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# ABSTRACT

## Sources of Risk in Currency Returns*

We quantify the sources of risk in currency returns as a first step toward understanding the returns reported for the carry trade. To do this, we develop and estimate an empirical model of exchange rate dynamics using daily data for four currencies relative to the US dollar: the Australian dollar, the British pound, the Swiss franc, and the Japanese yen. The model includes (i) Gaussian shocks with stochastic variance, (ii) jumps up and down in the exchange rate, and (iii) jumps in the variance. We identify these components using data on exchange rates and at-the-money implied variances. We find that the probability of a jump depreciation (appreciation) in the exchange rate is increasing in the domestic (foreign) interest rate. The probability of jumps in variance is increasing in the variance but not related to interest rates. Many of the jumps in exchange rates are associated with macroeconomic and political news, but jumps in variance are not. Overall, jumps account for 25% of total currency risk over horizons of one to three months.

JEL Classification: C58, F31 and G12
Keywords: Bayesian MCMC, carry trades, exchange rates, implied volatility and jumps

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# 1 Introduction

The time variation and high magnitude of currency (FX) carry trade returns have attracted a lot of recent attention. The properties of these returns reflect the risk premia – a covariation of the pricing kernel with risks that currencies are exposed to. Our objective in this paper is to quantify these risks as a first step towards understanding risk premia in currency markets.

We develop an empirical model of exchange rate dynamics that is rich enough to capture a number of important properties in the data. We estimate the model with Bayesian MCMC using a joint dataset of currency returns and short-term at-the-money-implied volatilities. We use daily data from 1986 to 2010 (the options data start in 1994) on four spot exchange rates: Australian dollar, Swiss franc, British pound, and Japanese yen. One of the key advantages of our estimation methodology is that it provides estimates of the conditional distribution of currency returns, as well as estimates of realised shocks. This feature allows us to link "big" shocks, or jumps, to important macro-finance events and thereby illuminate the potential economic channels that are responsible for crash risk in currencies.

Our model has three key elements. First, it is well-documented that currency returns are heteroscedastic (e.g., Baillie and Bollerslev, 1989; Engel and Hamilton, 1990; Engle, Ito, and Lin, 1990; Jorion, 1988). Casual observation of time-series variation in option-implied exchange rate volatility also confirms this point. We capture this feature of the data with a standard stochastic volatility component in our model.

Second, there is also strong empirical evidence that daily changes in exchange rates are not conditionally Gaussian (as would approximately be the case in a model with only stochastic volatility). To account for this feature of the data, our model includes jump risks in exchange rates. We allow the probability of these jumps to be time-varying, in order to capture the variation in conditional skewness that has been previously documented (e.g., Johnson, 2002; Carr and Wu, 2007; Bakshi, Carr, and Wu, 2008; Brunnermeier, Nagel, and Pedersen, 2008).

Third, changes in the at-the-money implied volatility of a typical exchange rate exhibit unconditional skewness of 1 and kurtosis of 10 or more. To accommodate this property, our model allows for jumps in the variance of Gaussian shocks to exchange rates. The importance of such jumps for modelling equity returns has been emphasized in Broadie, Chernov, and Johannes (2007); Duffie, Pan, and Singleton (2000); Eraker, Johannes, and Polson (2003), among others. To our knowledge, our paper is the first to investigate the role of jumps in the volatility of exchange rates.

A jump in an exchange rate is qualitatively different from a jump in its variance. Almost by definition, large jumps are relatively rare events. Therefore, when there is a direct jump in the exchange rate, one doesn't necessarily expect there to be many subsequent jumps in the near future. By contrast, when there is a jump in the variance of the Gaussian shock to an exchange rate, one expects there to be many large subsequent moves in the exchange rate. We use our model and empirical analysis to determine whether these qualitative distinctions lead to materially quantitative differences.

Statistical tests strongly favour both jumps in exchange rates and in their variances. The two types of jumps arise via different mechanisms. The probability of a jump in the variance of currency returns is positively related to the variance itself. Thus, this component belongs to the class of self-exciting processes. The probability of a jump up in the exchange rate, which corresponds to a depreciation of the US dollar, is positively related to the domestic (US) interest rate. The probability of a jump down, which corresponds to an appreciation of the US dollar, is positively related to the foreign interest rate.

Although jumps in currencies and in variance are alternative channels for large currency returns, we find that economically they are quite distinct. We can connect most of the jumps in FX to important macro or political announcements. In contrast, jumps in variance cluster at the moments of high uncertainty in the markets, which are captured by comments on current events, political speculation and overall anxiety about upcoming events.

We use entropy (a generalized measure of variance) of exchange rate returns to measure the amount of risk associated with currency positions and to decompose this risk into the contributions from different sources of shocks (Alvarez and Jermann, 2005; Backus, Chernov, and Martin, 2011). Appropriately scaled entropy is equal to the variance of an exchange rate return if it is normally distributed, but otherwise includes high-order cumulants. Therefore, entropy is a convenient measure that captures both normal and tail risk in one number. We find that, depending on the currency, the time-series average of the joint contribution of the three types of jumps can be as high as 25% of the total risk and on individual days this contribution can be up to 40%. Jumps in variance contribute about a third to the average contribution and can be as high as 15% of the total risk on individual days. Also, the contribution of jumps in variance to the total risk increases with investment horizon.

Given the large contribution of jumps to the overall risk, it is natural to ask whether the jump risk is priced. The full answer to this question requires an explicit model of the pricing kernel and the use of assets, such as out-of-the-money options, that are particularly sensitive to jumps for estimation. While such analysis is outside of the scope of this paper, we carry out a limited option valuation exercise. We select representative implied volatility smiles for currencies with positive and negative interest rate differential. Such smiles exhibit positive and negative skewness, respectively, in the data. Our model can replicate the same sign of skewness even when we assume zero premiums for jump risk. However, these theoretical smiles cannot match the curvature of the smile observed in the data, even after accounting for statistical uncertainty. In our view, this initial evidence suggests that jumps risk may be priced.

**Related Literature**

We limit our discussion of related literature to papers that highlight the importance of jumps for understanding the properties of exchange rate returns. One exception are the works of Brandt and Santa-Clara (2002) and Graveline (2006). These papers are early antecedents of our paper in terms of methods and research questions. These authors also estimate a time-series model of exchange rates using the time-series of FX and implied variance. However, they do not allow for jumps.

Our paper is most closely related to recent empirical papers that investigate whether the high returns to carry trades can be explained as compensation for jump, or crash, risk. Jurek (2009) analyzes the returns on carry trade portfolios in which the exposure to currency crashes is hedged with options. He concludes that exposure to currency crashes account for 15% to 35% of the excess returns on unhedged carry trade portfolios. Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011) investigate whether carry trade returns reflect a "peso problem" (i.e., a low probability event that did not occur in the sample). They use carry returns hedged with options to argue that any such peso event must be a modest negative return on the carry trade combined with an extremely large value of the stochastic discount factor (i.e., the marginal utility of a representative investor must be very high in the, as yet, unobserved peso state). Jordà and Taylor (2009) propose to manage the risk of carry positions by conditioning on macro information instead of options, but the resulting strategy still yields a very high Sharpe ratio. The common thread in these papers is that they provide indirect evidence on the magnitude of jumps risk. Our paper aims to complement this previous work with a formal statistical model and analysis.

Farhi, Fraiberger, Gabaix, Ranciere, and Verdelhan (2009) use an explicit model of exchange rates that allows for both normal and jump risks. Under the model assumptions, short-dated at-the-money options are not exposed to crash risk. Therefore, hedged carry trades are exposed to normal risk alone. In contrast, carry trades are exposed to both types of risk. This property allows the authors to quantify the contribution of jump risk by observing returns on hedged and unhedged portfolios. However, similar to the aforementioned papers, the authors do not test the assumed model directly.

Our paper is also related to the option pricing literature, which has focused on modeling the risk-adjusted (risk-neutral) distribution of exchange rates. By construction, these papers do not consider risk premia. However, the shock structures under the risk-adjusted and actual (true) distributions are usually modelled to be similar. Bates (1996) considers option prices on the Deutsche Mark and is the earliest paper that argues for the inclusion of jumps. He considers a single normally distributed jump in FX with a constant probability. Carr and Wu (2007) distinguish jumps up and down in FX and also allow for time-varying jump probabilities controlled by unobservable states. Bakshi, Carr, and Wu (2008) extend the Carr-Wu model to a triangle of currencies (GBP, JPY, and USD) and estimate it using 2.25 years of data on exchange rates and option prices. Our analysis provides additional economic intuition, as time variation in jump probabilities are driven by observable interest rates. None of these papers consider jumps in variance.

There is also an important literature that attempts to explain the behaviour of exchange rates in macro-founded equilibrium.[1] Our paper is silent about the prices of risk, but it may have implications for how to best model the fundamentals in an equilibrium setting. Gourio, Siemer, and Verdelhan (2010) and Guo (2007) propose production-based models with recursive preferences. Productivity is allowed to experience a disastrous decline with time-varying probability. Farhi and Gabaix (2008) consider a pure exchange economy with additive preferences and a similar assumption of time-varying probability of a disaster in consumption. Disasters are modelled as jumps down, and all three papers allow unobservable processes to drive disaster probabilities. Exchange rates inherit these properties. Our results suggest that it may also be important to allow for jumps in the volatility of these processes and for the process driving probability of jumps in consumption to be related to interest rates in equilibrium.

Our results speak also to the frictions-based equilibrium model of Plantin and Shin (2011). These authors focus on endogenously generated dynamics of a carry trade. A carry trade gets started in a high-liquidity environment, such as accommodative monetary policy. It is self-enforcing because of the speculators' belief that others will join the trade. The trade crashes when the speculators hit funding constraints. As a result, extended periods of slow appreciations of a high interest rate currency are randomly interrupted by endogenous crashes. Because our analysis is implemented at the daily frequency, we are able to capture, perhaps in reduced form, related phenomena.

# 2 Preliminaries

This section motivates our analysis and highlights properties of the data that our model is designed to capture.

## 2.1 Excess Returns

Let $r_t$ be the continuously-compounded domestic (e.g. USD) interest rate, $\tilde{r}_t$ be the analogous foreign (e.g. GBP) interest rate, and $S_t$ be the exchange rate expressed as units of domestic currency per unit of foreign currency. Then borrowed $\exp(-r_t)$ units of the domestic currency buys $1/S_t \cdot \exp(-r_t)$ units of the foreign currency at time $t$, which grows at the foreign risk free interest rate to $1/S_t \cdot \exp(\tilde{r}_t - r_t)$ units at time $t+1$, and can be

[1] Examples include, but not limited to Bekaert (1996); Backus, Gavazzoni, Telmer, and Zin (2010); Bansal and Shaliastovich (2010); Colacito (2009); Colacito and Croce (2010).

exchanged for $S_{t+1}/S_t \cdot \exp(\tilde{r}_t - r_t)$. Then the amount borrowed in domestic currency (with interest) can be repaid. Thus, the log excess return to investing in the foreign currency is

$$y_{t+1} = (s_{t+1} - s_t) - (r_t - \tilde{r}_t),$$

where $s_t = \ln S_t$. In this paper, we will always treat USD as the domestic currency.

Figures 1 - 4 display the time series of log excess returns, $y_{t+1}$ (panel (a)), and implied volatilities (panel (b)) for the currencies we consider in this paper. We have selected four currencies - Australian Dollar (AUD), Swiss Franc (CHF), British Pound (GBP), and Japanese Yen (JPY) based on the availability of daily data, and cross-sectional and time-series variation in the interest rate differential. We use one-month LIBOR to proxy for interest rates. Using one-month rather than overnight rates implicitly assumes a flat term structure at the very short end of the LIBOR curve and allows us to abstract from potential high-frequency idiosyncratic effects associated with fixed-income markets. Because we treat USD as a domestic currency, the movements up correspond to depreciation in the USD.

## 2.2 Properties of Excess Returns

We provide summary statistics of daily log excess returns and changes in the one-month at-the-money implied volatility in Table 1. Means are close to zero at daily frequency. Therefore, these summary statistics inform us primarily about the properties of shocks.

All currencies have volatility of about 10% per year. There is evidence of substantial kurtosis (AUD and JPY are the most notable in this regard), which is suggestive of non-normalities. Skewness of all currencies is mild. It turns out that this is a manifestation of time-varying and sign-switching conditional skewness. We produce a rough estimate of conditional skewness by computing a six-month rolling window. The time-series of these estimates are displayed in panels (a) of Figures 1 - 4. Depending on the currency, conditional skewness ranges from -2 to 2. Thus, excess returns are not only fat-tailed, but also asymmetric with the degree of asymmetry changing over time.

The implied volatility is itself quite variable at about 60% per year (the number in the table multiplied by $\sqrt{252}$) and highly non-normal with skewness and kurtosis much higher than that of the currency returns themselves. The implied volatility from the short-dated options should be very close to the true volatility of exchange rates (which is unobservable) and therefore its properties provide insight into the features that a realistic model of variance must require.

As a reference, we report the same summary statistics for S&P 500 whose risks were thoroughly studied in the literature. The index returns are more volatile and exhibit much stronger departures from normality as compared to currencies. In particular, negative unconditional skewness is evident (in fact, a measure of conditional skewness becomes positive rarely). In contrast, changes in VIX, a cousin of implied variance display weaker non-normalities than currencies. These statistics suggest that a model of currency risks could be substantively different from that of equity risks even though one clearly has to use similar building blocks.

### 2.3 Risks and Expected Excess Returns

We can generically represent excess returns as:

$$y_{t+1} = E_t(y_{t+1}) + \text{shocks}. \tag{2.1}$$

Most of the research is focused on conditional expected excess returns $E_t(y_{t+1})$. For example, if currencies do not carry a risk premium, then uncovered interest rate parity (UIP) holds and $E_t(y_{t+1}) = 0$. However, Bilson (1981), Fama (1984), and Tryon (1979) establish that the regression

$$s_{t+1} - s_t = a_1 + a_2(f_t - s_t) + \text{shocks}, \tag{2.2}$$

where $f_t$ is is the log of the one-month forward exchange rate, typically yields estimates of $a_2$ of approximately $-2$. If covered interest rate parity (i.e., no-arbitrage) holds, then the log forward exchange rate is given by $f_t = s_t + r_t - \tilde{r}_t$, therefore this result is equivalent to:

$$y_{t+1} = a_1 + (a_2 - 1)(r_t - \tilde{r}_t) + \text{shocks}, \tag{2.3}$$

with a slope coefficient of about $-3$. Subsequent research has extended the specification of risk premiums $E_t(y_{t+1})$ (e.g., Beber, Breedon, and Buraschi, 2010; Bekaert and Hodrick, 1992; Lustig, Roussanov, and Verdelhan, 2011; Menkhoff, Sarno, Schmeling, and Schrimpf, 2011, among others).

Our objective in this paper is to carefully model all the shocks that drive currency returns and their relative contributions to the overall risk. To measure shocks, we need to model conditional means as well. We use a simple specification that encompasses the UIP regressions result by allowing for linear dependence on the domestic and foreign interest rates, and includes variance of FX returns as an extra variable.[2] Because we are working with daily returns, the magnitude of the drift term is much smaller than the higher order moments and so any omitted variables that might affect expected returns are not likely to introduce much bias in our results.[3] As such, to avoid overfitting, we did not include any other variables in the drift of the exchange rate. Moreover, as noted in Cheung, Chinn, and

[2]This addition can be supported in various theoretical settings (Bacchetta and van Wincoop, 2006; Brennan and Xia, 2006). Empirical work with such a term includes Bekaert and Hodrick (1993), Bekaert (1995), Brandt and Santa-Clara (2002), Domowitz and Hakkio (1985), Lustig, Roussanov, and Verdelhan (2011), and Menkhoff, Sarno, Schmeling, and Schrimpf (2011).

[3]A recent literature suggests improving inference about conditional mean of excess returns by considering portfolios of currencies (e.g., Barroso and Santa-Clara, 2011; Lustig, Roussanov, and Verdelhan, 2011; Lustig and Verdelhan, 2007; Menkhoff, Sarno, Schmeling, and Schrimpf, 2011).

Pascual (2005), it is notoriously difficult for any model to beat a random walk for exchange returns in terms of forecasting.

While our focus is on careful modelling of currency risks, our conclusions should have implications for expected excess returns, that is premiums for bearing these risks. The expected excess returns depend on how the risks that affect currencies covary with investors' marginal rate of substitution, a.k.a. the pricing kernel. To this end, our model can be used to construct portfolios that isolate jump risks and serve as inputs to traditional factor models that examine the pricing of these risks. Moreover, our extensive analysis of the shocks to currency returns provides useful guidance for specifying shocks to fundamentals in equilibrium models.

# 3 Empirical Model

We start by presenting our empirical model in Section 3.1. Section 3.2 discusses how we arrived at the assumed functional forms.

## 3.1 Currency Dynamics

In this paper we model each exchange rate in isolation from others. A large fraction of currency analysis, such as UIP regressions or equilibrium modelling is conducted on a currency-by-currency basis. This approach is able to identify the normal and non-normal shocks, and how they should be modelled. However, we cannot say which fraction of shocks can be explained by common variation in the exchange rates, and which fraction is country-pair specific. The important question of modelling the joint distribution of currency risks goes hand-in-hand with modelling of the pricing kernel and we leave this investigation for future research.[4]

We model log excess FX returns as

$$y_{t+1} \equiv (s_{t+1} - s_t) - (r_t - \tilde{r}_t) = \mu_t + v_t^{1/2} w_{t+1}^s + z_{t+1}^u - z_{t+1}^d \,, \tag{3.1}$$

where $w_{t+1}^s$ is a standard Gaussian shock (i.e. zero mean and unit variance), $z_{t+1}^u$ is a jump up (i.e. depreciation of USD) and the negative of $z_{t+1}^d$ is a jump down (i.e. appreciation of USD). The conditional spot variance is $v_t$ and the jump intensities of $z_{t+1}^u$ and $z_{t+1}^d$ are $h_t^u$ and $h_t^d$ respectively. The discussion of $\mu_t$ is postponed until we have further described these three shocks.

[4]Lustig, Roussanov, and Verdelhan (2011); Sarno, Schneider, and Wagner (2011) perform such modelling allowing normal shocks only. Bakshi, Carr, and Wu (2008) model a triangle of currencies (GBP, JPY, and USD) allowing for jumps in FX.

The conditional spot variance $v_t$ is assumed to follow a mean-reverting "square-root" process,

$$v_{t+1} = (1-\nu)v + \nu v_t + \sigma_v v_t^{1/2} w_{t+1}^v + z_{t+1}^v \,, \tag{3.2}$$

which itself can jump with intensity $h_t^v$.[5] The shocks to excess returns $w^s$ and to conditional spot variance $w^v$ have a correlation coefficient corr $(w^s, w^v) = \rho$. Finally, to ensure positivity of the variance when jumps are present, we only allow for upward jumps so that $z_{t+1}^v$ has non-negative support.

The jump arrival rate is controlled by a Poisson distribution. The assumed jump intensities imply that the number of jumps takes non-negative integer values $j$ with probabilities

$$\mathrm{Prob}(j_{t+1}^k = j) = e^{-h_t^k}(h_t^k)^j/j! \,, \quad k = u, d, v \,. \tag{3.3}$$

We allow all of the jump intensities to depend on the domestic and foreign interest rates, as well as on the conditional spot variance,

$$h_t^k = h_0^k + h_r^k r_t + \tilde{h}_r^k \tilde{r}_t + h_v^k v_t \,, \quad k = u, d, v \,. \tag{3.4}$$

For a given number of jumps per period, the magnitude of a jump size is assumed to be random with a Gamma distribution,

$$z_t^k | j \sim \mathcal{G}\mathrm{amma}(j, \theta_k) \,, \quad k = u, d, v \,. \tag{3.5}$$

Intuitively, because we consider daily data, a Bernoulli distribution is a very good approximation to our model as it is reasonable to assume no more than one jump per day. Then, the probability of a jump is $1 - e^{-h_t^k} \approx h_t^k$ and the distribution of the jump size is exponential with mean parameter $\theta_k$.[6]

We complement our data on exchange rate rates with variances implied from option prices. In this respect we follow the rich options literature that highlights the importance of using information in options for model estimation (e.g., see Aït-Sahalia and Kimmel, 2007; Brandt and Santa-Clara, 2002; Chernov and Ghysels, 2000; Jones, 2003; Pan, 2002; Pastorello, Renault, and Touzi, 2000). Many authors use implied variance in empirical work by interpreting it as a very accurate approximation of the risk-adjusted expectation of the average future variance realized over an option's lifetime. This is certainly true for models with stochastic volatility only. If this is the case, one can derive $\alpha_{iv}$ and $\beta_{iv}$ as explicit functions of risk-adjusted parameters (e.g., Chernov, 2007, and Jones, 2003). The one-for-one

[5] In continuous time, the Feller condition $\sigma_v^2 \leq 2v$ ensures that the variance stays positive if there are no jumps. A formal modelling of this process in discrete time is achieved via a Poisson mixture of Gamma distributions (e.g., Gourieroux and Jasiak, 2006; Le, Singleton, and Dai, 2010). We use a direct discretization of the continuous-time counterpart so that the model parameters can be easily interpreted. We ensure that the variance stays positive at the estimation stage by a careful design of the simulation strategy.

[6] Our choice of the variance jump size distribution is frequently used when modelling variance to ensure its positivity as discussed above. The model of variance is also capable of generating quite rapid variance declines after jumps. A jump leads to a large deviation from the long-run mean $v$, and mean-reversion controlled by parameter $\nu$ ensures that the variance is pulled back.

relationship between implied variance and risk-adjusted expected variance may break down in the presence of jumps. For example, Chernov (2007) has to assume that the risk-adjusted mean of jumps in FX is equal to zero to retain the simple relationship. Importance of careful accounting for jumps is manifested more clearly in the literature on model-free implied variance, such as VIX for S&P 500, where analytic expressions are feasible. Martin (2011) shows that, in the presence of jumps, VIX is equal to risk-adjusted expected variance plus additional terms reflecting the higher order risk-adjusted cumulants of returns.

We treat the Black-Scholes implied variance of a short-term (one-month) at-the-money option, $IV_t$, as a noisy and biased observation of the conditional spot variance $v$. Such a view allows us to avoid the aforementioned difficulties in explicit connection between implied variance and risk-adjusted expected future variance. The cost of such approach is our inability to estimate risk-adjusted parameters of the model. Specifically,

$$IV_t = \alpha_{iv} + \beta_{iv} v_t + \sigma_{iv} v_t \sqrt{\lambda_t}\, \varepsilon_t \,, \tag{3.6}$$

where $IV_t$ is expressed in daily terms, $\varepsilon_t$ is $\mathcal{N}(0,1)$ and $\lambda_t$ is $\mathcal{IG}(\nu/2, \nu/2)$, so the product $\sqrt{\lambda_t}\varepsilon_t$ is $t_\nu$−distributed (Cheung, 2008; Jacquier, Polson, and Rossi, 2004).[7] We have considered a version of (3.6) with non-zero loadings on $r_t$ and $\tilde{r}_t$, but this specification did not find empirical support.[8]

The model implies that expected log excess return is equal to

$$E_t\left[y_{t+1}\right] = \mu_t + \underbrace{h_t^u \theta_u}_{E_t\left[z_{t+1}^u\right]} - \underbrace{h_t^d \theta_d}_{E_t\left[z_{t+1}^d\right]} \,. \tag{3.7}$$

As discussed in the previous section, we assume that

$$\mu_t = \mu_0 + \mu_r r_t + \tilde{\mu}_r \tilde{r}_t + \mu_v v_t. \tag{3.8}$$

The resulting expected excess return is

$$E_t\left[y_{t+1}\right] = \mu_0^* + \mu_r^* r_t + \tilde{\mu}_r^* \tilde{r}_t + \mu_v^* v_t \tag{3.9}$$

where

$$\mu_0^* = \mu_0 + h_0^u \theta_u - h_0^d \theta_d \,, \tag{3.10a}$$
$$\mu_r^* = \mu_r + h_r^u \theta_u - h_r^d \theta_d \,, \tag{3.10b}$$
$$\tilde{\mu}_r^* = \tilde{\mu}_r + \tilde{h}_r^u \theta_u - \tilde{h}_r^d \theta_d \,, \tag{3.10c}$$
$$\mu_v^* = \mu_v + h_v^u \theta_u - h_v^d \theta_d \,. \tag{3.10d}$$

[7] Jones (2003) makes a strong case for heteroscedastic measurement errors in implied variance. His specification sets $\lambda_t = 1$. Cheung (2008) generalizes the specification to the Student $t$−error. We tried using a normal error with volatility $\sigma_{iv}$, a normal error with volatility $\sigma_{iv} v_t$, and the Student $t$−error described above. We find that heavy-tailed $t_3$ works very well.

[8] The error specification in (3.6) is very flexible. Therefore, it could be the case that the contribution of interest rates to the variation in implied variance cannot be empirically distinguished from the error, if the former is reasonably small.

Thus, our risk premium encompasses the UIP regressions which set

$$\tilde{\mu}_r^* = -\mu_r^*\,, \tag{3.11}$$

$$\mu_v^* = 0\,. \tag{3.12}$$

We conclude with a discussion of our approach to modelling interest rates. We do not need an explicit model of interest rates to estimate our model of FX excess returns if we are willing to assume that one-day $r_t$ and $\tilde{r}_t$ can be reasonably proxied with short-term yields. We view this feature as a strength of our approach because explicitly modelling the behaviour of spot interest rates entails a massive effort. There is a separate literature dedicated to this task and the state-of-the-art models rely on five factors for capturing the interest rate dynamics. These studies are typically conducted with monthly or quarterly data, so they do not take into account the higher-frequency movements in interest rates which are susceptible to jumps themselves (e.g., Johannes, 2004; Piazzesi, 2005). Moreover, interest rates and currencies have low conditional correlation and variability in interest rates is much smaller than that in currencies. In summary, elaborate modelling and estimation of interest rates does not appear to be worthwhile in our case.

Nonetheless, we use the estimated model to compute some useful objects (expectations of future variance, or expected excess returns over multiple horizon) that depend on the distribution of interest rates. In order to obtain reasonable quantities, we assume the simplest possible model for the interest rates:

$$r_{t+1} = (1-b_r)a_r + b_r r_t + \sigma_r r_t^{1/2} w_{t+1}^r, \tag{3.13a}$$

$$\tilde{r}_{t+1} = (1-\tilde{b}_r)\tilde{a}_r + \tilde{b}_r r_t + \tilde{\sigma}_r \tilde{r}_t^{1/2} \tilde{w}_{t+1}^r. \tag{3.13b}$$

As in the case with the variance process, a square root process for interest rates is subject to caveats in discrete time. We calibrate the models to match the mean, variance and serial correlation of the respective observed short-term interest rates. Our computations with reasonable variation in parameters confirm our intuition that they have minimal impact on the role of normal and non-normal currency risks.

## 3.2 Qualitative Features of the Model

In this section we explain how we arrived at the specified functional form of the model. We evaluated too many models to provide a detailed account of our analysis, so we briefly summarize the results that led us to the above specification. Our initial specifications were motivated by the well-developed literature on equity returns (Andersen, Benzoni, and Lund, 2002; Chernov, Gallant, Ghysels, and Tauchen, 2003; Eraker, Johannes, and Polson, 2003; Eraker, 2004; Jones, 2003) and some of the few models of currencies (Bates, 1996; Johnson, 2002; Jorion, 1988; Maheu and McCurdy, 2008). The salient features of equity data are presence of substantial moves up and down and a pronounced negative skewness in the return distribution. Therefore, jumps in equity returns are often modelled via a single compound Poisson process with a normally distributed size of non-zero mean. However, in contrast to equity returns, currency returns have very mild skewness over long samples, which suggests a zero-mean normal distribution for jump sizes.

Further, Bates (1996), Campa, Chang, and Reider (1998), Carr and Wu (2007), and Johnson (2002) emphasize the time-varying and sign-switching nature of the risk-adjusted skewness of exchange rates. The key to modelling this feature successfully is to allow the conditional expected jump to vary over time. A single jump process with a zero mean jump size implies a zero conditional expected jump. Two jump processes have a potential to generate the requisite variation either via time-varying jump intensities, or time-varying jump size distributions, or both. We do not explore time-varying jump means as such specifications do not allow for tractable option valuation in the affine framework, and we eventually want our model to be used for option analysis. As can be seen from the expression for the currency risk premium (3.9), the conditional jump expectation is $h_t^u\theta_u - h_t^d\theta_d$, and is capable of producing the needed variation. We have also considered normally distributed jump sizes in excess returns with means of the jump size distribution having opposing signs. However, because normal distributions have infinite support, it was hard to distinguish empirically the down and up components. The exponential distribution does not have this issue because the support is on the positive line.

Another interesting feature of our specification is that we allow not only for two different Poisson processes in currency returns, but also for a third one in the variance. Our starting point was again in the equity literature where all jumps in returns and variance are guided by the same (or at least correlated) Poisson processes. First, we found that the model with correlated Poisson processes poorly fitted the data. Second, it was very hard to establish what such a model was lacking.

# 4 Empirical Approach

We employ the Bayesian MCMC approach to estimate the model. This method was successfully implemented in many applications (see Johannes and Polson, 2009 for a review). For our purposes, the key advantage of this approach over other methodologies is that estimation of unobserved variance and jump times and sizes is a natural by-product of the procedure. Online appendix describes all the details of the implementation.

It is worth pointing out how we distinguish jumps and normal shocks in the model. Formally, all shocks are discontinuous in our discrete-time formulation. We think of jumps as relatively infrequent events with relatively large variance. We use priors on jump arrival and jump size parameters to express this view.

It proved to be extremely fruitful to use option implied variances in our estimation. Ignoring information in option prices made it very hard to settle on a particular model. Parameters were estimated imprecisely and the algorithm had poor convergence properties – both are manifestations of the data being not sufficiently informative about the model. We had a similar experience when estimating the most general model, even when using the options data. Complicated dependencies of jump intensities on state variables, and the sheer number of separate Poisson processes was too much for the available data.

As such, we pursue the following model selection strategy. First, we treat implied variances as observed spot variances and estimated the model of variance (3.2). At this stage we select the best model by checking the significance of parameters on the basis of both confidence intervals and Bayes odds ratios. Specifically, the parameters of concern are the ones controlling the jump intensity in (3.4) for $k = v$. It turns out that, regardless of the currency, only the loading on variance is significant. In other words, the probability of jumps in the variance is affected by the variance itself. Thus, jumps in the variance are self-exciting (Hawkes, 1971).[9] Pinning down the model of variance is an extremely useful step in our estimation procedure.

Second, we use the lessons from the estimation exercise on the basis of implied variance alone to guide us in a formal search in the context of our full model. That is, we take the model (3.1), (3.2) and combine it with equation (3.6) that recognizes implied variances as noisy observations of the spot variance. As a benchmark, we estimate the stochastic variance model with no jumps. Next, we estimate a model with jumps in variance but no jumps in exchange rates ($h_t^u = h_t^d = 0$). We refer to this model as stochastic variance with jumps.

Finally, we allow for the full model with jumps in both exchange rates and variance. Here, we focus on the significance of the parameters controlling the jump intensities in (3.4) for $k = u$ and $d$. We are not reporting all the details here, but we find that $\tilde{h}_r^u$, $h_v^u$, $h_r^d$ and $h_v^d$ are insignificant. Thus, the probability of jumps up in the exchange rate is driven by the domestic rates only, and the probability of jumps down in the exchange rate is driven by the foreign rate only. We also test if some interesting parameters, or combinations of parameters, are equal to zero. First, we can test the UIP regression restrictions on the risk premia in Eq. (3.11) (whether interest rates affect the risk premium as a differential) and Eq. (3.12) (whether the variance affects the risk premium). As we noted earlier, the behaviour of the FX skewness is dramatically affected by expected effect of jumps, which is equal to $h_t^u\theta_u - h_t^d\theta_d$. Here we are interested in testing whether $\theta_u = \theta_d = \theta$, $h_0^u = h_0^d = h_0$, and $h_r^u = \tilde{h}_r^d = h_r$. These hypotheses are interesting because if they cannot be jointly rejected then expected jump would be equal to $\theta h_r(r_t - \tilde{r}_t)$. Thus, the excess return asymmetries will be directly driven by the interest rate differential as noted in Brunnermeier, Nagel, and Pedersen (2008). The final version of this model that incorporates all the unrejected null hypotheses is referred to as the preferred.

[9]The recent literature on equity returns also finds support for self-exciting jumps. See, for example, Aït-Sahalia, Cacho-Diaz, and Laeven (2011); Carr and Wu (2011); Nowotny (2011); Santa-Clara and Yan (2010).

We implement a series of informal diagnostics and specification tests to establish the preferred model. The diagnostics test the null hypothesis that the shocks to the observable excess return, $w^s$, and implied variance, $\varepsilon$, should be normal under the null of a given model. We can construct the posterior distribution of these shocks and evaluate how they change from model to model and whether they are normal. Online appendix describes the procedure.

One has to exhibit caution when interpreting the evidence on normality of $\varepsilon$. The variance of the error term in the implied variance equation (3.6), $\sigma_{iv}^2 v_t^2 \lambda_t$ is very flexible. If a model is misspecified, $\lambda_t$ will adjust so that the $\varepsilon$ is close to a normal variable. Therefore, diagnostics of $\varepsilon$ are not enough. We should be tracking the size of the variance of the error term. A better specified model should have smaller variance. We keep track of the time-series average of this variance – which we refer to as $IVvar$ – and report its posterior distribution.

Bayes odds ratios offer a formal specification test of the models. The test produces a number that measures the relative odds of two models given the data (the posterior distribution of the null model is in the denominator of the ratio). Following Kass and Raftery (1995), we interpret a log odds ratio that is greater than 3 as strong evidence against the null. Odds ratios do not necessarily select more complex models because the ratios contain a penalty for using more parameters (so-called automatic Occam's razor). Online appendix details the computations.

# 5 Results

We start by highlighting statistical properties of the estimated models. Next, we study economic implications.

## 5.1 Statistical Properties of Currency Risks

Tables 2 - 5 report the parameter estimates and Tables 6 - 9 report the corresponding model diagnostics. Table 10 displays the results of specification tests on the basis of Bayes odds ratios. Table 11 summarises parameters of the calibrated interest rate processes.

The results exhibit a lot of commonalities across the different currencies. As we move from models with stochastic variance to stochastic variance with jumps, we observe a change in two key parameters: both the persistence $\nu$ of variance and the long-run mean of its conditionally normal component $v$ decline. Taking AUD as an example, $\nu$ declines from 0.9943 to 0.9855. This seemingly small change translates into drop in the half-life of the conditionally normal component, $\log 2/(1-\nu)$, from 122 to 48 days. The high persistence of variance in the model without jumps is a sign of misspecification. Variance has to take high values occasionally to generate the observed exchange rates in the data. In the absence of jumps, variance builds up to the high values gradually via the high persistent channel. Additionally, in the case of GBP only, the volatility of variance $\sigma_v$ declines significantly from 0.0321 to 0.0272. High $\sigma_v$ helps the misspecified model with stochastic variance in generating high values of variance. The diagnostics support this interpretation. $IVvar$ drops by 50% across all currencies; this change is statistically significant. As expected, diagnostics for $\varepsilon$ show that it is close to a normal variable for both models because of the flexibility in $\lambda_t$. Bayes odds ratios strongly favour stochastic variance with jumps.

Continuing with AUD, its volatility ($\sqrt{v}$) declines from 0.70% to 0.53% per day (11.19% to 8.43% per year). This happens because the total variance has contributions from the regular and jump components in the model with stochastic variance with jumps. When there are no jumps in FX, the long-run variance is equal to $v_J = [(1-\nu)v + h_0^v\theta_v]/[1-\nu-h_v^v\theta_v]$. See Appendix A.1 for more details. This expression produces the average volatility of 0.65%, much closer to the figure in the model with stochastic variance.

To aid in interpreting the parameters controlling jumps in variance, consider the impact of a jump in variance. Suppose the current variance is at its long-run mean and the variance jumps by the average amount $\theta_v$. Then in the case of AUD, the resulting volatility will move from 0.65% to $(v_J+\theta_v)^{1/2} = 0.90\%$, a nearly 40% increase in volatility (this increase ranges from 20% to 40% for the different currencies). The average jump intensity is equal to $h_0^v + h_v^v v_J = 0.0053$ jumps per day, or 1.34 per year (this number ranges from 1.34 to 2.61 for the the different currencies). Jumps in variance are self-exciting, so that a jump increases the likelihood of another jump. When the variance jumps by $\theta_v$, intensity changes to 1.71 for AUD (the range is from 1.71 to 3.41 for all the currencies).

Also note that $\rho$, the "leverage effect," has the same sign as the average interest rate differential. It is positive for JPY and CHF, and negative for AUD and GBP. This result is consistent with the analysis in Brunnermeier, Nagel, and Pedersen (2008) and the common wisdom among market participants that investors who are long carry are essentially short volatility. For example, consider the position of a carry trade investor who borrows money in USD and invests in AUD. This investor can loose money when the AUD depreciates against the USD. We estimate that $\rho$ is negative for this currency pair, so the volatility of this exchange rate tends to increase during times when the AUD depreciates.

The preferred version of the full model is the one with all of the aforementioned restrictions imposed ($\theta_u = \theta_d = \theta$, $h_0^u = h_0^d = h_0$, and $h_r^u = \tilde{h}_r^d = h_r$). That is, the size of the jumps in FX up and down are symmetric and their intensities have numerically identical functional form (but they depend on different interest rates). As a result, the overall structure of jump arrivals differs from the one used in popular models of S&P 500 returns, where jumps in variance and the index are simultaneous.

Parameters reflecting the average jump size have a different interpretation as compared to jump in variance. The latter is a jump in the level of the variable, while the former is the jump in return. Therefore, it is scale-free: it is not daily or annual, it reflects by how much the return changes at the moment of the jump. Thus, on average, AUD returns increase (decline) by 1.69% when there is a jump up (down). Average intensities of down and up jumps are similar to each other for a given currency and, overall, are lower than those of variances: they range from 0.44 to 1.33 jumps per year (we use sample averages of interest rates to compute average $h_t^u$ and $h_t^d$).

The diagnostics of residuals $w^s$ indicate that the major improvement in moving from stochastic variance with jumps to the preferred model comes from a statistically significant drop in kurtosis from roughly 4 to 3.5 across all currencies. The absolute value of skewness of $w$ experiences a significant drop for all currencies except for GBP, where it was insignificantly different from zero in the model with stochastic variance with jumps already. Serial correlation is slightly negative for all currencies except for GBP (where it is zero in the model with stochastic variance with jumps already), and the change from one model to another is insignificant. $IVvar$ does not change appreciably because we did not change our model for variance. Bayes odds ratios strongly favor the preferred model. In summary, the preferred is clearly a superior model, but there are some residual non-normalities left in the fitted shocks to exchange rates. We leave improvements to future research.

The expected excess return in (3.9) can be simplified for the preferred model to

$$E_t(y_{t+1}) = \mu_0 + (\mu_r + h_r\theta)r_t + (\tilde{\mu}_r - h_r\theta)\tilde{r}_t + \mu_v v_t. \tag{5.1}$$

Thus, by testing if $\mu_r = -\tilde{\mu}_r$ and $\mu_v = 0$, we test the UIP regression specification (3.11) - (3.12) of currency excess returns across all three models. For all currency pairs, we cannot reject that $\mu_r = -\tilde{\mu}_r$ at the conventional significance levels. Moreover, $\mu_r \approx -3$ for all currencies that is consistent with our earlier discussion of UIP regression results. In addition, the loading on the variance $\mu_v$ is significantly negative in all currencies except for JPY which has a significantly positive estimate. The tiny serial correlation of the residuals $w^s$ suggests that this model is adequate in capturing conditional mean of excess returns and, therefore, potentially omitted variables cannot affect materially our conclusions about the structure of currency risks.

The probability of USD depreciation, $h_t^u$, depends positively on the US interest rate. This result seems to contradict basic intuition about the relationship between changes in FX and interest rates. It is important to note that this connection is applicable to jumps only. Parameter estimates and expression (5.1) imply that, all else equal, the expected excess return on the USD is higher when the difference between the U.S. and foreign interest rate is higher. However, when the interest rate differential is positive, our model says that the probability of a large depreciation in the USD (a jump up) is higher than the probability of a large appreciation (a jump down).

### 5.2 Jumps and Events

In this section we study the economic properties of the documented jumps. We ask basic questions regarding the structure of the jump components, examine whether jumps can be related to important economic events, and gauge their impact on the overall risk of carry trades. Our discussion is based on Figures 1 - 4 and Table 12.

For each exchange rate, the figures display the time series of data (excess returns and implied volatilities) complemented with the estimated unobservable model components: spot volatility $v_t^{1/2}$ and jumps. Regarding the latter, the model produces an estimate of a jump size and an ex-post probability that a jump actually took place on a specific day. However, all of this information is not easy to digest as there are a lot of small jumps and jumps with small ex-post probability of taking place. Thus, to simplify the reporting, we stratify the jump probabilities by assigning them a value of one if their actual value is 0.5 or higher, and zero otherwise. Then we plot the estimated jumps sizes on the days with the assigned value of one. Our strategy yields 219 jumps overall across all currencies. Approximately 25% of these jumps take place simultaneously in at least two currencies. We call such jumps international. There are only 8 episodes when FX and variance jumped at the same time. We overlay the plots of the estimated jumps sizes with the state-dependent ex-ante jump probabilities $h_t^v$, $h_t^u$, and $h_t^d$.

To interpret the plots better, we have to reference the jump magnitudes against the summary statistics available in Table 1. Let us use JPY in Figure 4 as an example. Table 1 tells us that the volatility of JPY is 0.7% per day and the mean is approximately zero. Thus, a "regular" excess return can be within the range of $\pm 2\%$ ($\mu \pm 3\sigma$). The upper left panel of Figure 4 shows that there are quite a few days when excess returns are outside of this range. In practice, the volatility is time-varying and unobserved. Therefore, the "regular" range is time-varying also and uncertain. The estimation procedure takes this uncertainty into account by producing ex-post probability of a jump taking place. We arbitrarily select the level of uncertainty about jumps that we are comfortable with by discarding the jumps with such probabilities less than 50%. The bottom left panel confirms this by showing the estimated jumps in excess returns. Their magnitude ranges from 2% to 6% in absolute value. Interestingly, the larger jumps coincide with spikes in the moving-window estimates of skewness across all currencies.

However, not all of the big spikes in excess returns are attributable to jumps in FX. For example, on October 28, 2008, the plot of excess returns shows a big drop of 5.5%. The model tells us that there were no jumps on that day. The model is capable of generating such a big move via a normal component because of the jump in variance. Volatility jumped by $(v_{t-1} + z_t^v)^{1/2} - v_{t-1}^{1/2} = 0.18\%$ (2.9% annualized), on average, on each of three days between October 22 and 24. Each day the jump in variance was increasing the probability of a jump the following day. Over these three days volatility moved from 1.35% (21.5% annualized) to 1.8% (28% annualized). To put this number into a perspective, the long-run volatility mean is $v_J^{1/2} = 0.66\%$ (10.4% annualized). Thus, the increase in volatility over these three days was roughly equal to the average level of volatility. Moreover, there is no significant news associated with either October 22-24 or October 28. Thus, we attribute these events to pure uncertainty in the markets.

GBP generates large movements via jumps in variance in the most transparent way. The exchange rate itself exhibits only 11 jumps throughout the sample, none of which take place after 2000. In fact, even the famous “Black Wednesday” – the GBP devaluation on September 16, 1992 – is attributed primarily to a jump in variance on September 8. The volatility moved from 0.93% (14.7% annualised) to 1.13% (17.9% annualised), then it drifted up to 1.15% (18.3% annualised) on the day of the crash. These values of volatility are high, as the average volatility of GBP is $v_J^{1/2} = 0.55\%$. Nonetheless, this level of volatility is still insufficient to generate the whole drop of -4.10%. Of course, these rough computations assume that $v_t$ is known with certainty. It is not, and a small deviation in the estimate may be able to attribute the whole return to a normal shock in FX. This is why the estimated probability of a jump is only 26% on this day (and the estimated jump size is -0.42%).

As a next step, we check if the jumps we detect are related to economic, political or financial events. For each day a currency has experienced a jump, we check if there were significant news. This strategy is effectively opposite to the one employed in studies of the news impact on financial assets (see, e.g., Andersen, Bollerslev, Diebold, and Vega, 2003 for FX). Usually one measures news surprise by computing a standardised difference between an announcement's expectation and realisation and then checks, usually at intra-day frequency, if the surprise had an impact on values of financial assets. Our approach does not require us taking a stand on measuring a surprise. In addition, we are careful in distinguishing announcements, a clear public release of a fact, from uncertainty: anticipation, comments on the current economic situation and overall market anxiety that is sometimes evident in the news. Table 12 provides a summary of the types of events associated with jumps. Online appendix provides a jump-by-jump description of all events.

Consistent with Figures 1 - 4, we see that there are many more jumps in variance than in the exchange rates. Almost all jumps are associated with important events, however there is a critical difference between jumps in FX and those in variance. The former is most commonly associated with announcements and the latter is related to uncertainty. We document a lot of common jumps across the currencies, particularly jumps in variance. Thus, it appears promising to extend the existing research on common and currency-specific factors affecting risk premiums (e.g., Lustig, Roussanov, and Verdelhan, 2010) by allowing for common and currency-specific jump risk.

The plots of jump intensities provide a partial insight into why jumps in variance are so prominent. Probabilities of jumps in FX are moving together with interest rates, which experienced secular decline in our sample in all countries. As we highlighted earlier, the probability of a jump in variance is primarily driven by the variance itself. This probability went through a couple of cycles of high and low values as volatility is less persistent than interest rates.

### 5.3 Decomposing the total risk

Are these risks important quantitatively? Jumps in FX and variance should affect the tail events the most. The properties of tails are captured by high-order moments or cumulants. Intuitively, jumps in FX affect the conditional odd cumulants of exchange rate returns as they generate asymmetries directly. Jumps in variance can lead to both large negative and positive outcomes, so they must have an impact on even cumulants. Armed with this intuition, we measure the total risk corresponding to investment horizon $n$ using entropy of changes in FX:

$$L_t(S_{t+n}/S_t) = \log E_t(e^{s_{t+n}-s_t}) - E_t(s_{t+n} - s_t). \tag{5.2}$$

Entropy is a loaded term. Our use of entropy is similar to that of Backus, Chernov, and Martin (2011) and Backus, Chernov, and Zin (2011), who characterise entropy of the pricing kernel, and the closest to the way it is used in Martin (2011) who uses entropy of equity index returns. Entropy's connection to cumulants of log FX returns makes it attractive for our purposes. Specifically, the definition (5.2) implies that

$$\begin{aligned} L_t(S_{t+n}/S_t) &= k_t(1; s_{t+n} - s_t) - \kappa_{1t}(s_{t+n} - s_t) \\ &= \kappa_{2t}(s_{t+n} - s_t)/2! + \kappa_{3t}(s_{t+n} - s_t)/3! + \kappa_{4t}(s_{t+n} - s_t)/4! + \ldots, \end{aligned} \tag{5.3}$$

where $k_t(1; s_{t+n} - s_t)$ is the conditional cumulant-generating function of $s_{t+n} - s_t$ evaluated at the argument equal to 1, $\kappa_{jt}(s_{t+n} - s_t)$ is the $j$th conditional cumulant of $s_{t+n} - s_t$, and we used the fact that $k_t(1; s_{t+n} - s_t)$ is equal to the infinite sum of $\kappa_{jt}(s_{t+n} - s_t)/j!$ over $j$ starting with $j = 1$. The significance of the property (5.3) is that if currency changes are normally distributed, then entropy is equal to a half of their variance (the first term in the sum). All the higher-order terms arise from non-normalities. Thus, entropy captures tail behaviour of returns in a compact form. As a result, we view entropy as a natural generalisation of variance as a risk measure. For this reason, Alvarez and Jermann (2005) explicitly refer to $L_t$ as generalised variance.

We decompose the total risk of currency returns into the contribution of the jump and normal components. Appendix A.2 explains how we compute the full entropy. We can compute the individual components by zeroing out the rest.[10] Figures 5 and 6 display the contributions of these components for the investment horizons of 1 months ($n = 21$) and 3 months ($n = 63$). We overlay these contributions with a time-series plot of entropy. We scale entropy to ensure that it is equal to variance in the normal case and to adjust for the horizon. Finally, to make the number easily interpretable, we take the square-root

[10]If two variables $x_s$ and $y_s$ are conditionally independent, then $L_t(x_s y_s) = L_t(x_s) + L_t(y_s)$. Therefore, our decomposition approach correctly separates the contributions of the two jumps in currencies. Because probability of the jump in variance depends on the variance itself, the normal shock to variance and the jump are conditionally independent only over one period, $n = 1$. When $n > 1$ our procedure attributes all the covariance terms, which are positive because of the estimated functional form of jump probabilities, to the jump in variance. We think that this approach is sensible because the presence of these covariance terms is due to jumps.

and express it in percent. Thus, we plot $\sqrt{2L_t/n}$. Finally, Table 13 supports the plots by reporting summary statistics of the relative contributions of the different components.

We start by characterising the contribution of the different components at a given point in time. We see that the regular, or normal, risk is the most prominent regardless of the horizon. The average total contribution of jumps at a one-month horizon ranges from 11.03 % for GBP to 20.19% for AUD. The risk of jump in FX (up or down) [the range is between 6.82% and 9.17%] is higher than that of the jump in variance [the range is between 3.76% and 4.83%] and has higher time-variation at a one-month horizon (GBP is the exception as jumps in variance contribute 4.37% as compared to 2.98% for jumps up and 3.68% for jumps down). Therefore, in the short-term the risk of the jump in variance has the smallest contribution to the overall currency risk. However, this conclusion changes as we extend the investment horizon to three months. The average total contribution of jumps at this horizon increases – the range is from 17.71% for GBP to 25.19% for JPY. In this case individual contribution of jumps in variance [the range is 8.94% to 11.48%] is higher than those of jumps in FX [the range is 2.78% to 8.57%] (in the case of GBP the contribution of the jump in variance is greater than the sum of jumps up and down).

The contribution of jumps in FX declines towards the end of our sample thereby making the contribution of jumps in variance more important. We connect this result to the secular decline in the probability of FX jumps that we highlighted earlier. This effect diminishes expected contribution of such jumps to the overall risk. In contrast, the probability of jumps in variance is less persistent and, therefore, exhibits mean-reversion in our sample.

The time-series variation in total risk resembles the time-series variation in the spot variance $v_t$. This is not surprising because $L_t$ is a linear function of $v_t$ (Appendix A.2). Thus, whenever spot variance moves, especially jumps, we observe a clear move in entropy. We conclude that the risk of jumps in variance are at least as important as the risk of jumps in FX, but the two types of jumps serve a different purpose.

### 5.4 Preliminary evidence of priced jump risk

The large amount of jump risk prompts a natural question of whether this risk is priced. Answering this question has important implications for the carry trade literature. In particular, one should be able to attribute a specific fraction of carry risk premium to compensation for bearing crash risk.

The full answer to this question requires an explicit model of the pricing kernel and the use of assets that are sensitive to jump risk for estimation. In this regard, out-of-the-money options are particularly informative about the price of jump risks (i.e., the covariance of the pricing kernel with jumps). However, such analysis is outside of the scope of this paper. Instead, we provide a back-of-the-envelope computation, which we view as preliminary evidence of priced jump risk.

Our idea is to explore the shape of the implied volatility smile that is derived from our model. If the jump risk is not priced, then we would be able to replicate the smile without additional assumptions about risk premiums. To capture the diversity of the smile shapes, we consider examples of a currency with a positive average interest rate differential (GBP) and a negative one (JPY). The reason we focus on the interest rate differential is that our model connects it to asymmetry of the conditional distribution of an exchange rate. Indeed, the third conditional cumulant can be obtained by taking the third derivative of the cumulant-generating function of log currency returns (provided in Appendix A.2). For example,

$$\kappa_{3t}(s_{t+1} - s_t) = 6\theta^3 h_r(r_t - \tilde{r}_t)$$

in the preferred model.

For both currencies we pick a day in which the variance and interest rate differential are roughly equal to the sample averages: November 12, 2007 (GBP) and April 20, 2004 (JPY). Figure 7 displays the implied volatility smiles observed on these days. Consistent with our expectations, GBP exhibits positive asymmetry (defined as the difference between implied volatility corresponding to moneyness less than one and the one with moneyness greater than one, with moneyness symmetric around at-the-money), and JPY exhibits negative asymmetry. The solid black lines in Figure 7 are the option-implied volatilities that correspond to our model estimates with no risk premia. The model can generate both the smile and the correct direction of asymmetry. However, the level and curvature of the smile that are implied by the model cannot match those observed in the data. The natural question is whether the omitted risk premiums are responsible for this disparity.

Before we conclude that the disparity is due to risk premia, we first evaluate whether statistical uncertainty about parameter values and the unobserved spot variance could account for the difference in levels and curvatures. The theoretical implied volatility is a function of observable states, option contract characteristics (strike and time to maturity), model parameters, and the unobservable variance: $IV_t^{1/2} = F(S_t, r_t, \tilde{r}_t, K, T-t, \Theta, v_t)$. The solid black lines in Figure 7 display $F(S_t, r_t, \tilde{r}_t, K, T-t, \hat{\Theta}, \hat{v}_t)$. We can take the uncertainty about these estimated values into account by computing confidence bounds. One of the by-products of our estimation procedure is a set of draws $\{\Theta^{(g)}, v_t^{(g)}\}_{g=1}^{250{,}000}$ from the posterior distribution $p(\Theta, v_t|\text{full dataset})$. We obtain the corresponding set of draws from the posterior distribution of implied volatilities by evaluating $F(S_t, r_t, \tilde{r}_t, K, T-t, \Theta^{(g)}, v_t^{(g)})$. The blue dashed lines in Figure 7 display the (2.5%, 97.5%) posterior coverage interval for theoretical implied volatilities.

The only case where the posterior interval covers observed implied volatilities corresponds to six-month options on GBP. Thus, statistical uncertainty, on its own, cannot explain the gap between the model and the data. The fact that the curvature is much more pronounced in the data suggests that jump risk premiums are present (variance risk premiums may be helpful in adjusting the level of the smile but not its curvature). See, for instance, Backus, Chernov, and Martin (2011) who provide a detailed discussion of how risk adjustment in the jump parameters affects the shape of the smile (Figure 3). While by no means conclusive, this initial evidence warrants further more detailed investigation of the magnitude of jump risk premiums.

# 6 Conclusion

We explore sources of risk affecting currency returns. We find a large time-varying component that is attributable to jump risk. The most interesting part of this finding is that jumps in currency variance play an important role, especially at long (quarterly) investment horizons, yet there is no obvious link between macroeconomic fundamentals and these jumps. We interpret this evidence as a manifestation of economic uncertainty.

We see at least two important directions in which our analysis can be extended. First, we should use prices of financial assets associated with currencies (e.g., bonds, options) to estimate the pricing kernel. This would allow us to characterize how the risks documented in this paper are valued in the marketplace. Second, existing research presents evidence of common and currency-specific factors affecting risk premiums. Our evidence suggests that common jump risks are shared across the different currencies. It would be useful to establish a model of joint currency behaviour that explicitly incorporates common and country-specific jump components and how they contribute to risk premiums.

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**Table 1**
**Properties of excess log currency and S&P 500 returns and changes in implied volatility**

| | | Mean | Std Dev | Skewness | Kurtosis | Nobs |
|---|---|---|---|---|---|---|
| AUD | Excess return | 0.0186 | 0.7435 | -0.3870 | 13.7202 | 6332 |
| | $\Delta\sqrt{IV}$ | 0.0109 | 3.7661 | 0.9077 | 9.7290 | 3933 |
| CHF | Excess return | 0.0057 | 0.7232 | 0.1194 | 4.7841 | 6521 |
| | $\Delta\sqrt{IV}$ | 0.0073 | 3.8057 | 0.9966 | 9.8095 | 3823 |
| GBP | Excess return | 0.0096 | 0.6197 | -0.2337 | 5.6832 | 6521 |
| | $\Delta\sqrt{IV}$ | 0.0142 | 4.0001 | 1.3884 | 44.2683 | 3823 |
| JPY | Excess return | 0.0003 | 0.6950 | 0.3626 | 8.0878 | 6393 |
| | $\Delta\sqrt{IV}$ | -0.0045 | 4.8277 | 1.0395 | 10.7764 | 3934 |
| S&P 500 | Excess return | 0.0090 | 1.1803 | -1.3584 | 32.9968 | 6521 |
| | $\Delta\sqrt{VIX}$ | 0.0089 | 5.8997 | 0.5096 | 6.7502 | 3914 |

Notes. Descriptive statistics for daily log currency and log S&P 500 excess returns and changes in implied volatility, in percent, per day: AUD return from September 25, 1986, to December 31, 2010, and AUD IV from December 6, 1995, to December 31, 2010; CHF return from January 3, 1986, to December 31, 2010, and CHF IV from May 8, 1996, to December 31, 2010; GBP return from January 3, 1986, to December 31, 2010, and GBP IV from May 8, 1996, to December 31, 2010; JPY return from July 2, 1986, to December 31, 2010, and GBP IV from December 5, 1995, to December 31, 2010; S&P 500 return from January 3, 1986, to December 31, 2010 and VIX from January 2, 1996, to December 31, 2010. The interest rates used to compute returns are one-month LIBOR rates. Source: Bloomberg.

**Table 2**
**AUD Parameter Estimates**

| | SV ($\theta = 0$, $\theta_v = 0$) | SVJ ($\theta = 0$) | Preferred |
|---|---|---|---|
| $\mu_0$ | -0.0004 | 0.0003 | 0.0014 |
| | (-0.0181,0.0173) | (-0.0175, 0.0182) | (-0.0166, 0.0194) |
| $\mu_r$ | -2.4893 | -2.5200 | -2.7643 |
| | (-3.5895, -1.3910) | (-3.6170, -1.4190) | (-3.8613, -1.6801) |
| $\mu_v$ | -0.0150 | -0.0152 | -0.0147 |
| | (-0.0247, -0.0053) | (-0.0249, -0.0056) | (-0.0244, -0.0051) |
| $v$ | 0.4968 | 0.2819 | 0.2819 |
| | (0.2903, 0.8984) | (0.2101, 0.3728) | (0.2110, 0.3734) |
| $\nu$ | 0.9943 | 0.9855 | 0.9857 |
| | (0.9925, 0.9961) | (0.9837, 0.9873) | (0.9838, 0.9875) |
| $\sigma_v$ | 0.0391 | 0.0343 | 0.0342 |
| | (0.0379, 0.0404) | (0.0330, 0.0357) | (0.0329, 0.0356) |
| $\rho$ | -0.2924 | -0.2770 | -0.2839 |
| | (-0.3279, -0.2563) | (-0.3156, -0.2378) | (-0.3237, -0.2435) |
| $\theta_v$ | | 0.3864 | 0.3837 |
| | | (0.3392, 0.4406) | (0.3367, 0.4362) |
| $\theta$ | | | 1.6910 |
| | | | (1.5208, 1.8779) |
| $h_0^v$ | | 0.0037 | 0.0036 |
| | | (0.0029, 0.0040) | (0.0028, 0.0040) |
| $h_v$ | | 0.0038 | 0.0038 |
| | | (0.0031, 0.0040) | (0.0031, 0.0040) |
| $h_0$ | | | 0.0017 |
| | | | (0.0000, 0.0038) |
| $h_r$ | | | 0.1737 |
| | | | (0.1177, 0.1992) |
| $\alpha_{iv}$ | 0.0033 | 0.0027 | 0.0033 |
| | (0.0009, 0.0064) | (-0.0002, 0.0056) | (0.0005, 0.0063) |
| $\beta_{iv}$ | 1.0006 | 1.0021 | 1.0022 |
| | (0.9958, 1.0054) | (0.9983, 1.0059) | (0.9984, 1.0061) |

Notes. The estimates correspond to daily excess currency returns, in percent. The 95% confidence intervals are reported in parentheses. The preferred model is:

$$\begin{aligned}
y_{t+1} &= \mu_0 + \mu_r(r_t - \tilde{r}_t) + \mu_v v_t + v_t^{1/2} w_{t+1}^s + z_{t+1}^u - z_{t+1}^d \\
v_{t+1} &= (1-\nu)v + \nu v_t + \sigma_v v_t^{1/2} w_{t+1}^v + z_{t+1}^v \\
h_t^u &= h_0 + h_r r_t,\ h_t^d = h_0 + h_r \tilde{r}_t,\ h_t^v = h_0^v + h_v v_t \\
z_t^u | j &\sim \mathcal{G}\text{amma}(j,\theta),\ z_t^d | j \sim \mathcal{G}\text{amma}(j,\theta),\ z_t^v | j \sim \mathcal{G}\text{amma}(j,\theta_v)
\end{aligned}$$

**Table 3**
**CHF Parameter Estimates**

| | SV ($\theta = 0$, $\theta_v = 0$) | SVJ ($\theta = 0$) | Preferred |
|---|---|---|---|
| $\mu_0$ | 0.0340 | 0.0353 | 0.0324 |
| | (0.0150, 0.0531) | (0.0163, 0.0543) | (0.0132, 0.0516) |
| $\mu_r$ | -2.9851 | -3.0674 | -3.2501 |
| | (-4.3345, -1.6354) | (-4.4174, -1.7169) | (-4.5952, -1.8996) |
| $\mu_v$ | -0.0198 | -0.0199 | -0.0199 |
| | (-0.0333, -0.0064) | (-0.0335, -0.0065) | (-0.0334, -0.0064) |
| $v$ | 0.5088 | 0.3502 | 0.3427 |
| | (0.3136, 0.8364) | (0.2564, 0.4741) | (0.2507, 0.4639) |
| $\nu$ | 0.9891 | 0.9789 | 0.9785 |
| | (0.9863, 0.9919) | (0.9758, 0.9818) | (0.9755, 0.9815) |
| $\sigma_v$ | 0.0388 | 0.0337 | 0.0337 |
| | (0.0373, 0.0404) | (0.0321, 0.0352) | (0.0321, 0.0352) |
| $\rho$ | 0.0875 | 0.0856 | 0.0856 |
| | (0.0480, 0.1271) | (0.0439, 0.1273) | (0.0416, 0.1298) |
| $\theta_v$ | | 0.2205 | 0.2145 |
| | | (0.1845, 0.2622) | (0.1804, 0.2550) |
| $\theta$ | | | 1.3582 |
| | | | (1.1771, 1.5744) |
| $h_0$ | | 0.0037 | 0.0037 |
| | | (0.0029, 0.0040) | (0.0028, 0.0040) |
| $h_v$ | | 0.0145 | 0.0144 |
| | | (0.0131, 0.0150) | (0.0130, 0.0150) |
| $h_0$ | | | 0.0051 |
| | | | (0.0011, 0.0078) |
| $h_r$ | | | 0.2175 |
| | | | (0.0615, 0.2973) |
| $\alpha_{iv}$ | 0.0061 | 0.0041 | 0.0056 |
| | (0.0009, 0.0113) | (-0.0013, 0.0093) | (0.0003, 0.0104) |
| $\beta_{iv}$ | 0.9919 | 0.9934 | 0.9946 |
| | (0.9753, 1.0091) | (0.9795, 1.0071) | (0.9802, 1.0096) |

Notes. The estimates correspond to daily excess currency returns, in percent. The 95% confidence intervals are reported in parentheses. The preferred model is:

$$
\begin{aligned}
y_{t+1} &= \mu_0 + \mu_r(r_t - \tilde{r}_t) + \mu_v v_t + v_t^{1/2} w_{t+1}^s + z_{t+1}^u - z_{t+1}^d \\
v_{t+1} &= (1-\nu)v + \nu v_t + \sigma_v v_t^{1/2} w_{t+1}^v + z_{t+1}^v \\
h_t^u &= h_0 + h_r r_t,\ h_t^d = h_0 + h_r \tilde{r}_t,\ h_t^v = h_0^v + h_v v_t \\
z_t^u|j &\sim \mathcal{G}\text{amma}(j,\theta),\ z_t^d|j \sim \mathcal{G}\text{amma}(j,\theta),\ z_t^v|j \sim \mathcal{G}\text{amma}(j,\theta_v)
\end{aligned}
$$

**Table 4**
**GBP Parameter Estimates**

| | SV ($\theta = 0$, $\theta_v = 0$) | SVJ ($\theta = 0$) | Preferred |
|---|---|---|---|
| $\mu_0$ | 0.0348 | 0.0360 | 0.0337 |
| | (0.0128, 0.0565) | (0.0138, 0.0584) | (0.0116, 0.0556) |
| $\mu_r$ | -3.0632 | -3.1692 | -3.1897 |
| | (-4.4127, -1.7209) | (-4.5046, -1.8351) | (-4.5138, -1.8673) |
| $\mu_v$ | -0.1326 | -0.1377 | -0.1341 |
| | (-0.1928, -0.0720) | (-0.1986, -0.0773) | (-0.1952, -0.0731) |
| $v$ | 0.3773 | 0.2227 | 0.2180 |
| | (0.1855, 0.8002) | (0.1631, 0.2989) | (0.1619, 0.2909) |
| $\nu$ | 0.9941 | 0.9810 | 0.9809 |
| | (0.9919, 0.9963) | (0.9786, 0.9834) | (0.9786, 0.9833) |
| $\sigma_v$ | 0.0321 | 0.0272 | 0.0273 |
| | (0.0311, 0.0332) | (0.0262, 0.0283) | (0.0263, 0.0284) |
| $\rho$ | -0.1341 | -0.1295 | -0.1303 |
| | (-0.1713, -0.0965) | (-0.1692, -0.0896) | (-0.1709, -0.0895) |
| $\theta_v$ | | 0.1953 | 0.1959 |
| | | (0.1728, 0.2206) | (0.1731, 0.2211) |
| $\theta$ | | | 1.1680 |
| | | | (0.9593, 1.4127) |
| $h_0^v$ | | 0.0038 | 0.0038 |
| | | (0.0033, 0.0040) | (0.0033, 0.0040) |
| $h_v$ | | 0.0121 | 0.0121 |
| | | (0.0110, 0.0125) | (0.0110, 0.0125) |
| $h_0$ | | | 0.0012 |
| | | | (0.0001, 0.0020) |
| $h_r$ | | | 0.1223 |
| | | | (0.0634, 0.1491) |
| $\alpha_{iv}$ | 0.0109 | 0.0063 | 0.0089 |
| | (0.0063, 0.0155) | (0.0009, 0.0109) | (0.0039, 0.0137) |
| $\beta_{iv}$ | 0.9905 | 0.9951 | 0.9940 |
| | (0.9855, 0.9955) | (0.9905, 0.9996) | (0.9895, 0.9985) |

Notes. The estimates correspond to daily excess currency returns, in percent. The 95% confidence intervals are reported in parentheses. The preferred model is:

$$
\begin{aligned}
y_{t+1} &= \mu_0 + \mu_r(r_t - \tilde{r}_t) + \mu_v v_t + v_t^{1/2} w_{t+1}^s + z_{t+1}^u - z_{t+1}^d \\
v_{t+1} &= (1-\nu)v + \nu v_t + \sigma_v v_t^{1/2} w_{t+1}^v + z_{t+1}^v \\
h_t^u &= h_0 + h_r r_t,\ h_t^d = h_0 + h_r \tilde{r}_t,\ h_t^v = h_0^v + h_v v_t \\
z_t^u | j &\sim \mathcal{G}\text{amma}(j,\theta),\ z_t^d | j \sim \mathcal{G}\text{amma}(j,\theta),\ z_t^v | j \sim \mathcal{G}\text{amma}(j,\theta_v)
\end{aligned}
$$

**Table 5**
**JPY Parameter Estimates**

| | SV ($\theta = 0$, $\theta_v = 0$) | SVJ ($\theta = 0$) | Preferred |
|---|---|---|---|
| $\mu_0$ | 0.0253 | 0.0253 | 0.0203 |
| | (0.0064, 0.0441) | (0.0062, 0.0443) | (0.0018, 0.0389) |
| $\mu_r$ | -3.1861 | -3.2046 | -3.4590 |
| | (-4.4200, -1.9526) | (-4.4540, -1.9531) | (-4.6992, -2.2266) |
| $\mu_v$ | 0.0151 | 0.0152 | 0.0152 |
| | (0.0054, 0.0248) | (0.0055, 0.0249) | (0.0054, 0.0248) |
| $v$ | 0.4816 | 0.3143 | 0.3012 |
| | (0.2926, 0.8111) | (0.2328, 0.4223) | (0.2207, 0.4079) |
| $\nu$ | 0.9896 | 0.9762 | 0.9771 |
| | (0.9868, 0.9924) | (0.9730, 0.9794) | (0.9739, 0.9802) |
| $\sigma_v$ | 0.0496 | 0.0438 | 0.0436 |
| | (0.0476, 0.0516) | (0.0419, 0.0458) | (0.0417, 0.0455) |
| $\rho$ | 0.3681 | 0.3505 | 0.3631 |
| | (0.3316, 0.4040) | (0.3098, 0.3902) | (0.3205, 0.4047) |
| $\theta_v$ | | 0.3917 | 0.3771 |
| | | (0.3313, 0.4622) | (0.3198, 0.4447) |
| $\theta$ | | | 1.2351 |
| | | | (1.0847, 1.4071) |
| $h_0^v$ | | 0.0037 | 0.0037 |
| | | (0.0031, 0.0040) | (0.0029, 0.0040) |
| $h_v^v$ | | 0.0077 | 0.0076 |
| | | (0.0068, 0.0080) | (0.0067, 0.0080) |
| $h_0$ | | | 0.0052 |
| | | | (0.0034, 0.0060) |
| $h_r$ | | | 0.4447 |
| | | | (0.3133, 0.4984) |
| $\alpha_{iv}$ | 0.0140 | 0.0116 | 0.0159 |
| | (0.0086, 0.0193) | (0.0062, 0.0169) | (0.0099, 0.0214) |
| $\beta_{iv}$ | 1.0052 | 1.0083 | 1.0248 |
| | (0.9871, 1.0248) | (0.9916, 1.0256) | (1.0069, 1.0431) |

Notes. The estimates correspond to daily excess currency returns, in percent. The 95% confidence intervals are reported in parentheses. The preferred model is:

$$
\begin{aligned}
y_{t+1} &= \mu_0 + \mu_r(r_t - \tilde{r}_t) + \mu_v v_t + v_t^{1/2} w_{t+1}^s + z_{t+1}^u - z_{t+1}^d \\
v_{t+1} &= (1-\nu)v + \nu v_t + \sigma_v v_t^{1/2} w_{t+1}^v + z_{t+1}^v \\
h_t^u &= h_0 + h_r r_t,\ h_t^d = h_0 + h_r \tilde{r}_t,\ h_t^v = h_0^v + h_v v_t \\
z_t^u|j &\sim \mathcal{G}\text{amma}(j,\theta),\ z_t^d|j \sim \mathcal{G}\text{amma}(j,\theta),\ z_t^v|j \sim \mathcal{G}\text{amma}(j,\theta_v)
\end{aligned}
$$

**Table 6**
**Model diagnostics for AUD**

| | SV ($\theta = 0$, $\theta_v = 0$) | SVJ ($\theta = 0$) | Preferred |
|---|---|---|---|
| $skewness^C$ | -0.3080 | -0.3074 | -0.2004 |
| | (-0.3308, -0.2860) | (-0.3304, -0.2855) | (-0.2408, -0.1599) |
| $kurtosis^C$ | 4.1472 | 4.0822 | 3.4892 |
| | (4.0677, 4.2366) | (4.0006, 4.1810) | (3.3802, 3.6055) |
| $autocorrelation^C$ | -0.0281 | -0.0271 | -0.0324 |
| | (-0.0311, -0.0252) | (-0.0303, -0.0241) | (-0.0406, -0.0242) |
| $skewness^{IV}$ | 0.0402 | 0.0303 | 0.0310 |
| | (-0.0373, 0.1181) | (-0.0466, 0.1070) | (-0.0459, 0.1080) |
| $kurtosis^{IV}$ | 3.0618 | 3.0385 | 3.0375 |
| | (2.9103, 3.2314) | (2.8902, 3.2034) | (2.8896, 3.2033) |
| $autocorrelation^{IV}$ | 0.1043 | 0.0634 | 0.0637 |
| | (0.0749, 0.1336) | (0.0331, 0.0937) | (0.0334, 0.0940) |
| $IVvar$ | 0.0064 | 0.0034 | 0.0034 |
| | (0.0041, 0.0122) | (0.0021, 0.0070) | (0.0021, 0.0070) |

Notes. Posterior means and 95% confidence intervals (reported in parentheses) for the residuals from the currency return return and from the IV equations. Superscript $C$ stands for the residuals from the currency return equation, superscript $IV$ stands for the residuals from the $IV$ equation.

**Table 7**
**Model diagnostics for CHF**

| | SV ($\theta = 0$, $\theta_v = 0$) | SVJ ($\theta = 0$) | Preferred |
|---|---|---|---|
| $skewness^C$ | 0.1178 | 0.1282 | 0.0586 |
| | (0.0994, 0.1365) | (0.1078, 0.1486) | (0.0182, 0.0983) |
| $kurtosis^C$ | 3.9497 | 3.9438 | 3.4333 |
| | (3.8825, 4.0198) | (3.8919, 4.0011) | (3.3373, 3.5405) |
| $autocorrelation^C$ | -0.0203 | -0.0198 | -0.0272 |
| | (-0.0227, -0.0179) | (-0.0226, -0.0170) | (-0.0352, -0.0192) |
| $skewness^{IV}$ | 0.0224 | 0.0201 | 0.0210 |
| | (-0.0574, 0.1022) | (-0.0585, 0.0985) | (-0.0573, 0.0995) |
| $kurtosis^{IV}$ | 3.0648 | 3.0399 | 3.0406 |
| | (2.9091, 3.2378) | (2.8887, 3.2097) | (2.8890, 3.2094) |
| $autocorrelation^{IV}$ | 0.0777 | 0.0565 | 0.0564 |
| | (0.0459, 0.1094) | (0.0247, 0.0883) | (0.0246, 0.0881) |
| $IV var$ | 0.0010 | 0.0006 | 0.0006 |
| | (0.0007, 0.0017) | (0.0004, 0.0011) | (0.0004, 0.0011) |

Notes. Posterior means and 95% confidence intervals (reported in parentheses) for the residuals from the currency return and from the IV equations. $C$ stands for the residuals from the currency return equation, superscript $IV$ stands for the residuals from the $IV$ equation.

**Table 8**
**Model diagnostics for GBP**

| | SV ($\theta = 0$, $\theta_v = 0$) | SVJ ($\theta = 0$) | Preferred |
|---|---|---|---|
| $skewness^C$ | -0.0407 | -0.0211 | -0.0232 |
| | (-0.0606, -0.0202) | (-0.0436, 0.0012) | (-0.0609, 0.0143) |
| $kurtosis^C$ | 3.9181 | 3.8540 | 3.4947 |
| | (3.8427, 4.0061) | (3.7784, 3.9423) | (3.4006, 3.5969) |
| $autocorrelation^C$ | 0.0009 | 0.0006 | -0.0027 |
| | (-0.0024, 0.0040) | (-0.0038, 0.0047) | (-0.0094, 0.0037) |
| $skewness^{IV}$ | 0.0352 | 0.0212 | 0.0215 |
| | (-0.0443, 0.1146) | (-0.0565, 0.0995) | (-0.0568, 0.0998) |
| $kurtosis^{IV}$ | 3.0710 | 3.0293 | 3.0296 |
| | (2.9160, 3.2461) | (2.8798, 3.1972) | (2.8786, 3.1977) |
| $autocorrelation^{IV}$ | 0.0791 | 0.0510 | 0.0510 |
| | (0.0483, 0.1096) | (0.0204, 0.0814) | (0.0204, 0.0815) |
| $IVvar$ | 0.0011 | 0.0004 | 0.0004 |
| | (0.0007, 0.0019) | (0.0003, 0.0008) | (0.0003, 0.0008) |

Notes. Posterior means and 95% confidence intervals (reported in parentheses) for the residuals from the currency return and from the IV equations. $C$ stands for the residuals from the currency return equation, superscript $IV$ stands for the residuals from the $IV$ equation.

**Table 9**
**Model diagnostics for JPY**

| | SV ($\theta = 0$, $\theta_v = 0$) | SVJ ($\theta = 0$) | Preferred |
|---|---|---|---|
| $skewness^C$ | 0.3348 | 0.3360 | 0.1298 |
| | (0.3060, 0.3650) | (0.3038, 0.3668) | (0.0799, 0.1800) |
| $kurtosis^C$ | 4.8254 | 4.7148 | 3.6054 |
| | (4.7109, 4.9645) | (4.5982, 4.8361) | (3.4829, 3.7445) |
| $autocorrelation^C$ | -0.0146 | -0.0140 | -0.0221 |
| | (-0.0176 -0.0116) | (-0.0174, -0.0108) | (-0.0312, -0.0131) |
| $skewness^{IV}$ | 0.0568 | 0.0278 | 0.0311 |
| | (-0.0210, 0.1349) | (-0.0495, 0.1054) | (-0.0465, 0.1087) |
| $kurtosis^{IV}$ | 3.0707 | 3.0430 | 3.0423 |
| | (2.9175, 3.2420) | (2.8940, 3.2100) | (2.8923, 3.2098) |
| $autocorrelation^{IV}$ | 0.1042 | 0.0758 | 0.0768 |
| | (0.0733, 0.1349) | (0.0443, 0.1070) | (0.0453, 0.1083) |
| $IVvar$ | 0.0061 | 0.0029 | 0.0037 |
| | (0.0036, 0.0125) | (0.0017, 0.0059) | (0.0021, 0.0078) |

Notes. Posterior means and 95% confidence intervals (reported in parentheses) for the residuals from the currency return and from the IV equations. $C$ stands for the residuals from the currency return equation, superscript $IV$ stands for the residuals from the $IV$ equation.

**Table 10**
**Log-Bayes-Odds Ratios**

| | AUD | CHF | GBP | JPY |
|---|---|---|---|---|
| SVJ/SV | 22.03 | 52.05 | 44.50 | 34.89 |
| Preferred/SVJ | 26.36 | 18.77 | 13.43 | 61.22 |

Notes. Formal model comparison. We compare the SV ($\theta = 0$, $\theta_v = 0$), SVJ ($\theta = 0$) and the preferred models pairwise. In the first row, we consider the SV and SVJ models and quantify evidence against the SV model; in the second row, we consider the SVJ and the preferred models and quantify evidence against the SVJ model.

**Table 11**
**Calibration of the interest rates**

| | AUD | CHF | GBP | JPY |
|---|---|---|---|---|
| $a_r$ | 0.0181 | 0.0184 | 0.0184 | 0.0182 |
| $\tilde{a}_r$ | 0.0291 | 0.0121 | 0.0269 | 0.0077 |
| $b_r$ | 0.9999 | 0.9997 | 0.9997 | 0.9998 |
| $\tilde{b}_r$ | 0.9991 | 0.9995 | 0.9998 | 0.9997 |
| $\sigma_r$ | 0.0012 | 0.0016 | 0.0016 | 0.0014 |
| $\tilde{\sigma}_r$ | 0.0035 | 0.0030 | 0.0018 | 0.0027 |

Notes. We calibrate processes for domestic (US)

$$r_{t+1} = (1 - b_r)a_r + b_r r_t + \sigma_r r_t^{1/2} w_{t+1}^r$$

and foreign interest rates

$$\tilde{r}_{t+1} = (1 - \tilde{b}_r)\tilde{a}_r + \tilde{b}_r \tilde{r}_t + \tilde{\sigma}_r \tilde{r}_t^{1/2} \tilde{w}_{t+1}^r$$

Parameters correspond to daily interest rates in percent. There are four versions of the parameters corresponding to the US interest rate. This is because the foreign data have different starting dates, and we calibrated the US rate in the matching samples.

**Table 12**
**Summary of events associated with jumps**

| Type of event/uncertainty | Jump Up | Jump Down | Jump in Vol |
|---|---|---|---|
| | | **AUD** | |
| Trade | 1 | 1 | – |
| Macro-Economic | 2 | 2 | 9 |
| Intervention | – | 4 | 1 |
| Monetary policy | 2 | 7 | 10 |
| Spillover from financial markets | 1 | 5 | 20 |
| Other | 1 | 3 | 3 |
| Total | 6 | 21 | 34 |
| International | 2 | 5 | 17 |
| | | **CHF** | |
| Trade | 6 | – | – |
| Macro-Economic | 1 | 2 | 11 |
| Intervention | 6 | 2 | 2 |
| Monetary policy | 1 | 1 | 10 |
| Spillover from financial markets | 4 | 1 | 18 |
| Other | 2 | – | 12 |
| Total | 17 | 6 | 45 |
| International | 9 | 3 | 23 |
| | | **GBP** | |
| Trade | – | – | 1 |
| Macro-economic | 1 | 2 | 13 |
| Intervention | 2 | 1 | 3 |
| Monetary policy | 2 | 2 | 17 |
| Spillover from financial markets | 1 | – | 19 |
| Other | – | 2 | 14 |
| Total | 5 | 6 | 56 |
| International | 3 | 1 | 25 |
| | | **JPY** | |
| Trade | 11 | 3 | 6 |
| Macro-Economic | 4 | 3 | 11 |
| Intervention | 8 | 4 | 4 |
| Monetary policy | 2 | – | 16 |
| Spillover from financial markets | 7 | 1 | 15 |
| Other | 3 | 3 | 7 |
| Total | 34 | 13 | 52 |
| International | 12 | 4 | 19 |

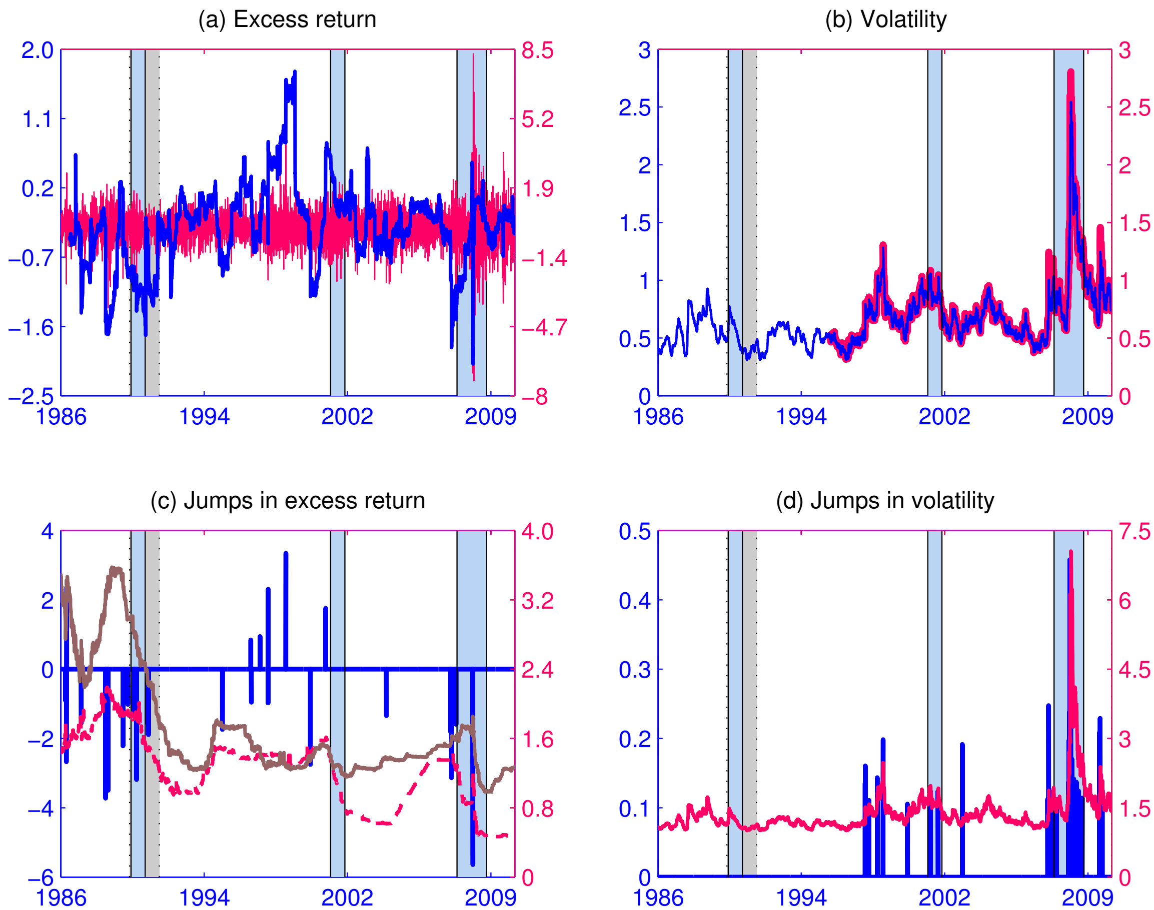
Notes: We match each jump in the preferred model with economic, political or financial events. If we cannot attribute a jump to a specific event then we indicate type of uncertainty dominating FX markets on that date. We compute how many jumps correspond to every economic source of risk. We distinguish trade, intervention, and monetary policy events (inflation, interest rate policy, monetary union) from events connected to other macro-economic factors (growth, employment, sales, payroll, etc.) We group episodes of dramatic movements in stock and commodity markets under the "Spillover from financial markets." All remaining episodes fall under rubric "Other". We report total number of jumps in prices (up and down) and volatility in the row "Total". We provide the number of jumps that occur simultaneously in two or more currencies in the row "International". Every jump episode can be generated by multiple sources of economic uncertainty. In such a case, we assign the jump to every important source of risk. Thus in our table the number in the row "Total" can be lower than the columnwise sum of the inputs.

**Table 13**
**Decomposition of the total risk**

| | Jump Up | | Jump Down | | Jump in Vol | | Normal | |
|---|---|---|---|---|---|---|---|---|
| | $n = 21$ | $n = 63$ | $n = 21$ | $n = 63$ | $n = 21$ | $n = 63$ | $n = 21$ | $n = 63$ |
| | | | | **AUD** | | | | |
| Mean | 6.90 | 6.37 | 9.17 | 8.40 | 4.12 | 10.18 | 79.81 | 75.05 |
| Std | 3.70 | 2.97 | 5.22 | 4.13 | 1.10 | 1.99 | 9.60 | 8.38 |
| Min | 0.31 | 0.38 | 0.60 | 0.74 | 1.58 | 4.45 | 55.17 | 56.33 |
| Max | 15.84 | 12.41 | 26.24 | 21.34 | 7.14 | 14.73 | 97.45 | 94.32 |
| | | | | **CHF** | | | | |
| Mean | 6.82 | 6.66 | 5.48 | 5.37 | 3.76 | 8.94 | 83.94 | 79.04 |
| Std | 2.56 | 2.03 | 1.82 | 1.50 | 0.27 | 0.39 | 4.32 | 3.34 |
| Min | 1.02 | 1.31 | 0.98 | 1.26 | 3.13 | 8.01 | 66.78 | 69.49 |
| Max | 16.34 | 11.88 | 12.04 | 10.55 | 4.84 | 9.91 | 94.87 | 89.38 |
| | | | | **GBP** | | | | |
| Mean | 2.98 | 2.78 | 3.68 | 3.45 | 4.37 | 11.48 | 88.96 | 82.29 |
| Std | 1.51 | 1.19 | 1.68 | 1.38 | 0.80 | 1.35 | 3.73 | 3.42 |
| Min | 0.18 | 0.23 | 0.25 | 0.32 | 2.64 | 7.79 | 74.21 | 72.60 |
| Max | 8.84 | 5.83 | 9.87 | 8.45 | 7.70 | 15.48 | 96.87 | 91.58 |
| | | | | **JPY** | | | | |
| Mean | 9.10 | 8.57 | 5.80 | 5.43 | 4.83 | 11.19 | 80.27 | 74.82 |
| Std | 4.18 | 3.32 | 3.53 | 2.96 | 0.71 | 1.08 | 7.56 | 6.10 |
| Min | 0.93 | 1.23 | 0.69 | 0.91 | 3.05 | 7.96 | 54.70 | 57.34 |
| Max | 22.04 | 18.05 | 17.68 | 13.89 | 6.82 | 13.61 | 65.20 | 89.67 |

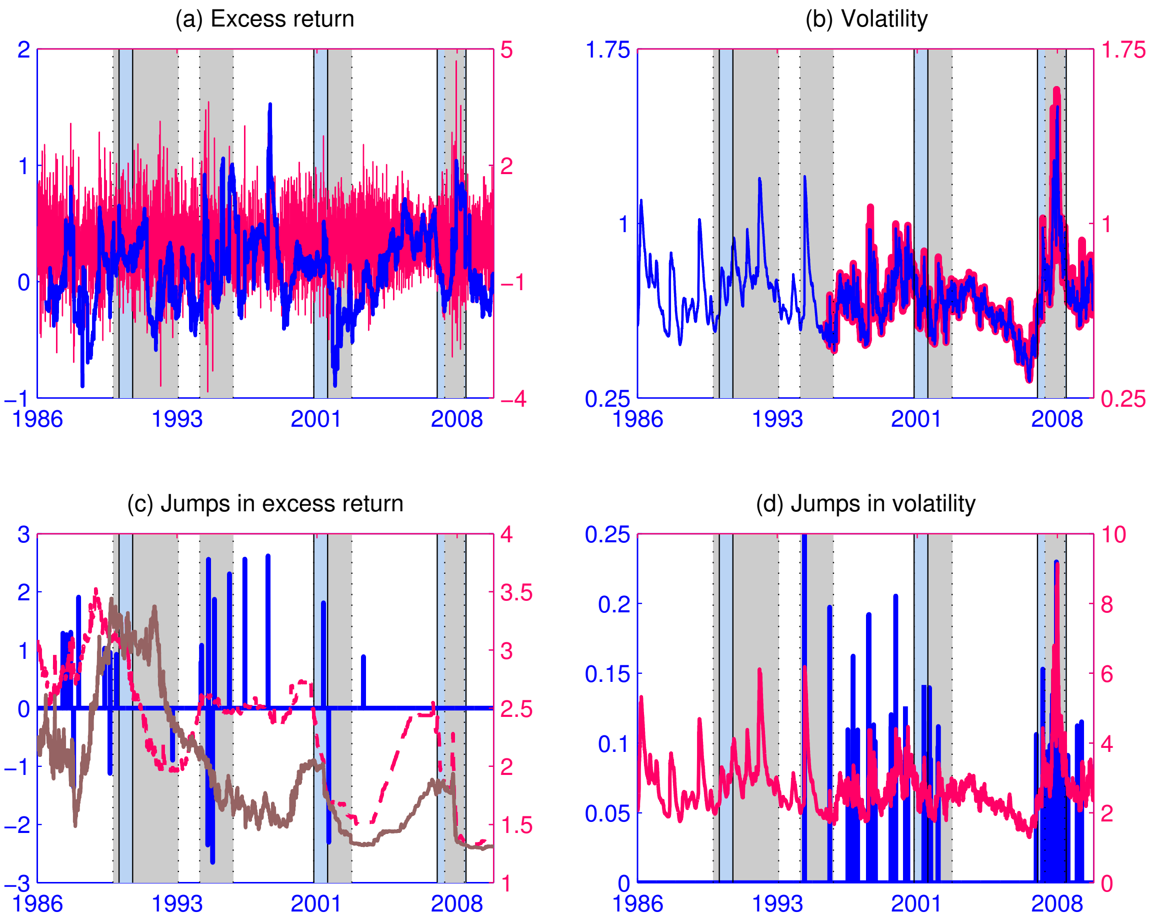
Notes. We report summary statistics of the percentage contribution of the different risks to the total risk of currency returns (horizon $n = 21$ and $n = 63$ days).

Figure 1: **AUD data and estimated states**



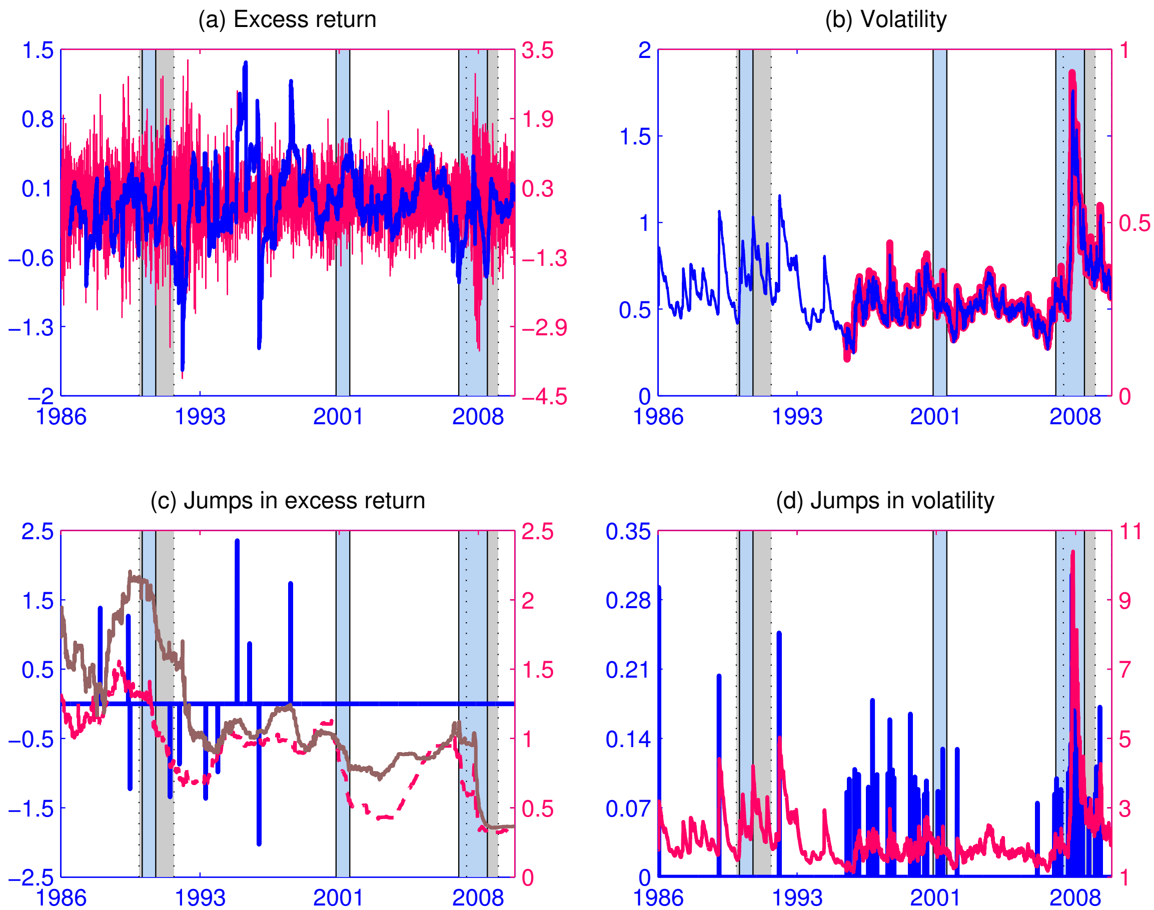

Panel (a): thin red line displays observed log currency returns; thick blue line displays the conditional 6-months skewness of the log currency returns. Panel (b): thick red line shows observed one-month at-the-money implied volatility; thin blue line shows the estimated spot volatility $\sqrt{v_t}$. The bottom panels display estimated jump sizes in returns (c) and volatility (d) with jump intensities. Panel (c): brown solid line is the intensity of the jump down; the red dashed line is the intensity of the jump up; blue bars are jumps themselves. Panel (d): the red line is the intensity of the jump in volatility; blue bars are jumps. We say that there was a jump at time $t$ if the estimated probability of a jump on that day was above 50%. Gray vertical bars with the dashed border indicate recessions in Australia; light blue vertical lines with the the thin solid border indicate recessions in the US.

Figure 2: **CHF data and estimated states**



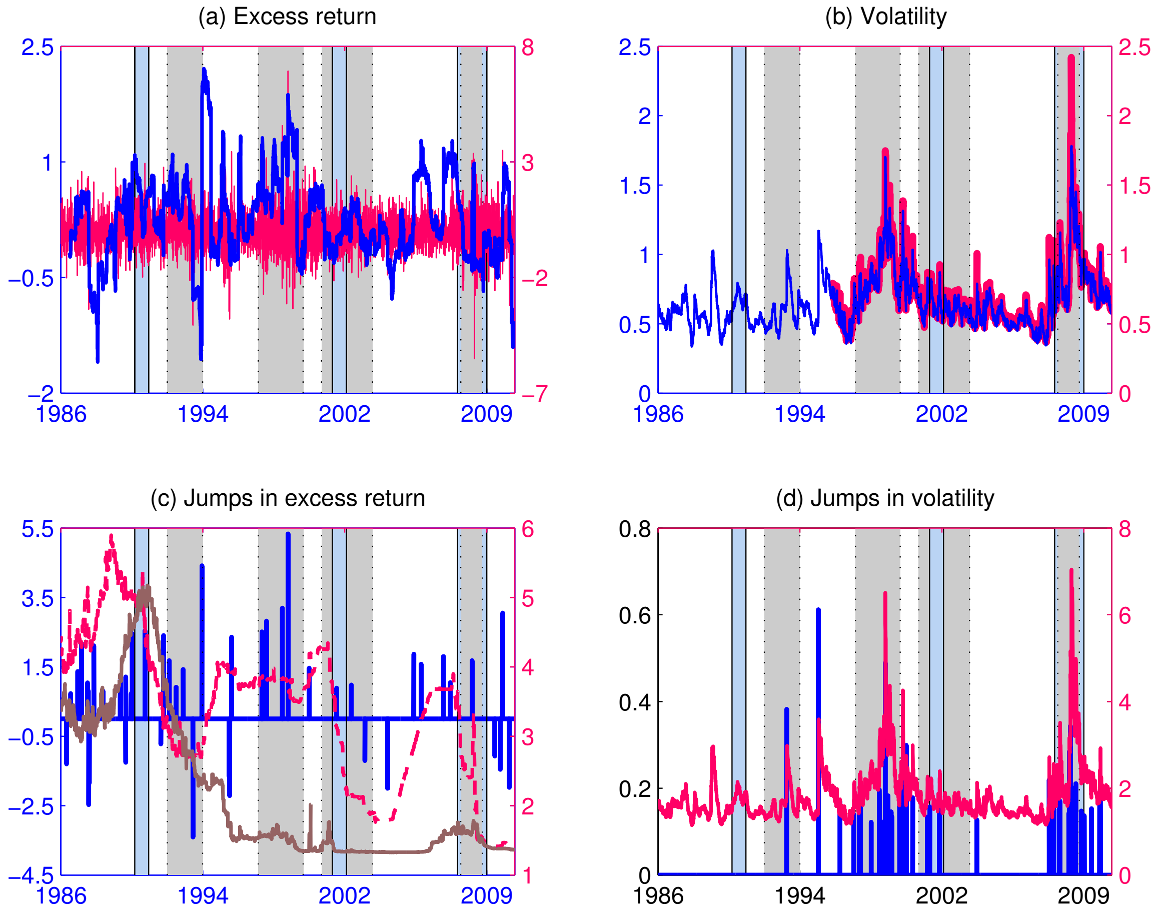

Notes. Panel (a): thin red line displays observed log currency returns; thick blue line displays the conditional 6-months skewness of the log currency returns. Panel (b): thick red line shows observed one-month at-the-money implied volatility; thin blue line shows the estimated spot volatility $\sqrt{v_t}$. The bottom panels display estimated jump sizes in returns (c) and volatility (d) with jump intensities. Panel (c): brown solid line is the intensity of the jump down; the red dashed line is the intensity of the jump up; blue bars are jumps themselves. Panel (d): the red line is the intensity of the jump in volatility; blue bars are jumps. We say that there was a jump at time $t$ if the estimated probability of a jump on that day was above 50%. Gray vertical bars with the dashed border indicate recessions in Switzerland; light blue vertical lines with the the thin solid border indicate recessions in the US.

Figure 3: **GBP data and estimated states**



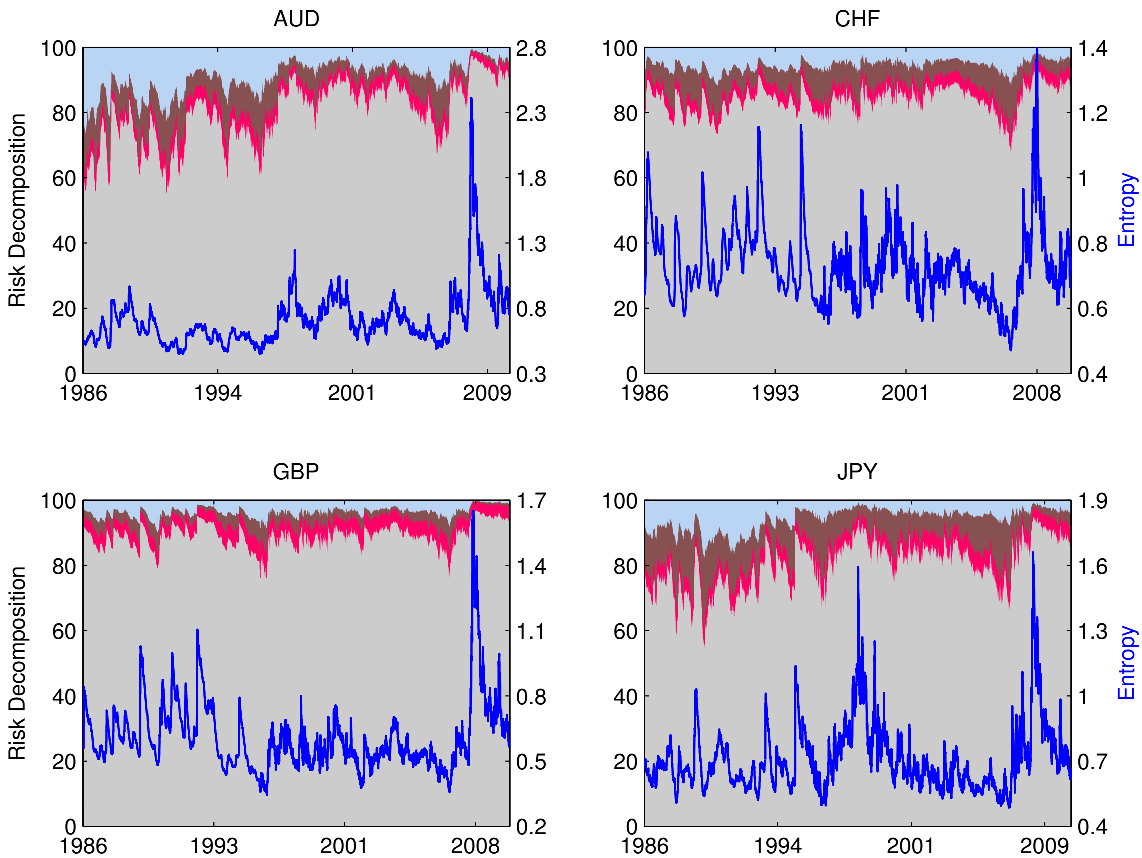

Notes. Panel (a): thin red line displays observed log currency returns; thick blue line displays the conditional 6-months skewness of the log currency returns. Panel (b): thick red line shows observed one-month at-the-money implied volatility; thin blue line shows the estimated spot volatility $\sqrt{v_t}$. The bottom panels display estimated jump sizes in returns (c) and volatility (d) with jump intensities. Panel (c): brown solid line is the intensity of the jump down; the red dashed line is the intensity of the jump up; blue bars are jumps themselves. Panel (d): the red line is the intensity of the jump in volatility; blue bars are jumps. We say that there was a jump at time $t$ if the estimated probability of a jump on that day was above 50%. Gray vertical bars with the dashed border indicate recessions in the UK; light blue vertical lines with the the thin solid border indicate recessions in the US.

Figure 4: **JPY data and estimated states**




Notes. Panel (a): thin red line displays observed log currency returns; thick blue line displays the conditional 6-months skewness of the log currency returns. Panel (b): thick red line shows observed one-month at-the-money implied volatility; thin blue line shows the estimated spot volatility $\sqrt{v_t}$. The bottom panels display estimated jump sizes in returns (c) and volatility (d) with jump intensities. Panel (c): brown solid line is the intensity of the jump down; the red dashed line is the intensity of the jump up; blue bars are jumps themselves. Panel (d): the red line is the intensity of the jump in volatility; blue bars are jumps. We say that there was a jump at time $t$ if the estimated probability of a jump on that day was above 50%. Gray vertical bars with the dashed border indicate recessions in Japan; light blue vertical lines with the the thin solid border indicate recessions in the US.

Figure 5: **Conditional decomposition of the total risk for monthly returns**

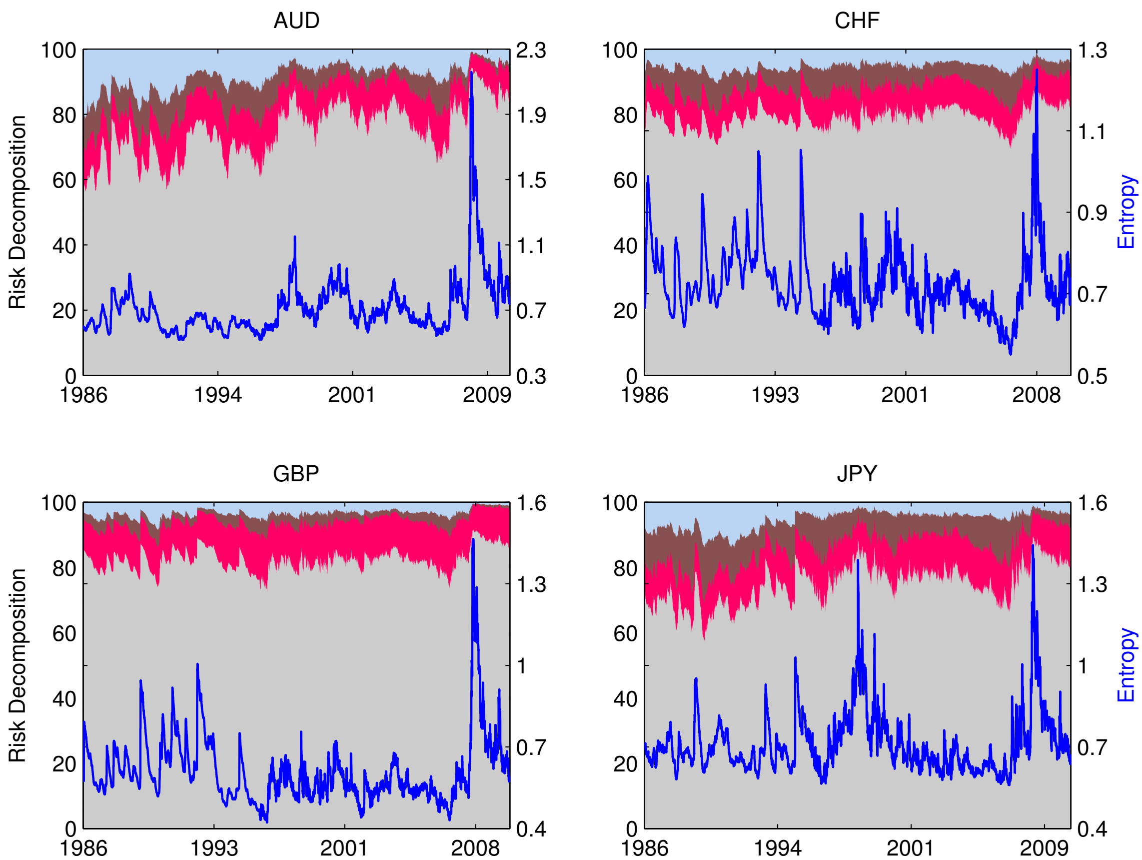



Notes. We display cumulative contribution of the different risks to the total risk of excess returns (the left axis). We measure the total amount of risk using entropy

$$L_t(S_{t+n}/S_t) = \kappa_{2t}(s_{t+n} - s_t)/2! + \kappa_{3t}(s_{t+n} - s_t)/3! + \kappa_{4t}(s_{t+n} - s_t)/4! + \dots,$$

where $\kappa_{jt}(s_{t+n} - s_t)$ is the $j$th cumulant of log FX returns. Investment horizon is $n = 21$ days. The contribution of the down jumps in FX is displayed in light blue, contribution of the up jumps in FX is in brown, and the contribution of the jumps in variance is in red. Gray area is the contribution of the normal shocks. The blue line shows $\sqrt{2L_t/n}$ in percent (the right axis). This quantity has an interpretation of one-period volatility in the case of normally distributed returns.

Figure 6: **Conditional decomposition of the total risk for quarterly returns**

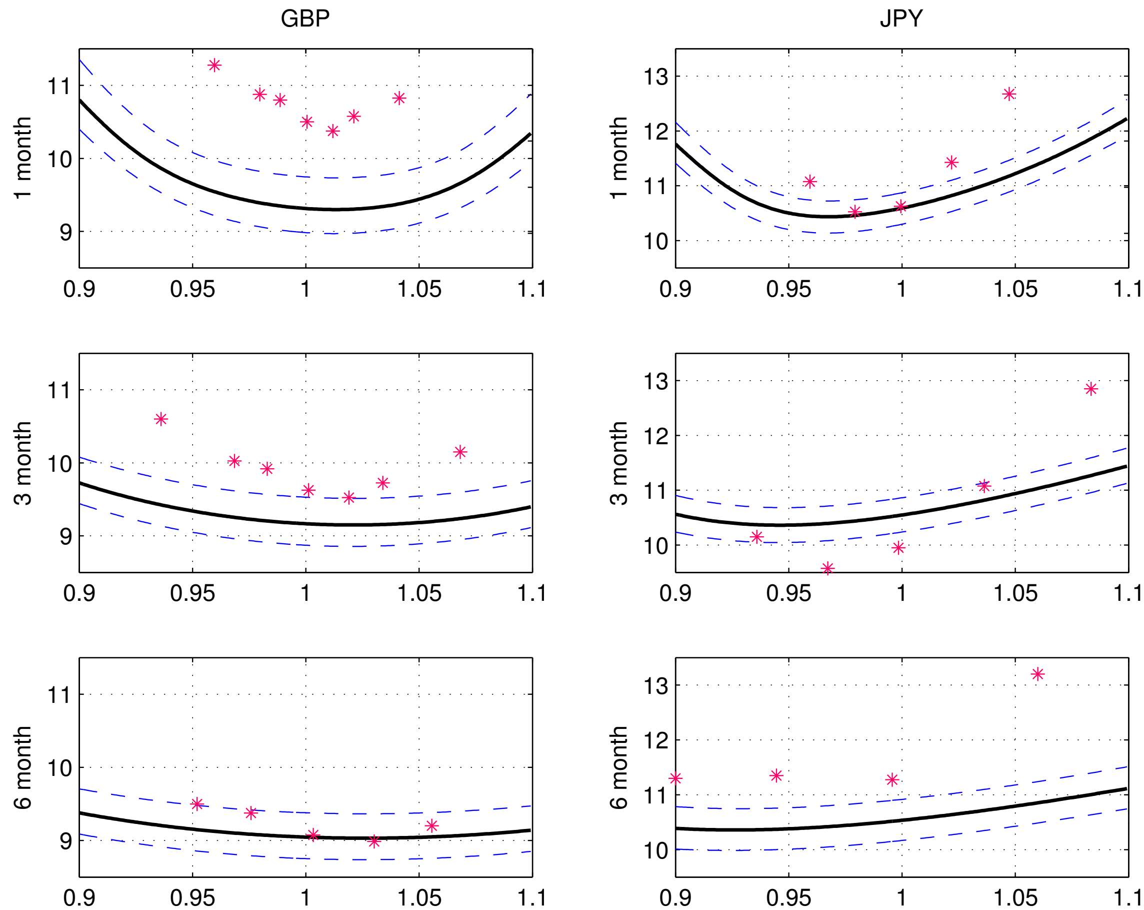



Notes. We display cumulative contribution of the different risks to the total risk of excess returns (the left axis). We measure the total amount of risk using entropy

$$L_t(S_{t+n}/S_t) = \kappa_{2t}(s_{t+n} - s_t)/2! + \kappa_{3t}(s_{t+n} - s_t)/3! + \kappa_{4t}(s_{t+n} - s_t)/4! + \dots,$$

where $\kappa_{jt}(s_{t+n} - s_t)$ is the $j$th cumulant of log FX returns. Investment horizon is $n = 63$ days. The contribution of the down jumps in FX is displayed in light blue, contribution of the up jumps in FX is in brown, and the contribution of the jumps in variance is in red. Gray area is the contribution of the normal shocks. The blue line shows $\sqrt{2L_t/n}$ in percent (the right axis). This quantity has an interpretation of one-period volatility in the case of normally distributed returns.

Figure 7: **Implied volatility**




Notes. We check the ability of the model to generate the implied volatility (IV) smiles and varying IV skewness. We select a currency with a positive average interest rate differential (GBP) and a negative one (JPY). For both currencies we pick a day with an approximately average variance and interest rate differential: November 12, 2007 (GBP) and April 20, 2004 (JPY). The asterisks indicate observed implied volatilities. The solid black lines depict theoretical implied volatilities evaluated at the estimated parameters and spot variance under the assumption that investors do not demand compensation for the variance or jump risks. The dashed blue lines show the 95% posterior coverage intervals for the theoretical smiles.

# A Appendix

## A.1 Expected future variance

We do not consider the most general model to streamline the presentation. We focus on the empirically relevant case where intensity of jumps in variance depends on variance only, and jumps up (down) in FX depend on domestic (foreign) interest rate only. We start by computing expectation of the variance process in (3.2). Conditional expectation $E_t(v_{t+i}) \equiv v_{t,i}$ can be computed via a recursion. Note that $v_{t,0} = v_t$. Suppose we know $v_{t,i-1}$. Then

$$\begin{aligned} v_{t,i} &= (1-\nu)v + \nu v_{t,i-1} + \sigma_v E_t(E_{t+i-1}(v_{t+i-1}^{1/2} w_{t+i}^v)) + E_t(E_{t+i-1} z_{t+i}^v) \\ &= (1-\nu)v + \nu v_{t,i-1} + \theta_v h_0^v + \theta_v h_v^v v_{t,i-1} = (1-\nu)v + \theta_v h_0^v + (\nu + \theta_v h_v^v) v_{t,i-1}. \end{aligned}$$

We can solve this recursion analytically:

$$\begin{aligned} v_{t,i} &= [(1-\nu)v + \theta_v h_0^v](1 + (\nu + \theta_v h_v^v)) + (\nu + \theta_v h_v^v)^2 v_{t,i-2} \\ &= [(1-\nu)v + \theta_v h_0^v](1 - (\nu + \theta_v h_v^v)^i)/(1 - (\nu + \theta_v h_v^v)) + (\nu + \theta_v h_v^v)^i v_t. \end{aligned}$$

Next, we can compute expectation of average future $v$ :

$$\begin{aligned} & E_t\left(\sum_{i=1}^n v_{t+i}\right)/n = 1/n \sum_{i=1}^n E_t v_{t+i} = 1/n \sum_{i=1}^n v_{t,i} \\ =\ & 1/n \sum_{i=1}^n [(1-\nu)v + \theta_v h_0^v](1 - (\nu + \theta_v h_v^v)^i)/(1 - (\nu + \theta_v h_v^v)) + 1/n \sum_{i=1}^n (\nu + \theta_v h_v^v)^i v_t \\ =\ & \frac{(1-\nu)v + \theta_v h_0^v}{1 - (\nu + \theta_v h_v^v)} \left[1 - \frac{\nu + \theta_v h_v^v}{n} \frac{1 - (\nu + \theta_v h_v^v)^n}{1 - (\nu + \theta_v h_v^v)}\right] + \frac{\nu + \theta_v h_v^v}{n} \frac{1 - (\nu + \theta_v h_v^v)^n}{1 - (\nu + \theta_v h_v^v)} v_t \\ \equiv\ & \underbrace{\frac{(1-\nu)v + \theta_v h_0^v}{1 - (\nu + \theta_v h_v^v)} [1 - \beta_n]}_{\alpha_n} + \beta_n v_t. \end{aligned}$$

Similarly, we can obtain conditional expectations of future interest rates:

$$r_{t,i} \equiv E_t(r_{t+i}) = a_r(1 - b_r^i) + b_r^i r_t,$$

and the expectations of average future interest rates:

$$\begin{aligned} & E_t\left(\sum_{i=1}^n r_{t+i}\right)/n = 1/n \sum_{i=1}^n E_t r_{t+i} = 1/n \sum_{i=1}^n r_{t,i} \\ =\ & a_r\left[1 - \frac{b_r}{n} \frac{1 - b_r^n}{1 - b_r}\right] + \frac{b_r}{n} \frac{1 - b_r^n}{1 - b_r} r_t \end{aligned}$$

and the similar expression holds for expectations associated with $\tilde{r}_t$.

Now, we can characterize the variance of excess returns:

$$v_t^y \equiv var_t(y_{t+1}) = v_t + 2h_t^u\theta_u^2 + 2h_t^d\theta_d^2.$$

Therefore, the conditional expectation of this variance can be computed on the basis of our results for the variance of the normal component $v_t$ and the expectations of interest rates:

$$v_{t,i}^y \equiv E_t(v_{t+i}^y) = v_{t,i} + 2\theta_u^2 h_0^u + 2\theta_u^2 h_r^u E_t(r_{t+i}) + 2\theta_d^2 h_0^d + 2\theta_d^2 \tilde{h}_r^d E_t(\tilde{r}_{t+i}).$$

This expression implies that the unconditional expectation, or long-run mean, of the variance is:

$$\begin{aligned} v_J = \lim_{i\to\infty} v_{t,i}^y &= [(1-\nu)v + \theta_v h_0^v]/(1-(\nu+\theta_v h_v^v)) \\ &+ 2\theta_u^2 h_0^u + 2\theta_u^2 h_r^u a_r + 2\theta_d^2 h_0^d + 2\theta_d^2 \tilde{h}_r^d \tilde{a}_r. \end{aligned}$$

When there are no jumps, that is, $\theta_v = 0$, $\theta_u = 0$, and $\theta_d = 0$, then $v_J = v$.

Next, we compute $E_t(\sum_{i=1}^n v_{t+i}^y)/n$

$$\begin{aligned} & E_t\left(\sum_{i=1}^n v_{t+i}^y\right)/n = 1/n\sum_{i=1}^n E_t v_{t+i}^y = 1/n\sum_{i=1}^n v_{t,i}^y \\ = \; & \alpha_n + 2\theta_u^2 h_0^u + 2\theta_u^2 h_r^u a_r \left[1 - \frac{b_r}{n}\frac{1-b_r^n}{1-b_r}\right] + 2\theta_d^2 h_0^d + 2\theta_d^2 \tilde{h}_r^d \tilde{a}_r \left[1 - \frac{\tilde{b}_r}{n}\frac{1-\tilde{b}_r^n}{1-\tilde{b}_r}\right] \\ + \; & \beta_n v_t + 2\theta_u^2 h_r^u \frac{b_r}{n}\frac{1-b_r^n}{1-b_r} r_t + 2\theta_d^2 \tilde{h}_r^d \frac{\tilde{b}_r}{n}\frac{1-\tilde{b}_r^n}{1-\tilde{b}_r}\tilde{r}_t. \end{aligned}$$

### A.2 Computing entropy

Entropy of currency changes over a horizon of $n$ days is equal to:

$$L_t(S_{t+n}/S_t) = \log E_t(e^{x_{t,n}}) - E_t(x_{t,n}) = k_t(1; x_{t,n}) - \kappa_{1t}(x_{t,n}),$$

where $x_{t,n} = \log(S_{t+n}/S_t) = \sum_{i=t}^{t+n}(s_{i+1} - s_i)$, $k_t(s; x_{t,n})$ is a cumulant-generating function of $x_{t,n}$, and $\kappa_{1t}(x_{t,n})$ is the first cumulant of $x_{t,n}$. Thus, we need to compute the cumulant-generating function of $x_{t,n}$ :

$$k_t(s; x_{t,n}) = \log E_t e^{s x_{t,n}}.$$

The first cumulant can be recovered as $\partial k_t(s; x_{t,n})/\partial s$ at $s = 0$. Denote the the drift of log currency changes by $\bar{\mu}_t = \mu_t + (r_t - \tilde{r}_t)$.

Guess

$$k_t(s;x_{t,n}) = A(n) + B_v(n)v_t + B_r(n)r_t + \tilde{B}_r(n)\tilde{r}_t.$$

Then

$$\begin{aligned}
& A(n) + B_v(n)v_t + B_r(n)r_t + \tilde{B}_r(n)\tilde{r}_t \\
= \; & k(s;x_{t,n}) = \log E_t[e^{sx_{t,1}}E_{t+1}e^{sx_{t+1,n-1}}] \\
= \; & \log E_t[e^{sx_{t,1}}e^{A(n-1)+B_v(n-1)v_{t+1}+B_r(n-1)r_{t+1}+\tilde{B}_r(n-1)\tilde{r}_{t+1}}] \\
= \; & A(n-1) + \log E_t e^{sx_{t,1}+B_v(n-1)v_{t+1}} + \log E_t e^{B_r(n-1)r_{t+1}+\tilde{B}_r(n-1)\tilde{r}_{t+1}} \\
= \; & A(n-1) + s\bar{\mu}_t + B_v(n-1)((1-\nu)v + \nu v_t) \\
+ \; & B_r(n-1)((1-b_r)a_r + b_r r_t) + \tilde{B}_r(n-1)((1-\tilde{b}_r)\tilde{a}_r + \tilde{b}_r\tilde{r}_t) \\
+ \; & \log E_t e^{s(1-\rho^2)^{1/2}v_t^{1/2}w_{t+1}^s + s\rho v_t^{1/2}w_{t+1}^v + sz_{t+1}^u + sz_{t+1}^d + B_v(n-1)\sigma_v v_t^{1/2}w_{t+1}^v + B_v(n-1)z_{t+1}^v} \\
+ \; & \log E_t e^{B_r(n-1)r_t^{1/2}\sigma_r w_{t+1}^r + \tilde{B}_r(n-1)\tilde{r}_t^{1/2}\tilde{\sigma}_r\tilde{w}_{t+1}^r} \\
= \; & A(n-1) + s\bar{\mu}_t + B_v(n-1)((1-\nu)v + \nu v_t) \\
+ \; & B_r(n-1)((1-b_r)a_r + b_r r_t) + \tilde{B}_r(n-1)((1-\tilde{b}_r)\tilde{a}_r + \tilde{b}_r\tilde{r}_t) \\
+ \; & s^2 v_t/2 + v_t s\rho\sigma_v B_v(n-1) + B_v^2(n-1)\sigma_v^2 v_t/2 + h_t^u((1-s\theta_u)^{-1} - 1) + h_t^d((1+s\theta_d)^{-1} - 1) \\
+ \; & h_t^v((1 - B_v(n-1)\theta_v)^{-1} - 1) + B_r^2(n-1)\sigma_r^2 r_t/2 + \tilde{B}_r^2(n-1)\tilde{\sigma}_r^2\tilde{r}_t/2 \\
= \; & A(n-1) + s(\mu + (\mu_r+1)(r_t - \tilde{r}_t) + \mu_v v_t) + B_v(n-1)((1-\nu)v + \nu v_t) \\
+ \; & B_r(n-1)((1-b_r)a_r + b_r r_t) + \tilde{B}_r(n-1)((1-\tilde{b}_r)\tilde{a}_r + \tilde{b}_r\tilde{r}_t) \\
+ \; & s^2 v_t/2 + v_t s\rho\sigma_v B_v(n-1) + B_v^2(n-1)\sigma_v^2 v_t/2 + s\theta_u(h_0^u + h_r^u r_t)/(1-s\theta_u) - s\theta_d(h_0^d + \tilde{h}_r^d\tilde{r}_t)/(1+s\theta_d) \\
+ \; & (h_0^v + h_v^v v_t)B_v(n-1)\theta_v/(1 - B_v(n-1)\theta_v) + B_r^2(n-1)\sigma_r^2 r_t/2 + \tilde{B}_r^2(n-1)\tilde{\sigma}_r^2\tilde{r}_t/2.
\end{aligned}$$

Collect terms, match them with the corresponding terms in the first line, solve for the coefficients:

$$\begin{aligned}
A(n) &= A(n-1) + s\mu + B_v(n-1)(1-\nu)v + s\theta_u h_0^u/(1-s\theta_u) - s\theta_d h_0^d/(1+s\theta_d) \\
&+ h_0^v B_v(n-1)\theta_v/(1-\theta_v B_v(n-1)) + B_r(n-1)(1-b_r)a_r + \tilde{B}_r(n-1)(1-\tilde{b}_r)\tilde{a}_r \\
B_v(n) &= s\mu_v + B_v(n-1)\nu + s^2/2 + s\rho\sigma_v B_v(n-1) + B_v^2(n-1)\sigma_v^2/2 \\
&+ h_v^v B_v(n-1)\theta_v/(1 - B_v(n-1)\theta_v), \\
B_r(n) &= s(\mu_r+1) + B_r(n-1)b_r + s\theta_u h_r^u/(1-s\theta_u) + B_r^2(n-1)\sigma_r^2/2, \\
\tilde{B}_r(n) &= -s(\mu_r+1) + \tilde{B}_r(n-1)\tilde{b}_r - s\theta_d\tilde{h}_r^d/(1+s\theta_d) + \tilde{B}_r^2(n-1)\tilde{\sigma}_r^2/2.
\end{aligned}$$

To compute initial conditions for the above recursion, write down the cumulant generating function of a one-period return:

$$k_t(s;x_{t,1}) \;=\; s\bar{\mu}_t + s^2 v_t/2 + (h_0^u + h_r^u r_t)\frac{s\theta_u}{1-s\theta_u} - (h_0^d + \tilde{h}_r^d\tilde{r}_t)\frac{s\theta_d}{1+s\theta_d}.$$

Therefore,

$$
\begin{aligned}
A(1) &= s\mu + h_0^u \frac{s\theta_u}{1 - s\theta_u} - h_0^d \frac{s\theta_d}{1 + s\theta_d}, \\
B_v(1) &= s\mu_v + s^2/2, \\
B_r(1) &= s(\mu_r + 1) + s\theta_u h_r^u/(1 - s\theta_u), \\
\tilde{B}_r(1) &= -s(\mu_r + 1) - s\theta_d \tilde{h}_r^d/(1 + s\theta_d).
\end{aligned}
$$