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EXTERNALITIES AND
MACROPRUDENTIAL REGULATION**

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ABSTRACT

Aggregate Investment Externalities and Macroprudential Regulation*

Evidence suggests that banks tend to lend a lot during booms, and very little during recessions. We propose a simple explanation for this phenomenon. We show that, instead of dampening productivity shocks, the banking sector tends to exacerbate them, leading to excessive fluctuations of credit, output and asset prices. Our explanation relies on three ingredients that are characteristic of modern banks' activities. The first ingredient is moral hazard: banks are supposed to monitor the small and medium sized enterprises that borrow from them, but they may shirk on their monitoring activities, unless they are given sufficient informational rents. These rents limit the amount that investors are ready to lend them, to a multiple of the banks' own capital.

The second ingredient is the banks' high exposure to aggregate shocks: banks' assets have positively correlated returns. Finally the third ingredient is the ease with which modern banks can reallocate capital between different lines of business. At the competitive equilibrium, banks offer privately optimal contracts to their investors but these contracts are not socially optimal: banks' decisions of reallocating capital react too strongly to aggregate shocks. This is because banks do not internalize the impact of their decisions on asset prices. This generates excessive fluctuations of credit, output and asset prices. We examine the efficacy of several possible policy responses to these properties of credit markets, and derive a rationale for macroprudential regulation.

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1 Introduction

There is now a large consensus among economists that prudential regulation of banks should also be envisaged from a systemic, or global perspective, and not only from a microeconomic point of view. The notion of macroprudential regulation, that was coined at the Bank for International Settlements (BIS) in the late 1970s, and repeatedly put forward by Borio (2003, 2010), has now become a buzzword in banking economics. However, it remains quite imprecise, since it does not rely on a universally accepted conceptual framework. Even if one restricts attention to academic publications, the motivations for macroprudential regulation are still broad and somewhat vague.¹

A first strand of the literature, that includes Lorenzoni (2008), Jeanne and Korinek (2011), Korinek (2009) and Bianchi (2011) builds upon the financial accelerator mechanism identified by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). In that framework, firms and households tend to borrow too much, because they do not internalize the impact of their borrowing decisions on asset prices, and more specifically on downward spirals that occur during recessions. When borrowers make losses, they may indeed become credit constrained and be forced to sell assets, provoking a decrease in asset prices. This in turn reinforces credit constraints, leading to further asset sales and downward price spirals. This is the well-known debt-deflation mechanism identified by Fisher (1933) in his study of the Great Depression. The objective of macroprudential regulation is then to curb excessive borrowing so as to decrease the frequency and cost of banking crises. Other analyses such as Diamond and Rajan (2010) or Hansen, Kashyap and Stein (2010) rely on similar mechanisms such as the tendency of banks to issue too many short term deposits, in order to satisfy the demand of investors for (quasi-)safe assets.

Another strand of the literature relies on network externalities (see Allen, Babus and Carletti, 2010) or herding behavior of banks (see Acharya, 2009) to explain why banks should be regulated from a systemic viewpoint and not only on an individual, institution by institution, basis.

We follow here a different (but complementary) route, focusing on the notion of credit cycles. Indeed, empirical evidence shows that banks tend to lend a lot during booms, and very little

¹One line of argument suggests that banking regulation should have a macroeconomic component, e.g. by making bank equity requirements dependent on macroeconomic variables such as GDP growth (Repullo et al. (2009), Brunnermeier et al. (2009)), indicating that countercyclical bank equity buffers could dampen output volatility.

during recessions. We propose a simple explanation for this phenomenon and show that credit markets are dysfunctional. We argue that, instead of dampening productivity shocks, the banking sector tends to exacerbate them, leading to excessive fluctuations of credit, output and asset prices. Our explanation relies on three simple ingredients that are characteristic of modern banks' activities.

The first ingredient is moral hazard: banks are supposed to monitor the small and medium sized enterprises that borrow from them, but they may shirk on their monitoring activities, unless they are given sufficient informational rents. These rents limit the amount that investors are ready to lend them, to a multiple of the banks' own capital. The second ingredient is the banks' high exposure to aggregate shocks: banks' assets have positively correlated returns. Finally the third ingredient is the ease with which modern banks can reallocate capital between different lines of business.

At the competitive equilibrium of the financial sector, banks offer privately optimal contracts to their investors but these contracts are not socially optimal: banks' decisions of reallocating capital react too strongly to aggregate shocks. This is because banks do not internalize the impact of their decisions on asset prices. This generates excessive fluctuations of credit, output and asset prices.

We examine the efficacy of several possible policy responses to this dysfunctionality of credit markets, and show that it can provide a rationale for macroprudential regulation.

The rest of the paper is organized as follows. Section 2 discusses how our approach relates to previous literature. Section 3 presents our model. Section 4 presents two simple benchmarks (the frictionless economy and the rigid economy) in which no public intervention is warranted: in both cases the competitive equilibrium leads to an efficient allocation of resources. Section 5 characterizes optimal contracts for financing banks and shows that they imply recourse to short term financing. Section 6 characterizes the competitive equilibrium. Section 7 shows that this competitive equilibrium is constrained inefficient and justifies some form of macroprudential regulation. Section 8 concludes.

2 Relation to the literature

Our work is related to several strands of the literature.

2.1 Sectoral Shocks

The role of sectoral shocks in macroeconomic fluctuations has been an important theme in the literature over the last three decades (Long and Plosser (1983), Horvath (1998, 2000), Dupor (1999), Conley and Dupor (2003), Carvalho (2009), Acemoglu et al. (2010) and Shea (2002)). While this literature centered around whether sectoral shocks would translate into aggregate shocks, it has also highlighted that sectoral shocks are important in explaining aggregate fluctuations (Horvath (2000)). The triggering event of the recent crisis in the US subprime market can also be interpreted as a negative sectoral productivity shock as real returns on invested capital in the housing market declined. This sectoral shock has spilled over to other sectors in a dramatic way (see e.g. Shleifer and Vishny (2010)).

This literature has also identified the mechanisms by which shocks in one sector spill over to other sectors and the degree of factor substitutability has turned out to be crucial (Dupor (1999), Horvath (2000)).² In our paper shocks to one sector spill over to the rest of the economy because banks reallocate capital across sectors, as discussed in the next subsection.

2.2 Capital Reallocation and its Limits

An important line of research has documented that the amount of reallocation of existing capital is considerable. Eisfeldt and Rampini (2006) indicate³ that reallocation of existing capital comprises about one quarter of total investment.⁴ In parallel to the empirical literature several theories have been offered why reallocation of capital to its most productive use is impeded and thus may be suboptimal. Apart from physical reallocation costs, informational or contractual

²Another line of research has developed models in which there are strategic complementarities across firms so that shocks to some firms can induce cascade effects (Jovanovic (1987) and Durlauf (1993)). Gabaix (2009) shows that idiosyncratic firm-level fluctuations can explain part of aggregate shocks if the firm size distribution is fat-tailed. Acemoglu, Ozdaglar and Tahbaz-Salehi (2010) characterize the conditions under which small shocks can create cascade effects in supply networks.

³They also establish that the reallocation of productive assets across firms is procyclical while the benefits of reallocation are countercyclical. They conclude that the cost or frictions involved in reallocating capital are countercyclical.

⁴Earlier studies have found similar magnitudes (Ramey and Shapiro (1998), Maksimovic and Phillips (2001)). Caballero and Hammour (2005) examine whether reallocation shocks lead to lower aggregate output.

frictions have been studied. Eisfeldt (2004) shows that adverse selection in the market for existing assets reduces reallocation, in particular in bad times.⁵ Eisfeldt and Rampini (2010) examine a model in which managers have private information about the productivity of capital under their control. Reallocation requires paying large bonuses to unproductive managers in order to reveal the productivity to enable value-increasing reallocation. In particular in bad times this may be too costly to investors and the investor may forgo reallocation. Another channel of capital mobility limits has been identified by Azariadis and Kaas (2009) who focus on limited enforcement of loans when borrowers can default but in such cases are denied access to future loans.

We focus on capital reallocation across sectors. The extent of capital reallocation in our model depends on the degree of substitutability, captured through adjustment costs, on the severity of moral hazard in banks and on the exposure of the banking system to aggregate shocks. We obtain excessive capital mobility. The intuitive reason is pecuniary externalities that are detailed in the next subsection.

2.3 Pecuniary Externalities and Financial Fragility

Our paper is part of a growing literature that highlights the role of pecuniary externalities in generating excessive phenomena in financial markets. Welfare reducing pecuniary externalities occur when agents facing credit frictions act atomistically and do not internalize market price reactions which a social planner facing the same credit frictions would take into account.⁶

The main focus of the recent literature has been on overborrowing and insufficient insurance. In Caballero and Krishnamurthy (2003) and Lorenzoni (2008) entrepreneurs invest too much because they cannot insure against the risk of future binding constraints. In Korinek (2011), financially constrained bankers take on insufficient insurance against binding future constraints as insurance has to be bought from risk-averse households which make it costly. Bianchi (2011) provides a quantitative assessment of macroeconomic and welfare implications of overborrowing and allows for the evaluation of the benefits of policy measures to correct these externalities.

⁵Shleifer and Vishny (1992) examine how expected values of assets impact on the debt capacity of a firm.

⁶Suarez and Sussman (1997) develop a dynamic extension of the Stiglitz-Weiss model of lending under moral hazard. They establish a revision mechanism that also relies on pecuniary externalities. In booms firms start producing more which decreases prices which, in turn, creates liquidity shortages next period. As a consequence, the propensity to default raises and the economy turns into a bust.

Lack of insurance does not play a role in our model. The intuitive reason for the excess volatility of bank lending is as follows. When investment return prospects for all banks become more favorable, they buy additional capital. The opposite occurs when negative aggregate events make selling capital more attractive for all banks.

2.4 Financial Intermediaries and Macroeconomic Shocks

The role of the financial sector and its potentially amplifying impact on business cycle fluctuations has been an enduring theme in economics over the last decades. Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and others have focused on credit constraints faced by non-financial borrowers and have provided conceptual foundations for the so called financial accelerator. The idea is that credit constraints arising from asymmetric information between borrowers and lenders amplify and increase the persistence of even small and transitory exogenous shocks.

The role of the balance sheet of financial intermediaries in amplifying macroeconomic shocks has long been recognized in the empirical macroeconomic literature.⁷ The empirical literature has also stressed that well-capitalized banks can better absorb macroeconomic shocks.⁸ Typically, the volatility of bank lending is much higher than the volatility of GDP. For instance, Meh and Moran (2010) report that bank lending growth is over four times as volatile as GDP growth in the US. Adrian and Shin (2010) find that leverage of investment banks is strongly procyclical. Jimenez et al (2011) provide evidence of credit cycles.

Our analysis provides a rationale for why the volatilities of bank lending and capital prices are excessive and how these volatilities are affected by the characteristics of banks. In particular, the more severe the moral hazard problem in banking, the higher the volatility of bank lending. Moreover, higher inside bank equity capital in the economy reduces the fluctuations of bank financing and smooths macroeconomic shocks.

⁷See e.g. Bernanke and Lown (1991) and Peek and Rosengren (1995).

⁸Theoretical foundations have been rare in the previous century. In recent years, however, a flourishing literature has identified the ways in which banks' balance sheets transmit aggregate shocks. An entire strand of DSGE modelling frameworks which we cannot summarize in this paper including the banking sector has been developed to quantify the mutual feedbacks between the financial health of banks and real economic activity. A canonical framework of how financial intermediation interacts with aggregate economic activity and a discussion of part of the literature are given in Gertler and Kiyotaki (2011).

3 The Model

We consider a simple three-period economy ($t = 0, 1, 2$). Initially there is a single physical good that can be transformed into capital at $t = 0$. It can also be consumed at $t = 0$ and $t = 2$. The total amount of physical good that is available in the economy in $t = 0$ is normalized to 1. The consumption good at $t = 0$ is taken as a numeraire. There are three classes of agents: bankers, entrepreneurs and investors. All agents live for three periods from $t = 0$ to $t = 2$. They are risk-neutral and can consume in $t = 0$ and $t = 2$. They do not discount future consumption. The details of the model are set out in the next subsections.

3.1 Agents

There is a continuum of bankers with measure 1. Each banker is endowed with some amount e of the good (his “wealth”) which varies across bankers. The aggregate endowment of bankers in $t = 0$ is denoted by E with $0 < E < 1$.

There is a continuum of investors with measure 1. Aggregate endowments of investors are given by $1 - E$ as total endowments in the economy are normalized to 1. Finally, there is a continuum of entrepreneurs with measure 1. They only play a passive role in our model.

Because of risk neutrality, social welfare is simply measured by aggregated expected consumption $U = C_0 + \mathbb{E}(C_2)$ where C_t denotes aggregate consumption in period t .

3.2 Technologies

The economy comprises two sectors or technologies, denoted by FS (the frictionless sector) and BS (the banking sector), respectively. Investments in the FS and the BS entail the formation of a capital good that can be used for production of the consumption good that becomes available at $t = 2$.

In the BS there is a constant returns technology (the banking technology) that is subject to macroeconomic risk. Specifically, if an amount k is invested by a bank in $t = 0$, the output in $t = 2$ is $\eta \tilde{R}k$, where \tilde{R} is an idiosyncratic return and η is a macroeconomic shock with

$$\eta = \begin{cases} h & \text{(high) with prob. } q \\ l & \text{(low) with prob. } 1 - q \end{cases}, \quad (1)$$

whereby q and $1 - q$ are the probabilities of high and low productivity shocks, respectively, and l and h are real numbers that satisfy $0 < l < 1 < h$. We denote by \bar{R} the expectation of the idiosyncratic return \tilde{R} , which is i.i.d. across banks. The expected output in $t = 2$ per unit of investment in the BS is thus $m\bar{R}$, where $m = qh + (1 - q)l$. Without loss of generality, m is normalized to one. The uncertainty about the aggregate shock is resolved in $t = 1$, where all the market participants observe η and learn whether it is high or low.

The technology of the FS exhibits decreasing marginal returns at the aggregate level. Specifically, if an amount X is invested in period $t = 0$, the output in $t = 2$ is $F(X)$ with $F(0) = 0$, $F'(X) > 0$ and $F''(X) < 0$. $F(\cdot)$ is assumed to fulfill the Inada conditions, i.e. $\lim_{X \rightarrow 0} F'(X) = \infty$ and $\lim_{X \rightarrow 1} F'(X) = 0$. These two conditions ensure that some but not all of the resources are invested in the FS in $t = 0$. Note that the technology shock is sectoral: it only impacts the sector financed by banks. The analysis could be easily extended to technology shocks that impact both sectors.

3.3 Entrepreneurs

Entrepreneurs operate the technologies but they only play a passive role in our model. Those operating in the frictionless sector are directly financed by investors. Those operating in the banking sector must be monitored, and therefore have to be financed by banks.⁹ Because our focus is on the macroeconomic impact of shocks to the banking sector, we do not model entrepreneurs explicitly. However, as we assume that markets in the FS are perfectly competitive, it is useful to think of entrepreneurs in this sector as being a continuum of agents. Each agent has access to an indivisible project of size one that delivers an amount x of consumption good in $t = 2$. The productivity x is distributed according to some continuous and differentiable distribution function $G(x)$ on $[0, \infty)$. Then, if an amount X is invested in the sector, the marginal entrepreneur with productivity \tilde{x} who just receives funds is given by

$$X = \int_{\tilde{x}}^{\infty} dG(x).$$

Total output is given by

$$F(X) = \int_{\tilde{x}}^{\infty} x dG(x)$$

and the marginal productivity is

$$F'(X) = \tilde{x}.$$

⁹The costs of monitoring are set to zero.

3.4 Capital Allocation

In period $t = 0$ some amount C_0 of the good is consumed and the rest is transformed into capital and allocated between the two technologies: K to the banking technology and X to the frictionless technology. The aggregate resource constraint amounts to $C_0 + K + X = 1$. Upon observing macroeconomic events in $t = 1$ the scale of investments in the BS and the FS can be adjusted by reallocating capital between the two sectors. We denote by p_η the price for 1 unit of capital used at date 1 in the FS in state η , in terms of claims on consumption good at $t = 2$. The price p_η is the interim rate of return on capital in the FS.

As of date 0, the (ex-ante) rate of return on capital is $\mathbb{E}[p_\eta] \equiv 1 + r$ where $r \geq 0$ can be interpreted as the interest rate. Note that, given risk neutral preferences with no discounting, either r or C_0 must be zero.

Capital is traded against claims on period-two consumption. There are no defaults nor contract enforcement problems. In period $t = 2$ no further trade takes place. The consumption good at $t = 0$ is taken as a numeraire. We assume that financial markets are complete and frictionless. Given risk neutral preferences, the contingent price paid at $t = 0$ against delivery of the good at $t = 2$ in state h (l) is simply equal to the discounted probability $\frac{q}{1+r}$ ($\frac{1-q}{1+r}$) of this state, where $r \geq 0$ is the interest rate. In all interior allocations ($0 < C_0 < 1$) the interest rate is necessarily zero.

At the interim period $t = 1$ capital goods are sold or bought by bankers and entrepreneurs. We assume that investments in the FS are observable. Hence, claims on investment returns in the frictionless sector can be used as a means of payment in the market for capital goods in $t = 1$. The rate of return p_η thus represents the amount of consumption good at $t = 2$ that is exchanged for one unit of capital at $t = 1$ in state η .

3.5 Banks

Each banker faces the sequence of decisions and events illustrated in Figure 1.

We denote by $k_\eta = (1 + \alpha_\eta)k$ the capital invested by the typical bank after the adjustment decision, where α_η depends on the macroeconomic shock η and satisfies $\alpha_\eta \geq -1$. A value $\alpha_\eta > 0$ characterizes additional investment in the BS and $\alpha_\eta < 0$ expresses disinvestments. We can interpret $\alpha_\eta = \frac{k_\eta - k}{k}$ as the rate of growth of credit in the BS. Capital can be bought or sold for a promised repayment p_η . Moreover, investment adjustments involve additional costs $\frac{c}{2}\alpha_\eta^2 k$ where c is a positive constant ($c > 0$) that measures the relative ease of reallocating capital

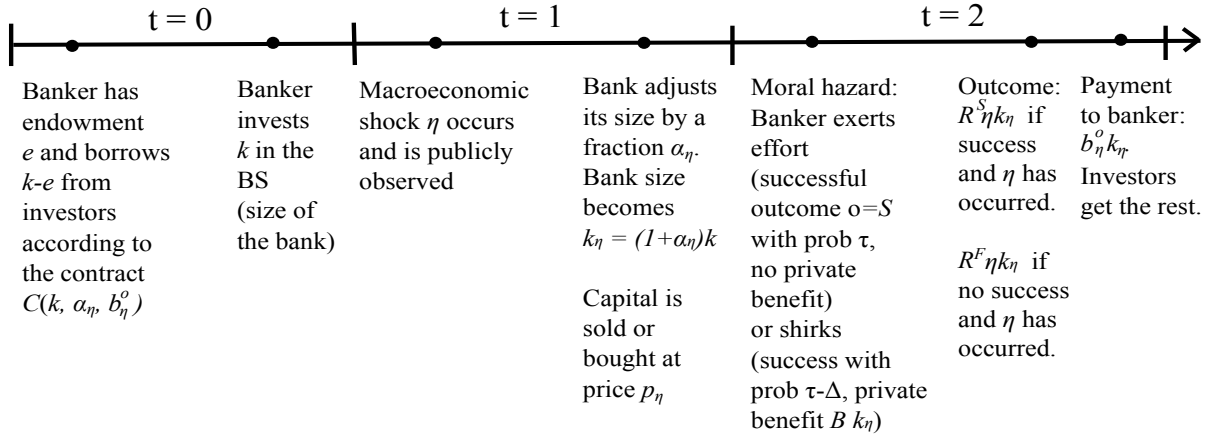


Figure 1

across sectors.¹⁰ Adjustment costs reduce output in the BS and thus are incurred at $t = 2$, and are deducted from gross returns on investment.

The banker's investments are subject to moral hazard as in Holmström and Tirole (1997). The project outcome is either a success ($o = S$) or a failure ($o = F$), and therefore the idiosyncratic return \tilde{R} is either R^S or R^F . If the banker exerts effort, or equivalently, if he chooses a project with high probability of success, he has no private benefit and $o = S$ occurs with probability τ . If the banker shirks or equivalently chooses a project with lower prospects of success, he receives a private benefit $Bk_\eta > 0$ and $o = S$ only occurs with probability $\tau - \Delta$. B is measured in terms of the consumption good. The banker receives a payment $b_\eta^o k_\eta$ when the macroeconomic shock η and the project outcome o have occurred. The contract between the banker and investors is denoted by $C(k, \alpha_\eta, b_\eta^o)$.

Figure 2 represents the random structure of returns. The capital adjustment and reallocation decisions are made by each banker after the realization of the macroeconomic shock. The probability of success is τ when the banker exerts effort, and only $\tau - \Delta$ when the banker shirks. The term in brackets at the branches represents the impact of shirking on the probabilities of success and failure.

¹⁰The case $c = 0$ introduces a variety of subtleties and is available upon request.

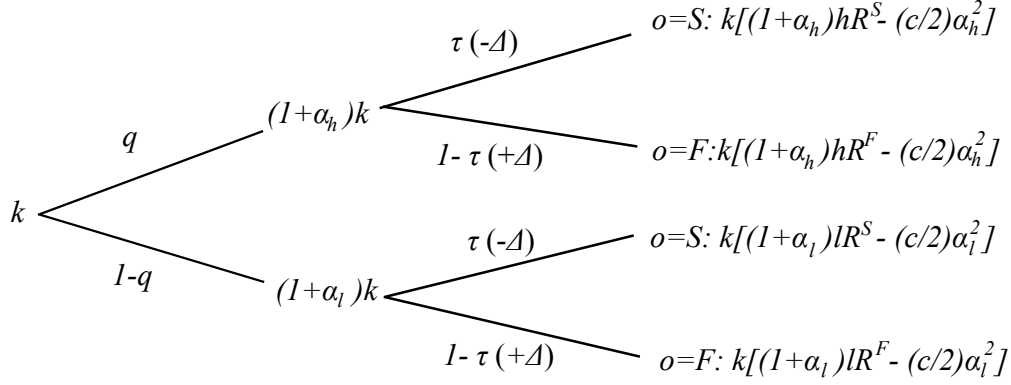


Figure 2

3.6 Social Welfare

In this risk neutral economy, social welfare is measured by aggregate expected consumption¹¹

$$U = C_0 + \mathbb{E}[C_2] = 1 - K - X + K\mathbb{E}[(1 + \alpha_\eta)\eta\bar{R} - \frac{c}{2}\alpha_\eta^2] + \mathbb{E}[F(X - \alpha_\eta K)].$$

This aggregate consumption is shared across agents according to the following aggregate rules:

- Investors get $C_0 + (1 + r)(1C_0 - -E)$, where r is the interest rate.
- Bankers get $K\mathbb{E}[b_\eta^o(1 + \alpha_\eta)]$.
- Entrepreneurs get $\mathbb{E}[F(X - \alpha_\eta K) - p_\eta(X - \alpha_\eta K)]$, where $p_\eta = F'(X - \alpha_\eta K)$ is the marginal productivity of capital in the FS and thus the price of capital in state η .

At equilibrium, the sum of these three terms coincides with U . This is because the expected rate of return on capital in the banking sector has to be equal to $1 + r$, as the risk neutral investors have to be indifferent between investments in the BS and the FS, i.e.

$$(1 + r)(K - E) = K\mathbb{E}[(1 + \alpha_\eta)(\eta\bar{R} - b_\eta^o) - \frac{c}{2}\alpha_\eta^2 - p_\eta\alpha_\eta].$$

¹¹In general, the private benefits of bankers also enter social welfare. Throughout the paper, we will focus on circumstances in which shirking is inefficient and will be avoided by paying the banker a higher amount if the project outcome is a success.

$K - E$ is the amount of resources offered by investors to the BS. Therefore¹²

$$U = (1+r)C_0 + \mathbb{E}[C_2] = (1+r)(1-K-X) + (1+r)(K-E) + K\mathbb{E}[b_\eta^o(1+\alpha_\eta) + p_\eta\alpha_\eta] + \mathbb{E}[F(X - \alpha_\eta K)],$$

and after simplification,

$$U = (1+r)(1-E) + K\mathbb{E}[b_\eta^o(1+\alpha_\eta)] + \mathbb{E}[F(X - \alpha_\eta K) - p_\eta(X - \alpha_\eta K)], \quad (2)$$

where we have used the fact that $\mathbb{E}[p_\eta] = 1+r$.

4 Two Simple Benchmarks

This section examines two cases where public intervention is not justified: the frictionless economy (no moral hazard) and the rigid economy (no capital reallocations).

4.1 The Frictionless Economy

In the absence of moral hazard, which is equivalent to setting $B = 0$, supply and demand for capital in both sectors, the BS and the FS, are not subject to frictions. The competitive equilibrium of this economy leads to a first best social optimum. To establish uniqueness and social efficiency of the competitive equilibrium, we note first that the rate of return on capital in state η in $t = 1$ must be equal to the marginal return of investment in both technologies:

$$p_\eta = F'(X - K\alpha_\eta) = \eta\bar{R} - c\alpha_\eta. \quad (3)$$

Moreover, the investment decisions at date 0 are determined by the equality of expected returns on capital in these two technologies:

$$\mathbb{E}[p_\eta] = \mathbb{E}[\eta\bar{R}(1 + \alpha_\eta) - \frac{c}{2}\alpha_\eta^2 - \alpha_\eta p_\eta].$$

Combining these two properties, we see that

$$\mathbb{E}[\eta\bar{R} - c\alpha_\eta] = \mathbb{E}[\eta\bar{R}(1 + \alpha_\eta) - \frac{c}{2}\alpha_\eta^2 - \alpha_\eta(\eta\bar{R} - c\alpha_\eta)],$$

which leads, after simplification, to a fundamental relation between α_h and α_l , namely:

$$\mathbb{E}[\alpha_\eta^2 + 2\alpha_\eta] = 0. \quad (4)$$

¹²Recall that $rC_0 = 0$ (either $r = 0$ or $C_0 = 0$) by the complementarity slackness condition.

This condition will play an important role in the sequel, as it is also satisfied in the competitive equilibrium of the economy with moral hazard and capital reallocations. It characterizes a decreasing curve in the (α_l, α_h) plane, limited by two extreme points:

- $(-1, \frac{1}{\sqrt{q}} - 1)$, which corresponds to the complete liquidation of banking assets in the bad state,¹³ and
- $(0, 0)$, which corresponds to a complete absence of adjustment at date 1.

It is easy to see that this curve can be parameterized by the variance of α_η , namely $\sigma^2 = q(1 - q)(\alpha_h - \alpha_l)^2$. Indeed condition (4) is equivalent to

$$\sigma^2 + \bar{\alpha}^2 + 2\bar{\alpha} = 0,$$

with $\bar{\alpha} := \mathbb{E}[\alpha_\eta]$. This implies¹⁴ that $\bar{\alpha} = \sqrt{1 - \sigma^2} - 1$ and

$$\begin{cases} \alpha_h &= \sqrt{1 - \sigma^2} - 1 + \sigma \sqrt{\frac{1-q}{q}} \\ \alpha_l &= \sqrt{1 - \sigma^2} - 1 - \sigma \sqrt{\frac{q}{1-q}}. \end{cases}$$

When σ increases from 0 to $\sqrt{1 - q}$, α_l decreases from 0 to -1, and α_h increases from 0 to $(\frac{1}{\sqrt{q}} - 1)$. To determine the first best allocation, we need a second condition, that is given by the equality of supply and demand for capital in the banking sector. It is natural to focus on the case where the banking technology is expected to be profitable even when there are no capital reallocations, namely:

Assumption 1

$$\bar{R} > 1.$$

Under Assumption 1, the first-best allocation involves no consumption at $t = 0$ ($C_0 = 0$ and $K + X = 1$) and a positive interest rate r , that can be deduced from condition (3), transformed by using the fact that $X = 1 - K$:

$$1 + r = \mathbb{E} [F'[1 - K(1 + \alpha_\eta)]] = \mathbb{E} [\eta \bar{R} - c \alpha_\eta]. \quad (5)$$

¹³In order to ensure that output in the BS is non-negative we can restrict $\alpha_l \geq \underline{\alpha}_l > -1$ where $\underline{\alpha}_l$ is defined by $l\bar{R}(1 + \underline{\alpha}_l)k - \frac{c}{2}\underline{\alpha}_l^2 k = 0$, which yields $\underline{\alpha}_l = \frac{l\bar{R} - \sqrt{l\bar{R}(l\bar{R} + 2c)}}{c}$.

¹⁴The other solution of equation (4) is not applicable as it violates the boundary conditions of α_l or α_h .

Let us define the supply of capital to the banking sector as the function $S(p)$ such that

$$F'[1 - S(p)] = p. \quad (6)$$

$S(\cdot)$ is an increasing function such that $S(0) = 0$ and $S(\infty) = 1$. Condition (5) just means that

$$K(1 + \alpha_\eta) = S(\eta\bar{R} - c\alpha_\eta),$$

which gives the second condition that we need to characterize the first-best allocation

$$K = \frac{S(h\bar{R} - c\alpha_h)}{1 + \alpha_h} = \frac{S(l\bar{R} - c\alpha_l)}{1 + \alpha_l}. \quad (7)$$

This allocation is Pareto optimal, as it maximizes total surplus U given by (2). Thus we obtain:

Proposition 1

In the frictionless economy there is a unique competitive equilibrium characterized by conditions (4) and (7). This competitive equilibrium is Pareto optimal.

4.2 The Rigid Economy

As a second benchmark, let us consider the case where capital adjustments are impossible ($c = \infty$) but moral hazard is present. We will show that, in this case also, the competitive allocation cannot be Pareto improved and that bank regulation is not needed.

In fact, macroeconomic shocks do not play any role in this rigid economy. Since agents are all risk neutral and no adjustments can be made at $t = 1$, the macroeconomic shock has no impact on utilities. The rigid economy is thus a simple extension of the static model of Holmström and Tirole (1997). We assume that investment in the BS is socially beneficial only if the banker exerts effort:

Assumption 2

$$\bar{R} + B - \Delta(R^S - R^F) < 1.$$

Note that Assumptions 1 and 2 taken together imply that $R^S - R^F > \frac{B}{\Delta}$. Effort is exerted in state η if and only if payments to the banker satisfy

$$\begin{aligned} \Delta(b_\eta^S - b_\eta^F)k &\geq kB \\ \text{or } b_\eta^S - b_\eta^F &\geq \frac{B}{\Delta}. \end{aligned} \quad (8)$$

Since the banker is protected by limited liability ($b_\eta^o \geq 0, o = S, F$), it is easy to see that the optimal contract is such that $b_\eta^S \equiv \frac{B}{\Delta}, b_\eta^F \equiv 0$ for $\eta \in \{h, l\}$. Under Assumption 2, the optimal contract between investors and the banker consists of paying the latter a bonus $\frac{B}{\Delta}k$ (proportional to the size k of the bank) in case of success, and nothing in case of failure. This contract minimizes the informational rents of the banker per unit of investment. The optimal size of the bank is the maximum k that satisfies the break-even constraint of investors

$$(k - e)(1 + r) \leq k(\bar{R} - b),$$

where $b = \frac{\tau B}{\Delta}$ represents the expected payment to the banker per unit of asset. This condition is equivalent to a (market imposed) capital ratio¹⁵

$$\frac{e}{k} \geq \rho_0(r),$$

where the capital ratio $\rho_0(r) = 1 - \frac{\bar{R} - b}{1 + r}$ is an increasing function of r . This is because Assumptions 1 and 2 imply that $\bar{R} > b$.

Note that this contract can be implemented by a simple capital structure whereby the bank issues in $t = 0$ riskless deposits with face value $D_R^0 = lkR^F$ (corresponding to the minimum liquidation value of the bank's assets) and issues in $t = 1$ if $\eta = h$ has occurred, additional debt D_R^1 with face value $k(hR^F - lR^F)$. Moreover, the bank issues in $t = 0$ (outside) equity that pays out a dividend $k[\eta R^S - \eta R^F - \frac{B}{\Delta}]$ in case of success.¹⁶ The banker's payment $\frac{kB}{\Delta}$ is only paid in case of success. An alternative is to give the banker some (inside) equity. In this case, total dividend payments amount to $k\eta[R^S - R^F]$ of which a fraction $\theta_\eta = \frac{B}{\Delta\eta(R^S - R^F)}$ is given to the banker as inside equity in $t = 1$. Assumption 2 implies that $\theta_\eta \leq 1$.

The equilibrium interest rate is then given by the equality of supply and demand for banking

¹⁵Our banks are financed by sophisticated investors, not by uninformed retail depositors. Thus there is no need for microprudential regulation.

¹⁶We assume that $l(R^S - R^F) - \frac{B}{\Delta} \geq 0$.

capital:

$$K = \frac{E}{\rho_0(r)} = S(1+r). \quad (9)$$

When E is small (specifically $E < E^* := S(1)\rho_0(0)$) this equilibrium interest rate is zero and $C_0 > 0$. When E is large (specifically $E > E^{**} = \frac{b}{\bar{R}}S(\bar{R})$) the moral hazard constraint ceases to bind and social welfare is maximal. Social welfare in this case is given by

$$U = \mathbb{E}(C_2) = \max_X \{F(X) + (1-X)\bar{R}\}$$

where $K = 1 - X = S(\bar{R})$.

Note that, when $E \leq E^{**}$, social welfare could be increased by transferring wealth ex-ante from investors to bankers. We will systematically rule out such transfers because they would require giving taxation powers to banking supervisors, which would face strong political opposition. Instead we will focus on traditional regulatory instruments such as capital or liquidity ratios that do not involve payment flows between supervisors and banks. The following proposition summarizes the three possible types of equilibrium. We assume that the moral hazard problem in the BS is sufficiently severe:

Assumption 3

$$\bar{R} < 1 + b.$$

Proposition 2

Assume $\bar{R} < 1 + b$. Then the rigid economy has a unique equilibrium, characterized as follows:

- When $E < E^* = S(1)\rho_0(0)$: $K = \frac{E}{\rho_0(0)}$, $X = 1 - S(1) - C_0$, $C_0 > 0$ and $r = 0$.
- When $E^* \leq E \leq E^{**} = \frac{b}{\bar{R}}S(\bar{R})$: $K = \frac{E}{\rho_0(r)} = S(1+r)$, $C_0 = 0$.
- When $E > \frac{b}{\bar{R}}S(\bar{R})$: $K = S(\bar{R}) = 1 - X$, $C_0 = 0$.

As final comments to this section, note that:

- There is no need for maturity transformation: bankers find it optimal to choose the maturity of their debt to be equal to the maturity of their assets ($t = 2$),
- Any regulatory intervention on the size of banks would be socially wasteful: reducing the size of banks would decrease social surplus, while forcing them to lend more would generate shirking.

5 Privately Optimal Contracts

5.1 The Contractual Problem

In the economy with capital adjustments, contracts between each banker and its financiers have to specify α_h and α_l . The optimal payments to the banker follow the same logic as in the rigid economy, with the only difference that they will in general depend on the macroeconomic shock η . Assuming for instance that private benefits per unit of assets (obtained by the banker who shirks) are still B (independently of η), the optimal payments to the banker are $\frac{B}{\Delta}k_\eta$ in case of success (and zero in case of failure). This means that the expectation of the rent that investors have to give to the banker is $b\mathbb{E}(k_\eta) = bk\mathbb{E}(1 + \alpha_\eta)$, where $b = \frac{\tau B}{\Delta}$. Thus the optimal banking contract solves

$$\max_{\{k, \alpha_h, \alpha_l\}} bk\mathbb{E}(1 + \alpha_\eta)$$

subject to the investors' participation constraint

$$(k - e)(1 + r) \leq k\mathbb{E}[(1 + \alpha_\eta)(\eta\bar{R} - b) - \frac{c}{2}\alpha_\eta^2 - p_\eta\alpha_\eta]$$

and the banker's participation constraint

$$bk\mathbb{E}[1 + \alpha_\eta] \geq e(1 + r).$$

Since returns to scale in BS are constant, the aggregate allocation (K, α_h, α_l) solves the same program where individual banks' capital e is replaced by aggregate bank capital E . We focus on the case where bank capital is scarce (E is small). Then, as we show below, the second constraint does not bind and $C_0 > 0$, which implies $r = 0$. Note that, by absence of arbitrage opportunities, we must have then

$$\mathbb{E}[p_\eta] = 1.$$

5.2 Characterization of the Optimal Contract

The following Proposition characterizes the solution of the contractual problem, given the vector of capital prices and the banker's initial wealth.

Proposition 3

Given the vector $p = (p_h, p_l)$ of capital prices and the banker's initial wealth e , the optimal

banking contract is characterized by

$$k(p, e) := \frac{e}{\rho(p)}, \text{ where} \quad (10)$$

$$\rho(p) = 1 - \mathbb{E}[(1 + \alpha_\eta(p))(\eta\bar{R} - b) - \frac{c}{2}\alpha_\eta^2(p) - p_\eta\alpha_\eta(p)] \quad (11)$$

and $(\alpha_h(p), \alpha_l(p))$ maximize

$$\frac{b\mathbb{E}[1 + \alpha_\eta(p)]e}{1 - \mathbb{E}[(1 + \alpha_\eta(p))(\eta\bar{R} - b) - \frac{c}{2}\alpha_\eta^2(p) - p_\eta\alpha_\eta(p)]}. \quad (12)$$

Proposition 3 follows directly from the fact that investors' participation constraint binds: solving for k with $r = 0$ and plugging it into the objective function yields the expression (12). The values (α_h, α_l) are chosen to maximize this expression. The solution of the maximization problem will be given in the next section. We note that adjustment decisions $(\alpha_h(p), \alpha_l(p))$ are independent of the scale of banks. Moreover, the size k of the bank is proportional to the banker's wealth e , a property that will make aggregation easy.

Proposition 3 implies that there is a "market-imposed" capital ratio for banks:

$$\frac{e}{k} \geq \rho(p).$$

This equity ratio is a function of the capital prices p_h and p_l .

5.3 Implementation of the Optimal Contract

We now show that the optimal banking contract can be implemented by a certain capital structure, involving a combination of short term debt, long term debt and some insurance against aggregate shocks.

The role of short term debt is to "discipline" bankers, in the spirit of Jensen (1983) and Diamond and Rajan (2000). Since their bonus is proportional to the size of their bank, bankers would not spontaneously reduce this size by the factor $\alpha_l < 0$ in the state l . To force them to do so, investors can impose a debt repayment of an amount equal to the net proceeds of liquidation in this state:

$$D_{ST} = -k\alpha_l p_l > 0.$$

Of course, this short term debt increases the needs for funds in the good state h . In that state the bank must then be allowed to issue new debt for the amount

$$D_1^h = D_{ST} + k\alpha_h p_h.$$

This particular debt structure is needed to force the bankers to implement the reallocation decisions (α_l, α_h) that are optimal for investors. There is some degree of freedom for the rest of the implementation since the Modigliani-Miller logic applies to the total payments that investors must receive¹⁷ at date 2. When state η has occurred, these payments are

$$\begin{aligned} P_\eta^S &= k(1 + \alpha_\eta)\left[\eta R^S - \frac{B}{\Delta}\right] - \frac{c}{2}\alpha_\eta^2 k && \text{in case of success, and} \\ P_\eta^F &= k(1 + \alpha_\eta)\eta R^F - \frac{c}{2}\alpha_\eta^2 k && \text{in case of failure,} \end{aligned}$$

while the banker receives

$$\begin{aligned} b_\eta^S &= k(1 + \alpha_\eta)\frac{B}{\Delta} && \text{in case of success, and} \\ b_\eta^F &= 0 && \text{in case of failure.} \end{aligned}$$

A natural choice of securities that implements these payments is a combination of debt and equity such that dividends are paid only in case of success. Total debt repayment is then P_η^F , while dividends $\delta_\eta = P_\eta^S - P_\eta^F$ are paid in case of success. The banker receives a bonus $k(1 + \alpha_\eta)\frac{B}{\Delta}$ in case of success and nothing in case of failure. Note that the total repayment to initial debtholders is not the same in the two states. In state l , this total repayment is $P_l^F = k(1 + \alpha_l)lR^F - \frac{c}{2}\alpha_l^2 k$. This is the amount that can be paid back in any case and thus can be promised as repayment for a long-term debt, issued in $t = 0$ and repaid in $t = 2$. The latter amount is denoted by D_{LT} . However if state h occurs, new debt D_1^h is issued at date 1 and the borrowing capacity of the bank increases, so that total repayment to initial debtholders at date 2 in case of failure becomes

$$P_h^F - D_1^h = k \left[(1 + \alpha_h)hR^F - \alpha_h p_h - \frac{c}{2}(\alpha_h)^2 + \alpha_l p_l \right].$$

In general, this amount is higher than P_l^F . Thus, some insurance against macroeconomic shocks must be included in the package of securities that implements the optimal banking contract. One example of such an insurance mechanism is a convertible bond with a face value $C^N = (P_h^F - D_1^h) - P_l^F$ that has to be repaid if it is not converted to equity. If state l occurs, this bond is converted into equity, so that total repayment to debtholders is reduced to P_l^F , which corresponds to the volume of long term debt issued at $t = 0$.

To wrap up, the optimal contract can be implemented as follows:

- at date 0, the bank issues **outside** equity for an amount O , short term debt for an amount

¹⁷The contract only specifies the payments to the bankers. The payments to investors can be split arbitrarily between different types of securities.

D_{ST} , long term debt for an amount D_{LT} and convertible bonds for an amount C^N .

- if state l occurs, the bank liquidates at date 1 a fraction $|\alpha_l|$ of its assets, and uses the net proceeds to repay its short term debt:

$$D_{ST} = -k\alpha_l p_l > 0.$$

Each convertible bond is converted into y shares. At date 2, long term debt $D_{LT} = k(1 + \alpha_l)lR^F - \frac{c}{2}(\alpha_l)^2 k$ is repaid and in case of success, a total amount of dividends $k[1 + \alpha_l][l(R^S - R^F) - \frac{B}{\Delta}]$ is paid to initial shareholders, and new shareholders (who initially held convertible debt). In case of success, the banker receives $k(1 + \alpha_l)\frac{B}{\Delta}$.

- if state h occurs, the bank issues new debt for an amount $D_1^h = D_{ST} + k\alpha_h p_h$. It increases its size by a factor α_h . At date 2, total payment to debt holders is

$$P_h^F = D_{LT} + D_1^h + C^N = k(1 + \alpha_h)hR^F - \frac{c}{2}(\alpha_h)^2 k$$

where

$$C^N = k \left[(1 + \alpha_h)hR^F - (1 + \alpha_l)lR^F - \alpha_h p_h - \frac{c}{2}(\alpha_h)^2 + \alpha_l p_l + \frac{c}{2}(\alpha_l)^2 \right]$$

is the face value of the convertible bond.

Finally, if the bank succeeds, the banker receives a bonus $k(1 + \alpha_h)\frac{B}{\Delta}$ and shareholders receive dividends $k[1 + \alpha_h][h(R^S - R^F) - \frac{B}{\Delta}]$.

6 Competitive Equilibrium

6.1 Definition

In this section we characterize the competitive equilibrium of the economy. For this purpose we recall that we have denoted by C_0 the aggregate consumption at $t = 0$, by K the aggregate amount of capital invested in $t = 0$ in the BS and by X the aggregate amount invested in the FS in $t = 0$. Henceforth, the variables α_h, α_l and k always correspond to the privately optimal values, given by Proposition 3.

Definition 1

A competitive equilibrium in the economy with moral hazard and capital reallocation is an array $\Sigma = \{C_0, K, X, \alpha_h, \alpha_l, p_h, p_l\}$ such that:

(i) A banker with wealth e obtains $k - e$ from investors with

$$k = \frac{e}{\rho(p)}.$$

Aggregate investments in the BS in $t = 0$ are

$$K = \frac{E}{\rho(p)}.$$

(ii) Investments in the BS are adjusted in $t = 1$ by $(\alpha_h(p), \alpha_l(p))$ defined in Proposition 3.

(iii) The price of capital at $t = 1$ in state η is $p_\eta = F'(X - \alpha_\eta(p)K)$, $\eta \in \{h, l\}$.

(iv) $\mathbb{E}[p_\eta] = qp_h + (1 - q)p_l = 1 + r$.

(v) $K + X + C_0 = 1$.

(vi) $C_0 \geq 0$, $K \geq 0$, $X \geq 0$.

(vii) $rC_0 = 0$.

Thus at date 0, K is invested in the banking technology, X in the frictionless technology and $C_0 = 1 - K - X$ is consumed. At date 1, the aggregate shock η is revealed, and investments are adjusted to $K_\eta = K(1 + \alpha_\eta)$ and $X_\eta = X - \alpha_\eta K$. The price of capital is determined by its marginal productivity in the frictionless technology. We will focus on interior solutions, in which $C_0 > 0$ and the expected rate of return on capital in equilibrium is equal to 1 (i.e. $r = 0$).

Finally, at date 2, the average return on bank assets is $\eta\bar{R}$, while the frictionless sector produces $F(X_\eta)$. Aggregate output in state η is thus

$$Y_\eta = \eta\bar{R}K_\eta + F(X_\eta) - \frac{c}{2}\alpha_\eta^2 K. \quad (13)$$

6.2 Existence and Uniqueness

To obtain a handy characterization of the competitive equilibrium (and an existence theorem) let us reformulate Proposition 3. Because the banking technology has constant returns to scale, the optimal contract can be characterized at the aggregate level. That is, (K, α_h, α_l) is the solution of

$$\max_{\{K, \alpha_h, \alpha_l\}} Kb\mathbb{E}[1 + \alpha_\eta] \quad (14)$$

subject to

$$K\mathbb{E}[(1 + \alpha_\eta)(\eta\bar{R} - b) - \frac{c}{2}\alpha_\eta^2 - p_\eta\alpha_\eta] = K - E.$$

Denoting by \mathcal{L} the associated Lagrangian, and by ν the multiplier, we can characterize this solution by the first order conditions:

$$\frac{\partial \mathcal{L}}{\partial K} = b\mathbb{E}[1 + \alpha_\eta] + \nu\mathbb{E}[(1 + \alpha_\eta)(\eta\bar{R} - b) - \frac{c}{2}\alpha_\eta^2 - p_\eta\alpha_\eta] - \nu = 0 \quad (15)$$

$$\frac{\partial \mathcal{L}}{\partial \alpha_\eta} = KPr(\eta)[b + \nu(\eta\bar{R} - b - c\alpha_\eta - p_\eta)] = 0, \quad (16)$$

where $Pr(\eta)$ denotes the probability that η occurs, which is q for $\eta = h$ and $1 - q$ for $\eta = l$. From the second condition we derive

$$p_\eta = \eta\bar{R} - b - c\alpha_\eta + \frac{b}{\nu}. \quad (17)$$

Plugging this expression into the first condition yields

$$1 - \frac{b}{\nu}(1 + \bar{\alpha}) = \mathbb{E}[(\eta\bar{R} - b)(1 + \alpha_\eta) - \frac{c}{2}\alpha_\eta^2 - \alpha_\eta(\eta\bar{R} - b - c\alpha_\eta + \frac{b}{\nu})], \quad (18)$$

with $\bar{\alpha} = \mathbb{E}[\alpha_\eta]$. After simplifications we obtain

$$\frac{b}{\nu} = 1 - \bar{R} + b - \frac{c}{2}\mathbb{E}[\alpha_\eta^2], \quad (19)$$

and thus¹⁸

$$p_\eta = (\eta - 1)\bar{R} + 1 - c\alpha_\eta - \frac{c}{2}\mathbb{E}[\alpha_\eta^2]. \quad (20)$$

Now the participation constraint of investors gives

$$K - E = K[1 - \frac{b}{\nu}(1 + \bar{\alpha})], \quad (21)$$

or

$$\frac{E}{K} = (1 + \bar{\alpha})[1 - \bar{R} + b - \frac{c}{2}\mathbb{E}[\alpha_\eta^2]]. \quad (22)$$

Finally the fourth equilibrium condition gives:

$$\mathbb{E}[p_\eta] = 1. \quad (23)$$

¹⁸Note that Condition (19) implies $\nu > 1$.

Using the expression of p_η obtained above we get

$$1 = \mathbb{E}[(\eta - 1)\bar{R} + 1 - c\alpha_\eta - \frac{c}{2}\mathbb{E}[\alpha_\eta^2]], \text{ and thus} \quad (24)$$

$$\bar{\alpha} + \frac{1}{2}\mathbb{E}[\alpha_\eta^2] = 0. \quad (25)$$

As above, it is convenient to parameterize the equilibrium by the variance of α_η , namely $\sigma^2 = q(1 - q)(\alpha_h - \alpha_l)^2$. The above condition implies that $2\bar{\alpha} + \bar{\alpha}^2 + \sigma^2 = 0$ like in the frictionless economy (see Subsection 4.1). Thus $\bar{\alpha} = \sqrt{1 - \sigma^2} - 1$ and

$$\begin{cases} \alpha_h = \sqrt{1 - \sigma^2} - 1 + \sigma\sqrt{\frac{1-q}{q}} \\ \alpha_l = \sqrt{1 - \sigma^2} - 1 - \sigma\sqrt{\frac{q}{1-q}}. \end{cases} \quad (26)$$

The prices of capital can be expressed as

$$\begin{cases} p_h = (h - 1)\bar{R} + 1 - c\sigma\sqrt{\frac{1-q}{q}} \\ p_l = (l - 1)\bar{R} + 1 + c\sigma\sqrt{\frac{q}{1-q}}. \end{cases} \quad (27)$$

When σ increases from 0 to $\sqrt{1 - q}$, α_l decreases from 0 to -1 and α_h increases from 0 to $\frac{1}{\sqrt{q}} - 1$. Thus we always have:

$$-1 < \alpha_l < 0 < \alpha_h.$$

We now determine the equilibrium value of σ .

The final equilibrium conditions are¹⁹:

$$p_\eta = F'(X - \alpha_\eta K), \quad \eta = h, l, \quad (28)$$

where K is such that

$$\frac{E}{K} = (\sqrt{1 - \sigma^2})[1 - \bar{R} + b + c\sqrt{1 - \sigma^2} - c]. \quad (29)$$

Using the definition of $S(p)$ in equation (6) and condition (28) yields $1 - S(p_\eta) = X - \alpha_\eta K$. We can eliminate X between these conditions and obtain

¹⁹These conditions imply that $p_l < p_h$ (indeed, $\alpha_l < \alpha_h$ and F' is decreasing). Since $\mathbb{E}[p_\eta] = 1$ this implies $p_l < 1 < p_h$.

$$(\alpha_h - \alpha_l)K = S(p_h) - S(p_l), \quad (30)$$

where $S(\cdot)$ is the supply function of bank capital defined above.

This provides the last condition on σ :

$$\begin{aligned} & \frac{1}{\sqrt{q(1-q)}} \frac{E\sigma}{\sqrt{1-\sigma^2}[1-\bar{R}+b+c\sqrt{1-\sigma^2}-c]} \\ &= S\left((h-1)\bar{R}+1-c\sigma\sqrt{\frac{1-q}{q}}\right) - S\left((l-1)\bar{R}+1+c\sigma\sqrt{\frac{q}{1-q}}\right), \end{aligned} \quad (31)$$

or alternatively

$$\begin{aligned} E &= \sqrt{q(1-q)} \left(\frac{1}{\sigma^2} - 1\right) \left[1 - \bar{R} + b + c\sqrt{1-\sigma^2} - c\right] \\ &\quad \cdot \left\{ S\left((h-1)\bar{R}+1-c\sigma\sqrt{\frac{1-q}{q}}\right) - S\left((l-1)\bar{R}+1+c\sigma\sqrt{\frac{q}{1-q}}\right) \right\}. \end{aligned} \quad (32)$$

The righthand side of this equation is a function of σ that we denote $H(\sigma)$. It is the product of three strictly decreasing functions of σ , which are positive on $(0, 1)$. Therefore H is strictly decreasing on $(0, 1)$. Moreover, H is continuous on $(0, 1)$ with $H(0) = +\infty$ and $H(1) = 0$. This guarantees the existence of a unique solution σ^E to the equation $H(\sigma) = E$. However, we need to check that $C_0 = 1 - K - X > 0$. This is satisfied when σ^E is large enough, or equivalently when E is smaller than some threshold E_1^* . Like in the rigid economy characterized by Proposition 2 there is a second threshold E_1^{**} above which moral hazard ceases to be binding. Thus we have established:

Proposition 4

When bank capital is scarce ($E \leq E_1^*$) the unique competitive equilibrium is such that $C_0 > 0$ and $r = 0$. It can be parameterized by the variance of credit growth: $\sigma^2 = q(1-q)(\alpha_h - \alpha_l)^2$.

- σ^E is the unique solution of $H(\sigma) = E$.
- $\bar{\alpha}^E = \sqrt{1 - (\sigma^E)^2} - 1$.
- $p_h^E = (h-1)\bar{R} + 1 - c\sigma^E\sqrt{\frac{1-q}{q}}$, $p_l^E = (l-1)\bar{R} + 1 + c\sigma^E\sqrt{\frac{q}{1-q}}$.
- $\frac{E}{K^E} = \sqrt{1 - (\sigma^E)^2}[1 - \bar{R} + b + c\sqrt{1 - (\sigma^E)^2} - c]$.

Proposition 4 and the properties of equation (32) yield immediately the following Corollary.

Corollary 1

When E , the capital of the banking industry, increases,

- the standard deviation σ^E of bank credit growth decreases and its mean $\overline{\alpha^E}$ increases.
- p_h^E increases and p_l^E decreases (capital prices vary more).
- $\frac{E}{K^E}$ increases.

Another immediate consequence of equation (32) and Proposition 4 is

Corollary 2

An increase of the severity of moral hazard in banking, i.e. an increase of b ,

- increases σ^E and lowers $\overline{\alpha^E}$,
- decreases α_l^E and increases α_h^E if σ^E is not very large,
- decreases p_h^E and increases p_l^E .

7 Social Efficiency and Regulation

In this section we explore the scope for regulation.

Given that all agents are risk neutral, social efficiency is simply measured by expected aggregate consumption (remember that we focus on the case where $r = 0$):

$$U = K\mathbb{E}[\eta\bar{R}(1 + \alpha_\eta) - \frac{c}{2}\alpha_\eta^2 - 1] + \mathbb{E}[F(X - \alpha_\eta K)] - X + 1. \tag{33}$$

7.1 The Competitive Allocation is Inefficient

We first show that capital reallocation at the competitive equilibrium is inefficient. In particular, we show in the next proposition that, starting from the competitive equilibrium, characterized in Proposition 4, the expected social surplus U can be increased by raising α_l and adjusting α_h . Such an intervention would force banks to invest more during recessions and less

in booms if $\Delta\alpha_h < 0$.²⁰ We will later discuss which type of regulatory tools can achieve this objective.

Proposition 5

Starting from the equilibrium allocation $(K^E, X^E, \alpha_h^E, \alpha_l^E)$ it is possible to increase α_l^E by $\Delta\alpha_l > 0$ and to adjust α_h^E by $\Delta\alpha_h$ in such a way that, after investors revise their positions, and capital prices adjust, social welfare is increased.

The proof of Proposition 5 is given in the appendix.

The main intuition for the result is as follows. When banks reallocate capital they exert a negative pecuniary externality on other banks as capital prices decline in downturns and raise in boom periods. Bankers aim at maximizing the expected scale of banks in order to obtain the maximal expected rents. When a negative shock occurs, informational rents, i.e. the bonuses for bankers, are particularly costly for investors. Hence, capital reallocation is comparatively more attractive to motivate investors to finance a larger expected size of the bank. As a consequence, banks sell capital in downturns excessively. The opposite – but to a lesser degree – occurs in boom periods. The associated negative pecuniary externalities on other banks translates into welfare losses. As a consequence, by increasing α_l and adjusting α_h , pecuniary externalities are reduced when banks adjust their balance sheets. As downward and upward adjustments are limited, capital prices will fall less in downturns and will increase less in upturns which limits the pecuniary externalities. This is welfare improving.

7.2 One-sided Intervention

We next investigate whether welfare gains can be achieved by increasing α_l alone. In particular, we show in the next proposition that, starting from the competitive equilibrium (characterized in Proposition 4), it is possible to increase the expected social surplus U by increasing α_l , i.e. “forcing” banks to invest more during recessions. As we explain later, a similar improvement of social welfare may also be obtained by decreasing α_h , i.e. restricting bank lending during booms.

²⁰ $\Delta\alpha_h < 0$ does not need to hold in all circumstances.

Proposition 6

Starting from the equilibrium allocation $(K^E, X^E, \alpha_h^E, \alpha_l^E)$ it is possible to increase α_l^E by $\Delta\alpha_l > 0$ in such a way that, after investors revise their positions, and capital prices adjust, social welfare is increased.

The proof is given in the appendix. Proposition 6 shows that it is sufficient to limit excessive downsizing in downturns to achieve welfare gains.

7.3 Macroprudential Regulation

Now the question is: how to force banks to lend more (or rather to reduce lending by a lesser amount) in case of a recession? The answer is a corollary of the implementation of the optimal contract that we proposed in Section 5.3: when investors cannot directly control banks' investments decisions, they have to rely on short term debt as a way to force bankers to downsize in the bad state. Since social surplus is improved by increasing α_l , imposing a regulatory upper bound on short term debt (i.e. a liquidity ratio in the spirit of the Net Stable Funding Ratio of Basel III) would do the job.

An alternative, indirect, way for the regulator to increase social welfare would be to impose a binding capital requirement in state h , i.e. contingent on a boom.²¹ It is easy to see that it would lead to the same effects than the restriction on short term finance discussed above. Note that in our model there is no need for microprudential regulation, since banks are financed by sophisticated investors who can impose a limit to bank leverage. If we introduced as well retail deposits, and thus a rationale for microprudential regulation, Proposition 4 could be viewed as establishing the need for a macroprudential regulation in the form of additional capital requirements only enforced in case of a boom. This is again in the spirit of the Basel III proposals.

8 Ramifications and Conclusion

Banks play a dual role in allocating capital in the economy. They lend and reallocate capital across sectors if new information about the productivity of capital arrives. Our main insight is that banks reallocate capital excessively. As a consequence, fluctuations of credit, output

²¹This is possible only if $\Delta\alpha_h < 0$ improves welfare.

and capital prices are excessive. This justifies macroprudential regulation. Numerous further implications and extensions could be pursued. We provide several examples.

Implications

First, as investors hold claims on capital in the FS, the excess volatility of capital prices translates into excess volatility of asset prices. Therefore, our paper provides an explanation why the volatility of asset prices is higher than the volatility of sectoral productivity shocks would suggest. Second, one of our results states that investors will force banks to strongly downsize when negative shocks occur. If banks were buying – voluntarily or forced by regulation – financial securities that provide fresh resources in bad times at the expense of payments in the good state, investors would increase short-term debt to ensure that the same amount of downsizing takes place. Hence, our model implies that banks with insurance against negative macroeconomic shocks have even more short-term debt on their balance sheets.

We have focused on the case where bank capital is scarce. When bank capital is less scarce, investors may find it attractive to invest all resources into the BS and the FS and defer consumption to the end. In such a setup with positive interest rates, macroprudential regulation of the sort described in the last section continues to be welfare improving if moral hazard still binds.²² In addition, macroprudential regulation will have additional repercussions as it affects interest rates. For instance, if additional capital requirements are imposed in a boom, the interest rate is likely to rise, which may be an important information for monetary policy makers.

Dynamic Extensions

The model and its mechanism could be embedded in a dynamic macroeconomic model in which payments to bankers constitute the inside equity for the next period. Some bankers may retire, but the others will try to obtain new funding from investors who ponder over reinvesting part of their wealth. Such an extension would create an intertemporal linkage and volatility cycles of bank credit. The intuitive reason is as follows. Suppose that in a particular period a negative macroeconomic shock causes excessive downsizing and thus excessively low returns on inside equity. As a consequence (see Corollary 1), with low levels of inside equity in the next period, the volatility of bank credit increases further and downsizing upon negative macroeconomic shocks becomes even more pronounced. Proceeding with this kind of logic suggests the existence

²²The analysis however becomes much more cumbersome.

of volatility cycles of bank lending. In such circumstances, macroprudential regulation in a particular period does not only avoid excessive volatility of bank lending in this period, but it also reduces the volatility in future periods. A thorough investigation of these intertemporal linkages is left to future research.

Appendix A

Proof of Proposition 5:

We change α_t^E by some small $\Delta\alpha_l > 0$ and we choose an adjustment $\Delta\alpha_h$ such that investors will continue to provide $K^E - E$ at $t = 0$ to banks. The intervention $\Delta\alpha_l$ and $\Delta\alpha_h$ will impact on the prices of capital p_η and the investment X at date 0 in the frictionless technology. This is because in interior solutions ($C_0 > 0$, $K > 0$, $X > 0$) competition for funds in $t = 0$ imposes that the expected returns on investment in the FS and the BS are equal to 1:

$$\mathbb{E}[p_\eta] = 1. \quad (34)$$

We choose the changes $\Delta\alpha_l > 0$ and $\Delta\alpha_h$ such that the investors continue to offer $K^E - E$ at $t = 0$. Hence,

$$\mathbb{E}[(1 + \alpha_\eta)(\eta\bar{R} - b) - \frac{c}{2}\alpha_\eta^2 - p_\eta\alpha_\eta] = 1 - \frac{E}{K^E}. \quad (35)$$

As α_l and α_h are determined by a social planner, and K^E just fulfills the participation constraint of the investors, the bankers have no possibility to increase their rents. Now:

$$p_\eta = F'(X - \alpha_\eta K^E) \Rightarrow \mathbb{E}[\Delta p_\eta] = \mathbb{E}[F''(X - \alpha_\eta K^E)(\Delta X - K^E \Delta\alpha_\eta)] = 0. \quad (36)$$

Thus

$$\Delta X = K^E \frac{\mathbb{E}[F''(X - \alpha_\eta K^E)\Delta\alpha_\eta]}{\mathbb{E}[F''(X - \alpha_\eta K^E)]} \quad (37)$$

and

$$\Delta p_\eta = K^E F''(X - \alpha_\eta K^E) \left[\frac{\mathbb{E}[F''(X - \alpha_\eta K^E)\Delta\alpha_\eta]}{\mathbb{E}[F''(X - \alpha_\eta K^E)]} - \Delta\alpha_\eta \right]. \quad (38)$$

Totally differentiating condition (35), we get:

$$\mathbb{E}[(\eta\bar{R} - b - c\alpha_\eta - p_\eta)\Delta\alpha_\eta - \alpha_\eta\Delta p_\eta] = 0. \quad (39)$$

Using the equilibrium condition (17), we see that

$$\eta\bar{R} - b - c\alpha_\eta^E - p_\eta^E = -\frac{b}{\nu} < 0. \quad (40)$$

Thus the previous condition (39) simplifies to:

$$\frac{b}{\nu}\mathbb{E}[\Delta\alpha_\eta] = -\mathbb{E}[\alpha_\eta^E \Delta p_\eta]. \quad (41)$$

The impact on social welfare is

$$\Delta U = K^E \mathbb{E}[\{\eta \bar{R} - c\alpha_\eta - F'(X - \alpha_\eta K^E)\} \Delta \alpha_\eta] + \mathbb{E}[F'(X - \alpha_\eta K^E) - 1] \Delta X. \quad (42)$$

The last term is equal to zero, while the first term can be simplified by inserting the equilibrium values of α_η :

$$\Delta U = K^E b \left(1 - \frac{1}{\nu}\right) \mathbb{E}[\Delta \alpha_\eta] = (1 - \nu) K^E \mathbb{E}[\alpha_\eta^E \Delta p_\eta]. \quad (43)$$

Since $\nu > 1$, we just have to check that $\mathbb{E}[\alpha_\eta^E \Delta p_\eta] < 0$.

Step 1:

We first show that $\Delta p_l > 0$.

Suppose to the contrary that $\Delta p_l < 0$. This would imply that

$$\Delta X - K^E \Delta \alpha_l > 0$$

and thus

$$\Delta X > K^E \Delta \alpha_l > 0.$$

Since $\Delta p_h > 0$, as $\mathbb{E}[\Delta p_\eta] = 0$, and thus $\Delta X - K^E \Delta \alpha_h < 0$, $\Delta \alpha_h$ has to be positive.

Now recall that $\Delta \alpha_l > 0$. Hence we obtain $\mathbb{E}[\Delta \alpha_\eta] > 0$ and $\mathbb{E}[\alpha_\eta^E \Delta p_\eta] = q \alpha_h^E \Delta p_h + (1 - q) \alpha_l^E \Delta p_l > 0$ which is a contradiction to Condition (41).

We conclude that $\Delta p_l > 0$.

Step 2:

From $\Delta p_l > 0$ and thus $\Delta p_h < 0$ we obtain

$$\mathbb{E}[\alpha_\eta^E \Delta p_\eta] < 0$$

and thus $\Delta U > 0$ and the proposition follows. ■

Proof of Proposition 6:

Increasing α_l^E by $\Delta\alpha_l > 0$ will impact on bankers' choice of α_h^E , the capital prices p_η , investment X at date 0 in the FS and investment K in the BS.

Step 1:

Given a predetermined value $\alpha_l^R = \alpha_l^E + \Delta\alpha_l$, with $\Delta\alpha_l > 0$ and small, the bankers' problem written at the aggregate level is,

$$\max_{\{\alpha_h, K\}} Kb \{q(1 + \alpha_h) + (1 - q)(1 + \alpha_l^R)\}$$

subject to

$$K \left[q \left\{ (1 + \alpha_h)(h\bar{R} - b) - \frac{c}{2}\alpha_h^2 - p_h\alpha_h \right\} + (1 - q) \left\{ (1 + \alpha_l^R)(l\bar{R} - b) - \frac{c}{2}(\alpha_l^R)^2 - p_l\alpha_l \right\} \right] = K - E.$$

Denoting by \mathcal{L} the associated Lagrangian, and by λ the multiplier, we obtain the first order conditions:

$$\frac{\partial \mathcal{L}}{\partial K} = b\mathbb{E}[1 + \alpha_\eta] + \lambda\mathbb{E}[(1 + \alpha_\eta)(\eta\bar{R} - b) - \frac{c}{2}\alpha_\eta^2 - p_\eta\alpha_\eta] - \lambda = 0 \quad (44)$$

$$\frac{\partial \mathcal{L}}{\partial \alpha_h} = qK[b + \lambda(h\bar{R} - b - c\alpha_h - p_h)] = 0. \quad (45)$$

From the second condition we obtain,

$$p_h = h\bar{R} - b - c\alpha_h + \frac{b}{\lambda}.$$

Inserting this expression and the equilibrium condition

$$p_l = \frac{1 - qp_h}{1 - q}$$

into the first condition yields

$$1 - \frac{b}{\lambda}(1 + \bar{\alpha}) = \mathbb{E}[(\eta\bar{R} - b)(1 + \alpha_\eta) - \frac{c}{2}\alpha_\eta^2] - \alpha_l^R - q(\alpha_h - \alpha_l^R)(h\bar{R} - b - c\alpha_h + \frac{b}{\lambda}).$$

After rearranging terms we obtain

$$\begin{aligned} \frac{b}{\lambda} (1 + q\alpha_h + (1 - q)\alpha_l^R - q\alpha_h + q\alpha_l^R) &= 1 + \alpha_l^R + b [1 + q\alpha_h + (1 - q)\alpha_l^R - q\alpha_h + q\alpha_l^R] \\ &\quad - \bar{R} [qh + qh\alpha_h + (1 - q)l + (1 - q)l\alpha_l^R - qh\alpha_h + qh\alpha_l^R] \\ &\quad + \frac{c}{2} [q\alpha_h^2 + (1 - q)(\alpha_l^R)^2 - 2q\alpha_h^2 + 2q\alpha_l^R\alpha_h]. \end{aligned}$$

After simplification we obtain

$$\frac{b}{\lambda} = 1 + b - \bar{R} + \frac{c}{2} \left[\frac{(\alpha_l^R)^2 - q(\alpha_h - \alpha_l^R)^2}{1 + \alpha_l^R} \right].$$

We denote

$$\psi(\alpha_l^R, \alpha_h) := \frac{(\alpha_l^R)^2 - q(\alpha_h - \alpha_l^R)^2}{1 + \alpha_l^R}.$$

Hence, we obtain

$$p_h = (h - 1)\bar{R} + 1 + \frac{c}{2} (\psi(\alpha_l^R, \alpha_h) - 2\alpha_h). \quad (46)$$

For given (p_h, α_l^R) , equation (46) uniquely determines α_h . The solution is denoted by $\alpha_h(p_h, \alpha_l^R)$.

Step 2:

For every choice α_l^R we can define an equilibrium as an array $\Sigma = \{C_0, K, X, \alpha_h, p_h, p_l | \alpha_l^R\}$ such that

$$(i) \quad K = \frac{E}{\rho(p | \alpha_l^R)}$$

$$\begin{aligned} \text{where } \rho(p | \alpha_l^R) &= 1 - q \left\{ (1 + \alpha_h(p_h, \alpha_l^R)) (h\bar{R} - b) - \frac{c}{2} \alpha_h^2(p_h, \alpha_l^R) - p_h \alpha_h(p_h, \alpha_l^R) \right\} \\ &\quad - (1 - q) \left\{ (1 + \alpha_l^R)(l\bar{R} - b) - \frac{c}{2} (\alpha_l^R)^2 - p_l \alpha_l^R \right\} \end{aligned}$$

$$(ii) \quad \text{Investments in the BS are adjusted in } t = 1 \text{ by } \alpha_h(p_h, \alpha_l^R), \alpha_l^R.$$

$$(iii) \quad \begin{aligned} p_h &= F'(X - \alpha_h(p_h, \alpha_l^R)K) \\ p_l &= F'(X - \alpha_l^R K) \end{aligned}$$

$$(iv) \quad p_l = \frac{1 - qp_h}{1 - q}$$

$$(v) \quad K + X + C_0 = 1$$

$$(vi) \quad C_0 > 0, K > 0, X > 0.$$

Step 3:

For $\alpha_l^R = \alpha_l^E$ the equilibrium defined in Step 2 coincides with the equilibrium without intervention.

Step 4:

We next show that for $\alpha_l^R = \alpha_l^E + \Delta\alpha_l$ we have $\Delta p_l > 0, \Delta p_h < 0$.

Suppose that this is not true and $\Delta p_l < 0, \Delta p_h > 0$. Then, the following lemma holds.

Lemma 1

$$\Delta K < 0.$$

The proof of the lemma will be given at the end of this proof. With Lemma 1 we obtain,

$$\Delta X - \Delta\alpha_l K - \alpha_l^E \Delta K > 0$$

$$\Delta X - \Delta\alpha_l K > 0$$

From

$$\Delta X < \Delta\alpha_h K + \alpha_h^E \Delta K$$

we obtain

$$\Delta X < \Delta\alpha_h K$$

Hence, $\Delta\alpha_h > \Delta\alpha_l > 0$. Then, from (46) we obtain

$$\Delta p_h = -c\Delta\alpha_h + \frac{c}{2} [\psi(\alpha_l^E + \Delta\alpha_l, \alpha_h^E + \Delta\alpha_h) - \psi(\alpha_l^E, \alpha_h^E)].$$

As $\Delta\alpha_h > \Delta\alpha_l > 0$ we observe that

$$\psi(\alpha_l^E + \Delta\alpha_l, \alpha_h^E + \Delta\alpha_h) < \psi(\alpha_l^E, \alpha_h^E).$$

Hence, $\Delta p_h < 0$, which is a contradiction. We conclude that $\Delta p_l > 0$, $\Delta p_h < 0$.

Step 5:

With $\Delta p_l > 0$, $\Delta p_h < 0$, and thus less volatility of capital prices and more favorable prices for reallocation decisions of banks, similar arguments as in Proposition 5 establish that social welfare will increase. ■

Proof of Lemma 1:

Suppose $\Delta p_l < 0$ and $\Delta p_h > 0$. This implies

$$\Delta X - K\Delta\alpha_l - \Delta K\alpha_l^E > 0 \tag{47}$$

$$\Delta X - K\Delta\alpha_h - \Delta K\alpha_h^E < 0. \tag{48}$$

As prices for bankers are less favorable in the bad state and the good state, their expected payoff has to decline. Otherwise his choices $(K^E, \alpha_h^E, \alpha_l^E)$ could not be optimal. Hence,

$$(K^E + \Delta K) (q(1 + \alpha_h^E + \Delta\alpha_h) + (1 - q)(1 + \alpha_l^E + \Delta\alpha_l)) < K^E (q(1 + \alpha_h^E) + (1 - q)(1 + \alpha_l^E))$$

which yields

$$\Delta K(1 + \bar{\alpha}^E) + (K^E + \Delta K) (q\Delta\alpha_h + (1 - q)\Delta\alpha_l) < 0.$$

As $\Delta\alpha_l$, ΔK and $\Delta\alpha_h$ are small, we neglect all terms with $\Delta K\Delta\alpha_l$ and $\Delta K\Delta\alpha_h$. From (47) and (48) we obtain

$$\begin{aligned} K^E \Delta\alpha_l + \Delta K \alpha_l^E &< K^E \Delta\alpha_h + \Delta K \alpha_h^E \\ \text{or } K^E \Delta\alpha_h &> K^E \Delta\alpha_l + \Delta K (\alpha_l^E - \alpha_h^E). \end{aligned}$$

Hence, it must hold

$$\begin{aligned} \Delta K(1 + \bar{\alpha}^E) + \Delta\alpha_l K^E + q\Delta K(\alpha_l^E - \alpha_h^E) &< 0 \\ \text{or } \Delta K (1 + q\alpha_h^E + (1 - q)\alpha_l^E + q\alpha_l^E - q\alpha_h^E) + \Delta\alpha_l K^E &< 0 \end{aligned}$$

which yields

$$\Delta K(1 + \alpha_l^E) + \Delta\alpha_l K^E < 0.$$

As $\alpha_l^E > -1$ and $\Delta\alpha_l > 0$ we obtain

$$\Delta K < 0.$$

■

Appendix B - List of Notation

Symbol *Meaning*

FS	Frictionless sector with a technology that exhibits decreasing marginal returns
BS	Banking sector that is subject to macroeconomic risk
e	Wealth of an individual banker
E	The aggregate wealth of bankers in $t = 0$
U	The social welfare
C_t	Consumption in period t with $t = 0, 2$
k	The investment of a single bank in the BS in $t = 0$
K	The aggregate amount of investment in the BS in $t = 0$
\tilde{R}	The idiosyncratic return of the the BS
\bar{R}	The expectation of the idiosyncratic return \tilde{R}
η	Macroeconomic shock with $\eta = h$ (high) with probability q and $\eta = l$ (low) with probability $1 - q$
m	The expectation of the macroeconomic shock, i.e. $m = qh + (1 - q)l = 1$
X	The aggregate amount of investment in the FS in $t = 0$
$F(X)$	Output of the FS in $t = 2$, if X is invested in the FS in $t = 0$
$G(x)$	The distribution function of x where x is the productivity of an agent in the FS
p_η	The (interim) rate of return on capital when macroeconomic shock is η
r	The interest rate for capital with expected rate of return on capital $\mathbb{E}[p_\eta] = 1 + r$
α_η	The adjustment to the investment in the BS in $t = 1$. $\alpha_\eta > 0$ means additional investment in the BS and $\alpha_\eta < 0$ denotes disinvestment
$c(\alpha)$	Adjustment cost for reallocating capital across sectors
o, S, F	o denotes the project outcome with $o = S$ indicating success and $o = F$ indicating a failure
τ	The probability with which $o = S$ occurs if the banker exerts effort
$\tau - \Delta$	The probability with which $o = S$ occurs if the banker shirks
b_η^o	Payment to the banker in $t = 2$ per unit of investment in the BS ($o = S, F$) if η occurs
$C(k, \alpha_\eta, b_\eta^o)$	The financial contract between a banker and investor
B	Private benefits per unit of investment in the BS obtained by the banker by shirking
R^o	The idiosyncratic return \tilde{R} for project outcome $o = S, F$
σ	The standard deviation of credit growth α_η
$\bar{\alpha}$	The expectation of α_η
$\underline{\alpha}_l$	$\underline{\alpha}_l > -1$ is the lower bound of α_l such that output in the BS is non-negative

$S(p)$	The supply of capital to the BS, as a function of capital prices p
b	The expected payment to the banker per unit of asset, with $b = \frac{\tau B}{\Delta}$
$\rho_0(r)$	The market imposed capital ratio ($\frac{e}{k}$) in the rigid economy
D_R^0	The repayment amount of the riskless deposits issued in the rigid economy in $t = 0$
D_R^1	The repayment amount of the riskless deposits issued in the rigid economy in $t = 1$
θ_η	The fraction of the total dividends paid to the banker as inside equity in the rigid economy
$\rho(p)$	The market imposed capital ratio for privately optimal contracts
D_{ST}	The amount of short term debt (repaid in $t = 1$) issued for privately optimal contracts
D_{LT}	The amount of long term debt (repaid in $t = 2$) issued for privately optimal contracts
C^N	The amount of convertible bonds issued for privately optimal contracts where each convertible bond is converted to y shares when state l occurs
D_1^h	The new debt issued in $t = 1$ for privately optimal contracts if state h occurs
P_η^o	The total payment to debt holders in $t = 2$ for privately optimal contracts where $\eta = h, l$ and $o = S, F$
Y_η	The aggregate output in state η
ν	The Lagrangian multiplier of the maximization problem in the competitive equilibrium
$Pr(\eta)$	The probability that η occurs
$H(\sigma)$	A decreasing function of σ , which gives a unique solution σ^E , that determines the unique competitive equilibrium
I^E	The parameter values of the competitive equilibrium, where $I = K, X, \alpha_\eta, p_\eta, \sigma$
$\Delta\alpha_\eta$	The change from the equilibrium capital adjustment as a result of intervention
Δp_η	The change from the equilibrium capital prices as a result of intervention
ΔI	The change from the equilibrium capital allocations as a result of intervention where $I = K, X$
λ	The Lagrangian multiplier of the maximization problem with one sided intervention
α_l^R	The capital adjustment after one sided intervention with $\alpha_l^R = \alpha_l^E + \Delta\alpha_l$
$\psi(\alpha_l^R, \alpha_h)$	A function defined as $\psi(\alpha_l^R, \alpha_h) := \frac{(\alpha_l^R)^2 - q(\alpha_h - \alpha_l^R)^2}{1 + \alpha_l^R}$

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