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INTERMEDIARIES, TRANSPORT COSTS AND INTERLINKED TRANSACTIONS

Mélanie Lefèvre and Joe Tharakan

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Mélanie Lefèvre, CREPP, Université de Liège
Joe Tharakan, Université de Liège, CORE and CEPR

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Centre for Economic Policy Research
77 Bastwick Street, London EC1V 3PZ, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

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Mélanie Lefèvre
CREPP
Université de Liège
7, Bd Rectorat (Bat 31)
B-4000 Liège
BELGIUM

Email: Melanie.Lefevre@ulg.ac.be

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=173791

Joe Tharakan
Department of Economics
Université de Liège
7, Bd Rectorat (B31)
B-4000 Liège
Belgium

Email: j.tharakan@ulg.ac.be

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=136639

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Intermediaries, transport costs and interlinked transactions*

Mélanie Lefèvre[†] and Joe Tharakan[‡]

October 4, 2011

Abstract

Transport costs play a key role in agricultural markets in developing countries and are one of the causes of poverty amongst farmers that are geographically isolated. Another characteristic of agricultural markets is that they often involve interlinked transactions. However, the existing theoretical literature on interlinked transactions does not take into account the existence of transport costs. This paper develops a model of input-output interlinked contracts between geographically dispersed farmers and a trader, whether this trader is for-profit or non-profit. We derive implications of imposing either uniform or mill pricing policies, as opposed to spatial price discrimination. Impact on profit, farmers' income, level of production, social welfare and regional disparities are investigated.

1 Introduction

In most developing countries, poverty is located in rural areas. In a recent report, the World Bank (2008) estimated that 75% of poor people in developing countries live in rural areas. Most of them depend directly or indirectly on agriculture for their livelihoods. Rural areas are characterized by bad road infrastructure and bad access to markets. In 30 selected developing countries, Roberts et al. (2006) estimate that only 57% of rural people live within two kilometers of an all-season road, and only 30% in sub-saharian Africa.

Smallholder farming or family farming constitutes about 80 percent of African agriculture. 500 million of such farms provides an income to about two-thirds of the 3 billion rural people in the world (FAO, 2008). However, what we observe in recent decades is that small-scale agriculture has suffered, globalization and agro-industrialization cause small farms to go out of business (Reardon and Barrett, 2000). Small farmers' access to land has been shown to decrease over time (Jayne et al., 2003). In South Asia and subsaharian Africa, the number of

*We are grateful to Axel Gautier, Knud Munk and Pierre Pestieau for useful comments and suggestions.
†CREPP, HEC-ULg, Université de Liège, Bd du Rectorat 7, Bat 31, B-4000 Liège, Belgium, email: Melanie.Lefevre@ulg.ac.be.
‡Université de Liège, CORE and CEPR.

poor people in rural areas is still increasing and is expected to stay above the number of urban poor, at least until 2040 (World Bank, 2008). The prevalence of hunger is still greater in rural than in urban areas (Von Braun et al., 2004) and rural children are nearly twice as likely to be underweight as urban ones (United Nations, 2010).

Evidence suggests that agricultural activities are more effective than nonagricultural ones in reducing poverty among the poorest (see for example Christiaensen et al. (2010), Byerlee, de Janvry, and Sadoulet (2009), Ligon and Sadoulet (2008) or Bravo-Ortega and Lederman (2005)). Improving agricultural productivity leads to an increase in food availability and decreases food prices, but also boosts rural incomes that generate demand for local products. This contributes to reducing poverty and results in broadbased socio-economic development in rural areas. This dynamic explains why agricultural growth is four times more effective in reducing poverty than growth in other sectors (FAO, 2008). Indeed, the recent decline in poverty rates seems to be mainly due to improved conditions in rural areas (Ravallion, Chen and Sangraula, 2007). A 10 percent increase in agricultural productivity is associated with a 7.2 percent reduction in poverty in Africa and with a 4 to 12 percent in India (Von Braun et al., 2004).

Several studies have shown that a major factor which hampers the necessary increase of productivity of farmers in developing countries are high transport costs. Inadequate transport infrastructure and large distances between areas of production and areas of consumption diminish both input use and production. For instance, in the milk sector in Kenya, Stall et al. (2002) have found that an 10 additional kilometers between the farmer and Nairobi decreases the probability of using concentrate feeding by more than 1%. Holloway et al. (2000) found that, in Ethiopia, each additional minute walk to the collection center reduces the marketable quantity of milk by 0.06 litres per day. In a region where milk yields per day are less than 4 liters, this is of considerable importance. In Madagascar, Stifel and Minten (2008) found a strong negative relationship between productivity and isolation. Reduction of transport costs is shown to increase the use of various inputs, such as fertilizer, as well as rice production per acre. Indeed, an increase of one percent in transport costs decreases probability of using fertilizer by 0.06% and reduces production per acre by 0.17%. Similar or stronger effects have been found in Bangladesh (Ahmed and Hossain, 1990) and India (Binswanger et al., 1993). Evidence also suggests that transport costs reduce income. In Nepal, Jacoby (2000) found that the two main components of rural income, namely farm profit and labor earnings, decrease with transport costs. He showed that a 10% increase in travel time reduces the maximal profit that can be earned on a hectare of land by 2.2% and reduces agricultural wage by about 0.5%.

Agricultural transactions in developing countries often rely on intermediaries, such as private traders, retailers, agribusinesses, cooperatives or food processing companies, which possess an advantage over farmers. This advantage can take different forms. For example, it can be the ability to transport the good at a lower cost (by the use of more efficient transport devices, such as trucks, or a transformation of the product that reduces the volume and/or perishability of the product, etc.). This cost advantage often requires an important fixed cost, which cannot be borne by each farmer alone. Several examples of such intermediaries can be found in various sectors in developing countries.[1] Contrary to the farmers, because of their advantage, these intermediaries have a better access to markets. Hence a question is whether grants or subsidies to these intermediaries or simply helping them set up, would be a way forward in reducing poverty in rural areas. Local farmers would benefit from the cost advantage of this intermediary.

An additional element is that evidence in various countries and sectors shows that contracts between intermediaries and farmers often involve interlinked transactions (see for instance Warning and Key (2002) for an analysis of groundnut sector in Senegal; Jayne, Yamano and Nyoro (2004) for examples in cash crops production in Kenya; Simmons, Winters and Patricks (2005) for examination of various Indonesian sectors; or Key and Runsten (1999) for a look at the Mexican frozen vegetable industry). While these interlinked contracts have been shown to be efficient, it has also been shown that any efficiency gain is completely appropriated by the trader (Gangopadhyay and Sengupta, 1987). This means that interlinked contracts do not allow farmers to benefit from the intermediary's cost advantage. Extrapolating this result to our setting, this would imply that helping intermediaries obtain an advantage would only lead to increase their profitability and would have little effect on improving the poverty amongst farmers in rural areas. However, this result from the interlinked contracts literature has been obtained in a context in which the trader is a profit maximizer and sets a different contract for each farmer.

In practice, we observe that there is a lot of diversity regarding the nature of the intermediaries (IFAD, 2003). In agriculture and livestock sector in developing countries, intermediaries are not necessarily profit maximizers and we often observe local producers' associations, NGOs, cooperatives or even public organizations set up trading structures whose aim is to improve farmers' living conditions and income (Chau et al., 2009). In addition, a recent report from the United Nations encourages farmers to develop cooperative structures, in order to realize economies of scale (De Schutter, 2010: 19).

The interlinked contracts literature has been analyzed in a non-spatial context situations in which a for profit trader deals with heterogeneous agents. It assumes that this trader is able to offer different contracts to different agents (Gangopadhyay and Sengupta, 1987). In our spatial context, this would correspond to assuming that the intermediary perfectly price discriminates between spatially dispersed farmers. Spatial price discrimination is only one possible pricing policy. For instance, in the Senegalese milk sector, some processing units, such as *La Laiterie du Berger* (LdB) in Richard-Toll, organize milk collection and pay all the farmers the same price, independent of the distance. This corresponds to uniform pricing. In others, such as *Le Fermier* in Kolda, farmers are responsible for transport, such that the ones who are located far from the processing unit receive a considerably lower net price than the closer ones. This corresponds to mill pricing.

The choice of particular pricing policy may be important for farmers, as transport costs represent a high part of the price received. In Kolda, where the price received by the producers ranges between 75 and 150 CFA, transport by bicycle costs between 20 and 25 CFA per liter

[1] Such as maize, beans, roots and tubers in Malawi and Benin (Fafchamps and Gabre-Madhin, 2006), mandarin in Nepal (Pokhrel and Thapa, 2007), cashews in Mozambique (McMillan et al., 2003), etc.

(Dia, 2002: 53-54). Motorized transport is even more costly, according to one of the managers of the LdB (personal interview, 2009); average transport cost in their collection area is 100 CFA per liter, while farmers receive 200 CFA per liter. Mill pricing, where farmers have to support transport cost is disadvantageous for those located far away. Uniform pricing may seem fairer, as all the producers receive the same price. However, the closest ones may receive a lower net price than if they were themselves responsible for the transport. As both pricing rules are observed in practice, one may ask what drives the choice among them. Given the diversity in the types of intermediaries, the question is whether this matters for the choice of pricing policy. A priori, the optimal pricing policy is not obvious, neither from the trader point of view, nor from a social welfare perspective.

In this paper, we develop a simple contract model where interlinkage arises because of the trader's advantage in transport costs.[2] In our model, the heterogeneity among farmers is due to their spatial dispersion. We compare the outcomes of different policies (discriminatory, mill and uniform pricing) in terms of social welfare and regional disparities in order to arrive at some policy recommendations as to the type of spatial pricing policy that should be used. In particular, we consider a benevolent policy maker who wants to reach the social optimum, characterized by efficient production and maximized income of farmers, but is unable to impose a complex tax and subsidy scheme. Assuming that regulation concerning pricing policy is the only instrument available for such a public authority, we look at what pricing policy it should impose. Alternatively, our results regarding the optimal pricing policy can be seen as the pricing policy that an external donor which helps set up agricultural intermediaries with a view of reducing rural poverty should impose as a condition to those intermediaries.

The paper is structured as follows. The next section describes the model and its assumptions. Sections 3, 4 and 5 develop the interlinked transaction model for a for-profit trader in the cases of spatial price discrimination, uniform pricing, and mill pricing. Section 6 extends the model to the case of a non-profit organization. Section 7 discusses the implications of pricing policy choice to profit, farmers' income, social welfare, level of production, regional differences among farmers and so on. Finally, section 8 concludes.

2 Model

We analyze the impact of transport costs and interlinked transactions on poverty in the following theoretical framework. Geographical locations are represented along a linear segment of size $r + R$. A final good market is located at the origin 0. We consider one agricultural good whose price p is set on this market. We assume that the different agents in our model do not have an impact on this price.[3] This good is consumed at location 0 which can be assumed to

[2] Other rationales for the existence of such transactions have been analyzed, from rationed or imperfect rural credit (Gangopadhyay and Sengupta, 1987; Chakrabarty and Chaudhuri, 2001), output market price uncertainty (Chaudhuri and Gupta, 1995), risk aversion (Basu, 1983; Basu, Bell and Bose, 2000), unobservable tenant effort (Braverman and Stiglitz, 1982; Mintra, 1983) to the inability to collude (Motiram and Robinson, 2010).

[3] This can be the case for example because we are in a small open economy and the price of this good is determined on world markets.

be an urban center. At a distance r from this urban center, there is a rural area which has a geographical extend R. Farmers are uniformly distributed over this area. Each farmer produces the agricultural good according to the same production function $f(k)$, where k is the quantity of input he uses. This input is sold at price i on the market at location 0. The production function has the usual properties: $f(.)$ is continuously differentiable, $f(0) = 0$, $f_k = \frac{df}{dk} > 0$, $\lim_{k\to 0} f_k = \infty$, $\lim_{k\to\infty} f_k = 0$ and $\frac{d^2f}{dk^2} < 0$. Farmers are assumed to be profit maximizers. A farmer located at x facing prices $p_F(x)$ and $i_F(x)$ maximizes his income $y(x, p_F(x), i_F(x))$ by using the optimal quantity $k(x, p_F(x), i_F(x))$ (for simplicity, as long as it does not cause any confusion, shortcut notations $y(x)$ and $k(x)$ will be used):

$$\max_{k(x)} \; y(x) = p_F(x)f(k(x)) - i_F(x)k(x) \tag{2.1}$$

The existence of an interior solution to this problem is guaranteed by the above assumptions regarding the production function. The choice of input quantity satisfies the following necessary condition:

$$\frac{df}{dk} = \frac{i_F(x)}{p_F(x)} \tag{2.2}$$

To transport the agricultural good to the market, farmers face high transport costs. These costs are assumed to be linear in distance for the output. To simplify the analysis we assume that transport costs are negligible for the input and set them equal to zero. A farmer located at a distance x from the market faces a transport cost τx and hence this farmer can obtain a net price $p_F(x) = p - \tau x$ for the good he produces. All farmers are assumed to be able to sell profitably on the market which implies the following restriction, $p > \tau r + \tau R$.

An intermediary is located at r.[4] This trader offers interlinked contracts to the geographically dispersed producers. There is an input-output interlinked relationship between them: on the one hand he buys the output from the farmers and, on the other hand, sells them an input necessary for their production. Prices for both input and output are simultaneously fixed in the contract between the trader and the farmer. The trader sells the agricultural output from the farmers and buys input for them on the market located in 0, at market price p and i. The intermediary is assumed to have a cost advantage. Here, we assume that the trader has an advantage to transport the good between r and 0. Transport costs for the trader are given by $t(x) = \theta r + \tau(x - r)$ per unit of output transported with $\theta < \tau$.

The sequence is the following. In a first step, the trader proposes a contract $(p_C(x), i_C(x))$ to each farmer located on the segment $[r, r + R]$.[5] Each farmer can individually accept or reject the contract. In a second step, the farmer chooses his optimal quantity of input, which determines his level of production. If he has accepted the contract, he faces prices $(p_C(x), i_C(x))$ and chooses optimal input use $k^*(x) = k(x, p_C(x), i_C(x))$. If he rejects the contract, he sells

[4]In developing countries, poor infrastructures in rural area reduces the incentive for firms to locate within the rural area. In this context, the location at the beginning of the rural area is involved by better access to roads, electricity, water and so on at this point.

[5]Strictly speaking, as shown in Lefèvre and Tharakan (2011), sufficient condition for this to be profitable for the trader is that the trader's cost advantage is large enough compared to the size of the farmer's area, that is $\tau r - \theta r > \sqrt{5}\tau(R/2)$.

his production directly to the final market. The same applies to the purchase of inputs. In this case, he chooses the optimal amount of inputs k^0 as a function of market prices (p, i) as well as of the transport cost he has to support, that is $k^0(x) = k(x, p - \tau x, i)$. In a last step, output is produced and is sold on the market, directly by the farmer (if he has rejected the contract) or via the trader (if the farmer has accepted the contract).

This means that the trader's problem can be characterized as follows:

$$\max_{p_C(x), i_C(x)} \Pi = \int_r^{r+R} (p - \theta r - \tau(x - r) - p_C(x)) f(k^*(x)) + (i_C(x) - i) k^*(x) dx \tag{2.3}$$

subject to the incentive compatibility constraint given by the input demand equation (2.2) and the following individual rationality constraint:

$$y(x) \equiv p_C(x) f(k^*(x)) - i_C(x) k^*(x) \geq y^0(x) \equiv (p - \tau x) f(k^0(x)) - i k^0(x) \tag{2.4}$$

One of the questions we will be looking at in the following sections is whether without state intervention the different outcomes are socially optimal. Due to the farmer's cost disadvantage compared to the trader, his standalone production is not socially efficient. Indeed, the efficient (first-best) input use $k^{\#}(x)$ maximizes the sum of trader's profit and farmer's incomes $\int_r^{r+R} (p - \theta r - \tau(x - r)) f(k(x)) - i k(x) dx$ and satisfies

$$\frac{df}{dk} = \frac{i}{p - \theta r - \tau(x - r)} \tag{2.5}$$

Given $\theta < \tau$ and the concavity of production function, this implies that $k^0(x) < k^{\#}(x)\ \forall x$.

In the following sections we will look at different ways in which the trader can set contracts with farmers who are geographically dispersed.

3 Spatial price discrimination

The trader proposes a contract $(p_D(x), i_D(x))$ to the farmer located in x. This contract can be different, depending on the location of the farmer and the difference in two farmers' contracts does not necessarily represent the difference in transport costs between them. Each farmer can individually accept or refuse the contract proposed. Hence, to maximize total profit, the trader chooses a contract which maximizes the profit he makes at each location. We assume arbitrage between farmers is impossible[6] which implies that farmers' contracts are independent.

From equations (2.3) and (2.4), the trader's problem may be written as:

$$\max_{p_D(x), i_D(x)} \pi(x) = (p - \theta r - \tau(x - r)) f(k^*(x)) - i k^*(x) - (p_D(x) f(k^*(x)) - i_D(x) k^*(x)) \tag{3.1}$$

[6] For instance, the trader may be able to impede arbitrage between locations by monitoring the quantity provided by each farmer. An alternative justification may be that the trader is able to test output quality while the farmer is not. If a farmer deals with another to take advantage of the contract difference between them, he faces the risk to see his own product refused by the trader because the other farmer's product has a low quality. If this risk is high enough, no farmer has an interest in making such a deal.

$$\text{s.t. } g(x) \equiv p_D(x) f(k^*(x)) - i_D(x) k^*(x) - y^0(x) \geq 0 \tag{3.2}$$

The Lagrangian is given by:

$$\mathcal{L} = (p - \theta r - \tau(x-r)) f(k^*(x)) - i k^*(x) + (\lambda(x) - 1)(p_D(x) f(k^*(x)) - i_D(x) k^*(x)) - \lambda(x) y^0(x) \tag{3.3}$$

Noting that at equilibrium $\frac{df}{dk} = \frac{i_D(x)}{p_D(x)}$ and applying the envelop theorem to the income of the farmer, the Kuhn-Tucker conditions can be written as:

$$\frac{\partial \mathcal{L}}{\partial p_D(x)} = \left((p - \theta r - \tau(x-r)) \frac{i_D(x)}{p_D(x)} - i \right) \frac{\partial k^*(x)}{\partial p_D(x)} + (\lambda(x) - 1) f(k^*(x)) = 0 \tag{3.4}$$

$$\frac{\partial \mathcal{L}}{\partial i_D(x)} = \left((p - \theta r - \tau(x-r)) \frac{i_D(x)}{p_D(x)} - i \right) \frac{\partial k^*(x)}{\partial i_D(x)} + (\lambda(x) - 1)(-k^*(x)) = 0 \tag{3.5}$$

$$\lambda(x) \geq 0; \; g(x) \geq 0; \; \lambda(x) g(x) = 0 \tag{3.6}$$

From (3.5),

$$\lambda(x) - 1 = \left((p - \theta r - \tau(x-r)) \frac{i_D(x)}{p_D(x)} - i \right) \frac{\partial k^*(x)}{\partial i_D(x)} \frac{1}{k^*(x)} \tag{3.7}$$

Substituting (3.7) in (3.4) we have:

$$\left((p - \theta r - \tau(x-r)) \frac{i_D(x)}{p_D(x)} - i \right) \left(\frac{\partial k^*(x)}{\partial p_D(x)} + \frac{\partial k^*(x)}{\partial i_D(x)} \frac{f(k^*(x))}{k^*(x)} \right) = 0 \tag{3.8}$$

If the second term was equal to zero, it can be shown that $y(x) = 0$ so that $g(x) < 0$, which contradicts (3.6).[7] Thus, the first term has to be equal to zero, that is:

$$\frac{i_D(x)}{p_D(x)} = \frac{i}{p - \theta r - \tau(x-r)} \tag{3.9}$$

Equation (3.9) characterizes the optimal contract $(p_D(x), i_D(x))$. This contract induces the farmer to increase his level of input (as well as his level of output) with respect to the levels he would have chosen in the standalone case, even though he receives the same income, as it is stated in the following proposition.

Proposition 1. ***Under spatial price discrimination, the profit maximizing contract is characterized by a binding individual rationality constraint for each farmer*** *($y(x) = y^0(x)\ \forall x$)* ***and farmer's production choices are the efficient ones:*** $k^*(x) = k^{\#}(x) > k^0(x)$.

[7]Indeed, implicit function theorem applied to equation (2.2) gives (omitting the argument (x)): $\frac{dk}{dp_F} = -\frac{\frac{df}{dk}}{p_F \frac{d^2 f}{dk^2}}$ and $\frac{dk}{di_F} = -\frac{-1}{p_F \frac{d^2 f}{dk^2}}$. By the envelop theorem, we know that $\frac{dk}{dp_F} = \left.\frac{\partial k}{\partial p_F}\right|_{k=k^*}$ and $\frac{dk}{di_F} = \left.\frac{\partial k}{\partial i_F}\right|_{k=k^*}$ that we call $\frac{\partial k^*}{\partial p_F}$ and $\frac{\partial k^*}{\partial i_F}$. Thus, $\frac{\partial k^*}{\partial p_F} = -\frac{\frac{df}{dk}}{p_D \frac{d^2 f}{dk^2}}$ and $\frac{\partial k^*}{\partial i_F} = -\frac{-1}{p_F \frac{d^2 f}{dk^2}}$, which implies $-\frac{\partial k^*/\partial p_F}{\partial k^*/\partial i_F} = \frac{df}{dk}$. By (2.2), this is equal to $\frac{i_F}{p_F}$. If the second term in (3.8) was equal to zero, we would have $-\frac{\partial k^*/\partial p_F}{\partial k^*/\partial i_F} = \frac{f(k^*)}{k^*}$, thus $\frac{f(k^*)}{k^*} = \frac{i_F}{p_F} \Leftrightarrow p_F = \frac{i_F k^*}{f(k^*)}$. Substituting in the farmer's income would give $y(x) = p_F f(k^*) - i_F k^* = 0$.

Proof of proposition 1: As the ratio of input price to output price is given by (3.9), this tells us, by using (2.2) and comparing it to (2.5), that the farmer will choose the efficient level of input: $k^*(x) = k^\#(x)$. Given that $\tau > \theta$ and that $f(k)$ is strictly concave and using (2.2) with respectively $(p_F(x), i_F(x)) = (p_D(x), i_D(x))$ and $(p_F(x), i_F(x)) = (p - \tau x, i)$, we have that $k^*(x) > k^0(x)$. Substituting (3.9) in (3.7) gives $\lambda(x) = 1$. From (3.6), this implies that the individual rationality constraint is binding: $g(x) \equiv p_D(x) f(k^*(x)) - i_D(x) k^*(x) - y^0(x) = 0$. $\square$

Substituting (3.9) in the binding participation constraint $g(x) = 0$ gives:

$$p_D(x) = (p - \theta r - \tau(x - r)) \underbrace{\frac{(p - \tau x) f(k^0(x)) - i k^0(x)}{(p - \theta r - \tau(x - r)) f(k^*(x)) - i k^*(x)}}_{\eta_D(x)} \tag{3.10}$$

$$i_D(x) = i \underbrace{\frac{(p - \tau x) f(k^0(x)) - i k^0(x)}{(p - \theta r - \tau(x - r)) f(k^*(x)) - i k^*(x)}}_{\delta_D(x)} \tag{3.11}$$

Proposition 2. ***Under spatial price discrimination, the profit maximizing contract is characterized by*** $p_D(x) < p - \theta r - \tau(x - r)$***,*** $i_D(x) < i$ ***and*** $\eta_D(x) = \delta_D(x) < 1$***: the trader "loses" on the input trading and "gains" on the output trading.***

Proof of proposition 2: As $k^*(x) = k^\#(x)$ (proposition 1), $\eta_D(x) = \delta_D(x)$ may be written as:

$$\eta_D(x) = \delta_D(x) = \frac{\max_k \, (p - \tau x) f(k) - ik}{\max_k \, (p - \theta r - \tau(x - r)) f(k) - ik}$$

Using the envelop theorem and since by assumption $\theta < \tau$, this implies that $\eta_D(x) = \delta_D(x) < 1$. Using this result with (3.10) and (3.11) this implies that $p_D(x) < p - \theta r - \tau(x - r)$ and $i_D(x) < i$. $\square$

Gangopadhyay and Sengupta (1987) obtain similar results. They analyzed interlinked contracts when input market is characterized by an imperfection, such that the farmer faces a higher input price than the firm. They show that the trader has an interest to "subsidize" the input and "tax" the output, and that this type of contract allows him to appropriate himself all the efficiency gain (i.e. farmers are pushed down to their reservation income). In our context, the difference between the trader and the farmer lies in (output) transport costs, and the previous analysis shows that their results remain valid in this context.

Evidence also suggests that in interlinked contracts the input is sold at a discount. For instance, La laiterie du Berger in Sénégal sells cattle feed to farmers at 50% of the market price (personal interview, 2009). In Kenya, British American Tobacco Ltd delivers input to farmers at prices that are "*in most cases lower than the Nairobi wholesale prices for similar products*",while Kenya Tea Development Agency Ltd supplies bags of fertilizer at a price "*significantly lower than the wholesale price in Nairobi and much lower than the retail price offered to the smallholders by the village-level stockists*" (IFAD, 2003). Sometimes, input is even given for free (Koo, 2011; IFAD, 2003).

It can be easily seen, as it is done in Gangopadhyay and Sengupta (1987), that, if there was no difference between the trader and the farmer (i.e. $\tau = \theta$), the optimal contract would be characterized by $\eta_D(x) = \delta_D(x) = 1$, and the role of the trader would be irrelevant. If he has no cost advantage, the trader is not able to organize the production in a more efficient way than farmers do.

Proposition 3. ***Under spatial price discrimination,*** $p_D(x) < p - \tau x$ ***and*** $i_D(x) < i$***: each farmer "loses" on the output trading and "gains" on the input trading.***

Proof of proposition 3: From (3.10), $p_D(x) < p - \tau x$ if

$$f(k^0(x)) - \frac{i}{p - \tau x} k^0(x) < f(k^*(x)) - \frac{i}{p - \theta r - \tau(x - r)} k^*(x)$$

From (2.2), (2.5) and proposition 1, this is equivalent to

$$f(k^0(x)) - \left.\frac{df}{dk}\right|_{k(x)=k^0(x)} k^0(x) < f(k^{\#}(x)) - \left.\frac{df}{dk}\right|_{k(x)=k^{\#}(x)} k^{\#}(x)$$

This is true provided that the production elasticity $\frac{df}{dk}\frac{k}{f(k)}$ is constant or decreasing in k. The result $i_D(x) < i$ follows from proposition 2. □

When involved in the interlinked transaction, each farmer receives a price for the output which is lower than the net price he would have received in the stand alone situation. This "loss" on the output trading is compensated by a "gain" on the input trading, such that, as proposition 1 states, each farmer gets an income $y(x)$ from the contract which is exactly equal to his reservation income $y^0(x)$.

The results show that farmers are treated differently depending on their location. On the one hand, farmers located far from the market receive a lower price for their output, but on the other hand they also pay a lower price for input. Moreover, those farmers receive a smaller share of the net price received by the trader on the market for the output and pay a lower part of the input price. Indeed, from (3.10) and (3.11), it can be shown[8] that $p_D(x)$, $i_D(x)$, and $\eta_D(x) = \delta_D(x)$ are decreasing in x. Contract prices $p_D(x)$ and $i_D(x)$ are increasing with the output market price p. An illustration of this is Strohm and Hoeffler (2006) who report that Deepa Industries in Kenya paid a higher price to potatoes producers than originally agreed because the market price had risen. We also have that $\eta_D(x) = \delta_D(x)$ increase with p which means that trader's mark-up on the output and discount on the input are lower when p is higher.

[8] First derivative of $\eta_D(x)$ with respect to x is negative if $f(k^*(x))[(p - \tau x) f(k^0(x)) - ik^0(x)] < f(k^0(x))[(p - \theta r - \tau(x-r)) f(k^*(x)) - ik^*(x)]$. As $\theta < \tau$, a sufficient condition is $k^0(x)/f(k^0(x)) > k^*(x)/f(k^*(x))$ which is ensured by the concavity of the production function and the fact that $k^*(x) > k^0(x)$ from proposition 1. As $\eta_D(x)$ is decreasing in x, it follows that $p_D(x)$ and $i_D(x)$ are also decreasing in x since $\frac{\partial p_D(x)}{\partial x} = \frac{\partial \eta_D(x)}{\partial x}(p - \theta r - \tau(x - r)) - \tau \eta_D(x) < 0$ and $\frac{\partial i_D(x)}{\partial x} = \frac{\partial \eta_D(x)}{\partial x} i < 0$.

4 Uniform pricing

Under uniform pricing policy, the trader is constrained to propose the same contract (p_U, i_U) to all farmers (where p_U and i_U are independent of x). Each farmer can individually accept or refuse the contract proposed.

The trader's problem can be written as:

$$\max_{p_U,i_U} \Pi = \int_r^{r+R} (p - \theta r - \tau(x-r))f(k^*) - ik^* - (p_U f(k^*) - i_U k^*)dx$$

$$\text{s.t. } g(x) \equiv p_U f(k^*) - i_U k^* - y^0(x) \geq 0 \ \forall x$$

Note that k^* is the same for all farmers, independent of their location (see (2.2) where $p_F(x) = p_U$ and $i_F(x) = i_U$ are independent of x). As farmers are distributed on the interval $[r, r+R]$, there is a continuum of participation constraints $g(x)$ with $x \in [r, r+R]$. The satisfaction of the constraint for the first farmer (located at r) is sufficient to ensure that it is satisfied for all farmers located further (in $x \in]r, r+R]$). Indeed, as k^* is constant for all x and $y^0(x)$ is strictly decreasing in x, $g(x)$ is strictly increasing in x.

Thus, we can replace the continuum of constraints $g(x) \geq 0$ by the unique constraint $g(r) \geq 0$ (see for instance Bolton and Dewatripont, 2005: 82). The problem is now the following:

$$\max_{p_U,i_U} \Pi = R\left(\left(p - \theta r - \tau\frac{R}{2}\right) f(k^*) - ik^* - (p_U f(k^*) - i_U k^*)\right)$$

$$\text{s.t. } g(r) \equiv p_U f(k^*) - i_U k^* - y^0(r) \geq 0$$

The Lagrangian is given by:

$$\mathcal{L} = R\left(\left(p - \theta r - \tau\frac{R}{2}\right) f(k^*) - ik^*\right) + (\lambda - R)(p_U f(k^*) - i_U k^*) - \lambda y^0(r) \tag{4.1}$$

Noting that at equilibrium $\frac{df}{dk} = \frac{i_U}{p_U}$ and applying the envelop theorem to the income of the farmer, the Kuhn-Tucker conditions can be written as:

$$\frac{\partial \mathcal{L}}{\partial p_U} = R\left(\left(p - \theta r - \tau\frac{R}{2}\right)\frac{i_U}{p_U} - i\right)\frac{\partial k^*}{\partial p_U} + (\lambda - R) f(k^*) = 0 \tag{4.2}$$

$$\frac{\partial \mathcal{L}}{\partial i_U} = R\left(\left(p - \theta r - \tau\frac{R}{2}\right)\frac{i_U}{p_U} - i\right)\frac{\partial k^*}{\partial i_U} + (\lambda - R)(-k^*) = 0 \tag{4.3}$$

$$\lambda \geq 0; \ g(r) \geq 0; \ \lambda g(r) = 0 \tag{4.4}$$

From (4.3),

$$\lambda - R = R\left(\left(p - \theta r - \tau\frac{R}{2}\right)\frac{i_U}{p_U} - i\right)\frac{\partial k^*}{\partial i_U}\frac{1}{k^*} \tag{4.5}$$

Substituting (4.5) in (4.2) we have:

$$R\left(\left(p - \theta r - \tau\frac{R}{2}\right)\frac{i_U}{p_U} - i\right)\left(\frac{\partial k^*}{\partial p_U} + \frac{\partial k^*}{\partial i_U}\frac{f(k^*)}{k^*}\right) = 0 \tag{4.6}$$

As R is strictly positive, the first and/or second term between brackets has to be equal to zero. If the last term was equal to zero, it can be shown[9] that $y(x) = 0 \; \forall x$ so that $g(x) < 0 \; \forall x$, which contradicts (4.4). Thus, the first term has to be equal to zero, that is:

$$\frac{i_U}{p_U} = \frac{i}{p - \theta r - \tau \frac{R}{2}} \tag{4.7}$$

Equation (4.7) characterizes the optimal contract (p_U, i_U). This contract implies that each farmer receives the same income from the contract as the standalone income of the first farmer.

Proposition 4. ***Under uniform pricing, the profit maximizing contract is characterized by a binding individual rationality constraint for the first farmer (the one located in r). If*** $\tau r - \theta r > \tau(R/2)$***, the optimal choice of input is characterized by*** $k^*(x) > k^0(x)$ ***for all farmers. When*** $\tau r - \theta r \leq \tau(R/2)$***, the trader is not able to make positive profit.***

Proof of proposition 4: Substituting (4.7) in (4.5) gives $\lambda = R$. From (4.4), this implies that the individual rationality constraint is binding: $g(r) \equiv p_U f(k^*) - i_U k^* - y^0(r) = 0$. If $\tau r - \theta r > \tau(R/2)$, given that that $f(k)$ is strictly concave and using (2.2) with respectively $(p_F(x), i_F(x)) = (p_U, i_U)$ and $(p_F(x), i_F(x)) = (p - \tau x, i)$, we have that $k^* > k^0(x)$. If $\tau r - \theta r \leq \tau(R/2)$, we have that $k^* \leq k^0(x)$. Given that $g(r) = 0$, the profit is $\Pi = R\left(\left(p - \theta r - \tau\frac{R}{2}\right) f(k^*) - ik^* - [(p - \tau r) f(k^0(r)) - ik^0(r)]\right)$. From $k^* \leq k^0(r)$ and $\tau r \leq \theta r + \tau(R/2)$, we have that $\Pi \leq 0$. □

Contrary to the spatial price discrimination case, when the trader is able to operate profitably under uniform pricing, all the farmers except the first one see an increase in their income with respect to their stand alone situation. Higher income due to the contract is consistent with empirical evidence. Indeed, Warning and Key (2002) have estimated an increase in gross agricultural income of 207000 CFA for Senegalese peanut producers that have accepted a contract with "arachide de bouche". Similarly, Simons et al. (2005) have found that the contracts for seed corn in East Java and for broilers in Lombok made significant contributions to farmers' capital returns.

Substituting (4.7) in the binding participation constraint $g(r) = 0$ gives:

$$p_U = \left(p - \theta r - \tau \frac{R}{2}\right) \underbrace{\frac{(p - \tau r) f(k^0(r)) - ik^0(r)}{\left(p - \theta r - \tau\frac{R}{2}\right) f(k^*) - ik^*}}_{\eta_U} \tag{4.8}$$

$$i_U = i \underbrace{\frac{(p - \tau r) f(k^0(r)) - ik^0(r)}{\left(p - \theta r - \tau\frac{R}{2}\right) f(k^*) - ik^*}}_{\delta_U} \tag{4.9}$$

[9] Indeed, we have shown (see footnote 4) that $-\frac{\partial k^*(x)/\partial p_F}{\partial k^*(x)/\partial i_F} = \frac{f(k^*(x))}{k^*(x)}$, implies $y(x) = 0$.

Proposition 5. ***Under uniform pricing, when*** $\tau r - \theta r > \tau(R/2)$***, the profit maximizing contract is characterized by*** $p_U < p - \theta r - \tau(R/2)$***,*** $i_U < i$ ***and*** $\eta_U = \delta_U < 1$***: the trader "loses" on the input trading and "gains" on average on the output trading.***

Proof of proposition 5: Note, from (4.7) and (2.5), that $k^* = k^{\#}(r + (R/2))$. Thus, $\eta_U = \delta_U$ may be written as:

$$\eta_U = \delta_U = \frac{\max_k (p - \tau r) f(k) - ik}{\max_k (p - \theta r - \tau(R/2)) f(k) - ik}$$

Using the envelop theorem, $\tau r - \theta r > \tau(R/2)$ implies that $\eta_U = \delta_U < 1$. Using this result with (4.8) and (4.9) implies that $p_U < p - \theta r - \tau(R/2)$ and $i_U < i$. □

Propositions 4 and 5 imply that, only if there exists a *sufficient* difference in transport costs ($\tau r - \theta r > \tau(R/2)$), the trader is able to make positive profit. In this case, he "loses" on the input trading and "gains" on the output trading, as the average net price he receives on the market is higher than the price he pays each farmer, similarly to what happens in the spatial price discrimination case. However, if his cost advantage is too small, he is not able to profitably induce farmers to organize production in a more efficient way. This result is in contrast with the result obtained under price discrimination, where the trader is able to exploit his cost advantage, even if the advantage is very small.

As it was the case with spatial price discrimination, when the trader's cost advantage is large enough, contract prices under uniform pricing p_U and i_U are increasing with the output market price p. The same applies for $\eta_U = \delta_U$, which means that farmers receive a higher share of trader's gain on the output transaction, but pay a higher share of input price, when p is higher.

5 Mill pricing

Under a mill pricing policy, the trader pays the same mill price to all farmers. He has to propose the same contract (p_M, i_M) to all farmers (where p_M and i_M are independent of x) but farmers have to support the transport costs. Thus, the net price received by the farmer for the output is $p_F(x) = p_M - \tau(x - r)$.

From equations (2.3) and (2.4), the trader's problem may be written as:

$$\max_{p_M), i_M} \Pi = \int_r^{r+R} (p - \theta r - p_M) f(k^*(x)) + (i_M - i) k^*(x) dx$$

$$\text{s.t. } g(x) \equiv (p_M - \tau(x - r)) f(k^*(x)) - i_M k^*(x) - y^0(x) \geq 0 \ \forall x$$

As farmers are distributed on the interval $[r, r + R]$, there is a continuum of participation constraints $g(x)$ with $x \in [r, r+R]$. However, as shown by the following lemma, the satisfaction of the participation constraint for the the last farmer (located at $r + R$) is sufficient to ensure that the participation constraint is satisfied for all farmers.

Lemma 1. ***Under mill pricing, if the production function is homogeneous*** $g(r+R) \geq 0$ ***is sufficient to ensure that*** $g(x) \geq 0 \ \forall x$***.***

Proof of lemma 1: See Appendix A.

Using lemma 1, the problem can be written as:

$$\max_{p_M, i_M} \Pi = \int_r^{r+R} (p - \theta r - p_M) f(k^*(x)) + (i_M - i) k^*(x) dx$$

$$\text{s.t. } g(r+R) \equiv (p_M - \tau R) f(k^*(r+R)) - i_M k^*(r+R) - y^0(r+R) \geq 0$$

The Lagrangian is given by:

$$\mathcal{L} = \lambda \left((p_M - \tau R) f(k^*(r+R)) - i_M k^*(r+R) - y^0(r+R) \right) + \int_r^{r+R} (p - \theta r - p_M) f(k^*(x)) + (i_M - i) k^*(x) dx \tag{5.1}$$

Noting that at equilibrium $\frac{df}{dk} = \frac{i_M}{p_M - \tau(x-r)}$ and applying the envelop theorem to the income of the farmer, the Kuhn-Tucker conditions can be written as:

$$\frac{\partial \mathcal{L}}{\partial p_M} = \lambda f(k^*(r+R)) + \int_r^{r+R} \left((p - \theta r - p_M) \frac{i_M}{p_M - \tau(x-r)} + i_M - i \right) \frac{\partial k^*}{\partial p_M} - f(k^*(x)) dx = 0 \tag{5.2}$$

$$\frac{\partial \mathcal{L}}{\partial i_M} = -\lambda k^*(r+R) + \int_r^{r+R} \left((p - \theta r - p_M) \frac{i_M}{p_M - \tau(x-r)} + i_M - i \right) \frac{\partial k^*}{\partial i_M} + k^*(x) dx = 0 \tag{5.3}$$

$$\lambda \geq 0; \; g(r+R) \geq 0; \; \lambda g(r+R) = 0 \tag{5.4}$$

From (5.4), we have two possible cases: either last farmer's participation constraint is binding at the equilibrium ((p_M^{**}, i_M^{**}) is constrained), or, neither constraint is binding at the equilibrium ((p_M^*, i_M^*) is unconstrained).

Whether the optimum is constrained or unconstrained depends on the parameters of the model. In the following section, under some additional assumptions, we show how the output price p influences the optimal prices. In particular, we show that the outcome with mill pricing policy differs from other policies when the output price is very low.

5.1 Model with a specific production function

In what follows, we assume the trader has a sufficiently large transport cost advantage and we use a particular production function to derive some characteristics of the equilibrium.

Assumption 1. $\tau r - \theta r > \tau(R/2)$.

Assumption 2. $f(k) = 2\sqrt{k}$.

Proposition 6. ***Under mill pricing and assumptions 1 and 2, the profit maximizing contract is characterized by $i_M < i$. The trader "loses" on the input trading.***

Proof of proposition 6: See Appendix B.

Corollary 1. ***Under mill pricing and assumptions 1 and 2, the profit maximizing interlinked contract implies that each farmer increases the quantity of inputs he uses, and hence increases his production, compared to his stand alone alternative.***

Proof of corollary 1: The participation constraint has to be satisfied for all x. As the production function is homogeneous, that means $i_M k^*(x) - ik^0(x) \geq 0$ (see also appendix A). From proposition 6, $i_M < i$, which implies $k^*(x) > k^0(x)$ for the participation constraints to be satisfied. □

This result, as the one obtained under discriminatory and uniform pricing, is consistent with empirical evidences. In the Indian poultry sector, Ramaswami et al. (2006) have found that contract production is more efficient than noncontract one and that the efficiency surplus is largely appropriated by the processor. In Ethiopia, Tadesse and Guttormsen (2009) have estimated that producers of haricot bean who are in relational (interlinked) contract supply about 27% more than farmers in spot markets.

As under discrimination and uniform pricing policies, it can be shown that the price given to the farmer for his output is increasing with the market output price, whether the optimal contract is constrained or not (i.e. $dp^*_M/dp > 0$ and $dp^{**}_M/dp > 0$). However, in contrast to those policies, when mill pricing is characterized by an unconstrained optimum, the input price the farmer has to pay is decreasing with p (i.e. $di^*_M/dp < 0$) . Formal proofs are given in appendix C.1, C.2 and D.1.

Whether the optimum is constrained or unconstrained depends on the parameters of the model. However it is possible to establish some tendency. In particular, it can be shown that the constrained outcome (p^{**}_M, i^{**}_M) takes place when the market output price p is large and the unconstrained one (p^*_M, i^*_M) when p is small. This means that, when p is large enough, the last farmer participation constraint is binding and this producer is pushed down to his reservation income. However, when p is small, we can have a situation where all the farmers get a positive surplus with respect to their stand alone situation. In a context where agricultural output prices are often driven down by international competition, this result is particularly interesting. This means that when the output price p is very low, farmers located furthest from the market can benefit from contracting with a trader, even if he maximizes his profit. In contrast, under the two other pricing policies (discrimination or uniform pricing), there is always at least one farmer who is pushed down to his reservation income, for any p.

Proposition 7. ***Under mill pricing and assumptions 1 and 2, when*** p ***is large enough, the profit maximizing contract is characterized by*** $g(r+R) = 0$ ***and*** $g(x) > 0$ ***for*** $x \in [r, r+R[$ ***at the optimum. A sufficient condition for that is*** $p > 2\tau r + \tau R - \theta r$. ***When*** p ***is small, the contract may be characterized by*** $g(x) > 0 \ \forall x$ ***at the optimum.***

Proof of proposition 7: See Appendix E.

Appendix E shows that there exists a $\bar{p}$ such that the optimum is unconstrained for $p < \bar{p}$ and constrained for $p > \bar{p}$. From the sufficient condition for the optimum to be constrained, we know that $\bar{p} \leq 2\tau r + \tau R - \theta r < \infty$, that is, it is always possible to find a p such that the optimum is constrained. However, depending on other parameters it may be the case that $\bar{p} < \tau(r+R)$ which implies that the optimum is always constrained for the acceptable values of p.

6 The case of a non-profit trader

Most of the literature about spatial price policies and/or about interlinked transactions assumes that the trader is a profit maximizing firm. However in agriculture and livestock sector in developing countries, several local producers' associations and NGOs try to increase the income and welfare of producers. In order to take this into account, we look at the case in which the trader is a non-profit organization. When the trader defends the interest of farmers, the objective function can take different forms. Following the literature, we look at two different cases: a first case where the trader maximizes the sum of farmers's incomes and a second case where the trader maximizes total earnings (i.e. the sum of profit and farmers' income) assuming that the profit can be distributed to members (see for instance Royer, 2001).

6.1 Spatial price discrimination

When the trader maximizes total farmers' income, subject to a non-negative profit and participation of all farmers, his problem is the following:

$$\max_{p_D(x), i_D(x)} \; y(x) = p_D(x) f(k^*(x)) - i_D(x) k^*(x)$$

$$\text{s.t. } p_D(x) f(k^*(x)) - i_D(x) k^*(x) \geq y^0(x)$$

$$\text{and } (p - \theta r - \tau(x-r)) f(k^*(x)) - i k^*(x) - (p_D(x) f(k^*(x)) - i_D(x) k^*(x)) \geq 0$$

The Lagrangian is given by:

$$\mathcal{L} = \mu(x)\left((p - \theta r - \tau(x-r)) f(k^*(x)) - i k^*(x)\right) + (1 - \mu(x) + \lambda(x))(p_D(x) f(k^*(x)) - i_D(x) k^*(x))$$

Noting that at equilibrium $\frac{df}{dk} = \frac{i_D(x)}{p_D(x)}$ and applying the envelop theorem to the income of the farmer, the Kuhn-Tucker conditions can be written as:

$$\frac{\partial \mathcal{L}}{\partial p_D(x)} = \mu(x)\left((p - \theta r - \tau(x-r))\frac{i_D(x)}{p_D(x)} - i\right)\frac{\partial k^*(x)}{\partial p_D(x)} + (1 - \mu(x) + \lambda(x)) f(k^*(x)) = 0 \quad (6.1)$$

$$\frac{\partial \mathcal{L}}{\partial i_D(x)} = \mu(x)\left((p - \theta r - \tau(x-r))\frac{i_D(x)}{p_D(x)} - i\right)\frac{\partial k^*(x)}{\partial i_D(x)} + (1 - \mu(x) + \lambda(x))(-k^*(x)) = 0 \quad (6.2)$$

$$\lambda(x) \geq 0; \; g(x) \geq 0; \; \lambda(x) g(x) = 0 \quad (6.3)$$

$$\mu(x) \geq 0; \; \pi(x) \geq 0; \; \mu(x)\pi(x) = 0 \quad (6.4)$$

Optimal contract is characterized by $\mu(x) = 1$, $\lambda(x) = 0$ and:

$$\frac{i_D(x)}{p_D(x)} = \frac{i}{p - \theta r - \tau(x-r)} \quad (6.5)$$

This price ratio is the same as for the profit maximizing trader (see (3.9)). This implies that the optimal level of input chosen by each farmer $k^*(x)$ is the same, whether the trader is profit maximizer or not. The same applies for the output production level $f(k^*(x))$. Under this pricing policy, both non-profit and for-profit traders induce farmers to choose the efficient level of inputs. This means that maximizing farmers' income does not lead to any efficiency loss compared to maximizing profit.

From (6.4), and $\mu(x) = 1$, we know that, at the optimum, the profit is equal to zero (the constraint on profit is binding). Substituting (6.5) in $\pi(x) = 0$ gives:

$$p_D(x) = p - \theta r - \tau(x - r); \quad i_D(x) = i$$

Comparing these expressions to the for-profit case ((3.10) and (3.11)), we see that when the trader is an income maximizer, he does not make any gain nor loss neither on the input nor on the output. The trader transfers the net prices he faces to the farmers.

It has to be noted that when the trader maximizes farmers's income under price discrimination, while he has the possibility of setting for different farmers different contracts which do not reflect the difference in transport costs, it is not optimal for him to do so. The optimal contracts for different farmers will reflect the differences in transport costs and hence are the same as the optimal contracts under mill pricing.

The price ratio (6.5) is the result of profit maximization or farmers' income maximization or even the maximization of the sum of profit and farmers' income. Indeed, assume a cooperative maximizes such a function. The problem is then the following:

$$\max_{p_D(x), i_D(x)} (p - \theta r - \tau(x - r))f(k^*(x)) - ik^*(x)$$

$$\text{s.t. } p_D(x)f(k^*(x)) - i_D(x)k^*(x) \geq y^0(x)$$

$$\text{and } (p - \theta r - \tau(x - r))f(k^*(x)) - ik^*(x) - (p_D(x)f(k^*(x)) - i_D(x)k^*(x)) \geq 0$$

The Lagrangian is given by (for ease of notation, we drop the argument (x)):

$$\mathcal{L} = (1 + \mu)\left((p - \theta r - \tau(x - r))f(k^*) - ik^*\right) + (\lambda - \mu)(p_D f(k^*) - i_D k^*) - \lambda y^0$$

The Kuhn-Tucker first-order conditions imply that $\lambda(x) = \mu(x) = 0$ and that the optimal contract is characterized by (6.5). As p_D and i_D enter the objective function only through k^* and not separately, there exists a continuum of p_D and i_D (which satisfy (6.5)) which maximizes this objective function. This leads us to the following proposition:

Proposition 8. ***Under spatial price discrimination, when total earnings are maximized, the optimal contract is defined by*** $(p_D(x), i_D(x)) = (\psi_D(x)(p - \theta r - \tau(x - r)), \psi_D(x)i)$ ***where*** $\psi_D(x) \in [\eta_D(x), 1]$***,*** $\eta_D(x)$ ***is defined in*** (3.10)***. If*** $\psi_D(x) = \eta_D(x)$***, then*** $g(x) = 0$ ***and the profit is maximized. If*** $\psi_D(x) = 1$***, then*** $\pi(x) = 0$ ***and farmer's income is maximized.***

Whether the trader maximizes profit, farmers' income or total earnings, the optimal contract is always defined by the same price ratio, implying that the first-best outcome is reached. Only the distribution of the efficiency gain between agents is different. Profit maximization and income maximization can be seen as two particular cases of the total earnings maximization. If profit is maximized, the trader acquires all the efficiency gain, while if income is maximized, it is acquired by the farmers.

6.2 Uniform pricing

Under uniform pricing, as k^* is independent of x, the trader's problem, when he maximizes total farmers' income subject to non-negative profit and participation of all farmers, is the following:

$$\max_{p_U, i_U} R(p_U f(k^*) - i_U k^*)$$

$$\text{s.t } p_U f(k^*) - i_U k^* \geq y^0(r)$$

$$\text{and } R\left(\left(p - \theta r - \tau \frac{R}{2}\right) f(k^*) - ik^* - (p_U f(k^*) - i_U k^*)\right) \geq 0$$

The Lagrangian is given by:

$$\mathcal{L} = \mu R\left(\left(p - \theta r - \tau \frac{R}{2}\right) f(k^*) - ik^*\right) + (R - \mu R + \lambda)(p_U f(k^*) - i_U k^*)$$

Noting that at equilibrium $\frac{df}{dk} = \frac{i_U}{p_U}$ and applying the envelop theorem to the income of the farmer, the Kuhn-Tucker conditions can be written as:

$$\frac{\partial \mathcal{L}}{\partial p_U} = \mu R\left(\left(p - \theta r - \tau \frac{R}{2}\right) \frac{i_U}{p_U} - i\right) \frac{\partial k^*}{\partial p_U} + (R - \mu R + \lambda) f(k^*) = 0 \tag{6.6}$$

$$\frac{\partial \mathcal{L}}{\partial i_U} = \mu R\left(\left(p - \theta r - \tau \frac{R}{2}\right) \frac{i_U}{p_U} - i\right) \frac{\partial k^*}{\partial i_U} + (R - \mu R + \lambda)(-k^*) = 0 \tag{6.7}$$

$$\lambda \geq 0;\ g(r) \geq 0;\ \lambda g(r) = 0 \tag{6.8}$$

$$\mu \geq 0;\ \Pi \geq 0;\ \mu \Pi = 0 \tag{6.9}$$

The optimal contract is characterized by $R - \mu R + \lambda = 0$ and:

$$\frac{i_U}{p_U} = \frac{i}{p - \theta r - \tau \frac{R}{2}} \tag{6.10}$$

This price ratio is the same as for the profit maximizing trader (see (4.7)). This implies that the optimal level of input chosen by farmers, k^*, is the same, whether the trader is a profit maximizer or an income maximizer. Hence, as it was the case for the for-profit, if he has a sufficient cost advantage, the income maximizer trader is able to induce each farmer to increase his production with respect to his stand-alone alternative.

From (6.8), (6.9) and $R - \mu R + \lambda = 0$, we know that, an optimum exists only if $\tau r - \theta r \geq \tau(R/2)$. At the optimum, the profit is equal to zero (the constraint on profit is binding). Substituting (6.10) in $\Pi = 0$ gives:

$$p_U = p - \theta r - \tau(R/2); \quad i_U = i$$

Comparing these expressions to the for-profit case ((4.8) and (4.9)), we see that when the trader is an income maximizer, he does not make any gain nor loss neither on the input nor on the output.

It has to be noted that with those prices, when $\tau r - \theta r < \tau(R/2)$ the trader is not able to induce participation of the first farmers. Only if the difference in transport cost is large enough all farmers accept the contract proposed by the income-maximizer trader.

Here again, the price ratio (6.10) also corresponds to total earnings maximization. Indeed, assuming a cooperative maximizes the sum of profit and farmers' income, the problem is the following:

$$\max_{p_U, i_U} R\left(\left(p - \theta r - \tau \frac{R}{2}\right) f(k^*) - ik^*\right)$$

$$\text{s.t } p_U f(k^*) - i_U k^* \geq y^0(r)$$

$$\text{and } R\left(\left(p - \theta r - \tau \frac{R}{2}\right) f(k^*) - ik^* - (p_U f(k^*) - i_U k^*)\right) \geq 0$$

The Lagrangian is given by:

$$\mathcal{L} = (1+\mu)R\left(\left(p - \theta r - \tau \frac{R}{2}\right) f(k^*) - ik^*\right) + (\lambda - \mu)(p_U f(k^*) - i_U k^*) - \lambda y^0(r)$$

The Kuhn-Tucker first-order conditions imply that $\lambda = \mu$ and that the optimal contract is characterized by (6.10). As p_U and i_U enter the objective function only through k^* and not separately, there exists a continuum of p_U and i_U (which satisfy (6.10)) which maximizes this objective function. This leads us to the following proposition:

Proposition 9. ***Under uniform pricing and assumption 1, when total earnings are maximized, the optimal contract is defined by*** $(p_U, i_U) = (\psi_U(p - \theta r - \tau(R/2)), \psi_U i)$ ***where*** $\psi_U \in [\eta_U, 1]$***,*** η_U ***is defined in*** (4.8)***. If*** $\psi_U = \eta_U$***, then*** $g(r) = 0$ ***and the profit is maximized. If*** $\psi_U = 1$***, then*** $\Pi = 0$ ***and total farmers' income is maximized.***

6.3 Mill pricing

When the trader maximizes total farmers' income subject to non-negative profit and participation of all farmers, his problem is the following[10]:

$$\max_{p_M, i_M} \int_r^{r+R} ((p_M - \tau(x-r))f(k^*(x)) - i_M k^*(x))dx$$

$$\text{s.t. } \int_r^{r+R} (p - \theta r - \tau(x-r))f(k^*(x)) - ik^*(x) - ((p_M - \tau(x-r))f(k^*(x)) - i_M k^*(x))dx \geq 0$$

$$\text{and } p_M f(k^*(r)) - i_M k^*(r) \geq y^0(r)$$

$$\text{and } (p_M - \tau R)f(k^*(r+R)) - i_M k^*(r+R) \geq y^0(r+R)$$

[10]Under the assumptions needed for both extreme participation constraints to be sufficient to ensure that participation constraint is satisfied for all farmers (see appendix A).

As it was shown that under spatial price discrimination the optimal pricing policy is a mill pricing policy, the result regarding the optimal mill pricing one is immediate:

$$p_D(x) = p - \theta r - \tau(x-r) = p_F(x) = p_M - \tau(x-r) \Leftrightarrow p_M = p - \theta r$$

$$i_D(x) = i = i_F(x) = i_M \Leftrightarrow i_M = i$$

$$\Pi = 0$$

Proposition 10. ***Under mill pricing, when farmers' income is maximized, the optimal contract is characterized by*** $p_M = p - \theta r$ ***and*** $i_M = i$***. This equilibrium is equivalent to the one obtained from spatial price discrimination, in terms of farmer's income as well as input choice and output production.***

Contrary to the two other pricing policies, under mill pricing, the optimal contract will be different depending on whether the trader maximizes farmers' income or profit. Only income maximization corresponds to total earnings maximization. Indeed, assuming a cooperative maximizes the sum of profit and farmers' income, the problem is the following[11]:

$$\max_{p_M, i_M} W = \int_r^{r+R} (p - \theta r - \tau(x-r))f(k^*(x)) - ik^*(x)dx$$

$$\text{s.t.} \int_r^{r+R} (p - \theta r - \tau(x-r))f(k^*(x)) - ik^*(x) - ((p_M - \tau(x-r))f(k^*(x)) - i_M k^*(x))dx \geq 0$$

$$\text{and } p_M f(k^*(r)) - i_M k^*(r) \geq y^0(r)$$

$$\text{and } (p_M - \tau R)f(k^*(r+R)) - i_M k^*(r+R) \geq y^0(r+R)$$

The Lagrangian is given by:

$$\begin{aligned}
\mathcal{L} = &(1+\mu)\int_r^{r+R} \left((p - \theta r - \tau(x-r))f(k^*(x)) - ik^*(x)\right) dx \\
&- \mu \int_r^{r+R} ((p_M - \tau(x-r))f(k^*(x)) - i_M k^*(x))dx \\
&+ \lambda_1(p_M f(k^*(r)) - i_M k^*(r) - y^0(r)) + \lambda_2((p_M - \tau R)f(k^*(r+R)) - i_M k^*(r+R) - y^0(r+R))
\end{aligned}$$

From the Kuhn-Tucker first-order conditions, either $\Pi = 0$ or $\mu = 0$. In the first case, the problem is similar to the total farmers's income maximization case; while in the second, it is similar to profit maximization. As the first case is similar to non-profit spatial price discrimination, which has been shown to provide the highest sum of profit and farmers' income, the first case is preferred to the latter one. Contrary to other policies, under mill pricing, only

[11] Under the assumptions needed for both extreme participation constraints to be sufficient to ensure that participation constraint is satisfied for all farmers (see appendix A).

one contract leads to maximized total earnings and this contract corresponds to farmers' income maximization.

While imposing a mill pricing policy to a for-profit trader leads to a loss of efficiency, this is not the case for a non-profit trader. Indeed, the latter induces farmers to produce efficiently, and all the efficiency gain (compared to the stand alone farmers' situation) is acquired by the farmers.

7 Policy implications

As explained before, remote farmers in developing countries are characterized by lower production, input use and income than less isolated ones. Improving their living conditions may contribute to reducing rural poverty and boost socio-economic development in rural areas. In this context, we look at the measures that can be adopted by policy makers to increase farmers' production, input use and income. Similarly, we look at the conditions an external donor should impose to the intermediaries he promotes, when his aim is reducing rural poverty.

It has been shown that, whatever the pricing policy chosen, the use of an interlinked contract by an intermediary who has a sufficient transport cost advantage incites each farmer to increase the level of input he uses, compared to what he used in the stand-alone case and hence to increase his production. However, farmers are not always able to improve their income, as all the efficiency gain can be acquired by the trader. A policy maker or an external donor may use appropriate regulations on the spatial pricing policy to be used in the interlinked contract in order to improve farmers' livelihoods.

Particular attention has to be given to the observed diversity regarding the nature of the intermediaries. As policy recommendations are different for non-profit traders and for-profit traders, we analyze them separately.

7.1 For-profit trader

If discrimination is possible and costless, in a laissez-faire situation, the for-profit trader will certainly choose to discriminate, as it leads to the highest profit. In this situation, first-best efficient optimum is reached. However, no farmer's poverty is reduced, as they all get the same income as in their stand-alone initial situation. While the presence of a trader who has a transport cost advantage is beneficial from an efficiency point of view, it is not from a poverty reduction one.

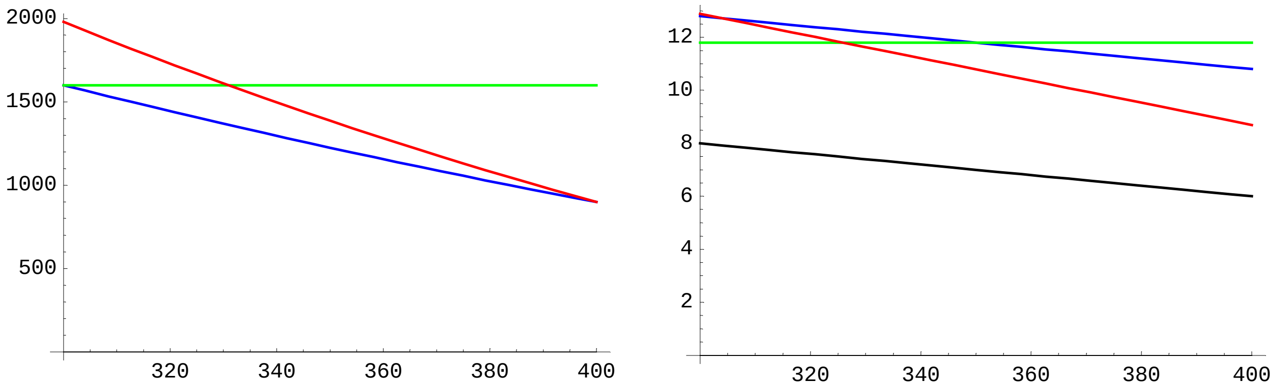
A policy maker whose aim is to reduce rural poverty may want to tax trader's profit in order to distribute it among farmers. However, it is possible that the public authorities in some developing countries do not have the power of doing so. In what follows, we assume the policy maker is only able to impose a pricing policy.

If the trader's transport cost advantage is large enough, imposing uniform pricing allows to reduce poverty amongst the poorest farmers, while richer ones are not worse off. Indeed, under this policy, only the farmer the closest to the market, that is, the one who has the highest initial income, is not able to increase his revenue. All the others are able to obtain a positive surplus from the contract, and hence to increase their income. Equality among farmers is ensured, as they all receive the same income and produce the same quantity. However, if the difference in transport cost between the trader and the farmers is small, imposing uniform pricing does not allow the trader to make a positive profit and to exploit his cost advantage to increase production.

Assuming that trader has a sufficiently large cost advantage, imposing him to use mill pricing also increases the revenue of most of the farmers. But, contrary to the uniform pricing, farmers far from the trader, who were already poor, gain less than the one close to the trader. Mill pricing increases inequality among farmers, with respect to their stand alone situation, but also with respect to a situation where the trader is allowed to spatially discriminate.

Figure 1: Comparison of spatial pricing policies

(a) Farmer's income $y(x)$ (b) Output produced $f(k(x))$



* Blue: spatial price discrimination, green: uniform pricing, red: mill pricing, black: stand-alone alternative. Note that income under spatial price discrimination is equivalent to reservation income $y^0(x)$. Choice of the parameters: $r = 300$, $R = 100$, $p = 700$, $\tau = 1$, $i = 100$ and $\theta = 0.2$. Note that parameters are such that the mill pricing contract is constrained, indeed the sufficient condition (see proposition 7) $p > 2\tau r + \tau R - \theta r$ is satisfied.

The previous discussion is illustrated by figure 1, which represents the evolution of farmers' income and output produced with distance, under the three pricing policies. Both uniform and mill pricing policies have positive effects on the income of most of farmers. Hence, if the policy maker is concerned only by farmers' revenue, spatial price discrimination should be prohibited. The choice of the profit-maximizing trader among the two remaining policies is not obvious. As illustrated in Appendix F.4, the trader tends to prefer mill pricing when the output price p is large. However, when p is small, situations may occur where trader's profit is higher under uniform pricing. This is particularly true when r is large or $\tau - \theta$ is large, that is, when the trader has an important cost advantage compared to farmers.

Producers' organizations in developing countries and NGOs argue that prices for agricultural goods are too low and claim that they remain low due to "unfair" international competition caused by subsidized exports from industrialized countries. This is seen as one of the reasons which keeps small producers in poverty (see for instance Oxfam (2002) or CFSI (2007) on milk sector). In a context in which p is very low, imposing mill pricing may result in increasing all farmers' income, including the most distant one. Numerical simulations also show that, when p is small, mill pricing may be preferred to uniform pricing by a majority of farmers.[12] In the same way, the sum of all farmer's incomes may be higher under mill pricing. [13] If the policy-maker's objective is to choose a policy that increases farmers' total income and/or is preferred by the majority of them, then imposing mill pricing in a context of low output price is optimal. When the output price is large, however, uniform pricing is preferred by a majority of farmers and leads to a higher total farmers's income, but is never able to increase the income of all the farmers.

A policy maker may also be interested in improving social welfare, defined as the sum of total farmers' income and trader's profit. Whatever the pricing policy, it is given by:

$$W = \int_{r}^{r+R} (p - \theta r - \tau(x - r))f(k^*(x)) - ik^*(x)dx \tag{7.1}$$

This expression is maximized when $k^*(x)$ is equal to $k^{\#}(x)$ as defined by (2.5). From proposition 1, spatial price discrimination leads to the highest social welfare. Numerical simulations show that social welfare is higher under uniform pricing than mill pricing (see Appendix F.3).[14]

7.2 Non-profit trader

Whatever the pricing policy used by the non-profit trader, he uses his cost advantage to improve all the farmer's income, compared to their stand-alone initial revenue. As in the for-profit case, the non-profit intermediary who has a sufficiently large cost advantage incites all farmers to increase the quantity of input they use as well as their level of production. If if the difference in transport cost between the trader and the farmers is small, using a uniform pricing policy hampers the non-profit trader to use his advantage to increase farmers' income. Under mill pricing policy however, even if the trader has a small advantage, he is able to promote increased production and input use.

In a laissez-faire situation, it is optimal for the non-profit trader to use mill pricing. Even if the trader is allowed to price discriminate he will choose to charge prices which reflect the difference in transport costs. Hence, in this case mill pricing and discrimination pricing are equivalent. If the trader's advantage is large enough, imposing uniform pricing would serve the equality objective as in this case all farmers get the same income. Moreover, this policy allows to reduce relatively more poverty of the poorest (i.e. the most distant) farmers, compared to the less remote ones. With a mill pricing policy, most distant farmers receive a lower price for their output, hence get a lower income. However, the difference in the price received exactly

[12] That is, the median farmer located in $r + R/2$ has a higher income under mill than under uniform pricing, see appendix F.1.

[13] See appendix F.2.

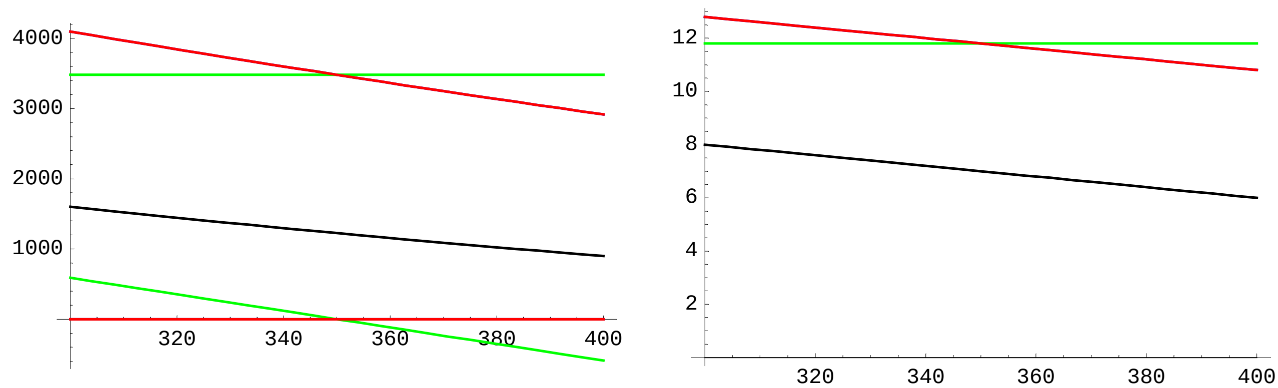
[14] One may argue that social welfare has to take into account transport costs, as they create a revenue for somebody. First, this is not necessarily the case as they are not always paid to some agent, but may represent a loss of time for the trader and/or the farmer. They may also consist in the loss of a fraction of the output, due for instance to perishability. Second, even if transport costs are taken into account in the social welfare function (i.e. $W' = \int_r^{r+R}(p - \theta r)f(k^*(x)) - ik^*(x)dx$), numerical simulations show that uniform pricing still dominates mill pricing (that is: $W'_U > W'_M$).

reflects the difference in transport cost, and each farmer receive the same additional income from the contract, compared to his initial income. This discussion is illustrated by figure 2.

Figure 2: Farmers' income maximization

(a) Farmer's income $y(x)$ (up) and trader's profit $\pi(x)$ (down)

(b) Output produced $f(k(x))$



* Green: uniform pricing, red: mill pricing and spatial price discrimination, black: stand-alone alternative. Choice of the parameters: $r = 300$, $R = 100$, $p = 700$, $\tau = 1$, $i = 100$ and $\theta = 0.2$.

From a social welfare perspective, when the trader is a non-profit trader mill pricing is preferred to uniform pricing since this leads to the highest social welfare. This implies that the policy recommendation regarding spatial pricing to be used depends crucially on the nature of the trader: in the case of a for-profit trader, the highest social welfare was shown to be reached under uniform pricing rather than mill pricing. Hence, if the policy maker is not able to distinguish the trader's type, or to impose different policies to different traders, no unambiguous policy recommendation regarding spatial pricing can be made. Regarding external aid donors, if the recipient of this aid is a non-profit organization, there should be no need to condition their aid to the use of a particular pricing given that the trader would adopt the pricing policy which maximizes social welfare. The situation is different if the recipient of this aid is for-profit. Indeed, it has been numerically shown that imposing uniform pricing contributes to a higher social welfare, compared to mill pricing policy.

8 Conclusions

In this paper, we develop a model of input-output interlinked contracts between a trader and geographically dispersed farmers, and analyze the welfare implications of the spatial pricing by this trader. We look at three different spatial price policies, namely spatial price discrimination, uniform pricing and mill pricing.

We assume an agricultural output market that is characterized by large transport costs. The intermediary has a (transport) cost advantage over the farmers from whom it buys their production. This cost difference leads to an input-output interlinked contract between the intermediary and the farmer. A first result is that the use of an interlinked contract by a trader who has a sufficient transport cost advantage leads to an increase of the farmer' production, independently of the type of pricing policy used by the intermediary.

If the intermediary is able to perfectly discriminate contracts between farmers, this would be his preferred option. This allows him to push all the farmers' incomes down to their stand-alone initial income and hence appropriate all the efficiency gain generated by the contract. If this is the case, the presence of the intermediary, while improving agricultural efficiency, does not directly help to reduce rural farmers poverty. In practice discriminatory pricing might not be feasible and other pricing policies exist, such as uniform pricing, where the trader bears the transport costs and concludes the same contract with all farmers, or mill pricing, where farmers are in charge of transport, and receive the same price at the mill. If the trader's cost advantage is large enough, we show that in both cases, most of the farmers get a positive surplus from the contract, while the trader is still able to make a profit. In the mill pricing case, under some conditions we can have a situation in which all farmers, including those located the furthest from the market, see an increase in their income.

We show that imposing a uniform pricing policy to the trader leads to a reduction of isolated farmers' poverty. Providing the same income to all farmers, uniform pricing favors relatively more isolated farmers, since they are the ones who initially receive a lower income. Moreover, it leads to a higher social welfare, defined as the sum of trader's profit and farmers' income, than mill pricing. When the output market price is large enough, uniform pricing is also preferred to mill pricing by a majority of farmers, and it leads to higher total farmers' income.

In developing countries, agricultural market prices are often driven down by international competition. If output market prices are very low, imposing mill pricing may be the optimal policy. Indeed, it may increase all farmers' income, including the closest and the most distant one. This is not possible under uniform pricing, whatever the output market price. Moreover, when the output market price is low, total farmers' income, as well as median farmer's income may be higher under mill than under uniform pricing.

Since, in developing countries, there are several examples of NGOs and farmers' associations setting up intermediaries, we also look at a situation in which the trader maximizes total farmers' income. For any pricing rule, such a trader is able to reach at least the same social welfare level as the for-profit does. We also show that it is optimal, from the non-profit firm's point of view, to use a mill pricing scheme, also implying that, in this case, social welfare is the highest under mill pricing. From this we can conclude that the optimal pricing policy depends on the nature of the trader. Given that both types of trader exists, a priori no policy recommendation regarding the type of pricing policy can be made.

Under mill pricing, maximizing social welfare implies that the profit of intermediary will be nil, while under the other pricing policies, its profit can be strictly positive. Under mill pricing, only one vector of prices is optimal for social welfare. Under discriminatory and uniform pricing, there is a continuum of prices which will maximize social welfare. Within this range, some prices favor more the farmers while others favor more the trader.

We also generalize the result found in Gangopadhyay and Sengupta (1987) that the trader has an interest in giving a discount to the farmer on the input price. If the trader's cost advantage is sufficiently large, this is true for all three pricing policies considered.

The model developed here gives potential avenues for future research. First, in certain cases, the choice of the size of the collection area may be important to the trader. In that case, rather than considering the number of farmers as being fixed, the number of participants may constitute a choice variable for the trader. A possible extension of the our model would consider how the number of suppliers is endogenously chosen. This would also allow to analyze the impact of pricing policy choice on the inclusion of isolated farmers in a collection area. Secondly, it may be interesting to see whether besides the pricing policies considered in this paper, other spatial pricing policies such as two part pricing would improve social welfare.

References

Ahmed, R. and Hossain, M. (1990), Developmental Impact of Rural Infrastructure in Bangladesh, *IFPRI Research Report*, No. 83, Washington DC.

Basu, K. (1983), The emergence of isolation and interlinkage in rural markets, *Oxford Economic Papers*, 35, pp. 262-280.

Basu, K., Bell, C. and Bose, P. (2000), Interlinkage, limited liability and strategic interaction, *Journal of Economic Behavior and Organization*, Vol. 42, pp. 445-462.

Bell, C. (1988), *Credit markets and interlinked transactions*, in Chenery, H.B. and Srinivasan, T.N. (eds.), *Handbook of Development Economics*, North-Holland, Amsterdam, pp. 763-830.

Binswanger, H., Khandker, S. and Rosenzweig, M. (1993), How Infrastructure and Financial Institutions Affect Agricultural Output and Investment in India, *Journal of Development Economics*, Vol. 41, pp. 337-366.

Braverman, A. and Stiglitz, J.E. (1982), Sharecropping and the interlinking of agrarian markets, *American Economic Review*, 72, pp. 695-715.

Bolton, P. and Dewatripont, M. (2005), *Contract theory*, MIT Press.

Bravo-Ortega, C. and Lederman, D. (2005), Agriculture and National Welfare around the World: Causality and International Heterogeneity since 1960, World Bank, *Policy Research Working Paper Series*, 3499.

Byerlee D., de Janvry A. and Sadoulet, E. (2009), Agriculture for Development: Toward a New Paradigm, *Annual Review of Resource Economics*, Vol. 1, No. 1, pp. 15-35.

CFSI (2007), *Campagne "Lait: l'Europe est vache avec l'Afrique"*, Communiqué de presse, February 2007, `http://www.cfsi.asso.fr/upload/cpdp20022007.pdf`.

Chakrabarty, D. and Chaudhuri, A. (2001), Formal and informal sector credit institutions and interlinkage, *Journal of Economic Behavior and Organization*, Vol. 46, pp. 313-325.

Chau, N.H, Goto, H. and Kanbur, R. (2009), Middlemen, Non-Profits and Poverty, *CEPR Discussion Paper*, No. 7459.

Chaudhuri, S. and Gupta, M.R. (1995), Price uncertainty and credit-product interlinkage: An extension of the analysis of Gangopadhyay and Sengupta, *Journal of International Trade and Economic Development*, 4(1), pp. 93-113.

Christiaensen, L., Demery, L. and Kuhl, J. (2011), The (Evolving) Role of Agriculture in Poverty Reduction: An Empirical Perspective, *Journal of Development Economics*, Vol 96, No 2, pp. 239-254.

De Schutter, O. (2010), *Report submitted by the Special Rapporteur on the right to food*, United Nations, General Assembly, A/HRC/16/49, `http://www.srfood.org/images/stories/pdf/officialreports/20110308_a-hrc-16-49_agroecology_en.pdf`.

Dia, D. (2002), *Le transport rural: une contrainte majeure au développement de la production laitière dans le département de Kolda*, Mémoire de DEA, Université Cheikh Anta Diop, Dakar.

FAFCHAMPS, M. and GABRE-MADHIN, E. (2006), Agricultural Markets in Benin and Malawi, *African Journal of Agricultural and Resource Economics*, Vol 1, No 1, pp 67-94.

FAO (2008), *The State of Food Insecurity in the World 2008: High food prices and food security - threats and opportunities*, Rome, FAO.

GANGOPADHYAY, S. and SENGUPTA, K. (1987), Small Farmers, Moneylenders and Trading Activity, *Oxford Economic Papers*, New Series, Vol. 39, No. 2, pp. 333-342.

GREENHUT, M.L., MAI, C.C. and NORMAN, G. (1986), Impacts on Optimum Location of Different Pricing Strategies, Market Structures and Customer Distributions over Space, *Regional Science and Urban Economics*, 16, pp. 329-351.

HOLLOWAY, G. et al. (2000), Agroindustrialization through institutional innovation: Transaction costs, cooperatives and milk-market development in the east-African highlands, *Agricultural Economics*, 23, pp. 279-288.

HSU, S.K. (1997), The Agro-industry: A Neglected Aspect of the Location Theory of Manufacturing, *Journal of Regional Science*, 37(2), pp. 259-274.

IFAD (2003), *Agricultural Marketing Companies as Sources of Smallholder Credit in Eastern and Southern Africa: Experiences, Insights and Potential Donor Role*, International Fund for Agricultural Development.

JACOBY, H.G. (2000), Access to Markets and the Benefits of Rural Roads, *The Economic Journal*, 110 (July), pp. 713-737.

JACOBY, H.G. and MINTEN, B. (2009), On measuring the benefits of lower transport costs, *Journal of Development Economics*, 89, pp. 28-38.

JAYNE T.S., YAMANO, T. and NYORO, J. (2004), Interlinked credit and farm intensification: evidence from Kenya, *Agricultural Economics*, 31, pp. 209-218.

JAYNE et al. (2003), Smallholder income and land distribution in Africa: implications for poverty reduction strategies, *Food Policy*, 28, pp. 253-275.

KEY, N. and RUNSTEN, D. (1999), Contract Farming, Smallholders, and Rural Development in Latin America: The Organization of Agroprocessing Firms and the Scale of Outgrower Production, *World Development*, Vol. 27, No. 2, pp. 381-401.

KOO, H., HUANG, C. and KAN, K. (2011), Interlinked Contracts: An Empirical Study, *Economica*, forthcoming.

LEFÈVRE, M. and THARAKAN, J. (2011), Can contract farming improve smallholder farmers' participation to the market?, mimeo.

LIGON, E. and SADOULET, E. (2008), *Estimating the Effects of Aggregate Agricultural Growth on the Distribution of Expenditures*, Background Paper for the World Development Report 2008.

MCMILLAN, M.S., WELCH, K. and RODRIK, D. (2003), When Economic Reform Goes Wrong: Cashew in Mozambique, *Brookings Trade Forum*, pp. 97-151.

MITRA, P. (1983), A theory of interlinked rural transactions, *Journal of Public Economics*, 20, pp.169-191.

MOTIRAM, S. and ROBINSON, J.A. (2010), Interlinking and Collusion, *Review of Development Economics*, 14(2), pp. 282-301.

OXFAM (2002), Milking the CAP, How Europe's dairy regime is devastating livelihoods in the developing world, *Oxfam briefing paper*, No 34.

POKHREL, D.M. and THAPA, G.B. (2007), Are marketing intermediaries exploiting mountain farmers in Nepal? A study based on market price, marketing margin and income distribution analyses, *Agricultural Systems*, No 94, pp. 151-164.

RAMASWAMI, R., BIRTHAL, P.S. and JOSHI, P.K (2006), Efficiency and Distribution in Contract Farming: The Case of Indian Poultry Growers, *MTDI Discussion paper*, No 91, International Food Policy Research Institute (IFPRI).

RAVALLION, M., CHEN, S. and SANGRAULA (2007), New Evidence on the Urbanization of Global Poverty, *World Bank Policy Research Working Paper*, 4199.

REARDON, T. and BARRETT, C. B. (2000), Agroindustrialization, globalization, and international development An overview of issues, patterns, and determinants, *Agricultural Economics*, 23, pp. 195-205.

ROBERTS, P., KC, S. and RASTOGI, C. (2006), Rural Access Index: A Key Development Indicator, *Transport paper*, TP-10, The World Bank.

ROYER, J. S. (2001), Agricultural Marketing Cooperatives, Allocative Efficiency, and Corporate Taxation, *Journal of Cooperatives*, Vol. 16, pp. 1-13.

SIMMONS, P., WINTERS, P. and PATRICKS, I. (2005), An analysis of contract farming in East Java, Bali, and Lombok, Indonesia, *Agricultural Economics*, 33, supplement, pp. 513-525.

STAAL, S. J. et al. (2002), Location and uptake: integrated household and GIS analysis of technology adoption and land use, with application to smallholder dairy farms in Kenya, *Agricultural Economics*, 27, pp. 295-315.

STIFEL, D. and MINTEN, B. (2008), Isolation and agricultural productivity, *Agricultural Economics*, 39, pp. 1-15.

STROHM, K. and HOEFFLER, H. (2006), Contract farming in Kenya: Theory, Evidence from selected Value Chains, and Implications for Development Cooperation, Report prepared for PSDA and the Contract Farming Task Force in the Ministry of Agriculture, Kenya.

TADESSE, G. and GUTTORMSEN, A. (2009), Commercializing smallholder's through interlinked contracts: prospects and challenges in the rift valleys of Ethiopia, Paper presented at the Nordic Conference in Development Economics, Oscarsborg, Drøbak, June 18-19, 2009.

UNITED NATIONS (2010), *The Millennium Development Goals Report*, United Nations, New York.

VON BRAUN, J., SWAMINATHAN, M. S. and ROSEGRANT, M. W. (2004), *Agriculture, food security, nutrition and the Millenium Development Goals*, International Food Policy Research Institute (IFPRI), Washington DC.

WARNING, M. and KEY, N. (2002), The Social Performance and Distributional Consequences of Contract Farming: An Equilibrium Analysis of the Arachide de Bouche Program in Senegal, *World Development*, Vol. 30, No. 2, pp. 255-263.

WORLD BANK (2008), *World development report 2008, Agriculture for Development*, The World Bank, `http://siteresources.worldbank.org/INTWDR2008/Resources/WDR_00_book.pdf`.

Appendices

A Proof of lemma 1

Using the envelop theorem, we have for a participation constraint at location x

$$\frac{\partial g(x,p_M,i_M)}{\partial x} = -\tau\left(f(k^*(x)) - f(k^0(x))\right) \lesseqqgtr 0 \Leftrightarrow k^*(x) \gtreqqless k^0(x) \qquad \text{(A.1)}$$

If the production function is homogeneous of degree h, then, using Euler's theorem, the farmer's income is given by $y(x) = i_M k^*(x)\left(\frac{1}{h}-1\right)$. Farmer's alternative income is given by $y^0(x) = ik^0(x)\left(\frac{1}{h}-1\right)$. A participation constraint at $\tilde{x}$ which is binding for a (p_M, i_M) i.e. $g(\tilde{x}, p_M, i_M) = 0$ implies $i_M k^*(\tilde{x}) - ik^0(\tilde{x}) = 0$ or equivalently $k^*(\tilde{x}) = \frac{i}{i_M}k^0(\tilde{x})$.

(i) If $i_M \leq i$, we have $\left.\frac{\partial g(x,p_M,i_M)}{\partial x}\right|_{x=\tilde{x}} \leq 0$. This means that $g(r+R, p_M, i_M) \geq 0$ implies $g(x, p_M, i_M) \geq 0$ for all $x \in [r; r+R]$. On the contrary if $i_M > i$, then we have $\left.\frac{\partial g(x,p_M,i_M)}{\partial x}\right|_{x=\tilde{x}} > 0$ which means that $g(r, p_M, i_M) \geq 0$ implies $g(x, p_M, i_M) \geq 0$ for all $x \in [r; r+R]$. The couple of constraints $g(r) \geq 0$ and $g(r+R) \geq 0$ is thus sufficient to ensure that $g(x) \geq 0\ \forall x$.

(ii) Suppose $g(r) = 0$ at the optimum. For other constraints to be satisfied, this implies $\left.\frac{\partial g(x,p_M,i_M)}{\partial x}\right|_{x=r} \geq 0$. Suppose also that $g(r+R) > 0$. Thus $\left.\frac{\partial g(x,p_M,i_M)}{\partial x}\right|_{x=r} > 0$ and, from (i), $i_M > i$ (otherwise $\left.\frac{\partial g(x,p_M,i_M)}{\partial x}\right|_{x=\tilde{x}}$ would be negative and $\tilde{x}$ could not be equal to r). From (A.1), this would imply $k^*(r) < k^0(r)$. As production function is concave, using (2.2), it would imply $\frac{i_M}{p_M} > \frac{i}{p-\tau r}$ or $\frac{p-\tau r}{i} < \frac{p_M}{i_M}$. Subtracting $\frac{\tau(x-r)}{i}$ on both sides and given that $i_M > i$, this would give $\frac{p-\tau x}{i} < \frac{p_M - \tau(x-r)}{i_M}$, thus $k^*(x) < k^0(x)\ \forall x$. Trader's profit would be given by:

$$\Pi = \int_r^{r+R} (p - \theta r - \tau(x-r) f(k^*(x)) - ik^*(x) - y(x) dx \qquad \text{(A.2)}$$

The trader could always increase his profit by replicating farmers' stand alone situations (that is, proposing a contract where $p_M = p - \tau r$ and $i_M = i$, each farmer using exactly $k^0(x)$ and getting his reservation income $y^0(x)$). In this case the profit is given by $\int_r^{r+R}(p - \theta r - \tau(x - r)f(k^0(x)) - ik^0(x) - y^0(x)dx$. This is always higher than (A.2). Indeed, from participation constraints, $y(x) \geq y^0(x)$, and, given our assumptions on $f(k)$, the function $(p - \theta r - \tau(x - r))f(k(x)) - ik(x)$ is concave in $k(x)$ and maximized in $k^{\#}(x)$ defined by (2.5). Comparing with (2.2) we see that $k^{\#}(x) > k^0(x)$. Thus, $k^{\#}(x) > k^0(x) > k^*(x)$, implying that $k^0(x)$ and $k^*(x)$ lie in the increasing part of the function, thus $(p - \theta r - \tau(x - r)f(k^0(x)) - ik^0(x) > (p - \theta r - \tau(x - r)f(k^*(x)) - ik^*(x) \quad \forall x$.

As trader's profit could always be increased, the situation where $g(r) = 0$ and $g(r+R) > 0$ cannot characterize the optimum. The only way for first farmer's participation constraint to be binding is that all farmers' participation constraint are binding. As first farmer's participation constraint is never the only one to be binding at the equilibrium, it can be said that $g(r+R) \geq 0$ is sufficient to ensure that $g(r) \geq 0$.

From (i) and (ii), $g(r+R) \geq 0$ is sufficient to ensure that $g(x) \geq 0\ \forall x$. □

B Proof of proposition 6

Suppose the solution to trader's problem is characterized by $g(r) = 0$. From Appendix A (ii), this implies $g(x) = 0\ \forall x$ and $p_M = p - \tau r$ and $i_M = i$. Isolating λ in (5.3) and substituting in (5.2), when $f(k) = 2\sqrt{k}$, then replacing p_M by $p - \tau r$ and i_M by i gives:

$$i = i\frac{6(p-\tau r)^2 - 3(p-\tau r)\tau R + \tau^2 R^2}{6(p-\theta r - \tau R)(p-\tau r) + 2\tau^2 R^2}$$
$$\Leftrightarrow 6(p-\tau r)(\theta r - \tau r + \tau(R/2)) = \tau^2 R^2$$

Under assumption 1, this is impossible as $\theta r - \tau r + \tau(R/2) < 0$ while other terms are strictly positive. The supposition that $g(r) = 0$ is absurd, thus $g(r) > 0$.

From Appendix A (i), $g(r) > 0$ and $g(r+R) \geq 0$ imply $g(x) > 0$ for $x \in [r, r+R[$. Optimum is either (p_M^{**}, i_M^{**}) where $g(r+R) = 0$ and $g(x) > 0$ for $x \in [r, r+R[$, either (p_M^*, i_M^*) where $g(x) > 0$ for $x \in [r, r+R]$.

Knowing that $g(r+R) = 0$ at the constrained optimum (p_M^{**}, i_M^{**}), for other constraints to be satisfied, this implies $\left.\frac{\partial g(x,p_M,i_M)}{\partial x}\right|_{x=r+R} < 0$. Then, by (A.1), $i_M^{**} < i$.

At the unconstrained optimum (p_M^*, i_M^*) we have that $\frac{\partial i_M^{FOCp_M}(p_M)}{\partial p_M} \geq \frac{\partial i_M^{FOCi_M}(p_M)}{\partial p_M}$ where $i_M^{FOCp_M}(p_M)$ (resp. $i_M^{FOCi_M}(p_M)$) is implicitly given by $\frac{\partial \Pi}{\partial p_M} = 0$ (resp. $\frac{\partial \Pi}{\partial i_M} = 0$). This is the consequence of the second order condition for a maximum, $\frac{\partial^2 \Pi}{\partial p_M^2}\frac{\partial^2 \Pi}{\partial i_M^2} - \left(\frac{\partial^2 \Pi}{\partial p_M \partial i_M}\right)^2 \geq 0$, and using the implicit function theorem, $\frac{\partial i_M^{FOCp_M}}{\partial p_M} = -\frac{\partial^2\Pi/\partial p_M^2}{\partial^2\Pi/\partial p_M \partial i_M}$ and $\frac{\partial i_M^{FOCi_M}}{\partial p_M} = -\frac{\partial^2\Pi/\partial p_M \partial i_M}{\partial^2\Pi/\partial i_M^2}$. This implies that all points (p_M, i_M) on $\frac{\partial \Pi}{\partial p_M} = 0$ with $i_M > i_M^*$ will lead to $\frac{\partial \Pi}{\partial i_M} < 0$.

Under assumption 2, at point (p_M', i), where $p_M' = \frac{1}{2}(p - \theta r + \tau(R/2))$ and is the solution to $\frac{\partial \Pi}{\partial p_M} = 0$ when $i_M = i$, we have that $\frac{\partial \Pi}{\partial i_M} = \frac{R}{4i^2}\left((p - \theta r + \frac{\tau R}{12})\tau R - (p-\theta r)^2\right)$. This is negative if $p > \theta r + \frac{(3+2\sqrt{3})}{6}\tau R$ which is guaranteed by our assumptions $p > \tau r + \tau R$ and $\tau r - \theta r > \tau R/2$. As a consequence $i > i_M^*$. $\square$

C Mill pricing: unconstrained optimum

In the unconstrained case $(p_M^*, i_M^*))$, $\lambda = 0$. Replacing in (5.2) and (5.3) when $f(k) = 2\sqrt{k}$ and simplifying gives:

$$(p - \theta r - p_M) - \frac{i}{i_M}\left(p_M - \tau\frac{R}{2}\right) = 0 \tag{C.1}$$

$$(p - \theta r - p_M)\left(p_M - \tau\frac{R}{2}\right) + \left(\frac{1}{2} - \frac{i}{i_M}\right)\left(p_M^2 - p_M\tau R + \frac{\tau^2 R^2}{3}\right) = 0 \tag{C.2}$$

C.1 Proof of $0 < dp_M^*/dp < 1$

Taking total derivatives of (C.1) and (C.2), equalizing them to zero and rearranging:

$$-\left(1 + \frac{i}{i_M}\right)\frac{dp_M}{dp} + \frac{i}{i_M^2}\left(p_M - \frac{\tau R}{2}\right)\frac{di_M}{dp} = -1 \tag{C.3}$$

$$\left(\frac{p-\theta r-p_M}{p_M-\frac{\tau R}{2}}-2\frac{i}{i_M}\right)\frac{dp_M}{dp}+\frac{\frac{i}{i_M^2}\left(p_M^2-p_M\tau R+\frac{\tau^2R^2}{3}\right)}{p_M-\frac{\tau R}{2}}\frac{di_M}{dp}=-1 \tag{C.4}$$

Using Cramer's rule on this two-equations system, we can calculate dp_M/dp as:

$$\frac{dp_M}{dp}=\frac{-\frac{\frac{i}{i_M^2}\left(p_M^2-p_M\tau R+\frac{\tau^2R^2}{3}\right)}{p_M-\frac{\tau R}{2}}+\frac{i}{i_M^2}\left(p_M-\frac{\tau R}{2}\right)}{-\left(1+\frac{i}{i_M}\right)\frac{\frac{i}{i_M^2}\left(p_M^2-p_M\tau R+\frac{\tau^2R^2}{3}\right)}{p_M-\frac{\tau R}{2}}-\frac{i}{i_M^2}\left(p_M-\frac{\tau R}{2}\right)\left(\frac{p-\theta r-p_M}{p_M-\frac{\tau R}{2}}-2\frac{i}{i_M}\right)}$$

$$\Leftrightarrow\frac{dp_M}{dp}=\frac{\frac{\tau^2R^2}{12}}{\left(1+\frac{i}{i_M}\right)\left(p_M^2-p_M\tau R+\frac{\tau^2R^2}{3}\right)+\left(p_M-\frac{\tau R}{2}\right)^2\left(\frac{p-\theta r-p_M}{p_M-\frac{\tau R}{2}}-2\frac{i}{i_M}\right)}$$

From (C.1), $i/i_M=(p-\theta r-p_M)/(p_M-\tau R/2)$, thus:

$$\Leftrightarrow\frac{dp_M}{dp}=\frac{\frac{\tau^2R^2}{12}}{\left(1+\frac{i}{i_M}\right)\left(p_M^2-p_M\tau R+\frac{\tau^2R^2}{3}\right)+\left(p_M-\frac{\tau R}{2}\right)^2\left(-\frac{i}{i_M}\right)}$$

$$\Leftrightarrow\frac{dp_M}{dp}=\frac{1}{\frac{12}{\tau^2R^2}\left(p_M-\frac{\tau R}{2}\right)^2+1+\frac{i}{i_M}} \tag{C.5}$$

From this expression, $0<\frac{dp_M}{dp}<1$. □

C.2 Proof of $di_M^*/dp<0$

Using Cramer's rule on the two-equations system (C.3)-(C.4), we can calculate di_M/dp as:

$$\frac{di_M}{dp}=\frac{\left(1+\frac{i}{i_M}\right)+\left(\frac{p-\theta r-p_M}{p_M-\frac{\tau R}{2}}-2\frac{i}{i_M}\right)}{-\left(1+\frac{i}{i_M}\right)\frac{\frac{i}{i_M^2}\left(p_M^2-p_M\tau R+\frac{\tau^2R^2}{3}\right)}{p_M-\frac{\tau R}{2}}-\frac{i}{i_M^2}\left(p_M-\frac{\tau R}{2}\right)\left(\frac{p-\theta r-p_M}{p_M-\frac{\tau R}{2}}-2\frac{i}{i_M}\right)}$$

From (C.1), $i/i_M=(p-\theta r-p_M)/(p_M-\tau R/2)$, thus:

$$\frac{di_M}{dp}=\frac{p_M-\frac{\tau R}{2}}{-\left(1+\frac{i}{i_M}\right)\frac{i}{i_M^2}\left(p_M^2-p_M\tau R+\frac{\tau^2R^2}{3}\right)-\frac{i}{i_M^2}\left(p_M-\frac{\tau R}{2}\right)^2\left(-\frac{i}{i_M}\right)}$$

$$\Leftrightarrow\frac{di_M}{dp}=-\frac{p_M-\frac{\tau R}{2}}{\frac{i}{i_M^2}\left(\left(p_M-\frac{\tau R}{2}\right)^2+\left(1+\frac{i}{i_M}\right)\frac{\tau^2R^2}{12}\right)} \tag{C.6}$$

As p_M is necessarily larger than $\frac{\tau R}{2}$ (otherwise participation constraints could not be satisfied), $\frac{di_M}{dp}$ is strictly negative. □

D Mill pricing: constrained optimum

In the constrained case $(p_M^{**}, i_M^{**}))$, $\lambda \geq 0$. Isolating λ in (5.3) and substituting in (5.2), when $f(k) = 2\sqrt{k}$, gives:

$$i_M = i\frac{6p_M^2 - 3p_M\tau R + \tau^2 R^2}{6pp_M - 6p_M\tau R + 2\tau^2 R^2 - 6\theta r p_M} \tag{D.1}$$

Binding participation constraint $g(r+R) = 0$ gives:

$$i_M = i\frac{(p_M - \tau R)^2}{(p - \tau r - \tau R)^2} \tag{D.2}$$

Prices (p_M^{**}, i_M^{**}) are given by the intersection between the curves (D.1) and (D.2), provided $\tau R \leq p_M^{**} \leq p - \tau r$. Simplifying:

$$\begin{aligned} \Leftrightarrow h(p_M) \equiv (-6p + 6(\tau R + \theta r))\, p_M^3 + \left(6p^2 - 12p\tau r + 2(3\tau^2 r^2 - 4\tau^2 R^2 + 6\tau R(\tau r - \theta r))\right) p_M^2 \\ + \left(-3p^2\tau R + 6p\tau r\tau R + \tau R(\tau^2 r^2 + 7\tau^2 R^2 + 6\tau R(-\tau r + \theta r))\right) p_M \\ + \left(p^2\tau^2 R^2 - 2p\tau^2 R^2(\tau r + \tau R) + \tau^2 R^2(\tau^2 r^2 + 2\tau r\tau R - \tau^2 R^2)\right) = 0 \end{aligned}$$

(a) $h(p_M)$ is a polynomial of degree three in p_M where the leading coefficient is strictly negative. This implies that $h(p_M)$ is decreasing at both ends. (b) Evaluated at $p_M = \tau R$, $h(\tau R) > 0$. (c) The first derivative of $h(p_M)$, evaluated at τR is strictly positive. This implies that τR lies in an increasing part of $h(p_M)$. (d) If $\tau r - \theta r > \tau R/2$ holds, then $h(p - \tau r) < 0$ holds for $p > \tau(r + R)$. Elements (a) to (d) are sufficient to ensure that $h(p_M)$ has one unique root between τR and $p - \tau r$, i.e. that the constrained optimum exists.

D.1 Proof of $dp_M^{**}/dp > 0$

(i) $h(p_M) = 0$ is a polynomial of degree three in p_M where the leading coefficient is strictly negative, $h(p_M)$ is decreasing at both ends. We have already shown that $h(\tau R) > 0$ and $h(p - \tau r) < 0$. As p_M^{**} lies between τR and $p - \tau r$ this is sufficient to ensure that $h(.)$ is decreasing at p_M^{**}, that is $\left.\frac{\partial h}{\partial p_M}\right|_{p_M = p_M^{**}} < 0$.

(ii) First derivative of $h(.)$ with respect to p is given by $-6p_M^3 + p_M^2(12p - 12\tau r) + p_M(-6p\tau R + 6\tau r\tau R) + 2p\tau^2 R^2 - 2\tau^2 R^2(\tau r + \tau R)$. As $\tau R < p_M^{**} < p - \tau r$, it is strictly positive at p_M^{**}. That is, $\left.\frac{\partial h}{\partial p}\right|_{p_M = p_M^{**}} > 0$.

(iii) From implicit function theorem, $\frac{dp_M^{**}}{dp} = -\left.\frac{\partial h}{\partial p}\right|_{p_M = p_M^{**}} \Big/ \left.\frac{\partial h}{\partial p_M}\right|_{p_M = p_M^{**}}$. From (i) and (ii), this is strictly positive. □

E Proof of proposition 7

Here we show that (1) when the equilibrium is constrained, then it is still constrained when p increases, (2) when the equilibrium is constrained, then it may become unconstrained when

p decreases, (3) when the equilibrium is unconstrained, then it is still unconstrained when p decreases, (4) when the equilibrium is unconstrained, then it may become constrained when p increases. Statements (1), (2), (3) and (4) ensure that there exists a $\bar{p}$ such that the equilibrium is unconstrained for $p < \bar{p}$ and constrained for $p > \bar{p}$. Moreover, we show that (5) a sufficient condition for the equilibrium to be constrained is $p > 2\tau r + \tau R - \theta r$.

Proof of statement (1): The constrained maximum is relevant when unconstrained one does not satisfy the constraint. Suppose that the unconstrained maximum $(p^*_M(p_0), i^*_M(p_0))$ does not satisfy the constraint for a given p_0: $g(r+R, p^*_M, i^*_M, p_0) = \frac{(p^*_M(p_0)-\tau R)^2}{i^*_M(p_0)} - \frac{(p_0-\tau r-\tau R)^2}{i} < 0$.

$$\frac{\partial g(r+R,p^*_M,i^*_M,p)}{\partial p} = \left(2(p^*_M(p) - \tau R)\frac{dp^*_M}{dp} i^*_M(p) - \frac{di^*_M}{dp}(p^*_M(p) - \tau R)^2\right)\frac{1}{i^*_M(p)^2} - \frac{2(p-\tau r-\tau R)}{i}.$$

Using (C.5) and (C.6), $\frac{\partial g(r+R,p^*_M,i^*_M,p)}{\partial p} = \frac{2(p^*_M(p)-\tau R)}{i}\frac{\left(\frac{i}{i^*_M(p)}+(p^*_M(p)-\tau R)(p^*_M(p)-\tau(R/2))\frac{6}{\tau^2R^2}\right)}{1+\frac{i}{i^*_M(p)}+(p^*_M(p)-\tau(R/2))^2\frac{12}{\tau^2R^2}} - \frac{2(p-\tau r-\tau R)}{i}$.

As the second ratio is lower that 1, it can be said that $\frac{\partial g(r+R,p^*_M,i^*_M,p)}{\partial p} < \frac{2(p^*_M(p)-\tau R)}{i} - \frac{2(p-\tau r-\tau R)}{i}$.

This is strictly negative for all $p \geq p_0$ as $p^*_M(p) < p - \tau r$ for those values of p. Indeed, we know that this is satisfied at $p = p_0$, otherwise $(p^*_M(p_0), i^*_M(p_0))$ would satisfy the constraint. This is sure as we know that $i^*_M(p_0) \leq i$ (see proposition 6). If this is true at p_0, it is also true at any $p > p_0$ because $p^*_M(p)$ increases less rapidly with p than $p - \tau r$ does (see appendix C.1: $dp^*_M/dp < 1$.)

As $\partial g(r+R, p^*_M, i^*_M, p)/\partial p < 0$ for all $p \geq p_0$, if $(p^*_M(p_0), i^*_M(p_0))$ does not satisfy the constraint, then $(p^*_M(p_1), i^*_M(p_1))$ does not satisfy the constraint neither, for any $p_1 > p_0$. □

Proof of statement (2): Suppose that the unconstrained maximum $(p^*_M(p_0), i^*_M(p_0))$ does not satisfy the constraint for a given p_0: $g(r+R, p^*_M, i^*_M, p_0) < 0$. We have shown (see proof of statement (1)) that, at $p = p_0$, $\partial g(r+R, p^*_M, i^*_M, p)/\partial p < 0$. Thus there may exist a $p' < p_0$ such that $g(r+R, p^*_M, i^*_M, p') > 0$, that is, $(p^*_M(p'), i^*_M(p'))$ satisfies the constraint. Note that the existence of such a such a p' that satisfies the assumption $p' > \tau r + \tau R$ is not ensured. Depending on the other parameters, such a p' may or may not exist. □

Proof of statement (3): Suppose that the unconstrained maximum $(p^*_M(p_0), i^*_M(p_0))$ satisfies the constraint for a given p_0: $g(r+R, p^*_M, i^*_M, p_0) \geq 0$.

(i) Suppose that $p^*_M(p_0) < p_0 - \tau r$. In this case, we have shown (see proof of statement (1)) that $g(r+R, p^*_M, i^*_M, p)$ is decreasing in p. This is sufficient to ensure that $g(r+R, p^*_M, i^*_M, p_1) \geq 0$ for any $p_1 < p_0$, thus that $(p^*_M(p_1), i^*_M(p_1))$ satisfies the constraint for any $p_1 < p_0$.

(ii) Suppose that $p^*_M(p_0) \geq p_0 - \tau r$. As, from appendix C.1, $0 < dp^*_M/dp < 1$, this implies $p^*_M(p_1) \geq p_1 - \tau r$ for any $p_1 < p_0$. Given that $i^*_M(p) \leq i$ (see proposition 6), this is sufficient for $(p^*_M(p_1), i^*_M(p_1))$ to satisfy the constraint.

(i) and (ii) ensure that if $(p^*_M(p_0), i^*_M(p_0))$ satisfies the constraint, $(p^*_M(p_1), i^*_M(p_1))$ still satisfies the constraint, for any $p_1 < p_0$. □

Proof of statement (4): Suppose that the unconstrained maximum $(p^*_M(p_0), i^*_M(p_0))$ satisfies the constraint for a given p_0: $g(r+R, p^*_M, i^*_M, p_0) \geq 0$.

(i) Suppose that $p^*_M(p_0) < p_0 - \tau r$. In this case, we have shown (see proof of statement (1)) that $g(r+R, p^*_M, i^*_M, p)$ is decreasing in p. This is sufficient to ensure that there exists a $p' > p_0$ such that $g(r+R, p^*_M, i^*_M, p') < 0$.

(ii) Suppose that $p_M^*(p_0) \geq p_0 - \tau r$. From appendix C.1, $dp_M^*/dp < 1$. Thus it exists a $p'' > p_0$ such that $p_M^*(p'') < p'' - \tau r$. From (i), there exists a $p' > p'' > p_0$ such that $(p_M^*(p'), i_M^*(p'))$ does not satisfy the constraint.

(i) and (ii) ensure that if $(p_M^*(p_0), i_M^*(p_0))$ satisfies the constraint, there exists a $p' > p_0$ such that $(p_M^*(p'), i_M^*(p'))$ does not satisfy the constraint anymore. $\square$

Proof of statement (5): To establish that the optimum is constrained, it is sufficient to determine that the unconstrained optimum, does not satisfy the constraint, i.e. $\frac{(p_M^* - \tau R)^2}{i_M^*} - \frac{(p - \tau r - \tau R)^2}{i} < 0$. Isolating i_M^* in (C.1) and substituting in the previous expression gives $\frac{(p_M^* - \tau R)^2}{p_M^* - \tau(R/2)} < \frac{(p - \tau r - \tau R)^2}{p - \theta r - p_M^*}$. A sufficient condition for that is:

$$\frac{(p_M^* - \tau R)^2}{p_M^* - \tau R} < \frac{(p - \tau r - \tau R)^2}{p - \theta r - p_M^*}$$

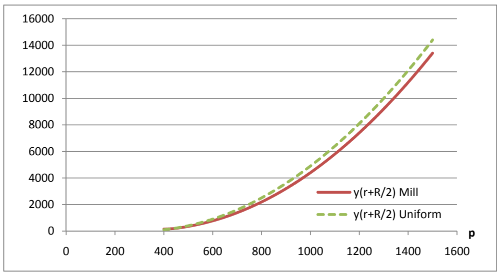
$$\Leftrightarrow (p_M^* - \tau R)(p - \theta r - p_M^*) - (p - \tau r - \tau R)^2 < 0$$

$$\Leftrightarrow w(p_M^*) \equiv -p_M^{*2} + (p - \theta r + \tau R)p_M^* - (p - \theta r)\tau R - (p - \tau r - \tau R)^2 < 0$$

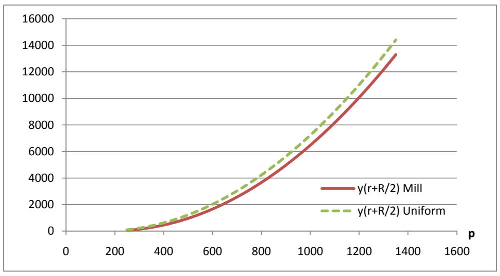
$w(p_M^*)$ is a polynomial of degree two in p_M^* where the leading coefficient is strictly negative. The discriminant is given by $(p - \theta r + \tau R)^2 - 4((p - \theta r)\tau R) - 4(p - \tau r - \tau R)^2$. This is a polynomial of degree two in p where the leading coefficient is strictly negative. It is thus negative before the first root ($p_1 = \frac{1}{3}(2\tau r + \theta r) + \tau R$) and after the second one ($p_2 = 2\tau r + \tau R - \theta r$). A negative discriminant is sufficient for $w(p_M^*)$ to be negative. Thus $p > 2\tau r + \tau R - \theta r$ is a sufficient condition for the unconstrained optimum to be excluded, hence for the optimum to be constrained. $\square$

F Numerical simulations

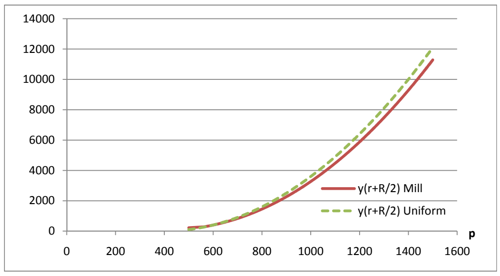
F.1 Median farmer's income under uniform and mill pricing



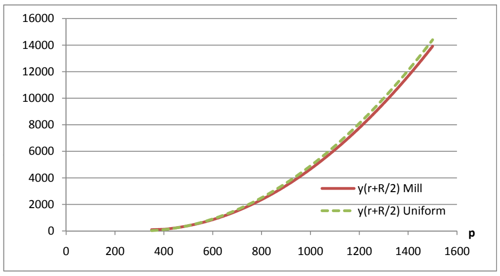
$r = 300,\ R = 100,\ \tau = 1,\ i = 100,\ \theta = 0.2$



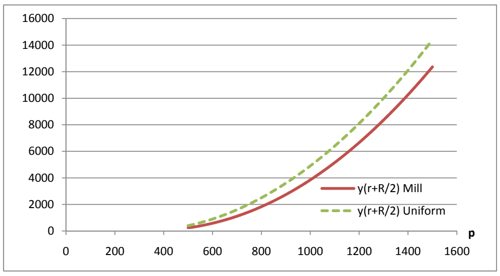
$r = 150,\ R = 100,\ \tau = 1,\ i = 100,\ \theta = 0.2$



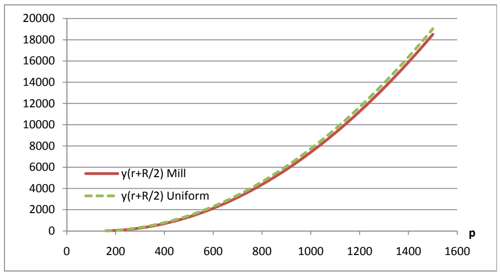
$r = 400,\ R = 100,\ \tau = 1,\ i = 100,\ \theta = 0.2$



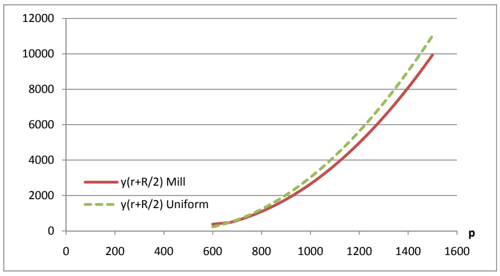
$r = 300,\ R = 50,\ \tau = 1,\ i = 100,\ \theta = 0.2$



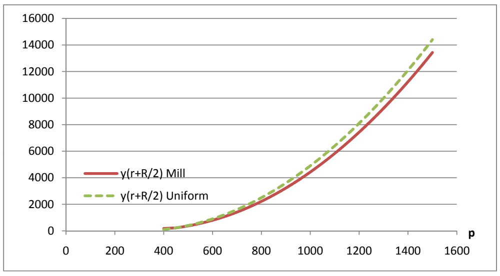
$r = 300,\ R = 200,\ \tau = 1,\ i = 100,\ \theta = 0.2$



$r = 300,\ R = 100,\ \tau = 0.4,\ i = 100,\ \theta = 0.2$



$r = 300,\ R = 100,\ \tau = 1.5,\ i = 100,\ \theta = 0.2$



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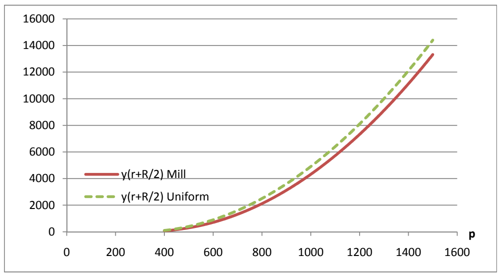

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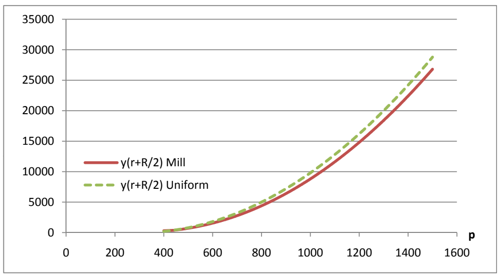
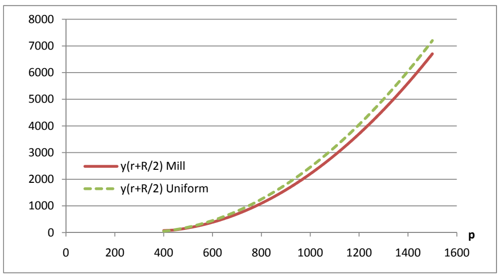


$r = 300,\ R = 100,\ \tau = 1,\ i = 50,\ \theta = 0.2$



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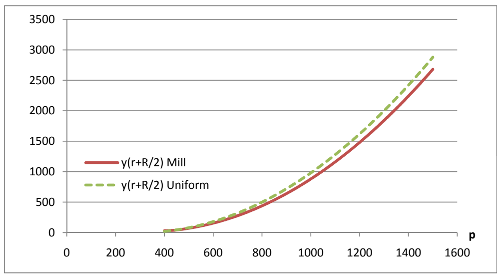
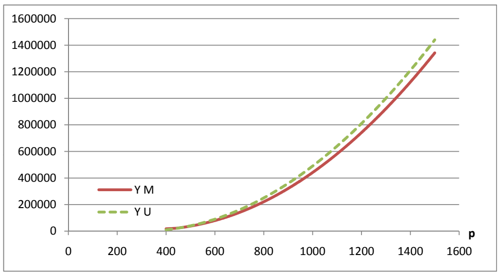
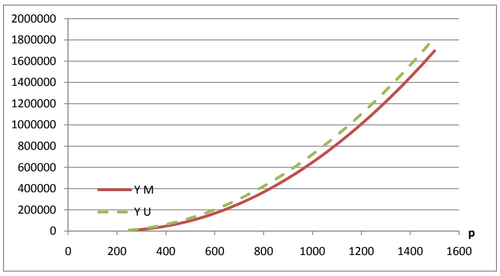


$r = 300,\ R = 100,\ \tau = 1,\ i = 500,\ \theta = 0.2$

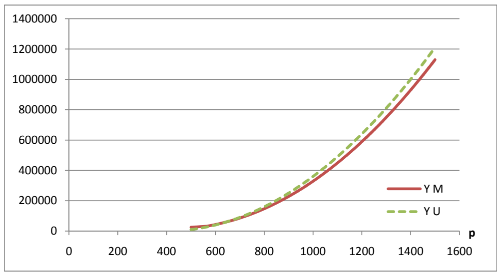
F.2 Total farmers income under uniform and mill pricing



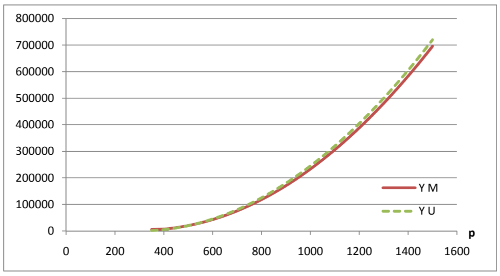
$r = 300,\ R = 100,\ \tau = 1,\ i = 100,\ \theta = 0.2$



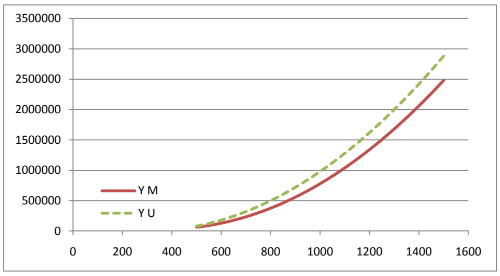
$r = 150,\ R = 100,\ \tau = 1,\ i = 100,\ \theta = 0.2$



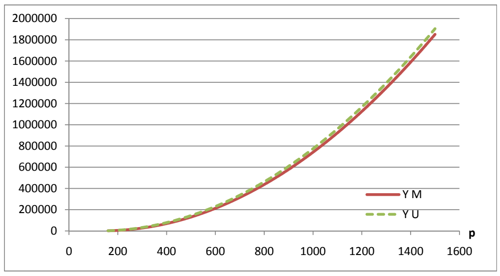
$r = 400,\ R = 100,\ \tau = 1,\ i = 100,\ \theta = 0.2$



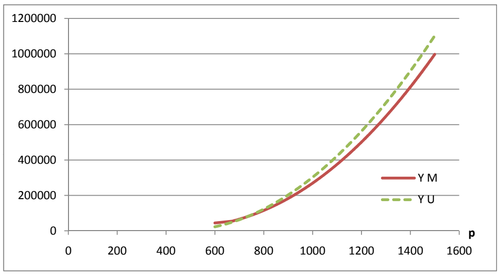
$r = 300,\ R = 50,\ \tau = 1,\ i = 100,\ \theta = 0.2$



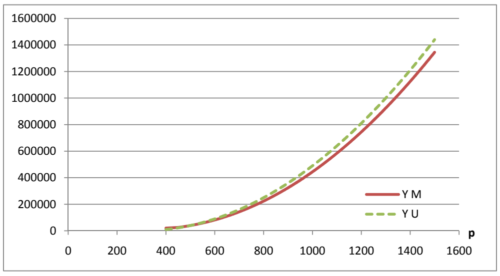
$r = 300,\ R = 200,\ \tau = 1,\ i = 100,\ \theta = 0.2$



$r = 300,\ R = 100,\ \tau = 0.4,\ i = 100,\ \theta = 0.2$



$r = 300,\ R = 100,\ \tau = 1.5,\ i = 100,\ \theta = 0.2$



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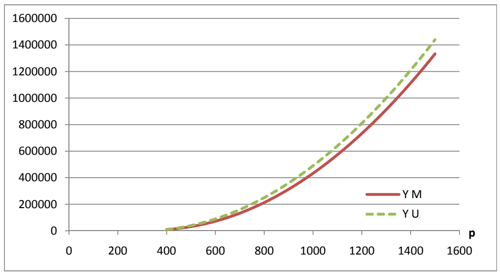

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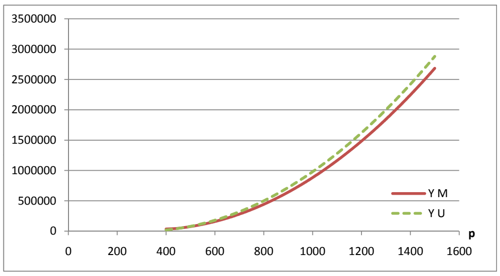

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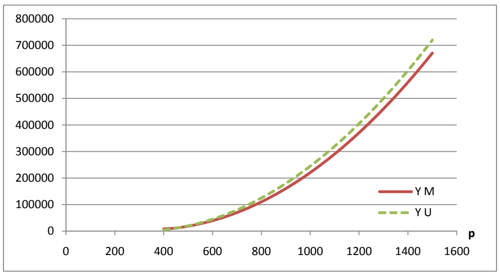

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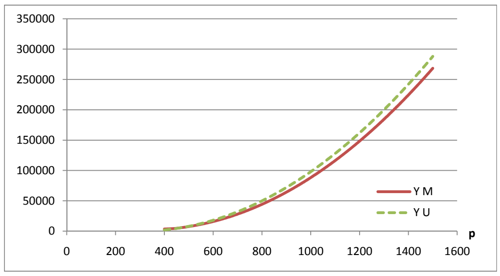
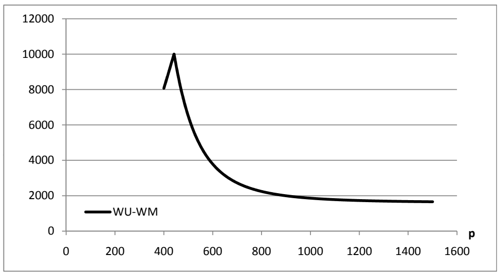
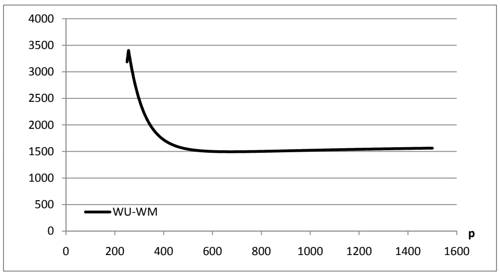


$r = 300,\ R = 100,\ \tau = 1,\ i = 500,\ \theta = 0.2$

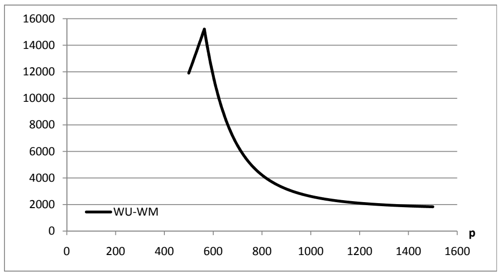
F.3 Social welfare comparison: $W_U - W_M$



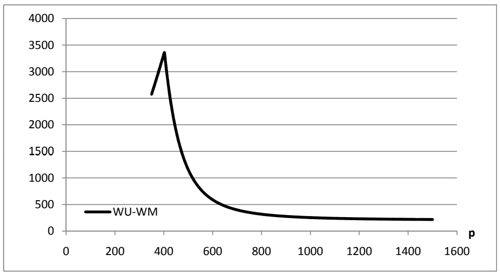
$r = 300,\ R = 100,\ \tau = 1,\ i = 100,\ \theta = 0.2$



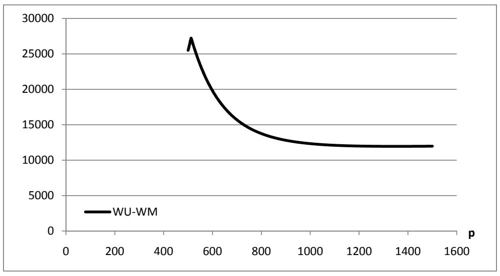
$r = 150,\ R = 100,\ \tau = 1,\ i = 100,\ \theta = 0.2$



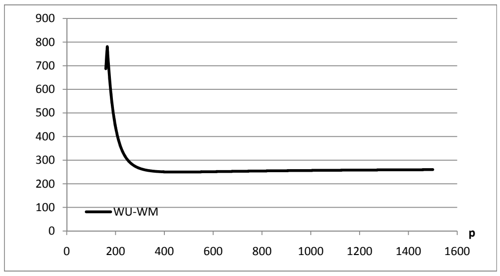
$r = 400,\ R = 100,\ \tau = 1,\ i = 100,\ \theta = 0.2$



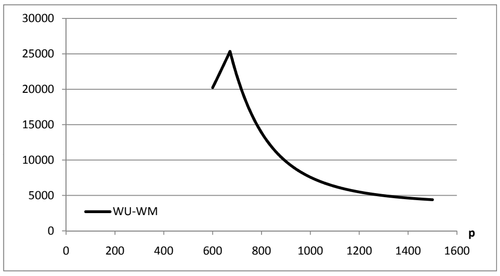
$r = 300,\ R = 50,\ \tau = 1,\ i = 100,\ \theta = 0.2$



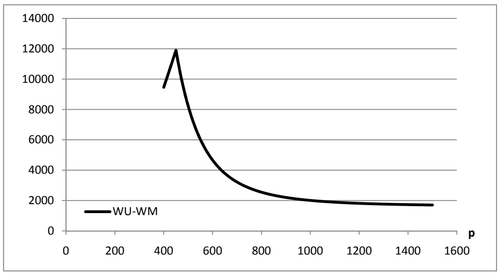
$r = 300,\ R = 200,\ \tau = 1,\ i = 100,\ \theta = 0.2$



$r = 300,\ R = 100,\ \tau = 0.4,\ i = 100,\ \theta = 0.2$



$r = 300,\ R = 100,\ \tau = 1.5,\ i = 100,\ \theta = 0.2$



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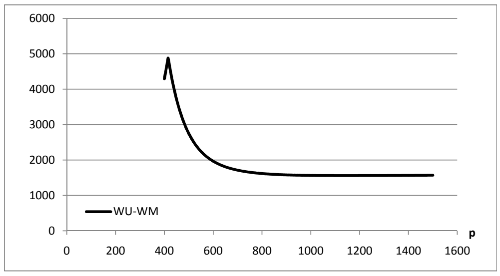

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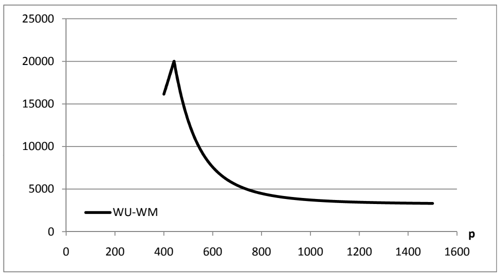

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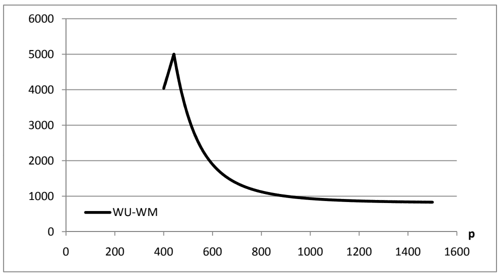

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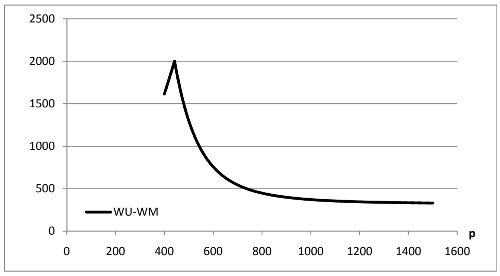
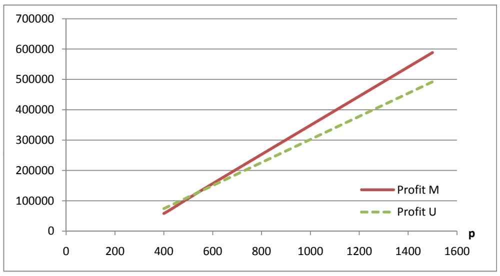
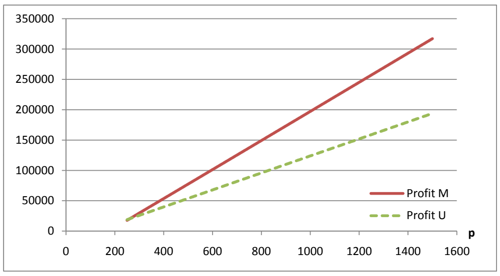


$r = 300,\ R = 100,\ \tau = 1,\ i = 500,\ \theta = 0.2$

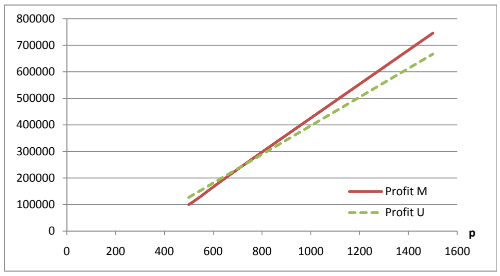
F.4 Trader's profit under uniform and mill pricing



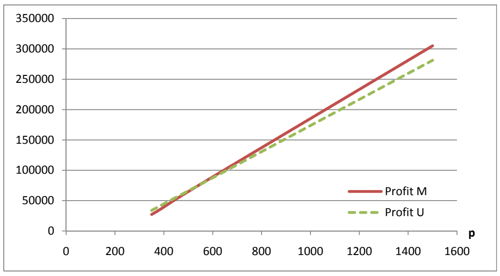
$r = 300,\ R = 100,\ \tau = 1,\ i = 100,\ \theta = 0.2$



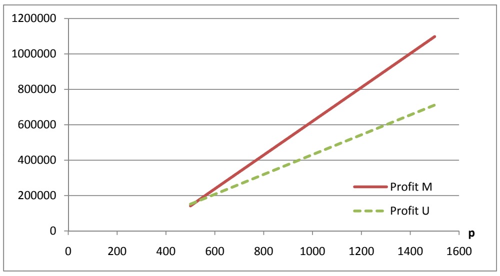
$r = 150,\ R = 100,\ \tau = 1,\ i = 100,\ \theta = 0.2$



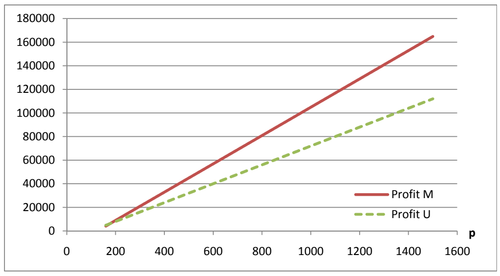
$r = 400,\ R = 100,\ \tau = 1,\ i = 100,\ \theta = 0.2$



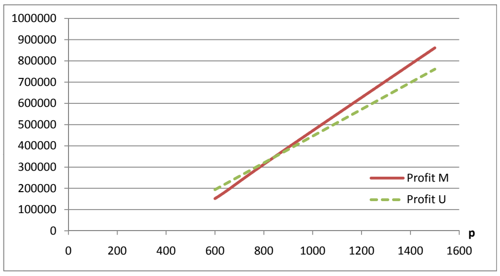
$r = 300,\ R = 50,\ \tau = 1,\ i = 100,\ \theta = 0.2$



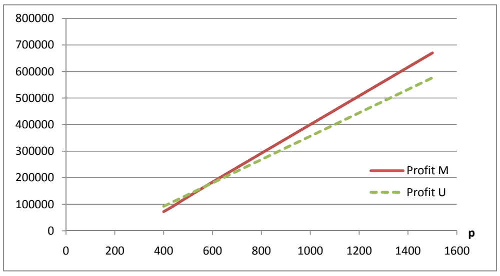
$r = 300,\ R = 200,\ \tau = 1,\ i = 100,\ \theta = 0.2$



$r = 300,\ R = 100,\ \tau = 0.4,\ i = 100,\ \theta = 0.2$



$r = 300,\ R = 100,\ \tau = 1.5,\ i = 100,\ \theta = 0.2$



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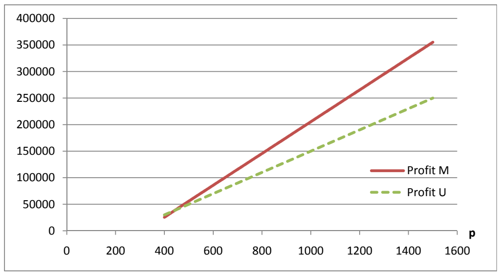

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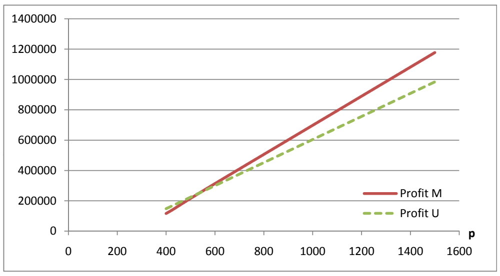

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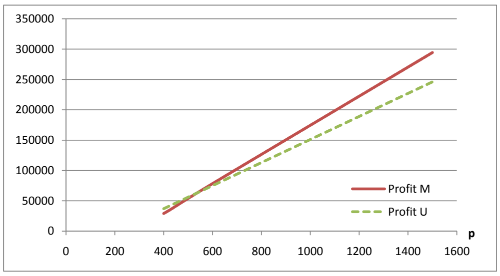
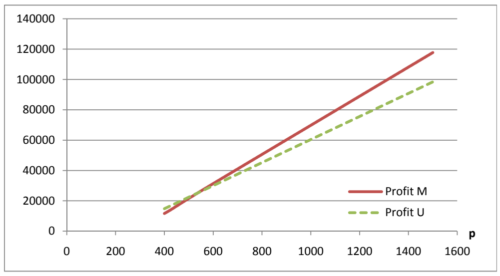


$r = 300,\ R = 100,\ \tau = 1,\ i = 200,\ \theta = 0.2$



$r = 300,\ R = 100,\ \tau = 1,\ i = 500,\ \theta = 0.2$