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ABSTRACT

Key Player Policies When Contextual Effects Matter

We consider a model where the criminal decision of each individual is affected by not only her own characteristics, but also by the characteristics of her friends (contextual effects). We determine who the key player is, i.e. the criminal who once removed generates the highest reduction in total crime in the network. We generalize the intercentrality measure proposed by Ballester et al. (2006) by taking into account the change in contextual effects following the removal of the key player. We also provide an example that shows how the new formula can be calculated in practice.

JEL Classification: A14, D85, K42 and Z13
Keywords: contextual effects, crime, key players and peer effects

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1 Introduction

Important differences in crime rates are commonly observed across different social groups and/or locations displaying otherwise identical economic fundamentals (Glaeser et al., 1996). One of the main explanations put forward to account for this phenomenon is the presence of *social multiplier effects* on individual crime decisions. That is, as the fraction of individuals participating in a criminal behavior increases, the impact on others is multiplied through social interactions or networks (see, in particular, Sah, 1991; Kleiman, 2009; Glaeser et al., 1996; Rasmussen, 1996; Schrag and Scotchmer, 1997; Calvó-Armengol and Zenou, 2004). Empirically, recent research has shown the importance of peer and social multiplier effects in crime (see e.g., Kling et al., 2005; Damm and Dustmann, 2008; Bayer et al., 2009; Patacchini and Zenou, 2012).

Ballester et al. (2006, 2010) have argued that concentrating efforts by targeting "key players", i.e. criminals who once removed generate the highest possible reduction in aggregate crime level in a network, can have large effects on crime because of these feedback effects or "social multipliers". That is, as the fraction of individuals participating in a criminal behavior increases, the impact on others is multiplied through social networks. Thus, criminal behaviors can be magnified, and interventions can become more effective. Based on a peer-effect model, Ballester et al. (2006, 2010) have proposed a measure (the intercentrality measure) that determines the key player in each network. However, in their model, contextual effects are not taken into account, i.e. only each individual's characteristics affect her effort but not the characteristics of her friends. In the present paper, we extend this intercentrality measure to include contextual effects and show that the formula proposed by Ballester et al. (2006, 2010) is not correct in that case.

Contextual effects are important, especially for the empirical measure of peer effects in crime. In the standard linear-in-means models, Manski (1993, 2000) has put forward the importance of the *reflection problem*, which is the difficulty of separating the contextual effect from the endogenous peer effect on own behavior. Recent empirical papers have used the network topology to separate these two effects and to show the importance of contextual effects in education (Calvó-Armengol et al., 2009; Lin, 2010), obesity (Cohen-Cole and Fletcher, 2008) and crime (Patacchini and Zenou, 2012). In particular, Cohen-Cole and Fletcher (2008) have replicated the very influential study of Christakis and Fowler (2007) to show that, when contextual effects are introduced in the empirical analysis, then all the endogenous effects of friends' obesity on own obesity found in Christakis and Fowler disappear.

In the present paper, we develop a network model[1] where contextual effects are taken into account. We calculate the Nash equilibrium of this game and propose a new intercentrality measure that determines the key player in a network. This measure captures two effects. The first effect is a pure *contextual effect*, which is due to the change in the context (own and friends' characteristics) when the key player is removed from the network while the network is kept unchanged. The second effect is a pure *network effect*, which captures the change in crime effort due to the network structure change after the removal of the key player. We also propose a simple example of a network with four individuals to illustrate all our results.

There are two recent papers that empirically determine key players in soccer and crime. Using data from UEFA Euro 2008 Tournament, Sarangi and Unlu (2011) evaluate a player's contribution to her team and relates her effort to her salaries, they show that key players regardless of their field position have significantly higher market values than other players. In this paper, the authors do not take into account contextual effects because they have no information on the characteristics of the player and thus use the standard intercentrality measure of Ballester et al. (2006). Using data from adolescents in the United States (AddHealth data), Liu et al. (2011) determine the characteristics of the key players in crime. They include contextual effects in their analysis and show that key players are more likely to be a male, have less educated parents, are less attached to religion and feel socially more excluded than the average criminal. They also show that contextual effects matter since the key player may be different when the intercentrality measure used is the one given by Ballester et al. (2006) or our formula.

The rest of the paper unfolds as follows. In the next section, we present our network model and determine the Nash equilibrium of this game. In Section 3, we expose the intercentrality measure of Ballester et al. (2006), which characterizes the key player when contextual effects are not considered. In Section 4, we determine our new intercentrality measure and illustrate it with a simple example. Finally, Section 5 concludes.

[1] There is a growing literature on networks in economics. See, in particular, Goyal (2007) and Jackson (2008).

2 The model

2.1 The game

A *network* $g \in \mathcal{G}$ is a set of ex-ante identical delinquents $N = \{1, \ldots, n\}$ and a set of *links* between them. We assume $n \geq 2$. The set of i's direct contacts is: $N_i(g) = \{j \neq i \mid g_{ij} = 1\}$ and the cardinality of this set is denoted by $n_i(g) = |N_i(g)|$.[2] The $n-$square adjacency matrix $\mathbf{G}$ of a network g keeps track of the direct connections in this network. By definition, delinquents i and j are directly connected in g if and only if $g_{ij} = 1$, (denoted by ij), and $g_{ij} = 0$ otherwise. Links are taken to be reciprocal, so that $g_{ij} = g_{ji}$ (*undirected* graphs/networks). By convention, $g_{ii} = 0$. Thus $\mathbf{G}$ is a *symmetric* $(0, 1)-$matrix.[3]

We consider the following utility function for each delinquents i who chooses effort y_i:

$$u_i(\mathbf{y}, g) = \alpha_i \, y_i - \frac{1}{2} y_i^2 + \phi \sum_{j=1}^{n} g_{ij} y_i y_j \tag{1}$$

where $\phi > 0$ measures the strength of complementarities. We have indeed:

$$\sigma_{ij} = \frac{\partial^2 u_i(\mathbf{y}, g)}{\partial y_i \partial y_j} = \begin{cases} -1 & \text{if} \quad i = j \\ \phi g_{ij} & \text{if} \quad i \neq j \end{cases} \tag{2}$$

This implies that we have *local strategic complementarities* since if j is linked with i, then if q_j increases $\frac{\partial u_i(\mathbf{y},g)}{\partial y_i}$ is increased because of peer effects. This is exactly the same utility function as in Ballester et al. (2006, 2010) and Calvó-Armengol et al. (2009) with one crucial difference: the ex ante heterogeneity α_i of each agent i is not given by $\alpha_i = x_i$ but by:

$$\alpha_i = x_i + \frac{1}{g_i} \sum_{j=1}^{n} g_{ij} \, x_j \tag{3}$$

where $g_i = \sum_{j=1}^{n} g_{ij}$ is the total number of links of individual i. In other words, the ex ante heterogeneity of individual i is not only determined by her own heterogeneity x_i (like e.g. her race, age, education, etc.) but also by the average heterogeneity of her friends (i.e. the average race of her friends, the average age of her friends, the average education of her friends, etc.). The heterogeneity α_i described in (3) is usually referred to as the *contextual effect* (Manski, 1993, 2000). What is crucial for our analysis is that the *structure of the network* determines this ex ante heterogeneity α_i.

[2] Vectors and matrices are in bold while scalars are in normal letters.

[3] All our results hold if we consider a *weighted directed* network, which implies a *weighted asymmetric* $\mathbf{G}$ matrix. Likewise

2.2 The Bonacich network centrality measure

Let $\mathbf{G}^k$ be the kth power of $\mathbf{G}$, with coefficients $g_{ij}^{[k]}$, where k is some integer. The matrix $\mathbf{G}^k$ keeps track of the indirect connections in the network: $g_{ij}^{[k]} \geq 0$ measures the number of paths of length $k \geq 1$ in $\mathbf{g}$ from i to j.[4] In particular, $\mathbf{G}^0 = \mathbf{I}$.

Given a scalar $\phi \geq 0$ and a network $\mathbf{g}$, we define the following matrix:

$$\mathbf{M}(\mathbf{g}, \phi) = [\mathbf{I}-\phi\mathbf{G}]^{-1} = \sum_{k=0}^{+\infty} \phi^k \mathbf{G}^k.$$

where $\mathbf{I}$ is the identity matrix. These expressions are all well-defined for low enough values of ϕ.[5] The parameter ϕ is a decay factor that scales down the relative weight of longer paths. If $\mathbf{M}(\mathbf{g}, \phi)$ is a non-negative matrix, its coefficients $m_{ij}(\mathbf{g}, \phi) = \sum_{k=0}^{+\infty \phi k} g_{ij}^{[k]}$ count the number of paths in $\mathbf{g}$ starting from i and ending at j, where paths of length k are weighted by ϕ^k. Observe that since $\mathbf{G}$ is symmetric then $\mathbf{M}$ is also symmetric.

Definition 1 *Consider a network* $\mathbf{g}$ *with adjacency* $n-$*square matrix* $\mathbf{G}$ *and a scalar* ϕ *such that* $\mathbf{M}(\mathbf{g}, \phi) = [\mathbf{I}-\phi\mathbf{G}]^{-1}$ *is well-defined and non-negative. Given a vector* $\mathbf{u} \in \mathbb{R}_+^n$*, the Katz-Bonacich* $\mathbf{u}$*-weighted centrality of parameter* ϕ *in* $\mathbf{g}$ *is defined as:*

$$\mathbf{b}_{\mathbf{u}}(\mathbf{g}, \phi) = \sum_{k=0}^{+\infty} \phi^k \mathbf{G}^k \mathbf{u} = [\mathbf{I}-\phi\mathbf{G}]^{-1} \mathbf{u} \tag{4}$$

An element of the vector $\mathbf{b}_{\mathbf{u}}(\mathbf{g}, \phi)$ is denoted by $b_{\mathbf{u}}(\mathbf{g}, \phi)$. Observe that, by definition, the Katz-Bonacich centrality of a given node is zero when the network is empty. It is also null when $\phi = 0$, and is increasing and convex with ϕ.

2.3 Nash equilibrium

Denote by $\omega(\mathbf{G})$ the largest eigenvalue of $\mathbf{G}$. We have the following result:

[4] A *path* lof length k from i to j is a sequence $\langle i_0, ..., i_k \rangle$ of players such that $i_0 = i$, $i_k = j$, $i_p \neq i_{p+1}$, and $g_{i_p i_{p+1}} > 0$, for all $0 \leq k \leq k-1$, that is, players i_p and i_{p+1} are directly linked in $\mathbf{g}$. In fact, $g_{ij}^{[k]}$ accounts for the total weight of all paths of length k from i to j. When the network is un-weighted, that is, $\mathbf{G}$ is a $(0,1)-$matrix, $g_{ij}^{[k]}$ is simply the number of paths of length k from i to j.

[5] When ϕ is smaller than the norm of the inverse of the largest eigenvalue of $\mathbf{G}$ (Debreu and Herstein, 1953)

Proposition 1 *Consider a game where the utility function of each agent i is given by (1) with $\boldsymbol{\alpha} > \mathbf{0}$ (i.e., $\alpha_i > 0$, for all $i \in N$) and $\boldsymbol{\alpha} \neq \alpha\mathbf{1}$. Each α_i is defined by (3). If $\phi\omega(\mathbf{G}) < 1$, then this game has a unique Nash equilibrium in pure strategies $\mathbf{y}^*$, which is interior and given by:*

$$\mathbf{y}^* = \mathbf{b}_{\boldsymbol{\alpha}}(g, \phi) \tag{5}$$

This proposition is a direct application of Theorem 1 in Calvó-Armengol et al. (2009) and says that, at the Nash equilibrium, each delinquent i's effort is equal to her weighted Bonacich centrality.

3 Finding the key player when there are no contextual effects

We would like now to expose the "key player" policy. The planner's objective is to find the key player, i.e. the delinquent who once removed generates the highest possible reduction in aggregate delinquency level. Formally, the planner's problem is the following:

$$\max\{y^*(g) - y^*(g^{[-i]}) \mid i = 1, ..., n\},$$

where $y^*(g) = \sum_i y_i^*(g)$ is the total level of crime in network g and $g^{[-i]}$ is network g without individual i. When the original delinquency network g is fixed, this is equivalent to:

$$\min\{y^*(g^{[-i]}) \mid i = 1, ..., n\} \tag{6}$$

From Ballester et al. (2006), we can define a new network centrality measure $d(g, \phi)$ that solves (6). Remember that the Bonacich centrality of node i is $b_{\boldsymbol{\alpha},i}(g, \phi) = \sum_{j=1}^{n} \alpha_j m_{ij}(g, \phi)$, and counts the *total* number of paths in g starting from i weighted by the α_j of each linked node j. Let $b_{\boldsymbol{\alpha},i}(g, \phi)$ be the centrality of delinquent i in network g, $b_{\boldsymbol{\alpha}}(g, \phi)$ the *total* centrality in network g (i.e. $b_{\boldsymbol{\alpha}}(g, \phi) = \sum_{i=1}^{n} b_{\boldsymbol{\alpha},i}(g, \phi)$) and $b_{\boldsymbol{\alpha}}(g^{[-i]}, \phi)$ the *total* centrality in $g^{[-i]}$.

Definition 2 *Assume that $\alpha_i = x_i$ for all i (no contextual effects). Then, for all networks g and for all i, the intercentrality measure of delinquent i is:*

$$d_i(g, \phi) = b_{\boldsymbol{\alpha}}(g, \phi) - b_{\boldsymbol{\alpha}}(g^{[-i]}, \phi) = \frac{b_{\boldsymbol{\alpha},i}(g, \phi) \sum_{j=1}^{n} m_{ji}(g, \phi)}{m_{ii}(g, \phi)} \tag{7}$$

The intercentrality measure $d_i(g,\phi)$ of delinquent i is the sum of i's centrality measures in g, and i's contribution to the centrality measure of every other delinquent $j \neq i$ also in g. It accounts both for one's exposure to the rest of the group and for one's contribution to every other exposure.

The following result (Theorem 3 in Ballester et al., 2006) establishes that when $\alpha_i = x_i$, intercentrality captures, in an meaningful way, the two dimensions of the removal of a delinquent from a network, namely, the direct effect on delinquency and the indirect effect on others' delinquency involvement.

Proposition 2 *Assume that the utility function of each delinquent i is given by (1) for $\alpha_i = x_i$. Then, a player i^* is the key player that solves (6) if and only if i^* is a delinquent with the highest intercentrality in g, that is, $d_{i^*}(g,\phi) \geq d_i(g,\phi)$, for all $i = 1, ..., n$.*

4 Finding the key player when contextual effects matter

4.1 A motivating example

Let us now show with a simple example that, when $\alpha_i = x_i + \frac{1}{g_i} \sum_{j=1}^{n} g_{ij}\, x_j$, Proposition 2 is not correct. Consider the following symmetric undirected network with four individuals (i.e. $n = 4$):

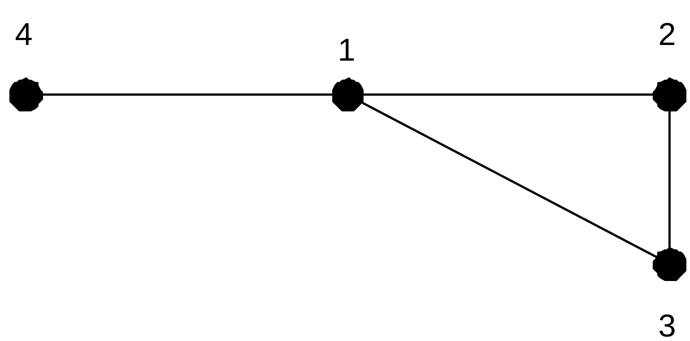

Figure 1: A bridge network with 4 deliquents

The adjacency matrix $\mathbf{G}$ of this network is given by:

$$\mathbf{G} = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Assume $\phi = 0.3$ and that[6]

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \end{pmatrix}$$

so that

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{pmatrix} = \begin{pmatrix} 0.4 \\ 0.4 \\ 0.45 \\ 0.5 \end{pmatrix} \tag{8}$$

It is straightforward to see that, using Proposition 1, we obtain:

$$\begin{pmatrix} y_1^* \\ y_2^* \\ y_3^* \\ y_4^* \end{pmatrix} = \begin{pmatrix} b_{\boldsymbol{\alpha},1}(g,\phi) \\ b_{\boldsymbol{\alpha},2}(g,\phi) \\ b_{\boldsymbol{\alpha},3}(g,\phi) \\ b_{\boldsymbol{\alpha},4}(g,\phi) \end{pmatrix} = \begin{pmatrix} 1.4004 \\ 1.1881 \\ 1.2265 \\ 0.92016 \end{pmatrix}$$

so that the total crime level is given by:

$$y^* = y_1^* + y_2^* + y_3^* + y_4^* = b_\alpha(g,\phi) = 4.735$$

Individual 1 has the highest weighted Bonacich and thus provides the highest crime effort. If we look at the formula in Definition 2, it says that the delinquent that the planner wants to remove is:

$$d_{i^*}(g,\phi) = b_{\boldsymbol{\alpha}}(g,\phi) - b_{\boldsymbol{\alpha}}^{[-i]}(g,\phi) = \frac{b_{\boldsymbol{\alpha},i}(g,\phi)\sum_{j=1}^{n} m_{ji}(g,\phi)}{m_{ii}(g,\phi)}$$

[6] The spectral radius of this graph is: 2.17 and thus the condition $\phi\mu_1(\mathbf{G}) < 1$ is satisfied since $2.17 \times 0.3 = 0.651 < 1$.

Let us remove delinquent 1. We have now a network with three delinquents where we have deleted the first column and first row in $\mathbf{G}$ to obtain:

$$\mathbf{G}^{[-1]} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

What is important is that the αs also change. Denote by $\boldsymbol{\alpha}^{[-1]}$ the $(n-1)\times 1$ vector after the removal of delinquent 1. Then, $(\alpha_2, \alpha_3, \alpha_4)$ are not anymore equal to $(0.4, 0.45, 0.5)$ but to:

$$\boldsymbol{\alpha}^{[-1]} = \begin{pmatrix} \alpha_2^{[-1]} \\ \alpha_3^{[-1]} \\ \alpha_4^{[-1]} \end{pmatrix} = \begin{pmatrix} 0.5 \\ 0.5 \\ 0.4 \end{pmatrix} \tag{9}$$

Using the same decay factor, $\phi = 0.3$, we obtain:[7]

$$\begin{pmatrix} y_2^* \\ y_3^* \\ y_4^* \end{pmatrix} = \begin{pmatrix} b_{\boldsymbol{\alpha},2}(g^{[-1]}, \phi) \\ b_{\boldsymbol{\alpha},3}(g^{[-1]}, \phi) \\ b_{\boldsymbol{\alpha},4}(g^{[-1]}, \phi) \end{pmatrix} = \begin{pmatrix} 0.71429 \\ 0.71429 \\ 0.4 \end{pmatrix}$$

so that the total effort is now given by:

$$y^{*[-1]} = y_2^* + y_3^* + y_4^* = b_{\boldsymbol{\alpha}}^{[-1]}(g, \phi) = 1.8286$$

Thus, player 1's contribution is

$$y^* - y^{*[-1]} = b_{\boldsymbol{\alpha}}(g, \phi) - b_{\boldsymbol{\alpha}}^{[-1]}(g, \phi) = \mathbf{1}^\top \mathbf{M} \boldsymbol{\alpha} - \mathbf{1}^\top \mathbf{M}^{[-1]} \boldsymbol{\alpha}^{[-1]} = 2.9064 \tag{10}$$

If we perform the same procedure for the other players, we obtain delinquent 2's contribution

$$y^* - y^{[-2]*} = 2.4301,$$

delinquent 3's contribution

$$y^* - y^{[-3]*} = 2.686\,2,$$

[7] Since individual 4 is now isolated, we have:

$$y_4^* = \alpha_4 = 0.4$$

and delinquent 4's contribution

$$y^{*}-y^{[-4]*}=1.735$$

Since delinquent 1 has the highest contribution, she is the key player.

Let us now check if the formula (7) works, i.e., if

$$d_{1^{*}}(g,\phi)=\frac{b_{\boldsymbol{\alpha},1}(g,\phi)\sum_{j=1}^{j=4}m_{j1}(g,\phi)}{m_{11}(g,\phi)}=2.9064$$

Let us go back to the initial network with four individuals. It is easily verified that (with $\phi=0.3$):

$$\mathbf{M}=[\mathbf{I}-\phi\mathbf{G}]^{-1}=\begin{pmatrix} 1.5317 & 0.65646 & 0.65646 & 0.45952 \\ 0.65646 & 1.3802 & 0.61101 & 0.19694 \\ 0.65646 & 0.61101 & 1.3802 & 0.19694 \\ 0.45952 & 0.19694 & 0.19694 & 1.1379 \end{pmatrix} \tag{11}$$

so that

$$m_{11}(g,\phi)=1.5317$$

and

$$\begin{aligned}\sum_{j=1}^{j=3}m_{j1}(g,\phi) &= m_{11}(g,\phi)+m_{21}(g,\phi)+m_{31}(g,\phi)+m_{41}(g,\phi)\\ &= 1.5317+0.65646+0.65646+0.45952\\ &= 3.3041\end{aligned}$$

Therefore,

$$d_{1^{*}}(g,\phi)=\frac{b_{\boldsymbol{\alpha},1}(g,\phi)\sum_{j=1}^{j=3}m_{j1}(g,\phi)}{m_{11}(g,\phi)}=\frac{1.4004\times 3.3041}{1.5317}=3.0209 \tag{12}$$

which is clearly different than (10). This is because in (12) contextual effects are not taken into account. while they are in (10). Interestingly,

$$d_{2^{*}}(g,\phi)=2.8442,\ d_{3^{*}}(g,\phi)=2.5278 \text{ and } d_{4^{*}}(g,\phi)=1.6103$$

This shows that the ranking in terms of intercentralities (1, 2, 3, 4) differs than the one in terms of contribution to crime reduction (1, 3, 2, 4).

To see the importance of contextual effects, consider the same example but without contextual effects so that

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \boldsymbol{\alpha} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{pmatrix} = \begin{pmatrix} 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \end{pmatrix}$$

We still assume $\phi = 0.3$. It is then straightforward to see that, using Proposition 1, we obtain:

$$\begin{pmatrix} y_1^* \\ y_2^* \\ y_3^* \\ y_4^* \end{pmatrix} = \begin{pmatrix} b_{\boldsymbol{\alpha},1}(g,\phi) \\ b_{\boldsymbol{\alpha},2}(g,\phi) \\ b_{\boldsymbol{\alpha},3}(g,\phi) \\ b_{\boldsymbol{\alpha},4}(g,\phi) \end{pmatrix} = \begin{pmatrix} 0.66521 \\ 0.60377 \\ 0.68068 \\ 0.59958 \end{pmatrix}$$

so that the total activity level is given by:

$$y^* = y_1^* + y_2^* + y_3^* + y_4^* = b_\alpha(g,\phi) = 2.549$$

Individual 3 has now the highest weighted Bonacich and thus provides the highest crime effort. In other words, when there are no contextual effects, individual 3 is the most active criminal while it is individual 1 who has the highest weighted Bonacich centrality when contextual effects are taken into account. This is because individual 1 has not only a central position but she is also linked to individual 4 who has the highest α in the network.

Let us now calculate the key player when there are no contextual effects. It is easily verified that individual 1's contribution is $y^* - y^{[-1]*} = 1.435$ while the contribution of the other individuals is equal to:

$$y^* - y^{[-2]*} = 1.244$$

$$y^* - y^{[-2]*} = 1.146$$

$$y^* - y^{[-2]*} = 0.988$$

Thus, criminal 1 is still the key player but her contribution is much lower. It is easily checked that the formula (7) is now correct so that, for each $i = 1, 2, 3, 4$, we have:

$$y^* - y^{[-i]*} = \frac{b_{\boldsymbol{\alpha},i}(g,\phi)\sum_{j=1}^{n} m_{ji}(g,\phi)}{m_{ii}(g,\phi)}$$

4.2 A new formula for the key player

Let us start with some notations. For simplicity and without loss of generality, we focus on the removal of delinquent 1. It could be any delinquent k; it suffices to replace 1 by k. Let

$$\boldsymbol{\alpha}^{[1]} = \begin{pmatrix} \alpha_1 \\ \boldsymbol{\alpha}^{[-1]} \end{pmatrix}$$

and let $\mathbf{M}^{[1]}$ be the $n \times n$ matrix such each element of this matrix is

$$m_{ij}^{[1]} = \frac{m_{i1} m_{1j}}{m_{11}}$$

It is easily verified that

$$\mathbf{M}^{[1]} = \begin{pmatrix} m_{11} & m_{12} & ... & m_{1n} \\ m_{21} & \frac{m_{21} m_{12}}{m_{11}} & ... & \frac{m_{21} m_{1n}}{m_{11}} \\ ... & ... & ... & ... \\ m_{n1} & \frac{m_{n1} m_{12}}{m_{11}} & ... & \frac{m_{n1} m_{1n}}{m_{11}} \end{pmatrix}$$

so that

$$\mathbf{M}^{[1]} \boldsymbol{\alpha}^{[1]} = \begin{pmatrix} \alpha_1 m_{11} + \alpha_2^{[-1]} m_{12} + ... + \alpha_n^{[-1]} m_{1n} \\ \frac{m_{21} m_{11}}{m_{11}} \alpha_1 + \alpha_2^{[-1]} \frac{m_{21} m_{12}}{m_{11}} + ... + \alpha_n^{[-1]} \frac{m_{21} m_{1n}}{m_{11}} \\ ... \\ \frac{m_{n1} m_{11}}{m_{11}} \alpha_1 + \alpha_2^{[-1]} \frac{m_{n1} m_{12}}{m_{11}} + ... + \alpha_n^{[-1]} \frac{m_{n1} m_{1n}}{m_{11}} \end{pmatrix}$$

and

$$\begin{aligned} \mathbf{1}^{\top} \mathbf{M}^{[1]} \boldsymbol{\alpha}^{[1]} &= \frac{\left(\alpha_1 m_{11} + \alpha_2^{[-1]} m_{12} + ... + \alpha_n^{[-1]} m_{1n} \right) \sum_{j=1}^{n} m_{j1}}{m_{11}} \\ &= \frac{b_{\boldsymbol{\alpha}^{[1]}} (g, \phi) \sum_{j=1}^{n} m_{j1}}{m_{11}} \end{aligned}$$

In other words,

$$\mathbf{1}^{\top} \mathbf{M}^{[1]} \boldsymbol{\alpha}^{[1]} = d_1(g, \phi) = \frac{b_{\boldsymbol{\alpha}^{[1]},1}(g, \phi) \sum_{j=1}^{n} m_{j1}(g, \phi)}{m_{11}(g, \phi)}$$

which is the definition of intercentrality (see Definition 2 and (7)) for $\boldsymbol{\alpha}^{[1]}$, that is when $\alpha_1 = x_1 + \frac{1}{g_1} \sum_{j=1}^{n} g_{1j} \, x_j$ and $\alpha_k = x_k + \frac{1}{g_k^{[-1]}} \sum_{j=1}^{n} g_{kj}^{[-i]} \, x_j$, for $k \neq 1$.

Let $\mathbf{M}^{[-1]}$ be the $(n-1)\times(n-1)$ inverse matrix after the removal of player 1, i.e.,

$$\mathbf{M}^{[-1]} = \left[\mathbf{I} - \phi\mathbf{G}^{[-1]}\right]^{-1}$$

Note that when $i = 1$ or $j = 1$, then $m_{ij}^{[1]} = \frac{m_{11}m_{1j}}{m_{11}} = m_{1j}$ (for $i = 1$) or $m_{ij}^{[1]} = \frac{m_{i1}m_{11}}{m_{11}} = m_{i1}$ (for $j = 1$), and thus $m_{ij}^{[1]} = m_{ij}$, that is,

$$\mathbf{M} - \mathbf{M}^{[1]} = \begin{pmatrix} 0 & \mathbf{0} \\ \mathbf{0} & \mathbf{M}^{[-1]} \end{pmatrix} \tag{13}$$

where the first $\mathbf{0}$ is a $1\times(n-1)$ row vector and the second $\mathbf{0}$ is a $(n-1)\times 1$ column vector. Indeed,

$$\begin{aligned}
\mathbf{M} - \mathbf{M}^{[1]} &= \begin{pmatrix} m_{11} & m_{12} & \dots & m_{1n} \\ m_{21} & m_{22} & \dots & m_{2n} \\ \dots & \dots & \dots & \dots \\ m_{n1} & m_{n2} & \dots & m_{nn} \end{pmatrix} - \begin{pmatrix} m_{11} & m_{12} & \dots & m_{1n} \\ m_{21} & \frac{m_{21}m_{12}}{m_{11}} & \dots & \frac{m_{21}m_{1n}}{m_{11}} \\ \dots & \dots & \dots & \dots \\ m_{n1} & \frac{m_{n1}m_{12}}{m_{11}} & \dots & \frac{m_{n1}m_{1n}}{m_{11}} \end{pmatrix} \\
&= \begin{pmatrix} 0 & 0 & \dots & 0 \\ 0 & m_{22} - \frac{m_{21}m_{12}}{m_{11}} & \dots & m_{2n} - \frac{m_{21}m_{1n}}{m_{11}} \\ \dots & \dots & \dots & \dots \\ 0 & m_{n2} - \frac{m_{n1}m_{12}}{m_{11}} & \dots & m_{nn} - \frac{m_{n1}m_{1n}}{m_{11}} \end{pmatrix} \\
&= \begin{pmatrix} 0 & 0 & \dots & 0 \\ 0 & m_{22}^{[-1]} & \dots & m_{2n}^{[-1]} \\ \dots & \dots & \dots & \dots \\ 0 & m_{n2}^{[-1]} & \dots & m_{nn}^{[-1]} \end{pmatrix} = \begin{pmatrix} 0 & \mathbf{0} \\ \mathbf{0} & \mathbf{M}^{[-1]} \end{pmatrix}
\end{aligned}$$

We can also show that

$$\mathbf{1}^{\top}\mathbf{M}^{[-1]}\boldsymbol{\alpha}^{[-1]} = \mathbf{1}^{\top} \begin{pmatrix} 0 & \mathbf{0} \\ \mathbf{0} & \mathbf{M}^{[-1]} \end{pmatrix} \boldsymbol{\alpha}^{[1]} \tag{14}$$

We have:

$$\begin{pmatrix} 0 & \mathbf{0} \\ \mathbf{0} & \mathbf{M}^{[-1]} \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \boldsymbol{\alpha}^{[-1]} \end{pmatrix} = \begin{pmatrix} 0 & 0 & ... & 0 \\ 0 & m_{22}^{[-1]} & ... & m_{2n}^{[-1]} \\ ... & ... & ... & ... \\ 0 & m_{n2}^{[-1]} & ... & m_{nn}^{[-1]} \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2^{[-1]} \\ ... \\ \alpha_n^{[-1]} \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ \alpha_2^{[-1]} m_{22}^{[-1]} + ... + \alpha_n^{[-1]} m_{2n}^{[-1]} \\ ... \\ \alpha_2^{[-1]} m_{n2}^{[-1]} + ... + \alpha_n^{[-1]} m_{nn}^{[-1]} \end{pmatrix}$$

Thus

$$\begin{aligned} \mathbf{1}^\top \begin{pmatrix} 0 & \mathbf{0} \\ \mathbf{0} & \mathbf{M}^{[-1]} \end{pmatrix} \boldsymbol{\alpha}^{[1]} &= \alpha_2^{[-1]} \sum_{j=2}^{n} m_{j2}^{[-1]} + \alpha_3^{[-1]} \sum_{j=2}^{n} m_{j3}^{[-1]} + ... + \alpha_n^{[-1]} \sum_{j=2}^{n} m_{jn}^{[-1]} \\ &= \mathbf{1}^\top \mathbf{M}^{[-1]} \boldsymbol{\alpha}^{[-1]} \end{aligned}$$

The equilibrium outcome of the game before the removal of player 1 is given by:

$$\mathbf{y}^* = [\mathbf{I} - \phi \mathbf{G}]^{-1} \boldsymbol{\alpha} = \mathbf{M}\boldsymbol{\alpha} = \mathbf{b}_{\boldsymbol{\alpha}}(g, \phi)$$

and the total equilibrium effort before the removal of player 1 is equal to:

$$y^* = \mathbf{1}^\top [\mathbf{I} - \phi \mathbf{G}]^{-1} \boldsymbol{\alpha} = \mathbf{1}^\top \mathbf{M}\boldsymbol{\alpha} = \mathbf{1}^\top \mathbf{b}_{\boldsymbol{\alpha}}(g, \phi) = b_{\boldsymbol{\alpha}}(g, \phi)$$

What is important is that $\boldsymbol{\alpha}$ now depends on $\mathbf{G}$. Let $\boldsymbol{\alpha}^{[-1]}$ be the $(n-1) \times (n-1)$ vector after removing delinquent 1 from the network. Note that $\boldsymbol{\alpha}$ and $\boldsymbol{\alpha}^{[-1]}$ may be completely different. When we remove delinquent 1, the equilibrium outcome of the game is:

$$\mathbf{y}^{*[-1]} = \left[\mathbf{I} - \phi \mathbf{G}^{[-1]}\right]^{-1} \boldsymbol{\alpha}^{[-1]} = \mathbf{M}^{[-1]} \boldsymbol{\alpha}^{[-1]} = \mathbf{b}_{\boldsymbol{\alpha}^{[-1]}}\left(g^{[-1]}, \phi\right)$$

and the total equilibrium effort after the removal of player 1 is equal to:

$$y^{*[-1]} = \mathbf{1}^\top \left[\mathbf{I} - \phi \mathbf{G}^{[-1]}\right]^{-1} \boldsymbol{\alpha}^{[-1]} = \mathbf{1}^\top \mathbf{M}^{[-1]} \boldsymbol{\alpha}^{[-1]} = \mathbf{1}^\top \mathbf{b}_{\boldsymbol{\alpha}^{[-1]}}\left(g^{[-1]}, \phi\right) = b_{\boldsymbol{\alpha}^{[-1]}}\left(g^{[-1]}, \phi\right).$$

Let us calculate the contribution of delinquent 1 when contextual effects matter. We have:

$$\begin{aligned} y^* - y^{*[-1]} &= \mathbf{1}^\top \mathbf{M}\boldsymbol{\alpha} - \mathbf{1}^\top \mathbf{M}^{[-1]} \boldsymbol{\alpha}^{[-1]} \\ &= \mathbf{1}^\top \begin{pmatrix} 0 & \mathbf{0} \\ \mathbf{0} & \mathbf{M}^{[-1]} \end{pmatrix} \boldsymbol{\alpha}^{[1]} \end{aligned}$$

where we use (14) for the last equality. Now using (13), we obtain:

$$\begin{aligned} y^* - y^{*[-\mathbf{1}]} &= \mathbf{1}^\top \mathbf{M}\boldsymbol{\alpha} - \mathbf{1}^\top \left(\mathbf{M} - \mathbf{M}^{[1]}\right) \boldsymbol{\alpha}^{[1]} \\ &= \mathbf{1}^\top \mathbf{M}\boldsymbol{\alpha} - \mathbf{1}^\top \mathbf{M}\boldsymbol{\alpha}^{[1]} + \mathbf{1}^\top \mathbf{M}^{[1]} \boldsymbol{\alpha}^{[1]} \\ &= \underbrace{b_{\boldsymbol{\alpha}}(g,\phi) - b_{\boldsymbol{\alpha}^{[1]}}(g,\phi)}_{\text{Total Bonacich centralities computed for network } g} + \underbrace{\frac{b_{\boldsymbol{\alpha}^{[1]}}(g,\phi) \sum_{j=1}^{n} m_{j1}}{m_{11}}}_{\text{Intercentrality of player 1 for fixed } \boldsymbol{\alpha}^{[1]}} \end{aligned}$$

In this formula, the first effect is the *contextual effect*, which is is due to the change in the contextual effect $\boldsymbol{\alpha}$ (from $\boldsymbol{\alpha}$ to $\boldsymbol{\alpha}^{[1]}$) after the removal of the key player while the network g remains unchanged. The second effect is the *network effect*, which captures the change in the network structure when the key player is removed. The latter corresponds to the standard inter-centrality measure of Ballester et al. (2006) (see 7) for $\boldsymbol{\alpha}^{[1]}$. If there were no contextual effects so $\boldsymbol{\alpha}$ did not change after the removal of delinquent 1 (i.e., $\boldsymbol{\alpha} = \boldsymbol{\alpha}^{[1]}$), we would have had $b_{\boldsymbol{\alpha}}(g,\phi) - b_{\boldsymbol{\alpha}^{[1]}}(g,\phi) = 0$, and we would be back to the standard formula defined in (7).

We can define a new measure of intercentrality for the removal of delinquent i when contextual effects matter as follows:

$$\begin{aligned} \delta_{\boldsymbol{\alpha}^{[i]}}(g,\phi) &= b_{\boldsymbol{\alpha}}(g,\phi) - b_{\boldsymbol{\alpha}^{[i]}}(g,\phi) + d_{\boldsymbol{\alpha}^{[i]}}(g,\phi) \\ &= b_{\boldsymbol{\alpha}}(g,\phi) - b_{\boldsymbol{\alpha}^{[i]}}(g,\phi) + b_{\boldsymbol{\alpha}^{[i]}}(g,\phi) \sum_{j=1}^{n} m_{j1}/m_{11} \end{aligned} \tag{15}$$

Proposition 3 *Assume that the utility function of each delinquent i is given by (1) where $\alpha_i = x_i + \frac{1}{g_i}\sum_{j=1}^{n} g_{ij}\, x_j$. Then, player i^* is the key player that solves (6) if and only if i^* is a delinquent with the highest intercentrality in g, that is, $\delta_{\boldsymbol{\alpha}^{[i^*]}}(g,\phi) \geq \delta_{\boldsymbol{\alpha}^{[i]}}(g,\phi)$, for all $i = 1, ..., n$.*

4.3 Back to the example

Let us illustrate this last result with the network described in Figure 1. For $\phi = 0.3$ and $(x_1, x_2, x_3, x_4) = (0.1, 0.2, 0.3, 0.4)$, we showed that the removal of delinquent led to

$$3.0209 = d_{1^*}(g,\phi) \neq y^* - y^{*[-1]} = 2.9064.$$

This meant that the formula proposed in (7) was not correct when contextual effects were taken into account. Let us now show that the new formula is correct so that $y^{*}-y^{*[-1]}=b_{\boldsymbol{\alpha}}\left(g,\phi\right)-b_{\boldsymbol{\alpha}^{[1]}}\left(g,\phi\right)+d_{\boldsymbol{\alpha}^{[1]}}(g,\phi)$.

Let us first calculate $b_{\boldsymbol{\alpha}}\left(g,\phi\right)-b_{\boldsymbol{\alpha}^{[1]}}\left(g,\phi\right)$. We have:

$$b_{\boldsymbol{\alpha}}\left(g,\phi\right)-b_{\boldsymbol{\alpha}^{[1]}}\left(g,\phi\right)=\mathbf{1}^{\top}\mathbf{M}\boldsymbol{\alpha}-\mathbf{1}^{\top}\mathbf{M}\boldsymbol{\alpha}^{[\mathbf{1}]}$$

It is easily verified that (using (8) and (11))

$$\mathbf{y}=\left[\mathbf{I}-\phi\mathbf{G}\right]^{-1}\boldsymbol{\alpha}=\mathbf{M}\,\boldsymbol{\alpha}=\begin{pmatrix} y_1^{*}\\ y_2^{*}\\ y_3^{*}\\ y_4^{*}\end{pmatrix}=\begin{pmatrix} b_{\alpha,1}(g,\phi)\\ b_{\alpha,2}(g,\phi)\\ b_{\alpha,3}(g,\phi)\\ b_{\alpha,4}(g,\phi)\end{pmatrix}=\begin{pmatrix} 1.4004\\ 1.1881\\ 1.2265\\ 0.92016\end{pmatrix}$$

so that the total activity level before the removal of the key player is given by:

$$y^{*}=b_{\boldsymbol{\alpha}}(g,\phi)=\mathbf{1}^{\top}\mathbf{M}\boldsymbol{\alpha}=\ 4.735$$

We also have:

$$\boldsymbol{\alpha}^{[\mathbf{1}]}=\begin{pmatrix}\alpha_1\\ \boldsymbol{\alpha}^{[-\mathbf{1}]}\end{pmatrix}=\begin{pmatrix}\alpha_1\\ \alpha_2{}^{[-\mathbf{1}]}\\ \alpha_3{}^{[-\mathbf{1}]}\\ \alpha_4{}^{[-\mathbf{1}]}\end{pmatrix}=\begin{pmatrix}0.4\\ 0.5\\ 0.5\\ 0.4\end{pmatrix}$$

so that (using (11)), we obtain:

$$b_{\boldsymbol{\alpha}^{[1]}}\left(g,\phi\right)=\mathbf{1}^{\top}\mathbf{M}\boldsymbol{\alpha}^{[\mathbf{1}]}=4.9628$$

As a result,

$$b_{\boldsymbol{\alpha}}\left(g,\phi\right)-b_{\boldsymbol{\alpha}^{[1]}}\left(g,\phi\right)=\mathbf{1}^{\top}\mathbf{M}\boldsymbol{\alpha}-\mathbf{1}^{\top}\mathbf{M}\boldsymbol{\alpha}^{[\mathbf{1}]}=-\ 0.2278$$

What is interesting here is that, when contextual effects matter, removing a delinquent from the network can increase the effect on total crime when only contextual effects matter (no network effect). This is because, when 1 is removed, the average contextual effect of 3 and 4 increases from $\alpha_2=0.4$ and $\alpha_3=0.45$ (before the removal of 1) to $\alpha_2^{[-1]}=0.5$ and $\alpha_3^{[-1]}=0.5$ (after the removal of 1).

We now need to calculate $b_{\boldsymbol{\alpha}^{[1]}}(g,\phi)\sum_{j=1}^{n} m_{j1}/m_{11}$. We have

$$\mathbf{M}^{[1]} = \begin{pmatrix} 1.5317 & 0.65646 & 0.65646 & 0.45952 \\ 0.65646 & 0.28135 & 0.28135 & 0.19694 \\ 0.65646 & 0.28135 & 0.28135 & 0.19694 \\ 0.45952 & 0.19694 & 0.19694 & 0.13786 \end{pmatrix},$$

$$\mathbf{M}^{[1]}\boldsymbol{\alpha}^{[1]} = \begin{pmatrix} 1.4529 \\ 0.62271 \\ 0.62271 \\ 0.43589 \end{pmatrix} \text{ and } \mathbf{1}^{\top}\mathbf{M}^{[1]}\boldsymbol{\alpha}^{[1]} = 3.1342$$

Thus

$$b_{\boldsymbol{\alpha}^{[1]}}(g,\phi)\sum_{j=1}^{n} m_{j1}/m_{11} = \mathbf{1}^{\top}\mathbf{M}^{[1]}\boldsymbol{\alpha}^{[1]} = 3.1342$$

As a result,

$$b_{\boldsymbol{\alpha}}(g,\phi) - b_{\boldsymbol{\alpha}^{[1]}}(g,\phi) + b_{\boldsymbol{\alpha}^{[1]}}(g,\phi)\sum_{j=1}^{n} m_{j1}/m_{11} = -0.2278 + 3.1342 = 2.9064$$

Therefore, we have:

$$2.9064 = \delta_{\boldsymbol{\alpha}^{[1^*]}}(g,\phi) = y^* - y^{*[-1]} = 2.9064$$

5 Concluding remarks

In the present paper, we consider a model where the criminal decision of each individual is affected by not only her own characteristics, but also by the characteristics of her friends (contextual effects). We characterize the Nash equilibrium of this game and determine who the key player is, i.e. the criminal who once removed generates the highest reduction in total crime in the network. We show that the formula proposed by Ballester et al. (2006) is not correct and give another one that highlights two effects. The first effect is a pure contextual effect, which is due to the change in the context (own and friends' characteristics) when the key player is removed from the network while the network is kept unchanged. The second effect is a pure network effect, which captures the change in crime effort due to the network structure change after the removal of the key player. We also propose a simple example of a network with four individuals to illustrate all our results. We believe that this result is important, especially for the empirical determination of the key player in real-world networks where contextual effects matter.

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