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ABSTRACT

Criminal Networks: Who is the Key Player?*

We analyze delinquent networks of adolescents in the United States. We develop a dynamic network formation model showing who the key player is, i.e. the criminal who once removed generates the highest possible reduction in aggregate crime level. We then structurally estimate our model using data on criminal behaviors of adolescents in the United States (AddHealth data). Compared to other criminals, key players are more likely to be a male, have less educated parents, are less attached to religion and feel socially more excluded. We also find that, even though some criminals are not very active in criminal activities, they can be key players because they have a crucial position in the network in terms of betweenness centrality.

JEL Classification: A14, D85, K42 and Z13
Keywords: Bonacich centrality, crime policies and dynamic network formation

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1 Introduction

There are 2.3 million people behind bars at any moment of time in the United States and that number continues to grow. It is the highest level of incarceration per capita in the world. Moreover, since the crime explosion of the 1960s, the prison population in the United States has multiplied fivefold, to one prisoner for every hundred adults–a rate unprecedented in American history and unmatched anywhere in the world.[1] Even as the prisoner head count continues to rise, crime has not stopped falling, and poor people and minorities still bear the brunt of both crime and punishment.

One possible way to reduce crime is to detect, apprehend, convict, and punish criminals. This is what has been done in the United States and all of those actions cost money, currently about $200 billion per year nationwide. This "brute force" policy does not seem to work well since, for example, the cost of prison in California is higher than the cost of education[2] and crime rates do not seem to decrease.

In his recent book published in 2009, Mark Kleiman argues that simply locking up more people for lengthier terms is no longer a workable crime-control strategy. But, says Kleiman, there has been a revolution in controlling crime by means other than brute-force incarceration: substituting swiftness and certainty of punishment for randomized severity, concentrating enforcement resources rather than dispersing them, communicating specific threats of punishment to specific offenders, and enforcing probation and parole conditions to make community corrections a genuine alternative to incarceration. As Kleiman shows, "zero tolerance" is nonsense: there are always more offenses than there is punishment capacity.[3]

Is there an alternative to brute force? In this paper, we argue that concentrating efforts by targeting "key criminals", i.e. criminals who once removed generate the highest possible reduction in aggregate crime level in a network, can have large effects on crime because of the feedback effects or "social multipliers" at work (see, in particular, Sah, 1991; Kleiman, 1993, 2009; Glaeser et al., 1996; Rasmussen, 1996; Schrag and Scotchmer, 1997; Verdier and Zenou, 2004). That is, as the fraction of individuals participating in a criminal behavior increases, the impact on others is multiplied through social networks. Thus, criminal behaviors can be magnified, and interventions can become more effective. The impacts from social

[1] See Cook and Ludwig (2010) and the references therein.

[2] For example, "Three Strikes" is a law in California passed in 1994 that mandates extremely long prison terms (between 29 years and life) for anyone previously convicted in two serious of violent felonies (including residential burglary) who is convicted of a third felony, even something as minor as a petty theft.

[3] In the standard crime literature (Becker, 1968; Garoupa, 1997; Polinsky and Shavell, 2000), punishment is seen as an effective tool for reducing crime.

networks may also be particularly important for adolescents because this developmental period overlaps with the initiation and continuation of many risky, unhealthy, and delinquent behaviors and is a period of maximal response to peer pressure (Thornberry et al., 2003; Warr, 2002).

It is indeed well-established that delinquency is, to some extent, a group phenomenon, and the source of crime and delinquency is located in the intimate social networks of individuals (see e.g. Sutherland, 1947; Sarnecki, 2001; Warr, 2002; Haynie, 2001; Patacchini and Zenou, 2008; 2011). Delinquents often have friends who have themselves committed several offences, and social ties among delinquents are seen as a mean whereby individuals exert an influence over one another to commit crimes. In fact, not only friends but also the *structure* of social networks matters in explaining individual's own delinquent behavior. This suggests that the underlying structural properties of friendship networks must be taken into account to better understand the impact of peer influence on delinquent behavior and to address adequate and novel delinquency-reducing policies.

Following Ballester et al. (2006), we first propose a theoretical model of criminal networks. Building on the Beckerian incentives approach to delinquency, we develop a model where peer effects matter so that criminals are directly influenced by their friends. Individuals decide non-cooperatively their crime effort and we show that, in equilibrium, each criminal effort is equal to her Katz-Bonacich centrality.[4] The Katz-Bonacich centrality measure is an index of *connectivity* that not only takes into account the number of direct links a given delinquent has but also all his indirect connections. In our delinquency game, the network payoff interdependence is restricted to direct network mates. But, because clusters of direct friends overlap, this local payoff interdependence spreads all over the network. In equilibrium, individual decisions emanate from all the existing network chains of direct and indirect contacts stemming from each player, a feature characteristic of Katz-Bonacich centrality.

We then consider different policies that aim at reducing the total crime activity in a delinquent network. The standard policy tool to reduce aggregate delinquency relies on the deterrence effects of punishment (Becker, 1968). By uniformly hardening the punishment costs borne by all delinquents, the distribution of delinquency efforts shifts to the left and the average (and aggregate) delinquency level decreases. This homogeneous policy tackles average behavior explicitly and does not discriminate among delinquents depending on their relative contribution to the aggregate delinquency level. To this "brute force" policy, we propose a *targeted policy* that discriminates among delinquents depending on their relative

[4]Due to Katz (1953) and extended by Bonacich (1987).

network location, and removes a few suitably selected targets from this network, alters the whole distribution of delinquency efforts, not just shifting it. To characterize the network optimal targets, we use a new measure of network centrality, the *intercentrality measure*, proposed by Ballester et al. (2006, 2010) and Ballester and Zenou (2012). This measure solves the planner's problem that consists in finding and getting rid of the *key player*, i.e., the delinquent who, once removed, leads to the highest aggregate delinquency reduction. We show that the key player is, precisely, the individual with the highest intercentrality in the network.

These models (theory and policy) developed by Ballester et al. (2006) have two main drawbacks: the decision to become criminal is not considered and the network is fixed and taken as given. We extend these models to incorporate these two aspects. First, we add a first stage where individuals decide whether to become criminal or not. Then, in the second stage, those who become criminals play the effort game of Ballester et al. (2006) while the individuals who find it optimal not to be criminal obtain a fixed utility level. We are able to fully characterize the subgame perfect equilibrium of this game and to show that observable and unobservable characteristics of individuals play a major role in the crime decision.

Second, and this is a much more important extension, we endogeneize the way individuals form links. Indeed, so far, we assumed that individuals took the structure of the network as given. This is referred to as the *invariant case*. This is a strong assumption, especially for the key player policy, since once a criminal has been removed from the network, we assumed that no one could create new links. This assumption could be justified in the short run where policy makers take "by surprise" criminals in a network but, in the long run, criminals may re-organize themselves by creating new links and thus changing the network structure. As a result, we now develop a dynamic network formation model where, at each period of time, a criminal is chosen at random and optimally decides with whom she wants to form a link, anticipating the criminal effort game all criminals in the network will play after the new link has been added. There is a trade off since having one more link is always beneficial (due to local complementarities) but there is an individual-specific cost of forming a new link. We study the evolution of the network over time where, at each period of time, both link formation and Nash equilibrium in criminal efforts are determined. We have a Markov chain and the network converges to an equilibrium (or an absorbing state) when no criminal has an incentive to create a new link. We then study the key player policy in this dynamic network formation model. In this case, the planner will compare the total crime effort in the equilibrium network with and without removing one criminal and the key player will be the one who once removed reduces the most total crime effort in equilibrium. Compared to the invariant case, the planner now considers the effect on removing the key player not only in the short run but also in the long run, allowing other criminals to form new links with individuals they were not linked with when the key player was removed.

Using the AddHealth data of adolescents in the United States, we then test the different results of our theoretical analysis. To be consistent with our theoretical models, there are basically three parts in our empirical analysis: (*i*) we test our static model to investigate if there are peer effects in crime; (*ii*) we test our extended static model to examine what drives the decision to become criminal, (*iii*) we estimate and simulate our dynamic model to determine who is the key player.

(*i*) We first test whether or not there are peer effects in crime in the static model. While the potential benefits of leveraging social networks to reduce criminal behaviors are substantial so are the empirical difficulties of uncovering how social networks form, operate and the strength of network effects on outcomes. These difficulties are partly due to the lack of theoretical models that can help us understand the way these feedback effects operate. They are also due to the lack of network data, as well as to the fact that social networks are formed purposefully and connected individuals share environmental influences. These features of social networks complicate the estimation of causal impacts of networks and reduce the ability to suggest policies to reduce bad behaviors and encourage good behaviors. It is often difficult to disentangle whether the observation of two friends skipping school or smoking with other adolescents is due to both facing low punishment regimes, or because they influence each other to pursue risky behaviors, or because they choose to be friends based on their common interest in pursuing risky behaviors.

We tackle the econometric issues in the estimation of peer effects in crime by extending the recent method of Liu and Lee (2010). Using an instrumental variable approach as well as network fixed effects, we estimate the first-order conditions of our theoretical model to evaluate the intensity of peer effects and the role of centrality in crime. We also provide an over-identifying restriction (OIR) test to show that our instruments constructed using the social adjacency matrix are valid for the data we use. Hence the adjacency matrix of social interaction can be considered as exogenous. If we consider an average group of 4 best friends (linked to each other in a network), we find that a standard deviation increase in the level of delinquent activity of each of the peers translates into a roughly 17 percent increase of a standard deviation in the individual level of activity. This is a strong effect, especially given our long list of controls. We also test this model by type of crime. We find that the impact of peer effects on crime are much higher (almost double) for more serious crimes than for petty crimes.

(*ii*) We then estimate the determination to become criminal. To address the endogenous participation (or selection) problem, from an econometric viewpoint, we consider a type-2 Tobit model. As in the theory, in the econometrics model, we have two equations. The first equation (*the participation equation*) determines whether an agent will become or not a criminal. For those individuals who decide to be criminals, the second equation (*the outcome equation*) determines for an agent the effort level she decides to exert as a function of her own characteristics, the characteristics and efforts of her direct friends. Interestingly, we find that the individuals' characteristics that affect the crime decision vary with the type of crime committed. For example, we find that female teenagers are more likely to commit petty crimes and much less likely to commit serious crimes than male teenagers. Similarly, blacks are more likely to commit serious crime than whites while there are no statistically differences between blacks and whites for petty crimes.

(*iii*) Finally, we test the key player policy using our dynamic network formation model. For that, using the dynamic model, we *structurally* estimate all the parameters of the model to determine the key player. We find that it is *not* straightforward to determine which delinquent should be removed from a network by only observing his or her criminal activities or position in the network. In other words, the key player is often not the criminal who has the highest Katz-Bonacich centrality or the highest betweenness centrality. Once we have determined the key player for each network, we can analyze his/her main characteristics. Compared to other criminals, "key" criminals are less likely to be a female, are less religious, belong to families whose parents are less educated and have the perception of being socially more excluded. They also feel that their parents care less about them, are less likely to come from families where both parents are married and have more troubles getting along with the teachers. An interesting feature is that key players are more intelligent (i.e. higher mathematics scores) than the average criminal and are more likely to have friends who are older (i.e. in higher grades), more religious and whose are more educated. Also, even though key players do not have a better self-esteem of themselves or are not more physically developed or are not more urbanites than other criminals, their friends do.

The rest of the paper unfolds as follows. In the next section, we discuss the related literature and explain our contribution. The theory section (Section 3) is divided into two subsections: the static models (Section 3.1) and the dynamic ones (Section 3.2). All proofs of the theoretical part can be found in Appendix 1 while examples illustrating the different theoretical results are exposed in Appendix 2. Our data are described in Section 4 while the estimation and empirical results of the impact of peer effects on crime are provided in Section 5. Section 6 details the empirical analysis of the key player and gives the results. In

Section 7, we discuss some policy implications of our results. Finally, Section 8 concludes.

2 Related literature

Our paper lies at the intersection of different literatures. We would like to expose them in order to highlight our contribution.

Theories of crime with social interactions There is a growing theoretical literature on the social aspects of crime. Sah (1991) was one of the first who develops a social interaction crime model where the social setting affects the individual perception of the costs of crime, and is thus conducive to a higher or a lower sense of impunity. Glaeser et al. (1996) propose a model where criminals are located on a circle where some of them are conformists (i.e. copy what their neighbors do) while others decide by themselves their criminal activities. They show that criminal interconnections act as a social multiplier on aggregate crime. Calvó-Armengol and Zenou (2004), Ballester et al. (2006, 2010), Patacchini and Zenou (2008, 2012) were the first to embed criminal activities in a general social network. They study the effect of the structure of the network on crime. They show that the location in the social network of each criminal not only affects her direct friends but also friends of friends of friends, etc.[5] Ballester et al. (2006, 2010) also study the policy implications of the network models of crime. They show that a key-player policy, which consists in removing from the network the criminal who reduces the most total crime, can be more efficient than standard punishment policies in reducing crime. Compared to this literature, we have the following contributions. In all these models of crime with social interactions, the network is fixed and taken as given. We relax this assumption by considering a dynamic network formation model where criminals not only decide how much effort they put into crime but with whom they want to form links with. Furthermore, the key-player policy proposed by Ballester et al. (2006) assumes that the network is invariant, i.e. when a criminal is removed from a network nobody can form new links. This limits the policy implications of the model since, in that case, it is just a short-run policy and, eventually, criminals can adapt their behavior in order to avoid to be targeted by this policy. We relax this assumption by allowing criminals to form friendship relations (i.e. new links) after the removal of the key player.

[5] Other related contributions on the social aspects of crime include Silverman (2004), Verdier and Zenou (2004), Calvó-Armengol et al. (2007), Ferrer (2010).

Theories of network formation There is an important literature on dynamic network formation.[6] The first approach is to consider a random network formation (looking at stochastically stable networks) and to study how emerging networks match real-world networks (Ehrhardt et al., 2006; 2008; Vega-Redondo, 2006; Hofbauer and Sandholm, 2007; Feri, 2007; Staudigl, 2011). While sharing some common features with this literature, our model is quite different since *agents do not create links randomly but in a strategic way, i.e. they maximize their utility function.* From the economic literature, there are also dynamic network formation models with strategic interactions. Bala and Goyal (2000), Watts (2001), Jackson and Watts (2002a), Dutta et al. (2005) are prominent papers of this literature. Our dynamic network formation model is different than the ones developed in these papers in the sense that we consider both dynamic models of network formation and optimal actions from agents. This allows us to give a microfoundation of the network formation process as equilibrium actions transform into equilibrium utility functions.

There are also some papers that, as in our framework, combine both network formation and endogenous actions. These papers include Bramoullé et al. (2004), Cabrales et al. (2011), Calvó-Armengol and Zenou (2004), Galeotti and Goyal (2010), Goyal and Vega-Redondo (2005), Goyal and Joshi (2003), Jackson and Watts (2002b). Most of these models are, however, static and the network formation process is different. König et al. (2011) is the closest to ours since it is both a dynamic network formation model and players optimally chooses effort. They impose, however, that individuals have to delete one of their links with some probability, which leads to steady-state networks that have very specific properties (like e.g. a diameter of two). In our model, individuals do not delete links but bear a cost of forming links.

Empirical aspects of criminal behavior with social interactions There is a also growing empirical literature suggesting that peer effects are very strong in criminal decisions. Case and Katz (1991), using data from the 1989 NBER survey of youths living in low-income Boston neighborhoods, find that the behaviors of neighborhood peers appear to substantially affect criminal activities of youth behaviors. They find that the direct effect of moving a youth with given family and personal characteristics to a neighborhood where 10 percent more of the youths are involved in crime than in his or her initial neighborhood is to raise the probability the youth will become involved in crime by 2.3 percent. Ludwig et al. (2001) and Kling et al. (2005) explore this last result by using data from the Moving to Opportunity (MTO) experiment that relocates families from high- to low-poverty neighborhoods. They

[6] See Goyal (2007) and Jackson (2008) for overviews on network theory.

find that this policy reduces juvenile arrests for violent offences by 30 to 50 percent for the control group. This also suggests very strong social interactions in crime behaviors. Patacchini and Zenou (2008, 2012) find that peer effects in crime are strong, especially for petty crimes. Damm and Dustmann (2008) investigate the following question: Does growing up in a neighborhood in which a relatively high share of youth has committed crime increase the individual's probability of committing crime later on? To answer this question, Damm and Dustmann exploit a Danish natural experiment that randomly allocates parents of young children to neighborhoods with different shares of youth criminals. With area fixed effects, their key results are that one standard deviation increase in the share of youth criminals in the municipality of initial assignment increases the probability of being charge with an offense at the age 18-21 by 8 percentages point (or 23 percent) for men. This neighborhood crime effect is mainly driven by property crime. Finally, Bayer et al. (2009) consider the influence that juvenile offenders serving time in the same correctional facility have on each other's subsequent criminal behavior. They also find strong evidence of learning effects in criminal activities since exposure to peers with a history of committing a particular crime increases the probability that an individual who has already committed the same type of crime recidivates with that crime.[7] Compared to this literature, we use the network structure as a source of identification of peer effects in crime. In particular, we improve the identification strategy of peer effects proposed by Bramoullé et al. (2009) and Lee et al. (2010) by addressing the case of a non-row-normalized matrix of social interactions. More importantly, we structurally estimate a dynamic model of network formation with and without the removal of a criminal, which allows us to determine who the key player is in each network of adolescent friendship. To the best of our knowledge, this is the first paper that empirically tests the importance of key players in criminal activities (or any other activity).

This paper's contributions To sum-up, we have the following main contributions:

(*i*) We provide an explicit crime model where ex ante heterogeneous individuals decide whether to become criminal or not and, if criminal, how much effort to put in criminal activities;

(*ii*) We provide a dynamic network formation model without and with a key-player policy.

(*iii*) We improve the identification strategy of peer effects proposed by Bramoullé et al. (2009) and Lee et al. (2010) by addressing the case of a non-row-normalized matrix of social interactions;

[7] Building on the binary choice model of Brock and Durlauf (2001), Sirakaya (2006) identifies social interactions as the primary source of recidivist behavior in the United States.

(iv) We provide estimates of criminal outcomes that separate peer effects from contextual and correlated effects;

(v) We structurally estimate our dynamic network formation model to determine the key player in each network of adolescent friendships;

(vi) We identify the characteristics of the key players, study the significant differences between key players and criminals and see if other measures of centrality can explain why some key players are not the most active criminals in a network.

3 Theoretical frameworks

3.1 Static models with exogenous networks

We expose a network model of crime where the network is taken as given. We will analyze the impact of network structure and peer effects on criminal outcomes.

3.1.1 Model and Nash equilibrium

We develop a network model of peer effects, where the network reflects the collection of active bilateral influences.

The network $N = \{1, \ldots, n\}$ is a finite set of agents in a connected network $g \equiv g_N$. We keep track of social connections in a delinquency network g through its adjacency matrix $\boldsymbol{G} = [g_{ij}]$,[8] where $g_{ij} = 1$ if i nominates j ($j \neq i$) as i's friend and $g_{ij} = 0$, otherwise. We set $g_{ii} = 0$. For the ease of the presentation, we focus on directed networks so that $\boldsymbol{G}$ can be asymmetric.[9]

Preferences Delinquents in network g decide how much effort to exert. We denote by y_i the delinquency effort level of delinquent i in network g and by $\boldsymbol{y} = (y_1, ..., y_n)'$ the population delinquency profile in network g. Each agent i selects an effort $y_i \geq 0$, and obtains a payoff $u_i(\boldsymbol{y}, g)$ that depends on the effort profile $\boldsymbol{y}$ and on the underlying network

[8] Matrices and vectors are in bold while scalars are in normal letters.

[9] All our theoretical results also hold under undirected network since the (a)symmetry of the adjacency matrix $\boldsymbol{G}$ does not play any role in the proof of our theoretical results.

g, in the following way:

$$u_i(\boldsymbol{y}, g) = \underbrace{(a_i + \eta + \epsilon_i)\, y_i}_{\text{Proceeds}} - \underbrace{\frac{1}{2} y_i^2}_{\text{moral cost of crime}} - \underbrace{p \cdot f \cdot y_i}_{\text{cost of being caught}} + \underbrace{\phi \sum_{j=1}^{n} g_{ij} y_i y_j}_{\text{positive peer effects}} \tag{1}$$

where $\phi > 0$. This utility has a standard cost/benefit structure (as in Becker, 1968). The proceeds from crime are given by $(a_i + \eta + \epsilon_i)\ y_i$ and are increasing in own effort y_i. The costs of committing crime are captured by the probability to be caught $0 < p < 1$ times the fine $f \cdot y_i$, which increases with own effort y_i, as the severity of the punishment increases with one's involvement in crime. Also, as it now quite standard (see e.g. Verdier and Zenou, 2004; Conley and Wang, 2006), individuals have a *moral* cost of committing crime equals to $\frac{1}{2}y_i^2$, which is also increasing in own crime effort y_i. Finally, the new element in this utility function is the last term $\phi \sum_{j=1}^{n} g_{ij} y_i y_j$, which reflects the influence of friends' behavior on own action. The peer effect component can also be heterogeneous, and this *endogenous heterogeneity* reflects the different locations of individuals in the friendship network g and the resulting effort levels.

Let us now comment in more detail this utility function. In (1), η denotes the the unobservable network characteristics, e.g., the prosperous level of the neighborhood/network g (i.e. more prosperous neighborhoods lead to higher proceeds from crime) and ϵ_i is an error term, which captures the unobservable characteristics of individual i and other uncertainty in the proceeds from crime. Both η and ϵ_i are observed by the delinquents (when choosing effort level)[10] but not by the econometrician. Also, in (1), a_i denotes the *exogenous heterogeneity* that captures the *observable* differences between individuals. In this model, a_i captures the fact that individuals differ in their ability (or productivity) of committing crime. Indeed, for a given effort level y_i, the higher a_i, the higher the productivity and thus the higher the proceeds from crime $a_i y_i$. Observe that a_i is assumed to be deterministic, perfectly *observable* by all individuals in the network and corresponds to the observable characteristics of individual i (e.g. sex, race, age, parental education, etc.) and to the observable *average* characteristics of individual i's best friends, i.e. average level of parental education of i's friends, etc. (*contextual effects*). To be more precise, a_i can be written as:

$$a_i = \sum_{m=1}^{M} \beta_{1m} x_i^m + \frac{1}{g_i} \sum_{m=1}^{M} \sum_{j=1}^{n} \beta_{2m} g_{ij} x_j^m \tag{2}$$

[10] We will assume that ϵ_i is not observable by the delinquents when making link-formation decisions in the dynamic network formation model described in Section 3.2.

where x_i^m is a set of M variables accounting for observable differences in individual, neighborhood and school characteristics of individual i, β_{1m}, β_{2m} are parameters, and $g_i = \sum_{j=1}^{n} g_{ij}$ is the connectivity of individual i.

To summarize, the utility function can be written as:

$$u_i(\boldsymbol{y}, g) = (a_i + \bar{\eta} + \epsilon_i) y_i - \frac{1}{2} y_i^2 + \phi \sum_{j=1}^{n} g_{ij} y_i y_j, \tag{3}$$

where $\bar{\eta} = \eta - pf$. So when a delinquent i exerts some effort in crime, the proceeds from crime depends on ability a_i, the expected marginal cost of being caught pf, how prosperous is the neighborhood/network η and on some random element ϵ_i, which is specific to individual i. In other words, a_i is the observable part (by the econometrician) of i's characteristics while ϵ_i captures the unobservable individual characteristics and other uncertainty in the proceeds from crime. Note that the utility (1) is concave in own decisions, and displays decreasing marginal returns in own effort levels.

The Katz-Bonacich network centrality For each network g with adjacency matrix $\boldsymbol{G} = [g_{ij}]$, the kth power of $\boldsymbol{G}$ given by $\boldsymbol{G}^k = \boldsymbol{G}^{(k \overset{times}{\dots})} \boldsymbol{G}$ keeps track of direct and indirect connections in g. More precisely, the (i, j)th cell of $\boldsymbol{G}^k$ gives the number of paths of length k in g between i and j. In particular, $\boldsymbol{G}^0 = \boldsymbol{I}$. Note that, by definition, a path between i and j needs not to follow the shortest possible route between those agents. For instance, if $g_{ij} = g_{ji} = 1$, the sequence $ij \to ji \to ij$ constitutes a path of length three in g between i and j.

Definition 1 (Katz, 1953; Bonacich, 1987) *Given a vector $\boldsymbol{u} \in \mathbb{R}_+^n$, and $\phi \geq 0$ a small enough scalar, the vector of Bonacich centralities of parameter ϕ in network g is defined as:*

$$\boldsymbol{b}_{\boldsymbol{u}}(g, \phi) = (\boldsymbol{I} - \phi \boldsymbol{G})^{-1} \boldsymbol{u} = \sum_{p=0}^{\infty} \phi^p \boldsymbol{G}^p \boldsymbol{u}. \tag{4}$$

Because we are focusing on directed networks, we follow the approach of Wasserman and Faust (1994, pages 205-210) who state that: "centrality indices for directional relations generally focus on choices made". In the language of graph theory, in a directed graph, a link has two distinct ends: a head (the end with an arrow) and a tail. Each end is counted separately. The sum of head endpoints count toward the *indegree* and the sum of tail endpoints count toward the *outdegree*. Formally, as stated above, we denote a link from i to j as $g_{ij} = 1$ if i has nominated j as her friend, and $g_{ij} = 0$, otherwise. The *indegree* of student i, denoted by $\vec{g}_i$, is the number of nominations student i receives from other students, that is $\vec{g}_i = \sum_j g_{ji}$. This is the *column-sum* of $\boldsymbol{G}$ corresponding to i. The *outdegree* of student i, denoted by $\bar{g}_i$, is the number of friends student i nominates, that is $\bar{g}_i = \sum_j g_{ij}$. This is the *row-sum* of $\boldsymbol{G}$ corresponding to i. In the following, we measure the Bonacich centrality in terms of *outdegrees*. This is consistent with our data where individuals nominate each other since if individual i nominates j but j does not, it is then very possible that j is a role model for i. In other words, i is learning from j even though j does not consider i as her best friend.

Nash equilibrium

We now characterize the Nash equilibrium of the game where agents choose their effort level $y_i \geq 0$ simultaneously. At equilibrium, each agent maximizes her utility (1) and we obtain the following best-reply function for each $i = 1, ..., n$:

$$y_i = \phi \sum_{j=1}^{n} g_{ij} y_j + a_i + \bar{\eta} + \epsilon_i, \tag{5}$$

where a_i is defined by (2). Denote by $\mu_1(\boldsymbol{G})$ the spectral radius of $\boldsymbol{G}$, by $\alpha_i = a_i + \bar{\eta} + \epsilon_i$, with corresponding non-negative vector $\boldsymbol{\alpha}$, we have:

Proposition 1 *If $\phi\mu_1(\boldsymbol{G}) < 1$, the peer effect game with payoffs (1) has a unique Nash equilibrium in pure strategies given by:*

$$\boldsymbol{y}^* \equiv \boldsymbol{y}^*(g) = \boldsymbol{b}_{\boldsymbol{\alpha}}(g, \phi). \tag{6}$$

This results shows that the Bonacich centrality is the right network index to account for equilibrium behavior when the utility functions are linear-quadratic. In (1), the local payoff interdependence is restricted to direct network contacts. At equilibrium, though, this local payoff interdependence spreads all over the network through the overlap of direct friendship clusters. The Bonacich centrality precisely reflects how individual decisions feed into each other along any direct and indirect network path. Furthermore, the condition $\phi\mu_1(\boldsymbol{G}) < 1$ stipulates that local complementarities must be small enough than own concavity, which prevents multiple equilibria to emerge and, in the same time, rules out corner solutions (i.e., negative or zero solutions).[11] This condition also guarantees that $(\boldsymbol{I} - \phi\boldsymbol{G})$ is invertible and its series expansion well defined. Observe that

$$\boldsymbol{b}_{\boldsymbol{\alpha}}(g, \phi) = (\boldsymbol{I} - \phi\boldsymbol{G})^{-1} \boldsymbol{\alpha} = \sum_{p=0}^{\infty} \phi^p \boldsymbol{G}^p \boldsymbol{\alpha} \tag{7}$$

[11] See Ballester et al. (2006) for a formal proof of this result.

where $\boldsymbol{\alpha} = \boldsymbol{a} + \bar{\eta}\boldsymbol{l}_n + \boldsymbol{\epsilon}$ and where $\boldsymbol{l}_n$ is an n-dimensional vector of ones. To simplify notation, we drop the subscript $\boldsymbol{\alpha}$ in $\boldsymbol{b}_{\boldsymbol{\alpha}}(g,\phi)$ and just write $\boldsymbol{b}(g,\phi)$ whenever there is no ambiguity. Let $b_i(g,\phi)$ be the ith entry of $\boldsymbol{b}(g,\phi)$. From Proposition 1, we have, for each individual i, $y_i^* = b_i(g,\phi)$. Observe that, from (5), it is easy to show that:

$$u_i(\boldsymbol{y}^*, g) = \alpha_i y_i^* - \frac{1}{2} y_i^{*2} + \phi \sum_{j=1}^{n} g_{ij} y_i^* y_j = \frac{1}{2} y_i^{*2} = \frac{1}{2} \left[b_i(g,\phi) \right]^2 . \tag{8}$$

3.1.2 Finding the key player in the static model

We would like now to expose the "key player" policy. The planner aims at finding the key player, i.e. the delinquent who once removed generates the highest possible reduction in aggregate delinquency level. We are assuming that, once a person is removed from the network, the links of the remaining players do not change and that a_i in (8) does not depend on the adjacency matrix. Formally, the planner's problem is the following:

$$\max\{Y^*(g) - Y^*(g^{[-i]}) \mid i = 1, ..., n\},$$

where $Y^*(g) = \sum_i y_i^*(g)$ is the total level of crime in network g and $g^{[-i]}$ is network g without individual i. When the original delinquency network g is fixed, this is equivalent to:

$$\min\{Y^*(g^{[-i]}) \mid i = 1, ..., n\}. \tag{9}$$

From Ballester et al. (2006, 2010) and Ballester and Zenou (2012),[12] we now define a new network centrality measure $\boldsymbol{d}(g,\phi)$ that will happen to solve this program. Let $\boldsymbol{M}(g,\phi) = (\boldsymbol{I} - \phi\boldsymbol{G})^{-1}$. Its entries $m_{ij}(g,\phi)$ count the number of walks in g starting from i and ending at j, where walks of length k are weighted by ϕ^k. The Bonacich centrality of node i is $b_i(g,\phi) = \sum_{j=1}^{n} \alpha_j m_{ij}(g,\phi)$, and counts the *total* number of paths in g starting from i weighted by the α_j of each linked node j. Let $b_i(g,\phi)$ be the centrality of i in network g, $B(g,\phi)$ the *total* centrality in network g (i.e. $B(g,\phi) = \boldsymbol{l}_n' \boldsymbol{M} \boldsymbol{\alpha}$) and $B(g^{[-i]},\phi) = \boldsymbol{l}_n' \boldsymbol{M}^{[-i]} \boldsymbol{\alpha}^{[-i]}$ the *total* centrality in $g^{[-i]}$, where $\boldsymbol{\alpha}^{[-i]}$ is a $(n-1)\times 1$ column vector in which α_i has been removed and $\boldsymbol{M}^{[-i]} = (\boldsymbol{I} - \phi\boldsymbol{G}^{[-i]})^{-1}$ is a $(n-1)\times(n-1)$ matrix in which the ith row and ith column corresponding to i has been removed from $\boldsymbol{M}$. Finally, let $\boldsymbol{\alpha}^{[i]}$ be a $(n\times 1)$ column vector where all entries but i are defined as $\boldsymbol{\alpha}^{[-i]}$ while entry i contains the initial α_i and let $\boldsymbol{M}^{[i]}$ be the $n \times n$ matrix such that each element is

$$m_{jk}^{[i]} = \frac{m_{ji} m_{ik}}{m_{ii}}$$

[12] Ballester and Zenou (2012) extend the definition of the key player proposed by Ballester et al. (2006, 2010) to account for contextual effects.

so that $B(g^{[i]},\phi) = \boldsymbol{l}_n'\boldsymbol{M}\boldsymbol{\alpha}^{[i]}$ and $\boldsymbol{l}_n'\boldsymbol{M}^{[i]}\boldsymbol{\alpha}^{[i]} = b_{\boldsymbol{\alpha}^{[i]},i}(g,\phi)\sum_{j=1}^n m_{ji}(g,\phi)/m_{ii}(g,\phi)$. Ballester and Zenou (2012) have proposed the following definition:

Definition 2 *For all networks g and for all i, the intercentrality measure of delinquent i is:*

$$\begin{aligned} d_i(g,\phi) &= B(g,\phi) - B(g^{[-i]},\phi) \\ &= \boldsymbol{l}_n'\boldsymbol{M}\boldsymbol{\alpha} - \boldsymbol{l}_n'\boldsymbol{M}\boldsymbol{\alpha}^{[i]} + \boldsymbol{l}_n'\boldsymbol{M}^{[i]}\boldsymbol{\alpha}^{[i]} \\ &= B(g,\phi) - B(g^{[i]},\phi) + \frac{b_{\boldsymbol{\alpha}^{[i]},i}(g,\phi)\sum_{j=1}^n m_{ji}(g,\phi)}{m_{ii}(g,\phi)}. \end{aligned} \tag{10}$$

The intercentraity measure (10) highlights the fact that when a delinquent is removed from a network, two effects are at work. The first effect is the *contextual effect*, which is due to the change in the contextual effect $\boldsymbol{\alpha}$ (from $\boldsymbol{\alpha}$ to $\boldsymbol{\alpha}^{[i]}$) after the removal of the key player while the network g remains unchanged. The second effect is the *network effect*, which captures the change in the network structure when the key player is removed. More generally, the intercentrality measure $d_i(g,\phi)$ of delinquent i accounts both for one's exposure to the rest of the group and for one's contribution to every other exposure.

The following result establishes that intercentrality captures, in an meaningful way, the two dimensions of the removal of a delinquent from a network, namely, the direct effect on delinquency and the indirect effect on others' delinquency involvement.[13]

Proposition 2 *A player i^* is the key player that solves (9) if and only if i^* is a delinquent with the highest intercentrality in g, that is, $d_{i^*}(g,\phi) \geq d_i(g,\phi)$, for all $i = 1,...,n$.*

Observe that this result is true for both *undirected* networks (*symmetric* adjacency matrix) and *directed* networks (*asymmetric* adjacency matrix). It is also true for adjacency matrices with weights (i.e. values different than 0 and 1) and self-loops (delinquents have a link with themselves). An illustrative example for Proposition 2 can be found in Appendix 2 (Example 1).

3.1.3 Is the key player always the more active criminal?

Definition 2 specifies a clear relationship between the Bonacich centrality and the intercentrality measures. Holding $b_i(g,\phi)$ fixed, the intercentrality $d_i(g,\phi)$ of player i decreases

[13] As in Ballester et al. (2010), we could also identify a *key group* that reduces the most aggregate delinquency in each network by characterizing the *optimal group removal* from the network. Because in the empirical analysis our networks have relatively small sizes (see Section 4), the key group policy is less relevant and, therefore, we will mainly focus on the key player policy.

with $m_{ii}(g, \phi)$ of i's Bonacich centrality due to self-loops, and increases with the fraction of i's centrality amenable to out-walks. As a result, it should be clear from Definition 2 that the key player is very likely to be the criminal with the highest Bonacich centrality (i.e. the most active criminal in the network) but not necessary. In Example 1 provided in Appendix 2, the key player was criminal 1 and was also the most active criminal, i.e. the criminal with the highest Bonacich centrality. In Appendix 2, we provide another example (Example 2) where, even if the αs are identical for all individuals, there can be key players (highest inter-centrality measures) who are not the most active criminals (highest Katz-Bonacich centrality measures).

To summarize, the individual Nash equilibrium efforts of the delinquency-network game are proportional to the *equilibrium Bonacich centrality* network measures, while the key player is the delinquent with the highest *intercentrality measure*. As the previous example illustrates, these two measures need not to coincide.

3.1.4 Selection issues: The decision to become criminal

So far, we have assumed that the delinquency network was given. In some cases, though, delinquents may have opportunities outside the delinquency network. Here, we expand the model and endogenize the delinquency decision where individuals take a binary decision on whether being delinquent or not.

Model and equilibrium Formally, we consider the following two-stage game. Fix an initial network g connecting agents. In the first stage, each agent $i \in \bar{N} = \{1, ..., \bar{n}\}$ decides whether to become a delinquent or not. This is a simple binary decision. These decisions are simultaneous. Let $\psi_i \in \{0, 1\}$ denote i's decision, where $\psi_i = 1$ (resp. $\psi_i = 0$) stands for becoming a delinquent (resp. not becoming a delinquent), and denote by $\boldsymbol{\psi} = (\psi_1, ..., \psi_n)$ the corresponding population binary decision profile. We assume that agents not committing crime obtains some fixed utility level (a nonnegative scalar) $\bar{u} > 0$. The idea, here, is that all students who are not committing crime enjoy the same utility level $\bar{u}$. The payoff for delinquents is determined in the second stage of the game. As before, denote by $N = \{1, ..., n\}$, with $n < \bar{n}$, the set of players who decided to participate (i.e. to become criminals). In the second stage, delinquents decide their effort level, which depends on the first-stage outcome.

Definition 3 *The extended game is a two stage game where:*

- *In stage 1, each player $i = 1, ..., \bar{n}$ decides whether to participate ($\psi_i = 1$) or not ($\psi_i = 0$) to the crime market.*
- *In stage 2, the n persons who decided to be criminal in the first stage play the game in g_N.*
- *The final utilities are:*

$$u_i(\boldsymbol{\psi}, \boldsymbol{y}, g_N) = \begin{cases} (a_i + \bar{\eta} + \epsilon_i)y_i - \frac{1}{2}y_i^2 + \phi \sum_{j=1}^{n} g_{ij} y_i y_j & \text{if } \psi_i = 1 \\ \bar{u} & \text{otherwise} \end{cases}.$$

We study the *subgame perfect equilibrium in pure strategies* of this extended game.

Definition 4 *The set N is supported in equilibrium if there exists a $\bar{u}$ and a subgame perfect equilibrium where the set of players who decide to participate is N, given the utility $\bar{u}$ of not being a criminal. N is also called an (equilibrium) participation pool of the game with $\bar{u}$.*

The following result characterizes the class of sets that can be supported by some $\bar{u}$.

Proposition 3 *Let $N \subseteq \bar{N}$ and $\phi \mu_1(\boldsymbol{G}) < 1$. Then, the set N is supported in equilibrium by the outside option $\bar{u}$ if and only if:*

$$\max_{j \in \bar{N} \backslash N} b_j(g_{N \cup \{j\}}, \phi) \leq 2\sqrt{\bar{u}} \leq \min_{i \in N} b_i(g_N, \phi). \tag{11}$$

Given that the utility $\bar{u}$ is fixed, it is clear that this two-stage game is *supermodular,* in the sense that the payoffs of player i are increasing with respect to participation decisions of other agents. Formally, for all $N \subseteq T \subseteq \bar{N}$ and $i \in \bar{N} \backslash T$, it is clear that:

$$b_i(g_{N \cup \{i\}}, \phi) \leq b_i(g_{T \cup \{i\}}, \phi)$$

because the right-hand side measures a higher number of walks. This property ensures the existence of equilibrium for any utility $\bar{u}$. This is clearly due the local complementarity in crime activities.

Proposition 3 means that some individuals will not commit crime while others will. Indeed, each individual i has the option of either not participating in delinquent activities and getting a fixed utility $\bar{u} > 0$ or participating and obtaining an utility equals to $\frac{1}{2}\left[b_i(g_N, \phi)\right]^2$, which depends on her position in the delinquent network g_N. So the participation decision depends on the comparison of these two options (i.e. utilities). Observe that $\bar{u}$ is exogenous and that the Bonacich centrality $b_i(g_N, \phi)$ is defined by the vector $\boldsymbol{b}(g, \phi) = (\boldsymbol{I} - \phi \boldsymbol{G})^{-1} \boldsymbol{\alpha}$, where $\alpha_i = a_i + \epsilon_i + \bar{\eta}$ (see (7)), which is increasing in both a_i (individuals' observable characteristics such as gender, race, parents' education, etc.) and ϵ_i (individuals' unobservable characteristics such as genes, moral ethics, etc.). Since $a_i + \epsilon_i$ is a realization of continuous random variable, we can rank individuals according to their characteristics $a_1 + \epsilon_1 < \cdots < a_{\bar{n}-n} + \epsilon_{\bar{n}-n} < a_{\bar{n}-n+1} + \epsilon_{\bar{n}-n+1} < \cdots < a_{\bar{n}} + \epsilon_{\bar{n}}$. As a result, we can write condition (11) as follows:

$$b_{\bar{n}-n}(g_{N \cup \{\bar{n}-n\}}, \phi) \leq 2\sqrt{\bar{u}} \leq b_{\bar{n}-n+1}(g_N, \phi). \tag{12}$$

Instead of Bonacich centralities, Proposition 3 can be written in terms of exogenous characteristics as follows:

Proposition 4 *Rank individuals such that* $a_1 + \epsilon_1 < \cdots < a_{\bar{n}-n} + \epsilon_{\bar{n}-n} < a_{\bar{n}-n+1} + \epsilon_{\bar{n}-n+1} < \cdots < a_{\bar{n}} + \epsilon_{\bar{n}}$. *Assume that* $a_{\bar{n}-n} + \epsilon_{\bar{n}-n}$ *is sufficiently low compared to* $a_{\bar{n}-n+1} + \epsilon_{\bar{n}-n+1}$. *Then all individuals with characteristics belonging to* $\{a_1 + \epsilon_1, ..., a_{\bar{n}-n} + \epsilon_{\bar{n}-n}\}$ *will find it optimal not to become delinquents while all individuals with characteristics belonging to* $\{a_{\bar{n}-n+1} + \epsilon_{\bar{n}-n+1}, ..., a_{\bar{n}} + \epsilon_{\bar{n}}\}$ *will become criminal.*

If, for example, a_i is an *inverse* measure of parental education and ϵ_i an *inverse* measure of how strict the parents are in terms of moral ethics, then this proposition means that those with high-educated parents and/or strong moral ethics are less likely to commit crime. In other words, for these students, whatever their friends do, the cost of committing crime is too high and will therefore never commit crime. For others, who, for example, do not have a strong moral ethics against crime, peers do matter. We assume that this decision whether to become criminal or not is made once and for all and is irreversible. If we assume that the potential crime benefit is a relatively small component of the crime decision (in our data, we are dealing with teenager delinquents committing mostly petty crimes; see Section 4), then Proposition 4 means that the decision to become a criminal for each individual i is mainly based her own characteristics $a_i + \epsilon_i$. As a result, when a key-player policy is implemented, it will only affect active criminals but not the ones who have already decided not to become criminals. In other words, in our model, individuals decide whether or not to be criminal mainly based on their observable and unobservable characteristics but then, if they become criminals, peer effects matter a lot because they belong to criminal networks and are therefore mainly friends with other criminals. In Appendix 2, we provide an example (Example 3) that illustrates Proposition 4.

3.2 Dynamic network formation models

So far, we have assumed that the network was fixed and that, when the key player was removed, no new links were formed. This means that, when the key player i is removed from network g, the remaining network becomes $g^{[-i]}$ where the ith row and ith column in $\boldsymbol{G}$ has been removed. In other words, we had an *invariant assumption* on the reduced network $g^{[-i]}$. This assumptions is realistic in the short run but clearly not in the long run.

We would like now to develop a *dynamic model* where both network formation and effort decisions take place. Remember that we want to model students in grades 7-12 (see Section 4) who have to decide whether to commit crime or not. Following Proposition 4 and the discussion after this proposition, we assume here that individuals make the decision to become criminal or not once and for all before the dynamic network formation game takes place. In other words, as stated in Proposition 4, we assume that, for some individuals, their observable characteristics a_i (gender, race, parents' education, etc.) and unobservable characteristics ϵ_i (how strict is their parents' education, their own ethics, etc.) are such that they will never commit a crime, whatever their friends do. Similarly, other individuals have a a_i and a ϵ_i, which are high enough, so that they will always commit a crime, whatever their friends do. However, for the latter, their crime effort (i.e. how often they will commit a crime) will depend on peer effects as modeled by the utility function (1). The decision to become criminal or not is taken before the dynamic network formation game starts and thus we focus on the way crime and link formation evolves over time for a network of only criminals. As we will see below, in our dataset (see Section 4), we only observe the network at one point in time (1994/1995) where crime decisions have already taken place. This will correspond to our network at $t = 0$. Then, we will analyze how the network will evolve over time both with and without a key player policy. So it seems reasonable to assume that the crime decision has been taken by each individual before $t = 0$ and that this decision will not change afterwards. What will change after $t = 0$ is the structure of the network as well as the crime effort each criminal will provide. This will allow us to have a tractable framework with an already complicated model where both link formation and crime effort are taken into account. Let us now describe this dynamic model.

3.2.1 The model

At each period of time t, a person is chosen at random among the n criminals in the network g_t, and has to decide whether or not to create a link and, in case of link formation, with whom she wants to create this link. Then there is a random shock and all individuals decide how much crime effort to provide. We analyze this dynamic network game and characterizes the equilibrium network. Let us now describe in more detail what happens within each period. We call this game the *morning-afternoon game*.

At each period of time t, the timing of the game is as follows.

In the *morning* of day t, with *equal probability*, an agent (say, agent i) is chosen and makes a link-formation decision under uncertainty as she does not know the realization of a random shock ϵ_i. The shock is individual specific and *i.i.d* across individuals such that such that $\mathrm{E}[\epsilon_i] = 0$ and $\mathrm{Var}[\epsilon_i] = \sigma_\epsilon^2 > 0$. We assume that the agent i who initiates a link formation with agent j pays the cost c_i and that she is *myopic*, i.e. agent i only maximize expected utility at the end of day t. Note that the chosen agent i is only allowed to create a new link with an agent (say, agent j) if there does not exist a link from i to j at the beginning of day t. Agent i is not allowed to delete an existing link. She is also not allowed to create a link pointing from j to i. This means that we focus on *directed networks* so that, in terms of the adjacency matrix $\boldsymbol{G} = [g_{ij}]$, agent i is only allowed to change a non-diagonal element of the ith row of $\boldsymbol{G}$ to one if that element was zero at the beginning of day t.[14] Agent i can also choose to keep the adjacency matrix unchanged.[15]

At *noon* of day t, a new network is formed, which is denoted by $g_t = g_t^{(i,j)}$ with the adjacency matrix $\boldsymbol{G}_t = \boldsymbol{G}_t^{(i,j)}$. If $j \neq i$, then $\boldsymbol{G}_t^{(i,j)} = \boldsymbol{G}_{t-1} + e_i e_j'$, where e_i is the ith column of the identity matrix. If $j = i$, then $\boldsymbol{G}_t^{(i,j)} = \boldsymbol{G}_{t-1}$.

In the *afternoon* of day t, the random shock ϵ_i is realized and its value becomes complete information for all agents. As in Section 3.1.1, all agents in the (new) network simultaneously choose their effort level to maximize their utility at time t. In particular, agent i (the agent who is randomly chosen in the morning of day t to make a link-formation decision) will choose an effort level $y_{i,t} \equiv y_{i,t}(g_t^{(i,j)})$ to maximize the utility

$$u_{i,t}(\boldsymbol{y}_t, g_t^{(i,j)}) = (a_i + \bar{\eta} + \epsilon_i) y_{i,t} - \frac{1}{2} y_{i,t}^2 + \phi \sum_{k=1}^{n} g_{ik,t} y_{i,t} y_{k,t} - c_i (\bar{g}_{i,t} - \bar{g}_{i,t-1}) \tag{13}$$

where $\bar{g}_{i,t} = \displaystyle\sum_{k=1}^{n} g_{ik,t}$ and c_i is the marginal cost of a link. Observe that utility (13) is exactly as (3), apart from the fact that we add the link formation cost.

[14] Remember that the *outdegree* of individual i, denoted by $\bar{g}_i$, is the number of friends individual i nominates, that is $\bar{g}_i = \sum_j g_{ij}$. This is the *row-sum* of $\boldsymbol{G}$ corresponding to i. As stated in Section 3.1.1, we only consider outdegrees because it is an indication of role models.

[15] All our results are robust to *undirected networks* since a link proposal from i to j will always be accepted by j because the latter does not pay the cost of the link and there are strategic complementarities. We find it more convenient to present the model and results for the directed network case.

For each period of time t, we solve, as usual, the model backwards. As in section 3.1.1, the unique Nash equilibrium of the "afternoon game" (assuming that $\phi\mu_1(\boldsymbol{G}_t(i,j)) < 1$; see Proposition 1) is such that

$$y_{i,t}^* = \phi \sum_{k=1}^{n} g_{ik,t} y_{j,t}^* + a_i + \bar{\eta} + \epsilon_i \tag{14}$$

or in vector form $\boldsymbol{y}_t^* = (\boldsymbol{I} - \phi \boldsymbol{G}_t)^{-1} (\boldsymbol{a} + \bar{\eta} \boldsymbol{l}_n + \boldsymbol{\epsilon})$. Let $\boldsymbol{M}_t \equiv \boldsymbol{M}_t^{(i,j)} = (\boldsymbol{I} - \phi \boldsymbol{G}_t^{(i,j)})^{-1}$, with $\boldsymbol{m}_{i,t} \equiv \boldsymbol{m}_{i,t}^{(i,j)}$ being $\boldsymbol{M}_t$'s ith row and $m_{ij,t} \equiv m_{ij,t}^{(i,j)}$ being its (i,j)th entry. Then $y_{i,t}^* = \boldsymbol{m}_{i,t}(\boldsymbol{a} + \bar{\eta} \boldsymbol{l}_n + \boldsymbol{\epsilon})$.

For the "morning game", as $\boldsymbol{\epsilon}$ is unobservable in the morning, the chosen agent i makes her link formation decision by maximizing her expected utility

$$\begin{aligned} \mathrm{E}[u_{i,t}(\boldsymbol{y}_t^*, g_t^{(i,j)})] &= (a_i + \bar{\eta}) \,\mathrm{E}(y_{i,t}^*) + \mathrm{E}(\epsilon_i y_{i,t}^*) - \frac{1}{2} \mathrm{E}[(y_{i,t}^*)^2] \\ &\quad + \phi \sum_{k=1}^{n} g_{ik,t} \mathrm{E}(y_{i,t}^* y_{k,t}^*) - c_i(\bar{g}_{i,t} - \bar{g}_{i,t-1}), \end{aligned} \tag{15}$$

where $\mathrm{E}(y_{i,t}^*) = \boldsymbol{m}_{i,t}(\boldsymbol{a} + \bar{\eta} \boldsymbol{l}_n)$, $\mathrm{E}(\epsilon_i y_{i,t}^*) = \sigma_\epsilon^2 m_{ii,t}$, $\mathrm{E}[(y_{i,t}^*)^2] = [\mathrm{E}(y_{i,t}^*)]^2 + \sigma_\epsilon^2 \boldsymbol{m}_{i,t} \boldsymbol{m}_{i,t}'$ and $\mathrm{E}(y_{i,t}^* y_{k,t}^*) = \mathrm{E}(y_{i,t}^*) \mathrm{E}(y_{k,t}^*) + \sigma_\epsilon^2 \boldsymbol{m}_{i,t} \boldsymbol{m}_{k,t}'$. It is easily seen, as in Section 3.1.1 (see (8)), that

$$\begin{aligned} \mathrm{E}[u_{i,t}(\boldsymbol{y}_t^*, g_t^{(i,j)})] &= \frac{1}{2} [\mathrm{E}(y_{i,t}^*)]^2 + \sigma_\epsilon^2 m_{ii,t} - \frac{1}{2} \sigma_\epsilon^2 \boldsymbol{m}_{i,t} \boldsymbol{m}_{i,t}' \\ &\quad + \phi \sigma_\epsilon^2 \sum_{k=1}^{n} g_{ik,t} \boldsymbol{m}_{i,t} \boldsymbol{m}_{k,t}' - c_i(\bar{g}_{i,t} - \bar{g}_{i,t-1}). \end{aligned} \tag{16}$$

If chosen, an agent i will *not* create a link with any agent j if and only if

$$\mathrm{E}[u_{i,t}(\boldsymbol{y}_t^*, g_t^{(i,i)})] \geq \max_{j \neq i} \mathrm{E}[u_{i,t}(\boldsymbol{y}_t^*, g_t^{(i,j)})], \tag{17}$$

that is, i's expected utility of not creating a link is higher than that of creating a link with any agent j. Let

$$\kappa_{i,t}(g_t^{(i,j)}) = \frac{1}{2} \{\mathrm{E}[y_{i,t}^*(g_t^{(i,j)})]\}^2 + \sigma_\epsilon^2 m_{ii,t}^{(i,j)} - \frac{1}{2} \sigma_\epsilon^2 \boldsymbol{m}_{i,t}^{(i,j)} \boldsymbol{m}_{i,t}^{(i,j)\prime} + \phi \sigma_\epsilon^2 \sum_{k=1}^{n} g_{ik,t}^{(i,j)} \boldsymbol{m}_{i,t}^{(i,j)} \boldsymbol{m}_{k,t}^{(i,j)\prime}. \tag{18}$$

Using (16), the inequality (17) gives the lower bound of c_i:

$$\underline{c}_{i,t} = \max_j \kappa_{i,t}(g_t^{(i,j)}) - \kappa_{i,t}(g_t^{(i,i)}), \tag{19}$$

such that if $c_i \geq \underline{c}_{i,t}$ then agent i will have no incentive to create a new link at period t.

3.2.2 Convergence of the link formation process

Let us now determine the convergence and the equilibrium of this dynamic formation model. Let time be measured at countable dates $t = 0, 1, 2, ...$ and consider a *discrete time Markov chain* for the network formation process $(g_t)_{t=0}^{\infty}$ with $g_t = (N, L_t)$ comprising the set of delinquents $N = \{1, ..., n\}$ together with a set of links L_t at time t between them. $(g_t)_{t=0}^{\infty}$ is a collection of random variables g_t, indexed by time t on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where Ω is the countable state space of all networks with n nodes, $\mathcal{F}$ is the σ-algebra $\sigma(\{g_t : t = 0, 1, 2, ...\})$ generated by the collection of g_t, and $\mathbb{P} : \mathcal{F} \to [0, 1]$ is a countably additive, non-negative measure on $(\Omega, \mathcal{F})$ with total mass $\sum_{g \in \Omega} \mathbb{P}(g) = 1$. At every time $t \geq 0$, links can be created or not according to the game described above.

Definition 5 *Consider a discrete time Markov chain $(g_t)_{t=0}^{\infty}$ on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Consider a network $g_t = (N, L_t)$ at time t with delinquents $N = \{1, ..., n\}$ and links L_t. Let $g_t^{(i,j)}$ be the network obtained from g_{t-1} by the addition of the edge $ij \notin L_{t-1}$ between agents $i, j \in N$. Let $\boldsymbol{u}_t(\boldsymbol{y}_t^*, g_t) = (u_{1,t}(\boldsymbol{y}_t^*, g_t), ..., u_{n,t}(\boldsymbol{y}_t^*, g_t))$ denote the profile of Nash equilibrium payoffs of the delinquents in g_t following from the payoff function (13) with parameter $\phi < 1/\mu_1(\boldsymbol{G}_t)$. Then the delinquent j is a best response of delinquent i if $u_{i,t}(\boldsymbol{y}_t^*, g_t^{(i,j)}) \geq u_{i,t}(\boldsymbol{y}_t^*, g_t^{(i,k)})$ for all $k \in N \backslash \mathcal{N}_{i,t-1}$, where $\mathcal{N}_{i,t} = \{j \in N : ij \in L_t\}$ is the neighborhood of individual $i \in N$ at t. The set of delinquent i's best responses is denoted by $BR_{i,t}$.*

The key question is how individuals choose among their potential linking partners. This definition shows that, at every t, an agent i, selected uniformly at random from the set N, enjoys an opportunity of updating her current links. If an agent i receives such an opportunity, then she initiates a link to agent j which increases her equilibrium payoff the most. Agent j is said to be the best response of agent i given the network g_t. In our framework, agent j always accepts the link proposal because she does not pay the cost of the link and her utility always increases following the creation of the link because of local complementarities. If the utility of not creating a link is higher than that of creating a link with individual i's best response, then a link is not created and the network is unchanged. In the previous section, we have shown that this is the case if $c_i > \underline{c}_i$, where $\underline{c}_i$ is defined by (19).

Definition 6 *We define the network formation process $(g_t)_{t=0}^{\infty}$ with $g_t = (N, L_t)$, as a sequence of networks $g_0, g_1, ...$ in which, at every time $t = 0, 1, 2, ...$, a delinquent $i \in N$ is uniformly selected at random. This delinquent i initiates a link to a best response delinquent*

$j \in BR_{i,t}$. The link is created if $BR_{i,t} \neq \emptyset$ and $u_{i,t}(\boldsymbol{y}_t^*, g_t^{(i,j)}) \geq u_{i,t}(\boldsymbol{y}_t^*, g_t^{(i,i)})$. No link will be created otherwise. If $BR_{i,t}$ is not unique, then i randomly selects one delinquent in $BR_{i,t}$.

In this framework, the newly established link affects the overall network structure and thus the centralities and payoffs of all other delinquents in the network. The formation of links thus can introduce large, unintended and uncompensated externalities.

We now analyze in more detail the network formation process $(g_t)_{t=0}^{\infty}$ defined above, where g_t is the random variable realized at time $t \geq 0$. Let us show that the network formation process $(g_t)_{t=0}^{\infty}$ introduced in Definition 6 induces a Markov chain on a finite state space Ω. Ω contains all unlabeled graphs with n nodes. Therefore, the number of states is finite and the transition between states can be represented with a transition matrix $\boldsymbol{T}$. Let us show that $(g_t)_{t=0}^{\infty}$ is a *Markov chain*. The network g_{t+1} is obtained from g_t by adding a link to L_t or doing nothing. Thus, the probability of obtaining g_{t+1} depends only on g_t and not on the previous networks $g_{t'}$ for $t' < t$, that is

$$\mathbb{P}\left(g_{t+1}|g_0, g_1, ..., g_t\right) = \mathbb{P}\left(g_{t+1}|g_t\right).$$

The number of possible networks g_t is finite for any time t and the transition probability from a network g_t to g_{t+1} does not depend on t. Therefore, $(g_t)_{t=0}^{\infty}$ is a finite state, discrete time, homogeneous Markov chain. Moreover, the transition matrix $\boldsymbol{T} = [T_{ij}]$ is defined by

$$T_{ij} = \mathbb{P}\left(g_{t+1} = g_j \mid g_t = g_i\right) \text{ for any } g_i, g_j \in \Omega.$$

Let us explain what kind of equilibrium concept we are using for this dynamic network formation model. As above, define $\mathrm{E}[u_{i,t}(\boldsymbol{y}_t^*, g_t^{(i,j)})]$ as the expected utility at time t for i to create a link with j, for all j such that $j \neq i$ and the (i,j)th element of the adjacency matrix $\boldsymbol{G}_{t-1}$ is zero and $\mathrm{E}[u_{i,t}(\boldsymbol{y}_t^*, g_t^{(i,i)})]$ as the expected utility at time t for i not to create a link (see 16).

Definition 7 *Consider the network formation process $(g_t)_{t=0}^{\infty}$ with $g_t = (N, L_t)$ described in Definition 6, where, at each period of time t, the morning-afternoon game described in Section 3.2.1 is played. We say that the network g_0 at time $t = 0$ converges to an equilibrium network g_T at time $t = T$ when each of the n delinquents in the network g_T has no incentive to create a new link at time $t = T$, that is,* $\mathrm{E}[u_{i,t}(\boldsymbol{y}_t^*, g_t^{(i,i)})] > \max_j \mathrm{E}[u_{i,T}(\boldsymbol{y}_t^*, g_t^{(i,j)})]$ *for all $i = 1, ..., n$.*

This definition says that the first time period for which each delinquent has no incentive to create a new link is T. This means that at times $T+1$, $T+2$, etc., there are also no incentive for them to create a new link. In that case, the network $g_T = (N, L_T)$ is an equilibrium network. In terms of Markov chain, this means that the equilibrium network g_T is an *absorbing state*. From now on, when use the word "equilibrium" or "equilibrium network" in the dynamic network formation model, we refer to the equilibrium concept defined in Definition 7.

3.2.3 Finding the key player in the dynamic model

We will now perform a key-player policy in the dynamic network formation model. At $t = 0$, before the dynamic network formation game described above starts, the planner will choose the key player i^* in the following way. The planner will compare the (expected) total crime that will emerge in equilibrium (Definition 7) when she does not remove a delinquent and when she does. As in Section 3.1.2, the key player i^* will be the delinquent who reduces the most the total (expected) crime.

To be more precise, we can calculate the total expected crime in equilibrium when no delinquent is removed. Let the adjacency matrix for the equilibrium network g_T be denoted by $\boldsymbol{G}_T$. The equilibrium effort level is given by:

$$\boldsymbol{y}_T^* = (\boldsymbol{I} - \phi \boldsymbol{G}_T)^{-1}(\boldsymbol{a} + \bar{\eta}\boldsymbol{l}_n + \boldsymbol{\epsilon}).$$

The expected equilibrium effort outcome is equal to:

$$\mathrm{E}(\boldsymbol{y}_T^*) = (\boldsymbol{I} - \phi \boldsymbol{G}_T)^{-1}(\boldsymbol{a} + \bar{\eta}\boldsymbol{l}_n).$$

Then the total expected crime for the equilibrium network is: $\sum_{j=1}^{n} \mathrm{E}(y_{j,T}^*)$.

Let us now calculate the total expected crime of the equilibrium network after removing a delinquent at $t = 0$. Let the adjacency matrix for the equilibrium network $g_T^{[-i]}$ without delinquent i be denoted by $\boldsymbol{G}_T^{[-i]}$. In that case, the equilibrium effort level is given by:

$$\boldsymbol{y}_T^*(g_T^{[-i]}) = (\boldsymbol{I} - \phi \boldsymbol{G}_T^{[-i]})^{-1}(\boldsymbol{a} + \bar{\eta}\boldsymbol{l}_{n-1} + \boldsymbol{\epsilon})$$

and the expected equilibrium effort outcome is equal to:

$$\mathrm{E}[\boldsymbol{y}_T^*(g_T^{[-i]})] = (\boldsymbol{I} - \phi \boldsymbol{G}_T^{[-i]})^{-1}(\boldsymbol{a} + \bar{\eta}\boldsymbol{l}_{n-1}).$$

Then the total expected crime for the equilibrium network is: $\sum_{j=1, j\neq i}^{n} \mathrm{E}[y_{j,T}^*(g_T^{[-i]})]$.

The planner's objective to find the key player is to generate the highest possible reduction in aggregate delinquency level by picking the appropriate delinquent. Formally, the planner's problem is now:

$$\max_i \left\{ \sum_{j=1}^{n} \mathrm{E}(y_{j,T}^{*}) - \sum_{j=1, j\neq i}^{n} \mathrm{E}[y_{j,T}^{*}(g_T^{[-i]})] \mid i = 1, ..., n \right\}.$$

This is equivalent to:

$$\min_i \left\{ \sum_{j=1, j\neq i}^{n} \mathrm{E}[y_{j,T}^{*}(g_T^{[-i]})] \mid i = 1, ..., n \right\}. \tag{20}$$

We would like now to test the different models exposed in Section 3. For that, we will first describe the data, then expose the econometric methodologies and finally comment the empirical results.

4 Data description

Our analysis is made possible by the use of a unique database on friendship networks from the National Longitudinal Survey of Adolescent Health (AddHealth).[16]

The AddHealth database[17] has been designed to study the impact of the social environment (i.e. friends, family, neighborhood and school) on adolescents' behavior in the United States by collecting data on students in grades 7-12 from a nationally representative sample of roughly 130 private and public schools in years 1994-95. Every pupil attending the sampled schools on the interview day is asked to compile a questionnaire (in-school data) containing questions on respondents' demographic and behavioral characteristics, education, family background and friendship. This sample contains information on roughly 90,000 students. A subset of adolescents selected from the rosters of the sampled schools, about 20,000 individuals, is then asked to compile a longer questionnaire containing more sensitive individual and household information (in-home and parental data).[18]

[16] This research uses data from Add Health, a program project designed by J. Richard Udry, Peter S. Bearman, and Kathleen Mullan Harris, and funded by a grant P01-HD31921 from the National Institute of Child Health and Human Development, with cooperative funding from 17 other agencies. Special acknowledgment is due Ronald R. Rindfuss and Barbara Entwisle for assistance in the original design. Persons interested in obtaining data files from Add Health should contact Add Health, Carolina Population Center, 123 W. Franklin Street, Chapel Hill, NC 27516-2524 (addhealth@unc.edu). No direct support was received from grant P01-HD31921 for this analysis.

[17] The AddHealth website *http://www.cpc.unc.edu/projects/addhealth* describes survey design and data in details.

[18] The subjects of the in-home sub-sample are

For the purposes of our analysis, the most interesting aspect of the AddHealth data is the information on friendships. Indeed, the friendship information is based upon actual friends nominations. Pupils were asked to identify their best friends from a school roster (up to five males and five females).[19] Friendship relationships, however, are not always reciprocal. We construct the geometric structure of the friendship networks using the *outdegree* of each student i, denoted by $\bar{g}_i = \sum_j g_{ij}$, which is the number of friends student i nominates. This is the *row-sum* of $\boldsymbol{G}$ corresponding to i. We prefer to use *outdegree* (instead of *indegree*) because if individual i nominates j but j does not, it is then very possible that j is a role model for i. In other words, i is learning from j even though j does not consider i as her best friend.[20] For each school, we thus obtain all the networks of (best) friends. By matching the identification numbers of the friendship nominations to respondents' identification numbers, one can obtain information on the characteristics of nominated friends.

The in-home questionnaire contains an extensive set of questions on juvenile delinquency, that are used to construct our dependent variable. Specifically, the AddHealth contains information on 15 delinquency items.[21] The survey asks students how often they participate in each of these activities during the past year.[22] Each response is coded using an ordinal scale ranging from 0 (i.e. never participate) to 1 (i.e. participate 1 or 2 times), 2 (participate 3 or 4 times) up to 3 (i.e. participate 5 or more times). Non-criminal individuals are defined as those who report never participating in any delinquent activity. This data allow us to get valid information on 6,993 (criminal and non-criminal) students distributed over 1,596

interviewed again in 1995–96 (wave II), in 2001–2 (wave III), and again in 2007-2008 (wave IV). In this paper, we do not use this panel information.

[19] The limit in the number of nominations is not binding (even by gender). Less than 1% of the students in our sample show a list of ten best friends.

[20] As highlighted by Wasserman and Faust (1994), centrality indices for directional relationships generally focus on choices made.

[21] Namely, paint graffiti or signs on someone else's property or in a public place; deliberately damage property that didn't belong to you; lie to your parents or guardians about where you had been or whom you were with; take something from a store without paying for it; get into a serious physical fight; hurt someone badly enough to need bandages or care from a doctor or nurse; run away from home; drive a car without its owner's permission; steal something worth more than $50; go into a house or building to steal something; use or threaten to use a weapon to get something from someone; sell marijuana or other drugs; steal something worth less than $50; take part in a fight where a group of your friends was against another group; act loud, rowdy, or unruly in a public place.

[22] Respondents listened to pre-recorded questions through earphones and then they entered their answers directly on laptop computers. This administration of the survey for sensitive topics minimizes the potential for interview and parental influence, while maintaining data security.

networks, with network size ranging between 2 and 1,050 individuals.[23] This sample is used to investigate the decision to commit crime (see Section 3.1.4). Our analysis on peer effects in crime focusses on networks of criminals only, once selection into crime activity has been controlled for. Because the strength of peer effects may vary with network size (see Calvó-Armengol et al., 2009), the large variation in network size forces us to exclude networks at the extremes of the network size distribution. Indeed, our theoretical model assumes homogenous peer effects in crime ϕ, and we thus need to estimate only one peer-effect parameter.[24] Our selected sample consists of 1,297 criminals distributed over 150 networks, with network size range between 4 and 77 individuals.[25] The mean and the standard deviation of network size are roughly 9 and 12 pupils.[26] To derive quantitative information on crime activity using qualitative answers to a battery of related questions, we calculate an index of delinquency involvement for each respondent.[27] The delinquency index ranges between 1.51 and 11.04, with mean equal to 2.35 and standard deviation to 1.09.

Table A.2 in Appendix 3 provides a description of the control variables used in our study and Table A.3 collects the summary statistics distinguishing between criminals and non criminals.[28] As expected, Table A.3 shows that delinquent students are less likely to be a female, to be religious, to be attached to the school and to come from less educated families than non-delinquent students. They are also more likely to reside in urban areas. In Section 5.7, we look more closely at these associations, uncovering some interesting differences between different types of crimes. Looking at our selected sample of delinquents (last two columns), we find that, on average, criminal adolescents feel that adults care about them but have troubles getting along with the teachers. About 70 percent of our delinquent adolescents live in a household with two married parents, although about 25 percent come from a single parent family. The most popular occupation of the father is a manual one (roughly 30 percent)

[23] The large reduction in sample size with respect to the original AddHealth sample is mainly due to missing values in variables and to the network construction procedure. Indeed, roughly 20% of the students do not nominate any friends and another 20% cannot be correctly linked (for example because the identification code is missing or misreported).

[24] From an empirical point of view, the estimation of heterogenous peer effects requires the exclusion of network fixed effects, which are an important aspect of our identification strategy, allowing us to control for unobserved factors.

[25] Our results, however, do not depend crucially on these network size thresholds. They remain qualitatively unchanged when slightly moving the network size window.

[26] On average, delinquents declare having 1.33 delinquent friends with a standard deviation of 1.29.

[27] This is a standard factor analysis, where the factor loadings of the different variables are used to derive the total score.

[28] Information at the school level, such as school quality and teacher/pupil ratio, is unnecessary given our fixed effects estimation strategy.

and 21 percent of them have parents who work in a professional/technical occupations. The average parental education is high school graduate and almost 60 percent of our adolescents live in urban areas. The performance at school, as measured by mathematics score, is slightly above the average. To the extent to which mathematics score is a good proxy for individual ability or intelligence, this suggests that criminals are more "able" individuals. On average, our criminals declare themselves being slightly more intelligent than their peers and their level of physical development appears to be slightly higher than that of other boys/girls of the same age.[29]

5 Peer effects and network centrality

Let us now begin the test of the first part of our theoretical framework, namely the impact of peers on own criminal activities when the network is exogenously given (Section 3.1). We want to provide an appropriate estimate of peer effects in crime (ϕ). We first present our empirical model and estimation strategy. We use the architecture of networks to identify peer effects as described in Bramoullé et al. (2009) and Lee et al. (2010) but we consider the case of a non-row-normalized $\boldsymbol{G}$ and we highlight the methodological improvements that are achieved in our context. Our estimation method follows the 2SLS and GMM strategies proposed by Lee (2007a) and refined by Liu and Lee (2010) to capture the impact of centrality in networks. To be more specific, we will begin by explaining the empirical issues than hinder the identification of peer effects and show to what extent it is possible to tackle each of these issues with the AddHealth dataset.

5.1 Empirical model

Let $\bar{r}$ be the total number of networks in the sample (150 in our dataset), n_r be the number of individuals in the rth network g_r, and $n = \sum_{r=1}^{\bar{r}} n_r$ be the total number of sample observations. Defining the ex ante heterogeneity $a_{i,r}$ of each individual in network g_r as

$$a_{i,r} = \boldsymbol{x}'_{i,r}\boldsymbol{\beta}_1 + \frac{1}{\bar{g}_{i,r}} \sum_{j=1}^{n_r} g_{ij,r}\boldsymbol{x}'_{j,r}\boldsymbol{\beta}_2,$$

[29] When reading these summary information, one need to keep in mind that we deal here with juvenile delinquency, where some of the offences recorded as crimes (such as paint graffiti or lie to the parents) are quite minor.

where $\boldsymbol{x}_{i,r} = (x_{i,r}^1, \cdots, x_{i,r}^m)'$ and $\bar{g}_{i,r} = \sum_{j=1}^{n_r} g_{ij,r}$. The empirical model corresponding to (5) can be written as:

$$y_{i,r} = \phi \sum_{j=1}^{n_r} g_{ij,r} y_{j,r} + \boldsymbol{x}_{i,r}' \boldsymbol{\beta}_1 + \frac{1}{\bar{g}_{i,r}} \sum_{j=1}^{n_r} g_{ij,r} \boldsymbol{x}_{j,r}' \boldsymbol{\beta}_2 + \bar{\eta}_r + \epsilon_{i,r}, \tag{21}$$

for $i = 1, \cdots, n_r$ and $r = 1, \cdots, \bar{r}$, where $\bar{\eta}_r = \eta_r - pf$ and $\epsilon_{i,r}$'s are i.i.d. innovations with zero mean and variance σ^2 for all i and r.

5.2 Identification strategy

The identification of endogenous peer effects (ϕ in model (21)) raises different challenges.

Reflection problem In linear-in-means models, simultaneity in behavior of interacting agents introduces a perfect collinearity between the expected mean outcome of the group and its mean characteristics. Therefore, it is difficult to differentiate between the effect of peers' effort choice and peers' characteristics that do impact on their own effort choice (the so-called *reflection problem*; see Manski, 1993). Basically, the reflection problem arises because, in the standard approach, individuals interact in groups, that is individuals are affected by all individuals belonging to their group and by nobody outside the group. In other words, members in each group form a complete graph. In the case of social networks, instead, this is nearly never true since the reference group has individual-level variation. Take individuals i and k such that $g_{ik} = 1$. Then, individual i is directly influenced by $\sum_{j=1}^{n} g_{ij} y_j$ while individual k is directly influenced by $\sum_{j=1}^{n} g_{kj} y_j$, and there is little chance for these two values to be the same unless the network is complete (i.e. everybody is linked with everybody). Formally, as shown by Bramoullé et al. (2009), social effects are identified (i.e. no reflection problem) if $\boldsymbol{I}$, $\boldsymbol{G}$ and $\boldsymbol{G}^2$ are linearly independent, where $\boldsymbol{G}^2$ keeps track of indirect connections of length 2 in g.[30] In other words, if i and j are friends and j and k are friends, it does not necessarily imply that i and k are also friends. Because of these intransitivities, $\boldsymbol{G}^2\boldsymbol{X}$, $\boldsymbol{G}^3\boldsymbol{X}$, etc. are not collinear with $\boldsymbol{GX}$ and they act as valid instruments for $\boldsymbol{Gy}$ (under the situation that $\boldsymbol{X}$ is relevant). Intuitively, $\boldsymbol{G}^2\boldsymbol{X}$ represents the vector of the friends' friends attributes of each agent in the network. The architecture of social networks implies that these attributes will affect her outcome only through their effect on her friends' outcomes. Even in linear-in-means models, the Manski's (1993) reflection

[30] For example, complete networks do not satisfy this condition. In our dataset, where 150 networks are considered (see above in the data section), many of them have different sizes but none of them are complete and all satisfy the condition that guarantees the identification of social effects. Note that, even when networks are all complete, Lee (2007) shows that identification can be achieved by exploring strengths of interactions across networks of different sizes.

problem is thus eluded.[31] Peer effects in social networks are thus identified and can be estimated using 2SLS (Lee 2007; Lin, 2010). In Appendix 4, we detail in a more technical way the identification of model (21). In particular, we highlight the difference between the case with row-normalized $\boldsymbol{G}$ (Bramoullé et al., 2009) and our case with non-row-normalized $\boldsymbol{G}$.

Endogenous network formation/correlated effects Although this setting allows us to solve the reflection problem, the estimation results might still be flawed because of the presence of unobservable factors affecting both individual and peer behavior. It is indeed difficult to disentangle the endogenous peer effects from the correlated effects, i.e. effects arising from the fact that individuals in the same network tend to behave similarly because they face a common environment. If individuals are not randomly assigned into networks, this problem might originate from the possible sorting of agents. If the variables that drive this process of selection are not fully observable, potential correlations between (unobserved) network-specific factors and the target regressors are major sources of bias. Observe that our particularly large information on individual (observed) variables should reasonably explain the process of selection into groups. However, a number of papers have treated the estimation of peer effects with correlated effects (e.g., Clark and Loheac, 2007; Lee, 2007; Calvó-Armengol et al., 2009; Lin, 2010; Lee et al., 2010). This approach is based on the use of *network fixed effects* and extends Lee (2003) 2SLS methodology after the removal of network fixed effects. Network fixed effects can be interpreted as originating from a two-step model of link formation where agents self-select into different networks in a first step with selection bias due to specific network characteristics and, then, in a second step, link formation takes place within networks based on observable individual characteristics only. An estimation procedure alike to a panel within group estimator is thus able to control for these correlated effects. One can get rid of the network fixed effects by subtracting the network average from the individual-level variables.[32] As detailed in the next section, this paper follows this approach.

[31] These results are formally derived in Bramoullé et al. (2009) (see, in particular, their Proposition 3) and used in Calvó-Armengol et al. (2009) and Lin (2010). Cohen-Cole (2006) presents a similar argument, i.e. the use of out-group effects to achieve the identification of the endogenous group effect in the linear-in-means model (see also Weinberg et al., 2004; Laschever, 2009). See Durlauf and Ioannides (2010) and Blume et al. (2011) for an overview on these issues.

[32] Bramoullé et al. (2009) also deal with this problem in the case of a row-normalized $\boldsymbol{G}$ matrix. In their Proposition 5, they show that if the matrices $\boldsymbol{I}$, $\boldsymbol{G}$, $\boldsymbol{G}^2$ and $\boldsymbol{G}^3$ are linearly independent, then by subtracting from the variables the network average social effects are again identified and one can disentangle endogenous effects from correlated effects. In our dataset this condition of linear independence is always satisfied.

Individual unobserved factors The richness of the information provided by the AddHealth questionnaire on adolescents' behavior allows us to find observable individual variables as well as proxies for typically unobserved individual characteristics that may be correlated with our variable of interest. Specifically, to control for differences in leadership propensity across adolescents, we include an indicator of self-esteem and an indicator of the level of physical development compared to the peers, and we use mathematics score as an indicator of ability. Also, we attempt to capture differences in attitude towards education, parenting and more general social influences by including indicators of the student's school attachment, relationship with teachers, parental care and social inclusion.

To summarize, our identification strategy is based on the assumption that any troubling source of heterogeneity (if any), which is left unexplained by our unusually large set of observed characteristics can be captured at the network level, and thus taken into account by the inclusion of network fixed effects.

To be more precise, we allow link formation (as captured by our matrix $\boldsymbol{G}$) to be correlated with observed individual characteristics[33] and contextual effects ($\boldsymbol{G}^{*}\boldsymbol{X}$, where $\boldsymbol{G}^{*}$ is row-normalized from $\boldsymbol{G}$) and unobserved network characteristics (captured by the network fixed effects). The presence of other remaining unobserved effect is very unlikely in our case given our set of controls that includes behavioral factors and, most importantly, because we deal with quite small networks (see Section 4). In our empirical study, we provide a statistic test to support this claim.

Deterrence effects So far, we have dealt with issues that are common to the identification of any kind of peer effects. There is, however, something that is specific to crime: How deterrence effects (pf in our theoretical model) are measured? The identification of deterrence effects on crime is an equally difficult empirical exercise because of the well-known potential simultaneity and reverse causality issues (Levitt, 1997), which cannot be totally solved using our network-based empirical strategy. Network fixed effects also prove useful in this respect. Because in our sample, networks are within schools, the use of network fixed effects also accounts for differences in the strictness of anti-crime regulations across schools (i.e. differences in the expected punishment for a student who is caught possessing illegal drug, stealing school property, verbally abusing a teacher, etc.). As mentioned above, they account for any kind of school level heterogeneity. As a result, instead of directly estimating deterrence effects (i.e. to include in the model specification observable measures of deterrence, such as local police expenditures or the arrest rate in the local area), we focus our

[33] As long as the link formation process between two individuals does not involve the characteristics of any third individual.

attention on the estimation of peer effects in crime, accounting for network fixed effects.

5.3 Econometric methodology

Let $\boldsymbol{y}_r = (y_{1,r}, \cdots, y_{n_r,r})'$, $\boldsymbol{X}_r = (\boldsymbol{x}_{1,r}, \cdots, \boldsymbol{x}_{n_r,r})'$, and $\boldsymbol{\epsilon}_r = (\epsilon_{1,r}, \cdots, \epsilon_{n_r,r})'$. Denote the $n_r \times n_r$ sociomatrix by $\boldsymbol{G}_r = [g_{ij,r}]$, and the row-normalized $\boldsymbol{G}_r$ by $\boldsymbol{G}_r^*$. Then model (21) can be written in matrix form as:

$$\boldsymbol{y}_r = \phi \boldsymbol{G}_r \boldsymbol{y}_r + \bar{\boldsymbol{X}}_r \boldsymbol{\beta} + \bar{\eta}_r \boldsymbol{l}_{n_r} + \boldsymbol{\epsilon}_r,$$

where $\bar{\boldsymbol{X}}_r = (\boldsymbol{X}_r, \boldsymbol{G}_r^* \boldsymbol{X}_r)$ and $\boldsymbol{\beta} = (\boldsymbol{\beta}_1', \boldsymbol{\beta}_2')'$. For a sample with $\bar{r}$ networks, stack up the data by defining $\boldsymbol{y} = (\boldsymbol{y}_1', \cdots, \boldsymbol{y}_{\bar{r}}')'$, $\bar{\boldsymbol{X}} = (\bar{\boldsymbol{X}}_1', \cdots, \bar{\boldsymbol{X}}_{\bar{r}}')'$, $\boldsymbol{\epsilon} = (\boldsymbol{\epsilon}_1', \cdots, \boldsymbol{\epsilon}_{\bar{r}}')'$, $\boldsymbol{G} = \mathrm{D}(\boldsymbol{G}_1, \cdots, \boldsymbol{G}_{\bar{r}})$, $\boldsymbol{\iota} = \mathrm{D}(\boldsymbol{l}_{n_r}, \cdots, \boldsymbol{l}_{n_{\bar{r}}})$ and $\bar{\boldsymbol{\eta}} = (\bar{\eta}_1, \cdots, \bar{\eta}_{\bar{r}})'$, where $\mathrm{D}(\boldsymbol{A}_1, \cdots, \boldsymbol{A}_K)$ is a block diagonal matrix in which the diagonal blocks are $m_k \times n_k$ matrices $\boldsymbol{A}_k$'s. For the entire sample, the model is

$$\boldsymbol{y} = \boldsymbol{Z}\boldsymbol{\theta} + \boldsymbol{\iota}\bar{\boldsymbol{\eta}} + \boldsymbol{\epsilon}, \tag{22}$$

where $\boldsymbol{Z} = (\boldsymbol{G}\boldsymbol{y}, \bar{\boldsymbol{X}})$ and $\boldsymbol{\theta} = (\boldsymbol{\phi}, \boldsymbol{\beta}')'$.

We treat $\bar{\boldsymbol{\eta}}$ as a vector of unknown parameters. When the number of networks $\bar{r}$ is large, we have the incidental parameter problem. Let $\boldsymbol{J} = \mathrm{D}(\boldsymbol{J}_1, \cdots, \boldsymbol{J}_{\bar{r}})$, where $\boldsymbol{J}_r = \boldsymbol{I}_{n_r} - \frac{1}{n_r} \boldsymbol{l}_{n_r} \boldsymbol{l}_{n_r}'$. In order to avoid the incidental parameter problem, the network fixed effect can be eliminated by the transformation $\boldsymbol{J}$ such that

$$\boldsymbol{J}\boldsymbol{y} = \boldsymbol{J}\boldsymbol{Z}\theta + \boldsymbol{J}\boldsymbol{\epsilon}. \tag{23}$$

Let $\boldsymbol{M} = (\boldsymbol{I} - \phi \boldsymbol{G})^{-1}$. The equilibrium outcome vector $\boldsymbol{y}$ in (22) is given by the reduced form equation

$$\boldsymbol{y} = \boldsymbol{M}(\bar{\boldsymbol{X}}\boldsymbol{\delta} + \boldsymbol{\iota}\bar{\boldsymbol{\eta}}) + \boldsymbol{M}\boldsymbol{\epsilon}. \tag{24}$$

It follows that $\boldsymbol{G}\boldsymbol{y} = \boldsymbol{G}\boldsymbol{M}\bar{\boldsymbol{X}}\boldsymbol{\delta} + \boldsymbol{G}\boldsymbol{M}\boldsymbol{\iota}\bar{\boldsymbol{\eta}} + \boldsymbol{G}\boldsymbol{M}\boldsymbol{\epsilon}$. $\boldsymbol{G}\boldsymbol{y}$ is correlated with ϵ because $\mathrm{E}[(\boldsymbol{G}\boldsymbol{M}\boldsymbol{\epsilon})'\boldsymbol{\epsilon}] = \sigma^2 \mathrm{tr}(\boldsymbol{G}\boldsymbol{M}) \neq 0$. Hence, in general, (23) cannot be consistently estimated by OLS.[34] If $\boldsymbol{G}$ is row-normalized such that $\boldsymbol{G} \cdot \boldsymbol{l}_n = \boldsymbol{l}_n$, the endogenous social interaction effect can be interpreted as an average effect. With a row-normalized $\boldsymbol{G}$, Lee et al. (2010) have proposed a partial-likelihood approach for the estimation based on the transformed model (23). However, for this empirical study, we are interested in the aggregate endogenous effect instead of

[34] Lee (2002) has shown that the OLS estimator can be consistent in the scenario where each cross-sectional unit is influenced by many neighbors whose influences are uniformly small when there are many separate networks. However, in the current data, the number of neighbors are limited, and hence that result does not apply.

the average effect. Hence, row-normalization is not appropriate. Furthermore, we are also interested in the centrality of networks that are captured by the variation in row sums (out-degrees) in the adjacency matrix $\boldsymbol{G}$. Row-normalization could eliminate such information. However, as $\boldsymbol{G}$ is not row-normalized in this empirical study, the (partial) likelihood function for (23) could not be derived, and alternative estimation approaches need to be considered.

In this paper, we estimate (23) by the 2SLS and generalized method of moments (GMM) approaches proposed by Liu and Lee (2010). The conventional instrumental matrix for the estimation of (23) is $\boldsymbol{Q}_1 = \boldsymbol{J}(\boldsymbol{G}\bar{\boldsymbol{X}}, \bar{\boldsymbol{X}})$ (*finite-IV 2SLS*). For the case that the adjacency matrix $\boldsymbol{G}$ is not row-normalized, Liu and Lee (2010) have proposed to use additional instruments (IVs) $\boldsymbol{J}\boldsymbol{G}\boldsymbol{\iota}$ so that $\boldsymbol{Q}_2 = (\boldsymbol{Q}_1, \boldsymbol{J}\boldsymbol{G}\boldsymbol{\iota})$ (*many-IV 2SLS*). The additional IVs $\boldsymbol{J}\boldsymbol{G}\boldsymbol{\iota}$ are based on the row sums of $\boldsymbol{G}$ and thus use the information on centrality of a network. Those additional IVs could help model identification when the conventional IVs are weak and improve upon the estimation efficiency of the conventional 2SLS estimator based on $\boldsymbol{Q}_1$. The number of such instruments depends on the number of networks. If the number of networks grows with the sample size, so does the number of IVs. The 2SLS could be asymptotic biased when the number of IVs increases too fast relative to the sample size (see, e.g., Bekker, 1994; Bekker and van der Ploeg, 2005; Hansen et al., 2008). Liu and Lee (2010) have shown that the proposed many-IV 2SLS estimator has a properly-centered asymptotic normal distribution when the average group size is large relative to the number of networks in the sample. As detailed in Section 4, in this empirical study, we have a number of small networks. Liu and Lee (2010) have proposed a bias-correction procedure based on the estimated leading-order many-IV bias. The bias-corrected many-IV 2SLS estimator (*bias-corrected 2SLS*) is properly centered, asymptotically normally distributed, and efficient when the average group size is sufficiently large. It is thus the more appropriate estimator in our case study.

The 2SLS approach can be generalized to the GMM with additional quadratic moment equations (*finite-IV GMM, many-IV GMM*). While the IV moments use the information of the main regression function of (24) for estimation, the quadratic moments explore the correlation structure of the reduced form disturbances. Liu and Lee (2010) have shown that the many-IV GMM estimators can be consistent, asymptotically normal, and efficient when the sample size grows fast enough relative to the number of networks. Liu and Lee (2010) have also suggested a bias-correction procedure for the many-IV GMM estimator based on the estimated leading order many-instrument bias. The bias-corrected many-IV GMM estimator (*bias-corrected GMM*) is shown to be more efficient than the corresponding 2SLS estimator. Appendix 5 details the derivation and asymptotic properties of both the 2SLS and GMM estimators.

Our identification strategy and the validity of the moment conditions employed by the 2SLS and GMM estimators rest on the exogeneity of the adjacency matrix $\boldsymbol{G}$ (conditional on covariates and network fixed effects). We test the exogeneity of $\boldsymbol{G}$ using the over-identifying restrictions (OIR) test as described in Lee (2007a). If the OIR test cannot reject the null hypothesis that the moment conditions are correctly specified, then it provides evidence that $\boldsymbol{G}$ can be considered as exogenous. If the number of moment restrictions is fixed, the OIR test statistic given by the 2SLS (or GMM) objective function evaluated at the 2SLS (or GMM) estimator follows a chi-squared distribution with degrees of freedom equal to the number of over-identifying restrictions (Lee, 2007b, Proposition 2). However, the OIR test might not be robust in the presence of a large number of moment restrictions.[35] Hence, we only consider the OIR test for the 2SLS and GMM estimator based on IV matrix $\boldsymbol{Q}_1$.

5.4 Estimation results: all crimes

Table 1a collects the estimation results of model (21) when using the different estimators discussed in the previous section.

As explained above, for the estimation of ϕ, we pool all the networks together by constructing a block-diagonal network matrix with the adjacency matrices from each network on the diagonal block. Hence we implicitly assume that the ϕ in the empirical model is the same for all networks. The difference between networks is controlled for by network fixed effects. Indeed, the estimation of ϕ for each network might be difficult (in terms of precision) for the small networks. Furthermore, it is a crucial empirical concern to control for unobserved network heterogeneity by using network fixed effects.

For equation (7) to be well-defined, ϕ needs to be in absolute value smaller than the inverse of the largest eigenvalue of the block-diagonal network matrix $\boldsymbol{G}$ (Proposition 1). In our case, the largest eigenvalue of $\boldsymbol{G}$ is 3.11. Furthermore our theoretical model postulates that $\phi \geq 0$. As a result, we can accept values within the range $[0, 0.32)$. Table 1a (first row) shows that all our estimates of ϕ are within this parameter space. The p-value of the OIR test is larger than conventional significance levels, which means we cannot reject the null hypothesis that the moment conditions based on an exogenous $\boldsymbol{G}$ are valid. This evidence provides further confidence on *the exogeneity of network structure* (conditional on controls and network fixed effects).

[35] For the case with independent observations, Chao et al. (2010) have proposed an OIR test that is robust to many IVs. However, no robust OIR test with many IVs is available when observations are spatially correlated.

As explained above, we focus on the bias-corrected many-IV GMM estimator.[36] In a group of two friends, a standard deviation increase in the level of delinquent activity of the friend translates into a roughly 5 percent increase of a standard deviation in the individual level of activity. If we consider an average group of 4 best friends (linked to each other in a network), a standard deviation increase in the level of delinquent activity of each of the peers translates into a roughly 17 percent increase of a standard deviation in the individual level of activity. This is a non-negligible effect, especially given our long list of controls.

[*Insert Table* 1*a here*]

5.5 Estimation results: Petty versus serious crimes

We would like now to investigate whether peer effects are stronger in petty crimes than in more serious crimes. The literature is unclear about this issue. For instance, Ludwig et al. (2000) find that neighborhood effects are large and negative for violent crimes but have a mild positive effect on property crimes. In contrast, Glaeser et al. (1996) show that social interactions have a large effect on petty crimes, a moderate effect on more serious crimes and a negligible effect on very violent crimes.

To investigate this issue, we split the reported offences between *petty crimes* and *more serious crimes*. The first group (*type-1 crimes or petty crimes*) encompasses the following offences: (*i*) paint graffiti or sign on someone else's property or in a public place; (*ii*) lie to the parents or guardians about where or with whom having been; (*iii*) run away from home; (*iv*) act loud, rowdy, or unruly in a public place; (*v*) take part in a group fight; (*vi*) damage properties that do not belong to you; (*vii*) steal something worth less than $50. The second group (*type-2 crimes or more serious crimes*) consists of (*i*): taking something from a store without paying for it; (*ii*) hurting someone badly enough to need bandages or care from a doctor or nurse; (*iii*) driving a car without its owner's permission; (*iv*) stealing something worth more than $50; (*v*) going into a house or building to steal something; (*iv*) using or threatening to use a weapon to get something from someone; (*vii*) selling marijuana or other drugs; (*viii*) getting into a serious physical fight.

We obtain a sample of 1,099 petty criminals distributed over 132 networks and a sample of 545 more serious criminals distributed over 75 networks. Petty crime networks have a

[36]The first stage partial F-statistics (see Stock et al., 2002, and Stock and Yogo, 2005, for its statistical properties) reveal that our instruments in the linear moment conditions are quite informative. Hence, the different types of GMM estimators deliver similar results as their 2SLS counterparts with a better precision in the estimation of ϕ.

minimum of 4 individuals and a maximum of 73 (with mean equals to 8.33 and standard deviation equals to 10.74), whereas the range for more serious crime networks is between 4 and 38 (with mean equals to 7.27 and standard deviation equals to 6.64).

We estimate model (21) for different types of crimes. The results for *directed networks*[37] are contained in Tables 1b and 1c. All estimates are within the acceptable parameter space $[0, 1/\mu_1(\boldsymbol{G}))$, which is $[0, 0.32)$ for type-1 crimes and $[0, 0.42)$ for type-2 crimes. In terms of magnitude, it appears that the impact of peer effects on crime are much higher (almost double) for more serious crimes than for petty crimes. Indeed, we find that in a group of two friends, a standard deviation increase in the level of delinquent activity of the friend translate into a roughly 4 percent and 8 percent increase of a standard deviation in the individual level of activity for petty crimes and more serious crimes, respectively.

[*Insert Tables* 1*b and* 1*c here*]

5.6 Endogenous participation in criminal activities: Econometric issues

In this section, we deal with the estimation of the model when the endogenous crime decision is taken into account. We would like to write down an econometric model corresponding to the theoretical model developed in Section 3.1.4, in particular Proposition 4. Remember from Section 3.1.4 that, what mattered in the decision of whether to become criminal or not was each individual i's own observable and unobservable characteristics, a_i and ϵ_i. This decision was made once and for all and was irreversible. This implied, in particular, that the decision of becoming a criminal did not depend on what others would do. However, if someone become criminal, then peer effects will matter because she will belong to a criminal network and will be friend with mostly other criminals.

Let us address this endogenous participation (or selection) problem, from an econometric viewpoint, by considering a type-2 Tobit model. As in the theory, in the econometrics model, we consider two equations. The first equation (*the participation equation*) determines whether an agent will become or not a criminal among the $\bar{n}$ individuals in the economy. This corresponds to (12) (and Proposition 4). For those individuals who decide to be criminals, the second equation (*the outcome equation*), which determines for an agent i the effort level she decides to exert as a function of her own characteristics, the characteristics and efforts of her direct friends. This corresponds to (5).

[37] As stated above, the results for undirected networks are available upon request.

Based on (12), Proposition 4 says that the observable and unobservable characteristics of each individual i are the key determinant of the crime decision. To make this statement explicit, we assume that the participation equation is given by:

$$\boldsymbol{y}_{1\bar{n}}^{*} = \boldsymbol{X}_{\bar{n}}\boldsymbol{\gamma} + \boldsymbol{\epsilon}_{1\bar{n}}, \tag{25}$$

where $\boldsymbol{y}_{1\bar{n}}^{*} = (y_{1,1}^{*}, \cdots, y_{\bar{n},1}^{*})'$, $\boldsymbol{X}_{\bar{n}} = (\boldsymbol{x}_1, \cdots, \boldsymbol{x}_{\bar{n}})'$, and $\boldsymbol{\epsilon}_{1\bar{n}} = (\epsilon_{1,1}, \cdots, \epsilon_{\bar{n},1})'$. An agent i will become a criminal if $y_{i,1}^{*} > 0$ and will not become a criminal if $y_{i,1}^{*} \leq 0$. Since $y_{i,1}^{*} \leq 0$ is equivalent to $\boldsymbol{x}_i'\boldsymbol{\gamma} + \epsilon_{i,1} \leq 0$, this corresponds to the ranking in terms of observable and unobservable characteristics we made in Proposition 4 so that individuals i with a sufficiently low $\boldsymbol{x}_i'\boldsymbol{\gamma} + \epsilon_{i,1}$ will *not* be a criminal.

Without loss of generality, we assume the first n (of the $\bar{n}$) individuals will choose to become criminals. For those who become criminals,

$$\boldsymbol{y}_1^{*} = \boldsymbol{X}\boldsymbol{\gamma} + \boldsymbol{\epsilon}_1,$$

where $\boldsymbol{y}_1^{*} = (y_{1,1}^{*}, \cdots, y_{n,1}^{*})' > 0$, $\boldsymbol{X} = (\boldsymbol{x}_1, \cdots, \boldsymbol{x}_n)'$ and $\boldsymbol{\epsilon}_1 = (\epsilon_{1,1}, \cdots, \epsilon_{n,1})'$, and their crime effort levels in vector form are given by the following outcome equation (see (5)):

$$\boldsymbol{y} = \phi\boldsymbol{G}\boldsymbol{y} + \boldsymbol{X}\boldsymbol{\beta}_1 + \boldsymbol{G}^{*}\boldsymbol{X}\boldsymbol{\beta}_2 + \boldsymbol{\iota}\bar{\boldsymbol{\eta}} + \boldsymbol{\epsilon}_2, \tag{26}$$

where $\boldsymbol{y} = (y_1, \cdots, y_n)'$ and $\boldsymbol{\epsilon}_2 = (\epsilon_{1,2}, \cdots, \epsilon_{n,2})'$. The reduced form equation is therefore:

$$\boldsymbol{y} = (\boldsymbol{I} - \phi\boldsymbol{G})^{-1}(\boldsymbol{X}\boldsymbol{\beta}_1 + \boldsymbol{G}^{*}\boldsymbol{X}\boldsymbol{\beta}_2 + \boldsymbol{\iota}\bar{\boldsymbol{\eta}}) + (\boldsymbol{I} - \phi\boldsymbol{G})^{-1}\boldsymbol{\epsilon}_2.$$

Let $\boldsymbol{M} = (\boldsymbol{I} - \phi\boldsymbol{G})^{-1}$ and the ith row of $\boldsymbol{M}$ be $\boldsymbol{m}_i$. Then $y_i = \boldsymbol{m}_i(\boldsymbol{X}\boldsymbol{\beta}_1 + \boldsymbol{G}^{*}\boldsymbol{X}\boldsymbol{\beta}_2 + \boldsymbol{\iota}\bar{\boldsymbol{\eta}}) + \boldsymbol{m}_i\boldsymbol{\epsilon}_2$. Given the participation decisions of the $\bar{n}$ individuals in the first stage, the expected crime effort of criminal i is equal to:

$$\mathrm{E}[y_i|\boldsymbol{y}_1^{*} > 0] = \boldsymbol{m}_i(\boldsymbol{X}\boldsymbol{\beta}_1 + \boldsymbol{G}^{*}\boldsymbol{X}\boldsymbol{\beta}_2 + \boldsymbol{\iota}\bar{\boldsymbol{\eta}}) + \boldsymbol{m}_i\mathrm{E}\left[\boldsymbol{\epsilon}_2|\boldsymbol{y}_1^{*} > 0\right] \tag{27}$$

The term $\mathrm{E}\left[\boldsymbol{\epsilon}_2|\boldsymbol{y}_1^{*} > 0\right] = \mathrm{E}\left[\boldsymbol{\epsilon}_2|\boldsymbol{\epsilon}_1 > -\boldsymbol{X}\boldsymbol{\gamma}\right]$ is the *correction term* for the sample selection.

Suppose, for $i, j = 1, \cdots, n$,

$$\begin{bmatrix} \epsilon_{i,1} \\ \epsilon_{i,2} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix}\right),$$

and $\mathrm{E}(\epsilon_{i,1}|\epsilon_{j,2}) = 0$ for $i \neq j$. Then, $\mathrm{E}\left[\boldsymbol{\epsilon}_2|\boldsymbol{\epsilon}_1 > -\boldsymbol{X}\boldsymbol{\gamma}\right] = \sigma_{12}\boldsymbol{\lambda}$, where $\boldsymbol{\lambda} = \left[\lambda(\boldsymbol{x}_1'\boldsymbol{\gamma}), ..., \lambda(\boldsymbol{x}_n'\boldsymbol{\gamma})\right]'$ and $\lambda(\boldsymbol{x}_i'\boldsymbol{\gamma}) = f(\boldsymbol{x}_i'\boldsymbol{\gamma})/\Phi(\boldsymbol{x}_i'\boldsymbol{\gamma})$ (where $f(.)$ is the density of a standard normal random variable and $\Phi(.)$ is the distribution function of a standard normal random variable). By substituting this value into (27), we obtain, in matrix notation,

$$\mathrm{E}[\boldsymbol{y}|\boldsymbol{y}_1^* > 0] = \boldsymbol{M}(\boldsymbol{X}\boldsymbol{\beta}_1 + \boldsymbol{G}^*\boldsymbol{X}\boldsymbol{\beta}_2 + \boldsymbol{\iota}\bar{\boldsymbol{\eta}}) + \sigma_{12}\boldsymbol{M}\boldsymbol{\lambda}. \tag{28}$$

So we can now implement the Heckman's two-step approach. In the first step, using the bias-corrected IV estimator, we estimate $\boldsymbol{\gamma}$ from the participation equation (25) using a simple *probit* model to obtain $\hat{\boldsymbol{\gamma}}$. In the second step, we estimate

$$\boldsymbol{y} = \phi\boldsymbol{G}\boldsymbol{y} + \boldsymbol{X}\boldsymbol{\beta}_1 + \boldsymbol{G}^*\boldsymbol{X}\boldsymbol{\beta}_2 + \boldsymbol{\iota}\bar{\boldsymbol{\eta}} + \sigma_{12}\hat{\boldsymbol{\lambda}} + \boldsymbol{\epsilon}. \tag{29}$$

where $\hat{\boldsymbol{\lambda}} = [\lambda(\boldsymbol{x}_1'\hat{\gamma}), ..., \lambda(\boldsymbol{x}_n'\hat{\gamma})]'$ by *finite-IV 2SLS*.[38] The details of the two-step approach is given in Appendix 6.

5.7 Endogenous participation in criminal activities: Empirical results

In Table 2, we report the probit estimation results for the participation equation (25), for all crimes and different types of crimes separately. This table shows what individual characteristics affect the decision to become criminal. Interestingly, these characteristics vary with the type of crime committed. For example, other things being equal, female teenagers have the same probability to become criminal than male teenagers in general (column 2). They are, however, more likely to commit petty crimes (column 3) and much less likely to commit serious crimes (column 4) than male teenagers. Similarly, blacks are more likely to commit serious crime than whites while there are no statistically differences between blacks and whites for petty crimes. Family background variables (in particular, parental education and the social structure of families, like for example living with two married parents or with only one parent) show a significant and negative correlation with the propensity to commit serious crime only. If mathematics score is taken as a proxy for ability, then delinquents appear to be more able than non-delinquents. Interestingly, students that commit serious crimes do not consider themselves as more intelligent than their peers. Not surprisingly, religious practice is negatively correlated with the propensity to commit any type of crime. We also find that students that are physically more developed, who do not feel too much attached to the school, who have trouble getting along with teachers and feel that adults

[38] Because the estimated sample selection correction term introduces heteroskedasticity and correlation into the error term of (29), many-IV 2SLS and GMM approaches are not applicable here.

do not care much about them are more likely to be criminal. This is also true for students residing in urban areas.

In Table 3, we report the estimation results of our peer-effect crime model (21) with and without endogenous participation for all crimes and by types of crime. For each panel (all crimes, type-1 crimes, type-2 crimes), the results reported in the first column are the same as in Tables 1a-1c, whereas the results reported in the second column are obtained when the decision to select into criminal activities is taken into account (equation (29)). We find that the results are virtually the same,[39] pointing to the fact that an endogenous selection into crime activity is not a matter of concern.

[*Insert Table* 2 *and* 3 *here*]

6 Who is the key player?

Let us now calculate empirically who is the key player in each of our real-world networks. For this part, we will mainly use the dynamic network formation model developed in Section 3.2 and determine the key player as we did in Section 3.2.3, by solving the program given by (20). We will *structurally estimate* this model. Let us explain in detail how we do it.

6.1 Description of the procedure: A structural approach

6.1.1 Determining the key player without endogenous participation

We first show how to determine the key player for each network g_r $(r = 1, \cdots, \bar{r})$ as described in Section 3.2.3 where we do not take into account the endogenous criminal decision. We deal with the case with endogenous participation in the next subsection. To simplify notation, we drop the subscript r when there is no ambiguity. To determine the key player, we need first to estimate the total expected crime in equilibrium when a delinquent is removed, i.e. $\sum_{j=1, j\neq i}^{n} \mathrm{E}[y^{*}_{j,T}(g_T^{[-i]})]$. Remember that the expected equilibrium crime effort when i has been removed is given by $\mathrm{E}[\boldsymbol{y}^{*}_{T}(g_T^{[-i]})] = (\boldsymbol{I} - \phi \boldsymbol{G}_T^{[-i]})^{-1}(\bar{\boldsymbol{X}}\boldsymbol{\beta} + \bar{\eta}\boldsymbol{l}_{n-1})$ and $\sum_{j=1, j\neq i}^{n} \mathrm{E}[y^{*}_{j,T}(g_T^{[-i]})]$ is

[39] We show the similarity using the 2SLS-1 estimator, as the heteroschedasticity introduced by the correction in the error term renders the other estimators inappropriate.

the sum of each element of the vector $\mathrm{E}[\boldsymbol{y}_T^*(g_T^{[-i]})]$. We estimate $\mathrm{E}[\boldsymbol{y}_T^*(g_T^{[-i]})]$ as

$$\widehat{\mathrm{E}[\boldsymbol{y}_T^*(g_T^{[-i]})]} = (\boldsymbol{I} - \hat{\phi}\boldsymbol{G}_T^{[-i]})^{-1}(\bar{\boldsymbol{X}}\hat{\boldsymbol{\beta}} + \widehat{\overline{\eta}}\boldsymbol{l}_{n-1}), \tag{30}$$

where $\hat{\phi}$ and $\hat{\boldsymbol{\beta}}$ are the estimates obtained from the bias-corrected many-IV GMM estimation procedure in Section 5, and $\bar{\eta}$ can be estimated by the average of the estimation residuals. The only parameter undetermined from the bias-corrected many-IV GMM estimation is c_i, the marginal cost of forming links. Let us assume that the network observed in the AddHealth data is in *equilibrium* ($t = 0$), as defined by Definition 7), that is, no one has an incentive to create a new link at day $t = 1$, and denote this network by $\boldsymbol{G}_0$ (i.e. the adjacency matrix at day $t = 0$). For each i, $\underline{c}_i = \underline{c}_{i,1}$ is defined by (19).

Let $\hat{\boldsymbol{M}}_t^{(i,j)} = (\boldsymbol{I} - \hat{\phi}\boldsymbol{G}_t^{(i,j)})^{-1}$, with $\hat{\boldsymbol{m}}_{i,t}^{(i,j)}$ being $\hat{\boldsymbol{M}}_t^{(i,j)}$'s ith row and $\hat{m}_{ij,t}^{(i,j)}$ being its (i,j)th entry. As $\mathrm{E}(y_{i,t}^*) = \boldsymbol{m}_{i,t}^{(i,j)}(\bar{\boldsymbol{X}}\boldsymbol{\beta} + \bar{\eta}\boldsymbol{l}_n)$, from (18) $\kappa_{i,t}(g_t^{(i,j)})$ can estimated by:

$$\hat{\kappa}_{i,t}(g_t^{(i,j)}) = \frac{1}{2}[\hat{\boldsymbol{m}}_{i,t}^{(i,j)}(\bar{\boldsymbol{X}}\hat{\boldsymbol{\beta}} + \boldsymbol{\iota}\widehat{\overline{\boldsymbol{\eta}}})]^2 + \hat{\sigma}_\epsilon^2 \hat{m}_{ii,t}^{(i,j)} - \frac{1}{2}\hat{\sigma}_\epsilon^2 \hat{\boldsymbol{m}}_{i,t}^{(i,j)}\hat{\boldsymbol{m}}_{i,t}^{(i,j)\prime} + \hat{\phi}\hat{\sigma}_\epsilon^2 \sum_{k=1}^{n} g_{ik,t}^{(i,j)}\hat{\boldsymbol{m}}_{i,t}^{(i,j)}\hat{\boldsymbol{m}}_{k,t}^{(i,j)\prime}.$$

Hence, in the simulations, we estimate $\underline{c}_i$ by

$$\widehat{\underline{c}}_i = \max_j \hat{\kappa}_{i,1}(g_1^{(i,j)}) - \hat{\kappa}_{i,1}(g_1^{(i,i)}). \tag{31}$$

Observe that even if $c_i = \underline{c}_i$ for all i so that nobody will want to form a link, it will be possible that links will be formed after the removal of the key player since $\underline{c}_i(g) \neq \underline{c}_i(g^{[-i]})$.

We will compare the determination of the key player that solves (20), i.e.

$$\min_i \left\{ \sum_{j=1, j\neq i}^{n} \mathrm{E}[y_{j,T}^*(g_T^{[-i]})] \mid i = 1, ..., n \right\}$$

with the key player calculated in the invariant case, i.e. the network is given and there is no link formation. To be consistent with (20), in the invariant network case, we will calculate the key player as the one whose removal leads to the largest *expected* total crime reduction[40], or equivalently, the key player i^* will be the one that solves:

$$\min_i \left\{ \sum_{j=1, j\neq i}^{n} \mathrm{E}[y_{j,0}^*(g_0^{[-i]})] \mid i = 1, ..., n \right\} \tag{32}$$

[40] Note this is different from the definition of the key player in Section 3.1.2.

6.1.2 Determining the key player with endogenous participation

Let us now take into account the endogenous criminal decision in the dynamic formation model exposed in Section 3.2.3. The timing is now as follows. The decision to become criminal takes place before the existence of the network observed in the AddHealth data. There are $\bar{n}$ persons making this decision based on their own characteristics (see equation (25)). The decision to become criminal is made once and for all and is irreversible. Among them, n individuals choose to become criminals. These n criminals then play the morning-afternoon game described in Section 3.2.1 to build a network until it converges to a stable network. This is the network we observe in the AddHealth data at $t = 0$ (1994/1995 in the data), which we denote by $\boldsymbol{G}_0$. Then the planner removes the key player so that the network is now $\boldsymbol{G}_0^{[-i]}$. Then, the remaining criminals play again the morning-afternoon game until the network converges to an equilibrium network after T periods, which we denote by $\boldsymbol{G}_T^{[-i]}$. As in the case without endogenous participation, we can determine the key player by minimizing the total expected crime crime that emerges in this stable network.

Here, for the estimation of these networks, we proceed as before by estimating ϕ and $\boldsymbol{\beta}$ from the two-step estimation procedure described in Section 5 while η can be estimated by the average of the estimation residuals. We also estimate $\underline{c}_i$ by (31) described above. Finally, to evaluate the expected utility, we need to estimate the variance of $\boldsymbol{\epsilon}$ in (29), which include some additional noise due to the estimated selection bias correction term. The variance of $\boldsymbol{\epsilon}$ can be estimated by a similar approach as Lee and Trost (1978).[41]

6.1.3 Simulation results

We would like to comment the results of the simulations, which use the methodologies described in Sections 6.1.1 and 6.1.2. We have run many simulations and we can obviously not comment all of them. First, we will focus on the simulation results of the key player for the case without endogenous participation (Section 6.1.1) as both results are quite similar.[42] Second, for the case of all crimes, as described in Section 4, we have 1,297 criminals distributed over 150 networks. For the dynamic structural model, we only retain networks that satisfy the eigenvalue condition $\phi\mu_1(\boldsymbol{G}) < 1$.[43] Fortunately, only 5 networks do not satisfy

[41] See Appendix 5 for details.

[42] The simulation results for the case with endogenous participation are available upon request.

[43] In fact, this condition needs to be satisfied for all networks during the dynamic link-formation process. Since the eigenvalue of a network (and thus the largest eigenvalue condition) changes over time, we use a sufficient condition: $\phi\sqrt{\sum_{i,j} g_{ij} + n - 1} < 1$, which uses an upper bound on the largest eigenvalue (see Corrolary 1 page 1409 in Ballester et al., 2006). The main advantage of this new condition is that one does

this condition and thus we end up with 1,038 criminals distributed over 145 networks. The average network is size 7 with a minimum of 4 and a maximum of 64 delinquents. Figure 1 displayed the distribution of these 145 networks by their size. The distribution is clearly very skewed to the left, meaning that small networks are over-represented. We have also run the simulations for type-1 crimes or petty crimes (only 4 networks out of 132 networks do not satisfy the eigenvalue condition) and type-2 crimes or more serious crimes (only 5 networks out of 75 networks do not satisfy the eigenvalue condition).[44]

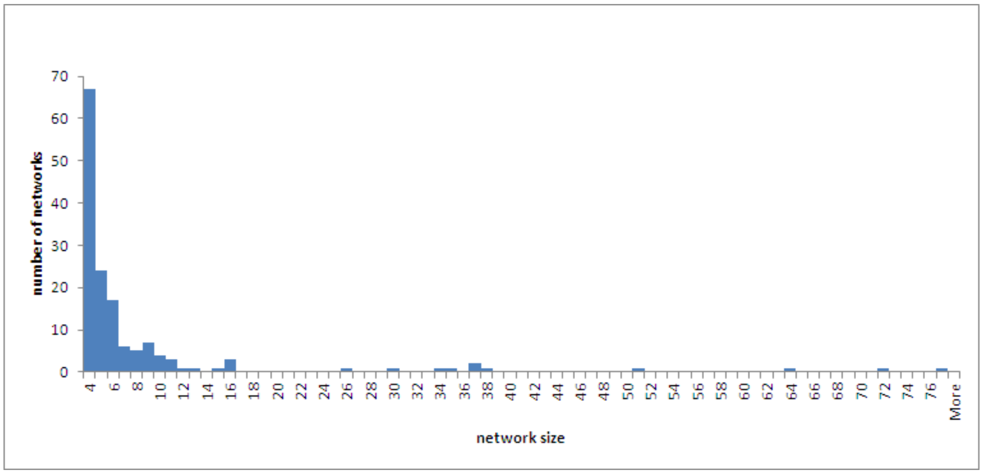

Figure 1: Distribution of networks by size for all crimes

Because of space constraints, in Tables 4a, 4b, 4c, and 4d, we report the simulation results only for *all crimes* and for the first 20 networks for the expected utility case.[45] Table 4a displays the simulation results for the first 20 networks when the cost $c_i = \underline{\hat{c}}_i$ (given by equation (31)), which means that the network observed in the AddHealth is in equilibrium. Table 4b displays the same results when the cost $c_i = \underline{\hat{c}}_i - 0.05$. Table 4c displays the same results when $c_i = 0$. Finally, Table 4d compares the three different cases ($c_i = \underline{\hat{c}}_i$, $c_i = \underline{\hat{c}}_i - 0.05$, $c_i = 0$) for small networks.[46]

Looking at Table 4a, we can see that, even for the first 20 networks, there are important variations in network sizes (see also Figure 1). Some networks only have 4 criminals while others have 51 or 64 criminals. If we compare columns 2 (which gives the delinquent who

not need to calculate the largest eigenvalue of the network at each period of time.

[44]Because some criminals conduct both type-1 and type-2 crimes, the sum of the networks for type-1 and type-2 crimes is not equal to the total number of networks for all crimes.

[45]The simulation results for the 145 networks for all crimes, the 128 networks for type-1 crimes (petty crimes) and for the 70 networks for type-2 crimes (more serious crimes) are available upon request.

[46]Because it takes much more time to converge to an equilibrium network, we only focus on networks of small size ($n_r \leq 10$) when $c_i = \underline{\hat{c}}_i - 0.05$ and $c_i = 0$.

has the highest betweenness centrality in the network), 3 (which gives the delinquent who has the highest Bonacich centrality in the network, i.e. the most active delinquent), 4 (which gives the key player when other delinquents cannot create links after the removal of the key player, i.e. the key player calculated in Section 3.1.2 and determined by (10); this is referred to as the invariant case), and 5 (which gives the key player when other delinquents play the dynamic network formation game described in Section 3.2.1 after the removal of the key player, i.e. the key player calculated in Section 6.1.1; this is referred to as the dynamic network formation case), one can see that, in most cases, the persons are different. In other words, only in 20 percent of the cases (4 out of 20 networks), these four columns are identical and this is only true for very small networks (less than 6 delinquents). This means that it is *not* straightforward to determine which delinquent should be removed from a network by only observing his or her criminal activities or position in the network. If we compare columns 4 and 5, we see that in 70 percent of the cases (14 out of 20 networks), the key players are same in the invariant and in the dynamic network formation cases, meaning that the invariant assumption is relatively good, especially for small networks. In large networks, this is less true since individuals have more possibilities to form new links after the removal of the key player.

If we now look at columns 6 (which gives the total crime effort in the initial network observed in the AddHealth data), 7 (which gives the total expected crime effort in the convergent network of the dynamic network formation game when no delinquent is removed), 8 (which gives the total expected crime effort in the invariant case when the key player is removed), and 9 (which gives the total expected crime effort in the convergent network of the dynamic network formation game when the key player is removed), we see that the total crime effort in equilibrium is nearly the same for the invariant and for the dynamic network formation cases for small networks. This is less true for large networks, even though the difference is not very large.

If we now compare the density and the diameter (maximum distance) of the network before and after the removal of the key player (columns 10, 11, 12 and 13), it can be seen that networks tend to be denser with a smaller diameter after the removal of the key player. Observe that, even if the diameter is 1, the density is not always equal to 1. Indeed, suppose there are 4 delinquents in a network such 1 and 2 are friends and 1, 2 and 3 are all friends to 4. After delinquent 4 is removed, individuals 1 and 2 are still friends but 3 is now isolated since he/she has no friend. In that case, the density is *not* equal to 1 but the diameter is 1. Finally, the number of days it takes for the network to converge to its equilibrium value is always zero before the removal of the key player (this is by definition since $\underline{\hat{c}}_i$ is the cost that ensures that the network is in equilibrium) and can take up to 5 days after the removal of the key player.

Table 4b displays the results for lower values of c_i. The results are relatively similar as compared to the case when $c_i = \underline{\hat{c}}_i$ with, however, two main differences. First, because it is less costly to form links, the total expected crime in the dynamic model before and after the removal of the key player (columns 7 and 9) is much higher. The same applies for the density of the network (columns 11 and 13). Second, the number of days it takes for the network to converge to an equilibrium network is much higher (see last column) since it can take up to 21 days to converge for a network with 9 delinquents. Table 4c displays the same results when there is no cost of link formation. The results are even stronger and it takes sometimes up to 63 days for the network to converge to an equilibrium network. Observe that, even if $c_i = 0$, the network does not always converge to the complete network. This is because of contextual effects. Indeed, when someone creates a new link, the average characteristics of his or her friends change (contextual effects) and thus, in some cases, it can be better not to form a link even with local complementarities. This is because some characteristics take negative values and the negative contextual effect can outweigh the positive effort effect.

[*Insert Tables* 4*a*, 4*b and* 4*c here*]

Finally, Table 4d compares the three different cases: $c_i = \underline{\hat{c}}_i$, $c_i = \underline{\hat{c}}_i - 0.05$ and $c_i = 0$. In nearly 70 percent of the cases (11 out of 16 networks), the key player is the same. This implies that, in 30 percent of the networks, depending on the cost value, the delinquent the planner wants to remove changes. For example, in network 11 (which has 9 delinquents), the key player is delinquent 3 when the cost is very high (i.e. $c_i = \underline{\hat{c}}_i$) while it is delinquent 8 when the cost is lower or even zero. In terms of policy implications, this means that in networks where the cost of forming delinquent friendships is relatively low (for example, for people living in the same neighborhood), the person to target would be different than in networks where this cost is high (for example, for people belonging to a gang). The same types of comments apply for the reduction in total crime after the removal of the key player. Consider again network 11. When $c_i = \underline{\hat{c}}_i$, the crime is reduced by 15 percent (from 38.188 in the initial AddHealth data to 33.208) while for $c_i = \underline{\hat{c}}_i - 0.05$, it decreased by 12 percent. Even more interestingly, when $c_i = 0$, total crime *increases* by 20 percent (from 38.188 to 46.009). Indeed, since the cost of forming links is zero, it takes 48 days for the network to converge after the removal of the key player (see last column of Table 4d). As a result, many new links are formed and this generates a lot of interaction between criminals, which end up committing many crimes. This result has two policy implications. First, if the planner would *not* have removed the key player, then total crime would have been much more higher and equal to 56.666 when $c_i = 0$ (see column 7 in Table 4c). Indeed, 56.666 is the (expected) total crime of the network that, starting at the network observed in the AddHealth data in 1994/1995, converges to an equilibrium network after 63 days when no criminal is removed while 46.009 is the (expected) total crime of the network that converges to an equilibrium network after 48 days after the removal of the key player. So the right comparison is between 56.666 and 46.009, which is the way we calculated the key player. In that case, there is a crime reduction of 23 percent. Second, if we had removed a delinquent at random, the reduction in crime would have been less important. Indeed, to calculate the key player, for each network, we compare the (expected) total crime when no player is removed in the dynamic network formation game and the (expected) total crime when one person is removed. The key player is the one that reduces the most the (expected) total crime. In the case of network 11 with $c_i = 0$, the maximum possible reduction is 23 percent (Table 4c). Interestingly, in some cases, the (expected) total crime after the removal of the key player can even be lower than the initial total crime observed in the AddHealth data. Take, for example, network 12 with 6 delinquents. The initial total crime is 12.962, the (expected) total crime of the dynamic game without the removal of any criminal is 15.327 and with the removal of the key player is 11.637. In other words, total crime decreases by 11 percent compared to the initial total crime and by nearly 32 percent compared to the (expected) total crime of the dynamic game without the removal of any criminal.

[*Insert Table* 4*d here*]

6.2 Individual characteristics of key players

We would like now to identify the characteristics of the key player in our networks. For that, we use the results of the simulations above that identify for network the key player in the dynamic network formation game developed in Section 6.1.2, where the structural estimation procedure is explained in Section 6.1.1. As explained in Section 6.1.3, we consider 1,038 criminals distributed over 145 networks for *all crimes* and identify the key players in each of these 145 networks when $c_i = \underline{\hat{c}}_i$. As an illustration, the identity of the key players for the first 20 networks was given in Table 4a. The cases when $c_i = \underline{\hat{c}}_i - 0.05$ and $c_i = 0$ (whose results for the first 20 networks of size less than 10 delinquents were given in Tables 4b and 4c) were analyzed only as robustness checks. Indeed, it seems reasonable to assume that the criminal networks observed in the AddHealth data in 1994/1995 are *equilibrium* networks and thus, to guarantee that this is true, we need to set the cost c_i of forming links to be equal to $\hat{\underline{c}}_i$.

Once we have identified the key player for each network, we can draw his or her "profile" by comparing the characteristics of these key players with those of the other criminals in the network.[47] Table 5 displays the results only for the variables whose differences in means between these two samples are statistically significant.

Compared to other criminals, "key" criminals are less likely to be a female, are less religious, belong to families whose parents are less educated and have the perception of being socially more excluded. They also feel that their parents care less about them, are less likely to come from families where both parents are married and have more troubles getting along with the teachers. An interesting feature is that key players are more intelligent (i.e. higher mathematics scores) than the average criminal and are more likely to have friends who are older (i.e. in higher grades), more religious and whose parents are more educated. Also, even though key players do not have a better self-esteem of themselves, are not more physically developed, are not more urbanites than other criminals, their friends do.[48]

[*Insert Table* 5 *here*]

An interesting and important question that was highlighted by our theoretical framework and that we seek to investigate empirically is whether the key player is always the criminal with the highest crime level (or equivalently with the highest Bonacich centrality in the network). We have shown in the theoretical section that, in some cases, it is not the case (see Section 3.1.3) because the two measures (Bonacich versus inter-centrality) differ substantially in their foundation. Whereas the equilibrium-Bonacich centrality index (defined in (4)) derives from strategic individual considerations, the intercentrality measure (defined in (10)) solves the planner's optimality collective concerns. In particular, the equilibrium Bonacich centrality measure fails to internalize all the network payoff externalities delinquents exert on each other, while the intercentrality measure internalizes them all.

As a result, for each of our 145 networks, we investigate whether the key player is also the most active criminal in the network (i.e. has the highest Bonacich centrality). We find that in 56 out of 145 networks (almost 40%), it is not the case. This interesting (and unexpected) result is important for policy purposes since it means that, in some cases, we should not always target the most active criminals in a network.

[47] Since the results on key players for undirected networks are relatively similar, we will not discuss them. They are available upon request.

[48] Summary statistics of the other characteristics of the key players, as well as the ones of their best friends are not reported for brevity. They remain available upon request.

In Table 6, we compare the characteristics of key players who are the most active criminals in the network with key players who are not. As in Table 5, Table 6 only shows variables whose differences in means between these two samples are statistically significant. Interestingly, there are very few characteristics that are significantly different between the two types of criminals, indicating that it is difficult to distinguish between them by only observing their characteristics. We find that the typical key player who is the most active criminal is more likely to be a male and is less likely to feel socially integrated and to live in bad neighborhoods. He or she has friends who are more likely to reside in urban residential areas.

[*Insert Table* 6 *here*]

6.3 Petty versus serious crimes

We repeat our structural estimations for key players for different types of crimes. There are 935 criminals distributed over 128 networks for *type-1 crimes* or *petty crimes* (the average network size is 8.33 and the minimum and maximum sizes are 4 and 73, respectively) and 404 criminals distributed over 70 networks for *type-2 crimes* or *more serious crimes* (the average network size is 7.27 and the minimum and maximum sizes are 4 and 38, respectively). Using the same procedure as in Section 6.1.3, we identify the key players in each of these networks when $c_i = \hat{\underline{c}}_i$. Although the results of this exercise need to be taken with caution because of the small sample size of students committing the more serious offences (404 of them), we report our findings in Tables 7 and 8.

Tables 7 and 8 have the same structure as Table 5 but draw a profile of the key player for petty and more serious crimes. Let us start with petty crimes. As compared to other criminals, a key player is more likely to be a male, to have higher mathematics scores, to be more physically developed and to reside in urban areas. He or she also seems to be a student who has troubles getting along with teachers and does not feel attached to the school. His/her parents are more likely to have a managerial occupation and less likely to work in a manual occupation than parents of other criminals. He/she feels that parents and adults, in general, do not care very much about him/her. This last aspect is not true for his/her friends. Indeed, compared to friends of other criminals, the friends of key players feel more often that their parents care very much about them, are socially more integrated and feel more attached to the school. They are also older, more religious and consider themselves slightly more intelligent than the average.

If we compare these characteristics with those of a key player committing more serious crime networks, then five key variables do not appear here: mathematics score, parental care, social inclusion, residential urban area, parent occupation manual. In other words, these five characteristics express the difference between key players and other criminals for petty crime but not for more serious crimes. Also, the distinctive features of friends of key players in petty crime networks are many more than the ones for key players in more serious crime networks.

[*Insert Tables* 7 *and* 8 *here*]

Table 9 compares the characteristics of key players for different types of crimes. It is interesting to see that they differ quite a lot. In particular, key players committing more serious crimes are more likely to be African Americans and have friends who are themselves African Americans, have a lower self-esteem, better parental care and better relation with teachers than key players committing petty crimes.

[*Insert Table* 9 *here*]

Finally, Tables 10 and 11 have the same structure as Table 6 but for different types of crimes. As in Table 6, we still find a sizable number of networks where the most "harmful" criminal (the key player) is not the individual with the highest Bonacich centrality, i.e. the most active criminal (37 percent for petty crimes and 40 percent for serious crimes). Interestingly, the differences in characteristics between key players who are and who are not the most active criminals are to be found mainly in more serious crimes. For these types of crimes, the key players who are not the most active criminals are less attached to religion, have worse parental care and feel less attached to schools, have a worse relationship with teachers and are more likely to reside in urban areas.

[*Insert Tables* 10 *and* 11 *here*]

6.4 Key players and network topology

As in Section 3.1.3, let us now investigate the characteristics of these key players in terms of other network centrality measures (i.e. other than Bonacich centrality).

So far, we have used the Bonacich centrality to capture the importance of network structure. This is because this measure has a precise behavioral foundation, coming from our theoretical model. However, it counts *all* path connecting one person to others, not the shortest ones. Let us thus consider two other standard measures of centrality that are based on shortest paths, i.e. *closeness* and *betweenness* centralities, and a measure of cliquishness of each node, i.e. the *clustering coefficient.* They are all defined in Appendix 7 and used in the theory section (3.1.3.)

Table 12 provides information on the distributions of these measures for the key players in our networks and compares them with the Bonacich centrality (which is equal to the crime level of each individual). Looking at the betweenness centrality (which takes into account the number of shortest paths going through each individual), we observe that at least 50 percent of our key players has a betweenness centrality equal to zero (i.e. the median is equal to 0). This indicates that there are few shortest paths that go through them. However, if we consider the upper tail of this distributions, that is if we look at the key players with the highest betweenness centrality, we see that a larger portion of them are key players who are not the most active criminals. Indeed, above the 90th percentile of the distribution of the whole sample, 71 percent of key players are those who are not the most active criminals. This suggests that, even though some criminals do not commit much crime, they can be key players because they have a crucial position in the network in terms of betweenness centrality (for example, in the network described in Section 3.1.3, individual 1 who bridges two otherwise separated networks is not the most active criminal but is the key player and has the highest betweenness centrality).

When looking at the closeness centrality (how each individual is close in terms of shortest paths to all individuals in the network he/she belongs to), the results are quite different. Indeed, many key players are now quite central since the median is equal to 0.33. We also find that less active key players tend to be less concentrated in the upper tail of the closeness distribution (27 percent in the 90th percentile). Finally, the distribution of the clustering coefficient (which indicates if friends of friends are also friends) shows that key players do not operate in particularly tight networks (at least 75 percent of key players show a clustering coefficient equal to 0) and more and less active key players appear equally distributed in the upper 90 percent tail. The last column simply shows that in the large majority of the cases, key players who are the most active criminals in the network they belong to are also among the most active criminals in the overall sample.

[*Insert Table* 12 *here*]

In the lower panels of Table 12, we perform the same analysis for petty and serious crimes. We find that, for petty crimes, key players have a higher betweenness centrality than for more serious crimes. Indeed, if we look at *p90* (lower 90 percent of the distribution), we see that, among key players, at least 90 percent of them has a betweenness centrality less than 0.09 for serious crimes while, for petty crimes, this value is 0.33. Moreover, for petty crimes, the most active key players seem also to be the more central ones in terms of betweenness centrality while, for serious crimes, the most central key players in terms of betweenness centrality tend to be the less criminal ones.

We also find that the clustering coefficient can be helpful to understand why individuals who are not the most active criminals can be key players among delinquents that commit similar crimes. Indeed, more than 70 percent of the key players with the highest clustering coefficient (in the upper 90 percent tail) are not the most active criminals. This finding suggest that, even though some criminals do not commit much crime, they can be key players because they operate in tighter networks of best friends.

The distribution of key players in terms of closeness centrality is instead not very different between petty and serious crimes and does not show anymore (i.e. when the crimes committed are more homogeneous) that more active key players are over-represented in the upper 90 percent tail.

Finally, in Table 13, we investigate the role of network characteristics (see Appendix 7 for all the definitions) in describing the differences between key players who are the most active criminals and those who are not. In terms of statistical significance, the differences are not very pronounced. If we only look at the qualitative evidence, then we see that, for all crimes, the network diameter, network betweenness and the average distance are smaller in networks where the key player is also the most active criminal. An interesting suggestive result is that crime networks tend to be *assortative*, i.e. "popular" criminals tend to be friends with other "popular" criminals.

[*Insert Table* 13 *here*]

7 Policy implications

We would like now to discuss the implications of our key-player policy. The first key issue is *when* a key-player policy should be implemented and when it should not. Indeed, at the policy level, we assume that targeting potential criminals based on key-player characteristics does not impose any administrative costs. However, given the complex nature of some of these characteristics (e.g., school attachment, trouble relationship with teachers, social inclusion, religious practice, residential building quality, parental care,...), one should expect that tagging key-players is likely to be a source of important costs for the government. Therefore, it is not clear that targeting the relative key player rather that adopting a selection at random in the delinquency framework is optimal under strong targeting administrative costs. In order to answer this question, for each of the 145 networks (all crimes), we have calculated the reduction in total crime,[49] following the removal of the key player. Figure 2 (the horizontal axis is the index of the network and the vertical axis is the total crime reduction in percent) displays the results. One can see that there are very large variations in crime reduction between different networks. Indeed, for some networks, total crime is reduced by less than 5 percent while, for other networks, the reduction in crime can be as high as 35 percent. As a result, one could argue that for the networks for which the key player only reduces crime by less than say 5 or even 10 percent, then it could be better not to implement the key-player policy given the costs while, for the networks for which the key player reduces crime by more than 10 percent, implementing a key-player policy could offset the administrative costs associated with this policy.

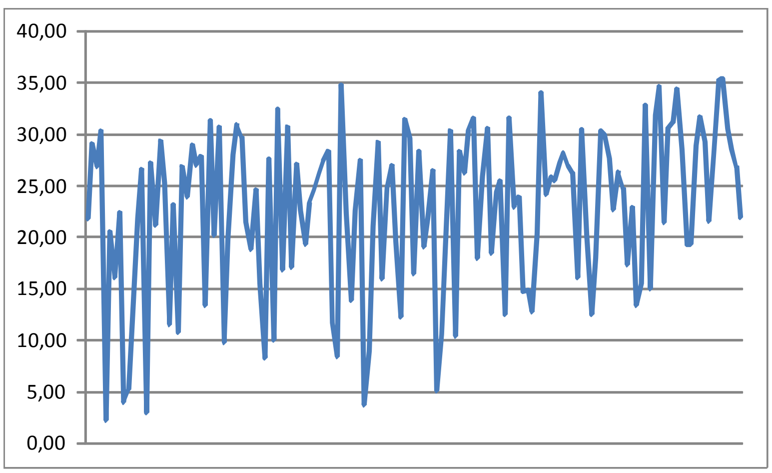

Figure 2: Distribution of networks by reduction in crime after the removal of the key player

In order to better understand this issue, we have reported in Figure 3 the relationship between crime reduction and network size (the horizontal axis is the number of nodes and the vertical axis is the total crime reduction in percent). Not surprisingly, the crime reduction is much more important in small networks than in large networks. This is because we remove only one key player and in large networks the effect is clearly lower than in small networks. A way to capture the size effect is to fix an objective in terms of crime reduction (say 10 percent) and analyze how many key players need to be removed in order to reach this objective. We

[49] This reduction is calculated as: [Expected total crime of the converged network (before the removal of the key player) - Expected total crime of the converged network (after the removal of the key player)] / Expected total crime of the converged network (before the removal of the key player).

have performed this exercise[50] by removing one by one key players[51] and show that this number increases with the size of the network, even though there are large variations for networks of same size. For the small networks (less than 10 delinquents), one key player is often enough (see Figure 2) while for large networks (more than 40 delinquents), more than three key players can be necessary to reach the objective of 10 percent reduction in crime. In that case, a planner would compare the costs of removing more than one key player with its benefits as compared to targeting a criminal at random. It should be clear, however, that it is indeed cheaper to implement a key-player policy for small networks since it is easier to figure out the structure of smaller networks and the crime reduction effects are larger.

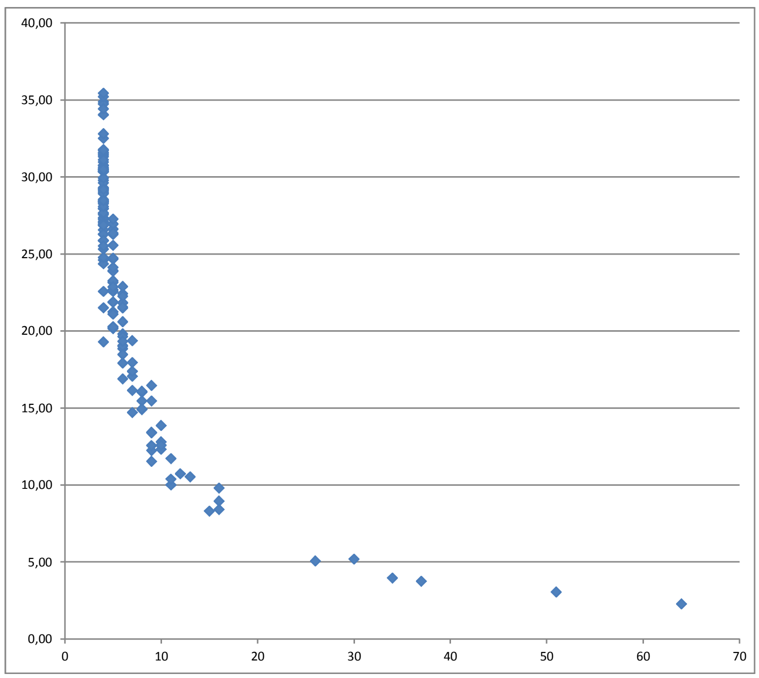

Figure 3: Crime reduction and network size

To further investigate this issue, in Table 14a, for the first 20 networks, we have reported the *maximum* crime reduction (key player policy, column 6) and the *average* crime reduction from a random-target policy (column 7) when removing a delinquent from the network. One can see that there are very large differences in crime reduction between implementing a

[50] Not reported here to save space but available upon request.

[51] Ballester et al. (2010) have shown that the approximation error of implementing a greedy algorithm that deletes one by one key players is small compared to deleting all key players at once, the latter being an NP-hard problem.

key-player policy and the average crime reduction. This is even true for small networks. For example, in network 1 (six delinquents), by implementing a key-player policy, the crime reduction is more 5 points more than the average crime reduction.

[*Insert Table* 14*a here*]

Figure 4 plots the average crime reduction for all the 145 networks when a key-player (blue curve) and a random-target policy (red curve) is implemented. To plot this figure, we have put together network of the same size and calculate the average crime reduction for this size of networks under the two policies. For example, for all networks of size 4 (horizontal axis), the average crime reduction is 29.94 percent on average when the key-player policy (vertical axis blue curve) is implemented while it is 23.86 percent when a random-target policy is implemented (vertical axis red curve). While Figure 4 focusses on networks of size less than 16, Table 4b gives the results for all network sizes. This confirmed what we obtained in Table 14a since the difference in crime reduction between these two policies can be extremely large, especially for big networks where implementing a random-target policy can backfire by increasing rather than decreasing crime.

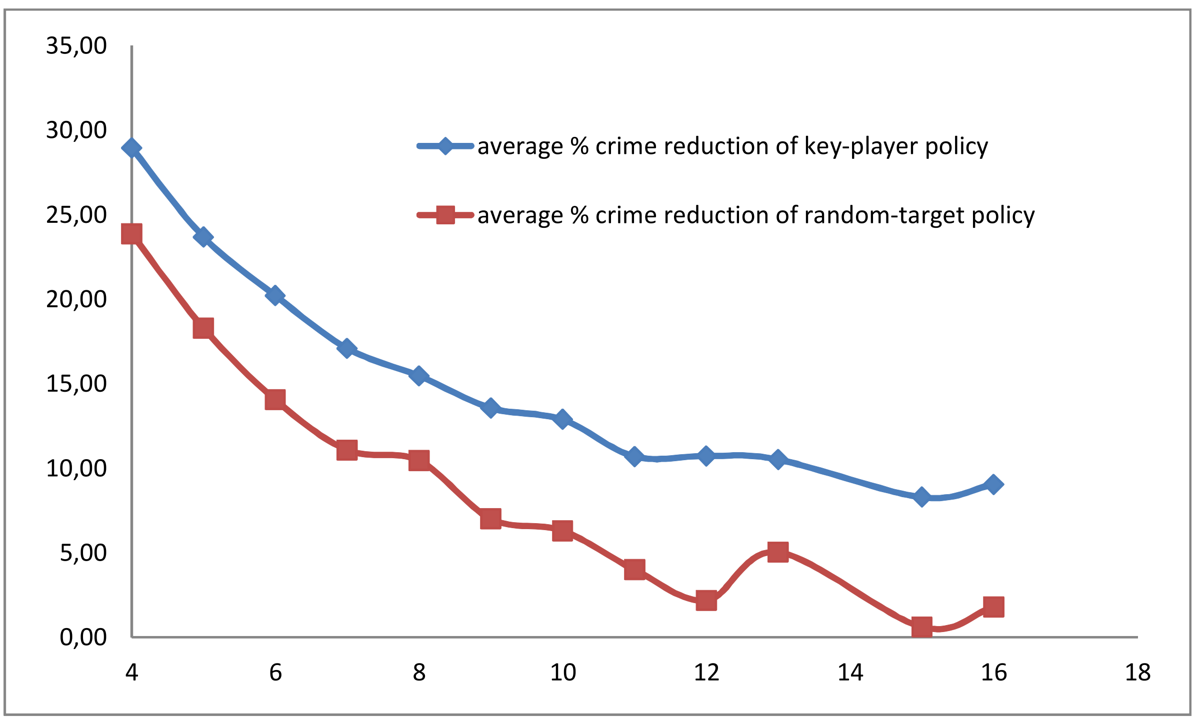

Figure 4: Difference between a key-player and a random-target policy

This implies that, if the administrative costs of implementing a key-player policy are large, then the latter should be implemented when the crime reduction difference between a key-player and a random-target policy is relatively high. We believe that we obtain these results because targeting key players generate large *multiplier and amplifying effects* as opposed to a random-target policy.

[*Insert Table* 14*b here*]

The second key issue is *how* to implement a key-player policy in the real-world. Indeed, in the previous section, we have shown that the characteristics of key players can be used to target some criminals if the information on networks is not known. However, several characteristics that can be used by the government to identify the key-player can be endogenous to the teenager (e.g., religious practice, social inclusion,...). Therefore, as long as he/she has some information regarding these characteristics, the teenager can change his/her behavior in order not to be targeted as a key player. Furthermore, policies that explicitly tie punishment to individual characteristics (e.g., family background and religious attachment) can raise a whole host of other legal and equity issues. In other words, if one targets certain individuals for greater scrutiny (e.g., racial or religious profiling)[52] and making punishments discriminatory (e.g., based on attributes rather than crimes), then this could create legal issues.

First, in our description of the characteristics of the key player, there are some characteristics that are truly *exogenous* and cannot be changed. These include (Table 5): mathematics scores (it is difficult to improve of their math scores for those who have difficulties at school), physical development, gender, parent education. Furthermore, even if a planner targets a key player based on his/her observable characteristics such as mathematics scores, physical development, gender, parent education, it does not mean that the key player is easy to replace. Indeed, if this person is removed based on his/her observable characteristics, it is possible that he/she has unobservable characteristics (such as leadership capacity, etc.) that are crucial in becoming the key player. In other words, even if someone has exactly the same observable characteristics, it does not mean that he/she can be the next key player since he/she may have different unobservable characteristics.

Second, it is clear that targeting students in terms of race or religious attributes could raise some legal issues. But here we put forward characteristics that are less controversial such as mathematics scores, physical development or parent education. Also, we are dealing with juvenile crime within schools. So for example, the head of a school could target students who have the lowest decile of math scores, who are more physically developed, etc. In the

[52] There is an important literature on racial and religious profiling. For an overview of this literature and a discussion on policy issues, see Risse and Zeckhauser (2004) and Durlauf (2006).

specific school context studied here, targeting students (key players) does not mean removing them from schools and isolated them on the basis of their characteristics. On the contrary, here, a key-player policy means to invest *extra resources* on the students. It is well known that juvenile crime is often committed after school (say between 3pm and 5pm). The head of the school can then "occupy" these kids (key players) by providing them some activities (sports or art) during this time.

There is indeed a small literature that discussed and tested policies aiming at "neutralizing" disruptive kids because of negative peer interaction effects they have on other kids. Lazear (2001) proposed a model showing that class size can be an issue if some kids are disruptive. Using our results, we could define the key player as the most disruptive student in a classroom, i.e., the student who once removed generates the highest possible increase in total education activity (as measured by the grades of the students). "Removing" the key player would mean here to put this student in another class or investing special resources (like having an extra teacher) on him/her. It is also often suggested that one way to reduce juvenile crime is to lengthen the school day or school year and/or to provide activities for young people when school is not in session. The implicit notion behind such program-oriented solutions to juvenile crime is a belief in the importance of *incapacitation*—that, as Jacob and Lefgren (2003) put it: "idle hands are the devil's workshop" and that keeping kids busy will keep them out of trouble. Advocates of after-school and other youth programs frequently claim that juvenile violence peaks in the after-school hours on school days and in the evenings on nonschool days.

All the potential effects of school attendance on crime are likely to be relevant to changes in compulsory schooling, while the effects of in-service days and teacher strikes are likely to be limited to incapacitation and social interactions (Lochner, 2011). Any social interaction effects are likely to be magnified in the latter cases due to the universal nature of the policy. Using our framework, we could recommend the same policies to reduce juvenile crime (i.e. lengthen the school day or school year and/or to provide activities for young people when school is not in session) by targeting "key players" instead of encompassing everybody. In their conclusion, Jacob and Lefgren (2003) suggest that summer youth employment programs or smaller, neighborhood-based after-school programs, that provide structured activities for adolescents but do not substantially increase their concentration, may be the best way to reduce juvenile crime. We could apply the same type of programs to "key players" that we could identify using our framework. Targeting these "key players", i.e. delinquents who once removed generate the highest possible reduction in aggregate delinquent level in a network, can have large effects on crime because of the feedback effects or "social multipliers" at work.

There is an important debate, especially in Canada, on youth pre-sentence reports (PSRs). The pre-sentence report is a report prepared by a probation officer, typically at the request of a judge. The report is used by judges to assist in determining sentencing outcomes. Within the past five years, 9 of Canada's 13 provincial and territorial jurisdictions have adopted some form of *risk assessment* in the preparation of youth pre-sentence reports; a number of others are following this trend (Hannah-Moffat and Maurutto, 2003). To do so, *actuarial tools* have been used in sentencing wherein dispositions are determined on the basis of how closely a young offender matches some profile of likely offences. PSRs have typically constructed a narrative profile of offenders, including demographic data on age and gender, evidence of previous contact with the police or the courts, and information about current and previous charges and convictions. For example, PSRs in Manitoba and Saskatchewan, in an effort to ensure transparency, include a subsection identifying the precise tool used, and the final *numerical risk score* is reported (Maurutto and Hannah-Moffat, 2007). Interestingly, studies report as high as an 80 percent correlation between PSR recommendations and dispositions (Hagan 1975; Boldt et al., 1983). A more recent study by Bonta et al., (2005) reports 87.4 percent satisfaction with PSRs for judges and found that 68 percent of judges found them useful. One could easily use the key-player methodology to determine the key characteristics that induce criminals to recidivate in order to calculate this numerical risk score.

Finally, it is important to remember that all our results (especially in terms of characteristics of key player) are only valid for the AddHealth data, that is for juvenile/adolescent crimes where the network is composed of *social relationships* (best friends). Even for this network, we have seen that the characteristics of key players vary a lot between different types of crimes. This means that we cannot transpose our results in terms of individual and network characteristics of the key players to other types of crime (like e.g. drunk driving), other types of networks (like e.g. drug networks) and other types of criminals (like e.g. adult criminals). We can, however, use the same methodology to identify key players in other contexts. Our methodology requires knowing the network, which is something that police usually knows quite well. For example, Sarnecki (2001) provides a comprehensive study of co-offending relations and corresponding network structure for football hooligans and right-wing extremists in Stockholm. Baker and Faulkner (1993) reconstruct the structure of conspiracy networks for three well-known cases of collusion in the heavy electrical equipment industry in the U.S. In all these cases, one may directly use the available data to determine the key player. In some other cases, though, ad hoc information gathering programs have to be implemented. Interestingly, Costebander and Valente (2003) show that centrality measures based on connectivity (rather than betweenness) are robust to mispecifications in sociometric data, and thus open the door to estimations of centrality measures with incomplete samples of network data. This, obviously, reduces the cost of identifying the key player.[53]

8 Concluding remarks

In this paper, we analyze delinquent networks of adolescents in the United States. We first develop a static model of peer effects where it is shown that the individual crime efforts is proportional to their Bonacich centrality. We also consider the decision to become criminal before joining the criminal network. We then develop a dynamic network formation model where delinquents decide how much effort to exert in criminal activities and with whom they want to form friendship links with. We are able to determine who the key player is, i.e. the criminal who once removed generates the highest possible reduction in aggregate crime level in equilibrium.

We test these models for adolescents in the United States (AddHealth data). We first show that peer effects are important in crime. If we consider an average group of 4 best friends in a network, we find that a standard deviation increase in the level of delinquent activity of each of the peers translates into a roughly 17 percent increase of a standard deviation in the individual level of activity. We also find that the impact of peer effects on crime are much higher (almost double) for more serious crimes than for petty crimes. Concerning the decision to become criminal, we find that the individuals' characteristics that affect the crime decision vary with the type of crime committed. For example, we find that female teenagers are more likely to commit petty crimes and much less likely to commit serious crimes than male teenagers. Similarly, blacks are more likely to commit serious crime than whites while there are no statistically differences between blacks and whites for petty crimes.

We then structurally estimate our model to determine who the key player is. We find that it is *not* straightforward to determine which delinquent should be removed from a network by only observing his or her criminal activities or position in the network. In other words, the key player is often not the criminal who has the highest Bonacich centrality or the highest

[53] The idea behind these results is that these measures take into account *all* walks in the network. Thus, generally the centrality of a player is not determined only by his/her direct links but by the complete structure of the network. In this sense, the probability that a missing link affects the choice of the most central/intercentral player is smaller than with other type of measures.

betweenness centrality. We also draw a "profile" of the key player in terms of individual and friends' characteristics and show that no specific characteristic can be singled out. Indeed, compared to other criminals, "key" criminals are less likely to be a female, are less religious, belong to families whose parents are less educated and have the perception of being socially more excluded. They also feel that their parents care less about them, are less likely to come from families where both parents are married and have more troubles getting along with the teachers. Those characteristics, however, vary with the type of crime committed.

To summarize, our analysis allows us to identify the key player and can help policy makers identify the most "harmful" individual in crime networks. It can also be extended to identify the *group* of the most "harmful" criminals if a given crime reduction is targeted.

We believe that our key-player policy has more general policy implications and can be applied to contexts other than crime and education. For example, the financial market is very connected and can be considered as a network where links could be loans between banks (Leitner, 2005; Cohen-Cole et al., 2011). A key-player policy would be to identify the key bank that needs to be bailed out in order for the system to resist a financial crisis. We could also apply the key player policy to the issue of adoption of a new technology in developing countries. There is indeed strong evidence of social learning (Conley and Udry, 2010). One could therefore identify key players and target them so that their influence on others will be crucial in the adoption of a new technology. Another application of a key player policy could be the political world. There is evidence that personal connections amongst politicians have a significant impact on the voting behavior of U.S. politicians (Cohen and Malloy, 2010). One could identify "key politicians" who should be promoted within the party because they would have a significant impact on election outcomes.

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Appendices (Not for publication)

Appendix 1: Proofs of theoretical results

Proof of Proposition 1. Apply Theorem 1, part b, in Calvó-Armengol et al. (2009) to our problem. ▮

Proof of Proposition 2. Apply Theorem 3 in Ballester et al. (2006) to our problem. ▮

Proof of Proposition 3. The conditions $\phi\mu_1(\boldsymbol{G}) < 1$ implies that $b_j(g_{N\cup\{j\}}, \phi)$ is well-defined for all $N \subset N$ and $j \in \bar{N}\backslash N$. Given that $\mu_1(g_{N\cup\{j\}}) \geq \mu_1(g_N)$, $b_i(g_N, \phi)$ is also well-defined for all $i \in N$. On the other hand, by Proposition 1, this also implies the uniqueness of the Nash equilibrium in the second stage game defined, respectively, by g_N and $g_{N\cup\{j\}}$, for all $j \in \bar{N}\backslash N$:

$$y_i^*(g_N) = b_i(g_N, \phi) \text{ for all } i \in N \tag{33}$$

$$y_i^*(g_{N\cup\{j\}}) = b_i(g_{N\cup\{j\}}, \phi) \text{ for all } i \in N \tag{34}$$

Now, uniqueness in the second-stage allows us to concentrate on the pure strategy Nash equilibria of the whole game where no agent j outside a sustainable N would be willing to enter the game in the network g_N to obtain $u_j\left(y^*(g_{N\cup\{j\}}), g_{N\cup\{j\}}\right)$; and no agent $i \in N$ would be better off by obtaining $\bar{u}$, rather that $u_i\left(y^*(g_N), g_N\right)$. Formally, a set N is supported by $\bar{u}$ at equilibrium if and only if:

$$\max_{j\in\bar{N}\backslash N} u_j\left(y^*(g_{N\cup\{j\}}), g_{N\cup\{j\}}\right) \leq \bar{u} \leq \min_{i\in N} u_i\left(y^*(g_N), g_N\right).$$

The result follows by using (33) and (34) and (8). ▮

Appendix 2: Illustrative examples of the theoretical results

Example 1 (Intercentrality measure: who is the key player?)

To illustrate Proposition 2 and formula (10), consider the following symmetric undirected network with four delinquents (i.e. $n = 4$):

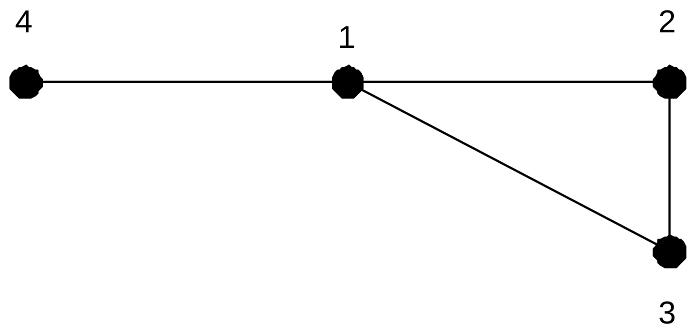

Figure A1: A network with 4 criminals

The adjacency matrix is then given by:

$$\boldsymbol{G} = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Assume $\phi = 0.3$, $\beta_1 = \beta_2 = 1$ and that[54]

$$\boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \end{pmatrix}$$

so that

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{pmatrix} = \begin{pmatrix} 0.4 \\ 0.4 \\ 0.45 \\ 0.5 \end{pmatrix} \tag{35}$$

[54]The spectral radius of this graph is: 2.17 and thus the condition $\phi\mu_1(\mathbf{G}) < 1$ is satisfied since $2.17 \times 0.3 = 0.651 < 1$.

It is then straightforward to see that, using Proposition 1, we obtain:

$$\begin{pmatrix} y_1^* \\ y_2^* \\ y_3^* \\ y_4^* \end{pmatrix} = \begin{pmatrix} b_1(g,\phi) \\ b_2(g,\phi) \\ b_3(g,\phi) \\ b_4(g,\phi) \end{pmatrix} = \begin{pmatrix} 1.4004 \\ 1.1881 \\ 1.2265 \\ 0.92016 \end{pmatrix}$$

so that the total activity level is given by:

$$Y^*(g) = \boldsymbol{l}_4' \boldsymbol{M} \boldsymbol{\alpha} = y_1^*(g) + y_2^*(g) + y_3^*(g) + y_4^*(g) = B(g,\phi) = 4.735$$

Individual 1 has the highest weighted Bonacich and thus provides the highest crime effort. If we look at the formula in Definition 2, it says that the delinquent that the planner wants to remove is:

$$d_{i^*}(g,\phi) = B(g,\phi) - B(g^{[-i]},\phi)$$

Let us remove delinquent 1. The network becomes:

Figure A2: The network when criminal 1 has been removed

We have now a network with three delinquents where we have deleted the first column and first row in $\boldsymbol{G}$ to obtain:

$$\boldsymbol{G}^{[-1]} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

What is important is that the αs also change. Indeed, $(\alpha_2, \alpha_3, \alpha_4)$ are not anymore given by (35) but by:

$$\boldsymbol{\alpha}^{[-1]} = \begin{pmatrix} \alpha_2^{[-1]} \\ \alpha_3^{[-1]} \\ \alpha_4^{[-1]} \end{pmatrix} = \begin{pmatrix} 0.5 \\ 0.5 \\ 0.4 \end{pmatrix} \tag{36}$$

Using the same decay factor, $\phi = 0.3$, we obtain:[55]

$$\begin{pmatrix} y_2^* \\ y_3^* \\ y_4^* \end{pmatrix} = \begin{pmatrix} b_2(g^{[-1]}, \phi) \\ b_3(g^{[-1]}, \phi) \\ b_4(g^{[-1]}, \phi) \end{pmatrix} = \begin{pmatrix} 0.71429 \\ 0.71429 \\ 0.4 \end{pmatrix}$$

so that the total crime effort is now equal to:

$$Y^*(g^{[-1]}) = y_2^*(g^{[-1]}) + y_3^*(g^{[-1]}) + y_4^*(g^{[-1]}) = B(g^{[-1]}, \phi) = 1.8286$$

Thus, player 1's contribution is

$$B(g, \phi) - B(g^{[-1]}, \phi) = \boldsymbol{l}_4' \boldsymbol{M} \boldsymbol{\alpha} - \boldsymbol{l}_4' \boldsymbol{M}^{[-1]} \boldsymbol{\alpha}^{[-1]} = 4.735 - 1.8286 = 2.9064 \tag{37}$$

Doing the similar exercise for individuals 2, 3, 4, we obtain:

$$\begin{aligned} B(g, \phi) - B(g^{[-2]}, \phi) &= 2.4301, \\ B(g, \phi) - B(g^{[-3]}, \phi) &= 2.6862, \\ B(g, \phi) - B(g^{[-4]}, \phi) &= 1.735. \end{aligned}$$

Criminal 1 is the key player since her contribution to total crime is the highest one.

Let us now check if the formula (10) is correct, i.e.,

$$\begin{aligned} d_1(g, \phi) &= B(g, \phi) - B(g^{[-1]}, \phi) \\ &= \boldsymbol{l}_4' \boldsymbol{M} \boldsymbol{\alpha} - \boldsymbol{l}_4' \boldsymbol{M} \boldsymbol{\alpha}^{[1]} + \boldsymbol{l}_4' \boldsymbol{M}^{[1]} \boldsymbol{\alpha}^{[1]} \\ &= B(g, \phi) - B(g^{[i]}, \phi) + \frac{b_{\boldsymbol{\alpha}^{[1]},1}(g, \phi) \sum_{j=1}^{n} m_{j1}(g, \phi)}{m_{11}(g, \phi)} \end{aligned}$$

It is easily verified that

$$\boldsymbol{M} = \begin{pmatrix} 1.5317 & 0.65646 & 0.65646 & 0.45952 \\ 0.65646 & 1.3802 & 0.61101 & 0.19694 \\ 0.65646 & 0.61101 & 1.3802 & 0.19694 \\ 0.45952 & 0.19694 & 0.19694 & 1.1379 \end{pmatrix},$$

[55] Since individual 4 is now isolated, we have:

$$y_4^* = \alpha_4 = 0.4$$

$$\boldsymbol{M}^{[1]}=\begin{pmatrix} 1.5317 & 0.65646 & 0.65646 & 0.45952 \\ 0.65646 & 0.28135 & 0.28135 & 0.19694 \\ 0.65646 & 0.28135 & 0.28135 & 0.19694 \\ 0.45952 & 0.19694 & 0.19694 & 0.13786 \end{pmatrix}$$

and

$$\boldsymbol{\alpha}^{[1]}=\begin{pmatrix} \alpha_1 \\ \alpha_2^{[-1]} \\ \alpha_3^{[-1]} \\ \alpha_4^{[-1]} \end{pmatrix}=\begin{pmatrix} 0.4 \\ 0.5 \\ 0.5 \\ 0.4 \end{pmatrix}$$

As a result, $\boldsymbol{l}_4'\boldsymbol{M}\boldsymbol{\alpha}^{[1]}=4.9628$ so that

$$\boldsymbol{l}_4'\boldsymbol{M}\boldsymbol{\alpha}-\boldsymbol{l}_4'\boldsymbol{M}\boldsymbol{\alpha}^{[1]}=B(g,\phi)-B(g^{[1]},\phi)=4.735-4.9628=-0.2278$$

and

$$\boldsymbol{l}_4'\boldsymbol{M}^{[1]}\boldsymbol{\alpha}^{[1]}=\frac{b_{\boldsymbol{\alpha}^{[1]},1}(g,\phi)\sum_{j=1}^{n}m_{j1}(g,\phi)}{m_{11}(g,\phi)}=3.1342$$

We thus obtain:

$$\begin{aligned} d_1(g,\phi) &= B(g,\phi)-B(g^{[1]},\phi)+\frac{b_{\boldsymbol{\alpha}^{[1]},1}(g,\phi)\sum_{j=1}^{n}m_{j1}(g,\phi)}{m_{11}(g,\phi)} \\ &= -0.2278+3.1342=2.9064 \end{aligned} \tag{38}$$

When comparing (37) and (38), we see that the values are the same and thus:

$$d_1(g,\phi)=B(g,\phi)-B(g^{[-1]},\phi)=2.9064$$

Example 2 (Is the key player always the most active player?)

Consider the network g in the following figure with eleven criminals.

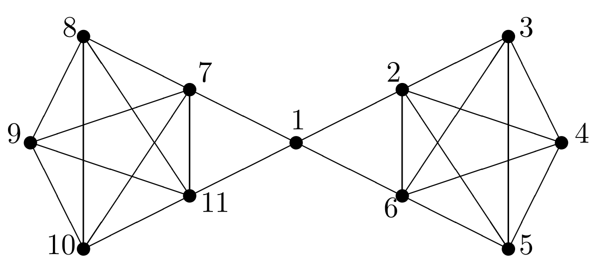

Figure A3: A bridge network with 11 criminals

We distinguish three different types of equivalent actors in this network, which are the following:

Type	Criminals
1	1
2	2, 6, 7 and 11
3	3, 4, 5, 8, 9 and 10

From a macro-structural perspective, type-1 and type-3 criminals are identical: they all have four direct links, while type -2 criminals have five direct links each. From a micro-structural perspective, though, criminal 1 plays a critical role by bridging together two closed-knit (fully intraconnected) communities of five criminals each. By removing delinquent 1, the network is maximally disrupted as these two communities become totally disconnected, while by removing any of the type-2 criminals, the resulting network has the lowest aggregate number of network links.

We identify the key player in this network of criminals. If the choice of the key player were solely governed by the *direct* effect of criminal removal on aggregate crime, type-2 criminals would be the natural candidates. Indeed, these are the ones with the highest number of direct connections. But the choice of the key player needs also to take into account the *indirect* effect on aggregate delinquency reduction induced by the network restructuring that follows the removal of one delinquent from the original network. Because of his communities' bridging role, criminal 1 is also a possible candidate for the preferred policy target.

In order to focus on the role of location in the network, in this example, we assume that criminals are ex identical so that $\boldsymbol{\alpha} = \boldsymbol{l}_3$ and thus $\boldsymbol{b}(g, \phi) = (\boldsymbol{I} - \phi \boldsymbol{G})^{-1} \boldsymbol{l}_3$ and $y_i^* = b_i(g, \phi)$ while $d_i(g, \phi) = B(g, \phi) - B(g^{[-i]}, \phi)$. We take $\phi = 0.2$. The following table computes, for criminals of types 1, 2 and 3, the value of delinquency centrality measures $b_i(g, \phi)$ (or equivalently efforts y_i^*) and intercentrality measures $d_i(g, \phi)$ for different values of ϕ. In each column, a variable with a star identifies the highest value.[56]

Table A1a: Key player versus Bonacich centrality in a bridge network

Player Type	1	2	3
$y_i = b_i$	8.33	9.17*	7.78
d_i	41.67*	40.33	32.67

First note that type-2 delinquents display the highest Bonacich centrality measure. These delinquents have the highest number of direct connections. Besides, they are directly

[56] We can compute the highest possible value for ϕ compatible with our definition of centrality measure (i.e. the inverse of the largest eigenvalue of G), which is equal to $\widehat{\phi} = \frac{2}{3+\sqrt{41}} \simeq 0,213$.

connected to the bridge delinquent 1, which gives them access to a very wide and diversified span of indirect connections. Altogether, they are the most central delinquents (in terms of Bonacich centrality). Second, the most active delinquents are not the key players. Because indirect effects matter a lot ($\phi = 0.2$), eliminating delinquent 1 has the highest joint direct and indirect effect on aggregate delinquency reduction. Indeed, when ϕ is not too low, delinquents spread their know-how further away in the network and establishing synergies with delinquents located in distant parts of the social setting. In this case, the optimal targeted policy is the one that maximally disrupts the delinquency network, thus harming the most its know-how transferring ability.

In Table A1a, we have shown that the key player is not the most active criminal (i.e. does have the highest Bonacich centrality). To further understand this result, let us analyze the characteristics of all criminals in terms of network position, as well as those of the network described in Figure A3. For that, we will first use some measures of centrality other than Bonacich. Indeed, over the past years, social network theorists have proposed a number of centrality measures to account for the variability in network location across agents (Wasserman and Faust, 1994).[57] While these measures are mainly geometric in nature, our theory provides a behavioral foundation to the Bonacich centrality measure (and only this one) that coincides with the unique Nash equilibrium of a non-cooperative peer effects game on a social network. Let us now calculate for the network given in Figure 3 the other individual centrality measures, namely: degree, closeness, betweenness centralities as well as the clustering coefficient. Their mathematical definitions are given in Appendix 7. We obtain:

Table A1b: Characteristics of criminals in a network where the most active criminal is not the key player

Player type	1	2	3
Degree centrality	0.4	0.5	0.4
Closeness centrality	0.625	0.555	0.416
Betweenness centrality	0.555	0.2	0
Clustering coefficient	0.33	0.7	1

Even if player 1 is not the most active criminal (she has the lowest degree centrality and the lowest clustering coefficient), it is now even easier to understand why she is the key

[57] See Borgatti (2003) for a discussion on the lack of a systematic criterium to pick up the "right" network centrality measure for each particular situation.

player: she has the highest closeness and betweenness centralities. Observe that criminal 3 has a betweenness centrality equals to zero because there are no shortest path between two criminals that go through her.

Let us now examine the characteristics of the network described in Figure A3 where the key player is not the most active criminal. We will consider standard network characteristics, which are also all defined in Appendix 7. We obtain the following results:

Table A1c: Characteristics of the network
in which the most active criminal is not the key player

Network Characteristics	
Average Distance	2.11
Average Degree	4.36
Diameter	4
Density	0.211
Asymmetry	0.125
Clustering	0.805
Degree centrality	7.78×10^{-3}
Closeness centrality	0.323
Betweenness Centrality	0.47556
Assortativity	-3.49×10^{-16}

We see from Table A1c that the network described in Figure A3 has a low average distance and low diameter (small-world properties), a very high clustering (0.805) and a weak dissortativity. Furthermore, it is not very dense nor asymmetric while having average values of centralities measures.

Example 3 (Selection issue: who will be criminals?) Consider a population of 5 individuals, i.e., $\bar{N} = \{1, ..., 5\}$, where 3 of them become criminals, i.e., $N = \{3, 4, 5\}$. Assume, for simplicity, that $\epsilon_i = 0$ for all $i = 1, ..., 5$, $\bar{\eta} = 0$, $\phi = 0.2$, and

$$a_1 = 0.5 < a_2 = 1 < a_3 = 8 < a_4 = 9 < a_5 = 10$$

We would like to illustrate Propositions 3 and 4. The upper panel of Figure A4 describes the network of 3 criminals.

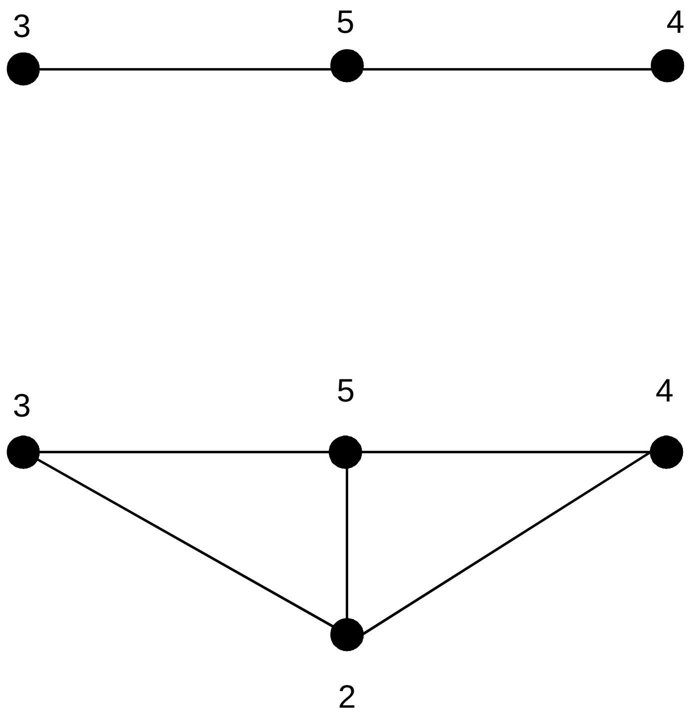

Figure A4: The decision to become criminal for 3 individuals

It is easily verified that:[58]

$$\begin{pmatrix} y_3^*(g_N) \\ y_4^*(g_N) \\ y_5^*(g_N) \end{pmatrix} = \begin{pmatrix} b_3(g_N, 0.2) \\ b_4(g_N, 0.2) \\ b_5(g_N, 0.2) \end{pmatrix} = \begin{pmatrix} 10.913 \\ 11.913 \\ 14.566 \end{pmatrix}$$

so that the right-hand side of condition (11), i.e., $2\sqrt{\bar{u}} \leq \min_{i \in N} b_i(g_N, \phi) = b_3(g_N, \phi)$, can be written as:

$$2\sqrt{\bar{u}} \leq 10.913$$

We also need to check the left-hand side of condition (11). For that, we take the individual with the highest as among the non-criminals, that is individual 2, and construct the best possible network for this individual to be connected with criminals 3, 4, and 5, and we obtain the network displayed in the lower panel of Figure A4. For this network, it is easily verified

[58] It can be verified that $\phi = 0.2 < 1/\sqrt{2} = 0.707$ (condition on the largest eigenvalue), which guarantees that the Nash equilibrium in efforts in the second stage is interior, unique and well-defined.

that:[59]

$$\begin{pmatrix} y_2^*\left(g_{N\cup\{2\}}\right) \\ y_3^*\left(g_{N\cup\{2\}}\right) \\ y_4^*\left(g_{N\cup\{2\}}\right) \\ y_5^*\left(g_{N\cup\{2\}}\right) \end{pmatrix} = \begin{pmatrix} b_2\left(g_{N\cup\{2\}}, 0.2\right) \\ b_2\left(g_{N\cup\{2\}}, 0.2\right) \\ b_4\left(g_{N\cup\{2\}}, 0.2\right) \\ b_5\left(g_{N\cup\{2\}}, 0.2\right) \end{pmatrix} = \begin{pmatrix} 10.156 \\ 13.563 \\ 14.563 \\ 17.656 \end{pmatrix}$$

As a result, $b_2(g_{N\cup\{2\}}, 0.2) = \max_{j\in\bar{N}\setminus N} b_j(g_{N\cup\{j\}}, \phi) \leq 2\sqrt{\bar{u}}$ can be written as:

$$10.156 \leq 2\sqrt{\bar{u}}$$

Putting these two inequalities together, condition (11) can be written as:

$$10.156 \leq 2\sqrt{\bar{u}} \leq 10.913$$

Hence, any value of $\bar{u}$ such that $25.786 \leq \bar{u} \leq 29.773$ satisfies condition (11) and thus the network displayed in the upper panel of Figure A4 is an equilibrium criminal network with endogenous participation.

[59] It can be verified that $\phi = 0.2 < \left(1+\sqrt{17}\right)/2 = 0.39$, which guarantees that the Nash equilibrium in efforts in the second stage is interior, unique and well-defined.

Appendix 3: Data appendix

Table A2: Description of Variables

	Variable definition
Individual socio-demographic variables	
Female	Dummy variable taking value one if the respondent is female.
Religion practice	Response to the question: "In the past 12 months, how often did you attend religious services", coded as 1= never, 2= less than once a month, 23= once a month or more, but less than once a week, 4= once a week or more. Coded as 5 if the previous is skipped because of response “none” to the question: “What is your religion?”
Student grade	Grade of student in the current year.
Black or African American	Race dummies. “White” is the reference group.
Other races	“
Mathematics score	Score in mathematics at the most recent grading period, coded as4= D or lower, 3= C, 2=B, 1=A.
Self esteem	Response to the question: "Compared with other people your age, how intelligent are you", coded as 1= moderately below average, 2= slightly below average, 3= about average, 4= slightly above average, 5= moderately above average, 6= extremely above average.
Physical development	Response to the question: "How advanced is your physical development compared to other boys/girls your age", coded as 1= I look younger than most, 2= I look younger than some, 3= I look about average, 4= I look older than some, 5= I look older than most
Family background variables	
Household size	Number of people living in the household.
Two married parent family	Dummy taking value one if the respondent lives in a household with two parents (both biological and non biological) that are married.
Single parent family	Dummy taking value one if the respondent lives in a household with only one parent (both biological and non biological).
Parent education	Schooling level of the (biological or non-biological) parent who is living with the child, distinguishing between "never went to school", "not graduate from high school", "high school graduate", "graduated from college or a university", "professional training beyond a four-year college", coded as 1 to 5. We consider only the education of the father if both parents are in the household.
Parent occupation manager	Parent occupation dummies. Closest description of the job of (biological or non-biological) parent that is living with the child is manager. If both parents are in the household, the occupation of the father is considered. “none” is the reference group
Parent occupation professional/technical	”
Parent occupation office or sales worker	”
Parent occupation manual	”
Parent occupation military or security	”
Parent occupation farm or fishery	”
Parent occupation other	”
Protective factors	
School attachment	Response to the question: "You feel like you are part of your school coded as 5= strongly agree, 4= agree, 3=neither agree nor disagree, 2= disagree, 1= strongly disagree.
Relationship with teachers	Response to the question: “How often have you had trouble getting along with your teachers?” 0= never, 1= just a few times, 2= about once a week, 3= almost everyday, 4=everyday
Social inclusion	Response to the question: "How much do you feel that adults care about you, coded as 5= very much, 4= quite a bit, 3= somewhat, 2= very little, 1= not at all
Parental care	Dummy taking value one if the respondent reports that the (biological or non-biological) parent that is living with her/him or at least one of the parents if both are in the household cares very much about her/him
Residential neighborhood variables	
Residential building quality	Interviewer response to the question "How well kept is the building in which the respondent lives", coded as 4= very poorly kept (needs major repairs), 3= poorly kept (needs minor repairs), 2= fairly well kept (needs cosmetic work), 1= very well kept.
Residential area urban	Interviewer description of the residential immediate area or street (one block, both sides). Dummy taking value 1 if the area is “urban” and 0 otherwise (“suburban”, “industrial properties - mostly wholesale” , “rural area”, “other type”)

Table A3: Summary statistics

		Non criminals (population) N = 6,993		Criminals (population) N = 5,522		Criminals (selected sample) N = 1,297	
		Network size range 2-1,050				Network size range 4-150	
VARIABLES	range min-max	mean	sd	mean	sd	mean	sd
Female	0-1	0.55	0.50	0.50	0.50	0.49	0.50
Religion practice	1-5	3.85	1.36	3.66	1.39	3.73	1.40
Student grade	7-12	9.41	1.66	9.63	1.58	9.01	1.56
Black or African American	0-1	0.19	0.39	0.20	0.40	0.22	0.42
Other races	0-1	0.14	0.35	0.16	0.36	0.05	0.23
Mathematics score	1-4	2.14	0.99	2.36	1.04	2.19	1.01
Self esteem	1-6	4.03	1.10	3.90	1.08	4.00	1.09
Physical development	1-5	3.10	1.09	3.26	1.11	3.34	1.11
Household size	1-11	4.57	1.48	4.60	1.51	4.39	1.33
Two married parent family	0-1	0.73	0.44	0.71	0.45	0.71	0.45
Single parent family	0-1	0.23	0.42	0.25	0.43	0.25	0.43
Parent education	0-5	3.15	1.08	3.10	1.09	3.23	1.10
Parent occupation manager	0-1	0.11	0.31	0.10	0.30	0.12	0.32
Parent occupation professional/technical	0-1	0.18	0.38	0.19	0.39	0.21	0.41
Parent occupation office or sales worker	0-1	0.11	0.31	0.11	0.32	0.11	0.31
Parent occupation manual	0-1	0.34	0.47	0.34	0.47	0.30	0.46
Parent occupation military or security	0-1	0.03	0.18	0.03	0.16	0.02	0.14
Parent occupation farm or fishery	0-1	0.02	0.15	0.01	0.11	0.02	0.13
Parent occupation other	0-1	0.13	0.34	0.14	0.34	0.14	0.35
School attachment	1-5	4.07	0.88	3.88	0.98	4.06	0.92
Relationship with teachers	0-4	0.52	0.77	0.92	0.96	1.08	1.01
Social inclusion	1-5	4.52	0.76	4.40	0.79	4.46	0.76
Parental care	0-1	0.94	0.24	0.91	0.28	0.92	0.28
Residential building quality	1-4	1.53	0.80	1.58	0.80	1.53	0.81
Residential area urban	0-1	0.67	0.47	0.72	0.45	0.61	0.49

Variables in italics denote statistically significant differences in means between criminals and non-criminals at least at the 10 percent significance level

Appendix 4: Identification of network models with non-row-normalized adjacency matrices

Consider the following model

$$\begin{aligned} \boldsymbol{y}_r &= \phi_0 \boldsymbol{G}_r \boldsymbol{y}_r + \boldsymbol{X}_r \boldsymbol{\beta}_{10} + \boldsymbol{G}_r^* \boldsymbol{X}_r \boldsymbol{\beta}_{20} + \eta_r \boldsymbol{l}_{n_r} + \boldsymbol{\epsilon}_r \\ &= [\boldsymbol{G}_r \boldsymbol{y}_r, \boldsymbol{X}_r, \boldsymbol{G}_r^* \boldsymbol{X}_r, \boldsymbol{l}_{n_r}]\vartheta + \boldsymbol{\epsilon}_r, \end{aligned} \tag{39}$$

where $\boldsymbol{G}_r^*$ is the row-normalized $\boldsymbol{G}_r$ and $\boldsymbol{\vartheta} = (\phi_0, \boldsymbol{\beta}_{10}', \boldsymbol{\beta}_{20}', \eta_r)'$. To achieve model identification based only on the reduced form regression equation, we need that the deterministic part of the right hand side variables, $[\mathrm{E}(\boldsymbol{G}_r \boldsymbol{y}_r), \boldsymbol{X}_r, \boldsymbol{G}_r^* \boldsymbol{X}_r, \boldsymbol{l}_{n_r}]$, have full column rank, where

$$\mathrm{E}(\boldsymbol{G}_r \boldsymbol{y}_r) = \boldsymbol{G}_r \boldsymbol{X}_r \boldsymbol{\beta}_{10} + \phi_0 \boldsymbol{G}_r \boldsymbol{M}_r \boldsymbol{G}_r \boldsymbol{X}_r \boldsymbol{\beta}_{10} + \boldsymbol{G}_r \boldsymbol{M}_r \boldsymbol{G}_r^* \boldsymbol{X}_r \boldsymbol{\beta}_{20} + \eta_r \boldsymbol{G}_r \boldsymbol{M}_r \boldsymbol{l}_{n_r}. \tag{40}$$

First, we consider the case that $\boldsymbol{G}_r$ is row-normalized such that $\boldsymbol{G}_r = \boldsymbol{G}_r^*$. In this case, (40) can be simplified as

$$\mathrm{E}(\boldsymbol{G}_r \boldsymbol{y}_r) = \boldsymbol{G}_r^* \boldsymbol{X}_r \boldsymbol{\beta}_{10} + \boldsymbol{G}_r^* \boldsymbol{M}_r \boldsymbol{G}_r^* \boldsymbol{X}_r (\phi_0 \boldsymbol{\beta}_{10} + \boldsymbol{\beta}_{20}) + \frac{\eta_r}{1-\phi_0} \boldsymbol{l}_{n_r}.$$

To illustrate the challenges in identification, we consider three cases. (1) $\boldsymbol{\beta}_{10} = \boldsymbol{\beta}_{20} = 0$. This is the case when there is no relevant exogenous variables in the model. In this case, $\mathrm{E}(\boldsymbol{G}_r \boldsymbol{y}_r) = \frac{\eta_r}{1-\phi_0} \boldsymbol{l}_{n_r}$. Hence, the model is not identified because $[\frac{\eta_r}{1-\phi_0} \boldsymbol{l}_{n_r}, \boldsymbol{X}_r, \boldsymbol{G}_r^* \boldsymbol{X}_r, \boldsymbol{l}_{n_r}]$ does not have full column rank. (2) $\phi_0 \boldsymbol{\beta}_{10} + \boldsymbol{\beta}_{20} = 0$. In this case, $\mathrm{E}(\boldsymbol{G}_r \boldsymbol{y}_r) = \boldsymbol{G}_r^* \boldsymbol{X}_r \beta_0 + \frac{\eta_r}{1-\phi_0} \boldsymbol{l}_{n_r}$. The model can not be identified due to perfect collinearity. This corresponds to the case where the endogenous effect and exogenous effect exactly cancel out. Lee et al. (2010) have shown, in this case, the reduced form of (39) becomes a simple regression model with (spatially) correlated disturbances. In the reduced form, there are neither endogenous nor contextual effects. Interactions go through unobservables (disturbances) instead of observables. (3) $\phi_0 \boldsymbol{\beta}_{10} + \boldsymbol{\beta}_{20} \neq 0$. For this case, Bramoullé et al. (2009) and Lee et al. (2010) have derived some sufficient conditions for model identification, which are simpler to interpret. The identification can still be hard to achieve when the network is dense. For example, the "reflection problem", where the endogenous effects can not be identified from the contextual effects due to perfect collinearity, exists in the model of Manski (1993), which has the linear-in-mean specification such that $\boldsymbol{G}_r = \frac{1}{n_r} \boldsymbol{l}_{n_r} \boldsymbol{l}_{n_r}'$. When $\boldsymbol{G}_r = \frac{1}{(n_r-1)}(\boldsymbol{l}_{n_r} \boldsymbol{l}_{n_r}' - \boldsymbol{I})$ and networks are of the same size such that $n_r = n/\bar{r}$, the model still can not be identified (see Moffitt, 2001). On the other hand, when $\boldsymbol{G}_r = \frac{1}{(n_r-1)}(\boldsymbol{l}_{n_r} \boldsymbol{l}_{n_r}' - \boldsymbol{I})$ and there are variations in network sizes, Lee (2007a) has shown that the model can be identified because the endogenous effect is stronger in small networks than in large networks. However, the identification can be weak when the all networks are large.

Row-normalization of $\boldsymbol{G}_r$ has some limitations. First, as in the structural model in this paper, one may be interested in the aggregate influence rather than average influence of the peers. Second, for some network structures, it is impossible to row normalize the adjacency matrix $\boldsymbol{G}_r$. For example, for an asymmetric $\boldsymbol{G}_r$, where agent i's outcome affects peers' outcomes but he/she is not affected by peers, the ith row of $\boldsymbol{G}_r$ would be all zeros. It would be impossible to normalize the ith row of $\boldsymbol{G}_r$ to sum to one. Finally, normalization may eliminate some useful information of the network structure. For the undirected friendship network, $\boldsymbol{G}_r$ will be a symmetric matrix. It should not be row-normalized because row-normalization would destroy the symmetry property.

Indeed, $\boldsymbol{G}_r\boldsymbol{M}_r\boldsymbol{l}_{n_r}$ is the measure of centrality in Bonacich (1987). The ith entry of $\boldsymbol{G}_r\boldsymbol{M}_r\boldsymbol{l}_{n_r}$ is the (weighted) sum of direct and indirect connections of agent i with others in the network. When $\boldsymbol{G}_r$ is not row-normalized, the entries of $\boldsymbol{G}_r\boldsymbol{M}_r\boldsymbol{l}_{n_r}$ in general is not all the same. The variation of this centrality measure in a network provides useful information for model identification. Even for the case that $\boldsymbol{\beta}_{10} = \boldsymbol{\beta}_{20} = 0$, with non-row-normalized $\boldsymbol{G}_r$, $[\mathrm{E}(\boldsymbol{G}_r\boldsymbol{y}_r), \boldsymbol{X}_r, \boldsymbol{G}_r^*\boldsymbol{X}_r, \boldsymbol{l}_{n_r}] = [\eta_r\boldsymbol{G}_r\boldsymbol{M}_r\boldsymbol{l}_{n_r}, \boldsymbol{X}_r, \boldsymbol{G}_r^*\boldsymbol{X}_r, \boldsymbol{l}_{n_r}]$ can still have full column rank. Hence, the model can be identified.

Under a certain regularity condition, $\boldsymbol{M}_r = \sum_{j=0}^{\infty}(\phi_0\boldsymbol{G}_r)^j$. It follows that $\boldsymbol{G}_r\boldsymbol{M}_r\boldsymbol{G}_r\boldsymbol{X}_r = \sum_{j=0}^{\infty}(\phi_0\boldsymbol{G}_r)^j\boldsymbol{G}_r^2\boldsymbol{X}_r$, $\boldsymbol{G}_r\boldsymbol{M}_r\boldsymbol{G}_r^*\boldsymbol{X}_r = \sum_{j=0}^{\infty}(\phi_0\boldsymbol{G}_r)^j\boldsymbol{G}_r\boldsymbol{G}_r^*\boldsymbol{X}_r$ and $\boldsymbol{G}_r\boldsymbol{M}_r\boldsymbol{l}_{n_r} = \sum_{j=0}^{\infty}(\phi_0\boldsymbol{G}_r)^j\boldsymbol{G}_r\boldsymbol{l}_{n_r}$. Hence, from (40) we can use terms like $\boldsymbol{G}_r\boldsymbol{l}_{n_r}$ as IVs for the endogenous effect in addition to the "traditional" IVs like $\boldsymbol{G}_r\boldsymbol{X}_r$, $\boldsymbol{G}_r^2\boldsymbol{X}_r$ and/or $\boldsymbol{G}_r\boldsymbol{G}_r^*\boldsymbol{X}_r$ to help model identification and improve estimation efficiency (Liu and Lee, 2010).

Appendix 5: 2SLS and GMM estimators

2SLS Estimation From the reduced form equation (24), $\mathrm{E}(\boldsymbol{Z}) = [\boldsymbol{G}\boldsymbol{M}(\bar{\boldsymbol{X}}\boldsymbol{\delta}_0+\boldsymbol{\iota}\bar{\boldsymbol{\eta}}), \bar{\boldsymbol{X}}]$. The best (in terms of efficiency) instrumental matrix for $\boldsymbol{J}\boldsymbol{Z}$ in (23) is given by

$$\boldsymbol{F} = \boldsymbol{J}\mathrm{E}(\boldsymbol{Z}) = \boldsymbol{J}[\boldsymbol{G}\boldsymbol{M}\bar{\boldsymbol{X}}\boldsymbol{\delta}_0 + \boldsymbol{G}\boldsymbol{M}\boldsymbol{\iota}\bar{\boldsymbol{\eta}}, \bar{\boldsymbol{X}}], \tag{41}$$

which is an $n\times(2m+1)$ matrix, where m is the dimension of $\boldsymbol{X}$. However, this instrumental matrix is infeasible as it involves unknown parameters ϕ_0, $\boldsymbol{\delta}_0$ and $\bar{\eta}$. Note that $\boldsymbol{F}$ can be considered as a linear combination of the IVs in $\boldsymbol{Q}_0 = \boldsymbol{J}(\boldsymbol{G}\boldsymbol{M}\bar{\boldsymbol{X}}, \boldsymbol{G}\boldsymbol{M}\boldsymbol{\iota}, \bar{\boldsymbol{X}})$. Furthermore, as $\boldsymbol{M} = (\boldsymbol{I}-\phi_0\boldsymbol{G})^{-1} = \sum_{j=0}^{\infty}(\phi_0\boldsymbol{G})^j$ when $|\phi_0\mu_1(\boldsymbol{G})|<1$, $\boldsymbol{G}\boldsymbol{M}\bar{\boldsymbol{X}}$ and $\boldsymbol{G}\boldsymbol{M}\boldsymbol{\iota}$ can be approximated by linear combinations of $(\boldsymbol{G}\bar{\boldsymbol{X}}, \boldsymbol{G}^2\bar{\boldsymbol{X}}, \cdots)$ and $(\boldsymbol{G}\boldsymbol{\iota}, \boldsymbol{G}^2\boldsymbol{\iota}, \cdots)$ respectively, and, hence, $\boldsymbol{Q}_0$ can be approximated by a linear combination of $\boldsymbol{Q}_\infty = \boldsymbol{J}(\boldsymbol{G}\bar{\boldsymbol{X}}, \boldsymbol{G}^2\bar{\boldsymbol{X}}, \cdots, \boldsymbol{G}\boldsymbol{\iota}, \boldsymbol{G}^2\boldsymbol{\iota}, \cdots, \bar{\boldsymbol{X}})$.

For the estimation of (23), let $\boldsymbol{Q}_K = \boldsymbol{J}(\boldsymbol{G}_x^{(\boldsymbol{p})}, \boldsymbol{G}_{\boldsymbol{\iota}}^{(\boldsymbol{p})}, \bar{\boldsymbol{X}})$ be an $n\times K$ submatrix of $\boldsymbol{Q}_\infty$, where $\boldsymbol{G}_x^{(\boldsymbol{p})} = (\boldsymbol{G}\bar{\boldsymbol{X}}, ..., \boldsymbol{G}^{\boldsymbol{p}}\bar{\boldsymbol{X}})$ and $\boldsymbol{G}_{\boldsymbol{\iota}}^{(\boldsymbol{p})} = (\boldsymbol{G}\iota, ..., \boldsymbol{G}^{\boldsymbol{p}}\iota)$ for some p that increases as n increases. As $\boldsymbol{\iota}$ has $\bar{r}$ columns, the number of IVs in $\boldsymbol{Q}_K$ is large if the number of groups $\bar{r}$ is large. In general, more valid IVs would improve the efficiency of the estimator. However, the IV-based estimator could be asymptotically biased in the presence of many IVs.

Let $\boldsymbol{P}_K = \boldsymbol{Q}_K(\boldsymbol{Q}_K'\boldsymbol{Q}_K)^{-1}\boldsymbol{Q}_K'$. The many-IV 2SLS estimator is $\hat{\boldsymbol{\theta}}_{2sls} = (\boldsymbol{Z}'\boldsymbol{P}_K\boldsymbol{Z})^{-1}\boldsymbol{Z}'\boldsymbol{P}_K\boldsymbol{y}$. Let $\boldsymbol{e}_j$ denote the jth column of an identity matrix. Liu and Lee (2010) have shown that, under some regularity assumptions, if $K/n\to 0$ then $\sqrt{n}(\hat{\boldsymbol{\theta}}_{2sls} - \boldsymbol{\theta}_0 - \boldsymbol{b}_{2sls}) \xrightarrow{d} N(0, \sigma_0^2(\lim_{n\to\infty}\frac{1}{n}\boldsymbol{F}'\boldsymbol{F})^{-1})$, where $\boldsymbol{b}_{2sls} = \sigma_0^2\mathrm{tr}(\boldsymbol{P}_K\boldsymbol{G}\boldsymbol{M})(\boldsymbol{Z}'\boldsymbol{P}_K\boldsymbol{Z})^{-1}\boldsymbol{e}_1 = O_p(K/n)$. The term $\boldsymbol{b}_{2sls}$ is a bias due to the presence of many IVs. When $K^2/n\to 0$, the bias term $\sqrt{n}\boldsymbol{b}_{2sls}$ converges to zero so that $\sqrt{n}(\hat{\boldsymbol{\theta}}_{2sls} - \boldsymbol{\theta}_0) \xrightarrow{d} N(0, \sigma_0^2(\lim_{n\to\infty}\frac{1}{n}\boldsymbol{F}'\boldsymbol{F})^{-1})$. Hence, the sequence of IV matrices $\{\boldsymbol{Q}_K\}$ gives the asymptotically best IV estimator as the variance matrix attains the efficiency lower bound for the class of IV estimators.

To correct for the many-instrument bias in $\hat{\boldsymbol{\theta}}_{2sls}$, we can adjust the many-IV 2SLS estimator by the estimated leading order bias. The bias-corrected many-IV 2SLS is given by $\hat{\boldsymbol{\theta}}_{c2sls} = (\boldsymbol{Z}'\boldsymbol{P}_K\boldsymbol{Z})^{-1}\boldsymbol{Z}'\boldsymbol{P}_K\boldsymbol{Y} - \hat{\boldsymbol{b}}_{2sls}$, where $\hat{\boldsymbol{b}}_{2sls}$ is a consistent estimator of $\boldsymbol{b}_{2sls}$. Liu and Lee (2010) have shown that, if $K/n\to 0$, then $\sqrt{n}(\hat{\boldsymbol{\theta}}_{c2sls} - \boldsymbol{\theta}_0) \xrightarrow{d} N(0, \sigma_0^2(\lim_{n\to\infty}\frac{1}{n}\boldsymbol{F}'\boldsymbol{F})^{-1})$.

Note that the number of IVs K is proportional to the number of groups $\bar{r}$. Hence, $K^2/n\to 0$ implies $\bar{r}^2/n = \bar{r}/\bar{m}\to 0$, where $\bar{m}$ is the average group size. So for asymptotic efficiency of the many-IV 2SLS estimator, the average group size needs to be large relative to the number of groups. On the other hand, $K/n\to 0$ implies $\bar{r}/n = 1/\bar{m}\to 0$. So for the bias-corrected many-IV 2SLS to be properly centered and asymptotically efficient, we only need the average group size to be large.

To summarize, the 2SLS estimators considered in the empirical studies of this paper are:

(i) *Finite-IV 2SLS*: $\hat{\boldsymbol{\theta}}_{2sls-1} = (\boldsymbol{Z}'\boldsymbol{P}_1\boldsymbol{Z})^{-1}\boldsymbol{Z}'\boldsymbol{P}_1\boldsymbol{y}$, where $\boldsymbol{P}_1 = \boldsymbol{Q}_1(\boldsymbol{Q}_1'\boldsymbol{Q}_1)^{-1}\boldsymbol{Q}_1'$ and $\boldsymbol{Q}_1$ contains the linearly independent columns of $\boldsymbol{J}(\boldsymbol{G}\bar{\boldsymbol{X}}, \bar{\boldsymbol{X}})$.

(ii) *Many-IV 2SLS*: $\hat{\boldsymbol{\theta}}_{2sls-2} = (\boldsymbol{Z}'\boldsymbol{P}_2\boldsymbol{Z})^{-1}\boldsymbol{Z}'\boldsymbol{P}_2\boldsymbol{y}$, where $\boldsymbol{P}_2 = \boldsymbol{Q}_2(\boldsymbol{Q}_2'\boldsymbol{Q}_2)^{-1}\boldsymbol{Q}_2'$ and $\boldsymbol{Q}_2$ contains the linearly independent columns of $\boldsymbol{J}(\boldsymbol{G}\bar{\boldsymbol{X}}, \bar{\boldsymbol{X}}, \boldsymbol{G}\boldsymbol{\iota})$.

(iii) *Bias-corrected 2SLS*: $\hat{\boldsymbol{\theta}}_{c2sls-2} = (\boldsymbol{Z}'\boldsymbol{P}_2\boldsymbol{Z})^{-1}[\boldsymbol{Z}'\boldsymbol{P}_2\boldsymbol{y} - \tilde{\sigma}_n^2\mathrm{tr}(\boldsymbol{P}_2\boldsymbol{G}\tilde{\boldsymbol{M}})\boldsymbol{e}_1]$, where $\tilde{\boldsymbol{M}} = (\boldsymbol{I} - \tilde{\phi}\boldsymbol{G})^{-1}$, and $\tilde{\sigma}^2, \tilde{\phi}$ are $\sqrt{n}$-consistent initial estimators of σ_0^2, ϕ_0.

GMM Estimation The 2SLS approach can be generalized to the GMM with additional quadratic moment equations. While the IV moments use the information of the main regression function of the reduced form equation for estimation, the quadratic moments can explore the correlation structure of the reduced form disturbances. Let $\boldsymbol{\epsilon}(\theta) = \boldsymbol{J}(\boldsymbol{y} - \boldsymbol{Z}\theta)$ with $\boldsymbol{\theta} = (\phi, \boldsymbol{\delta}')'$. The IV moments $\boldsymbol{g}_l(\boldsymbol{\theta}) = \boldsymbol{Q}_K'\boldsymbol{\epsilon}(\boldsymbol{\theta})$ are linear in $\boldsymbol{\epsilon}$ at $\boldsymbol{\theta}_0$. The quadratic moment is given by $\boldsymbol{g}_q(\boldsymbol{\theta}) = \boldsymbol{\epsilon}'(\boldsymbol{\theta})\boldsymbol{U}\boldsymbol{\epsilon}(\boldsymbol{\theta})$ where $\boldsymbol{U} = \boldsymbol{J}\boldsymbol{G}\boldsymbol{M}\boldsymbol{J} - \mathrm{tr}(\boldsymbol{J}\boldsymbol{G}\boldsymbol{M})\boldsymbol{J}/\mathrm{tr}(\boldsymbol{J})$. At $\boldsymbol{\theta}_0$, $\mathrm{E}[\boldsymbol{g}_q(\boldsymbol{\theta}_0)] = 0$, because $\mathrm{E}(\boldsymbol{\epsilon}'\boldsymbol{J}\boldsymbol{U}\boldsymbol{J}\boldsymbol{\epsilon}) = \sigma_0^2\mathrm{tr}(\boldsymbol{J}\boldsymbol{U}) = 0$.[60] The vector of combined linear and quadratic empirical moments for the GMM estimation is given by $\boldsymbol{g}(\boldsymbol{\theta}) = [\boldsymbol{g}_l'(\boldsymbol{\theta}), \boldsymbol{g}_q'(\boldsymbol{\theta})]'$.

In order for asymptotic inference to be robust, we do not impose the normality assumption for the following results. For any $n \times n$ matrix $\boldsymbol{A} = [a_{ij}]$, let $\boldsymbol{A}^{\boldsymbol{s}} = \boldsymbol{A} + \boldsymbol{A}'$ and $\mathrm{vec}_D(\boldsymbol{A}) = (a_{11}, \cdots, a_{nn})'$. In general, μ_3 and μ_4 denote, respectively, the third and fourth moments of the error term. The variance matrix of $\boldsymbol{g}(\boldsymbol{\theta}_0)$ is given by

$$\boldsymbol{\Omega} = \boldsymbol{\Omega}(\boldsymbol{Q}_K, \boldsymbol{U}, \sigma_0^2, \mu_3, \mu_4) = \begin{pmatrix} \sigma_0^2\boldsymbol{Q}_K'\boldsymbol{Q}_K & \mu_3\boldsymbol{Q}_K'\boldsymbol{\omega} \\ \mu_3\boldsymbol{\omega}'\boldsymbol{Q}_K & (\mu_4 - 3\sigma_0^4)\boldsymbol{\omega}'\boldsymbol{\omega} + \sigma_0^4\boldsymbol{\Delta} \end{pmatrix},$$

where $\boldsymbol{\omega} = \mathrm{vec}_D(\boldsymbol{U})$ and $\boldsymbol{\Delta} = \frac{1}{2}\mathrm{vec}(\boldsymbol{U}^{\boldsymbol{s}})'\mathrm{vec}(\boldsymbol{U}^{\boldsymbol{s}})$. The optimal many-IV GMM estimator is given by $\hat{\boldsymbol{\theta}}_{gmm} = \arg\min \boldsymbol{g}'(\theta)\boldsymbol{\Omega}^{-1}\boldsymbol{g}(\theta)$.

The optimal weighting matrix $\boldsymbol{\Omega}^{-1}$ involves unknown parameters σ_0^2, μ_3 and μ_4. In practice, with consistent initial estimators $\tilde{\sigma}^2$, $\tilde{\mu}_3$ and $\tilde{\mu}_4$, $\boldsymbol{\Omega}$ can be estimated as $\tilde{\boldsymbol{\Omega}} = \boldsymbol{\Omega}(\tilde{\sigma}^2, \tilde{\mu}_3, \tilde{\mu}_4)$. Let $\boldsymbol{D} = \mathrm{E}[\frac{\partial}{\partial\theta'}\boldsymbol{g}_q(\theta_0)] = -\sigma_0^2\mathrm{tr}(\boldsymbol{U}^{\boldsymbol{s}}\boldsymbol{G}\boldsymbol{M})\boldsymbol{e}_1'$ and $\boldsymbol{B}^{-1} = \boldsymbol{B}^{-1}(\boldsymbol{U}, \sigma_0^2, \mu_3, \mu_4) = (\mu_4 - 3\sigma_0^4)\boldsymbol{\omega}'\boldsymbol{\omega} + \sigma_0^4\boldsymbol{\Delta} - \frac{\mu_3^2}{\sigma_0^2}\boldsymbol{\omega}'\boldsymbol{P}_K\boldsymbol{\omega}$. Liu and Lee (2010) have shown that, if $K^{3/2}/n \to 0$, the feasible optimal many-IV GMM estimator $\hat{\boldsymbol{\theta}}_{gmm} = \arg\min_{\theta\in\Theta} \boldsymbol{g}'(\theta)\tilde{\boldsymbol{\Omega}}^{-1}\boldsymbol{g}(\theta)$ has the asymptotic distribution

$$\sqrt{n}(\hat{\boldsymbol{\theta}}_{gmm} - \boldsymbol{\theta}_0 - \boldsymbol{b}_{gmm}) \xrightarrow{d} N(0, [\sigma_0^{-2}(\lim_{n\to\infty}\frac{1}{n}\boldsymbol{F}'\boldsymbol{F})^{-1} + \lim_{n\to\infty}\frac{1}{n}\bar{\boldsymbol{D}}'\boldsymbol{B}\bar{\boldsymbol{D}}]^{-1}), \tag{42}$$

[60] Liu and Lee (2010) have shown that the quadratic moment $\boldsymbol{g}_{\boldsymbol{2}}(\theta) = \boldsymbol{\epsilon}'(\theta)\boldsymbol{U}\boldsymbol{\epsilon}(\theta)$ is the best (in terms of efficiency of the GMM estimator) under normality.

where $\boldsymbol{b}_{gmm} = (\sigma^{-2}\boldsymbol{Z}'\boldsymbol{P}_K\boldsymbol{Z} + \check{\boldsymbol{D}}'\boldsymbol{B}\check{\boldsymbol{D}})^{-1}\mathrm{tr}(\boldsymbol{P}_K\boldsymbol{G}\boldsymbol{M})\boldsymbol{e}_1 = O(K/n)$, $\check{\boldsymbol{D}} = \boldsymbol{D} - \frac{\mu_3}{\sigma_0^2}\boldsymbol{\omega}'\boldsymbol{P}_K\boldsymbol{Z}$, and $\bar{\boldsymbol{D}} = \boldsymbol{D} - \frac{\mu_3}{\sigma_0^2}\boldsymbol{\omega}'\boldsymbol{F}$.

As the asymptotic bias $\boldsymbol{b}_{gmm}$ is $O(K/n)$, the asymptotic distribution of the GMM estimator will be centered at $\boldsymbol{\theta}_0$ only when $K^2/n \to 0$. With the consistently estimated leading order bias $\hat{\boldsymbol{b}}_{gmm}$, Liu and Lee (2010) have shown that, if $K^{3/2}/n \to 0$, the feasible bias-corrected many-IV GMM estimator $\hat{\boldsymbol{\theta}}_{cgmm} = \hat{\boldsymbol{\theta}}_{gmm} - \hat{\boldsymbol{b}}_{gmm}$ is properly centered and has the asymptotic normal distribution as given in (42).

The asymptotic variance matrix of the many-IV GMM estimator can be compared with that of the many-IV 2SLS estimator. As $\bar{\boldsymbol{D}}'\boldsymbol{B}\bar{\boldsymbol{D}}$ is nonnegative definite, the asymptotic variance of the many-IV GMM estimator is relatively smaller than that of the 2SLS estimator. The many-IV GMM estimator with additional quadratic moments improves efficiency upon the 2SLS estimator.

To summarize, the GMM estimators considered in the empirical studies of this paper are:

(i) *Finite-IV GMM*: $\hat{\boldsymbol{\theta}}_{gmm-1} = \arg\min_{\theta\in\Theta} \tilde{\boldsymbol{g}}_1'(\theta)\tilde{\boldsymbol{\Omega}}_1^{-1}\tilde{\boldsymbol{g}}_1(\theta)$, where $\tilde{\boldsymbol{g}}_1(\theta) = [\boldsymbol{Q}_1, \tilde{\boldsymbol{U}}\boldsymbol{\epsilon}(\boldsymbol{\theta})]'\boldsymbol{\epsilon}(\boldsymbol{\theta})$, $\tilde{\boldsymbol{\Omega}}_1 = \boldsymbol{\Omega}(\boldsymbol{Q}_1, \tilde{\boldsymbol{U}}, \tilde{\sigma}^2, \tilde{\mu}_3, \tilde{\mu}_4)$, $\tilde{\boldsymbol{U}} = \boldsymbol{J}\boldsymbol{G}\tilde{\boldsymbol{M}}\boldsymbol{J} - \mathrm{tr}(\boldsymbol{J}\boldsymbol{G}\tilde{\boldsymbol{M}})\boldsymbol{J}/\mathrm{tr}(\boldsymbol{J})$, $\tilde{\boldsymbol{M}} = (\boldsymbol{I} - \tilde{\phi}\boldsymbol{G})^{-1}$, and $\tilde{\sigma}^2, \tilde{\mu}_3, \tilde{\mu}_4, \tilde{\phi}$ are $\sqrt{n}$-consistent initial estimators.

(ii) *Many-IV GMM*: $\hat{\boldsymbol{\theta}}_{gmm-2} = \arg\min_{\theta\in\Theta} \tilde{\boldsymbol{g}}_2'(\theta)\tilde{\boldsymbol{\Omega}}_2^{-1}\tilde{\boldsymbol{g}}_2(\theta)$, where $\tilde{\boldsymbol{g}}_2(\theta) = [\boldsymbol{Q}_2, \tilde{\boldsymbol{U}}\boldsymbol{\epsilon}(\boldsymbol{\theta})]'\boldsymbol{\epsilon}(\boldsymbol{\theta})$, $\tilde{\boldsymbol{\Omega}}_2 = \boldsymbol{\Omega}(\boldsymbol{Q}_2, \tilde{\boldsymbol{U}}, \tilde{\sigma}^2, \tilde{\mu}_3, \tilde{\mu}_4)$, $\tilde{\boldsymbol{U}} = \boldsymbol{J}\boldsymbol{G}\tilde{\boldsymbol{M}}\boldsymbol{J} - \mathrm{tr}(\boldsymbol{J}\boldsymbol{G}\tilde{\boldsymbol{M}})\boldsymbol{J}/\mathrm{tr}(\boldsymbol{J})$, $\tilde{\boldsymbol{M}} = (\boldsymbol{I} - \tilde{\phi}\boldsymbol{G})^{-1}$, and $\tilde{\sigma}^2, \tilde{\mu}_3, \tilde{\mu}_4, \tilde{\phi}$ are $\sqrt{n}$-consistent initial estimators.

(iii) *Bias-corrected GMM*: $\hat{\boldsymbol{\theta}}_{cgmm-2} = \hat{\boldsymbol{\theta}}_{gmm-2} - \tilde{\boldsymbol{b}}_{gmm}$, where $\tilde{\boldsymbol{b}}_{gmm} = [\tilde{\sigma}^{-2}\boldsymbol{Z}'\boldsymbol{P}_2\boldsymbol{Z} + \tilde{\tilde{\boldsymbol{D}}}'\tilde{\boldsymbol{B}}\tilde{\tilde{\boldsymbol{D}}}]^{-1}\mathrm{tr}(\boldsymbol{P}_2\boldsymbol{G}\tilde{\boldsymbol{M}})\boldsymbol{e}_1$, where $\tilde{\boldsymbol{B}} = \boldsymbol{B}(\tilde{\boldsymbol{U}}, \tilde{\sigma}^2, \tilde{\mu}_3, \tilde{\mu}_4)$, $\tilde{\tilde{\boldsymbol{D}}} = -\tilde{\sigma}^2\mathrm{tr}(\tilde{\boldsymbol{U}}^s\boldsymbol{G}\tilde{\boldsymbol{M}})\boldsymbol{e}_1' - \frac{\tilde{\mu}_3}{\tilde{\sigma}^2}\mathrm{vec}_D(\tilde{\boldsymbol{U}})'\boldsymbol{P}_2\boldsymbol{Z}$, $\tilde{\boldsymbol{M}} = (\boldsymbol{I} - \tilde{\phi}\boldsymbol{G})^{-1}$, and $\tilde{\sigma}^2, \tilde{\mu}_3, \tilde{\mu}_4, \tilde{\phi}$ are $\sqrt{n}$-consistent initial estimators.

Appendix 6: Estimation of selection-bias corrected outcome equation

From (12), the infeasible selection-bias corrected outcome equation

$$\boldsymbol{y} = \phi \boldsymbol{G}\boldsymbol{y} + \boldsymbol{X}\boldsymbol{\beta}_1 + \boldsymbol{G}^*\boldsymbol{X}\boldsymbol{\beta}_2 + \boldsymbol{\iota}\bar{\boldsymbol{\eta}} + \sigma_{12}\boldsymbol{\lambda} + \boldsymbol{\epsilon}^*,$$

where $\boldsymbol{\epsilon}^* = \boldsymbol{\epsilon}_2 - \sigma_{12}\boldsymbol{\lambda}$ is the bias-corrected disturbances such that $\mathrm{E}(\boldsymbol{\epsilon}^*|\boldsymbol{y}_1^* > 0) = 0$. The conditional variance of $\boldsymbol{\epsilon}^*$ is

$$\mathrm{Var}(\boldsymbol{\epsilon}^*|\boldsymbol{y}_1^* > 0) = \mathrm{E}(\boldsymbol{\epsilon}^*\boldsymbol{\epsilon}^{*\prime}|\boldsymbol{y}_1^* > 0) = \sigma_2^2\boldsymbol{I} - \sigma_{12}^2\boldsymbol{A},$$

where $\boldsymbol{A} = \mathrm{D}(\lambda(\boldsymbol{x}_1'\gamma)\boldsymbol{x}_1'\gamma + \lambda^2(\boldsymbol{x}_1'\gamma), \cdots, \lambda(\boldsymbol{x}_n'\gamma)\boldsymbol{x}_n'\gamma + \lambda^2(\boldsymbol{x}_n'\gamma))$, and $\boldsymbol{A}$ is evaluated at the true parameter vector of $\boldsymbol{\gamma}$.

For a two-stage estimation, let $\hat{\boldsymbol{\gamma}}$ be the probit MLE. The feasible outcome equation for the second stage estimation is

$$\boldsymbol{y} = \phi \boldsymbol{G}\boldsymbol{y} + \boldsymbol{X}\boldsymbol{\beta}_1 + \boldsymbol{G}^*\boldsymbol{X}\boldsymbol{\beta}_2 + \boldsymbol{\iota}\bar{\boldsymbol{\eta}} + \sigma_{12}\hat{\boldsymbol{\lambda}} + \boldsymbol{\epsilon},$$

where $\boldsymbol{\epsilon} = \boldsymbol{\epsilon}_2 - \sigma_{12}\hat{\boldsymbol{\lambda}} = \boldsymbol{\epsilon}^* - \sigma_{12}(\hat{\boldsymbol{\lambda}} - \boldsymbol{\lambda})$. After we eliminate the fixed group effect by the projector $\boldsymbol{J}$, the outcome equation becomes

$$\boldsymbol{J}\boldsymbol{y} = \phi \boldsymbol{J}\boldsymbol{G}\boldsymbol{y} + \boldsymbol{J}\bar{\boldsymbol{X}}\boldsymbol{\beta} + \sigma_{12}\boldsymbol{J}\hat{\boldsymbol{\lambda}} + \boldsymbol{J}\boldsymbol{\epsilon},$$

where $\bar{\boldsymbol{X}} = (\boldsymbol{X}, \boldsymbol{G}^*\boldsymbol{X})$ and $\boldsymbol{\beta} = (\boldsymbol{\beta}_1', \boldsymbol{\beta}_2')'$. Let $\boldsymbol{Q} = \boldsymbol{J}[\boldsymbol{G}\bar{\boldsymbol{X}}, \bar{\boldsymbol{X}}, \boldsymbol{\lambda}]$ be an (infeasible) IV matrix and $\hat{\boldsymbol{Q}}' = \boldsymbol{J}[\boldsymbol{G}\bar{\boldsymbol{X}}, \bar{\boldsymbol{X}}, \hat{\boldsymbol{\lambda}}]$. Let $\boldsymbol{P} = \boldsymbol{Q}(\boldsymbol{Q}'\boldsymbol{Q})^{-1}\boldsymbol{Q}'$, $\hat{\boldsymbol{P}} = \hat{\boldsymbol{Q}}(\hat{\boldsymbol{Q}}'\hat{\boldsymbol{Q}})^{-1}\hat{\boldsymbol{Q}}'$, $\boldsymbol{Z} = [\boldsymbol{G}\boldsymbol{y}, \bar{\boldsymbol{X}}, \boldsymbol{\lambda}]$, and $\hat{\boldsymbol{Z}} = [\boldsymbol{G}\boldsymbol{y}, \bar{\boldsymbol{X}}, \hat{\lambda}_n]$. Then the 2SLS estimator of $\boldsymbol{\theta} = (\boldsymbol{\phi}, \boldsymbol{\beta}', \sigma_{12})'$ is given by:

$$\hat{\boldsymbol{\theta}} = (\hat{\boldsymbol{Z}}'\hat{\boldsymbol{P}}\hat{\boldsymbol{Z}})^{-1}\hat{\boldsymbol{Z}}'\hat{\boldsymbol{P}}\boldsymbol{y} = \boldsymbol{\theta} + (\hat{\boldsymbol{Z}}'\hat{\boldsymbol{P}}\hat{\boldsymbol{Z}})^{-1}\hat{\boldsymbol{Z}}'\hat{\boldsymbol{P}}\boldsymbol{\epsilon}.$$

As, by the mean value theorem,

$$\frac{1}{\sqrt{n}}\hat{\boldsymbol{Z}}'\hat{\boldsymbol{P}}\boldsymbol{\epsilon} = \frac{1}{\sqrt{n}}\hat{\boldsymbol{Z}}'\hat{\boldsymbol{P}}[\boldsymbol{\epsilon}^* - \sigma_{12}(\hat{\boldsymbol{\lambda}} - \boldsymbol{\lambda})] = \frac{1}{\sqrt{n}}\hat{\boldsymbol{Z}}'\hat{\boldsymbol{P}}\boldsymbol{\epsilon}^* - \sigma_{12}\frac{1}{\sqrt{n}}\hat{\boldsymbol{Z}}'\hat{\boldsymbol{P}}\boldsymbol{A}(\bar{\gamma})\boldsymbol{X}(\hat{\boldsymbol{\gamma}} - \boldsymbol{\gamma})$$

the asymptotic variance of $\hat{\boldsymbol{\theta}}$ is $\mathrm{Avar}\sqrt{n}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) = \mathrm{plim}(\frac{1}{n}\boldsymbol{Z}'\boldsymbol{P}\boldsymbol{Z})^{-1}\boldsymbol{\Sigma}(\frac{1}{n}\boldsymbol{Z}'\boldsymbol{P}\boldsymbol{Z})^{-1}$, where

$$\boldsymbol{\Sigma} = \frac{1}{n}\boldsymbol{Z}'\boldsymbol{P}[\sigma_2^2\boldsymbol{I} - \sigma_{12}^2\boldsymbol{A} + \sigma_{12}^2\boldsymbol{A}\boldsymbol{X}(\boldsymbol{X}_{\bar{n}}'\boldsymbol{\Lambda}_{\bar{n}}\boldsymbol{X}_{\bar{n}})^{-1}\boldsymbol{X}'\boldsymbol{A}']\boldsymbol{P}\boldsymbol{Z},$$

and $\boldsymbol{\Lambda}_{\bar{n}} = \mathrm{D}(\frac{f^2(\boldsymbol{x}_1'\gamma)}{\Phi(\boldsymbol{x}_1'\gamma)(1-\Phi(\boldsymbol{x}_1'\gamma))}, \cdots, \frac{f^2(\boldsymbol{x}_{\bar{n}}'\gamma)}{\Phi(\boldsymbol{x}_{\bar{n}}'\gamma)(1-\Phi(\boldsymbol{x}_{\bar{n}}'\gamma))})$.

To estimate σ_2^2, we consider $\mathrm{E}(\boldsymbol{\epsilon}^*\boldsymbol{\epsilon}^{*\prime}|\boldsymbol{y}_1^* > 0) = \sigma_2^2\boldsymbol{I} - \sigma_{12}^2\boldsymbol{A}$. Let $\hat{\boldsymbol{\epsilon}}^* = \boldsymbol{J}\boldsymbol{y} - \boldsymbol{J}\hat{\boldsymbol{Z}}\hat{\boldsymbol{\theta}}$. Then σ_2^2 can be estimated by $\hat{\sigma}_2^2 = \hat{\boldsymbol{\epsilon}}^{*\prime}\hat{\boldsymbol{\epsilon}}^*/\mathrm{tr}(\boldsymbol{J}) + \hat{\sigma}_{12}^2\frac{1}{n}\sum_{i=1}^n[\lambda(\boldsymbol{x}_i'\gamma)\boldsymbol{x}_i'\gamma + \lambda^2(\boldsymbol{x}_i'\gamma)]$.

Appendix 7: Individual centrality measures and network characteristics

The simplest index of connectivity of individual i in network g is the number of direct friends divided by the maximum possible number of friends individual i can have (i.e. $n-1$ individuals if everyone is directly connected to individual i), i.e. *degree centrality*:

$$\delta_i(g) = \frac{\bar{g}_i}{n-1}, \tag{43}$$

where $\bar{g}_i = \sum_{j=1}^{n} g_{ij}$.

The standard measure of *closeness centrality* of individual i in network g is given by:

$$c_{1i}(g) = \frac{n-1}{\sum_j d(i,j)} \tag{44}$$

where $d(i,j)$ is the geodesic distance (length of the shortest path)[61] in network g between individuals i and j. As a result, the closeness centrality of individual i is the inverse of the sum of geodesic distances from i to the $n-1$ other individuals (i.e. the reciprocal of its "farness") divided by $n-1$, which is the maximum possible distance between two individuals in the network. Compared to degree centrality, the closeness measure takes into account not only direct connections among individuals but also indirect connections. However, compared to the Bonacich centrality, the closeness measure assumes a weight of one to each indirect connection, whereas the Bonacich centrality uses weights that depend on the strength of social interaction within the network.

The closeness centrality is not very informative for a network that is not strongly connected. As $d(i,j) = \infty$ if node j is not reachable from i, the closeness centrality of node i will be zero if there is some node in the network that is not reachable from i. To tackle this problem, we propose an alternative closeness centrality measure

$$c_{2i}(g) = \frac{1}{n-1} \sum_{j=1, j\neq i}^{n} \frac{1}{d(i,j)},$$

which is used in the empirical studies of this paper.

For a directed network g, the *betweenness centrality* measure of agent i can be defined as:

$$f_i(g) = \frac{1}{(n-1)(n-2)} \sum_{j=1}^{n} \sum_{k=1, k\neq j}^{n} \frac{a_{jk}(i)}{a_{jk}} \tag{45}$$

[61] The length of a shortest path is the smallest k such that there is at least one path of length k from i to j. Therefore we can find the length by computing $\boldsymbol{G}$, $\boldsymbol{G}^2$, $\boldsymbol{G}^3$, ..., until we find the first k such that the (i,j)th entry of $\boldsymbol{G}^k$ is not zero.

where j and l denote two given agents in g, $a_{jl}(i)$ is the number of shortest paths between j and l through i in g, a_{jl} is the number of shortest paths between j and l in g and $(n-1)(n-2)$ is the total number of links in a complete directed network. Note that betweenness centrality, as the degree and closeness centrality measures, is a parameter-free index while the Bonacich centrality is not since it depends on the decay factor ϕ.

The *clustering coefficient* of individual i in network g is given by:

$$\psi_i(g) = \frac{\sum_{l \in N_i(g)} \sum_{k \in N_i(g)} g_{lk}}{n_i(g)\left[n_i(g) - 1\right]} \qquad \text{for all } i \in \{i \in N \mid n_i(g) \geq 2\} \tag{46}$$

where N is the set of nodes in network g, $N_i(g) = \{j \neq i \mid g_{ij} = 1\}$ is the set of i's direct contacts and $n_i(g)$, its size (or cardinality of this set). $\psi_i(g)$ gives us the percentage of an individual's links who are linked to each other. This is an indication of the percentage of transitive triads[62] around individual i. It thus measures the probability with which two of i's friends are also friends.

Unit centralities in a network can have large or small variance. Network, where one unit (or a low number of units) has (have) much higher centrality than other units is highly centralized. On the other hand, if unit centrality measures do not differ significantly, the centrality of a network is low.

From these individual measures we can compute the corresponding measures at the network level using the definition provided by Freeman (1979). In our notation, the Freeman (1979)'s network index for a given network g is

$$C^A(g) = \frac{\sum_{i=1}^{n}(C_{i^*}^A - C_i^A)}{\max \sum_{i=1}^{n}(C_{i^*}^A - C_i^A)}$$

where $C_{i^*}^A$ is the largest value of C_i^A for any individual in the network and $\max \sum_{i=1}^{n}(C_{i^*}^A - C_i^A)$ is the maximum possible sum of differences in unit centrality for a network of n individuals. The network index is thus a number between 0 and 1, being 0 if all units have equal value, and 1, when one unit completely dominates all other units. The individual degree, closeness $(c_{1i}(g))$ and betweenness centrality measures then lead to the network degree, closeness and betweenness centrality measures. For the alternative individual closeness centrality measure $c_{2i}(g)$, the corresponding network closeness centrality is given by $\frac{1}{n}\sum_i c_{2i}(g)$. Let us finally introduce other widely used network characteristics.

The *average degree* is the total number of links divided by n (i.e. $\frac{1}{n}\sum_i \bar{g}_i$).

[62] A triad is the subgraph on three individuals, so that when studying triads, one has to consider the threesome of individuals and all the links between them. A triad involving individuals i, j, k is transitive if whenever $i \to j$ and $j \to k$, then $i \to k$.

The *average distance* of a network (also known as the *average path length*) is defined as the average number of steps along the shortest paths for all possible pairs of network nodes (i.e. $\sum_{(i,j)\in C(g)} d(i,j)/c(g)$, where $c(g)$ is the size of $C(g) = \{(i,j)|i \neq j, d(i,j) \neq \infty\}$).

The *diameter* of a network is the largest (shortest) distance between any two nodes in the network given by $\max_{(i,j)\in C(g)} d(i,j)$. The diameter is set to be zero if $d(i,j) = \infty$, for all i, j. It thus provides an upper-bound measure of the size of the network.

Network *density* is simply the fraction of ties present in a network over all possible ones (it is the average degree divided by $n-1$). It ranges from 0 to 1 as networks get denser.

Network *asymmetry* is measured using the variance of connectivities given by $\frac{1}{n-1}\max(\bar{g}_i)/\min(\bar{g}_i)$. Note for directed G in the data, $\min(\bar{g}_i)$ could be zero and thus the measure is undefined. Hence, we report the network asymmetry of the corresponding undirected G.

Network redundancy or *clustering* is the fraction of all transitive triads over the total number of triads given by $\sum_i n_i(g)[n_i(g)-1]\psi_i(g)/\sum_j n_j(g)[n_j(g)-1]$. It measures the probability with which two of i's friends know each other.

Finally, network *assortativity* measures the correlation patterns among high-degree nodes. If high-degree nodes tend to be connected to other high-degree nodes, then the network is said to be positive assortative. The degree of assortativity of the network g is computed as: $\sum_i \sum_j (\bar{g}_i - m)(\bar{g}_j - m)/\sum_i (\bar{g}_i - m)^2$, where m is the average degree in network g.

Table 1a: 2SLS and GMM Estimations
All Crimes

	2SLS-1	2SLS-2	C2SLS	GMM-1	GMM-2	CGMM
Peer effects (φ)	0.0399***	0.0282***	0.0357***	0.0505***	0.0396***	0.0462***
	(0.0150)	(0.0118)	(0.0118)	(0.0132)	(0.0111)	(0.0111)
Female	-0.3007***	-0.2977***	-0.2996***	-0.3008***	-0.2969***	-0.2986***
	(0.0605)	(0.0605)	(0.0605)	(0.0606)	(0.0605)	(0.0605)
Student grade	-0.0328	-0.0357	-0.0338	-0.0294	-0.0320	-0.0304
	(0.0329)	(0.0328)	(0.0328)	(0.0329)	(0.0328)	(0.0328)
Black or African American	0.0489	0.0417	0.0464	0.0559	0.0491	0.0530
	(0.1489)	(0.1487)	(0.1488)	(0.1490)	(0.1488)	(0.1489)
Other races	-0.0095	-0.0149	-0.0114	-0.0107	-0.0202	-0.0177
	(0.1539)	(0.1538)	(0.1538)	(0.1540)	(0.1538)	(0.1539)
Religion Practice	-0.0177	-0.0174	-0.0176	-0.0173	-0.0168	-0.0170
	(0.0257)	(0.0257)	(0.0257)	(0.0257)	(0.0257)	(0.0257)
Parental Education	0.0094	0.0101	0.0097	0.0080	0.0083	0.0079
	(0.0352)	(0.0351)	(0.0352)	(0.0352)	(0.0352)	(0.0352)
Mathematics score	0.0749**	0.0741**	0.0746**	0.0750**	0.0739**	0.0743**
	(0.0340)	(0.0340)	(0.0340)	(0.0341)	(0.0340)	(0.0341)
Self esteem	0.0065	0.0067	0.0066	0.0065	0.0067	0.0066
	(0.0308)	(0.0308)	(0.0308)	(0.0309)	(0.0308)	(0.0309)
Physical development	0.0570**	0.0573**	0.0571**	0.0585**	0.0597**	0.0596**
	(0.0271)	(0.0271)	(0.0271)	(0.0272)	(0.0271)	(0.0271)
Parental care	-0.1738	-0.1782	-0.1754	-0.1719	-0.1781	-0.1755
	(0.1165)	(0.1164)	(0.1164)	(0.1166)	(0.1164)	(0.1165)
School attachment	-0.0457	-0.0452	-0.0455	-0.0453	-0.0440	-0.0443
	(0.0350)	(0.0349)	(0.0349)	(0.0350)	(0.0350)	(0.0350)
Relationship with teachers	0.2544***	0.2533***	0.2540***	0.2561***	0.2555***	0.2561***
	(0.0327)	(0.0327)	(0.0327)	(0.0327)	(0.0327)	(0.0327)
Social inclusion	-0.0953**	-0.0941**	-0.0949**	-0.0965**	-0.0955**	-0.0961**
	(0.0433)	(0.0433)	(0.0433)	(0.0433)	(0.0433)	(0.0433)
Residential building quality	0.0873**	0.0871**	0.0872**	0.0871**	0.0865**	0.0866**
	(0.0416)	(0.0416)	(0.0416)	(0.0416)	(0.0416)	(0.0416)
Residential area urban	-0.2017**	-0.2045**	-0.2027**	-0.1946**	-0.1951**	-0.1935**
	(0.0854)	(0.0854)	(0.0854)	(0.0855)	(0.0854)	(0.0854)
Household size	-0.0235	-0.0237	-0.0236	-0.0233	-0.0236	-0.0234
	(0.0244)	(0.0244)	(0.0244)	(0.0244)	(0.0244)	(0.0244)
Two married parent family	-0.2844*	-0.2873*	-0.2854*	-0.2803*	-0.2824*	-0.2810*
	(0.1606)	(0.1605)	(0.1605)	(0.1608)	(0.1606)	(0.1607)
Single parent family	-0.2629	-0.2648	-0.2636	-0.2614	-0.2633	-0.2624
	(0.1621)	(0.1620)	(0.1620)	(0.1623)	(0.1621)	(0.1622)
Parental occupation dummies	yes	yes	yes	yes	yes	yes
Network fixed effects	yes	yes	yes	yes	yes	yes
First stage F statistic	52.924***	36.670***				
OIR test p-value	0.205			0.189		

Notes: Dependent variable: Index of delinquency. The number of observations is 1,297 individuals over 150 networks. Estimated coefficients and Standard errors (in parentheses) are reported. *,**,*** denote statistical significance at the 10, 5 and 1 percent level.

Table 1b: 2SLS and GMM Estimations
Type-1 Crimes

	2SLS-1	2SLS-2	C2SLS	GMM-1	GMM-2	CGMM
Peer effects (φ)	0.0459***	0.0321***	0.0388***	0.0486***	0.0379***	0.0436***
	(0.0152)	(0.0124)	(0.0124)	(0.0134)	(0.0115)	(0.0115)
Female	-0.3066***	-0.3035***	-0.3050***	-0.3069***	-0.3041***	-0.3054***
	(0.0705)	(0.0704)	(0.0704)	(0.0705)	(0.0704)	(0.0705)
Student grade	-0.0377	-0.0417	-0.0398	-0.0372	-0.0410	-0.0394
	(0.0378)	(0.0376)	(0.0376)	(0.0377)	(0.0376)	(0.0376)
Black or African American	-0.0704	-0.0820	-0.0763	-0.0684	-0.0780	-0.0734
	(0.1714)	(0.1710)	(0.1711)	(0.1713)	(0.1710)	(0.1711)
Other races	-0.0611	-0.0713	-0.0663	-0.0604	-0.0721	-0.0682
	(0.1820)	(0.1817)	(0.1818)	(0.1820)	(0.1817)	(0.1818)
Religion Practice	0.0153	0.0155	0.0154	0.0154	0.0157	0.0156
	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)
Parental Education	0.0247	0.0261	0.0254	0.0243	0.0252	0.0246
	(0.0423)	(0.0422)	(0.0422)	(0.0423)	(0.0422)	(0.0422)
Mathematics score	0.0695*	0.0692*	0.0694*	0.0696*	0.0693*	0.0694*
	(0.0401)	(0.0401)	(0.0401)	(0.0402)	(0.0401)	(0.0401)
Self esteem	0.0287	0.0287	0.0287	0.0283	0.0277	0.0277
	(0.0361)	(0.0361)	(0.0361)	(0.0361)	(0.0361)	(0.0361)
Physical development	0.0693**	0.0698**	0.0696**	0.0696**	0.0708**	0.0706**
	(0.0317)	(0.0317)	(0.0317)	(0.0317)	(0.0317)	(0.0317)
Parental care	-0.1933	-0.1987	-0.1960	-0.1928	-0.1988	-0.1964
	(0.1359)	(0.1357)	(0.1358)	(0.1360)	(0.1358)	(0.1358)
School attachment	-0.0548	-0.0528	-0.0538	-0.0549	-0.0524	-0.0532
	(0.0407)	(0.0406)	(0.0406)	(0.0407)	(0.0406)	(0.0406)
Relationship with teachers	0.2314***	0.2301***	0.2307***	0.2319***	0.2312***	0.2317***
	(0.0395)	(0.0395)	(0.0395)	(0.0395)	(0.0395)	(0.0395)
Social inclusion	-0.1412***	-0.1391***	-0.1401***	-0.1413***	-0.1391***	-0.1399***
	(0.0495)	(0.0494)	(0.0494)	(0.0495)	(0.0494)	(0.0494)
Residential building quality	0.0594	0.0602	0.0598	0.0594	0.0604	0.0600
	(0.0486)	(0.0485)	(0.0485)	(0.0486)	(0.0485)	(0.0486)
Residential area urban	-0.1462	-0.1502	-0.1483	-0.1442	-0.1448	-0.1432
	(0.1002)	(0.1001)	(0.1001)	(0.1002)	(0.1001)	(0.1002)
Household size	-0.0247	-0.0251	-0.0249	-0.0246	-0.0251	-0.0249
	(0.0284)	(0.0284)	(0.0284)	(0.0285)	(0.0284)	(0.0284)
Two married parent family	-0.4614**	-0.4659**	-0.4637**	-0.4595**	-0.4609**	-0.4590**
	(0.1975)	(0.1972)	(0.1973)	(0.1976)	(0.1973)	(0.1974)
Single parent family	-0.4627**	-0.4649**	-0.4638**	-0.4620**	-0.4627**	-0.4617**
	(0.2007)	(0.2004)	(0.2005)	(0.2007)	(0.2005)	(0.2006)
Parental occupation dummies	yes	yes	yes	yes	yes	yes
Network fixed effects	yes	yes	yes	yes	yes	yes
First stage F statistic	52.274**	38.247**				
OIR test p-value	0.528			0.580		

Notes: Dependent variable: Index of delinquency. The number of observations is 1,099 individuals over 132 networks. Estimated coefficients and Standard errors (in parentheses) are reported. *,**,*** denote statistical significance at the 10, 5 and 1 percent level.

Table 1c: 2SLS and GMM Estimations
Type-2 Crimes

	2SLS-1	2SLS-2	C2SLS	GMM-1	GMM-2	CGMM
Peer effects (φ)	0.0453	0.0336	0.0609**	0.0770**	0.0553**	0.0762***
	(0.0399)	(0.0286)	(0.0287)	(0.0317)	(0.0258)	(0.0257)
Female	-0.3142**	-0.3126**	-0.3163**	-0.3102**	-0.3054**	-0.3077**
	(0.1379)	(0.1378)	(0.1380)	(0.1383)	(0.1380)	(0.1383)
Student grade	-0.0276	-0.0279	-0.0273	-0.0201	-0.0195	-0.0187
	(0.0744)	(0.0744)	(0.0745)	(0.0747)	(0.0745)	(0.0747)
Black or African American	0.0425	0.0423	0.0428	0.0421	0.0396	0.0407
	(0.3062)	(0.3061)	(0.3065)	(0.3073)	(0.3066)	(0.3073)
Other races	-0.1041	-0.1219	-0.0807	-0.1101	-0.1522	-0.1232
	(0.3366)	(0.3339)	(0.3343)	(0.3351)	(0.3334)	(0.3342)
Religion Practice	-0.0252	-0.0253	-0.0252	-0.0229	-0.0230	-0.0229
	(0.0554)	(0.0554)	(0.0555)	(0.0556)	(0.0555)	(0.0556)
Parental Education	0.0870	0.0874	0.0865	0.0866	0.0880	0.0868
	(0.0722)	(0.0722)	(0.0723)	(0.0725)	(0.0723)	(0.0725)
Mathematics score	0.0430	0.0429	0.0430	0.0459	0.0466	0.0465
	(0.0714)	(0.0714)	(0.0715)	(0.0717)	(0.0715)	(0.0717)
Self esteem	-0.0031	-0.0028	-0.0035	-0.0009	0.0002	-0.0007
	(0.0637)	(0.0637)	(0.0638)	(0.0640)	(0.0638)	(0.0640)
Physical development	0.1222***	0.1229***	0.1212***	0.1211***	0.1223***	0.1210***
	(0.0580)	(0.0579)	(0.0580)	(0.0582)	(0.0580)	(0.0582)
Parental care	-0.1770	-0.1817	-0.1706	-0.1714	-0.1808	-0.1723
	(0.2335)	(0.2332)	(0.2335)	(0.2341)	(0.2335)	(0.2340)
School attachment	-0.1767***	-0.1773***	-0.1759***	-0.1776***	-0.1788***	-0.1776***
	(0.0693)	(0.0693)	(0.0693)	(0.0695)	(0.0693)	(0.0695)
Relationship with teachers	0.2741***	0.2738***	0.2744***	0.2730***	0.2727***	0.2734***
	(0.0635)	(0.0634)	(0.0635)	(0.0637)	(0.0635)	(0.0637)
Social inclusion	0.0196	0.0205	0.0184	0.0143	0.0154	0.0138
	(0.0874)	(0.0873)	(0.0875)	(0.0877)	(0.0875)	(0.0877)
Residential building quality	0.0852	0.0844	0.0863	0.0841	0.0824	0.0838
	(0.0810)	(0.0810)	(0.0811)	(0.0813)	(0.0811)	(0.0813)
Residential area urban	-0.1682	-0.1701	-0.1657	-0.1742	-0.1798	-0.1779
	(0.1946)	(0.1945)	(0.1948)	(0.1952)	(0.1947)	(0.1952)
Household size	-0.0436	-0.0430	-0.0443	-0.0422	-0.0406	-0.0416
	(0.0447)	(0.0447)	(0.0448)	(0.0449)	(0.0448)	(0.0449)
Two married parent family	-0.2383	-0.2376	-0.2392	-0.2330	-0.2302	-0.2316
	(0.2736)	(0.2735)	(0.2739)	(0.2746)	(0.2739)	(0.2746)
Single parent family	-0.3043	-0.3043	-0.3043	-0.3061	-0.3048	-0.3052
	(0.2776)	(0.2776)	(0.2779)	(0.2786)	(0.2780)	(0.2786)
Parental occupation dummies	yes	yes	yes	yes	yes	yes
Network fixed effects	yes	yes	yes	yes	yes	yes
First stage F statistic	11.865	15.497				
OIR test p-value	0.149			0.115		

Notes: Dependent variable: Index of delinquency. The number of observations is 545 individuals over 75 networks. Estimated coefficients and Standard errors (in parentheses) are reported. *,**,*** denote statistical significance at the 10, 5 and 1 percent level.

Table 2: First step Heckman selection model

	All crimes	Type-1 crimes	Type-2 crimes
Female	-0.0276	0.1258***	-0.3873***
	(0.0353)	(0.0337)	(0.0315)
Student grade	0.0590***	0.0718***	-0.0175*
	(0.0114)	(0.0109)	(0.0103)
Black or African American	0.0599	-0.0448	0.1838***
	(0.0480)	(0.0451)	(0.0427)
Other races	0.1126**	0.1257**	0.1517***
	(0.0533)	(0.0512)	(0.0470)
Religion Practice	-0.0331**	-0.0233*	-0.0379***
	(0.0132)	(0.0126)	(0.0118)
Parental Education	-0.0077	0.0204	-0.0389**
	(0.0193)	(0.0182)	(0.0171)
Mathematics score	0.0611***	0.0435**	0.1051***
	(0.0185)	(0.0175)	(0.0163)
Self esteem	-0.0318*	-0.0232	-0.0419***
	(0.0174)	(0.0166)	(0.0155)
Physical development	0.0863***	0.0771***	0.0736***
	(0.0160)	(0.0152)	(0.0143)
Parental care	-0.0985	-0.1010	-0.0444
	(0.0719)	(0.0678)	(0.0609)
School attachment	-0.0431**	-0.0336*	-0.0833***
	(0.0199)	(0.0187)	(0.0172)
Relationship with teachers	0.3109***	0.2820***	0.2627***
	(0.0227)	(0.0207)	(0.0183)
Social inclusion	-0.0439*	-0.0693***	-0.0957***
	(0.0242)	(0.0231)	(0.0213)
Residential building quality	0.0182	0.0152	0.0923***
	(0.0234)	(0.0222)	(0.0209)
Residential area urban	0.1116***	0.1105***	0.1114***
	(0.0394)	(0.0376)	(0.0358)
Household size	0.0129	0.0054	0.0238**
	(0.0125)	(0.0118)	(0.0111)
Two married parent family	0.0069	0.0834	-0.2588***
	(0.0941)	(0.0879)	(0.0835)
Single parent family	-0.0291	0.0442	-0.2345***
	(0.0954)	(0.0891)	(0.0848)
Parental occupation manager	0.1276	0.1258	0.0002
	(0.0860)	(0.0818)	(0.0777)
Parent occupation professional/technical	0.2065***	0.2061***	-0.0294
	(0.0794)	(0.0752)	(0.0713)
Parent occupation manual	0.1970**	0.2170***	0.0745
	(0.0832)	(0.0789)	(0.0745)
Parent occupation office or sales worker	0.1105	0.1347**	0.0313
	(0.0700)	(0.0662)	(0.0632)
Parent occupation military or security	0.0045	-0.0113	-0.0566
	(0.1211)	(0.1159)	(0.1111)
Parent occupation farm or fishery	-0.1137	-0.0788	-0.2638*
	(0.1486)	(0.1442)	(0.1456)
Parental occupation other	0.1790**	0.1568**	0.1000
	(0.0794)	(0.0750)	(0.0715)

Notes: Dependent variable: Probability to commit crime of a given type. The number of observations is 6,993 individuals over 1,596 networks. Probit estimation result. Marginal effects and standard errors (in parentheses) are reported. *,**,*** denote statistical significance at the 10, 5 and 1 percent level.

Table 3: 2SLS-1 Estimation results with/without endogenous participation

	All Crimes		Type-1 Crimes		Type-2 Crimes	
Peer effects (φ)	0.0399***	0.0387	0.0459***	0.0455***	0.0453	0.0428
	(0.0150)	(0.0145)	(0.0152)	(0.0153)	(0.0399)	(0.0466)
Inverse Mills Ratio		2.1522*		1.5215		3.8481
		(1.2006)		(1.5416)		(2.4440)
Female	-0.3007***	-0.3334***	-0.3066***	-0.2249**	-0.3142***	-1.2035**
	(0.0605)	(0.0723)	(0.0705)	(0.1154)	(0.1379)	(0.6155)
Student grade	-0.0328	0.0158	-0.0377	0.0112	-0.0276	-0.0625
	(0.0329)	(0.0468)	(0.0378)	(0.0649)	(0.0744)	(0.1009)
Black or African American	0.0489	0.1086	-0.0704	-0.0906	0.0425	0.5342
	(0.1489)	(0.1679)	(0.1714)	(0.1809)	(0.3062)	(0.5307)
Other races	-0.0095	0.0810	-0.0611	0.0244	-0.1041	0.2378
	(0.1539)	(0.1764)	(0.1820)	(0.2116)	(0.3366)	(0.4765)
Religion Practice	-0.0177	-0.0429	0.0153	0.0012	-0.0252	-0.1023
	(0.0257)	(0.0333)	(0.0300)	(0.0354)	(0.0554)	(0.0946)
Parental Education	0.0094	0.0025	0.0247	0.0401	0.0870	-0.0070
	(0.0352)	(0.0408)	(0.0423)	(0.0474)	(0.0722)	(0.1200)
Mathematics score	0.0749	0.1248**	0.0695	0.0977*	0.0430	0.2664
	(0.0340)	(0.0500)	(0.0401)	(0.0522)	(0.0714)	(0.1834)
Self esteem	0.0065	-0.0249	0.0287	0.0098	-0.0031	-0.1201
	(0.0308)	(0.0397)	(0.0361)	(0.0431)	(0.0637)	(0.1152)
Physical development	0.0570**	0.1348**	0.0693**	0.1238**	0.1222**	0.2963**
	(0.0271)	(0.0563)	(0.0317)	(0.0662)	(0.0580)	(0.1423)
Parental care	-0.1738	-0.2476	-0.1933	-0.2535	-0.1770	-0.2262
	(0.1165)	(0.1521)	(0.1359)	(0.1648)	(0.2335)	(0.3451)
School attachment	-0.0457	-0.0773	-0.0548	-0.0746	-0.1767	-0.3398
	(0.0350)	(0.0466)	(0.0407)	(0.0493)	(0.0693)	(0.1544)
Relationship with teachers	0.2544***	0.4917***	0.2314***	0.4092**	0.2741***	0.8091**
	(0.0327)	(0.1469)	(0.0395)	(0.1907)	(0.0635)	(0.3681)
Social inclusion	-0.0953	-0.1289	-0.1412	-0.1866	0.0196	-0.1811
	(0.0433)	(0.0551)	(0.0495)	(0.0717)	(0.0874)	(0.1869)
Residential building quality	0.0873	0.1054**	0.0594	0.0709	0.0852	0.3004
	(0.0416)	(0.0496)	(0.0486)	(0.0534)	(0.0810)	(0.1800)
Residential area urban	-0.2017**	-0.1075	-0.1462	-0.0689	-0.1682	0.0894
	(0.0854)	(0.1128)	(0.1002)	(0.1349)	(0.1946)	(0.3022)
Household size	-0.0235	-0.0138	-0.0247	-0.0215	-0.0436	0.0114
	(0.0244)	(0.0287)	(0.0284)	(0.0305)	(0.0447)	(0.0721)
Two married parent family	-0.2844**	-0.2890	-0.4614**	-0.4072**	-0.2383	-0.7699
	(0.1606)	(0.1858)	(0.1975)	(0.2177)	(0.2736)	(0.5465)
Single parent family	-0.2629	-0.3035	-0.4627**	-0.4382**	-0.3043	-0.7940
	(0.1621)	(0.1898)	(0.2007)	(0.2155)	(0.2776)	(0.5404)
Parental occupation dummies	yes	yes	yes	yes	yes	yes
Network fixed effects	yes	yes	yes	yes	yes	yes
First stage F statistic	52.924	52.752	52.274	52.183	11.865	11.910
OIR test p-value	0.205	0.276	0.528	0.543	0.149	0.654

Notes: Dependent variable: Index of delinquency. Estimated coefficients and Standard errors (in parentheses) are reported. *,**,*** denote statistical significance at the 10, 5 and 1 percent level.

Table 4a: Dynamic network formation and key players for all crimes when $c_i = \underline{c}_i$

Network (# nodes)	Highest Between	Highest Bonacich	KP Invariant	KP Dynamics	T crime Initial	ET crime Dynamics	ET crime Invariant KP	ET crime Dynamics KP	Density Initial (Diameter)	Density Dynamics (Diameter)	Density Invariant KP (Diameter)	Density Dynamics KP (Diameter)	# days Before (after KP)
1 (6)	3	1	3	3	11.235	11.216	8.718	8.783	0.167 (1)	0.167 (1)	0.200 (1)	0.250 (1)	0 (1)
2 (4)	2	4	3	3	7.732	7.762	5.489	5.489	0.333 (3)	0.333 (3)	0.500 (2)	0.500 (2)	0 (0)
3 (5)	2	2	2	2	12.631	12.616	9.222	9.222	0.300 (3)	0.300 (3)	0.333 (1)	0.333 (1)	0 (0)
4 (4)	1	1	1	1	8.117	8.138	5.532	5.663	0.250 (2)	0.250 (2)	0.167 (1)	0.500 (1)	0 (2)
5 (64)	28	57	35	62	148.653	148.204	144.184	144.858	0.0303 (10)	0.0306 (10)	0.0294 (8)	0.030 (10)	0 (0)
6 (6)	6	5	6	6	13.137	13.077	10.379	10.379	0.167 (1)	0.167 (1)	0.200 (1)	0.200 (1)	0 (0)
7 (7)	1	3	5	5	15.191	15.223	12.468	12.756	0.143 (2)	0.143 (2)	0.167 (1)	0.333 (2)	0 (5)
8 (6)	6	4	5	5	11.378	11.297	8.797	8.797	0.400 (4)	0.400 (4)	0.400 (3)	0.400 (3)	0 (0)
9 (34)	8	30	24	30	74.768	75.0138	69.923	72.055	0.0615 (7)	0.0615 (7)	0.057 (7)	0.063 (7)	0 (0)
10 (30)	30	13	26	26	79.651	79.665	75.546	75.546	0.0402 (3)	0.0402 (3)	0.037 (3)	0.037 (3)	0 (0)
11 (9)	4	5	4	3	38.188	37.939	32.819	33.208	0.125 (2)	0.125 (2)	0.054 (1)	0.125 (2)	0 (0)
12 (6)	5	5	5	5	12.962	13.023	10.156	10.156	0.167 (3)	0.167 (3)	0.150 (1)	0.150 (1)	0 (0)
13 (5)	2	1	1	1	11.657	11.593	8.524	8.524	0.250 (4)	0.250 (4)	0.333 (3)	0.333 (3)	0 (0)
14 (51)	6	4	39	13	142.113	141.823	137.131	137.523	0.039 (10)	0.039 (10)	0.038 (10)	0.040 (10)	0 (0)
15 (4)	1	1	1	1	14.524	14.467	10.493	10.493	0.250 (1)	0.250 (1)	0.167 (1)	0.167 (1)	0 (0)
16 (5)	3	4	5	5	9.556	9.5605	7.233	7.547	0.300 (3)	0.300 (3)	0.333 (2)	0.417 (2)	0 (1)
17 (4)	2	3	4	4	7.366	7.367	5.119	5.217	0.417 (3)	0.417 (3)	0.50 (2)	0.667 (2)	0 (1)
18 (4)	3	1	3	3	10.884	10.847	7.226	8.106	0.333 (2)	0.333 (2)	0.167 (1)	0.667 (2)	0 (3)
19 (9)	9	6	6	1	23.911	23.736	20.528	20.945	0.139 (5)	0.139 (5)	0.107 (5)	0.161 (4)	0 (1)
20 (5)	3	1	2	5	14.123	14.109	10.787	10.868	0.350 (3)	0.350 (3)	0.417 (3)	0.333 (1)	0 (0)

Table 4b: Dynamic network formation and key players for all crimes when $c_i = \underline{c}_i - 0.05$ (Small networks only)

Network (# nodes)	Highest Between	Highest Bonacich	KP Invariant	KP Dynamics	T crime Initial	ET crime Dynamics	ET crime Invariant KP	ET crime Dynamics KP	Density Initial (Diameter)	Density Dynamics (Diameter)	Density Invariant KP (Diameter)	Density Dynamics KP (Diameter)	# days Before (after KP)
1 (6)	3	1	3	3	11.235	11.627	8.718	9.056	0.167 (1)	0.333 (2)	0.200 (1)	0.400 (1)	5 (4)
2 (4)	2	4	3	4	7.732	8.363	5.488	5.671	0.333 (3)	0.750 (2)	0.500 (2)	0.333 (1)	5 (1)
3 (5)	2	2	2	2	12.631	13.674	9.223	9.223	0.300 (3)	0.550 (3)	0.333 (1)	0.333 (1)	5 (0)
4 (4)	1	1	1	1	8.117	8.852	5.532	5.663	0.250 (2)	0.667 (2)	0.167 (1)	0.500 (1)	5 (2)
6 (6)	6	5	6	4	13.137	14.382	10.379	11.200	0.167 (1)	0.367 (3)	0.200 (1)	0.200 (1)	6 (0)
7 (7)	1	3	5	5	15.191	17.913	12.469	12.756	0.143 (2)	0.405 (2)	0.167 (1)	0.333 (2)	11 (5)
8 (6)	6	4	5	5	11.378	12.202	8.798	9.553	0.400 (4)	0.600 (2)	0.400 (3)	0.650 (2)	6 (5)
11 (9)	4	5	4	8	38.188	44.114	32.820	34.078	0.125 (2)	0.417 (2)	0.054 (1)	0.250 (2)	21 (6)
12 (6)	5	5	5	5	12.962	14.352	10.156	11.045	0.167 (3)	0.367 (2)	0.150 (1)	0.350 (2)	6 (4)
13 (5)	2	1	1	3	11.657	12.710	8.524	8.696	0.250 (4)	0.600 (2)	0.333 (3)	0.250 (2)	7 (0)
15 (4)	1	1	1	1	14.524	15.516	10.493	10.493	0.250 (1)	0.583 (2)	0.167 (1)	0.167 (1)	4 (0)
16 (5)	3	4	5	5	9.556	10.933	7.235	7.548	0.300 (3)	0.550 (3)	0.333 (2)	0.417 (2)	5 (1)
17 (4)	2	3	4	4	7.366	7.885	5.119	5.216	0.417 (3)	0.833 (2)	0.50 (2)	0.667 (2)	5 (1)
18 (4)	3	1	3	3	10.884	11.866	7.227	8.108	0.333 (2)	0.667 (3)	0.167 (1)	0.667 (2)	4 (3)
19 (9)	9	6	6	1	23.911	26.383	20.525	20.945	0.139 (5)	0.264 (3)	0.107 (5)	0.161 (4)	9 (1)
20 (5)	3	1	2	5	14.123	15.010	10.789	11.211	0.350 (3)	0.550 (2)	0.417 (3)	0.500 (2)	4 (2)
21 (4)	2	3	2	2	7.348	7.752	4.775	5.3858	0.417 (3)	0.667 (2)	0.167 (1)	0.500 (2)	3 (2)
22 (5)	3	4	1	1	11.644	12.686	8.874	8.874	0.200 (2)	0.450 (3)	0.250 (1)	0.250 (1)	5 (0)
23 (4)	2	3	1	3	10.107	11.170	7.121	7.763	0.250 (2)	0.667 (2)	0.333 (2)	0.667 (1)	5 (2)
24 (4)	3	1	3	2	8.501	8.855	6.210	6.452	0.250 (1)	0.417 (1)	0.167 (1)	0.500 (1)	2 (1)

Table 4c: Dynamic network formation and key players for all crimes when $c_i = 0$ (Small networks only)

Network (# nodes)	Highest Between	Highest Bonacich	KP Invariant	KP Dynamics	T crime Initial	ET crime Dynamics	ET crime Invariant KP	ET crime Dynamics KP	Density Initial (Diameter)	Density Dynamics (Diameter)	Density Invariant KP (Diameter)	Density Dynamics KP (Diameter)	# days Before (after KP)
1 (6)	3	1	3	3	11.235	12.578	8.718	9.329	0.167 (1)	0.833 (1)	0.200 (1)	0.800 (1)	20 (12)
2 (4)	2	4	3	4	7.732	8.724	5.488	5.671	0.333 (3)	1 (1)	0.500 (2)	0.333 (1)	8 (1)
3 (5)	2	2	2	2	12.631	14.579	9.223	10.222	0.300 (3)	1 (1)	0.333 (1)	0.8333 (2)	14 (6)
4 (4)	1	1	1	1	8.117	8.960	5.532	5.789	0.250 (2)	1 (1)	0.167 (1)	0.833 (2)	9 (4)
6 (6)	6	5	6	6	13.137	16.298	10.379	11.587	0.167 (1)	1 (1)	0.200 (1)	0.550 (2)	25 (7)
7 (7)	1	3	5	5	15.191	18.981	12.469	15.175	0.143 (2)	0.643 (2)	0.167 (1)	1 (1)	21 (25)
8 (6)	6	4	5	5	11.378	13.007	8.798	9.983	0.400 (4)	1 (1)	0.400 (3)	1 (1)	18 (12)
11 (9)	4	5	4	8	38.188	56.666	32.820	46.009	0.125 (2)	1 (1)	0.054 (1)	1 (1)	63 (48)
12 (6)	5	5	5	5	12.962	15.327	10.156	11.637	0.167 (3)	1 (1)	0.150 (1)	0.350 (2)	6 (4)
13 (5)	2	1	1	3	11.657	13.438	8.524	9.291	0.250 (4)	1 (1)	0.333 (3)	0.750 (1)	15 (6)
15 (4)	1	1	1	3	14.524	15.961	10.493	11.000	0.250 (1)	1 (1)	0.167 (1)	0.333 (1)	9 (1)
16 (5)	3	4	5	1	9.556	11.970	7.235	8.877	0.300 (3)	1 (1)	0.333 (2)	1 (1)	14 (7)
17 (4)	2	3	4	4	7.366	8.059	5.119	5.403	0.417 (3)	1 (1)	0.50 (2)	1 (1)	7 (3)
18 (4)	3	1	3	3	10.884	12.202	7.227	8.195	0.333 (2)	1 (1)	0.167 (1)	1 (1)	8 (5)
19 (9)	9	6	6	1	23.911	36.306	20.525	29.565	0.139 (5)	1 (1)	0.107 (5)	1 (1)	62 (48)
20 (5)	3	1	2	5	14.123	15.080	10.789	11.342	0.350 (3)	0.650 (1)	0.417 (3)	0.583 (1)	6 (3)
21 (4)	2	3	2	3	7.348	7.840	4.775	5.558	0.417 (3)	0.750 (1)	0.167 (1)	0.667 (1)	4 (2)
22 (5)	3	4	1	1	11.644	13.665	8.874	9.855	0.200 (2)	1 (1)	0.250 (1)	1 (1)	16 (9)
23 (4)	2	3	1	1	10.107	11.229	7.121	7.872	0.250 (2)	0.750 (2)	0.333 (2)	0.667 (2)	6 (2)
24 (4)	3	1	3	3	8.501	8.980	6.210	6.466	0.250 (1)	0.500 (1)	0.167 (1)	0.333 (1)	3 (1)

Table 4d: Dynamic network formation and key players for all crimes and with cost changes (Small networks only)

Network (# nodes)	KP Invariant	KP Dynamics $c_i = \underline{c}_i$	KP Dynamics $c_i = \underline{c}_i$ -0.05	KP Dynamics $c_i = 0$	T crime Initial	ET crime Dynamics KP $c_i = \underline{c}_i$	ET crime Dynamics KP $c_i = \underline{c}_i$ -0.05	ET crime Dynamics KP $c_i = 0$	Density Initial (Diameter)	Density Dynamics KP (Diameter) $c_i = \underline{c}_i$	Density Dynamics KP (Diameter) $c_i = \underline{c}_i$ -0.05	Density Dynamics KP (Diameter) $c_i = 0$
1 (6)	3	3	3	3	11.235	8.783	9.056	9.329	0.167 (1)	0.250 (1)	0.400 (1)	0.800 (1)
2 (4)	3	3	4	4	7.732	5.489	5.671	5.671	0.333 (3)	0.500 (2)	0.333 (1)	0.333 (1)
3 (5)	2	2	2	2	12.631	9.222	9.223	10.222	0.300 (3)	0.333 (1)	0.333 (1)	0.8333 (2)
4 (4)	1	1	1	1	8.117	5.663	5.663	5.789	0.250 (2)	0.500 (1)	0.500 (1)	0.833 (2)
6 (6)	6	6	4	6	13.137	10.379	11.200	11.587	0.167 (1)	0.200 (1)	0.200 (1)	0.550 (2)
7 (7)	5	5	5	5	15.191	12.756	12.756	15.175	0.143 (2)	0.333 (2)	0.333 (2)	1 (1)
8 (6)	5	5	5	5	11.378	8.797	9.553	9.983	0.400 (4)	0.400 (3)	0.650 (2)	1 (1)
11 (9)	4	3	8	8	38.188	33.208	34.078	46.009	0.125 (2)	0.125 (2)	0.250 (2)	1 (1)
12 (6)	5	5	5	5	12.962	10.156	11.045	11.637	0.167 (3)	0.150 (1)	0.350 (2)	0.350 (2)
13 (5)	1	1	3	3	11.657	8.524	8.696	9.291	0.250 (4)	0.333 (3)	0.250 (2)	0.750 (1)
15 (4)	1	1	1	3	14.524	10.493	10.493	11.000	0.250 (1)	0.167 (1)	0.167 (1)	0.333 (1)
16 (5)	5	5	5	1	9.556	7.547	7.548	8.877	0.300 (3)	0.417 (2)	0.417 (2)	1 (1)
17 (4)	4	4	4	4	7.366	5.217	5.216	5.403	0.417 (3)	0.667 (2)	0.667 (2)	1 (1)
18 (4)	3	3	3	3	10.884	8.106	8.108	8.195	0.333 (2)	0.667 (2)	0.667 (2)	1 (1)
19 (9)	6	1	1	1	23.911	20.945	20.945	29.565	0.139 (5)	0.161 (4)	0.161 (4)	1 (1)
20 (5)	2	5	5	5	14.123	10.868	11.211	11.342	0.350 (3)	0.333 (1)	0.500 (2)	0.583 (1)

Table 5: Who is the Key Player?
-Significant Differences-
All crimes

	All Criminals		Key Player Criminals		
	Mean	St. dev	Mean	St. dev	t-test
Individual characteristics					
Female	0.53	0.50	0.23	0.42	0.0000
Religion practice	3.65	1.41	3.28	1.57	0.0078
Parent education	3.23	1.06	3.01	1.14	0.0279
Mathematics score	2.18	1.00	2.53	1.05	0.0003
Parental care	0.93	0.26	0.80	0.40	0.0002
School attachment	4.12	0.87	3.71	1.07	0.0000
Relationship with teachers	0.99	0.92	1.79	1.22	0.0000
Social inclusion	4.47	0.74	4.23	0.86	0.0018
Residential building quality	1.51	0.79	1.70	0.96	0.0226
Two married parent families	0.74	0.44	0.61	0.49	0.0020
Single parent family	0.22	0.42	0.30	0.46	0.0706
Parent occupation manager	0.11	0.31	0.17	0.38	0.0704
Parent occupation military or security	0.02	0.14	0.00	0.00	0.0000
Parent occupation other	0.16	0.37	0.11	0.31	0.0673
Friends' characteristics					
Religious practice	2.52	1.98	3.02	1.80	0.0025
Student grade	6.42	4.33	7.64	3.85	0.0006
Parental education	2.30	1.66	2.61	1.54	0.0279
Mathematics score	1.54	1.24	1.87	1.24	0.0033
Self esteem	2.84	1.99	3.28	1.76	0.0066
Physical development	2.44	1.76	2.69	1.52	0.0810
Parental care	0.65	0.46	0.75	0.42	0.0152
School attachment	2.90	1.99	3.35	1.74	0.0055
Social inclusion	3.12	2.09	3.65	1.83	0.0019
Residential building quality	1.05	0.89	1.19	0.83	0.0621
Residential area urban	0.43	0.48	0.55	0.48	0.0033
Household size	3.13	2.22	3.48	1.97	0.0474
Single parent families	0.14	0.31	0.23	0.39	0.0105
N.obs.	893		145		

Notes: T-test for differences in means with unequal variances had been performed. P-values are reported

Table 6: Key Player versus Bonacich Centrality
-Significant Differences-
All crimes

	Key Player Most Active Criminal		Key Player Not the Most Active Criminal		
	Mean	St. dev	Mean	St. dev	t-test
Individual characteristics					
Female	0.12	0.33	0.30	0.46	0.0080
Social inclusion	3.98	0.86	4.39	0.82	0.0053
Residential building quality	1.91	1.03	1.57	0.89	0.0459
Friends' characteristics					
Residential area urban	0.67	0.46	0.48	0.48	0.0231
N.obs.	56		89		

Notes: T-test for differences in means with unequal variances had been performed. P-values are reported

Table 7: Who is the Key Player?
-Significant Differences-
Petty crimes

	All Criminals		Key Player Criminals		
	Mean	St. dev	Mean	St. dev	t-test
Individual characteristics					
Female	0.54	0.50	0.24	0.43	0.0000
Mathematics score	2.17	1.00	2.44	1.01	0.0049
Physical development	3.33	1.09	3.55	1.06	0.0325
Parental care	0.93	0.25	0.74	0.44	0.0000
School attachment	4.11	0.88	3.69	1.09	0.0001
Relationship with teachers	0.99	0.94	1.62	1.16	0.0000
Social inclusion	4.48	0.73	4.14	0.88	0.0001
Residential area urban	0.56	0.50	0.65	0.48	0.0523
Parent occupation manager	0.11	0.31	0.18	0.38	0.0463
Parent occupation manual	0.33	0.47	0.22	0.41	0.0065
Friends' characteristics					
Student grade	6.53	4.39	7.66	3.95	0.0034
Religion practice	2.29	1.65	2.69	1.57	0.0086
Mathematics score	1.52	1.21	1.81	1.17	0.0108
Self esteem	2.85	2.00	3.24	1.76	0.0224
Parental care	0.65	0.46	0.75	0.42	0.0170
School attachment	2.90	1.99	3.29	1.76	0.0251
Social inclusion	3.12	2.09	3.62	1.86	0.0063
Residential area urban	0.41	0.47	0.54	0.47	0.0047
Single parent family	0.15	0.31	0.23	0.38	0.0181
Parent occupation professional/technical	0.14	0.31	0.20	0.36	0.0646
N.obs.	807		128		

Notes: T-test for differences in means with unequal variances had been performed. P-values are reported

Table 8: Who is the Key Player?
-Significant Differences-
More serious crimes

	All Criminals		Key Player Criminals		
	Mean	St. dev	Mean	St. dev	t-test
Individual characteristics					
Female	0.44	0.50	0.23	0.42	0.0004
Physical development	3.25	1.11	3.69	1.04	0.0023
School attachment	3.98	0.95	3.68	1.05	0.0271
Relationship with teachers	1.16	1.04	1.97	1.35	0.0000
Parent occupation manager	0.11	0.31	0.03	0.17	0.0022
Parent occupation military or security	0.01	0.09	0.00	0.00	0.0833
Friends' characteristics					
School attachment	2.74	1.95	3.17	1.78	0.0721
Social inclusion	3.07	2.11	3.53	1.96	0.0828
Parent occupation military or security	0.01	0.07	0.00	0.00	0.0718
Parent occupation farm or fishery	0.02	0.13	0.00	0.00	0.0115
N.obs.	334		70		

Notes: T-test for differences in means with unequal variances had been performed. P-values are reported

Table 9: Key Player for Petty and Serious Crimes
-Significant Differences-

	Key Player Petty Crime		Key Player More Serious Crime		
	Mean	St. dev	Mean	St. dev	t-test
Individual characteristics					
Black or African American	0.17	0.38	0.31	0.47	0.0308
Self esteem	4.04	1.08	3.73	1.11	0.0542
Parental care	0.74	0.44	0.90	0.30	0.0033
Relationship with teachers	1.62	1.16	1.97	1.35	0.0723
Social inclusion	4.14	0.88	4.47	0.70	0.0041
Parent occupation manager	0.18	0.38	0.03	0.17	0.0002
Parent occupation military or security	0.03	0.17	0.00	0.00	0.0451
Friends' characteristics					
Female	0.40	0.43	0.26	0.40	0.0315
Black or African American	0.13	0.32	0.24	0.43	0.0539
Relationship with teachers	0.70	0.71	1.05	1.03	0.0132
Parent occupation manager	0.11	0.29	0.05	0.19	0.0825
Parent occupation military or security	0.02	0.14	0.00	0.00	0.0575
Parental occupation farm or fishery	0.02	0.11	0.00	0.00	0.1027
N.obs.	128		70		

Notes: T-test for differences in means with unequal variances had been performed. P-values are reported

Table 10: Key Player versus Bonacich centrality
-Significant Differences-
Petty crimes

	Key Player Most Active Criminal		Key Player Not the Most Active Criminal		
	Mean	St. dev	Mean	St. dev	t-test
Individual characteristics					
Residential building quality	1.94	1.11	1.43	0.76	0.0073
Friends' characteristics					
Mathematics score	1.57	1.11	1.95	1.20	0.0690
Relationship with teachers	0.57	0.58	0.78	0.76	0.0789
N.obs.	47		81		

Notes: T-test for differences in means with unequal variances had been performed. P-values are reported

Table 11: Key Player versus Bonacich centrality
-Significant Differences-
More Serious crimes

	Key Player Most Active Criminal		Key Player Not the Most Active Criminal		
	Mean	St. dev	Mean	St. dev	t-test
Individual characteristics					
Religion practice	3.96	1.26	3.40	1.41	0.0884
Parental care	0.96	0.19	0.86	0.35	0.1056
School attachment	3.92	1.01	3.52	0.99	0.1049
Relationship with teachers	2.43	1.45	1.67	1.20	0.0256
Residential area urban	0.61	0.50	0.81	0.40	0.0775
Friends' characteristics					
Other races	0.00	0.00	0.05	0.18	0.1031
Relationship with teachers	1.31	1.15	0.87	0.91	0.0951
Parental occupation professional/technical	0.23	0.42	0.07	0.20	0.0610
N.obs.	28		42		

Notes: T-test for differences in means with unequal variances had been performed. P-values are reported

Table 12: Key Players and Network Topology
Individual centrality measures

	All crimes			
	Betweenness	Clustering	Closeness	Bonacich
percentiles				
p50	0	0	0.33	2.49
p75	0.02	0	0.50	3.51
p90	0.20	0.17	0.75	5.20
p95	0.33	0.50	0.83	5.83
p99	0.42	1	1	6.99
min	0	0	0	1.51
max	0.5	1	1	9.39
	Key Players Not the Most Active Criminals			
>p90	71%	58%	27%	21%
	Petty crimes			
percentiles	Betweenness	Clustering	Closeness	Bonacich
p50	0	0	0.33	2.78
p75	0.05	0	0.57	3.63
p90	0.33	0.17	0.75	5.51
p95	0.33	0.5	0.83	6.23
p99	0.5	1	1	7.31
min	0	0	0	1.61
max	0.5	1	1	7.60
	Key Players Not the Most Active Criminals			
>p90	40%	72%	54%	8%
	More serious crimes			
percentiles	Betweenness	Clustering	Closeness	Bonacich
p50	0	0	0.40	2.50
p75	0	0	0.67	4.21
p90	0.09	0.21	0.67	6.70
p95	0.17	1	0.75	7.62
p99	0.5	1	1	11.37
min	0	0	0	1.69
max	0.5	1	1	11.37
	Key Players Not the Most Active Criminals			
>p90	71%	71%	50%	14%

Table 13: Key Players and Network Topology
Network centrality measures

All crimes					
	Key Player Most Active Criminal		Key Player Not the Most Active Criminal		
Network characteristics	Mean	St. dev	Mean	St. dev	t-test
Diameter	2.52	1.43	2.59	1.47	0.7534
Average distance	1.43	0.44	1.44	0.41	0.8879
Average degree	1.02	0.32	1.03	0.29	0.8402
Density	0.23	0.10	0.24	0.10	0.6293
Asymmetry	0.68	0.24	0.67	0.23	0.9666
Network clustering	0.07	0.15	0.05	0.12	0.4527
Network degree	0.12	0.12	0.13	0.12	0.7331
Network closeness	0.30	0.12	0.30	0.14	0.5119
Assortativity	1.60×10^{-17}	1.81×10^{-16}	5.27×10^{-18}	1.06×10^{-16}	0.6892
Network betweeness-	0.13	0.12	0.15	0.12	0.3584
N.obs.	56		89		

Petty crimes					
	Key Player Most Active Criminal		Key Player Not the Most Active Criminal		
Network characteristics	Mean	St. dev	Mean	St. dev	t-test
Diameter	2.53	1.40	2.62	1.53	0.7482
Average distance	1.44	0.44	1.45	0.42	0.8793
Average degree	1.04	0.33	1.03	0.29	0.8316
Density	0.25	0.10	0.23	0.10	0.3710
Asymmetry	0.68	0.22	0.66	0.22	0.6797
Network clustering	0.05	0.14	0.06	0.12	0.7413
Network degree	0.13	0.12	0.12	0.10	0.4249
Network closeness	0.31	0.13	0.29	0.13	0.3919
Assortativity	2.94×10^{-17}	8.27×10^{-17}	7.00×10^{-18}	1.26×10^{-16}	0.2275
Network betweeness	0.16	0.13	0.14	0.12	0.3715
N.obs.	47		81		

More serious crimes					
	Key Player Most Active Criminal		Key Player Not the Most Active Criminal		
Network characteristics	Mean	St. dev	Mean	St. dev	t-test
Diameter	2.36	1.03	2.21	1.16	0.5900
Average distance	1.41	0.31	1.34	0.35	0.4370
Average degree	1.04	0.33	0.98	0.23	0.4221
Density	0.25	0.10	0.25	0.09	0.8746
Asymmetry	0.73	0.20	0.76	0.20	0.1606
Network clustering	0.06	0.14	0.05	0.11	0.7377
Network degree	0.13	0.13	0.16	0.13	0.4338
Network closeness	0.32	0.12	0.30	0.12	0.5638
Assortativity	2.15×10^{-17}	9.09×10^{-17}	1.17×10^{-17}	8.32×10^{-17}	0.6481
Network betweeness	0.18	0.15	0.13	0.13	0.1589
N.obs.	28		42		

Notes: T-test for differences in means with unequal variances had been performed. P-values are reported

Table 14a: Crime reduction when different policies are implemented (for $c_i = \underline{c}_i$)

Network (# nodes)	KP Dynamics	T crime Initial	ET crime Dynamics	ET crime Dynamics KP	Reduction Crime KP (%)	Reduction Crime AVERAGE (%)
1 (6)	3	11.235	11.236	8.783	21.80	16.71
2 (4)	3	7.732	7.740	5.489	29.08	21.78
3 (5)	2	12.631	12.621	9.222	26.93	17.612
4 (4)	1	8.117	8.130	5.663	30.35	24.64
5 (64)	62	148.653	148.204	144.858	2.26	-3.27
6 (6)	6	13.137	13.069	10.379	20.58	14.46
7 (7)	5	15.191	15.208	12.756	16.13	11.84
8 (6)	5	11.378	11.337	8.797	22.40	13.98
9 (34)	30	74.768	75.014	72.055	3.94	-9.82
10 (30)	26	79.651	79.665	75.546	5.17	1.53
11 (9)	3	38.188	37.831	33.208	12.22	7.12
12 (6)	5	12.962	12.990	10.156	21.81	12.88
13 (5)	1	11.657	11.612	8.524	26.59	17.76
14 (51)	13	142.113	141.823	137.523	3.032	-3.49
15 (4)	1	14.524	14.420	10.493	27.23	25.55
16 (5)	5	9.556	9.581	7.547	21.23	13.36
17 (4)	4	7.366	7.3782	5.217	29.30	23.25
18 (4)	3	10.884	10.850	8.106	25.29	22.319
19 (9)	1	23.911	23.670	20.945	11.51	4.848
20 (5)	5	14.123	14.138	10.868	23.13	18.92

Table 14b: Average crime reduction when different policies are implemented by network size (for $c_i = \underline{c}_i$)

Network size	Average % crime reduction of a key play policy	Average % crime reduction of a random target policy
4	28.94	23.86
5	23.67	18.29
6	20.21	14.05
7	17.08	11.07
8	15.46	10.44
9	13.56	7.00
10	12.87	6.29
11	10.68	3.99
12	10.72	2.18
13	10.51	5.02
15	8.28	0.60
16	9.04	1.79
26	5.05	0.96
30	5.17	1.53
34	3.94	-9.82
37	3.73	0.23
51	3.03	-3.49
64	2.26	-3.27