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GREAT MODERATION OR GREAT MISTAKE: CAN RISING CONFIDENCE IN LOW MACRO-RISK EXPLAIN THE BOOM IN ASSET PRICES?

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ABSTRACT

Great Moderation or Great Mistake: Can rising confidence in low macro-risk explain the boom in asset prices?*

The fall in US macroeconomic volatility from the mid-1980s coincided with a strong rise in asset prices. Recently, this rise, and the crash that followed, have been attributed to overconfidence in a benign macroeconomic environment of low volatility. This paper introduces learning about the persistence of volatility regimes in a standard asset pricing model. It shows that the fall in US macroeconomic volatility since the mid-1980s only leads to a relatively small increase in asset prices when investors have full information about the highly persistent, but not permanent, nature of low volatility regimes. When investors infer the persistence of low volatility from empirical evidence, however, Bayesian learning can deliver a strong rise in asset prices by up to 80%. Moreover, the end of the low volatility period leads to a strong and sudden crash in prices.

JEL Classification: D83, E32, E44 and G12
Keywords: asset prices, Great Moderation, macroeconomic risk

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"*From the Great Moderation to the Great Conflagration: The decline in volatility led the financial institutions to underestimate the amount of risk they faced, thus essentially (though unintentionally) reintroducing a large measure of volatility into the market.*"

Thomas F. Cooley, Forbes.com, 11 December 2008

"*The stress-tests required by the authorities over the past few years were too heavily influenced by behavior during the Golden Decade. [...] The sample in question was, with hindsight, most unusual from a macroeconomic perspective. The distribution of outcomes for both macroeconomic and financial variables during the Golden Decade differed very materially from historical distributions.*"

Andrew Haldane, Bank of England, 13 February 2009

"*But what matters is how market participants responded to these benign conditions. They are faced with what is, in essence, a complex signal-extraction problem. But whereas many such problems in economics involve learning about first moments of a distribution, this involves making inferences about higher moments. The longer such a period of low volatility lasts, the more reasonable it is to assume that it is permanent. But as tail events are necessarily rarely observed, there is always going to be a danger of underestimating tail risks.*"

Charles Bean, European Economic Association, 25 August 2009

"*The remaining question is whether the relaxation in financial prudence could have been triggered by false expectations of a perennially smooth economic environment that policymakers could have avoided in words and deeds.*"

Jean-Claude Trichet, European Central Bank, 5 September 2008

1 Introduction

The fall in macroeconomic volatility in the United States and other countries from the mid-1980s, later coined the "Great Moderation", coincided with a strong rise in asset prices. After the economic crisis that started in 2007, both policy-makers and academics attributed part of this rise, and the subsequent fall in prices, to overconfidence in the benign macroeconomic environment of the "golden decade" (Haldane 2009). According to this argument, in their attempt to infer the distribution of future shocks on the basis of observed data, investors overestimated the persistence of a low volatility environment, thus bidding up the price of assets beyond their fundamental value. This paper introduces learning about the persistence of volatility regimes in a standard asset pricing model. It shows that the fall in US macroeconomic volatility since the mid-1980s only leads to a relatively small increase in asset prices when investors have full information about the highly persistent, but not permanent, nature of low volatility regimes. When investors optimally infer the persistence of low volatility from empirical evidence using Bayes' rule, however, the model can deliver a much stronger rise in asset prices, although still smaller than observed in the data. Moreover, depending on the learning scheme, the end of the low volatility period leads to a strong and sudden crash in prices.

Previous studies have found that a fall in macroeconomic volatility of the magnitude observed in the United States between the late 1980s and the early 1990s would essentially have to be permanent to explain a significant proportion of the subsequent boom in equity prices (Lettau et al 2008). However, while some authors have attributed the Great Moderation to stuctural changes in developed economies that are indeed very persistent, or potentially permanent, such as central bank independence, the increase in world trade, or the development of new financial products to diversify risk, others have pointed to its transitory origins, such as an unusually long period of small exogenous shocks ("good luck") that hit Western economies during this period (see section II for more detail). Moreover, similar uncertainty about the origins and persistence of the Great Moderation can be found in statements by market participants. After the economic crisis that started in 2007, both policymakers and academics have attributed the boom in asset prices and their subsequent crash to the overconfidence of investors in a benign macroeconomic environment of low volatility (Bean 2009, Cooley 2008, Haldane 2009, Trichet 2008). For example, Haldane (2009) argues that data availability was such that the high volatility period preceding the Great Moderation was often neglected in the estimation of quantitative asset pricing models. Similarly, Bean (2009) attributes part of the boom and bust in asset prices to rising investor confidence that the low volatility environment would be permanent.
This paper looks at the behaviour of asset prices in an environment where investors have to infer the persistence of changes in macro-volatility from the data. Specifically, we interpret the economic experience of the US economy after the World War II as consisting of realisations of high and low volatility regimes, whose transition probabilities are unknown to investors. This allows us to analyse the behaviour of asset prices when investors use optimal Bayesian learning rules to infer the persistence of periods of low macro-volatility. Specifically, we study an economy where investors update their priors about transition probabilities in line with observed realisations of high and low volatility regimes according to Bayes' rule. In a standard specification where agents have a beta prior and thus attach positive probabilities to the whole range of persistence values (Sargent and Cogley 2008), the model delivers a boom and bust in asset prices of between 30 and 45 percent. With a two-point prior that captures the debate about the nature of the Great Moderation as either an unusually long sequence of small shocks ("good luck") or permanent

structural change ("good policy"), both boom and bust are about twice as large. Interestingly, the uncertainty around mean transition probabilities implied by learning increases the boom in asset prices, due to a strong Jensen's inequality effect not present in a full-information version of the model. As a robustness exercise, we also look at non-optimal, "adaptive" learning schemes, where investors use simple statistical rules to update their inference about volatility on the basis of observed data. This ad hoc learning results in strong overvaluation of assets, relative to the prices implied by full information about the data generating process, but does not yield a strong crash after the end of the Great Moderation (which we identify with the beginning of the economic crisis in 2007).

This paper is most related to the literatures on asset pricing with time-varying volatility, and with learning about features of the economic environment. After earlier papers on the effect of changes in economic volatility for asset prices in stationary environments (Bonomo and Garcia (1994, 1996) and Drifil and Sola (1998), more recently Bansal and Lundblad (2002)), Lettau et al (2008) ask whether a persistent change to a low macro-volatility regime can help explain the boom in US asset prices of the 1990s and early 2000s. They find that the low volatility environment would essentially have to be permanent to explain the data.[1]
Most papers that incorporate learning into asset pricing models look at environments where agents learn about the mean growth rate of output or consumption. For example, Zeira (1999) looks at asset price behaviour when investors continuously update their priors about the length of high productivity regimes. Cogley and Sargent (2008) assume that after the Great Depression, investors had pessimistic priors about the probability of transitions from a high to a low-growth state. Using a learning mechanism that is identical to one of those analysed in our study, they show how this may explain a sustained fall over time from an initially high equity premium, as learning leads to rising confidence in high growth. More recently, Adam and Marcet (2010) show how learning about an unknown process for cum-dividend equity returns introduces a self-referential element in equity prices that leads to persistent bubbles and occasional crashes. More related to this paper is a growing number of contributions that study learning about risk. Branch and Evans (2010) employ self-referential adaptive learning about asset prices and

[1]Lower macro-volatility is only one item on a long list of potential reasons behind the asset price boom of the 1990s and 2000s. Others are a lower equity premium (Blanchard (1993), Jagannathan, McGrattan, and Scherbina (2000), Fama and French (2002)), higher long-run growth (Jagannathan, McGrattan, and Scherbina (2000), Fama and French (2002), Campbell and Shiller (2004), although Siegel (1999) finds no evidence for this), stronger intangible investment in the 1990s (Hall (2000)) saving during the 1990s by the baby boom generation (Abel (2003)), redistribution of rents towards owners of capital (Jovanovic and Rousseau (2003)) or reduced costs of stock market participation and diversification (Heaton and Lucas (2000), Siegel (1999), Calvet, Gonzalez-Eiras, and Sodini (2003)).

return volatility in order to explain high frequency booms and busts in asset prices. Weitzman (2007) adopts a consumption-based asset-pricing model and replaces rational expectations with Bayesian learning about consumption growth rate volatility, which allows him to solve a number of asset pricing puzzles.

Most relevant for this paper are two studies that link the asset price boom and bust of 1990s and 2000s to learning about regime changes in key parameters of the economic environment. Boz and Mendoza (2010) study a partial equilibrium model where investors face an exogenous leverage constraint that follows a two-state Markov process with unknown transition probabilities. Assuming Bayesian learning as in Cogley and Sargent (2008), the authors show that with little prior information, the observation of a string of high leverage periods can lead to overoptimism about their persistence and thus a boom in asset prices, leverage and consumption which crashes abruptly once the economy switches back to a tighter constraint. While one of our learning mechanisms also follows Cogley and Sargent (2008), we analyse exogenous changes in macro-volatility, rather than in regimes of financial regulation. This focus is similar to that of Lettau et al. (2008) who also study the asset price effect of changes in macro volatility-regimes under limited information about the environment. Particularly, while knowing all parameters of the environment, including the persistence of volatility regimes, agents in their model ignore whether the economy is currently in a high or low volatility regime.[2] Rather than explicitly incorporating learning, they then calculate asset prices given the sequence of posterior state probabilities implied by an econometric regime-switching model estimated on post-war consumption data for the US. Asset prices are thus, essentially, weighted averages of full information prices. So the model-implied prices are always <u>lower</u> than those that would prevail in the most benign low-volatility regime with full information.

Our work differs from these studies, and the literature more generally, in several respects. First, based on our reading of the academic literature and the business press (see section II), we assume that agents were certain that the US economy had experienced a change in aggregate volatility with the Great Moderation, but were <u>uncertain</u> about its persistence. On the basis of this assumption, we are the first to systematically model the intuition, found in policy statements and the popular press, that increasing confidence in the persistent, or permanent, nature of the Great Moderation contributed to the boom and bust in asset prices in the 1990s and 2000s. Second, we show how Bayesian learning schemes that capture this intuition in a standard asset pricing model imply a boom in asset prices, and a subsequent bust if the economy returns to a regime of high volatility, of around 35 to 80 percent, much stronger than without learning, but

[2] Lettau et al (2008) also have two states of different mean growth, leaving four states of the economy in total.

smaller than in the data. Ad-hoc statistical learning schemes, on the other hand, can lead to an even stronger boom in prices, but do not explain a sudden fall once the economy returns to a high volatility regime. Finally, our work shows how, contrary to uncertainty about dividend realisations, uncertainty about the persistence of volatility regimes increases asset prices above certainty values. The reason for this can, as we demonstrate, be found in a Jensen's inequality effect that results from the strongly non-linear relationship between certainty prices and regime persistence.

The rest of the paper is organised as follows. To motivate our approach in more detail, section II reviews the main empirical facts on the Great Moderation as well as the debate about its causes among academics and market participants. Section III presents the model. Section IV presents the result for a standard Bayesian model of learning about transition probabilities between volatility regimes. Finally, section V shows how the results change when we make a different prior assumption designed to capture the suspicion about a possibly permanent Great Moderation, and for non-Bayesian learning schemes.

2 The Great Moderation, its uncertain cause and persistence, and the boom in asset prices

2.1 Asset Prices and the Great Moderation: Stylized Facts

Figure 1 and figure 2 present the time series of real GDP and consumption growth rates and their corresponding volatilities (computed as the standard deviation over 10-quarter rolling windows). Both series exhibit a significant and abrupt fall in volatility, which persisted until the beginning of the current crisis. The timing of the drop, however, differs: while GDP volatility declined around the middle of the 1980s, the fall occurred somewhat later for consumption growth, at the beginning of the 1990s.

Enter Figure 1 and 2 about here

Using quarterly data from $1952Q2$ to $2010Q2$, table 1 and 2 quantify this decline in volatility for different subperiods.[3] The end dates of the first subperiod are $1984Q1$ for GDP and $1992Q1$ for consumption[4], and the second period ends with the start of the financial crisis in 2007.

[3]See the Data Appendix for a more detailed description of the data series.

[4]McConnell and Perez-Quiros (2000) provide evidence that 1984Q1 was the break date for the GDP growth series and Lettau et al. (2008) provide evidence that 1992Q1 was the break date for the aggregate consumption growth series.

Moments of GDP growth

Date	*Mean* (%)	*StDev* (%)
$1952Q2 : 1983Q4$	0.53	1.1
$1984Q1 : 2006Q4$	0.51	0.51
$2007Q1 : 2010Q2$	-0.16	0.90

Table 1: The table reports the mean and standard deviation of the real GDP growth rate. Output is defined in real per-capita terms. GDP and the population data are taken from Bureau of Economic Analysis. The data are quarterly and span the period $1952Q2 - 2010Q2$.

Moments of Consumption Growth

Date	*Mean* (%)	*StDev* (%)
$1952Q2 : 1991Q4$	0.57	0.82
$1992Q1 : 2006Q4$	0.61	0.36
$2007Q1 : 2010Q2$	-0.19	0.50

Table 2: The table reports the mean and standard deviation of the real consumption growth rate. Consumption is defined in real per-capita terms. Consumption and population data are taken from BEA. The data are quarterly and span the period $1952Q2 - 2010Q2$.

Whereas there is almost no change in mean growth across the first two subperiods, there is a significant fall in volatility of more than 50 percent for both aggregate output and consumption growth. In the third sub-sample that covers the recent crisis, we observe a sharp decrease in mean growth for both GDP and consumption and a strong rise in volatility.

Enter Figure 3 and 4 about here

Figure 3 shows how the decline in macroeconomic volatility coincided with a strong rise in asset prices and a fall in the US dividend-price ratio for the $S\&P$ 500. Importantly, this fall was much less abrupt than the decline in volatility itself. The exact magnitude of the rise in US stock market valuation depends on the measure that is used to quantify payouts to shareholders. Figure 4 compares the price-dividend ratio (the solid line) to two other measures used in the literature. First, when measured relative to net earnings (the dotted line), apart from a lower absolute level, the time path of stock prices is very similar. The same is not true, however, when stock prices are measured relative to a dividend measure that includes payouts to shareholders via repurchases of stocks (the dashed line), which are attractive to firms because of the preferential tax treatment of capital gains relative to high incomes in the US. Specifically, the importance of repurchases has increased steadily after 1982, when SEC rule 10b-18 clarified the conditions under which firms could avoid an SEC investigation for market manipulation after a share repurchase, to reach a magnitude similar to dividend payments around the turn of the century.[5] Thus, the boom in asset prices between the early 1980s and the early 2000s is around half as strong when dividends are adjusted to include share repurchases. In correspondance to the previous tables, table 3 shows average stock price valuation measures for three subperiods, choosing 1995Q1, the period identified by Lettau et al (2008) as a structural break in the price-dividend ratio, as the start of the second subperiod. The price-dividend ratio more than doubled across the first two periods, while the price-earnings ratio increased more than 90 percent. The rise in the adjusted price-dividend ratio, for which data end in 2003, is with 60 percent about half as strong as that in the unadjusted measure. Both price-dividend and price-earnings ratios fell back to levels seen in the 1960s and 1970s with the start of the recent crisis.

The aim of this paper is to identify the contribution of rising confidence in the Great Moderation for the evolution of US asset prices over the last 30 years, rather than to replicate the exact magnitude of their observed rise in the data. We thus do not choose a preferred valuation ratio among the three measures discussed in this section. Rather we note that, as shown in figure 4, the rise in price-dividend and price-earnings ratios between the mid-1980s and the recent crisis was around 200 percent. The boom in a measure of the price-dividend ratio adjusted for share repurchases was, however, significantly lower. With the caveat that data on share repurchases are not available on a consistent basis for the whole period and that their cyclical nature makes averages over previous periods an imperfect guide to the latter part of the sample, a reasonable lower bound for the magnitude of the asset price boom should be around 100 percent.

2.2 Uncertainty about Origin and Persistence of the Great Moderation

By the second half of the 1990s, both the academic literature (Kim and Nelson (1999), McConnell and Perez-Quiros (1997, 2000)) and the business press had noticed a break in the volatility properties of US output growth around the middle of the preceding decade. Somewhat later, a similar decline in volatility was documented for a broader set of US macro-economic variables (Blanchard and Simon (2001), Stock and Watson (2005)), as well as for other industrial countries (Stock and Watson 2003). However, although the Great Moderation itself had become a stylised

[5]See Grullon and Michaely (2002) for details. Note that the share repurchase data are only available between 1971 and 2003.

US Equity Prices

$Date$	$Mean\ \frac{p}{d}$	$Mean\ \frac{p}{e}$	$Mean\ \frac{p}{d^{adj}}$
$1952Q2 : 1994Q4$	27.49	15.54	22.24
$1995Q1 : 2006Q4$	62.25	30.03	35.90
$2007Q1 : 2010Q2$	46.48	21.18	

Table 3: The table reports means of the price-dividend ratio $\frac{p}{d}$ and the price-earnings ratio $\frac{p}{e}$ for the $S\&P$ 500. $\frac{p}{d^{adj}}$ is the price-dividend ratio adjusted for share repurchases using the data by Boudoukh et al (2007). Their last available data relate to the year 2003, and the calculations are based on the assumption that repurchases are zero prior to 1971, as suggested by figure 4. As the sample of US firms in Boudoukh et al (2007) is slightly broader than that underlying the measures for PD and PE ratios, which are taken from Robert Shiller's homepage, the adjusted PD ratio is calculated as $PD^{adj} = PD\frac{PD^{adj\star}}{PD^{\star}}$, where a $\star$ denotes the measures presented in their paper.

fact, there was no consensus about its causes. While some authors explained the phenomenon by changes in the structure of industrial economies, such as financial innovation (Dynan et al 2006), improved inventory management, or financial and trade liberalisation (see Wachter (2006) for a brief summary), the two perhaps most prominent hypotheses competed under the heading of "Good Policy or Good Luck?". Specifically, following the seminal article by Stock et al (2003), several studies[6] used time-varying VAR models to find that a string of unusually small shocks, rather than changes in their transmission to main macroeconomic variables or in the conduct of monetary policy, were at the root of the decline in macro-volatility. Against this, both academics (Benati et al 2008) and policymakers (Tucker 2005, Bernanke 2004) argued that reduced-form models were likely to mistakenly take effects of improved monetary policy, such as more stable but unobserved inflation expectations, for changes in the variance-covariance-properties of exogenous economic shocks. For example, Bernanke (2004) argued that "some of the benefits of improved monetary policy may easily be confused with changes in the underlying environment". Importantly, the lack of consensus about the causes of the observed fall in macro-volatility left it unclear whether the phenomenon was likely to be permanent, as suggested by structural change or possibly improved policy environments, or transitory, in line with the "good luck" hypothesis.

How did market participants perceive the Great Moderation and its effect on prices? Investment analysts explicitly attributed part of the observed fall in the equity risk premium since

[6] Primiceri (2005), Sims and Zha (2006), and Canova, Gambetti, and Pappa (2007).

the late 1980s to the decline in macro-volatility. For example, it was noted in Goldman Sachs research (2002) that an estimated 8 percentage point fall in the risk premium since the 1970s was "underpinned by dramatic improvements in the economic environment. Inflation fell sharply, and the volatility of GDP growth, inflation and interest rates all declined significantly." (p. 2). But while investors acknowledged the effect of the Great Moderation on asset prices, they were also aware of the uncertain persistence of this low-volatility environment and thus, of the decline in equity premia. For example, regarding risk premia in fixed income securities, Unicredit analysts (2006) argued that "the ongoing deterioration in surprise risk should be seen as one of the arguments behind the declining risk premium. Whether this is due to a more effective central bank policy, a major improvement in the forecast ability of economic observers around the globe, sheer luck or maybe a mix of all three factors can't finally be answered." (p. 10). Researchers at JP Morgan (2005), on the other hand, attribute most of the fall in volatility to a changed orientation of policymakers towards a "Stability Culture" which, however, they see as uncertain to persist.

We draw three conclusions from this evidence: first, the fall of macro-volatility since the mid-1980s was accepted as a stylised fact, and widely seen as a contributing factor to higher asset prices during the 1990s and 2000s. Second, as shown by Lettau et al (2008), standard asset pricing models predict significantly higher asset prices during periods of low volatility only when the fall in volatility is permanent, or extremely persistent. Finally, during the Great Moderation it was exactly this persistence that investors were uncertain about. Therefore, this paper puts learning about the persistence of volatility changes at the center of its analysis. Particularly, we study if rising confidence in the persistence of low volatility can explain the strong and gradual rise in asset prices during the Great Moderation. Second, we look at the bust in asset prices implied by an end of the benign environment of low macro-volatility, which we compare to the fall in asset prices observed after the beginning of the recent crisis. And finally, we analyse how the asset price dynamics with learning compare to those in a full information version of the model.

3 The model

This section adds learning about the persistence of volatility regimes to a standard asset pricing model with recursive preferences as in Epstein and Zin (1989, 1991) or Weil (1989).

3.1 Preferences

We consider an endowment economy with an infinitely-lived representative agent who solves the following problem

$$\max_{C_t, S_t} \; U_t \tag{1}$$

$$s.t. \; S_t P_t + C_t = S_{t-1} P_t + D_t \tag{2}$$

$$S_{-1} \; given \tag{3}$$

where U_t denotes an expected utility index at time t, C_t denotes consumption, S_t are the agent's stockholdings, P_t is the stock price and D_t are dividends. Preferences U_t are as in Epstein and Zin (1989, 1991) or Weil (1989)

$$U(C_t) = [(1-\beta) C_t^{\frac{1-\gamma}{\alpha}} + \beta (E_t U_{t+1}^{1-\gamma})^{\frac{1}{\alpha}}]^{\frac{\alpha}{1-\gamma}}$$

where E_t is the mathematical expectation with respect to the agent's subjective probability distribution conditional on period t information, $\alpha = \frac{1-\gamma}{1-\frac{1}{\psi}}$, γ is the coefficient of relative risk aversion, and ψ the elasticity of intertemporal substitution.
The first-order condition associated with this problem is

$$P_t = E_t^s [M_{t+1}(P_{t+1} + D_{t+1})] \tag{4}$$

where M_{t+1} is the stochastic discount factor, which, with Epstein-Zin preferences, equals

$$M_{t+1} = (\beta (\frac{C_{t+1}}{C_t})^{-\frac{1}{\psi}})^{\alpha} R_{w,t+1}^{\alpha-1}$$

Here, $R_{w,t+1}^{\alpha-1}$ is the return on the aggregate wealth portfolio of the representative agent, equal to the aggregate consumption flow.

3.2 The Processes for Consumption and Dividend Growth

We choose a simple and transparent way of modelling an economy that goes through periods of low and high macro-volatility by assuming that log consumption follows an exogenous random walk with drift

$$g_t = \Delta \ln C_t = \bar{g} + \varepsilon_t$$

where $\bar{g}$ is constant mean consumption growth.[7] Shocks ε_t are independently normally distributed, and their variance follows a two-state Markov process

[7]Previous studies have looked at the time-variation in $\bar{g}$. Here, we assume $\bar{g}$ to be constant over time, and instead concentrate on changes over time in the variance of shocks ε_t.

$$\varepsilon_t \sim N(0, \sigma_t^2), \quad \sigma_t^2 \in \{\sigma_l^2, \sigma_h^2\}$$

The transition probabilities for the Markov process are

$$\begin{aligned}
\Pr(\sigma_{t+1}^2 &= \sigma_l^2 \mid \sigma_t^2 = \sigma_l^2) = F_{ll} \\
\Pr(\sigma_{t+1}^2 &= \sigma_h^2 \mid \sigma_t^2 = \sigma_h^2) = F_{hh}
\end{aligned}$$

which yields the transition probability matrix as

$$\mathbf{F} = \begin{bmatrix} F_{ll} & 1 - F_{ll} \\ 1 - F_{hh} & F_{hh} \end{bmatrix}$$

Following Mehra and Prescott (1985), and in line with the endowment nature of the economy, it is common to assume that dividend flows equal consumption flows. To capture the higher empirical volatility of dividends, we follow Campbell (1986), Abel (1999), Bansal and Yaron (2004) or Lettau et al (2008), and use a generalised version of the standard model where shocks to dividend growth are a multiple of those to consumption

$$\Delta lnD_t = \overline{g} + \lambda \varepsilon_t \quad \lambda \geq 1$$

Dividends thus follow the same volatility pattern as consumption, but are on average more volatile.

3.3 Full Information Price-Dividend Ratios

We can use the first-order condition for share holdings to express the price-dividend ratio $p_t = \frac{P_t^D}{D_t}$ as

$$p_t = (\rho_t)^{1-a} E_t[\beta^a (\frac{C_{t+1}}{C_t})^{-\frac{\alpha}{\psi}+\alpha-1} (\rho_{t+1}+1)^{\alpha-1} (p_{t+1}+1) \frac{D_{t+1}}{D_t}] \tag{5}$$

where $\rho_t = \frac{P_t^C}{C_t}$ is the price of a claim to aggregate consumption relative to its flow return and $\frac{P_t^C}{C_t}$ equals $\frac{P_t^D}{D_t}$ whenever $\lambda = 1$. When the agent knows the true structure of uncertainty, given the random walk nature of consumption and dividends, the price-dividend and price-consumption ratios are functions only of the volatility state, $p_t = p(\sigma_t^2)$, and thus non-random conditional on σ_t^2. Thus, we can simplify (5) by taking expectations across realisations of log-normal consumption and dividend growth conditional on σ_{t+1}^2, which gives a recursive expression for the price-dividend and ratios.[8]

Note that in the special case when $\lambda = 1$, both consumption and dividend growth follow the same log-normal distribution. With $\psi = \frac{1}{\gamma}$ (CRRA preferences), this yields an analytical solution to the vector of price-dividend ratios p as

$$p = \beta F(1+p)e^{(-\gamma+1)\bar{g}}(e^{(-\gamma+1)^2 2\sigma^2}) \tag{10}$$
$$= \mathbb{F}\beta F e^{(-\gamma+1)\bar{g}}(e^{(-\gamma+1)^2 2\sigma^2}) \tag{11}$$

where $\sigma^2 = [\sigma_l^2; \sigma_h^2]$ is the vector of volatilities, and $\mathbb{F} = [I - \beta F e^{(-\gamma+1)\bar{g}}(e^{(-\gamma+1)^2 2\sigma^2})]^{-1}$.

3.4 Learning and Subjective Beliefs

To study whether a long spell of σ_l can lead to a boom in asset prices by increasing the confidence in the persistence of a low-volatility environment, we assume that the representative agent does not know the full probabilistic structure of the economy. Specifically, the agent knows that log-changes of dividends are normally distributed with mean $\bar{g}$ but learns about the transition probabilities between volatility states F_{hh} and F_{ll} from observed transitions between high and low volatility. The agent thus knows the structure of the model and all parameter values except the true transition probabilities, F_{hh} and F_{ll}, which she aims to infer on the basis of the history of volatility-states $\Sigma^t = \{\sigma_t^2, \sigma_{t-1}^2, ..., \sigma_2^2, \sigma_1^2\}$. Thus, we assume that every period, the agent observes a dividend realization and the distribution that this specific realization is drawn from, parameterised by σ_t^2. We believe that this assumption captures well the evidence, contained in Section II, that market participants were certain of a change in the volatility around the mid-1980s, but uncertain about its persistence.

Our benchmark version of the model follows Sargent and Cogley (2008) and assumes that

[8]Specifically, for $F_{ij} = 1 - F_{ii}$ and $i, j \in \{h, l\}, i \neq j$, $p(\sigma_t^2)$ is defined by

$$p(\sigma_t^2 = \sigma_i^2) = \rho_i^{1-\alpha}\beta^\alpha e^{\left(-\frac{\alpha}{\psi}+\alpha\right)\bar{g}} \tag{6}$$
$$\left(F_{ii}e^{\frac{\left(-\frac{\alpha}{\psi}+\alpha-1+\lambda\right)^2}{2}\sigma_i^2}(1+\rho_i)^{\alpha-1}(1+p_i) + F_{ij}e^{\frac{\left(-\frac{\alpha}{\psi}+\alpha-1+\lambda\right)^2}{2}\sigma_j^2}(1+\rho_j)^{\alpha}(1+p_j)\right) \tag{7}$$

where $\rho = \frac{P_t^C}{C_t}$ follows

$$\rho^\alpha(\sigma_t^2 = \sigma_i^2) = \rho_i^{1-\alpha}\beta^\alpha e^{\left(-\frac{\alpha}{\psi}+\alpha\right)\bar{g}} \tag{8}$$
$$\left(F_{ii}e^{\frac{\left(-\frac{\alpha}{\psi}+\alpha\right)^2}{2}\sigma_i^2}(1+\rho_i)^{\alpha-1} + F_{ij}e^{\frac{\left(-\frac{\alpha}{\psi}+\alpha\right)^2}{2}\sigma_j^2}(1+\rho_j)^{\alpha}\right) \tag{9}$$

.

the agent has independent beta-binomial prior distributions about F_{hh} and F_{ll}

$$f_0(F_{hh}, F_{ll}) \propto f_0(F_{hh}) f_0(F_{ll})$$

with

$$\begin{aligned} f_0(F_{hh}) &= f(F_{hh} \mid \Sigma^0) = beta(n_0^{hh}, n_0^{hl}) \propto F_{hh}^{n_0^{hh}-1}(1-F_{hh})^{n_0^{hl}-1} \\ f_0(F_{ll}) &= f(F_{ll} \mid \Sigma^0) = beta(n_0^{ll}, n_0^{lh}) \propto F_{ll}^{n_0^{ll}-1}(1-F_{ll})^{n_0^{lh}-1}. \end{aligned}$$

where Σ^0 denotes a prior belief about frequencies n_0^{ij} of transitions from state i to state j. The agent updates this prior on the basis of the likelihood function L for the history of volatility states Σ^t conditional on F_{hh} and F_{ll}, which is the product of two independent binomial density functions, thus

$$L(\Sigma^t \mid F_{hh}, F_{ll}) \propto L(\Sigma^t \mid F_{hh}) L(\Sigma^t \mid F_{ll})$$

where

$$\begin{aligned} L(\Sigma^t \quad | \quad F_{hh}) &= binomial(F_{hh}, F_{hl}) \propto F_{hh}^{n_t^{hh}-n_0^{hh}}(1-F_{hh})^{n_t^{hl}-n_0^{hl}} \\ L(\Sigma^t \quad | \quad F_{ll}) &= binomial(F_{ll}, F_{lh}) \propto F_{ll}^{n_t^{ll}-n_0^{ll}}(1-F_{ll})^{n_t^{lh}-n_0^{lh}} \end{aligned}$$

Here, n_t^{ij} is a "counter" that equals the number of transitions from state i to state j up to time t plus the prior frequencies n_0^{ij}. The posterior kernel is the product of the beta prior and the binomial likelihood function,

$$f(F_{hh}, F_{ll} \quad | \quad \Sigma^t) \propto \underbrace{L(\sigma_t^2 \mid F_{hh}, F_{ll})}_{Likelihood} \cdot \underbrace{f(F_{hh}, F_{ll} \mid \Sigma^{t-1})}_{Prior}$$

which after normalizing by $M(\Sigma^t) = \int\int F_{hh}^{n_t^{hh}-1}(1-F_{hh})^{n_t^{hl}-1} F_{hh}^{n_t^{hh}-1}(1-F_{hh})^{n_t^{hl}-1} dF_{hh}F_{ll}$ yields the posterior density function as the product of independent *beta* distributions

$$\begin{aligned} f(F_{hh} \quad | \quad \Sigma^t) &= beta(n_t^{hh}, n_t^{hl}) \propto F_{hh}^{n_t^{hh}-1}(1-F_{hh})^{n_t^{hl}-1} \\ f(F_{ll} \quad | \quad \Sigma^t) &= beta(n_t^{ll}, n_t^{lh}) \propto F_{ll}^{n_t^{ll}-1}(1-F_{ll})^{n_t^{lh}-1} \end{aligned}$$

Note that in this context, the counters n_t^{ij} are sufficient statistics for the posterior.

Let $p(\sigma_t^2, F)$ denote the price-dividend ratio when the transition probability matrix is F. Following Cogley and Sargent (2008), p_t^{BL}, the vector of price-dividend ratios under Bayesian learning about transition probabtilities can then be written as

$$p_t^{BL} = \int p(\sigma_t^2, F) f(F, \Sigma^t) dF \tag{12}$$

where $f(F, \Sigma^t)$ is the posterior distribution of F[9]. Note that for given F_{hh}, F_{ll}, $p(\sigma_t^2, F)$ is described by the same pair of equations as under full information ((9), (7)). And the law of iterated expectations implies that we can compute $p(\sigma_t^2, F)$ as a fixed point of these two equations. p_t^{BL} can then easily be calculated by numerical integration across the independent beta posteriors for F_{hh}, F_{ll}.

4 Quantitative Results for the Benchmark Economy

4.1 The exercise

This section presents the results of numerical simulations to answer the two main questions of this paper: Can learning about the persistence of the Great Moderation explain the observed boom and bust in US asset prices? And can increasing confidence in this persistence lead to an overvaluation of assets, and a larger fall in prices at the end of the low-volatility period, relative to the case of full information? To answer these questions, we analyse a scenario that is similar to the economic experience of the US after World War II. In particular, we interpret this experience as a long realisation of high volatility followed by the Great Moderation that ends with the recent crisis. Our data generation process thus consists of three sequences of shocks corresponding to three subperiods of different consumption growth volatility σ_t^2. Specifically, our analysis starts with a high volatility regime in 1952Q2. Since in our highly stylised model, there is no distinction between consumption and GDP, we use a starting date for the Great Moderation at the beginning of 1984, as suggested by the fall in GDP volatility, but also look at later dates as suggested by the consumption growth series. In line with the observed rise in volatility in figure 1, we locate the end of the Great Moderation at the beginning of 2007, the starting year of the crisis. To compute the fall in asset prices around this end of the Great Moderation we also make the stronger assumption that the economy returned to the high volatility environment observed before the Great Moderation. This assumption is largely heuristical. It allows us to isolate the crash in asset prices implied by the end of the Great Moderation from other factors that this paper abstracts from.

[9]For a derivation of equation (12) see Appendix B.

4.2 Parameter choice

4.2.1 Preferences

As shown by Bansal and Yaron (2004), for a rise in consumption volatility to increase asset prices with Epstein-Zin preferences, the intertemporal elasticity of substitution ψ has to be greater than unity. Thus, we follow Lettau et al (2008) and set $\psi = 1.5$. For our statements about the size of boom and bust to be interesting, the model has to deliver a level of asset prices that is approximately equal to the data in the period before the Great Moderation. Rather than changing parameters across different learning rules to target asset prices exactly, however, we choose $\beta = 0.9935$ to target an interest rate of 2 percent p.a. (which varies very little across different specifications), and set $\gamma = 30$ which yields equity prices that are, on average across the versions of the model we analyse, close to US data, but not exactly equal to it for any particular specification.

4.2.2 The Process for Consumption and Dividends

Apart from the transition matrix $\mathbf{F}$, the consumption process in this model is characterised by three parameters: constant mean growth $\overline{g}$, and the standard deviations in the two subperiods σ_h, σ_l, which we estimate directly from quarterly data on US personal real per capita consumption expenditure, using the subperiods from table 1. This yields mean growth of 0.6 percent per quarter and standard deviations of 0.82 and 0.37 percent respectively.

When agents learn about the persistence of regimes from the observed transitions between low and high volatility periods, the matrix $\mathbf{F}$ that defines the underlying data generating process has no relevance for the equilibrium asset prices in the economy. However, to obtain benchmark values of asset prices in the absence of uncertainty about transition probabilities and without learning, we use a particularly simple ex-post estimate of $\mathbf{F}$, which we denote as the "full-information" transition probability matrix $\mathbf{F}^{FI}$. Specifically, we choose F_{ll}^{FI}, F_{hh}^{FI} such that the expected durations of high and low volatility regimes equal the subperiods identified from US data. So $F_{ii}^{FI} = 1 - \frac{1}{T_i}$, where T_l, T_h are the durations of the Great Moderation and the high-volatility period preceding it, which yields

$$\mathbf{F}^{FI} = \begin{bmatrix} 0.989 & 1 - 0.989 \\ 1 - 0.992 & 0.992 \end{bmatrix}$$

It is interesting to note that these transition probabilities are almost identical to those in

Parameter Values for the Benchmark Model		
Preferences		
β	0.9935	Discount Factor
γ	30	Relative Risk Aversion
ψ	1.5	Elasticity of Intertemporal Substitution
Endowment Process		
$\bar{g}$	0.0059	Mean of Consumption Growth
σ_l	0.0037	Low Standard Deviation of Consumption Growth
σ_h	0.0082	High Standard Deviation of Consumption Growth
λ	4.5	Leverage

Table 4: Parameter values in the benchmark model.

Lettau et al (2008), based on a more sophisticated estimated Markov process on the same data.[10]

Unless otherwise mentioned, we set $\lambda = 4.5$ as suggested by Lettau et al (2008) on the basis of the relative volatility of US consumption and dividends. Table 4 summarises the parameters of preferences and the endowment process for the benchmark model.

4.2.3 Learning Parameters

When agents learn about transition probabilities, the only remaining free parameters are those describing their beta prior distribution $f_0(F_{hh}, F_{ll})$. To be as agnostic as possible about the information agents have at the beginning of the scenario we analyse, we choose an uninformative prior distribution with initial parameters $n_0^{ij} = 1, \forall i, j$, for which the beta distribution coincides with the uniform distribution on $[0, 1]$. Note how the assumption of an uninformative prior, together with that of independence of F_{hh}, F_{ll}, implies that the increase in the persistence estimate during the course of the high volatility regime does not lead agents to assume any prior persistence for the low volatility regime. In other words, although agents have significantly changed their views about the durability of one of the two regimes, which they estimate to be highly persistent by the early 1980s, they continue to expect that any move to low volatility

[10] Their point estimates are

$$\mathbf{F} = \begin{bmatrix} 0.991 & 1 - 0.991 \\ 1 - 0.994 & 0.994 \end{bmatrix}$$

Their process is more complex, however, as they also include uncertainty about mean growth.

is, essentially, a short-lived outlier. To see to which degree the results depend on this, we also investigate the implications of an alternative assumption, that agents have a moderately persistent prior for the low volatility regime. Specifically, in this alternative case, we assume that $n_0^{ll} = 1.5, n_0^{lh} = 0.5$, so agents have the same amount of information as in the benchmark case, but with a moderate mean persistence of 0.75.[11]

4.3 Asset Price Dynamics with Learning

4.3.1 Rising Posterior Mean Persistence Gradually Increases Asset Prices

Figure 5 presents the time path of the PD ratio in US data in the upper panel. The bottom panel depicts both the PD ratio with learning from an uninformed prior (solid lines) and when agents take as certain the ex-post, full-information transition probability matrix F^{FI}. As a result of the calibration, and independently of learning, the model delivers realistic levels of asset prices, and thus a realistic equity premium, before the beginning of the Great Moderation. The model with full information delivers a small jump in prices of around 15% in 1985, but no sustained asset price boom during the Great Moderation. With learning, however, although the model is not able to replicate the hump-shape in PD ratios or their almost three-fold rise until 2007, we see a strong and gradual rise in prices of more than thirty percent as agents increase their persistence estimate of the Great Moderation. And importantly, when the low volatility period comes to an end at the end of 2007, the model predicts a strong fall in prices of 22%. Again, this is larger than in the full-information case, as the move back to the high-volatility regime does not only lead to a "switch" in the conditional probabilities (from the top to the bottom rows of the matrix $\mathbf{F}$), but also reduces the estimated persistence of the low-volatility regime through the addition of an observed regime change.

Enter Figure 5 and 6 about here

Figure 6 presents the same results for a moderately persistent prior at the beginning of the Great Moderation. The time path of price-dividend ratios has a shape very similar to that in figure 5, but the magnitudes are larger, with a boom of 42%, and a fall in asset prices at the end of the Great Moderation of 32 percent. Table 5 summarises the results.

Figure 7 illustrates, for the case of an uninformative prior, the learning dynamics underlying the path of asset prices by showing how the posterior probability distributions evolve during

[11] This is equivalent to giving equal weight to an uninformative prior and a distribution with the same amount of information and a mean equal to the posterior mean persistence for the high volatility regime.

Asset Prices - Benchmark Model

	Boom	Overvaluation	Bust
Full Info	15%	0	15%
Uninformed prior	32%	11%	22%
Moderately persistent prior	42%	20%	32%

Table 5: "Boom" denotes the increase in prices until the end of the Great Moderation relative to the high-volatility regime preceding it (computed over windows of five years). "Overvaluation" is the overvaluation at the end of the Great Moderation relative to the prices under full information. And "Bust" is the fall in prices in the first period after the Great Moderation.

the Great Moderation. Starting from the initial uniform distribution, probability mass becomes more and more concentrated at values close to 1, leading to the rise in asset valuation in figure 5 as agents predict low volatility to last longer on average.

Enter Figure 7 about here

4.3.2 Uncertainty about Transition Probabilities Increases Price Levels above their Full-Information Value

By construction of our ex-post estimate $\mathbf{F}^{FI}$, mean persistence at the end of the Great Moderation under learning is with 0.989 almost exactly equal to the persistence in the full-information case. In other words, the increase in mean persistence alone cannot explain the boom in prices under learning, where price-dividend-ratios rise significantly above the full information value at the end of the low-volatility period. The reason for this is an additional variance effect on prices that arises from the uncertainty about transition probabilities and is absent under full information. To understand this effect, it is important to note that certainty PD ratios are a strongly convex function of persistence at high values of F_{ll}. Under learning, where persistence values are dispersed around their mean value, this convexity implies a strong positive Jensen's inequality effect on PD ratios. For the simplified case of identical conditional distributions ($f(F_{hh}) = f(F_{ll})$), figure 8 illustrates this by showing how asset prices change as a function of both the mean and variance of the beta-distributed transition probabilities. The solid lines depict the value of PD ratios at high and low volatility in the absence of uncertainty, as a function of persistence $F_{hh} = F_{ll}$. As persistence rises, high-volatility prices fall, since agents are less willing to pay for assets whose payoffs they anticipate to remain volatile with a larger probability. Interestingly, low-volatility prices initially fall slightly, but rise strongly for high values of persistence above 0.995. The remaining lines in figure 8 show that this non-linearity of the certainty price leads to an increase in the level of prices as priors about persistence become looser.

Enter Figure 8 and 9 about here

Figure 8 does not explain why PD ratios under certainty are a convex function of persistence in the first place. Figure 9 gives a partial answer by plotting the diagonal and off-diagonal elements of the present discounted value matrix $V = \sum_{i=0}^{\infty} \beta^i \mathbf{F}^i = [I - \beta \mathbf{F}]^{-1}$ as a function of persistence $F_{hh} = F_{ll}$. As the figure shows, for other than very high persistence, the geometrically declining probability of remaining in the same state for $1, 2, ..., n$ periods leads to entries in V that are close to $\frac{1}{2}$, and thus, asset prices that differ little between regimes. Thus, it is the geometric nature of present discounted probabilities that leads to the highly non-linear relationship between asset prices and persistence in figure 8.

4.3.3 Sensitivity of the Results to Alternative Parameter Choices

This section briefly presents the benchmark results with an unchanged uninformative prior but alternative values of risk-aversion, leverage, and the starting date of the Great Moderation, summarised in table 6. The assumption of high risk-aversion was made to target price-dividend ratios that are close to those observed in the period before the Great Moderation. With lower risk aversion ($\gamma = 20$), the boom and bust in asset prices are only marginally reduced relative to the benchmark case, but the level of asset prices is about a quarter higher. When the relative volatility of log-dividend growth is reduced to $\lambda = 2.5$, both the boom and the bust are only about half as strong as in the benchmark calibration of $\lambda = 4.5$ that followed Lettau et al (2008) and their estimates of the relative volatility of dividends observed in post-war US data. Finally, with a later beginning of the Great Moderation in 1992, as suggested by the data for US consumption growth volatility, the boom is, with 25%, 7 percentage points smaller than in the benchmark case, although the bust is almost of the same magnitude.

This section has shown how, with learning about the transition probabilities between volatility regimes, a temporary moderation in macro-volatiltiy can lead to a gradual rise in asset prices by between 30 and 45 percent. This boom in prices is due both to an increase in mean persistence as agents observe low volatility persist, and to a convexity effect. Particularly, with uncertainty about transition probabilities around a mean that is almost identical to their full-information value, the positive probability of persistence values beyond 0.995, which, according

Asset Prices - Benchmark Model

	Boom	Overvaluation	Bust
Benchmark			
Uninformed prior	32%	11%	22%
Sensitivity			
$\gamma = 20$	30%	9%	18%
$\lambda = 2.5$	17%	6%	11%
Start of GM 1992Q1	26%	9%	19%

Table 6: "Boom" denotes the increase in prices until the end of the Great Moderation relative to the high-volatility regime preceding it (computed over windows of 5 years). "Overvaluation" is the overvaluation at the end of the Great Moderation relative to the prices under full information. And "Bust" is the fall in prices in the first period after the Great Moderation.

to figure 8, would warrant a significantly higher low-volatility price, strongly increases the observed price-dividend ratios during low-volatility regimes relative to the case of full information. By increasing low-volatility prices relative to an environment with full information, uncertainty about the value of regime persistence thus has a fundamentally different effect in this model from that of uncertainty about dividend realisations, which decreases prices.

5 Asset Prices under Alternative Learning Schemes

The previous section used a framework that is standard in the learning literature, with beta priors and Markov transitions as in Cogley and Sargent (2008) or Boz and Mendoza (2010), to investigate the effect of uncertainty and learning about the transition probabilities between volatility regimes on asset prices. This section looks at two alternative learning mechanisms. First, we replace the continuous beta prior with a discrete prior distribution that puts strictly positive mass on a persistence value of 1. This is in order to investigate a key-feature of the Great Moderation: as low-volatility persisted, market participants were increasingly suspicious of whether "this time it's different", and low volatility might in fact be a permanent feature of the post-1980s economy, rather than just a temporary regime. As a second alternative, we investigate asset price dynamics under ad hoc, or "recursive" learning schemes, where agents ignore the two-regime nature of the world and compute a best guess for average volatility using different statistics of the observed history.

5.1 "This Time it's Different": Learning when Low-Volatility is Suspected to be Permanent

Section II showed how statements by both policymakers and market participants linked the uncertainty about the durability of the Great Moderation to that about its potential causes, some of which were permanent and others temporary in nature. Our benchmark learning scheme, based on a standard mechanism in Cogley and Sargent (2008), has the advantage of being transparent and imposing relatively little structure on the agent's view about the economy apart from her knowledge of the two-regime nature of the data generating process. In this section, we propose an alternative learning scheme that tries to explicitly capture the uncertainty about the "permanent vs. transitory" character of the Great Moderation. Thus, we assume, as previously, that the agent updates a prior about transition probabilities between two volatility regimes on the basis of the observed history of volatility-states $\Sigma^t = \{\sigma_t^2, \sigma_{t-1}^2, ..., \sigma_2^2, \sigma_1^2\}$.. However, in contrast to the previous section, we assume that the prior about transition probabilities is a two point distribution. Specifically, agents attach a small prior probability of $\widehat{p}$ to $F_{ll} = 1$, or a permanent Great Moderation, and probability $1 - \widehat{p}$ to a persistent value in "normal times" of $F_{ll} = F_{ll}^0 < 1$. Note that, after a switch to low volatility, the conditional probability of observing T consecutive low-volatility periods declines with T if $F_{ll} = F_{ll}^0$, but equals 1 if $F_{ll} = 1$. More specifically, after a switch to low volatility, the likelihood of observing σ_l^N, a sequence of N additional low-variance periods, when $F_{ll} = F_{ll}^0$ is simply

$$L(\sigma_l^N|\sigma_t^2 = \sigma_l^2, F_{ll} = F_{ll}^0)) = P(\sigma_{t+1}^2 = \sigma_l, \sigma_{t+2}^2 = \sigma_l, ..., \sigma_{t+N}^2 = \sigma_l|\sigma_t^2 = \sigma_l^2, F_{ll} = F_{ll}^0) = (F_{ll}^0)^N$$

where $P(A|B, F_{ll})$ denotes the probability of event A conditional on event B and persistence F_{ll}. The posterior probability of a permanent great moderation, denoted $P(F_{ll} = 1|\sigma_l^N)$, thus increases with N according to Bayes' Rule

$$\begin{aligned} P(F_{ll} = 1|\sigma_l^N) &= \frac{P(F_{ll} = 1 \wedge \sigma_l^N)}{P(F_{ll} = 1 \wedge \sigma_l^N) + P(F_{ll} = F_{ll}^0 \wedge \sigma_l^N)} \\ &= \frac{\widehat{p}}{\widehat{p} + F_{ll}^{0\ N}(1 - \widehat{p})} \end{aligned} \tag{13}$$

We focus on the same scenario as in the previous section, designed to capture the experience of the US economy after World War II. Moreover, for simplicity, we abstract from uncertainty about transition probabilities in high-volatility times, and set F_{hh} equal to its ex-post estimate with probability 1.[12] The vector of price dividend ratios under Bayesian learning about a 'permanent

[12] As illustrated by figure 8, the effect of rising persistence of the current regime on asset prices during high-volatility regimes is much weaker, and more linear, than in low-volatility times. So the effect of uncertainty and learning on high-volatility asset prices is small, which justifies our simplification.

vs. transitory' Great Moderation, denoted p_t^{PT}, is then described by equations similar to (9) and (7). With $\lambda = 1$, this yields

$$p_{it}^{PT} = \beta e^{\left(-\frac{a}{\psi}+a\right)\bar{g}} \left(\mathbf{P}_{ii,t} e^{\frac{\left(-\frac{\alpha}{\psi}+\alpha\right)^2}{2}\sigma_i^2} (1 + p_{i,t+1}^{PT})^a + \mathbf{P}_{ij,t} e^{\frac{\left(-\frac{\alpha}{\psi}+\alpha\right)^2}{2}\sigma_j^2} (1 + p_{j,t+1}^{PT})^a \right)^{1/a} \tag{14}$$

where once more $i, j \in \{h, l\}$,$\mathbf{P}_{hj,t} = F_{hj},\ j = h, l$ and $\mathbf{P}_{lj,t}$ is the probability of moving from low volatility to regime j given the period t posterior probability of the change to low volatility being permanent in equation (13). Note, however, that price-dividend ratios under this learning schemes are not simply fixed points to equation (14). Rather, the representative agent anticipates that, should low volatility persist in the next period, the probability of a permanent change increases, as does the price-dividend ratio. Thus, we have to compute the whole path of price-dividend ratios jointly.[13]

To implement this model quantitatively, we set the conditional probability of the Great Moderation being permanent to $\widehat{p} = 1\%$. For the transition probability F_{ll}^0 in "normal times", the ex-post estimate $\mathbf{F}_{ll}^{FI}$ seems, at first sight, a natural choice. However, this estimate is based on the entire sequence of low-volatility periods during the Great Moderation. Its high persistence of 0.99 thus leaves little room for the suspicion, observed in the middle of the 1990s, that low-volatility might actually be permanent. Therefore, we set F_{ll}^0 such that the likelihood of 48 consecutive observations (12 years) of low-volatility is with 10 % sufficiently low to explain the debate about a potentially permanent Great Moderation in the second half of the 1990s. This results in a value of $F_{ll}^0 = 0.87$.

Enter Figure 10 about here

Figure 10 shows the time path that results from this learning scheme, as compared to US data. The rising posterior probability of a permanent moderation in macro-volatility over the

[13]This is done as follows: first, calculate the price dividend ratio for permanently low volatility $p(F_{ll} = 1)$ as a fixed point to (14) for $F_{ll} = 1$. Second, calculate the path of posterior probabilities $P(F_{ll} = 1|\sigma_l^N)$ as N rises, and the associated transition probabilities $\mathbf{P}_{Li,t}$. Once $P(F_{ll} = 1|\sigma_l^N)$ is close enough to 1, say after $\overline{N}$ low volatility periods, we know that $p_{l,t+\overline{N}}^{PT} = p(F = \mathbf{1})$. Third, we know that the high-volatility price-dividend ratio p_{ht}^{PT} is constant through time by the assumption of perfect information of F_{hh} and the fact that past low-volatility realisations have no impact on the prior $\widehat{p}$ for future switches to low-volatility. Choose a value for $p_{hs}^{PT}, s = t, t+1, ...$ and calculate the sequence of price-dividend ratios at low volatility p_{ls}^{PT} for $s = \overline{N}-1, \overline{N}-2, ...t$ by backward induction using (14), given $p_{l,t+\overline{N}}^{PT} = p(F = \mathbf{1})$ and the sequence $\mathbf{P}_{Li,t+s}, s = 1, ..., \overline{N}-1$. Fourth, calculate p_{ht-1}^{PT}, the price-dividend ratio at high volatility in period $t-1$, from F_{hh} and the values of p_{lt}^{PT} and p_{ht}^{PT} using (14). If $p_{ht-1}^{PT} = p_{ht}^{PT}$, we have found an equilibrium price sequence. If not, set $p_{hs}^{PT} = p_{ht-1}^{PT}, s = t, t+1, ...,$ and iterate.

Asset Prices - Learning about a Transitory vs. Permanent GM

	Boom	Overvaluation	Bust
No Learning	3%	0	3%
Learning	77%	79%	84%

Table 7: "Boom" denotes the increase in prices until the end of the Great Moderation relative to the high-volatility regime preceding it (computed over windows of five years). "Overvaluation" is the overvaluation at the end of the Great Moderation relative to the prices without learning. And "Bust" is the fall in prices in the first period after the Great Moderation (which can be higher than the "Boom", as the latter is calculated as the difference in averages over 20 quarters).

course of the Great Moderation leads to an S-shaped increase in prices. Importantly, the magnitude of the boom in table 7 is with 77% more than twice as large as in the benchmark learning scheme above. Moreover, the observed end of the Great Moderation comes with a strong bust in asset prices of 84%, as agents update the probability of being in a permanently more benign macroeconomic environment to zero. The reason for this stronger effect, relative to learning with a standard beta prior, is that the effect of additional information on the posterior probability is concentrated at the maximum persistence value of 1. Given the non-linearity of asset prices in figure 8, this leads to an effect that is stronger than the one resulting from a model where the increase in persistence affects the probabilities attached to all values in $[0, 1]$.

Enter Figure 11 about here

Figure 11 illustrates how the results depend on the assumptions about the conditional prior probability $\widehat{p}$ and the value of F^0_{ll}. Specifically, when the prior probability $\widehat{p}$ equals 0.1% (second panel of figure 11), the rise in prices is delayed, and the Great Moderation comes to an end before the posterior converges to 1. So the rise in prices is "cut off", and the resulting boom is with 56% somewhat smaller. Unsurprisingly, as the transition probabilities of the Great Moderation in normal times become highly persistent (third and fourth panel), the boom in prices all but vanishes because a long sequence of low volatility periods provides less reason to suspect a permanent change in the data generating process. Finally, the effect of lower risk aversion γ or leverage λ is very similar under this learning scheme to that in the benchmark economy[14].

[14]The results are available from the authors upon request.

5.2 Ad hoc learning

It has been argued by Haldane (2009), for example, that overconfidence in a low volatility environment may arise when agents base their inferences about the future predominantly on recent observations of small shocks. This over-reliance on the recent past is not captured by the optimal nature of the learning schemes considered so far, but in line with a large number of studies where agents follow ad hoc learning rules that map observations into estimates of parameters of interest (see for example Evans and Honkapohja (1999)). To see whether non-optimal learning rules can deliver a boom and bust in asset prices similar to those observed in US data, we assume that the representative agent knows the mean dividend growth $\bar{g}$ and observes the history of shocks $\Omega_t = \{\varepsilon_s\}_{s=0}^t$. But she ignores, or chooses to ignore, the two-stage nature of the data generating Markov process in her estimate about future macro-volatility. Rather, she uses simple ad hoc rules that map observed histories into estimates $\widehat{\sigma_{t+1}^2}$ of the variance of future shocks σ^2

$$\widehat{\sigma_{t+1}^2} = G(\Omega_t)$$

where $G : R^t \longrightarrow R^+$. Specifically, we consider three simple mappings G

$$G^{OLS} = \frac{1}{N}\sum_{s=0}^{t}(\varepsilon_s)^2 \tag{15}$$

$$G^{CG} = \xi(\varepsilon_t)^2 + (1-\xi)G_{t-1}^{CG} = \sum_{s=0}^{t}\xi(1-\xi)^{t-s}\varepsilon_t^2,\ 0 < \xi < 1 \tag{16}$$

$$G^{CW} = \frac{1}{n}\sum_{s=t-n}^{t}(\varepsilon_s)^2 \tag{17}$$

Thus, under G^{OLS} agents simply compute their best guess of the future variance as an average over the entire history of shocks. G^{CG} describes a simple "constant-gain" learning rule: the agent computes the variance as a weighted average of his best guess in the previous period and the squared shock today. Relative to G^{OLS}, this overweighs more recent observations, as the weight on more distant observations decays geometrically at the rate $1-\xi$. Finally, G^{CW} uses windows of the n most recent observations to compute the variance.

To implement the three ad-hoc learning rules quantitatively, we choose a window length of 20 years ($n = 80$), and a constant gain parameter of 3 percent. Figure 12 presents, for each of the three rules, averages over 120 realisations of the time path of asset prices, together with full-information prices. With OLS learning, the fall in the variance estimate for consumption growth is relatively slow. Moreover, since each estimate weighs all past periods equally, the variance estimate remains an average across high and low volatility periods, resulting in a relatively small

Asset Prices - Ad hoc Learning

	Boom	Overvaluation	Bust
Full Info	15%	0	15%
OLS	22%	4%	0%
Constant Gain	73%	48%	2%
Constant Window	82%	57%	1%

Table 8: "Boom" denotes the relative increase in prices until the end of the Great Moderation relative to the high-volatility regime preceding it (computed over windows of 5 years). "Overvaluation" is the overvaluation at the end of the Great Moderation relative to the prices under full information. And "Bust" is the fall in prices in the first period after the Great Moderation.

rise in prices. Nevertheless, the price rises above that under full information, as agents are more willing to pay for an asset with average volatility, compared to one whose payoff transits between periods of high and low volatility with moderate persistence.

Enter Figure 12 about here

With constant gain learning, the contribution of past periods to the variance estimate falls geometrically over time. This implies that the estimate of consumption variability during the Great Moderation falls faster, and further, than with OLS learning. The boom in prices is thus steeper and stronger, amounting to around 70 % at the end of the Great Moderation, far above that implied by full information. When agents compute their estimate of the consumption growth variance as an average across a window of constant length, asset prices reach an even higher plateau than under constant gain learning, although their path is slightly more convex, as estimates adjust more slowly at the beginning of a new regime.
Under all three ad-hoc learning rules, the fall in prices at the end of the Great Moderation is relatively slow: only as information about a change in volatility accumulates do agents adjust their estimates. Contrary to their Bayesian counterparts, recursive, ad hoc learning rules are thus not able to deliver sudden crashes in prices. Table 8 summarises the results for the benchmark case.

6 Conclusion

From a review of both academic and investment research, we conclude that, first, the "Great Moderation" in macro-volatility was perceived to be an important factor behind the asset price boom of the 1990s and 2000s, and, second, that academics and investors alike were uncertain about the origins and persistence of the new low-volatility environment. Using different learning mechanisms, we modelled this uncertainty explicitly in an asset pricing model with time-varying volatility. The results confirmed the intuition of policymakers (Bean 2009, Haldane 2009) that increasing confidence in a benign macroeconomic environment may have led to a strong and gradual increase in asset prices above values that are consistent with ex-post estimates of the persistence of volatility regimes. In particular, we find that Bayesian learning can lead to an asset price boom of around 30 to 45 percent in our benchmark learning model based on Cogley and Sargent (2008). This increase results from both an increase in posterior mean persistence and a pure Jensen's inequality effect that increases asset prices with uncertain transition probabilities above certainty levels. A similar learning scheme with a two-point prior distribution that highlights the uncertainty about a permanent vs. transitory Great Moderation leads to an even stronger boom in asset prices of almost 80 percent. The end of the low-volatility period, which we identified with the beginning of the recent crisis, leads to a strong crash in prices in both Bayesian models. Finally, ad hoc, or statistical, learning rules also predict a strong boom in prices, but do not predict a strong crash at the end of the Great Moderation period, as they react much more slowly to information than Bayesian learning schemes.

Future research could extend this study in several directions. For example, although mean growth during the Great Moderation was essentially the same as during the preceding period, it should be interesting to include time variation in the mean growth of the economy. Also, one could analyse an alternative scenario where agents directly form expectations about future prices, rather than the distribution of dividends as in the model studied here. Adam and Marcet (2010) show how this can lead to self-fulfilling bubbles and crashes in asset prices, as a rise in prices is sustained by generating expectations of rises in the future. When learning about volatility, this self-referential mechanism is less clear, as higher expected volatility primarily feeds into the level of prices, and not into their second moment. An in-depth analysis of this issue should be done in future work.

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7 Appendix

7.1 Data Appendix

Consumption is quantified as the *Total Real Personal Consumption Expenditures* measured in quantity index [index numbers, 2005 = 100]. The data are quarterly, seasonally adjusted and their source is the National Economic Accounts of the Bureau of Economic Analysis (BEA).

GDP is quantified as the *Real Gross Domestic Product*, measured in 2005-chained dollars. The data are quarterly, seasonally adjusted and their source is the National Economic Accounts of the Bureau of Economic Analysis (BEA).

Population is quantified as the *Midperiod Population* of each quarter. The data source is the National Economic Accounts of the Bureau of Economic Analysis (BEA).

Asset Price is quantified as the average *S&P 500 Stock Price Inde*x of each quarter. The data source is Robert Shiller's homepage. The original data are monthly averages of daily closing prices.

Dividend is quantified as the original quarterly *Dividend Payment* reported in the Robert Shiller's homepage.

Price-Earning Ratio is quantified as the *Cyclically Adjusted Price Earnings Ratio* (P/E10), known also as the *CAPE*. The data source is the Robert Shiller's homepage.

7.2 Appendix B

7.2.1 Solving for the equilibrium price numerically with Bayesian learning about Transition Probabilities

The Bayesian agent enters each period with a prior. He observes the realization of the exogenous process and he updates the counters

$$\begin{aligned} n_{t+1}^{ij} &= n_t^{ij} + 1 \; if \; s_{t+1} = j \; and \; s_t = i \\ n_{t+1}^{ij} &= n_t^{ij} \qquad if \qquad otherwise. \end{aligned}$$

The posterior density function is

$$f(F_{hh}, F_{ll} \mid \Sigma^t) = beta(n_t^{hh}, n_t^{hl}) * beta(n_t^{ll}, n_t^{lh}). \tag{18}$$

We would like to calculate

$$p_t = \int p(S_t, F) f(F \mid \Sigma^t) dF$$

which can be also expressed as

$$\int p(S_t, F) f(F \mid \Sigma^t) dF = E_{\Sigma^t}[p(F)]. \tag{19}$$

Therefore, equation (2) can be approximated as

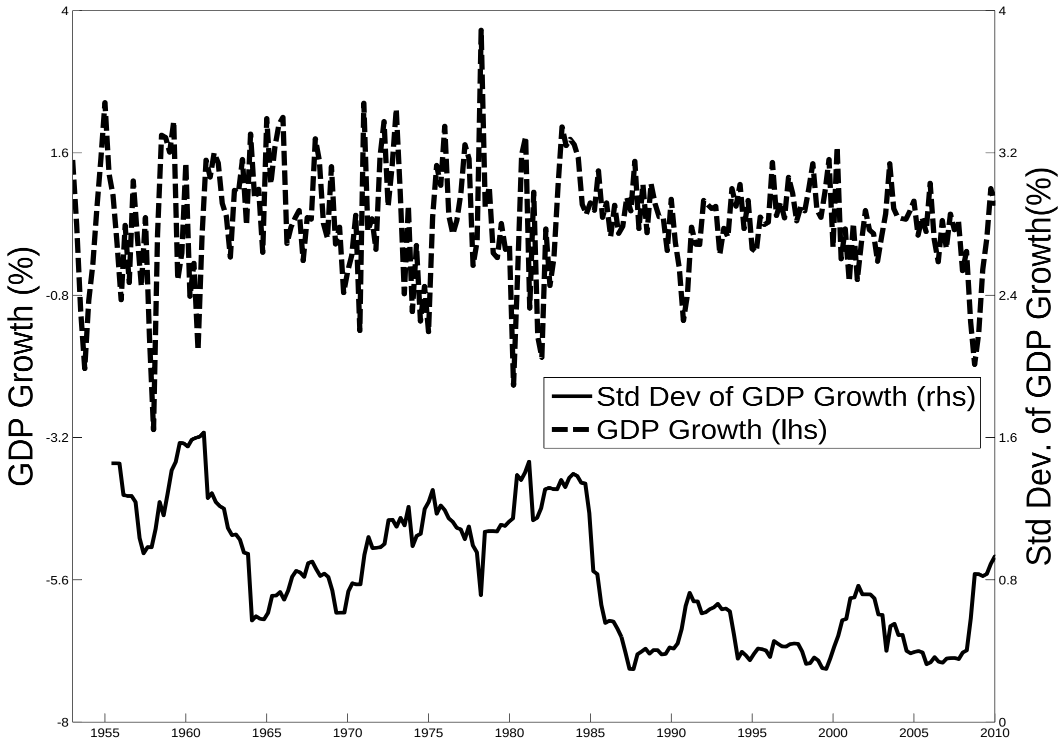
$$E_{\Sigma^t}[p(F)] \approx \frac{\sum_{i=1}^{n} p(S_t, F_i)}{n} \tag{20}$$

In order to compute equation (20) at each time t we generate a sample of $n = 3000$ transition probability matrixes, F, as random observations from equation (18) . Then, we approximate the price function by its sample average, so

$$p_t \approx \frac{\sum_{i=1}^{n} p(S_t, F_i)}{n}$$

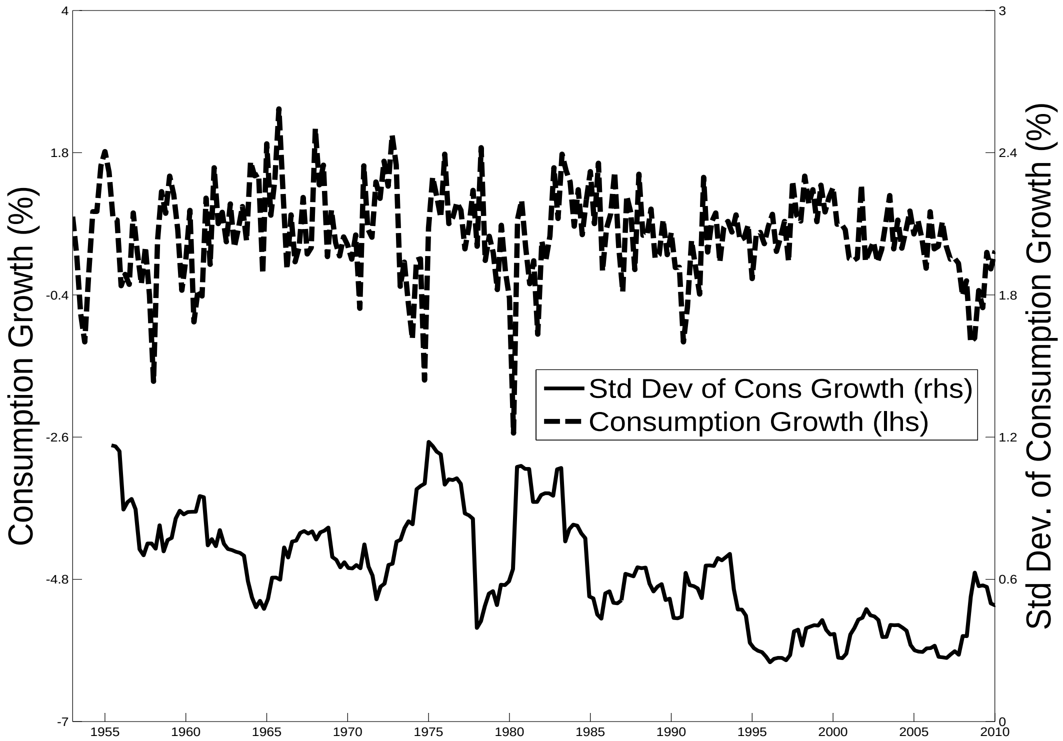
8 Figures

Figure 1: GDP growth



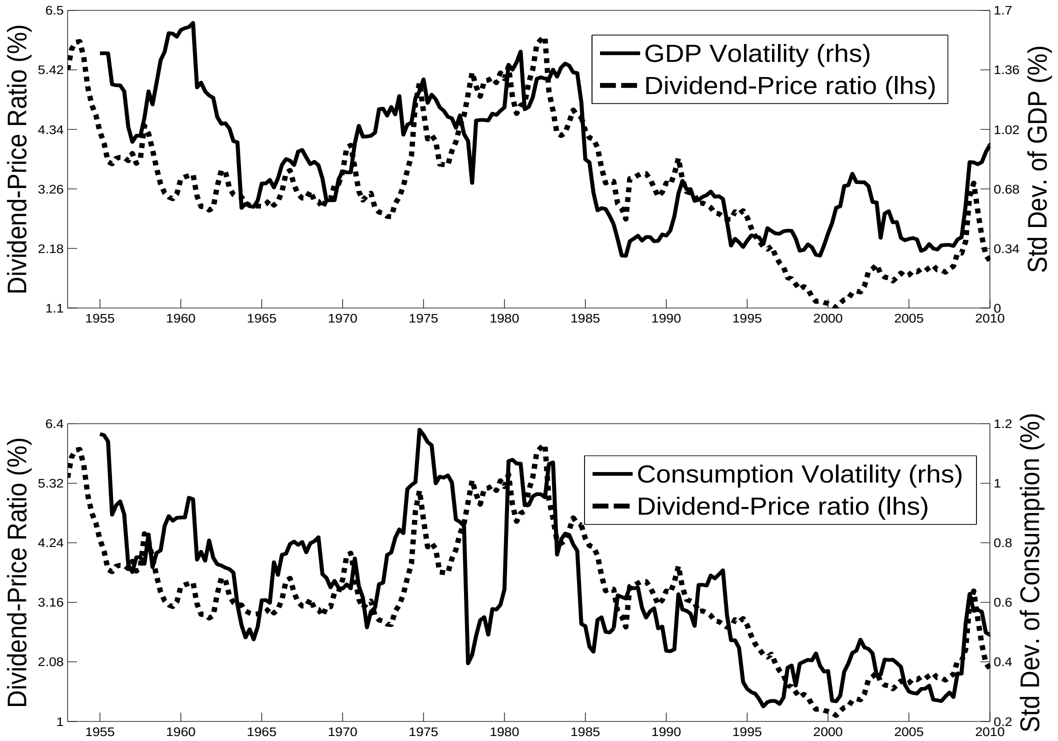
The figure plots the growth rate of real GDP and its standard deviation estimated in 10-quarter rolling windows. Output is defined in per-capita terms, calculated as the ratio of real gross domestic product, measured in 2005 dollars, over the total population. The data are quarterly and span the period $1952Q2 - 2010Q2$. The data are taken from the BEA.

Figure 2: Consumption growth



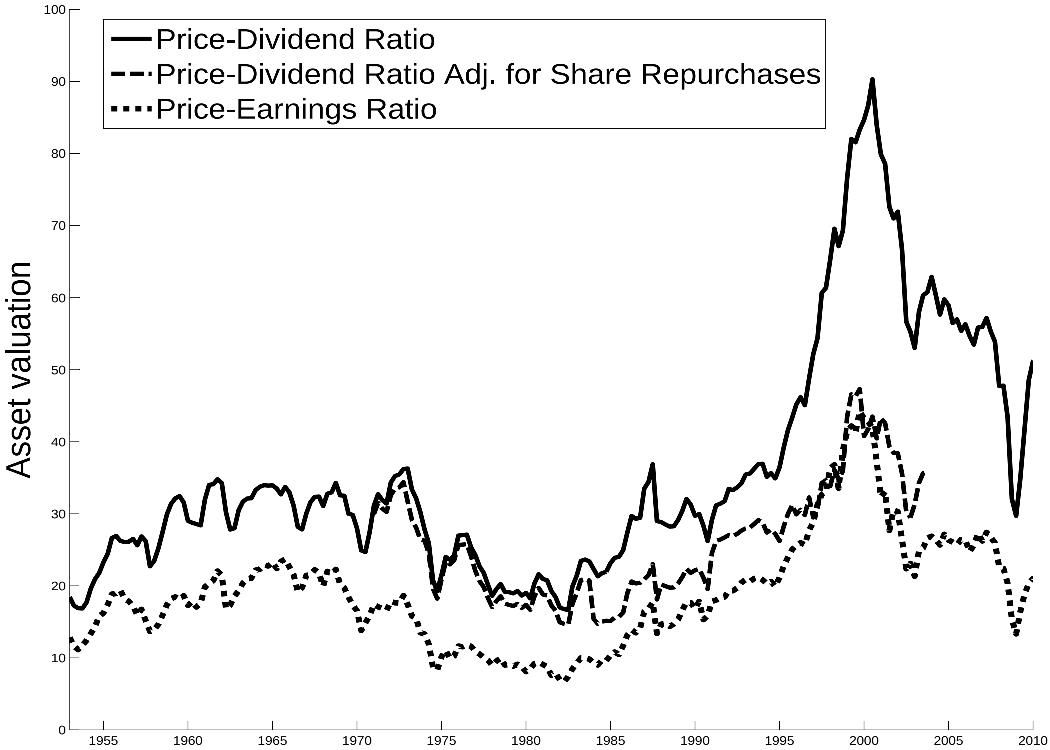
The figure plots the growth rate of real consumption and its standard deviation estimated in 10-quarter rolling windows. Consumption is defined in per-capita terms, calculated as the ratio of total real personal consumption expenditures, measured in quantity index [index numbers, 2005 = 100], over the total population. The data are quarterly and span the period $1952Q2 - 2010Q2$. The data are taken from the BEA.

Figure 3: Macro-Volatility and Asset Prices



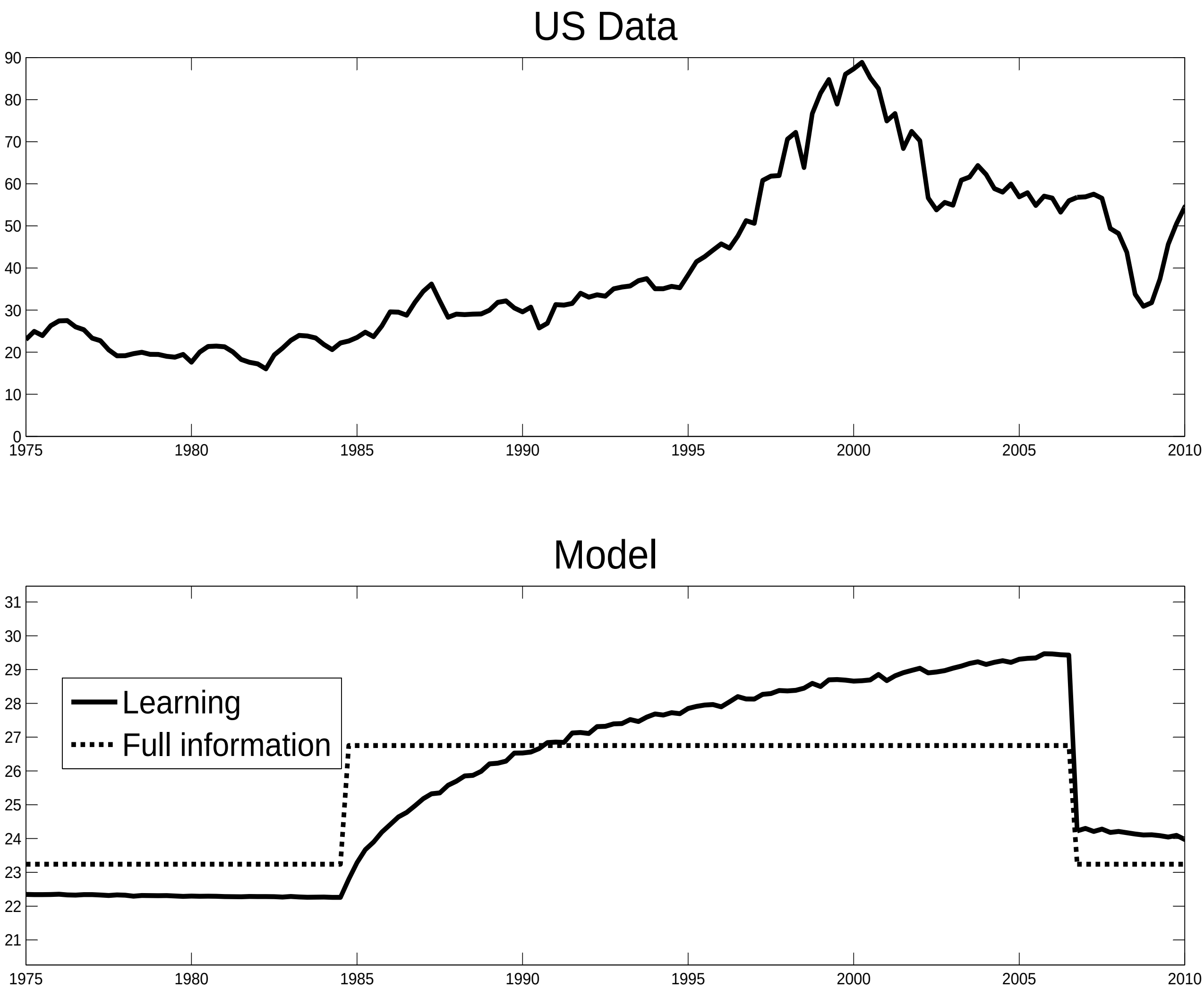
The figure plots the dividend price ratio together with the standard deviation of the real GDP growth rate (first subplot) and the standard deviation of the real consumption growth rate (second subplot), estimated in 10-quarter rolling windows. GDP and consumption are defined as in figures 1 and 2. The financial data are taken from the Robert Shiller's homepage and the rest of the data from the BEA.

Figure 4: Stock Market Valuation Measures



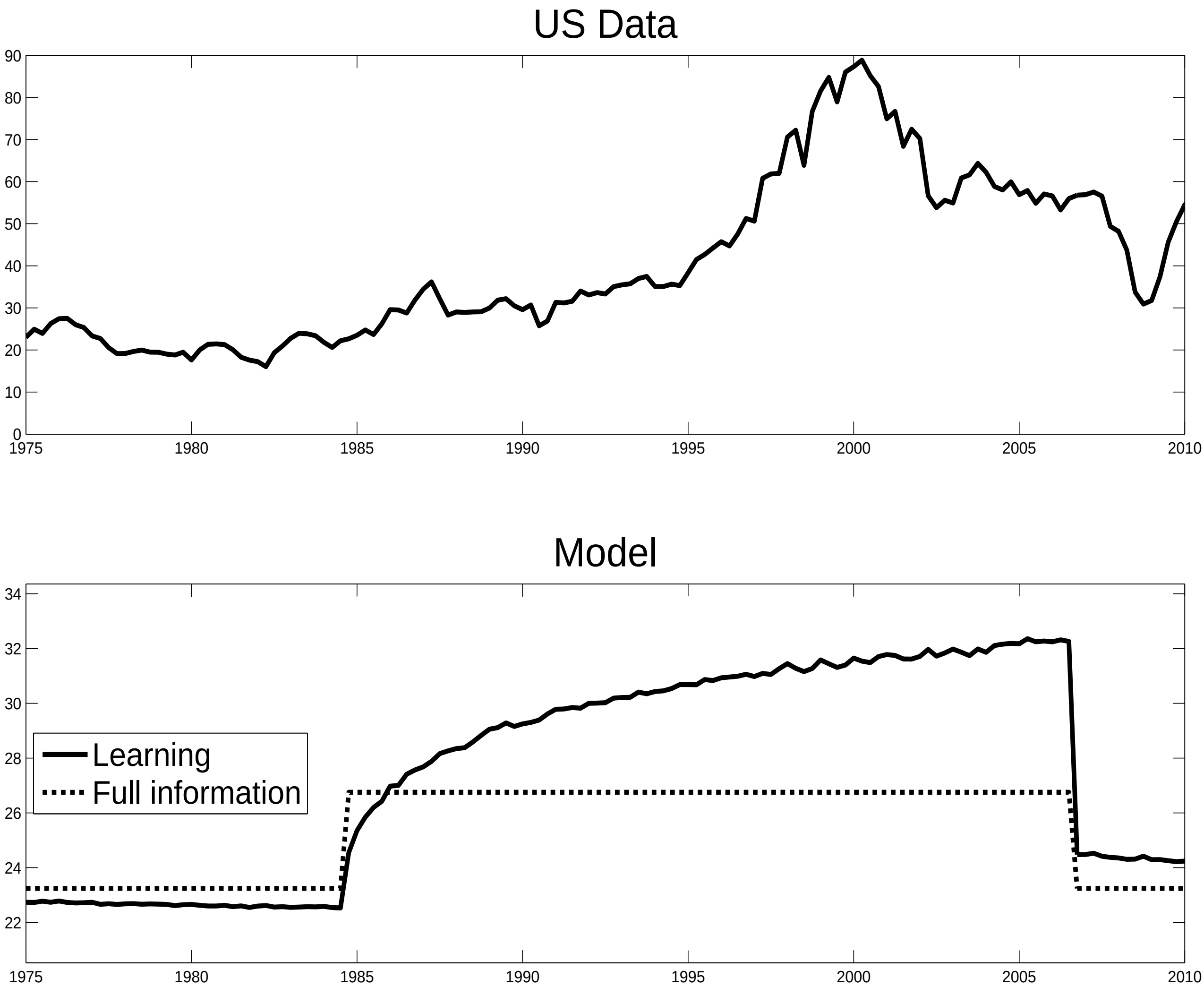
The figure plots the price-dividend and the price-earnings ratio for the $S\&P$ 500, as well as the price-dividend ratio adjusted for share repurchases using the data by Boudoukh et al (2007), which is available between 1971 and 2003. As their sample of US firms is slightly broader than that underlying the measures for PD and PE ratios, which are taken from Robert Shiller's homepage, the adjusted PD ratio is calculated as $PD^{adj} = PD\frac{PD^{adj\star}}{PD^{\star}}$, where a $\star$ denotes the measures presented in Boudoukh et al (2007). The data on prices are monthly, those on dividends and on the price-earnings ratio are quarterly, and those on the adjusted price-dividend ratio is yearly. We calculate quarterly estimates for the prices by taking quarterly averages over the monthly data, and using the yearly observation for the adjusted series in all 4 quarters.

Figure 5: Price-Dividend Ratios: Benchmark Model



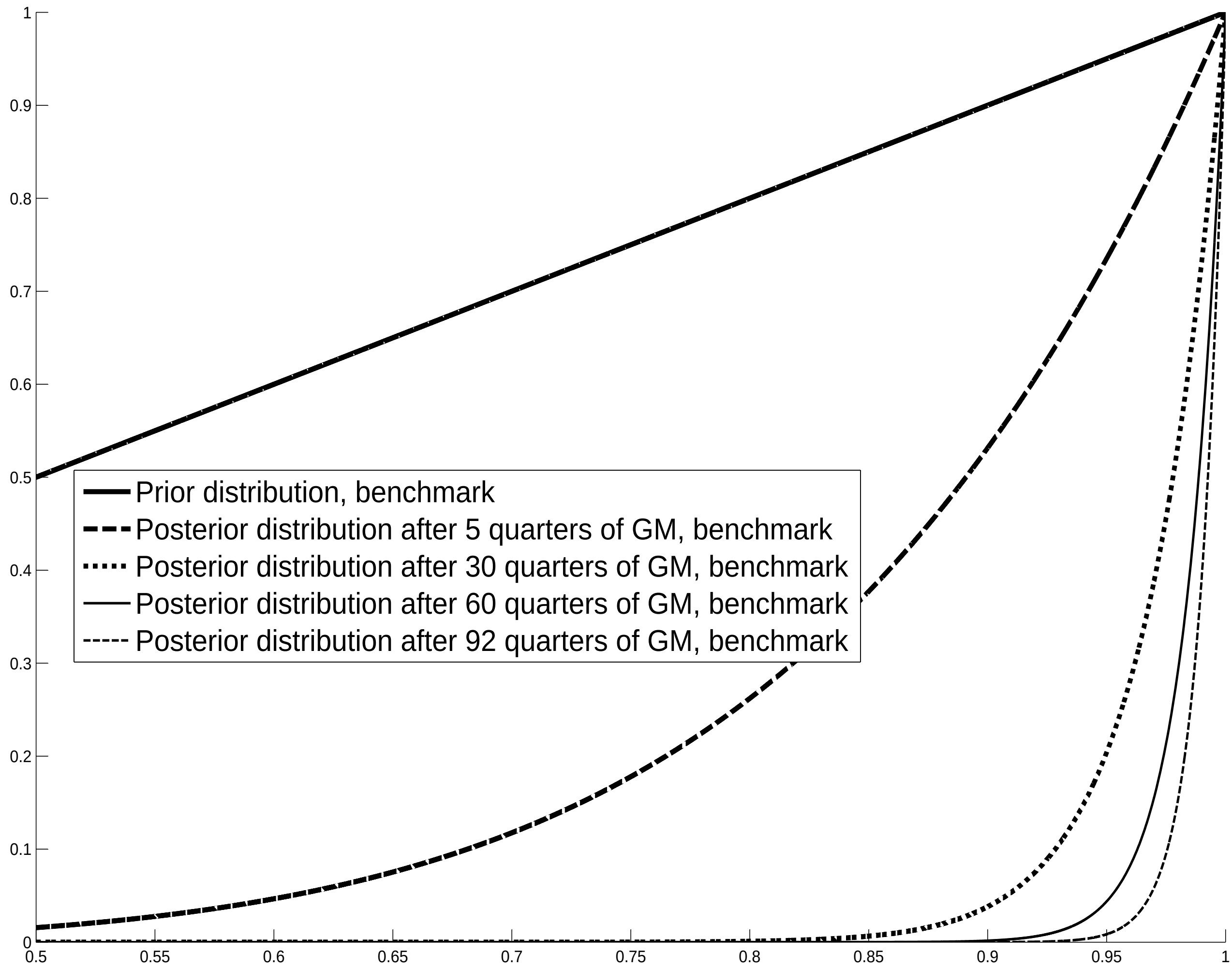
The figure plots the price dividend ratio in US data (upper Panel), and in the model (lower panel), for the benchmark calibration of the model.

Figure 6: Price-Dividend Ratios with Moderately Persistent Prior



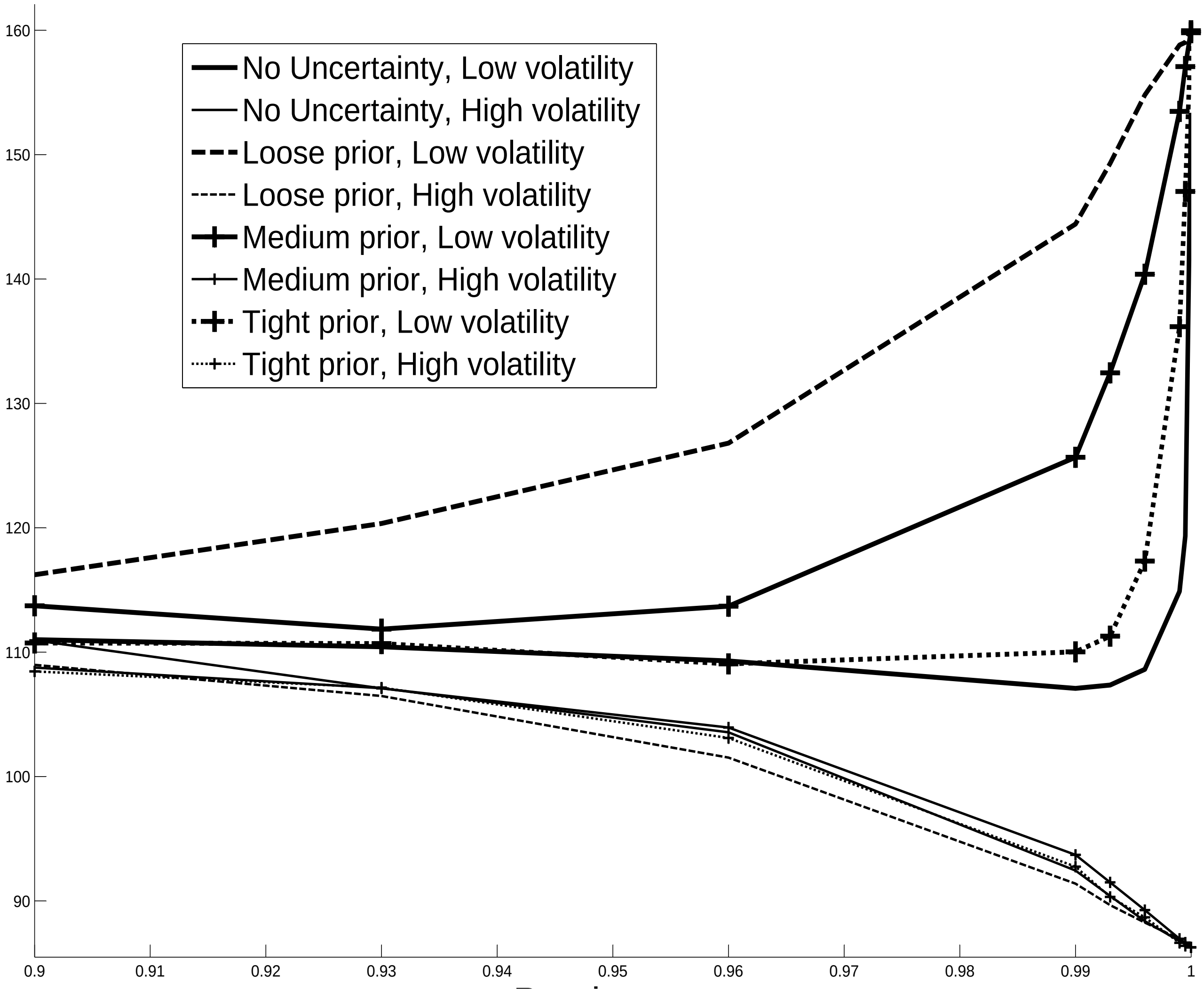
The figure plots the price dividend ratio in US data (upper Panel), and under learning about transition probabilities with beta priors (lower panel), for a moderately persistent prior with mean persistence of 0.75.

Figure 7: Posterior Distributions



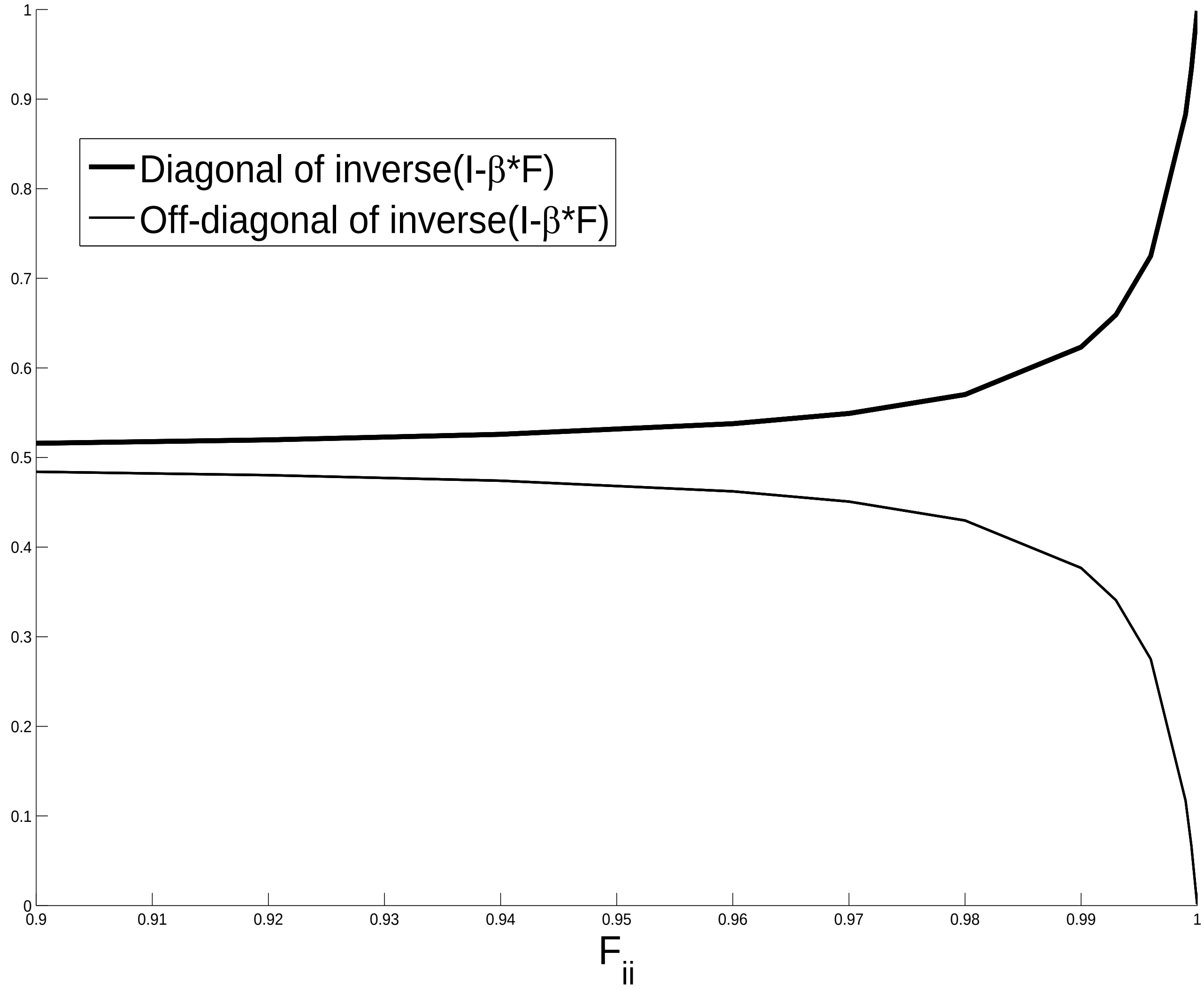
The figure plots the cumulative posterior distributions for the transition probability F_{ll} after an increasing number of observations on the Great Moderation, starting from an uninformative uniform prior.

Figure 8: Price-Dividend Ratios as a Function of Persistence and Prior Tightness



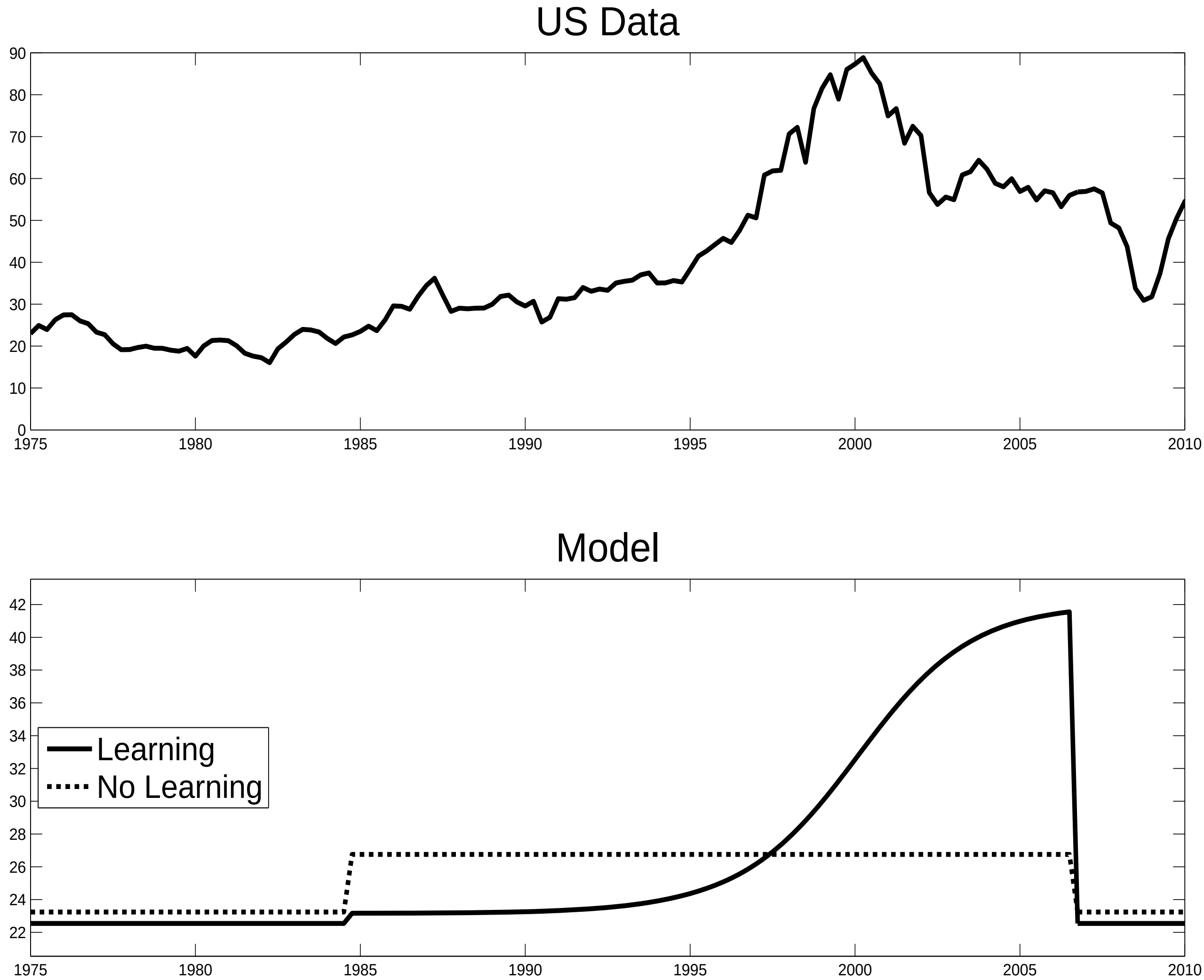
For the simplified case of symmetric transition probabilities ($F_{ll} = F_{hh}$), the figure plots the price dividend ratio as a function of persistence for different values of the tightness of priors for the benchmark calibration of the model.

Figure 9: Behind the Non-Linear Asset Price-Persistence Relation



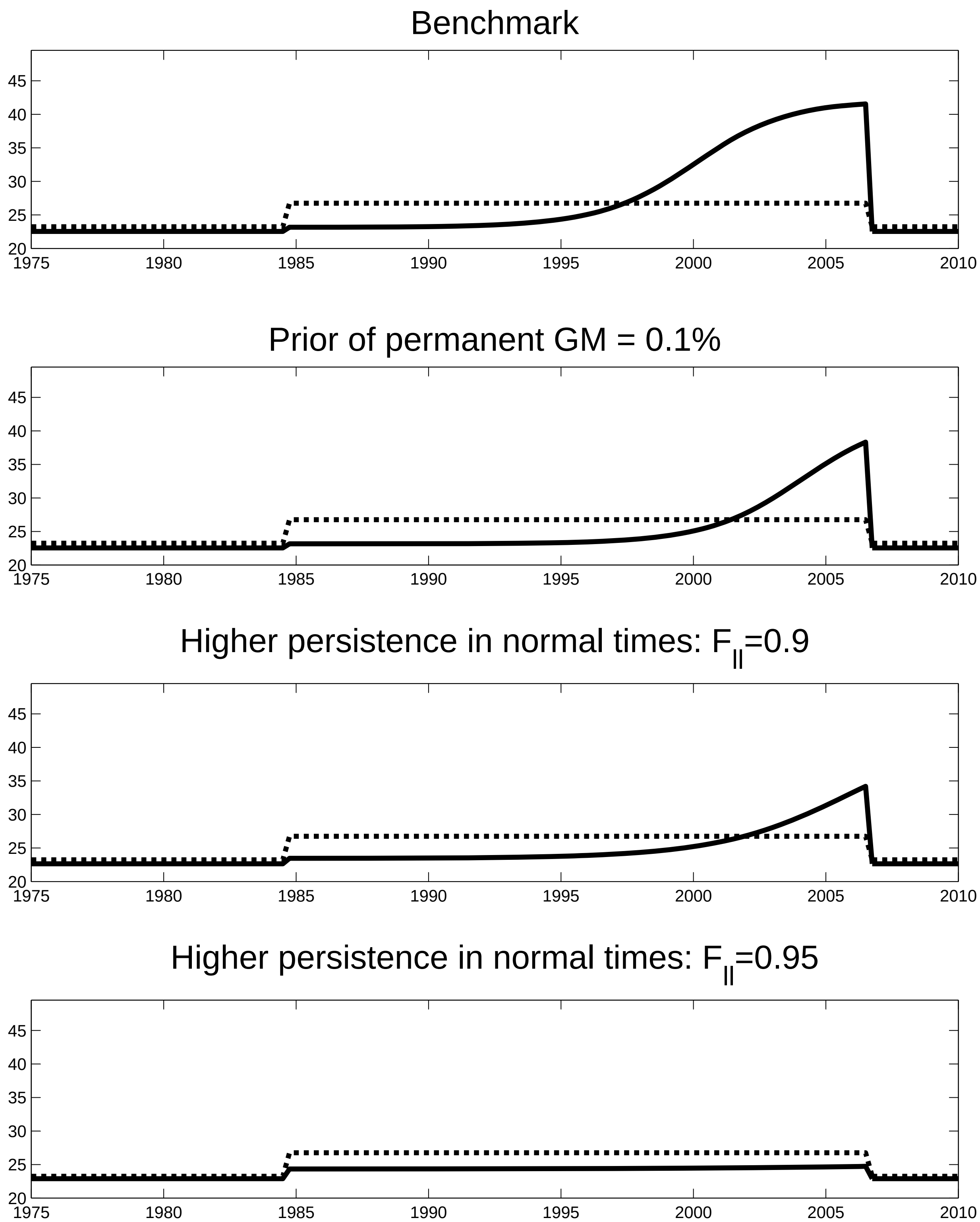
For the case of symmetric transition probabilities $F_{hh} = F_{ll}$, the figure depicts the diagonal and non-diagonal elements of the present discounted value matrix $(I - \beta\mathbb{F})^{-1}$.

Figure 10: Price-Dividend Ratios: Learning about a Permanent vs. Transitory Great Moderation



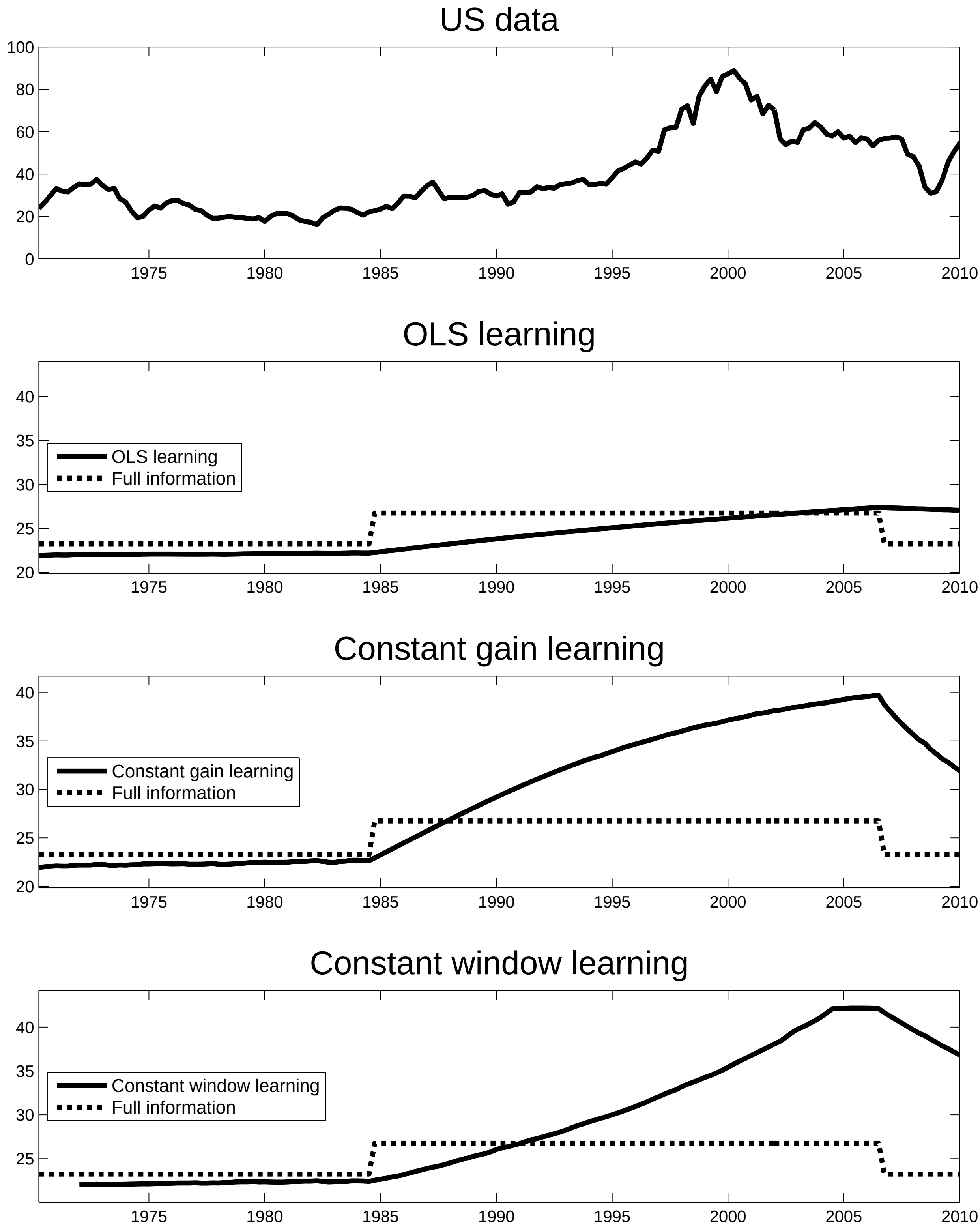
The figure plots the price dividend ratio in US data (upper panel), and under learning about a permanent vs. transitory Great Moderation (lower panel), for the benchmark calibration of the model.

Figure 11: Learning about a Permanent vs. Transitory Great Moderation with Different Priors



The figure shows the time-path of dividends with learning about a permanent vs. transitory Great Moderation with different prior probabilities.

Figure 12: Price-Dividend Ratios - Ad Hoc Learning



The figure plots the price dividend ratio in US data (upper Panel), and under three ad hoc learning rules: OLS (second panel), constant gain (third panel), and constant window (bottom panel), for the benchmark calibration of the model. The full information prices correspond to the case of high persistence.