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**Sylvester C W Eijffinger, CentER and European Banking Center, Tilburg University and CEPR**
**Rob Nijskens, CentER and European Banking Center, Tilburg University**

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Centre for Economic Policy Research
77 Bastwick Street, London EC1V 3PZ, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

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# ABSTRACT

## Complementing Bagehot: illiquidity and insolvency resolution*

During the recent financial crisis, central banks have provided liquidity and governments have set up rescue programmes to restore confidence and stability, often against the LLR principle advocated by Bagehot. Using a model of a systemic bank suffering from liquidity shocks, we find that the unregulated bank keeps too much liquidity and monitors too little. A central bank can alleviate the liquidity problem, but induces moral hazard. Therefore, we introduce an additional authority that is able to bail out the bank either by injecting capital at a fixed return or by receiving an equity claim. This authority faces a trade-off: demanding a fixed premium increases investment but worsens moral hazard. Request for an equity claim by the fiscal authority reduces excessive risk taking at the expense of investment. This resembles the current situation on financial markets, in which banks take less risk but also provide less credit to the economy

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Sylvester C W Eijffinger
Department of Economics
CentER Tilburg University
PO Box 90153
5000 LE Tilburg
THE NETHERLANDS

Email: s.c.w.eijffinger@uvt.nl

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Rob Nijskens
Graduate Programme in Economics
CentER Tilburg University
PO Box 90153
5000 LE Tilburg
THE NETHERLANDS

Email: R.G.M.Nijskens@uvt.nl

For further Discussion Papers by this author see:
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# 1 Introduction

The financial crisis of 2008 and 2009 has shown the inability of banking regulation and supervision to cope with large shocks to the financial system. To begin with, central banks around the world have had to provide substantial amounts of liquidity to alleviate liquidity shortages and to prevent the interbank market from breaking down completely. They have provided this liquidity on very generous terms, letting virtually every bank access their facilities. Among the many banks that received liquidity assistance, several were in fact insolvent. This goes against the principle advocated by Bagehot (1873): insolvent banks should not be provided with liquidity. However, as these banks constituted a risk for the financial system as a whole, central banks have had no choice but to save them.

In addition to the liquidity provision by central banks, governments around the world have constructed very large rescue packages consisting of capital injections into banks, all-out nationalizations, explicit guarantees on bank lending and purchases of troubled assets. Halfway through 2009, total resources committed in these packages amounted to €5 trillion or 18.8% of GDP for 11 large western countries[1], whereas actual outlays were €2 trillion (Panetta et al., 2009) at that time. For some smaller countries, like the Netherlands, Denmark or Belgium, recapitalisation efforts and debt guarantees even amount to around 30% of GDP (Levy and Schich, 2010). Nevertheless, this large-scale intervention has turned out to be absolutely necessary to restore confidence and stability, and is therefore something that has to be taken into account when analyzing bank resolution.

Naturally, the academic literature on the Lender of Last Resort (LLR) and bank bailouts has increased tremendously after the recent financial crisis. Traditionally this literature has focused on the principle proposed by Bagehot (1873): a central bank acting as a Lender of Last Resort should provide liquidity freely to illiquid (but solvent) banks, against good collateral and at a penalty rate[2]. Many authors have since then analyzed, adjusted and criticized this principle.

[1]Australia, Canada, France, Germany, Italy, Japan, the Netherlands, Spain, Switzerland, the United Kingdom and the United States.

[2]A good overview of two decades of research on LLR and closure policy has been provided by Freixas and Parigi (2008).

A major criticism on Bagehot's principle is that with modern, well-functioning financial markets a Lender of Last Resort is not necessary anymore: a solvent bank in need of liquidity can go to the interbank market (Goodfriend and King, 1988; Kaufman, 1991). However, the recent financial crisis showed that it is very hard for market participants to distinguish liquidity from solvency problems, which leads to coordination failures. This kind of failures has been analyzed by, among others, Rochet and Vives (2004) and Freixas et al. (2004). In Rochet and Vives (2004), interbank market participants are not willing to lend to the bank anymore when its fundamentals fall below a certain threshold. Although the bank may still be solvent in this case, the interbank market will see it as insolvent. This suggests a role for the CB as an LLR, providing enough liquidity to increase confidence in financial markets. However, regulators also face similar problems in determining whether they should assist a bank or not (Goodhart, 1988), since banks are often better informed about the quality of their assets than regulators are. Because of the inability to discriminate between liquidity and solvency problems, banks may be inefficiently closed, or left open (Boot and Thakor, 1993; Rochet, 2004). Freixas et al. (2004) thoroughly examine this issue, assuming that the CB cannot determine ex ante whether the bank is only illiquid or also insolvent. Their model finds that a CB providing LLR support is optimal under 3 conditions: insolvent banks are not detected by the market (as in Rochet and Vives (2004)), it is costly for banks to screen borrowers, and interbank market spreads are high. This resembles crisis episodes with inefficient supervision, such as the recent financial crisis.

Also, moral hazard by the bank may ensue: as it may be provided with liquidity even if it is insolvent, it can take on more risky investments than it would otherwise do. To deal with this moral hazard problem the central bank can levy a penalty rate on its liquidity provision. However, in most financial crises this has not been the case. During the recent financial crisis, for instance, the Fed and the ECB lent freely, but not at a penalty rate (the Fed even lends at a discount). The literature has also addressed this issue, and indeed several authors found that penalty rates may even increase moral hazard. Repullo (2005), for instance, finds that the existence of a lender of last resort in itself does not create moral hazard, but the introduction of a penalty rate does. More recently, Castiglionesi and Wagner (2011) have considered liquidity provision to a bank that can become insolvent, but can influence the likelihood of this by exerting effort. This effort will be lower when penalty rates are charged, as the bank will have a lower incentive to avoid insolvency: the cost difference between illiquidity and insolvency will be lower under penalty rates.

Finally, the literature has recently considered the effect of systemic shocks on LLR practices. Rochet (2004), for instance, analyzes the optimal LLR policy when banks choose their exposure to macroeconomic shocks. Banks with an exposure above a certain threshold are perceived as too risky and should not receive liquidity assistance. However, he also finds a time inconsistency in this threshold rule, leading to ex post regulatory forbearance. More recently, Acharya and Yorulmazer (2007, 2008) have considered the interaction between banks to gauge the effect of regulation on systemic stability. Systemic problems arise when banks' investments are correlated, which leads to a time inconsistency problem for the central bank: the central bank would want to let these banks fail, but is not able to credibly commit to this policy.

We will, however, not focus on interactions between multiple banks. Instead, we model a bank that is systemic by nature (i.e. vital for the financial system) and study the interaction between this bank and multiple regulators. Repullo (2000) studies this interaction in the context of the lender of last resort function, while Kahn and Santos (2005) additionally consider the authority to close the bank. In both models regulator's choices are based on imperfectly observable information. Both analyses find that multiple regulators may improve forbearance situations, where the CB should be the LLR in case of small shocks, and that the DIF should fulfil this role in case of large shocks.

Additional to this notion of two regulators, we also incorporate the idea of prompt corrective action, as recently analyzed by Kocherlakota and Shim (2007). This means that there is one authority who should decide, at a certain threshold level of liquidity problems, whether the bank remains open or should be closed. In case it remains open, the closure authority should provide capital or guarantees to make sure liquidity provision is warranted. This resembles the recent crisis, in which central banks (as providers of liquidity) and fiscal authorities (by providing capital or guarantees) have both acted vigorously and at the same time. It is thus imperative to perform a simultaneous analysis of liquidity provision and solvency regulation. The analytical model in this paper will provide a framework for doing this. Furthermore, our analysis incorporates two principles regarding lender of last resort practices. One is the abovementioned principle of Bagehot, stating that central banks should only provide liquidity to solvent banks. The other, complementing Bagehot's doctrine, is the idea that bailout assistance (e.g. capital injections or loan guarantees) should be made costly for banks (Eijffinger, 2008), as a punishment for threatening financial stability.

The results of our analysis indicate that without any safety net, banks take excessive risk and hoard too much liquidity. The introduction of a central bank providing liquidity can decrease excessive liquidity hoarding but also leads to engagement in moral hazard by banks. To alleviate the moral hazard problem we extend the rescue measures to comprise also assistance by a fiscal authority, which can be made costly for banks. Ultimately we find that the regulators face a trade-off. Injecting capital into the bank at a fixed premium only worsens the moral hazard problem, although the bank invests more in productive assets. Demanding a share in bank equity claims, on the other hand, can decrease moral hazard at the expense of investment. This reflects the post-crisis situation: due to conditions on bailout assistance by governments, banks faced harsher funding requirements and thus extended less credit for risky investment. On the other hand, they also monitored more than before the crisis. In the next section, we will introduce our analytical framework in an informal way, before setting up our formal model.

## 2 Methodology

We model a bank that is systemically important by nature (i.e. because it is central in the financial system) and thus generates externalities if it would fail. This bank chooses its portfolio and the extent to which it monitors these investments. At an intermediate stage, the bank can suffer from a liquidity shock, and (especially in crisis times) it is difficult to obtain short term funding. Therefore, we also model a central bank (CB), that can perform the function of a Lender of Last Resort (LLR) as advocated by Bagehot[3]. However, this CB cannot observe all choices made by the bank, and thus will have to base its policy on its anticipation of these choices.

[3]This CB can be seen as an institution with a more general mandate for supervision and macroprudential policy, or financial stability.

Our approach is thus game theoretic in nature: there is strategic interaction between the bank, choosing its investment and monitoring, and the CB that sets a LLR policy. Other approaches have been taken in the literature, i.e. by Philippon and Schnabl (2009) and Bhattacharya and Nyborg (2010), that employ mechanism design to tackle regulatory questions. However, these studies do not consider liquidity and solvency at the same time. Rather, they focus on the problem of debt overhang that is more general in corporate finance, and a specific problem in banking. While they answer a very interesting question (are equity injections, asset purchases or debt guarantees the optimal intervention), this method is not very suitable in capturing the strategic interaction between banks and regulators that we want to investigate.

Instead, our approach more resembles that of Repullo (2000, 2005) and Kahn and Santos (2005), in which the CB sets a certain threshold for the liquidity shock, beyond which it will not assist the bank anymore. We then introduce an additional authority into the game: the Fiscal Authority (FA), which disposes over a solvency instrument. This instrument will be called "capital", as this FA can provide funds to the bank in return for either a senior debt or a preferred equity claim. The FA can be seen as a representative of the Treasury or Ministry of Finance; while it is independent[4], it is concerned more with maximizing social welfare than the central bank.

Let us now consider the three players in our model in turn. First, there is a systemic bank that has a given liability structure consisting of deposits and capital; capital is provided by the bank owner, while deposits are fully insured by an exogenous deposit insurance fund (DIF). The bank chooses its investment and monitoring effort. The investment is risky but productive, while its counterpart, liquid reserves, is riskless but not productive. However, liquid reserves can protect against liquidity shocks, which we discuss below. These reserves are called liquid since they represent investment in a storage technology[5]. Furthermore, monitoring of investments increases the probability of success, but comes at a cost to the bank owner. Little monitoring can thus be considered as moral hazard on the part of the banker.

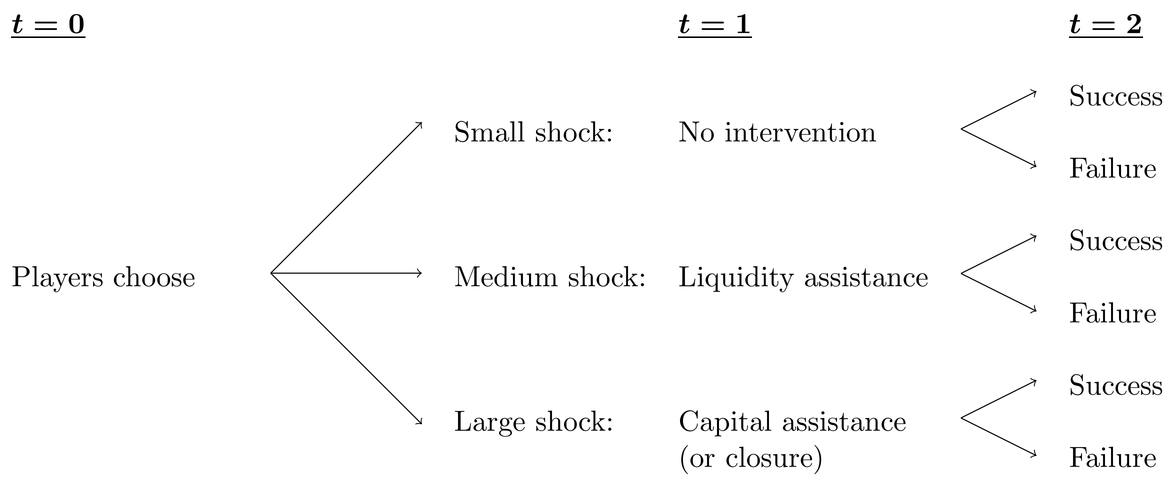
As stated above, the bank is subject to a stochastic liquidity shock at an intermediate

[4]Although time inconsistency problems may be of concern to some, we have seen that several governments (eg. the Dutch one) have been tough in providing bailout assistance.

[5]These can also be viewed as making use of existing credit lines, for example on the interbank market or at the central bank

stage. This liquidity shock leads to depositors withdrawing a fraction of their deposits (i.e. because they face an exogenous need for liquidity)[6]. When the shock is small, the bank can resolve it with its own reserves. However, when the shock is of medium size, the bank will apply for emergency liquidity at the CB. Finally, when the shock is large, the CB cannot resolve it on its own and the FA will step in. This sequence is depicted in figure 1.

Figure 1: Sequence of events




As mentioned above, when the liquidity shock is of medium size the bank will turn to the central bank (CB) for assistance. The CB minimizes a loss function that consists of the benefits and costs of providing emergency liquidity. As soon as the shock is too large to warrant a liquidity injection, the CB will stop providing liquidity.

Beyond a certain threshold (determined endogenously), the shock is considered to be large. In this case the Fiscal Authority (FA), who is the third player in our model, will decide on the bank's fate. It will be able to inject capital to improve the bank's solvency position, so the CB will be able to provide more liquidity to the bank. However, as we have seen during the crisis, the involvement of government in rescuing banks has caused a lot of public indignation. To capture this phenomenon, we assume that the FA demands a premium return on its assistance.

[6]Taking the credit crisis as a reference point, this kind of liquidity shock is very similar to investors in asset-backed securities selling their claims back to the bank. Banks were obliged to return the money, which led to severe liquidity problems. We can see this as analogous to deposit withdrawals, be it by retail depositors or wholesale investors (Rochet and Vives, 2004).

The FA can demand two types of repayment. First, it can set an ex ante premium on its support; this premium depends positively on the importance the FA attaches to preventing bank failure. Second, it can demand a stake in period 2 bank value, effectively becoming an equity claimant in the bank. This FA assistance is the innovation we add to existing LLR models; it resembles prompt corrective action (PCA) as in Kocherlakota and Shim (2007). However, our authority is not maximizing social welfare. Instead, it is an authority with a mandate to resolve problems threatening the financial system. Its independence is a necessary prerequisite to deal with time inconsistency in bank resolution.

Table 1 summarizes the players in our model and their choice variables.

Table 1: Overview of players and their choices

| Player | Choices |
|---|---|
| Bank | Investment & Monitoring |
| Central Bank | LLR policy |
| Fiscal Authority | Capital injection and its return structure |

Finally, we like to recall that we explicitly exclude both penalty rates and ambiguity in our model. As we have noted in section 1, penalty rates have not been applied in recent financial crises, and certainly not in the most recent one. Furthermore, several authors have argued that penalty rates can increase moral hazard instead of reducing it, especially when banks are close to insolvency.

Another issue that we have not yet mentioned is the notion of ambiguity. Although Bagehot stated that the CB should always provide liquidity to banks, many authors argue instead that a bank should face some uncertainty about whether it will receive liquidity. This so-called "creative ambiguity" doctrine is analyzed by, among others, Freixas (1999), Goodhart and Huang (1999) and Cordella and Levy-Yeyati (2003), with contrasting results. While Freixas (1999) finds that ambiguity may have its merits in some cases (by reducing moral hazard), it also provides a rationale for a Too-Big-to-Fail (TBTF) policy: the CB will always assist large banks with liquidity, which can be detrimental to welfare if the bank is insolvent. Essentially the same result is found by Goodhart and Huang (1999), although their TBTF policy is motivated by contagion concerns. Cordella and Levy-Yeyati (2003) also conclude that commitment to an unambiguous LLR policy can lead to an increase in bank charter value, compensating the possible moral hazard effect of having an LLR. Additional to these arguments, we also argue that many financial crises (including the most recent one) have shown that ambiguity has not been applied. Every large or otherwise important financial institution[7] has been assisted by either the central bank, fiscal authorities or both. Therefore, we abstract from this notion in our analysis. Let us now move to the formal specification of our model.

## 3 The Model

We start this section with a brief summary of section 2. We consider an economy with risk-neutral agents and three dates: $t = 0, 1, 2$. In this economy, we find one systemically important bank that operates under limited liability and will choose how much to invest in risky assets and how much liquid reserves to keep. Additionally, the bank can choose to what extent it will monitor its risky investments, thereby affecting the return structure of these assets. Furthermore, the economy also contains two regulatory agencies: a central bank (CB) fulfilling the role of Lender of Last Resort (LLR) and a fiscal resolution authority (FA) that, in case of a bank failure, has to decide on the failure resolution procedure. This authority disposes over a solvency instrument that can be used to increase the bank's capital. In return, the FA will ask either a fixed premium or an equity claim on bank value.

The bank starts at $t = 0$ with an exogenously given capital structure consisting of equity and deposits. We normalize the size of the bank to one[8], so we can denote the share of capital with $k$ and the share of deposits with $1-k$. As we have mentioned, deposits are fully insured, which means they are riskless, and thus yield a return of one at $t = 2$. To abstract completely from deposit insurance issues, we assume that the bank pays no deposit insurance premium. Equity and deposits can be invested in a risky, illiquid asset or in liquid reserves. The share of reserves will be called $l$, which provides a riskless return of one on the fraction $l$. This implies that the riskless interest rate in our model is equal to zero. This definition leaves $1-l$

[7] A notable exception being Lehman Brothers.

[8] Since we have assumed that there is only one bank and thus bank failure is costly for society, we may abstract from letting bank size determine bank closure policy.

to be invested in the risky asset. This asset provides a random gross return $\tilde{R}$ per unit of investment in period 2, with

$$\tilde{R} = \begin{cases} R(p) & \text{with probability } p \\ 0 & \text{with probability } 1-p, \end{cases}$$

where $p \in [0,1]$ is the success probability of investment, which increase with the efforts of the bank to monitor this investment. The assumptions on $R(p)$ are

$$R'(p) < 0, \quad R''(p) \leq 0, \quad R(p) \geq 1 \; \forall \, p \in [0,1], \quad R(1) + R'(1) < 0. \tag{1}$$

This return function is also used by Repullo (2005) and Cordella and Levy-Yeyati (2003), and implies that there are decreasing returns to monitoring of investments. It also allows us to analyze moral hazard in a continuous manner. Expected return on investments $E(\tilde{R}) = pR(p)$ will be maximized at $\hat{p} \in (0,1)$ where $\hat{p}$ is defined by $R(\hat{p}) + \hat{p}R'(\hat{p}) = 0$[9]. Furthermore, $E(\tilde{R})$ is greater than one, and investments are illiquid since they cannot be sold before $t = 2$. Given the above assumptions, we can write expected bank value at the end of period 2, denoted by V, as follows:

$$V = p[(R(p) - 1)(1 - l) + k]. \tag{2}$$

### 3.1 A liquidity shock

The liquidity shock $x$ is uniformly distributed on the interval $(0,1)$ with cumulative density $F(x) = x$ and probability density $f(x) = 1$. The size of the shock is public information when it occurs at $t = 1$. Taking into account that we have two regulatory agencies, we can distinguish three cases:

1. $x \leq \frac{l}{1-k} = \underline{x}$, in which the liquidity shock can be resolved using liquid reserves;

2. $\underline{x} < x \leq \overline{x}$, in which the bank is illiquid and will apply for emergency lending at the LLR. $\overline{x}$ is a threshold that is determined by the Central Bank, as described below; and

[9]Note that, for $p = 0$, $\frac{dpR(p)}{dp} = R(0) > 0$ and, for $p = 1$, $R(1) + R'(1) < 0$. The second order condition for a maximum is $\frac{d^2pR(p)}{dp^2} = 2R'(p) + pR''(p) < 0$ for all $p > 0$. This suffices for an interior maximum at $\hat{p}$.

3. $\overline{x} < x$, in which the solvency of the bank is insufficient to warrant LLR borrowing and the fiscal authority will have to take a closure/continuation decision.

In case 1, the shock is small and the bank can repay the withdrawn deposits using its liquid reserves $l$. Note that we assume there is no interbank market; the bank's only liquidity comes from the amount of liquid reserves it has kept at $t = 0$[10].

In case 2, when $\underline{x} < x < \overline{x}$, the bank cannot finance the liquidity shortage by itself, so it has to apply for emergency liquidity from the Central Bank (CB) at an amount of $x(1-k)-l$. The CB will ask a repayment $R_{CB} = 1$ (we assume no penalty rate) at $t = 2$ and will only lend to solvent banks. This means it sets a threshold for $x$, called $\overline{x}$, above which it will not lend to the bank. We will elaborate on this in section 3.2.

In the third case, when $x > \overline{x}$, the bank cannot borrow from the CB. The bank will enter into a prompt corrective action programme by the fiscal authority (FA). The FA has two ways to assist the bank, by providing capital to increase the solvency position of the bank: its new capital ratio will become $k + k_{FA}$, where $k_{FA}$ denotes the share that the FA contributes. As described in section 2, following bailout assistance the FA decides upon the conditions on which this capital will be provided. In the next section, we will explain this procedure in more detail.

## 3.2 Regulator's objectives

As stated above, we have assumed the existence of two regulatory authorities: a central bank (CB) and a financial resolution authority (FA). These authorities are given a mandate for financial stability by the government, who explicitly delegates this responsibility to these authorities to alleviate problems of time inconsistency. Thus instead of focusing on maximizing social welfare, both the CB and the FA will have a loss function that they should minimize. This reflects common arrangements in the institutional design of central banks, but also in that of financial supervisors and resolution authorities (Mayes, 2009).

In fulfilling its role of LLR, the central bank (CB) will want to minimize the social cost of a bank's failure. This is reflected in the bankruptcy cost $C$, which may represent a breakdown of i.e. payment systems, interbank lending or the general channeling of funds to productive

[10] This assumption can be justified since we are focusing on crisis management. In the financial crisis the interbank market nearly broke down (Allen et al., 2009; Diamond and Rajan, 2009). Massive intervention by central banks seemed to be the only way to get it going again.

uses. The CB will therefore provide liquidity up until a certain threshold[11]. This follows from the generally accepted principle stated by Bagehot (1873): central banks should not lend to banks that are both illiquid and insolvent. In determining this so-called solvency threshold, the CB takes into account an expected cost of $(1-p)[\alpha C + (x(1-k) - l)]$ when it supports the bank with emergency liquidity. When it does not support the bank, the CB incurs the certain loss $\alpha C$. In these expressions, $\alpha$ is the weight the regulator attaches to the bankruptcy cost. This can be interpreted as the political or reputational cost to the central bank and is assumed to be greater than zero[12].

Comparing the two above expressions, $-(1-p)[\alpha C+(x(1-k)-l)] \geq -\alpha C$, we can deduce the solvency threshold for the CB at $t=1$, denoted by $\overline{x}$:

$$x \leq \overline{x} \equiv \frac{\frac{p}{1-p}\alpha C + l}{1-k}. \tag{3}$$

Otherwise stated: the bank will apply for an amount of $x(1-k)-l$ and the CB will only provide liquidity when (3) holds. This means that the certain cost of a bank failure at $t=1$ is greater than the expected cost of failure at $t=2$. In this case the bank is considered to be solvent ($x \leq \overline{x}$), but illiquid.

In the case when $x > \overline{x}$, the FA will have to decide whether it assists the bank or not. As stated above, the FA can assist the bank by increasing the bank's capital. The FA will provide this capital $k_{FA}$ to make sure that the CB alleviates the bank's liquidity problem completely. Due to this capital injection (which may also be seen as a debt guarantee) the CB's new solvency threshold $\overline{x}$ will thus become a function of $k_{FA}$:

$$x \leq \overline{\overline{x}}(k_{FA}) = \frac{\frac{p}{1-p}\alpha C + l + k_{FA}}{1-k}. \tag{4}$$

and $k_{FA}$ will be such that $x = \overline{\overline{x}}(k_{FA})$. This means that at $t=1$, the FA injects capital such that the solvency threshold and the realization of the shock are equalized, which makes the bank solvent again from the CB's viewpoint.

[11] Depositors get $1-k$ back in case of insolvency, but this is dealt with by the DIF (a separate authority). We assume that deposit insurance is provided exogenously, so it is not part of the loss functions.

[12] $\alpha > 1$ in Kahn and Santos (2005), but Repullo (2000) assumes $\alpha < 1$ and Repullo (2005) assumes $\alpha = 1$. We will not yet make any assumptions other than $\alpha > 0$. The same holds for $\beta$ in the case of the fiscal authority.

However, the FA will also have to weigh the benefits of injecting capital against the costs. When it assists the bank the FA demands a premium that depends on the weight it places on bankruptcy cost, which is denoted by $\beta$. The FA's premium is denoted by $R_{FA}(\beta)$, with properties $R'_{FA}(\beta) > 0$ and $R_{FA}(0) = 1$. Note that $\beta$ is of similar nature as the CB's $\alpha$, but it need not be equal to $\alpha$. This reflects the political relation between the CB and the FA; they may have different responsibilities regarding financial stability. The net expected costs of providing capital are $p(R_{FA}(\beta) - 1)k_{FA}(x) - (1-p)(\beta C + k_{FA}(x))$. When it does not assist the bank, the FA will incur a certain cost $\beta C$. It follows that the maximum amount of capital that the FA is willing to provide will be determined by $p(R_{FA}(\beta) - 1)k_{FA}(x) - (1-p)(\beta C + k_{FA}(x)) \geq -\beta C$, or

$$k_{FA} \leq \overline{k}_{FA} \equiv \frac{p\beta C}{1 - pR_{FA}(\beta)}. \tag{5}$$

Substituting this expression for $\overline{k}_{FA}$ into the new CB threshold $\overline{\overline{x}}$, we arrive at a maximum for this threshold:

$$\overline{\overline{x}}_{max} = \frac{\frac{p}{1-p}\alpha C + l + \frac{p\beta C}{1-pR_{FA}(\beta)}}{1-k}. \tag{6}$$

As we can see, this depends positively on $\beta$, which can be interpreted as the weight the FA attaches to financial stability.

In the case that the FA will ask for an equity claim on the bank's value, the FA will require a share $g$ of $V$ at $t = 2$ in case of success. Again, it will incur the bankruptcy cost $\beta C$ and lose its investment $x(1-k) - l$ in case of failure. However, when it does not provide assistance, it will incur the cost $\beta C$ with certainty. The FA will then choose the repayment $g$ such that it breaks even in expectation, equating the expected loss when it assists the bank with the certain loss under no assistance. Furthermore, it again requires at least the premium $R_{FA}(\beta)$ on its investment, leading to the following expression:

$$g \geq \frac{R_{FA}(\beta)k_{FA}(x) - p\beta C}{V}, \tag{7}$$

where we will ultimately assume that this will hold with equality, as the FA will just need to break even to be willing to provide capital.

The first possibility of government assistance or bailout, where the FA injects capital, is a stylized representation of the situation in which a bank is recapitalized or provided with guarantees on its borrowing, at a certain price that is set ex ante. The second possibility, with a required period 2 return of $g$, can be seen as the government providing funds with a preferred equity claim, which is determined ex post. In the extreme ($g = 1$) this will lead to a nationalization of the bank.

These measures have been used extensively in crisis management during the last 2 years. Of course, these measures have not been free for banks: regulators have set a premium on the rates to be paid for access to these facilities, as the government has taken over part of the risk from the bank. This is for instance represented by the $g$ in our model, which contains the abovementioned risk premium $R_{FA}$. Bailout assistance thus comes at a cost for the bank owner.

### 3.3 The bank's objective

Taking the liquidity shock and the regulatory system into account, the bank owner will maximize total bank value at $t = 2$. The choice variables for the bank owner are the effort put into monitoring, embodied in the probability of success $p$, and the amount investment $1 - l$. The probability of success, which increases with monitoring effort at $t = 0$, can be interpreted as the inverse of the amount of risk taken.

Using the properties of the liquidity shock and the aforementioned conditions $\overline{x}$ and $g$ set by the regulatory authorities, we can refine the bank's objective function. We assume that there is no time discounting. The bank owner will maximize its $t = 2$ payoff, denoted by $\Pi_2$, under different regimes and different realizations of the shock $x$:

$$\Pi_2 = \begin{cases} \int_0^{\underline{x}} V dF(x) - k & \text{without any safety net,} \\ \int_0^{\overline{x}} V f(x) dF(x) - k & \text{when CB acts as LLR,} \\ \int_0^{\overline{x}} V dF(x) + \int_{\overline{x}}^{\overline{\overline{x}}} (V - (R_{FA}(\beta) - 1)k_{FA}) dF(x) - k & \text{when FA provides } k_{FA}(x), \\ \int_0^{\overline{x}} V dF(x) + \int_{\overline{x}}^{1} (V(1-g) dF(x) - k & \text{when FA acts as LLR.} \end{cases}$$

We can see that expected bank value is not only varying with $p$ and $l$, but also with $\overline{x}$, $\overline{\overline{x}}$, $k_{FA}$ and $g$. This indicates that it depends on the choices made by the bank owner as well as those made by the regulators. In the next section we will characterize this interdependence.

# 4 Liquidity or liquidation

To summarize the previous sections, we can systematically go through the sequence of events. We let the bank simultaneously choose its risk $p$ (determined by its monitoring effort) and its portfolio of risky investments $1 - l$ at $t = 0$, taking into account the possibility of liquidity shocks at $t = 1$ and responses by the CB and the FA. At $t = 1$, the liquidity shock realizes and it is observable. If $x \leq \underline{x}$, the bank pays depositors out of its liquidity reserves. If $\underline{x} < x \leq \overline{x}$, the bank applies for liquidity and the CB will provide it. Finally, if $x > \overline{x}$, the CB is not willing to provide liquidity and the FA will take action, leading to either a premium repayment by or an equity claim on the bank. Finally, at $t = 2$ returns on $1 - l$ realize and assistance has to be repaid.

## 4.1 Social welfare maximization

As a benchmark, we first analyze the socially efficient solution to the problem of choosing optimal investment and risk taking in our economy with one bank. In this case, the bank hypothetically chooses risk, investment and the regulatory instruments such that the social value of bank investments is maximized. This means that these choices also incorporate the externalities from bank failure; this assumption will not hold in a private bank setting. The gains to society are the total profit on bank investments at $t = 2$ minus the potential bank failure costs. These costs are comprised of DIF costs and bankruptcy costs $C$, and realize when the investment fails and the DIF has to pay depositors $(1-k)-l$. The problem to solve is thus:

$$\max_{p,l} V - pk - (1 - p)((1 - k) - l + C). \tag{8}$$

The first order conditions for (8) are:

$$R(p^{sw}) + p^{sw} R'(p^{sw}) = -\frac{c}{1-l} \tag{9}$$

$$1 - p^{sw} R(p^{sw}) = 0. \tag{10}$$

For $c$ not too large and $R(p)$ sufficiently concave, equation (9) holds and (10) does not; $1 - p^{sw} R(p^{sw})$ is negative. This means that it is socially optimal to set $l^{sw} = 0$ and invest all funds in the risky asset; with this knowledge, monitoring effort (and thus $p$) is chosen to maximize the expected return on these investments.

### 4.2 Bank optimization without regulation

Let us now consider the case of a private bank choosing an optimal portfolio, and analyze whether it reaches the socially efficient allocation. We assume that there are no regulatory authorities, such as a Lender of Last Resort or a fiscal authority, which may provide assistance. There is also no interbank market, as mentioned above. The bank thus has to cope with liquidity shocks on its own, which means that the bank fails if the sudden demand for liquidity is larger than the bank's liquid assets. In case of failure, the returns at $t = 2$ are zero, since effectively $g = 1$ when there is no FA. The bank's expected value is thus equal to

$$\int_0^{\underline{x}} V dF(x) - k \tag{11}$$

and the bank maximizes this by choosing investment $1 - l$ and monitoring $p$, which leads us to the following result.

**Proposition 1:** *The bank monitors less than is socially optimal (it engages in moral hazard), but also invests less in productive assets than is optimal. An increase in capital can alleviate the moral hazard problem, but also leads to less productive investment.*

Proof: see appendix ■.

The bank owner thus generates too little productive investment compared to the socially efficient case, and takes too much risk while doing so. The investment decision follows from the assumption that there is no safety net in the form of a central bank able to provide emergency liquidity; the bank has to reserve part of its funds to cope with liquidity shocks. As it has to keep more liquidity on its balance sheet, the bank tries to make up for the foregone investment returns by taking more risk. This means the bank owner ”gambles” for a higher return in the case of success, which is harmful to social welfare.

### 4.3 The Central Bank as the Lender of Last Resort

Conventionally, liquidity problems (that are of a temporary nature) can be alleviated by a Central Bank (CB) acting as the Lender of Last Resort (LLR). The bank owner then chooses risk-taking and the amount of investment in this new situation by setting $p$ and $l$, with equilibrium values $p^\ell$ and $l^\ell$ (where $\ell$ denotes that we are dealing with the possibility of liquidity provision). As in Repullo (2005) and Kahn and Santos (2005), bank and CB play a simultaneous Bayesian Nash game in the determination of $p$ and $\overline{x}$. In this game, the CB can only observe the choice of $l$ (from the bank's balance sheet) when it has to make a liquidity provision decision at $t = 1$; this observation of $l$ is not verifiable. The CB does not know the choice of $p$ at this moment[13]. However, the CB can form a belief about $p^\ell$ through its knowledge of $l$ and $k$. The threshold can be written as

$$\overline{x} = \frac{p^\ell}{1-p^\ell}\frac{\alpha C}{1-k} + \frac{l}{1-k}, \tag{12}$$

with equilibrium value $\overline{x}^l = \overline{x}(p^\ell, l^\ell)$. This threshold shows that the CB only faces downside risk; the bank gets the upside. We can also see that the threshold depends only on the bank's actual choice of $l$; it doesn't change directly with the actual choice of $p$. Instead, it is determined by $p^\ell$, the equilibrium value of $p$.

Furthermore, if $x > \overline{x}$ the bank finds itself in a crisis situation and it will be taken over completely by the fiscal authority. The depositors will be compensated by the DIF, and the remaining parts of the bank will be sold by the FA at $t = 2$. The bank owner will thus get a zero return in this case; we will relax this assumption in the next section.

[13] One could say that if the CB knows the form of the function $R(\cdot)$, it can infer the choice of $p$ perfectly ex post. However, we assume that the CB does not exactly know what the monitoring technology of the bank looks like.

At $t = 0$, the bank will take all this into account while choosing $p$ and $l$. Its new objective is thus

$$\max_{p,l} \int_0^{\overline{x}^\ell} V dF(x) - k \tag{13}$$

taking into account the equilibrium decision by the CB. The following result obtains.

**Proposition 2:** *With a central bank acting as the lender of last resort the bank engages in moral hazard, but also invests more in the productive asset. An increase in capital can counteract both these effects.*

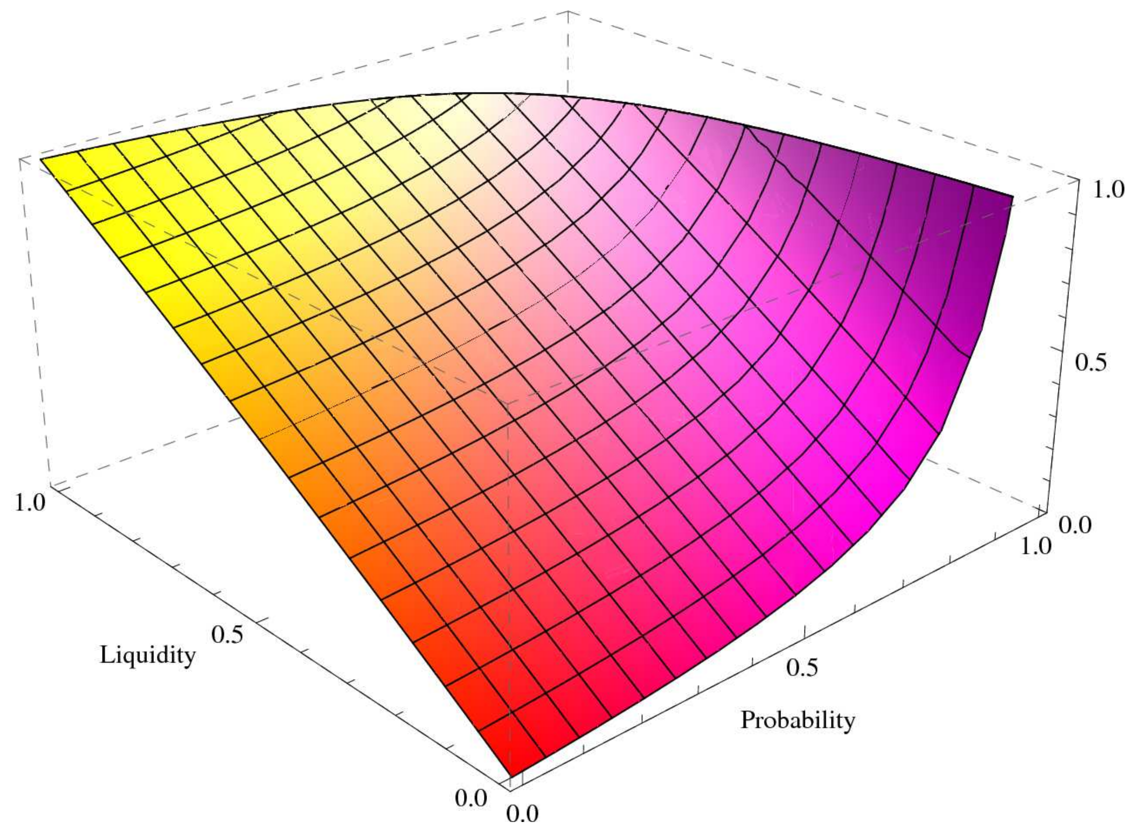
Proof: see appendix ■.

The bank thus invests more in productive assets than in the situation without a liquidity provider: a positive development. However, it also takes more risks when doing so, which is worse from a social point of view. This may reflect a moral hazard effect caused by the introduction of a safety net: since there is a Lender of Last Resort, the bank takes more risk.

To illustrate this phenomenon, we have calibrated our model using reasonable parameter values. We have specified the returns as a concave decreasing function of $p$, namely $R(p) = 3 - 2p^2$ (satisfying the assumptions from section 3.3), and the cost of bankruptcy is set to 0.10 or 10% of the bank's balance sheet (Repullo, 2005). $\alpha$ is set to 1 (Cordella and Levy-Yeyati, 2003) and the capital ratio $E$ is assumed to be at the minimum Basel II requirement, which is 8% of risk weighted assets. We assume that the risky asset gets a 100% weight.

Figure 2 shows that investment and the solvency threshold are indeed negatively related, as an increase in investment means a decrease in liquidity buffers. We also see that the probability of success and the solvency threshold are positively related. This means that an increase in investment should be met with an increase in its success probability to keep the threshold at the same level. The bank will thus face a trade-off between investment and risk-taking if it wants to induce the CB to set the optimal solvency threshold. In the end, this leads to a higher $l$ but a lower $p$: there is more productive investment (and less liquidity), but this goes with increased moral hazard.

Figure 2: The optimal solvency threshold $\overline{x}$




In section 2 we have stated plausible reasons to abstract from penalty rates and the "creative ambiguity" principle. Instead, we focus on a situation in which the regulator will bail out the bank by injecting capital (as the bank is a systemic one). At the same time, the regulator can determine what cost will be attached to this assistance. We will analyze this situation in the next section.

### 4.4 The possibility of bailout

After analyzing the case where a bank goes simply bankrupt when a crisis occurs (i.e. when $x > \overline{x}$), we will now have the Fiscal Authority assist the bank in cases of severe distress. As mentioned above, the FA injects capital $k_{FA}$ into the bank to improve its solvency. The repayment of this capital can be structured in two different ways: the FA either sets an ex ante premium that has to be repaid by the bank, or it will demand a share $g$ in the bank's final value. These options reflect senior debt and preferred equity, respectively.

### 4.4.1 Senior debt assistance

We assume that the fiscal authority gets supervisory information from the central bank. Therefore, the bank and the FA, just as the bank and the CB, play a simultaneous Bayesian Nash game. This means that the FA can only condition $k_{FA}$ on $l$ and the realization of $x$, but not on $p$ (only on its equilibrium value $p^d$).We will assume additionally that the CB and the FA observe each other's actions, but take them for granted; there is no ex ante cooperation between the CB and the FA. The only way in which the FA can influence the CB's actions is by injecting capital $k_{FA}$. This makes the bank more solvent from the CB's viewpoint, thereby increasing the CB's solvency threshold to $\overline{\overline{x}}$. The FA will then require the bank to repay this assistance at a premium $R_{FA}(\beta)$. The capital injection relates to the new CB threshold $\overline{\overline{x}}$ as follows:

$$x \leq \overline{\overline{x}} \equiv \frac{\frac{p^d}{1-p^d}\alpha C + l + k_{FA}}{1-k}. \tag{14}$$

As the FA only injects capital when the shock $x$ has been observed, it can provide just enough capital to make $x = \overline{\overline{x}}$ hold:

$$k_{FA}(x) = x(1-k) - \left(\frac{p^d}{1-p^d}\alpha C + l\right) \tag{15}$$

Weighing costs and benefits, the FA will not provide $k_{FA}$ larger than $\frac{p\beta C}{1-pR_{FA}(\beta)}$, so the new CB threshold $\overline{\overline{x}}$ will not be higher than $\overline{\overline{x}}_{max}$.

These reaction functions of the regulators are known by the bank ex ante, and it will take them into account when determining $p^d$ and $l^d$. This means that the bank's objective is as follows:

$$\max_{p,l} \quad \int_0^{\overline{x}} V dF(x) + \int_{\overline{x}}^{\overline{\overline{x}}_{max}} (V - p(R_{FA}(\beta) - 1)k_{FA}(x))dF(x) - k. \tag{16}$$

stating that the bank maximizes its value at $t = 2$, taking into account that it will have to pay a premium on capital assistance when the shock is higher than $\overline{x}$. This form of assistance can have effects on both monitoring and risk taking. The following result obtains:

**Proposition 3:** *Having a Fiscal Authority providing solvency assistance in the form of debt capital, additional to a CB providing liquidity, leads to an increase in moral hazard, but also more investment in productive assets.*

Proof: see appendix ■.

This means that the FA policy of injecting capital to increase the solvency threshold again has opposite effects: as $\beta$ increases above zero, which means the FA will take action, we see that investment increases ($l$ decreases) but $p$ decreases. In other words, this policy still creates moral hazard, although it promotes socially productive investment. This may be improved by letting the FA provide a different kind of incentives to the bank, by claiming an equity stake instead of a fixed premium on its debt assistance. We will address this situation in the next section.

#### 4.4.2 The Fiscal Authority as an equity claimant

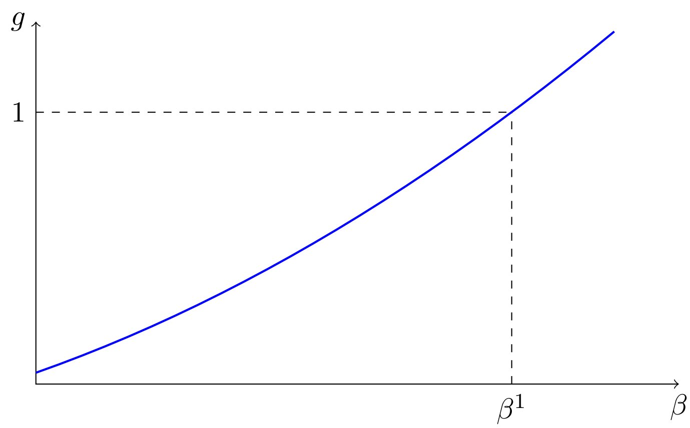
As before, the CB will be willing to provide liquidity as long as $x$ does not exceed $\overline{x}$. Above this threshold, the FA will now provide liquidity (instead of capital), which means it indeed acts as an LLR. We have assumed that, in this case, it stipulates its required return as a share in the bank's value at $t = 2$; this is denoted by $g$.

The bank again chooses risk-taking and the amount of investment in this new situation by setting $p$ and $l$, with equilibrium values $p^e$ and $l^e$. The $e$ indicates that we have added the possibility providing equity capital to the bank. As before, $g$ is determined by the following equation, where we can see it depends on the bank's actual choice of $l$, but only on its *equilibrium* choice of $p$, which is $p^e$:

$$g \geq \frac{R_{FA}(\beta)k_{FA}(x) - p^e \beta C}{V(p^e, l)}. \tag{17}$$

For the bank, this $g$ will be a function of the expectation of $x$, conditional on $x > \overline{x}$ and ,

Figure 3: The FA's required return $g$ at $(p^e, l^e)$




Nationalization at $g(\beta^1) = 1$

where $\overline{x}$ is determined in a similar manner as in section 4.3 and $\overline{x}^e$ is its equilibrium value. We thus find an expected government share of:

$$E(g) = \int_{\overline{x}^e}^{1} \frac{R_{FA}(\beta)k_{FA}(x) - p^e \beta C}{V(p^e, l)} dF(x). \tag{18}$$

Examining the properties of $E(g)$ leads us to the following useful result, illustrated in figure 3:

**Lemma 1:** *There exists a level of $\beta$, called $\beta^1$, for which $E(g) = 1$. This means that the bank is nationalized, as the FA appropriates all of the bank value at $t = 2$.*

Proof: see appendix ∎.

The bank's objective in the case of bailout possibility is thus as follows:

$$\max_{p,l} \quad \int_0^{\overline{x}} V dF(x) + \int_{\overline{x}}^{1} V(1-g) dF(x) - k \tag{19}$$

Solving this objective, making use of Lemma 1, leads to the following result:

**Proposition 4:** *Having a Fiscal Authority providing solvency assistance in the form of equity capital, additional to a CB providing liquidity, reduces the bank's moral hazard while decreasing productive investment.*

Proof: see appendix ■.

Our conclusion from this analysis is that the introduction of an FA that can claim part of bank value at $t = 2$ (decreasing $\beta$ below $\beta^1$) has opposing effects: the probability of success $p$ increases, but productive investment decreases as $l$ rises. An FA that does not completely nationalize the bank ($g = 1$) in case of a large shock, but leaves something from the bank owner will thus face a trade-off. A lower $\beta$ (becoming more lenient) can increase the probability of success, but will also lead to more liquidity hoarding by the bank. The choice of an FA regime (i.e. determining a $\beta$) thus depends on the preference of society for liquidity versus investment gains. Of course, the same holds for the choice of a CB regime. In the next section, we summarize the different liquidity and bailout possibilities.

### 4.5 Wrapping up

Table 2: Effects on monitoring and investment under different regimes relative to social welfare maximization

| Regime | Monitoring | Investment |
|---|---|---|
| No regulation | - | - |
| CB as LLR | - | + |
| FA as debt claimant | - | + |
| FA as equity claimant | + | - |

Table 2 summarizes the different situations analyzed in the previous sections. As expected, no regulation or safety net will cause the bank to gamble and hoard too much liquidity. A central bank can improve on this, but the moral hazard problem will be more severe. When the government or fiscal authority injects capital into the bank to provide more security for central bank lending, it will only exacerbate the moral hazard problem if it does this in the form of a debt contract. A final solution may be to let the fiscal authority demand an equity claim on the bank's value in case capital has to be provided. Although this alleviates the moral hazard problem, it also causes the bank to hoard more liquidity. The difference between these two results lies in the marginal benefits of these choices: the marginal benefit from monitoring is positive in the case of an equity injection, but the marginal benefit from investing in productive assets declines. This is because keeping liquid reserves influences the probability of arriving in the equity injection situation, which can be very costly for the bank.

Regulatory authorities thus face a trade-off when establishing regulation in the form of a safety net. They have to decide whether they attach more value to an increase in investment, or to a decrease in risk taking. This seems to be realistic: the nationalization, bailout and guarantee efforts by governments have led banks to mitigate their risk taking, while at the same time they have cut back on (risky) lending to entrepreneurs.

# 5 Conclusion

The recent financial crisis has provoked governments and central banks to supply unusually large amounts of capital and liquidity to banks. Regard for systemic stability is the main motivation with which this support to the financial system has been provided. However, the risk for financial stability (ultimately leading to the financial crisis) has arisen because of excessive risk taking by individual institutions that were central to the system. Since they thus posed a risk for the financial system as a whole, regulators had no choice but to prevent them from failing.

Because of the enormous costs that are associated with financial system failure, but also with its prevention, it is necessary to complement Bagehot's (1873) principle for an LLR with new measures. In our analytical model, we have thus simultaneously allowed for liquidity provision (by a central bank) and capital assistance (by a fiscal authority) to examine how they interact with a bank facing a crisis.

We have assessed this interaction for a systemic bank suffering from liquidity shocks, with which it can only cope by keeping liquid reserves. There is no interbank market in our model, reflecting a crisis situation in which the interbank market does not function well. We find that being in this situation without any regulation leads a bank to hoard too much liquid assets and take too much risk, compared to the socially efficient situation.

The introduction of a liquidity provider in the form of a central bank (CB) should alleviate this problem. This CB has no information other than the bank's investment level. It cannot observe the bank's choice of risk ex ante and can thus not condition its Lender of Last Resort

(LLR) policy upon this information. We find that this measure indeed induces a higher investment level. However, the introduction of a safety net also increases moral hazard as found by Freixas (1999).

To improve the situation, we set up a second regulator in the form of a fiscal authority (FA) that is responsible for the bank closure decision. It can also decide to give the bank a capital injection if it deems the bank solvent. This FA has the same information as the CB. We find that capital provision in return for a fixed premium causes the bank to invest more in productive assets, but also increases the moral hazard problem. However, when the FA can demand an equity claim on bank value, it can alleviate the moral hazard problem at the expense of productive investment. The FA thus faces a trade-off in choosing the regime under which it assists the bank.

We must conclude that an additional regulatory authority with responsibility for solvency is not a completely satisfactory solution for curbing excessive risk taking. This result is in line with the situation after the crisis: although banks take less risk, they provide less credit to the economy partly due to the terms of their rescue packages. Furthermore, relative effects of CB and government policies are also likely to play a role: central banks continue to provide liquidity to stimulate lending, while banks are hoarding liquid reserves as the government induces them to reduce risk.

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## Appendix: Proofs of propositions

**Proof of Proposition 1:**

The bank simultaneously chooses optimal values $p = p^n$ and $l = l^n$ to maximize its objective in equation (11). The choice of $p^n$ is given by the following first order condition (FOC):

$$R(p^n) + p^n R'(p^n) = 1 - \frac{k}{1 - l^n}, \tag{20}$$

which holds since $l > 0$: if $l = 0$, $\underline{x} = 0$ and the bank would always fail. The bank would thus choose $l^n > 0$ to receive a positive payoff at $t = 2$. Next, for $l^n$ the following FOC holds:

$$l^n = \frac{1}{2}\left[1 + \frac{k}{R(p^n - 1)}\right] \tag{21}$$

where we have used $\frac{\partial \underline{x}}{\partial(1-l)} = -\frac{1}{1-k}$. Under the assumptions on $R(p)$ these FOCs also fulfill the second order conditions for a maximum.

We can deduce from equations (20) and (21) that the bank takes more risk than is desirable from a social perspective. This follows from our assumption that the bank invests with leverage (i.e. $1 - k > l > 0$), which means $1 - l^n > k$ and thus $R(p^n) + p^n R'(p^n) > 0$. As $R(p^n) + p^n R'(p^n)$ is decreasing in $p$, we see that $p^n < p^{sw}$. Furthermore, we can state that $l^n > 0 = l^{sw}$, which follows from assuming that $k > 0$ and $R(p^n) > 1$ (otherwise it would not be profitable to invest in the risky asset).

$$l^n - l^{sw} = \frac{1}{2}\left[1 + \frac{k}{R(p^n - 1)}\right] > 0. \tag{22}$$

Finally, the capital effect can be deduced from equations (20) and (21): $\frac{\partial p^n}{\partial k} > 0$ and $\frac{\partial l^n}{\partial k} > 0$ at $R(p^n) > 1$, a condition that should hold in equilibrium.

**Proof of Proposition 2:**

The corresponding FOC w.r.t. $p$ and $l$ are:

$$R(p^\ell) + p^\ell R'(p^\ell) = 1 - \frac{k}{1 - l^\ell} \tag{23}$$

$$l^\ell = \frac{1}{2}\left[1 + \frac{k}{R(p^\ell) - 1} - \frac{p^\ell}{1 - p^\ell}\alpha C\right] \tag{24}$$

where we can see that $p^\ell$ and $l^\ell$ are determined in a similar way as $p^n$ and $l^n$.

However, we also see that $l^n \neq l^\ell$ when $\alpha > 0$, which means that $\overline{x}^l > \underline{x}$. To determine the relative size of $l^n$ and $l^\ell$, we note that a decrease in $\alpha$ means that the Central Bank cares very little about bank failure. Analogously, when $\alpha = 0$ the CB will never intervene as it will not incur any political cost from failure. This is equivalent to the earlier situation without a Lender of Last Resort. It is thus straightforward to perform comparative statics regarding $\alpha$ by taking the total derivative of $l^\ell$ w.r.t. $\alpha$ (note that $\frac{\partial p}{\partial \alpha} = 0$):

$$\frac{dl^\ell}{d\alpha} = \frac{\partial l^\ell}{\partial \alpha} + \frac{\partial l^\ell}{\partial p^\ell} \underbrace{\frac{\partial p^\ell}{\partial \alpha}}_{=0} = -\frac{C}{2} \frac{p^\ell}{1 - p^\ell} < 0 \tag{25}$$

This expression indicates that $l^\ell$ increases when $\alpha$ decreases: $l^\ell \to l^n$ when $\alpha \to 0$. This means that $l^\ell < l^n$, and thus that $1 - l^e ll > 1 - l^n$.

To compare $p^\ell$ with $p^n$, we again consider what happens as $\alpha \to 0$ by performing comparative statics w.r.t $\alpha$:

$$\frac{dp^\ell}{d\alpha} = \underbrace{\frac{\partial p^\ell}{\partial \alpha}}_{=0} + \frac{\partial p^\ell}{\partial l^\ell} \frac{\partial l^\ell}{\partial \alpha} = \left( \frac{-k/(1-l)^2}{R'(p) + pR''(p)} \right) \left( \frac{-C}{2} \frac{p^\ell}{1 - p^\ell} \right) < 0 \tag{26}$$

where the inequality holds because of the assumptions on $R(p)$. As we have found that $l^\ell < l^n$, we must also conclude that $p^\ell < p^n$ because of equation (26).

**Proof of Proposition 3:**

Equation (16) is maximized according to the following FOC, where we have used $f(x) = 1$ and we have not explicitly written out all the partial derivatives to save space:

$$FOC_p^c := \int_0^{\overline{\overline{x}}_{max}} \frac{\partial V}{\partial p}(p^c, l^c) dF(x) - \int_{\overline{x}}^{\overline{\overline{x}}_{max}} (R_{FA}(\beta) - 1) k_{FA}(x) dF(x) = 0 \tag{27}$$

$$\begin{aligned} FOC_l^c := &\int_0^{\overline{\overline{x}}_{max}} \frac{\partial V}{\partial l}(p^c, l^c) dF(x) - \int_{\overline{x}}^{\overline{\overline{x}}_{max}} p^c (R_{FA}(\beta) - 1) \frac{\partial k_{FA}}{\partial l}(p^c, l^c) dF(x) \\ &+ p^c (R_{FA}(\beta) - 1) k_{FA}(\overline{x}) \frac{\partial \overline{x}}{\partial l^c}(p^c, l^c) \\ &+ (V(p^c, l^c) - p^c (R_{FA}(\beta) - 1) k_{FA}(\overline{\overline{x}}_{max})) \frac{\partial \overline{\overline{x}}_{max}}{\partial l^c}(p^c, l^c) = 0 \end{aligned} \tag{28}$$

Given our assumptions on $R_{FA}$, we can see from these equations that the case $\beta = 0$ (when $R_{FA} = 1$ and $\overline{x} = \overline{\overline{x}}$) corresponds with the situation without an FA. Therefore, we can perform comparative statics on $\beta$ to see how the bank's choice of $p$ and $l$ change when we an FA is introduced. The most straightforward way to do this is to apply the Implicit Function

Theorem, where FOC denotes the respective first order conditions:

$$\frac{dp^c}{d\beta} = -\left(\frac{\partial FOC_p^c}{\partial p}\right)^{-1} \frac{\partial FOC_p^c}{\partial \beta} \tag{29}$$

$$\frac{dl^c}{d\beta} = -\left(\frac{\partial FOC_l^c}{\partial l}\right)^{-1} \frac{\partial FOC_l^c}{\partial \beta} \tag{30}$$

Applying the Envelope Theorem and using our assumptions on $R(p)$, $R_{FA}(\beta)$ and $\overline{\overline{x}}_{max}$ we know that $\frac{\partial FOC_p}{\partial p} < 0$ and $\frac{\partial FOC_p}{\partial \beta} < 0$, so $\frac{dp^c}{d\beta} < 0$. Furthermore, our specification of $k_{FA}$ also guarantees that, at $\beta = 0$ (the case we want to consider), $\frac{\partial FOC_l}{\partial \beta} > 0$ and $\frac{\partial FOC_l}{\partial l} > 0$, meaning that $\frac{dl^c}{d\beta} < 0$.

**Proof of Lemma 1:**

Following our assumptions on $R_{FA}(\beta)$, we can conclude that there is one value of $\beta$ that makes $E(g) = 1$. We call this value $\beta^1$. Looking at our expression for $E(g)$,

$$E(g) = \int_{\overline{x}}^{1} \frac{R_{FA}(\beta)(x(1-k) - l) - p\beta C}{V(p^e, l)} dF(x), \tag{31}$$

we can also see that $\frac{\partial g}{\partial \beta} > 0$ for some $\beta^1 > 0$ and $R'_{FA}(\beta^1)$ not too large.

**Proof of Proposition 4:**

Equation 19 is optimized according to the following FOCs:

$$FOC_p^e := R(p^e) + p^e R'(p^e) - (1 - \overline{x}^e) = 0 \tag{32}$$

$$FOC_l^e := \int_0^{\overline{x}} \frac{\partial V}{\partial l} dF(x) + \int_{\overline{x}}^{1} \frac{\partial V}{\partial l} dF(x) = 0 \tag{33}$$

where $\overline{x}^d = \overline{x}(p^e, l^e)$ and $g^d = g(p^e, l^e)$. It is not straightforward to write an explicit solution for both $p^e$ and $l^e$ from these conditions. To gauge the effect of having the possibility of bailout on $l$ and $p$, let us again perform comparative statics. Since the introduction of a bailout possibility means that $g < 1$ (as opposed to $g = 1$, were the bank is nationalized completely), our analysis should focus on the effect of this change.

Knowing the properties of $g$ and using Lemma 1, we can analyze what happens if we go from $g = 1$ (the CB case) to $g < 1$ by determining the reactions of $p^e$ and $l^e$ when $\beta$ decreases below $\beta^1$. To this end we apply the Implicit Function Theorem to equations (32) and (33):

$$\frac{dp^e}{d\beta} = -\left(\frac{\partial FOC_p^e}{\partial p}\right)^{-1} \frac{\partial FOC_p^e}{\partial \beta} \tag{34}$$

$$\frac{dl^e}{d\beta} = -\left(\frac{\partial FOC_l^e}{\partial l}\right)^{-1} \frac{\partial FOC_l^e}{\partial \beta} \tag{35}$$

Let us now determine the sign of these derivatives at $\beta^1$. Again applying the Envelope Theorem and using our assumptions on $R(p)$ and the expressions for $g$ and $\overline{x}$ we know that $\frac{\partial FOC_p}{\partial p} < 0$ and $\frac{\partial FOC_p}{\partial \beta} < 0$, so $\frac{dp^e}{d\beta} < 0$. Furthermore, noting that $\frac{\partial g}{\partial \beta} > 0$ at $\beta = \beta^1$, we can also conclude that $\frac{\partial FOC_l}{\partial l} > 0$ and $\frac{\partial FOC_l}{\partial \beta} > 0$, which means that also $\frac{dl^e}{d\beta} < 0$.