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ABSTRACT

Fiscal Multipliers Over the Business Cycle*

This paper develops a theory characterizing the effects of fiscal policy on unemployment over the business cycle. The theory is based on a model of equilibrium unemployment in which jobs are rationed in recessions. Fiscal policy in the form of government spending on public-sector jobs reduces unemployment, especially during recessions: the fiscal multiplier---the reduction in unemployment rate achieved by spending one dollar on public-sector jobs---is positive and countercyclical. Although the labor market always sees vast flows of workers and a great deal of matching, recessions are periods of acute job shortage without much competition for workers among recruiting firms. Hence hiring in the public sector reduces unemployment effectively because it does not crowd out hiring in the private sector much. An implication is that empirical studies should control for the state of the economy when fiscal policies are implemented to estimate accurately the amplitude of fiscal multipliers in recessions.

JEL Classification: E24, E32, E62 and J64
Keywords: business cycle, fiscal multiplier, job rationing, matching frictions and unemployment

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1 Introduction

This paper develops a theory characterizing the effects of fiscal policy on unemployment over the business cycle.[1] Such a theory is lacking from the macroeconomics literature because: (i) existing dynamic stochastic general equilibrium (DSGE) models used to study fiscal policy do not capture the job shortage that explains most of recessionary unemployment; and (ii) fiscal policy in these models does not stimulate aggregate demand for labor, and is bound to be ineffective in recessions.

In standard real business cycle (RBC) and new Keynesian (NK) models, the labor market always clears and there is no unemployment [Aiyagari et al., 1992; Baxter and King, 1993; Christiano and Eichenbaum, 1992; Christiano et al., 2011; Galí et al., 2007; Woodford, 2011]. These models are of little help to determine the effectiveness of fiscal policy at reducing unemployment.

Some RBC and NK models incorporate the model of unemployment of Pissarides [2000], in which frictions impede matching of recruiting firms with unemployed workers and wages are set by Nash bargaining between firms and workers [Andolfatto, 1996; Bruckner and Pappa, forthcoming; Merz, 1995; Monacelli et al., 2010]. These models have some unemployment but no recessions, because there are no labor market fluctuations. The absence of recessions results from the flexibility of bargained wages, which adjust fully to productivity shocks [Shimer, 2005]. Hence, these models cannot characterize the effects of fiscal policy in recessions.

[1] This paper focuses on unemployment because, even though employment is not an exact measure of social welfare, the social cost of recessions ultimately comes from the increase in unemployment they generate: Sullivan and von Wachter [2009] document an increase in mortality caused by job displacements; von Wachter et al. [2009] document long-term earning losses following mass layoffs, which could reflect long-term losses in human capital; Hawton and Platt [2000] survey the medical literature studying suicidal behavior among unemployed workers; and Brenner and Mooney [1983] survey the medical literature measuring the health costs of unemployment. Accordingly, the first goal of policymakers in recessions is to reduce unemployment [for example, Romer and Bernstein, 2009].

Following Hall [2005], some NK models combine the Pissarides [2000] model of unemployment with rigid wages to generate labor market fluctuations and recessions [Blanchard and Galí, 2010; Gertler et al., 2008]. But all unemployment is frictional in these models as their labor market converges to full employment when matching frictions become arbitrarily small, or equivalently, when unemployed workers exert an arbitrarily large effort to search for jobs [Landais et al., 2010; Michaillat, forthcoming]. This property is at odd with the long queues of jobseekers in front of factory gates during the Great Depression. Thus these models are inadequate to study recessionary unemployment because they do not capture the job shortage observed in recessions.

Fiscal policy in these models typically consists of government purchases of consumption goods [Ramey, 2011].[2] Government consumption reduces the amount of goods available for private consumption, stimulating labor supply through a negative wealth effect (in the basic model with lump-sum taxes). But when jobs are rationed in recessions, unemployment remains high irrespective of labor supply [Landais et al., 2010]. Hence this policy is bound to be ineffective in recessions, and the analysis of government consumption is not informative of the scope for unemployment-reducing fiscal policy.

To study how fiscal policy can reduce recessionary unemployment, this paper proposes a barebone model with: (1) equilibrium unemployment as in Pissarides [2000]; (2) recessions driven by real wage rigidity as in Hall [2005]; (3) job rationing in recessions resulting from the combination of wage rigidity and diminishing marginal returns to labor as in Michaillat [forthcoming]; and (4) fiscal policy in the form of public-sector employment. The choice of public employment is

[2] There are a few exceptions: Finn [1998] or Cavallo [2005] consider government purchases of hours in the public sector; Baxter and King [1993] or Leeper et al. [2010] consider government investments.

guided by theory, as public employment stimulates aggregate labor demand, which is depressed in recessions; by empirical results, as government spending on public employment seems to have a higher output multiplier than government spending on investment or consumption goods [Pappa, 2010]; and by the relevance of the analysis for governments, who often use public employment to reduce unemployment in recessions: the Roosevelt administration hired millions of unemployed workers to build dams, bridges, and roads during the Great Depression [Fishback et al., 2003; Fleck, 1999]; the American Jobs Act presented by the Obama administration to Congress in 2011 proposes to spend $140 billion to hire teachers and other public-sector workers.

The central result of the paper is that in a model with job rationing, the *public-employment multiplier*—the reduction in unemployment rate achieved by spending one dollar on public-sector jobs—is positive and countercyclical. Hence fiscal policy in the form of hiring in the public sector reduces unemployment, especially in recessions. This result hinges on the nature of unemployment in recessions: although the labor market always sees vast flows of workers and a great deal of matching activity, recessions are periods of acute job shortage without much competition for workers among recruiting firms. Hence hiring in the public sector reduces unemployment effectively because it does not crowd out hiring in the private sector much.

To understand the scope for fiscal policy, I underline necessary conditions for fiscal policy to reduce unemployment effectively. First, public employment is not effective in search-and-matching models without job rationing. Second, fiscal policies stimulating labor supply, such as monitoring unemployed workers' job search, have virtually no effect on unemployment when jobs are rationed in recessions. I also show that the *wage-subsidy multiplier*—the reduction in unemployment rate achieved by spending one dollar on a wage subsidy—is positive and countercyclical. A wage subsidy is effective here because technology shocks drive fluctuations and recessions arise from rigid wages being too high relative to labor productivity. But a wage subsidy may not be effective if demand shocks drive fluctuations, firms face a downward-sloping goods demand, and prices are completely sticky in the short run. In that case firm's labor demand is fully determined by goods demand and production function so that a wage subsidy is ineffective. On the contrary, public employment would reduce unemployment in any model with job rationing, independent of the source of fluctuations.

A critical implication of the paper is that available estimates of the fiscal multiplier do not apply in recessions, because the fiscal multiplier fluctuates widely over the business cycle while the estimation methods used in the literature average the effect of fiscal policy over all possible states of the economy [Parker, 2011].

The paper proceeds as follows. Section 2 presents a generic model of equilibrium unemployment with a public sector. Section 3 proves that the welfare-maximizing centralized allocation in this model is constant, irrespective of the stochastic process driving economic fluctuations. Hence unemployment fluctuations over the business cycle in the decentralized economy are always inefficient. Section 4 introduces recessions and job rationing into the model to prove that the public-employment multiplier is positive and countercyclical. Section 5 examines the scope for (i) public employment in alternative models, and (ii) alternative fiscal policies in a model with job rationing. Section 6 concludes by discussing implications and limitations of the paper. Derivations and proofs are collected in the Appendix.

2 Model of Equilibrium Unemployment with a Public Sector

This section presents a generic model of equilibrium unemployment. Risk-averse unemployed workers search for a vacant job indiscriminately in the private and public sectors. Some frictions impede matching of jobseekers with vacancies. Labor market fluctuations are driven by technology, which follows a stochastic process $\{a_t\}_{t=0}^{+\infty}$.[3] The time-t elements of the worker's choice, the firm's choice, and government policies are function of the history of technology levels $a^t \equiv (a_0, a_1, \ldots, a_t)$ and of initial employments (g_{-1}, l_{-1}) in the public and private sectors.

2.1 Labor market

There is a unit mass of workers in the labor market. The labor market is composed of two sectors: a private sector with l_t workers, and a public sector with g_t workers. Public employment follows a stochastic process $\{g_t\}_{t=0}^{+\infty}$, whose time-t element is measurable with respect to (a^t, g_{-1}, l_{-1}). Aggregate employment is $n_t = l_t + g_t$. Jobseekers apply to jobs randomly, not directing their search toward any of the two sectors.[4]

At the end of period $t-1$, a fraction s of the n_{t-1} existing worker-job matches is exogenously destroyed. Workers who lose their job apply for a new job immediately. At the beginning of period t, $u_t = 1-(1-s)\cdot n_{t-1}$ unemployed workers search for a job with effort $e_t \in [0,1]$. Those who find a job participate in production in period t with the $(1-s)\cdot n_{t-1}$ incumbent workers.

[3]Empirical evidence suggests that recessions are driven by aggregate-activity shocks and not by reallocation shocks [Abraham and Katz, 1986; Blanchard and Diamond, 1989]. Hence I assume a stable matching function and introduce, in line with the literature, aggregate technology shocks.

[4]Models of the labor market do not usually include a public sector. Holmlund and Linden [1993], Quadrini and Trigari [2007], Hörner et al. [2007], and Gomes [2010] are exceptions that add a public sector to a search-and-matching model to study the macroeconomic impact of public-employment policies.

By posting a total of v_t vacancies, a representative firm hires workers in the private sector and the government hires workers in the public sector. The number h_t of matches made in the period is given by a matching function $h_t = h(e_t \cdot u_t, v_t)$ of aggregate search effort $e_t \cdot u_t$ and vacancies v_t, with the restriction that $h(e_t \cdot u_t, v_t) \leq \min\{u_t, v_t\}$. To simplify the analysis, I specify a Cobb-Douglas matching function: $h(e_t \cdot u_t, v_t) = \omega_h \cdot (e_t \cdot u_t)^{\eta} \cdot v_t^{1-\eta}$, where ω_h and $\eta \in (0,1)$ are parameters.

Labor market conditions are summarized by labor market tightness $\theta_t \equiv v_t / (e_t \cdot u_t)$. The matching technology is such that not all unemployed workers can find a job and not all vacancies can be filled. A jobseeker finds a job with probability $f(\theta_t) = h(e_t \cdot u_t, v_t)/(e_t \cdot u_t) = h(1, \theta_t)$ per unit of search effort; hence a jobseeker searching with effort e_t finds a job with probability $e_t \cdot f(\theta_t)$. Vacancies in the public and private sectors are filled with the same probability $q(\theta_t) = h(e_t \cdot u_t, v_t)/v_t = h(1/\theta_t, 1)$. In a tight market it is easy to find jobs—the per-unit job-finding probability $f(\theta_t)$ is high—but difficult to find workers—the job-filling probability $q(\theta_t)$ is low.

Keeping a vacancy open has a per-period cost $r \cdot a_t$ in units of private good, where r captures resources spent recruiting workers.[5] I assume away randomness at the firm level: a worker is hired with certainty by opening $1/q(\theta_t)$ vacancies and spending $r \cdot a_t / q(\theta_t)$.

[5]As in Pissarides [2000], the cost of opening a vacancy is proportional to technology a_t. This assumption can be justified on the grounds that recruiting is a labor-intensive activity so the cost to maintain a vacancy open depends on wages, which are nearly proportional to technology in practice. This assumption also simplifies derivations.

2.2 Wage

The wage is set once worker and firm have matched. Since search costs are sunk at the time of matching, there are always mutual gains from trade. There is no compelling theory of wage determination in such an environment [Hall, 2005]. Hence I assume that the equilibrium wage in the private sector follows a stochastic process $\{w_t\}_{t=0}^{+\infty}$, whose time-t element is measurable with respect to (a^t, g_{-1}, l_{-1}). I impose that the wage process $\{w_t\}_{t=0}^{+\infty}$ be privately efficient: worker-firm pairs exploit all opportunities for mutual improvement. Private efficiency guarantees that the wage never causes the destruction of a match generating a positive bilateral surplus, a reasonable equilibrium requirement when rational workers and firms engage in long-term interactions [Barro, 1977]. Below, I consider as special cases of this wage process a rigid wage and the outcome of Nash bargaining. Finally, I assume that the government sets the public-sector wage at the level of the private-sector wage.

2.3 Worker

Workers have utility that depends on consumption c_t of private good produced in the private sector, consumption p_t of public good produced in the public sector, and job-search effort e_t, of the form $\chi \cdot \ln(p_t) + \ln(c_t) - k(e_t)$, where $k(e_t) = \omega_k \cdot e_t^{1+\kappa} / (1+\kappa)$. χ, ω_k, and $\kappa > 0$ are parameters. Job-search effort is not observable. Employed workers earn a wage w_t, and unemployed workers receive unemployment insurance (UI) benefits $b_t \cdot w_t$. Workers neither borrow nor save, so consumption is w_t when employed and $b_t \cdot w_t$ when unemployed. The benefit rate follows a stochastic process $\{b_t\}_{t=0}^{+\infty}$, whose time-t element is measurable with respect to (a^t, g_{-1}, l_{-1}).

Given a benefit rate, wage,and labor market tightness $\{b_t, w_t, \theta_t\}_{t=0}^{+\infty}$, the *representative worker's problem* is to choose job-search effort $\{e_t\}_{t=0}^{+\infty}$ to maximize the expected utility

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \left\{(1-n_t^s) \cdot \ln(b_t \cdot w_t) + n_t^s \cdot \ln(w_t) - \left[1-(1-s) \cdot n_{t-1}^s\right] \cdot k(e_t)\right\},$$

subject to the law of motion of the probability n_t^s to be employed in period t

$$n_t^s = (1-s) \cdot n_{t-1}^s + \left[1-(1-s) \cdot n_{t-1}^s\right] \cdot e_t \cdot f(\theta_t).$$

$\mathbb{E}_0$ is the mathematical expectation conditional on time-0 information, δ is the discount factor.[6] The first-order condition with respect to search effort e_t simplifies to

$$\left\{\frac{k'(e_t)}{f(\theta_t)} - \delta \cdot (1-s) \cdot \mathbb{E}_t \left[\frac{k'(e_{t+1})}{f(\theta_{t+1})}\right]\right\} + \kappa \cdot \delta \cdot (1-s) \cdot \mathbb{E}_t \left[k(e_{t+1})\right] = \ln\left(\frac{1}{b_t}\right). \tag{1}$$

2.4 Firm

The private sector is composed of a representative firm producing a private good sold to workers taking price as given. The firm's production function satisfies

$$y_t = a_t \cdot y(l_t) = a_t \cdot l_t^{\alpha}, \tag{2}$$

[6]The law of motion is an application of the total probability theorem. The probability n_t^s to have a job at t is the sum of the probability $e_t \cdot f(\theta_t)$ to find a job in t conditional on being jobless in $t-1$, which occurs with probability $1-n_{t-1}^s$, plus the probability $e_t \cdot f(\theta_t)$ to find a job in t conditional on losing a job at the end of $t-1$, which occurs with probability $s \cdot n_{t-1}$, plus the probability of keeping a job at the end of $t-1$, which occurs with probability $(1-s) \cdot n_{t-1}^s$.

where y_t is output of private good, a_t is technology, l_t is employment in the firm, and α is a parameter. The firm is owned by risk-neutral entrepreneurs with the same discount factor δ as workers. Given technology, wage, and labor market tightness $\{a_t, w_t, \theta_t\}_{t=0}^{+\infty}$, the *representative firm's problem* is to choose employment $\{l_t\}_{t=0}^{+\infty}$ to maximize expected profit

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \left\{ a_t \cdot y(l_t) - w_t \cdot l_t - \frac{r \cdot a_t}{q(\theta_t)} \cdot [l_t - (1-s) \cdot l_{t-1}] \right\}.$$

Since endogenous layoffs and quits never occur, $l_t - (1-s) \cdot l_{t-1} \geq 0$ is the number of hires in period t.[7] The first-order condition with respect to employment l_t is

$$a_t \cdot y'(l_t) = w_t + \frac{r \cdot a_t}{q(\theta_t)} - \delta \cdot (1-s) \cdot \mathbb{E}_t \left[\frac{r \cdot a_{t+1}}{q(\theta_{t+1})} \right], \tag{3}$$

which implies that the firm hires labor until until marginal product of labor $a_t \cdot y'(l_t)$ equals marginal cost of labor, which is the sum of wage w_t, plus hiring cost $r \cdot a_t / q(\theta_t)$, minus discounted cost of hiring next period $\delta \cdot (1-s) \cdot \mathbb{E}_t [r \cdot a_{t+1} / q(\theta_{t+1})]$.

2.5 Equilibrium

Wages follow an exogenous stochastic process and cannot equalize labor supply and labor demand. Instead, labor market tightness θ_t equilibrates labor demand $(l_t + g_t)$ to labor supply n_t^s each period:

$$l_t + g_t = n_t^s = n_t. \tag{4}$$

[7] There are no endogenous separations because the wage process is privately efficient. As in Michaillat [forthcoming], I can derive a sufficient condition for the wage process to be privately efficient.

Given a technology process $\{a_t\}_{t=0}^{+\infty}$, a wage process $\{w_t\}_{t=0}^{+\infty}$, a government policy $\{b_t, g_t\}_{t=0}^{+\infty}$, an *equilibrium with public sector* is a collection of stochastic processes $\{e_t,\ l_t,\ \theta_t\}_{t=0}^{+\infty}$ that solve the worker's problem (1), the firm's problem (3), and satisfy the equilibrium condition (4).

3 Efficient Allocation

This section solves the problem of a benevolent social planner who faces the technological constraints and labor market frictions present in the decentralized economy. The resulting allocation underscores that the welfare cost of recessions arises from the reduction in output and the increase in inequality due to unemployment.

3.1 Definitions

An *allocation* is a collection of stochastic processes $\{g_t, l_t, \theta_t, e_t, c_t^l, c_t^g, c_t^u, y_t, p_t\}_{t=0}^{+\infty}$ for public and private employment; labor market tightness; job-search effort; consumption in private jobs, public jobs, and unemployment; and output of private and public good; whose time-t element are measurable with respect to (a^t, l_{-1}, g_{-1}). A *feasible allocation* is an allocation that satisfies (i) the production constraint (2) for the private good; (ii) a production constraint for the public good:

$$p_t = \omega_p \cdot a_t \cdot y(g_t), \tag{5}$$

where ω_p is a parameter that scales productivity in the public sector relative to that in the private sector; (iii) the resource constraint in the economy, which imposes that the private good be either consumed or allocated to recruiting:

$$y_t = \left[l_t \cdot c_t^l + g_t \cdot c_t^g + (1-n_t) \cdot c_t^u\right] + \frac{r \cdot a_t}{q(\theta_t)} \cdot [n_t - (1-s) \cdot n_{t-1}]; \tag{6}$$

and (iv) the law of motion for aggregate employment

$$(1-s) \cdot n_{t-1} + u_t \cdot e_t \cdot f(\theta_t) = n_t, \tag{7}$$

where $n_t = l_t + g_t$. The *efficient allocation* is the feasible allocation that maximizes expected utility

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \left\{\chi \cdot \ln(pt) + \left[l_t \cdot \ln(c_t^l) + g_t \cdot \ln(c_t^g) + (1-n_t) \cdot \ln(c_t^u)\right] - [1-(1-s) \cdot n_{t-1}] \cdot k(e_t)\right\}.$$

3.2 Properties

The first critical property of the efficient allocation is that there is perfect insurance against unemployment risk. Since workers are risk averse, it is optimal that all workers consume the same amount c_t of private good each period: $c_t = c_t^l = c_t^g = c_t^u$.

The second critical property of the efficient allocation, formalized in Proposition 1, is that labor market variables (public and private employment, job-search effort, and labor market tightness) are constant over time, independent of the technology process. Therefore there are no cyclical fluctuations on the labor market in the efficient allocation. In other words, unemployment fluctuations over the business cycle are socially inefficient.

PROPOSITION 1. *In the efficient allocation, labor market variables* $\{g_t, l_t, \theta_t, e_t\}_{t=0}^{+\infty}$ *are deter-*

ministic, independent of the technology process $\{a_t\}_{t=0}^{+\infty}$. *These variables remain constant over time for an appropriate choice of initial employments* l_{-1} *and* g_{-1}. *The ratios of private consumption to technology* $\{c_t/a_t\}_{t=0}^{+\infty}$, *private output to technology* $\{y_t/a_t\}_{t=0}^{+\infty}$, *and public output to technology* $\{p_t/a_t\}_{t=0}^{+\infty}$ *share the same property.*

I give an overview of the proof to illuminate the key economic mechanisms behind this property. First, the resource and production constraints imply that the ratios of private consumption, private output, and public output to technology are only function of current labor market variables:

$$c_t/a_t = y(l_t) - s \cdot \frac{r}{q(\theta_t)} \cdot n_t$$

$$y_t/a_t = y(l_t)$$

$$p_t/a_t = \omega_p \cdot y(g_t).$$

Therefore, these ratios remain constant over time if labor market variables do. Labor market variables in the efficient allocation are four stochastic processes $\{g_t, l_t, \theta_t, e_t\}_{t=0}^{+\infty}$ characterized by four equations, which I present below. Since hiring a worker has the same marginal cost in the private and public sector, the marginal benefit from a worker must be equal in both sectors:

$$\omega_p \cdot \chi \cdot \frac{c_t/a_t}{p_t/a_t} = \frac{y'(l_t)}{y'(g_t)} = \left[\frac{g_t}{l_t}\right]^{1-\alpha}.$$

In particular, it is always optimal to employ some workers in the public sector as long as the public good is valuable ($\chi > 0$) and the government is productive ($\omega_p > 0$). The ratios p_t/a_t and c_t/a_t are only function of current labor market variables, so the efficient ratio of public to private employment g_t/l_t is only function of current labor market variables. Intuitively, the trade-off between private and public employment remains unchanged when technology falls because the productivities of public and private jobs fall in concert. Next, employments g_t and l_t, search effort e_t, and labor market tightness θ_t, are related by the law of motion of aggregate employment

$$(1-s)\cdot(l_{t-1}+g_{t-1})+[1-(1-s)\cdot(l_{t-1}+g_{t-1})]\cdot e_t\cdot f(\theta_t)=(l_t+g_t),$$

which does not involve technology. The marginal cost of effort must equal its marginal benefit:

$$k'(e_t)=\frac{\eta}{1-\eta}\cdot r\cdot\theta_t\cdot\frac{1}{c_t/a_t}.$$

The efficient search effort e_t is only function of current labor market variables. Intuitively when technology falls, the marginal benefit of search remains unaffected because the vacancy-posting cost, expressed in utility terms, is unchanged (the vacancy-posting cost expressed in private-good terms falls, but the marginal utility of private-good consumption increases by the same amount). Finally, efficient labor market tightness θ_t satisfies

$$(1-\eta)\cdot y'(l_t)=\frac{r}{q(\theta_t)}-\delta\cdot(1-s)\cdot\mathbb{E}_t\left[\frac{c_t/a_t}{c_{t+1}/a_{t+1}}\cdot\left\{\frac{r}{q(\theta_{t+1})}-\eta\cdot r\cdot\frac{\kappa}{1+\kappa}\cdot e_{t+1}\cdot\theta_{t+1}\right\}\right].$$

This relationship is similar to that in the canonical model of equilibrium unemployment, but for the factor $\kappa/(\kappa+1)\cdot e_{t+1}$ [Pissarides, 2000, Chapter 1]. It says that the marginal benefit from having workers search for jobs must equal the marginal benefit from having them produce goods. It depends only on labor market variables and not on technology. Intuitively, when technology falls public and private sectors become less productive, but recruiting also becomes less costly so that the trade-off between production and search remains unchanged. This last equation completes the characterization of the efficient allocation. This system of four equations does not involve the technology process $\{a_t\}_{t=0}^{+\infty}$. Thus, the efficient allocation is deterministic, and it remains constant over time for an appropriate choice of initial employments l_{-1} and g_{-1}.

3.3 Implementation

Since job-search effort is not observable, the government cannot provide unemployment insurance contingent on search effort. As effort enters negatively in the utility function, unemployed workers would have no incentive to search if they were fully insured against unemployment risk. Hence full insurance would imply no effort and no employment, which is suboptimal. Because of moral hazard, the government must provide incentives to search by reducing the consumption of unemployed workers. Thus the efficient allocation, which provides full insurance against unemployment risk, cannot be implemented in a decentralized economy. In fact two inefficiencies arise in the decentralized economy: the government does not provide perfect unemployment insurance and unemployment may be above its efficient level. This paper focuses on the reduction of unemployment. It complements the work of Landais et al. [2010], who study optimal unemployment insurance in a similar model.

4 Fiscal Multiplier

This section specializes the model of Section 2 to capture two key properties of recessions: (i) unemployment is higher in recessions; and (ii) jobs are rationed in recessions, as there is an acute job shortage irrespective of job search and matching frictions. The government uses fiscal policy in the form of spending on public-sector jobs to reduce unemployment. This policy is widely used in practice. In the US, public employment represents 16% of aggregate employment, and Figure 1 suggests that it is countercyclical. More systematically, the correlation between public employment and unemployment rate is 0.66 in the US for the 1970–2007 period [Gomes, 2010]. And since World War II, 75% of the US government consumption of goods and services consists of public-sector wages [Cavallo, 2005]. To guide the design of fiscal policy, I study the public-employment multiplier—the reduction in unemployment achieved by spending one unit of private good on public employment—over the business cycle.

4.1 Job rationing

I introduce recessions and job rationing in the generic model of Section 2 by making two assumptions on the production function and wage schedule, as in Michaillat [forthcoming].

ASSUMPTION 1. The production function has diminishing marginal returns to labor: $\alpha < 1$.

This assumption yields a downward-sloping demand for labor in the price θ-quantity n diagram, which has important macroeconomic implications. This assumption is motivated by the observation that, at business cycle frequency, some production inputs are slow to adjust.

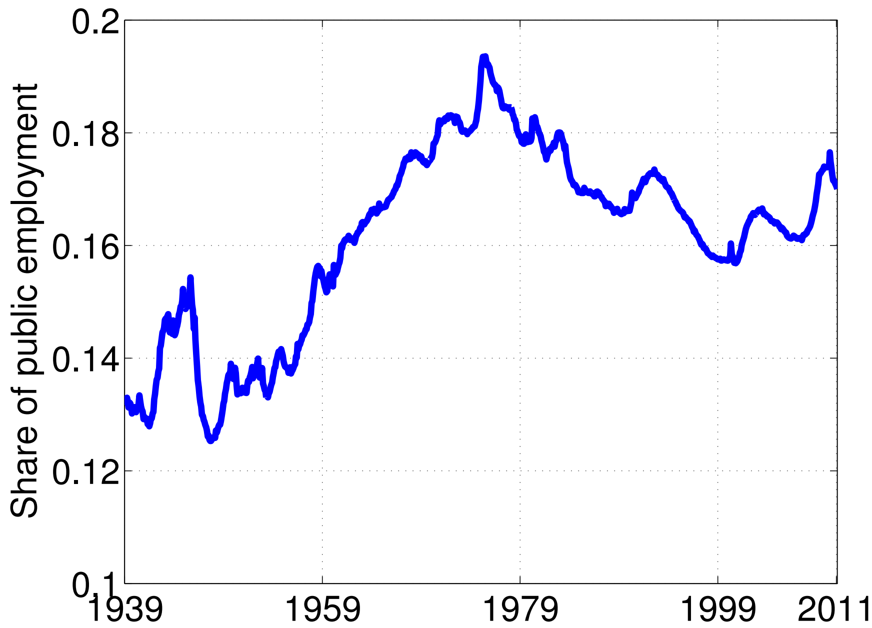

Figure 1: Share of public employment in total employment in the US, 1939–2011.

Notes: The data are seasonally-adjusted, monthly data from the Current Employment Survey (CES) collected by the Bureau of Labor Statistics (BLS) for the 1939–2011 period. Employment in the public sector is the employment level in the government super sector. Total employment is the employment level in the total nonfarm super sector.

I use the simple wage schedule from Blanchard and Galí [2010]: $w_t = w(a_t) = \omega \cdot a_t^{\gamma}$. The parameter γ captures wage rigidity over the business cycle. If $\gamma = 0$, wages are completely fixed over the cycle. If $\gamma = 1$, wages are proportional to technology and fully flexible over the cycle.

ASSUMPTION 2. The wage schedule is rigid: $\gamma < 1$.

Wages are rigid in the sense that they adjust only partially to technology shocks. This rigidity generates unemployment fluctuations over the business cycle [Hall, 2005]. Historical, ethnographic, and empirical studies document and explain the sources of wage rigidity [Bewley, 1999; Campbell and Kamlani, 1997; Doeringer and Piore, 1971; Jacoby, 1984; Kahn, 1997; O'Brien, 1989].

4.2 Static environment

A first step in the study of the public-employment multiplier over the business cycle is to characterize the equilibrium of the model in a static environment with no aggregate shocks ($a_t = a$ for all t), a constant government policy ($b_t = b$ and $g_t = g$ for all t), and a labor market in steady state (inflows to and outflows from unemployment are equal).[8] I focus on four labor market variables: effort e, private employment l, labor market tightness θ, and total employment n.

First, the utility-maximizing job-search effort e is related to labor market tightness θ by applying first-order condition (1) to a static environment:

$$[1-\delta\cdot(1-s)]\cdot\frac{k'(e)}{f(\theta)}+\kappa\cdot\delta\cdot(1-s)\cdot k(e)=\ln\left(\frac{1}{b}\right). \tag{8}$$

Equation (8) implicitly defines an increasing supply of effort $e(\theta)$, since the per-unit job-finding probability $f(\theta)$ increases with θ. In steady state inflows into unemployment $s\cdot n$ equal outflows from unemployment $[1-(1-s)\cdot n]\cdot e(\theta)\cdot f(\theta)$, determined by unemployed workers' search effort. Hence labor supply $n^s(\theta)$ can be expressed as a function of labor market tightness θ:

$$n^s(\theta)=\frac{e(\theta)\cdot f(\theta)}{s+(1-s)\cdot e(\theta)\cdot f(\theta)}. \tag{9}$$

$n^s(\theta)$ is a labor supply because it gives steady-state employment when jobseekers optimally choose search effort for a given tightness θ. Labor supply $n^s(\theta)$ increases with θ, because (i) higher θ increases the optimal provision of effort $e(\theta)$, increasing mechanically labor supply; and (ii) higher

[8] In search-and-matching models, comparing static environments delivers the same qualitative predictions as the study of a stochastic environment [Michaillat, forthcoming; Pissarides, 2009].

θ increases mechanically labor supply by increasing the per-unit job-finding probability $f(\theta)$.[9]

Second, the profit-maximizing private employment l is related to labor market tightness θ by applying first-order condition (3) to a static environment:

$$y'(l) = \frac{w}{a} + [1 - \delta \cdot (1 - s)] \frac{r}{q(\theta)}. \tag{10}$$

Equation (10) implicitly defines the demand for labor in the private sector $l(\theta; a)$. Under Assumption 1, $y'(l)$ decreases in l. Thus labor demand $l(\theta; a)$ decreases with labor market tightness θ, since the job-filling probability $q(\theta)$ decreases in θ. Intuitively when the labor market is slack, it is easy and cheap for firms to recruit, stimulating hiring. Under Assumption 2, the normalized wage $w/a = \omega \cdot a^{\gamma - 1}$ decreases with technology a, so labor demand $l(\theta; a)$ increases with a. Intuitively when technology is low, wages are relatively high, depressing hiring. I define aggregate labor demand as the sum of the labor demands in the private and public sectors:

$$n^d(\theta; a, g) = g + l(\theta; a). \tag{11}$$

Finally, labor market tightness θ acts as a price equilibrating labor supply and labor demand:

$$n^s(\theta) = n^d(\theta; a, g) \equiv n(a, g). \tag{12}$$

Equation (12) implicitly defines equilibrium labor market tightness $\theta(a, g)$, and equilibrium em-

[9]The labor supply curve can be interpreted as a Beveridge curve in this model with endogenous job search because it captures the condition that inflows into and outflows from unemployment are balanced on the labor market.

ployment $n(a,g)$. When technology a decreases, equilibrium labor market tightness $\theta(a,g)$ and equilibrium employment $n(a,g)$ decrease because aggregate labor demand $n^d(\theta;a,g)$ decreases.

For a given technology a and public employment g, the labor market equilibrium can be represented by a simple labor supply-labor demand diagram [Landais et al., 2010]. The equilibrium is depicted in Figure 2 in a price θ-quantity n diagram. This figure plots labor demand curves for high (left panel) and low (right panel) technology. Equilibrium employment $n(a,g)$ is given by the intersection of the downward-sloping labor demand curve $n^d(\theta;a,g)$ with the upward-sloping labor supply curve $n^s(\theta)$. Tightness $\theta(a,g)$ acts as a price equalizing labor supply and labor demand. If labor supply is above labor demand, a reduction in θ increases labor demand n^d by reducing recruiting costs; reduces labor supply n^s by reducing the per-unit job-finding probability as well as search efforts; until labor supply equals labor demand. When technology decreases in a recession, labor demand shifts inwards. The new equilibrium has higher unemployment and lower tightness.

Jobs are rationed in recessions in the sense that the labor market fails to clear and some unemployment remains even as unemployed workers exert arbitrarily large search efforts. This job shortage is depicted in Figure 2. After a negative technology shock, marginal product of labor falls but the rigid wage adjusts downwards only partially, so labor demand shifts inwards. If the adverse shock is large enough, the marginal product of the least productive workers falls below the wage. It is unprofitable for firms to hire these workers even if recruiting is costless at $\theta = 0$: labor demand cuts the x-axis at $n^R < 1$. Even if workers search infinitely hard, shifting labor supply outwards and pushing tightness θ to 0, firms never hire more than $n^R < 1$ workers: jobs are rationed.

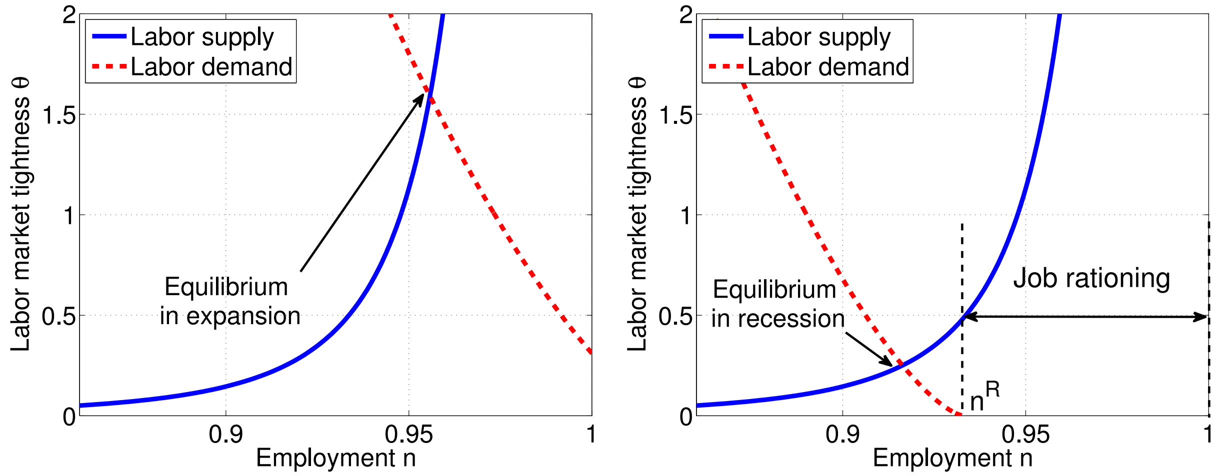

Figure 2: Labor market equilibrium in a price θ-quantity n diagram

4.3 Public-employment multiplier

In a static environment parameterized by technology a and public employment g, equilibrium is described by (12). Differentiating this equilibrium condition with respect to g yields

$$\frac{\partial n^s}{\partial \theta} \cdot \frac{\partial \theta}{\partial g} = \frac{\partial n^d}{\partial \theta} \cdot \frac{\partial \theta}{\partial g} + \frac{\partial n^d}{\partial g} = \frac{\partial n}{\partial g}.$$

I define the price-elasticities ε^s and ε^d of labor supply and labor demand:

$$\varepsilon^s = \frac{\theta}{n^s} \cdot \frac{\partial n^s}{\partial \theta} > 0,$$
$$\varepsilon^d = -\frac{\theta}{n^d} \cdot \frac{\partial n^d}{\partial \theta} > 0.$$

ε^d is normalized to be positive. The effect of public employment g on aggregate employment n is

$$\frac{\partial n}{\partial g} = \frac{\partial n^d}{\partial g} \cdot \frac{1}{1 + \left(\varepsilon^d / \varepsilon^s\right)}. \tag{13}$$

This formula says that the increase dn in aggregate employment following a small increase dg in public employment equals the direct increase in labor demand dn^d, attenuated by a factor $1/\left[1 + \left(\varepsilon^d/\varepsilon^s\right)\right] < 1$ that captures the reduction in labor demand due to the equilibrium increase in labor market tightness. In Figure 2, an increase dg in public employment leads to an outward shift in labor demand of amplitude dn^d. The shift in labor demand is accompanied by an increase in equilibrium labor market tightness to equilibrate labor supply with the higher level of labor demand. This increase in tightness leads to a movement along the labor demand curve that reduces employment by a factor $1/\left[1 + \left(\varepsilon^d/\varepsilon^s\right)\right] < 1$. This factor depends on the ratio of elasticities e^d/e^s because it measures the relative slope of labor demand and labor supply, which determines the reduction in employment due to the equilibrium adjustment in tightness. Finally, the public-employment multiplier λ_g is the increase in aggregate employment achieved by spending one unit of private good on public employment. The multiplier is obtained by dividing (13) by the per-period marginal cost of an increase in public employment $x_g = w(a) + [1 - \delta \cdot (1 - s)] \cdot r \cdot a/q(\theta)$:

$$\lambda_g = \frac{1}{x_g} \cdot \frac{\partial n^d}{\partial g} \cdot \frac{1}{1 + \left(\varepsilon^d / \varepsilon^s\right)}. \tag{14}$$

Studying the multiplier λ_g requires to study $\partial n/\partial g$. Labor demand (11) implies that $\partial n/\partial g = 1 + \partial l/\partial g$, so it is crucial to study the effect $\partial l/\partial g$ of public employment on private employment.

Using $\partial n^d/\partial g = 1$ as is evident from (11), formula (13) implies that

$$\frac{\partial l}{\partial g} = -\frac{1}{1+\left(\varepsilon^s/\varepsilon^d\right)}. \tag{15}$$

PROPOSITION 2. *$\partial l/\partial g < 0$ so $\partial n/\partial g < 1$. Under Assumption 1, $\partial l/\partial g > -1$ so $\partial n/\partial g > 0$.*

Proposition 2 shows that public employment crowds out private employment: $\partial l/\partial g < 0$. To recruit new workers in the public sector the government posts more vacancies, which raises equilibrium labor market tightness. Crowding out occurs because it becomes more costly for private firms to recruit workers, so they reduce employment. Intuitively the government recruits from the same pool of unemployed workers as private firms; hiring more workers in the public sector increases competition for unemployed workers, making recruiting more expensive for private firms; as private firms face higher marginal costs of labor, they reduce employment. Critically, Proposition 2 shows that when there are diminishing marginal returns to labor, public jobs crowd out private jobs strictly less than one-for-one: $|\partial l/\partial g| < 1$. In that case public employment increases total employment: $\partial n/\partial g = 1 + \partial l/\partial g > 0$ and the public-employment multiplier λ_g is positive.

Next, I study how the multiplier λ_g fluctuates over the business cycle, assuming public employment adjusts automatically to economic conditions such that the share g/n of public jobs in the labor market remains constant. Formally, I assume that public employment g is an implicit function of technology a: $g(a) = \zeta \cdot n(a, g(a))$, $\zeta \in [0,1)$. This assumption simplifies derivations because all equilibrium variables become implicit functions of a only. The central question is how crowding out $\partial l/\partial g$ varies over the business cycle. To answer this question, I compute price-elasticities ε^s and ε^d and perform comparative statics with respect to a. Using (8), I infer that the elasticity ε^e_θ of optimal search effort $e(\theta)$ with respect to labor market tightness θ is

$$\varepsilon^e_\theta = \frac{\theta}{e} \cdot \frac{\partial e}{\partial \theta} = \frac{1-\eta}{K(a)},$$

where $K(a) = \kappa \cdot \{1 + s \cdot [\delta \cdot (1-s)] / [1 - \delta \cdot (1-s)] \cdot n/u\}$ is procyclical. Labor supply (9) implies

$$\varepsilon^s = u \cdot [\varepsilon^e_\theta + (1-\eta)] = u \cdot (1-\eta) \cdot \left[1 + \frac{1}{K(a)}\right].$$

Thus the price-elasticity ε^s of labor supply is countercyclical. Next, I calculate the price-elasticity ε^d of labor demand. Equation (10) implies that the elasticity ε^l_θ of optimal private employment $l(\theta; a)$ with respect to tightness θ is

$$\varepsilon^l_\theta = \frac{\theta}{l} \cdot \frac{\partial l}{\partial \theta} = -\frac{\eta}{1-\alpha} \cdot [1 - \Omega(a)],$$

where $\Omega(a) = w(a)/mpl(a)$ characterizes the wedge between real wage $w(a)$ and marginal product of labor $mpl(a) = a \cdot y'(l)$ in the private sector. $\Omega(a)$ is countercyclical because in recessions the rigid wage is high relative to marginal productivity. Aggregate labor demand (11) implies

$$\varepsilon^d = (1-\zeta) \cdot \varepsilon^l_\theta = -\eta \cdot \frac{1-\zeta}{1-\alpha} \cdot [1 - \Omega(a)].$$

Hence, the price-elasticity ε^d of labor demand is procyclical.

PROPOSITION 3. *Assume that* $g(a) = \zeta \cdot n(a, g(a))$. *Under Assumptions 1 and 2:*

$$\frac{\partial\left[\partial l/\partial g\right]}{\partial a} < 0.$$

Proposition 3 says that when technology a falls in recessions, the amplitude $|\partial l/\partial g|$ of crowding out of private jobs by public jobs falls. Hence when unemployment is high, crowding out is low and public employment reduces unemployment effectively. Intuitively when jobs are rationed in recessions, competition among employers to recruit workers is weak: in particular, competition for workers from the public sector does not hinder job creation by private firms. Without much crowding out of the private sector, a public-employment policy is effective in recessions. But in expansions, crowding out is important and public employment is ineffective at reducing unemployment. Proposition 3 implies that the public-employment multiplier λ_g is countercyclical.

COROLLARY 1. *Assume that* $g(a) = \zeta \cdot n(a, g(a))$. *Under Assumptions 1 and 2,* $d\lambda_g/da < 0$.

Corollary 1 says that when technology a falls in recessions, the public-employment multiplier λ_g increases. As a result, spending one dollar on public-sector jobs reduces unemployment more effectively in recessions. The result comes mostly from the decrease in the crowding-out of private employment by public employment in recessions. In addition, it becomes cheaper to hire labor in recessions as wages and recruiting costs fall.

Figure 2 illustrates the mechanism behind Proposition 3. In recessions, an outward shift in aggregate labor demand following an increase in public employment leads to a small increase in equilibrium tightness θ and thus a small decrease in private employment $l(\theta; a)$. Crowding out is

Table 1: Parameter values in simulations (weekly frequency)

	Interpretation	Value	Source
δ	Discount factor	0.999	Corresponds to 5% annually
ζ	Share of public employment	0.16	CES, 2000–2010
s	Separation rate	0.94%	JOLTS, 2000–2010
b	UI benefit rate	60%	Pavoni and Violante [2007]
η	Effort-elasticity of matching	0.7	Petrongolo and Pissarides [2001]
γ	Real wage flexibility	0.5	Pissarides [2009], Haefke et al. [2008]
r	Recruiting cost	0.23	Barron et al. [1997], Silva and Toledo [2009]
ω_h	Effectiveness of matching	1.06	JOLTS, 2000–2010
α	Marginal returns to labor	0.67	Matches labor share of 0.66
ω	Steady-state real wage	0.71	Matches unemployment of 5.9%
κ	Elasticity of disutility of effort	0.59	Matches Krueger and Mueller [2010]
ω_k	Disutility of effort	47.9	Matches search intensity of 0.085

limited so the increase in aggregate employment is large. But in expansions, an outward shift in aggregate labor demand leads to a large increase in equilibrium tightness and a large decrease in private employment. Crowding out is important, the increase in aggregate employment is small.[10]

Collecting these results, I can provide an expression of the public-employment multiplier:

$$\lambda_g = \frac{1-\alpha}{1-\alpha+(1-\zeta)\cdot\frac{\eta}{(1-\eta)}\cdot\frac{K(a)}{1+K(a)}\cdot\frac{1-\Omega(a)}{u}}\cdot\frac{1}{mpl(a)}. \tag{16}$$

4.4 Calibration and numerical illustration

I calibrate the model to US data to quantity the cyclical fluctuations of the public-sector multiplier. I calibrate all parameters at a weekly frequency as summarized in Table 1. Following the strategy

[10]The crowding-out of private employment by public employment results from the competition for workers between private firms and government. In Quadrini and Trigari [2007] and Gomes [2010], the public sector influences the private sector though wages: higher wages in the public sector increase the outside option of workers, and leads through wage bargaining to higher wages in the private sector (and therefore lower private employment).

in Landais et al. [2010], I calibrate as many parameters as possible from micro-evidence for the US, and macro-data for the US for the December 2000–June 2010 period:[11] $\delta = 0.999$, $s = 0.0094$, $r = 0.32 \cdot \omega$, $\eta = 0.7$, $\gamma = 0.5$, $b = 60\%$. I calibrate the share of public employment in total employment ζ using seasonally-adjusted, monthly data from the Current Employment Survey (CES) collected by the BLS for the 2000-2010 period. Public employment is the employment level in the government super sector. Total employment is the employment level in the total nonfarm super sector. I find that on average $\zeta = 0.16$.

I normalize steady-state technology $\overline{a} = 1$. I target the average vacancy-unemployment ratio $\overline{v}/\overline{u} = 0.47$ and unemployment rate $\overline{u} = 5.9\%$ in the US for 2000–2010. I target the conventional labor share $\overline{ls} = 0.66$. Krueger and Mueller [2010] provide evidence on job-search intensity of unemployed workers in the US, modeling job-search intensity as time allocated to job-search activities.[12] In the American Time Use Surveys (ATUS) from 2003 to 2007, unemployed workers devote 41 minutes to job search on weekdays on average. Assuming a workday of 8 hours, the fraction of time spent on job search is $41/(8 \cdot 60) = 0.085$. I set $\overline{e} = 0.085$. I target labor market tightness $\overline{\theta} = \overline{v}/(\overline{u} \cdot \overline{e}) = 0.47/0.085 = 5.53$. The last target is the elasticity of job-search effort to UI benefit rate b, estimated by Krueger and Mueller [2010] at -1.6. I set $\overline{\varepsilon}^e_b = -1.6$.

I calibrate the six remaining parameters: production function parameters α, ω_p, utility function parameters ω_k, κ, effectiveness of matching ω_h, wage level ω, to match these steady-state

[11] This is the longest period during which the Job Openings and Labor Turnover Survey (JOLTS), collected by the BLS, is available. This survey is critical because it contains monthly series for job openings, job separations, and new hires. As in Landais et al. [2010], I use JOLTS series for all nonfarm industries. These series are appropriate because they includes all jobs in the (nonfarm) private sector and the public sector (federal, state, and local government).

[12] Job search activities include contacting a potential employer, calling visiting an employment agency, reading and replying to job advertisements, and job interviewing.

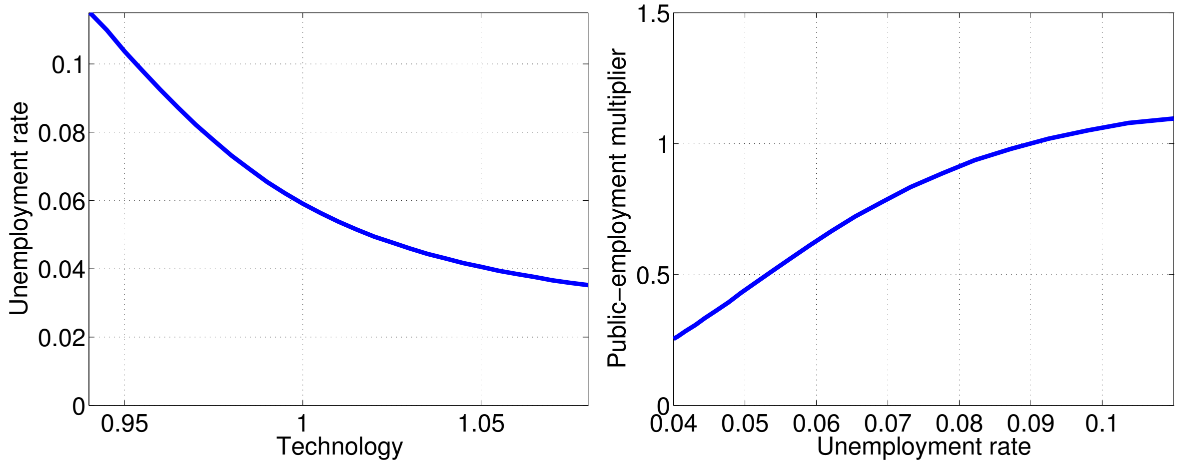

Figure 3: Public-employment multiplier

Notes: The left panel shows the relationship between technology and unemployment in the calibrated model. The right panel represents a measure of the public-employment multiplier, computed by multiplying equation (16) by the GDP, which is $a \cdot y(l) + w \cdot g - (r \cdot a)/q(\theta) \cdot (s \cdot n)$. The graph represents the increase (in percentage point) of employment rate achieved by spending 1% of GDP on public employment. Calibration is in Table 1.

targets estimated in the data. To calibrate matching effectiveness ω_h, I use the steady-state relationship $\omega_h = s/(1-s) \cdot [(1-\overline{u})/(\overline{e} \cdot \overline{u})] \cdot \overline{\theta}^{\eta-1}$, which yields $\omega_h = 1.06$. I calibrate the production function parameter α using the steady-state relation $\overline{ls} \equiv \left(\overline{w} \cdot \overline{l}\right)/\overline{y} = \omega \cdot \overline{l}^{1-\alpha}$, which implies that the firm's first-order condition (10) is $\overline{ls} \cdot \left([1-\delta \cdot (1-s)] \cdot 0.32/q(\overline{\theta}) + 1\right) = \alpha$, yielding $\alpha = 0.67$. The steady-state wage ω is related to steady-state labor share by $\omega = \overline{ls} \cdot \overline{l}^{\alpha-1}$, so $\omega = 0.71$ and recruiting cost $r = 0.32 \cdot \omega = 0.23$. In steady state, the elasticity of job search with respect to benefits (keeping labor market tightness constant) can be derived from the worker's first-order condition (8): $\overline{\varepsilon}^e_b = \partial \ln(e)/\partial \ln(b) \approx (\kappa + \overline{u})/[\kappa \cdot (1+\kappa)] \cdot 1/\ln(b)$, where the approximation is valid when $\delta \approx 1$. Solving for κ, I find $\kappa = 0.59$. Finally, I rewrite (8) as $\omega_k \cdot \left[(1-\delta \cdot (1-s)) \cdot \left\{\overline{e}^{\kappa}/f(\overline{\theta})\right\} + \delta \cdot (1-s) \cdot \{\kappa/(\kappa+1)\} \cdot \overline{e}^{\kappa+1}\right] = \ln(1/b)$ to calibrate $\omega_k = 47.9$.

Figure 3 illustrates numerically the theoretical results of Corollary 1. The left panel illustrates how unemployment fluctuates with technology. Recessions correspond to periods of low technology and high unemployment. The right panel plots the public-employment multiplier for a series of technology levels. I compute this multiplier by multiplying (16) by the Gross Domestic Product (GDP), which is $a \cdot y(l) + w \cdot g - (r \cdot a)/q(\theta) \cdot (s \cdot n)$. The graph represents the increase, measured in percentage points, of employment rate achieved by spending 1% of GDP on public employment. The cyclical fluctuations of the fiscal multiplier predicted in theory are large: the multiplier increases nearly fourfold from 0.3 to 1.1 when the unemployment rate increases from 4% to 11%.

4.5 Preliminary empirical evidence

The key theoretical result of this paper, proved in Corollary 1, is that the public-employment multiplier is countercyclical. Unfortunately empirical evidence about the behavior of fiscal multipliers over the business cycle is lacking, because studies estimating multipliers do not usually account for the state of the economy when fiscal measures are enacted [Parker, 2011]. Furthermore it is difficult to obtain precise estimates of recessionary fiscal multipliers, because few recessions occur in available data [Parker, 2011]. Finally, my theoretical results concern government spending on public-sector jobs whereas most studies lump together government spendings on goods and services, which include both spendings on public-sector jobs and spendings on goods purchases. The estimated effects of government spending on goods and services are comparable to my theoretical results only if fiscal expansions consist mostly of creation of public-sector jobs, which is plausible in the US as public-sector wages represent about 75% of government spending on goods and services and public employment is quite countercyclical [Cavallo, 2005; Gomes, 2010]. While more empirical work is required to reach a consensus about the effects of fiscal policies over the business cycle, recent studies suggest that fiscal multipliers may be countercyclical as in Corollary 1.

The most compelling evidence comes from Auerbach and Gorodnichenko [2010], who use a regime-switching structural vector autoregression (SVAR) model to capture fluctuations in fiscal multipliers over the business cycle. They find that fiscal multipliers are much larger in recessions than in expansions. Nakamura and Steinsson [2011] investigate the effect of government spending in US states during military build-ups on state employment, and find that the effect is roughly twice larger in high-unemployment than in low-unemployment periods. Last, Shoag [2011] shows that the positive effect of state government spending, instrumented using state pension-plan returns, on state unemployment is larger during times of high unemployment.[13]

Quantitatively, Figure 3 suggests that the public-employment multiplier is in the 0.3–1.1 range, and is 0.6 for an unemployment rate of 6%. Monacelli et al. [2010] use a SVAR framework to estimate the effects of fiscal policy on the US labor market for 1954–2006. They find that an increase in government spending on goods and services by 1% of GDP increases the employment rate by 0.6 percentage point at the peak, in line with my simulations. Using a SVAR with alternative identification restrictions, Pappa [2010] studies the transmission of fiscal shocks in the labor market for several developed economies. In US data for 1970–2008, she finds that the peak response to a 1% increase in government spending on public employment is a 0.15% increase in employment.[14] On average, employment rate is 94% and spending on public employment is 10% of GDP in the

[13]On the other hand Canova and Pappa [forthcoming], using a SVAR framework with additional restrictions to capture recessionary periods, argue that fiscal multipliers in recessions are unlikely to be larger than those obtained on average over the business cycle. They identify recessions in the data as periods when government deficit is large and the nominal interest cannot respond to shocks (because it is at the zero lower bound).

[14]This result also holds at the state level in the US. Using state-level data for the US for 1969–2001, Pappa [2009] finds that for a typical state, a 1% increase in government spending on public employment leads to a 0.15% increase in employment at the peak.

US for the 1970–2008 period. Therefore the employment rate increases by 1.4 percentage points in response to an increase of spending on public employment by 1% of GDP. This public-employment multiplier of 1.4 is in line, albeit slightly above, my numerical results.[15]

5 Robustness

This section delimits the domain where fiscal policy can effectively reduce recessionary unemployment. First, public employment is not effective in search-and-matching models without job rationing such as (i) the canonical Pissarides [2000] model with Nash bargaining over wages; and (ii) the Hall [2005] model with rigid wages. Second, fiscal policies stimulating labor supply, such as monitoring unemployed workers' job search, have virtually no effect on unemployment when jobs are rationed in recessions. Last, other fiscal policies stimulating labor demand are effective in recessions: the *wage-subsidy multiplier*—the reduction in unemployment rate achieved by spending one unit of private good on a wage subsidy—is also positive and countercyclical.

5.1 Public employment in the absence of job rationing

Proposition 2 implies that the public-employment multiplier is positive in a model with job rationing. But fiscal policy in the form of public employment is not effective in any model of unemployment. In standard search-and-matching models, such as the Pissarides [2000] model with

[15] My model assumes that labor force participation does not respond to fiscal policy, such that an increase in employment translates into a reduction in unemployment. This assumption is supported by Monacelli et al. [2010], who find that fiscal policy does not affect labor force participation. Bruckner and Pappa [forthcoming] point out, however, that the response of labor market variables to fiscal policy seems to change over time in the US. For 1964–2009 they find that, in response to an increase in government spending, employment increases but unemployment also increases because of an increase in labor force participation. Their result holds in a number of OECD countries.

Nash bargaining or the Hall [2005] model with rigid wages, public jobs crowd out private jobs one-for-one and the public-employment multiplier is nil. In these models there is no job rationing. Infinitely many jobs are available, and the only hindrance to full employment is not a lack of jobs but matching frictions [Michaillat, forthcoming]. Hence public employment cannot reduce unemployment and simply replaces private jobs one-for-one with public jobs.

To capture the main features of the Pissarides [2000] model, I modify the model of Section 4.1 by assuming that (i) the production function is linear: $\alpha = 1$; and (ii) wages are determined by Nash bargaining. In a static environment parameterized by technology a and public employment g, the equilibrium $\{e(a,g), l(a,g), \theta(a,g)\}$ on the labor market is characterized by three equations: (8), (10), and (12) that can be conveniently rewritten as:

$$l + g = \frac{e \cdot f(\theta)}{s + (1-s) \cdot e \cdot f(\theta)}. \tag{17}$$

To determine equilibrium wage $w(a,g)$ I solve in the Appendix the bargaining problem between worker and firm using the generalized Nash solution, which adds an extra equilibrium condition:

$$\frac{1-\beta}{\beta} \cdot \ln\left(\frac{1}{b}\right) = [1 - \delta \cdot (1-s)] \cdot \frac{r}{q(\theta)} \left[1 + \frac{1-\beta}{\beta} \cdot \ln\left(\frac{1}{b}\right)\right] + \delta \cdot (1-s) \cdot \frac{\kappa}{1+\kappa} \cdot e \cdot r \cdot \theta. \tag{18}$$

The equilibrium system of four equations {(8), (10), (17), (18)} and four variables $\{e, l, \theta, w/a\}$ is independent of technology a. Thus fluctuations in technology do not lead to any variations in labor market variables $\{e, l, \theta\}$. Only the equilibrium wage is fully flexible and varies in proportion to technology. Consequently there are no periods when unemployment is above average, so there are no recessions. Furthermore, public employment has no effect on aggregate employment. Variables $\{e, \theta\}$ are fully determined by equations $\{$(8), (18)$\}$, independent of public employment g. Equilibrium condition (17) implies that aggregate employment $n = l + g$ is also independent of public employment g. When public employment increases by dg, it reduces private employment by $dl = -dg$. Hence the public-employment multiplier $\lambda_g = 0$. On Figure 2 labor demand is horizontal because the production function is linear so labor demand (10) is fully elastic. Hence labor demand is unaffected by an increase in public employment, as is equilibrium employment.

To capture the main features of the Hall [2005] model, I modify the model of Section 4.1 by assuming that the production function is linear: $\alpha = 1$. I maintain rigid wages. This model generates large employment fluctuations but does not exhibit job rationing [Michaillat, forthcoming]. The expression (16) for the public-employment multiplier applies to this model by plugging in $\alpha = 1$. It implies that the multiplier $\lambda_g = 0$. In Figure 2, labor demand (10) is fully elastic and horizontal.

5.2 Fiscal policies stimulating labor supply

To stimulate employment, governments often try to improve matching on the labor market by monitoring unemployed workers' job search: a placement agency was implemented in Germany by the "Hartz reform" from 2003 to 2005 to advise and monitor jobseekers [Fahr and Sunde, 2009]; monitoring has been implemented in the US under the Temporary Assistance for Needy Families (TANF) program started in 1996, which specifies that unemployed recipients must participate at least to 20 hours of monitored job search per week [Pavoni and Violante, 2007]; and the "New Deal", a program for all unemployed workers introduced in the UK in 1998, includes job-search monitoring during compulsory regular meetings with an adviser [Van Reenen, 2003].

The Appendix adds job-search monitoring to the model of Section 4.1. Unemployed workers are monitored with probability π_t, and those who are not searching when monitored receive only a fraction $z_t < 1$ of UI benefits. By increasing the monitoring probability π_t, the government increases search incentives and reduces unemployment by stimulating labor supply. In a static environment, the utility-maximizing choice of search effort e becomes

$$[1-\delta\cdot(1-s)]\cdot\frac{k'(e)}{f(\theta)}+\delta\cdot(1-s)\cdot\kappa\cdot k(e)=\ln(1/b)+\pi\cdot\ln(1/z)\cdot\left[\delta\cdot(1-s)+\frac{1-\delta\cdot(1-s)}{f(\theta)}\right].$$

Compared to (8), there is an additional term on the right-hand side that is positive for a monitoring probability $\pi > 0$. If $\pi > 0$, effort $e(\theta)$, labor supply $n^s(\theta)$, and equilibrium employment $n(a,g)$ are higher. I assume that the per-period marginal cost of monitoring u unemployed workers is $x_\pi = u\cdot m$ in units of private good, where m is a parameter.[16] In a static environment parameterized by technology a and monitoring π, the labor market equilibrium is characterized by a condition similar to (12), saying that tightness acts as a price equilibrating labor demand and labor supply:

$$n^d(\theta;a)=n^s(\theta;\pi)\equiv n(a,\pi).$$

I differentiate this equilibrium condition with respect to π, and proceed as in Section 4.3. The

[16]The resource cost of monitoring includes administrative costs, time spent interviewing jobseekers, or time spent verifying worker's search efforts.

effect of monitoring on aggregate employment is

$$\frac{\partial n}{\partial \pi} = \frac{\partial n^s}{\partial \pi} \cdot \frac{1}{1 + \left(\varepsilon^s / \varepsilon^d\right)}. \tag{19}$$

The *monitoring multiplier* λ_π, which is the increase in aggregate employment achieved by spending one unit of private good on monitoring, is given by

$$\lambda_\pi = \frac{1}{x_\pi} \cdot \frac{\partial n^s}{\partial \pi} \cdot \frac{1}{1 + \left(\varepsilon^s / \varepsilon^d\right)}. \tag{20}$$

While public employment stimulates aggregate labor demand n^d, monitoring stimulates labor supply n^s. As a consequence the monitoring multiplier (20) is increasing in the ratio of elasticities $\varepsilon^d / \varepsilon^s$, whereas the public-employment multiplier (14) is increasing in the ratio of elasticities $\varepsilon^s / \varepsilon^d$. Section 4.3 shows that ratio $\varepsilon^s / \varepsilon^d$ and public employment multiplier are countercyclical. Since the ratio $\varepsilon^d / \varepsilon^s$ is procyclical, the monitoring multiplier tends to be procyclical, and all the more so as the marginal monitoring cost x_π is countercyclical. The behavior of $\partial n^s / \partial \pi$ is more difficult to characterize, but the Appendix proves that the monitoring multiplier λ_π increases with technology when technology is below some threshold. As technology falls below this threshold, the monitoring multiplier decreases. In particular, the monitoring multiplier is low in recessions.

In recessions, increasing unemployed workers' search effort through higher monitoring has little desirable effect on unemployment because jobs are rationed and job search does not matter much. In the same way, having unemployed workers search harder by reducing the generosity of unemployment insurance has little desirable effect on unemployment in Landais et al. [2010]. On the other hand in expansions, an increase in search efforts reduces unemployment significantly. Thus the monitoring multiplier decreases when technology decreases. This theoretical result mirrors Proposition 3 in Landais et al. [2010], which proves that the macro-elasticity of unemployment with respect to net reward from work is procyclical.

To verify that the monitoring multiplier is procyclical for a plausible range of technology and unemployment, Figure 4 plots the multiplier as a function of unemployment in the calibrated model. As in Figure 3, the multiplier is multiplied by GDP to represent the increase, measured in percentage point, of employment rate achieved by spending 1% of GDP on monitoring. Cyclical fluctuations of the multiplier are large: it decreases nearly sixfold, from 1.8 to 0.3, when the unemployment rate increases from 4% to 11%. Quantitatively, this decline is similar to the decline of the macro-elasticity of unemployment plotted in Figure 3 in Landais et al. [2010]. This is because both monitoring multiplier and macro-elasticity capture the increase in aggregate employment achieved by stimulating labor supply in a model of equilibrium unemployment with job rationing.

5.3 Wage and recruiting subsidies

Empirical evidence suggests that tax cuts are expansionary [Romer and Romer, 2010]. In the model of Section 4.1 pre-tax wages are rigid so a wage subsidy, which could be implemented as a payroll tax cut, necessarily reduces unemployment. This section analyzes the effect of a wage subsidy on employment over the business cycle, captured by a wage-subsidy multiplier. This section shows that the two key findings of Section 4—a positive and countercyclical fiscal multiplier—also hold for a wage subsidy. In fact, there is a tight link between public-employment and wage-subsidy

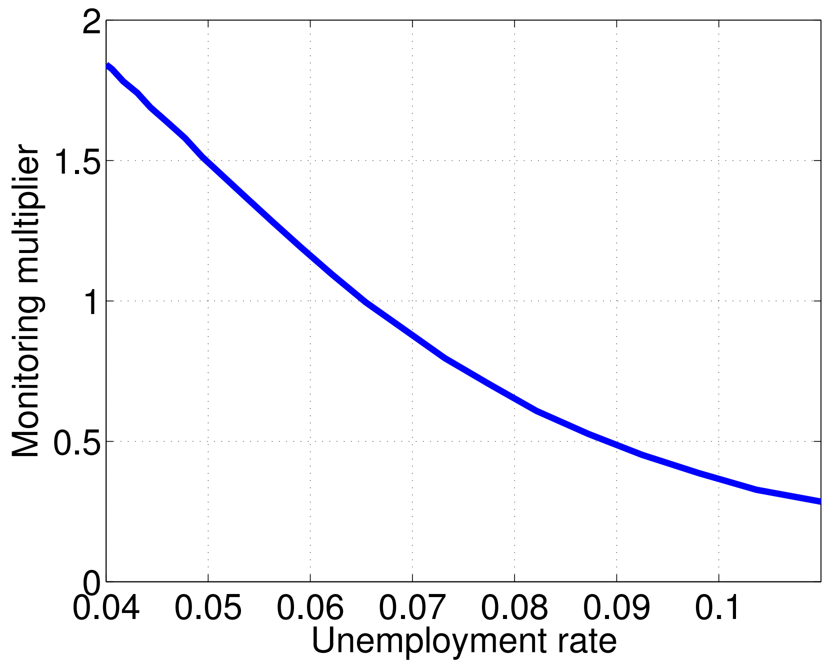

Figure 4: Monitoring multiplier

Notes: This graph represents the monitoring multiplier computed by multiplying equation (A20) by the GDP, which is $a \cdot y(l) + w \cdot g - (r \cdot a)/q(\theta) \cdot (s \cdot n)$. The graph represents the increase (in percentage point) of employment rate achieved by spending 1% of GDP on monitoring job search. Calibration is in Table 1. In addition, I set the monitoring cost to $m = 0.63$ and the punishment to $z = 43\%$ (details of the calibration are in the Appendix).

multipliers because both policies stimulate aggregate labor demand.

This analysis has policy implications because payroll tax cuts on the employer's side are a common measure to stimulate employment. In the US, the Hiring Incentives to Restore Employment Act was approved in 2010 to exempt businesses from Social Security payroll taxes for newly hired worker [Kitao et al., 2010]; the American Jobs Act presented to Congress in 2011 also includes payroll tax cuts for firms from 6.2% to 3.1%, for a cost of $70 billion.

The government provides a wage subsidy τ to reduce the marginal cost of labor. In a static environment the profit-maximizing level of private employment l becomes

$$y'(l) = (1-\tau)\cdot\frac{w}{a} + [1-\delta\cdot(1-s)]\cdot\frac{r}{q(\theta)}.$$

The wage-subsidy multiplier λ_τ is the increase in aggregate employment achieved by spending one unit of private good on a wage subsidy. The per-period marginal cost of a wage subsidy is $x_\tau = w(a)\cdot l$. Since a wage subsidy stimulates aggregate labor demand like public employment, the methodology of Section 4.3 remains valid and the wage-subsidy multiplier is

$$\lambda_\tau = \frac{1}{x_\tau}\cdot\frac{\partial n^d}{\partial\tau}\cdot\frac{1}{1+\left(\varepsilon^d/\varepsilon^s\right)}. \tag{21}$$

In the Appendix, I show that $\left(\partial n^d/\partial\tau\right) = \partial l/\partial\tau = 1/(1-\alpha)\cdot(w(a)\cdot l)/mpl(a) = \left[\left(\partial n^d/\partial g\right)/x_g\right]\cdot x_\tau/(1-\alpha)$. Comparing (21) to (14), it becomes clear that the wage-subsidy multiplier λ_τ is related to the public-employment multiplier λ_g by

$$\lambda_\tau = \frac{1}{1-\alpha}\cdot\lambda_g.$$

Since the methodology of Section 4.3 applies to any fiscal policy stimulating labor demand, the multipliers for these policies are all very similar: differences only come from the ratio of the direct effect of the policy on labor demand (say, $\partial n^d/\partial\tau$) to the marginal cost of the policy (say, x_τ). These ratios are proportional for public employment and wage subsidy, so λ_τ and λ_g are proportional.

The cyclical fluctuations of the wage-subsidy multiplier λ_τ are therefore identical to those of the public-employment multiplier λ_g. The key finding from Corollary 1 remains: the wage-subsidy multiplier is countercyclical. A wage subsidy reduces the marginal cost of labor, which leads firms to increase employment; higher aggregate employment increases labor market tightness and recruiting costs until a new equilibrium is reached, at which point the new marginal cost of labor equals the marginal product of labor. In recessions when jobs are rationed, recruiting costs are low and do not vary much with employment so a wage subsidy triggers a large increase in employment; in expansions, recruiting costs are high and increase rapidly with employment so a wage subsidy only achieves a small increase in employment; thus, a wage subsidy is more effective in recessions.

The Appendix relates the wage-subsidy multiplier λ_τ to a *recruiting-subsidy multiplier*, also commonly used in practice [Kluve, 2006]. I assume that the government covers a fraction $\vartheta \in [0,1]$ of the recruiting cost $r \cdot a/q(\theta)$ paid by firms for each new hire. The relation between wage-subsidy and recruiting-subsidy multipliers is very simple: $\lambda_\vartheta = [1 - \delta \cdot (1-s)]/s \cdot \lambda_\tau$. If $\delta \approx 1$, then $\lambda_\vartheta \approx \lambda_\tau$. These subsidies are broadly equivalent. They have the same effect on employment because they both consist in a transfer to firms for each marginal hire. The way the transfer is paid (through wages or recruiting costs) is irrelevant: what matters is that firms face a lower marginal cost of labor. The recruiting-subsidy multiplier is obviously countercyclical.

6 Towards the Design of Optimal Fiscal Policies

I conclude by discussing the implications of the paper for the design of fiscal policies, as well as the limitations to overcome to characterize fully welfare-maximizing fiscal policies. Designing optimal fiscal policies requires joint empirical and theoretical efforts to measure the effectiveness of fiscal policies in the data, and design fiscal policies maximizing social welfare subject to the government's budget constraint. This section is articulated along the empirical and theoretical challenges to designing optimal fiscal policies.

6.1 Estimating fiscal multipliers

On the empirical front, a critical implication of this paper is that the empirical estimates of fiscal multipliers reported in the literature, which are averages over all possible states of the economy, do not apply in recessions because government spending and tax multipliers vary widely over the business cycle.[17] Since recessionary multipliers are very different from average multipliers, average multipliers fail to characterize the effectiveness of a fiscal expansion in recessions. They do not convey much useful information to policymakers. Future studies should aim to estimate multipliers over the business cycle instead of average multipliers. As pointed out by Parker [2011], estimating state-dependent multipliers may require to upgrade our empirical methods to move beyond the linear dynamics characterized by VAR and linearized DSGE models.

[17] Ramey [2011] provides a comprehensive survey of this literature. There is also a large literature in labor economics that estimates the effectiveness of active labor market policies, designed to help unemployed workers find a new job [Card et al., 2009; Heckman et al., 1999]. These papers study microeconomic instead of macroeconomic outcomes: they estimate how much the unemployment duration of a treated unemployment worker decreases, and not how much local unemployment falls when a program is implemented. Hence these results are not directly relevant for the design of macroeconomic policy: as pointed out by Landais et al. [2010] in the context of unemployment insurance, these micro-estimates may differ greatly from macro-estimates because of general-equilibrium effects arising in a labor market with job rationing.

6.2 Characterizing optimal fiscal policies

On the theoretical front, this paper proposes an equilibrium unemployment framework in which jobs are rationed in recessions. The framework can be used to study the effects of government spending and taxes on unemployment over the business cycle. The cyclical behavior of multipliers suggests that a desirable fiscal policy likely stimulates aggregate labor demand in recessions, for instance through public employment. However, the multipliers provide only a partial picture of the welfare effects of fiscal policies. Characterizing optimal fiscal policy requires to maximize social welfare subject to the government's budget constraint. This paper is a first attempt at studying fiscal policy in recessions theoretically, and my simple model should be extended to study welfare-maximizing fiscal policies over the business cycle.[18]

First, the only source of recessions in the model are technology shocks. Future work should explore how other shocks, in particular aggregate demand shocks or financial disturbances, influence the effectiveness of fiscal policies. For instance if firms were constrained by a low and inelastic goods demand in recessions, then a wage subsidy would be completely ineffective.

Second, the rigid wage schedule specified in the model, while theoretically and empirically

[18] A large literature studies welfare-maximizing labor market policies in search-and-matching models, but there are no aggregate shocks and no business cycles in these models. Boone et al. [2007] solve for optimal unemployment insurance when job-search effort is unobservable, and examine whether the optimal policy involves monitoring of search effort and benefit sanctions. Kitao et al. [2010] analyze the effects of various hiring subsidies to stimulate hiring on equilibrium unemployment in the model of Mortensen and Pissarides [1994]. Mortensen and Pissarides [2002] investigate the effects of a linear payroll tax, a job destruction tax and unemployment compensation on labor market variables and characterize the optimal policy, which corrects search externalities. Hungerbühler et al. [2006] study the optimal tax policy in a search-and-matching model with a continuum of unobservable productivity levels. Gomes [2010] studies optimal public-employment policy in the Pissarides [2000] model. Gomes [2010] does examine the properties of optimal public employment when the economy is subject to technology shocks. However as acknowledged by the author, there are no recessions in his model: after a 1% negative productivity shock, the unemployment rate increases at most by 0.05 percentage points, 10 times less than in the data. This lack of amplification in a search-and-matching model with Nash bargaining was first pointed out by Shimer [2005]: this model is unable to generate enough fluctuations in unemployment in response to technology shocks because bargained wages are too flexible.

valid, does not explain where wage rigidity comes from. It is critical to design a micro-founded wage-setting mechanism to understand the effects of labor market policies on employment through wages. For instance this mechanism could explain the influence of a wage subsidy on the after-tax wage paid by employers, or the influence of public-sector wages on private-sector wages. Another issue to resolve is whether real or nominal wages are rigid. The econometric studies used to calibrate the model exploit micro-data for wages received by existing workers and new hires to show that real wages are somewhat rigid over the business cycle [Haefke et al., 2008; Martins et al., 2010; Pissarides, 2009]. But ethnographic and econometric evidence suggests that nominal wages may be more rigid than real wages [Bewley, 1999; Campbell and Kamlani, 1997; Card and Hyslop, 1996; Kahn, 1997]. If that is the case, monetary policy could be used instead of fiscal policy to tackle unemployment. If wages exhibit some nominal rigidity, some inflation would erode real wages and stimulate employment [Akerlof et al., 1996].

Finally to characterize the welfare-maximizing amount of employment in the public sector, it is necessary to quantify the welfare effects of government production. A large literature studies the impact of government consumption and government capital on economic outcomes and social welfare, with mixed results [Aschauer, 1989; Evans and Karras, 1994; Garcia-Milà and McGuire, 1992; Holtz-Eakin, 1993; Ratner, 1983]. More work is needed in this direction, for instance to assess the interaction of government capital with private production through better infrastructure, better institution, or better education.

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Appendix – Not for Publication

A Proofs

A.1 Notations

I assume that independent of technology a, the government follows constant policies. So the UI benefit rate b remain constant, and the ratio of public to aggregate employment $\zeta = g/n$. Accordingly a fully characterize the state of the economy in a static environment. I define the following functions:

- Labor supply: $n^s(e,\theta)$ is increasing in e and θ, and is defined by

$$n^s(e,\theta) = \frac{e \cdot f(\theta)}{s + (1-s) \cdot e \cdot f(\theta)}.$$

- Labor demand: for $\alpha < 1$, $l^d(\theta,a)$ is increasing in a, decreasing in θ, and is defined by

$$l^d(\theta,a) = \left[\frac{1}{\alpha}\left(\omega \cdot a^{\gamma-1} + \frac{r}{q(\theta)}\right)\right]^{1/(\alpha-1)}.$$

- Effort supply: $e^s(\theta,b)$ is increasing in θ and Δv, and is defined implicitly by

$$[1-\delta\cdot(1-s)] \cdot \frac{k'(e^s}{f(\theta)} + \kappa \cdot \delta \cdot (1-s) \cdot k(e^s) = \ln\left(\frac{1}{b}\right).$$

- Equilibrium labor market tightness: $\theta(a)$ is defined implicitly by

$$\frac{1}{1-\zeta} \cdot l^d(\theta,a) = n^s\left(e^s(\theta),\theta\right). \tag{A1}$$

- Equilibrium effort: $e(a)$ is defined by

$$e(a) = e^s(\theta(a)).$$

- Equilibrium private employment: $l(a)$ is defined by

$$l(a) = l^d(\theta(a),a).$$

- Equilibrium public employment: $g(a)$ is defined by

$$g(a) = \frac{\zeta}{1-\zeta} \cdot l(a).$$

- Equilibrium aggregate employment: $n(a)$ is defined by

$$n(a) = n^s(e(a), \theta(a)).$$

- Equilibrium unemployment: $u(a)$ is defined by

$$u(a) = 1 - (1-s) \cdot n(a).$$

- Equilibrium marginal product of labor: $mpl(a)$ is defined by

$$mpl(a) = a \cdot y'(l(a)) = w(a) + [1 - \delta \cdot (1-s)] \cdot \frac{r \cdot a}{q(\theta(a))}.$$

- Wedge between wage and marginal product of labor: $\Omega(a)$ is defined by

$$\Omega(a) == \frac{w(a)}{mpl(a)} = 1 - [1 - \delta \cdot (1-s)] \cdot \frac{r}{q(\theta(a))} \cdot l(a)^{1-\alpha}, \quad \text{(A2)}$$

- A measure of the elasticity of job-search effort to labor market condition:$K(a)$ is defined by

$$K(a) = \kappa \cdot \left[1 + s \cdot \frac{\delta \cdot (1-s)}{1 - \delta \cdot (1-s)} \cdot \frac{n(a)}{u(a)}\right]. \quad \text{(A3)}$$

A.2 Some preliminary comparative-static results

LEMMA A1. *If* $\gamma \in [0,1)$ *and* $\alpha \in (0,1)$*, equilibrium variables satisfy:*

$$\frac{\partial \theta}{\partial a} > 0, \quad \frac{\partial e}{\partial a} > 0, \quad \frac{\partial n}{\partial a} > 0, \quad \frac{\partial l}{\partial a} > 0, \quad \frac{\partial u}{\partial a} < 0.$$

Proof. Effort and labor supply satisfy:

$$\frac{de^s}{d\theta} > 0, \quad \frac{\partial n^s}{\partial \theta} > 0, \quad \frac{\partial n^s}{\partial e} > 0. \quad \text{(A4)}$$

If $\gamma \in [0,1)$ and $\alpha \in (0,1)$, then labor demand satisfy:

$$\frac{\partial l^d}{\partial \theta} < 0, \frac{\partial l^d}{\partial a} > 0.$$

Differentiating equilibrium condition (A1) with respect to a yields:

$$(1-\zeta)\cdot\left[\frac{\partial n^s}{\partial e}\cdot\frac{\partial e^s}{\partial\theta}+\frac{\partial n^s}{\partial\theta}\right]\cdot\frac{\partial\theta}{\partial a}=\frac{\partial l^d}{\partial a}+\frac{\partial l^d}{\partial\theta}\cdot\frac{\partial\theta}{\partial a}$$

$$\frac{\partial\theta}{\partial a}=\underbrace{\frac{\partial l^d}{\partial a}}_{+}\cdot\left[(1-\zeta)\cdot\left\{\underbrace{\frac{\partial n^s}{\partial e}}_{+}\cdot\underbrace{\frac{\partial e^s}{\partial\theta}}_{+}+\underbrace{\frac{\partial n^s}{\partial\theta}}_{+}\right\}-\underbrace{\frac{\partial l^d}{\partial\theta}}_{-}\right]^{-1}.$$

Thus $\frac{\partial\theta}{\partial a}>0$. I conclude by using the comparative statics (A4) and noting that $e(a)=e^s(\theta(a))$, $n(a)=n^s(e(a),\theta(a))$, $u(a)=1-(1-s)\cdot n(a)$, and $l(a)=(1-\zeta)\cdot n(a)$. □

A.3 Proof of Proposition 1

The proof first derives the first-order necessary conditions for the efficient allocation. It then follows the argument highlighted in Section 3.2. Since workers are risk-averse, the efficient allocation provides full insurance against unemployment: $c_t^l=c_t^g=c_t^u=c_t$. Accordingly, the Lagrangian of the government's problem is

$$\begin{aligned}L=\sum_{t=0}^{+\infty}\delta^t\cdot\Big\{&\chi\cdot\ln(a_t\cdot\omega_p\cdot y(g_t))+\ln(c_t)-[1-(1-s)n_{t-1}]\cdot k(e_t)\\&+\mu_t^1\cdot\{a_t\cdot y(l_t)-c_t-\frac{r\cdot a}{q(\theta_t)}\cdot\{n_t-(1-s)\cdot n_{t-1}\}\\&+\mu_t^2\cdot\{(1-s)\cdot(1-e_t\cdot f(\theta_t))\cdot n_{t-1}+e_t\cdot f(\theta_t)-n_t\}\\&+\mu_t^3\cdot\{n_t-l_t+g_t\}\Big\}.\end{aligned}$$

where $\{\mu_t^1,\mu_t^2,\mu_t^3\}_{t=0}^{+\infty}$ are Lagrange multipliers. The first-order conditions with respect to $\{c_t,e_t,\theta_t,n_t,l_t,g_t\}_{t=0}^{+\infty}$ yield respectively

$$1/c_t=\mu_t^1 \tag{A5}$$

$$u_t\cdot k'(e_t)=\mu_t^2\cdot u_t\cdot f(\theta_t) \tag{A6}$$

$$\mu_t^2\cdot e_t\cdot q(\theta_t)\cdot u_t=r\cdot a\cdot\frac{\eta}{1-\eta}\cdot\frac{1}{f(\theta_t)}\cdot\mu_t^1\cdot h_t \tag{A7}$$

$$\mu_t^1\cdot\frac{r\cdot a_t}{q(\theta_t)}+\mu_t^2=\mu_t^3+\delta\cdot(1-s)\cdot\mathbb{E}_t\left[\mu_{t+1}^1\cdot\frac{r\cdot a_{t+1}}{q(\theta_{t+1})}+\mu_{t+1}^2\cdot[1-e_{t+1}\cdot f(\theta_{t+1})]+k(e_{t+1})\right] \tag{A8}$$

$$\mu_t^3=\mu_t^1\cdot a_t\cdot y'(l_t) \tag{A9}$$

$$\mu_t^3=\chi\cdot a_t\cdot\omega_p\cdot\frac{y'(g_t)}{p_t}. \tag{A10}$$

where I simplify the notations by using

$$u_t = 1-(1-s)\cdot n_{t-1}$$
$$h_t = n_t-(1-s)\cdot n_{t-1}.$$

Noting that $e_t \cdot f(\theta_t) = h_t/u_t$, the first-order condition (A7) with respect to θ_t yields

$$\frac{\mu_t^2}{\mu_t^1} = \frac{\eta}{1-\eta}\cdot\frac{r\cdot a_t}{q(\theta_t)}. \tag{A11}$$

Combining (A11) with the first-order condition (A5) with respect to c_t and the first-order condition (A6) with respect to e_t yields

$$k'(e_t) = \frac{\eta}{1-\eta}\cdot r\cdot\theta_t\cdot\frac{1}{c_t/a_t},$$

which determines optimal job-search effort e_t. We assume that ω_k is large enough such that there is an interior solution for the optimal effort and $e_t < 1$.[19] Combining the first-order conditions (A9) and (A10) with respect to l_t and g_t respectively yields

$$\chi\cdot\omega_p\cdot\frac{c_t/a_t}{p_t/a_t} = \frac{y'(l_t)}{y'(g_t)},$$

which determines optimal public employment g_t. Combining the first-order conditions (A8) and (A9) with respect to n_t and l_t respectively, dividing by $a_t\cdot\mu_t^1$, yields:

$$y'(l_t) = \frac{r}{q(\theta_t)} + \frac{\mu_t^2}{\mu_t^1 a_t} - \delta(1-s)\mathbb{E}_t\left[\frac{\mu_{t+1}^1}{\mu_t^1 a_t}\left[\frac{ra_{t+1}}{q(\theta_{t+1})} + \frac{\mu_{t+1}^2}{\mu_{t+1}^1}\left[1-e_{t+1}f(\theta_{t+1})\right] + \frac{k(e_{t+1})}{\mu_{t+1}^1}\right]\right] \tag{A12}$$

Using the isoelasticity of the disutility of effort and equation (A11):

$$k(e_t) = e_t\cdot k'(e_t)\cdot\frac{1}{\kappa+1}$$
$$k(e_t) = e_t\cdot f(\theta_t)\cdot\frac{1}{\kappa+1}\cdot\mu_t^2. \tag{A13}$$

[19]The optimal effort $e_t < 1$ if

$$\omega_k > \frac{\eta}{1-\eta}\cdot\frac{1}{c_t/a_t}\cdot r\cdot\theta_t.$$

The right-hand side of this inequality is a constant because, as we will see, the efficient allocation is constant over time, independent of the technology process $\{a_t\}$.

Combining (A11), (A12), and (A13), and multiplying by $(1-\eta)$, yields

$$(1-\eta)\cdot y'(l_t) = \frac{r}{q(\theta_t)} - \delta\cdot(1-s)\cdot\mathbb{E}_t\left[\frac{c_t/a_t}{c_{t+1}/a_{t+1}}\cdot\left\{\frac{r}{q(\theta_{t+1})} - \eta\cdot r\cdot\frac{\kappa}{1+\kappa}\cdot e_{t+1}\cdot\theta_{t+1}\right\}\right]$$

This last equation determines optimal private employment l_t, and completes the characterization of the efficient allocation in a stochastic environment.

These 3 first-order conditions, together with the production constraints (2) and (5), the resource constraint (6), and the law of motion of employment (7), constitute the system of 7 equations that characterize the efficient allocations. As explained in Section 3.2, this system does not involve the technology stochastic process $\{a_t\}_{t=0}^{+\infty}$. If the efficient allocation (the solution to this system of 7 equations) is unique, then the efficient allocation is independent of the realizations of the stochastic process for technology: the efficient allocation is deterministic. If initial employment l_{-1} and g_{-1} are chosen adequately to be at their steady-state values, then the labor market variables in the efficient allocation remain constant over time.

A.4 Proof of Proposition 2

In a static environment parameterized by technology a, the equilibrium $\{\theta(a), e(a), l(a), g(a)\}$ on the labor market is characterized in Section A.1. I now consider a marginal change dg in public employment. The remaining equilibrium variables $\{\theta, e, l\}$ evolve according to the system of three equations: (17), (8), and (10). I report these three equations here for convenience:

$$\begin{aligned}
l+g &= \frac{e\cdot f(\theta)}{s+(1-s)\cdot e\cdot f(\theta)}\\
\ln(1/b) &= [1-\delta\cdot(1-s)]\cdot\frac{k'(e)}{f(\theta)} + \kappa\cdot\delta\cdot(1-s)\cdot k(e)\\
y'(l) &= \frac{w}{a} + [1-\delta\cdot(1-s)]\,\frac{r}{q(\theta)}.
\end{aligned}$$

I log-linearize this system to obtain the effect of a marginal change in dg in public employment. I start by log-linearizing the worker's optimality condition:

$$
\begin{aligned}
\ln(1/b) &= [1-\delta\cdot(1-s)]\cdot\frac{k'(e)}{f(\theta)} + \kappa\cdot\delta\cdot(1-s)\cdot k(e) \\
0 &= (1-\delta\cdot(1-s))\cdot\left[\frac{k'(e)}{f(\theta)}\right]\left[\kappa\check{e}-(1-\eta)\cdot\check{\theta}\right] + \kappa\cdot\delta\cdot(1-s)\cdot k(e)\cdot[1+\kappa]\check{e} \\
0 &= \left[\kappa\check{e}-(1-\eta)\cdot\check{\theta}\right] + \check{e}\cdot\kappa\cdot\frac{\delta\cdot(1-s)}{1-\delta\cdot(1-s)}\cdot f(\theta)\frac{(1+\kappa)k(e)}{k'(e)} \\
(1-\eta)\cdot\check{\theta} &= \kappa\check{e} + \check{e}\cdot\kappa\cdot\frac{\delta\cdot(1-s)}{1-\delta\cdot(1-s)}\cdot f(\theta)\cdot e \\
(1-\eta)\cdot\check{\theta} &= \kappa\check{e}\left[1+\frac{\delta\cdot(1-s)}{1-\delta\cdot(1-s)}\cdot f(\theta)\cdot e\right]
\end{aligned}
$$

Hence, the log-linear system becomes:

$$(1-\zeta)\cdot\check{l}+\zeta\cdot\check{g} = u(a)\cdot\left[\check{e}+(1-\eta)\cdot\check{\theta}\right] \tag{A14}$$

$$K(a)\cdot\check{e} = (1-\eta)\cdot\check{\theta} \tag{A15}$$

$$(\alpha-1)\cdot\check{l} = \eta\cdot[1-\Omega(a)]\cdot\check{\theta}$$

where $\Omega(a)$ characterizes the wedge between real wage $w(a)$ and marginal product of labor $mpl(a) = a\cdot y'(l)$ in the private sector, and is defined by (A2) and $K(a)$ characterizes the elasticity of job-search effort with respect to labor market conditions and is defined by (A3).

To derive the public-employment multiplier, it is convenient to rewrite the log-linear system as:

$$
\begin{aligned}
\check{e} &= (1-\eta)\cdot\frac{1}{K(a)}\cdot\check{\theta} \\
\check{\theta} &= -\frac{(1-\alpha)}{\eta}\cdot\frac{1}{1-\Omega(a)}\cdot\check{l} \\
\zeta\cdot\check{g} &= -\left[u(a)\cdot\left\{\left(\frac{1}{K(a)}+1\right)(1-\alpha)\cdot\frac{(1-\eta)}{\eta}\cdot\frac{1}{1-\Omega(a)}\right\}+(1-\zeta)\right]\check{l}.
\end{aligned}
$$

Since

$$\frac{dl}{dg} = \frac{l}{g}\cdot\frac{d\ln(l)}{d\ln(g)} = \frac{1-\zeta}{\zeta}\cdot\frac{\check{l}}{\check{g}},$$

we infer that

$$\frac{dl}{dg} = -\frac{1-\zeta}{1-\zeta+(1-\alpha)\cdot\frac{(1-\eta)}{\eta}\cdot\frac{1+K(a)}{K(a)}\cdot\frac{u(a)}{1-\Omega(a)}}. \tag{A16}$$

Clearly, $-1 < \partial l/\partial g < 0$ (note that $1-\Omega(a) > 0$).

A.5 Proof of Proposition 3

Using (A16), I infer that

$$\frac{dn}{dg} = 1 + \frac{dl}{dg} = 1 - \frac{1-\zeta}{1-\zeta+(1-\alpha)\cdot\frac{(1-\eta)}{\eta}\cdot\frac{1+K(a)}{K(a)}\cdot\frac{u(a)}{1-\Omega(a)}}$$
$$= \frac{(1-\alpha)\cdot\frac{(1-\eta)}{\eta}\cdot\frac{1+K(a)}{K(a)}\cdot\frac{u(a)}{1-\Omega(a)}}{1-\zeta+(1-\alpha)\cdot\frac{(1-\eta)}{\eta}\cdot\frac{1+K(a)}{K(a)}\cdot\frac{u(a)}{1-\Omega(a)}}$$
$$= \frac{1-\alpha}{1-\alpha+(1-\zeta)\cdot\frac{K(a)}{1+K(a)}\cdot\frac{\eta}{(1-\eta)}\cdot\frac{1-\Omega(a)}{u(a)}}.$$

I can now express the public-employment multiplier λ_g, which is the increase dn in total employment that can be achieved by spending one unit of private good on public employment. The cost of a marginal increase dg in public employment is $dg\cdot[w+(1-\delta\cdot(1-s))\cdot r\cdot a/q(\theta)] = dg\cdot mpl(a)$. To conclude, the employment multiplier is given by

$$\lambda_g = \frac{1-\alpha}{1-\alpha+(1-\zeta)\cdot\frac{K(a)}{1+K(a)}\cdot\frac{\eta}{(1-\eta)}\cdot\frac{1-\Omega(a)}{u(a)}}\cdot mpl(a)^{-1}.$$

Using Lemma A1, the fact that $q'(\theta)<0$, as well as the definitions in Section A.1, I infer:

$$\frac{dK(a)}{da}>0,\quad \frac{d[1-\Omega(a)]}{da}>0,\quad \frac{du(a)}{da}<0,\quad \frac{dmpl(a)}{da}>0.$$

Hence, $d\lambda_g/da<0$.

B Solution to the Worker's Problem

The Lagrangian of the worker's problem described in Section 2.3 is

$$\mathbb{E}_0\sum_{t=0}^{+\infty}\delta^t\Big\{(1-u_t)\cdot\ln(w_t)+u_t\cdot[k(e_t)+\ln(b_t\cdot w_t)+e_t\cdot f(\theta_t)\cdot\ln(1/b_t)]$$
$$+A_t\cdot\left[u_t\cdot e_t\cdot f(\theta_t)+(1-s)\cdot n^s_{t-1}-n^s_t\right]\Big\},$$

where n^s_t is the probability to be employed in period t after history a^t, I define the probability of being unemployed at the beginning of period t: $u_t\equiv 1-(1-s)\cdot n^s_{t-1}$, and $\{A_t\}$ is a stochastic process for Lagrange multipliers. The first-order condition with respect to effort e_t gives (after dividing by $u_t \cdot f(\theta_t)$):

$$\frac{k'(e_t)}{f(\theta_t)} = \ln(1/b_t) + A_t.$$

The first-order condition with respect to the probability of being employed n_t^s yields:

$$A_t = \delta \cdot (1-s) \cdot \mathbb{E}_t \left[k(e_{t+1}) + [A_{t+1} + \ln(1/b_{t+1})] \cdot [1 - e_{t+1} \cdot f(\theta_{t+1})]\right]$$

Merging the two first-order conditions together yields:

$$\frac{k'(e_t)}{f(\theta_t)} = \ln(1/b_t) + \delta(1-s)\mathbb{E}_t \left[k(e_{t+1}) + \frac{k'(e_{t+1})}{f(\theta_{t+1})} \cdot [1 - e_{t+1} f(\theta_{t+1})]\right]$$

With the isoelastic disutility of effort, $e \cdot k'(e) = (1+\kappa) \cdot k(e)$, and:

$$\frac{k'(e_t)}{f(\theta_t)} = \ln(1/b_t) + \delta(1-s)\mathbb{E}_t \left[-\kappa k(e_{t+1}) + \frac{k'(e_{t+1})}{f(\theta_{t+1})}\right]$$

Thus, the optimal effort function therefore satisfies the Euler equation (1).

C Equilibrium in a Pissarides [2000] Model

In a Pissarides [2000] model, the stochastic process for wages $\{w_t\}_{t=0}^{+\infty}$ is determined using the generalized Nash solution to the bargaining problem faced by a firm-worker pair. The equilibrium system described in Section 2.5 includes one more equation to characterize the wage process.

Let $\mathcal{W}_t$ denote the equilibrium surplus to a worker from being employed after the matching process in period t. Worker's optimal search behavior satisfies $k'(e_t)/f(\theta_t) = \mathcal{W}_t$.[20] Rewriting the first-order condition (1) determining worker's optimal search effort as

$$\frac{k'(e_t)}{f(\theta_t)} = \ln\left(\frac{1}{b_t}\right) + \delta \cdot (1-s) \cdot \mathbb{E}_t \left[\frac{k'(e_{t+1})}{f(\theta_{t+1})} - \kappa \cdot k(e_{t+1})\right],$$

I infer the the worker's share of the match surplus satisfies

$$\mathcal{W}_t = \ln\left(\frac{1}{b_t}\right) + \delta \cdot (1-s) \cdot \mathbb{E}_t \left[\mathcal{W}_{t+1} - \kappa \cdot k(e_{t+1})\right]. \tag{A17}$$

In equilibrium, the firm's surplus from an established relationship is simply given by the hiring

[20] Let $\mathcal{E}_t$ and $\mathcal{U}_t$ be the worker's continuation values from being employed and unemployed respectively at time t. Then the worker maximizes $-k(e_t) + e_t \cdot f(\theta_t) \cdot \mathcal{E}_t + [1 - e_t \cdot f(\theta_t)] \cdot \mathcal{U}_t$ over effort e_t. The first-order condition with respect to e_t is $k'(e_t) = f(\theta_t) \cdot [\mathcal{E}_t - \mathcal{U}_t] = f(\theta_t) \cdot \mathcal{W}_t$.

cost since a firm can immediately replace a worker at that cost during the matching period:

$$\mathcal{F}_t = \frac{r \cdot a_t}{q(\theta_t)}.$$

Assume that wages are continually renegotiated. Then the wage bargained in the current period only influences payoffs in the current period. Accordingly, since the firm's utility is simply its profits, a wage w_t brings a utility $-w_t$ to the firm (or its owners) and

$$\frac{d\mathcal{F}_t}{dw_t} = -1.$$

Since workers would consume exactly the wage w_t in the current period (there are no saving and no borrowing), a wage brings utility $\ln(w_t)$ to the employed worker in the current period and

$$\frac{d\mathcal{W}_t}{dw_t} = \frac{1}{w_t}.$$

The generalized Nash solution to the bargaining problem faced by a firm-worker pair is the wage w_t that maximizes

$$\mathcal{W}_t(w_t)^{\beta} \cdot \mathcal{F}_t(w_t)^{1-\beta},$$

β is the worker's bargaining power. The first-order condition of this maximization problem implies that the worker's surplus each period is related to the firm's surplus

$$\mathcal{W}_t = \frac{\beta}{1-\beta} \cdot \frac{\mathcal{F}_t}{w_t} = \frac{\beta}{1-\beta} \cdot \frac{r \cdot a_t}{q(\theta_t)} \cdot \frac{1}{w_t}. \tag{A18}$$

Plugging the expression (A18) for the worker's surplus, which reflect surplus sharing, into the recursive equation (A17) for the worker's surplus, which reflects optimal job search, I derive a new relation between equilibrium variables $\{\theta_t, e_t, w_t\}$ and technology $\{a_t\}$ imposed by setting wages using the Nash bargaining solution:

$$\frac{\beta}{1-\beta} \cdot \frac{r}{q(\theta_t)} \cdot \frac{a_t}{w_t} = \ln\left(\frac{1}{b_t}\right) + \delta \cdot (1-s) \cdot \mathbb{E}_t\left[\frac{\beta}{1-\beta} \cdot \frac{r}{q(\theta_{t+1})} \cdot \frac{a_{t+1}}{w_{t+1}} - \kappa \cdot k(e_{t+1})\right].$$

It is useful to express the disutility from job search in equilibrium as a function of other equilibrium variables:

$$
\begin{aligned}
k(e_t) &= e_t \cdot \frac{k'(e_t)}{1+\kappa} \\
k(e_t) &= e_t \cdot \frac{1}{1+\kappa} \cdot \frac{k'(e_t)}{f(\theta_t)} \cdot f(\theta_t) \\
k(e_t) &= e_t \cdot \frac{1}{1+\kappa} \cdot \mathcal{W}_t' \cdot f(\theta_t) \\
k(e_t) &= \frac{\beta}{1-\beta} \cdot e_t \cdot \frac{1}{1+\kappa} \cdot \frac{r \cdot a_t}{q(\theta_t)} \cdot \frac{1}{w_t} \cdot f(\theta_t) \\
k(e_t) &= \frac{1}{1+\kappa} \cdot \frac{\beta}{1-\beta} \cdot e_t \cdot r \cdot \theta_t \cdot \frac{a_t}{w_t}.
\end{aligned}
$$

Combining both equations yield

$$
\beta \cdot \frac{r}{q(\theta_t)} \cdot \frac{a_t}{w_t} = (1-\beta)\ln\left(\frac{1}{b_t}\right) + \beta \cdot \delta \cdot (1-s) \cdot \mathbb{E}_t\left[\frac{a_{t+1}}{w_{t+1}} \cdot \left\{\frac{r}{q(\theta_{t+1})} - \frac{\kappa}{1+\kappa} \cdot e_{t+1} \cdot r \cdot \theta_{t+1}\right\}\right].
$$

In a static environment, this wage equation becomes:

$$
\frac{1-\beta}{\beta} \cdot \ln\left(\frac{1}{b}\right) \cdot \frac{w}{a} = [1-\delta \cdot (1-s)]\frac{r}{q(\theta)} + \delta \cdot (1-s) \cdot \frac{\kappa}{1+\kappa} \cdot e \cdot r \cdot \theta.
$$

Combining this equation with the firm's optimal employment decision (10) yields

$$
\begin{aligned}
\frac{1-\beta}{\beta} \cdot \ln\left(\frac{1}{b}\right) \cdot \left[1 - [1-\delta \cdot (1-s)] \cdot \frac{r}{q(\theta)}\right] &= [1-\delta \cdot (1-s)] \cdot \frac{r}{q(\theta)} + \delta \cdot (1-s) \cdot \frac{\kappa}{1+\kappa} \cdot e \cdot r \cdot \theta \\
\frac{1-\beta}{\beta} \cdot \ln\left(\frac{1}{b}\right) &= [1-\delta \cdot (1-s)] \cdot \frac{r}{q(\theta)}\left[1 + \frac{1-\beta}{\beta} \cdot \ln\left(\frac{1}{b}\right)\right] + \delta \cdot (1-s) \cdot \frac{\kappa}{1+\kappa} \cdot e \cdot r \cdot \theta.
\end{aligned}
$$

D The Effect of Job-Search Monitoring on Employment

D.1 A model with monitoring

Monitoring provides higher incentives to search for a job. In equilibrium, jobseekers exert more effort, which makes makes recruiting cheaper for firms, reduces the marginal cost of labor, leading firms to hire more workers and produce more. Unemployed workers who are searching for a job when monitored receive UI benefits $b_t \cdot w_t$. Unemployed workers who are not searching for a job when monitored receive a punishment $z_t \cdot b_t \cdot w_t$, with $z_t < 1$. In practice, the punishment may be an income-assistance program granted to unemployed workers irrespective of their search effort. Each period an unemployed worker faces a probability π to be monitored. Given that an unemployed worker searches with probability e, there are three case: (1) with probability $1-\pi$, the unemployed worker is not monitored and receives UI benefits $b_t \cdot w_t$; (2) with probability $\pi \cdot e$, the unemployed worker is monitored while searching and receives UI benefits $b_t \cdot w_t$; and (3) with probability $\pi \cdot (1-e)$, the unemployed worker is monitored while not searching and receives a sanction $z_t \cdot b_t \cdot w_t$. Monitoring u unemployed workers for a fraction π of the time has a cost $\pi \cdot u \cdot m$ in units of private goods, where m is a parameter.[21]

D.2 Solution to the worker's problem with placement agency

Given the government policy $\{\pi_t, b_t, z_t\}_{t=0}^{+\infty}$, labor market tightness and wage processes $\{\theta_t, w_t\}_{t=0}^{+\infty}$, the *worker's problem* is to choose a stochastic process for search effort $\{e_t\}_{t=0}^{+\infty}$ to maximize the expected utility

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \{(1-u_t) \cdot \ln(w_t) + u_t \cdot [-k(e_t) + \ln(b_t \cdot w_t) + e_t \cdot f(\theta_t) \cdot \ln(1/b_t) - (1-e_t) \cdot \pi_t \cdot \ln(1/z_t)]\},$$

subject to the same law of motion of the probability n_t^s to be employed in period t for all t:

$$n_t^s = \left[1 - (1-s) \cdot n_{t-1}^s\right] \cdot e_t \cdot f(\theta_t) + (1-s) \cdot n_{t-1}^s.$$

I define the probability of being unemployed at the beginning of period t: $u_t \equiv 1 - (1-s) n_{t-1}^s$. The Lagrangian of the worker's problem is

$$\mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \Big\{ (1-u_t) \cdot \ln(w_t) + u_t \cdot [-k(e_t) + \ln(b_t \cdot w_t) + e_t \cdot f(\theta_t) \cdot \ln(1/b_t) - (1-e_t) \cdot \pi_t \cdot \ln(1/z_t)] + A_t \left[u_t \cdot e_t \cdot f(\theta_t) + (1-s) \cdot n_{t-1}^s - n_t^s\right] \Big\}.$$

where n_t^s is the probability to be employed in period t after history a^t, and $\{A_t\}$ is a stochastic process for the Lagrange multiplier. The first-order conditions with respect to effort e_t yields

$$\frac{k'(e_t)}{f(\theta_t)} = \left[\ln\left(\frac{1}{b_t}\right) + \frac{\pi_t}{f(\theta_t)} \cdot \ln\left(\frac{1}{z_t}\right)\right] + A_t. \tag{A19}$$

The Lagrange multiplier A_t captures the influence of future labor market conditions. The first-order condition with respect to the probability of being employed n_t^s yields

$$A_t = \delta(1-s)\mathbb{E}_t \left[k(e_{t+1}) + A_{t+1}\left[1 - e_{t+1} f(\theta_{t+1})\right] + \left[1 - e_{t+1} f(\theta_{t+1})\right] \ln\left(\frac{1}{b_{t+1}}\right) + (1-e_{t+1})\pi_{t+1} \ln\left(\frac{1}{z_{t+1}}\right)\right]$$

[21] The resource cost of monitoring includes administrative costs, time spent interviewing jobseekers, or time spent verifying worker's search efforts.

Substituting for A_t and A_{t+1} using (A19), with the isoelastic disutility function $e \cdot k'(e)/(1+\kappa) = k(e)$, I obtain the following optimality condition:

$$\frac{k'(e_t)}{f(\theta_t)} = \ln\left(\frac{1}{b_t}\right) + \frac{\pi_t}{f(\theta_t)} \ln\left(\frac{1}{z_t}\right) + \delta(1-s)\mathbb{E}_t\left[\frac{k'(e_{t+1})}{f(\theta_{t+1})} - \kappa k(e_{t+1}) - \pi_{t+1}\frac{1-f(\theta_{t+1})}{f(\theta_{t+1})}\ln\left(\frac{1}{z_{t+1}}\right)\right]$$

In a static environment:

$$[1-\delta\cdot(1-s)]\cdot\frac{k'(e)}{f(\theta)} + \delta\cdot(1-s)\cdot\kappa\cdot k(e) = \ln\left(\frac{1}{b}\right) + \pi\cdot\ln\left(\frac{1}{z}\right)\cdot\left[\delta\cdot(1-s) + \frac{1-\delta\cdot(1-s)}{f(\theta)}\right].$$

D.3 Monitoring multiplier

I assume that that there is no job-search monitoring, $\pi = 0$, in the static environment. This assumption captures the existing state of government intervention on the labor market in the US, and simplifies derivations.[22]

In a static environment parameterized by technology a, given policies $\{g, \pi\}$, the equilibrium $\{\theta, e, l\}$ on the labor market is characterized by three equations: (17), (8), and (10). I report these three equations here for convenience:

$$l + g = \frac{e\cdot f(\theta)}{s + (1-s)\cdot e\cdot f(\theta)}$$
$$\ln(1/b) + \pi\cdot\ln(1/z)\left[\delta\cdot(1-s) + \frac{1-\delta\cdot(1-s)}{f(\theta)}\right] = [1-\delta\cdot(1-s)]\cdot\frac{k'(e)}{f(\theta)} + \kappa\cdot\delta\cdot(1-s)\cdot k(e)$$
$$y'(l) = \frac{w}{a} + [1-\delta\cdot(1-s)]\frac{r}{q(\theta)}.$$

I now consider a marginal increase in the monitoring probability π, keeping replacement rates b, z and public employment g constant. I log-linearize this system to obtain the effect of a marginal change $d\pi$ in monitoring probability, around the steady-state value of $\pi = 0$. I start by log-linearizing the worker's optimality condition:

$$d\pi\cdot\ln(1/z)\left[\delta\cdot(1-s) + \frac{1-\delta\cdot(1-s)}{f(\theta)}\right] = [1-\delta\cdot(1-s)]\cdot\frac{k'(e)}{f(\theta)}\cdot\left[\kappa\check{e} - (1-\eta)\cdot\check{\theta}\right] + \kappa\cdot\delta\cdot(1-s)\cdot k(e)\cdot[1+\kappa]\check{e}$$

[22] The OECD reports expenditure on public employment service, including placement services, for the US for 1985–2008. This expenditure influences the administration capacity to monitor job search effectively. On average only 0.047% of GDP is spent on this service, explaining why many states have switched from in-person filing and regular job-search reporting to reporting by telephone [Grubb, 2011].

Hence, the log-linear system becomes:

$$(1-\zeta)\cdot\check{l} = u(a)\cdot\left[\check{e}+(1-\eta)\cdot\check{\theta}\right]$$

$$d\pi\cdot\ln(1/z)\left[\delta\cdot(1-s)+\frac{1-\delta\cdot(1-s)}{f(\theta)}\right] = (1-\delta\cdot(1-s))\cdot\frac{k'(e)}{f(\theta)}\left[-(1-\eta)\cdot\check{\theta}\right]$$
$$+\kappa\check{e}\cdot\left[(1-\delta\cdot(1-s))\cdot\frac{k'(e)}{f(\theta)}+\delta\cdot(1-s)\cdot k(e)\cdot[1+\kappa]\right]$$

$$(\alpha-1)\cdot\check{l} = \eta\cdot[1-\Omega(a)]\cdot\check{\theta}$$

where $\zeta = g/n$ is the share of public employment in total employment, $u(a) = 1-(1-s)\cdot n(a)$ is unemployment, $\Omega(a)$ is characterized by (A2) and $K(a)$ is characterized by (A3). To derive the monitoring multiplier, I rewrite the log-linear system as:

$$\check{e} = \frac{(1-\zeta)}{u(a)}\cdot\check{l}-(1-\eta)\cdot\check{\theta}$$

$$-(1-\eta)\check{\theta} = (1-\alpha)\cdot\frac{1-\eta}{\eta}\cdot\frac{1}{1-\Omega(a)}\cdot\check{l}$$

$$d\pi\cdot\ln(1/z)\left[\delta\cdot(1-s)+\frac{1-\delta\cdot(1-s)}{f(\theta)}\right] = (1+\kappa)\cdot\ln(1/b)\left[-(1-\eta)\cdot\check{\theta}\right]$$
$$+\kappa\cdot\check{l}\cdot\frac{1-\zeta}{u(a)}\cdot[\ln(1/b)+\delta\cdot(1-s)\cdot k(e)]$$

We infer that

$$\frac{dn}{d\pi} = \frac{dl}{d\pi} = l\cdot\frac{\check{l}}{d\pi} = \frac{l\cdot u\cdot\ln(1/z)\left[\delta\cdot(1-s)+\frac{1-\delta\cdot(1-s)}{f(\theta)}\right]}{(1+\kappa)\cdot\ln(1/b)\left[(1-\alpha)\cdot\frac{1-\eta}{\eta}\cdot\frac{u(a)}{1-\Omega(a)}\right]+\kappa\cdot(1-\zeta)\cdot[\ln(1/b)+\delta\cdot(1-s)\cdot k(e)]}$$

I can now express the monitoring multiplier λ_π, which is the increase dn in total employment that can be achieved by spending one unit of private good on monitoring job search. The cost of a marginal increase $d\pi$ in monitoring is $d\pi\cdot m\cdot u(a)$. Thus the monitoring multiplier is given by

$$\lambda_\pi = \frac{\ln(1/z)}{m}\cdot\frac{l\cdot\left[\delta\cdot(1-s)+\frac{1-\delta\cdot(1-s)}{f(\theta)}\right]}{(1+\kappa)(1-\alpha)\cdot\frac{1-\eta}{\eta}\frac{u(a)}{1-\Omega(a)}\ln(1/b)+\kappa(1-\zeta)\left[\ln(1/b)+\delta(1-s)\cdot k(e)\right]}. \quad \text{(A20)}$$

I can now characterize the cyclicality of the monitoring multiplier over the business cycle.

PROPOSITION A1. *Assume* $\alpha < 1$ *and* $\gamma < 1$. *Also assume* $\delta = 1$. *Then there exists* $a^* > 0$ *such that for all* $a < a^*$, $d\lambda_\pi/da > 0$.

The assumption on the discount factor δ is a technical assumption to simplify the algebra. The assumption that $\gamma < 1$ is necessary to obtain fluctuations in unemployment and other variables.

With $\gamma = 1$, $d\lambda_\pi/da = 0$. The assumption that $\alpha < 1$ is necessary to obtain a downward-sloping aggregate demand for labor.

Proof. I assume that $\delta = 1$. I start from the above expression for the monitoring multiplier λ_π. Notice that:

$$\frac{1}{1-\Omega(a)} = \frac{1}{1-\delta\cdot(1-s)}\cdot\frac{q(\theta)}{r}\cdot l^{\alpha-1} = \frac{1}{s\cdot r}\cdot q(\theta)\cdot l^{\alpha-1}.$$

Using the optimal search condition (8) and the isoelasticity of the disutility of effort $k(e) = e\cdot k'(e)/(1+\kappa)$, I find that

$$\begin{aligned}\ln(1/b)+\delta\cdot(1-s)\cdot k(e) &= [1-\delta\cdot(1-s)]\cdot\frac{k'(e)}{f(\theta)}+(1+\kappa)\cdot\delta\cdot(1-s)\cdot k(e)\\ &= \frac{1+\kappa}{n}\cdot k(e).\end{aligned}$$

Rearranging this equation I infer that

$$\begin{aligned}k(e) &= \ln(1/b)\cdot\frac{n}{\kappa+u}\\ \ln(1/b)+\delta\cdot(1-s)\cdot k(e) &= \ln(1/b)\cdot\frac{1+\kappa}{\kappa+u}.\end{aligned}$$

Therefore,

$$\lambda_\pi = Q\cdot\frac{(\kappa+u)\cdot l\cdot\left[(1-s)+\frac{s}{f(\theta)}\right]}{(\kappa+u)\cdot(1-\alpha)\cdot\frac{1-\eta}{\eta}\cdot u\cdot\frac{1}{s\cdot r}\cdot q(\theta)\cdot l^{\alpha-1}+\kappa\cdot(1-\zeta)}.$$

where

$$Q = \frac{\ln(1/z)}{m}\cdot\frac{1}{(1+\kappa)\cdot\ln(1/b)} > 0$$

is a constant that depends on the parameters of the model. Multiplying numerator and denominator by $e\cdot f(\theta)/l = s/(u\cdot(1-\zeta))$ yields

$$\lambda_\pi = Q\cdot\frac{(\kappa+u)\cdot[(1-s)\cdot f(\theta)+s]\cdot e}{(\kappa+u)\cdot\frac{1-\alpha}{1-\zeta}\cdot\frac{1-\eta}{\eta}\cdot\frac{q(\theta)}{r}\cdot l^{\alpha-1}+\kappa\cdot\frac{s}{u}}.$$

First step. I determine how $N = (\kappa+u)\cdot[(1-s)\cdot f(\theta)+s]\cdot e$ responds to a marginal change da in technology. Using the same logic as for the derivation of equations (A14) and (A15), I infer that the log-deviations $\check{e}$ and $\check{n}$ of equilibrium effort and aggregate employment are related to the log-deviation $\check{\theta}$ of labor market tightness by

$$\check{e} = (1-\eta)\cdot\frac{1}{K(a)}\cdot\check{\theta} \tag{A21}$$

$$\check{n} = u\left[\check{e} + (1-\eta)\cdot\check{\theta}\right] = (1-\eta)\cdot\frac{1+K(a)}{K(a)}\cdot u\cdot\check{\theta} \tag{A22}$$

$$\check{u} = -\frac{1-u}{u}\check{n} = -(1-u)\cdot(1-\eta)\cdot\frac{1+K(a)}{K(a)}\cdot\check{\theta} \tag{A23}$$

where the last equality holds because $\check{u} = -(1-u)/u\cdot\check{n}$ by definition of unemployment. I infer that the log-deviation of the numerator of λ_π is

$$\check{N} = \check{e} + (1-\eta)\cdot\frac{(1-s)\cdot f(\theta)}{s+(1-s)\cdot f(\theta)}\cdot\check{\theta} + \frac{u}{u+\kappa}\cdot\check{u}$$
$$\check{N} = (1-\eta)\cdot\check{\theta}\cdot\left\{\left[\frac{1}{K(a)} + \frac{(1-s)\cdot f(\theta)}{s+(1-s)\cdot f(\theta)}\right] - \frac{u\cdot(1-u)}{u+\kappa}\cdot\left[1+\frac{1}{K(a)}\right]\right\}$$

Note that since $e \leq 1$,

$$\frac{(1-s)\cdot f(\theta)}{s+(1-s)\cdot f(\theta)} = \frac{(1-s)\cdot e\cdot f(\theta)}{e\cdot s+(1-s)\cdot e\cdot f(\theta)}$$
$$\frac{(1-s)\cdot f(\theta)}{s+(1-s)\cdot f(\theta)} > \frac{(1-s)\cdot e\cdot f(\theta)}{s+(1-s)\cdot e\cdot f(\theta)} = (1-s)\cdot n = 1-u > (1-u)\cdot\frac{u}{u+\kappa}$$

Furthermore it is clear that $u\cdot(1-u)/(u+\kappa) < 1$. Hence $\check{N}/\check{\theta} > 0$. But as showed by Lemma A1, $\check{\theta}/\check{a} > 0$. Thus, $\check{N}/\check{a} > 0$ and clearly $dN/da > 0$.

Second step. I determine how

$$Q = (\kappa+u)\cdot\frac{1-\alpha}{1-\zeta}\cdot\frac{1-\eta}{\eta}\cdot\frac{q(\theta)}{r}\cdot l^{\alpha-1} + \kappa\cdot\frac{s}{u}$$

responds to a marginal change da in technology. Notice that $\check{l} = \check{n}$. Also notice that since $\delta = 1$, $K(a) = \kappa/u$. Using (A21), (A22), (A23), I find

$$\begin{aligned}
\frac{dQ}{\check{\theta}} &= \kappa \cdot \frac{s}{u} \cdot \left[(1-u) \cdot (1-\eta) \cdot \frac{u+\kappa}{\kappa} \right] \\
&\quad - \left[(\kappa+u) \cdot \frac{1-\alpha}{1-\zeta} \cdot \frac{1-\eta}{\eta} \cdot \frac{q(\theta)}{r} \cdot l^{\alpha-1} \right] \\
&\quad \cdot \left[\eta + (1-\alpha) \cdot (1-\eta) \cdot \frac{u+\kappa}{\kappa} \cdot u + \frac{u}{\kappa+u} \cdot (1-u) \cdot (1-\eta) \cdot \frac{u+\kappa}{\kappa} \right] \\
\frac{\kappa}{(u+\kappa)\cdot(1-u)} \cdot \frac{\theta}{1-\eta} \cdot \frac{dQ}{d\theta} &= \kappa \cdot \frac{s}{u} - \left[\frac{1-\alpha}{1-\zeta} \cdot \frac{1-\eta}{\eta} \cdot \frac{q(\theta)}{r} \cdot l^{\alpha-1} \right] \\
&\quad \cdot \left[\frac{\kappa}{1-u} \cdot \frac{\eta}{1-\eta} + (1-\alpha) \cdot (u+\kappa) \cdot \frac{u}{1-u} + u) \right]
\end{aligned}$$

Unfortunately, I cannot sign $dQ/d\theta$ for any u (or any underlying a). However Lemma A1 says that $1/u$ is increasing with a while $q(\theta), l^{\alpha-1}, 1/(1-u), u, u(u+\kappa)/(1-u)$ are all decreasing with a. Hence the right-hand side of the equation above is increasing with a. When $a \to 0$, $n \to 0$, $u \to 1$, $\theta \to 0$, so the right-hand side $\to -\infty$. Accordingly, there exists $a^* > 0$ such that for all $a < a^*$, the right-hand side is negative, and $dQ/d\theta < 0$. Since, from Lemma A1, $d\theta/da > 0$, I infer that there exists $a^* > 0$ such that for all $a < a^*$, $dQ/da < 0$. In that case $d(N/Q)/da > 0$ and $d\lambda_\pi/da > 0$. □

D.4 Calibration

Transfers from the government. If unemployed workers do not actively search for a job, they lose their entitlement to unemployment benefits. I assume that in that case, the only social assistance provided is food stamps.[23] Using a detailed state-level data, Pavoni and Violante [2007] compute that in 1996 on a monthly basis, the median monthly allotment of food stamps for a family of four was \$397. At the same time, the median monthly wage paid to a worker with at most a high-school diploma (who would be eligible for food stamps) is \$1,540. Hence, I set $z \times b = 397/1,540 = 26\%$. Hence, $z = 0.26/0.60 = 43\%$.

Monitoring cost function. The US Department of Education and the US Department of Health and Human Services jointly sponsored a large-scale evaluation of welfare-to-work policies, the National Evaluation of Welfare-to-Work Strategies, based on data pertaining to over 40,000 individuals followed for 5 years in the 1991–1999 period, in 11 separate locations [Hamilton et al.,

[23] As explained by Pavoni and Violante [2007], once unemployment insurance benefits and other job search assistance programs expire, households remain virtually without any other form of benefits and have the right to the maximum allotment of food stamps. The Food Stamp program provides monthly coupons to eligible low-income families, which can be used to purchase food.

2001]. In particular, in three locations, Atlanta, GA, (1,441 treated individuals), Grand Rapids, MI (1,557 individuals treated individuals), and Riverside, CA (3,384 individuals treated individuals), implemented labor force attachment programs in which monitored job search was the first mandatory activity. In the three sites considered, these welfare-to-work program requires unemployed workers to participate to a minimum of 20 hours of monitored job search per week. Given a normal work week of 40 hours, this corresponds to a monitoring probability of $\pi = 0.5$, as they are monitored half of the time. The average cost per unemployed workers per month of participation across the three sites is $1/3 \cdot [416 + (259 + 1025) + 759] = 820$ in 1999 $.[24] The average weekly earning for production workers from the CES for 1999 in 1999 $ is 464$, which gives an average monthly earning of 1856$. Hence the monitoring cost for a monitoring probability of $\pi = 0.5$ is $820/1856 = 44\%$ of average earning. Hence the marginal monitoring cost m satisfies $m \cdot 0.5 = 0.44 \cdot \omega$, which implies $m = 0.88 \cdot \omega = 0.63$.

E The Effect of Hiring Subsidies on Employment

E.1 Wage-subsidy multiplier

In a static environment parameterized by technology a, given policies $\{g, b, \tau\}$, the equilibrium $\{\theta, e, l\}$ on the labor market is characterized by three equations: (17), (8), and (10) (modified to take the wage subsidy into account). I report these three equations here for convenience:

$$
\begin{aligned}
l + g &= \frac{e \cdot f(\theta)}{s + (1-s) \cdot e \cdot f(\theta)} \\
\ln(1/b) &= [1 - \delta \cdot (1-s)] \cdot \frac{k'(e)}{f(\theta)} + \kappa \cdot \delta \cdot (1-s) \cdot k(e) \\
y'(l) &= (1-\tau) \cdot \frac{w}{a} + [1 - \delta \cdot (1-s)] \cdot \frac{r}{q(\theta)}.
\end{aligned}
$$

I now consider a marginal increase in the wage subsidy, keeping public employment g and replacement rate b constant. I log-linearize this system to obtain the effect of a marginal change in wage subsidy $d\tau$, around an equilibrium with $\tau = 0$ (so that the equilibrium is comparable to the equilibria around which I performed the previous policy experiments):

$$
\begin{aligned}
(1-\zeta) \cdot \check{l} &= u(a) \cdot \left[\check{e} + (1-\eta) \cdot \check{\theta}\right] \\
K(a) \cdot \check{e} &= (1-\eta) \cdot \check{\theta} \\
(\alpha - 1) \cdot \check{l} &= \eta \cdot [1 - \Omega(a)] \cdot \check{\theta} - \Omega(a) \cdot d\tau
\end{aligned}
$$

[24] These costs include: operating costs (overheads, space rental, case managers expenditure, job search facilitation, classroom instruction), costs of support service to enable participation (for instance, childcare for single parents).

where $\zeta = g/n$ is the share of public employment in total employment, $\Omega(a)$ is characterized by (A2) and $K(a)$ is characterized by (A3). I rewrite the log-linear system as:

$$\check{e} = (1-\eta)\cdot\frac{1}{K(a)}\cdot\check{\theta}$$
$$\check{\theta} = \frac{1-\zeta}{u(a)\cdot(1-\eta)\cdot(1+\frac{1}{K(a)})}\cdot\check{l}$$
$$\Omega(a)\cdot d\tau = \left[(1-\alpha)+\eta\cdot[1-\Omega(a)]\,\frac{1-\zeta}{u(a)\cdot(1-\eta)(1+\frac{1}{K(a)})}\right]\cdot\check{l}.$$

Since $dn/d\tau = dl/d\tau = l\cdot\check{l}/d\tau$, I infer that

$$\frac{dn}{d\tau} = l\cdot\frac{\Omega(a)}{(1-\alpha)+(1-\zeta)\cdot\frac{1-(1-\tau)\cdot\Omega(a)}{u(a)}\cdot\frac{\eta}{1-\eta}\cdot\frac{K(a)}{1+K(a)}}.$$

I can now express the wage-subsidy multiplier λ_τ, which is the increase dn in total employment that can be achieved by spending one dollar on a wage subsidy. The cost of a marginal increase in wage subsidy is $d\tau\cdot l\cdot w(a)$. Hence, the multiplier is given by

$$\lambda_\tau = \frac{1}{(1-\alpha)+(1-\zeta)\cdot\frac{1-\Omega(a)}{u(a)}\cdot\frac{\eta}{1-\eta}\cdot\frac{K(a)}{K(a)+1}}\cdot mpl(a)^{-1} = \frac{1}{1-\alpha}\cdot\lambda_g.$$

E.2 Equivalence of wage subsidy with recruiting subsidy

In this section, I briefly relate a wage subsidy to a recruiting subsidy, also commonly used in OECD countries. I assume that the government covers a fraction $\vartheta\in[0,1]$ of the recruiting cost $r\cdot a/q(\theta)$ paid by the firm for each new hire. In a static environment parameterized by technology a, given policies $\{g,b,\vartheta\}$, the equilibrium $\{\theta,e,l\}$ on the labor market is characterized by three equations: (17), (8), and (10) (modified to take the recruiting subsidy into account). I report these three equations here for convenience:

$$l+g = \frac{e\cdot f(\theta)}{s+(1-s)\cdot e\cdot f(\theta)}$$
$$\ln(1/b) = [1-\delta\cdot(1-s)]\cdot\frac{k'(e)}{f(\theta)}+\kappa\cdot\delta\cdot(1-s)\cdot k(e)$$
$$y'(l) = \frac{w}{a}+(1-\vartheta)\cdot[1-\delta\cdot(1-s)]\,\frac{r}{q(\theta)}.$$

I now consider a marginal increase in the recruiting subsidy, keeping public employment g and replacement rate b constant. I log-linearize this system to obtain the effect of a marginal change in wage subsidy $d\vartheta$, around an equilibrium with $\vartheta=0$ (so that the equilibrium is comparable to the equilibria around which I performed the previous policy experiments):

$$\begin{aligned}(1-\zeta)\cdot\check{l} &= u(a)\cdot\left[\check{e}+(1-\eta)\cdot\check{\theta}\right]\\ K(a)\cdot\check{e} &= (1-\eta)\cdot\check{\theta}\\ (\alpha-1)\cdot\check{l} &= [1-\Omega(a)]\cdot\left[\eta\cdot\check{\theta}-d\vartheta\right]\end{aligned}$$

where $\zeta = g/n$ is the share of public employment in total employment, $\Omega(a)$ is characterized by (A2) and $K(a)$ is characterized by (A3). It is convenient to rewrite the log-linear system as:

$$\begin{aligned}\check{e} &= (1-\eta)\cdot\frac{1}{K(a)}\cdot\check{\theta}\\ \check{\theta} &= \frac{1-\zeta}{u(a)\cdot(1-\eta)\cdot(1+\frac{1}{K(a)})}\cdot\check{l}\\ [1-\Omega(a)]\cdot d\vartheta &= \left[(1-\alpha)+\eta\cdot[1-\Omega(a)]\frac{1-\zeta}{u(a)\cdot(1-\eta)(1+\frac{1}{K(a)})}\right]\cdot\check{l}.\end{aligned}$$

Since $dn/d\vartheta = dl/d\vartheta = l\cdot\check{l}/d\vartheta$, I infer that

$$\frac{dn}{d\vartheta} = l\cdot\frac{[1-\Omega(a)]}{(1-\alpha)+(1-\zeta)\cdot\frac{1-\Omega(a)}{u(a)}\cdot\frac{\eta}{1-\eta}\cdot\frac{K(a)}{1+K(a)}}.$$

I can now express the recruiting-subsidy multiplier λ_ϑ, which is the increase dn in total employment that can be achieved by spending one dollar on a recruiting subsidy. The cost of a marginal increase in wage subsidy is $d\vartheta\cdot[r\cdot a/q(\theta)]\cdot[s\cdot l]$, which can rewritten $d\vartheta\cdot[mpl(a)\cdot\{1-\Omega(a)\}]\cdot[s/(1-\delta\cdot(1-s))]\cdot l$. Hence, the multiplier is given by

$$\lambda_\vartheta = \frac{[1-\delta\cdot(1-s)]/s}{(1-\alpha)+(1-\zeta)\cdot\frac{1-\Omega(a)}{u(a)}\cdot\frac{\eta}{1-\eta}\cdot\frac{K(a)}{1+K(a)}}\cdot mpl(a)^{-1}.$$

Hence, wage-subsidy and recruiting-subsidy multipliers are related by a very simple relationship:

$$\lambda_\vartheta = \frac{1-\delta\cdot(1-s)}{s}\cdot\lambda_\sigma.$$

If $\delta\to 1$, $1-\delta\cdot(1-s)\to s$ and $\lambda_\vartheta\to\lambda_\sigma$.