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WITH LIMITED COMMITMENT AND  
INCOMPLETE MARKETS**

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## ABSTRACT

### International Capital Flows with Limited Commitment and Incomplete Markets\*

Recent literature has proposed two alternative types of financial frictions, i.e., limited commitment and incomplete markets, to explain the patterns of international capital flows between developed and developing countries observed in the past two decades. This paper integrates both types of frictions into a two-country overlapping-generations framework to facilitate a direct comparison of their effects. In our model, limited commitment distorts the investment made by agents with different productivity, which creates a wedge between the interest rates on equity capital vs. credit capital; while incomplete markets distort the investment among projects with different riskiness, which creates a wedge between the risk-free rate and the mean rate of return to risky capital. We show that the two approaches are observationally equivalent with respect to their implications for international capital flows, production efficiency, and aggregate output.

JEL Classification: E44 and F41

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# 1 Introduction

According to the conventional neoclassical theory, capital should flow “downhill” from the rich country where the marginal return on capital is low to the poor country where the marginal return on capital is high. Meanwhile, there would be no difference between gross and net capital flows because capital flows would be unidirectional. The recent empirical patterns of international capital flows, however, are in stark contrast to these predictions (Lane and Milesi-Ferretti, 2001, 2007a,b). First, capital in the net term flows “uphill” from poor to rich countries (Prasad, Rajan, and Subramanian, 2006, 2007). Second, financial capital flows from poor to rich countries, while foreign direct investment (FDI, hereafter) flows in the opposite direction (Ju and Wei, 2010). Third, despite its negative net positions of international investment since 1986, the U.S. has been receiving a positive net investment income until 2005 (Gourinchas and Jeanne, 2009; Hausmann and Sturzenegger, 2007; Higgins, Klitgaard, and Tille, 2007).

A booming literature is offering explanations for these facts. The main idea of this literature is that financial markets suffer from imperfections and that international differences in these imperfections drive international capital flows. One line of research focuses on financial market imperfections in the form of limited commitment, (Antras and Caballero, 2009; Antras, Desai, and Foley, 2009; Aoki, Benigno, and Kiyotaki, 2009; Buera and Shin, 2010; Caballero, Farhi, and Gourinchas, 2008; Ju and Wei, 2010; Smith and Valderrama, 2008; Song, Storesletten, and Zilibotti, 2011). These models distinguish between different types of individuals in an economy, which are more or less productive in the use of real capital. Financial markets serve to channel the savings of the less productive types to the more productive types to improve aggregate production efficiency. In a world with limited commitment, however, the more productive types cannot borrow enough to reach the optimal allocation of capital. Another line of research focuses on the risk-sharing function of financial markets. With incomplete financial markets, idiosyncratic investment risks are not fully insurable (Angeletos and Panousi, 2011; Mendoza, Quadrini, and Rios-Rull, 2009; Sandri, 2010). Individuals over-invest in safe but less productive assets and under-invest in risky but more productive assets, which also distorts aggregate production efficiency.<sup>1</sup> The differences in these two approaches make it difficult to compare results across models.

The purpose of this paper is to integrate both approaches into one model to allow a direct comparison of the results and to facilitate the understanding of the two approaches.

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<sup>1</sup>Another line of research focuses on the risk-sharing investors can achieve by diversifying their portfolios globally (Devereux and Sutherland, 2009; Tille and van Wincoop, 2008, 2010). These models can explain “uphill” capital flows, but they fail to distinguish between financial capital and FDI flows.

We develop a tractable, two-country, overlapping-generations model, which embeds both types of financial market imperfections, and compare their respective implications for international capital flows and real output. In the presence of limited commitment, the more productive individuals obtain a rate of return on their equity capital which exceeds the social return to investment; while, due to the constraint on aggregate credit demand, the less productive individuals obtain a loan rate which falls short of the social return. Thus, limited commitment drives a wedge between interest rates. (Matsuyama, 2004) In the presence of incomplete markets (Angeletos and Panousi, 2011), individuals over-invest in the risk-free sector and under-invest in the risky sector, due to precautionary motive and risk aversion, respectively. As a result, the risk-free interest rate is below while the mean rate of return to risky investment is above the social rate of return. This way, incomplete markets also drive a wedge between interest rates, albeit one of a different kind.

Either way, the more severe the financial market distortion, the larger the interest rate wedge. Therefore, international differences in financial market distortions lead to international differences in interest rates, and these differences determine the patterns of international capital flows. Equating more severe distortions with a lower degree of financial market development, the less financially developed country has a lower return on loans and higher return on equity in the steady state than the more financially developed country under limited commitment. Similarly, the less developed country has a lower risk-free interest rate and a higher mean rate of return on risky investment in the steady state under incomplete markets. Either way, the financially less developed country is poorer in terms of per-capita output. With full capital mobility and limited commitment, the less financially developed country exports the savings of less productive individuals to and imports the foreign direct investment of more productive individuals from the more developed country. With full capital mobility and incomplete markets, savers in the less developed country invest in risk-free projects in the more developed country, while savers in the more developed invest in risky projects in the less developed countries. Furthermore, the more developed country receives net capital inflows, due to its larger credit market capacity and/or its better risk-sharing mechanism. Intuitively, the more developed country exports its superior financial services through two-way capital flows and receives a positive net investment income. Thus, the patterns of international capital flows in the two models are consistent with empirical observations.

Our model differs from the existing literature in the following aspects. While Angeletos and Panousi (2011); Buera and Shin (2010); Carroll and Jeanne (2011); Sandri (2010); Song, Storesletten, and Zilibotti (2011) address “uphill” financial capital flows, we also

explain the composition of capital flows in terms of financial investment and foreign direct investment. Caballero, Farhi, and Gourinchas (2008); Mendoza, Quadrini, and Rios-Rull (2009) analyze the joint determination of financial investment and FDI in an endowment-economy model, while endogenous capital accumulation is crucial in our model to analyze the output implications of financial integration. Caballero, Farhi, and Gourinchas (2008) assume that foreign direct investors from the more financially developed country have an advantage in capitalizing the return on investment in the host country and Mendoza, Quadrini, and Rios-Rull (2009) assume that investors from the more financially developed country can insure their foreign direct investment using the better risk-sharing opportunities in their home country. We do not need these extra assumptions.

The rest of the paper is organized as follows. Section 2 sets up the model under IFA and shows how financial frictions distort interest rates and aggregate output. Section 3 shows the patterns of international capital flows. Section 5 uses a numerical calibration of the model to study the dynamic responses to a financial crisis in the more developed country and how they change due to financial integration. Section 6 concludes and the appendix collects relevant proofs.

## 2 The Model Under International Financial Autarky

The world economy consists of two countries, N (North) and S (South), which are fundamentally identical except in the level of financial development as specified later. Variables in country  $i \in \{S, N\}$  are denoted with the superscript  $i$ . There is a tradeable final good, which is taken as the numeraire; there are two nontradeable intermediate goods, A and B, and the price of intermediate good  $k \in \{A, B\}$  in period  $t$  and country  $i$  is denoted by  $V_t^{i,k}$ . In this section, we assume that international capital flows are not allowed and both countries are under international financial autarky, IFA.

Agents live for two periods, young and old. The population size of each generation is normalized to one in each country. Each generation consists of two types of agents, *entrepreneurs* and *households*, of mass  $\eta$  and  $1 - \eta$ , respectively.

Agents have the preference over consumption in both periods of life. Consider agent  $j$  born in period  $t$ , where  $j \in \{e, h\}$  denotes its identity as entrepreneur or household. If the agent's second-period consumption is stochastic, its expected life-time utility is,

$$U_t^{i,j} = (1 - \beta) \ln c_{y,t}^{i,j} + \beta \ln [E_t(c_{o,t+1}^{i,j})^{1-\gamma}]^{\frac{1}{1-\gamma}}, \quad (1)$$

where  $c_{y,t}^{i,j}$  and  $c_{o,t+1}^{i,j}$  denote its consumption when young and when old;  $E_t$  is the expectation operator;  $\beta \in (0, 1]$  denotes the relative weight of utility from consumption

when old;  $\gamma \geq 0$  denotes the coefficient of relative risk aversion.<sup>2</sup> If the agent's second-period consumption is deterministic, its lifetime preference function is simplified as  $U_t^{i,j} = (1 - \beta) \ln c_{y,t}^{i,j} + \beta \ln c_{o,t+1}^{i,j}$ .

Young agents are endowed with one unit of labor, which they supply inelastically to aggregate production. Final goods are produced instantaneously with a Cobb-Douglas technology using the amounts  $M_t^{i,A}$  and  $M_t^{i,B}$  of intermediate goods and labor  $L = 1$ . All inputs are rewarded with their respective marginal products. To summarize,

$$Y_t^i = \left( \frac{M_t^{i,A}}{\frac{\alpha}{2}} \right)^{\frac{\alpha}{2}} \left( \frac{M_t^{i,B}}{\frac{\alpha}{2}} \right)^{\frac{\alpha}{2}} \left( \frac{L}{1 - \alpha} \right)^{1 - \alpha}, \text{ where } \alpha \in (0, 1), \quad (2)$$

$$\omega_t^i L = (1 - \alpha) Y_t^i \quad V_t^{i,A} M_t^{i,A} = \frac{\alpha}{2} Y_t^i, \quad V_t^{i,B} M_t^{i,B} = \frac{\alpha}{2} Y_t^i. \quad (3)$$

$Y_t^i$  and  $\omega_t^i$  denote aggregate output of final goods and the wage rate, respectively; the two intermediate goods have the same factor share of  $\frac{\alpha}{2}$  in aggregate production.

Young agents can use final goods and invest them in the production of intermediate goods, which takes one period to complete. All agents have the same technology to produce intermediate good A, but only entrepreneurs can produce intermediate good B.

Each agent can operate only one productive project in sector A. An agent who invests  $i_t^i$  units of final goods in period  $t$  produces  $e^{\epsilon_{t+1}} i_t^i$  units of intermediate good A in period  $t + 1$ , where the productivity shock  $\epsilon_{t+1}$  is idiosyncratic and follows a logarithmic normal distribution with the mean of  $-\frac{\sigma^2}{2}$  and the variance of  $\sigma^2$ . The idiosyncratic risk washes out at the aggregate level and the aggregate rate of transformation in sector A is  $E_t e^{\epsilon_{t+1}} = e^{E_t \epsilon_{t+1} + \frac{\text{Var}(\epsilon_{t+1})}{2}} = 1$ . At the individual level, each agent can insure himself against the idiosyncratic productivity risk through ex ante risk-sharing arrangements offered by financial markets. Doing so, the agent receives a state-contingent transfer in period  $t + 1$ ,  $\Gamma_{t+1}^i i_t^i V_{t+1}^A$ . The risk-sharing factor  $\Gamma_{t+1}^i \equiv 1 - e^{\epsilon_{t+1}} + e^{(1-\lambda^i)\epsilon_{t+1}} - e^{-\frac{\lambda^i(1-\lambda^i)}{2}\sigma^2}$  depends on a country-specific index of market-completeness  $\lambda^i \in [0, 1]$ . A negative value of  $\Gamma_{t+1}^i$  represents a payment made by the agent. If  $\lambda^i = 1$ , financial markets are complete and the rate of return to investment in sector A is deterministic after risk-sharing,

$$\Gamma_{t+1}^i + e^{\epsilon_{t+1}} = 1.$$

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<sup>2</sup>For the analytical tractability, we implicitly set the elasticity of intertemporal substitution at unity. With this preference function, we can distinguish between the coefficient of relative risk aversion and the elasticity of intertemporal substitution, which is useful in our numerical exercise in section 5. By setting  $\gamma = 1$ , we revert to the conventional preference function where CRRA is equal to the inverse of EIS,  $U_t^{i,j} = (1 - \beta) \ln c_{y,t}^{i,j} + \beta E_t \ln c_{o,t+1}^{i,j}$ . Our analytical results in sections 2 and 3 are unaffected. See Selden (1978) and Kocherlakota (1990) for further discussion on this preference function.

If  $\lambda^i \in [0, 1)$ , financial markets are incomplete and the rate of return to investment in sector A after risk-sharing is still affected by idiosyncratic shocks,

$$\Gamma_{t+1}^i + e^{\epsilon_{t+1}} = 1 + e^{(1-\lambda^i)\epsilon_{t+1}} - e^{-\frac{\lambda^i(1-\lambda^i)}{2}\sigma^2},$$

with mean of 1, and variance of  $e^{[(1-\lambda^i)\sigma]^2} - 1$ . In other words, uninsured idiosyncratic risk follows a logarithmic normal distribution with the mean of  $-\frac{[(1-\lambda^i)\sigma]^2}{2}$  and the variance of  $[(1-\lambda^i)\sigma]^2$ .<sup>3</sup> The expected value of the risk-sharing factor is zero in period  $t$ ,

$$E_t \Gamma_{t+1}^i \equiv 1 - e^{E_t \epsilon_{t+1} + \frac{\text{Var}(\epsilon_{t+1})}{2}} + e^{(1-\lambda^i)E_t \epsilon_{t+1} + (1-\lambda^i)^2 \frac{\text{Var}(\epsilon_{t+1})}{2}} - e^{-\frac{\lambda^i(1-\lambda^i)}{2}\sigma^2} = 0.$$

Subsequently,  $\lambda^i$  is an indicator of financial development in country  $i$ .

Only entrepreneurs can produce intermediate good B one-to-one using final goods.

**Assumption 1.**  $\eta \in (0, 0.5)$ .

Assumption 1 ensures that, in equilibrium, the aggregate saving of entrepreneurs is less than the socially efficient investment size in sector B. Entrepreneurs finance their investment using their own savings as equity and loans from households at the gross loan rate of  $R_t^{i,h}$ . For each unit of final good invested in sector B and period  $t$ , an entrepreneur receives the revenue  $V_{t+1}^{i,B}$  in period  $t+1$ . Due to limited commitment, he can pledge only a fraction  $\theta^i$  of his future revenue as collateral for the loan he takes. Thus, his maximum amount of borrowing in period  $t$  is  $\frac{\theta^i V_{t+1}^{i,B}}{R_t^{i,h}}$ , and the parameter  $\theta^i$  describes the severity of the borrowing constraint in country  $i$ . If  $\theta^i = 1$ , the borrowing constraint is slack; if  $\theta^i < 1$ , the borrowing constraint may be binding. Thus,  $\theta^i$  is another indicator of financial development in country  $i$ .

Let  $\psi_t^i$  denote an entrepreneur's investment-equity ratio in sector B. From the entrepreneur's point of view, each unit of equity earns its marginal revenue  $V_{t+1}^{i,B}$  in period  $t+1$ . Each unit of investment financed by borrowing yields the excess of its marginal revenue over the loan rate to the entrepreneur, i.e., the net rate of return  $V_{t+1}^{i,B} - R_t^{i,h}$ . The equity rate is defined as the entrepreneur's gross rate of return per unit of equity capital,

$$R_t^{i,e} \equiv V_{t+1}^{i,B} + (\psi_t^i - 1)(V_{t+1}^{i,B} - R_t^{i,h}) \geq R_t^{i,h}, \quad (4)$$

where  $(V_{t+1}^{i,B} - R_t^{i,h})(\psi_t^i - 1)$  captures the leverage effect. The equity rate should be no less than the loan rate; otherwise, the entrepreneur would rather lend than borrow. Thus, the inequality in (4) is the participation constraint for the entrepreneur. If  $R_t^{i,h} < V_{t+1}^{i,B}$ , the

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<sup>3</sup>We model idiosyncratic productivity risk as in Angeletos (2007) and incomplete markets as in Angeletos and Panousi (2011).



entrepreneur borrows to the limit,

$$\psi_t^i = \frac{1}{1 - \frac{\theta^i V_{t+1}^{i,B}}{R_t^i}} > \frac{1}{1 - \theta^i}, \quad \text{so that} \quad R_t^{i,e} = \frac{(1 - \theta^i)}{1 - \frac{\theta^i V_{t+1}^{i,B}}{R_t^{i,h}}} V_{t+1}^{i,B} > V_{t+1}^{i,B},$$

otherwise, the entrepreneur does not borrow to the limit

$$\psi_t^i \leq \frac{1}{1 - \theta^i}, \quad \text{so that} \quad R_t^{i,e} = V_{t+1}^{i,B} = R_t^{i,h}.$$

As production in sector B is deterministic, the equity rate,  $R_t^{i,e}$ , and the loan rate,  $R_t^{i,h}$ , are both risk free.

An agent of type  $j$  born in country  $i$  and period  $t$  receives a labor income  $\omega_t^i$ , consumes  $c_{y,t}^{i,j}$ , makes a risky investment  $i_t^{i,j}$  in sector A, and a risk-free investment  $d_t^{i,j} = \omega_t^i - i_t^{i,j} - c_{y,t}^{i,j}$  in equity or loans in sector B. In period  $t+1$ , he gets the safe return from sector B,  $R_t^{i,j} d_t^{i,j}$ , produces  $e^{\epsilon_{t+1}} i_t^{i,j}$  units of intermediate good A, and receives the risk-sharing transfer  $\Gamma_{t+1}^i i_t^{i,j} V_{t+1}^{i,A}$ . The agent consumes the total wealth,

$$c_{o,t+1}^{i,j} = V_{t+1}^{i,A} i_t^{i,j} (\Gamma_{t+1}^i + e^{\epsilon_{t+1}}) + R_t^{i,j} d_t^{i,j},$$

and exits from the economy.

In the following, we focus on three special cases. Let  $\bar{\theta} \equiv 1 - 2\eta$  define a threshold value. First, for  $\lambda^i = 1$  and  $\theta^i \in (\bar{\theta}, 1]$ , idiosyncratic risk in sector A is fully insured and the borrowing constraints in sector B are slack. The equilibrium allocation is unconstrained so that investment in the two sectors is efficient and so is aggregate output. Second, for  $\lambda^i = 1$  and  $\theta^i \in [0, \bar{\theta})$ , idiosyncratic risk in sector A is completely insured, while the borrowing constraints in sector B are binding. Thus, limited commitment in sector B distorts cross-sector investment, leading to inefficiently low aggregate output. Third, for  $\lambda^i < 1$  and  $\theta^i = 1$ , idiosyncratic risk in sector A is partially insured while the borrowing constraints in sector B are slack. Here, incomplete markets in sector A distort cross-sector investment, leading to inefficiently low aggregate output.

## 2.1 Unconstrained Equilibrium

In this subsection, we derive the reference case of an unconstrained equilibrium, i.e.,  $\lambda^i = 1$  and  $\theta^i \geq \bar{\theta}$ . Idiosyncratic risk is completely insured so that the investment in sector A has a deterministic rate of return,  $V_{t+1}^{i,A4}$ . A market equilibrium in country  $i$  is a set of allocations of agents  $j \in \{h, e\}$ , investment and consumption choices and rates of return,

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<sup>4</sup>See subsection 2.2 for a formal proof.

$\{i_t^{i,j}, d_t^{i,j}, R_t^{i,j}, c_{y,t}^{i,j}, c_{o,t}^{i,j}\}$ , and aggregate variables,  $\{Y_t^i, M_t^{i,A}, M_t^{i,B}, \omega_t^i, V_t^{i,A}, V_t^{i,B}\}$ , satisfying equations (2), (3), and (5a)-(5d).

$$R_t^{i,h} = R_t^{i,e} = V_{t+1}^{i,A} = V_{t+1}^{i,B}, \quad (5a)$$

$$c_{y,t}^{i,j} = (1 - \beta)\omega_t^i, \quad c_{o,t+1}^{i,j} = \beta\omega_t^i R_t^{i,j}, \quad (5b)$$

$$d_t^{i,e} = \beta\omega_t^i, \quad i_t^{i,e} = 0, \quad d_t^{i,h} = \frac{1 - 2\eta}{2(1 - \eta)}\beta\omega_t^i, \quad i_t^{i,h} = \frac{1}{2(1 - \eta)}\beta\omega_t^i \quad (5c)$$

$$M_{t+1}^{i,A} = (1 - \eta)i_t^{i,h} + \eta i_t^{i,e} = \frac{\beta\omega_t^i}{2}, \quad M_{t+1}^{i,B} = (1 - \eta)d_t^{i,h} + \eta d_t^{i,e} = \frac{\beta\omega_t^i}{2}. \quad (5d)$$

Since the two intermediate goods enter symmetrically into the Cobb-Douglas production function (2) and the aggregate rate of transformation is equal to unity in the two sectors, the rates of return equalize across individuals as well as across sectors; see equations (5a). With logarithmic preferences (1), young agents consume a constant fraction  $(1 - \beta)$  of their labor incomes  $\omega_t^i$  and invest the rest for the rate of return  $R_t^{i,j}$  in period  $t + 1$ ; see equations (5b). The symmetry of two intermediate goods sectors implies that aggregate savings  $\beta\omega_t^i$  are allocated equally in the two sectors and equations (5d) specify the aggregate output of the two intermediate goods. Individual investment in the two sectors can be easily derived in equations (5c).

Let  $\rho \equiv \frac{\alpha}{1 - \alpha}$ . In period  $t + 1$ , the aggregate revenue from producing intermediate goods,  $V_{t+1}^{i,A}M_{t+1}^{i,A} + V_{t+1}^{i,B}M_{t+1}^{i,B} = \alpha Y_{t+1}^i = \rho\omega_{t+1}^i$ , is distributed to entrepreneurs and households as the return to their savings,

$$\beta\omega_t^i[(1 - \eta)R_t^{i,h} + \eta R_t^{i,e}] = \rho\omega_{t+1}^i \Rightarrow (1 - \eta)R_t^{i,h} + \eta R_t^{i,e} = \Psi_t^i. \quad (6)$$

where  $\Psi_t^i \equiv \frac{V_{t+1}^{i,A}M_{t+1}^{i,A} + V_{t+1}^{i,B}M_{t+1}^{i,B}}{\beta\omega_t^i} = \frac{\rho\omega_{t+1}^i}{\beta\omega_t^i}$  denotes the social rate of return to aggregate investment and its steady-state value is  $\Psi_{IFA}^i = \frac{\rho}{\beta}$ .

Let  $\chi_{t+1}^i \equiv \frac{V_{t+1}^{i,A}}{V_{t+1}^{i,B}}$  denote the relative intermediate goods price in period  $t + 1$ . Let  $X_{IFA}$  denote the steady-state value of any particular variable  $X_t$  under IFA. Lemma 2.1 characterizes the reference case.

**Lemma 2.1.** *Let  $\lambda^i = 1$  and  $\theta^i \in (\bar{\theta}, 1]$ . The model dynamics can be characterized by the dynamics of wages,  $\omega_{t+1}^i = \left(\frac{\omega_t^i}{\Psi_{IFA}^i}\right)^\alpha$ . There exists a unique and stable non-zero steady state in country  $i$  with the wage at  $\omega_{IFA}^i = \Psi_{IFA}^{-\rho}$ .*

*Private and social rates of return coincide,  $R_t^{i,h} = R_t^{i,e} = \Psi_t^i = V_{t+1}^{i,A} = V_{t+1}^{i,B}$ . In the steady state,  $R_{IFA}^{i,h} = R_{IFA}^{i,e} = \Psi_{IFA}^i$ .*

*Aggregate savings  $\beta\omega_t^i$  are equally invested in the two sectors,  $M_{t+1}^{i,A} = M_{t+1}^{i,B} = \frac{\beta\omega_t^i}{2}$ . The relative intermediate goods price is  $\chi_{IFA}^i = 1$ .*

## 2.2 Equilibrium with Limited Commitment

For  $\lambda^i = 1$ , idiosyncratic risk is completely insured so that the investment in sector A and period  $t$  has a deterministic rate of return,  $V_{t+1}^{i,A}$ . If the borrowing constraints are binding, a market equilibrium in country  $i$  under IFA is a set of allocation of agents  $j \in \{h, e\}$ , consumption and investment choices and rates of return  $\{i_t^{i,j}, d_t^{i,j}, R_t^{i,j}, c_{y,t}^{i,j}, c_{o,t}^{i,j}\}$ , and aggregate variables,  $\{Y_t^i, M_t^{i,A}, M_t^{i,B}, \omega_t^i, V_t^{i,A}, V_t^{i,B}\}$ , satisfying equations (2), (3), and (7a)-(7e).

$$R_t^{i,h} \frac{(1-\eta)}{\eta} d_t^{i,h} = \theta^i V_{t+1}^{i,B} \left[ \frac{(1-\eta)}{\eta} d_t^{i,h} + d_t^{i,e} \right], \quad (7a)$$

$$R_t^{i,h} = V_{t+1}^{i,A}, \quad R_t^{i,e} = \frac{(1-\theta^i) V_{t+1}^{i,B}}{1 - \frac{\theta^i V_{t+1}^{i,B}}{R_t^{i,h}}} > R_t^{i,h}, \quad (7b)$$

$$c_{y,t}^{i,j} = (1-\beta)\omega_t^i, \quad c_{o,t+1}^{i,j} = \beta\omega_t^i R_t^{i,j}, \quad (7c)$$

$$d_t^{i,e} = \beta\omega_t^i, \quad i_t^{i,e} = 0, \quad d_t^{i,h} + i_t^{i,h} = \beta\omega_t^i, \quad (7d)$$

$$M_{t+1}^{i,A} = (1-\eta)i_t^{i,h} + \eta i_t^{i,e}, \quad M_{t+1}^{i,B} = (1-\eta)d_t^{i,h} + \eta d_t^{i,e}. \quad (7e)$$

Given the relative population mass of households and entrepreneurs,  $\frac{(1-\eta)}{\eta}$ , and the lending of each household,  $d_t^{i,h}$ , an entrepreneur obtains a loan of  $\frac{(1-\eta)}{\eta} d_t^{i,h}$  in equilibrium. He finances his total investment in sector B using his own capital,  $d_t^{i,e}$ , and the loan. Equation (7a) specifies his borrowing constraint. Investing in sector A and lending to entrepreneurs are perfect substitutes for households. The binding borrowing constraints have a general equilibrium effect causing the equity rate to be higher than the loan rate; see equations (7b). Young agents consume a constant fraction  $(1-\beta)$  of their labor income and invests the rest at the rate of return  $R_t^{i,j}$ . Old agents consume the entire return on investment; see equations (7c). Since the rate of return in sector A equals the loan rate, while the equity rate is higher than the loan rate, entrepreneurs only invest in sector B, while households invest directly in sector A and in loans to entrepreneurs; see equations (7d). Equations (7e) describe aggregate output of intermediate goods.

The binding borrowing constraints keep aggregate credit demand and aggregate investment in sector B inefficiently low. This has three consequences. First, the cross-sector investment distortion keeps the relative intermediate goods price below unity; second, the loan rate falls below the social rate of return to clear the credit market, while the equity rate rises above the social rate of return due to the leverage effect; third, the investment distortion keeps aggregate output inefficiently low. We define an indicator of production efficiency,  $\Lambda_t^i \equiv \frac{2\sqrt{\chi_{t+1}^i}}{1+\chi_{t+1}^i}$ . The model solutions under international financial autarky are

summarized as follows,

$$\chi_{t+1}^i = \chi_{IFA}^i \equiv 1 - \frac{\bar{\theta} - \theta^i}{1 - \eta} < 1, \text{ and } \frac{\partial \chi_{IFA}^i}{\partial \theta^i} > 0, \quad (8a)$$

$$R_t^{i,h} = V_{t+1}^{i,A} = \Psi_t^i \left[ 1 - \frac{(1 - \chi_{IFA}^i)}{2} \right] < \Psi_t^i, \quad (8b)$$

$$V_{t+1}^{i,B} = \frac{V_{t+1}^{i,A}}{\chi_{t+1}^i} = \Psi_t^i \left[ 1 + \frac{\frac{1}{\chi_{IFA}^i} - 1}{2} \right] > \Psi_t^i, \quad (8c)$$

$$\Lambda_t^i = \Lambda_{IFA}^i \equiv \frac{2\sqrt{\chi_{IFA}^i}}{1 + \chi_{IFA}^i} < 1, \quad \frac{\partial \Lambda_{IFA}^i}{\partial \theta^i} = \frac{\partial \chi_{IFA}^i}{\partial \theta^i} \frac{(1 - \chi_{IFA}^i)\Lambda_{IFA}^i}{2\chi_{IFA}^i(1 + \chi_{IFA}^i)} > 0, \quad (8d)$$

$$\omega_{t+1}^i = \left( \frac{\Lambda_{IFA}^i \omega_t^i}{\Psi_{IFA}^i} \right)^\alpha, \quad (8e)$$

$$R_t^{i,e} = \Psi_t^i \left[ 1 + \frac{(1 - \eta)(1 - \chi_{IFA}^i)}{2\eta} \right] > \Psi_t^i. \quad (8f)$$

The relative intermediate goods price,  $\chi_{t+1}^i$ , and the indicator of production efficiency  $\Lambda_t^i$  are time-invariant and positively related to  $\theta^i$ . Aggregate output is proportional to the wage rate,  $Y_t^i = \frac{\omega_t^i}{(1-\alpha)}$ . Thus, the model dynamics can be characterized by the dynamics of wages. In view of equation (8e) and with  $\alpha \in (0, 1)$ , there exists a unique and stable steady state with the wage at  $w_{IFA}^i = \left( \frac{\Lambda_{IFA}^i}{\Psi_{IFA}^i} \right)^\rho$ . Obviously, as long as  $\theta^i \in [0, \bar{\theta})$ , the relative intermediate goods price is less than unity, reflecting the distortion of the binding borrowing constraints. It justifies our definition of the threshold value.

According to equation (8d),  $\frac{\partial \Lambda_{IFA}^i}{\partial \theta^i} > 0$ , and  $\Lambda_{IFA}^i$  reaches the maximum of one, when the borrowing constraint is weakly binding at  $\theta^i = \bar{\theta}$ . The relative intermediate goods price is a key variable measuring the distortion on investment composition. According to equation (8a),  $\frac{\partial \chi_{IFA}^i}{\partial \theta^i} > 0$  and  $\chi_{IFA}^i$  reaches the maximum of one, for  $\theta^i = \bar{\theta}$ . Thus, the higher the relative intermediate goods price, the smaller the output distortion. Alternatively, in the country with a higher  $\theta^i$ , entrepreneurs are less credit constrained so that cross-sector investment allocation is more efficient. Thus, the relative intermediate goods price is higher and so is the loan rate and aggregate output, while the equity rate is lower. Proposition 2.1 summarizes the case where the borrowing constraints are binding.

**Proposition 2.1.** *Let  $\lambda^i = 1$  and  $\theta^i \in [0, \bar{\theta})$ . There exists a unique and stable non-zero steady state in country  $i$  with the wage at  $\omega_{IFA}^i = \left( \frac{\Lambda_{IFA}^i}{\Psi_{IFA}^i} \right)^\rho$ .*

*Limited commitment creates a wedge between the private and social rates of return,  $R_t^{i,h} = V_{t+1}^{i,A} < \Psi_t^i < V_{t+1}^{i,B} < R_t^{i,e}$ . In the steady state, the loan rate rises and the equity rate falls in  $\theta^i$ .*

*Limited commitment distorts the investment made by agents with different productivity. The deviation of  $\chi_{IFA}^i$  from unity reflects the output distortion, which declines in  $\theta^i$ .*

## 2.3 Equilibrium with Incomplete Markets

For simplicity, we set  $\theta^i = 1$  such that the borrowing constraints are slack in sector B and the equity rate coincides with the loan rate. As we do not have to distinguish the equity rate and the loan rate here, let  $R_t^i \equiv R_t^{i,h} = R_t^{i,e}$  define the rate of return in sector B which is risk-free. For  $\lambda^i < 1$ , idiosyncratic risk is partially insured so that the investment return in sector A is still stochastic after risk sharing. Since all agents face the same risk-free interest rate  $R_t^i$  and have the same production technology in sector A with risky return, we do not need to distinguish between entrepreneurs and households in this subsection.

Suppose that an agent invests a fraction  $\phi_t^i$  of its total savings into sector A. Let  $\hat{\xi}_{t+1}^i \equiv (1 - \phi_t^i)R_t^i + \phi_t^i(\Gamma_{t+1}^i + e^{\epsilon_{t+1}})V_{t+1}^{i,A}$  and  $\xi_t^i \equiv \left[ E_t(\hat{\xi}_{t+1}^i)^{1-\gamma} \right]^{\frac{1}{1-\gamma}}$  denote the ex post rate of return and the risk-adjusted expected rate of return to its portfolio investment, respectively. The agent's utility maximization problem is reformulated as

$$\max_{c_{y,t}^i, \phi_t^i} (1 - \beta) \ln c_{y,t}^i + \beta \ln(\omega_t^i - c_{y,t}^i) + \beta \ln \xi_t^i.$$

Define the Sharpe ratio as  $\zeta_t^i \equiv \frac{\ln V_{t+1}^{i,A} - \ln R_{t+1}^i}{(1-\lambda^i)\sigma}$  to measure the excess mean return per unit of uninsured risk in sector A. A market equilibrium in country  $i$  is a set of allocation of agents' consumption and investment choices  $\{\phi_t^i, c_{y,t}^i, c_{o,t}^i\}$ , and aggregate variables,  $\{Y_t^i, M_t^{i,A}, M_t^{i,B}, \omega_t^i, R_t^i, \zeta_t^i, V_t^{i,A}, V_t^{i,B}\}$ , satisfying equations (2), (3), (9a)-(9d).

$$R_t^i = V_{t+1}^{i,B}, \quad (9a)$$

$$c_{y,t}^i = (1 - \beta)\omega_t^i, \quad c_{o,t+1}^i = \beta\omega_t^i \hat{\xi}_{t+1}^i, \quad (9b)$$

$$\phi_t^i \approx \frac{\ln V_{t+1}^{i,A} - \ln R_{t+1}^i}{\gamma[(1 - \lambda^i)\sigma]^2} \approx \frac{\chi_{t+1}^i - 1}{\gamma[(1 - \lambda^i)\sigma]^2}, \quad (9c)$$

$$M_{t+1}^{i,A} = \phi_t^i \beta \omega_t^i, \quad M_{t+1}^{i,B} = (1 - \phi_t^i) \beta \omega_t^i. \quad (9d)$$

The risk-free interest rate is equal to the marginal return in sector B; see equation (9a). As shown in Angeletos (2007), the optimal choices of individual consumption and investment portfolio are the solutions to the standard Samuelson-Merton problem, as in equations (9b)-(9c). Equations (9d) specify the aggregate output of the two intermediate goods.

The fraction of aggregate savings  $\phi_t^i \beta \omega_t^i$  invested in sector A and period  $t$  requires a mean rate of return  $V_{t+1}^{i,A}$  and the remaining investment in sector B  $(1 - \phi_t^i) \beta \omega_t^i$  requires a risk-free rate of return  $R_t^i = V_{t+1}^{i,B}$ . In period  $t + 1$ , the aggregate revenue from producing intermediate goods,  $V_{t+1}^{i,A} M_{t+1}^{i,A} + V_{t+1}^{i,B} M_{t+1}^{i,B} = \alpha Y_{t+1}^i = \rho \omega_{t+1}^i$ , is distributed to all agents,

$$\beta \omega_t^i [\phi_t^i V_t^{i,A} + (1 - \phi_t^i) R_t^i] = \rho \omega_{t+1}^i \Rightarrow \phi_t^i V_t^{i,A} + (1 - \phi_t^i) R_t^i = \Psi_t^i. \quad (10)$$

In the case of incomplete markets, agents invest too much in the risk-free sector B and too little in the risky sector A. This has four consequences. First, the cross-sector

investment distortion keeps the relative intermediate goods price above unity; second, the risk-free interest rate is depressed below the social rate of return; third, the Sharpe ratio is positive, reflecting the risk premium per unit of uninsured idiosyncratic risk in sector A; fourth, the cross-sector investment distortion keeps aggregate output inefficiently low. The model solutions under IFA are summarized as follows,

$$\chi_{t+1}^i \approx \chi_{IFA}^i \equiv \sqrt{1 + \gamma[(1 - \lambda^i)\sigma]^2} > 1, \text{ and } \frac{\partial \chi_{IFA}^i}{\partial \lambda^i} < 0, \quad (11a)$$

$$R_t^i = V_{t+1}^{i,B} \approx \Psi_t^i \left[ 1 + \frac{\chi_{IFA}^i - 1}{2} \right] < \Psi_t^i, \quad (11b)$$

$$V_{t+1}^{i,A} = V_{t+1}^{i,B} \chi_{t+1}^i \approx \Psi_t^i \left[ 1 - \frac{1 - \chi_{IFA}^i}{2} \right] > \Psi_t^i, \quad (11c)$$

$$\Lambda_t^i \approx \Lambda_{IFA}^i \equiv \frac{2\sqrt{\chi_{IFA}^i}}{1 + \chi_{IFA}^i} < 1, \quad \frac{\partial \Lambda_{IFA}^i}{\partial \lambda^i} = \frac{\partial \chi_{IFA}^i}{\partial \lambda^i} \frac{(1 - \chi_{IFA}^i)\Lambda_{IFA}^i}{2\chi_{IFA}^i(1 + \chi_{IFA}^i)} > 0, \quad (11d)$$

$$\omega_{t+1}^i = \left( \frac{\Lambda_{IFA}^i}{\Psi_{IFA}^i} \omega_t^i \right)^\alpha \text{ where } \Lambda_{IFA}^i = \frac{2\sqrt{\chi_{IFA}^i}}{1 + \chi_{IFA}^i} < 1, \quad (11e)$$

$$\phi_t^i \approx \frac{1}{\chi_{IFA}^i + 1} < \frac{1}{2}, \quad \zeta_t^i \approx \zeta_{IFA}^i \equiv \sqrt{\gamma \left( 1 - \frac{2}{\chi_{IFA}^i + 1} \right)} > 0, \quad (11f)$$

$$\xi_t^i \approx R_t^i \left[ 1 + \frac{(\zeta_t^i)^2}{2\gamma} \right] \approx \Psi_t^i \left[ 1 - \frac{1 - \frac{1}{\chi_{IFA}^i}}{4} \right] < \Psi_t^i. \quad (11g)$$

The relative intermediate goods price,  $\chi_{t+1}^i$ , and the efficiency indicator  $\Lambda_t^i$  are time-invariant and positively related to  $\lambda^i$ . Aggregate output is proportional to the wage rate,  $Y_t^i = \frac{\omega_t^i}{(1-\alpha)}$ . Thus, the model dynamics can be characterized by the dynamics of wages. In view of equation (11e) and with  $\alpha \in (0, 1)$ , there exists a unique and stable steady state with the wage at  $w_{IFA}^i = \left( \frac{\Lambda_{IFA}^i}{\Psi_{IFA}^i} \right)^\rho$ .

According to equation (11d),  $\frac{\partial \Lambda_{IFA}^i}{\partial \lambda^i} > 0$  and  $\Lambda_{IFA}^i$  reaches the maximum of one in the case of complete markets  $\lambda^i = 1$ . The relative intermediate goods price is a key variable measuring the distortion in investment composition. According to equation (11a),  $\frac{\partial \chi_{IFA}^i}{\partial \lambda^i} < 0$  and  $\Lambda_{IFA}^i$  reaches the minimum value of one, for  $\lambda^i = 1$ . Thus, the smaller the relative intermediate goods price, the smaller the output distortion. In other words, in the country with a higher  $\lambda^i$ , a larger fraction of idiosyncratic risk is insured so that cross-sector investment allocation is less distorted. The risk-free interest rate is higher and so is aggregate output, while the relative intermediate goods price is lower and so is the Sharpe ratio. Proposition 2.2 summarizes the case of incomplete markets.

**Proposition 2.2.** *Let  $\lambda^i \in [0, 1)$  and  $\theta^i = 1$ . There exists a unique and stable non-zero steady state in country  $i$  with the wage at  $w_{IFA}^i = \left( \frac{\Lambda_{IFA}^i}{\Psi_{IFA}^i} \right)^\rho$ .*

*Incomplete markets create a wedge between the private and social rates of return,  $R_t^i = V_{t+1}^{i,B} < \xi_t^i < \Psi_t^i < V_{t+1}^{i,A}$ . In the steady state, the risk-free rate and the risk-adjusted rate of portfolio return rise in  $\lambda^i$ , while the Sharpe ratio falls in  $\lambda^i$ .*

*Incomplete markets distort investment among sectors with different riskiness. The deviation of  $\chi_{IFA}^i$  from unity reflects the output distortion, which declines in  $\lambda^i$ .*

### 3 The Model Under Full Capital Mobility

Under full capital mobility, agents can lend and make direct investments abroad. The two countries are initially in the steady state under IFA before capital mobility is allowed in period  $t = 0$ . We investigate the patterns of capital flows, interest rate, and output in the presence of limited commitment and incomplete markets, respectively.

#### 3.1 The Equilibrium with Limited Commitment Only

We assume  $\lambda^S = \lambda^N = 1$  and  $0 \leq \theta^S < \theta^N \leq \bar{\theta}$  such that idiosyncratic risk in sector A is fully insured, the borrowing constraints in sector B are binding in both countries under IFA as well as under full capital mobility, and country N is more financially developed than country S. Financial capital flows refer to the size of household lending abroad, while FDI flows refer to the size of investment made by entrepreneurs abroad. Let  $\Phi_t^i$  and  $\Omega_t^i$  denote the aggregate outflows of financial capital and FDI from country  $i$  in period  $t$ , respectively, with negative values indicating capital inflows.

With capital mobility, net credit supply in country  $i$  is  $(1 - \eta)(\beta\omega_t^i - i_t^{i,A}) - \Phi_t^i$ , and aggregate equity capital invested in country  $i$  is  $\eta\beta\omega_t^i - \Omega_t^i$ . Assuming that entrepreneurs borrow in the country where they invest in the production of intermediate good B, FDI flows raise aggregate credit demand in the host country and reduce it in the source country. With these changes, the analysis in subsection 2.2 carries through due to the linearity of intermediate goods production and the borrowing constraints. Financial capital flows equalize loan rates and FDI flows equalize equity rates across the border. Credit and equity markets clear in each country as well at the world level. FDI flows directly affect aggregate output of intermediate good B in each country. To summarize,

$$R_t^{S,h} = R_t^{N,h} = R_t^{*,h}, \quad R_t^{S,e} = R_t^{N,e} = R_t^{*,e}, \quad \Phi_t^S + \Phi_t^N = \Omega_t^S + \Omega_t^N = 0, \quad (12a)$$

$$\Phi_t^i = (1 - \eta)(\beta\omega_t^i - i_t^{i,h}) - (\psi_t^i - 1)(\eta\beta\omega_t^i - \Omega_t^i), \quad M_{t+1}^{i,B} = \psi_t^i(\eta\beta\omega_t^i - \Omega_t^i). \quad (12b)$$

The remaining conditions for market equilibrium are same as under IFA.

At the world level, aggregate revenue of intermediate goods in period  $t+1$  is distributed

to households and entrepreneurs as the return to their savings in period  $t$ ,

$$[(1 - \eta)R_t^{*,h} + \eta R_t^{*,e}] \sum_{i \in \{S, N\}} \beta \omega_t^i = \sum_{i \in \{S, N\}} (v_{t+1}^{i,A} M_{t+1}^{i,A} + v_{t+1}^{i,B} M_{t+1}^{i,B}) = \sum_{i \in \{S, N\}} \rho \omega_{t+1}^i. \quad (13)$$

Let  $\omega_t^w \equiv \frac{\omega_t^S + \omega_t^N}{2}$  denote the world average wage in period  $t$  and  $\Psi_t^w \equiv \frac{\omega_{t+1}^w}{\omega_t^w} \frac{\rho}{\beta}$  denote the social rate of return at the world level. Equation (13) is simplified as

$$(1 - \eta)R_t^{*,h} + \eta R_t^{*,e} = \Psi_t^w. \quad (14)$$

**Lemma 3.1.** *Under full capital mobility, the relative intermediate goods price is time-invariant and there exists a unique and stable steady state.*

Let  $X_{FCM}$  denote the steady-state value of variable  $X$  under full capital mobility. Define  $\wp_{IFA}^i \equiv \frac{R_{IFA}^{i,e}}{\Psi_{IFA}^i} = 1 + \frac{(1-\eta)(1-\chi_{IFA}^i)}{2\eta}$ ,  $\Delta\chi_{FCM}^i \equiv \chi_{FCM}^i - \chi_{IFA}^i$ , and  $\mathcal{Z}_{FCM}^i \equiv \frac{\Delta\chi_{FCM}^i R_{IFA}^{i,e}}{\Delta\chi_{FCM}^i + \frac{1-\theta^i}{(1-\eta)\wp_{IFA}^i}}$ . The model solutions under full capital mobility are,

$$R_t^{i,e} = \frac{\omega_{t+1}^w}{\omega_t^w} (R_{IFA}^{i,e} - \mathcal{Z}_{FCM}^i), \quad (15a)$$

$$R_t^{i,h} = \frac{\omega_{t+1}^w}{\omega_t^w} \left( R_{IFA}^{i,h} + \frac{\eta}{1-\eta} \mathcal{Z}_{FCM}^i \right). \quad (15b)$$

$$\chi_{t+1}^i = \chi_{FCM}^i = \frac{(1-\theta^i)R_t^{i,h}}{R_t^{i,e}} + \theta^i, \quad (15c)$$

$$\Phi_t^i = (1-\eta)\beta\omega_t^i \left[ 1 - \frac{\omega_{t+1}^i}{\omega_t^i} \frac{R_{IFA}^{i,h}}{R_t^{*,h}} \right] \quad (15d)$$

$$\Omega_t^i = \eta\beta\omega_t^i \left[ 1 - \frac{\omega_{t+1}^i}{\omega_t^i} \frac{R_{IFA}^{i,e}}{R_t^{*,e}} \right] \quad (15e)$$

$$\Omega_t^i + \Phi_t^i = \beta\omega_t^i \left\{ 1 - \frac{\omega_{t+1}^i}{\omega_t^i} \left[ \eta \frac{R_{IFA}^{i,e}}{R_t^{*,e}} + (1-\eta) \frac{R_{IFA}^{i,h}}{R_t^{*,h}} \right] \right\} \quad (15f)$$

$$\omega_{t+1}^i = (\chi_{FCM}^i)^{\frac{\rho}{2}} (R_t^*)^{-\rho}. \quad (15g)$$

In the steady state under full capital mobility, interest rates and capital flows are,

$$R_{FCM}^{i,e} = R_{IFA}^{i,e} - \mathcal{Z}_{FCM}^i, \quad (16a)$$

$$R_{FCM}^{i,h} = R_{IFA}^{i,h} + \frac{\eta}{1-\eta} \mathcal{Z}_{FCM}^i, \quad (16b)$$

$$\Phi_{FCM}^i = (1-\eta)\beta\omega_{FCM}^i \left( 1 - \frac{R_{IFA}^{i,h}}{R_{FCM}^{*,h}} \right) = \eta\beta\omega_{FCM}^i \frac{\mathcal{Z}_{FCM}^i}{R_{FCM}^{*,h}}, \quad (16c)$$

$$\Omega_{FCM}^i = \eta\beta\omega_{FCM}^i \left( 1 - \frac{R_{IFA}^{i,e}}{R_{FCM}^{*,e}} \right) = -\eta\beta\omega_{FCM}^i \frac{\mathcal{Z}_{FCM}^i}{R_{FCM}^{*,e}}, \quad (16d)$$

$$\Phi_{FCM}^i + \Omega_{FCM}^i = \eta\beta\omega_{FCM}^i \mathcal{Z}_{FCM}^i \frac{(R_{FCM}^{*,e} - R_{FCM}^{*,h})}{R_{FCM}^{*,e} R_{FCM}^{*,h}}. \quad (16e)$$



**Proposition 3.1.** *In the steady state, the world interest rates are  $R_{FCM}^{*,h} \in (R_{IFA}^{S,h}, R_{IFA}^{N,h})$  and  $R_{FCM}^{*,e} \in (R_{IFA}^{N,e}, R_{IFA}^{S,e})$ . Full capital mobility raises (reduces) the relative price in country S (N),  $\chi_{IFA}^S < \chi_{FCM}^S < \chi_{FCM}^N < \chi_{IFA}^N$ ; financial capital flows from country S to country N, FDI flows in the opposite direction, and net capital flows from country S to country N,  $\Phi_{FCM}^S > 0 > \Omega_{FCM}^S$  and  $\Phi_{FCM}^S + \Omega_{FCM}^S > 0$ . Gross international investment return sums up to zero in each country,  $\Phi_{FCM}^i R_{FCM}^{*,h} + \Omega_{FCM}^i R_{FCM}^{*,e} = 0$ .*

*Proof.* The proof follows immediately from equations (16a)-(16e).  $\square$

In the steady state under IFA, aggregate output is higher in country N than in country S. Under full capital mobility, country N, which is more financially developed, imports financial capital and exports FDI; overall, it has net capital inflows. Note that international capital flows lead to the partial convergence of the relative intermediate goods price in the two countries, implying that capital mobility improves investment composition in country S which is less financially developed. Since the rate of return to its foreign assets (FDI outflows) is higher than the interest rate it pays for its foreign liabilities (financial capital inflows),  $R_{FCM}^{*,e} > R_{FCM}^{*,h}$ , country N receives a positive net international investment income,  $\Phi_{FCM}^N (R_{FCM}^{*,h} - 1) + \Omega_{FCM}^N (R_{FCM}^{*,e} - 1) = 0 - (\Phi_{FCM}^N + \Omega_{FCM}^N) > 0$ , despite its negative international investment position,  $\Phi_{FCM}^N + \Omega_{FCM}^N < 0$ . This way, our model results are consistent with the three recent empirical facts.

### 3.2 The Equilibrium with Incomplete Markets Only

We assume  $0 \leq \lambda^S < \lambda^N < 1$  and  $\theta^S = \theta^N = 1$  such that idiosyncratic risk in sector A is partially insured, the borrowing constraints in sector B are slack in both countries under IFA as well as under full capital mobility, and country N is more financially developed than country S. Financial capital flows refer to the size of risk-free lending abroad, while FDI flows refer to the size of risky investment made in sector B abroad. Let  $\Phi_t^i$  and  $\Omega_t^i$  denote the aggregate outflows of financial capital and FDI from country  $i$  in period  $t$ , respectively, with negative values indicating capital inflows.

Agents optimally chooses between the risk-free investment in sector B domestically and the risk-free lending abroad. In equilibrium, financial capital flows equalize the risk-free interest rate globally. In the meantime, agents optimally chooses between investing in sector A domestically and abroad. By assumption, agents obtain risk sharing in the country where they invest in the production of intermediate good A. In equilibrium, FDI flows equalize the Sharpe ratio globally. See the proof of lemma 3.2 for details.

In the steady state under IFA, the risk-free interest rate is higher in country N, while the Sharpe ratio is higher in country S. Thus, similar as in the setting with limited

commitment only, financial capital flows from country S to country N, while FDI flows in the opposite direction,  $\Phi_t^S > 0 > \Omega_t^S$ . To summarize,

$$\Phi_t^S + \Phi_t^N = \Omega_t^S + \Omega_t^N = 0, \quad R_t^S = R_t^N = R_t^*, \quad \zeta_t^S = \zeta_t^N = \zeta_t^*. \quad (17a)$$

$$\beta\omega_t^S = \frac{M_{t+1}^{S,A} + \Omega_t^S}{\phi_t^S} = M_{t+1}^{S,A} + M_{t+1}^{S,B} + \Phi_t^S + \Omega_t^S, \quad (17b)$$

$$\beta\omega_t^N = \frac{M_{t+1}^{N,A}}{\phi_t^N} + \frac{\Omega_t^N}{\phi_t^S} = M_{t+1}^{N,A} + M_{t+1}^{N,B} + \Phi_t^N + \Omega_t^N. \quad (17c)$$

The remaining conditions for market equilibrium are same as under IFA.

**Lemma 3.2.** *Under full capital mobility, the relative intermediate goods price is time-invariant and there exists a unique and stable steady state.*

The cross-border equalization of the risk-free interest rate,  $R_t^N = R_t^S = R_t^*$ , and that of the Sharpe ratio,  $\zeta_t^N = \zeta_t^S = \zeta_t^*$ , jointly imply that of the mean rate of portfolio return,  $R_t^N e^{\frac{(\zeta_t^N)^2}{\gamma}} = R_t^S e^{\frac{(\zeta_t^S)^2}{\gamma}} = R_t^* e^{\frac{(\zeta_t^*)^2}{\gamma}}$ . At the world level, aggregate revenue of intermediate goods in period  $t + 1$  is distributed to agents as the return to their savings in period  $t$ ,

$$R_t^* e^{\frac{(\zeta_t^*)^2}{\gamma}} \beta(\omega_t^N + \omega_t^S) = \rho(\omega_{t+1}^N + \omega_{t+1}^S), \quad \Rightarrow \quad R_t^* \left[1 + \frac{(\zeta_t^*)^2}{\gamma}\right] \approx \frac{\rho\omega_{t+1}^w}{\beta\omega_t^w}. \quad (18)$$

Define an auxiliary variable  $\mathcal{Z}_{FCM}^i \equiv \frac{\Delta\chi_{FCM}^i R_{IFA}^i}{\Delta\chi_{FCM}^i + \frac{2\chi_{IFA}^i(\chi_{IFA}^i - 1)}{\chi_{IFA}^i + \chi_{FCM}^i - 2}}$ . The model solutions under full capital mobility are,

$$R_t^i = \frac{\omega_{t+1}^w}{\omega_t^w} (R_{IFA}^i - \mathcal{Z}_{FCM}^i), \quad (19a)$$

$$\Phi_t^S = -\Phi_t^N = (1 - \phi_t^S)\beta\omega_t^S \left[1 - \frac{\omega_{t+1}^S}{\omega_t^S} \frac{R_{IFA}^S}{R_t^*} \frac{\chi_{IFA}^S - 1}{\chi_{IFA}^S - \frac{\chi_{FCM}^S}{\chi_{IFA}^S}}\right] \quad (19b)$$

$$\Omega_t^S = -\Omega_t^N = \phi_t^S \beta\omega_t^S \left[1 - \frac{\omega_{t+1}^S}{\omega_t^S} \frac{R_{IFA}^S}{R_t^*} \frac{\chi_{IFA}^S(\chi_{IFA}^S - 1)}{\chi_{FCM}^S(\chi_{FCM}^S - 1)}\right] \quad (19c)$$

$$\Omega_t^S + \Phi_t^S = -(\Omega_t^N + \Phi_t^N) = \beta\omega_t^S \left[1 - \frac{\omega_{t+1}^S}{\omega_t^S} \frac{R_{IFA}^S}{R_t^*} \frac{1 + \frac{1}{\chi_{FCM}^S}}{1 + \frac{1}{\chi_{IFA}^S}}\right] \quad (19d)$$

$$\omega_{t+1}^i = (\chi_{FCM}^i)^{-\frac{\rho}{2}} (R_t^*)^{-\rho}. \quad (19e)$$

In the steady state under full capital mobility, interest rates and capital flows are,

$$R_{FCM}^i = R_{IFA}^i - \mathcal{Z}_{FCM}^i, \quad (20a)$$

$$\Phi_{FCM}^S = -\beta\omega_{FCM}^S \frac{\Delta\chi_{FCM}^S(\chi_{FCM}^S + \chi_{IFA}^S)}{2[(\chi_{IFA}^S)^2 - 1]} \quad (20b)$$

$$\Omega_{FCM}^S = \beta\omega_{FCM}^S \frac{\Delta\chi_{FCM}^S(\chi_{FCM}^S + \chi_{IFA}^S)}{2\chi_{FCM}^S[(\chi_{IFA}^S)^2 - 1]}, \quad (20c)$$

$$\Phi_{FCM}^S + \Omega_{FCM}^S = -\beta\omega_{FCM}^S \frac{\Delta\chi_{FCM}^S(\chi_{FCM}^S + \chi_{IFA}^S)(\chi_{FCM}^S - 1)}{2\chi_{FCM}^S[(\chi_{IFA}^S)^2 - 1]}. \quad (20d)$$

**Proposition 3.2.** *In the steady state, the world risk-free interest rates are  $R_{FCM}^* \in (R_{IFA}^S, R_{IFA}^N)$ . Full capital mobility reduces (raises) the relative price in country S (N),  $\chi_{IFA}^N < \chi_{FCM}^N < \chi_{FCM}^S < \chi_{IFA}^S$ ; financial capital flows from country S to country N, FDI flows in the opposite direction, and net capital flows from country S to country N,  $\Phi_{FCM}^S > 0 > \Omega_{FCM}^S$  and  $\Phi_{FCM}^S + \Omega_{FCM}^S > 0$ . Gross international investment return sums up to zero in each country,  $\Phi_{FCM}^S R_{FCM}^* + \Omega_{FCM}^S V_{FCM}^{S,A} = 0$ .*

*Proof.* The proof follows immediately from equations (20a)-(20d).  $\square$

In the steady state under IFA, aggregate output is higher in country N than in country S. Under full capital mobility, country N, which is more financially developed, imports financial capital and exports FDI; overall, it has net capital inflows. Note that international capital flows lead to the partial convergence of the relative intermediate goods price in the two countries, implying that capital mobility improves investment composition in country S which is less financially developed. Since the rate of return to its foreign assets (FDI outflows) is higher than the interest rate it pays for its foreign liabilities (financial capital inflows),  $V_{FCM}^{S,A} = R_{FCM}^* \chi_{FCM}^S > R_{FCM}^*$ , country N receives a positive net international investment income,  $\Phi_{FCM}^N (R_{FCM}^* - 1) + \Omega_{FCM}^N (V_{FCM}^{S,A} - 1) = 0 - (\Phi_{FCM}^N + \Omega_{FCM}^N) > 0$ , despite its negative international investment position,  $\Phi_{FCM}^N + \Omega_{FCM}^N < 0$ . This way, our model results are compatible with the empirical evidence noted above.

## 4 Model Comparison

Although limited commitment and incomplete markets feature different aspects of financial market imperfections, they have the same qualitative distortions on aggregate investment, production efficiency, and interest rates under IFA in our framework; see Proposition 2.1 and 2.2. In particular, comparing equations (8a)-(8e) with (11a)-(11e), we show that the analytical solutions to major endogenous variables under IFA are identical. Here, the relative intermediate goods price,  $\chi_{t+1}^i = \frac{V_{t+1}^{i,A}}{V_{t+1}^{i,B}}$ , is the key variable reflecting such distortions. Since the price of intermediate goods in the sector with financial frictions is higher than in the other sector,  $\chi_{t+1}^i$  is smaller than unity in the settings with only limited commitment and larger than unity in the setting with incomplete markets. Proposition 4.1 establishes a result of analytical equivalence between the two model settings.

**Proposition 4.1.** *For  $1 + \sqrt{1 + \gamma[(1 - \lambda^i)\sigma]^2} \leq \frac{1}{\eta}$ , there exists a one-to-one mapping between  $\theta^i$  and  $\lambda^i$  such that the steady-state output and investment are same across the two alternative model settings and so are the loan rate and the risk-free interest rate. In particular, this mapping takes the following form,  $1 - \frac{\bar{\theta} - \theta^i}{1 - \eta} = \frac{1}{\sqrt{1 + \gamma[(1 - \lambda^i)\sigma]^2}}$ .*

For the empirically relevant values of  $\gamma$  and  $\sigma$ , the condition  $1 + \sqrt{1 + \gamma[(1 - \lambda^i)\sigma]^2} \leq \frac{1}{\eta}$  holds, if the population share of entrepreneurs,  $\eta$ , is sufficiently small.<sup>5</sup> Intuitively, when the population share of entrepreneurs becomes larger, the entrepreneurial sector as a whole becomes less borrowing constrained in equilibrium, which weakens the macroeconomic importance of financial frictions.

Under full capital mobility, the steady-state patterns of capital flows, relative prices, interest rates, aggregate output are also qualitatively identical across the two model settings; see Proposition 3.1 and 3.2. Furthermore, section 5 shows in a numerical example that the model dynamics with respect to a financial crisis are also qualitatively identical across the two settings.

## 5 A Dynamic Analysis of Financial Crisis

So far, we have shown that the steady-state patterns of international capital flows under the two alternative settings are qualitatively identical. In the following, we compare numerically the transitional dynamics of the two model settings in the case of a financial crisis. Mendoza and Quadrini (2010) develop a two-country model with infinitely-lived agents, idiosyncratic endowment risk, and incomplete markets. A financial crisis is featured in their model as an unexpected decline in bank equity. In our framework, a financial crisis is modeled as an unexpected decline in  $\theta^N$  and  $\lambda^N$ , respectively. The purpose of this analysis is to investigate whether international financial integration magnifies or dampens the economic responses to financial crisis in country N under our two alternative settings. We also compare our results with those in Mendoza and Quadrini (2010).

The parameter values are chosen only for illustration purpose as follows. The population share of entrepreneurs is set at  $\eta = 10\%$ , implying that the threshold value  $\bar{\theta} = 1 - 2\eta = 0.8$ ; the share of labor income in aggregate output is  $1 - \alpha = 64\%$ ; the lifetime share of utility from consumption when old is  $\beta = 0.4$ ; the standard deviation of idiosyncratic risk is set at  $\sigma = 0.8$  and the coefficient of relative risk aversion  $\gamma = 10$ .

According to Proposition 4.1, there exists a one-to-one mapping between the two key parameters,  $\theta^i$  and  $\lambda^i$ , such that the steady-state aggregate output in country  $i$  under IFA is same across the two alternative settings. Thus, in order to make the model economy

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<sup>5</sup>The value of  $\gamma$  is set around 10 in the macro-finance literature to generate plausible risk premia. Angeletos and Panousi (2011) choose  $\sigma = 0.5$  in their numerical exercise, consistent with the preferred value in Bitler, Moskowitz, and Vissing-Jørgensen (2005); Moskowitz and Vissing-Jørgensen (2002). In order to check how tight the condition in Proposition 4.1 is, we choose the upper bounds of these parameters as  $\gamma = 15$  and  $\sigma = 1$ . Given  $\lambda^i \in [0, 1]$ , as long as the population share of entrepreneurs is  $\eta \leq 0.2$ , this condition holds.

under the two settings comparable, we choose the values of  $\theta^i$  and  $\lambda^i$ , according to this one-to-one mapping. In the setting with only limited commitment, we set  $\theta^N = 0.75$  and  $\theta^S = 0.25$ , while keeping  $\lambda^N = \lambda^S = 1$ ; in the setting with only incomplete markets, we set  $\lambda^N = 0.86$  and  $\lambda^S = 0.06$ , while keeping  $\theta^N = \theta^S = 1$ .

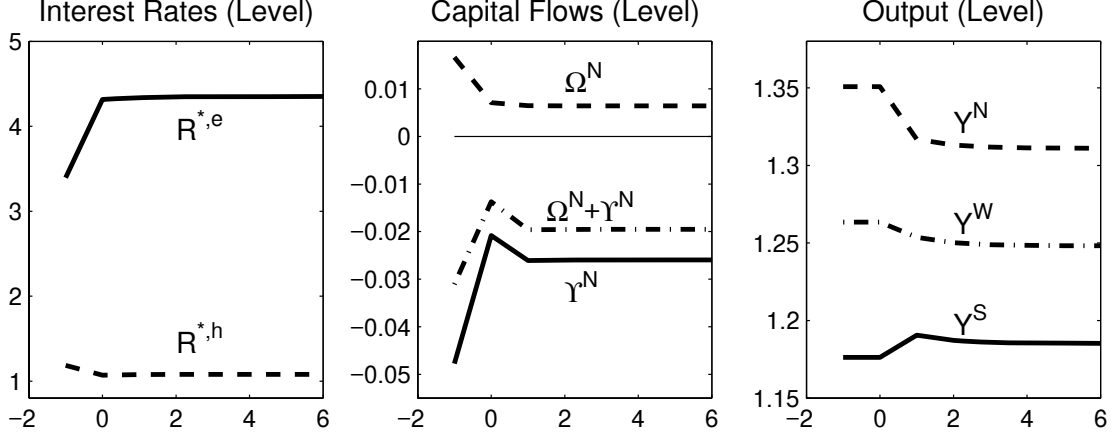


Figure 1: Financial Crisis in the Setting with Limited Commitment

Let us first look at the setting with only limited commitment. Suppose that the world economy is initially in the steady state under full capital mobility before  $\theta^N$  falls permanently from 0.75 to 0.5 in period  $t = 0$ . Figure 1 show the dynamic responses of variables in levels.

From period  $t = 0$  on, entrepreneurs in country N are subject to tighter borrowing constraints. The decline in aggregate credit demand forces the loan rate in country N,  $R_t^{N,h}$  to fall to clear the credit market. Meanwhile, it also makes aggregate investment tilted more towards sector A so that the price of intermediate good B rises in country N. The rise in the price of intermediate good B,  $V_{t+1}^{N,B}$ , and the decline in the loan rate,  $R_t^{N,h}$ , tend to raise the equity rate while the decline in the investment-equity ratio,  $\psi_t^N$ , tends to reduce the equity rate. Overall, the first effect dominates the last effect so that the equity rate in country N,  $R_t^{N,h}$ , rises. The rise in the equity rate induces entrepreneurs in country N to reduce their FDI outflows,  $\Omega_t^N$ , and the decline in the loan rate induces households in country S to reduce their foreign lending, leading to a decline in financial capital inflows,  $\Upsilon_t^N$ . Thus, both FDI outflows and financial capital inflows shrink. Since the aggregate credit capacity in country N falls, net capital inflows,  $\Omega_t^N + \Upsilon_t^N$ , also shrink. The decline in effective aggregate credit demand directly worsens cross-sector investment composition and the decline in net capital inflows reduces further the size of aggregate domestic investment in country N. Thus, aggregate output in country N,  $Y_t^N$ , declines.

Let us look at country S. By reducing their foreign lending, households save more

domestically and the rise in credit supply pushes down the loan rate,  $R_t^{S,h}$ , justifying the global equalization of the loan rate; the decline in the loan rate reduces the borrowing costs for entrepreneurs and the equity rate,  $R_t^{S,e}$  rises in country S, justifying the global equalization of the equity rate. Aggregate output is affected by two opposite forces. First, the decline in net capital outflows raises the size of aggregate domestic investment, which tends to raise aggregate output. Second, the decline in FDI inflows and financial capital outflows worsens the cross-sector investment composition, which tends to reduce aggregate output. The overall impact depends on the relative magnitude of the two forces. According to von Hagen and Zhang (2011), for  $\theta^S$  close to zero, the composition effect dominates the size effect so that aggregate output falls; otherwise, aggregate output rises. Given the chosen parameter values, aggregate output in country S,  $Y_t^S$  rises.

Mendoza and Quadrini (2010) ask whether financial integration amplifies or dampens the impacts of financial crisis on country N. In their model, unrecovered loans reduce bank equity unexpectedly, leading to the lending contraction and the decline in the demand for productive assets. Since financial integration creates larger financial markets, credit contraction and the negative impacts on asset prices are spread among countries that are financially integrated. Thus, in comparison with financial autarky, the responses of asset price are smaller under financial integration. However, given their model setting, the financial crisis does not affect aggregate output.

In order to see whether financial integration magnifies or dampens the impacts of the financial crisis in our model, we conduct a counterfactual experiment where country N is initially in the steady state under IFA before  $\theta^N$  declines in period  $t = 0$  and country N stays under financial autarky permanently upon as well as after the crisis. Figure 2 shows the dynamic responses of variables in country N in the percentage deviations from their steady-state levels under full capital mobility versus under IFA, respectively. Obviously, country S is not affected by the crisis under IFA.

Equity capital in our model can be interpreted as an asset held by entrepreneurs and thus, the asset price is by definition the inverse of the equity rate. As in Mendoza and Quadrini (2010), we find that the asset price responds less to the financial crisis under full capital mobility than under IFA. Our model setting allows the endogenous responses of aggregate output. The financial crisis only worsens the cross-sector investment composition under IFA, while the decline in net capital inflows further reduces the size of aggregate domestic investment under full capital mobility. Thus, financial integration magnifies the output responses. Intuitively, country N responds to financial crisis through the price channel (interest rates) and the quantity channel (investment and output). Financial

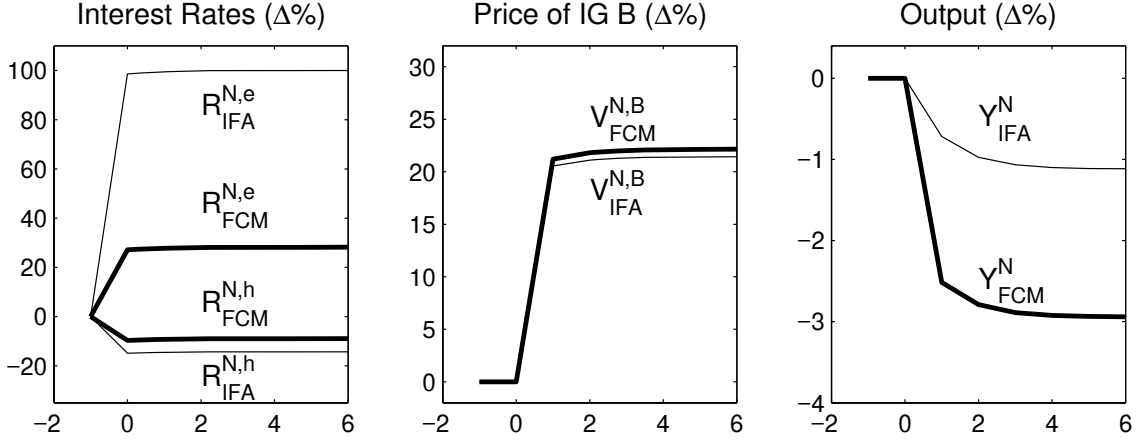


Figure 2: The Role of Financial Integration in the Setting with Limited Commitment

integration magnifies the quantity effect while dampens the price effect.<sup>6</sup>

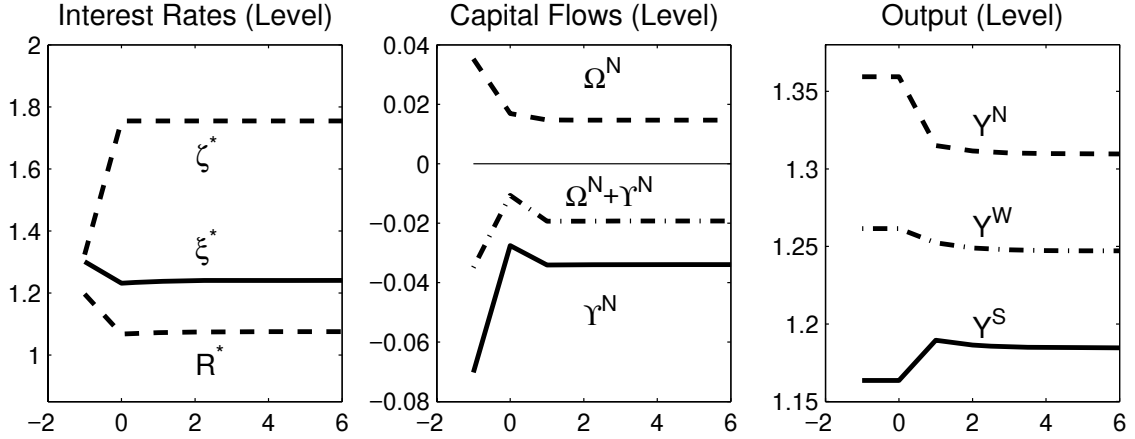


Figure 3: Financial Crisis in the Setting with Incomplete Markets

Now let us consider the setting with only incomplete markets. Suppose that the world economy is initially in the steady state under full capital mobility before  $\lambda^N$  falls permanently from 0.86 to 0.56 in period  $t = 0$ . As mentioned above, the value of  $\lambda^N$  is chosen consistently with  $\theta^N$  such that aggregate output in country N under IFA is same across the two model settings. Figure 3 show the dynamic responses of variables in levels.

The dynamic mechanism is essentially similar as in the setting with only limited commitment. A decline of  $\lambda^N$  implies that financial markets insure a smaller fraction of idiosyncratic risk in sector A. Since individual investors in country N have to bear a larger fraction of idiosyncratic risk, they reduce their risky investment in sector A and

<sup>6</sup>Although output in country N declines more dramatically to financial crisis in percentage points under financial integration than under IFA, aggregate output in country N is always higher under financial integration than under IFA, thanks to net capital inflows.

raise their risk-free investment in sector B. Thus, the mean rate of return in sector A,  $V_t^{N,A}$ , rises and the risk-free rate,  $R_t^N$  falls. The Sharpe Ratio,  $\zeta_t^N$ , rises to accommodate the increase in the riskiness of the investment in sector A. However, the risk-adjusted expected rate of portfolio return,  $\xi_t^N$ , still declines. The rise in the Sharpe ratio induces investors in country N to reduce their FDI outflows,  $\Omega_t^N$ , and the decline in the risk-free rate in country N induces investors in country S to reduce their risk-free lending abroad, implying a decline in the financial capital inflows,  $\Upsilon_t^N$ . Thus, both FDI outflows and financial capital inflows shrink. Since the risk-sharing capacity in country N falls, net capital inflows,  $\Omega_t^N + \Upsilon_t^N$ , also shrink. The decline in the risk-sharing capacity directly worsens the cross-sector investment composition and the decline in net capital inflows reduces the size of aggregate domestic investment in country N. Thus, aggregate output in country N,  $Y_t^N$ , declines.

Let us look at country S. First, by reducing their foreign risk-free lending, individuals invest more domestically so that the risk-free rate,  $R_t^S$ , falls, justifying the global equalization of the risk-free rate. Second, the decline in FDI inflows reduces the investment in sector A, implying a rise in the mean rate of return to the risky investment,  $V_t^{S,A}$ . Both factors lead to a rise in the Sharpe ratio,  $\zeta_t^S$ , justifying the global equalization of the Sharpe ratio. Aggregate output is affected by two opposite forces. First, the decline in net capital outflows raises the size of aggregate domestic investment, which tends to raises aggregate output. Second, the decline in FDI inflows and financial capital outflows worsens the cross-sector investment composition, which tends to reduces aggregate output. The overall impact depends on the relative magnitude of the two forces. Similar as the analysis in von Hagen and Zhang (2011), under certain parameter values, for  $\lambda^S$  close to zero, the composition effect dominates the size effect so that aggregate output falls; otherwise, aggregate output rises. Given the chosen parameter values, aggregate output rises in country S.

Figure 4 shows the dynamic responses of variables in country N in the percentage deviations from their steady-state levels under full capital mobility versus under IFA, respectively. As in the setting with only limited commitment, financial integration helps country N spread out the impacts of financial crisis on the risk-free rate as well as on the Sharpe ratio (risk-adjusted risk premium) across the world. However, due to the decline in the net capital inflows, aggregate output responds more under full capital mobility than under IFA. Again, financial integration magnifies the quantity effect while dampens the price effect.

To sum up, a financial crisis either in the form of a tightening of collateral constraints or in the form of a worsening of the risk-sharing capacity of the financial market system in



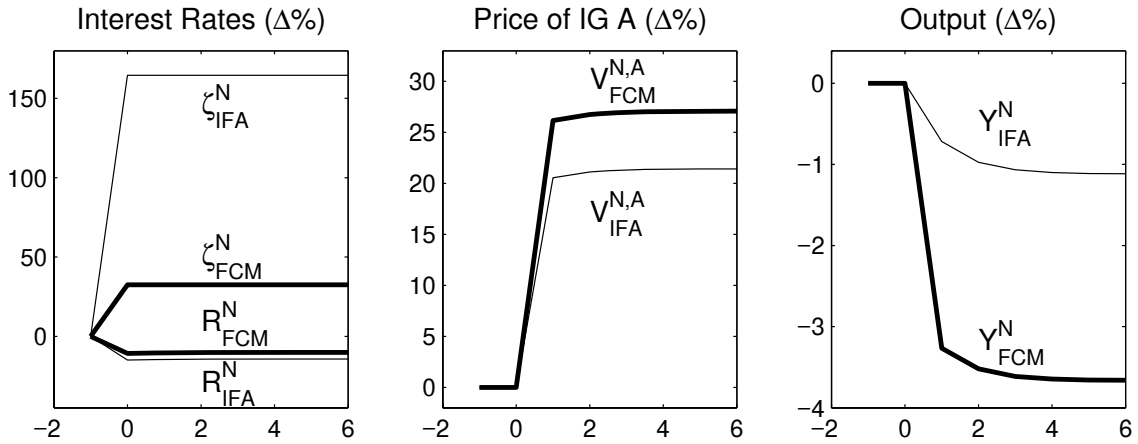


Figure 4: The Role of Financial Integration in the Setting with Incomplete Markets

the more advanced country produces a recession in the more financially developed country. World output falls and current account imbalances shrink as capital flows are reduced. These patterns are in line with the observed macroeconomic developments following the financial crisis that started in the US in 2007. Output may expand in the less financially developed country. This is interesting, because the expansion of the Chinese economy in particular following the crisis has generally been attributed to the country's fiscal expansion. Our model suggests that financial integration dampens the interest rate effects and amplifies the output effects in the country where the crisis originates. In addition, the responses of interest rates and aggregate output with respect to the financial crisis are qualitatively similar under the two alternative settings.

## 6 Conclusion

Although limited commitment and incomplete markets in our model economy capture different aspects of financial market imperfections and distort aggregate investment in different dimensions, they generate qualitatively identical distortions of interest rates and production efficiency under international financial autarky. Financial integration ameliorates these distortions in the less financially developed country. In particular, the steady-state patterns of international capital flows and the transitional dynamics following a financial crisis are also qualitatively identical across the two alternative model settings. Integrating both types of frictions into our model, we have shown that there is a one-to-one mapping from the severity of limited commitment to the degree of market incompleteness. Thus, the two approaches are observationally equivalent in their macroeconomic consequences.

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# A Proofs

## Proof of Proposition 2.1

*Proof.* According to equations (7b), the relative price is rewritten as  $\chi_{t+1}^i \equiv \frac{V_{t+1}^{i,A}}{V_{t+1}^{i,B}} = \frac{R_t^{i,h}}{V_{t+1}^{i,B}}$ . Combining equations (3), (7a), (7d), and (7e), we get the lending of individual household  $d_t^{i,h} = \frac{\theta^i}{\theta^i+1}\beta\omega_t^i$ . Combining it with equations (7a) and (7d), we get

$$\chi_{t+1}^i = \theta^i \left[ 1 + \frac{\eta}{1-\eta} \frac{d_t^{i,e}}{d_t^{i,e}} \right] = \frac{\eta + \theta^i}{1-\eta} = 1 - \frac{\bar{\theta} - \theta^i}{1-\eta} \quad (21)$$

Thus, the relative price is time invariant  $\chi_{t+1}^i = \chi_{IFA}^i$  and positively related with  $\theta^i$ , under IFA. According to equations (7b), the equity rate of entrepreneurs in sector B is rewritten as  $R_t^{i,e} = R_t^{i,h} \frac{1-\theta^i}{\chi_{t+1}^i - \theta^i}$ . Combining it with equation (6) and using equation (21) to substitute away  $\theta^i$  with  $\chi_{IFA}^i$ , we get the solution to the loan rate as specified in equation (8b). Plugging it back to equation (6), we get the solution to the equity rate as specified in equation (8f). The price of intermediate good B is obtained by definition. Given the Cobb-Douglas production function as specified in equation (2),  $(\omega_{t+1}^i)^{1-\alpha} (V_{t+1}^{i,A})^{\frac{\alpha}{2}} (V_{t+1}^{i,B})^{\frac{\alpha}{2}} = 1$  and the dynamic equation of wages is obtained as specified in equation (8e).  $\square$

## Proof of Proposition 2.2

*Proof.* A second-order Taylor approximation of  $\ln \zeta_t^i$  around  $\sigma = 0$  gives,

$$\ln \zeta_t^i = \frac{\ln E_t(\hat{\zeta}_{t+1}^i)^{1-\gamma}}{1-\gamma} \approx E_t \ln \hat{\zeta}_{t+1}^i + \frac{(1-\gamma)\text{Var}_t \ln \hat{\zeta}_{t+1}^i}{2} \quad (22)$$

$$\approx \phi_t^i \ln V_{t+1}^{i,A} + (1-\phi_t^i) \ln V_{t+1}^{i,B} - \frac{(\phi_t^i)^2}{2} \gamma [(1-\lambda^i)\sigma]^2 \quad (23)$$

The agent chooses  $\phi_t^i$  to maximize the ex ante risk-adjusted rate of portfolio return,  $\ln \zeta_t^i$ , and the first order condition gives the optimal portfolio choice,

$$\phi_t^i \approx \frac{\ln V_{t+1}^{i,A} - \ln V_{t+1}^{i,B}}{\gamma[(1-\lambda^i)\sigma]^2} = \frac{\ln \chi_{t+1}^i}{\gamma[(1-\lambda^i)\sigma]^2} \approx \frac{\chi_{t+1}^i - 1}{\gamma[(1-\lambda^i)\sigma]^2}. \quad (24)$$

Let  $\xi_t^i \equiv \frac{\chi_{t+1}^i - 1}{(1-\lambda^i)\sigma}$  denote the Sharpe Ratio. Plugging the solution of  $\phi_t^i$  into equation (22),

$$\ln \zeta_t^i \approx \ln V_{t+1}^{i,B} + \frac{(\xi_t^i)^2}{2\gamma}, \quad \Rightarrow \quad \zeta_t^i \approx V_{t+1}^{i,B} e^{\frac{(\xi_t^i)^2}{2\gamma}} \approx V_{t+1}^{i,B} \left[ 1 + \frac{(\xi_t^i)^2}{2\gamma} \right]. \quad (25)$$

Using equations (3), (9d), and (26), we get the relative price as a constant depending on the degree of market completeness,

$$\chi_{t+1}^i = \frac{V_{t+1}^{i,A}}{V_{t+1}^{i,B}} = \frac{M_{t+1}^{i,B}}{M_{t+1}^{i,A}} = \frac{1-\phi_t^i}{\phi_t^i} \quad \Rightarrow \quad \chi_{t+1}^i = \chi_{IFA}^i \approx \sqrt{1 + \gamma[(1-\lambda^i)\sigma]^2} > 1 \quad (26)$$

Use  $(\chi_{IFA}^i)^2 - 1 = \gamma[(1-\lambda^i)\sigma]^2$  to substitute away  $\gamma[(1-\lambda^i)\sigma]^2$  from equation (26), we get the portfolio choice  $\phi_t^i \approx \frac{\chi_{IFA}^i - 1}{(\chi_{IFA}^i)^2 - 1} = \frac{1}{\chi_{IFA}^i + 1}$ .

Plugging the solution to  $\phi_t^i$  and  $V_{t+1}^{i,A} = \chi_{IFA}^i R_t^i$  into equation (10), we solve the risk-free interest rate as specified in equation (11b). Other variables can be solved as in the proof of Proposition 2.1.  $\square$

### Proof of Lemma 3.1

*Proof.* The proof consists of three steps. First, we prove that equation (15a) is the solution to the equity rate under full capital mobility. Define  $\Delta\chi_{t+1}^i \equiv \chi_{t+1}^i - \chi_{IFA}^i$ . If the borrowing constraint is binding, it holds under IFA and under full capital mobility,

$$\chi_{t+1}^i = \frac{R_t^{i,h}(1-\theta^i)}{R_t^{i,e}} + \theta^i, \quad \Rightarrow \quad \frac{\Delta\chi_{t+1}^i}{1-\theta^i} = \frac{R_t^{i,h}}{R_t^{i,e}} - \frac{R_{IFA}^{i,h}}{R_{IFA}^{i,e}}. \quad (27)$$

According to equation (6),  $(1-\eta)R_{IFA}^{i,h} + \eta R_{IFA}^{i,e} = \Psi_{IFA}$ . Substituting  $R_t^{i,h}$  and  $R_{IFA}^{i,h}$  with  $R_t^{i,e}$  and  $R_{IFA}^{i,e}$  using equation (14) and  $R_{IFA}^{i,h} = \frac{1}{(1-\eta)}(\Psi_{IFA} - \eta R_{IFA}^{i,e})$ , we solve the equity rate from equation (27). Plug in the solution to the equity rate in equation (14) to solve the loan rate  $R_t^{i,h}$ .

Second, we prove that  $\chi_{t+1}^i$  is constant under full capital mobility. Let us assume that  $\chi_{t+1}^i$  is time variant and so is the auxiliary variable  $Z_{t+1}^i$  defined in equation (15a). According to equation (15a), the equity rate equalization in country  $i$  and  $N$  implies that

$$R_{IFA}^{S,e} - Z_{t+1}^S = R_{IFA}^{N,e} - Z_{t+1}^N, \quad (28)$$

$$\Delta\chi_{t+1}^S = \frac{1-\theta^S}{1-\theta^N} \Delta\chi_{t+1}^N + \left( \frac{1}{p_{IFA}^N} - \frac{1}{p_{IFA}^S} \right) \frac{1-\theta^S}{1-\eta}, \quad (29)$$

$$\frac{\partial \Delta\chi_{t+1}^i}{\partial \Delta\chi_{t+1}^N} = \frac{1-\theta^i}{1-\theta^N} > 0. \quad (30)$$

Using equations (15a), (15e), and (29), we rewrite the condition,  $\Omega_t^S + \Omega_t^N = 0$ , into

$$\omega_{t+1}^S \Delta\chi_{t+1}^S \frac{p_{IFA}^S(1-\eta)}{1-\theta^S} + \omega_{t+1}^N \Delta\chi_{t+1}^N \frac{p_{IFA}^N(1-\eta)}{1-\theta^N} = 0 \quad (31)$$

Given the Cobb-Douglas production function,  $\omega_{t+1}^i = (\chi_{t+1}^i)^{\frac{\rho}{2}} (R_t^i)^{-\rho}$ . Combining it with the loan rate equalization,  $R_t^{i,h} = R_t^{*,h}$ , we simplify equation (31) as

$$\mathcal{K}_{t+1}^S + \mathcal{K}_{t+1}^N = 0, \quad \text{where } \mathcal{K}_{t+1}^i \equiv (\Delta\chi_{t+1}^i + \chi_{IFA}^i)^{\frac{\rho}{2}} \Delta\chi_{t+1}^i \frac{p_{IFA}^i(1-\eta)}{1-\theta^i}, \quad (32)$$

$$\frac{\partial \mathcal{K}_{t+1}^i}{\partial \Delta\chi_{t+1}^i} = (\chi_{t+1}^i)^{\frac{\rho}{2}-1} (\chi_{t+1}^i + \frac{\rho}{2} \Delta\chi_{t+1}^i) \frac{p_{IFA}^i(1-\eta)}{1-\theta^i} > 0. \quad (33)$$

Using equations (29) to substitute  $\Delta\chi_{t+1}^i$  with  $\Delta\chi_{t+1}^N$ , the left-hand side of equation (32) becomes a monotonically increasing function of  $\Delta\chi_{t+1}^N$ ,

$$\frac{\partial (\mathcal{K}_{t+1}^S + \mathcal{K}_{t+1}^N)}{\partial \Delta\chi_{t+1}^N} = \frac{\partial \mathcal{K}_{t+1}^S}{\partial \Delta\chi_{t+1}^S} \frac{\partial \Delta\chi_{t+1}^S}{\partial \Delta\chi_{t+1}^N} + \frac{\partial \mathcal{K}_{t+1}^N}{\partial \Delta\chi_{t+1}^N} > 0. \quad (34)$$

Suppose that  $\Delta\chi_{t+1}^N \geq 0$ . Equation (29) implies that  $\Delta\chi_{t+1}^i > 0$ . According to the definition of  $\mathcal{K}_{t+1}^i$ ,  $\Delta\chi_{t+1}^i > 0$  implies that  $\mathcal{K}_{t+1}^i > 0$ . Thus, the left-hand side of equation (32) is larger than zero, which contradicts equation (32). Thus, there exists a unique solution of  $\Delta\chi_{t+1}^N$  smaller than zero and time-invariant. Using equations (29), we can then solve  $\Delta\chi_{t+1}^S$ , accordingly.

Finally, we prove the existence of a unique and stable steady state under full capital mobility.  $\chi_{t+1}^i$  is time-invariant and so is  $\mathcal{Z}_{t+1}^i$ . Let  $R_{FCM}^{i,h} \equiv R_{IFA}^{i,h} + \frac{\eta}{1-\eta}\mathcal{Z}_{FCM}^i$  which is same across countries,  $R_{FCM}^{i,h} = R_{FCM}^{*,h}$ . Thus, the loan rate depends on the dynamics of the world-average wages, according to equation (15b). So is the wage in country  $i$ ,

$$\omega_{t+1}^i = \left( \frac{\omega_{t+1}^w}{\omega_t^w} R_{FCM}^{*,h} \right)^{-\rho} (\chi_{FCM}^i)^{\frac{\rho}{2}}.$$

The dynamics of the world-average wages are

$$\begin{aligned} \omega_{t+1}^w &= \frac{\omega_{t+1}^S + \omega_{t+1}^N}{2} = \left( \frac{\omega_{t+1}^w}{\omega_t^w} R_{FCM}^{*,h} \right)^{-\rho} \frac{(\chi_{FCM}^S)^{\frac{\rho}{2}} + (\chi_{FCM}^N)^{\frac{\rho}{2}}}{2}, \\ \omega_{t+1}^w &= \left( \frac{\omega_t^w}{R_{FCM}^{*,h}} \right)^\alpha \left[ \frac{(\chi_{FCM}^S)^{\frac{\rho}{2}} + (\chi_{FCM}^N)^{\frac{\rho}{2}}}{2} \right]^{1-\alpha} \end{aligned}$$

Given  $\alpha \in (0, 1)$ , the phase diagram of the world-average wage is concave. Thus, there exists a unique and stable steady state. Proportional to the wage, aggregate output in country  $i$  is determined by the world output dynamics.  $\square$

### Proof of Lemma 3.2

*Proof.* The proof consists of three steps.

First, we prove that FDI equalizes the Sharpe ratio across the border. An agent born in country  $i$  can choose between investing its single project domestically or abroad. According to the solution to the optimal portfolio choices in Angeletos (2007) and Angeletos and Panousi (2011), three factors determine the agent's optimal portfolio share of risky investment and the risk-adjusted rate of portfolio return, i.e., the mean rate of return in the risky sector,  $\ln V_{t+1}^{i,A}$ , the risk-free interest rate,  $\ln R_t^i$ , and the risk-sharing factor  $\lambda^i$ . By assumption, agents obtain risk sharing in the country where they make the risky investment. Given the world risk-free interest rate  $R_t^*$ , the portfolio share of risky investment and the risk-adjusted rate of portfolio return are  $\phi_t^{i,l} \approx \frac{\zeta_t^{i,l}}{\gamma(1-\lambda^l)\sigma}$  and  $\xi_t^{i,l} \approx R_t^* \left[ 1 + \frac{(\zeta_t^{i,l})^2}{2\gamma} \right]$ , if the agent born in country  $i$  makes the risky investment abroad in country  $l \neq i$ , where the Sharpe ratio is  $\zeta_t^{i,l} \equiv \frac{\ln V_{t+1}^{i,A} - \ln R_t^*}{(1-\lambda^l)\sigma}$ . In equilibrium, an agent is indifferent between investing the risky project domestically or abroad. Given  $R_t^i = R_t^l = R_t^*$ , the no-arbitrage condition  $\xi_t^{i,l} = \xi_t^i$  is simplified as the equalization of the Sharpe ratio,  $\zeta_t^l = \zeta_t^i$ , and the portfolio share is simplified as  $\phi_t^{i,l} = \phi_t^l$ .

Suppose that FDI flows are from country N to country S, i.e.,  $\Omega_t^N > 0 > \Omega_t^S$  and  $\Omega_t^N + \Omega_t^S = 0$ . The total savings of agents born in country N but making the risky investment abroad is  $\frac{\Omega_t^N}{\phi_t^S}$ , while the total savings of agents born in country N and making the risky investment domestically

is  $\frac{M_{t+1}^{N,A}}{\phi_t^N}$ . Thus, the aggregate savings of agents born in country N and in country S are specified as in equations (17b) and (17b), respectively.

Second, we prove by contradiction that  $\chi_{t+1}^i$  is time-invariant under full capital mobility. Assume that  $\chi_{t+1}^i$  is time-variant. The equalization of the Sharpe Ratio  $\zeta_{t+1}^S = \zeta_{t+1}^N = \zeta_{t+1}^*$  implies that  $\chi_{t+1}^S$  is linear and increasing in  $\chi_{t+1}^N$ ,

$$\frac{\chi_{t+1}^S - 1}{1 - \lambda^S} = \frac{\chi_{t+1}^N - 1}{1 - \lambda^N}. \quad (35)$$

Net capital flows sum up to zero at the world level,

$$\beta(\omega_t^S + \omega_t^N) = \frac{\rho}{2R_t^*} \left[ \omega_{t+1}^S \frac{(1 + \chi_{t+1}^S)}{\chi_{t+1}^S} + \omega_{t+1}^N \frac{(1 + \chi_{t+1}^N)}{\chi_{t+1}^N} \right], \quad (36)$$

$$2(\omega_{t+1}^S + \omega_{t+1}^N) = \left[ 1 + \frac{(\xi_{t+1}^S)^2}{\gamma} \right] \omega_{t+1}^S \frac{(1 + \chi_{t+1}^S)}{\chi_{t+1}^S} + \left[ 1 + \frac{(\xi_{t+1}^N)^2}{\gamma} \right] \omega_{t+1}^N \frac{(1 + \chi_{t+1}^N)}{\chi_{t+1}^N}. \quad (37)$$

Since  $\omega_{t+1}^i = (\chi_{t+1}^i)^{-\frac{\rho}{2}} (R_t^i)^{-\rho}$  and  $R_t^S = R_t^N = R_t^*$ , equation (37) can be rewritten as

$$2[(\chi_{t+1}^S)^{-\frac{\rho}{2}} + (\chi_{t+1}^N)^{-\frac{\rho}{2}}] = \left[ 1 + \frac{(\xi_{t+1}^S)^2}{\gamma} \right] [(\chi_{t+1}^S)^{-\frac{\rho}{2}} + (\chi_{t+1}^S)^{-\frac{\rho}{2}-1}] + \quad (38)$$

$$\left[ 1 + \frac{(\xi_{t+1}^N)^2}{\gamma} \right] [(\chi_{t+1}^N)^{-\frac{\rho}{2}} + (\chi_{t+1}^N)^{-\frac{\rho}{2}-1}]. \quad (39)$$

Since  $\frac{(\xi_{t+1}^i)^2}{\gamma} = \frac{(\chi_{t+1}^i - 1)^2}{(\chi_{IFA}^i)^2 - 1}$  is a function of  $\chi_{t+1}^i$ , given  $\chi_{IFA}^i$  as a constant. Thus, according to equation (39),  $\chi_{t+1}^S$  is an implicit function of  $\chi_{t+1}^N$  and it can be proved that  $\frac{\partial \chi_{t+1}^S}{\partial \chi_{t+1}^N} < 0$ . Thus, according to equations (35) and (39), there exists a unique and time-invariant solution to  $\chi_{t+1}^N$  and  $\chi_{t+1}^S$ .

Finally, we prove the existence of a unique and stable steady state under full capital mobility. The relative price and the Sharpe ratio are time invariant which are denoted by  $\chi_{FCM}^i$  and  $\xi_{FCM}^i$ , respectively. Define  $R_{FCM}^i \equiv \frac{\rho}{\beta} \frac{1}{1 + \frac{(\xi_{FCM}^i)^2}{\gamma}}$ , which is same across countries,  $R_{FCM}^i = R_{FCM}^*$ . Thus, the loan rate depends on the dynamics of the world-average wages,  $R_{t+1}^i = \frac{\omega_{t+1}^w}{\omega_t^w} R_{FCM}^*$  and so does the wage in country  $i$ ,

$$\omega_{t+1}^i = \left( \frac{\omega_{t+1}^w}{\omega_t^w} R_{FCM}^* \right)^{-\rho} (\chi_{FCM}^i)^{-\frac{\rho}{2}}.$$

The dynamics of the world-average wages are

$$\omega_{t+1}^w = \frac{\omega_{t+1}^S + \omega_{t+1}^N}{2} = \left( \frac{\omega_{t+1}^w}{\omega_t^w} R_{FCM}^* \right)^{-\rho} \frac{(\chi_{FCM}^S)^{-\frac{\rho}{2}} + (\chi_{FCM}^N)^{-\frac{\rho}{2}}}{2},$$

$$\omega_{t+1}^w = \left( \frac{\omega_t^w}{R_{FCM}^*} \right)^\alpha \left[ \frac{(\chi_{FCM}^S)^{-\frac{\rho}{2}} + (\chi_{FCM}^N)^{-\frac{\rho}{2}}}{2} \right]^{1-\alpha}$$

Given  $\alpha \in (0, 1)$ , the phase diagram of the world-average wage is concave. Thus, there exists a unique and stable steady state. Proportional to the wage, aggregate output in country  $i$  is determined by the world output dynamics.  $\square$

### Proof of Proposition 4.1

*Proof.* The relative intermediate goods price  $\chi_{IFA}^i$  reflects the distortion of two financial frictions on aggregate allocation under IFA. Let  $\chi_{IFA,LC}^i$  and  $\chi_{IFA,IM}^i$  denote the respective relative intermediate goods price under the model setting of limited commitment and that of incomplete markets. According to equations (8e) and (11e), the wage rate has the same functional form with respect to  $\chi_{IFA}^i$  and so does aggregate output. Obviously, the steady-state aggregate output is same across the two model settings and so are the loan rate in the setting of limited commitment and the risk-free interest rate in the setting of incomplete markets, as long as  $\chi_{IFA,LC}^i = \frac{1}{\chi_{IFA,IM}^i}$ . That is,  $1 - \frac{\bar{\theta} - \theta^i}{1 - \eta} = \frac{1}{\sqrt{1 + \gamma[(1 - \lambda^i)\sigma]^2}}$  and the solution to  $\theta^i$  is a function of  $\lambda^i$  in the form,  $\theta^i = \bar{\theta} - (1 - \eta) \left\{ 1 - \frac{1}{\sqrt{1 + \gamma[(1 - \lambda^i)\sigma]^2}} \right\}$ . A necessary condition for  $\theta^i \in [0, \bar{\theta})$  is  $1 + \sqrt{1 + \gamma[(1 - \lambda^i)\sigma]^2} \leq \frac{1}{\eta}$ .  $\square$