequent quarter. Thus, within each quarter, the contemporaneous value of GDP growth is not available, making nowcasts necessary.

As a particular nowcasting example, suppose that the value of interest is still y_{Tk+k}, but that now HF information up to period $Tk+1$ is available, e.g. observations of monthly industrial production on the first month of a given quarter becomes available. Then, the expression in (9) can be easily modified to take the new information into account:

$$\widehat{y}_{Tk+k|Tk+1} = \widetilde{c}(L^k)y_{Tk} + \delta_1(L)\widehat{x}_{1Tk+k-1|Tk+1} + ... + \delta_N(L)\widehat{x}_{NTk+k-1|Tk+1}, \qquad (12)$$

where $\widehat{x}_{iTk+j|Tk+1} = x_{iTk+j|Tk+1}$ for $j \leq T+1$. Similarly, the coefficients in (10) would be now obtained by projecting y_t on information dated $t-k+1$ or earlier and the direct forecast becomes

$$\widetilde{y}_{Tk+k|Tk+1} = \widetilde{c}(L^k)y_{Tk} + \widetilde{\delta}_1(L)x_{1Tk+1} + ... + \widetilde{\delta}_N(L)x_{NTk+1}. \qquad (13)$$

If time passes by and new HF information becomes available, say, in periods $Tk+1, Tk+2, \ldots$, the nowcast can be updated similar to the one-step ahead case.

2.3 U-MIDAS and MIDAS with exponential Almon lags

It is interesting to compare the U-MIDAS approach with the original MIDAS specification of Ghysels et al. (2005, 2006) with functional lag polynomials, see also Clements and Galvao (2008). Assuming for simplicity that $c(L^k) = 1$, $N = 1$, $\omega(L) = 1$, the U-MIDAS model in (8) simplifies to

$$y_t = \delta_1(L)x_{t-1} + \epsilon_t \qquad (14)$$

for $t = k, 2k, ..., Tk$. The original MIDAS model would be

$$y_t = \beta_1 B(L,\theta)x_{t-1} + \varepsilon_t, \qquad (15)$$

where the polynomial $B(L,\theta)$ is the exponential Almon lag following Lütkepohl (1981) with

$$B(L,\theta) = \sum_{j=0}^{K} b(j,\theta)L^j, \quad b(j,\theta) = \frac{\exp(\theta_1 j + \theta_2 j^2)}{\sum_{j=0}^{K}\exp(\theta_1 j + \theta_2 j^2)}. \qquad (16)$$

Therefore, the MIDAS specification of Ghysels et al. (2005, 2006) is nested in U-MIDAS, since it is obtained by imposing a particular dynamic pattern. The key advantage of the original MIDAS specification is that it allows for long lags with a limited number of parameters, which can be particularly useful in financial applications with a high mismatch between the sampling frequencies of y and x, e.g. when y is monthly and x is daily.

However, for macroeconomic applications with small differences in sampling frequencies, for example monthly and quarterly data, the specification in (15) can have several disadvantages. In particular, it could simply be the case that the restriction $\beta_1 B(L,\theta) = \delta_1(L)$ is not valid and that the Almon lag approximation might not be general enough. Additionally, if the impulse response function is relatively short-lived and only a few HF lags are needed to capture the weights, a linear unrestricted lag polynomial might suffice for estimation. Moreover, the model resulting from (15) is highly nonlinear in the parameters, so that it cannot be estimated by OLS. In summary, these considerations suggest that U-MIDAS should perform better than the original MIDAS as long as the aggregation frequency is small and U-MIDAS is not too heavily parameterized.

In general, it should be kept in mind that both MIDAS and U-MIDAS should be regarded as approximations to dynamic linear models such as the one discussed in Section 2.1. Since we do not know the true model in practice, we cannot expect one of the approaches to dominate with empirical data. However, given a known DGP, it might be useful to identify conditions under which MIDAS or U-MIDAS do better. Thus, we will consider both approaches in the simulations below.

3 Monte Carlo experiments

This Section presents a set of Monte Carlo experiments that focus on the in-sample and forecasting performance of alternative MIDAS regressions. We discuss, in turn, the basic simulation design, the models under comparison, and the results. Next, as a robustness check, we present results for alternative simulation designs.

3.1 The simulation design

The simulation design is closely related to that in Ghysels and Valkanov (2006). We modify the exercise in a way to discuss its use in macroeconomic forecasting, in particular forecasting quarterly GDP. As predictors for this variable, the empirical literature typically uses monthly and daily indicators, see Clements and Galvao (2008, 2009) and Andreou, Ghysels and Kourtellos (2010b), respectively. Additionally, we consider weekly data. Thus, we end up with the sampling frequencies $k = \{3, 12, 60\}$, which represent cases of data sampled at monthly and quarterly frequency ($k = 3$), at weekly and quarterly frequency ($k = 12$), or at daily and quarterly frequency ($k = 60$).

In each case, the DGP is given by the high-frequency VAR

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{pmatrix} \rho & \delta_l \\ \delta_h & \rho \end{pmatrix} \begin{pmatrix} y_{t-1} \\ x_{t-1} \end{pmatrix} + \begin{pmatrix} e_{y,t} \\ e_{x,t} \end{pmatrix}. \tag{17}$$

y_t is the LF variable and x_t is the HF variable, where t is the HF time index with $t = 1, \ldots, (T + ES) \times k$. T defines the size of the estimation sample we use in the in-sample analysis (expressed in the low-frequency unit, e.g. quarters in our example), while for the forecasting purposes we generate an additional number of observations, which defines our evaluation sample, ES (also expressed in the low-frequency unit). k denotes the sampling frequency of the LF variable y_t, whereas x_t is sampled with $k = 1$. We further assume that $\omega(L) = 1$. Thus, the LF variable y_t is available only for $t = k, 2k, \ldots, (T + ES) \times k$.

In the VAR (17), the shocks $e_{y,t}$ and $e_{x,t}$ are sampled independently from the normal distribution with mean zero for all $t = 1, \ldots, (T + ES) \times k$, and the variances are chosen such that the unconditional variance of y is equal to one. For the persistence parameter, we choose $\rho = \{0.1, 0.5, 0.9\}$, and following Ghysels and Valkanov (2006), we fix $\delta_l = \{0.1, 0.5, 1.0\}$ and $\delta_h = 0$. Thus, the DGP is recursive, as the HF variable affects the LF variable, but not vice versa. Later on in Section 3.6, we will also consider non-recursive DGPs. Overall, we cover a broad range of DGPs representing different degrees of persistence and correlation between the HF and the LF variable.

Initially, for the in-sample analysis, y_t and x_t are simulated for all $t = 1, \ldots, T \times k$, see Ghysels and Valkanov (2006). To estimate the MIDAS regressions, the available data is defined as y_t with $t = k, 2k, \ldots, T \times k$ and x_t with $t = 1, 2, \ldots, T \times k$, representing mixed-frequency data which is typical in empirical applications. The number of observations for the LF variable is $T = 100$. To compare the in-sample fit obtained with the different methods, we look at the in-sample MSE for each simulation, defined as $IS - MSE_r = \frac{1}{T}\sum_{t=1}^{T}(\widehat{y}_{t,r} - y_{t,r})^2$, with $r = 1, ..., R$. In this experiment, we fix the number of simulations at $R = 2000$.[4]

In order to conduct a forecast comparison, we also simulate both variables $ES \times k$ HF periods ahead for $t = T \times k + 1, \ldots, T \times k + ES \times k$. ES is set equal to $\frac{T}{2} = 50$. The final values of the LF variable, from $y_{T\times k+k}$ to $y_{T\times k+ES\times k}$, will be used as the actual values to be compared with the alternative forecasts. Regarding the information set available for forecasting, we assume that we know values up to period $(T + es - 1) \times k$, with $es = 1, ..., ES$, for the LF variable and $(T + es - 1) \times k + k - 1$ for the HF variable x_t. This yields forecasts of the LF variable k HF periods ahead for each date in the evaluation sample, $y_{T\times k+es\times k|T\times k+es\times k-1}$, conditional on HF information within the LF forecast period. The corresponding forecast error is $y_{T\times k+es\times k|T\times k+es\times k-1} - y_{T\times k+es\times k}$. The latter is used to compute the mean-squared error (MSE) over the evaluation sample for each replication r, as $MSE_r = \frac{1}{ES}\sum_{es=1}^{ES}(y_{T\times k+es\times k|T\times k+es\times k-1,r} - y_{T\times k+es\times k,r})^2$, where $r = 1, ...R$, and in our experiment $R = 500$.[5]

For the in- and out-of-sample analyses we report summary statistics based on the empirical distribution of the IS-MSEs and MSEs over all replications, i.e., its average,

[4] We consider the IS-MSE to be coherent with the forecasting analysis later where, following common practice, we adopt the squared-error loss function. For the in-sample analysis, we could as well look at the R^2, a more traditional measure of goodness-of-fit, but the results would be qualitatively similar since both measures are based on the residual sum of squares.

[5] The number of replications in this case is only 500 for computational issues. In fact, for each replication we need to estimate the models and then compute the forecast 50 times, one for each quarter of the evaluation sample. Therefore, even with 500 replications, we obtain 25000 forecasts.

median, and selected percentiles.

3.2 The models under comparison

We consider empirical MIDAS regressions that are based on estimated coefficients and possibly misspecified functional forms. In particular, we evaluate the following two types of models:

1. MIDAS with an autoregressive term as used in Clements and Galvao (2008, 2009). This follows from the fact that the HF VAR also implies an autoregressive term. In order to rule out periodic movements in the impulse response function from the HF variable x_t on the LF variable y_t, a common factor specification is imposed, yielding the model

$$y_{t\times k} = \beta_0 + \beta_1 y_{t\times k-k} + \beta_2(1-\beta_1 L^k)B(L,\theta)x_{t\times k-1} + \varepsilon_{t\times k}, \tag{18}$$

   where the polynomial $B(L,\theta)$ is the exponential Almon lag defined as in eq. (16). We set $K = 1.5 \times k$, which suffices to capture the true impulse response function as implied by the DGP. The model is estimated using NLS, with the additional coefficient λ and the common factor structure imposed. We apply the coefficient restrictions $-100 < \theta_1 < 5$ and $-100 < \theta_2 < 0$. As starting values might matter, we compute the residual sum of squares in each replication for alternative parameter pairs in the sets $\theta_1 = \{-0.5, 0.0, 0.5\}$ and $\theta_2 = \{-0.01, -0.1, -0.5, -1\}$. Note that when x_t is available until $T\times k+es\times k-1$, this is the final date used to estimate the coefficients of the model. Given the NLS estimates of the parameters, the forecast can be computed as

$$y_{T\times k+es\times k|T\times k+es\times k-1} = \widehat{\beta}_0 + \widehat{\beta}_1 y_{T\times k+es\times k-k} + \widehat{\beta}_2(1-\widehat{\beta}_1 L^k)B(L,\widehat{\theta})x_{T\times k+es\times k-1}. \tag{19}$$

2. U-MIDAS as introduced in Section 2. In particular, U-MIDAS is estimated without considering the Almon lag polynomial. Rather, we leave the lag polynomial of the indicator HF variable x_t unrestricted. Furthermore, we do not impose the common factor restriction as in Clements and Galvao (2008) for the autoregressive term. Thus, the model becomes

$$y_{t\times k} = \mu_0 + \mu_1 y_{t\times k-k} + \psi(L)x_{t\times k-1} + \varepsilon_{t\times k}, \tag{20}$$

   with lag polynomial $\psi(L) = \sum_{j=0}^{K} \psi_j L^j = \psi_0 + \psi_1 L + \ldots + \psi_K L^K$. The coefficients μ_0, μ_1 and $\psi(L)$ are estimated by OLS. To specify the lag order, we use the BIC with a maximum lag order of K to choose the lags. Given the selected BIC order $\widehat{k}$ and OLS estimated parameters $\widehat{\mu}_0, \widehat{\mu}_1$ and $\widehat{\psi}(L)$, the U-MIDAS forecast can be computed as

$$y_{T\times k+es\times k|T\times k+es\times k-1} = \widehat{\mu}_0 + \widehat{\mu}_1 y_{T\times k+es\times k-k} + \widehat{\psi}(L) x_{T\times k+es\times k-1}. \quad (21)$$

3.3 Monte Carlo in-sample comparison results

We summarize the results of the in-sample evaluation in Table 1. As anticipated, we compare the performance of the two different methods, U-MIDAS and MIDAS, based on their respective in-sample MSEs. The table reports summary statistics for the distribution of the in-sample MSE of U-MIDAS relative to that of MIDAS as described in the previous subsection for alternative parameter values and sampling frequencies. More in detail, we first compute the ratio IS-MSE(U-MIDAS)/IS-MSE(MIDAS) for each replication, then report the mean, the median and selected percentiles of the distribution of these ratios. Hence, median, mean, and percentile ratio values smaller than one indicate a superior performance of U-MIDAS.

The results can be summarized as follows. When the difference in sampling frequencies is large ($k = 12, 60$), restricted MIDAS outperforms U-MIDAS for almost each value of persistence and interrelatedness. Instead, U-MIDAS clearly outperforms MIDAS for $k = 3$, especially when the process is persistent, that is when $\rho = \{0.5, 0.9\}$. The distribution results tend to be very concentrated, especially when $\rho = 0.1$, while the values of the ratio are slightly more dispersed when the persistence is bigger. It is very interesting to notice that when $\rho = \{0.5, 0.9\}$, U-MIDAS models outperform systematically MIDAS models, since even the values correspondent to the 90th percentile are smaller than 1.

3.4 Monte Carlo forecast comparison results

The results of the Monte Carlo forecast experiments are summarized in Table 2. As in the in-sample analysis, we compare the forecasting performance of U-MIDAS and MIDAS based on their (out-of-sample) MSEs, computed over the 50 periods of the evaluation sample. The Table reports the summary statistics for the distribution of the MSE of U-MIDAS relative to that of MIDAS, as described in Section 3.3 for the in-sample analysis. Again, a median or mean value smaller than one indicates a superior performance of U-MIDAS.

The results confirm the evidence of the in-sample analysis: U-MIDAS forecasts perform better than MIDAS for $k = 3$ if the process is persistent. If the difference in sampling frequencies is large ($k = 12, 60$), MIDAS outperforms U-MIDAS for almost each value of persistence and interrelatedness. A likely reason for this pattern of results is that when k is large, U-MIDAS becomes heavily parameterized, notwithstanding BIC lag length selection, and imprecise estimation affects the forecasting accuracy. On the contrary, when $k = 3$, the number of U-MIDAS parameters is limited and their estimates precise, and the additional flexibility allowed by U-MIDAS yields a better forecasting performance

Table 1: In-sample MSE of U-MIDAS relative to in-sample MSE of MIDAS (DGP: recursive HF VAR)

rho	delta l	delta h	k	mean	relative performance: 10th prctile	25th prctile	median	75th prctile	90th prctile
0.1	0.10	0.00	3	1.01	0.97	0.99	1.00	1.02	1.04
0.1	0.10	0.00	12	1.01	0.97	0.99	1.00	1.02	1.05
0.1	0.10	0.00	60	1.01	0.98	0.99	1.00	1.02	1.04
0.1	0.50	0.00	3	1.01	0.98	0.99	1.00	1.03	1.04
0.1	0.50	0.00	12	1.01	0.97	0.99	1.00	1.02	1.04
0.1	0.50	0.00	60	1.01	0.97	0.99	1.00	1.02	1.04
0.1	1.00	0.00	3	1.01	0.98	0.99	1.00	1.02	1.04
0.1	1.00	0.00	12	1.01	0.97	0.99	1.00	1.02	1.04
0.1	1.00	0.00	60	1.01	0.97	0.99	1.00	1.02	1.04
0.5	0.10	0.00	3	0.92	0.85	0.89	0.93	0.96	0.99
0.5	0.10	0.00	12	1.00	0.94	0.97	1.00	1.03	1.06
0.5	0.10	0.00	60	1.00	0.94	0.97	1.00	1.03	1.06
0.5	0.50	0.00	3	0.92	0.85	0.89	0.93	0.96	0.99
0.5	0.50	0.00	12	1.00	0.94	0.97	1.00	1.03	1.05
0.5	0.50	0.00	60	1.00	0.94	0.97	1.00	1.03	1.06
0.5	1.00	0.00	3	0.92	0.85	0.88	0.92	0.96	0.99
0.5	1.00	0.00	12	1.00	0.94	0.97	1.00	1.03	1.06
0.5	1.00	0.00	60	1.00	0.94	0.97	1.00	1.03	1.06
0.9	0.10	0.00	3	0.91	0.83	0.87	0.92	0.96	0.99
0.9	0.10	0.00	12	1.02	0.89	0.95	1.01	1.08	1.14
0.9	0.10	0.00	60	1.14	0.97	1.04	1.13	1.23	1.32
0.9	0.50	0.00	3	0.92	0.83	0.87	0.92	0.96	1.00
0.9	0.50	0.00	12	1.02	0.89	0.95	1.01	1.08	1.14
0.9	0.50	0.00	60	1.13	0.97	1.04	1.13	1.23	1.31
0.9	1.00	0.00	3	0.92	0.83	0.87	0.92	0.96	0.99
0.9	1.00	0.00	12	1.02	0.90	0.95	1.01	1.08	1.15
0.9	1.00	0.00	60	1.13	0.97	1.04	1.13	1.22	1.31

Notes: Columns 1 to 4 show the parameter specification for the DGP in eq. (17). The entries of columns 5 to 10 report the performance of the IS-MSE(U-MIDAS)relative to IS-MSE(MIDAS). The ratio IS-MSE(U-MIDAS)/IS-MSE(MIDAS) is computed for each replication, then in column 5 the mean of the distribution of these ratios is reported, and in columns 6 to 10 the main percentiles (10th, 25th, 50th, 75th, 90th) of the distribution are reported.

Table 2: Out-of-sample MSE of U-MIDAS relative to MSE of MIDAS (DGP: recursive HF VAR)

					relative performance:				
rho	**delta l**	**delta h**	**k**	**mean**	**10th prctile**	**25th prctile**	**median**	**75th prctile**	**90th prctile**
0.1	0.10	0.00	3	1.01	0.96	0.99	1.01	1.03	1.07
0.1	0.10	0.00	12	1.01	0.97	0.99	1.01	1.03	1.06
0.1	0.10	0.00	60	1.01	0.97	0.99	1.01	1.03	1.06
0.1	0.50	0.00	3	1.01	0.96	0.99	1.01	1.03	1.07
0.1	0.50	0.00	12	1.01	0.97	0.99	1.01	1.03	1.06
0.1	0.50	0.00	60	1.01	0.97	0.99	1.01	1.04	1.07
0.1	1.00	0.00	3	1.01	0.97	0.99	1.01	1.03	1.07
0.1	1.00	0.00	12	1.01	0.96	0.99	1.01	1.04	1.07
0.1	1.00	0.00	60	1.01	0.97	0.99	1.01	1.04	1.07
0.5	0.10	0.00	3	0.94	0.83	0.89	0.94	0.98	1.04
0.5	0.10	0.00	12	1.05	0.97	1.00	1.05	1.09	1.13
0.5	0.10	0.00	60	1.05	0.97	1.00	1.04	1.10	1.14
0.5	0.50	0.00	3	0.93	0.82	0.88	0.93	0.98	1.02
0.5	0.50	0.00	12	1.05	0.97	1.01	1.05	1.09	1.13
0.5	0.50	0.00	60	1.05	0.96	1.00	1.05	1.09	1.15
0.5	1.00	0.00	3	0.94	0.83	0.89	0.94	0.99	1.04
0.5	1.00	0.00	12	1.05	0.97	1.00	1.04	1.08	1.14
0.5	1.00	0.00	60	1.05	0.97	1.01	1.05	1.09	1.14
0.9	0.10	0.00	3	0.91	0.80	0.86	0.91	0.96	1.02
0.9	0.10	0.00	12	1.09	0.93	1.01	1.08	1.18	1.25
0.9	0.10	0.00	60	1.23	1.03	1.11	1.21	1.32	1.45
0.9	0.50	0.00	3	0.91	0.81	0.85	0.91	0.97	1.01
0.9	0.50	0.00	12	1.07	0.91	0.98	1.07	1.16	1.24
0.9	0.50	0.00	60	1.24	1.05	1.13	1.23	1.34	1.46
0.9	1.00	0.00	3	0.92	0.81	0.86	0.92	0.98	1.03
0.9	1.00	0.00	12	1.08	0.92	0.99	1.07	1.15	1.23
0.9	1.00	0.00	60	1.22	1.04	1.11	1.21	1.31	1.44

Notes: Columns 1 to 4 show the parameter specification for the DGP in eq. (17). The entries of columns 5 to 10 report the performance of the out-of-sample MSE(U-MIDAS)relative to out-of-sample MSE(MIDAS). The MSE is calculated over an evaluation sample of 50 periods. The ratio MSE(U-MIDAS)/MSE(MIDAS) is computed for each replication, then in column 5 the mean of the distribution of these ratios is reported, and in columns 6 to 10 the main percentiles (10th, 25th, 50th, 75th, 90th) of the distribution are reported.

than MIDAS. An alternative explanation could be that BIC selects models that are too parsimonious when k is large, and therefore omits relevant regressors. We will discuss this possibility in the next subsection.

3.5 The role of BIC for lag length selection

In Sections 3.3 and 3.4, we select the number of lags to be included in the U-MIDAS models according to the BIC criterion, which is known to prefer rather parsimonious models, at least in finite samples. We now want to check whether a different selection criterion for the number of lags can influence the results. In particular, we assess whether the use of the AIC criterion, which puts a lower loss on the number of parameters than BIC, can improve the results, and in particular those for k large.

Table 3: Out-of-sample MSE of U-MIDAS relative to out-of-sample MSE of MIDAS (DGP: recursive HF VAR). AIC selection criterion

					relative performance:				
rho	**delta l**	**delta h**	**k**	**mean**	**10th prctile**	**25th prctile**	**median**	**75th prctile**	**90th prctile**
0.1	0.10	0.00	3	1.01	0.96	0.98	1.00	1.02	1.05
0.1	0.10	0.00	12	1.02	0.97	0.99	1.01	1.05	1.09
0.1	0.10	0.00	60	1.11	0.97	1.00	1.03	1.14	1.37
0.1	0.50	0.00	3	1.01	0.96	0.98	1.00	1.03	1.05
0.1	0.50	0.00	12	1.02	0.97	0.99	1.01	1.05	1.09
0.1	0.50	0.00	60	1.12	0.98	1.00	1.03	1.15	1.45
0.1	1.00	0.00	3	1.01	0.97	0.99	1.01	1.03	1.06
0.1	1.00	0.00	12	1.02	0.96	0.98	1.01	1.05	1.10
0.1	1.00	0.00	60	1.10	0.98	1.00	1.04	1.13	1.32
0.5	0.10	0.00	3	0.93	0.83	0.88	0.93	0.98	1.02
0.5	0.10	0.00	12	1.05	0.96	1.00	1.04	1.09	1.14
0.5	0.10	0.00	60	1.14	0.98	1.01	1.07	1.20	1.40
0.5	0.50	0.00	3	0.93	0.82	0.88	0.92	0.98	1.02
0.5	0.50	0.00	12	1.05	0.97	1.00	1.05	1.09	1.14
0.5	0.50	0.00	60	1.16	0.98	1.02	1.08	1.20	1.48
0.5	1.00	0.00	3	0.94	0.83	0.89	0.93	0.99	1.03
0.5	1.00	0.00	12	1.05	0.97	1.00	1.04	1.09	1.14
0.5	1.00	0.00	60	1.15	0.98	1.03	1.09	1.19	1.42
0.9	0.10	0.00	3	0.91	0.81	0.85	0.90	0.96	1.01
0.9	0.10	0.00	12	1.05	0.92	0.99	1.05	1.12	1.18
0.9	0.10	0.00	60	1.32	1.05	1.13	1.24	1.45	1.71
0.9	0.50	0.00	3	0.91	0.81	0.85	0.91	0.96	1.01
0.9	0.50	0.00	12	1.03	0.91	0.97	1.04	1.09	1.15
0.9	0.50	0.00	60	1.32	1.05	1.13	1.26	1.42	1.68
0.9	1.00	0.00	3	0.92	0.80	0.86	0.92	0.97	1.02
0.9	1.00	0.00	12	1.04	0.92	0.97	1.03	1.10	1.18
0.9	1.00	0.00	60	1.30	1.05	1.12	1.24	1.43	1.64

Notes: See Table 2.

The results in Table 3 imply that there are no gains from the switch in information criterion. As with BIC lag length selection, U-MIDAS outperforms MIDAS only for a small mismatch in sampling frequency ($k = 3$). Moreover, the results are even worse for $k = 60$, confirming that the main problem is the estimation of too heavily parameterized models rather than omitted regressors.[6]

3.6 An alternative DGP with non-recursive VAR structure

To check the robustness of the results we have obtained so far, we now consider an alternative HF data generating process that allows for reverse causality from y to x by setting $\delta_h \neq 0$. The values of δ_l and δ_h cannot be chosen freely but must be selected in order to ensure a non-explosive solution, which ensures stationarity of both y and x. To ensure a stable solution, the condition

$$\det \left[I_2 - \begin{pmatrix} \rho & \delta_l \\ \delta_h & \rho \end{pmatrix} z \right] \neq 0 \text{ for } |z| \leq 1 \tag{22}$$

must hold. In general, the solution is $z_{1,2} = \frac{1}{\rho^2 - \delta_h \delta_l} \left(\rho \pm \sqrt{\delta_h \delta_l}\right)$, depending heavily on the relative size of δ_l and δ_h. For the sake of simplicity, we assume that both processes are equally important for each other, so that $\delta_l = \delta_h = \delta$. This implies the solutions for the characteristic roots are

$$z_{1,2} = \frac{\rho \pm \delta}{(\rho + \delta)(\rho - \delta)}. \tag{23}$$

These roots lie outside the unit circle, if $1 + \delta > \rho$ and $1 - \delta > \rho$. Thus, if we further assume that the series y_t and x_t have a positive impact on each other, $\delta > 0$, only the restriction $1 - \delta > \rho$ is binding. Hence, depending on the selection of $\rho = \{0.1, 0.5, 0.9\}$, we select the ρ, δ couples

$$\begin{gathered} \{0.1, 0.1\}, \{0.1, 0.4\}, \{0.1, 0.8\}, \\ \{0.5, 0.1\}, \{0.5, 0.2\}, \{0.5, 0.4\}, \\ \{0.9, 0.01\}, \{0.9, 0.04\}, \{0.9, 0.08\}, \end{gathered} \tag{24}$$

with varying degrees of persistence and interrelatedness. The results on the relative forecasting performance of U-MIDAS and MIDAS can be found in Table 4.

Overall, the results are in line with those for the benchmark case. With a medium to high degree of persistence, the forecasting performance of U-MIDAS is better than that of MIDAS when $k = 3$, whereas in general MIDAS dominates for $k = \{12, 60\}$.

[6] For the sake of space, for all the robustness checks in this and the next two subsections, we only report the results on the forecasting performance. Similar conclusions however emerge from the in-sample analysis, which is available upon request.

Table 4: Out-of-sample MSE of U-MIDAS relative to out-of-sample MSE of MIDAS (DGP: non-recursive HF VAR)

					relative performance:				
rho	**delta l**	**delta h**	**k**	**mean**	**10th prctile**	**25th prctile**	**median**	**75th prctile**	**90th prctile**
0.1	0.10	0.10	3	1.01	0.96	0.99	1.01	1.03	1.07
0.1	0.10	0.10	12	1.01	0.97	0.99	1.01	1.03	1.06
0.1	0.10	0.10	60	1.01	0.97	0.99	1.01	1.03	1.06
0.1	0.40	0.40	3	1.01	0.97	0.99	1.01	1.03	1.06
0.1	0.40	0.40	12	1.01	0.96	0.99	1.01	1.03	1.06
0.1	0.40	0.40	60	1.01	0.97	0.99	1.01	1.03	1.06
0.1	0.80	0.80	3	1.02	0.96	0.99	1.01	1.04	1.08
0.1	0.80	0.80	12	1.01	0.97	0.99	1.01	1.04	1.07
0.1	0.80	0.80	60	1.01	0.97	0.99	1.01	1.04	1.07
0.5	0.10	0.10	3	0.94	0.83	0.89	0.94	0.99	1.04
0.5	0.10	0.10	12	1.05	0.97	1.00	1.05	1.09	1.13
0.5	0.10	0.10	60	1.05	0.97	1.00	1.04	1.09	1.14
0.5	0.20	0.20	3	0.94	0.83	0.89	0.94	0.98	1.03
0.5	0.20	0.20	12	1.05	0.97	1.01	1.05	1.09	1.12
0.5	0.20	0.20	60	1.05	0.96	1.00	1.05	1.09	1.14
0.5	0.40	0.40	3	0.98	0.90	0.93	0.98	1.03	1.07
0.5	0.40	0.40	12	1.04	0.96	1.00	1.04	1.07	1.12
0.5	0.40	0.40	60	1.04	0.96	1.00	1.03	1.07	1.11
0.9	0.01	0.01	3	0.91	0.81	0.86	0.91	0.96	1.02
0.9	0.01	0.01	12	1.09	0.93	1.01	1.08	1.18	1.25
0.9	0.01	0.01	60	1.23	1.03	1.11	1.21	1.32	1.45
0.9	0.04	0.04	3	0.93	0.83	0.88	0.92	0.98	1.03
0.9	0.04	0.04	12	1.06	0.91	0.98	1.06	1.14	1.22
0.9	0.04	0.04	60	1.22	1.04	1.11	1.20	1.32	1.44
0.9	0.08	0.08	3	0.99	0.93	0.96	0.99	1.02	1.05
0.9	0.08	0.08	12	0.99	0.92	0.96	0.99	1.03	1.06
0.9	0.08	0.08	60	1.00	0.93	0.97	1.00	1.03	1.06

Notes: See Table 2.

3.7 Using MIDAS as DGP

As a final robustness check, we now carry out Monte Carlo simulations using a MIDAS regression equation with exponential Almon lag as the data generating process. Thus, we are in a case that favours a priori the restricted MIDAS regression over the U-MIDAS. The DGP is

$$y_{t\times k+k} = \beta_0 + \beta_1 B(L,\theta) x_{t\times k+k-1} + \varepsilon_{t\times k+k}, \tag{25}$$

with the lag polynomial $B(L,\theta)$ defined as in (16). We set $k = \{3, 12, 60\}$ to mimic the design in the previous sections. When simulating $y_{t\times k+es\times k}$, we use $T = 100$ and the sets $\theta_1 = 0.7$ and $\theta_2 = \{-0.025, -0.05, -0.3\}$. The monthly indicator x_t is generated as an AR(1) process, with persistence equal to 0.9. Given these 9 different DGPs, we again use U-MIDAS and MIDAS as before to forecast $y_{t\times k+es\times k}$ and evaluate their performance by MSE. For estimating MIDAS and U-MIDAS, we set $K = 1.5 \times k$, which again suffices to capture the true impulse response functions.

To fix the starting values of θ_1 and θ_2, we compute the residual sum of squares in each replication for alternative parameter pairs in the sets $\theta_1 = \{-0.5, 0.0, 0.5\}$ and $\theta_2 = \{-0.01, -0.1, -0.5, -1\}$. As initial values, we then choose those θ_1 and θ_2 that minimize the residual sum of squares. Results on the forecasting performance can be found in Table 5.

Table 5: Out-of-sample MSE of U-MIDAS relative to out-of-sample MSE of MIDAS, (DGP: MIDAS)

				relative performance:				
theta1	**theta2**	**k**	**mean**	**10th prctile**	**25th prctile**	**median**	**75th prctile**	**90th prctile**
0.70	-0.025	3	0.95	0.86	0.91	0.96	1.01	1.04
0.70	-0.025	12	1.20	1.00	1.07	1.19	1.31	1.43
0.70	-0.025	60	1.27	1.06	1.15	1.25	1.37	1.48
0.70	-0.05	3	0.90	0.80	0.85	0.90	0.96	1.00
0.70	-0.05	12	1.09	0.98	1.03	1.09	1.15	1.21
0.70	-0.05	60	1.14	1.01	1.07	1.14	1.21	1.28
0.70	-0.3	3	0.98	0.91	0.95	0.98	1.01	1.04
0.70	-0.3	12	1.03	0.96	1.00	1.03	1.07	1.10
0.70	-0.3	60	1.03	0.96	0.99	1.03	1.07	1.10

Notes: Columns 1 to 3 show the parameter specification for the DGP in eq. (25). The entries of columns 4 to 9 report the performance of the out-of-sample MSE(U-MIDAS)relative to out-of-sample MSE(MIDAS). The MSE is calculated over an evaluation sample of 50 periods. The ratio MSE(U-MIDAS)/MSE(MIDAS) is computed for each replication, then in column 4 the mean of the distribution of these ratio is reported, and in columns 5 to 9 the main percentiles (10th, 25th, 50th, 75th, 90th) of the distribution are reported.

Interestingly, the Table highlights that, even in a set up favorable to restricted MIDAS, as long as the frequency mismatch is small ($k = 3$) and therefore the number of parameters to be estimated is low, U-MIDAS still yields a better forecasting performance than MIDAS. Restricted MIDAS is of course strongly outperforming in the case of very large discrepancies in the sampling frequency.

A likely cause for the detected pattern is the following. UMIDAS provides more flexibility than MIDAS, since as we have seen the latter is typically nested in the former. Hence, as long as the number of parameters is rather limited with respect to the sample size, as in the case $k = 3$, UMIDAS is a nesting model and as such it is not surprising that it forecasts slightly better than MIDAS. Computational problems for the NLS estimator of the MIDAS parameters can further add to the advantages of OLS based UMIDAS. Both positive UMIDAS features, nesting and simplicity of estimation, are however more than counterbalanced by the curse of dimensionality when k is large, thus making MIDAS the clear winner in that case.

3.8 Summary of the simulation results

As a general summary of the simulation results, we can say that as long as the dependent variable is sufficiently persistent and the frequency mismatch with the explanatory variables limited, there is strong evidence that the U-MIDAS specification is better than MIDAS. The gains are larger in in-sample analysis, where the estimation sample is longer, but are still present in the out-of-sample comparison.

4 Empirical application to US GDP growth

In this section, we assess the MIDAS and U-MIDAS methods in terms of their in-sample and nowcasting performance. Specifically, we consider nowcasting quarterly US GDP growth using a set of selected monthly indicators as in Clements and Galvao (2009) and Stock and Watson (2003). The indicators are the ten components of the composite leading indicator provided by the Conference Board, starting in January 1959 and updated to July 2011. Appendix A.2 provides a complete description of the data. In the empirical application, we adopt a recursive approach in nowcasting, with the first evaluation quarter fixed at 1985Q1 and the last one at 2011Q1, for a total of 105 evaluation samples. For each quarter, we compute three nowcasts, at the beginning of each month of the quarter, thereby including information up to the end of the previous month. The dataset is a final dataset, but we replicate the ragged-edge structure due to different publication lags of the monthly series. Thus, we take into account the different information sets available at each point in time in which the nowcasts are computed. Each month, the models are re-estimated using all the data available at that point in time. To specify the MIDAS regressions, we consider a maximum number of monthly lags of the indicators equal to $2k = 6$ months.

To compare the two mixed-frequency approaches, we first look at the in-sample performance as in the Monte Carlo experiment. We first compute the in-sample ratio MSE(U-MIDAS)/MSE(MIDAS) for each monthly indicator. Table 6 shows the results of the in-sample performance. In panel A, we report the in-sample results computed with data up to the fourth quarter 2006, in panel B those computed based on data up to the fourth quarter 2010, so that we can compare the performance before and during the recent financial crisis.

Table 6: Results for individual indicators. In-sample analysis for US GDP growth

Relative in-sample MSE (MSE(U-MIDAS) / MSE(MIDAS))

A. Sample 1959Q1 - 2006Q4				B. Sample 1959Q1 - 2011Q1			
	h$_m$				**h**$_m$		
	1	**2**	**3**		**1**	**2**	**3**
M2	0.97	0.99	1.00	M2	0.96	0.98	0.99
stock	0.87	0.99	0.99	stock	0.95	0.98	0.98
hours	0.73	0.84	0.81	hours	0.74	0.84	0.79
ordersn	0.98	0.92	1.01	ordersn	0.98	1.01	1.01
ordersc	0.96	1.01	1.00	ordersc	0.96	1.01	1.00
building	0.82	1.00	0.98	building	0.95	0.99	0.98
claims	0.97	1.00	1.00	claims	0.97	0.99	1.00
vendor	0.92	0.91	0.90	vendor	0.92	0.92	0.91
spread	1.00	1.01	1.01	spread	0.99	1.00	1.01
expect	0.96	0.98	0.96	expect	0.98	0.99	0.97
average	0.92	0.96	0.97	average	0.94	0.97	0.96

Notes: the table reports the performance of the IS-MSE(U-MIDAS)relative to IS-MSE(MIDAS). The ratio IS-MSE(U-MIDAS)/IS-MSE(MIDAS) is computed for each single indicator model, and is reported in the corresponding row. The last row reports the average across indicators. Since the models change for each nowcast horizon, we report the results for each of the three h_m. Panel A reports the results at December 2006 (before the financial crisis), Panel B at March 2011 (with the financial crisis included in the sample).

In the majority of cases, the MSE ratios are smaller than one, indicating a superior performance of the U-MIDAS with respect to MIDAS. Especially for one-month ahead nowcasts, all indicators show a relative in-sample MSE smaller or equal to one. The results on the relative in-sample performance of U-MIDAS and MIDAS also do not change substantially before and after the beginning of the crisis, if we compare panel A and B in Table 6.

We now consider the out-of-sample nowcasting performance. We first look at the relative performance for nowcasting GDP growth using the different monthly indicators in the two MIDAS approaches against a benchmark. In our exercise, the benchmark is an AR model, with lag length recursively selected according to the BIC criterion. To compute the MSE of each MIDAS model and the benchmark, we consider two evaluation samples. The first one starts in 1985Q1 and ends in 2006Q4, whereas the second again covers the recent crisis period, ending in 2011Q1, and again starting in 1985Q1. Over the evaluation samples, we compute the out-of-sample MSE of the U-MIDAS and MIDAS models using each single indicator relative to that of a benchmark. A relative MSE not greater than one indicates a superior performance of the MIDAS nowcasts. The results are reported in the upper part 1 of Table 7. In addition, we report a relative out-of-sample MSE of the U-MIDAS to the MIDAS model as in the in-sample exercise. The results are reported in the lower part 2 of Table 7.

The figures reported in the upper part of Table 7 indicate that the MIDAS models can outperform the AR benchmark only for a few indicators. In particular, according to the results in first sample period, depicted on the left side of the Table, the orders of consumption goods (ordersc), building permits (build), new claims for unemployment insurance (claims), the vendor performance diffusion index (vendor), and consumer expectations (expect) yield a relative MSE lower than one at some horizons.

If we focus on those relative MSE pairs with either U-MIDAS or MIDAS or both having a relative MSE smaller than one at the different horizons, we find 5 pairs with a superior performance of U-MIDAS, and 3 pairs, where restricted MIDAS is better. In general, there is no particular combination of an indicator and U-MIDAS which is clearly dominated by the combination of the same indicator and restricted MIDAS, as the results generally depend on the horizon.

The lower part of the Table, left side, shows the relative MSE between U-MIDAS and MIDAS. Note that although the majority of cases seems to indicate a dominance of restricted MIDAS (relative MSE greater than one), most of the nowcasts are not informative compared to a benchmark, as discussed in the upper part of the Table, and thus should not be considered in the comparison.

In the evaluation sample ending in 2011Q1, as shown on the right side of Table 7, we generally find lower relative MSEs over the AR benchmark compared to the earlier evaluation period. Thus, the recent crisis seems to matter for the nowcast performance of the different models. In particular, the indicator-based MIDAS regressions seem to improve in relative terms compared to the benchmark. If we again only focus on those relative MSE pairs with either U-MIDAS or MIDAS or both having a relative MSE smaller than one at the different horizons, we find 9 pairs with a superior performance of U-MIDAS, and 8 pairs, where restricted MIDAS is better. If we focus on the best performing models (highlighted in bold in Table 7), we see that when including the crisis period the best performer is an indicator model estimated with the restricted MIDAS, while the opposite is true for the sample without the crisis. In the more recent period, we can also find a few cases where either U-MIDAS or MIDAS dominate. For example, using restricted MIDAS with orders of consumption goods (ordersc) now seems to work better than U-MIDAS over all horizons, whereas building permits (build) work better as predictors using U-MIDAS.

Overall, there is mixed empirical evidence on the relative performance of U-MIDAS and MIDAS. The in-sample performance of U-MIDAS is better than the nowcast performance. However, neither U-MIDAS or MIDAS are dominating out-of-sample. Generally,

Table 7: Results for individual indicators relative to an AR benchmark. Nowcasting performance for US GDP growth

1. Relative out-of-sample MSE (MSE(model) / MSE(AR benchmark))

A. Sample 1985Q1 - 2006Q4					B. Sample 1985Q1 - 2011Q1				
			h_m					**h_m**	
		1	**2**	**3**			**1**	**2**	**3**
M2	midas	1.42	1.40	1.40	M2	midas	1.60	1.57	1.46
	u-midas	1.16	1.03	1.45		u-midas	1.28	1.18	1.42
stock	midas	1.13	1.16	1.08	stock	midas	1.12	1.03	0.93
	u-midas	1.24	1.23	1.19		u-midas	1.09	1.08	0.98
hours	midas	1.28	1.17	1.03	hours	midas	1.25	1.16	1.03
	u-midas	1.53	1.41	1.06		u-midas	1.39	1.30	0.89
ordersn	midas	1.36	1.13	1.11	ordersn	midas	1.18	1.06	0.97
	u-midas	1.56	1.35	1.29		u-midas	1.32	1.26	1.18
ordersc	midas	0.92	1.13	1.00	ordersc	midas	**0.71**	0.91	**0.79**
	u-midas	0.94	1.13	0.99		u-midas	0.74	0.92	0.81
building	midas	1.00	0.98	1.01	building	midas	1.02	0.91	0.88
	u-midas	1.01	0.98	**0.89**		u-midas	0.93	0.90	0.81
claims	midas	0.94	1.03	1.14	claims	midas	0.86	0.93	0.89
	u-midas	**0.89**	0.99	1.10		u-midas	0.81	0.90	0.86
vendor	midas	0.97	0.98	0.94	vendor	midas	1.01	1.01	0.95
	u-midas	0.97	**0.93**	1.04		u-midas	1.02	0.99	1.05
spread	midas	1.57	1.57	1.57	spread	midas	1.55	1.56	1.46
	u-midas	1.46	1.54	1.52		u-midas	1.43	1.53	1.43
expect	midas	1.01	0.96	1.01	expect	midas	0.94	**0.89**	0.86
	u-midas	1.00	0.98	1.10		u-midas	0.93	0.92	0.94

2. Relative out of sample MSE (MSE(U-MIDAS) / MSE(MIDAS))

A. Sample 1985Q1 - 2006Q4				B. Sample 1985Q1 - 2011Q1			
		h_m				**h_m**	
	1	**2**	**3**		**1**	**2**	**3**
M2	0.82	0.74	1.04	M2	0.80	0.75	0.97
stock	1.10	1.05	1.10	stock	0.97	1.04	1.05
hours	1.20	1.21	1.04	hours	1.12	1.11	0.86
ordersn	1.15	1.19	1.16	ordersn	1.12	1.19	1.22
ordersc	1.02	1.00	0.99	ordersc	1.03	1.01	1.02
building	1.01	1.00	0.89	building	0.91	0.98	0.91
claims	0.95	0.96	0.97	claims	0.94	0.96	0.96
vendor	1.00	0.95	1.11	vendor	1.01	0.97	1.10
spread	0.93	0.99	0.96	spread	0.92	0.98	0.98
expect	0.99	1.03	1.09	expect	0.99	1.03	1.08
average	1.02	1.01	1.03	average	0.98	1.00	1.01

Notes: The upper part of the Table reports the performance of the MSE(model) relative to MSE(benchmark), see part 1. The ratioMSE(model)/MSE(benchmark) is computed for each single indicator model, and is reported in the corresponding row. Panel A reports the results for the evaluation sample ending in December 2006 (before the financial crisis), Panel B in March 2011 (with the financial crisis included in the sample). The benchmark is a AR model with lag lenght selected according to the BIC criterion. The number in bold represents the best performance for each horizon. The lower part of the Table reports the performance of the MSE(U-MIDAS) relative to MSE(MIDAS) and the average across indicators, see part 2.

U-MIDAS is a strong competitor to restricted MIDAS, as it performs better for several high frequency indicators and forecast horizons. Finally, we also find that during the recent crisis, the information content of monthly indicators has increased compared to a benchmark without high-frequency information.

5 An application to euro area GDP growth

To provide additional evidence on the relative performance of MIDAS and UMIDAS we also conduct an evaluation for the Euro area GDP growth rate. The dataset in this case includes the same series as in Foroni and Marcellino (2011), extracted from the Eurostat database of Principal European Economic Indicators (PEEI) and updated at the end of May 2011. The complete list of the series included is in Appendix A.3. Quarterly GDP is available from 1996Q1 until 2010Q4, while the roughly 140 monthly indicators from January 1996 to at most May 2011 (depending on the publication delay, there is a different number of missing observations for each series at the end of the sample). Generally, the monthly series include consumer and producer price index by sector, industrial production and (deflated) turnover indexes by sector, car registrations, new orders received index, business and consumers surveys with their components, sentiment indicators, unemployment indices, monetary aggregates, interest and exchange rates. When analyzing the nowcasting performance, we adopt a recursive approach as for the US, with the first evaluation quarter fixed at 2003Q1 and the last one at 2010Q4, for a total of 32 evaluation samples.

As in the analysis for the US, we start with an in-sample evaluation, distinguishing the periods with and without the recent crisis. The results are shown in Table 8 that reports the average and percentiles of the distribution across indicators of the IS-MSE(U MIDAS) relative to IS-MSE(MIDAS).

The average ratios are always smaller than one, indicating a clear superior performance of the U-MIDAS with respect to the MIDAS approach. In most of the cases, the in-sample MSE obtained with the unrestricted model is 10% smaller than the corresponding one obtained with the restricted polynomial. Moreover, the results are stable across the different subsamples.

Results are different in the case of the nowcasting performance. Table 9 indicates that only for $h_m = 1$ more than half of the U-MIDAS models perform better than the correspondent restricted models, in the pre-crisis sample. If we consider also the crisis, the restricted MIDAS performs much better. This is due to the fact that the estimates of the parameters of the U-MIDAS are influenced substantially by the dramatic drop and subsequent recovery of the GDP in the quarters Q4 2008 and Q1 2009.

Overall the results are in line with those with simulated data and similar to those obtained for the US. It is also worth noting that for at least 25% of the indicators the U-MIDAS approach produces more precise nowcasts than MIDAS, with even larger values

Table 8: Results for individual indicators. In-sample analysis for Euro area GDP growth,

Relative in sample MSE (MSE(U-MIDAS) / MSE(MIDAS))

A. Sample 1996Q1 - 2006Q4				B. Sample 1996Q1 - 2010Q4			
	h$_m$				**h**$_m$		
	1	**2**	**3**		**1**	**2**	**3**
average	0.89	0.87	0.88	average	0.86	0.81	0.82
10th pctile	0.76	0.73	0.74	10th pctile	0.69	0.57	0.58
25th pctile	0.82	0.80	0.82	25th pctile	0.80	0.75	0.74
median	0.90	0.89	0.90	median	0.89	0.86	0.87
75th pctile	0.96	0.94	0.95	75th pctile	0.95	0.93	0.92
90th pctile	1.01	0.99	0.99	90th pctile	0.98	0.95	0.95

Notes: the table reports the performance of the IS-MSE(U MIDAS)relative to IS-MSE(MIDAS). The ratio IS-MSE(U MIDAS)/IS-MSE(MIDAS) is computed for each single indicator model, then in the second row the mean of the distribution of these ratio is reported, and in rows 3 to 7 the main percentiles (10th, 25th, 50th, 75th, 90th) of the distribution are reported. Since the models change for each nowcast horizon, we report the results for each of the three h_m. Panel A reports the results at December 2006 (before the financial crisis), Panel B at December 2010 (with the financial crisis included in the sample).

Table 9: Results for individual indicators. Nowcasting performance for Euro area GDP growth.

Relative out of sample MSE (MSE(U-MIDAS) / MSE(MIDAS))

A. Sample 2003Q1 - 2006Q4				B. Sample 2003Q1 - 2010Q4			
	h$_m$				**h**$_m$		
	1	**2**	**3**		**1**	**2**	**3**
average	1.03	1.03	1.00	average	1.09	1.03	0.99
10th pctile	0.79	0.79	0.79	10th pctile	0.90	0.73	0.76
25th pctile	0.88	0.94	0.91	25th pctile	1.01	0.92	0.90
median	0.99	1.00	0.99	median	1.09	1.06	1.01
75th pctile	1.08	1.09	1.07	75th pctile	1.17	1.14	1.07
90th pctile	1.34	1.29	1.21	90th pctile	1.31	1.25	1.16

Notes: the table reports the performance of the MSE(U MIDAS)relative to MSE(MIDAS). The ratio MSE(U MIDAS)/MSE(MIDAS) is computed for each single indicator model, then in the second row the mean of the distribution of these ratio is reported, and in rows 3 to 7 the main percentiles (10th, 25th, 50th, 75th, 90th) of the distribution are reported. Panel A reports the results for the evaluation sample ending in December 2006 (before the financial crisis), Panel B in December 2010 (with the financial crisis included in the sample).

in the pre-crisis period.

The evidence of a better performance of U-MIDAS in the pre-crisis sample emerges also from Table 10, where we look at the average and the median relative MSE performance for nowcasting quarterly GDP growth for the two different classes of models, against a benchmark. We consider as a benchmark an AR process, with lag length selected according to the BIC criterion. First, we estimate every individual model and compute the relative MSE with respect to the benchmark. Then, we take the average and the median across all the indicators of the relative MSE within a model class.

Table 10: Results for individual indicators relative to an AR benchmark. Nowcasting performance for Euro area GDP growth

Relative out of sample MSE (MSE(model) / MSE(AR benchmark))							
A. Sample 2003Q1 - 2006Q4				B. Sample 2003Q1 - 2010Q4			
	$\mathbf{h}_m$				$\mathbf{h}_m$		
	1	**2**	**3**		**1**	**2**	**3**
average u-midas	1.15	1.08	1.14	average u-midas	0.87	0.80	0.85
average midas	1.14	1.08	1.19	average midas	0.81	0.79	0.87
median u-midas	1.06	1.03	1.07	median u-midas	0.94	0.85	0.91
median midas	1.06	1.03	1.09	median midas	0.86	0.83	0.90

Notes: the table reports the average and the median performance of the MSE(model) relative to MSE(benchmark). The ratio MSE(model)/MSE(benchmark) is computed for each single indicator model, and then mean and median are computed for each class of models. Panel A reports the results for the evaluation sample ending in December 2006 (before the financial crisis), Panel B in December 2010 (with the financial crisis included in the sample). The benchmark is a AR model with lag lenght selected according to the BIC criterion.

Table 10 also highlights striking differences before and after the crisis. As already noticed in Foroni and Marcellino (2011), it is impossible to outperform a naive benchmark in the period up to 2006 on average across all indicators. However, during the crisis, the use of monthly information becomes very important, and both MIDAS approaches clearly outperform the benchmark.

Given the large set of monthly indicators under analysis, it is of interest to identify the best performing ones, and to assess whether the ranking changed substantially during the crisis. Table 11 reports the top ten best performing monthly indicators for nowcasting euro area GDP growth up to $\mathbf{h}_m = 2$, which also have a good performance at $\mathbf{h}_m = 3$, with details provided in Appendix A.4 The performance of MIDAS and UMIDAS is overall comparable, with UMIDAS better than MIDAS in four out of ten cases before the crisis and five out of ten case with the crisis sample included. The best indicators are overall rather stable over time, and include components of the business surveys such as IEOB, BCI and IPE. However, the monthly indicators are much more useful over the full sample, due to their improved performance with respect to the AR benchmark during the crisis period.

Table 11: Results for best individual indicators. Out-of-sample analysis for Euro area GDP growth,

Top 10 euro area indicators

A. Sample 2003Q1 - 2006Q4				B. Sample 2003Q1 - 2010Q4			
	$\mathbf{h}_m$				$\mathbf{h}_m$		
	1	2	3		1	2	3
BS IEOB u-midas	0.45	0.82	1.10	BS IPE midas	0.44	0.33	0.54
BS BCI midas	0.66	0.77	0.93	BS BCI u-midas	0.47	0.29	0.51
BS IPE midas	0.71	0.66	0.82	BS BCI midas	0.48	0.57	0.74
BS IEOB midas	0.71	0.88	0.99	MIG NDCOG IS PPI midas	0.50	0.86	0.95
BS IPT midas	0.73	0.84	0.92	BS IEOB u-midas	0.51	0.36	0.69
BS ICI midas	0.73	0.76	0.92	MIG NDCOG IS PPI u-midas	0.51	0.53	0.50
BS IOB midas	0.75	0.85	0.95	BS GES NY midas	0.52	0.55	0.57
BS IOB u-midas	0.75	0.83	1.05	MIG COG IS PPI midas	0.52	0.86	0.96
BS BCI u-midas	0.76	0.78	1.14	BS IPE u-midas	0.52	0.28	0.43
BS IPE u-midas	0.77	0.74	0.84	BS GES NY u-midas	0.55	0.61	0.58

Notes: the table reports the performance of the top ten monthly indicators for forecasting euro area GDP growth up to $h_m = 2$, with a good performance also at $h_m = 3$. Specifically, the values are the ratio MSE(U MIDAS) or MSE(MIDAS) relative to the AR benchmark. Panel A reports the results up to December 2006 (before the financial crisis), Panel B up to December 2010 (with the financial crisis included in the sample).

In summary, as for the US we observe a deterioration of the out of sample performance of U-MIDAS relative to MIDAS with respect to the in-sample evaluation. The difference is even more marked than for the US, likely due to the shorter estimation samples used in the recursive out of sample exercise for the euro area. As small samples are more problematic for heavily parameterized models, the short sample length might be more problematic for U-MIDAS than MIDAS, even when the specification of the former is based on the BIC criterion. A second explanation could be related to the very good performance of MIDAS during the crisis. Overall, there is mixed empirical evidence on the relative performance of U-MIDAS and MIDAS also for the euro area. However, generally, U-MIDAS is a strong competitor to restricted MIDAS, as it performs better for several high frequency indicators and forecast horizons.

6 Conclusions

In the recent literature, mixed-data sampling (MIDAS) regressions have turned out to be useful reduced-form tools for nowcasting low-frequency variables with high-frequency indicators. To avoid parameter proliferation in the case of long high-frequency lags, functional lag polynomials have been proposed. In this paper, we have discussed a variant of the MIDAS approach, which does not resort to functional lag polynomials, but rather to simple linear lag polynomials. Compared to the standard MIDAS approach in the literature, these polynomials are not restricted by a certain functional form, and we therefore call the approach unrestricted MIDAS (U-MIDAS).

We derive U-MIDAS regressions from a linear dynamic model, obtaining a simple and flexible specification to handle mixed-frequency data. It can be expected to perform better for forecasting than MIDAS as long as it is not too heavily parameterized, in particular, as long as the differences in sampling frequencies are not too large. We have shown that this is indeed the case by means of Monte Carlo simulations. U-MIDAS is particularly suited to provide macroeconomic nowcasts and forecasts of quarterly variables, such as GDP growth, given timely observations of monthly indicators like industrial production. On the other hand, when daily data is available, our simulation results indicate that MIDAS with functional lag polynomials are preferable to predict quarterly variables.

In the empirical applications for US and euro area GDP growth, we find that U-MIDAS provides a very good in-sample fit based on monthly macroeconomic indicators. The evidence is more mixed when looking at the out-of-sample nowcast performance. Nonetheless, the out-of-sample evidence suggests that U-MIDAS can outperform MIDAS with restricted polynomials for some of the high-frequency indicators. Overall, we do not expect one polynomial specification to be dominant in every case. As U-MIDAS might be a strong competitor, we rather suggest to consider it as an alternative to the existing MIDAS approaches.

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A Appendix

A.1 Identification of the disaggregate process

Let us consider the LF exact MIDAS model for y (equation (7)):

$$\begin{aligned} h(L^k)\omega(L)y_t &= b_1(L)\beta(L)\omega(L)x_{1t} + ... + b_N(L)\beta(L)\omega(L)x_{Nt} + q(L^k)u_{yt}, \\ t &= k, 2k, 3k, ... \end{aligned} \quad (26)$$

We want to determine what and how many HF models are compatible with this LF model, namely, whether the parameters of the generating mechanism of y at HF can be uniquely identified from those at LF. The following discussion is based on Marcellino (1998), to whom we refer for additional details.

To start with, assuming that y follows the model in (26), we try and identify the $a(L)$ polynomials that can have generated $h(L^k)$. This requires to analyze all the possible decompositions of $h(L^k)$ into $\beta(L)a(L)$.

We have said that at least one h_{si} for each s in (6) has to be such that $a(h_{si}) = 0$. The other $k-1$ h_{si}s can instead solve either $\beta(h_{si}) = 0$ or also $a(h_{si}) = 0$. Thus, for each s, there are $2^k - 1$ possible "distributions" of the h_{si}s as roots of $\beta(L)$ and $a(L)$. Hence, we obtain a total of $(2^k - 1)^h$ potential disaggregated AR components, which can be written as

$$\prod_m (1 - \frac{1}{h_m} L), \quad (27)$$

where the h_ms are the h_{si}s which are considered as roots of $a(l) = 0$. The possible degree of $a(L)$ ranges from h to hk, with $h < p$.

A simple but rather stringent sufficient condition for exact identification of the disaggregate process in our context is:

Proposition 1. All the roots of $a(l) = 0$ are distinct and positive, or distinct and possibly negative if k is even.

Proof If $a(l) = 0$ has distinct and positive roots, or distinct and possibly negative roots if k is even, then they coincide with those of $h(z) = 0$ raised to power of $1/k$, and this exactly identifies the AR component. Once, $a(L)$ is exactly identified, $\beta(L)$ is also unique. Finally, given $\beta(L)$ and since the aggregation operator $\omega(L)$ is known, the polynomials $b_j(L)$ can be also recovered, $j = 1, ..., N$. ■

To conclude, it is worth making a few comments on this result. First, Wei and Stram (1990) discuss more general sufficient a priori conditions for one disaggregate model to be identifiable from an aggregate one. Second, the hypothesis of no MA component at the disaggregate level can be relaxed. Marcellino (1998) shows that such an MA component can be uniquely identified if its order is smaller than $p-1$ and the condition in Proposition 1 holds. Third, the condition in Proposition 1 could be relaxed by imposing constraints on the $b_i(L)$ polynomials. Fourth, when y is multivariate the link between the disaggregate and aggregate models is much more complicated, see Marcellino (1999), even though conceptually the procedure to recover the disaggregate model is similar to the one we have proposed for the univariate case. Finally, notice that the identification problem does not emerge clearly within a Kalman filter approach to interpolation and forecasting, where the underlying assumption is that the disaggregate model is known.

A.2 Monthly US data

Name	**Monthly indicator**
M2	Real money supply M2
stock	Stock price index (500 common stocks)
hours	Average weekly hours in manufacturing
ordersn	Orders: non-defence capital goods
ordersc	Orders: consumer goods and materials
building	Building permits
claims	New claims for unemployment insurance
vendor	Vendor performance diffusion index
spread	Term spread (10 year - Federal Funds)
expect	Consumer confidence index (U Michigan)

A.3 Monthly euro area indicators

Monthly indicators
HICP - All items excluding energy and unprocessed food
HICP - All items excluding energy, food, alcohol and tobacco
HICP - All items excluding energy and seasonal food
HICP - All items excluding energy
HICP - All items excluding tobacco
HICP - All items
HICP - Food and non alcoholic beverages
HICP - Alcoholic beverages and tobacco
HICP - Clothing and footwear
HICP - Housing, water, electricity,gas and other fuels
HICP - Furnishings, household equipment and maintenance
HICP - Health
HICP - Transport
HICP - Communication
HICP - Recreation and culture
HICP - Education
HICP - Hotels, cafes and restaurants
HICP - Miscellaneous goods and services
HICP - Energy
HICP - Food
Producer price index - Electricity, gas, steam and air conditioning supply
Producer price index - Industry (except construction), sewerage, waste management and remediation activities
Producer price index - Mining and quarrying
Producer price index - Manufacturing
Producer price index - Manufacturing, for new orders
Producer price index - Electricity, gas, steam and air conditioning supply
Producer price index - Water collection, treatment and supply
Producer price index - Capital goods
Producer price index - Consumer goods
Producer price index - Durable consumer goods
Producer price index - Intermediate goods
Producer price index - Non-durable consumer goods
Producer price index - Energy
Business Climate Indicator
Consumer confidence indicator
Consumer surveys - Financial situation over the last 12 months
Consumer surveys - Financial situation over the next 12 months
Consumer surveys - General economic situation over the last 12 months

Consumer surveys - Major purchases over the next 12 months

Consumer surveys - Major purchases at present

Consumer surveys - Price trends over the last 12 months

Consumer surveys - Price trends over the next 12 months

Consumer surveys - Statement on financial situation of household

Consumer surveys - Savings over the next 12 months

Consumer surveys - Savings at present

Consumer surveys - Unemployment expectations over the next 12 months

Business surveys - Constructions - Assessment of order-book levels

Business surveys - Constructions - Employment expectations for the months ahead

Business surveys - Constructions - Price expectations for the months ahead

Business surveys - Constructions - Construction confidence indicator

Business surveys - Constructions - Factors limiting building activity - None

Business surveys - Constructions - Factors limiting building activity - Insufficient demand

Business surveys - Constructions - Factors limiting building activity - Weather conditions

Business surveys - Constructions - Factors limiting building activity - Shortage of labour

Business surveys - Constructions - Factors limiting building activity - Shortage of material and/or equipment

Business surveys - Constructions - Factors limiting building activity - Other

Business surveys - Constructions - Factors limiting building activity - Financial constraints

Business surveys - Industry - Industrial confidence indicator

Business surveys - Industry - Employment expectations for the months ahead

Business surveys - Industry - Assessment of export order-book levels

Business surveys - Industry - Assessment of order-book levels

Business surveys - Industry - Expectations for the months ahead

Business surveys - Industry - Production trend observed in recent months

Business surveys - Industry - Assessment of stocks of finished products

Business surveys - Industry - Selling price expectations for the months ahead

Business surveys - Retail - Assessment of stocks

Business surveys - Retail - Retail confidence indicator

Business surveys - Retail - Expected business situation

Business surveys - Retail - Employment

Business surveys - Retail - Orders placed with suppliers

Business surveys - Retail - Present business situation

Business survey - Services - Assessment of business climate

Business survey - Services - Evolution of demand expected in the months ahead

Business survey - Services - Evolution of demand in recent months

Business survey - Services - Services Confidence Indicator

Business survey - Services - Evolution of employment in recent months

Consumer confidence indicator

Economic sentiment indicator

Production index
Production index - Buildings
Production index - Civil engineering works
Production index - Construction
Production index - Mining and quarrying; manufacturing; electricity, gas, steam and air conditioning supply
Production index - Mining and quarrying; manufacturing
Production index - Mining and quarrying
Production index - Manufacturing
Production index - Manufacturing, for new orders
Production index - Electricity, gas, steam and air conditioning supply
Production index - Capital goods Production index - Consumer goods
Production index - Durable consumer goods
Production index - Intermediate goods
Production index - Non-durable consumer goods
Turnover index - domestic market - Mining and quarrying; manufacturing
Turnover index - non-domestic market - Mining and quarrying; manufacturing
Turnover index - total - Mining and quarrying; manufacturing
Turnover index - domestic market - Manufacturing
Turnover index - non-domestic market - Manufacturing
Turnover index - total - Manufacturing
Turnover index - domestic market - Manufacturing, for new orders
Turnover index - non-domestic market - Manufacturing, for new orders
Turnover index - total - Manufacturing, for new orders
Turnover index - domestic market - Capital goods
Turnover index - non-domestic market - Capital goods
Turnover index - total - Capital goods
Turnover index - domestic market - Consumer goods
Turnover index - non-domestic market - Consumer goods
Turnover index - total - Consumer goods
Turnover index - domestic market - Durable consumer goods
Turnover index - non-domestic market - Durable consumer goods
Turnover index - total - Durable consumer goods
Turnover index - domestic market - Intermediate goods
Turnover index - non-domestic market - Intermediate goods
Turnover index - total - Intermediate goods
Turnover index - domestic market - Non-durable consumer goods
Turnover index - total - Non-durable consumer goods
New orders received index - Manufacturing, for new orders
New orders received index - Manufacturing, for new orders (except heavy transport equipment)

A.4 Top performers monthly euro area indicators

Name	Monthly indicator
BS BCI	Business Climate Indicator
BS GES NY	Consumer Surveys - General economic situation over the next 12 months
BS ICI	Business Surveys - Industry - Industrial confidence indicator
BS IEOB	Business Surveys - Industry - Assessment of export order-book levels
BS IOB	Business Surveys - Industry - Assessment of order-book levels
BS IPE	Business Surveys - Industry - Assessment of order-book levels
BS IPT	Business Surveys - Industry - Production trend observed in recent months
MIG COG IS PPI	Industry - Producer price index: Consumer goods
MIG NDCOG IS PPI	Industry - Producer price index: Non-durable consumer goods