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Melvyn G Coles and
Ali Moghaddasi Kelishomi

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Melvyn G Coles, Institut d'Anàlisi Econòmica CSIC and CEPR
Ali Moghaddasi Kelishomi, University of Essex

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Centre for Economic Policy Research
77 Bastwick Street, London EC1V 3PZ, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

New Business Start-ups and the Business Cycle

This paper considers new business start-up activity within a stochastic equilibrium model of unemployment. The resulting job creation process is both natural and tractable, and generates equilibrium unemployment and vacancy dynamics which match the volatility and persistence observed in the data. The insight is that the standard Diamond/Mortensen/Pissarides matching framework works beautifully once the free entry of vacancies assumption is replaced by a model of business start-up activity. The approach is particularly important as it is demonstrated that a large part of net job creation in the U.S. economy can be attributed to new business start-ups.

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Melvyn G Coles
Institut d'Anàlisi Econòmica
Campus UAB
08139 Bellaterra
SPAIN

Email: mcole@essex.ac.uk

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Ali Moghaddasi Kelishomi
Department of Economics
University of Essex
Wivenhoe Park
Colchester CO4 3SQ

Email: amoghab@essex.ac.uk

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=174310

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1 Introduction

"There's only one strategy for growth we can have now – and that is ...doing everything possible to make it easier for people to start and to grow a business." David Cameron, March 2011.

This paper considers business start-up activity within a stochastic equilibrium matching framework. The approach is important as a large part of net job creation rates in the U.S. economy can be attributed to new business start-up activity. Following Kane (2010) and using annual firm level census data in the U.S., Figure 1 describes aggregate (net) job creation in the U.S by (i) new start-up firms (defined as firms aged less than one year) and (ii) existing firms (defined as firms which existed in the previous census year).[1]

There are two notable features to these data:

- net job creation by startups is large (around 3 million new jobs created each year) and is relatively inelastic over the cycle, while
- net job creation by existing firms is typically negative and recessions are characterised by large levels of net job destruction by existing firms.

At first sight these data seem surprising. Nevertheless it is obvious that start-up companies must generate a non-negative number of new jobs. Unless there is a rapidly expanding workforce, aggregation across all firms implies average net job creation by pre-existing firms (including firm deaths) must then (typically) be negative. Furthermore we already know from the original work by Davis and Haltiwanger (1992) that the job destruction process on existing firms generates relatively infrequent spells of large job destruction rates. Figure 1 confirms the latter insight and informs us that job creation rates by new business start-ups is relatively smooth over the cycle and that

[1] The figure uses the U.S. government dataset Business Dynamics Statistics (BDS), available at http://webserver03.ces.census.gov/index.php/bds/bds _database_list.

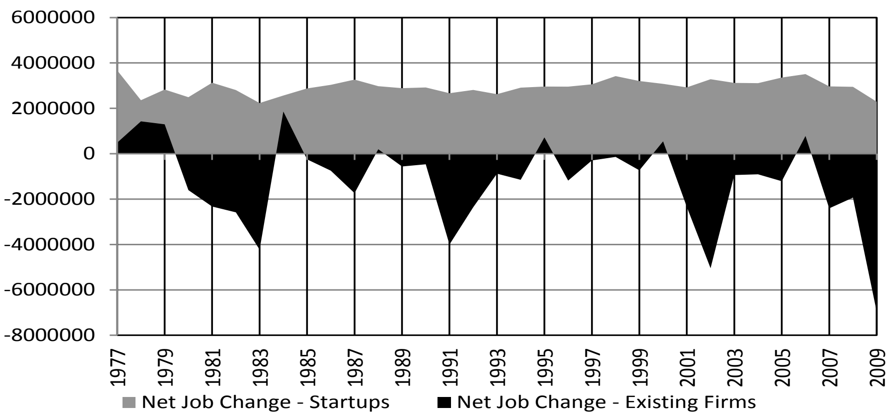

Figure 1: Net job Creation in U.S. by (i) Start-up firms and (ii) Existing firms

a large part of net job creation in the U.S. economy can be attributed to new business start-ups.

The aim of this paper is twofold. The first is to construct a tractable equilibrium framework with endogenous new business start-up activity which captures the essential properties of the data described above. Clearly a model designed with precisely these properties must then generate the observed net job creation/job destruction process. The second aim is to pass this process through the lens of the standard matching framework and ask whether the generated (unemployment,vacancy) dynamics are consistent with the data. This issue is important as, following Shimer (2005), there has been substantial debate on the relevance of the matching framework to explaining observed unemployment dynamics. Our principle finding is that the business start-up process identified here generates all relevant volatility and persistence measures of unemployment and vacancies as identified in the data by

Shimer (2005). The insight then is that the standard matching framework works beautifully once the free entry of vacancy assumption is replaced with a model of business start-up activity consistent with Figure 1. A useful byproduct of the analysis is that we also demonstrate why the previous literature, when adopting the free entry approach, struggles to match business cycle data.

Fonseca et al (2001) and Pissarides (2004) were the first to consider the link between entrepreneurial activity and equilibrium unemployment levels. Using a steady state framework, those papers establish that large cross-country variation in business start-up costs across OECD economies are correlated with large variations in observed employment levels. Here we extend that approach to a stochastic environment with idiosyncratic start-up costs: some entrepreneurs are lucky and have a good (i.e. profitable) business idea while others are less fortunate. An important literature has considered how illiquid innovators may need to match with venture capitalists in order to finance their start-up e.g. Silveira and Wright (2010).[2] We abstract from such financial frictions, however, by instead supposing that any (sufficiently) profitable idea always succeeds in being implemented, though the time to implementation may be random.

Gartner et al (2004) in the Handbook of Entrepreneurial Dynamics describe business start-ups as a three stage process. The first stage is "conception" where an entrepreneur (potentially) invests in a new business start-up. The second stage then describes the "start-up process". Depending on the efforts of the entrepreneur, this process describes the length of time it takes to bring a business into existence. The final stage describes the evolution of the firm once start-up has been successful. There is a theoretical literature on equilibrium firm size dynamics and labour turnover - for example Coles (2001), Moscarini and Postel-Vinay (2009,2010), Coles and Mortensen

[2] also see Michelacci and Suaraz (2004), Inderst and Mueller (2004), Michelacci (2007) who consider how intermediation costs affect business start-up activity.

(2011). To abstract from those complications, however, here we follow the standard Pissarides (2000) approach and assume each firm has only one job. In this approach, existing firms either continue to survive or suffer a firm destruction shock where, in the latter case, the employee re-enters the unemployment pool.

The central feature of the economy is that there are many independent entrepreneurs, each searching for profitable business ventures. The value of any such venture depends both on aggregate market conditions (such as aggregate productivity and the level of unemployment) and on an idiosyncratic component which depends on how good is the business idea. In the "conception stage", the entrepreneur decides whether to invest in the idea. Of course he/she does so as long as the business opportunity has positive net value. The "start-up" process then determines the amount of time it takes to develop the project into a workable technology. Here we shall assume the simplest case, that the time taken to develop a project is described by the exponential distribution with parameter α. Thus some projects are quickly brought to the market while others may take a long time to be completed. Once a project becomes a workable technology, the entrepreneur posts a vacancy and matching occurs in a frictional labor market.

The business start-up process is highly inertial: it not only takes time for entrepreneurs to identify profitable business ventures, it takes time to develop those ventures into new start-up companies. Indeed it is this mix of entrepreneurial search for profitable ventures and a time-to-build constraint which ensures business start-up rates are relatively inelastic over the cycle. But to see why this inertia is so central in explaining business cycle outcomes, it is worth stressing the following feature of the data: that the measured standard deviation of job separation rates from trend is much greater than the measured standard deviation of labour productivity. Figures 2 and 3 graph these measures as reported in Shimer (2005).

The job separation rate in the U.S. exhibits large variations, up to 22%

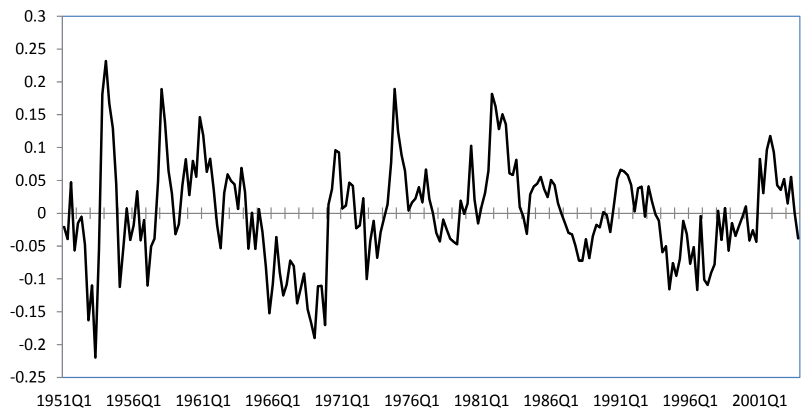

Figure 2: U.S. Separation Rates, in logs as deviations from an HP trend with $\lambda = 10^5$

from trend, compared to labor productivity variations, which peak at around 6%. Generating (endogenously) such large variations in the job separation rate given such small (exogenous) variations in labor productivity is a major theoretical challenge.[3]

In this paper we adopt a black-box approach to the job destruction process: both labour productivity and job destruction rates jointly follow an exogenous Markov process consistent with the volatility and persistence exhibited by the data (Figures 2,3).[4] The advantage of this two-shocks approach

[3]see Costain and Reiter (2007), Hagedorn and Manovskii (2008), Mortensen and Nagypal (2007) for a recent debate.

[4]This structure is very much in line with recent work which argues separation rates are strongly countercycical; e.g. Yashiv (2007), Elsby et al (2009), Fujita and Ramey (2007,

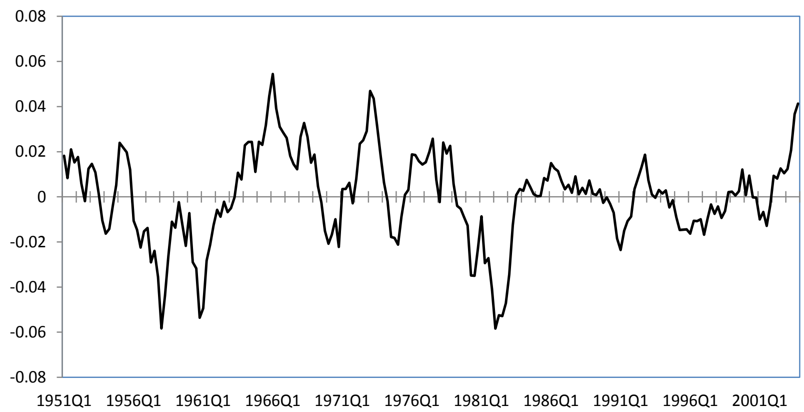

Figure 3: U.S. Labor Productivity, in logs as deviations from an HP trend with $\lambda = 10^5$

is that an aggregate productivity shock would seem a poor metaphor for the recent 2007 banking crisis. As financial failure would seem to play a central role in explaining large job (or firm) destruction rates in recessions (e.g. Kiyotaki and Moore (1997)), tying ones hands to a single source of shocks, so-called aggregate productivity shocks, would appear unduly restrictive.

Given large (exogenous to the model) job destruction shocks and free entry of vacancies, a pure job destruction shock implies vacancies V_t increase as unemployment U_t increases (as $V_t = \theta U_t$ given equilibrium market tightness θ). This is clearly inconsistent with the data where instead (unemployment, vacancy) dynamics are strongly negatively correlated. But this does not

2009)

imply that job separation shocks do not play a material role over the cycle: Figures 1 and 2 clearly demonstrate that variations in job destruction rates are indeed large. Replacing the free entry assumption with endogenous business start-up activity instead generates very different dynamics. The reason is vacancies here behave as a stock variable rather than as a "jump" variable (also see Fujita and Ramey (2007)). A large job destruction shock generates a rising tide of unemployed workers, some of whom are lucky and quickly re-match with the existing vacancy stock. As it takes time for entrepreneurs to create new jobs, the original stock of vacancies becomes depleted by the increasing number of job seekers. Thus vacancies naturally fall as unemployment increases to a (large) job destruction shock. Furthermore the added inertia generated by the business start-up process implies unemployment grows more and takes longer to recover. We show this simple change in specification for the job creation process, motivated by Figure 1, and an exogenous job destruction process, consistent with Figure 2, yields in a very natural way the volatility, correlations and persistence identified in the Shimer (2005) data.

There is a large related literature, much of which has addressed the so-called Shimer puzzle: that the "textbook" matching model cannot generate sufficient volatility in unemployment, vacancies and market tightness. Merz and Yashiv (2007) also drop the free entry assumption and consider a representative firm framework with strictly convex adjustment (hiring) costs. By slowing down the response of aggregate job creation rates to adverse job destruction shocks, convex adjustment costs also increase the variation and persistence of endogenous variables such as unemployment and vacancies over the cycle. Although their approach is quite different to ours, we identify an interesting mathematical link so that the estimates in Merz and Yashiv (2007) can be used to calibrate our model of business start-up activity.

Hagedorn and Manovskii (2011) also consider time-to-build constraints (but with free entry) and show that such constraints much improve the quantitative performance of the standard matching approach. Here, however, we introduce a much more tractable variation. Hagedorn and Manovskii (2011) assume a time-to build constraint of 3 months; i.e. an investment today becomes a vacancy in precisely 3 months time. Unfortunately in a stochastic business cycle framework, this yields a large state space as one must then keep track of all investments which have not yet finished the time-to-build phase. To achieve a finite state space, that paper has to assume any unfilled vacancy is immediately destroyed. We do not do this - instead unfilled vacancies continue search next period which then generates the vacancy stock dynamics described above.[5]

For ease of exposition, we adopt the standard Nash bargaining approach (consistent with a competitive search equilibrium (Moen (1997)). There has, however, been some debate on the role of sticky wages. With Nash bargaining, the wage responds positively to an increase in productivity which dampens the response of vacancy creation rates to such shocks. As with free entry the matching framework generates too little volatility, Shimer (2005), Hall (2005), Hall and Milgrom (2007) argue that sticky wages are necessary to increase that volatility. Mortensen and Nagypal (2007) and Pissarides (2010) instead argue that sticky wages neither generate enough volatility nor are they consistent with the data. Others have extended the bargaining approach to other wage setting mechanisms (see for example Gertler and Trigari (2009), Menzio and Shi (2010), Brugemann and Moscarini (2010)). This debate, however, becomes less directly relevant once the free entry of vacancies assumption is dropped. If here we were to impose a sticky wage then, ceteris paribus, we would then get too much volatility in vacancy creation rates over the cycle (as firm profits move more). But in our framework the vacancy creation elasticity is a modelling choice (rather than fixed at perfectly elastic). Reducing this elasticity to compensate for the introduction of sticky wages

[5] Our stochastic "time-to-build" approach has no meaningful impact in a free entry environment. If an investment which costs c only becomes a vacancy with probability α, this is equivalent to vacancies being created at expected cost c/α (which is trivial).

would then yield equivalent results.

The next section describes the model and section 3 characterises equilibrium. Section 4 calibrates the model to the data and evaluates the impact of replacing the free entry of vacancies assumption with so-called "Diamond entry" and a time to build constraint. Section 5 then compares results to an adjustment cost structure (e.g. Merz and Yashiv (2007)).

2 Model

The model uses a conventional equilibrium unemployment framework with discrete time and an infinite time horizon; e.g. Pissarides (2000). The main difference is the business start-up process which we describe first.

There is a fixed measure B of entrepreneurs in the economy who potentially invest in business ventures. The first stage in the business creation process is the "conception" stage. Given a business idea, the entrepreneur compares the investment cost x of that idea against its expected return. The expected return of the business idea depends on the state of the aggregate economy at time t, denoted Ω_t and is described in detail below. We let $J_t^P = J^P(\Omega_t)$ denote the expected return of an idea in state Ω_t. The investment cost x is considered as an idiosyncratic random draw from an exogenous cost distribution H. For tractability we assume this investment cost captures all of the idiosyncratic features associated with any given business venture - in other words, highly profitable ideas correspond to low realised values of x.

We assume each entrepreneur has one (independent) business idea in each period. Following Diamond (1982), each entrepreneur invests in their business idea if and only if it has positive value; i.e. when $J_t^P - x \geq 0$. This requires that an entrepreneur can manage multiple projects and there is no recall of a project if the entrepreneur does not immediately invest in it. As investment occurs whenever $x \leq J_t^P$ then, at the aggregate level, $i_t = BH(J_t^P)$ describes total period t investment in new projects. We refer to this investment process as Diamond-entry and note that a higher aggregate return J_t^P yields greater aggregate start-up investment i_t.

Should the entrepreneur decide to adopt a business idea, he/she pays the upfront investment cost x and then holds a *project* with expected value J_t^P. The project then enters the "start-up" process (its development phase). In an extended model one might consider the efforts an entrepreneur makes in trying to complete the start-up process. Here for simplicity the start-up process is described by the exponential distribution with parameter $\alpha \in [0, 1]$; i.e. the probability the start-up process is completed (and so creates a new business) after $k = 0, 1..$ periods is $P(k) = (1 - \alpha)^k \alpha$. Thus some business concepts are brought quickly to the market, while other concepts take longer. Assuming all projects face the same ex-ante completion probabilities, then the expected start-up time for any given project is $\alpha^{-1} - 1$. Thus the expected start-up time is zero when $\alpha = 1$. Although one might ascribe some per period development cost $d > 0$ during the start-up process, there is no loss in generality by putting $d = 0$ as these costs can be subsumed into x. Once the start-up phase has been completed, the entrepreneur has a new business which we assume creates one new job.

As each new business starts life with one unfilled job, all that remains is to describe how the entrepreneur goes about hiring a worker. To do this we adopt the standard matching framework (but without the free entry of vacancies assumption).

There is a unit mass of equally productive and infinitely lived workers. All workers and entrepreneurs are risk neutral and have the same discount factor $0 < \beta < 1$. Entrepreneurs are never unemployed (they are self-employed project managers) while workers switch between being employed and unemployed depending on the realised labour market outcomes.

Each period is characterised by the number I_t of projects currently in the "start-up" phase, the number v_t of vacancies (currently unfilled jobs) and the number u_t of unemployed workers (so that $1 - u_t$ describes the number employed). As the hiring process is frictional, the number m_t of new job-worker matches in period t is described by a matching function $m_t = m(u_t, v_t)$, where $m(.)$ is positive, increasing, concave and homogenous of degree one.

While unemployed a job seeker enjoys per period payoff $z > 0$. In period t, each job-worker match produces the same market output $p = p_t$, where aggregate productivity p_t evolves according to an exogenous AR1 process (described below). There are also job destruction shocks. δ_t describes the probability that any given job-worker match is destroyed. In the event of such a job destruction shock, the worker becomes unemployed and the job's continuation payoff is zero. This job destruction parameter, δ_t, also evolves according to an exogenous AR1 process (described below).

We next describe the sequence of events within each period t. Each period has 5 separate stages:

Stage I [new realisations]: given (p_{t-1}, δ_{t-1}) from the previous period, new values of p_t, δ_t are realised according to

$$\begin{aligned} \ln p_t &= \rho_p \ln p_{t-1} + \varepsilon_t \\ \ln \delta_t &= \rho_\delta \ln \delta_{t-1} + (1 - \rho_\delta) \ln \overline{\delta} + \eta_t \end{aligned}$$

where (ε_t, η_t) are white noise innovations drawn from the Normal distribution with mean zero and covariance matrix Σ, and $\overline{\delta} > 0$ is the long-run average job destruction rate;

Stage II [bargaining and production]: the wage w_t is determined by Nash bargaining. Production takes place so that a business with a filled job enjoys one period profit $p_t - w_t$ while an employed worker enjoys payoff w_t. Each unemployed worker enjoys payoff z;

Stage III [project investment]: entrepreneurs invest in new projects i_t and fraction $\alpha \in [0, 1]$ of all projects become new businesses, each of which holds one vacancy. If I_{t-1} denotes the stock of projects inherited from the previous period then $\alpha[I_{t-1} + i_t]$ describes new vacancy creation while

$$I_t = (1 - \alpha)[I_{t-1} + i_t] \tag{1}$$

determines the number of projects which continue into the next period. $\alpha = 1$ implies there is no time-to-build as all projects immediately become vacancies;

Stage IV [matching]: let u_t,v_t denote the stock of unemployed job seekers and vacancies at the start of this stage. Matching takes place so that $m_t = m(u_t, v_t)$ describes the total number of new matches;

Stage V [job destruction]: each vacancy and each filled job is independently destroyed with probability δ_t.

3 Markov Dynamics and Equilibrium.

This section describes the (Markov) equilibrium dynamics. Noting that u_t is defined as the number unemployed in period t immediately prior to the matching stage (stage IV), then u_t evolves according to:

$$u_t = u_{t-1} + \delta_{t-1}(1 - u_{t-1}) - (1 - \delta_{t-1})m_{t-1} \tag{2}$$

where $m_{t-1} = m(u_{t-1}, v_{t-1})$. The second term describes the stock of employed workers in period $t-1$ who become unemployed through a job destruction shock. The last term describes the match outflow where such matches are also subject to the period $t-1$ job destruction shock.

The vacancy dynamics are given by:

$$v_t = (1 - \delta_{t-1})[v_{t-1} - m_{t-1}] + \alpha[I_{t-1} + i_t]. \tag{3}$$

The first term describes those vacancies which survive (unfilled) from the previous matching event. The second term describes new vacancy creation through successful project development.

Equations (1)-(3) describe the dynamic evolution of the state variables $\{I_t, u_t, v_t\}$ which, clearly, are driven by new project investment i_t. The next step is to determine equilibrium i_t. To do this, we restrict attention to equilibria where all agents use Markov strategies.

Once (p_t, δ_t) are realised, it is useful to define the intermediate measure of vacancies

$$v_t' = (1 - \delta_{t-1})[v_{t-1} - m_{t-1}]$$

which is the number of surviving vacancies carried over from the previous period. Define the stage II state vector $\Omega_t = \{p_t, \delta_t, I_{t-1}, u_t, v_t',\}$ where I_{t-1} is the number of continuing projects inherited from the previous period, u_t is the number unemployed and v_t' is the continuing number of vacancies. Stage II determines wages according to a standard Nash bargaining procedure: below we show this yields a wage rule of the form $w_t = w^N(\Omega_t)$. Stage III then determines optimal investment in new projects: below we show this also takes the form $i_t = i(\Omega_t)$. As the matching and separation dynamics ensure Ω_t evolves as a first order Markov process, then Ω_t is indeed a sufficient statistic for optimal decision making in period t.

To determine equilibrium wage formation and new project investment, we next characterise the Bellman equations describing optimal behaviour. In period t and at the start of stage II with state vector Ω_t (i.e. prior to production and matching but after new p_t, δ_t have been realised) let:

$J_t^P = J^P(\Omega_t)$ denote the entrepreneur's expected value of a project;
$J_t^V = J^V(\Omega_t)$ denote the expected value of a vacancy;
$J_t^F = J^F(\Omega_t)$ denote the expected value of a filled job;
$V_t^U = V^U(\Omega_t)$ denote the worker's expected value of unemployment;
$V_t^E = V^E(\Omega_t)$ denote the worker's expected value of employment.

Letting $E[.|\Omega_t]$ denote the expectations operator in period t with current state vector Ω_t, the firms' value functions are defined recursively by:

$$J_t^P = \alpha J_t^V + (1-\alpha)\beta E\left\{J_{t+1}^P|\Omega_t\right\} \tag{4}$$

$$J_t^V = -c + \beta(1-\delta_t)E\left\{\frac{m(u_t,v_t)}{v_t}J_{t+1}^F + [1-\frac{m(u_t,v_t)}{v_t}]J_{t+1}^V|\Omega_t\right\} \tag{5}$$

$$J_t^F = p_t - w_t + \beta(1-\delta_t)E\{J_{t+1}^F|\Omega_t\} \tag{6}$$

where the interpretation is standard and follows the timing of the model described above.

The worker value functions are also defined recursively:

$$V_t^U = z + \beta E\left[V_{t+1}^U + (1-\delta_t)\frac{m(u_t,v_t)}{u_t}\left[V_{t+1}^E - V_{t+1}^U\right]|\Omega_t\right] \tag{7}$$

$$V_t^E = w_t + \beta E\left[V_{t+1}^E + \delta_{t+1}[V_{t+1}^U - V_{t+1}^E]|\Omega_t\right]. \tag{8}$$

To close the model all that remains is to determine the equilibrium investment and wage outcomes. As Diamond entry implies the reservation cost rule - invest if and only if cost $x \leq J_t^P$ - then equilibrium investment $i_t = i(\Omega_t)$ is given by:

$$i_t = BH(J_t^P), \tag{9}$$

where $J_t^P = J^P(\Omega_t)$.

Assuming workers have bargaining power $\phi \in [0,1]$, the axiomatic Nash bargaining approach closes the model with

$$(1-\phi)\left[V_t^E - V_t^U\right] = \phi\left[J_t^F - J_t^V\right].$$

Using the above equations, this condition determines the equilibrium wage $w_t = w(\Omega_t)$. The above thus yields a system of autonomous, first order difference equations determining (i) the evolution of Ω_t and (ii) the equilibrium value functions with corresponding investment rule $i_t = i(\Omega_t)$.

4 Calibration and Comparative Dynamics.

The central issue of interest is to compare the dynamic properties of (i) the standard textbook model with free entry of vacancies, (ii) business start-ups with no time to build $\alpha = 1$, and (iii) business start-ups with time to build $\alpha < 1$. Specifically we use previous work to calibrate the basic model framework to the US economy and assess the extent to which the implied unemployment and vacancy dynamics are consistent with:

(a) observed volatility over the business cycle;

(b) observed persistence over the cycle, and

(c) the Beveridge curve - the observed negative covariance between unemployment and vacancies.

4.1 Calibration Parameters.

Mortensen and Nagypal (2007) offers a careful critique of suitable calibration parameters and so we adopt their parameter values throughout (Table 1a). Specifically we assume each period corresponds to one month, a standard Cobb-Douglas matching function $m = Au^{\gamma}v^{1-\gamma}$ and the following Mortensen/Nagypal parameter values.

Table 1a: Mortensen/Nagypal Parameters

Parameter		Value
γ	elasticity parameter on matching function	0.6
ϕ	worker bargaining power	0.6
z	outside value of leisure	0.7
β	monthly discount factor	0.9967

Notice that bargaining is efficient in the sense that the Hosios condition is satisfied. As the productivity process for p_t (described below) ensures its (long run) mean value $\overline{p}$ is equal to one, production surplus $(\overline{p}-z)/z = 43\%$ is substantial. The monthly discount factor implies an annual discount rate of 4%.

The next step is to calibrate the stochastic process for $\{p_t, \delta_t\}$. Figures 2 and 3 in the Introduction describe the (quarterly) measures of aggregate productivity and separation rates as identified in Shimer (2005). As these data are only recorded quarterly while the model adopts a monthly time structure, we choose the autocorrelation parameters ρ_p, ρ_δ and covariance matrix Σ so that the implied process (p_t, δ_t), when reported at quarterly intervals, matches the first order autocorrelation and cross correlation implied by the data. Doing this yields:

Table 1b: (p_t, δ_t) Stochastic Process.

Parameter		Value
ρ_p	productivity autocorrelation	0.978
ρ_δ	separation autocorrelation	0.925
σ_p	st. dev. productivity shocks	0.0064
σ_δ	st. dev. separation shocks	0.031
$\rho_{p\delta}$	cross correlation	-0.60

The job destruction innovations are negatively correlated with the productivity innovations. Although the separation process is less persistent than the productivity process it has much greater variance. The long-run variance of these processes, as measured by $\sigma^2/(1-\rho)$, finds the long-run variance of productivity is small, being less than 1/7th the variance of separation shocks.

The framework is further calibrated to fit the long run turnover means. To ensure comparability of results, we follow Shimer (2005) who argues that (i) the mean job separation probability should equal 3.4% per month, (ii) the average duration of an unemployment spell is 2.2 months and thus the long run unemployment rate equals $u = 7\%$. We also note the average duration of vacancies is around 3 weeks (Blanchard and Diamond (1989)). For the free entry case, Table 1c describes the remaining parameter values so that the model fits these turnover means.

Table 1c: Turnover Parameters [free entry]

Parameter	Free Entry Case	Value
c	per period vacancy posting cost	0.17
A	scale parameter on matching function	0.594
$\overline{\delta}$	mean monthly job separation probability	0.034

Before describing the calibration results for the textbook model with free entry, we next describe how to calibrate the model with new job creation by business start-ups - for brevity "Diamond entry". For comparability we maintain the parameter values described in Tables 1a and 1b. As turnover with Diamond entry is different, however, we have to recalibrate the turnover parameters in Table 1c.

Note that the above value of c and the same turnover rates imply the value of a vacancy would, on average, be close to zero. As this would imply the value of a project is also close to zero, then investment costs x would necessarily have to be extremely small for entrepreneuurs to invest in their ideas. It seems more plausible to assume investment and development costs are significantly greater than the simple cost of posting a vacancy. For Diamond entry, we set $c = 0$ so that the value of a project is large and allows a non-trivial distribution of investment costs $H(.)$. As the investment rate $i_t = BH(J_t^P)$, one could in principle adopt functional form

$$i_t = B\,[J_t^p]^{\xi} \tag{10}$$

and ξ then describes the elasticity of project investment with respect to the value of a new project.

With no time to build constraint, $\alpha = 1$, then $J_t^P = J_t^V$ and (10) describe new vacancy creation. $\xi = \infty$ would describe infinitely elastic new vacancy creation (analogous to the free entry case) while $\xi = 0$ would imply perfectly inelastic (fixed) new vacancy creation. Inelastic new vacancy creation clearly slows down the response of vacancies to aggregate changes in profitability. In what follows we assume H is uniform and so $\xi = 1$; i.e. new vacancy creation is neither elastic nor inelastic (e.g. Fujita and Ramey (2007)). Table 1d reports the parameter values for A, B so that the model then fits the long run turnover means described above.

The final set of results described in Table 2 considers Diamond entry with time to build. Noting that the "expected" time to build is $\alpha^{-1} - 1$, then assuming a time to build constraint of 12 months as assumed in the RBC literature (e.g. Kydland and Prescott (1982)) requires $\alpha = 1/13$. Table 1d now reports the required values for A,B so that Diamond-entry also fits the long-run turnover means.

Table 1d: Turnover Parameters [Diamond Entry]

Parameter	Diamond entry	Value
c	vacancy posting cost	0
A	scale parameter on matching function	0.594
$\overline{\delta}$	mean monthly job separation probability	0.034
B (when $\alpha = 1$)	entrepreneurial activity	0.060
B (when $\alpha = 1/13$)	entrepreneurial activity	0.063

4.2 The Results

We report the results in two separate subsections. The first describes the implied volatility of unemployment, vacancies and the vacancy/unemployment ratio for the three calibrated models and compares them to the data. The second describes persistence and the the covariance of unemployment and vacancies over the cycle. In both sets of results, Column 1 describes the empirical measures taken directly from Shimer (2005). The other columns are the equivalent measures using data generated by the calibrated models, where column 2 describes the free entry case, column 3 is the Diamond entry case with no time-to-build ($\alpha = 1$), and the last column is Diamond entry with one year expected time-to-build.

4.3 Volatility in unemployment and vacancies.

Table 2a reports business cycle volatility measured as the standard deviation of unemployment (σ_u), of vacancies (σ_v) and of the vacancy/unemployment ratio ($\sigma_{v/u}$) from trend.[6]

Table 2a: volatility of unemployment and vacancies.

Volatility	Data	Free Entry	Diamond	Diamond+TTB
σ_u	0.19	0.08	0.15	0.18
σ_v	0.20	0.05	0.14	0.18
$\sigma_{v/u}$	0.38	0.07	0.28	0.36

A remarkable feature is that Diamond entry with time-to-build almost exactly matches observed volatility. With perfectly elastic new vacancy creation, the free entry case (column 2) explains around one third to one half of the observed volatility. By making new vacancy creation less elastic, Diamond entry with no time to build (column 3) increases volatility to around four fifths of the observed variation. But adding a time to build constraint yields the required level of volatility.

Figures 4,5 reveal the critical insight. They describe the impulse response function of unemployment and vacancies to a single separation innovation at date zero.

In Figure 4 with free entry of vacancies (FE), the impulse response function of unemployment to a job separation shock not only has the smallest

[6] The model-generated data was passed through an HP filter with parameter 10^5 and the standard deviations are measured as deviations from the trend.

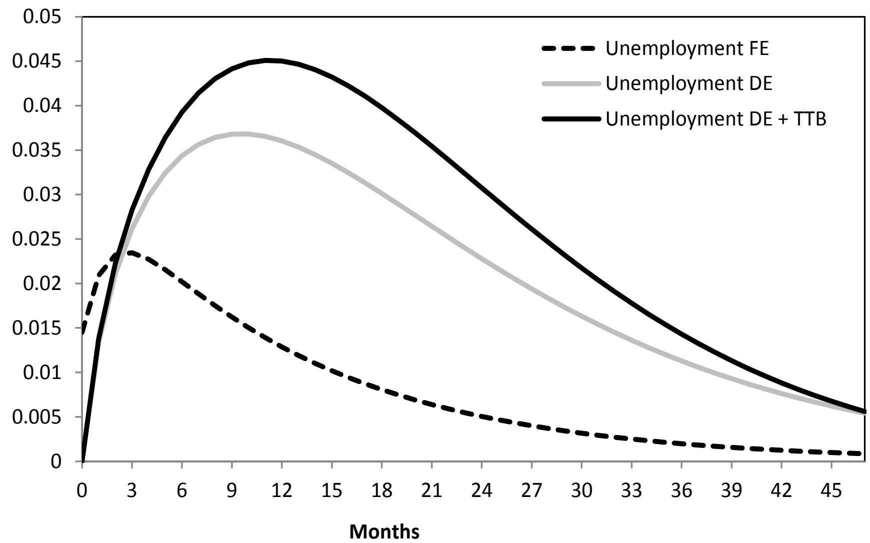

Figure 4: Impulse Response of Unemployment to a Separation Shock

increase in unemployment, unemployment also quickly recovers to its steady state value. Diamond entry with no time-to-build (DE) generates a larger unemployment peak and much more unemployment persistence. The time-to-build constraint (DE+TTB) generates an even higher unemployment peak and greater persistence. Figure 5, which describes the corresponding impulse response function for vacancies, reveals why.

Free entry of vacancies (FE) implies vacancies immediately increase in response to an increase in unemployment (with no productivity shock). This vacancy response ensures unemployment quickly recovers to its long run steady state value and the model demonstrates little persistence to the separation shock (the persistence observed is largely due to the separation process being an AR1 process given a single innovation).

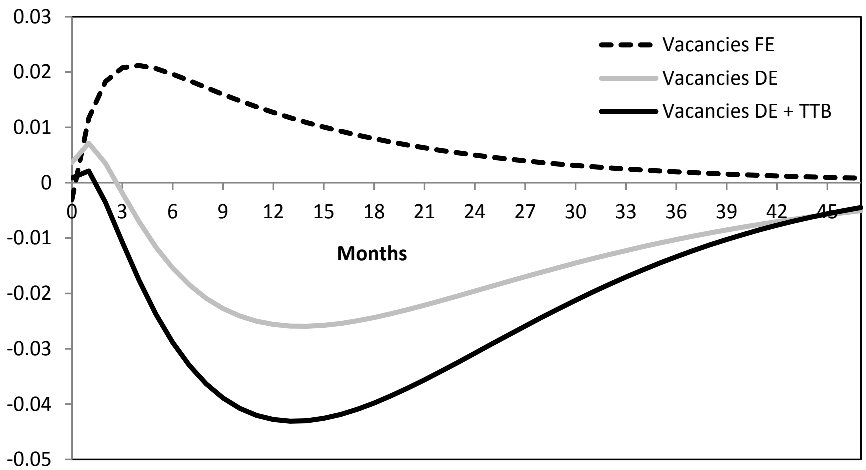

Figure 5: Impulse Response of Vacancies to a Separation Shock

With instead Diamond entry and no time to build (DE), Figure 5 demonstrates the vacancy stock initially increases to a job destruction shock. As it takes time for entrepreneurs to identify profitable business ventures, however, the vacancy response is more attenuated. Unemployment thus increases to a higher level than that implied by the case with free entry. The equilibrium dynamics find the vacancy stock overshoots, eventually falling below its long run value before converging. At first sight this dynamic suggests that new vacancy creation rates fall below their long-run steady state value. This is not correct: the economy's equilibrium response ensures that new vacancy creation and hiring always exceed their long run steady state values. Overshooting occurs, however, as the increasing number of job seekers cause the aggregate match rate to exceed the vacancy creation rates for a sufficiently long period that the vacancy stock falls below its long-run steady state value (before ultimately converging).

The introduction of the time-to-build constraint, $\alpha = 1/13$, instead implies vacancies immediately fall in response to the job separation shock. The separation innovation generates a rising tide of unemployed workers who rematch with the existing vacancy stock. As it takes time to increase new vacancy creation rates, the original stock of vacancies becomes depleted by the increased number of job seekers. The increased inertia implies unemployment grows more and it takes longer for the economy to recover. This in turn yields larger variations in unemployment, the vacancy stock and the vacancy/unemployment ratio over the cycle.

4.4 Persistence and covariance of Unemployment and Vacancy stocks.

Table 2b describe the persistence and covariance of unemployment and vacancies over the cycle.

Table 2b: persistence in unemployment and vacancies.

Serial Persistence	**Data**	**Free Entry**	**Diamond**	**Diamond+ttb**
autocorr(u)	0.94	0.85	0.95	0.96
autocorr(v)	0.94	0.77	0.96	0.96
corr (u,v)	-0.89	0.33	-0.93	-0.97

The first two rows describe the autocorrelation of vacancy and unemployment stocks. Both versions of Diamond entry generate the right degree of persistence, though the free entry case also does reasonably well in this dimension. The big difference, however, is how unemployment and vacancies covary over the cycle. Row 3 describes the Beveridge curve: how unemployment and vacancies covary over the cycle. The data find this response is strongly negative. The free entry case instead finds that vacancies and unemployment are positively correlated. This result arises as separation shocks are large (the free entry model obtains the required negative covariance between unemployment and vacancies when there are no separation shocks). In contrast, both Diamond entry models find that vacancies and unemployment covary negatively over the cycle. Indeed the negative covariance would seem a little too strong.

5 On the relationship with adjustment cost models

The previous section established why the free entry approach may not be a useful assumption when trying to explain the business cycle. Rather than adopt the business start-up approach developed here, however, an alternative is to introduce employment adjustment costs.

Assuming no time to build ($\alpha = 1$), the above approach implies new vacancy creation (by entrepreneurs)

$$i_t = BH(J_t^V) \tag{11}$$

with corresponding vacancy stock dynamics

$$v_t = (1 - \delta_{t-1})[v_{t-1} - m_{t-1}] + i_t. \tag{12}$$

Yashiv (2006) instead considers a general equilibrium RBC framework with a frictional labour market.[7] Assuming no adjustment costs on capital and constant returns to scale in production, suppose a representative firm with N_t employees chooses recruitment effort v to solve the programming

[7] Yashiv (2006) assumes cost function $c = c([\phi v_t + (1-\phi)h_t]/N_t]$ which is a weighted average of recruitment effort v and realised hires. The exposition here focusses on the two extreme cases.

problem:

$$\begin{aligned}\Pi(N_t,\Omega_t) &= \max_v \left[N_t\left[p_t - w_t - p_t c(\frac{v}{N_t})\right] + \beta E\left[\Pi(N_{t+1},\Omega_{t+1})|\Omega_t\right]\right]\\ \text{where } N_{t+1} &= (1-\delta_t)[N_t + v\frac{m_t}{v_t}]\end{aligned}$$

with aggregate matching $m_t = m(u_t, v_t)$ and, by construction, firm recruitment costs $p_t N_t c(v/N_t)$ are homogenous of degree 1 in (v, N). Assuming $c(.)$ is increasing and strictly convex, optimal recuitment effort is then given by the first order condition:

$$p_t c'(\frac{v}{N_t}) = \beta(1-\delta_t)\frac{m(u_t,v_t)}{v_t} E\left[\Pi_N(N_{t+1},\Omega_{t+1})|\Omega_t\right].$$

Given the constant returns to scale assumption then, in the context of our model, the corresponding vacancy creation rule can be written as

$$p_t c'(\frac{v}{N_t}) = \beta(1-\delta_t)\frac{m(u_t,v_t)}{v_t} E\left\{J^F_{t+1}|\Omega_t\right\}. \tag{13}$$

where the right hand side describes the expected value of a vacancy J^V_t. Note that the impact of job destruction shocks δ_t in (13) is to introduce "stochastic" discounting on the value of a filled job. As the return to recruitment effort depends on "queue" length $q_t \equiv m_t/v_t$, note that an increase in aggregate unemployment u_t directly increases the return to recruitment effort v.

Merz and Yashiv (2007) structurally estimate a closely related employment adjustment cost function and find it is well approximated by a cubic function. (13) thens yields the representative firm's vacancy creation rule $v = c_0 N_t[J^V_t/p_t]^{1/2}$ where $c_0 > 0$ is a parameter to be calibrated. Comparing this specification with equation (11) suggests rather than assume H is uniform (linear) instead set $\xi = 1/2$ so that new vacancy creation $i_t = B[J^V_t]^{1/2}$.

Tables 3a, 3b compare the calibrated outcomes of three cases: (i) the above adjustment cost structure with $c(.)$ cubic and noting firm aggregation implies aggregate $v_t = c_0[1-u_t][J_t^V/p_t]^{1/2}$ (ii) Diamond entry with $\xi = 1/2$ and $\alpha = 1$, (iii) Diamond entry with $\xi = 1/2$ and $\alpha = 1/13$ (one year expected time to build).[8]

Table 3a: volatility with an adjustment cost specification.

Volatility	Data	Adjustment Cost	Diamond $\xi = 1/2$, no TTB	Diamond $\xi = 1/2$ +TTB
σ_u	0.19	0.08	0.17	0.19
σ_v	0.20	0.04	0.17	0.21
$\sigma_{v/u}$	0.38	0.05	0.34	0.40

The adjustment cost specification (column 3) does surprisingly badly - we discuss this further below. Comparing the Diamond entry results with those in Table 2a establishes our calibrations are robust. Making new project investment less elastic (reducing ξ from one to 0.5) increases volatility: the economy adjusts even more slowly to adverse job destruction states. The change in measured volatility, however, is not large. The added time to build constraint yields volatility measures which are now (slightly) greater than those found in the data. Table 3b reports the corresponding persistence measures and confirms that the Diamond entry process has reduced form properties which are very close to the data. It also reveals the difficulty faced by the above adjustment cost specification.

Comparing with Table 2b which describes persistence in the free entry case, the introduction of adjustment costs clearly increases the persistence of vacancies over the cycle. It also increases the (counterfactual) positive covariance between unemployment and vacancies over the cycle. The reason

[8] This changes the value of B in our calibration but the rest of the parameters are set the same as in the benchmark calibration

is that, as in the free entry case, vacancies are a jump variable: in an adverse job destruction state with rising unemployment, vacancies in the model increase with unemployment as it is easier for firms to attract new employees.

Table 3b: persistence and correlations.

Persistence	**Data**	**Adjustment Cost**	**Diamond** $\xi = 1/2$, **no TTB**	**Diamond** $\xi = 1/2$ **+TTB**
autocorr(u)	0.94	0.88	0.96	0.96
autocorr(v)	0.94	0.89	0.96	0.97
corr (u,v)	-0.89	0.98	-0.95	-0.98

In contrast, the case of "Diamond entry" implies vacancies evolve as a stock variable with dynamics given by (12). Those dynamics match the data as, with inelastic new vacancy creation, the vacancy stock instead becomes depleted by a rising tide of new job seekers.

An important property in Merz and Yashiv (2007) is not just the addition of adjustment costs to the matching framework, it is that the cost of adjustment is on realised hires (say training costs of new recruits) rather than on recruitment effort. For example suppose instead a representative firm with N_t employees chooses hires h to solve the programming problem:

$$\begin{aligned} \Pi(N_t, \Omega_t) &= \max_h \left[N_t[p_t - w_t] - c(h) + \beta E\left[\Pi(N_{t+1}, \Omega_{t+1})|\Omega_t\right]\right] \\ \text{where } N_{t+1} &= (1-\delta_t)[N_t + h]. \end{aligned}$$

where $c(.)$ now describes the cost of training h new employees (rather than recruitment effort). With constant returns to scale, the first order condition describing optimal hires is

$$c'(h_t) = \beta(1-\delta_t)E[J^F_{t+1}|\Omega_t].$$

Comparing this condition with equation (13) identifies the critical restriction:

in the absence of matching frictions, the aggregate match rate h_t [say there is a unit measure of firms] no longer responds directly to fluctuations in aggregate unemployment. Of course this does not imply unemployment has no impact on equilibrium hiring rates - wages might still be determined by a Nash bargaining rule and lower wages (caused by higher unemployment) raise the value of a filled job J_t^F. Nevertheless this hire process ensures aggregate hiring rates do not respond directly to variations in unemployment. The Diamond entry approach identified here relies on a similar mechanism - an inelastic job creation process yields greater volatility over the cycle. But the above also establishes (with Diamond entry) that the corresponding vacancy stock dynamics generate equilibrium unemployment and vacancy dynamics which are consistent with the data.

6 Conclusion.

It is important to note the original Mortensen and Pissarides (1994) framework assumed only one source of aggregate variation - aggregate productivity shocks - and explained two margins of variation, both job creation and job destruction margins. It is perhaps surprising how well that model performs. Here instead there are two types of aggregate shocks - productivity shocks and job destruction shocks - and there is no attempt to explain the job destruction margin. Not surprisingly the more flexible structure allows a much improved fit of the business cycle data, though only when the free entry assumption is dropped in favour of a model of new business start-up activity. The advantage of this two-shocks approach is that an aggregate productivity shock would seem a poor metaphor for the recent (and ongoing) 2007 banking crisis. Indeed an extended framework might examine how financial distress is propagated across firms and how it leads to large job destruction levels (for example see Kiyotaki and Moore (1997) on credit chains and default). By adopting a statistical black-box approach to describe the job destruction process, this paper identifies a job creation process which is simple, natural and generates equilibrium unemployment and vacancy dynamics which match the business cycle frequencies.

Of course the restriction to one firm-one job is very strong. It is useful as it rules out on-the-job search where workers switch to higher productivity firms who may have several vacancies. The difficulty with an extension to on-the-job search is that the state space typically (but not always) includes the distribution of firm size which evolves endogenously (see for example Coles (2001), Menzio and Shi (2010), Moscarini and Postel-Vinay (2010), Coles and Mortensen (2011)). Clearly the on-the-job search literature is important and provides useful insights on turnover behaviour and aggregate productivity: see for example the discussions contained in Lentz and Mortensen (2008), Moscarini and Postel-Vinay (2009). Nevertheless by abstracting from inter-firm employment dynamics, this paper identifies a tractable equilibrium model of unemployment which is consistent with observed dynamics. It thus identifies a coherent direction for future research on policy and the business cycle.

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