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Johan N. M. Lagerlöf

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Johan N. M. Lagerlöf, University of Copenhagen and CEPR

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Centre for Economic Policy Research
77 Bastwick Street, London EC1V 3PZ, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

Does Cost Uncertainty in the Bertrand Model Soften Competition?*

Although naive intuition may indicate the opposite, the existing literature suggests that uncertainty about costs in the homogeneous-good Bertrand model intensifies competition: it lowers price and raises total surplus (but also makes profits go up). Those results, however, are derived under two assumptions that, if relaxed, conceivably could reverse the results. The present paper first shows that the results hold also if drastic innovations are possible. Next, the paper assumes asymmetric cost distributions, a possibility that is empirically highly plausible but which has been neglected in the previous literature. Using numerical methods it is shown that, under this assumption, uncertainty lowers price and raises total surplus even more than with identical distributions. However, if the asymmetry is large enough, industry profits are lower under uncertainty; this is in contrast to the known results and reinforces the notion that uncertainty intensifies competition rather than softens it.

JEL Classification: D43, D44 and L13
Keywords: asymmetric auctions, asymmetric firms, auctions with endogenous quantity, Bertrand competition, boundary value method, Hansen-Spulber model, information sharing, oligopoly and private information

Johan N. M. Lagerlöf
Department of Economics
University of Copenhagen
Oester Farimagsgade 5, Building 26
DK-1353 Copenhagen K
DENMARK

Email: johan.lagerlof@econ.ku.dk

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1 Introduction

The Bertrand model of price competition predicts that price equals marginal cost and that firms earn zero profit — a result which is often referred to as the Bertrand paradox, as it suggests that the presence of only two firms is sufficient to eliminate all market power and give rise to the perfectly competitive outcome. The paradox has prompted a number of scholars to study extensions and variations of Bertrand's original model, thereby identifying several model features that, if added to the standard set-up, resolves the paradox by providing some amount of market power to the firms. Examples of such features include product differentiation, capacity constraints, repeated interaction, and cost asymmetries between firms.

In an interesting paper, Spulber (1995) studies another, empirically very plausible, variation of the standard Bertrand model, namely to assume that each price-setting firm has private information about some characteristic of its production technology, a leading example of which is the firm's (constant) marginal cost.[1] Spulber shows that in that setting there is a unique and symmetric equilibrium price strategy, which is increasing in the own marginal cost. Importantly, the equilibrium price lies strictly between the marginal cost and the monopoly price, which means that the firms have some market power and earn a positive profit.[2]

What exactly is the model feature that gives rise to that outcome, thus solving the Bertrand paradox? An answer that naturally comes to mind is that it is asymmetric information (or uncertainty more generally), and this is exactly what Spulber (1995) suggests (p. 10, emphasis added):

> Asymmetric information thus plays an important role in imperfect competition. In [the model] studied here, the surgical precision that is required to price slightly below [...] higher cost rivals is eliminated by the lack of exact knowledge about the characteristics of the rivals. In the short run, with market structure fixed, *asymmetric information appears to reduce competition* [...].

It is not only Spulber himself who interprets his results in this fashion, so do several other authors. For example, Spiegel and Tookes (2008, p. 33, f.n.

[1] The model that Spulber studies has also found its way into textbooks — see Wolfstetter (1999, pp. 236-37) and Belleflamme and Peitz (2010, pp. 47-49). Arozamena and Weinschelbaum (2009) study a sequential version of Spulber's model and compare with the simultaneous-move version. Lofaro (2002) obtains a closed-form solution of Spulber's model by assuming a uniform distribution, and he then compares the price competition outcomes with the quantity competition outcomes. Abbink and Brandts (2007) test Spulber's model in the lab.

[2] The firm that draws the lowest marginal cost (and thus charges the lowest price) earns a positive profit ex post. The other firms earn a zero profit ex post, but their expected profit, at the stage before they have learned their cost parameter, is positive.

33) write that "Spulber (1995) also shows how, in Bertrand competition, not knowing rivals' costs implies equilibrium prices that are above marginal costs (i.e., information asymmetry softens product market competition)."[3]

But is it really asymmetric information (or uncertainty) that softens competition? When Spulber assumes uncertainty about the firms' marginal cost parameters, he implicitly also introduces the assumption that these parameters may differ from each other. This means that, in principle, the model feature that creates market power could be cost heterogeneity and not uncertainty. Moreover, it actually follows from relatively early work of Hansen (1988) that, at least for a special case of Spulber's (1995) model, it is indeed cost heterogeneity that softens product market competition. Uncertainty is, in Hansen's (1988) setting and given the presence of cost heterogeneity, not anti-competitive but pro-competitive — at least in the sense that it lowers expected price and raises expected consumer and total surplus. Uncertainty also makes expected industry profits go up, which of course can be thought of as providing the firms with more market power.[4]

However, Hansen's result is derived under certain assumptions that seem restrictive and which, if relaxed, conceivably could reverse his result. The goal of the present paper is therefore to further investigate under what circumstances cost uncertainty in the homogeneous-good Bertrand model is pro-competitive. In the first part of the paper I propose a model specification that yields a closed-form solution of the equilibrium price in Spulber (1995) and Hansen (1988). This specification is tractable and easy to work with but is still more general than other model versions yielding closed-form solutions that have been considered in the literature,[5] and it may therefore be of some interest in its own right. I then compare the equilibrium outcomes of this model to the ones of a complete

[3] See also Wolfstetter (1999), who presents two simple versions of the model, one with inelastic and one with elastic demand. He introduces the analysis of these models by stating that (p. 236): "A much simpler [relative to the model with capacity constraints] resolution of the Bertrand paradox can be found by introducing incomplete information." Yet another paper that refers (twice — on p. 638 and p. 646) to Spulber's (1995) result as a "resolution of the Bertrand paradox" is Abbink and Brandts (2007).

[4] There is also a related literature that studies firms' incentive to share information about their own marginal cost parameter under Bertrand competition with *differentiated* goods. Gal-Or (1986) analyses such a model with two firms and shows that not to share information is a dominant strategy for each duopolist. Raith (1996) considers the more general case with n firms and reports that in that environment Gal-Or's result can be reversed. As shown by Jin (2000), however, that particular finding in Raith's paper is based on an algebraic error: in fact, Gal-Or's result extends also to Raith's setting with an arbitrary number of firms. (Note that Vives (1999, Section 8.3.1) in his survey of the information sharing literature also reports the incorrect result.)

[5] Wolfstetter (1999), Lofaro (2002), and Belleflamme and Peitz (2010) all assume a uniform cost distribution on $[0, 1]$ together with linear demand and derive closed-form solutions. The specification used in the present paper is more general but includes theirs as a special case. (Abbink and Brandts (2007) assume, in their experimental study, a uniform distribution on $[0, 99]$; this is in practice also a special case of the model in the present paper, although with another scaling of the units in which output and cost are measured.)

information version of the same model. It turns out that the results are unaltered relative to Hansen's (1988). The analysis is not a special case of the one in Hansen, as it allows for (i) an arbitrary number of firms and (ii) the possibility that a firm makes a "drastic innovation" (which, as will be explained in Section 2, conceivably could imply that uncertainty indeed softens competition).[6] These results thus support the conclusion that uncertainty intensifies competition.

In the second part of the paper an asymmetric duopoly version of the Hansen-Spulber model is studied, using numerical methods (henceforth, the homogeneous-good Bertrand model with private information about costs will be referred to as the "Hansen-Spulber model"). To the best of my knowledge, this is the first investigation of an asymmetric version of that model.[7] The results of the numerical analysis suggest that uncertainty lowers expected price and raises expected consumer and total surplus *even more* when the firms are asymmetric compared to when they draw their costs from identical distributions. The results also show that, if the asymmetry is large enough, expected industry profits are *lower* under uncertainty. Moreover, it may be that one of the duopolists benefits from uncertainty while the other does not. These results about profit levels are at odds with the existing results in the literature on the Hansen-Spulber model. All in all, the results for the asymmetric model reinforce the notion that uncertainty intensifies competition rather than softens it, as here the firms lose market power also in the sense that aggregate profits decrease.

Section 2 of the paper starts out by discussing how we can disentangle the effect of uncertainty from the one of cost heterogeneity, by comparing Spulber's (1995) model to a complete information Bertrand model with heterogenous costs. Section 2 also provides a review of Hansen's (1988) model and his results. In Section 3, the specification yielding a closed-form solution is presented and analyzed, and a comparison with a complete information version of the same model is performed. Section 4 studies the asymmetric model and makes a similar comparison for that one. Finally, Section 5 concludes the paper and suggests some open questions for future work. Some of the formal proofs are relegated to two appendices.

[6]Dastidar (2006) has, under the assumption that drastic innovations are not possible, extended Hansen's (1988) result that uncertainty yields a lower expected price to an environment with an arbitrary number of firms.

[7]There is, however, a literature on asymmetric first-price auctions with a *fixed* (as opposed to variable) quantity. See, for example, Maskin and Riley (2000) and Kirkegaard (2009) for work that uses analytical methods to derive results for such models. And see Marshall et al. (1994), Bajari (2001), Li and Riley (2007), Gayle and Richard (2008) and Fibich and Gavish (2011) for work that uses numerical methods.

2 Hansen's (1988) results

As explained in the introduction, Spulber (1995) studies a standard one-shot, homogeneous-good Bertrand model with n ex ante identical firms, but adds the assumption that each firm has private information about some characteristic of its production technology, a leading example of which is the firm's (constant) marginal cost. He shows that in that environment the equilibrium price lies strictly above the marginal cost. Our first goal is to understand whether it is the uncertainty as such that creates market power for the firms. To that end, it is useful to note that when adding private information about the marginal cost, Spulber makes (by logical necessity) *two* assumptions:

A1. The firms' marginal cost parameters may differ from each other.

A2a. Each firm has private information about its own marginal cost parameter.

In order to assess the role of asymmetric information, we can compare the outcome of Spulber's model with the outcome of a benchmark that also makes assumption A1 but replaces A2a with[8]

A2b. The (possibly different) marginal cost parameters are common knowledge among the firms.

Assumptions A1 and A2b give rise to a standard variation of the Bertrand set-up, discussed in many textbooks. The equilibrium outcome of that model is that the lowest-cost firm wins the whole market and charges a price equal to the minimum of the monopoly price and the marginal cost of the firm with the second-lowest cost draw.[9] That is, also this model with complete information but cost heterogeneity gives rise to a market price above marginal cost and a positive profit for the lowest-cost firm. A first conclusion is thus that, also within Spulber's framework, asymmetric information is not *required* for the firms (or at least one firm) to have market power. A more interesting question, however, is whether the amount of market power in the model A1+A2b is less than that in the model A1+A2a. That is, does asymmetric information soften competition, as Spulber (1995) and the other authors cited in the introduction suggest?

Thanks to work of Hansen (1988), we actually know that, at least for a special case[10] of Spulber's (1995) model, the answer to the above question is "no".

[8] In fact, Spulber (1995) does compare these two models in Section III of his paper, but not in terms of their competitiveness.

[9] In order to sustain this behavior as part of a Nash equilibrium, some textbooks assume a particular sharing rule that says that the more efficient firm gets all the demand if both firms charge the same price. Making such an assumption is not necessary, however: if the less efficient firm uses a mixed strategy, then the outcome can be sustained as part of an equilibrium under a standard sharing rule; see Blume (2003).

[10] Hansen (1988) assumes that the firms have a constant returns to scale technology and that the uncertainty concerns each firm's marginal cost parameter, which is only one of the

Hansen's model is couched in terms of a procurement auction in which two firms bid for the right to serve a market with a downward-sloping demand, and within that framework he compares the outcomes of an open (descending) auction and a (first-price) sealed bid auction. The open auction is effectively a Bertrand game with complete information (i.e., A1+A2b), whereas the sealed bid auction is the same as Spulber's (1995) incomplete information model (i.e., A1+A2a). Hansen (1988) shows that the sealed bid auction yields a lower expected price than the open auction. It also yields a higher expected total surplus. Under a somewhat stronger assumption about the demand function, Hansen can also show that the sealed bid auction yields a higher expected profit for the firms and a higher expected consumer surplus, meaning that both consumers and firms are better off under incomplete information.

It is instructive to look at what broad arguments Hansen (1988) uses when proving the result that the sealed bid auction yields a lower expected price than the open auction. First he notes that in an open auction the equilibrium price strategies are the same regardless of whether the quantity is variable or fixed (or, in oligopoly language, whether demand is elastic or inelastic): In either case, the lowest-cost firm can win the whole market with a price that equals the marginal cost of the firm with the second-lowest cost. This yields equality (a) in (1):

$$\begin{aligned} & E\left[p \mid \text{open, variable}\right] \overset{(a)}{=} E\left[p \mid \text{open, fixed}\right] \\ & \overset{(b)}{=} E\left[p \mid \text{sealed, fixed}\right] \overset{(c)}{>} E\left[p \mid \text{sealed, variable}\right]. \end{aligned} \tag{1}$$

The argument that yields equality (a) relies critically on Hansen's assumption that the lowest-cost firm's optimal monopoly price always exceeds the marginal cost of the firm with the second-lowest cost (i.e., a "drastic innovation" must not be possible); without that assumption, the equality would be replaced by a "$<$"-sign and Hansen's proof would no longer be valid.

Next, Hansen invokes the revenue equivalence theorem, which in this model (and quite generally) says that in a fixed-quantity auction the expected revenue (which equals the expected price) is the same regardless of whether the auction is sealed-bid or open.[11] This is equality (b) in (1). Finally Hansen shows that, in a sealed bid auction, the equilibrium price must be lower when the quantity is variable compared to when it is fixed (inequality (c) in (1)). The intuition for this result is straightforward. In the sealed bid auction, if the firm raises its

possibilities that Spulber (1995) considers. Hansen (1988) also assumes a duopoly, whereas Spulber (1995) allows for an arbitrary number of firms. Finally, Hansen (1988) assumes that the support of the unknown marginal cost parameter is such that the monopoly cost always lies above the marginal cost of the second most efficient firm — an assumption that Spulber (1995) does not need to make, given the comparisons he makes in his paper.

[11] On the revenue equivalence theorem, see for example Klemperer (1999) or Krishna (2002).

price, it will have a higher profit if it still wins the market, but the probability of winning has decreased. The optimal price balances those two effects. However, the former (positive) effect is smaller when demand is downward-sloping, as the higher price then leads to a loss of sales. Therefore the expected price must be lower when demand is elastic.

Jointly, the three steps (a), (b), and (c) yield the desired result that the expected price in the sealed bid auction with a variable quantity is lower than the expected price in the open auction with a variable quantity — or, in other words, cost uncertainty in the Bertrand model intensifies competition in the sense that it lowers the expected market price. While step (c) appears to be quite robust, it has already been noted that step (a) relies critically on Hansen's assumption about the support of the cost distribution from which the firms draw their costs. If we allowed for the possibility that the winning firm has such a large cost advantage that it sometimes, under complete information, optimally charges its monopoly price, then the expected price in the open auction with a variable quantity would be lower than the expected price in the open auction with a fixed quantity. This could conceivably also reverse the result that cost uncertainty in the Bertrand model intensifies competition. I will investigate this question in Section 3. It is also clear from the above reasoning that Hansen's proof relies on the revenue equivalence theorem (step (b)). In an environment where that theorem does not hold, the proof will not be valid and it is again conceivable that the result could be reversed. In Section 4 I will study one such environment, namely a duopoly model where the firms' cost distributions are non-identical.

3 Symmetric Bertrand competition with, and without, private information about cost

There are n risk neutral and profit maximizing firms that compete à la Bertrand in a homogeneous product market. The firms are ex ante identical, they choose their prices simultaneously, and they interact only once. Market demand is given by $D(p) = 1 - p$, where p denotes price. The firm that charges the lowest price, denoted $p_{\min}$, sells the quantity $D(p_{\min})$, while all firms charging a higher price sell nothing and make a zero profit. If two or more firms have all chosen the lowest price, then these firms share the market equally between themselves.

Let c_i be firm i's (constant) marginal cost. The parameters c_i (for $i = 1, \ldots, n$) are independent draws from the cumulative distribution function

$$F(c_i) = 1 - (1 - c_i)^x, \tag{2}$$

with support $[0, 1]$ and with $x > 0$ being a parameter. The larger is the parameter x, the more probability mass is shifted toward relatively low values of c_i; I will therefore sometimes refer to x as the "efficiency parameter." The density function associated with $F(c_i)$ is denoted $f(c_i)$ [where $f(c_i) \equiv F'(c_i) = x(1-c_i)^{x-1}$].

I will solve, and then compare, two versions of this model: One where the cost draws are common knowledge and one where each firm's draw is private information of that firm.

3.1 Incomplete information

First consider the incomplete information model. That is, assume that the marginal cost c_i is the private information of firm i, although it is common knowledge that c_i is drawn from the distribution specified in equation (2). I will look for a symmetric equilibrium strategy $p^*(c)$ that is strictly increasing and differentiable. Denote the inverse of this function by $\chi(p)$, meaning that χ is the value of the cost draw that would give rise to the price p. A firm that has drawn the marginal cost c_i thus chooses its price p_i in order to maximize its expected profit,

$$E\pi_i = (p_i - c_i)(1-p_i)\left[1 - F(\chi(p_i))\right]^{n-1}. \tag{3}$$

In this expression, $(p_i - c_i)(1-p_i)$ is the profit the firm will earn if quoting the price p_i and if having no competitors; this is maximized at the monopoly price p_i^m [$\equiv (1+c_i)/2$]. The remaining part of the above expression is the probability that firm i's price p_i is lower than all the other firms' prices, given that all those other firms follow the strategy $p^*(c)$; firm i can make this probability larger by lowering its price. When choosing p_i, the firm trades off its desire to quote a low price in order to win the market against its desire to set a price close to p_i^m in order to earn a large profit in case it does win the market.

The first order condition associated with the firm's problem is given by $\partial E\pi_i/\partial p_i = 0$. By rearranging this and imposing symmetry, we obtain the following differential equation, which characterizes the equilibrium price $p^*(c)$:

$$\frac{\partial p^*}{\partial c} = (n-1)\,h(c)\,\frac{\left[p^*(c) - c\right]\left[1 - p^*(c)\right]}{1 + c - 2p^*(c)}, \tag{4}$$

where $h(c) \equiv f(c)/\left[1 - F(c)\right] = x/(1-c)$ is the hazard rate associated with the distribution function F. In addition, the equilibrium price $p^*(c)$ must satisfy the boundary condition $p^*(1) = 1$.[12]

[12] To see this, first note that a firm will never charge a price below its marginal cost (which also is its average cost); hence $p^*(1) \geq 1$. Moreover, a firm with a cost draw $c < 1$ will never charge a price above the monopoly price, $p^m = (1+c)/2$; therefore $p^*(c) < 1$ for any $c < 1$. By continuity of $p^*(c)$ we thus cannot have $p^*(1) > 1$. It follows that $p^*(1) = 1$.

Under our assumptions, the equilibrium price $p^*(c)$ has an analytical solution, and this is linear (or affine) in c. To find the solution, set $p^*(c) = A + Bc$, where A and B are unknown coefficients. Also note that the boundary condition implies that $B = 1 - A$. The differential equation now simplifies to[13]

$$1 - A = \frac{(n-1)\,x}{1-c} \frac{[A(1-c)]\,[(1-A)(1-c)]}{(1-2A)(1-c)},$$

which has the roots $A = 1$ and $A = [(n-1)\,x + 2]^{-1}$. The former root would imply $p^*(c) = 1$, which cannot be part of an equilibrium as it is not strictly increasing. However, the other root gives us the equilibrium price[14] (which is also illustrated in Figure 1):[15]

$$p^*(c) = \frac{1}{(n-1)\,x+2} + \frac{(n-1)\,x+1}{(n-1)\,x+2} c. \tag{5}$$

The intercept of this equilibrium price is decreasing in x and n, and the slope is increasing in both these variables. Hence, for any given cost draw, the equilibrium price drops as the number of firms or the efficiency parameter becomes larger. In particular, in the limit as either x or n approaches infinity, we obtain marginal cost pricing (in terms of Figure 1, the graph of the equilibrium price is approaching the 45-degree line).

We can now calculate the expected market price, expected profits, expected consumer surplus and expected total surplus, given the equilibrium price $p^*(c)$. To that end, let the kth lowest cost draw (or the kth order statistic) be denoted by $c_{(k)}$. Given that the firms all use the same (increasing) price strategy, the firm that charges the lowest price, and thus the one that serves the market, will be the firm with the lowest cost draw, $c_{(1)}$. The probability density function of $c_{(1)}$ is given by[16]

$$g_1\left[c_{(1)}\right] = nx\left[1 - c_{(1)}\right]^{nx-1}. \tag{6}$$

The price that the consumers must pay is the price charged by the firm with the lowest cost draw; denote this by p^{II} $[\equiv p^*\left(c_{(1)}\right)]$, where the superscript

[13] From $p = A + Bc$ and $B = 1 - A$ we obtain $p - c = A(1-c)$, $1 - p = (1-A)(1-c)$, and $1 + c - 2p = (1-2A)(1-c)$.

[14] In order to verify that the second order condition of the typical firm's problem is satisfied one can first, by using (2) and (5), calculate

$$[1 - F(\chi(p_i))]^{n-1} = \left[\frac{(n-1)\,x+2}{(n-1)\,x+1}(1-p_i)\right]^{x(n-1)}.$$

One can then check that the implied expression for the expected profit in (3) is strictly quasi-concave in p_i. Moreover, since this model is a special case of Spulber's (1995), it follows from his Proposition 2 that the equilibrium is unique.

[15] This closed-form solution is a generalization of the ones derived in the previous literature, which has used a uniform distribution on the interval $[0,1]$; see Wolfstetter (1999), Lofaro (2002), and Belleflamme and Peitz (2010). The closed-form solution derived by those authors is obtained by setting $x = 1$ in (5).

[16] This is straightforward to derive. It can also be found in most texts on order statistics; see, for example, David (1981, p. 8) or Balakrishnan and Rao (1998, p. 5).

"II" is short for incomplete information. Using (5), we have that the expected value of p^{II} is given by

$$E\left[p^{II}\right] = \frac{1 + \left[(n-1)x + 1\right] E\left[c_{(1)}\right]}{(n-1)x + 2}. \tag{7}$$

Similarly, expected industry profits and expected consumer surplus are, respectively,

$$E\left[\Pi^{II}\right] = \int_0^1 \left[p^{II} - c_{(1)}\right]\left(1 - p^{II}\right) g_1\left[c_{(1)}\right] dc_{(1)}, \tag{8}$$

$$E\left[S^{II}\right] = \int_0^1 \frac{\left(1 - p^{II}\right)^2}{2} g_1\left[c_{(1)}\right] dc_{(1)}. \tag{9}$$

Finally, using the above results, we have that expected total surplus is $E\left[W^{II}\right] = E\left[S^{II}\right] + E\left[\Pi^{II}\right]$. Using the above formulas together with the functional form (6), one obtains (after straightforward algebra — see Appendix A) the expressions summarized in the first column of Table 1.

3.2 Complete information

Consider now a Bertrand model that is identical to the one above, except that the cost draws of all firms are common knowledge. This is a model that is often analyzed in textbooks; as already discussed in Section 2, the equilibrium outcome is that the firm with the lowest cost draw serves the whole market and charges a price that equals either the second most efficient firm's marginal cost or, if that is lower, the monopoly price:

$$p^{CI} = \min\left\{c_{(2)}, \frac{1 + c_{(1)}}{2}\right\},$$

where the superscript "CI" is short for complete information

In order to calculate the expected market price and the other expressions that are required for the comparisons, we need the joint probability density function of $c_{(1)}$ and $c_{(2)}$. This is given by [see, for example, Gumbel (1958/2004, p. 53)]

$$g_{1,2}\left[c_{(1)}, c_{(2)}\right] = n(n-1) f\left[c_{(1)}\right] f\left[c_{(2)}\right]\left[1 - F\left(c_{(2)}\right)\right]^{n-2} \tag{10}$$

if $c_{(1)} \le c_{(2)}$ and 0 otherwise. With this at hand, we can write the expected market price as

$$\begin{aligned} E\left[p^{CI}\right] &= \int_0^1 \int_{c_{(1)}}^{\frac{1+c_{(1)}}{2}} c_{(2)} g_{1,2}\left[c_{(1)}, c_{(2)}\right] dc_{(2)} dc_{(1)} \\ &\quad + \int_0^1 \int_{\frac{1+c_{(1)}}{2}}^1 \frac{1 + c_{(1)}}{2} g_{1,2}\left[c_{(1)}, c_{(2)}\right] dc_{(2)} dc_{(1)}. \end{aligned}$$

Expected industry profits and expected consumer surplus are, respectively,

$$\begin{aligned}
E\left[\Pi^{CI}\right] &= \int_0^1 \int_{c_{(1)}}^{\frac{1+c_{(1)}}{2}} \left[c_{(2)} - c_{(1)}\right]\left[1 - c_{(2)}\right] g_{1,2}\left[c_{(1)}, c_{(2)}\right] dc_{(2)} dc_{(1)} \\
&\quad + \int_0^1 \int_{\frac{1+c_{(1)}}{2}}^1 \left[\frac{1+c_{(1)}}{2} - c_{(1)}\right]\left[1 - \frac{1+c_{(1)}}{2}\right] g_{1,2}\left[c_{(1)}, c_{(2)}\right] dc_{(2)} dc_{(1)},
\end{aligned}$$

$$\begin{aligned}
E\left[S^{CI}\right] &= \int_0^1 \int_{c_{(1)}}^{\frac{1+c_{(1)}}{2}} \frac{\left[1 - c_{(2)}\right]^2}{2} g_{1,2}\left[c_{(1)}, c_{(2)}\right] dc_{(2)} dc_{(1)} \\
&\quad + \int_0^1 \int_{\frac{1+c_{(1)}}{2}}^1 \frac{1}{2}\left[1 - \frac{1+c_{(1)}}{2}\right]^2 g_{1,2}\left[c_{(1)}, c_{(2)}\right] dc_{(2)} dc_{(1)}.
\end{aligned}$$

Finally, expected total surplus under complete information equals $E\left[W^{CI}\right] = E\left[S^{CI}\right] + E\left[\Pi^{CI}\right]$. Using these formulas one can calculate the expressions summarized in the second column of Table 1 (the algebra that is required is straightforward but tedious — see Lagerlöf (2012) for detailed derivations).

3.3 Comparison

Comparing the expressions from the incomplete information model with those from the complete information model, all of which are listed in Table 1, we have the following results.

Proposition 1. *With incomplete information instead of complete information in the symmetric model:*

- *the expected price is lower ($E\left[p^{II}\right] < E\left[p^{CI}\right]$);*
- *the expected consumer surplus is larger ($E\left[S^{II}\right] > E\left[S^{CI}\right]$);*
- *the expected total surplus is larger ($E\left[W^{II}\right] > E\left[W^{CI}\right]$);*
- *and the expected industry profits are larger ($E\left[\Pi^{II}\right] > E\left[\Pi^{CI}\right]$).*

Proof: See Appendix B.

The relationships reported in Proposition 1 are all in line with the ones found in Hansen (1988). Thus, at least in the model studied here, Hansen's assumption about the cost distribution — implying that the lowest-cost firm's optimal monopoly price always exceeds the second-lowest cost draw — does not matter for the results. The results in Proposition 1 also hold for any arbitrary number of firms, whereas Hansen assumed a duopoly market.

We can conclude that, also in this environment, asymmetric information is not anti-competitive but pro-competitive, at least in the sense that the firms' equilibrium mark-ups are (in expectation) smaller in an environment with asymmetric information. In addition, asymmetric information yields an outcome that is more efficient (in that expected total surplus is larger) and socially more desirable (in that both consumers and firms are better off).

The intuition for Hansen's results, and thus also for the ones in Proposition 1, is clearly explained in Klemperer (1999, p. 242). In the model with asymmetric information, the quantity traded if firm i wins the market depends on that firm's marginal cost (as opposed to the marginal cost of one of that firm's rivals). This has two consequences. First, it creates a stronger incentive for firm i to choose a low price, because a low price also has the effect of increasing the quantity sold if winning (not only the probability of winning). Second, it means that the quantity traded reflects the winning firm's cost of producing the good, which makes the environment with asymmetric information more productively efficient.

It should now be clear that the intuition suggested by Spulber (1995) in the quote in the introduction is misleading. Although it is true that a firm that lacks precise knowledge about its rival's cost will be unable to slightly undercut that firm's price, this does not necessarily mean that the first firm in that situation chooses a relatively high price. To choose a high price is costly, as it might mean that the firm does not win the market and thus makes a zero profit. On the other hand, to choose a relatively low price is also costly, as it lowers the profit the firm would earn if it did win the market. The firm trades those two effects off against each other. The relevant question, then, is whether the optimal tradeoff leads to a higher or a lower expected price in an environment where the firm does not know the rivals' costs. The intuition in the above paragraph suggests that the expected price should in fact be lower: With uncertainty (and only then), the traded quantity depends on the firm's own price, which means that the firm has an extra incentive to charge a low price and thus increase its demand.

4 Asymmetric firms

Both Hansen (1988) and Spulber (1995) assume that the firms draw their cost parameters from identical distributions. This means that, in a symmetric equilibrium of the incomplete information model, the firm with the lowest cost draw always serves the whole market, which obviously is beneficial for production efficiency. However, with asymmetric cost distributions also the equilibrium strategies will be asymmetric; in particular, a firm with a less favorable distribution will try to compensate for this by using a more aggressive pricing strategy, which means that the market will sometimes be served by a firm that has *not* drawn the lowest cost. In order to explore the consequences of this I will in this section study an asymmetric version of the Hansen-Spulber model, using numerical methods. I will confine attention to the case of a duopoly.

4.1 Model

There are two risk neutral and profit maximizing firms that compete à la Bertrand in a homogeneous product market. The firms choose their prices simultaneously and they interact only once. Market demand is given by $D(p) = 1-p$, where p denotes price. The firm that charges the lowest price, denoted $p_{\min}$, sells the quantity $D(p_{\min})$ at that price, while the other firm sells nothing and makes a zero profit. If the two firms have chosen the same price, then they share the market equally between themselves.

Firm i's (for $i = 1, 2$) constant marginal cost is denoted by c_i. The two cost parameters are drawn from two, not necessarily identical, distributions, $F_1(c_1)$ and $F_2(c_2)$, where

$$F_i(c_i) = 1 - (1 - c_i)^{x_i}$$

for $0 < x_1 \leq x_2$. That is, firm 2 is, from an ex ante perspective, the (weakly) more efficient firm. The two cost draws are independent, and the associated density functions are denoted $f_1(c_1)$ and $f_2(c_2)$.

Using numerical methods I will solve and compare two versions of this model: One where the cost draws are common knowledge and one where each firm's draw is private information of that firm.

4.2 Equilibrium characterization

For each firm i I will look for an equilibrium strategy $p_i^*(c_i)$ that is strictly increasing and differentiable. Denote the inverse of this function by $\chi_i(p_i)$, meaning that χ_i is the value of the cost draw that would give rise to the price p_i. Thus, if firm 1 has drawn the marginal cost c_1, it chooses its price p_1 in order to maximize its expected profit,

$$E\pi_1 = (p_1 - c_1)(1 - p_1)[1 - F_2(\chi_2(p_1))].$$

The first order condition of this problem can be written as

$$\begin{aligned}\frac{E\pi_1}{\partial p_1} &= (1 + c_1 - 2p_1)[1 - F_2(\chi_2(p_1))] - (p_1 - c_1)(1 - p_1) f_2[\chi_2(p_1)] \frac{d\chi_2}{dp_1} \\ &= 0.\end{aligned}$$

Rearranging, we have

$$\frac{d\chi_2}{dp_1} = \frac{(1 - \chi_2)(1 - 2p_1 + c_1)}{x_2(p_1 - c_1)(1 - p_1)}.$$

Now write $\chi_2 = c_2$, ignore the subindex on p_1, and add the analogous equation for firm 2. We then obtain the following pair of differential equations:

$$\begin{cases} \frac{dc_1}{dp} = \frac{(1-c_1)(1-2p+c_2)}{x_1(p-c_2)(1-p)} \\ \frac{dc_2}{dp} = \frac{(1-c_2)(1-2p+c_1)}{x_2(p-c_1)(1-p)} \end{cases} \qquad \underline{p} \le p \le 1. \tag{11}$$

The associated boundary conditions are

$$c_1\left(\underline{p}\right) = c_2\left(\underline{p}\right) = 0 \qquad c_1(1) = c_2(1) = 1. \tag{12}$$

Thus, $\underline{p}$ is the price each firm charges if its cost equals zero, and it is of course endogenous to the analysis.[17]

4.3 Numerical analysis

The differential equations (11) together with the equations in (12) constitute a nonlinear boundary value problem, in which the location of the left boundary $\underline{p}$ is unknown. This boundary value problem is of course related to the ones resulting from the first-price private-value auction models with a fixed quantity that are studied in the previous literature. The differences are that (i) here it is the left instead of the right boundary that is unknown and (ii) the right-hand sides of the differential equations (11) are more complex, due to the downward-sloping demand. The differential equations in this problem as well as the ones resulting from the fixed-quantity models are not particularly well-behaved; indeed, to solve the fixed-quantity model numerically using the standard methods has often proved difficult.[18] The added complexity of the present problem, due to the downward-sloping demand function, is likely to add further to these difficulties.

To overcome these problems, I will adopt an approach suggested recently by Fibich and Gavish (2011), which they call the boundary value method. Their starting point is the observation that a main difficulty with the standard formulation of the problem is that, as mentioned above, one of the boundary conditions is stated in terms of an endogenous variable (in the present application this variable is $\underline{p}$, the price that either one of the firms charges if having a zero marginal cost). To deal with this problem Fibich and Gavish rewrite the two differential equations so they become functions of the realized cost of one

[17] The equalities $c_1\left(\underline{p}\right) = c_2\left(\underline{p}\right) = 0$ must hold at an equilibrium. If they did not, then one of the firms would charge a price $p_i(0)$ that is strictly lower than the price charged by the rival for any cost realization of that firm. Therefore the firm charging $p_i(0)$ could raise this price and still win the market with probability one. The equalities $c_1(1) = c_2(1) = 1$ must hold because of the same reason as the boundary condition $p(1) = 1$ in Section 3.1 must hold (see f.n. 12).

[18] For example, Marshall et al. (1994, pp. 194-95) write: "[N]umerical solutions to the first order conditions for the existence of [a Bayesian Nash equilibrium of an asymmetric first price auction] are non-trivial to evaluate. Although these solutions belong to a class of 'two-point boundary value problems' for which there exist efficient numerical solution techniques, they all suffer from major pathologies at the origin."

of the firms, rather than the price. The boundary conditions are also rewritten accordingly. Doing this for the problem in (11) and (12) yields:

$$\begin{cases} \frac{dc_1}{dc_2} = \frac{x_2(p-c_1)(1-c_1)(1-2p+c_2)}{x_1(p-c_2)(1-c_2)(1-2p+c_1)} \\ \frac{dp}{dc_2} = \frac{x_2(p-c_1)(1-p)}{(1-c_2)(1-2p+c_1)} \end{cases} \qquad 0 \leq c_2 \leq 1 \tag{13}$$

with the boundary conditions

$$c_1(c_2 = 0) = 0 \qquad c_1(c_2 = 1) = p(1) = 1. \tag{14}$$

The advantage with this way of formulating the problem is that it is now defined on a fixed domain $c_2 \in [0, 1]$.

Following Fibich and Gavish I will solve the boundary value problem in (13) and (14) using fixed-point iterations. In particular I will use the following formulation, which is analogous to the one used in Fibich and Gavish (2011, eq. (21)):

$$\begin{cases} \left[\frac{d}{dc_2} + \frac{x_2\left(1-c_1^{(k)}\right)\left(1-2p^{(k)}+c_2\right)}{x_1\left(p^{(k)}-c_2\right)(1-c_2)\left(1-2p^{(k)}+c_1^{(k)}\right)}\right] c_1^{(k+1)} = \frac{x_2 p^{(k)}\left(1-c_1^{(k)}\right)\left(1-2p^{(k)}+c_2\right)}{x_1\left(p^{(k)}-c_2\right)(1-c_2)\left(1-2p^{(k)}+c_1^{(k)}\right)} \\ \left[\frac{d}{dc_2} - \frac{x_2\left(1-p^{(k)}\right)}{(1-c_2)\left(1-2p^{(k)}+c_1\right)}\right] p^{(k+1)} = \frac{-x_2 c_1^{(k+1)}\left(1-p^{(k)}\right)}{(1-c_2)\left(1-2p^{(k)}+c_1^{(k+1)}\right)} \end{cases}$$

with the boundary conditions

$$c_1(0)^{(k+1)} = 0 \qquad c_1^{(k+1)}(1) = p^{(k+1)}(1) = 1,$$

where $k = 0, 1, \ldots$ is the iteration number. In the numerical analysis presented in the paper I used the initial guess

$$c_1^{(0)}(c_2) = c_2 \qquad p^{(0)}(c_2) = (1 + 2c_2)/3.$$

Overall, using the boundary value method and the fixed-point iterations above to solve the problem in (13) and (14) appear to work well: The iterations converge and for the symmetric case the solution coincides with the known analytical solution.[19]

4.4 Results

It is useful to begin the presentation of the results by studying a figure that shows how the equilibrium prices in a situation with two asymmetric firms

[19] However, convergence occurs only for integer values of x_1 and x_2. Also, it seems to matter that the code is treating c_2 and not c_1 as the independent variable, as the code works well only for $x_2 \geq x_1$. This appears to be a problem that Fibich and Gavish (2011, p. 17) had in their analysis, too: "[...] we computed the equilibrium strategies with $F_1(v) = v$ and $F_2(v) = v^2$ by changing the independent variable from b to v_2 and solving the transformed nonlinear system using the fixed-point iterations. The same fixed-point iterations, however, would diverge if we choose v_1 as the independent variable. Further research is needed in order to eliminate the ad-hoc choice of the independent variable." The Matlab code that was used to generate the results reported in the present paper is available at www.johanlagerlof.org.

differ from the symmetric strategies discussed in Section 3. Figure 2a graphs the two asymmetric equilibrium price strategies in a diagram with cost on the horizontal axis and price on the vertical axis, assuming $x_1 = 1$ and $x_2 = 12$; these strategies are shown as the two curves in the diagram, with the lower one belonging to firm 1. As benchmarks, two straight lines are also indicated in the figure; they represent the symmetric equilibrium prices when both firms have the low ($x_1 = x_2 = 1$) and the high ($x_1 = x_2 = 12$) efficiency parameter, respectively. The effect of firm 2's obtaining an efficiency advantage is thus that firm 1's price drops more than firm 2's price. This is the same phenomenon as in asymmetric first price auctions with a fixed quantity: The firm that is less efficient compensates for this by adopting a more aggressive pricing strategy.

Figure 2b shows how the expected prices of firm 1 (dashed curve) and firm 2 (dotted curve), as well as the expected market price $E\left[p^{II}\right]$ (solid curve), change as firm 2's efficiency is gradually increased from $x_2 = 1$ to $x_2 = 20$, while keeping firm 1's efficiency parameter fixed at $x_1 = 1$. For $x_1 = x_2 = 1$, the numbers are of course the same as we obtained in the analysis of the symmetric model with $x = 1$ and $n = 2$.[20] As firm 2's efficiency parameter x_2 is increased, the expected price of each firm drops, as one would expect. For any $x_2 > 1$, firm 1's expected price is higher than firm 2's. This is because even though firm 1 uses a more aggressive pricing strategy, it tends to draw higher cost parameters than firm 2. The expected market price is of course (by construction) lower than each of the individual expected firm prices.

Figure 2c plots the probabilities that firm 2 has the lowest cost (dashed curve) and the lowest price (dotted curve), respectively, against firm 2's level of ex ante efficiency (as before keeping firm 1's efficiency parameter fixed at $x_1 = 1$). We see that both graphs are upward-sloping. However, because of the fact that firm 1 uses a more aggressive pricing strategy than firm 2 for all $x_1 < x_2$, the probability that firm 2 has the lowest price increases more slowly than the probability that it has the lowest cost. Hence the market is sometimes served by the firm that is ex post less efficient (in the sense that it has drawn the highest cost), which has an adverse effect on production efficiency. The probability that the market is served by the ex post less efficient firm is graphed in Figure 2d, which shows that the probability increases in the level of asymmetry, taking the approximate value of 18 percent at $x_2 = 20$.

Figures 3a-c compare the expected levels of market price, consumer surplus and total surplus, respectively, under incomplete information (dotted curves) and complete information (solid curves). It is immediately clear (Figure 3a)

[20] In particular, we know from Table 1 and eq. (5), respectively, that we should in this case have $E\left[p_1^*\right] = E\left[p_2^*\right] = 2/3$ and $E\left[p^{II}\right] = 5/9$, which is indeed the result of the numerical analysis.

that the expected market price is lower in the model with incomplete information also when the firms are asymmetric. Indeed, the difference in expected price becomes larger as the difference between the firms grows (not only in absolute but also in relative terms, as Figure 3d shows). We also see (Figures 3b-c and 3e-f) that privately informed firms has a positive effect on expected consumer and total surplus, with asymmetric as well as symmetric firms. Again, the extent to which there is such a positive effect is actually increasing in the level of asymmetry between the firms. For expected consumer surplus, this is unambiguously true regardless of whether we consider the absolute or relative difference. For expected total surplus the relative measure is not monotonically increasing in the level of asymmetry, although overall (comparing, for example, $x_2 = 1$ and $x_2 = 20$) there is indeed an increase.

The results reported in this section are so far all in line with the ones in Proposition 1 and the ones derived by Hansen (1988). This suggests that the conclusions drawn there are robust to the introduction of ex ante asymmetric firms: The fact that firms are privately informed about their costs intensifies competition, in the sense that it lowers the expected price level and raises the expected consumer and total surplus. Indeed, we found that the effects on those three variables become even stronger when the firms draw their cost parameters from different distributions.

However, the assumption of asymmetric firms does alter our previous result about the expected profit levels. Figures. 4a-c compare expected profit levels for firm 1, firm 2 and in the aggregate, respectively, under incomplete information (dotted curves) and complete information (solid curves). We see that while firm 1 benefits from incomplete information (as both firms do when $x_1 = x_2$), firm 2 is worse off for all $x_2 > x_1$. This result is quite intuitive. When firm 2 has the largest efficiency parameter, it is more likely to be the firm that has the lowest cost. When it does have the lowest cost, it is beneficial for firm 2 if the cost parameters are common knowledge, because then firm 2 can gain the whole market with probability one by just slightly undercutting firm 1's cost (or, if that is lower, charging the optimal monopoly price). Thus the firms' costs being common knowledge is good for firm 2's profit and bad for firm 1's profit. Figure 4c shows the expected aggregate profits. We see that if the difference between the firms' efficiency parameters is large enough, aggregate profits are lower under incomplete information, as for those parameter values the loss of firm 2 dominates the gain of firm 1.

5 Conclusions

The starting point for this paper was the observation that, contrary to what naive intuition might suggest, competition is *intensified* if we to the standard homogeneous-good Bertrand model add the assumption that firms have private information about their marginal costs (thereby obtaining what here is referred to as the Hansen-Spulber model). This has been known in the literature since Hansen (1988), although several authors writing after him appears to have overlooked this result. The proof of Hansen's result relies, however, on at least two assumptions that, if relaxed, conceivably could overturn the result.

The first assumption is that a firm cannot obtain a cost advantage that is so large that it implies a "drastic innovation" (i.e., it is never so large that the firm can charge its monopoly price without fear of being undercut). The first part of the present paper formulated a version of the Hansen-Spulber model that allows for a closed-form solution, and within that framework it was shown that Hansen's result holds also when we allow for the possibility that a firm makes a drastic innovation. As in Hansen's original model, uncertainty lowers expected price and raises expected consumer and total surplus. However, uncertainty also makes expected industry profits go up, which of course can be thought of as providing the firms with more market power.

The second assumption that is crucial for Hansen's proof is that the firms draw their costs from identical distributions. The second part of the present paper used numerical methods to investigate the consequences of relaxing that assumption. To the best of my knowledge, this is the first investigation of asymmetries in the Hansen-Spulber model that can be found in the literature, although there is a fairly large and growing literature on asymmetries in first price auctions with a fixed quantity.[21] The results of the numerical analysis suggest that uncertainty lowers expected price and raises expected consumer and total surplus *even more* when the firms are asymmetric compared to when they draw their costs from identical distributions. The results also show that, if the asymmetry is large enough, expected industry profits are *lower* under uncertainty. The results for the asymmetric model therefore reinforce the notion that uncertainty intensifies competition rather than softens it, as here the firms lose market power also in the sense that aggregate profits decrease.

At least two extensions of the present analysis suggest themselves as candidates for future work on this topic. First, a relatively straightforward exercise would be to run simulations with more than two firms, for example, one or two ex ante efficient "dominant" firms and in addition a "competitive fringe" of firms that are ex ante less efficient. Second, an interesting extension would be

[21] The Hansen-Spulber model is effectively an auction model with a *variable* quantity.

to assume that a regulator can choose a price cap that the firms are not allowed to exceed (similar to a reserve price in an auction). From a social welfare point of view, a price cap has the potential benefit of inducing firms to charge prices closer to their marginal costs, as long as these costs do not exceed the price cap; the drawback with a price cap, however, is that any gains from trade will be lost whenever all firms' marginal costs exceed the price cap. In this framework one could investigate at which level a price cap should be set in order to maximize total (or consumer) surplus. Moreover, given such an optimal price cap, we could again ask the question whether uncertainty is indeed pro-competitive.

Appendix A: Derivation of the expressions in the left column of Table 1

Here I derive the expressions that are reported in the left column of Table 1. I consider, in turn, expected price, expected industry profits, expected consumer surplus, and expected total surplus. First, however, I derive some preliminary results that will be used in the later derivations.

I will make use of the following two results (the first result can be verified by differentiating the right-hand side, and the second one follows from the first one):

$$\int c\,(1-c)^a\,dc = -\,(1-c)^{a+1}\,\frac{(a+1)\,c+1}{(1+a)\,(2+a)}, \tag{15}$$

$$\int_0^1 c\,(1-c)^a\,dc = \frac{1}{(1+a)\,(2+a)}. \tag{16}$$

Next, using (6), we have that the expected value of $c_{(1)}$ is given by

$$E\left[c_{(1)}\right] = \int_0^1 c_{(1)} g_1\left[c_{(1)}\right] dc_{(1)} = \int_0^1 c_{(1)} nx\left[1-c_{(1)}\right]^{nx-1} dc_{(1)} = \frac{1}{1+nx}, \tag{17}$$

where the last equality follows from (16). Finally, recall from Section 3 that the market price (i.e., the price charged by the firm with the lowest cost draw) equals

$$p^{II} = A + (1-A)\,c_{(1)}, \qquad \text{with } A = \frac{1}{(n-1)\,x+2}. \tag{18}$$

We are now ready to calculate the expressions in the left column of Table 1. Using (7) and (17), we have that the expected value of the market price equals

$$\begin{aligned} E\left[p^{II}\right] &= \frac{1+[(n-1)\,x+1]\,E\left[c_{(1)}\right]}{(n-1)\,x+2} = \frac{(1+nx)+(n-1)\,x+1}{[(n-1)\,x+2]\,(1+nx)} \\ &= \frac{2+(2n-1)\,x}{[(n-1)\,x+2]\,(1+xn)}. \end{aligned}$$

From (8) we have that expected industry profits equal

$$\begin{aligned}
E\left[\Pi^{II}\right] &= \int_0^1 \left[p^{II}-c_{(1)}\right]\left(1-p^{II}\right) g_1\left[c_{(1)}\right] dc_{(1)} \\
&= \int_0^1 A\left[1-c_{(1)}\right](1-A)\left[1-c_{(1)}\right] g_1\left[c_{(1)}\right] dc_{(1)} \\
&= A(1-A)\, nx \int_0^1 \left[1-c_{(1)}\right]^{nx+1} dc_{(1)} = \frac{A(1-A)\, nx}{nx+2} \\
&= \frac{nx\left[(n-1)x+1\right]}{(nx+2)\left[(n-1)x+2\right]^2},
\end{aligned}$$

where the second equality uses $p^{II}-c_{(1)} = A\left[1-c_{(1)}\right]$ and $1-p^{II} = (1-A)\left[1-c_{(1)}\right]$, the third one uses (6), and the last equality uses (18).

From (9) we have that expected consumer surplus equals

$$\begin{aligned}
E\left[S^{II}\right] &= \int_0^1 \frac{\left(1-p^{II}\right)^2}{2} g_1\left[c_{(1)}\right] dc_{(1)} \\
&= \int_0^1 \frac{(1-A)^2\left[1-c_{(1)}\right]^2}{2} g_1\left[c_{(1)}\right] dc_{(1)} \\
&= \frac{nx(1-A)^2}{2} \int_0^1 \left[1-c_{(1)}\right]^{nx+1} dc_{(1)} \\
&= \frac{nx(1-A)^2}{2(nx+2)} = \left[\frac{(n-1)x+1}{(n-1)x+2}\right]^2 \frac{nx}{2(nx+2)},
\end{aligned}$$

where the second equality uses $1-p^{II} = (1-A)\left[1-c_{(1)}\right]$, the third one uses (6), and the last equality uses (18).

Finally, using the above results, we have that expected total surplus is

$$\begin{aligned}
E\left[W^{II}\right] &= E\left[S^{II}\right] + E\left[\Pi^{II}\right] = \\
&\quad \left[\frac{(n-1)x+1}{(n-1)x+2}\right]^2 \frac{nx}{2(nx+2)} + \frac{nx\left[(n-1)x+1\right]}{(nx+2)\left[(n-1)x+2\right]^2} \\
&= \frac{nx\left[(n-1)x+1\right]^2 + 2nx\left[(n-1)x+1\right]}{2(nx+2)\left[(n-1)x+2\right]^2} \\
&= \frac{nx\left[(n-1)x+1\right]\left[(n-1)x+1+2\right]}{2(nx+2)\left[(n-1)x+2\right]^2} \\
&= \frac{nx\left[(n-1)x+1\right]\left[(n-1)x+3\right]}{2(nx+2)\left[(n-1)x+2\right]^2}.
\end{aligned}$$

Appendix B: Proof of Proposition 1

Here I derive the results reported in Proposition 1. First, however, I state and prove a lemma that will be used in the later derivations.

Lemma B1. *We have* $(n-1)x+2 < 2^{(n-1)x+1}$ *for all* $x>0$ *and* $n>1$.

Proof. We can write

$$(n-1)\,x+2<2^{(n-1)x+1}\Leftrightarrow \ln(y+2)<(y+1)\ln(2)\Leftrightarrow h(y)>0,$$

where

$$y\equiv(n-1)\,x \qquad \text{and} \qquad \xi(y)\equiv(y+1)\ln(2)-\ln(y+2)\,.$$

The result now follows from the facts that $\xi(0)=0$ and

$$\xi'(y)=\ln(2)-\frac{1}{y+2}>0$$

(where the last inequality holds because $\ln(2)>0.5$ and $y>0$). $\square$

Consider the comparison of expected prices. Using the expressions in Table 1 we have

$$E\left[p^{II}\right]<E\left[p^{CI}\right]\Leftrightarrow \frac{2\,(nx+1)-x}{(nx+1)\,[(n-1)\,x+2]}<\frac{(2n-1)\,x+1-nx\left(\frac{1}{2}\right)^{(n-1)x+1}}{(nx+1)\,[(n-1)\,x+1]}\Leftrightarrow$$

$$[2\,(nx+1)-x]\,[(n-1)\,x+1]<\left[(2n-1)\,x+1-nx\left(\frac{1}{2}\right)^{(n-1)x+1}\right][(n-1)\,x+2]\Leftrightarrow$$

$$\begin{aligned}
& [(n-1)\,x+2]\,nx\left(\frac{1}{2}\right)^{(n-1)x+1} \\
< \; & [(2n-1)\,x+1]\,[(n-1)\,x+2]-[2\,(nx+1)-x]\,[(n-1)\,x+1] \\
= \; & (n-1)\,x\,\{[(2n-1)\,x+1]-[2\,(nx+1)-x]\}+2\,[(2n-1)\,x+1]-[2\,(nx+1)-x] \\
= \; & -(n-1)\,x+2nx-x=nx\Leftrightarrow
\end{aligned}$$

$$[(n-1)\,x+2]\left(\frac{1}{2}\right)^{(n-1)x+1}<1\Leftrightarrow(n-1)\,x+2<2^{(n-1)x+1},$$

which we know holds for all $x>0$ and $n>1$ (see Lemma B1).

Next consider the comparison of expected industry profits in the two models. Using the expressions in Table 1 we have

$$E\left[\Pi^{CI}\right]<E\left[\Pi^{II}\right]\Leftrightarrow\frac{nx\left[(n-1)\,x+\left(\frac{1}{2}\right)^{(n-1)x+1}\right]}{(nx+2)\,[(n-1)\,x+1]\,[(n-1)\,x+2]}<\frac{nx\,[(n-1)\,x+1]}{(nx+2)\,[(n-1)\,x+2]^2}\Leftrightarrow$$

$$[(n-1)\,x+2]\left[(n-1)\,x+\left(\frac{1}{2}\right)^{(n-1)x+1}\right]<[(n-1)\,x+1]^2\Leftrightarrow$$

$$[(n-1)\,x+2]\left(\frac{1}{2}\right)^{(n-1)x+1}<[(n-1)\,x+1]^2-[(n-1)\,x+2]\,(n-1)\,x=1\Leftrightarrow$$

$$(n-1)\,x+2<2^{(n-1)x+1},$$

which we know holds for all $x>0$ and $n>1$ (see Lemma B1).

Now consider the comparison of expected consumer surplus in the two models. Using the expressions in Table 1 we have

$$E\left[S^{CI}\right] < E\left[S^{II}\right] \Leftrightarrow \frac{nx\left[(n-1)\,x + \left(\frac{1}{2}\right)^{x(n-1)+1}\right]}{2\left[x\,(n-1)+2\right](xn+2)} < \left[\frac{(n-1)\,x+1}{(n-1)\,x+2}\right]^2 \frac{nx}{2\,(nx+2)}.$$

By simplifying this inequality one can verify that it is equivalent to the inequality $E\left[\Pi^{CI}\right] < E\left[\Pi^{II}\right]$ above, which we saw holds for all $x > 0$ and $n > 1$.

Finally, the fact that $E\left[W^{CI}\right] < E\left[W^{II}\right]$ follows immediately from the results that $E\left[S^{CI}\right] < E\left[S^{II}\right]$ and $E\left[\Pi^{CI}\right] < E\left[\Pi^{II}\right]$.

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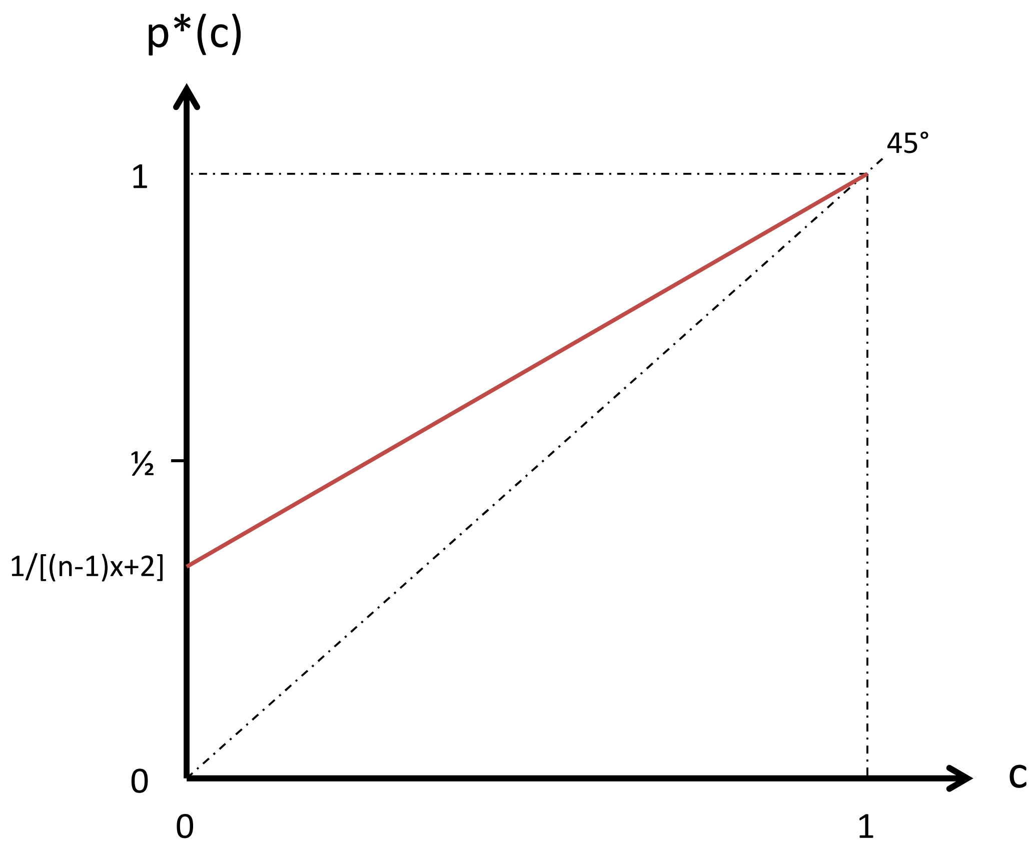


Figure 1: The equilibrium price strategy

Table 1. Expected price, profits, consumer surplus and total surplus. The model with incomplete information and the one with complete information.

	Incomplete info model	Complete info model
Expected price	$\frac{2(nx+1)-x}{(nx+1)[(n-1)x+2]}$	$\frac{(2n-1)x+1-nx\left(\frac{1}{2}\right)^{(n-1)x+1}}{(nx+1)[(n-1)x+1]}$
Expected industry profits	$\frac{nx[(n-1)x+1]}{(nx+2)[(n-1)x+2]^2}$	$\frac{nx\left[(n-1)x+\left(\frac{1}{2}\right)^{(n-1)x+1}\right]}{(nx+2)[(n-1)x+1][(n-1)x+2]}$
Expected consumer surplus	$\frac{nx[(n-1)x+1]^2}{2(nx+2)[(n-1)x+2]^2}$	$\frac{nx\left[(n-1)x+\left(\frac{1}{2}\right)^{x(n-1)+1}\right]}{2(nx+2)[x(n-1)+2]}$
Expected total surplus	$\frac{nx[(n-1)x+1][(n-1)x+3]}{2(nx+2)[(n-1)x+2]^2}$	$\frac{nx[(n-1)x+3]\left[(n-1)x+\left(\frac{1}{2}\right)^{x(n-1)+1}\right]}{2(nx+2)[(n-1)x+1][x(n-1)+2]}$

Fig. 2a: Equilibrium pricing strategies

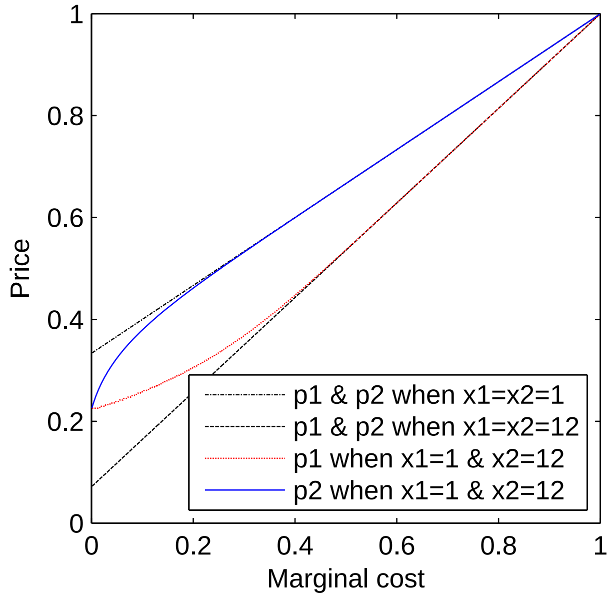


Fig. 2b: Expected prices

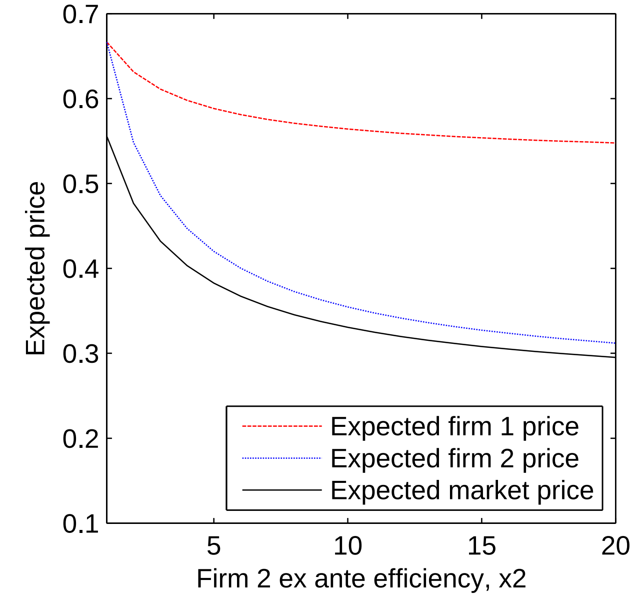


Fig. 2c: Prob firm 2 price, resp. cost, is lowest

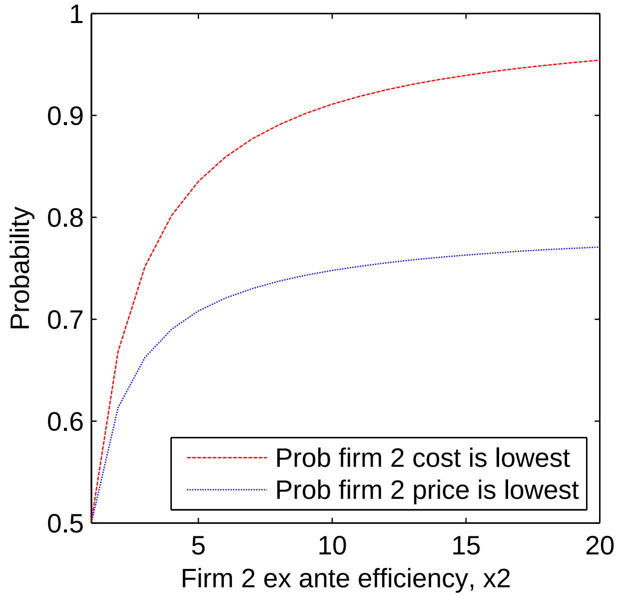
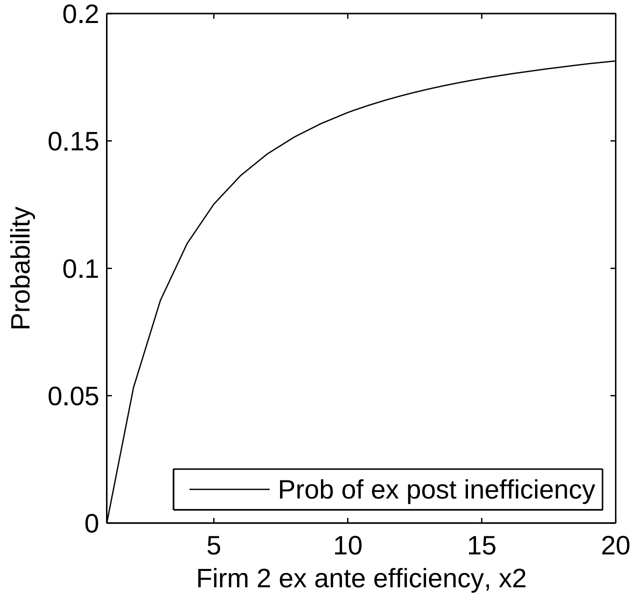


Fig. 2d: Prob of ex post inefficiency



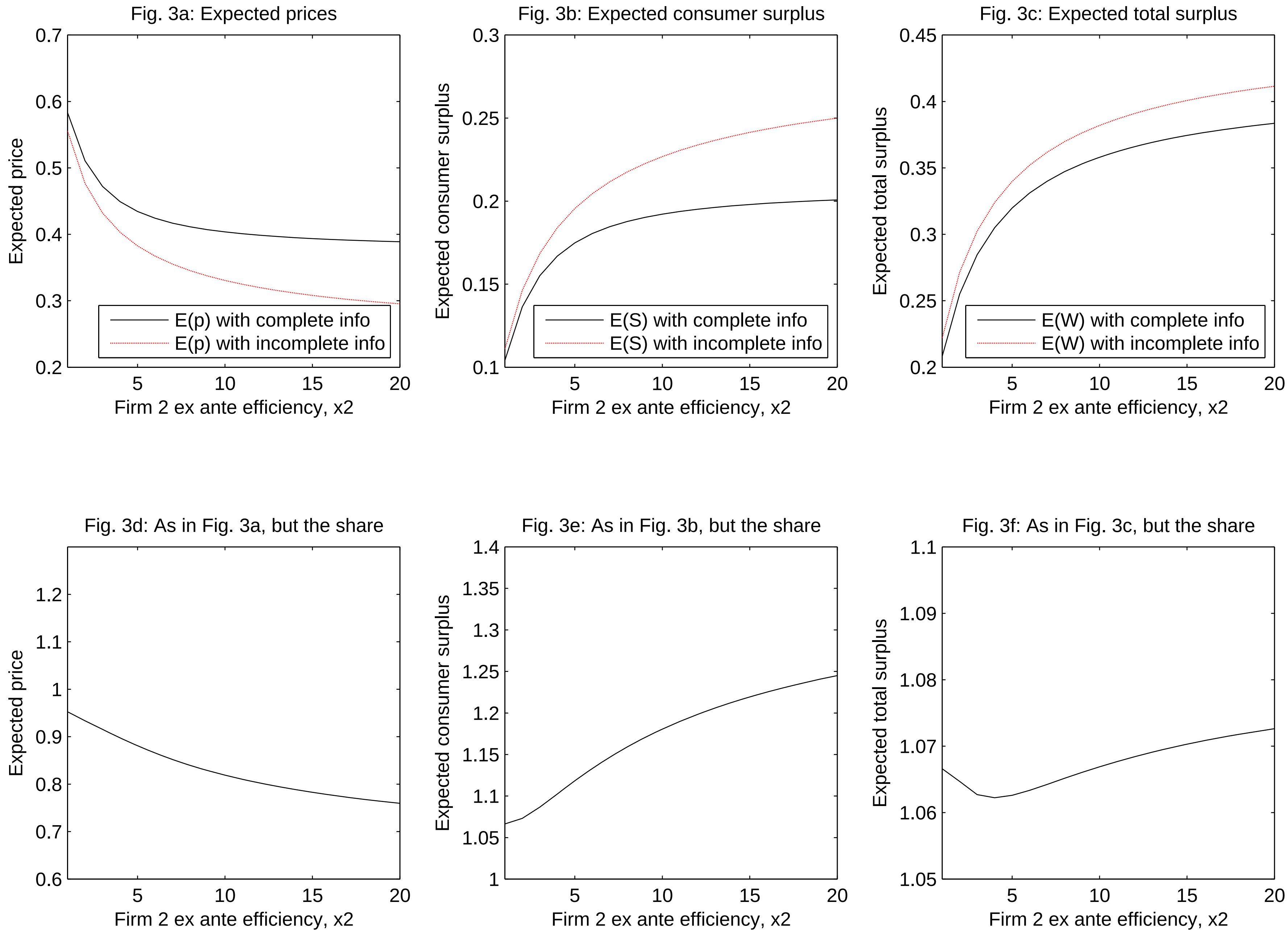


Fig. 3a: Expected prices
Expected price
0.7
0.6
0.5
0.4
0.3
0.2
E(p) with complete info
E(p) with incomplete info
5
10
15
20
Firm 2 ex ante efficiency, x2
Fig. 3b: Expected consumer surplus
Expected consumer surplus
0.3
0.25
0.2
0.15
0.1
E(S) with complete info
E(S) with incomplete info
5
10
15
20
Firm 2 ex ante efficiency, x2
Fig. 3c: Expected total surplus
Expected total surplus
0.45
0.4
0.35
0.3
0.25
0.2
E(W) with complete info
E(W) with incomplete info
5
10
15
20
Firm 2 ex ante efficiency, x2
Fig. 3d: As in Fig. 3a, but the share
Expected price
1.2
1.1
1
0.9
0.8
0.7
0.6
5
10
15
20
Firm 2 ex ante efficiency, x2
Fig. 3e: As in Fig. 3b, but the share
Expected consumer surplus
1.4
1.35
1.3
1.25
1.2
1.15
1.1
1.05
1
5
10
15
20
Firm 2 ex ante efficiency, x2
Fig. 3f: As in Fig. 3c, but the share
Expected total surplus
1.1
1.09
1.08
1.07
1.06
1.05
5
10
15
20
Firm 2 ex ante efficiency, x2

Fig. 4a: Exp firm 1 profits

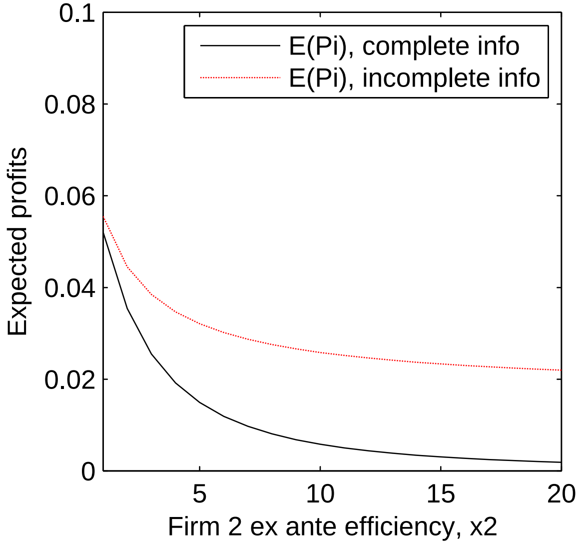


Fig. 4b: Exp firm 2 profits

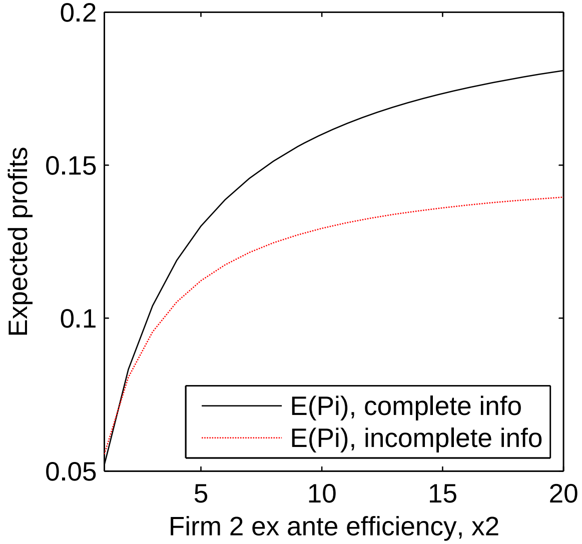


Fig. 4c: Exp aggregate profits