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ABSTRACT

When Bonds Matter: Home Bias in Goods and Assets*

This paper presents a model of international portfolios with real exchange rate and non financial risks that accounts for observed levels of equity home bias. A key feature is that investors can trade equities as well as domestic and foreign real bonds. Bonds matter: in equilibrium, investors structure their bond portfolio to hedge real exchange rate risk since relative bond returns are strongly correlated with real exchange rate movements. Equity home bias does not arise from the co-movements between relative stock returns and real exchange rates, but from the hedging properties of stock returns against other sources of risk, conditionally on bond returns. We estimate the optimal equity and bond portfolios implied by the model for G-7 countries and find strong empirical support for the theory. We are able to account for a significant share of the equity home bias and obtain a currency exposure of bond portfolios comparable to the data.

JEL Classification: F30, F41 and G11
Keywords: equity home bias, international portfolios and international risk sharing

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1 Introduction

The current international financial landscape exhibits two critical features. First, the last twenty years witnessed an unprecedented increase in cross-border financial transactions.[1] Second, despite this massive wave of financial globalization, international portfolios remain heavily tilted toward domestic assets. This is the well-known equity home bias (French and Poterba (1991), Tesar and Werner (1995) and Ahearne, Griever and Warnock (2004)). As of 2008, the share of US stocks in US investors' equity portfolios was 77.2%, despite the fact that US equity markets account for only 32% of world market capitalization.[2] The importance of these two features has not gone unnoticed, and spurred renewed interest for the theory of optimal international portfolio allocation.

Two important strands of literature aim to account for this observed equity home bias. In both approaches, investors depart from the perfectly diversified portfolio of frictionless general equilibrium models à la Lucas (1982) in order to insulate their consumption stream from additional sources of risk. Differences in equilibrium portfolio holdings across countries reflect the equilibrium hedging properties of relative equity returns. Generically, consider a risk-factor X that impacts *negatively* domestic investor's wealth relatively more than foreigner's. Equilibrium differences in equity holdings across countries will be proportional to the following hedge ratio:

$$\frac{\text{cov}\,(X, R)}{\text{var}\,(R)}, \tag{1}$$

where R denotes relative equity returns. Domestic equity bias arises when excess equity returns are *positively correlated* with X. In that case domestic equities constitute a good hedge against risk factor X.

The two strands of literature differ in the risk factor they consider. The first one, launched into orbit by the influential contribution of Obstfeld and Rogoff (2000), sets out to explore the link between the allocation of consumption expenditures and portfolios in general equilibrium models with stochastic endowments.[3] One popular approach, initially developed by Baxter et al. (1998) and extended by Coeurdacier (2009) and Obstfeld (2007), consists in characterizing the constant equity portfolio that –locally– reproduces an efficient market allocation through trades in claims to domestic and foreign equities. In this class of models, investors face real exchange rate risk and efficient risk-sharing requires them to hold different portfolios. The hedging demand for equities is proportional to $(1 - 1/\sigma)\text{cov}(\Delta \ln Q, R)/\text{var}(R)$ where σ is the coefficient of relative risk aversion and $\Delta \ln Q$ is the rate of change of the real exchange

[1] As a share of GDP of industrialized countries, gross foreign equity and direct investment positions have been multiplied by more than four between 1983 and 2003. See Lane and Milesi-Ferretti (2003).

[2] The equity home bias is a general phenomenon. See Coeurdacier and Rey (2011) for recent evidence. The share of home equities in other G7 countries portfolios in 2008 are as follows: 80.2% in Canada, 73.5% in Japan, 66% in France, 53% in Germany and 52% in Italy. All these countries account for less than 10% of world market capitalization.

[3] A chronological but non-exhaustive list of contributions –some of which precedes Obstfeld and Rogoff (2000)– includes Dellas and Stockman (1989), Baxter, Jermann and King (1998), Kollmann (2006), Obstfeld (2007), Heathcote and Perri (2007), Coeurdacier, Kollmann and Martin (2009), Collard, Dellas, Diba and Stockman (2007), Coeurdacier (2009) and Benigno and Nistico (2011).

rate (defined as the ratio of domestic to foreign price levels so that an increase represents an appreciation).[4]

For a coefficient of relative risk aversion σ above unity, domestic equity bias arises when excess equity returns ($R > 0$) are *positively* correlated with an appreciation of the domestic real exchange rate ($\Delta \ln Q > 0$). The reason is quite straightforward: with $\sigma > 1$, efficient risk sharing requires that domestic consumption expenditures increase as the domestic price increases, i.e. as the real exchange rate appreciates. If domestic equity returns are high precisely at that time, domestic equity provides the appropriate hedge against real exchange rate risk, and domestic investors optimally tilt their portfolio towards domestic equity.

This line of research faces a serious challenge, as shown by van Wincoop and Warnock (2010): for many countries, the empirical correlation between excess equity returns and the real exchange rate is close to zero. This casts a serious doubt on the ability of this class of models to account for observed levels of equity home bias.

The second important strand of literature focuses on non-financial income risk R^n. In that case, the hedge ratio takes the form $-\text{cov}(R^n, R)/\text{var}(R)$. If returns on domestic equities are high precisely when returns on non-financial wealth are low, then investors will favor domestic stocks. This line of research also faces an important empirical challenge as initially shown by Baxter and Jermann (1997). These authors find that financial and non-financial returns appear to be positively correlated -as would be the case in a standard one-good model-, suggesting that optimal portfolios should be biased *towards* foreign equity.[5]

This paper demonstrates that many of the results in this literature are not robust to the introduction of domestic and foreign real bonds. We establish this point by solving jointly for the optimal equity and bond portfolio in a generic environment with multiple sources of risk.[6] The key economic insight is that in most models of interest, as well as in the data, relative bond returns (whether nominal or real) are strongly positively correlated with real exchange rate fluctuations. As a consequence, it is optimal for investors to use bond holdings to hedge real exchange rate risks. In that sense, bonds matter! All that is left for equities is to hedge the impact of any *additional* source of risk on investors' wealth. Of course, the precise structure that these additional risk factors take matters for optimal portfolio holdings but the general portfolio structure is very robust and quite independent of the details of the model.

To establish these claims, we begin with a simple extension of Coeurdacier (2009) with one additional risk factor, so that risk sharing remains -locally- efficient. We show that this extended model can map many cases of interest studied in the literature: redistributive

[4] See Kouri and Macedo (1978), Krugman (1981) and the references in Adler and Dumas (1983) for an early derivation of this result under partial equilibrium.

[5] Other papers found more mixed results. See Bottazzi, Pesenti and van Wincoop (1996) and Julliard (2003).

[6] As we will see, bonds are redundant in the earlier models since risk-sharing is locally efficient with equities only. This creates an obvious and uninteresting indeterminacy which is lifted once additional sources of risk are introduced. That the economic environment is subject to more than one source of uncertainty strikes us as eminently realistic.

shocks, fiscal shocks, investment shocks, preference shocks, nominal shocks, etc.... Under the assumption (verified in the data) that bonds take care of the hedging of the real exchange rate, this simple extension delivers two important results. First, equilibrium equity holdings take a very simple form, that does *not* depend on the equilibrium correlation between equity returns and the real exchange rate. Second, the optimal equity portfolio does *not* depend upon the preferences of the representative household. Equivalently, optimal equity positions coincide with the equity positions of a *log-investor* who doesn't care about hedging the real exchange rate risk.

This result has important empirical implications. First, since equity positions are not driven by real exchange rate risk, equity home bias can only arise from hedging demands *other* than the real exchange rate. This simultaneously validates van Wincoop and Warnock (2010)'s result and establishes its limits. Equity home bias emerges in equilibrium if the correlation between the return on non-financial wealth and the return on equity is negative, *conditional on bond returns*, a generalization of both Baxter and Jermann (1997) and Heathcote and Perri (2007).[7] In recent and independent work, Engel and Matsumoto (2009) developed similar results in a specific model with nominal rigidities.

The model also provides tight predictions about equilibrium *bond* holdings. The equilibrium bond position reflects the balance of two effects: an optimal hedge for real exchange rate risk (for non-log investors), as well as a hedge for the implicit real exchange rate exposure arising from equilibrium equity holdings and non-financial wealth. In other words, investors want to hold domestic real bonds since these bonds have higher returns in states of the world where the price of domestic consumption increases (real exchange rate hedging). However, if equity returns and non-financial wealth are also higher in those states, investors optimally undo this implicit exposure by shorting the domestic currency bond. For plausible parameter values, these two effects will tend to generate small currency exposure of bond portfolios and it is possible for a country to have short or long domestic currency positions.

Next, we show how equilibrium portfolios can be constructed from observable data on bond returns, real exchange rates and the (unobservable) returns to financial and non-financial wealth. Simple regressions of real exchange rate fluctuations and the return on non-financial wealth on bond and financial returns are sufficient to back out equilibrium portfolios from the data. This makes an important link between recent theoretical work on international portfolios and data on asset returns. Importantly, this remains true in presence of multiple sources of risk. In that case, markets become incomplete (even locally). Yet, the theoretical portfolios can be constructed using the exact same empirical methodology. This is reassuring since it indicates that our empirical results do not depend on the assumed degree of completeness of financial markets. The intuition for this result is the following: the market structure influences the mapping between risk factors and equilibrium asset returns. Conditional on this mapping, however, equilibrium portfolios can always be recovered from estimated hedging factors.

We confront our theory to the data. We use quarterly data on market returns as well as

[7]See also Bottazzi et al. (1996).

non-financial and financial income for the G-7 countries since 1970 to ask wether data on asset prices are theoretically consistent with observed portfolios. Since returns on non-financial and financial wealth are not directly observed, we follow Campbell (1996) and Lustig and Nieuwerburgh (2008) and construct alternate measure of these returns. For all countries, and across most specifications, we find that the presence of bonds is key to obtaining more reasonable asset positions. Without bond trading, 'the international diversification puzzle is worse than you think' as Baxter and Jermann (1997) argued. In other words, the *unconditional* correlation between financial and non-financial returns is positive. However, once we allow for bond holdings, we find, as in the data, significant levels of equity home bias for all G-7 countries. Put differently, financial and non-financial returns are significantly *negatively* correlated, once investors are able to control their real exchange rate exposure with bonds. Regarding predicted bond positions, our empirical estimation predicts short but fairly small domestic currency positions for a reasonable degree of relative risk aversion. This is in line with recent empirical evidence on the (average) currency exposure of international portfolios for G-7 countries.[8]

Section 2 follows Coeurdacier (2009) and develops the basic model with equities only. Section 3 constitutes the theoretical core of the paper. It introduces bonds and an additional source of risk, then characterizes the efficient equity and bond positions under different risk structures. The model is then extended to the case of incomplete markets. Section 4 presents our empirical results. Section 5 concludes.

2 A Benchmark Model.

2.1 Goods and preferences.

Consider a two-period ($t = 0, 1$) endowment economy similar to Coeurdacier (2009). There are two symmetric countries, Home (H) and Foreign (F), each with a representative household. Each country produces one tradable good. Agents consume both goods with a preference towards the local good. In period $t = 0$, no output is produced and no consumption takes place, but agents trade financial claims (stocks and bonds). In period $t = 1$, country i receives an exogenous endowment y_i of good i. Countries are symmetric and we normalize $E_0(y_i) = 1$ for both countries, where E_0 is the conditional expectation operator, given date $t = 0$ information. Once stochastic endowments are realized in period 1, households consume using the revenues from their portfolio chosen in period 0 and their endowment received in period 1.

Country $i's$ representative household has standard CRRA preferences, with a coefficient

[8] On average over the period 2000-2004, G-7 countries hold short (but small) domestic currency debt positions with some heterogeneity across countries: the US, UK Italy and Japan, are short in their own currency debt, while Canada, Germany and France long but with smaller currency exposures than US, UK or Japan. See Lane and Shambaugh (2010a).

of relative risk aversion $\sigma \geq 1$ defined over a consumption index C_i:

$$U_i = E_0 \left[\frac{C_i^{1-\sigma}}{1-\sigma} \right], \tag{2}$$

For $i, j = H, F$, the consumption index C_i is given by:

$$C_i = \left[a^{1/\phi} c_{ii}^{(\phi-1)/\phi} + (1-a)^{1/\phi} c_{ij}^{(\phi-1)/\phi} \right]^{\phi/(\phi-1)},$$

where c_{ij} is country $i's$ consumption of the good from country j at date 1. ϕ is the elasticity of substitution between the two goods and $1 \geq a \geq 1/2$ captures preference for the home good (mirror-symmetric preferences).

The ideal consumer price index that corresponds to these preferences is, for $i = H, F$:

$$P_i = \left[a p_i^{1-\phi} + (1-a) p_j^{1-\phi} \right]^{1/(1-\phi)}, \tag{3}$$

where p_i denotes the price of country $i's$ good in terms of an arbitrary numeraire.

Resource constraints are given by:

$$c_{ii} + c_{ji} = y_i. \tag{4}$$

q denotes Home's terms of trade, i.e. the relative price of the Home tradable good in terms of the Foreign tradable good:

$$q \equiv \frac{p_H}{p_F},$$

so that an increase in q represents an improvement in Home's terms of trade.

2.2 Financial markets.

Trade in stocks and bonds occurs in period 0. In each country there is one Lucas tree. A share δ of the endowment in country i is distributed to stockholders as dividend (financial income), while a share $(1-\delta)$ is instead distributed to households of country i (non-financial income). At the simplest level, one can think of the share $1-\delta$ as representing 'labor income', but more generally, it captures the share of output that cannot be capitalized into financial claims.[9] In our symmetric setting, δ is common to both countries. The supply of each type of share is normalized at unity.

Agents can also trade Home and Foreign bonds. Our benchmark model assumes risk-free *real* bonds for the sake of simplicity. The important economic feature of these bonds for our results is the high correlation between relative bond returns and the real exchange rate, a feature shared by both nominal and real bonds.[10] Each bond is denominated in the

[9] This could be due to domestic financial frictions, capital income taxation or poor enforcement of property rights.

[10] This results from the well-known fact that most real exchange rate fluctuations are driven by the nominal exchange rate.

composite good of each country: buying one unit of the Home (Foreign) riskfree bond in period 0 yields one unit of the Home composite (Foreign) good at $t = 1$. Both bonds are in zero net supply.

Initially, each household fully owns the local stock, and has zero initial foreign assets. Country i household thus faces the following budget constraint at $t = 0$:

$$p_S S_{ii} + p_S S_{ij} + p_b b_{ii} + p_b b_{ij} = p_S,$$

where S_{ij} denotes the number of shares of stock j held by country i at the end of period 0, and b_{ij} represents country $i's$ holdings of $j's$ riskfree bond. By symmetry, p_S is the share price of both stocks, while p_b is the price of the both countries real bond.

Market clearing in asset markets for stocks and bonds requires:

$$S_{ii} + S_{ji} = 1; \;\; b_{ii} + b_{ji} = 0.$$

Symmetry of preferences and distributions of shocks also implies that equilibrium portfolios are symmetric and can be summarized by domestic holding of local equity: $S \equiv S_{HH} = S_{FF}$ and domestic holdings of the local bond $b \equiv b_{HH} = b_{FF}$. The vector (S, b) fully describes international portfolios. $S > \frac{1}{2}$ means that there is equity home bias on stocks, while $b < 0$ means that a country issues bonds denominated in its local composite good, and simultaneously invests in foreign bonds.

2.3 Characterization of world equilibrium.

We characterize first the equilibrium with *locally complete markets*. We say that markets are locally complete when the Backus and Smith (1993) international risk sharing condition holds as a first-order approximation. This will be the case as long as the set of (independent) assets returns spans the space shocks, a condition satisfied in our model.[11]

2.3.1 Budget constraints.

Recall that household i holds S shares of the local stock with dividend $\delta p_i y_i$, $1 - S$ shares of the foreign stock, with dividend $\delta p_j y_j$, b bonds denominated in the local good, with payment P_i and $-b$ bonds in the foreign good, with payment P_j. The period 1 budget constraints are thus:

$$P_i C_i = S\delta p_i y_i + (1 - S)\delta p_j y_j + P_i b - P_j b + (1 - \delta) p_i y_i \tag{5}$$

where the last term represents non-financial income.

Taking the difference between Home and Foreign implies:

$$P_H C_H - P_F C_F = [\delta\,(2S - 1) + (1 - \delta)](p_H y_H - p_F y_F) + 2b(P_H - P_F) \tag{6}$$

which trivially states that the difference in countries' consumption expenditures reflects the difference in their incomes.

[11] See appendix A.1 for a precise statement of the associated rank and spanning conditions.

2.3.2 Goods market equilibrium

After the realization of uncertainty in period 1, the representative consumer in country i maximizes $C_i^{1-\sigma}/(1-\sigma)$ subject to the period budget constraint (5) where $P_i C_i = p_i c_{ii} + p_j c_{ij}$.

Using the intratemporal allocation across goods together with market-clearing conditions (4), we get:

$$\frac{y_H}{y_F} = \frac{c_{ii} + c_{ji}}{c_{jj} + c_{ij}} = q^{-\phi}\Omega_a\left[\left(\frac{P_F}{P_H}\right)^{\phi}\frac{C_F}{C_H}\right] \tag{7}$$

where $\Omega_a(x) = \left[1 + x(\frac{1-a}{a})\right] / \left[x + (\frac{1-a}{a})\right]$. Without home bias in preferences ($a = 1/2$), this simplifies to $\frac{y_H}{y_F} = q^{-\phi}$: terms-of-trade are simply negatively related to relative supply (with a constant elasticity $1/\phi$) and thus independently on the portfolio allocation. As emphasized by Obstfeld (2007), the term $\Omega_a(.)$ captures the Keynesian transfer effects due to consumption home-bias: with $a > 0.5$, a reallocation of wealth towards the home country requires an improvement in the domestic terms of trade.

2.3.3 Log-linearization of the model and locally complete markets.

Denote $y \equiv y_H/y_F$ the relative output. We log-linearize the model around the symmetric mean where y equal unity, and use Jonesian hats ($\hat{x} \equiv log(x/\bar{x})$) to denote the log-deviation of a variable x from its mean value $\bar{x}$. Define the Home country real exchange rate as the foreign price of the domestic good, $Q \equiv P_H/P_F$, so that an increase in the real exchange rate represents a real appreciation. Using (3) gives:

$$\hat{Q} = \hat{\frac{P_H}{P_F}} = (2a-1)\hat{q}. \tag{8}$$

so that in this model, the real exchange rate always appreciates when the terms of trade improve.

As shown in appendix A.1, if a rank and spanning conditions are satisfied, markets are locally complete, that is, the competitive equilibrium replicates the efficient risk-sharing allocation up-to the first order.[12] This property turns out to simplify the portfolio problem: one just needs to find the portfolio that replicates *locally* the efficient allocation. In particular, the ratio of Home to Foreign marginal utilities of aggregate consumption is linked to the consumption-based real exchange rate by the familiar Backus and Smith (1993) condition (in log-linearized terms):[13]

$$-\sigma(\hat{C_H} - \hat{C_F}) = \hat{\frac{P_H}{P_F}} = (2a-1)\,\hat{q}. \tag{9}$$

[12]The spanning condition states that the dimensionality of the shocks is smaller than the number of independent available assets. The rank condition states that shock innovations do not leave asset pay-off unaffected.

[13]Under complete markets, the ratio of marginal utilities of one unit of numeraire $\left(C_i^{-\sigma}/P_i\right)/\left(C_j^{-\sigma}/P_j\right)$ is constant. Under locally complete markets, the same condition holds at the first-order.

Efficient risk sharing requires that relative consumption declines with an elasticity $1/\sigma$ when the real exchange rate appreciates. Log-linearizing equation (7) then substituting (9) and (8) gives:

$$\hat{y} = -\lambda \hat{q} \tag{10}$$

where $\lambda \equiv \phi\left(1-(2a-1)^2\right)+(2a-1)^2/\sigma > 0$ represents the equilibrium terms of trade elasticity of relative output. A relative increase in the supply of the home good ($\hat{y} > 0$) is always associated with a worsening of the terms of trade ($\hat{q} < 0$) with an elasticity $-1/\lambda$. Without home bias in preferences ($a = 1/2$), $\lambda = \phi$, the elasticity of substitution between Home and Foreign goods. When $a > 1/2$, there are deviations from purchasing power parity. An increase in relative output triggers a fall in the relative price level. Under locally complete markets, this requires an increase in domestic consumption expenditures (at a rate $1/\sigma$), increasing relative demand for the home good.[14]

If we denote (log) relative returns $\hat{R}^f$ for financial income, $\hat{R}^b$ for bond returns and $\hat{R}^n$ for non-financial income, we get from from (10):

$$\begin{aligned} \hat{R}^f &= \hat{q}+\hat{y} = (1-\lambda)\hat{q}, \\ \hat{R}^b &= \hat{Q} = (2a-1)\,\hat{q}, \\ \hat{R}^n &= \hat{q}+\hat{y} = (1-\lambda)\hat{q}. \end{aligned} \tag{11}$$

When $\lambda < 1$, an increase in Home relative output is associated with a decrease in Home financial (equity) and non-financial returns (relative to Foreign). This happens when either the elasticity of substitution between goods is low ($\phi < 1$) or the preference for the home good is sufficiently strong.[15] Note also that in that case, relative bond returns, relative financial and non-financial returns are all perfectly positively correlated.

Next, log-linearize equation (6) using (9) and (11) to obtain:

$$\begin{aligned} \hat{PC} &= \hat{P_H C_H} - \hat{P_F C_F} = \left(1-\frac{1}{\sigma}\right)(2a-1)\,\hat{q} \\ &= \left[\delta\,(2S-1)+(1-\delta)\right](1-\lambda)\hat{q} + 2b\,(2a-1)\,\hat{q}. \end{aligned} \tag{12}$$

The first equality is simply a restatement of the Backus-Smith condition in terms of relative consumption expenditures $\hat{PC}$. With locally complete markets, a shock that leads to an appreciation of the real exchange rate ($(2a-1)\,\hat{q} > 0$) induces an increase in relative consumption expenditures when $\sigma \geq 1$. The expression on the second line of equation (12) shows the change in relative income necessary to implement the efficient allocation of relative consumption expenditures.

2.4 Optimal Equity Portfolios.

Financial markets are locally complete when there exists a portfolio (S, b) such that equations (10) and (12) both hold for arbitrary realizations of the relative shocks $\hat{y}$. Since equity and

[14]See Obstfeld (2007).

[15]Specifically, when $\phi > 1$ and $\sigma > 1$ (the empirically plausible case), one needs: $a > \frac{1}{2}[1+(\frac{1-\phi}{\frac{1}{\sigma}-\phi})^{1/2}]$.

bonds returns are perfectly correlated, portfolios are indetermined: there are infinitely many combinations (S, b) that implement the locally efficient allocation. Consequently, we can ignore bonds and focus on the case where efficient risk sharing is implemented with equities only, as done in earlier literature.

Substituting $b = 0$ into (12) and using (10), the equilibrium equity portfolio position satisfies:

$$S = \frac{1}{2}\left[\frac{2\delta - 1}{\delta} + \frac{\left(1 - \frac{1}{\sigma}\right)(2a - 1)}{\delta(1 - \lambda)}\right] \tag{13}$$

When $\delta = 1$, this expression coincides with the equilibrium equity position of Coeurdacier (2009) and Obstfeld (2007). In the more general case where $\delta \leq 1$, the optimal equity portfolio has two components. The first term inside the brackets represents the equity position of a log-investor ($\sigma = 1$). As in Baxter and Jermann (1997), the domestic investor is already endowed with an implicit equity position equal to $(1 - \delta)/\delta$ through her exposure to non-financial income $(1 - \delta) y$. Offsetting this implicit equity holding and diversifying optimally requires an equity position $S = (2\delta - 1)/2\delta < 1/2$ for $\delta < 1$. As is well known since Baxter and Jermann (1997), this component of the optimal portfolio imparts a *foreign* equity bias.

The second component of the optimal equity portfolio represents a hedge against real exchange rate fluctuations. It only applies when $\sigma \neq 1$, i.e. when total consumption expenditures fluctuate with the real exchange rate. Looking more closely at the structure of this hedging component calls for a number of observations. First, it is a complex and non-linear function of the structure of preferences summarized by the parameters σ, ϕ and a. As Obstfeld (2007) and Coeurdacier (2009) note, for reasonable parameter values, this hedging demand can contribute to home equity bias only when $\lambda < 1$, i.e. when the terms of trade impact of relative supply shocks is large.[16] Using equations (11) and (8), this hedge component can be rewritten as in equation (1):

$$\frac{1}{2\delta}\left(1 - \frac{1}{\sigma}\right)\frac{\text{cov}\left(\hat{Q}, \hat{R}^f\right)}{\text{var}\left(\hat{R}^f\right)}.$$

As in the early partial equilibrium literature the optimal hedge component is simply a function of the covariance-variance ratio between excess equity returns and the real exchange rate.[17]

This model faces three main problems. First, the non-linearity in (13) implies that small changes in preferences can have a large impact on the hedging demand. This is most apparent if we consider the optimal portfolio in the neighborhood of $\lambda = 1$. As figure 1 makes clear, small and reasonable changes in σ, ϕ or a have a large and disproportionate impact on

[16] When $\lambda = 1$, this component is indeterminate since the relative return on equities is independent of the real exchange rate (and constant). This case is similar to Cole and Obstfeld (1991): perfect risk sharing is achieved through movements in the terms of trade and equity returns in both countries are perfectly correlated.

[17] See Kouri and Macedo (1978), Krugman (1981) and Adler and Dumas (1983).

optimal portfolio holdings, from large foreign bias ($S \ll 0$) to unrealistically high domestic bias ($S \gg 1$). To the extent that researchers don't know precisely what is the correct value of these parameters, the model does not provide enough guidance to pin down equity portfolios, and a-fortiori, to explain the home portfolio bias. As emphasized by Obstfeld (2007), and as figure 1 shows, things are even worse since the benchmark model cannot deliver home equity holdings between $S = 1 - 1/2\delta < 0.5$ and $S = 1$, thus excluding the relevant empirical range.

Second, given the constant income sharing rule δ, the model predicts a perfect correlation between financial returns $\hat{R}^f$ and non-financial returns $\hat{R}^n$. This tilts portfolios towards foreign equities, the first term in (13), as emphasized by Baxter and Jermann (1997). While this correlation might be positive, many papers found it pretty low.[18]

Third, the extent to which the model delivers equity home bias depends on the hedging properties of equities for real exchange risk, as captured by the covariance-variance ratio $\text{cov}\left(\hat{Q}, \hat{R}^f\right)/\text{var}\left(\hat{R}^f\right)$. In the case of the United States, van Wincoop and Warnock (2010) show that relative equity returns are poorly correlated with the real exchange rate. They find a covariance-variance ratio $\text{cov}\left(\hat{Q}, \hat{R}^f\right)/\text{var}\left(\hat{R}^f\right)$ equal to 0.32.[19] With such a low covariance-variance ratio, the model can deliver equity home bias ($S > 1/2$) only if the share of financial income in total income δ exceeds $1 - \text{cov}\left(\hat{Q}, \hat{R}^f\right)/\text{var}\left(\hat{R}^f\right) = 68$ percent, a number vastly in excess of any reasonable estimate.

3 Equity and Bond Equilibrium Portfolios

This section turns the basic model of section 2 into a theory of bond and equity holdings by introducing additional sources of risk. Section 3.1 adds exactly one source of risk. That case is particularly tractable since markets remain locally complete. Unlike the previous section, bonds and equities are now imperfect substitutes and the equilibrium portfolio holdings of both types of assets are well defined. Section 3.2 shows that this additional source of uncertainty can take many forms that map into a variety of models of interest. A general finding is that equities and bond holdings depend on their general relative hedging properties against both real exchange rate risk and the additional risk factor. Section 3.3 covers the general case with multiple additional risk factors. In that case, markets are incomplete even locally, yet we find that our portfolio characterization remains unchanged and all our results go through.

3.1 Equity and Bond Equilibrium Portfolios with Locally-Complete Markets.

Assume that a shock ε_i affects country i in period $t = 1$, assumed i.i.d. across countries. Denote $\varepsilon = \varepsilon_H/\varepsilon_F$ the relative shock and assume that $E_0(\varepsilon) = 1$ and $\hat{\varepsilon} \equiv \ln \varepsilon$ is not perfectly

[18]See Fama and Schwert (1977), Bottazzi et al. (1996), Julliard (2003) and Lustig and Nieuwerburgh (2008).

[19]Once controlling for forward markets, their estimated covariance-variance ratio falls to 0.005.

correlated with $\hat{y}$ so that it represents a genuine source of additional risk. To characterize optimal portfolio, we only need to specify how $\hat{\varepsilon}$ impacts financial returns $\hat{R}^f$, bond returns $\hat{R}^b$ and the return on non-financial wealth $\hat{R}^n$. By analogy with equation (11), we write:

$$\left\{\begin{array}{lclcl} \hat{R}^f & = & (1-\bar{\lambda})\hat{q} & + & \gamma_f\hat{\varepsilon} \\ \hat{R}^b & = & (2a-1)\hat{q} & + & \gamma_b\hat{\varepsilon} \\ \hat{R}^n & = & (1-\bar{\lambda})\hat{q} & + & \gamma_n\hat{\varepsilon} \end{array}\right. . \tag{14}$$

where $\bar{\lambda} > 0$ is possibly model dependent. The parameters γ_k can be positive or negative. They represent the impact of $\hat{\varepsilon}$ on financial (equity) returns, bond returns and non-financial returns. Different models will have different implications on what γ_k and $\bar{\lambda}$ should be and will be explored in more details in subsection 3.2. For the time being, the only restriction we impose on the model is $\gamma_f \neq 0$, that is, the new shock affects financial returns.

Under the assumption that markets remain *locally*-complete, the budget constraint (12) can be rewritten as:[20]

$$(1-\frac{1}{\sigma})(2a-1)\hat{q} = \delta\,(2S-1)\,\hat{R}_f + (1-\delta)\hat{R}^n + 2b\hat{R}^b. \tag{15}$$

Inspecting equation (15), there is a unique portfolio (S^*, b^*) such that it is satisfied for all realization of shocks $\hat{y}$ and $\hat{\varepsilon}$:

$$\begin{aligned} b^* &= \frac{1}{2}\frac{(2a-1)\left(1-\frac{1}{\sigma}\right)+(1-\delta)\left(1-\bar{\lambda}\right)\left(\gamma_n/\gamma_f-1\right)}{(2a-1)-\gamma_b/\gamma_f\left(1-\bar{\lambda}\right)}, \\ S^* &= \frac{1}{2}\left[1-\frac{1-\delta}{\delta}\frac{\gamma_n}{\gamma_f}-\frac{\gamma_b}{\gamma_f}\frac{2b^*}{\delta}\right]. \end{aligned} \tag{16}$$

This portfolio ensures that markets are indeed locally-complete. While expression (16) may look forbidding, it can be reinterpreted in terms of simple factor loadings.

To start with, let's rewrite the equilibrium bond and equity portfolios in terms of the equilibrium asset loadings on the real exchange rate $\hat{Q} = (2a-1)\,\hat{q}$ and the return to nonfinancial wealth $\hat{R}_n$. To do this, let's first manipulate equations (14) to eliminate $\hat{\varepsilon}$:

$$\begin{aligned} \hat{Q} &= (2a-1)\,\hat{q} = (2a-1)\,\psi\hat{R}_b - (2a-1)\,\psi\frac{\gamma_b}{\gamma_f}\hat{R}^f \qquad (17) \\ &\equiv \beta_{Q,b}\hat{R}^b + \beta_{Q,f}\hat{R}^f. \\ \hat{R}^n &= \left(1-\bar{\lambda}\right)\left(1-\frac{\gamma_n}{\gamma_f}\right)\psi\hat{R}^b + \left(\frac{\gamma_n}{\gamma_f}-\left(1-\bar{\lambda}\right)\left(1-\frac{\gamma_n}{\gamma_f}\right)\psi\frac{\gamma_b}{\gamma_f}\right)\hat{R}^f \qquad (18) \\ &\equiv \beta_{n,b}\hat{R}^b + \beta_{n,f}\hat{R}^f. \end{aligned}$$

where $\psi = \left[(2a-1)-\left(1-\bar{\lambda}\right)\gamma_b/\gamma_f\right]^{-1}$. The coefficients $\beta_{i,j}$ capture the loading of asset return j on factor i. They have the interpretation of covariance-variance ratios since they

[20] This requires that $(2a-1)\,\gamma_e \neq \gamma_b\left(1-\bar{\lambda}\right)$. This condition makes sure that our rank condition is satisfied, i.e. equity and bond excess returns are not collinear. See appendix A.1.

can be expressed as:

$$\beta_{n,i} = \frac{cov\left(\hat{R}^n, \hat{R}^i|\hat{R}^j\right)}{var\left(\hat{R}^i|\hat{R}^j\right)} \; ; \; \beta_{Q,i} = \frac{cov\left(\hat{Q}, \hat{R}^i|\hat{R}^j\right)}{var\left(\hat{R}^i|\hat{R}^j\right)},$$

where $i \neq j \in \{f, b\}$. Importantly, each loading is *conditional* on the other asset return. Since these loadings are expressed in terms of observables, they have an intuitive empirical counterpart, and can be readily estimated from a multivariate regression, independently of the specifics of the model and of the source of the shock $\hat{\varepsilon}$. This formulation will motivate our empirical analysis in section 4.

Next, we express the optimal portfolio in terms of the factor loadings:

$$\begin{aligned} b^* &= \frac{1}{2}\left(1 - \frac{1}{\sigma}\right)\beta_{Q,b} - \frac{1}{2}\left(1 - \delta\right)\beta_{n,b}, \\ S^* &= \frac{1}{2}\left[1 + \frac{1 - \frac{1}{\sigma}}{\delta}\beta_{Q,f} - \frac{1 - \delta}{\delta}\beta_{n,f}\right]. \end{aligned} \tag{19}$$

Consider the equilibrium bond portfolio b^* in the first line of equation (19). It contains two terms. The first term $\frac{1}{2}\left(1 - \frac{1}{\sigma}\right)\beta_{Q,b}$ captures the hedging of real exchange rate risk. When $\sigma > 1$, the household's relative consumption expenditures increase when the real exchange rate appreciates. If domestic bonds deliver a high return precisely when the currency appreciates, then domestic bonds constitute a good hedge against real exchange rate risk. The second term $-\frac{1}{2}\left(1 - \delta\right)\beta_{n,b}$ captures the hedging of non-financial income risk. When domestic bonds and the return to nonfinancial wealth conditionally on the equity return are positively correlated ($\beta_{n,b} > 0$), investors want to short the domestic bond to hedge the implicit exposure from their non-financial income. Equation (19) indicates that investors will go long or short in their domestic bond holdings depending on the relative strength of these two effects.

Let's now turn to the equilibrium equity position S^* in the bottom line of (19). The first term inside the brackets represents the symmetric risk-sharing equilibrium of Lucas (1982): $S = 1/2$. The following terms corresponds to the hedge portfolio similar to equation (1). The second term, $\frac{1-\frac{1}{\sigma}}{\delta}\beta_{Q,f}$, represents the hedging demand for domestic equity that arises from the correlation between equity returns and the real exchange rate, *conditional on the bond returns* . If this correlation is positive, domestic equities represent a good hedge against movements in real exchange rates. The last term, $-\frac{1-\delta}{\delta}\beta_{n,f}$, determines how equity portfolios are affected when non-financial wealth and financial wealth are conditionally correlated. In the case of Baxter and Jermann (1997) with $\beta_{Q,f} = 0$ and $\beta_{n,f} = 1$, the equilibrium equity position becomes $S = (2\delta - 1)/2\delta < 1/2$. This term makes clear that equity home bias can arise if $\beta_{n,f} < 0$. Importantly, what matters is the covariance-variance ratio between the returns to non-financial wealth and to financial wealth *conditional on the bond returns* $\hat{R}^b$. To our knowledge, this condition has not yet been empirically investigated in the literature.[21]

[21] Engel and Matsumoto (2009) also note that this is the relevant condition in presence of bond holdings, or forward exchange contracts.

To summarize, the model indicates that equity home bias can arise even if equities are a poor hedge for exchange rate risk, as long as non-financial wealth and equity returns are negatively conditionally correlated: $\beta_{n,f} < 0$. The model can also generate short positions in domestic bond market when $(1 - 1/\sigma)\, \beta_{Q,b} < (1 - \delta)\, \beta_{n,b}$.

We know from van Wincoop and Warnock (2010) that $\beta_{Q,f}$ is empirically small. In fact, these authors show that the covariance-variance ratio is very close to zero precisely after conditioning on the excess bond returns, or equivalently on forward rates. That is, $\beta_{Q,f} \simeq 0$. Going back to equation (17), we see that $\beta_{Q,f} = 0$ when $\gamma_b = 0$, i.e. when bond returns are almost unaffected by the $\hat{\varepsilon}$ risk factor. In this case, the equilibrium equity position simplifies further:

$$S^* = \frac{1}{2}\left(1 - \frac{1-\delta}{\delta}\beta_{n,f}\right) = \frac{1}{2}\left(1 - \frac{1-\delta}{\delta}\frac{\gamma_n}{\gamma_f}\right). \tag{20}$$

Contrary to most of the previous literature, in the empirically relevant case where $\beta_{Q,f} = 0$, the optimal equity portfolio S^* is *independent* of preference parameters such as the elasticity of substitution across goods ϕ, the degree of risk aversion σ or the 'tradability' of goods in consumption measured by a. Surprisingly, the complex and non-linear dependence of equilibrium equity portfolios on preferences parameters *disappears* once we introduce trade in bonds.

As a corollary, this implies that the optimal equity portfolio is the same as that of a log-investor ($\sigma = 1$). Since we know that log-investors do not care about fluctuations in the real exchange rate, it follows that what determines optimal equity holdings is *not* the correlation between equity returns and the real exchange rate. Our result is thus very different from much of the previous literature that emphasized the hedging properties of equity returns for real exchange rate risk. Instead, equity holdings insulate total wealth (both financial and non-financial) from the $\hat{\varepsilon}$ shocks.

To understand the structure of the optimal equity portfolio in equation (20), observe that the domestic investor is endowed with an implicit equity exposure through the impact of $\hat{\varepsilon}$ on the return to nonfinancial wealth $\hat{R}^n$, equal to $\gamma_n (1 - \delta) / \delta$. Offsetting this implicit equity exposure and diversifying optimally requires an equity position $S^* = 0.5 \left(1 - \gamma_n/\gamma_f (1 - \delta)/\delta\right)$.

3.2 Examples

The reduced form specification (14) nests many fully specified general equilibrium models which we now explore.

3.2.1 Redistributive shocks, or nominal shocks with preset prices.

In the model, the distribution of total income between its financial and non-financial components is controlled by the parameter δ. Variations in δ redistribute income from one to the other. If we interpret non-financial income as labor income, shocks to δ represent shocks to the labor share. Such fluctuations can occur in a model where capital and labor enter into the production function with a non-unit elasticity in presence of capital *and* labor augmenting productivity shocks or in presence of *biased* technical change in the sense of Young (2004).[22]

If we interpret ε_i as shocks to the share that his distributed as dividend, with $E_0(\varepsilon_i) = \delta$, one can verify that asset returns satisfy:

$$\begin{cases} \hat{R}^f & = & (1-\lambda)\hat{q} & + & \hat{\varepsilon} \\ \hat{R}^b & = & (2a-1)\hat{q} & & \\ \hat{R}^n & = & (1-\lambda)\hat{q} & - & \frac{\delta}{1-\delta}\hat{\varepsilon} \end{cases}. \tag{21}$$

This is a specific case of the general representation in (14) where $\gamma_f = 1$, $\gamma_b = 0$ and $\gamma_n = -\frac{\delta}{1-\delta}$. Substituting into equation (20) the optimal portfolio satisfies:[23]

$$S^* = 1 \; ; \; b^* = \frac{1}{2}(1 - \frac{1}{\sigma}) + \frac{1}{2}(2a-1)^{-1}(\lambda - 1). \tag{22}$$

Since purely redistributive shocks only affect the distribution of total output, but not its size, the optimal hedge is for the representative domestic household to hold all the domestic equity. This perfectly offsets the impact of the redistributive shocks on total income. Consequently, the equity portfolio exhibits full equity home bias. Equity home bias can arise here even if returns to financial wealth and returns to non-financial wealth are positively correlated unconditionally.[24]

The bond position is negative when $\lambda < 1 - (1 - \frac{1}{\sigma})(2a-1)$ and positive otherwise. A negative bond position (borrowing in domestic bonds and investing in foreign bonds) is possible only for sufficiently low values for λ. This condition echoes the condition for home equity bias in the equity only model of section 2.

The model with redistributive shocks has the exact same portfolio implications as the two period model of Engel and Matsumoto (2009) with preset prices and shocks to the money supply. In that paper, productivity shocks act as redistributive shocks since firms cannot adjust their prices but modify their profit margin, redistributing income between labor and dividends (see section 3.2.3).

3.2.2 Government expenditures or investment expenditures shocks.

Government expenditures represent another potential source of risk. Assume that in each country i, the government must finance period-1 government expenditures $E_{g,i}$ equal to $P_{g,i}G_i$, where G_i is the aggregate consumption index of the government and $P_{g,i}$ is the price index for government consumption, potentially different from the price index for private consumption. G_i is stochastic and symmetrically distributed, with $E_0(G_i) = \bar{G}$. Denote $E_g = (P_{g,H}G_H) / (P_{g,F}G_F)$ the ratio of Home to Foreign government expenditures and $\hat{E}_g$

[22]See also Ros-Rull and Santaeullia-Llopis (2010).

[23]Coeurdacier et al. (2009) obtain a similar result.

[24]Notice that this result does not depend upon the size of the redistributive shock: even a very small amount of redistributive variation leads to full equity home bias, as long as fluctuations in the share of nonfinancial income are of the first order.

the log deviation of relative government expenditures from their steady-state symmetric value of one.

Preferences of the government are similar to that of the households:

$$G_i = \left[a_g^{1/\phi}\left(g_{ii}\right)^{(\phi-1)/\phi} + (1-a_g)^{1/\phi}\left(g_{ij}\right)^{(\phi-1)/\phi}\right]^{\phi/(\phi-1)},$$

where g_{ij} denotes country i government's consumption of the good from country j and $a_g > 1/2$ represents the preference for the home good of the government (mirror-symmetric preferences) that may differ from the bias in household preferences ($a_g \neq a$).

Denote δ_g the share of government expenditures financed with taxes on financial income $T_{R,i} = \delta_g E_{g,i}$. Budget balance requires that taxes on nonfinancial income $T_i^w = (1-\delta_g)E_{g,i}$.[25]

Market-clearing conditions for both goods are now:

$$c_{ii} + c_{ji} + g_{ii} + g_{ji} = y_i. \tag{23}$$

Following the same steps as before, using intratemporal allocation across goods for governments and households, relative demand of Home over Foreign goods satisfies (in log-linearized terms):[26]

$$\hat{y} = s_c\hat{y}_c + s_g\hat{y}_g = -\bar{\lambda}\hat{q} + s_g(2a_g-1)\hat{E}_g, \tag{24}$$

where s_c (resp. $s_g = 1 - s_c$) is the steady-state ratio of consumption spending (resp. government spending) over GDP and $\bar{\lambda} = s_c\lambda + s_g\left[\phi(1-(2a_g-1)^2) + (2a_g-1)^2\right]$. Intuitively, the terms-of-trade $\hat{q}$ are decreasing with the relative supply of goods $\hat{y}$ (with an elasticity $1/\bar{\lambda}$) and increasing with relative government expenditure shocks (due to the presence of government home bias in preferences a_g), which act as relative demand shocks.

The net-of-taxes relative returns on assets can then be derived as:

$$\left\{\begin{array}{ccc} \hat{R}^f & = & (1-\bar{\lambda})\hat{q} + s_g\left(2a_g - 1 - \frac{\delta_g}{\delta}\right)\hat{E}_g \\ \hat{R}^b & = & (2a-1)\,\hat{q} \\ \hat{R}^n & = & (1-\bar{\lambda})\hat{q} + s_g\left(2a_g - 1 - \frac{1-\delta_g}{1-\delta}\right)\hat{E}_g \end{array}\right. . \tag{25}$$

Direct inspection of (25) reveals that in general markets are locally-complete and that the system is similar to (14) with:[27]

$$\hat{\varepsilon} = \hat{E}_g \ ; \ \gamma_f = s_g\left(2a_g - 1 - \frac{\delta_g}{\delta}\right) ; \ \gamma_b = 0 \ ; \ \gamma_n = s_g\left(2a_g - 1 - \frac{1-\delta_g}{1-\delta}\right). \tag{26}$$

[25] We restrict ourselves to cases where the *marginal* and *average* shares of taxes on financial and non-financial income in total fiscal revenues are the same (and equal to δ_g and $1-\delta_g$ respectively). What matters for equity portfolios is how *marginal* changes in government expenditures are financed, not how they are financed on average. So δ_g must be understood as the contribution of taxes on financial income to finance a marginal increase in government expenditures.

[26] See appendix (A.4) for details.

[27] The exception is the very peculiar case where $2a_g = 1 + \delta_g/\delta$. In that case, government expenditures do not modify equity returns conditionally on bond returns, and thus cannot be hedged perfectly. This rules out the case where government expenditures fall entirely on the domestic good ($a_g = 1$) and the fiscal incidence is equally distributed on financial and non-financial income ($\delta_g = \delta$). Note also that the bond return is unaffected because bond returns are not taxed. This ensures that $\gamma_b = 0$.

The impact of fiscal shocks on relative equity returns and non-financial incomes depends on the fluctuations in relative government expenditures $\hat{E}_g$, as well as the government preferences for the home good a_g, the steady state share of government expenditures in output s_g, and the relative fiscal incidence of the shocks δ_g/δ. Equilibrium portfolios are given by:[28]

$$\begin{aligned} b^* &= \frac{1}{2}s_c(1-\frac{1}{\sigma}) + \frac{1}{2}(2a-1)^{-1}(\bar{\lambda}-1)\frac{\delta_g-\delta}{2(1-a_g)\delta-(\delta-\delta_g)}, \\ S^* &= 1-\frac{(1-a_g)}{2(1-a_g)\delta-(\delta-\delta_g)}. \end{aligned} \tag{27}$$

Once again, portfolios are uniquely determined and the equity portfolio is independent from consumer preferences (ϕ, σ and a). While optimal equity portfolio are independent from household preferences, they depend on government preferences through a_g and δ_g.

When $a_g = 1$ (government expenditures are fully biased towards local goods), the equity portfolio is fully biased towards local stocks: $S^* = 1$.[29] From equation (24), a 1% increase in Home government expenditures raises Home dividends and Home non-financial income before taxes by s_g% holdings terms-of-trade (bond returns) constant. With a portfolio fully biased towards local equity, Home taxes also increase by s_g%. Such an equity portfolio insulates completely consumption expenditures from changes in government expenditures conditionally on bond returns.[30]

When $a_g < 1$, the optimal equity portfolio depends on the incidence of taxes. When $\delta_g = \delta$, *i.e* when increases in government expenditures fall on financial income proportionally to its share in gross GDP, the equity portfolio is the one of Baxter and Jermann (1997); in particular, investors exhibit foreign bias in equities:

$$S^* = \frac{1}{2}\frac{2\delta-1}{\delta}.$$

Conditionally on relative bond returns, shocks to Home government expenditures reduce Home equity returns and Home labor incomes in the same proportion, making financial and non financial incomes perfectly correlated.

When $\delta_g = 1$, *i.e* changes in government expenditures are entirely financed by taxes on financial incomes, the equilibrium equity portfolio becomes:

$$S^* = \frac{1}{2}\left[1+\frac{(2a_g-1)(1-\delta)}{1-\delta(2a_g-1)}\right]. \tag{28}$$

That equity portfolio always exhibits Home bias when $a_g > \frac{1}{2}$. Holding bond returns constant, an increase in Home government expenditures decreases dividends net of taxes at

[28]In this set-up, (15) needs to be slightly modified since private consumption in steady-state does not equal total consumption. (15) can be rewritten as follows: $s_c(1-\frac{1}{\sigma})(2a-1)\hat{q} = \delta\,(2S-1)\,\hat{R}_e + (1-\delta)\hat{R}^n + 2b\hat{R}^b$

[29]This is true except for the knife-edge case where equity and bonds have the same pay-offs, which occurs here when $\delta_g = \delta$. In that case, the portfolio is indeterminate. See footnote 27.

[30]Notice that in this case, government expenditures shocks act as redistributive shocks since $\gamma_n/\gamma_e = -\delta/(1-\delta)$.

Home and raises Home non-financial income by raising the relative demand for Home goods (see (25) for $\delta_g = 1$). Conditional relative equity and nonfinancial returns move in opposite directions and households favors local equities to hedge non-financial income. The mechanism is similar to the one in Heathcote and Perri (2007) and Coeurdacier, Kollmann and Martin (2010). Government expenditures play the same role as (endogenous) investment in these papers: for a given bond return, increases in Home investment raise Home non-financial income due to Home bias in investment spending, but decrease Home dividends, net of the financing of investment. This implies a negative conditional covariance between relative equity and nonfinancial income. In fact, the equity portfolios of (28) is identical to the one described in Heathcote and Perri (2007) and Coeurdacier et al. (2010) if we replace Home bias in government expenditures by Home bias in investment expenditures.

3.2.3 Other extensions: nominal shocks and quality shocks

The previous results hinge on the important assumption that bond returns provide a good hedge against fluctuations of the real exchange rate. This might not be true in two cases which we now discuss: first, when nominal shocks are important and the bonds available to investors are nominal. Second, in presence of shocks to the quality of goods or when new varieties are introduced as in Corsetti, Martin and Pesenti (2007) or Coeurdacier et al. (2009). In these two cases, while bond returns may be correlated with the real exchange rate measured by the statistician, the latter may differ from the *welfare-based* real exchange rate, the one that matters from the investor's point of view. We explore these two cases in more details in appendix A.5 and summarize the main findings here.

We show first that the case with nominal shocks fits the reduced form model of section 3.1.[31] With a *high* degree of price rigidities, nominal bonds track the real exchange rate and hedging of the real exchange rate is taken care of by the appropriate bond position. As shown by Engel and Matsumoto (2009), equities are essentially used to hedge fluctuations in the profit share of firms, as in our model with redistributive shocks. With a *low* degree of price rigidities, the nominal bond return differential fails to protect investors against real exchange rate risk, because of the inflation differentials across countries. Because holding bonds exposes investors to nominal risk, investors shift their bond position towards zero to insulate themselves and use equities to hedge real exchange rate exposure. While theoretically possible, we can safely argue that this example is not very relevant empirically. Indeed, it would contradict the empirical evidence we provide in section 4 since it would imply that nominal bond returns do not track well the real exchange rate, the exact opposite of our empirical findings.

Consider now the case of preference/quality/variety shocks. In that case, bond returns differential do not track the *welfare-based* real exchange rate (augmented for goods' quality)

[31] We introduce money (in the utility function) and price rigidities following Engel and Matsumoto (2009). Investors trade nominal bonds from both countries. Price rigidities are such that a constant fraction of firms have preset prices. Uncertainty is driven by shock to the money supply and shocks to the productivity of monopolistic firms. Hence, markets remain -locally- complete.

unlike equities since equity returns incorporate the adjustment for quality/preference.[32] As before, bonds have poor hedging properties and investors shift bond position towards zero. All risk-sharing is effectively done by equities. Again, while theoretically appealing, this is perhaps not so relevant empirically. It would require that the *welfare-based* real exchange rate is poorly correlated with the measured one since the latter is strongly correlated with bond returns.

3.3 The General Case: Bond and Equity Holdings under Incomplete Markets

Consider now the case with multiple source of uncertainties. While markets are not locally complete anymore, one can use Devereux and Sutherland's (2006) approach to characterize optimal equity and bond positions (see also Tille and van Wincoop (2010)). The key result is that the expression for equilibrium portfolios as a function of the loadings in equation (19) still holds when markets are incomplete.

To establish this result, consider a set-up similar to the one of section 3.1 with multiple sources of risk. The model can be summarized by the following (log-linearized) relationships:

$$\left\{ \begin{array}{ccccc} \hat{R}^f & = & \hat{q} + \hat{y} & + & \boldsymbol{\gamma}_f' \hat{\boldsymbol{\varepsilon}} \\ \hat{R}^b & = & (2a-1)\hat{q} + \hat{y} & + & \boldsymbol{\gamma}_b' \hat{\boldsymbol{\varepsilon}} \\ \hat{R}^n & = & \hat{q} + \hat{y} & + & \boldsymbol{\gamma}_n' \hat{\boldsymbol{\varepsilon}} \end{array} \right. ,$$

where $\hat{\boldsymbol{\varepsilon}}$ is now a N-dimensional vector of shocks and $\boldsymbol{\gamma}_i$ for $i = \{b, f, n\}$ is a conformable N-dimensional vector that controls the effect of $\hat{\boldsymbol{\varepsilon}}$ on returns (bond returns, financial and non-financial returns).

To these equations, we add the intratemporal allocation across goods and the budget constraint:

$$\begin{array}{rcl} \hat{y} & = & -\phi\hat{q} + (2a-1)[P_H\hat{C}_H - P_F\hat{C}_F - (1-\phi)\hat{Q}] \\ \hat{PC} & = & \delta\,(2S-1)\,\hat{R}^f + (1-\delta)\hat{R}_n + 2b\hat{R}^b. \end{array} \tag{29}$$

Appendix A.2 shows that the equilibrium portfolio (bond and equity) is unique and well-defined. As in previous cases, it is informative to project the equilibrium bond and equity portfolios returns onto of the real exchange rate $\hat{Q}$ and on return to non-financial wealth $\hat{R}^n$:

$$\left\{ \begin{array}{ccc} \hat{Q} & \equiv & \beta_{Q,b}\hat{R}^b + \beta_{Q,f}\hat{R}^f + u_Q \\ \hat{R}^n & \equiv & \beta_{n,b}\hat{R}^b + \beta_{n,f}\hat{R}^f + u_n \end{array} \right. , \tag{30}$$

where the loading factors $\beta_{i,j}$ are defined as before and u_i for $i = \{Q, n\}$ is a residual, orthogonal to $\hat{R}^j$ for $j = \{b, f\}$: $E\left[u_i\ \hat{R}^j\right] = 0$.

[32] We follow Coeurdacier et al. (2009) by adding preference shocks to the utility provided by Home goods and Foreign goods to the consumers of both countries. This can be interpreted as a worldwide demand shock for a given good as well as shock to the quality of the good.

Define m as the difference between the domestic and foreign stochastic discount factor: $m = C_H^{-\sigma}/P_H - C_F^{-\sigma}/P_F$. From the Euler equation of the investor problem, observe that m satisfies:

$$E\left[\hat{m}\ \hat{R}^i\right] = 0 \text{ for } i = f, b \tag{31}$$

Substituting (30) into the budget constraint (29) and using the definition of m, we obtain:

$$\begin{aligned} P_H\hat{C}_H - P_F\hat{C}_F &= -\frac{1}{\sigma}\hat{m} + (1-\frac{1}{\sigma})\hat{Q} \\ &= \delta\,(2S-1)\,\hat{R}^f + (1-\delta)[\beta_{n,b}\hat{R}^b + \beta_{n,f}\hat{R}^f + u_n] + 2b\hat{R}^b. \end{aligned}$$

Finally, using (31) to project the budget constraint on $\hat{R}^f$ and $\hat{R}^b$, we obtain:

$$\begin{aligned} b^* &= \frac{1}{2}\left(1-\frac{1}{\sigma}\right)\beta_{Q,b} - \frac{1}{2}\,(1-\delta)\,\beta_{n,b} \\ S^* &= \frac{1}{2}\left[1 - \frac{1-\delta}{\delta}\beta_{n,f} + \frac{1-\frac{1}{\sigma}}{\delta}\beta_{Q,f}\right]. \end{aligned} \tag{32}$$

This is the exact same expression as equation (19). Regardless of the number of risk factors, the optimal portfolio still uses equities and bonds to hedge the components of real exchange rate and non-financial wealth with which they are correlated.

4 Estimating Optimal Portfolios

We now turn to the empirical evidence. We estimate the importance of bond-hedging for equilibrium equity holdings of the G-7 countries. We do so by estimating the reduced-form loading factors $\beta_{Q,i}$ and $\beta_{n,i}$ for $i = f, b$. Equation (32) shows that this is all we need to characterize equilibrium portfolios.

4.1 The data.

We collect quarterly data for all G-7 countries over the period 1970:1-2008:3, stopping short of the global financial crisis.[33] We consider in turn each member of the G-7 as the Home country, aggregating the remaining countries into a 'Foreign country'.

4.1.1 The easy part: bond returns, real exchange rates, financial and nonfinancial income.

We measure gross real bond returns, R_i^b, as the ex-post gross return on 3-month domestic Treasury-bill converted in constant U.S. dollars.[34] The (log) of the real exchange rate Q_i

[33] See appendix B.1 for a detailed description of data sources.

[34] Short-term government bond yields and dollar nominal exchange rates are obtained from the *Global Financial Database*.

for country i is defined as the difference between the (log) of the consumer price index in country i, P_i, and the (log) of the consumer price index for the rest of the world, defined as a GDP-weighted average of the price indices of the remaining countries:[35]

$$\ln Q_i = \ln P_i - \sum_{j \neq i} \alpha_{ji} \ln P_j,$$

where α_{ji} represents the share of country $j's$ output in the rest of the world outside country i.[36] With this definition, an increase in Q_i represents a real appreciation of the currency of country i. Figure 2 reports the real exchange rate for the G-7 countries, normalized to 100 in 2001Q1.

Next, we decompose each country's gross domestic product into a financial and a nonfinancial component using National Income Account data.[37] All variables are converted in US dollars using nominal exchange rates. The decomposition of output Y by income satisfies:

$$Y = COMP + M + \Pi + D + T, \tag{33}$$

where $COMP$ refers to the compensation of employees, M to mixed income, Π to the net operating surplus, D to the consumption of fixed capital, and T to taxes minus subsidies on production and imports. According to the 1993 United Nations's System of National Accounts, the net operating surplus Π represents the profits of incorporated entities.[38] By contrast, mixed income M denotes income from self-employment as well as proprietary income.[39] In the model, nonfinancial income denotes the component of aggregate income that cannot be capitalized into financial claims. We follow Gollin (2002) and construct an empirical counterpart W as the sum of the compensation of employees $COMP$, plus a fraction ν of mixed income M:[40]

$$W = COMP + \nu M.$$

Financial income K is then defined as gross operating profits $\Pi + D$ plus the remainder of mixed income $(1 - \nu) M$, net of non-residential gross capital formation I:[41]

$$K = \Pi + D + (1 - \nu) M - I.$$

---

[35] Consumer Price Indices are from the *OECD Main Economic Indicators*.

[36] Formally, $\alpha_{ji} = Y_j / \sum_{j \neq i} Y_j$. We use the following weights: Canada (4.14%), France (7.90%), Germany (10.93%), Italy (7.60%), Japan (16.33%), U.K. (7.74%), U.S. (45.37%).

[37] Data is obtained from the OECD quarterly national income and from U.N. national account statistics.

[38] It is defined as "the surplus or deficit accruing from production before taking account of any interest, rent or similar charges payable on financial or tangible non-produced assets borrowed or rented by the enterprise, or any interest, rent or similar receipts receivable on financial or tangible non-produced assets *owned by the enterprise*."

[39] It is defined as "the surplus or deficit accruing from production by *unincorporated enterprises owned by households*; it implicitly contains an element of remuneration for work done by the owner, or other members of the household, that cannot be separately identified from the return to the owner as entrepreneur but it excludes the operating surplus coming from owner-occupied dwellings."

[40] ν is assumed equal to $COMP/(COMP + \Pi)$. The results are very robust to alternative measures of ν, including the polar cases where all mixed incomes are treated as nonfinancial income ($\nu = 1$) and all mixed incomes are treated as financial income ($\nu = 0$).

[41] We substract gross capital formation to compute the part of income that flows to owners of financial claims on capital. We adjust gross capital formation for residential investment since the latter does not reflect investment decisions of corporations but of households.

Using these measures, we construct estimates of the share of financial income δ as $K/(Y - T - I)$. Table 1 summarizes our estimates for the G-7 countries. These estimates range from 13.1 percent for Germany to 25.4 percent for Italy, with an unweighted average of 16.7 percent. For comparison, the table also reports the 'naïve' estimate of δ obtained as one minus the share of compensation of employees in output measured at factor prices, that is $1 - COMP/(Y - T)$. The naïve estimate is much higher, with an average of 41.3 percent.

In what follows, we normalize financial and nonfinancial income by population, and express them in constant U.S. dollars. Figure 2 reports nonfinancial income per capita for each country relative to the nonfinancial income of the remaining G-7 countries. Relative nonfinancial income exhibits marked fluctuations over the period. For instance, for the U.S., it fluctuates between 0.9 and 2.4 times per capita non financial income in the remaining countries. It is also strikingly correlated with the real exchange rate, also reported on the same figure.[42] This correlation is one of the reason that the unconditional correlation between non-financial returns and equity returns may be very different from the correlation conditional on bond returns.

4.1.2 The harder part: returns to financial and nonfinancial wealth

We now construct empirical counterparts to the return on financial and nonfinancial wealth since neither returns are directly observable.

Consider first the return to financial wealth, R^f, where we drop the country subscript i to ease notation. In general, that return is not equal to the return on aggregate equity R^e. In the model, the two are equal because financial wealth is entirely capitalized in the equity market. In practice, firms are financed through equity and corporate debt, among other instruments.[43] What is needed is an estimate of the return to the *firm value*. Our benchmark method looks at the liability side of the firms' balance sheet, using observable equity and corporate bond market data. Specifically, we construct the gross return to financial wealth, R^f, as a weighted average of the country's equity (R^e) and corporate debt (R^d) gross constant dollar returns, where the weight μ reflect the share of corporate debt in the total value of the firm. These weights are estimated for each country using balance sheet data for non-financial firms from Compustat.[44] Our measure of returns to financial wealth for each country is then:

$$r^f_{t+1} = \log(R^f_{t+1}) = \log\left[(1-\mu)R^e_{t+1} + \mu R^d_{t+1}\right]. \tag{34}$$

[42] The correlation ranges between 0.68 for Italy and 0.96 for Japan with an average of 0.85

[43] One might worry that equilibrium equity positions might differ if firms are able to issue debt as well as equity. We show in appendix A.3 that in the benchmark model where firms' financing decisions are irrelevant for the value of the firm, this is not the case. In this Modigliani-Miller limit case, the presence of corporate debt has no impact on equity portfolio decisions. In models with some departure from Modigliani-Miller, our results can remain valid as long as changes in the value accrued to debtholders versus shareholders does not interact with the fundamental shocks of the model.

[44] See appendix B.1 for details. The average share of debt in total liabilities is 67.1 percent (Canada), 75.2 percent (France), 75.3 percent (Germany), 76.2 percent (Italy), 70.7 percent (Japan), 59.2 percent (U.K.), 71.8 percent (U.S.). The country equity and corporate debt returns are obtained from the Global Financial Database. For Italy, we set $\mu = 0$ since we were unable to obtain a series of yields on Italian corporate debt.

As a robustness check, we also construct estimates of R^f under alternative assumptions in section 4.5.

Consider next the return to nonfinancial wealth, R^n. In a dynamic context, that return differs from the growth rate of real nonfinancial income per capita $\Delta \ln W$: the latter represents only the dividend component and not the total return on the corresponding asset.[45] To construct estimates of R^n, we follow the present-value method of Campbell and Shiller (1988), as detailed in Campbell (1996). Under the assumption that the dividend-price ratio on nonfinancial wealth is stationary, the return on that asset satisfies the following approximation:

$$r^n_{t+1} \equiv \ln \left(W_{t+1} + V^n_{t+1}\right) - \ln V^n_t = k + \phi^n_t - \rho\, \phi^n_{t+1} + \Delta w_{t+1}, \tag{35}$$

where lowercase letter to denote logs of variables (e.g. $r^n_{t+1} = \ln\left(R^n_{t+1}\right)$), V^n_t denotes nonfinancial wealth at time t, $\phi^n_t = \ln\left(W_t/V^n_t\right)$ is the log dividend-price ratio for nonfinancial wealth, ρ is a number slightly smaller than 1 and k is an unimportant constant.[46]

Using (35) to solve for ϕ_t forward, imposing the equilibrium condition that $\lim_{t\to\infty} \rho^t \left(r^n_t - \Delta w_t\right) = 0$, substituting back into equation (35), and taking conditional expectations, yields the usual present-value relationship:

$$r^n_{t+1} - E_t r^n_{t+1} = \left(E_{t+1} - E_t\right) \sum_{s=0}^{\infty} \rho^s \Delta w_{t+1+s} - \left(E_{t+1} - E_t\right) \sum_{s=1}^{\infty} \rho^s r^n_{t+1+s}. \tag{36}$$

This expression makes clear that the innovation to the return on nonfinancial wealth on the left hand side of the equation depends positively upon revisions to the path of future expected real nonfinancial income growth –the cash flow component represented by the first summation on the right hand side– and negatively upon revisions to the path of future expected real returns –the discount rate component represented by the second summation on the right hand side.

Our approach consists in constructing the empirical counterpart of equations (36) for each country using an empirical Vector-Auto-Regression (VAR) in first differences of the following form:[47]

$$\mathbf{Z}_{t+1} = \mathbf{A}\ \mathbf{Z}_t + \boldsymbol{\epsilon}_{t+1}$$

where $\mathbf{Z}_t = (\tilde{r}_t, \Delta w_t, \Delta k_t, \mathbf{x}'_t)'$. In this expression, $\tilde{r}_{t+s}$ represents a possible proxy for the return on nonfinancial wealth at time $t+s$, in the sense that $E_t \tilde{r}_{t+s} = E_t r^n_{t+s}$. This proxy is necessary to construct the second summation on the right hand side of (36). In practice, we will set $\tilde{r} = r^f$, that is, we will assume that expected financial and nonfinancial future returns are equal. Finally, $\mathbf{x}_t$ denotes a vector of additional controls used to forecast future factor income growth and future returns.

[45] See Baxter and Jermann (1997, p. 175)

[46] One can show that $\rho = 1/\left(1 + \exp\left(\phi\right)\right)$ where ϕ is the steady state value of the log dividend-price ratio. We will use the value of $\rho = 0.98$ in line with standard estimates in the literature. Our results are robust to changes in the value of ρ.

[47] Standard Akaike and Schwarz lag-selection criteria indicate that a VAR of order 1 is the preferred specification for all countries.

Our VAR specification first-differences financial and non-financial income. We discuss in detail in appendix B.2 why this is the appropriate empirical specification. In short, we find that while we cannot reject the null hypothesis that w and k are integrated processes, we do not find any statistical evidence of a cointegration relationship between the two.[48] Therefore, a stationary VAR in first-differences is appropriate.

Based on our reading of the literature on financial return predictability, we consider a comprehensive list of potential controls for future asset returns: consumption growth; the dividend-price ratio; the relative T-bill rate, defined as the difference between the yield on 3-month T-bill rate and a 4-quarter moving average; the term premium, defined as the spread between 10 year and 3 months government yields; the yield spread, defined as the spread between the yield on long-term corporate bonds and the yield on 10-year government bonds; cay, the fluctuations in US aggregate consumption-wealth ratio as measured by Lettau and Ludvigson (2001); and nxa, the Gourinchas and Rey (2007) measure of US external imbalances. In order to maintain a parsimonious and statistically significant representation, our selection of variables is as follows. First we exclude variables that appear integrated, based on Augmented-ADF tests, since this would violate our stationary VAR assumptions. Second, we select predictive variables based on the Least Angle Regression (LARS) approach of Efron, Hastie, Johnstone and Tibshirani (2004) applied to the financial return equation of the Vector Auto Regression. This selection algorithm efficiently selects a parsimonious subset of predictive variables. In our final specification, only a few predictive variables remain: consumption growth for Japan and the term premium for the U.S.

With estimates of $\mathbf{A}$ and $\boldsymbol{\epsilon}_{t+1}$ in hand, the empirical counterpart to $r^n_{t+1} - E_t r^n_{t+1}$ can be obtained from (36) as:

$$r^n_{t+1} - E_t r^n_{t+1} = \left(\mathbf{e}'_{\Delta w} - \rho \mathbf{e}'_{\tilde{r}} \mathbf{A}\right) \left(\mathbf{I} - \rho \mathbf{A}\right)^{-1} \boldsymbol{\epsilon}_{t+1}, \tag{37}$$

where $\mathbf{e}'_y$ is a row-vector that 'selects' variable y in $\mathbf{Z}$, i.e. such that $\mathbf{e}'_y \mathbf{Z} = y$ for $y = \Delta w$ or $\tilde{r}$. The first term, $\mathbf{e}'_{\Delta w} \left(\mathbf{I} - \rho \mathbf{A}\right)^{-1} \boldsymbol{\epsilon}_{t+1}$, captures the contribution of expected future non financial income growth (the first summation in equation (36)). The second term, $-\rho \mathbf{e}'_{\tilde{r}} \mathbf{A} \left(\mathbf{I} - \rho \mathbf{A}\right)^{-1} \boldsymbol{\epsilon}_{t+1}$, captures the contribution of expected future returns on nonfinancial wealth (the second summation in equation (36)). Figure 3 reports the return to nonfinancial wealth $r^n_{t+1} - E_t r^n_{t+1}$ for the U.S., together with the growth rate of nonfinancial income Δw. The correlation between the two series is high (0.66), but the striking fact is that the return innovation exhibits much more volatility.[49]

The last step consists in measuring bond, financial and non-financial returns relative to

[48] Our approach differs from Baxter and Jermann (1997) who estimate a Vector Error Correction Mechanism (VECM) on financial and nonfinancial income, imposing the cointegration relationship that $k - w$ is stationary. This assumption is appealing on theoretical grounds since the share of financial income is bounded between 0 and 1. It is, however, strongly rejected in the data, indicating a very persistent process for income shares, with no apparent error-correction term. Moreover, as discussed in appendix B.2, the assumption that $k - w$ is non-stationary is also strongly rejected by the data.

[49] The standard deviation of the return innovations is 3.09% vs. 1.01% for nonfinancial income growth.

the rest of the world. To this effect, we define the relative returns $\hat{r}^l_i$ of country i as follows:

$$\hat{r}^l_{i,t+1} = \left(r^l_{i,t+1} - E_t r^l_{i,t+1}\right) - \sum_{j \neq i} \alpha_{ji} \left(r^l_{j,t+1} - E_t r^l_{j,t+1}\right),$$

for $l \in \{b, n, f\}$, where α_{ji} is the output weight of country j in the rest of the world outside of country i.

4.2 Estimating the loadings on the real exchange rate

We are now in a position to estimate the key loading parameters in equation (30). We begin with the loadings on the real exchange rate, $\beta_{Q,j}$ for $j = f, b$. These unconditional moments can be estimated for each country by the following simple regression for each country i:

$$\Delta \ln Q_{i,t} \equiv c + \beta^i_{Q,b} \hat{r}^b_{i,t} + \beta^i_{Q,f} \hat{r}^f_{i,t} + u_{i,t}. \tag{38}$$

where $u_{i,t}$ captures the fluctuations in the real exchange rate that are not spanned by relative bond and financial returns.

Results of regression (38) for each countries are displayed in Table 2. Our empirical results confirm the results of van Wincoop and Warnock (2010) for all the countries considered in the sample: relative bond returns capture most of the variations of the real exchange rate. The coefficient on the relative bond returns in panel A, $\beta_{Q,b}$ is often not statistically different from one, between 0.89 for the U.K and 1.02 for Germany. The R^2 of the regression is also very strong, between 0.95 for Italy and 0.981 for the U.S. and France. Moreover, *conditional on bond returns*, the variance-covariance ratio for financial returns and the real exchange rate, $\beta_{Q,f}$ is almost never statistically different from zero.[50] Table 2 confirms that, from a theoretical standpoint, the model with $\gamma_b = 0$ provides a very reasonable approximation of the data.[51]

Panel B of the table reports the unconditional loading on the real exchange rate $\beta^{\text{unc}}_{Q,f}$ obtained from a regression *only* on the relative financial return $\hat{r}^f$. The coefficients are significantly positive for all countries, between 0.07 (Italy) and 0.69 (U.S.). This re-emphasizes the importance of properly conditioning on the relative bond returns. Finally, the last column of the table reports the results from a pooled regression with country fixed effects. This can be interpreted as an average loading for all G-7 countries. The estimates, $\beta_{Q,b} = 0.96$ and $\beta_{Q,f} = 0.01$ confirm the strong correlation between relative bond returns and real exchange rates.

4.3 Estimating the loading on the return to non-financial wealth

We now use the returns to non-financial wealth estimated for each country i to estimate the loadings of (relative) bond returns and (relative) returns to financial wealth by estimating

[50] The exceptions are the U.K. and Germany. Even for these two countries, $\beta_{Q,f}$ remains economically very small, less than 6 percent.

[51] Recall that when $\gamma_b = 0$, the theoretical model implies $\beta_{Q,b} = 1$ and $\beta_{Q,f} = 0$, which is consistent with our empirical estimates.

the following equation:

$$\hat{r}^n_{i,t} = c + \beta^i_{n,b}\hat{r}^b_{i,t} + \beta^i_{n,f}\hat{r}^f_{i,t} + v_{i,t}, \tag{39}$$

where $v_{i,t}$ is attributed both to the measurement error in the construction of the return on nonfinancial wealth, and to the fluctuations in relative nonfinancial income risk not spanned by relative bond returns and relative returns to financial wealth as in equation (30).

Results of the regression (39) for each countries are shown in Table 3. Panel B reports the estimate of the unconditional loading factor $\beta^{i,\text{unc}}_{n,f} = cov(\hat{r}^n_i, \hat{r}^f_i)/var(\hat{r}^f_i)$. This coefficient is positive and significant for all countries except Italy, with a pooled estimate of 0.26. This indicates that returns to non financial wealth are positively correlated with returns to financial wealth as in Baxter and Jermann (1997) and the international diversification puzzle would be 'worse than you think' in this earlier literature.

However, the loading factor conditional on bond returns $\beta^i_{n,f}$ reported in panel A is negative for all countries, often statistically so.[52] It varies between -0.005 (Germany) and -0.287 (France) with a pooled estimate of -0.139. As the previous analysis emphasized, this negative conditional loading indicates that in all these countries domestic equities constitute a good hedge against shocks to non financial wealth.

Moreover, the positive loadings of (relative) bond returns $\beta_{n,b} > 0$ implies that shorting the local currency bond, and going long in the foreign currency bond, constitutes a good hedge against fluctuations in returns to non-financial wealth (see equation (32)). This is not surprising: in our model, a (potentially large) part of relative non-financial income comoves with the real exchange rate (see figure 2), and we know that relative bond returns track almost perfectly the real exchange rate.

Going from the reduced form estimates to the structural parameters of the model requires taking a stand on the 'correct' model of the economy. A full-fledged structural estimation lies beyond what we attempt in this paper.[53]

4.4 Implied equity and bond portfolios

The previous estimates would allow us to back out the implied equity and bond positions using equations (32) if all countries were symmetric. Allowing for different country sizes,

[52]The loading factor is significantly negative for all countries but Germany at the 10% significance level.

[53]As an illustration, consider the reduced-form locally complete model of section 3.1. Equations 17 and 18 determine the mapping from the structural parameters of the model to the β_{ij}. It is easy to check that only three out of the four β_{ij} are independent, so the system can be inverted to recover $\bar{\lambda}$, γ_b/γ_f and $\gamma_n\gamma_f$ as a function of a, the preference for the home-good. Consider the pooled estimates reported in tables 2 and 3 and assume a reasonable value of a of 0.75 equal to one minus the share of imports. We infer the following estimates: $\gamma_b/\gamma_f = -\beta_{Qf}/\beta_{Qb} = -0.009$, $\gamma_n/\gamma_f = (\beta_{nf} + \beta_{Qb} - 1)/\beta_{Qb} = -0.168$ and $\bar{\lambda} = 1 - \beta_{nb}(2a-1)/(1-\beta_{nf}) = 0.506$. The estimate of $\bar{\lambda}$ indicates a relatively high elasticity of terms of trade to relative output, as in Corsetti, Dedola and Leduc (2008) and Kollmann (2006). Additional assumptions would be required to map these parameters back into the structural preference parameters such as the elasticity of substitution between goods ϕ, the coefficient of risk aversion σ, or the auxiliary parameters that control the loadings γ_i.

(32) must be rewritten as follows (see appendix (A.6)):

$$\left\{ \begin{array}{lcl} b^* & = & (1-\omega_i)\left(1-\frac{1}{\sigma}\right)\beta_{Q,b} - (1-\omega_i)(1-\delta)\,\beta_{n,b} \\ S^* & = & \omega_i + (1-\omega_i)\left(\frac{1-\frac{1}{\sigma}}{\delta}\beta_{Q,f} - \frac{1-\delta}{\delta}\beta_{n,f}\right) \end{array} \right. \tag{40}$$

where ω_i is the relative size of country i in world GDP.

The implied equity bias and bond portfolios are summarized in table 4 using the loading coefficients from our baseline estimates. As equation (40) indicates, the optimal bond position requires an estimate of the degree of risk aversion σ. We consider the plausible value of $\sigma = 2$ in our benchmark calibration. For the share of financial income δ, we use the average share of financial income across G7 countries in the more recent period (2000-2008): $\delta = 0.191$.

The model is very successful in predicting a significant degree of equity home bias for all countries when bond trading is allowed.

Consider first panel B, which excludes bonds, as in most of the literature. The baseline refers to the first term in equation (40), that is, a predicted portfolio share equal to the share in world output ω_i. The second term (Bias due to Q) reflects the contribution of the real exchange rate hedging component: $(1-\omega_i)(1-1/\sigma)/\delta\beta^{i,\text{unc}}_{Q,f}$. Given the positive unconditional correlation between financial returns and exchange rates ($\beta^{i,\text{unc}}_{Q,f} > 0$), this term is positive, indicating a potential source of home bias. The second term (Bias due to r^n) reflects the contribution of the non financial income hedging component: $-(1-\omega_i)(1-\delta)/\delta\beta^{i,\text{unc}}_{n,f}$. Since $\beta^{i,\text{unc}}_{n,f}$ is strongly positive, this term contributes negatively to the optimal equity portfolio and dominates the real exchange rate hedge. This results, as in Baxter and Jermann (1997) is a strong predicted foreign bias, $S^i - \omega_i$ ranging from -10 percent for the U.K. to -116 percent for Germany, in total contrast to the data.[54]

By contrast, Panel A shows that the estimated model accounts for a large share of observed equity home bias once we introduce bonds. The hedge portfolio is now dominated by the non-financial income component. This term is strongly positive since $\beta^i_{n,f} < 0$. The predicted equity portfolio (S) varies between 5.8% for Germany and 119% for France while available empirical evidence indicates a home equity position between 55% (Germany) and 85.6% (Canada) (averages over 2000-2008).[55] Except for Germany and to a lesser extent Japan, the equity bias predicted by the model is comparable to the amount of bias in the data.[56]

The last line (ΔS) reports the change in the predicted equity position between the equity only and the full model. In all cases, the predicted equity position increases substantially, moving the model closer to the data. For instance, in the case of the U.S., the model with equity only predicts a negligible share in domestic equity (0.83%) while the full model predicts 101.48% even exceeding the empirical estimate of 83.2%.

[54]The exception is Italy, where the unconditional loading is negative and therefore the model predicts more home bias than observed.

[55]Data are from Coeurdacier and Rey (2011).

[56]Using $\beta_{Q,f}$ and $\beta_{n,f}$ estimated on pooled data for all countries, we get equity portfolios ranging from 63% for Canada and Germany to 80% for the US, fairly close to the data.

Panel A also reports the model predictions for bond holdings. As for equities, we can decompose the predicted bond position into a real exchange rate hedge component ($(1-\omega_i)(1-1/\sigma)\beta_{Q,b}$) and a non financial income component ($-(1-\omega_i)(1-\delta)\beta_{n,b}$).[57] We find a strong positive demand for local currency bonds arising from real exchange rate hedging, given the positive loading factor $\beta_{Q,b}$, but an even stronger and negative loading factor for hedging non-financial income risk, given $\beta_{n,b}$.[58] While each of these component can be large relative to output, they offset each other and imply net currency exposure of bond portfolios of reasonable magnitude. Thus, the model predicts that countries should issue bond liabilities in their own currency, between 22 percent (Japan) and 50 percent (Canada or Italy) of their domestic output. Data regarding the net currency exposure of debt positions from Lane and Shambaugh (2010b) suggests that G7 countries are on average short in domestic currency (and long in foreign currency) although the positions are smaller than those predicted by the model. The average net currency bond exposure is $b = -6.3\%$ of GDP over 2000-2004.[59] There is some heterogeneity across countries: while US, UK, Japan and Italy are short in domestic currency, Germany, France and Canada are long. Our empirical counterpart of b ranges from -16.4% of GDP for the UK to +9.8% for France. As direct inspection of equation (40) shows, for higher values of σ the hedging of real exchange rate becomes progressively more important, reducing the size of predicted domestic currency bond positions.[60]

4.5 Using Different Measures of Returns to Financial and Non-Financial Wealth

A key element of our analysis is the construction of returns to financial and non financial wealth r^f and r^n. If these returns are incorrectly measured, one should be cautious when interpreting the loading factors and predicted portfolios. This section investigates the robustness of our results to various alternative measures of financial and non financial returns.

A first point of departure would be to construct returns to financial wealth using the same approach as for non financial returns, with national income data.[61] This approach yields the following expression for the return to financial wealth:

[57] The baseline is zero for the bond position.

[58] This term would only grow stronger relative to the real exchange rate hedge if we decrease the coefficient of risk aversion σ. In the limit where $\sigma = 1$, the real exchange rate hedge component disappears.

[59] In the data, countries often have leveraged external debt position. The counterpart of b in the model is $(b_{HH} - b_{HF})/2$ where b_{HH} denotes the net domestic currency debt exposure, that is, the difference between domestic currency denominated debt assets and domestic currency denominated debt liabilities– and b_{HF} denotes the net foreign currency debt exposure, that is, the difference between foreign currency debt assets and foreign currency debt liabilities. This counterpart generates the same wealth transfer towards the a country whose currency depreciates by 1% with respect to all other currencies than in our model.

[60] Bond position b becomes eventually positive for high values of σ. With $\sigma = 6.5$, the GDP weighted-average of our predictions for b matches the data, i.e. a GDP weighted currency exposure of -6.3% of GDP.

[61] Baxter and Jermann (1997) used such an approach.

$$r^f_{t+1} - E_t r^f_{t+1} = (E_{t+1} - E_t)\sum_{j=0}^{\infty}\rho^j \Delta k_{t+1+j} - (E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^j r^f_{t+1+s} \tag{41}$$

Using the same VAR specification as in section (4.1.2), the empirical estimates of the returns to financial wealth becomes:[62]

$$r^f_{t+1} - E_t r^f_{t+1} = \left(\mathbf{e}'_{\Delta k} - \rho \mathbf{e}'_{\tilde{r}}\mathbf{A}\right)\left(\mathbf{I} - \rho\mathbf{A}\right)^{-1}\boldsymbol{\epsilon}_{t+1}. \tag{42}$$

The returns on the firm thus obtained may be noisy and imperfectly estimated. Our second approach instruments the returns in equation (42) with the country's equity and corporate debt returns, and forcing the weights to sum to one. This is equivalent to choosing different weights $\hat{\mu}$ in equation (34), measuring the leverage implied by national accounts data. In other words, for each country, we run a first stage as follows:

$$r^f_t = (1 - \hat{\mu})r^e_t + \hat{\mu} r^d_t + \nu_t. \tag{43}$$

The predicted component $\left((1-\hat{\mu})r^e_t + \hat{\mu}r^d_t\right)$ of (43) becomes our proxy for returns to financial wealth. This method identifies the variations in financial wealth estimated from national accounts that are reflected in market returns and is therefore potentially more robust to measurement error.

A third approach approach simply sets $\mu = 0$, equating the return to financial wealth with observed equity returns:

$$r^f_t = r^e_t$$

This approach has the merit of simplicity, but as argued earlier, there are good theoretical reasons why equity returns may differ from the returns to the firm.[63]

Figure 4 reports the innovations to financial returns under these alternative measures for the United States. As can be seen from the figure, the return innovations tend to be positively correlated.[64] The equity return is also the more volatile, with a standard deviation of 8% per quarter against 3.5% for the projected NIPA return.

Lastly, we also consider a different approach to construct returns to non financial wealth, borrowing from Lustig and Nieuwerburgh (2008). The basic idea is to recover the unobserved innovation to non financial wealth from the joint behavior of consumption and market returns, under the assumption that aggregate consumption satisfies the first-order condition of

[62] The implementation still requires the use of observed market returns to form expectations of future returns. That is, in the above expression, $\tilde{r}$ denotes a proxy for future expected financial and non financial returns in the sense that $E_t\tilde{r}_{t+j} = E_t r^f_{t+j} = E_t r^n_{t+j}$. In practice, we use the returns on the firm constructed in the previous section as a proxy.

[63] We also set $\tilde{r} = r^e$ and re-run the LARS algorithm to select predictive variables. The following variables are added to the VAR: yield spread (Germany and U.S.), consumption growth and relative T-bill (Japan) and term spread (U.S.).

[64] The NIPA returns tend to have the lowest correlation with the other variables, between 0.15 and 0.21. All other measures are strongly correlated.

an optimizing representative household.[65] Because consumption growth is not strongly correlated with financial returns, this method infers that returns to non financial wealth must be strongly negatively correlated with financial returns. Indeed, the correlation between returns to non-financial wealth constructed in this manner and our benchmark financial returns is -0.55 for the United States.

Results of the regressions (38) and (39) are displayed in table 5 and table 6 for the different specifications and the different countries. Our empirical results confirm the previous results across all specifications: relative bond returns capture most of the variations of the real exchange rate and claims on financial income are not used to hedge real exchange rate changes (see table 5). Moreover, *conditional on bond returns*, the loadings of non-financial wealth on financial wealth are negative across all specifications and significantly so for most of the countries, implying home bias in our model (see table 6). This confirms the important role of bond holdings as an hedging instrument. Hence, qualitatively, results using these alternative measures of returns are very similar to our benchmark case. Quantitatively, the amount of equity bias generated by the model is very similar to our benchmark case when using the projection on market returns of financial returns estimated from national accounts (pooled estimate of $\beta_{n,f}$ equal to -0.14), or even larger when using the method of Lustig and Nieuwerburgh (2008) (pooled estimate of $\beta_{n,f}$ equal to -0.28). It falls a bit short compared to the data when using national accounts data or when using equity returns to compute financial returns (pooled estimates of $\beta_{n,f}$ equal to -0.07 and -0.09 respectively).[66] But even in these two specifications, the model can explain a significant share of equity home bias (above 40%). When looking at the US more specifically, the equity portfolio implied by the model are respectively 80% of domestic equity when using national account data and 63% when using equity returns.[67]

5 Conclusion

What drives equity home bias? This paper merges and improves upon two strands of literature. The first strand focused on risks to non-financial wealth. It concluded that home equity positions should be even more tilted towards foreign equity since non financial and financial returns appeared positively correlated. The second strand looked at frictions in goods markets and emphasized real exchange rate risks. In this class of models, efficient risk sharing requires holding securities delivering high returns when the domestic currency

[65] See appendix B.2 for details.

[66] One could also argue that these are noisier measure of financial returns causing attenuation bias on our estimates of the loadings ($\beta^i_{n,f}$ in particular).

[67] Like in our benchmark regression, the unconditional loadings (non-reported) for these two specifications are positive and highly significant ($\beta^{i,\text{unc}}_{n,f} > 0$) implying a very large foreign bias in the model without bonds. With the Lustig and Nieuwerburgh (2008) approach, the results are more complex. The unconditional loading becomes large and negative (-0.66 on pooled data) since –by construction- financial and non financial returns are strongly negatively correlated. However, such a large negative unconditional loading would imply an implausibly high level of home bias ($S = 3.76$). Nevertheless, the *conditional* loading remains largely unchanged and the predicted equity position stays close to the data ($S = 0.92$).

appreciates. For this class of models to explain the home portfolio bias we see in the data, domestic equity returns would need to be strongly positively correlated with real exchange rates. Alas, the data does not oblige and the correlation is too low. What this paper does is to show that both strands of the literature are related, but incomplete. It starts from the observation that relative bond returns (nominal or real) are strongly correlated with real exchange rates. It follows that, in a world where investors can trade both equities and bonds, they will hedge real exchange rate risk with the latter. And once this is achieved, the equilibrium equity position will be a function of the residual risks that investors face, namely the risk to their non-financial wealth, *conditional* on bond returns. The paper derives this prediction in a simple and fairly generic model that encompasses many models of interest. In this wide class of models, both with complete or incomplete markets, equity home bias will arise if non financial risk is negatively correlated with equity returns, after controlling for bond returns. The paper also characterizes equilibrium portfolios as a simple function of loading factors that can easily be estimated from data on real exchange rates and returns on bonds, financial and non-financial wealth. We implement this empirical strategy for the countries of the G-7 and show that under many reasonable specifications, the conditional correlation between financial and non-financial returns is such that it can empirically account for a significant share of the observed equity home bias. For most countries, the conditional correlation between financial and non financial returns is negative and economically significant. In other words, the international diversification puzzle is not so puzzling anymore! The model also makes reasonable predictions about bond positions and we find an implied currency exposure of bond portfolios broadly in line with the empirical evidence.

It is possible to interpret our results in a broader perspective. Nominal exchange rates present a deep source of puzzles in international finance. They are too volatile and largely uncorrelated with their fundamental determinants – the exchange rate disconnect puzzle. To the extent that nominal exchange rate movements drive real exchange rate fluctuations, real exchange rates too, do not behave as predicted in our models –the Mussa (1986) puzzle. For instance, relative real consumption is not correlated with real exchange rate movements as models of risk sharing predict –the Backus and Smith (1993) puzzle. In the context of international portfolios, this implies that real exchange rates fluctuations are both uncorrelated with relative financial returns, and that relative financial and non-financial returns are positively correlated, since a given change in the nominal exchange rate affects both returns in the same direction. Our paper shows that, once currency fluctuations are controlled for through the use of nominal or real bonds, the structure of international equity portfolios conforms to the predictions of standard portfolio models. This provides a qualified success for the theory, since an empirically successful theory of exchange rate fluctuations remains elusive.

Two obvious steps are left for future research. First, one would want to go back and enrich/discriminate among existing models to fully account for the loadings we obtain from the data. Such a model would be consistent both with observed portfolios (quantities) and with the covariance structure of exchange rates and asset returns (prices). Second, it would be interesting to extend the menu of assets beyond stocks and bonds. Models with a larger set of assets (housing for instance) could also deliver new insights on international risk sharing.

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	Canada	France	Germany	Italy	Japan	U.K.	U.S.	Euro	Average
δ	16.4	14.1	13.1	25.4	16.1	18.5	13.3	17.9	16.7
naïve-δ	39.9	39.9	40.5	50.9	43.5	36.7	37.8	42.9	41.3

Table 1: Estimates of the share of financial income in output δ (in percent), defined as the share of financial income $(\Pi + D + (1-\lambda)\,M - I)$ in output at product prices net of investment $(Y - T - I)$. The naïve share is estimated as one minus the share of compensation of employees $(COMP)$ in output at factor prices $(Y - T)$. Source: OECD Quarterly National Income and U.N. National Account Statistics. Authors' calculations.

	Canada	France	Germany	Italy	Japan	U.K.	U.S.	Pooled
				Panel A: Conditional Loadings				
$\beta_{Q,f}$	-0.019	-0.005	-0.030**	0.007	-0.002	0.059***	0.011	0.009**
(s.e.)	(0.017)	(0.014)	(0.015)	(0.006)	(0.018)	(0.016)	(0.016)	(0.004)
p-val	0.281	0.724	0.048	0.268	0.907	0.000	0.500	0.042
$\beta_{Q,b}$	1.007***	0.969***	1.019***	0.964***	1.008***	0.890***	0.981***	0.975***
(s.e.)	(0.023)	(0.017)	(0.019)	(0.017)	(0.023)	(0.028)	(0.017)	(0.007)
p-val	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
R^2	0.969	0.981	0.977	0.956	0.972	0.933	0.981	0.968
				Panel B: Unconditional Loadings				
$\beta^{\text{unc}}_{Q,f}$	0.579***	0.579***	0.594***	0.070**	0.639***	0.400***	0.691***	0.356***
(s.e.)	(0.041)	(0.041)	(0.044)	(0.027)	(0.042)	(0.034)	(0.048)	(0.016)
p-val	0.000	0.000	0.000	0.011	0.000	0.000	0.000	0.000
R^2	0.570	0.565	0.547	0.042	0.605	0.478	0.581	0.327
Obs.	154	154	154	154	154	154	154	1078

Table 2: Loadings on **real exchange rate changes**: $\Delta \ln Q_{i,t} = \beta^i_{Q,b}\hat{r}^b_{i,t} + \beta^i_{Q,f}\hat{r}^f_{i,t} + u_{i,t}$. Standard errors are in parenthesis. (***) (resp (**)) indicates significance at the 1% level (resp. 5%). Unconditional loadings impose $\beta_{Q,b} = 0$. Last column reports pooled fixed effect estimates. Constants are not reported. Sample: 1970:2 to 2008:3.

	Canada	France	Germany	Italy	Japan	U.K.	U.S.	Pooled
				Panel A: Conditional Loadings				
$\beta_{n,f}$	-0.171***	-0.287***	-0.005	-0.141***	-0.056	-0.091**	-0.235***	-0.139***
(s.e.)	(0.064)	(0.060)	(0.056)	(0.020)	(0.044)	(0.037)	(0.078)	(0.014)
p-val	0.008	0.000	0.923	0.000	0.211	0.014	0.003	0.000
$\beta_{n,b}$	1.259***	1.151***	1.110***	1.263***	0.950***	0.950***	1.227***	1.125***
(s.e)	(0.084)	(0.075)	(0.069)	(0.057)	(0.055)	(0.063)	(0.086)	(0.022)
p-val	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
R^2	0.740	0.705	0.803	0.769	0.826	0.684	0.703	0.742
				Panel B: Unconditional Loadings				
$\beta_{n,f}^{\text{unc}}$	0.574***	0.419***	0.675***	-0.058	0.552***	0.272***	0.619***	0.263***
(s.e.)	(0.064)	(0.061)	(0.059)	(0.040)	(0.047)	(0.044)	(0.077)	(0.021)
p-val	0.000	0.000	0.000	0.146	0.000	0.000	0.000	0.000
R^2	0.351	0.237	0.466	0.014	0.480	0.205	0.300	0.128
Obs.	153	153	153	153	153	153	153	1071

Table 3: Loadings on **nonfinancial returns**: $\hat{r}_{i,t}^{n} = \beta_{n,b}^{i}\hat{r}_{i,t}^{b} + \beta_{n,f}^{i}\hat{r}_{i,t}^{f} + v_{i,t}$. Standard errors are in parenthesis. (***) (resp (**)) indicates significance at the 1% level (resp. 5%). Unconditional loadings impose $\beta_{n,b} = 0$. Last column reports pooled fixed effect estimates. Constants are not reported. Sample: 1970:2 to 2008:3.

	Canada	France	Germany	Italy	Japan	U.K.	U.S.
			Panel A: With Bonds and Equity				
Equity							
Baseline (GDP weights)	4.14	7.90	10.93	7.60	16.33	7.74	45.37
Bias due to:							
Q	-4.72	-1.15	-7.11	1.60	-0.47	14.34	1.52
r^n	69.54	112.16	2.03	55.21	19.86	35.83	54.60
Total (S)	68.97	118.90	5.84	64.41	35.72	57.91	101.48
Data for (S) (2000-2008)	85.60	71.40	55.40	59.50	84.30	65.20	83.20
Bond							
Bias due to:							
Q	48.26	44.61	45.38	44.55	42.15	41.03	26.79
r^n	-97.70	-85.79	-80.06	-94.49	-64.36	-70.93	-54.25
Total (b)	-49.44	-41.19	-34.68	-49.94	-22.21	-29.90	-27.46
Data for (b) (2000-2004)	9.30	9.90	8.90	-2.70	-12.70	-16.40	-10.90
			Panel B: Equities Only				
Baseline (GDP weights)	4.14	7.90	10.93	7.60	16.33	7.74	45.37
Bias due to:							
Q	145.47	139.77	138.75	16.91	140.28	96.79	99.08
r^n	-233.63	-163.87	-255.35	22.73	-196.14	-106.67	-143.61
Total (S)	-84.01	-16.20	-105.68	47.23	-39.53	-2.15	0.83
ΔS	152.98	135.10	111.52	17.17	75.25	60.06	100.65

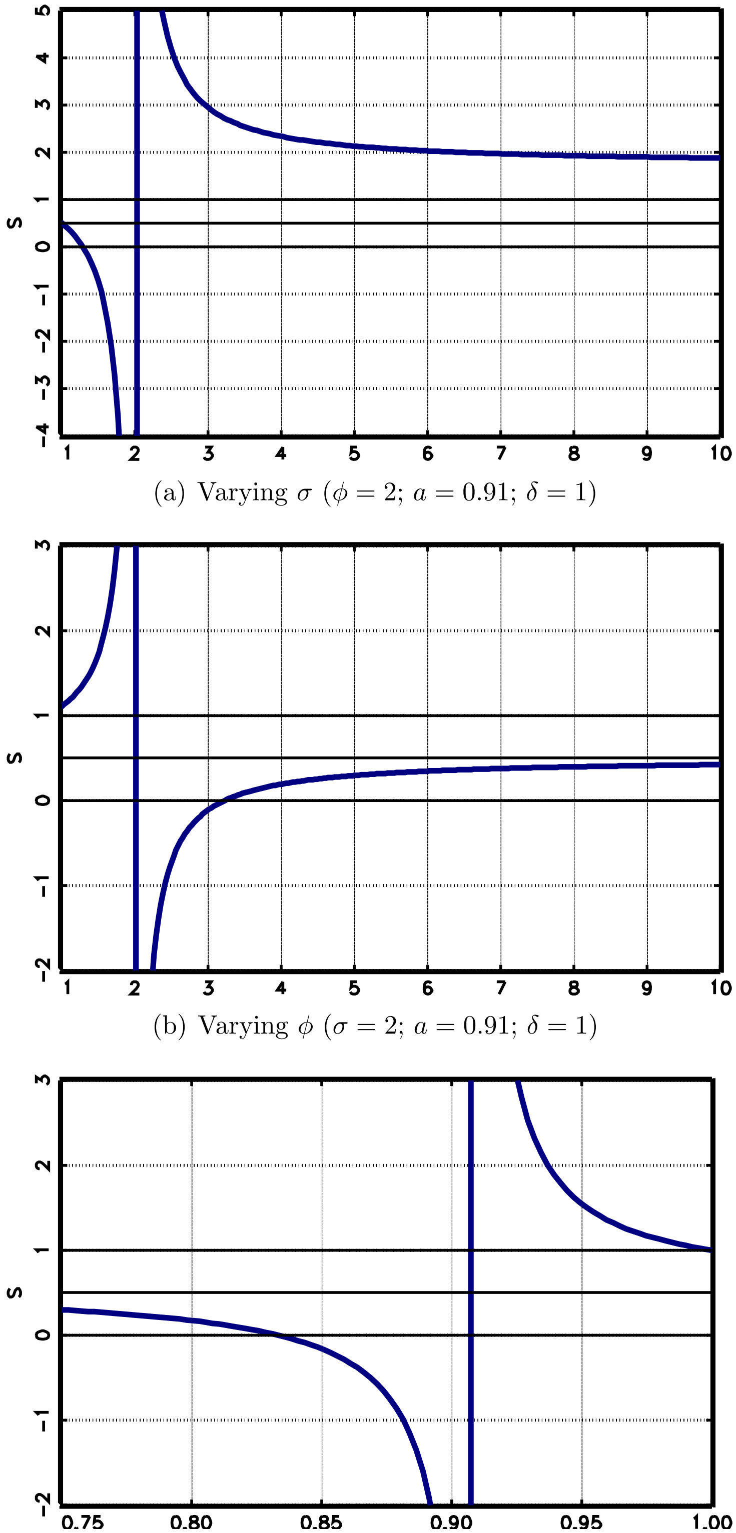
Table 4: Implied Portfolio Equity (S) and bond (b) position for G7 countries. Calculations are done under the assumption that $\delta = 0.19$ and $\sigma = 2$. (S) refers to the percentage of domestic stocks held by domestic residents (data for (S) are averaged over the period 2000-2008). ΔS refers to the difference between the implied S in a model with bonds and equity and the implied S with equities only. (b) refers to the net domestic currency exposure of bond portfolios (as a % of GDP). Data for (b) are computed from Lane and Shambaugh (2010b) and refers to the average between net debt assets in domestic currency and net debt liabilities in foreign currency as a % of GDP (averaged over 2000-2004): $b = \frac{b_{HH} - b_{HF}}{2}$.

	Canada	France	Germany	Italy	Japan	U.K.	U.S.	Pooled
				Financial returns estimated using national accounts				
$\beta_{Q,f}$	0.004	0.008	0.003	0.041***	0.011	0.017	0.015	0.012**
(s.e.)	(0.007)	(0.006)	(0.009)	(0.015)	(0.010)	(0.014)	(0.009)	(0.004)
p-val	0.533	0.161	0.727	0.007	0.281	0.226	0.101	0.001
$\beta_{Q,b}$	0.985***	0.958***	0.987***	0.929***	0.996***	0.940***	0.977***	0.972***
(s.e.)	(0.016)	(0.012)	(0.016)	(0.022)	(0.017)	(0.026)	(0.014)	(0.006)
p-val	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
R^2	0.969	0.981	0.977	0.958	0.972	0.928	0.982	0.968
				Projection of financial returns estimated using national accounts on market returns				
$\beta_{Q,f}$	-0.025	-0.011	-0.033**	0.005	-0.008	0.056**	0.012	0.003
(s.e.)	(0.019)	(0.015)	(0.014)	(0.006)	(0.020)	(0.022)	(0.018)	(0.004)
p-val	0.184	0.488	0.022	0.409	0.699	0.013	0.507	0.435
$\beta_{Q,b}$	1.013***	0.974***	1.024***	0.965***	1.013***	0.886***	0.979***	0.980***
(s.e.)	(0.024)	(0.018)	(0.019)	(0.017)	(0.024)	(0.035)	(0.020)	(0.007)
p-val	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
R^2	0.969	0.981	0.978	0.956	0.972	0.930	0.981	0.967
				Financial returns based on equity returns				
$\beta_{Q,f}$	-0.007	0.004	-0.005	0.011	-0.003	0.035***	0.005	0.006**
(s.e.)	(0.009)	(0.006)	(0.008)	(0.007)	(0.008)	(0.011)	(0.009)	(0.003)
p-val	0.469	0.536	0.506	0.125	0.722	0.002	0.547	0.039
$\beta_{Q,b}$	0.994***	0.960***	0.993***	0.960***	1.008***	0.920***	0.986***	0.977***
(s.e.)	(0.017)	(0.012)	(0.013)	(0.018)	(0.016)	(0.024)	(0.013)	(0.006)
p-val	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
R^2	0.969	0.981	0.977	0.956	0.972	0.932	0.981	0.968
				Nonfinancial returns estimated using the method of Lustig and Nieuwerburgh (2008)				
$\beta_{Q,f}$	-0.019	-0.005	-0.030	0.007	-0.002	0.059***	0.011	0.009**
(s.e.)	(0.017)	(0.013)	(0.015)	(0.006)	(0.018)	(0.016)	(0.016)	(0.004)
p-val	0.281	0.724	0.048	0.268	0.907	0.000	0.500	0.042
$\beta_{Q,b}$	1.007***	0.969***	1.019***	0.964***	1.008***	0.889***	0.981***	0.975***
(s.e.)	(0.023)	(0.017)	(0.019)	(0.017)	(0.023)	(0.028)	(0.017)	(0.007)
p-val	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
R^2	0.969	0.981	0.977	0.956	0.972	0.933	0.981	0.968
Obs.	154	154	154	154	154	154	154	1078

Table 5: Loadings on real exchange rate changes for alternative measures of returns: $\Delta \ln Q_{i,t} = \beta^i_{Q,b}\hat{r}^b_{i,t} + \beta^i_{Q,f}\hat{r}^f_{i,t} + u_{i,t}$. Standard errors are in parenthesis. (***) (resp (**)) indicates significance at the 1% level (resp. 5%). Last column reports pooled fixed effect estimates. Constants are not reported. Sample: 1970:2 to 2008:3.

	Canada	France	Germany	Italy	Japan	U.K.	U.S.	Pooled
				Financial returns estimated using national accounts				
$\beta_{n,f}$	-0.093***	-0.041	-0.078**	0.007	-0.103***	-0.030	-0.142***	-0.071***
(s.e.)	(0.026)	(0.028)	(0.032)	(0.058)	(0.022)	(0.032)	(0.045)	(0.013)
p-val	0.000	0.146	0.015	0.907	0.000	0.337	0.002	0.000
$\beta_{n,b}$	1.178***	0.904***	1.188***	1.179***	0.989***	0.876***	1.153***	1.063***
(s.e.)	(0.058)	(0.055)	(0.055)	(0.085)	(0.038)	(0.057)	(0.068)	(0.022)
p-val	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
R^2	0.750	0.664	0.810	0.690	0.846	0.673	0.704	0.726
				Projection of financial returns estimated using national accounts on market returns				
$\beta_{n,f}$	-0.167**	-0.214***	-0.022	-0.141***	-0.068	-0.058	-0.346***	-0.140***
(s.e.)	(0.070)	(0.069)	(0.052)	(0.018)	(0.049)	(0.051)	(0.090)	(0.014)
p-val	0.019	0.002	0.679	0.000	0.162	0.257	0.000	0.000
$\beta_{n,b}$	1.252***	1.079***	1.128***	1.262***	0.962***	0.919***	1.339***	1.130***
(s.e.)	(0.088)	(0.083)	(0.070)	(0.055)	(0.058)	(0.080)	(0.097)	(0.023)
p-val	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
R^2	0.738	0.680	0.803	0.780	0.826	0.674	0.713	0.742
				Financial returns based on equity returns				
$\beta_{n,f}$	-0.112**	-0.053**	0.009	-0.140***	-0.101***	-0.037	-0.073	-0.090***
(s.e.)	(0.045)	(0.023)	(0.066)	(0.025)	(0.027)	(0.028)	(0.047)	(0.015)
p-val	0.014	0.025	0.894	0.000	0.000	0.186	0.124	0.000
$\beta_{n,b}$	1.185***	1.067***	1.309***	1.283***	0.500***	0.917***	1.035***	1.014***
(s.e.)	(0.081)	(0.051)	(0.119)	(0.064)	(0.055)	(0.059)	(0.068)	(0.030)
p-val	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
R^2	0.622	0.771	0.490	0.727	0.356	0.657	0.655	0.541
				Nonfinancial returns estimated using the method of Lustig and Nieuwerburgh (2008)				
$\beta_{n,f}$	-0.189***	-0.085**	-0.216***	-0.361***	-0.123***	-0.275***	-0.193***	-0.287***
(s.e.)	(0.043)	(0.042)	(0.051)	(0.016)	(0.043)	(0.033)	(0.039)	(0.012)
p-val	0.000	0.044	0.000	0.000	0.005	0.000	0.000	0.000
$\beta_{n,b}$	-1.108***	-1.137***	-0.927***	-1.350***	-1.203***	-1.060***	-1.159***	-1.051***
(s.e.)	(0.056)	(0.052)	(0.064)	(0.047)	(0.054)	(0.057)	(0.043)	(0.018)
p-val	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
R^2	0.901	0.899	0.839	0.915	0.916	0.877	0.938	0.885
Obs.	154	154	154	154	154	154	154	1078

Table 6: Loadings on nonfinancial returns for alternative measure of returns: $\hat{r}^n_{i,t} = \beta^i_{n,b}\hat{r}^b_{i,t} + \beta^i_{n,f}\hat{r}^f_{i,t} + v_{i,t}$. Standard errors are in parenthesis. (***) (resp (**)) indicates significance at the 1% level (resp. 5%). Last column reports pooled fixed effect estimates. Constants are not reported. Sample: 1970:2 to 2008:3.



(a) Varying σ ($\phi = 2$; $a = 0.91$; $\delta = 1$)

(b) Varying ϕ ($\sigma = 2$; $a = 0.91$; $\delta = 1$)

(c) Varying a ($\sigma = 2$; $\phi = 2$; $\delta = 1$)

Figure 1: The *instability* of optimal equity positions as a function of preference parameters.

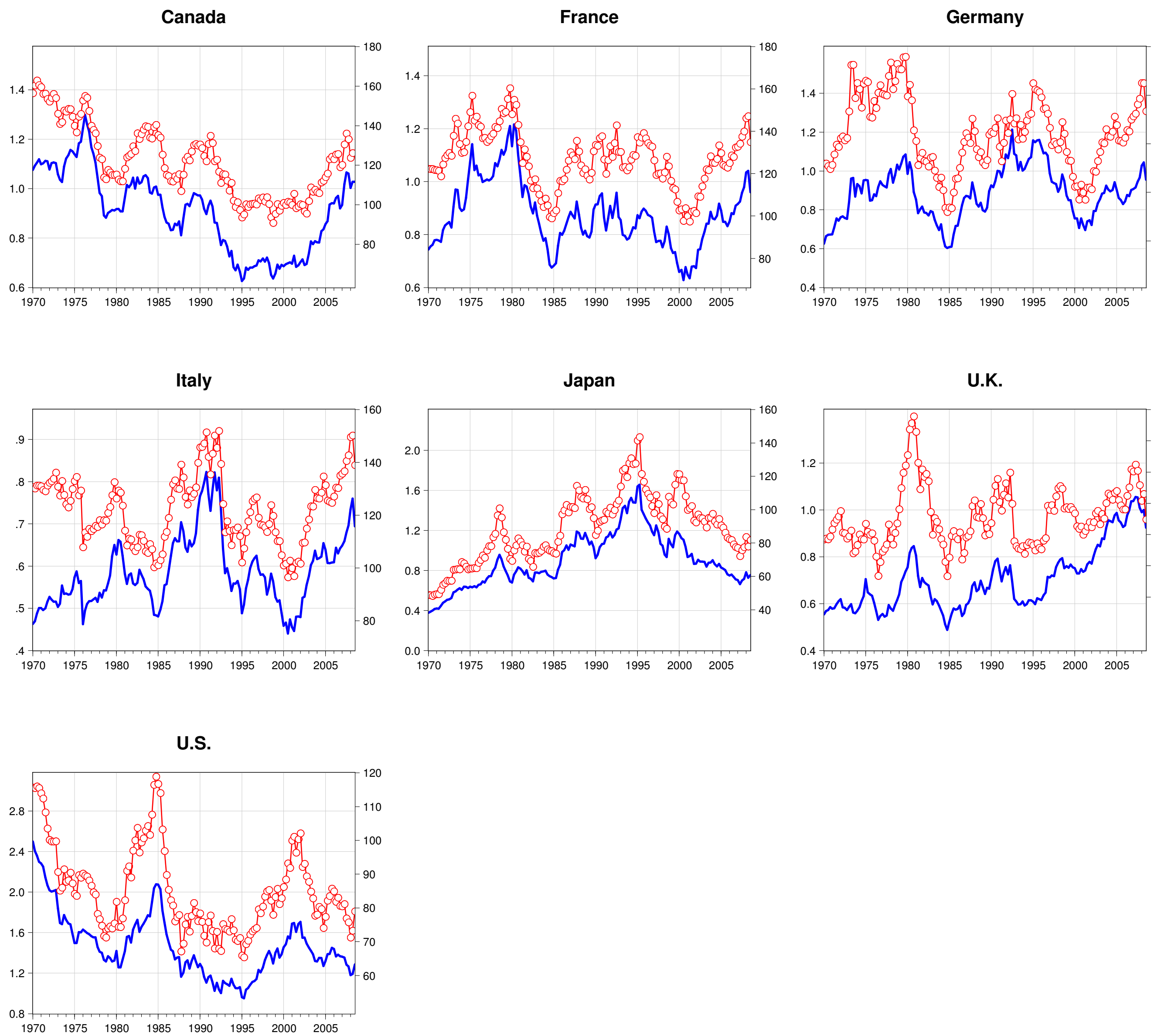


Figure 2: Relative nonfinancial income (left) [–] and real exchange rate [-o-] (100 in 2001Q1, right), G7 countries, 1970:1-2008:3. Data Sources: Global Financial Database, OECD Quarterly National Accounts and UN National Account Statistics. Authors' calculations.

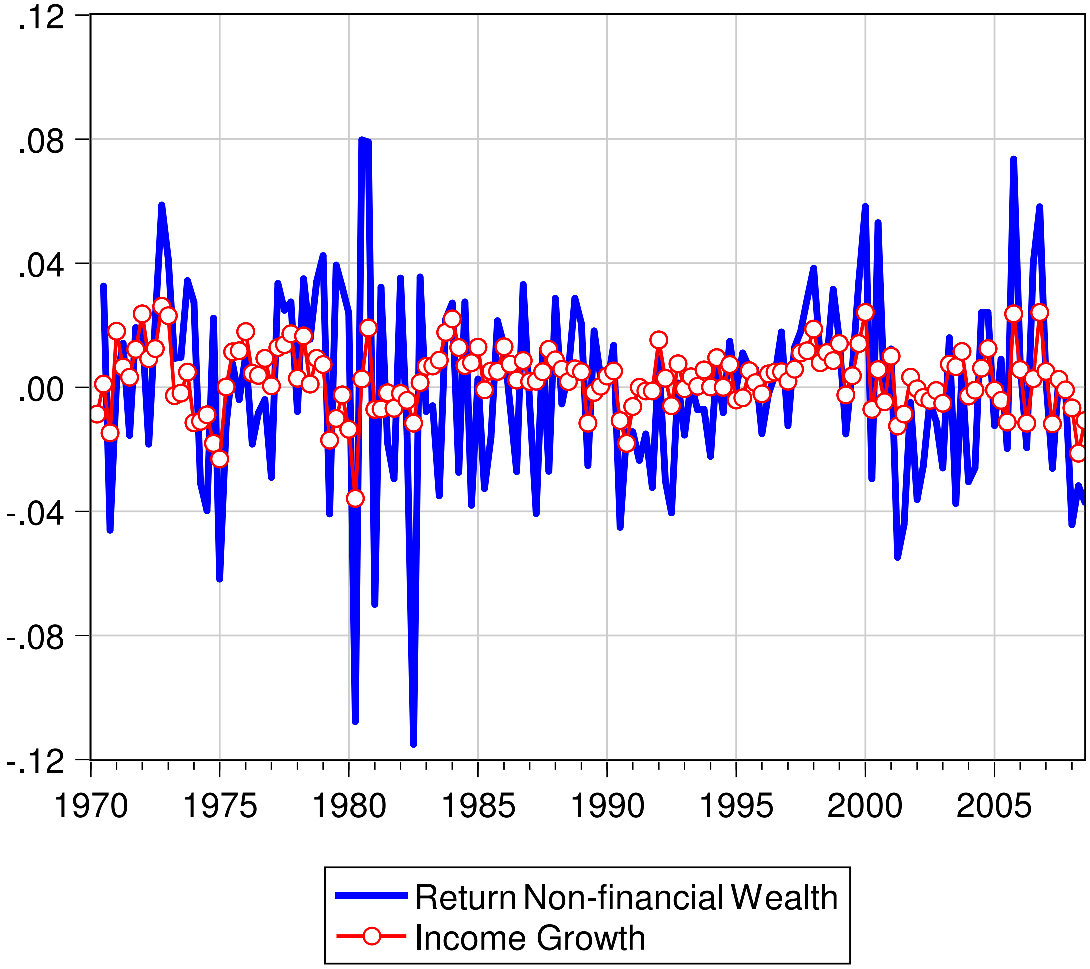


Figure 3: Innovations to returns on nonfinancial wealth $r^n_{t+1} - E_t r^n_{t+1}$, and nonfinancial income growth Δw, United States, 1970:1-2008:3.

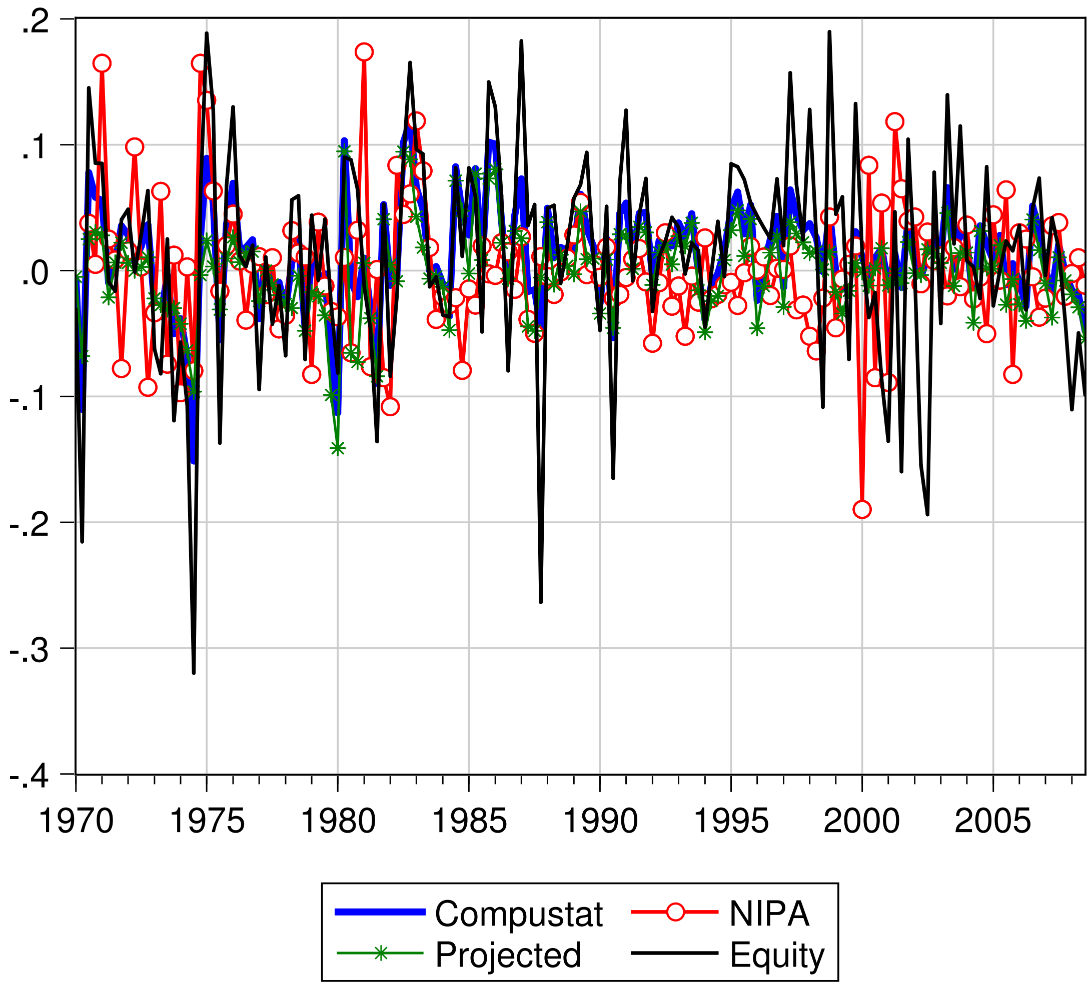


Figure 4: Innovations to returns on financial wealth. Compustat: weighted average of equity and corporate bond returns using share of debt in total assets measured from Compustat; NIPA: innovation to financial return constructed using equation (41); Projected: regresses NIPA returns on equity and corporate bond returns; Equity: S&P-500 Total Return Index. United States, 1970:1-2008:3.

Appendices

A Theoretical Derivations

A.1 Optimal portfolios with locally complete markets

We apply Devereux and Sutherland (2006) (see also Tille and van Wincoop (2010)) to characterize the equilibrium portfolio. We show that the zero-order (or static) portfolio is the one that locally replicates the efficient consumption allocation. The Devereux and Sutherland (2006) approach finds a portfolio that is consistent with 1) First-order approximation of non-portfolio equations (here intratemporal allocation across Home and Foreign goods and budget constraints in both countries) and 2) Second-order approximation of the Euler equations.

There are two non-portfolio equations. The first is the optimal intratemporal condition for the allocation of consumption across goods. In relative form this is equation (7). The log-linear first-order approximation is:

$$\hat{y} = \left[-\phi + (2a-1)^2(\phi - 1)\right]\hat{q} + (2a-1)\hat{PC} \tag{A.1}$$

where $\hat{PC}$ denotes relative consumption expenditures ($P_H\hat{C}_H - P_F\hat{C}_F$).

The second non-portfolio equation is the budget constraint. The log-linear first order approximation is:

$$P_H\hat{C}_H - P_F\hat{C}_F = (1-\delta)\hat{R}^n + \delta\,(2S-1)\,\hat{R}^f + 2b\hat{R}^b \tag{A.2}$$

where $\hat{R}^n$ denotes the return on nonfinancial wealth, $\hat{R}^f$ denotes excess financial (equity) returns and $\hat{R}^b$ excess bond returns.

The Euler equations for equity holdings in country $i = H, F$ is:

$$\lambda_{i,0} = E_0[\frac{C_i^{-\sigma}}{P_i}R_H^j]\ ;\ \lambda_{i,0} = E_0[\frac{C_i^{-\sigma}}{P_i}R_F^j] \tag{A.3}$$

where $\lambda_{i,0}$ denotes the Lagrange-multiplier of the budget constraint in period $t = 0$ in country $i = H, F$ and $j = f, b$. In relative terms across countries:

$$E_0\left[\left(\frac{C_H^{-\sigma}}{P_H} - \frac{C_F^{-\sigma}}{P_F}\right)R^j\right] = 0 \tag{A.4}$$

The second-order approximation of equation (A.4) yields:

$$cov(\hat{PC}, \hat{R}^j) = (1 - 1/\sigma)cov(\hat{Q}, \hat{R}^j);\ \text{ for } j = f, b \tag{A.5}$$

The optimal first-order portfolio is the pair (S^*, b^*) such that the first order non-portfolio conditions (A.1) and (A.2) and the second-order portfolio conditions (A.5) are satisfied for all realizations of the shocks. It is immediate that a portfolio $(S^*; b^*)$ such that $\hat{PC} = (1 - 1/\sigma)\hat{Q} = (1-1/\sigma)(2a-1)\hat{q}$ satisfies the two (second-order) Euler equation approximations. Let us a assume that it is possible to find such a portfolio. Then, this is the same thing as saying that relative consumption expenditures are linked to the real exchange by the expression (9) or equivalently that markets are (locally) complete.

If such a portfolio exists, it must also satisfy the first-order non-portfolio equations (A.1) and (A.2). These can be rewritten (see the equivalent expressions (10) and (12) in the benchmark model):

$$\hat{y} = -\lambda\hat{q} \tag{A.6}$$
$$(1-1/\sigma)(2a-1)\hat{q} = (1-\delta)\hat{R}^n + \delta\left(2S-1\right)\hat{R}^f + 2b\hat{R}^b \tag{A.7}$$

The portfolio choice only affects equation (A.6) because of its impact on equity and bond excess returns (through its impact on $\hat{q}$); so as long as asset returns are consistent with equation (A.6), then the first-order approximation of (A.1) is verified. The key question is wether one can verify (A.7) in all states of nature. Because we have two instruments (S and b), we must have at most two sources of risk. This is the **Spanning Condition**. Call $\hat{\varepsilon}_1$ and $\hat{\varepsilon}_2$ the two innovations (expressed in relative terms) arising from these two sources of risk and assume that our four endogenous variables $\{\hat{q};\hat{R}^n;\hat{R}_f;\hat{R}_b\}$ are driven by $\hat{\varepsilon}_1$ and $\hat{\varepsilon}_2$ according to the following expression in matrix form (where we assume that Home bond excess returns load perfectly on the real exchange rate, as in our benchmark case):[68]

$$\begin{pmatrix} \hat{q} \\ \hat{R}^n \\ \hat{R}_f \\ \hat{R}_b \end{pmatrix} = \begin{pmatrix} a_{1,q} & a_{2,q} \\ a_{1,n} & a_{2,n} \\ a_{1,R} & a_{2,R} \\ (2a-1)a_{1,q} & (2a-1)a_{2,q} \end{pmatrix} \begin{pmatrix} \hat{\varepsilon}_1 \\ \hat{\varepsilon}_2 \end{pmatrix}$$

Then, (A.7) is verified for all possible realizations of the shocks if and only if the following equality holds in matrix form (obtained from projections on the set of shocks $(\hat{\varepsilon}_1;\hat{\varepsilon}_2\ )$):

$$(1-1/\sigma)(2a-1)\begin{pmatrix} a_{1,q} \\ a_{2,q} \end{pmatrix} = (1-\delta)\begin{pmatrix} a_{1,n} \\ a_{2,n} \end{pmatrix} + \underbrace{\begin{pmatrix} a_{1,R} & a_{1,q} \\ a_{2,R} & a_{2,q} \end{pmatrix}}_{M} \begin{pmatrix} \delta\left(2S-1\right) \\ 2b(2a-1) \end{pmatrix}$$

The second condition for the portfolio to be unique and determined is that $\det\left(M\right) = \left(a_R^1 a_q^2 - a_R^2 a_q^1\right) \neq 0$. This is the **Rank Condition**. This is equivalent to assuming that Home excess equity and Home bond excess returns are not perfectly correlated. In that case, the equilibrium portfolio $(S^*;b^*)$ is unique and determined as follows:

$$\begin{pmatrix} \delta\left(2S^*-1\right) \\ 2b^*(2a-1) \end{pmatrix} = \begin{pmatrix} a_{1,R} & a_{1,q} \\ a_{2,R} & a_{2,q} \end{pmatrix}^{-1} \begin{pmatrix} \left[(1-1/\sigma)(2a-1)\right]a_{1,q} - (1-\delta)a_{1,n} \\ \left[(1-1/\sigma)(2a-1)\right]a_{2,q} - (1-\delta)a_{2,n} \end{pmatrix}$$

One can rewrite the same proof by changing the basis of shocks as in our examples by using a projection on $\hat{q}$ and $\hat{\varepsilon} = \hat{\varepsilon}_2$ (providing that $a_{1,q} \neq 0$, i.e. that $\hat{q}$ and $\hat{\varepsilon}$ are not collinear); We obtain the results of section 3 with an evident change of notation:

$$\begin{pmatrix} \hat{q} \\ \hat{R}^n \\ \hat{R}^f \\ \hat{R}^b \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ (1-\bar{\lambda}) & \gamma_n \\ (1-\bar{\lambda}) & \gamma_f \\ (2a-1) & \gamma_b \end{pmatrix} \begin{pmatrix} \hat{q} \\ \hat{\varepsilon} \end{pmatrix}$$

and the optimal portfolio satisfies:

$$\begin{pmatrix} \delta\left(2S^*-1\right) \\ 2b^*\left(2a-1\right) \end{pmatrix} = \begin{pmatrix} -\frac{\gamma_b}{\gamma_f}\frac{(2a-1)\left(1-\frac{1}{\sigma}\right)+(1-\delta)\left(1-\bar{\lambda}\right)\left(\gamma_n/\gamma_f-1\right)}{(2a-1)-\gamma_b/\gamma_f\left(1-\bar{\lambda}\right)} - \frac{\gamma_n}{\gamma_f}(1-\delta) \\ \frac{(2a-1)\left(1-\frac{1}{\sigma}\right)+(1-\delta)\left(1-\bar{\lambda}\right)\left(\gamma_n/\gamma_f-1\right)}{1-\gamma_b/\gamma_f\left(1-\bar{\lambda}\right)(2a-1)^{-1}} \end{pmatrix}$$

where the **rank condition** takes the form: $(2a-1)\,\gamma_f \neq \gamma_b\left(1-\bar{\lambda}\right)$.

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[68] We do not need additional assumptions on the stochastic properties except that they are not perfectly correlated.

A.2 Optimal portfolios with incomplete markets

As above, we use the Devereux and Sutherland (2006) approach to characterize the optimal equity and bond positions. To do so, we use the first-order approximations of the non-portfolio equations (see reduced-form of the model below) and the second order approximation of the Euler equations. This pins down a unique equilibrium portfolio.

Non portfolio equations:

- Intratemporal allocation across goods:

$$\hat{y} = -\phi\hat{q} + (2a-1)[P_H\hat{C}_H - P_F\hat{C}_F - (1-\phi)\hat{Q}]$$

- Budget constraint:

$$P_H\hat{C}_H - P_F\hat{C}_F = (1-\delta)\hat{R}^n + \delta\,(2S-1)\,\hat{R}^f + 2b\hat{R}^b$$

Relative returns on equities, bonds and nonfinancial wealth are expressed as follows:

$$\begin{cases} \hat{R}^f &= \hat{q} + \hat{y} + \boldsymbol{\gamma}_f'\hat{\boldsymbol{\varepsilon}} \\ \hat{R}^b &= (2a-1)\hat{q} + \hat{y} + \boldsymbol{\gamma}_b'\hat{\boldsymbol{\varepsilon}} \\ \hat{R}^n &= \hat{q} + \hat{y} + \boldsymbol{\gamma}_n'\hat{\boldsymbol{\varepsilon}} \end{cases},$$

where $\hat{\boldsymbol{\varepsilon}}$ is a N-dimensional vector of shocks and $\boldsymbol{\gamma}_i$ for $i = \{b, f, n\}$ is a $N \times 1$ vector that controls the impact of $\hat{\boldsymbol{\varepsilon}}$ on assets returns and non-financial wealth.

Portfolio equations: due to symmetry, we can write Euler equations in relative terms as follows for asset $i = \{f, b\}$:

$$E_0(mR^i) = 0 \text{ for } i = \{f, b\} \tag{A.8}$$

where m is the difference between stochastic discount factor across countries: $m = C_H^{-\sigma}/P_H - C_F^{-\sigma}/P_F$.

Using the budget constraint, the intratemporal condition can be rewritten as follows, where we introduce portfolio excess returns $\hat{\xi} = \delta\,(2S-1)\,\hat{R}^f + 2b\hat{R}^b = \begin{pmatrix} \delta\,(2S-1) & 2b \end{pmatrix}\binom{\hat{R}^f}{\hat{R}^b}$:

$$\hat{q} = q_y\hat{y} + \mathbf{q}_\varepsilon'\hat{\varepsilon} + q_\xi\hat{\xi}$$

where $q_y = \frac{[\phi(1-(2a-1)^2)+(2a-1)^2]^{-1}[(2a-1)(1-\delta)-1]}{1-[\phi(1-(2a-1)^2)+(2a-1)^2]^{-1}(2a-1)(1-\delta)}$, $\mathbf{q}_\varepsilon' = \frac{[\phi(1-(2a-1)^2)+(2a-1)^2]^{-1}(2a-1)(1-\delta)}{1-[\phi(1-(2a-1)^2)+(2a-1)^2]^{-1}(2a-1)(1-\delta)}\gamma_n'$ and $q_\xi = \frac{[\phi(1-(2a-1)^2)+(2a-1)^2]^{-1}(2a-1)(1-\delta)}{1-[\phi(1-(2a-1)^2)+(2a-1)^2]^{-1}(2a-1)(1-\delta)}$.

If we rewrite the reduced form model using Devereux and Sutherland (2006) notations, we get the following expression for the vector excess returns:

$$\begin{pmatrix} \hat{R}^f \\ \hat{R}^b \end{pmatrix} = \mathbb{R}_1\hat{\xi} + \mathbb{R}_2\begin{pmatrix} \hat{y} \\ \hat{\boldsymbol{\varepsilon}} \end{pmatrix}$$

where $\mathbb{R}_2 = \begin{pmatrix} 1+q_y & \mathbf{q}_\varepsilon' + \boldsymbol{\gamma}_e' \\ (2a-1)q_y & (2a-1)\mathbf{q}_\varepsilon' + \gamma_b' \end{pmatrix}$ and $\mathbb{R}_1 = \begin{pmatrix} q_\xi \\ (2a-1)q_\xi \end{pmatrix}$

The first-order approximation of the difference between stochastic discount factor across countries gives:

$$\hat{m} = D_1\hat{\xi} + \mathbf{D}_2\begin{pmatrix} \hat{y} \\ \hat{\boldsymbol{\varepsilon}} \end{pmatrix}$$

where D_1 is a scalar, $D_1 = 1 + [(1-\delta) + (2a-1)(1/\sigma - 1)]q_\xi$
and $\mathbf{D}_2 = \begin{pmatrix} (1-\delta)(1+q_y) + (2a-1)(1/\sigma-1)q_y & (1-\delta) + (2a-1)(1/\sigma-1)\mathbf{q}'_\varepsilon + (1-\delta)\gamma'_n \end{pmatrix}$ is a $1 \times N+1$ vector.

Following DS, we define $\tilde{\mathbb{R}}_2 = \mathbb{R}_1 \tilde{\mathbf{H}} + \mathbb{R}_2$ and $\tilde{\mathbf{D}}_2 = D_1 \tilde{\mathbf{H}} + \mathbf{D}_2$
with $\tilde{\mathbf{H}} = \left(\mathbf{1} - \begin{pmatrix} \delta(2S-1) & 2b \end{pmatrix} \mathbb{R}_1\right)^{-1} \begin{pmatrix} \delta(2S-1) & 2b \end{pmatrix} \mathbb{R}_2$

Then using the second-order approximation of the Euler equation, we get the following quadratic equation:

$$\tilde{\mathbb{R}}_2 \Sigma \tilde{\mathbf{D}}'_2 = 0$$

where Σ is the (N+1) x (N+1) variance-covariance matrix of the vector of innovations $\binom{\hat{y}}{\hat{\boldsymbol{\varepsilon}}}$.

Rearranging terms, this equation simplifies into the following expression for portfolios:

$$\begin{pmatrix} \delta(2S-1) \\ 2b \end{pmatrix} = \left(\mathbb{R}_2 \Sigma \mathbf{D}'_2 \mathbb{R}'_1 - D_1 \mathbb{R}_2 \Sigma \mathbb{R}'_2\right)^{-1} \mathbb{R}_2 \Sigma \mathbf{D}'_2$$

where we assume that the 2×2 matrix $[\mathbb{R}_2 \Sigma \mathbf{D}'_2 \mathbb{R}'_1 - D_1 \mathbb{R}_2 \Sigma \mathbb{R}'_2]$ is invertible (**Rank condition**). When this rank condition is satisfied, the equilibrium portfolio is unique and bond and equity excess returns are not collinear. Thus, there exists a unique decomposition such that:

$$\begin{aligned} \hat{Q} &\equiv \beta_{Q,b}\hat{R}^b + \beta_{Q,f}\hat{R}^f + u_Q \\ \hat{R}^n &\equiv \beta_{n,b}\hat{R}^b + \beta_{n,f}\hat{R}^f + u_n \end{aligned}$$

where u_i for $i = \{Q, n\}$ is orthogonal to $\hat{R}^j$ for $j = \{b, f\}$: $E\left[u_i \hat{R}^j\right] = 0$

A.3 Optimal portfolios with equity and corporate debt

Consider the benchmark model of section (3.1) under locally complete markets. Assume that firms in country i issue a given amount of corporate debt. We call D_i the debt payments that have to be paid in period $t = 1$ in country i (we preserve symmetry across countries, i.e $E_0(D_i) = D$ but results regarding equity home bias do not depend on this assumption).[69]

We call S_D the share of corporate debt in country i held by country i. Market clearing in the corporate debt market implies that country j holds a share $(1 - S_D)$.

In this environment, Modigliani-Miller theorem holds. This means that equilibrium firms values are independent of the amount of debt issued. In particular, the following log-linearized expressions for returns are still valid (see (14)):

$$\begin{cases} \hat{R}^f = \hat{q} + \hat{y} + \gamma'_f \hat{\varepsilon} \\ \hat{R}^b = (2a-1)\hat{q} + \hat{y} + \gamma'_b \hat{\varepsilon} \\ \hat{R}^n = \hat{q} + \hat{y} + \gamma'_n \hat{\varepsilon} \end{cases}$$

Note that $\hat{R}^f$ is *not* relative equity returns anymore (if D_i non-zero in some states) but relative financial returns, i.e cross country difference in the sum of the returns on equity and returns on corporate debt.

[69] We assume that debt issued is bounded above such that period $t = 1$ equity payments are strictly positive: $(\delta p_i y_i - D) > 0$ for all states.

We introduce R_i^n the non-financial returns in country i and R_i^f the financial returns in country i (sum of equity dividends $\left(R_i^f - D_i\right)$ and corporate debt payments D_i).

Budget constraint in country i can be written as:

$$P_i C_i = R_i^n + S\left(R_i^f - D_i\right) + S_{ij}\left(R_j^f - D_j\right) + S_D D_i + (1 - S_D) D_j + b(P_H - P_F) \tag{A.9}$$

$$= R_i^n + S\delta R_i^f + (1 - S) R_j^f + (S_D - S) D_i + (S - S_D) D_j + b(P_H - P_F) \tag{A.10}$$

Taking the difference across countries:

$$P_H C_H - P_F C_F = R_n + (2S - 1) R_f + (S_D - S)(D_i - D_j) + 2b(P_H - P_F)$$

with $R^n = R_H^n - R_F^n$ and $R^f = R_H^f - R_F^f$

In log-linearized terms (assuming locally-complete markets), this gives a similar equation to (15)

$$(1 - \frac{1}{\sigma})(2a - 1)\hat{q} = \delta (2S - 1) \hat{R}^f + (1 - \delta)\hat{R}^n + (S_D - S)\hat{D} + 2b\hat{R}^b$$

If we find a portfolio (S, S_D, b) such that the previous equation holds for arbitrary realizations of the shocks, markets are locally-complete and such a portfolio is the equilibrium one.

The portfolio $(S, S_D, b) = (S^*, S^*, b^*)$ obviously satisfies this condition and is unique, where (S^*, b^*) are the ones derived in (3.1) (see equations (16) and (19)). The intuition for the result is quite straightforward: the presence of corporate debt only redistributes income from shareholder to debt holders in some states (without any impact on total financial returns). By holding corporate debt and equity in the same proportion, investors insulate their consumption expenditures from this redistribution.

We can conclude that the presence of corporate leaves the degree of home bias unchanged as well as the expression in terms of factor loadings once we compute the aggregate financial returns $\hat{R}_f$ (equity returns plus corporate debt returns). We also obtain that the (equilibrium) home bias in corporate debt is equal to the one in equity. This so in this model because returns on financial wealth$\hat{R}_f$ are independent on the capital structure (i.e Modigliani-Miller holds) which makes the projection on the risk factors (and hence the portfolio) unchanged.

A.4 Detailed derivation for the model with government expenditures shocks. [Not for publication].

Market-clearing conditions for both goods:

$$c_{ii} + c_{ji} + g_{ii} + g_{ji} = y_i. \tag{A.11}$$

Relative demand of Home over Foreign goods by governments ($y_g = (g_{HH} + g_{FH}) / (g_{HF} + g_{FF})$) satisfies (in log-linearized terms):

$$\hat{y}_g = -\lambda_g \hat{q} + (2a_g - 1)\hat{E}_g \tag{A.12}$$

where $0 \leq \lambda_g = \phi(1 - (2a_g - 1)^2) + (2a_g - 1)^2 \leq \phi$ represents the impact of fluctuations in the terms of trade on relative government consumption, after controlling for relative expenditures $\hat{E}_g$.

Relative demand of Home over Foreign goods by consumers ($y_c = (c_{HH} + c_{FH}) / (c_{HF} + c_{FF})$) still satisfies equation (10) since the private allocation across goods has not changed:

$$\hat{y}_c = -\lambda \hat{q} \tag{A.13}$$

Equation (A.12) and (A.13) together with market clearing conditions of both goods (A.11) implies the following equilibrium on the goods market:

$$\hat{y} = s_c\hat{y}c + s_g\hat{y}_g = -\bar{\lambda}\hat{q} + s_g(2a_g - 1)\hat{E}_g \tag{A.14}$$

where s_c (resp. $s_g = 1 - s_c$) is the steady-state ratio of consumption spending (resp. government spending) over GDP and $\bar{\lambda} = s_c\lambda + s_g\lambda_g$.

Log-linearizing relative financial and non-financial incomes net-of-taxes and bond returns using (A.14) yields the system of equation derived in the paper:

$$\begin{cases} \hat{R}^f & = \hat{q}y - \frac{\delta_g}{\delta}\hat{E}_g = \quad (1-\bar{\lambda})\hat{q} + s_g(2a_g - 1 - \frac{\delta_g}{\delta})\hat{E}_g \\ \hat{R}^b & = \hat{Q} = \quad (2a-1)\hat{q} \\ \hat{R}^n & = \hat{q}y - \frac{1-\delta_g}{1-\delta}\hat{E}_g \quad (1-\bar{\lambda})\hat{q} + s_g(2a_g - 1 - \frac{1-\delta_g}{1-\delta})\hat{E}_g \end{cases}$$

A.5 Other extensions of the benchmark model. [Not for publication]

We provide two extensions of our benchmark model discussed in section 3.2.3: - nominal shocks (and nominal bonds) in presence of price rigidities as in Engel and Matsumoto (2009) - shocks to the quality of goods (or equivalently preferences shocks towards one good or changes in the number of varieties available to consumers) as in Corsetti et al. (2007) or Coeurdacier et al. (2009).

We do it in the benchmark model of section 3, i.e when markets are *locally* complete.

Nominal shocks in presence of price rigidities

We follow Engel and Matsumoto (2009) and add nominal shocks and price rigidities to the benchmark model. We suppose that prices are set one period in advance for a share ω of firms (where ω is common across countries to preserve symmetry). Firms do not know ex-ante if they will be able to adjust their price in period 1. All firms post a preset price, and a share $(1-\omega)$ will be able to adjust it. Given the symmetry of the model, firms in both countries will post the same preset the same price in period 0 denoted $\bar{p}$.

Money enters the utility function and the expected utility at date 0 of a representative agent in country i is:

$$U_i = E_0\left[\frac{C_i^{1-\sigma}}{1-\sigma} + \chi\log(\frac{m_i}{P_i})\right], \tag{A.15}$$

where m_i denotes money holdings in country i and χ is a positive parameter.[70]

Both countries experience symmetrically distributed nominal shocks to their money supply m_i. Bonds in country i are now nominal bonds that pay one unit of country i's currency. Due to the inflation risk, bonds returns differentials will be imperfectly correlated with real exchange rate changes. We introduce the *nominal* exchange rate s, defined as the number of Foreign currency units per unit of Home currency. A rise in s represents a nominal appreciation of the Home currency. All variables are expressed in Home currency terms and we denote $\hat{s}$ the deviations of the nominal exchange rate from its mean value of one ($E_0(s) = 1$).

To make our results easily comparable with Engel and Matsumoto (2009), we assume that in the steady state the share of revenues going to shareholders δ is related to the mark-up in a monopolistic competition Dixit-Stiglitz framework.[71]. Output in country i is produced using

[70] To simplify matters, we assume that consumption and real money balances are separable in the utility function and that preferences over real money balances are log. None of our results are affected by these assumptions.

[71] In such a set-up, the steady-state profit share δ equals to the inverse of the elasticity of substitution between varieties within a given country

labor l_i : $y_i = a_i l_i$ where a_i is a stochastic productivity shock with symmetric distribution across countries ($E_0(a_i) = 1$). This the second source of uncertainty. As in Engel and Matsumoto (2009), we have 'two relative shocks' ($\hat{a}$ and $\hat{m}$) and two non-collinear relative assets: markets will be locally-complete.

Assume that prices are set in the currency of the producer (PCP).[72] The price index in country H, expressed in domestic currency is:

$$P_H = \left[a p_H^{1-\phi} + (1-a) \left(\frac{p_F}{s} \right)^{1-\phi} \right]^{1/(1-\phi)}, \tag{A.16}$$

where p_i is the aggregate price over all producing firms in country i in country i's currency. Given that some firms do adjust and some do not, p_i is the CES weighted sum of preset prices $\bar{p}$ and adjusted prices in country i, denoted p_i^*.

The log-linearization of the Home country's real exchange rate $Q \equiv sP_H/P_F$ yields:

$$\hat{Q} = (2a-1)(\omega \hat{s} + (1-\omega)\hat{q^*}) \tag{A.17}$$

where $q^* = sp_H^*/p_F^*$ denotes the relative price of firms adjusting their price. Note that when $\omega = 1$, all prices are preset and the real exchange rate is perfectly correlated with the nominal one. When $\omega = 0$, we are in a flex-price model where the real exchange rate loads perfectly on the terms of trade equal to sp_H/p_F.

Under PCP, (10) still holds for relative private consumption ($\hat{y}_C$), where one should note that terms-of-trade are now equal to sp_H/p_F. Using market-clearing conditions, we get that relative total sales are equal to $(1-\lambda)(\omega\hat{s} + (1-\omega)\hat{q^*})$.

Due to price rigidities, output is (partly) demand determined and aggregate mark-ups will change in both countries. Changes in mark-up are formally equivalent to changes in the profit share δ which will potentially lead to some Home bias in equities.

One could solve the model to see how changes in productivity affect the profit share δ and relative flexible prices $\hat{q^*}$. This step is however not necessary. To express portfolios, we just need to rewrite asset returns and labor incomes as a function the Home terms-of-trade $\hat{q} = (\omega\hat{s} + (1-\omega)\hat{q^*})$ and changes in the profit share $\hat{\delta}$. The system of equations can be expressed as follows:

$$\left\{ \begin{array}{lcc} \hat{R}^f & = & (1-\lambda)\hat{q} + \hat{\delta} \\ \hat{R}^b & = & \hat{s} = \frac{1}{\omega}\left(\hat{q} + \frac{1-\omega}{\omega}\frac{\delta}{1-\delta}\hat{\delta} \right) \\ \hat{R}^n & = & ((1-\lambda)\hat{q} - \frac{\delta}{1-\delta}\hat{\delta} \end{array} \right. \tag{A.18}$$

For $\omega > 0$, this allows us to express equilibrium portfolios as in section 2.[73] This leads to the following equilibrium portfolios:

$$\left\{ \begin{array}{lcc} S & = & \frac{1}{2}\left[1 - \frac{(1-\delta)(\lambda-1)(1-\omega)-\omega}{\delta(\lambda-1)(1-\omega)+(1-\delta)\omega} - \frac{(1-1/\sigma)(2a-1)(1-\omega)}{\delta(\lambda-1)(1-\omega)+(1-\delta)\omega} \right] \\ b & = & \frac{1}{2}\omega\left[(1-1/\sigma)(2a-1) + (2S-1)(\lambda-1) \right] \end{array} \right. \tag{A.19}$$

When, $\omega = 1$, all firms have preset prices: $S = 1$ and $b = \frac{1}{2}(2a-1)(1-\frac{1}{\sigma}) + \frac{1}{2}(\lambda-1)$. The model is isomorphic to the one with redistributive shocks (where $\hat{\varepsilon} = \hat{\delta}$); indeed in that case bonds load perfectly on the real exchange rate.

[72] We adopt Producer-Currency-Pricing but as shown by Engel and Matsumoto (2009) results are very similar with Local-Currency-Pricing.

[73] With on slight difference: nominal bonds now load on the nominal exchange rate and not the real exchange rate $\hat{q} = \omega\hat{s} + (1-\omega)\hat{q^*}$

When ω is close to zero, we get close to a flex-price model with nominal shocks. In this case, bond positions converge towards zero as investors want to minimize their exposure to nominal risk.

In the extreme case of $\omega = 0$, nominal bonds do not load on the real exchange rate due to inflation risk, but equities do. Then, the model is equivalent to our reduced form model of section 2 with $\gamma_f = \gamma_w = 0$ and $\gamma_b \neq 0$. In that case, the portfolio satisfies:

$$b = 0 \; ; \; S = \frac{1}{2}\left[\frac{2\delta - 1}{\delta} - \frac{\left(1 - \frac{1}{\sigma}\right)(2a - 1)}{\delta\,(\lambda - 1)}\right] \tag{A.20}$$

In particular, as might have been expected, the bond return differential fails to protect investors against real exchange rate risk because of the inflation differentials across countries. This additional source of risk on bond returns shifts bond position towards zero. It then remains for equities to hedge real exchange rate exposure efficiently.

The case of changes in quality/preference shocks

We follow Coeurdacier et al. (2009) by adding preference shocks to the utility provided by Home goods and Foreign goods to the consumers of both countries. In that case, the aggregate consumption index C_i, for $i = H, F$, is now given by:

$$C_i = \left[a^{1/\phi}\left(\Psi_i c_{ii}\right)^{(\phi-1)/\phi} + (1-a)^{1/\phi}\left(\Psi_j c_{ij}\right)^{(\phi-1)/\phi}\right]^{\phi/(\phi-1)} \tag{A.21}$$

where Ψ_i, $i = H, F$ with $E_0(\Psi_i) = 1$ is an exogenous worldwide shocks to the (relative) preference for the country i good. Note that the shock Ψ_i can also have a more supply oriented interpretation, as a shock to the quality of good i. We denote $\Psi \equiv \frac{\Psi_H}{\Psi_F}$ the relative preference shocks and $\hat{\Psi}$ its deviation from its steady-state value of one.

As shown by Coeurdacier et al. (2009), the welfare-based real exchange rate in this case is equal to (up to the first-order):

$$\hat{Q} = (2a - 1)\left(\widehat{\frac{p_H}{p_F}} - \widehat{\frac{\psi_H}{\psi_F}}\right) = (2a - 1)\,\hat{q} \tag{A.22}$$

where we adjust the terms-of-trade for quality/preference shocks by scaling q as follows: $q = \frac{p_H/\psi_H}{p_F/\psi_F}$.

As shown by Coeurdacier et al. (2009), under the assumption of (locally) complete markets, intratemporal allocation across goods imply:

$$\widehat{\psi y} = -\lambda \hat{q} \tag{A.23}$$

where $\widehat{\psi y}$ are relative endowments adjusted for quality/preference shocks.

Relative equity returns and relative non-financial incomes still load perfectly on the real exchange rate adjusted for the quality/preference shocks (see also Coeurdacier et al. (2009)).

$$\hat{R}^f = (1 - \lambda)\hat{q} \tag{A.24}$$

$$\hat{R}^n = (1 - \lambda)\hat{q} \tag{A.25}$$

We assume that real bonds in each country pays one unit of the good **not** adjusted for quality:

$$\hat{R}^b = (2a - 1)\widehat{\frac{p_H}{p_F}} = (2a - 1)\left(\hat{q} + \hat{\psi}\right)$$

Then, introducing $\hat{\varepsilon} = (2a-1)\,\hat{\psi}$, we are back to our reduced form model (with $\gamma_w = \gamma_f = 0$) and equity and bond portfolios will be the same as in (A.20)). Bond returns differential fails to load on the welfare-based real exchange rate because of relative change in quality/preference shocks $\hat{\psi}$. This additional source of risk on bond returns shift bond position towards zero and risk-sharing is done by equities only.

A.6 Derivation of portfolios for countries of different sizes

We extend our benchmark model by allowing different country sizes. We assume that expected production in period $t = 1$ is not equal across countries: $E_0(y_H) = \overline{y_H}$ and $E_0(y_F) = \overline{y_F}$. We denote by ω_i the relative size of country i: $\omega_i = \frac{\overline{y_i}}{\overline{y_i}+\overline{y_j}}$, with $\omega_H + \omega_F = 1$.

Both countries also differ in their consumption Home bias:

$$C_i = \left[a_i^{1/\phi}\,(c_{ii})^{(\phi-1)/\phi} + (1-a_i)^{1/\phi}\,(c_{ij})^{(\phi-1)/\phi}\right]^{\phi/(\phi-1)}$$

We assume that is the non-stochastic equilibrium, the following relationship holds:

$$(1-a_H)\,y_H = (1-a_F)y_F$$

This ensure that in the trade balance is zero and terms-of-trade q are equal to unity in the non-stochastic equilibrium.

Keeping the same notations as in the symmetric case of our benchmark model with locally complete markets, log-linearization around the non-stochastic equilibrium implies the following relationships (note that potfolio implications are identical in the case of imperfect sapnning of the shocks):

$$\begin{aligned}
\hat{Q} &= (a_H + a_F - 1)\hat{q} \\
\hat{PC} &= (1-\frac{1}{\sigma})\hat{Q} = (1-\frac{1}{\sigma})(a_H + a_F - 1)\hat{q}
\end{aligned}$$

Log-linearization of the budget constraint in country i gives (using market clearing conditions in the asset market) for $i \neq j$:

$$\hat{P_iC_i} = (1-\delta)\hat{w}_i + \delta S_{ii}\hat{d}_i + \frac{\omega_j}{\omega_i}\delta\,(1-S_{jj})\,\hat{d}_j + \overline{b_{ii}}\hat{R}_i^b - \frac{\omega_j}{\omega_i}\overline{b_{jj}}\hat{R}_j^b$$

where $\hat{R}_i^b$ denotes the return on the bond of country i (specified below). $\overline{b_{ii}}$ denotes bonds i held by country i normalized by the mean output $\overline{y_i}$ of country i (in the benchmark model $\overline{y_i} = 1$ and $\overline{b_{ii}} = b_{ii}$)

Taking the difference across countries, we get:

$$\begin{aligned}
\hat{PC} &= (1-\frac{1}{\sigma})\hat{Q} \\
&= (1-\delta)\,(\hat{w_H} - \hat{w_F}) + \delta\hat{d_H}\left(S_{HH} - \frac{\omega_H}{\omega_F}(1-S_{HH})\right) \\
&\quad -\delta\hat{d_F}\left(S_{FF} - \frac{\omega_F}{\omega_H}(1-S_{FF})\right) + \left(1+\frac{\omega_H}{\omega_F}\right)\overline{b_{HH}}\hat{R}_H^b - \left(1+\frac{\omega_F}{\omega_H}\right)\overline{b_{FF}}\hat{R}_F^b
\end{aligned}$$

Rewrite portfolio as follows (assuming symmetry of shocks):

$$\begin{aligned} S_{ii} &= S = \omega_i + B^f(1-\omega_i) \\ \overline{b_{ii}} &= b = B^b(1-\omega_i) \end{aligned}$$

B^f and B^b are measures of the size of portfolio biases. Then, keeping the same notations:

$$(1-\frac{1}{\sigma})\hat{Q} = (1-\delta)\hat{R}_n + S\delta\hat{R}_f + B^b\hat{R}_b$$

Assuming the following loadings on $\hat{R}_n$ and $\hat{Q}$ (see main text)

$$\begin{aligned} \hat{Q} &\equiv \beta_{Q,b}\hat{R}_b + \beta_{Q,f}\hat{R}_f \\ \hat{R}_n &\equiv \beta_{n,b}\hat{R}_b + \beta_{n,f}\hat{R}_f \end{aligned}$$

Projection on $\hat{R}^n$ and $\hat{Q}$ gives B^f and B^b :

$$\begin{aligned} B^f &= -\frac{1-\delta}{\delta}\beta_{n,f} + \frac{1-\frac{1}{\sigma}}{\delta}\beta_{Q,f} \\ B^b &= \left(1-\frac{1}{\sigma}\right)\beta_{Q,b} - (1-\delta)\,\beta_{n,b} \end{aligned}$$

Using $S = \omega_i + B^f(1-\omega_i)$ and $b = B^b(1-\omega_i)$ gives equilibrium portfolios for countries of different sizes:

$$\left\{ \begin{aligned} b^* &= (1-\omega_i)\left(1-\tfrac{1}{\sigma}\right)\beta_{Q,b} - (1-\omega_i)(1-\delta)\,\beta_{n,b} \\ S^* &= \omega_i + (1-\omega_i)\left(\tfrac{1-\frac{1}{\sigma}}{\delta}\beta_{Q,f} - \tfrac{1-\delta}{\delta}\beta_{n,f}\right) \end{aligned} \right.$$

B Empirical Appendix

B.1 Data description. [Not for publication]

All data are quarterly, between 1970-Q1 and 2008-Q3.

- Government bond returns: gross return on 3-month domestic Treasury-bill, from Global Financial Database.
- Nominal exchange rates: from Global Financial Database.
- Consumer Price Index (CPI): from OECD Main Economic Indicators
- Gross Domestic Product: OECD Quarterly National Accounts, Seasonally adjusted, except as noted. Notes: Germany: data for West Germany before 1991:Q1. Japan: data before 1979 from 1999 OECD Statistical compendium and seasonally adjusted with X-12 method; Italy: data before 1979 from the 1999 OECD Statistical Compendium (seasonally adjusted).
- Compensation of Employees: OECD Quarterly National Accounts, Seasonally adjusted.except as noted. Notes: Japan: Japan: data before 1998 from 1999 OECD Statistical compendium and seasonally adjusted with X-12 method. Italy: data before 1979 from the 1999 OECD Statistical Compendium (seasonally adjusted). France: data before 1977 from the 1999 OECD Statistical Compendium (seasonally adjusted). Germany: data for West Germany before 1991:Q1.
- Mixed Income:
  - US: fraction of net operating surplus plus mixed income from OECD Quarterly National Accounts. Fraction calculated as the share of mixed income in (mixed income + gross operating surplus - consumption of fixed capital) annual data from UN National Income System of National Accounts. Ratio after 2007 is the average for 2004-2006.
  - UK: fraction of net operating surplus + mixed income. Fraction calculated as the share of net mixed income in (net mixed income + gross operating surplus - consumption of fixed capital); annual data from UN National Income System of National Accounts 1987-1994. Annual data from 2007 National Accounts Statistics Part III, pp836 for 1995-2005. Ratio after 2005 is the average for 2003-2005. Ratio before 1987 is the average for 1987-1989.
  - Japan: fraction of net operating surplus + mixed income. Fraction calculated as the share of net mixed income in (net mixed income + gross operating surplus - consumption of fixed capital); annual data from UN National Income System of National Accounts 1980-2003. Ratio after 2003 is the average for 2001-2003. Ratio before 1980 is the average for 1980-1982.
  - Italy: fraction of net operating surplus + mixed income. Fraction calculated as the share of net mixed income in (net mixed income + gross operating surplus - consumption of fixed capital); annual data from UN National Income System of National Accounts 1980-2003. Ratio after 2003 is the average for 2001-2003. Ratio before 1980 is the average for 1980-1982.
  - Germany: fraction of net operating surplus + mixed income. Uses data from West Germany before 1991:Q1. Fraction calculated as the share of net mixed income in (net mixed income + gross operating surplus - consumption of fixed capital); annual data from UN National Income System of National Accounts 1991-2002. Ratio after 2002 is the average for 2000-2002. Ratio before 1991 is the average for 1991-1993.

- France: fraction of net operating surplus + mixed income. Fraction calculated as the share of net mixed income in (net mixed income + gross operating surplus - consumption of fixed capital); annual data from UN National Income System of National Accounts 1978-2003. Ratio after 2003 is the average for 2001-2003. Ratio before 1978 is the average for 1978-1980.
- Canada: fraction of net operating surplus + mixed income. Fraction calculated as the share of net mixed income in (net mixed income + gross operating surplus - consumption of fixed capital); annual data from UN National Income System of National Accounts 1970-2005. Ratio for 2005-2007 from StatCan. Data after 2007 is the average for 2005-2007.

- Net Operating Surplus and mixed income: from OECD Quarterly National Accounts, seasonally adjusted, except as noted below.
  - France: before 1978 GDP minus compensation of employees, depreciation and indirect taxes.
  - Italy: before 1980 GDP minus compensation of employees, depreciation and indirect taxes.
  - Japan: Before 1998:Q3, net operating surplus + mixed income from OECD Statistical Compendium quarterly data, seasonally adjusted with X-12 routine. After 1998:Q3, defined as GDP minus compensation of employees, depreciation and indirect taxes.
  - United Kingdom: before 1988, GDP minus Compensation of employees, Depreciation and Indirect Taxes.

- Depreciation: OECD Quarterly National Accounts, Consumption of Fixed Capital. Seasonally adjusted, except as noted below.
  - France: Before 1978, calculated as fraction of GDP, where the fraction is computed annually as the ratio of consumption of fixed capital to GDP from 1999 OECD Annual National Accounts.
  - Germany: Data for West Germany before 1991:Q1.
  - Italy: Before 1980, calculated as fraction of GDP, where the fraction is computed annually as the ratio of consumption of fixed capital to GDP from 1999 OECD Annual National Accounts.
  - Japan: Before 1998:Q2 from the 1999 OECD Statistical Compendium. After 1998, calculated as fraction of GDP, where the fraction is computed annually as the ratio of consumption of fixed capital to GDP from United Nations system of national accounts annual data.
  - United Kingdom: Before 1988, calculated as fraction of GDP, where the fraction is computed annually as the ratio of consumption of fixed capital to GDP from 1999 OECD Annual National Accounts.

- Indirect Taxes: Taxes less Subsidies on Production and Imports from OECD Quarterly National Accounts, seasonally adjusted, except as noted below.
  - France: before 1978, from 1999 OECD Statistical Compendium.
  - Germany: before 1991 uses data for West Germany.
  - Italy: before 1980, calculated as fraction of GDP, where the fraction is computed annually as the ratio of indirect taxes to GDP from 1999 OECD Annual National Accounts.

  - Japan: before 1998:Q2, from OECD Statistical Compendium quarterly data, seasonally adjusted with X-12 routine. After 1998:Q2 calculated as fraction of GDP, where the fraction is computed annually as the ratio of indirect taxes to GDP from United Nations system of national accounts annual data.

- Gross Fixed Capital Formation: from OECD Quarterly National Accounts, seasonally adjusted, except as noted below.
  - France: before 1978, from 1999 OECD Statistical Compendium.
  - Germany: before 1991 uses data for West Germany
  - Italy: before 1980, from 1999 OECD Statistical Compendium.
  - Japan: before 1994 from ESRI National Accounts Office Total Fixed Investment.
- Residential Investment: OECD Quarterly National Accounts, seasonally adjusted, except as noted below.
  - Canada: before 1980, assumed to be 25% of total investment.
  - France: before 1978, from 1999 OECD Statistical Compendium.
  - Germany: before 1991 data for West Germany
  - Italy: before 1980, constructed backwards from the growth rate of total construction, from 1999 OECD Statistical Compendium.
  - Japan: before 1994, from ESRI National Accounts Office Residential Investment.
  - United States: before 1990, from 1999 OECD Statistical Compendium.
- Consumption: OECD Quarterly National Accounts, seasonally adjusted, except as noted below.
  - France: before 1978, from 1999 OECD Statistical Compendium.
  - Germany: before 1991 data for West Germany.
  - Italy: before 1980, from 1999 OECD Statistical Compendium.
  - Japan: before 1980, from OECD Statistical Compendium quarterly data, seasonally adjusted with X-12 routine.
- Equity Returns: Global Financial Database Total Return index.
- Corporate Bond Returns: quarterly holding return on corporate bond assuming a 10 year maturity. Yields on corporate debt from Global Financial Database.
- Compustat Weights: For each country and each available year, we construct the share of corporate debt as 1 minus the share of stockholder's equity in total assets for non-financial firms listed in Compustat North America (for the US and Canada) and Compustat Global (for France, Germany, Italy, Japan and the UK). Data start in 1970 for Canada and the US, 1987 for Japan and the UK, 1988 for Germany and France and 1989 for Italy.

B.2 Empirical Issues

B.2.1 VAR diagnostic tests

We specify our Vector Auto Regression in first differences for $\ln w$ and $\ln k$ (see section 4.1.2). This is empirically valid given that:

- w and k are integrated of order 1;
- w and k are *not* co-integrated.

We verify that these conditions are satisfied as follows (detailed results available upon request):

- we conduct Augmented Dickey-Fuller tests of unit roots for both variables. We cannot reject the null of a unit root, except for $\ln w$ in Japan.
- We perform Johansen tests of co-integration for (w, k). We find no cointegration relationship, except for Germany.
- Since theory suggests that the only correct co-integration vector is $w - k$ (see Baxter and Jermann (1997)), we directly test for stationarity for this variable. Using Augmented Dickey-Fuller tests, we cannot reject the null of a unit root, except for Germany (with a p-value of 5.3%).

We conclude from these diagnostic tests that a VAR in first difference is appropriate. Although theory suggests that $w - k$ should be stationary, this variable is extremely persistent even over long periods of time, suggesting that the correcting mechanism does not play an important role at least over the period we consider.

B.2.2 An alternative measure of the returns to non-financial wealth

Lustig and Nieuwerburgh (2008) propose an alternative approach to measuring the returns to human wealth. The key identification assumption consists in assuming that consumption choices are consistent with the choices of a representative agent faced with financial and non-financial wealth. In other words, aggregate consumption satisfies the Euler equation of the representative household when using the total return to the agent's wealth. Since this return is a combination of the return to financial wealth (observable) and non-financial wealth (non-observable), one can then back out the innovation to the return on non-financial wealth.

The Lustig and Nieuwerburgh (2008) method starts with the two equations below:

$$\begin{aligned} c_{t+1} - E_t c_{t+1} &= (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j r^m_{t+1+j} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \Delta c_{t+1+j} \\ E_t \Delta c_{t+1} &= \mu_m + \sigma^{-1} E_t r^m_{t+1} \end{aligned}$$

where the first equation is a log-linearization of the intertemporal budget constraint following Campbell (1993), under the assumption that $c_t - v^m_t$ is stationary, where c_t is log-consumption, v^m_t is log-total wealth and $r^m_t = \ln(R^m_t)$ is the return on total wealth: $V^m_{t+1} = R^m_{t+1}(V^m_t - C_t)$. ρ is related to the steady state consumption wealth ratio as $\rho = 1 - \exp(c - v^m)$. Crucially, V^m_t includes non-financial wealth. The second equation is the log-linearized form of the Euler equation that characterizes the slope of the consumption profile. σ is the coefficient of relative risk aversion (inverse of the intertemporal elasticity of substitution) and μ_m captures all variance-covariance terms, assumed constant.

Substituting the Euler equation into the budget constraint, one obtains an expression for the innovation to consumption:

$$c_{t+1} - E_t c_{t+1} = r^m_{t+1} - E_t r^m_{t+1} + \left(1 - \sigma^{-1}\right)\left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty}\rho^j r^m_{t+1+j} \tag{B.1}$$

The next step consists in writing the (log) return on total wealth as :

$$r^m_{t+1} = \left(1 - \kappa_t\right) r^f_{t+1} + \kappa_t r^n_{t+1} \tag{B.2}$$

where r^f_{t+1} is the return on financial wealth and r^n_{t+1} the return on non-financial wealth and κ_t is the share of nonfinancial wealth in total wealth (possibly time-varying). Following the usual steps, the innovation to the return on nonfinancial wealth satisfies:

$$r^n_{t+1} - E_t r^n_{t+1} = \left(E_{t+1} - E_t\right)\sum_{j=0}^{\infty}\rho^j \Delta w_{t+1+j} - \left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty}\rho^j r^n_{t+1+j} \tag{B.3}$$

The central idea is to recover $\left(E_{t+1} - E_t\right) r^n_{t+1+j}$ from the consumption innovations in (B.1). Substituting (B.2) and (B.3) into (B.1), and assuming constant portfolio shares ($\kappa_t = \kappa$), we obtain:

$$\begin{aligned} c_{t+1} - E_t c_{t+1} &= (1-\kappa)\left(r^f_{t+1} - E_t r^f_{t+1}\right) + \left(1 - \sigma^{-1}\right)(1-\kappa)\left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty}\rho^j r^f_{t+1+j} \\ &\quad + \kappa\left(E_{t+1} - E_t\right)\sum_{j=0}^{\infty}\rho^j \Delta w_{t+1+j} - \kappa\sigma^{-1}\left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty}\rho^j r^n_{t+1+j}. \end{aligned}$$

Assuming that financial returns are observed (as in the benchmark case), we can invert this expression to obtain an expression for the innovation to nonfinancial returns that does not involve expected future nonfinancial returns:[74]

$$\begin{aligned} r^n_{t+1} - E_t r^n_{t+1} &= (1-\sigma)\left(E_{t+1} - E_t\right)\sum_{j=0}^{\infty}\rho^j \Delta w_{t+1+j} - \sigma\left(\kappa^{-1} - 1\right)\left(r^f_{t+1} - E_t r^f_{t+1}\right) \\ &\quad - (\sigma - 1)\left(\kappa^{-1} - 1\right)\left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty}\rho^j r^f_{t+1+j} + \sigma\kappa^{-1}\left(c_{t+1} - E_t c_{t+1}\right). \end{aligned} \tag{B.4}$$

One can estimate the innovation to non-financial wealth in (B.4) using a Vector Autoregression of the form $\mathbf{z}_{t+1} = \mathbf{A}\mathbf{z}_t + \boldsymbol{\epsilon}_{t+1}$ with $\mathbf{z}'_t = \left(\Delta w_t, \Delta k_t, \Delta c_t, r^f_t, \mathbf{x}'_t\right)$ as:

$$\begin{aligned} r^n_{t+1} - E_t r^n_{t+1} &= (1-\sigma)\,\mathbf{e}'_{\Delta w}\left(\mathbf{I} - \rho\mathbf{A}\right)^{-1}\boldsymbol{\epsilon}_{t+1} \\ &\quad - \left(\kappa^{-1} - 1\right)\mathbf{e}'_{r^f}\left[\sigma + (\sigma - 1)\rho\mathbf{A}\left(\mathbf{I} - \rho\mathbf{A}\right)^{-1}\right]\boldsymbol{\epsilon}_{t+1} \\ &\quad + \sigma\kappa^{-1}\mathbf{e}'_{\Delta c}\boldsymbol{\epsilon}_{t+1} \end{aligned}$$

We implement this VAR estimation in section 4.5 under the assumption that $\sigma = 1$ and $\kappa = 1 - \delta$. In that case, the expression for the innovations simplifies substantially:

$$r^n_{t+1} - E_t r^n_{t+1} = -\left(\kappa^{-1} - 1\right)\mathbf{e}'_{r^f}\boldsymbol{\epsilon}_{t+1} + \kappa^{-1}\mathbf{e}'_{\Delta c}\boldsymbol{\epsilon}_{t+1}.$$

[74] A similar derivation can be obtained in the case where financial returns are not observed, using equation (41).