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Gianluca Benigno and Hande Küçük

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Gianluca Benigno, London School of Economics, CEP and CEPR
Hande Küçük, Research and Monetary Policy Department, Central Bank of Turkey and CEP

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Centre for Economic Policy Research
77 Bastwick Street, London EC1V 3PZ, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

Portfolio Allocation and International Risk Sharing*

We show that recent explanations of the consumption-real exchange rate anomaly which rely on goods and financial market frictions are not robust to introducing just one additional international asset. When portfolios are selected optimally, international trade in two nominal bonds implies a consumption-real exchange rate correlation that is too high compared to the data even when there are many shocks. Monetary policy specification plays a potentially important role for the degree of risk sharing provided by nominal bonds, both in the benchmark model with only tradable and non-tradable sector supply shocks and also in the model which allows for news or quality (i-pod) shocks.

JEL Classification: F31 and F41
Keywords: consumption-real exchange rate anomaly, incomplete financial markets, international risk sharing and portfolio choice

Gianluca Benigno
Department of Economics
London School of Economics
Houghton Street
London WC2A 2AE

Email: g.benigno@lse.ac.uk

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Hande Küçük
Research and Monetary Policy Department
Central Bank of Turkey
TCMB Idare Merkezi
Istiklal Caddesi No.10
Ulus, Ankara 06100
TURKEY

Email: handekucuk1@gmail.com

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www.cepr.org/pubs/new-dps/dplist.asp?authorid=169264

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1 Introduction

The last two decades have witnessed a dramatic increase in international capital flows. Lane and Milesi-Ferretti (2001, 2006) have documented the increase in gross holdings of cross-country bond and equities for various countries. Their analysis shows that gross external financial positions now exceed 100% of GDP for major industrialised countries.

Despite this massive wave of financial globalisation, international risk sharing remains low. Efficient risk sharing requires that consumption should be higher in the country where it is cheaper to consume, implying a positive correlation between relative consumption and real exchange rate (RER).[1] However, as first shown by Backus and Smith (1993) and Kollmann (1995), this is strongly rejected in the data. More recently, Obstfeld (2006) measures the degree of risk sharing by looking at averages of consumption growth and real exchange rates for various countries as in the original Backus and Smith (1993) paper. Using this metric, he finds a distinct negative relationship (i.e. faster consumption growth is associated with a real appreciation) in the data for the period going from 1991 to 2006 -the period of financial integration- suggesting a worsening rather than an improvement in international risk sharing. Table 1 displays data on financial globalisation, net foreign currency exposure and international risk sharing (measured by the correlation of relative consumption and the real exchange rate) for industrialised countries for 1991 and 2004.

While recent contributions (Benigno and Thoenissen, 2008 and Corsetti et al. 2008)[2] have successfully replicated the low degree of international risk sharing in the context of DSGE models, their analysis is based on a simple international financial market structure in which only a riskless bond is traded, a structure that is far from reflecting the recent trend in international financial integration.

Our contribution is to examine the extent to which a more plausible asset market structure is compatible with low international risk sharing as the current evidence suggests. We find that even in the case where we only allow for international trade in two nominal bonds, the so-called consumption real exchange anomaly is back.

It is well-known in the international risk sharing literature that specifying a model with incomplete financial markets is not sufficient to generate a negative correlation between relative consumption and real exchange rates even when international asset trade is restricted to a non-contingent bond (see Baxter and Crucini, 1995 and Chari et al., 2002). More importantly, Cole and Obstfeld (1991) show that terms of trade movements can provide consider-

[1] We define the real exchange rate as the price of the foreign consumption basket in units of the home consumption good, i.e. an increase implies a real depreciation of the home currency.

[2] Throughout this paper we frequently refer to these papers as BT and CDL, respectively.

	Financial Globalisation (A+L)/GDP		Net FX exposure as % of GDP		Cor(C-C^{US},Q) Hp-filtered		Cor(C-C^{US}, Q) First-differenced	
	1990	**2004**	**1990**	**2004**	**1970-1990**	**1991-2006**	**1970-1990**	**1991-2006**
Australia	95.5	218.4	-2.7	17.6	-0.26	-0.80	-0.13	-0.62
Austria	130.3	387.9	-7.8	-28.7	0.01	-0.65	-0.07	-0.19
Belgium	394.8	802.5	36.9	24.9	-0.17	-0.28	-0.12	0.02
Canada	122.1	211.1	-2.2	52.9	-0.53	-0.70	-0.16	-0.52
Denmark	195.8	398.7	-27.7	56.4	-0.08	-0.59	-0.24	-0.28
Finland	92.1	396.1	-23.5	50.3	-0.27	-0.53	-0.06	-0.63
France	128.5	415.1	17.3	37.0	-0.13	-0.37	-0.11	-0.27
Germany	118.6	325.6	18.7	19.2	-0.32	-0.28	-0.34	0.00
Greece	74.2	194.0	-9.6	10.4	-0.32	-0.76	-0.13	-0.57
Italy	73.9	222.5	-2.4	9.9	-0.12	-0.48	-0.04	-0.32
Japan	111.7	141.9	10.3	58.1	0.14	-0.23	0.19	-0.08
Netherlands	260.0	767.4	59.2	87.8	-0.45	0.59	-0.41	0.40
New Zealand	133.6	224.8	-27.0	-19.2	-0.15	-0.92	-0.18	-0.91
Norway	110.1	337.8	3.2	103.8	0.19	-0.39	0.01	-0.29
Portugal	85.3	404.0	13.2	2.1	-0.60	-0.19	-0.56	0.01
Spain	62.7	285.0	12.3	7.1	-0.64	-0.55	-0.45	-0.42
Sweden	147.8	422.8	-11.6	95.1	-0.55	-0.43	-0.28	-0.45
Switzerland	378.1	956.6	119.3	317.2	0.09	-0.29	0.06	-0.02
UK	349.0	713.3	52.1	99.5	-0.56	-0.05	-0.51	0.10
US	80.1	192.2	14.9	46.8				
Median	**120.4**	**362.8**	**6.8**	**41.9**	**-0.26**	**-0.43**	**-0.13**	**-0.28**

Table 1: International portfolios and relative consumption-real exchange rate correlations for selected industrial countries

Notes: The second column gives the sum of gross assets and liabilities as a share of GDP based on Lane and Milesi-Ferretti (2006) data. The third column contains the net foreign currency exposure as percent of GDP based on Lane and Shambaugh (2010) data. The last two columns report the correlation between relative consumption and real exchange rate in each country with respect to the U.S. for HP-filtered and first-differenced series. Consumption, exchange rates and prices are from OECD Outlook Database. Consumption is real private consumption index (2000=100) and real exchange rates are constructed using consumer price indices.

able insurance against supply shocks irrespective of the asset market structure. Therefore, it is important to start from a model which can account for the anomaly when there is trade in a single bond and analyse the implications of introducing a second internationally traded bond to this set-up.

We use a two-country, two-sector model with shocks to tradable and non-tradable sector productivity in each country along the lines of Benigno and Thoenissen (2008) and Corsetti et al. (2008). We first solve the model under the assumption that international asset trade is restricted to a non-contingent bond and review the different mechanisms that can account for the anomaly in this framework. These mechanisms rely on the strong wealth effects generated by uninsured country-specific supply shocks. In Benigno and Thoenissen (2008), a favourable supply shock in the domestic tradables sector increases the relative wealth of domestic agents and leads to higher consumption demand in the domestic country, which in turn raises the prices of domestic non-tradable goods relative to foreign, resulting in a real exchange rate appreciation. On the other hand, Corsetti et al. (2008) emphasise the role of low-substitutability between home and foreign goods. They show that the relative increase in domestic wealth following a favourable supply shock leads to a stronger increase in consumption of home goods due to home bias in consumption and increases the relative price of home goods. Since the assumed trade elasticity is very low, a rise in the relative price of home goods cannot generate substitution away from home goods to foreign goods, thus the income effect dominates the substitution effect and the terms of trade appreciates.

When we allow for international trade in domestic and foreign currency bonds, the above-mentioned wealth effect disappears and the anomaly returns.[3] Why does a seemingly small move away from one-bond to two-bonds bring the model much closer to complete consumption risk sharing despite the fact that markets are incomplete?[4]

First of all, relative consumption risk is affected more by tradable sector shocks than by non-tradable sector shocks. This is because the country that enjoys a rise in non-tradable sector productivity also experiences a fall in the price of non-tradable goods relative to the other country, which in turn reduces the value of home non-tradable output relative to foreign and offsets the effect of the non-tradable productivity shock on relative consumption.[5] Therefore, agents would want to use bonds mainly to hedge against relative consumption risk coming from tradable sector shocks. But whether they can do so, depends crucially on how relative bond returns are affected by non-tradable sector shocks.

If relative bond returns respond strongly to non-tradable sector shocks, a portfolio that insulates consumers from fluctuations in tradable sector output can make them more vulnerable to fluctuations in non-tradable output due to 'adverse valuation effects'. This in turn would limit the degree of risk sharing that can be provided by bonds. On the other hand, if relative bond returns are weakly related to non-tradable sector shocks, as is the case in most specifications of our model, agents can enjoy a high degree of risk sharing conditional on tradable sector shocks without increasing their exposure to non-tradable shocks, which brings the two bond economy closer to the complete markets economy.[6]

Although prices are flexible in the model, monetary policy specification has important implications for the portfolio allocation and the degree of risk sharing because it determines the nominal exchange rate, and relative bond returns are given by the surprises in the nominal exchange rate. This is not the case when international asset trade is restricted to a

[3]This result also holds when we allow for international trade in tradable sector equities instead of nominal bonds. The analysis with trade in equities is not included in the paper for space considerations. Results are available upon request.

[4]Markets are incomplete as there are two bonds and four independent sources of risk - shocks to tradable and non-tradable output in each country. We solve the optimal portfolio using the methodology developed by Devereux and Sutherland (2011).

[5]Cole and Obstfeld (1991) show that terms of trade adjustment can offset supply shocks when all goods are tradable, preferences are symmetric and trade elasticity is close to unity. In our model, we are far from the Cole and Obstfeld economy, therefore terms of trade does not ensure high risk sharing against tradable sector shocks.

[6]Ghironi et al.(2010) also focus on the role of valuation channel for international risk sharing. They show that valuation effects can dampen or amplify the response of consumption differential to productivity and government spending shocks in a two-country one-sector DSGE model where there is international trade in equity.

single bond, because in this set-up agents are not allowed to have portfolio holdings which would bring capital gains or losses depending on the realisations of the nominal exchange rate. We consider two simple monetary policy rules, domestic tradable price stabilisation and consumer price index (CPI) stabilisation, which imply different relative bond returns. Under the former, nominal exchange rate and relative bond returns are determined by the *terms of trade*, whereas under the latter they are given by the *real exchange rate*.[7]

We find that trade in bonds generally leads to higher risk sharing when relative bond returns are determined by the terms of trade as opposed to the real exchange rate. This is because the real exchange rate responds more strongly to non-tradable sector shocks, which prevents agents from choosing a portfolio that could insure them fully against the relative consumption risk coming from tradable sector shocks.[8] While the high risk sharing result is robust to different values of the trade elasticity when relative bond returns are equal to the terms of trade, this is not the case when relative bond returns are given by the real exchange rate. Our numerical results show that, under CPI stabilisation, the cross-correlation between relative consumption and real exchange rate can be high or low depending on the value of trade elasticity. But under domestic tradable price stabilisation, the correlation is almost perfect regardless of this parameter.

In light of these results, we enrich the shock structure in our two-sector model and consider demand shocks as well as supply shocks. Our focus is on the implications of this additional source of uncertainty on equilibrium portfolio allocation and, through that, on the international transmission of supply shocks. In other words, we explore whether the presence of demand shocks can generate enough market incompleteness such that the transmission of supply shocks can still be negative as in Benigno and Thoenissen (2008) and Corsetti et al. (2008) even under some endogenous portfolio choice. As demand shocks, we consider shocks to the predictable component of sectoral productivity shocks - 'news shocks' as in Beaudry and Portier (2004), Jaimovich and Rebelo (2008) and Colacito and Croce (2010) among others.[9]

Our numerical results show that only under certain parameter and policy settings can demand shocks reduce the degree of risk sharing implied by bonds without compromising the model's ability to match other business cycle facts. The intuition for how demand

[7]The real exchange rate consists of the terms of trade and the relative price of non-tradables.

[8]Because the real exchange rate includes the relative price of non-tradables, which is directly linked to the relative supply of non-tradables, it is affected more strongly by non-tradable sector shocks compared to the terms of trade.

[9]We want to stress that the demand shocks we consider work in a different way compared to Stockman and Tesar (1995) type 'taste shocks', which are basically shocks to the marginal utility of consumption. Heathcote and Perri (2007) show that these shocks can be used to generate a realistic negative correlation between relative consumption and real exchange rate but their explanation of the anomaly does not rely on market incompleteness.

shocks work is as follows. Demand shocks move the real exchange rate-adjusted relative consumption in the same direction as supply shocks, but affect relative bond returns in the opposite direction. Therefore, relative supply and demand shocks require different signs for optimal bond portfolios, which in turn limits the degree of risk sharing ensured by bonds.

This paper is closely related to the literature on country portfolios. Heathcote and Perri (2007), Kollmann (2006), Collard et al.(2007), Engel and Matsumoto (2009) and Coeurdacier et al.(2010) propose different models that can generate realistic portfolio positions under effectively complete markets. There is also a range of papers that analyse equilibrium portfolios under incomplete markets. Coeurdacier et al.(2007) specify an incomplete market model with supply, demand and redistributive shocks and trade in stocks and bonds to match the basic stylised facts on international portfolios. Hnatkovska (2010) analyses endogenous portfolio choice under incomplete markets in a model with tradable and non-tradable sectors and examines the dynamics of portfolio choice to reconcile the home bias in equity holdings with the high turnover and high volatility of international capital flows. Using different modelling frameworks, Coeurdacier and Gourinchas (2009) and Benigno and Nisticò (2009) also study endogenous bond and equity portfolios under incomplete markets. However, they mainly focus on different hedging motives behind equilibrium portfolio positions, e.g. whether home equity bias is driven by non-diversifiable labour income risk or real exchange rate risk, rather than analysing the implications of portfolio allocation for international risk sharing and consumption-real exchange rate anomaly.

The rest of the paper proceeds as follows. In sections 2 and 3, we lay out a two-country two-sector endowment model and solve the model analytically to show how the comovement of relative consumption and real exchange rates is affected by endogenous portfolio choice in the presence of anticipated and unanticipated shocks. Section 4 gives the quantitative results of a calibrated production model with capital accumulation. Section 5 concludes.

2 A two-country two-sector endowment economy

We first develop a basic two-country open economy endowment model. There is a home and a foreign country, each endowed with a tradable and a non-tradable good. Endowments in each country are stochastic. Households maximise utility over infinite horizon under different asset market configurations: complete markets where agents can trade in a full-set of state-contingent claims, incomplete markets where international asset trade is restricted to a single non-contingent bond and an intermediate case where both home and foreign currency bonds can be internationally traded. The structure of the model is related to the production economies described in BT, CDL and Stockman and Tesar (1995).

2.1 Preferences and good markets

The representative agent in home country maximises the expected present discounted value of the utility:

$$U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \frac{C_s^{1-\rho}}{1-\rho}, \tag{1}$$

where C is consumption and β is the intertemporal discount factor, with $0 < \beta < 1$.

C represents a consumption index defined over tradable C_T and non tradable C_N consumption:

$$C_t = \left[\gamma^{\frac{1}{\kappa}} C_{T,t}^{\frac{\kappa-1}{\kappa}} + (1-\gamma)^{\frac{1}{\kappa}} C_{N,t}^{\frac{\kappa-1}{\kappa}} \right]^{\frac{\kappa}{\kappa-1}}, \tag{2}$$

where κ is the elasticity of intratemporal substitution between C_N and C_T and γ is the weight that the households assign to tradable consumption. The tradable component of the consumption index is in turn a CES aggregate of home and foreign tradable consumption goods, C_H and C_F:

$$C_{T,t} = \left[\nu^{\frac{1}{\theta}} C_{H,t}^{\frac{\theta-1}{\theta}} + (1-\nu)^{\frac{1}{\theta}} C_{F,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{3}$$

where θ is the elasticity of intratemporal substitution between C_H and C_F and ν is the weight that the households assigns to home tradable consumption. We allow for a home bias in tradable goods by assuming $\nu > \frac{1}{2}$. We adopt a similar preference specification for the foreign country except that variables are denoted with an asterisk. The price indices corresponding to the consumption baskets defined above are given by:

$$P_t = \left[\gamma P_{T,t}^{1-\kappa} + (1-\gamma) P_{N,t}^{1-\kappa} \right]^{\frac{1}{1-\kappa}}, \quad P_{T,t} = \left[\nu P_{H,t}^{1-\theta} + (1-\nu) P_{F,t}^{1-\theta} \right]^{\frac{1}{1-\theta}}. \tag{4}$$

We assume that the law of one price holds, i.e. $P_{H,t}^* = P_{H,t}/S_t$,and $P_{F,t} = P_{F,t}^* S_t$, where S_t denotes the nominal exchange rate defined as the price of foreign currency in terms of domestic currency. The presence of non-tradable goods and home bias in tradables consumption leads to deviations from purchasing power parity. We define the real exchange rate as $Q = SP^*/P$.

Good market clearing requires $Y_{H,t} = C_{H,t} + C_{H,t}^*$, $Y_{F,t} = C_{F,t} + C_{F,t}^*$, $Y_{N,t} = C_{N,t}$ and $Y_{N,t}^* = C_{N,t}^*$ where C_H and C_F (C_F^* and C_H^*) should satisfy the intratemporal optimisation decisions of home (foreign) households. Endowments of tradable and non-tradable goods follow AR(1) processes of the form:

$$\log Y_{i,t} = (1-\delta_T)\log \bar{Y}_i + \delta_T \log Y_{i,t-1} + u_{i,t},\ i = H, F, \tag{5}$$

$$\log Y_{j,t} = (1-\delta_N)\log \bar{Y}_j + \delta_N \log Y_{j,t-1} + u_{j,t},\ j = N, N^*, \tag{6}$$

where $0 \leq \delta_T < 1$, $0 \leq \delta_N < 1$, $u_{H,t}, u_{F,t}, u_{N,t}, u_{N^*,t}$ are i.i.d. shocks symmetrically distributed over the interval $[-\epsilon, \epsilon]$ with $Var(u_H) = Var(u_F) = \sigma_T^2$ and $Var(u_N) = Var(u_{N^*}) = \sigma_N^2$.

2.2 Asset markets

We consider three different asset market structures to compare their implications for real exchange rate and relative consumption correlations.

2.2.1 Complete markets

Complete market set-up can be characterised either by assuming that agents in each country can trade in a complete set of state-contingent assets, as in Chari et al.(2002) or Heathcote and Perri (2007) for e.g., or by assuming that there are enough independent assets, bonds and equities, to span all the risks, as in Devereux and Sutherland (2011), Coeurdacier (2009) among others.[10] Here we follow the former approach and do not characterise equilibrium portfolios associated with the complete market equilibrium. We use the complete market set-up as a benchmark against which to compare the risk sharing implications of incomplete market models.

Assuming initial wealth levels are equal across countries, the following risk sharing condition holds under complete markets:

$$\frac{U_C(C_t^*)}{U_C(C_t)} = \frac{S_t P_t^*}{P_t}, \tag{7}$$

which states that the marginal utilities of consumption adjusted by the respective CPI's are equalised across countries for each date and state.

2.2.2 Incomplete markets: Non-contingent bond economy

In this setting, home and foreign agents hold an international bond, $B_{H,t}$, which pays in units of the home currency. The flow budget constraint of the representative home country

[10] The spanning condition with trade in equities and bonds requires that there are $n+1$ independent assets for n sources of uncertainty.

consumer is given by:

$$B_{H,t} = R_{H,t}B_{H,t-1} + P_{H,t}Y_{H,t} + P_{N,t}Y_{N,t} - P_tC_t, \tag{8}$$

where $R_{H,t}$ is the home country nominal interest rate. In this case, there is no portfolio choice problem. International trade in the non-contingent bond only allows for international borrowing and lending and does not provide any other hedging opportunity. This is the standard incomplete markets set-up used in the open economy macro literature.[11]

Maximisation of expected lifetime utility with respect to (8) implies the usual bond Euler equation for the home agent:

$$U_C(C_t) = \beta E_t U_C(C_{t+1})R_{H,t+1}\frac{P_t}{P_{t+1}}. \tag{9}$$

Foreign agent's optimal choice of home and foreign bonds is given by:

$$U_C(C_t^*) = \beta E_t U_C(C_{t+1}^*)R_{H,t+1}\tfrac{S_t}{S_{t+1}}\tfrac{P_t^*}{P_{t+1}^*}, \quad U_C(C_t^*) = \beta E_t U_C(C_{t+1}^*)R_{F,t+1}^*\tfrac{P_t^*}{P_{t+1}^*}. \tag{10}$$

where $R_{F,t}^*$ is the nominal interest rate on foreign bond expressed in terms of foreign currency. In the non-contingent bond economy, the risk sharing condition given by equation (7) no longer holds.

2.2.3 Incomplete markets: International trade in home and foreign currency bonds

In this set-up, agents in each country can trade in bonds denominated in home and foreign currency. Given that the number of independent assets that can be internationally traded is less than the number of shocks, the spanning condition is not satisfied, hence markets are incomplete. The flow budget constraint of the home agent in nominal terms is given by:

$$B_{H,t} + S_tB_{F,t} = R_{H,t}B_{H,t-1} + R_{F,t}^*S_tB_{F,t-1} + P_{H,t}Y_{H,t} + P_{N,t}Y_{N,t} - P_tC_t, \tag{11}$$

where $B_{H,t-1}$ is the home agent's holdings of internationally traded home bond and $B_{F,t-1}$ is the home agent's holdings of internationally traded foreign bond purchased at the end of period $t-1$ for holding into period t. $R_{H,t}$ and $R_{F,t}^*$ are the risk-free returns on home and foreign bonds.

[11] In Benigno and Thoenissen (2008), home agents can trade in both home currency and foreign currency-denominated bonds, while foreign agents can only trade in foreign currency-denominated bonds. Thus international asset trade is restricted to foreign bonds. This set-up has the same implications as our non-contingent bond economy set-up with international trade in home bonds.

Letting $\alpha_{H,t} \equiv B_{H,t}$, $\alpha_{F,t} \equiv S_t B_{F,t}$ and defining $NFA_t \equiv \alpha_{H,t} + \alpha_{F,t}$ as the total net claims of home agents on the foreign country at the end of period t (i.e. the net foreign assets of home agents) we can write (11) as a net foreign asset accumulation equation[12]:

$$NFA_t = NFA_{t-1} R_{H,t} + \alpha_{F,t-1} R_{x,t} + P_{H,t} Y_{H,t} + P_{N,t} Y_{N,t} - P_t C_t, \tag{12}$$

where $R_{x,t} = R_{F,t} - R_{H,t}$ is the excess return on foreign bond relative to home bond expressed in home currency units, with $R_{F,t} = R^*_{F,t} S_t / S_{t-1}$.[13]

Note that once α_F is determined, α_H, α^*_H and α^*_F will also be determined as $\alpha_H = NFA - \alpha_F$ by definition and $\alpha^*_H = -\alpha_H$, $\alpha^*_F = -\alpha_F$ from market clearing conditions. Thus, we only focus on α_F in what follows.

The main difference between the asset accumulation equations (12) and (8) is the excess return on the portfolio, $\alpha_{F,t-1} R_{x,t}$, which implies state-contingent valuation effects.

Consumers' first order conditions imply that as well as the Euler equations given by (9) and (10), there is also a home Euler equation for foreign bonds. Hence, the following optimal portfolio choice equations should hold in each country:

$$E_t \left[m_{t+1} R_{x,t+1} \right] = 0, \quad E_t \left[m^*_{t+1} R_{x,t+1} \frac{S_t}{S_{t+1}} \right] = 0, \tag{13}$$

where home and foreign stochastic discount factors are given by

$$m_{t+1} = \beta \frac{P_t}{P_{t+1}} \frac{C_{t+1}^{-\rho}}{C_t^{-\rho}}, \quad m^*_{t+1} = \beta \frac{P^*_t}{P^*_{t+1}} \frac{C_{t+1}^{*-\rho}}{C_t^{*-\rho}}, \tag{14}$$

respectively, and $R_{x,t+1}$ is the excess return on foreign nominal bond, taking home bond as a reference as defined above.

To solve the model in the presence of endogenous portfolio choice under incomplete markets, we use the approximation techniques proposed in Devereux and Sutherland (2011) and Tille and van Wincoop (2010). We approximate our model around the symmetric steady state in which steady-state inflation rates are assumed to be zero.

The second order approximation of the optimal portfolio choice equations in (13) together with the property of the model that expected excess returns are zero up to a first order

[12]Net foreign assets of home agent is defined as net claims of home country on foreign country assets, i.e. $NFA_t = \alpha_{F,t} - \alpha^*_{H,t}$. Since bonds are assumed to be in net zero supply $\alpha_{H,t} = -\alpha^*_{H,t}$. It follows that $NFA_t = \alpha_{H,t} + \alpha_{F,t}$.

[13]A similar budget constraint holds for the foreign agent, where foreign variables are denoted with an asterisk, *. Thus, $\alpha^*_{H,t-1}$ and $\alpha^*_{F,t-1}$ denote the foreign country's holdings of home and foreign bonds, expressed in units of home currency. Bonds are assumed to be in net zero supply in each country. Thus, equilibrium in asset market requires that total bond holdings of home and foreign agents should equal zero, i.e. $\alpha_{H,t} + \alpha^*_{H,t} = 0$ and $\alpha_{F,t} + \alpha^*_{F,t} = 0$.

approximation, gives an orthogonality condition between excess returns and the relative stochastic discount factors denominated in the same currency, which pins down optimal steady-state portfolios:

$$Cov_t\left[(\hat{m}_{t+1} - \hat{m}^*_{t+1} + \Delta\hat{S}_{t+1}), \hat{R}_{x,t+1}\right] = 0 + O(\epsilon^3), \tag{15}$$

where $O(\epsilon^3)$ is a residual which contains all terms of order higher than two. As shown by Devereux and Sutherland (2011), to evaluate (15) and determine the portfolio shares, it is sufficient to take a first-order approximation of the remaining equilibrium conditions for which the only aspect of portfolio behaviour that matters is the steady-state foreign bond portfolio, $\bar{\alpha}_F$.[14]

2.3 Policy rules

We close the model by considering two simple policy rules. Although prices are fully flexible in our model, the way we specify policy rules matters as long as we have a nominal asset. This is because the return differential between home and foreign bonds is given by the rate of (unexpected) nominal exchange depreciation, which is affected by the policy rule in a flexible price setting. Consequently, equilibrium portfolio shares will be affected, which will then feed back into the model (see Devereux and Sutherland, 2008 and De Paoli et al., 2010).

We focus on two cases: in the first one, policy authorities stabilise their own tradable prices ($P_{H,t} = 1$,and $P_{F^*,t} = 1$) and in the second one they stabilise domestic consumer prices ($P_t = 1$,and $P^*_t = 1$).[15] Nominal exchange rate is equal to the terms of trade in the former, while it is given by the real exchange rate in the latter.[16]

3 Relative consumption and real exchange rate under alternative asset markets

In this section we first describe the general equilibrium behaviour of relative consumption and real exchange rate in response to sectoral supply shocks under complete markets and illustrate the Backus-Smith-Kollmann condition. Next, we go over the mechanisms put forth by Benigno and Thoenissen (2008) and Corsetti et al.(2008) that can account for the

[14] The web appendix derives a partial equilibrium expression for optimal bond portfolio and discusses the hedging motives of investors.

[15] Benigno and Thoenissen (2008) close their model by assuming that monetary policy is characterised by CPI targeting whereas Corsetti et al. (2008) take the domestic CPI as numeraire, which are essentially equivalent.

[16] Having a nominal bond with a CPI targeting rule is equivalent to having a real bond (or CPI indexed bond) with any policy rule in terms of equilibrium portfolio and model solution.

consumption-real exchange rate anomaly when international asset trade is limited to a single non-contingent bond. Then, we analyse how the link between relative consumption and real exchange rate changes when we move from single bond economy to a two bond economy with endogenous portfolio choice.

3.1 Complete markets: Backus-Smith-Kollmann condition

Assuming CRRA preferences, log-linearisation of the risk sharing condition in (7) gives:

$$\hat{C}_t - \hat{C}_t^* = \frac{\hat{Q}_t}{\rho} + O(\epsilon^2), \tag{16}$$

which implies that consumption should be higher in the country where it is cheaper to consume. $O(\epsilon^2)$ is a residual that contains all terms of order higher than one. [17]

It is useful to characterise the full general equilibrium solution to relative consumption and real exchange rates under complete markets to compare it with the solution under different configurations of incomplete markets.

$$\begin{aligned} \hat{C}_t - \hat{C}_t^* &= \frac{\gamma\kappa(2\nu-1)}{\Gamma_1}(\hat{Y}_{H,t} - \hat{Y}_{F,t}) + \frac{\Gamma_2}{\Gamma_1}(\hat{Y}_{N,t} - \hat{Y}_{N,t}^*), & (17) \\ \hat{Q}_t &= \rho\left(\frac{\gamma\kappa(2\nu-1)}{\Gamma_1}(\hat{Y}_{H,t} - \hat{Y}_{F,t}) + \frac{\Gamma_2}{\Gamma_1}(\hat{Y}_{N,t} - \hat{Y}_{N,t}^*)\right), & (18) \end{aligned}$$

where $\Gamma_1 \equiv 4\theta\nu(1-\nu)(1+\gamma(\kappa\rho-1))+\kappa(2\nu-1)^2 > 0$ and $\Gamma_2 \equiv (1-\gamma)(\kappa(2\nu-1)^2+4\theta\nu(1-\nu)) > 0$ for all possible parameter values. Table 2 summarises the definitions of the parameters used in the discussion of analytical results.

$\Gamma_1 \equiv 4\theta\nu(1-\nu)(1+\gamma(\kappa\rho-1)) + \kappa(2\nu-1)^2 > 0.$
$\Gamma_2 \equiv (1-\gamma)(\kappa(2\nu-1)^2 + 4\theta\nu(1-\nu)) > 0.$
$\Gamma_3 \equiv (2\theta\nu-1)(\gamma\kappa\rho+1-\gamma) - \kappa(2\nu-1) > 0$ for $\theta > \theta_3^*$.
$\Gamma_4 \equiv (\gamma\kappa\rho+1-\gamma)\left(\theta^2 + (\theta-1)^2\frac{\sigma_T^2}{\sigma_N^2}\right).$
$\theta_1^* \equiv \frac{1}{2\nu} + \frac{\kappa}{1-\gamma}\frac{2\nu-1}{2\nu}.$
$\theta_2^* \equiv \frac{1+\kappa(2\nu-1)}{2\nu}$, $\theta_2^* < \theta_1^*$ for $1-\gamma < 1$.
$\theta_3^* \equiv \frac{1}{2\nu} + \frac{1}{2\nu}\frac{\kappa(2\nu-1)}{\gamma\kappa\rho+1-\gamma}$, $\theta_3^* < \theta_1^*$ for $\nu > \frac{1}{2}$.
$RV_1 \equiv \frac{\Gamma_1(1-\beta\delta_z)^2}{(2\nu-1)\Gamma_3\beta^2}.$
$\psi_c^{nc} \equiv \frac{4(1-\beta)\theta\nu}{\beta(1+2\nu(\theta-1))} > 0$ for $\theta > 1-1/2\nu$
$\psi_q^{nc} \equiv \frac{(1-\beta)}{\beta}\frac{(\kappa(2\nu-1)^2+4\nu(1-\nu)(1-\gamma)\theta)}{\gamma\kappa(1-\nu)(1+2\nu(\theta-1))} > 0$ for $\theta > 1-1/2\nu$

Table 2: Some parameter definitions.

[17] In the remainder of this section, we suppress the order notation unless the order of approximation needs to be particularly emphasised.

The only state variables of the complete market model are the exogenous state variables, i.e. the stochastic endowment processes in each sector and country. Net foreign asset accumulation does not matter for equilibrium dynamics under complete markets. Real exchange rate and relative consumption are perfectly correlated as can be seen from (17) and (18).

3.2 Incomplete markets: Non-contingent bond economy

Under incomplete markets, the risk sharing condition (7) holds not in levels, but in expected value. Combining the home and foreign Euler equations with respect to the international asset gives:

$$E_t(\Delta\hat{C}_{t+1} - \Delta\hat{C}^*_{t+1}) = \frac{1}{\rho}E_t\Delta\hat{Q}_{t+1}. \tag{19}$$

Deviations from the Backus-Smith-Kollmann condition are given by $\hat{C}_t - \hat{C}^*_t - \hat{Q}_t/\rho$. Country-specific shocks generate large fluctuations in relative wealth when there are significant deviations from this condition.

To simplify the analytical expressions we assume that shocks are permanent, i.e. $\delta_T = \delta_N = 1$, so that the general equilibrium solution for relative consumption and real exchange rate dynamics reads:[18]

$$\hat{C}_t - \hat{C}^*_t = \psi^{nc}_c \widehat{NFA}_{t-1} + \frac{\gamma(2\theta\nu - 1)}{1 + 2\nu(\theta - 1)}(\hat{Y}_{H,t} - \hat{Y}_{F,t}) + (1-\gamma)(\hat{Y}_{N,t} - \hat{Y}^*_{N,t}), \tag{20}$$

$$\hat{Q}_t = -\psi^{nc}_q \widehat{NFA}_{t-1} - \left[\frac{(1-\gamma)(2\theta\nu - 1) - \kappa(2\nu - 1)}{\kappa(1 + 2\nu(\theta - 1))}\right](\hat{Y}_{H,t} - \hat{Y}_{F,t}) + \frac{1-\gamma}{\kappa}(\hat{Y}_{N,t} - \hat{Y}^*_{N,t}) \tag{21}$$

where ψ^{nc}_c and ψ^{nc}_q are as defined in Table 2. $\psi^{nc}_c > 0$ and $\psi^{nc}_q > 0$ for $\theta > 1 - 1/2\nu$.

In an incomplete markets model, net foreign asset position is an endogenous state variable as reflected by the policy functions in (20) and (21). [19] Relative consumption and real exchange rate are positively related conditional on non-tradable sector shocks. However, they might move in opposite directions conditional on tradable sector shocks depending on the value of trade elasticity, θ, which in turn can account for the consumption-real exchange rate anomaly as shown by BT and CDL.

To illustrate how the transmission of tradable sector supply shocks changes with the trade elasticity, we decompose the real exchange rate into two components- the terms of

[18] Due to market incompleteness, there is a unit root in the net foreign assets. Although the non-stationarity inherent in incomplete market models creates problems for numerical simulations, it is not that important for the discussion of analytical solutions. The stationarity-inducing techniques described in Schmitt-Grohe and Uribe (2003) do not change the workings of the model. Hence, for simplicity we abstract from these when reporting the analytical results.

[19] For a sufficiently high elasticity of substitution between home and foreign goods ($\theta > 1 - 1/2\nu$), higher net foreign assets brought from previous period implies higher consumption at home country ($\psi^{nc}_c > 0$) and a more expensive home consumption basket ($\psi^{nc}_q > 0$).

trade, TOT_t, and the relative price of non-tradables across countries, P_t^N :

$$\hat{Q}_t = \gamma(2\nu - 1)\widehat{TOT}_t + (1-\gamma)\hat{P}_t^N, \tag{22}$$

where $\widehat{TOT} = \hat{P}_F^* + \hat{S} - \hat{P}_H$ and $\hat{P}^N = \hat{P}_N^* + \hat{S} - \hat{P}_N$.[20] Equation (22) shows clearly that in this model real exchange rates fluctuate due to the presence of home bias in consumption ($\nu > \frac{1}{2}$) and non-traded goods ($\gamma < 1$).

The general equilibrium solution for terms of trade and relative non-tradables price assuming permanent shocks are as follows:

$$\widehat{TOT}_t = -\psi_T^{nc}\widehat{NFA}_{t-1} + \frac{1}{1+2\nu(\theta-1)}(\hat{Y}_{H,t} - \hat{Y}_{F,t}), \tag{24}$$

where $\psi_T^{nc} > 0$ for $\nu > 1/2$ and $\theta > 1 - 1/2\nu$.[21]

$$\hat{P}_t^N = -\psi_N^{nc}\widehat{NFA}_{t-1} - \left[\frac{(2\theta\nu - 1) - \kappa(2\nu - 1)}{\kappa(1+2\nu(\theta-1))}\right](\hat{Y}_{H,t} - \hat{Y}_{F,t}) + \frac{1}{\kappa}(\hat{Y}_{N,t} - \hat{Y}_{N,t}^*), \tag{25}$$

where $\psi_N^{nc} > 0$ for $\theta > 1 - 1/2\nu$.

Using the analytical expressions given in equations (20) to (25), we can characterise five regions of trade elasticity, each of which implies a different transmission mechanism in response to tradable sector shocks on impact. Figure 1 illustrates these regions.

There are two regions of θ for which a positive tradable sector supply shock leads to an increase in relative consumption and a fall in real exchange rate - hence a negative conditional correlation on impact. These regions are region I, where $\theta < 1 - 1/2\nu$, and region V, where $\theta > \theta_1^*$ (See Table 2 for the definition of θ_1^*). In both of these regions, an unanticipated increase in the tradable endowment of the home country implies a large increase in the relative wealth of home agents, which in turn leads to higher consumption and higher prices in the home country. The main difference between the two regions is that in the former, the increase in relative wealth appreciates both the terms of trade and the relative price of non-tradables, while in the latter it only appreciates the relative non-tradables.[22]

[20] More often, non-tradable prices in each country are expressed relative to tradable prices, to highlight the Balassa-Samuelson effect:

$$\hat{Q}_t = (2\nu - 1)\widehat{TOT}_t + (1-\gamma)\widehat{RPN}_t \tag{23}$$

where the terms of trade is defined as above and the relative price of non-tradables is defined as $\widehat{RPN}_t \equiv (\hat{P}_{N,t}^* - \hat{P}_{T,t}^*) - (\hat{P}_{N,t} - \hat{P}_{T,t})$.

[21] Note that terms of trade is independent of non-tradable sector shocks because we assume, for ease of exposition, that the persistence of non-tradable endowments, δ_N, is equal to 1. As we show later, terms of trade is independent of non-tradable sector shocks also when $\gamma = 1$ or $\nu = \frac{1}{2}$ or $\kappa\rho = 1$ (utility is separable in tradable and non-tradable consumption).

[22] CDL shows that there is a sixth region, which gives a transmission mechanism similar to the one described by region I for high values of θ. The main idea is that if endowments are expected to reach a permanently higher level over time, demand exceeds supply in the short-run, increasing relative consumption and appreciating

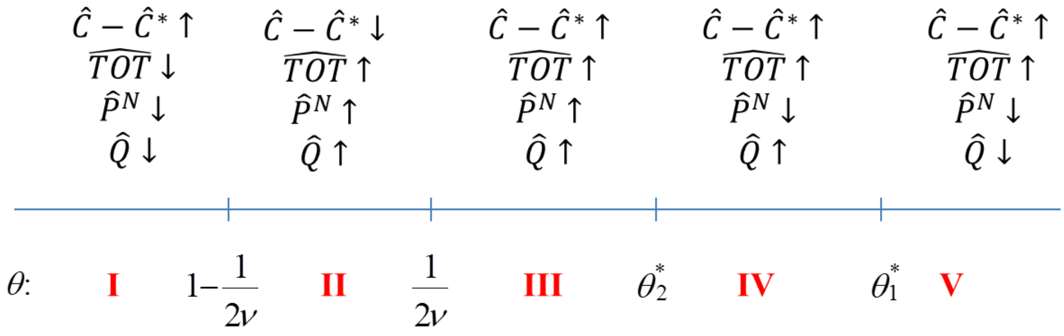

Figure 1: Impact responses to a positive tradable endowment shock with respect to trade elasticity, θ, for $\nu > \frac{1}{2}$.

Notes: $\theta_1^* \equiv \frac{1}{2\nu} + \frac{\kappa}{1-\gamma}\frac{2\nu-1}{2\nu}$ and $\theta_2^* = \frac{1+\kappa(2\nu-1)}{2\nu}$.

Figure 1 shows that there is another region, region II, given by $1 - 1/2\nu < \theta < 1/2\nu$, where relative consumption and real exchange rate are negatively correlated conditional on tradable endowment shocks. In this region, negative conditional correlation is due to the fact that relative consumption falls in response to a positive tradable sector shock while the real exchange rate depreciates. In what follows we focus our attention on regions I and V, which imply a positive relation between relative consumption and relative income.

Region I: Low trade elasticity

In this region, characterised by $\theta < 1-1/2\nu$, the mechanism that accounts for the consumption-real exchange rate anomaly is the one emphasised by Corsetti et al.(2008): Under incomplete markets, home agents become relatively wealthier following a positive home supply shock. Given that consumption is home biased, this positive wealth effect leads to a stronger increase in consumption of home goods, increasing the relative price of home goods. Since the price elasticity of tradables is very low, a rise in the relative price of home goods cannot generate substitution away from home goods to foreign goods, thus the income effect dominates the substitution effect and the terms of trade appreciates. The strong rise in relative home wealth also appreciates the relative price of non-tradables. In this region, 'negative transmission' of a positive supply shock does not rely on the presence of a non-tradable sector.

the terms of trade. Because in our set-up shocks bring endowment immediately to its permanent level, we do not get this region. But, we do get it in the production economy version of this two-sector model, which we show in the web appendix.

Region V: High trade elasticity

In this region, given by $\theta > \theta_1^*$, the mechanism that generates the conditional negative correlation between relative consumption and real exchange rates is the one emphasised by Benigno and Thoenissen (2008): In the absence of complete markets, a positive supply shock in the home tradable sector implies that home agents become relatively wealthier, which in turn increases the demand for non-tradables in the home country. Given the fixed supply of non-tradables, this increase in demand puts an upward pressure on the price of home non-tradables, more so if the elasticity of substitution between tradables and non-tradables, κ, is low so that the (negative) substitution effect on the demand for non-tradables is weaker than the (positive) income effect. The rise in the relative non-tradable price, in turn, appreciates the real exchange rate (See equations (25) and (21)). For this mechanism to yield an unconditional negative cross correlation between relative consumption and real exchange rate, it is crucial that tradable sector shocks are sufficiently larger than non-tradable sector shocks.

3.3 Incomplete markets: International trade in home and foreign currency bonds

Endogenous trade in bonds lets agents hedge ex-ante against the relative consumption risk caused by country-specific shocks. Given that there are two independent assets and four different sources of uncertainty (tradable and non-tradable sector shocks in each country), this asset market structure represents an incomplete market set-up. Therefore, we would expect the degree of risk sharing provided by trade in nominal bonds to fall somewhere in between the degree of risk sharing provided by trade in a single non-contingent bond and that provided by trade in a complete set of state-contingent claims. Then the main question is whether the two bond set-up is closer to the single bond or the complete market set-up.

To answer this question, we first solve for the optimal bond portfolio and characterise the policy functions for relative consumption and real exchange rate consistent with this portfolio position. Then we compare these policy functions with those obtained under the non-contingent bond and complete market set-ups.

3.3.1 Portfolio allocation and risk sharing under domestic tradable price stabilisation

Using the property of the model that expected excess returns are zero up to a first order approximation, we can write relative bond returns, $\hat{r}_{x,t+1}$, as the surprises in the nominal exchange rate:

$$\hat{r}_{x,t+1} = \hat{S}_{t+1} - E_t \hat{S}_{t+1} + O(\epsilon^2). \tag{26}$$

where $O(\epsilon^2)$ includes all terms that have order higher than one as defined before.

Therefore, loadings of the excess return on different sources of risk and the implied portfolio positions depend crucially on the behaviour of the nominal exchange rate, which in turn is determined by policy specification.

Assuming monetary policy in each country stabilises respective domestic tradable prices, excess return on foreign bonds is given by the terms of trade:

$$\hat{P}_{H,t} = \hat{P}^*_{F,t} = 0 \Rightarrow \hat{S}_t = \widehat{TOT}_t \Rightarrow \hat{r}_{x,t} = \widehat{TOT}_t - E_{t-1}\widehat{TOT}_t. \tag{27}$$

In this case, due to the monetary policy rule, nominal bonds act like bonds indexed to the domestic tradable price index.

To get the analytical solution for the bond portfolio, we characterise closed form expressions for the two components of the portfolio orthogonality condition, real exchange rate adjusted relative consumption and relative bond returns, in terms of the structural shocks and the excess return on portfolio $\tilde{\alpha}_F \hat{r}_{x,t}$. Assuming $\delta_T = 1$, $\delta_N = \delta < 1$ we get the following:[23]

$$\begin{aligned} \hat{C}_t - \hat{C}^*_t - \frac{\hat{Q}_t}{\rho} &= \psi_{rcq}\widehat{NFA}_{t-1} + \frac{\Gamma_3}{\kappa\rho(1+2\nu(\theta-1))}(\hat{Y}_{H,t} - \hat{Y}_{F,t}) \\ &+ \frac{(1-\beta)(1-\gamma)(\kappa\rho-1)}{(1-\beta\delta)\kappa\rho}(\hat{Y}_{N,t} - \hat{Y}^*_{N,t}) \\ &+ \frac{(1-\beta)\Gamma_1}{\kappa\rho\gamma(1-\nu)(1+2\nu(\theta-1))}\tilde{\alpha}_F \hat{r}_{x,t}, \end{aligned} \tag{28}$$

where $\psi_{rcq} > 0$ for $\theta > 1 - 1/2\nu$ and $\Gamma_3 \equiv (2\theta\nu - 1)(\gamma\kappa\rho + 1 - \gamma) - \kappa(2\nu - 1)$. $\Gamma_3 > 0$ for $\theta > \theta^*_3$ where θ^*_3 is defined as in Table 2. Note that $1 - 1/2\nu < \theta^*_3 < \theta^*_1$ (see Figure 1).

$$\begin{aligned} \hat{r}_{x,t} &= \widehat{TOT}_t - E_{t-1}\widehat{TOT}_t = \frac{1}{1+2\nu(\theta-1)}(u_{H,t} - u_{F,t}) \\ &+ \frac{\beta(1-\gamma)(1-\delta)(2\nu-1)(\kappa\rho-1)}{(1-\beta\delta)\Gamma_1}(u_{N,t} - u^*_{N,t}) \\ &- \frac{(1-\beta)(2\nu-1)}{\gamma(1-\nu)(1+2\nu(\theta-1))}\tilde{\alpha}_F \hat{r}_{x,t}. \end{aligned} \tag{29}$$

Consider first the zero-portfolio solution ($\tilde{\alpha}_F = 0$) for the real exchange rate adjusted relative consumption and excess returns to build intuition for the optimal bond position. The zero-

[23] We first consider the case with $\delta_N = \delta < 1$, instead of setting $\delta_N = 1$ as we do in the analysis of the non-contingent bond economy. We do this to understand how relative bond returns (terms of trade) responds to non-tradable shocks. Because when $\delta_N = 1$, terms of trade is independent of non-tradable sector shocks.

portfolio solution corresponds to the solution that would arise when agents can only trade in a single non-contingent bond. First note that for $\nu > 1/2$, hedging against non-tradable endowment shocks requires a short position in foreign bonds irrespective of the degree of substitutability between tradables and non-tradables or any other parameter. On the other hand, the optimal hedge against tradable endowment shocks depends crucially on the value of the trade elasticity in line with the arguments following Figure 1. For values of θ in region I, a positive tradable endowment shock leads to an increase in $\hat{C}_t - \hat{C}_t^* - \hat{Q}_t/\rho$ and a fall in $\hat{r}_{x,t}$, pulling the equilibrium portfolio towards a long position in foreign bonds. For values of θ that lie in region V, both $\hat{C}_t - \hat{C}_t^* - \hat{Q}_t/\rho$ and $\hat{r}_{x,t}$ increase following a positive tradable endowment shock, which makes it optimal to go short in foreign bonds.[24]

In what follows, to simplify algebra and facilitate the discussion of different cases, we focus on the case where both tradable and non-tradable endowment shocks have unit root, $\delta_T = \delta_N = \delta = 1$ as we do in the analysis in section (3.2). Solving equations (28), (29) and the portfolio orthogonality condition given in (15) under this assumption implies the following optimal bond portfolio:

$$\tilde{\alpha}_F = -\tilde{\alpha}_H = -\frac{\gamma(1-\nu)\Gamma_3}{(1-\beta)(\gamma\kappa\rho + (1-\gamma))}, \tag{30}$$

where $\Gamma_3 \gtrless 0$ for $\theta \gtrless \theta_3^*$. Therefore, the sign of the optimal bond portfolio depends on the value of the trade elasticity. For θ belonging to region I, optimal portfolio is long in foreign currency whereas for θ in region V, it is the opposite.[25] Although there are four shocks affecting each country and only two assets that can be internationally traded, optimal bond portfolio does not depend on the relative variance of different shocks. This is because under the assumption that $\delta = 1$, terms of trade is independent of non-tradable endowment shocks as shown in equation (29). Hence, agents can choose a portfolio to insure themselves perfectly against tradable sector shocks, without being subject to unwanted valuation effects conditional on non-tradable endowment shocks. [26] For more general parameter values, terms of trade loads on relative non-tradable income shocks, hence equilibrium portfolio becomes a complicated object that depends on the relative variance of tradable versus non-tradable income shocks. However, as we discuss below, even in this case, portfolios will be biased towards hedging against tradable income shocks as terms of trade loads weakly on

[24]Note that for $\theta = \theta_3^*$, $\Gamma_3 = 0$, i.e. there is perfect risk sharing conditional on tradable endowment shocks even under zero-portfolio. When $\nu = 1/2$, $\Gamma_3 = 0$ for $\theta = 1$. This is the knife-edge case described by Cole and Obstfeld: If $\nu = \frac{1}{2}$ and $\theta = 1$, fluctuations in the terms of trade ensures complete risk sharing conditional on tradable sector shocks irrespective of the assets that are traded.

[25]This follows from the fact that for $\nu > 1/2$, $1 - 1/2\nu < \theta_3^* < \theta_1^*$.

[26]The web appendix shows the decomposition of the equilibrium portfolio given in (30) in terms of the loadings of excess returns on relative non-financial income risk by sector and real exchange rate risk.

non-tradable income shocks even when tradable and non-tradable goods are complements in consumption.

Optimal portfolio allocation has important implications for the relative consumption and real exchange rate dynamics in response to tradable endowment shocks. The solution for relative consumption and real exchange rate in this case becomes:

$$\hat{C}_t - \hat{C}_t^* = \psi_c^{nc}\widehat{NFA}_{t-1} + \frac{\gamma\kappa(2\nu-1)}{\Gamma_1}(\hat{Y}_{H,t} - \hat{Y}_{F,t}) + (1-\gamma)(\hat{Y}_{N,t} - \hat{Y}_{N,t}^*), \tag{31}$$

$$\hat{Q}_t = -\psi_q^{nc}\widehat{NFA}_{t-1} + \frac{\rho}{\Gamma_1}\gamma\kappa(2\nu-1)(\hat{Y}_{H,t} - \hat{Y}_{F,t}) + \frac{(1-\gamma)}{\kappa}(\hat{Y}_{N,t} - \hat{Y}_{N,t}^*), \tag{32}$$

where ψ_c^{nc} and ψ_q^{nc} are as defined in Table 2. Comparison of equations (31) and (32) with equations (20) and (21), shows clearly that $\hat{C}_t - \hat{C}_t^*$ and $\hat{Q}_t$ are no longer negatively correlated conditional on tradable endowment shocks. Indeed, the response of $\hat{C}_t - \hat{C}_t^*$ and $\hat{Q}_t$ to tradable endowment shocks in this two bonds set-up is exactly the same as that under the complete market set-up given by equations (17) and (18).

Hence, when excess returns are given by the terms of trade, trade in two nominal bonds ensures perfect risk sharing across countries conditional on tradable endowment shocks for all possible values of θ.

How do the risk sharing implications of bonds change when the terms of trade loads on non-tradable endowment shocks, for e.g. when $\delta < 1$? A closer inspection of equation (29) suggests that even under general parameter values, the terms of trade loads more strongly on tradable sector shocks compared to non-tradable shocks. This is intuitive as the terms of trade is directly linked to relative supply of tradables whereas it is only indirectly affected by changes in the relative supply of non-tradables through the complementarity/substitutability between tradables and non-tradables. Thus, bonds would be mainly used to hedge against the risks they can span more effectively, implying high insurance in response to tradable income shocks, which implies high insurance overall.[27]

3.3.2 Portfolio allocation and risk sharing under consumer price stabilisation

When monetary policy in each country stabilises the respective consumer price index, excess return on foreign bonds is given by the real exchange rate:

$$\hat{P}_t = \hat{P}_t^* = 0 \Rightarrow \hat{S}_t = \hat{Q}_t \Rightarrow \hat{r}_{x,t} = \hat{Q}_t - E_{t-1}\hat{Q}_t. \tag{33}$$

[27] Numerical results for the endowment economy with stationary shocks ($\delta < 1$) show that when excess returns are given by the terms of trade, the cross-correlation between relative consumption and real exchange rate is robustly high (i.e. 0.999) regardless of the calibration of parameters.

In this case, nominal bonds act like CPI-indexed bonds because of the monetary policy specification. For $\delta_T = 1$ and $\delta_N = 1$, excess return on foreign bonds is given by:

$$\hat{r}_{x,t} = \hat{Q}_t - E_{t-1}\hat{Q}_t = -\left[\frac{(1-\gamma)(2\theta\nu-1) - \kappa(2\nu-1)}{\kappa(1+2\nu(\theta-1))}\right](u_{H,t} - u_{F,t}) \quad (34)$$
$$+\frac{1-\gamma}{\kappa}(u_{N,t} - u^*_{N,t}) - \frac{(1-\beta)[4\theta\nu(1-\nu)(1-\gamma) + \kappa(2\nu-1)^2]}{\gamma\kappa(1-\nu)(1+2\nu(\theta-1))}\tilde{\alpha}_F\hat{r}_{x,t}.$$

The other component of the portfolio orthogonality condition, real exchange rate adjusted relative consumption, is still given by equation (28), where $\tilde{\alpha}_F\hat{r}_{x,t}$ is suitably adapted to the new policy specification and $\delta = 1$ is imposed to make it compatible with (34).

To build intuition for the optimal bond position, we consider the zero-portfolio solution once again. As we established during our discussion of the non-contingent bond economy, real exchange rate appreciates in response to a positive supply shock in home tradables sector for θ taking values in regions I and V. For these values of θ, real exchange rate adjusted relative consumption also increases in response to the same shock. Therefore, hedging against the consumption risk coming from tradable sector shocks require a long position in foreign currency for values of θ in region I and region V (Figure 1).

The optimal hedge against non-tradable income shocks depends on whether tradable and non-tradable goods are substitutes or complements in consumption. Under the latter, i.e. $\kappa\rho < 1$, real exchange rate adjusted relative consumption falls in response to a positive non-tradable income shock (see equation (28)). Due to the complementarity between the two goods, demand for both goods increase following a positive non-tradable supply shock. Given that the supply of tradable goods is fixed, this leads to an excess demand for tradables, which appreciates the terms of trade and leads to a fall in real exchange rate adjusted consumption differential. On the other hand, the real exchange rate depreciates in response to an increase in relative home non-tradable income irrespective of any parameter specification (see equation (34)). Therefore, hedging against the consumption risk coming from non-tradable sector shocks requires a long position in foreign currency when $\kappa\rho < 1$, and a short position when $\kappa\rho > 1$.

Since $\hat{r}_{x,t}$ is a complicated expression even for permanent shocks, we impose the additional restriction that preferences for tradable goods are symmetric ($\nu = 1/2$) to be able to display analytical results for optimal portfolio allocation and show its implications for risk sharing. Note that for $\nu = 1/2$, real exchange rate movements are driven only by movements in the relative price of non-tradables, i.e. $\hat{Q}_t = (1-\gamma)\hat{P}^N_t$.

Evaluating the portfolio orthogonality condition using (28) and (34) under the parameter restrictions $\delta_T = \delta_N = 1$ and $\nu = 1/2$, we get the following optimal foreign bond position: [28]

$$\tilde{\alpha}_F = \frac{\gamma\left[(\theta-1)^2(\gamma\kappa\rho + (1-\gamma))\frac{\sigma_T^2}{\sigma_N^2} - \theta^2(1-\gamma)(\kappa\rho - 1)\right]}{2(1-\beta)(1-\gamma)\theta^2\rho}. \tag{35}$$

For the reasons discussed above, assuming complementarity between tradables and non-tradables, i.e. $\kappa\rho < 1$, is sufficient to have a long position in foreign bonds. If tradable sector shocks are sufficiently large compared to non-tradable sector shocks, optimal portfolio will still be a long position in foreign currency also for $\kappa\rho > 1$.[29]

Given the optimal portfolio allocation in (35), relative consumption and real exchange rate dynamics are as follows:

$$\begin{aligned} \hat{C}_t - \hat{C}_t^* &= \psi_c \widehat{NFA}_{t-1} \\ &+ \frac{1}{\Gamma_4}\left[\begin{array}{c} \gamma\kappa\rho\theta(\theta-1)(\hat{Y}_{H,t} - \hat{Y}_{F,t}) \\ + \left(\theta^2(1-\gamma) + (\theta-1)^2(\gamma\kappa\rho + 1 - \gamma)\frac{\sigma_T^2}{\sigma_N^2}\right)(\hat{Y}_{N,t} - \hat{Y}_{N,t}^*) \end{array}\right] \end{aligned} \tag{36}$$

$$\begin{aligned} \hat{Q}_t &= -\psi_q \widehat{NFA}_{t-1} \\ &+ \frac{\rho(1-\gamma)}{\Gamma_4}\left[-\theta(\theta-1)(\hat{Y}_{H,t} - \hat{Y}_{F,t}) + \theta^2(\hat{Y}_{N,t} - \hat{Y}_{N,t}^*)\right] \end{aligned} \tag{37}$$

where $\Gamma_4 \equiv (\gamma\kappa\rho + 1 - \gamma)\left(\theta^2 + (\theta-1)^2\sigma_T^2/\sigma_N^2\right) > 0$ for all possible parameter values. Equations (36) and (37) show that relative consumption and real exchange rate are negatively correlated conditional on tradable endowment shocks for all possible values of θ, given our parameter restrictions $\delta_T = \delta_N = 1$ and $\nu = 1/2$. This is because when relative bond returns are given by the real exchange rate, bonds are almost equally good in hedging against the relative consumption risks coming from tradable and non-tradable sector shocks. Therefore, optimal bond portfolio in this case is torn between hedging against tradable and non-tradable shocks, which in turn implies that the consumer cannot insure fully against any of these shocks. This gives rise to international wealth transfers that imply lower risk sharing compared to the case where relative bond returns are equal to the terms of trade.[30]

[28] To compare this foreign currency position with the one obtained under domestic tradable price stabilisation, impose $\nu = 1/2$ in equation (30):

$$\tilde{\alpha}_F = -\frac{\gamma(\theta-1)}{2(1-\beta)}$$

The optimal foreign bond position under domestic tradable price stabilisation is thus negative for $\theta > 1$.

[29] See the web appendix for a discussion of the loading factors that show the breakdown of the optimal portfolio according to different hedging motives.

[30] We should acknowledge that the parameter restrictions we impose here, particularly the restriction that $\nu = 1/2$, make it easier to get the negative comovement between real exchange rate and relative consumption conditional on tradable income shocks. This is because when $\nu = 1/2$, real exchange rates move only due to

3.4 Introducing demand shocks

In the previous section, we showed that even a small move away from the non-contingent bond set-up leads to very high risk sharing in response to supply shocks, especially when agents can have claims to the terms of trade. The insight from the analysis of endogenous trade in bonds under CPI-targeting is that we can limit the risk sharing implied by endogenous asset trade if excess returns load equally well on all sources of risks and different risks imply different portfolio positions. In this case, equilibrium portfolios will depend on the relative size of shocks and valuation effects will have the potential to impede risk sharing depending on the type of shock that hits the economy.

In this section, we introduce shocks to the anticipated component of tradable endowments - 'news shocks' that act as demand shocks in our two-sector endowment model and show how these shocks can change the risk sharing properties of nominal bonds conditional on supply shocks.[31] We present analytical results only for the case of tradable price targeting since this is the setting under which trade in two bonds brings the equilibrium close to that under complete markets. The intuition we build for this case can be used to understand the case of the CPI stabilisation.[32]

We assume that tradable endowment process in each country has a predictable component. $u_{H,t}$ and $u_{F,t}$ are unanticipated home and foreign tradable endowment shocks at time t, $z_{H,t}$ and $z_{F,t}$ are information that arrive at time t about the $t+1$ values of home and foreign tradable endowments. When there is positive news today, i.e. an increase in $u_{ZH,t}$, agents anticipate home tradable endowment to be higher in the next period. The formulation we use is similar to Colacito and Croce (2010):

$$\log Y_{i,t} = \delta_T \log Y_{i,t-1} + \log z_{i,t-1} + u_{i,t}, \tag{38}$$

$$\log z_{i,t} = \delta_z \log z_{i,t-1} + u_{Zi,t} \text{ for } i = H, F, \tag{39}$$

where $0 \leq \delta_T < 1$, $0 \leq \delta_z < 1$, $u_{H,t}, u_{F,t}, u_{ZH,t}, u_{ZF,t}$ are zero-mean i.i.d. shocks symmetrically distributed over the interval $[-\epsilon, \epsilon]$ with $Var(u_H) = Var(u_F) = \sigma_T^2$ and $Var(u_{ZH}) = Var(u_{ZF}) = \sigma_Z^2$. The stochastic processes for non-tradable endowments are still given by

relative non-tradable prices, which reflect the income effect more strongly. When $\nu > 1/2$ and $\theta > 1 - 1/2\nu$ such that terms of trade depreciates in response to tradable endowment shocks, it will be more difficult to get the real exchange rate to appreciate following the appreciation in relative non-tradable prices as there will be an offsetting effect coming from terms of trade. Nevertheless, numerical results show that this set-up can still generate a negative correlation between relative consumption and real exchange rate conditional on tradable sector shocks for $\nu > 1/2$ and θ in region V.

[31] Opazo (2006) looks at the role of expectation shocks in accounting for the Backus-Smith puzzle in a single bond economy with only tradable goods.

[32] We also derive analytical results for quality shocks that shift demand between home and foreign goods, 'i-pod' shocks as coined by Coeurdacier et al. (2007), which can be found in the web appendix.

equations (6).[33]

To understand how the presence of news shocks affects optimal portfolios, consider the general equilibrium expressions for the two components of the portfolio orthogonality condition given by (15), where we again assume that $\delta_N = \delta_T = 1$ for ease of exposition:

$$\begin{aligned} \hat{C}_t - \hat{C}_t^* - \tfrac{\hat{Q}_t}{\rho} &= \psi_{rcq}\widehat{NFA}_{t-1} \\ &+ \tfrac{\Gamma_3}{\kappa\rho(1+2\nu(\theta-1))}\left[(\hat{Y}_{H,t} - \hat{Y}_{F,t}) + \tfrac{\beta}{1-\beta\delta_z}(\hat{z}_{H,t} - \hat{z}_{F,t})\right] \\ &+ \tfrac{(1-\gamma)(\kappa\rho-1)}{\kappa\rho}(\hat{Y}_{N,t} - \hat{Y}_{N,t}^*) \\ &+ \tfrac{(1-\beta)\Gamma_1}{\gamma\kappa\rho(1-\nu)(1+2\nu(\theta-1))}\tilde{\alpha}_F\hat{r}_{x,t}, \end{aligned} \tag{40}$$

$$\begin{aligned} \hat{r}_{x,t} &= \widehat{TOT}_t - E_{t-1}\widehat{TOT}_t \\ &= \tfrac{1}{1+2\nu(\theta-1)}\Big((u_{H,t} - u_{F,t}) - \tfrac{\beta(2\nu-1)\Gamma_3}{(1-\beta\delta_z)\Gamma_1}(u_{ZH,t} - u_{ZF,t})\Big) \\ &\quad -\tfrac{(1-\beta)(2\nu-1)}{\gamma(1-\nu)(1+2\nu(\theta-1))}\tilde{\alpha}_F\hat{r}_{x,t}, \end{aligned} \tag{41}$$

where Γ_1 and Γ_3 are as defined before.[34] Note that the coefficients on unanticipated tradable and non-tradable shocks and the excess return on the portfolio ($\tilde{\alpha}\hat{r}_{x,t}$) are identical to the ones given in equations (28) and (29). Shocks to the anticipated component of tradable endowments affect the real exchange rate adjusted relative consumption in the same way as unanticipated shocks, only discounted by $\beta/(1-\beta\delta_z)$. In other words, for $\theta > \theta_3^*$ such that $\Gamma_3 > 0$, or for $\theta < 1 - 1/2\nu$, $\hat{C}_t - \hat{C}_t^* - \hat{Q}_t/\rho$ rises in response to an increase in both the anticipated and unanticipated components of tradable endowments.[35]

On the other hand, as shown by equation (41), the terms of trade responds differently to anticipated and unanticipated shocks. For $\nu > 1/2$ and $\theta > \theta_3^*$, a positive shock to the predictable component of tradables endowment, which increases the relative consumption gap in favour of home agents, appreciates the terms of trade. This is because after receiving the positive news about future endowment, home agents increase their demand for tradables in the current period. Given that the supply of tradables is still fixed when agents receive the news, this leads to an excess demand for tradables in the current period, which in turn appreciates the terms of trade as consumption is home biased. Since news about future supply conditions increase current demand and appreciate the terms of trade, news shock act as a demand shock.[36]

[33] We initially introduce 'news' only to the tradable sector, because trade in nominal bonds under domestic tradable price stabilisation ensures too much risk sharing conditional on tradable endowment shocks. In the numerical part, we consider news to both sectors.

[34] See Table 2 for a summary of parameter definitions.

[35] The extent to which anticipated shocks affect $\hat{C}_t - \hat{C}_t^* - \hat{Q}_t/\rho$ is determined by δ_z. As δ_z increases, $\beta/(1-\beta\delta_z)$ increases, amplifying the response of relative consumption to anticipated shocks.

[36] Note that when $\theta < 1 - 1/2\nu$, both anticipated and unanticipated endowment shocks work as demand shocks, because terms of trade appreciate following an unanticipated increase in tradable endowment in this region of θ.

As the real exchange rate adjusted consumption differential and excess returns are positively correlated conditional on unanticipated shocks but negatively correlated conditional on anticipated shocks, relative variance of the two shocks will determine the sign of optimal portfolio:

$$\tilde{\alpha}_F = -\frac{\gamma(1-\nu)\Gamma_3}{(1-\beta)(\gamma\kappa\rho+1-\gamma)}\left(1-\frac{\Gamma_3}{\Gamma_1}\frac{\beta^2(2\nu-1)\frac{\sigma_Z^2}{\sigma_T^2}}{(1-\beta\delta_z)^2}\right). \tag{42}$$

As shown in (42), the optimal bond portfolio in the presence of news shocks is the optimal bond portfolio given in (30) plus an expression that depends on the relative variance of anticipated shocks with respect to unanticipated shocks. Therefore, it is optimal to have a long position in foreign bonds rather than a short position if $\nu > 1/2$, $\theta > \theta_3^*$, and σ_Z^2/σ_T^2 is sufficiently high, i.e. $\sigma_Z^2/\sigma_T^2 > RV_1^*$, where RV_1^* is as defined in Table 2. This would then imply adverse valuation effects in the face of unanticipated endowment shocks and potentially impede risk sharing. In this case, endogenous trade in nominal bonds does not provide a perfect hedge against any of these shocks.

Table 3 gives the signs of the responses of relative consumption and real exchange rate to anticipated and unanticipated endowment shocks under certain parameter restrictions to illustrate how the introduction of demand shocks might affect the comovement of these variables through an adverse valuation channel.[37].

	$\hat{C}_t - \hat{C}_t^*$	$\hat{Q}_t$	$\tilde{\alpha}_F\hat{r}_{x,t}$
$\hat{Y}_{H,t} - \hat{Y}_{F,t}$	+	−	+
$\hat{z}_{H,t} - \hat{z}_{F,t}$	+	−	−
$\hat{Y}_{N,t} - \hat{Y}_{N,t}^*$	+	+	0
$\tilde{\alpha}_F\hat{r}_{x,t}$	+	−	

Table 3: Impact responses of relative consumption and real exchange rate to relative supply and demand (news) shocks with trade in two bonds

Notes: This table gives the sign of the impact responses of relative consumption, $\hat{C}_t - \hat{C}_t^*$, and real exchange rate, $\hat{Q}_t$, to the relative shocks described in the first column assuming that $\nu > 1/2$, $\theta > \theta_1^*$ and $\sigma_Z^2/\sigma_T^2 > RV_1^*$ so that $\tilde{\alpha}_F > 0$.

Under the zero-portfolio/non-contingent bond economy solution ($\tilde{\alpha}_F = 0$), the real exchange rate and the relative consumption are negatively correlated in response to both supply and demand shocks in the tradable sector. As shown in Table 3, $\hat{Q}_t$ is negatively related to $\tilde{\alpha}_F\hat{r}_{x,t}$, which means that the real exchange rate appreciates when the excess return on portfolio increases.

If the optimal bond portfolio is long in foreign bonds, i.e. $\tilde{\alpha}_F > 0$, a positive unanticipated

[37]Table 3 is constructed based on the general equilibrium expressions for $\hat{C}_t - \hat{C}_t^*$ and $\hat{Q}_t$ described in the web appendix

shock that depreciates the terms of trade implies a positive wealth transfer to the home agent, $\tilde{\alpha}_F \hat{r}_{x,t}(+)$ (See Table 3). This, in turn, appreciates the real exchange rate even more than it would under the non-contingent bond economy and increases the relative consumption gap in favour of the home country. Therefore, for sufficiently large news shocks, real exchange rate and relative consumption are negatively correlated conditional on both demand and supply shocks even with endogenous bond trade.[38]

4 Numerical analysis in a calibrated two-country, two-sector RBC model

In this section, we calibrate a two-country, two-sector production economy model with capital accumulation along the lines of BT and CDL and look at the quantitative implications of introducing a second internationally traded asset for optimal portfolios and relative consumption-real exchange rate correlation alongside standard business cycle moments.

4.1 The model

Each country specialises in the production of a tradable and a non-tradable intermediate good. Final goods are obtained by combining domestic and foreign tradable inputs with domestic non-tradable inputs. All trade between the two countries is in intermediate goods and final goods are only used for domestic consumption. Capital and labour are immobile across countries.

4.1.1 Consumers

The representative agent in the home economy maximises the expected present discounted value of utility,

$$U_t = E_t \sum_{s=t}^{\infty} \delta_s U(C_s, (1 - L_s)), \tag{43}$$

where utility now depends on leisure, $1 - L$, as well as consumption, C. The discount factor, δ_s, is endogenous:

$$\delta_{s+1} = \delta_s \beta(C_{As}, 1 - L_{As}), \ \delta_0 = 1, \tag{44}$$

where C_A is aggregate home consumption and L_A is aggregate leisure and $0 < \beta(C_A, 1 - L_A) < 1$. To achieve stationarity under incomplete market specification, we assume $\beta_C(C_A, 1 - L_A) \leq 0$ and $\beta_{1-L}(C_A, 1 - L_A) \leq 0$.

[38] Our simulations show that even if news shocks are not large enough to overturn the sign of the optimal portfolio, they still limit risk sharing conditional on unanticipated endowment shocks by changing the size of the optimal portfolio.

As before, we solve the model under alternative asset market structures. Consumer's first order conditions and net foreign asset accumulation equations under each market structure is as described in section (2.2), where marginal utility functions are adjusted accordingly, i.e. $U_C(C)$ is replaced by $U_C(C, 1-L)$ and net foreign asset accumulation equations are modified to account the fact that agents also spend their income on investment, $P_{H,t}X_t$. The optimal labour supply decision is given by:

$$w_t = \frac{u_{1-L}\left(C_t,(1-L_t)\right)}{u_C\left(C_t,(1-L_t)\right)}.$$

Similar equations hold for the foreign country.

4.1.2 Producers

Final good producers combine home and foreign intermediate goods, C_T and C_N, according to the CES function given by equation (2) to yield the final home consumption good $Y \equiv C$. Tradable intermediate inputs, C_T, are obtained by combining home and foreign intermediates according to (3). Price indices corresponding to final output and the output of tradable goods are given by (4).

Intermediate goods firm in each sector choose labour, capital and investment to maximise the expected discounted value of profits:

$$\max_{K_{i,t+1},L_{i,t},X_{i,t}} E_0 \sum_{t=0}^{\infty} \delta_t \frac{U_C(C_t,(1-L_t))}{U_C(C_0,(1-L_0))} \frac{P_0}{P_t} [P_{i,t}Y_{i,t} - P_t w_t L_{i,t} - P_{H,t}X_{i,t}], \tag{45}$$

subject to the production function in each sector,

$$Y_{i,t} = F(A_{i,t}, K_{i,t-1}, L_{i,t}) = A_{i,t} L_{i,t}^{\alpha_i} K_{i,t-1}^{1-\alpha_i}, \tag{46}$$

where the subscript i, for $i = H, N$ marks variables associated with tradable and non-tradable sectors. Y_i denotes the output in sector i, w_t is the real wage, $X_{i,t}$ denotes investment by intermediate firms producing sector i. A_i denotes sector-specific total factor productivity, L_i and K_i are labour and capital input used in sector i. It is assumed that investment is in units of the domestic tradable good, hence investment price in both sectors is given by P_H. Aggregate capital accumulation equation is:

$$K_t = (1-\delta)K_{t-1} + X_t. \tag{47}$$

Aggregate capital and investment are given simply by $K_t = K_{H,t} + K_{N,t}$ and $X_t = X_{H,t} + X_{N,t}$.

Intermediate firms' labour demand functions imply the following wage equation,

$$\alpha \frac{P_{H,t}}{P_t} A_t \left(\frac{K_{H,t-1}}{L_{H,t}} \right)^{1-\alpha} = w_t = \alpha_N \frac{P_{N,t}}{P_t} A_{N,t} \left(\frac{K_{N,t-1}}{L_{N,t}} \right)^{1-\alpha_N},$$

while optimal investment is determined simply by:

$$P_{H,t} = E_t m_{t+1} \left\{ P_{i,t+1} MPK_{i,t+1} + P_{H,t+1} (1-\delta) \right\}, \ i = H, N.,$$

where m_t is the stochastic discount factor of domestic agents defined as:

$$m_t = \frac{\beta(C_t, 1-l_t) U_C(C_{t+1}, 1-L_{t+1})}{U_C(C_t, 1-L_t)} \frac{P_t}{P_{t+1}}.$$

4.1.3 Market clearing

Market clearing for intermediate goods requires: $Y_{H,t} = C_{H,t} + C^*_{H,t} + X_t$, $Y_{F,t} = C^*_{F,t} + C_{F,t} + X^*_t$, $Y_{N,t} = C_{N,t}$ and $Y^*_{N,t} = C^*_{N,t}$. While for final goods we have $Y_t = C_t$ and $Y^*_t = C^*_t$.

Factor market clearing conditions are: $L_H + L_N = L$, $L_F + L^*_N = L^*$, $K_H + K_N = K$ and $K_F + K^*_N = K^*$. Asset market clearing is as described before for the endowment economy. We close the model by two different policy rules as before.

4.2 Calibration

We calibrate the model along the lines of BT and CDL assuming symmetry across countries. Our baseline calibration is given by Table 4.

In line with the international RBC literature, we assume that preferences are non-separable in consumption and leisure. We use the same specification as Backus et al.(1992) and CDL:

$$U(C, 1-L) = \frac{\left[C^\omega (1-L)^{1-\omega} \right]^{1-\rho} - 1}{1-\rho}, \quad 0 < \omega < 1, \ \rho > 0. \tag{48}$$

We calibrate the consumption share in utility, ω, such that at the steady-state agents devote one-third of time to work. Risk aversion parameter is equal to 2. We specify the endogenous discount factor as follows:

$$\beta(C, 1-L) = \frac{1}{1 + \psi[C^\omega (1-L)^{1-\omega}]}$$

where we set the Uzawa convergence parameter, ψ, such that the steady state discount factor, $\overline{\beta}$, is equal to 1/1.04, consistent with a real interest rate of 4% per year.

We set the trade elasticity equal to 2.5 in line with BT and assume an elasticity of

Parameter	Description	Baseline values
$\bar{\beta}$	Steady-state discount factor	0.96
ρ	Coefficient of constant relative risk aversion	2
ω	Consumption share in utility	0.34
θ	Elasticity of substitution between domestic and foreign goods	2.5
κ	Elasticity of substitution between tradables and non-tradables	0.44
ν	Preference for domestic goods in the production of tradables	0.72
γ	Preference for tradables in consumption	0.55
$\alpha = \alpha_N$	Labour share in production	0.67
δ	Depreciation rate of capital	0.10
Λ	Productivity shocks (persistence and spillovers)	$\begin{bmatrix} 0.88 & 0 & 0.22 & 0 \\ 0 & 0.88 & 0 & 0.22 \\ 0 & 0 & 0.30 & 0 \\ 0 & 0 & 0 & 0.30 \end{bmatrix}$
$V(u)$	Variance-covariance matrix of productivity shocks in percent	$\begin{bmatrix} 0.0376 & 0.0159 & 0.0072 & 0.0044 \\ 0.0159 & 0.0376 & 0.0044 & 0.0072 \\ 0.0072 & 0.0044 & 0.0051 & 0.0021 \\ 0.0044 & 0.0072 & 0.0021 & 0.0051 \end{bmatrix}$

Table 4: Baseline calibration

substitution between traded and non-traded goods, κ, of 0.44, as suggested by Stockman and Tesar (1995) and adopted by BT.[39] Our benchmark calibration implies traded and non-traded goods are complements as $\kappa\rho < 1$.

In calibrating the processes for tradable and non-tradable sector productivity shocks, we mainly rely on BT, who estimate these processes for the US relative to EU15 and Japan using annual data between 1979-2002. We calibrate the persistence of tradable sector productivity shocks slightly higher to 0.88 (BT calibration sets it to 0.84) while keeping the rest of the calibration as in their paper.[40]

4.3 Results with unanticipated productivity shocks

Table 5 reports various business cycle statistics for the baseline calibration under alternative asset markets. The model is able to generate a negative consumption-real exchange rate correlation of around -0.07 when trade is restricted to a single bond. Under complete markets, this correlation rises to 0.76, which shows that market incompleteness really matters

[39] We provide a sensitivity analysis with respect to θ and κ in the web appendix.

[40] The utility function used by BT following Stockman and Tesar (1995), implies a slightly higher volatility of relative consumption compared to the utility function we use here. This in turn yields somewhat lower consumption-real exchange rate correlations for a given shock calibration. To make-up for this difference between the two preference specifications, we slightly increase the persistence of tradable sector shocks to make the wealth effects of these shocks more important and to emphasise their mechanism. (See Baxter and Crucini (1995) and Baxter (1995) on how higher shock persistence makes market incompleteness more important in international RBC models).

in this set-up.[41] The mechanism that generates a negative correlation between relative consumption and real exchange rate for this calibration implies a negative correlation between the real exchange rate and terms of trade, which does not seem to be supported by the data.

	data	nc	2bonds (rx=Q)	2bonds (rx=TOT)	complete market
Std dev of GDP	1.57	1.80	1.82	1.85	1.85
Std dev rel. to GDP					
Real exchange rate (RER)	6.16	0.37	0.30	0.29	0.30
Terms of trade (TOT)	2.12	0.31	0.37	0.42	0.42
Rel. price of non-traded	1.46	0.92	0.82	0.80	0.81
Consumption	0.76	0.38	0.37	0.36	0.35
Investment	4.33	3.22	3.21	3.19	3.19
Hours worked	0.31	0.41	0.42	0.44	0.44
AR(1) coefficients					
Real exchange rate	0.67	0.44	0.54	0.39	0.36
GDP	0.50	0.50	0.50	0.51	0.51
Consumption	0.66	0.47	0.46	0.47	0.47
Cross corr btw H and F					
GDP	0.35	0.38	0.35	0.31	0.31
Consumption	0.06	0.69	0.72	0.83	0.84
Investment	0.07	0.29	0.26	0.24	0.24
Cross corr btw					
RER and Relative consumption	-0.45	**-0.07**	**0.19**	**0.74**	**0.76**
RER and Terms of trade	0.32	-0.16	-0.20	-0.10	-0.09
TOT and Relative consumption	-0.74*	0.65	0.41	0.41	0.44
Cross corr btw GDP and					
Real net exports	-0.26	0.09	0.27	0.43	0.44
Real exchange rate	-0.09	-0.37	-0.35	-0.22	-0.21
Consumption	0.78+	0.85	0.79	0.74	0.74
Investment	0.93+	0.97	0.97	0.97	0.97
Hours worked	0.86+	0.95	0.95	0.95	0.95
Foreign bond position/GDP	0.47		6.64	-6.47	

Table 5: Business cycle statistics with unanticipated shocks to sectoral TFP for $\theta = 2.5$.

Notes: Data column contains statistics calculated by BT. Data statistics marked by * are taken from CDL, while those marked by + are taken from Raffo (2010). Both the data and simulated moments are of annual frequency, logged and HP-filtered. Data for foreign bond position as a share of GDP is net foreign currency exposure as share of GDP for the US taken from Lane and Shambaugh (2010).

Whether trade in two bonds brings the equilibrium closer to the single bond or the complete market set-ups depends on the policy rule. Under the CPI stabilisation, the model implies a large *long* position in foreign bonds (around 6.6 times GDP) and a positive but low consumption-real exchange rate correlation around 0.19. On the other hand, under the domestic tradable price stabilisation, the model implies an equally large *short* position in foreign bonds, but a high consumption-real exchange rate correlation (0.74) that is very close to the correlation implied by complete markets (0.76). These results are in line with our discussion of the endowment economy.[42]

An apparent drawback of this calibration with $\theta = 2.5$ is the low volatility and persistence

[41] The fact that the consumption-real exchange rate correlation is below unity under complete markets is due to the non-separability of consumption and leisure in the utility function.

[42] Because of the non-separability of consumption and leisure in the utility function, hedging against fluctuations in relative marginal utilities of consumption requires hedging against fluctuations in relative labour supplies as well as relative consumption levels adjusted by the real exchange rate. It is optimal to have a long position in foreign bonds, if the excess on foreign bond is higher when consumption is lower in the home country and/or when total hours worked is higher in the home country.

of the real exchange rate across all asset markets. Also, the model for this calibration cannot account for the quantity puzzle, which refers to the failure of a general class of international RBC models to generate a higher cross-country correlation between GDP's compared to consumption levels. Comparing the second and fifth columns of Table 5 shows that market incompleteness goes in the right way as it reduces the cross-country consumption correlations with respect to complete markets, but it is not sufficient to account for the puzzle.[43] Furthermore, this calibration implies procyclical net exports contrary to the data.

The numerical results obtained from a calibrated two-sector RBC model confirm the intuition provided by the analytical results regarding the endowment model: A small deviation from a single bond economy brings the model closer to the complete market model and implies too much risk sharing compared to the Backus-Smith-Kollmann evidence. What is more, the implied portfolio positions are implausibly large. This result is robust to different values of the trade elasticity and other key parameters like the elasticity of substitution between tradables and non-tradables, the share of non-traded goods in the consumption of final goods, the degree of consumption home bias and the relative variance of non-tradable sector shocks with respect to tradable sector shocks. The web appendix provides sensitivity analysis with respect to these parameters under both policy rules.

In the presence of small and persistent news shocks to sectoral productivity, the consumption-real exchange rate correlation becomes more negative, -0.16, under the non-contingent bond economy without worsening the model's performance to fit other business cycle statistics.[44] Introducing news shocks improves the performance of the two bond model when excess returns are given by the terms of trade. This is because the terms of trade covaries negatively with the relative marginal utilities of consumption conditional on anticipated shocks, but positively conditional on unanticipated shocks. This tension makes the short position in foreign currency smaller and implies a negative consumption-real exchange rate correlation of -0.08. Hence, in the presence of small and persistent news shocks, trade in bonds that give claims to the terms of trade can no longer replicate the complete market outcome.

5 Conclusion

In this paper, we review and compare different mechanisms that rely on good market frictions and market incompleteness to account for the consumption-real exchange rate anomaly. We show that the performance of these models worsens considerably when we move away

[43] CDL shows that modelling a distribution sector can account for the quantity puzzle whether risk sharing is complete or not. It also increases the volatility of terms of trade and real exchange rate.

[44] Please refer to the web appendix for the details.

from a single bond economy and allow for ex-ante risk sharing in the form of home and foreign currency bonds. Irrespective of the value of trade elasticity, relative consumption-real exchange rate correlations increase dramatically to the values implied by complete markets when agents can trade in bonds which give claims to the terms of trade. Although trade in bonds leads to less risk sharing when relative bond returns are given by the real exchange rate, correlations implied by this asset-market and policy combination are much higher than that in the data. A common characteristic of optimal portfolios among different policies and trade elasticity values is that they are implausibly large. Therefore, two-sector models with sectoral productivity shocks fail in both generating realistic portfolio positions and a low degree of risk sharing when we allow for portfolio choice between two assets.

We explore the role of news shocks in generating meaningful market incompleteness in the presence of endogenous portfolio choice and show that only under certain trade elasticity and policy combinations anticipated and unanticipated shocks can create a significant tension on equilibrium bond portfolios and reduce the degree of risk sharing implied by bonds.

Our work suggests that allowing for more sources of uncertainty can potentially improve the performance of this class of models in accounting for the consumption-real exchange rate anomaly while generating realistic portfolio positions provided that they satisfy certain conditions. First of all, these additional shocks should imply a low correlation between relative consumption and real exchange rate in the zero-portfolio solution (non-contingent bond economy) to start with. Because, as long as optimal portfolios are chosen to minimise deviations from risk sharing as in our set-up and most of the recent portfolio literature, the unconditional correlation between relative consumption and real exchange rate in the presence of endogenous portfolio cannot be lower than the non-contingent bond economy outcome. Secondly, different shocks should pull portfolios in different directions. If hedging against all sources of uncertainty in the model requires a similar portfolio position, risk sharing would be high even if there are fewer assets than shocks. Finally, these additional shocks should be empirically relevant and should not have counterfactual implications for other business cycle statistics. Our experiments with other shocks such as quality shocks that shift demand between home and foreign goods ('i-pod shocks') and investment shocks suggest that finding shocks that satisfy these properties is a tedious task that might not have much value-added.

Nevertheless, one direction for further research might be to introduce portfolio choice in an estimated DSGE model with many shocks and look at the portfolio implications and consumption-real exchange rate correlations in such a set-up. Another direction is to introduce asset market imperfections alongside market incompleteness to limit asset trade and the degree of risk sharing as in Kollmann (2009).[45]

Finally, it might be important to consider alternative explanations of the anomaly that do not rely on market incompleteness, but on non-separable preferences. Raffo (2010), Karabarbounis (2010), Stathopoulos (2010) and Colacito and Croce (2010) are examples to papers that follow this approach without considering portfolio choice. These models suggest that relative consumption and real exchange rate can be negatively correlated under complete markets. This strand of literature can be reconciled with the general equilibrium portfolio literature that is successful in accounting for the observed portfolio positions in models which do not display large deviations from risk sharing.

[45] In a companion paper, we allow for portfolio transaction costs along the lines of Tille and van Wincoop (2010) and show that in certain cases they can be instrumental in matching the observed portfolios alongside a negative correlation between relative consumption and real exchange rate.

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