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ABSTRACT

Explaining the Black/White Employment Gap: The Role of Weak Ties*

The aim of this paper is to provide a new mechanism based on social interactions explaining why minority workers have worse labor-market outcomes than majority workers. Building on Granovetter's idea that weak ties are superior to strong ties for providing support in getting a job, we develop a social interaction model where workers can obtain a job through either their strong or weak ties. In this model, it is better to meet weak ties because a strong tie does not help in the state where all best friends are unemployed. But a weak tie can help leaving unemployment in any state because that person might be employed. So there is an asymmetry that is key to the model and that explains why some workers (blacks) may be stuck in poverty traps having little contact with weak ties (whites) that can help them escape unemployment.

JEL Classification: A14, J15 and Z13
Keywords: labor market, social networks and weak ties

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1 Introduction

Disparities between blacks and whites are persistent features of American society. Blacks in the United States are poorer than whites and have much lower employment rates. In 2008, the poverty rate for whites was 9.5 percent, compared with 23.7 percent for blacks, and the employment-to-population ratio for those aged 25-54 was 80.1 percent for whites, versus 73.9 percent for blacks (Hellerstein and Neumark, 2011). Understanding the causes of current racial inequality is a subject of intense debate. A wide variety of explanations have been put forward, which range from urban segregation (Kain, 1968; Massey and Denton, 1993; Ihlanfeldt and Sjoquist, 1998; Cutler et al., 1999; Iceland and Weinberg, 2002) to genetics (Rushton and Jensen, 2005) to personal and institutional discrimination (Darity and Mason, 1998; Altonji, and Blank, 1999; Pager, 2007; Charles and Guryan, 2011) to the cultural backwardness and dysfunctional families of minority groups (Reuter, 1945; Wilson, 2009; Heckman, 2011).

In the present paper, we propose an alternative explanation. Building on Granovetter (1973, 1974, 1983)'s idea that weak ties are superior to strong ties for providing support in getting a job,[1] we develop a model in which black workers have less connections to weak ties than whites. As underscored by Granovetter, in a close network where everyone knows each other, information is shared and so potential sources of information are quickly shaken down so that the network quickly becomes redundant in terms of access to new information. In contrast Granovetter stresses the *strength of weak ties* involving a secondary ring of acquaintances who have contacts with networks outside ego's network and therefore offer new sources of information on job opportunities.[2]

A number of studies for a range of countries have emphasized the popularity of using friends and family as a job search mechanism and indicate that they are an effective mechanism for obtaining job offers (Rees, 1966; Granovetter, 1974, 1979; Blau and Robins, 1990; Topa, 2001; Wahba and Zenou, 2005; Bayer et al., 2008; Bentolila et. al, 2010; Pellizzari,

[1]In his seminal papers, Granovetter (1973, 1974, 1983) defines *weak ties* in terms of lack of overlap in personal networks between any two agents, i.e. weak ties refer to a network of acquaintances who are less likely to be socially involved with one another. Formally, two agents A and B have a weak tie if there is little or no overlap between their respective personal networks. Vice versa, the tie is *strong* if most of A's contacts also appear in B's network.

[2]The existing empirical evidence lends some support to Granovetter (1995)'s ideas. Yakubovich (2005) uses a large scale survey of hires made in 1998 in a major Russian metropolitan area and finds that a worker is more likely to find a job through weak ties than through strong ones. These results come from a within-agent fixed effect analysis, so are independent of workers' individual characteristics. Using data from a survey of male workers from the Albany NY area in 1975, Lin et al. (1981) find similar results. Lai et al. (1998) and Marsden and Hurlbert (1988) also find that weak ties facilitate the reach to a contact person with higher occupational status, who in turn leads to better jobs, on average. See also Patacchini and Zenou (2008) who find evidence of the strength of weak ties in crime.

2010). The empirical evidence reveals that around 50 percent of individuals obtain or hear about jobs through friends and family (Holzer 1987, 1988; Montgomery, 1991; Addison and Portugal, 2001). Such methods have the advantage that they are relatively less costly and may provide more reliable information about jobs compared to other methods. Little is known, however, about the nature of job search methods across different ethnic groups and it is not clear how effective different methods are at linking job seekers to jobs for different ethnic groups. In particular, do the kinds of positive effects that have been found for friends and family hold across all ethnic groups in the labor market?

We investigate in more detail this issue in the present paper. To be more precise, we consider a dynamic model of the labor market in which dyad members do not change over time so that two individuals belonging to the same dyad hold a *strong tie* with each other. However, each dyad partner can meet other individuals outside the dyad partnership, referred to as *weak ties* or random encounters. By definition, weak ties are transitory and only last for one period. The process through which individuals learn about jobs results from a combination of a socialization process that takes place *inside* the family (in the case of strong ties) and a socialization process *outside* the family (in the case of weak ties).[3] Thus, information about jobs is essentially obtained through strong and weak ties and thus word-of-mouth communication.

We first characterize the steady-equilibrium employment rate and determine the time spent in a d_0 dyad (both workers are unemployed), in a d_1 dyad (one worker is employed and the other one is unemployed) and in a d_2 dyad (both workers are employed). We then show that the higher is the job-destruction rate or the lower is the job-information rate, the lower is the employment rate and more workers spend time in a d_0 dyad. We also show that by increasing the probability ω of meeting *new workers* (i.e. weak ties), the steady-state employment rate increases, formally demonstrating the Granovetter's informal idea of the *strength of weak ties* in finding a job. This is not a trivial result, since, by increasing ω, we have different and opposite effects on the job formation/destruction process. On the one hand, we increase the probability of getting out of unemployed dyads, while, on the other hand, we potentially give up the information of an employed partner in favor of a link with an unemployed one. We obtain this result because, in our model, it is indeed better to meet strong ties because a strong tie does you no good in state d_0 since your best friends are all unemployed. But a weak tie can do you good in any state because that person might be employed. So there is an *asymmetry* that is key to the model and that can explain why some workers (blacks) may be stuck in poverty traps (i.e. d_0 dyads) having little contact with weak ties that can help them leaving the d_0 dyad.

We then extend this framework by endogeneizing wages and social interactions. Using

[3]This idea was first put forward by Bisin and Verdier (2000, 2001) in the context of the transmission of a trait like, for example, religion or identity.

an efficiency wage approach, we show that, if workers use more their weak ties than strong ties to find a job, then they will receive higher wages if the elasticity of shirking employment with respect to weak ties is higher than that of non shirking. When workers choose social interactions, we show that when the job-destruction rate or the job-information rate increases, workers rely more on their weak ties. This means that, in downturn periods where jobs are destroyed at a faster rate, workers tend to spend more time with their weak ties because they know they will help them exit unemployment. In an economy where the flow of job information is faster, the same results occur.

Finally, we explicitly model black and white workers. We assume that strong ties are always of the same race while weak ties can be of either race. The main difference between black and white workers is that, because of racial discrimination, the former hear less about jobs than the latter. If, on top of that, they spend less time interacting with their weak ties (potentially whites), we show that black workers will be stuck in high unemployment 'traps' since they mostly exchange information with their strong ties, who are themselves likely not to possess much useful information about job opportunities. Since most blacks in the United States tend to be isolated from white workers (Ihlanfeldt and Sjoquist, 1998), then this model could explain why they have difficulty leaving unemployment. In our model, it is due to the fact that they mainly interact with their strong ties (other blacks) and very little interaction with their weak ties (whites) so that their information about jobs is limited since blacks tend to be more unemployed and have poorer social networks than whites (see, e.g. Wial, 1991). This is related to Putman (2007) who finds that higher levels of ethnic homogeneity are associated with higher level of trust.[4] In other words, blacks will not interact with whites (and vice versa) because they do not trust each other. In our framework, they do not interact with each other because they may choose not to interact with weak ties.

Related literature

There is a growing interest in theoretical models of peer effects and social networks (see e.g. Akerlof, 1997; Glaeser et al., 1996; Ballester et al., 2006; Calvó-Armengol et al., 2009; Goyal, 2007; Jackson, 2008; Ioannides, 2011), especially in the labor market. However, few models of social networks in the labor market are dynamic. Montgomery (1994) and Calvó-Armengol et al. (2007) propose a dynamic model of weak and strong ties but the former focuses on inequality while the latter on the interaction between crime and labor markets. Calvó-Armengol and Jackson (2004) have a more general network analysis (since they can encompass any network structure) but do not model racial issues.

There is also a theoretical literature on job search and social networks. See, in particular, Diamond (1981), Montgomery (1991), Mortensen and Vishwanath (1994), Calvó-Armengol (2004), Calvó-Armengol and Jackson (2004), Calvó-Armengol and Zenou (2005), Cahuc and

[4] Other studies have also found that socioeconomic diversity is associated with lower level of trust (Alesina and la Ferrera, 2002; Glaeser et al., 2000). See the literature review by Costa and Kahn (2003).

Fontaine (2009), and the literature surveyed by Ioannides and Loury (2004) and Topa (2011). However, these models have a very restrictive way of modelling the social network and racial issues are not taken into account.

To the best of our knowledge, this is the first paper that proposes a simple model showing the superiority of weak ties over strong ties and its implications for black and white workers. There are some empirical papers looking at the role of social networks for minority workers. Using data from the Quarterly Labour Force Survey, Frijters et. al (2005) and Battu et al. (2011) examine the job finding methods of different ethnic groups in the UK. Their empirical findings suggest that, though personal networks are a popular method of finding a job for the ethnic minorities, the foreign born and those who identify themselves as non-British, they are not necessarily the most effective either in terms of gaining employment or in terms of the level of job achieved. However, there are some important differences across ethnic groups with some groups losing out disproportionately from using personal networks. For the United States, Falcón and Melendez (1996) find that Latinos in Boston are more likely to use personal networks to gain employment relative to other job search methods. However, in an earlier study Falcón (1995) finds that Boston Latino's use of personal networks actually reduces their earnings. Green et al. (1999) also find an earnings penalty for Hispanics and Whites from utilizing informal job searches (personal networks) as opposed to formal approaches such as replying to advertisements. In a more recent paper, Mouw (2002), using longitudinal data, finds that Black workers who used personal contacts to find employment did no worse compared to where they used formal methods. The present paper investigates this issue from a theoretical perspective and shed light on the outcome and policy implications of the "narrow" social network of minority workers.

2 The model

Consider a population of individuals of size one.

Dyads We assume that individuals belong to mutually exclusive two-person groups, referred to as *dyads*. We say that two individuals belonging to the same dyad hold a *strong tie* to each other. We assume that dyad members do not change over time. A strong tie is created once and for ever and can never be broken. Thus, we can think of strong ties as links between members of the same family, or between very close friends.

Individuals can be in either of two different states: employed or unemployed. Dyads, which consist of paired individuals, can thus be in three different states,[5] which are the following:

[5]The inner ordering of dyad members does not matter.

(i) both members are employed −we denote the number of such dyads by d_2;

(ii) one member is employed and the other is unemployed (d_1);

(iii) both members are unemployed (d_0).

Aggregate state By denoting the employment rate and the unemployment rate at time t by $e(t)$ and $u(t)$, where $e(t), u(t) \in [0, 1]$, we have:

$$\begin{cases} e(t) = 2d_2(t) + d_1(t) \\ u(t) = 2d_0(t) + d_1(t) \end{cases} \tag{1}$$

The population normalization condition can then be written as

$$e(t) + u(t) = 1 \tag{2}$$

or, alternatively,

$$d_2(t) + d_1(t) + d_0(t) = \frac{1}{2} \tag{3}$$

Social interactions Time is continuous and individuals live for ever. We assume repeated random pairwise meetings over time. Matching can take place between dyad partners or not. At time t, each individual can meet a weak tie with probability $\omega(t)$ (thus $1 - \omega(t)$ is the probability of meeting her strong-tie partner at time t).[6] In Sections 2 to 5, we assume these probabilities to be constant and exogenous, not to vary over time and thus, they can be written as ω and $1 - \omega$. We endogeneize ω in Section 4.2 below. Observe that strong ties and weak ties are assumed to be substitutes, i.e. the more someone spends time with weak ties, the less he has time to spend with her strong tie.

We refer to matchings inside the dyad partnership as *strong ties*, and to matchings outside the dyad partnership as *weak ties* or random encounters. Within each matched pair, information is exchanged, as explained below. Observe that we assume symmetry within each dyad, that is if I meet a strong (or a weak) tie, then my strong (or weak) tie has to meet me. In the language of graph theory, this means that the network of relationships is *undirected* (Jackson, 2008).

Information transmission Each job offer is taken to arrive only to employed workers, who can then direct it to one of their contacts (through either strong or weak ties). This is a

[6] If each individual has one unit of time to spend with his friends, then $\omega(t)$ can also be interpreted as the percentage of time spent with weak ties.

convenient modelling assumption, which stresses the importance of on-the-job information.[7] The gist of the analysis would be preserved if this assumption is relaxed. To be more precise, employed workers hear of job vacancies at the exogenous rate λ while they lose their job at the exogenous rate δ. All jobs and all workers are identical (*unskilled labor*) so that all employed workers obtain the same wage. Therefore, employed workers, who hear about a job, pass this information on to their current matched partner, who can be a strong or a weak tie. Thus, information about jobs is essentially obtained through social networks.

This information transmission protocol defines a Markov process. The state variable is the relative size of each type of dyad. Transitions depend on labor market turnover and the nature of social interactions as captured by ω. Because of the continuous time Markov process, the probability of a two-state change is zero (small order) during a small interval of time t and $t + dt$. This means, in particular, that both members of a dyad cannot change their status at the same time. For example, two unemployed workers cannot find a job at the same time, i.e. during t and $t + dt$, the probability assigned to a transition from a d_0-dyad to a d_2-dyad is zero. Similarly, two employed workers (d_2-dyad) cannot both become unemployed, i.e. switch to a d_0-dyad during t and $t + dt$. This applies to all other dyads mentioned above.

Flows of dyads between states It is readily checked that the net flow of dyads from each state between t and $t + dt$ is given by:

$$\left\{ \begin{array}{l} \dot{d}_2(t) = h(e(t))d_1(t) - 2\delta d_2(t) \\ \dot{d}_1(t) = 2g(e(t))d_0(t) - [\delta + h(e(t))]\, d_1(t) + 2\delta d_2(t) \\ \dot{d}_0(t) = \delta d_1(t) - 2g(e(t))d_0(t) \end{array} \right. \tag{4}$$

where $h(e(t)) \equiv [1 - \omega + \omega\, e(t)]\, \lambda$ and $g(e(t)) \equiv \omega\, e(t)\lambda$.

Let us explain in details these equations. Take the first one. Then, the variation of dyads composed of two employed workers ($\dot{d}_2(t)$) is equal to the number of d_1-dyads in which the unemployed worker has found a job (through either her strong tie with probability $(1 - \omega)\lambda$ or her weak tie with probability $\omega e(t)\lambda$) minus the number of d_2-dyads in which one of the two employed workers has lost her job. In the second equation, the variation of dyads composed of one employed and one unemployed worker ($\dot{d}_1(t)$) is equal to the number of d_0-dyads in which one of the unemployed workers has found a job (only through her weak tie with probability $g(e(t))$ since her strong tie is unemployed and cannot therefore transmit

[7] There is strong evidence that firms rely on referral recruitment (Bartram et al. 1995; Barber et al., 1999; Mencken and Winfield, 1998; Pellizzari, 2010) and it is even common and encouraged strategy for firms to pay bonuses to employees who refer candidates who are successfully recruited to the firm (Berthiaume and Parsons, 2006).

any job information) minus the number of d_1-dyads in which either the employed worker has lost her job (with probability δ) or the unemployed worker has found a job with the help of her strong or weak tie (with probability $h(e(t))$) plus the number of d_2-dyads in which one the two employed has lost her job. Finally, in the last equation, the variation of dyads composed of two unemployed workers ($\overset{\bullet}{d}_0(t)$) is equal to the number of d_1-dyads in which the employed worker has lost her job minus the number of d_0-dyads in which one of the unemployed workers has found a job (only through her weak tie, with probability $g(e(t))$) These dynamic equations reflect the flows across dyads. Graphically,

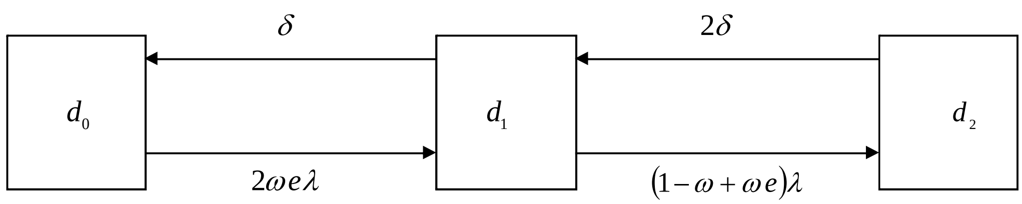

Figure 1: Flows in the labor market

Observe that the assumption stated above that both members of a dyad cannot lose their status at the same time is reflected in the flows described by (4). What is crucial in our analysis is that members of the same dyad (strong ties) always remain together throughout their life. So, for example, if a d_2-dyad becomes a d_0-dyad, the members of this dyad are exactly the same; they have just changed their employment status.

Taking into account (3), the system (4) reduces to a two-dimensional dynamic system in $d_2(t)$ and $d_1(t)$ given by:

$$\begin{cases} \overset{\bullet}{d}_2(t) = h(e(t))d_1(t) - 2\delta d_2(t) \\ \overset{\bullet}{d}_1(t) = 2g(e(t))\left(1/2 - d_2(t) - d_1(t)\right) - \left[\delta + h(e(t))\right] d_1(t) + 2\delta d_2(t) \end{cases}$$

where, using (1):

$$e(t) = 2d_2(t) + d_1(t)$$

3 Steady-state equilibrium and comparative statics analysis

3.1 Steady-state equilibrium

In a steady-state (d_2^*, d_1^*, d_0^*), each of the net flows in (4) is equal to zero. Setting these net flows equal to zero leads to the following relationships:

$$d_2^* = \frac{(1-\omega+\omega e^*)\lambda}{2\delta} d_1^* \tag{5}$$

$$d_1^* = \frac{2\omega e^* \lambda}{\delta} d_0^* \tag{6}$$

where

$$d_0^* = \frac{1}{2} - d_2^* - d_1^* \tag{7}$$

$$e^* = 2d_2^* + d_1^* \tag{8}$$

$$u^* = 1 - e^* \tag{9}$$

Definition 1 *A steady-state labor market equilibrium is a four-tuple $(d_2^*, d_1^*, d_0^*, e^*, u^*)$ such that equations (5), (6), (7), (8) and (9) are satisfied.*

Define $Z = (1-\omega)/\omega$, $B = \delta/(\lambda\omega)$. We have the following result.

Proposition 1

(*i*) *There always exists a steady-state equilibrium $\mathcal{U}$ where all individuals are unemployed and only d_0-dyads exist, that is $d_2^* = d_1^* = e^* = 0$, $d_0^* = 1/2$ and $u^* = 1$.*

(*ii*) *If*

$$\frac{\delta}{\lambda} < \frac{\omega + \sqrt{\omega(4-3\omega)}}{2} \tag{10}$$

there exists a steady-state equilibrium $\mathcal{I}$ where $0 < e^* < 1$ is defined by

$$e^* = \frac{B^2}{2d_0^*} - B - Z > 0, \tag{11}$$

$0 < u^* < 1$ by (9), and $0 < d_0^* < 1/2$ is the unique (feasible) solution of the following equation:

$$-\frac{Z}{B}d_0^{*2} - \frac{(1+Z)}{2}d_0^* + \left(\frac{B}{2}\right)^2 = 0 \tag{12}$$

Also, the other dyads are given by:

$$d_1^* = \frac{2e^*}{B}d_0^* \tag{13}$$

$$d_2^* = \frac{(Z+e^*)\,e^*}{B^2}d_0^* \tag{14}$$

If condition (10) holds, then an interior equilibrium always exists. Indeed, if the job-destruction rate δ is sufficiently low and/or the job-contact rate λ is sufficiently high, then an interior equilibrium exists. Otherwise, all workers will be unemployed and the steady-state equilibrium $\mathcal{U}$ will prevail. The latter is obviously uninteresting and, from now on, we only focus on the labor market equilibrium $\mathcal{I}$. For this equilibrium, we can, in fact, calculate explicitly e^* and d_0^*. We obtain:[8]

$$e^* = \frac{\sqrt{\lambda\left[\lambda + 4\delta\left(1-\omega\right)\right]} - 2\delta + 2\lambda\omega - \lambda}{2\lambda\omega} \tag{17}$$

$$d_0^* = \frac{\delta^2}{\lambda^2\omega + \lambda\omega\sqrt{\lambda\left[\lambda + 4\delta\left(1-\omega\right)\right]}} \tag{18}$$

3.2 Comparative statics results

We have the following comparative-statics results:

[8] To obtain (17) and (18), we proceed as follows. First, we plug the values of d_2^* and d_1^* from (5) and (6) into $e^* = 2d_2^* + d_1^*$ to obtain:

$$e^* = \left[\frac{(1-\omega+\omega e^*)\,\lambda}{\delta} + 1\right]\frac{2\omega e^*\lambda}{\delta}d_0^* \tag{15}$$

Then, we plug the values of d_2^* and d_1^* from (5) and (6) into $d_0^* = \frac{1}{2} - d_2^* - d_1^*$ to obtain:

$$d_0^* + \left[\frac{(1-\omega+\omega e^*)\,\lambda}{2\delta} + 1\right]\frac{2\omega e^*\lambda}{\delta}d_0^* = \frac{1}{2} \tag{16}$$

By solving simultaneously equations (15) and (16), we get (17) and (18).

Proposition 2 *Consider the steady-state equilibrium $\mathcal{I}$ and assume (10). Then, an increase in the job-destruction rate δ decreases the employment rate e^* and the time spent in dyad d_2^* but increases the time spent in a d_0^* dyad. If the elasticity of δ with respect to d_0^* is lower in absolute value than the elasticity of δ with respect to e^*, i.e.,*

$$\left|\frac{\partial d_0^*}{\partial \delta}\frac{\delta}{d_0^*}\right| < \left|\frac{\partial e^*}{\partial \delta}\frac{\delta}{e^*}\right| \tag{19}$$

then an increase in δ reduces the time spent in a d_1^* dyad .

The effect of δ, the job destruction rate, on the different endogenous variables is interesting and not straightforward. Consider first the effect of δ on the different dyads. When δ increases, workers lose their job at a faster rate and thus spend more time in the d_0^* dyad (where both workers are unemployed) and less time in the d_2^* dyad (where both workers are employed). The effect of δ on the time spent in the d_1^* dyad is, however, less clear. When δ increases, two opposite forces are at work. Remember from (7) that $d_1^* = \frac{1}{2} - d_2^* - d_0^*$. So, when δ increases, individuals spend less time in the d_2^* dyad, which means that they will spend more time in the d_1^* dyad, but, at the same time, they spend more time in the d_0^* dyad, which means that they will spend less time in the d_1^* dyad. The net effect is clearly ambiguous. A sufficient condition for the second effect to dominate the first one is given by condition (19), which just say that the effect on employment has to be large enough compared to the effect on the d_0^* dyad. Finally, consider the effect of δ on the employment rate e^*. Remember that $e^* = 2d_2^* + d_1^*$. When δ increases, there is a *negative* effect on both d_2^* and d_1^* since the two persons involve in the d_2^*-dyad and the employed person involved in the d_1^*-dyad have more chance of losing their jobs (see (4)). There is also a *positive* effect on d_1^* since if one of the persons involves in the d_2^*-dyad loses her job, then the d_2^*-dyad becomes a d_1^*-dyad (see (4)), which increases the number of d_1^*-dyads. In equilibrium, the negative effect dominates the positive one and the relationship between δ and e^* is negative.

Let us now focus on λ, the job-information rate. We have:[9]

Proposition 3 *Consider the steady-state equilibrium $\mathcal{I}$ and assume*

$$\lambda > \max\left\{\frac{2\delta}{\omega + \sqrt{\omega(4-3\omega)}}, 2\right\} \tag{20}$$

An increase in the job-information rate λ increases the employment rate e^* but decreases the time spent in a d_0^* dyad. Moreover, if the elasticity of λ with respect to e^* is higher in absolute value than the elasticity of λ with respect to d_0^*, i.e.,

$$\left|\frac{\partial e^*}{\partial \lambda}\frac{\lambda}{e^*}\right| > \left|\frac{\partial d_0^*}{\partial \lambda}\frac{\lambda}{d_0^*}\right|$$

[9] Observe that condition (20) is exactly the same condition (10) with the additional requirement that $\lambda \geq 2$.

then an increase in λ increases the time spent in dyad d_1^*. Finally, if

$$d_0^* > \frac{\delta^3}{(1-\omega)\,\omega\lambda\,(\lambda-2)\,(\delta^2-4\lambda^2\omega^2)} \tag{21}$$

an increase in λ raises the time spent in dyad d_2^*.

Interestingly, the effect of the job-information rate λ on the different endogenous variables is quite different than that of the job-destruction rate δ. When λ increases, people spend less time in a d_0^* dyad since both unemployed workers have a higher chance of meeting a weak time with a job information. This increases employment as long as λ is not too small. The effects on the time spent in a d_1^* dyad is less clear because unemployed workers tend to find a job at a faster rate, which both increases (mobility from a d_0^* to a d_1^* dyad) and decreases (mobility from a d_1^* to a d_2^* dyad) the time spent in a d_1^* dyad. Finally, the effect of λ on the time spent in a d_2^* dyad is positive only if the size of the d_0^* dyads is sufficiently large because there should be enough individuals moving first from a d_0^* to a d_1^* dyad and then from a d_1^* to a d_2^* dyad.

3.3 Social interactions

Let us study the impact of social interactions (captured by ω) on the different endogenous variables. We have the following important result.

Proposition 4 *Assume*

$$\frac{\delta}{\lambda} < \sqrt{\frac{\omega}{6}} \tag{22}$$

and consider steady-state equilibrium $\mathcal{I}$. Then, increasing the percentage of weak ties ω decreases both the number of d_0-dyads and the employment rate e^* in the economy, i.e.

$$\frac{\partial d_0^*}{\partial \omega} < 0 \qquad , \qquad \frac{\partial e^*}{\partial \omega} > 0$$

The effects of ω on d_1^* and on d_2^* are, however, ambiguous.

Proof. See the Appendix.

We show here that by increasing the probability of meeting *new workers* (i.e. weak ties), the steady-state employment rate increases. This is not a trivial result, since, by increasing ω, we have different and opposite effects on the job formation/destruction process. On the one hand, we increase the probability of getting out of unemployed dyads, while, on the other hand, we potentially give up the information of an employed partner in favor of a link with an unemployed one. This result is non trivial since strong and weak ties are substitutes.

In our model, it is indeed better to meet strong ties because a strong tie does you no good in state d_0 since your best friends are all unemployed. But a weak tie can do you good in any state because that person might be employed. So there is an *asymmetry* that is key to the model and that will explain (see below) why some workers (blacks) may be stuck in poverty traps (i.e. d_0 dyads) having little contact with weak ties that can help them leaving the d_0 dyad. This result is also interesting, since it solves a trade off between status-quo relations and new relations. It also formally demonstrates the Granovetter's informal idea of the *strength of weak ties* in finding a job.

Let us be more specific about this result. Here, individuals belong to mutually exclusive groups, the dyads, and weak tie interactions spread information across dyads. The parameter ω measures the proportion of social interaction that occurs outside the dyad, the inter-dyad interactions. When ω is high, the social cohesion between employed and unemployed workers is high and thus they are in close contact with each other. In this context, increasing ω induces more transitions from unemployment to employment and thus e^*, the employment rate in the economy, increases. This is true if (22) holds.[10] This condition (22) also guarantees that (10) holds, i.e. that an *interior* steady-state equilibrium $\mathcal{I}$ exists (see the Appendix). Condition (22) states that the job-destruction rate δ has to be sufficiently low while the job-contact rate λ and social interactions ω have to be sufficiently large. As a result, we are in a "reasonable" economy where jobs are not destroyed too fast and jobs are created at the sufficient high rate (otherwise we will end up with the steady-state equilibrium $\mathcal{U}$ where all workers are unemployed). Take our model and interpret the unit time as one quarter of a year. In the US, the sample average for the quarterly job destruction rate is 5.5% (Davis and Haltiwanger, 1992), thus $\delta = 0.055$. We know from most studies that $\lambda = 4$, which means that on average people hear from a job every three weeks. In that case, condition (22) is always satisfies even for very low values of ω, like e.g. $\omega = 0.01$.

Even though e^* decreases, the effect of ω on d_2^* and d_1^* is ambiguous. Indeed, from Figure 1, individuals leave dyad d_1 and enters dyad d_2 at rate $h(e) \equiv (1 - \omega + \omega e)\lambda$. Since

$$\frac{\partial\left[(1-\omega+\omega e)\lambda\right]}{\partial\omega} = (-1 + e + \omega\frac{\partial e}{\partial\omega})\lambda$$

is ambiguous (because $-1 + e < 0$), the effects mentioned above are also ambiguous. Now consider the effect of ω on d_0^*. This is clearly negative. Indeed, from Figure 1, one can see that individuals leave dyad d_0 at rate $2\omega e\lambda$. Since

$$\frac{\partial\left(2\omega e\lambda\right)}{\partial\omega} = 2\lambda\left(e + \omega\frac{\partial e}{\partial\omega}\right) > 0$$

then, when ω increases, there are fewer d_0-dyads.

[10]Even if (22) does not hold, it can still be true since (22) is a sufficient condition.

4 Extensions

We now consider two extensions of the basic model. First, we endogeneize wages by allowing firms to propose an incentive contract that prevents shirking. Second, we endogeneize social interactions so that workers decide how much time they spend with their weak and strong ties.

4.1 Endogenous wages

In this section, we endogenously determine the wage y. For that, we use an efficiency wage model (Shapiro and Stiglitz, 1984) where firms set wages to deter shirking. There are only two possible effort levels: either the worker shirks, exerts zero effort, $a = 0$, and contributes to zero production, or she does not shirk, provides full effort, $a > 0$, and contributes to 1 unit of production.

On the incentive mechanism of the efficiency wage model, there is plenty of empirical evidence. Basically, this model stipulates that employees are rational cheaters who anticipate the consequences of their actions and shirk when the marginal benefit exceeds the costs, and firms respond to this decision calculus by implementing monitoring and incentive pay policies (i.e. efficiency wage) that make shirking unprofitable. Cappelli and Chauvin (1991) find that higher wage premiums are associated with lower levels of shirking, as measured by disciplinary dismissals. Using data from the US National Longitudinal Survey of Youth (NLSY) in 1992, Goldsmith et al. (2000) find that receiving an efficiency wage enhances an individual's effort and that individuals providing a greater effort earn higher wages.[11] Recent research has used a natural experiment setting in which monitoring levels are exogenously varied across similar sites and substantial resources are devoted to tracking the behavior of employees. Fehr et al. (1996) were the first to use a natural experiment and show that higher wages indeed sharply reduce shirking. More recently, Nagin et al. (2002) propose another experiment by collecting data from a large telephone solicitation company. They show that a significant fraction of employees behave according to the predictions of the shirking model. Specifically, they find that these employees respond to a reduction in the perceived cost of opportunistic behavior by increasing the rate at which they shirk.[12]

As before, there is a stochastic process in employment status changes. However, firms cannot perfectly monitor workers, so there is a rate at which shirking is detected, denoted by θ (i.e. monitoring rate). If a worker is caught shirking, she is automatically fired. As a result, for non-shirkers, the stochastic process is as before and described by Figure 1. However, for

[11] See also Rebitzer (1995), who finds that high levels of supervision are associated with lower wage levels, and Strobl and Walsh (2007), which results indicate a positive relationship between monitoring and effort.

[12] There is a recent paper by Fehr and Goette (2007) using an experiment in a laboratory that shows that workers work more when wages are higher.

shirkers, it is as in Figure 1 with one difference: δ is replaced by $\delta + \theta$, since shirkers can lose their jobs either because there is a technological shock that leads to the destruction of the job or because the worker has been caught shirking and fired. The rest of the stochastic process is exactly the same as in the previous section.[13] In particular, the way workers find a job and transmit information within and outside the dyad is the same. As a result, the employment rate for non-shirkers, $e^*(\omega) = e^{NS}(\omega)$ is still given by (11), while that of shirkers $e^{S*}(\omega)$ is defined by (11), where δ is replaced by $\delta + \theta$.

We can now write the expected utilities. For a non shirker, her expected utility is equal to (assume $r = 0$):

$$EV^{NS} = e^*(\omega)\,(y - a) + [1 - e^*(\omega)]\, b \tag{23}$$

whereas, for a shirker, it is given by:

$$EV^{S} = e^{S*}(\omega) y + \left[1 - e^{S*}(\omega)\right] b \tag{24}$$

The trade off between shirking and non shirking is clear: shirkers do not provide effort a but spend more time unemployed. Let us calculate the efficiency wage. Firms know that workers have a zero discount rate, so they solve $EV^{NS} = EV^{S}$. By using (23) and (24), we easily obtain the following efficiency wage:

$$y^{eff} = b + a\,\frac{e^*(\omega)}{[e^*(\omega) - e^{S*}(\omega)]} \tag{25}$$

Equation (25) is also referred to as the non-shirking condition. This efficiency wage has the standard properties of the standard model (Shapiro and Stiglitz, 1984). Indeed, when b, a, or δ increases, or θ decreases, the efficiency wage has to increase in order to prevent shirking. What is new in the present model is the impact of social interactions ω on wages. The term $\frac{e^*(\omega)}{e^*(\omega) - e^{S*}(\omega)}$ captures the incentive aspect of the efficiency wage,[14] i.e. the amount necessary to prevent shirking. As in Shapiro and Stiglitz (1984), it is a function of employment (or unemployment) since unemployment acts as a worker discipline device. Denote $\Delta e^* \equiv \frac{e^*(\omega) - e^{S*}(\omega)}{e^*(\omega)} > 0$, i.e. the difference in employment rates between shirking and non-shirking behaviors, then

$$y^{eff} = b + \frac{a}{\Delta e^*}$$

Indeed, the higher the difference in employment rate between shirking and non-shirking behaviors, the less workers are induced to shirk, and the lower is the efficiency wage needed to reduce shirking. Define

$$\eta_\omega^{NS} \equiv \frac{\partial e^*(\omega)}{\partial \omega}\frac{\omega}{e^*(\omega)} > 0$$

[13]We assume that shirkers and non-shirkers never share a strong tie.

[14]Observe that, by definition, $e^*(\omega) > e^{S*}(\omega)$.

as the elasticity of non-shirking employment with respect to weak ties and

$$\eta_{\omega}^{S} \equiv \frac{\partial e^{S*}(\omega)}{\partial \omega} \frac{\omega}{e^{S*}(\omega)} > 0$$

as the elasticity of shirking employment with respect to weak ties. We have the following result:

Proposition 5 *Assume (22) and consider steady-state equilibrium $\mathcal{I}$. Then*

$$\frac{\partial y^{eff}}{\partial \omega} \gtreqless 0 \Leftrightarrow \eta_{\omega}^{NS} \lesseqgtr \eta_{\omega}^{S}$$

When interactions with weak ties ω increase, whether they shirk or not, workers are on average more employed over their lifecycle. However, if the responsiveness of employment to ω is higher for shirkers than non-shirkers, then firms need to increase the efficiency wage to deter shirking. This is an interesting result because it links social interactions and wages. In particular, it says that, if workers use more their weak ties than strong ties (strong ties than weak ties) to find a job, then they will receive higher wages if the elasticity of shirking (non-shirking) employment with respect to weak ties is higher than that of non shirking (shirking).

If we look at the empirical literature, the following relationship has been tested: do people who got their job through social contacts earn more or less than their peers who found a job using formal methods? The empirical results are not clear. Using data from across Europe and from three US cities (Boston, Atlanta and Los Angeles), Bentolila et al. (2010) found that, on average, people who obtained their job through social contacts find work more quickly but earned about 5 to 7 per cent less than their peers. Using the same data but looking at this relationship country by country, Pellizzari (2010) found that informal search channels lead to significantly better paying jobs in Austria, Belgium and the Netherlands, while the opposite is true in Greece, Italy, Portugal and the United Kingdom. In the other EU countries - and in the US - no significant wage difference is observed.

In all these studies, however, one cannot distinguish between weak and strong ties since social contacts are measured by "family, friends or other contacts". Our model predicts that the effects of strong and weak ties on wages are, in fact, different. There is some empirical evidence showing that the use of weak ties in job search tends to lead to higher wage outcomes (Granovetter, 1974), but the evidence is not very robust. Green et al. (1995) find that incomes are lower for those who use within-neighborhood ties, or ties to relatives, which, as in our model, tend to be strong rather than weak links. Green et al. (1999) find that the use of strong ties is negatively associated with annual earnings, especially for Hispanics. Bridges and Villemez (1986) also find that weak ties are linked to higher incomes than strong ties; however, the income effect of tie strength is greatly reduced when controls are added for education, experience, race, and gender.

Our model also provides a new mechanism explaining this relationship. Indeed, previous theoretical research on the role of contacts in the labor market emphasizes that people may have higher wages because they inform the employer about the worker (Saloner, 1985; Montgomery, 1991), because they allow workers to more effectively sample a given wage distribution (Mortensen and Vishwanath, 1994; Calvó-Armengol and Jackson, 2004; 2007), or because they provide a cheaper search channel (Holzer, 1988). Bentolila et al. (2010) propose another type of model that emphasizes the fact that workers may sacrifice their productive advantage so as to find a job more easily, which can explain why jobs found through social contacts exhibit a wage reduction rather than a premium. Our mechanism is different since it is based on the possibility of shirking behavior by workers and how the employment of shirkers and non-shirkers react to an increase in the use of weak ties in finding a job.

As in the standard efficiency wage model, we can close the model by modelling the behavior of firms. Consider M identical firms ($j = 1, ..., M$) in the economy. All firms produce the same composite good and sell it at a fixed market price p (this good is taken as the numeraire and its price p is set to 1). Firms only care of workers' productivity on the job and their main objective is to prevent shirking because it is very costly (workers produce nothing if they shirk).[15] On the contrary, each worker, whatever her location, contributes to one unit of production if he does not shirk (which will always be true in equilibrium). The production function of each firm j is: $F(l_j)$ and it is assumed that $F(\cdot)$ is twice differentiable, with $F(0) = 0$, $F'(\cdot) > 0$ and $F''(\cdot) \leq 0$, and it satisfies the Inada conditions, i.e. $F'(0) = +\infty$ and $F'(+\infty) = 0$.

Since all firms are identical, let us focus on a symmetric (steady-state) equilibrium in which each firm employs the same number of workers. This means that each firm j hires $L_j = L = e^*(\omega)/M$ workers, where $e^*(\omega)$ is given by (11). As a result, each firm adjusts employment until the marginal product of an additional worker equals the efficiency wage (25). We obtain:

$$b + a\frac{L^*\, M}{L^*\, M - e^S(\omega)} = F'(L^*) \tag{26}$$

Because of the assumptions made on the production function, it is easy to show that there exists a unique solution in L^*. Because $L^* = e^*(\omega)/M$, and $e^*(\omega)$ is given by (11), M will adjust so that $L^* = e^*(\omega)/M$ will be always true in equilibrium.

[15] Because ω has an impact on the efficiency wage, one could argue that firms could hire people depending on how they find a job. We assume that firms do not know if workers have found a job through their weak or strong ties. As a result, when deciding wage and employment, each firm takes ω as given.

4.2 Choosing social interactions

We would like now to extend the model so that ω is chosen by individuals and not exogenously defined as in the previous sections. The timing is as in the previous section. We assume that there is some cost of interacting with weak ties. Let c denotes the marginal cost of these interactions. The expected utility is now given by:

$$EV(\omega) = e^*(\omega)y + [1 - e^*(\omega)]\, b - c\,\omega$$

where $e^*(\omega)$ is defined by (11) or (17). Each individual optimally chooses ω that maximizes $EV(\omega)$. The first-order condition yields:

$$\frac{\partial EV(\omega)}{\partial \omega} = \frac{\partial e^*(\omega)}{\partial \omega}\,(y - b) - c = 0 \tag{27}$$

We have seen (see Proposition 4) that if (22) holds, then $\frac{\partial e^*(\omega)}{\partial \omega} > 0$. We have the following result:

Proposition 6 *Assume (22) and consider steady-state equilibrium $\mathcal{I}$. Then there exists a unique interior ω^* that maximizes $EV(\omega)$. Higher wages or lower unemployment benefits or lower interaction costs will increase the interactions with weak ties, i.e.*

$$\frac{\partial \omega^*}{\partial y} > 0 \qquad \frac{\partial \omega^*}{\partial b} < 0 \qquad \frac{\partial \omega^*}{\partial c} < 0$$

Furthermore, an increase in δ, the job-destruction rate or an increase in λ, the job-information rate (with $\lambda \geq 2$) induces workers to spend more time with their weak ties, i.e.,

$$\frac{\partial \omega^*}{\partial \delta} > 0 \qquad \frac{\partial \omega^*}{\partial \lambda} > 0$$

There is clear trade-off between the benefits of interacting with weak ties and the costs associated with it (see (27)). Indeed, workers want to interact with weak ties because it increases their probability of being employed (or, equivalently, the time they spend employed during their lifetime), i.e. $\frac{\partial e^*(\omega)}{\partial \omega} > 0$. Concerning the wage y and the unemployment benefit b, a higher y or b increases the value of employment and, since $e^*(\omega)$ and ω are positively related, workers will interact more with weak ties. Quite naturally, increasing the cost c of social interactions reduces the time spent with weak ties.

What is interesting and new here is the effect of the aggregate labor-market variables on social interactions. When δ or λ increases, workers spend more time with their weak ties because the cross effect of δ or λ on employment is positive, i.e. $\frac{\partial e^{*2}}{\partial \omega \partial \delta} > 0$ and $\frac{\partial e^{*2}}{\partial \omega \partial \lambda} > 0$. Indeed, when δ or λ increases, the positive effect of weak ties ω on employment e^* is even stronger and thus workers rely more on their weak ties. This is an interesting result since it shows that, in downturn periods where jobs are destroyed at a faster rate, workers tend to spend more time with their weak ties because they know they will help them exit unemployment. In an economy where the flow of job information is faster, the same results occur.

5 Explaining the black/white employment gap

We would like now to extend our model to incorporate black and white workers. We first start with a model where blacks and whites are totally separated so that they do not interact with each other and then analyze a more interesting model where they do interact with each other.

5.1 Segregation

It is well-documented that black and white families are highly segregated in both the social and physical space (see e.g. Borjas, 1998; Cutler et al., 1999; Hellerstein and Neumark, 2008). If, in our model, this is the case, so that blacks only meet black strong and weak ties while whites only meet whites, then our model will predict large differences between these two populations.

Consider a continuum of black and white workers whose mass is given by N_B and N_W, with $N_B + N_W = N$.[16] There is one key aspect that differentiates black from white workers: *labor-market discrimination*. Indeed, employed workers (black or white) now hear of job vacancies at the exogenous rate λ_j while they lose their job at the exogenous rate δ. We assume that the job-information rate is race specific so that $\lambda_W > \lambda_B$, i.e. whites hear more about job opportunities than blacks. Indeed, there is discrimination in the labor market so that when an employer advertises her job, she is more likely to give this information to a white than to a black worker. A way to justify this is that most employers are whites and tend to trust and favor more whites than blacks. Using nine years of personnel records from a regional grocery store chain in the United States, Giuliano and Ransom (2011) examine the role of manager ethnicity on the ethnic composition of employment at the firm's 73 stores. They find significant effects of manager ethnicity on hiring patterns, i.e. more Hispanics are hired under Hispanic managers than under non-Hispanic managers. In a comprehensive study of workplace segregation in the U.S., Hellerstein and Neumark (2008) document extensive segregation by race and by Hispanic ethnicity. There are also four studies that focus on black employment in the U.S. (Stoll et al., 2004; Carrington and Troske, 1998; Turner, 1997; Bates, 1994). All find that blacks are employed at greater rates in establishments with black supervisors, managers, or owners.

[16] Subscripts B and W stand for "Black" and "White".

Since blacks and whites are totally separated, we can use the model of Section 2. Because we do not anymore normalize the total population to 1, in the steady-state equilibrium $\mathcal{I}$, if $\delta < \lambda_j[\omega_j + \sqrt{\omega_j(4-3\omega_j)}]/2$, then e_j^*, d_{0j}^*, d_{1j}^* and d_{2j}^* are now respectively given by:[17]

$$e_j^* = \frac{\sqrt{\lambda_j\left[\lambda_j + 4\delta\left(1-\omega_j\right)\right]} - 2\delta + 2\lambda_j\omega_j - \lambda_j}{2\lambda_j\omega_j}$$

$$d_0^* = \frac{N_j\delta^2}{\lambda_j^2\omega_j + \lambda_j\omega_j\sqrt{\lambda_j\left[\lambda_j + 4\delta\left(1-\omega_j\right)\right]}}$$

$$d_{1j}^* = \frac{2e_j^*}{B_j}d_{0j}^*$$

$$d_{2j}^* = \frac{\left(Z + e_j^*\right)e_j^*}{B^2}d_{0j}^*$$

In that case, using Proposition 6, blacks will choose to interact less with their weak ties than whites, i.e. $\omega_B^* < \omega_W^*$, and using Proposition 1 and (17), black workers will experience a much higher unemployment rate than whites by spending much of their time in a d_0 dyad.

Let us now consider a more interesting model where there is some integration between black and white workers.

5.2 Integration

5.2.1 The model

As before, individuals belong to dyads. Quite naturally, we assume that *strong ties* are always of the same race (family, best friends). On the contrary, weak ties can be of either race and meeting them is random. Because the analysis gets quite complicated, we here assume that ω_j is exogenous. We will endogeneize it below. By denoting the employment level and the unemployment level of workers of type $j = B, W$ at time t by $E_j(t)$ and $U_j(t)$, we have:

$$\begin{cases} E_j(t) = 2d_{2j}(t) + d_{1j}(t) \\ U_j(t) = 2d_{0j}(t) + d_{1j}(t) \end{cases}$$

The population condition can then be written as:

$$E_j(t) + U_j(t) = N_j$$

As before, we denote the employment *rate* and the unemployment *rate* of workers of type $j = B, W$ at time t by $e_j(t)$ and $u_j(t)$, where $e_j(t), u_j(t) \in [0,1]$, $\forall j \in \{B, W\}$. We have:

$$e_j = \frac{E_j(t)}{N_j}\ ,\ \ u_j(t) = \frac{N_j - E_j(t)}{N_j}$$

[17]Remember that $Z_j \equiv \left(1-\omega_j\right)/\omega_j$ and $B_j \equiv \delta/\left(\lambda_j\omega_j\right)$.

which means that

$$u_j(t) = 1 - e_j(t)$$

or, alternatively,

$$d_{2j}(t) + d_{1j}(t) + d_{0j}(t) = \frac{N_j}{2} \tag{28}$$

As in the previous section, we assume that there is labor-market discrimination so that $\lambda_W > \lambda_B$. We further assume that $\omega_W > \omega_B$, i.e., white individuals spend more time with weak ties than blacks do ($\omega_W > \omega_B$). The following table (Table 1) from the GSS (General Social Survey) in the US indicates that indeed black respondents are somewhat more likely to spend time socializing with black friends than whites with white friends, confirming $1 - \omega_B > 1 - \omega_W$ or equivalently $\omega_W > \omega_B$. Indeed, in Table 2, one can see that the percentage of blacks who spend evening with friends almost daily is 4.34 percent (i.e. 142/3350) while for whites this number is equal to 2.73 percent (i.e. 573/20979). Interestingly, 11 percent of blacks and 9.7 percent of whites never spend evening with friends, indicating that blacks can be more isolated than whites.

Table 1: Socialization per race

Spend evening with friends	Race of respondent			
	White	Black	other	Total
almost daily	573	142	36	751
several times a week	4,013	663	152	4,828
several times a month	4,212	629	170	5,011
once a month	4,587	683	170	5,440
several times a year	4,065	515	114	4,694
once a year	1,490	280	40	1,810
never	2,039	438	84	2,561
Total	20,979	3,350	766	25,095

Moreover, there are plenty of evidence showing that inter-racial interactions are quite uncommon (see e.g. Sigelman et al., 1996). For example, in 1995, to the question: "What race are your close friends?", 87 percent of whites and 76 percent of blacks answered either "mostly my race", or "almost all my race", or "all my race" (Tuch et al., 1999). Also, using data from Toronto, Fong and Isajiw (2000) found that low-income minority workers are less likely to develop friendship ties with the majority group. Using data of students in US schools, Quillian and Campbell (2003) found that blacks are less likely to have white friends, especially in more segregated neighborhoods and schools.

5.2.2 Labor-market equilibrium

Each job offer is taken to arrive only to employed workers, who can then direct it to one of their contacts (through either strong or weak ties). Employed workers, who hear about a job, pass on this information on to their current matched partner, who can be a strong or a weak tie. White (black) employed workers pass the job information to their white (black) strong tie and to any (white or black) weak tie. Let us write the flows of dyads between states for black and white workers. For a worker of type $j = B, W$, they are given by:

$$\begin{cases} \overset{\bullet}{d}_{2j}(t) = h_j(t)\, d_{1j}(t) - 2\delta d_{2j}(t) \\ \overset{\bullet}{d}_{1j}(t) = 2g(t)d_{0j}(t) - [\delta + h(t)]\, d_{1j}(t) + 2\delta d_{2j}(t) \\ \overset{\bullet}{d}_{0j}(t) = \delta d_{1j}(t) - 2g(t)d_{0j}(t) \end{cases} \tag{29}$$

where

$$h_B(t) \equiv (1-\omega_B)\,\lambda_B + \omega_B \left[\frac{N_b}{N} e_B(t)\lambda_B + \frac{N_W}{N} e_W(t)\lambda_W\right]$$

$$h_W(t) \equiv (1-\omega_W)\,\lambda_W + \omega_W \left[\frac{N_b}{N} e_B(t)\lambda_B + \frac{N_W}{N} e_W(t)\lambda_W\right]$$

and

$$g_B(t) \equiv \omega_B \left[\frac{N_b}{N} e_B(t)\lambda_B + \frac{N_W}{N} e_W(t)\lambda_W\right]$$

$$g_W(t) \equiv \omega_W \left[\frac{N_b}{N} e_B(t)\lambda_B + \frac{N_W}{N} e_W(t)\lambda_W\right]$$

Let us explain the first equation since the interpretation of the other equations is similar. The variation of dyads composed of two employed workers of type j ($\overset{\bullet}{d}_{2j}(t)$) is equal to the number of $d_{1j}-$dyads in which the unemployed worker of type j has found a job through either her strong tie of type j with probability $(1-\omega_j)\lambda_j$ or her weak tie with probability $\omega_j \left[\frac{N_B}{N} e_B(t)\lambda_B + \frac{N_W}{N} e_W(t)\lambda_W\right]$ minus the number of $d_{2j}-$dyads in which one of the two employed workers has lost her job. It is important to understand why the probability of finding a job through a weak tie is $\omega_j \left[\frac{N_B}{N} e_B(t)\lambda_B + \frac{N_W}{N} e_W(t)\lambda_W\right]$ for workers of type j. A person of type $j = B, W$, who spends ω_j of her time with weak ties can meet a weak tie who is either an employed black worker who is aware of a job opportunity (this occurs with probability $\frac{N_B}{N} e_B(t)\lambda_B$) or an employed white worker who is aware of a job opportunity (this occurs with probability $\frac{N_W}{N} e_W(t)\lambda_W$).[18]

[18]As in Currarini et al (2009), we could have assumed *a preference bias* so that, for a black person, the probability of meeting an employed weak tie who is informed would be $\omega_B \left[\frac{N_B}{N} e_B(t)\lambda_B + m_B \frac{N_W}{N} e_W(t)\lambda_W\right]$, while, for whites, it would be: $\omega_W \left[m_W \frac{N_B}{N} e_B(t)\lambda_B + \frac{N_W}{N} e_W(t)\lambda_W\right]$, where $0 < m_B < 1$ and $0 < m_W < 1$. This would not change the main results of the anlysis but will rend the model much more cumbersome.

Taking into account (28), the system (29) reduces to a two-dimensional dynamic system in $d_2(t)$ and $d_1(t)$ given by:

$$\begin{cases} \overset{\bullet}{d}_{2j}(t) = h_j(t)d_{1j}(t) - 2\delta d_{2j}(t) \\ \overset{\bullet}{d}_{1j}(t) = 2g_j(t)\left[N_j/2 - d_{2j}(t) - d_{1j}(t)\right] - \left[\delta + h_j(t)\right] d_{1j}(t) + 2\delta d_{2j}(t) \end{cases}$$

where

$$N_j\, e_j(t) = 2d_{2j}(t) + d_{1j}(t)$$

In a steady-state $(d_{2j}^*, d_{1j}^*, d_{0j}^*)$, each of the net flows in (29) is equal to zero. Setting these net flows equal to zero leads to the following relationships for workers of type j:

$$d_{2j}^* = \frac{(1-\omega_j)\,\lambda_j + \omega_j\left(\frac{N_B}{N}e_B^*\lambda_B + \frac{N_W}{N}e_W^*\lambda_W\right)}{2\delta} d_{1j}^* \tag{30}$$

$$d_{1j}^* = \frac{2\omega_j\left(\frac{N_B}{N}e_B^*\lambda_B + \frac{N_W}{N}e_W^*\lambda_W\right)}{\delta} d_{0j}^* \tag{31}$$

$$d_{0j}^* = \frac{N_j}{2} - d_{2j}^* - d_{1j}^* \tag{32}$$

and

$$N_j e_j^* = 2d_{2j}^* + d_{1j}^* \tag{33}$$

$$u_j^* = 1 - e_j^* \tag{34}$$

The model is much more complicated now because e_B^* and e_W^* enter in each dyad of each type j of worker and, as a result, we cannot analyze the steady-state equilibrium separately for black and white workers. Define $M_{BW}^* \equiv \frac{N_B}{N}e_B^*\lambda_B + \frac{N_W}{N}e_W^*\lambda_W$ as the weak-tie meeting process. We have the following result.

Proposition 7

(*i*) *There always exists a steady-state equilibrium* $\mathcal{U}$ *where all individuals are unemployed and only* $d_{0j}-$*dyads exist, that is* $d_{2j}^* = d_{1j}^* = e_j^* = 0$, $d_{0j}^* = N_j/2$ *and* $u_j^* = 1$ *for* $j = B, W$.

(*ii*) *All the other steady-state equilibria are interior, that is* $0 < e_B^* < 1$ *and* $0 < e_W^* < 1$. *The equilibrium employment rate* e_j^* *and dyad* d_{0j}^* *of black and white workers are given by*

$$N_j e_j^* = \left[(1-\omega_j)\,\lambda_j + \omega_j M_{BW} + \delta\right] \frac{2\omega_j M_{BW}^*}{\delta^2} d_{0j}^* \tag{35}$$

$$d_{0j}^{*}+\left[\left(1-\omega_{j}\right)\lambda_{j}+\omega_{j}M_{BW}+2\delta\right]\frac{\omega_{j}M_{BW}^{*}}{\delta^{2}}d_{0j}^{*}=\frac{N_{j}}{2} \tag{36}$$

while dyads d_{1j}^{*} and d_{2j}^{*} are equal to:

$$d_{1j}^{*}=\frac{2\omega_{j}M_{BW}^{*}}{\delta}d_{0j}^{*} \tag{37}$$

$$d_{2j}^{*}=\left[\left(1-\omega_{j}\right)\lambda_{j}+\omega_{j}M_{BW}^{*}\right]\frac{\omega_{j}M_{BW}^{*}}{\delta^{2}}d_{0j}^{*} \tag{38}$$

Assume that $\lambda_W > \lambda_B$ and $\omega_W > \omega_B$. Then, if

$$\frac{\omega_{W}}{\omega_{B}}\frac{d_{0W}^{*}}{d_{0B}^{*}}\frac{N_{B}}{N_{W}}>\max\left\{1,\left(\frac{1-\omega_{B}}{1-\omega_{W}}\right)\frac{\lambda_{B}}{\lambda_{W}}\right\} \tag{39}$$

the employment rate (unemployment rate) of black workers is lower (higher) than that of whites, i.e. $e_B^* < e_W^*$ and $u_B^* > u_W^*$.

This proposition formally proves the intuition developed earlier. If black workers are sufficiently discriminated against in the labor market (i.e. $\lambda_W > \lambda_B$) and/or they interact mostly with their black strong ties (i.e. $1-\omega_B > 1-\omega_W$), they will have very little interaction with weak ties, especially whites and will end up experiencing high unemployment rate. Condition (39) gives an exact condition that guarantees that $e_B^* < e_W^*$. Weak ties are an important source of job information and when black individuals miss it, they end up having a higher unemployment rate than whites. This is a vicious circle since blacks experience a higher unemployment rate and mostly rely on other black workers who also experience a high unemployment rate, etc. Since jobs are mainly found through social networks via employed friends, black individuals are stuck in a bad labor market situation. As a result, when they found themselves in a d_0-dyad, they have nearly no chance of leaving it since the only way out is to meet an employed weak tie. As underscored by Granovetter (1973, 1974, 1983), in a close network where everyone knows each other, information is shared and so potential sources of information are quickly shaken down so that the network quickly becomes redundant in terms of access to new information. In contrast Granovetter stresses the *strength of weak ties* involving a secondary ring of acquaintances who have contacts with networks outside ego's network and therefore offer new sources of information on job opportunities. To summarize, when the time spent with weak ties is low, the social cohesion between employed and unemployed workers is also low and thus they are *not* in close contact with each other. Therefore, little interaction with weak ties induces more transitions from employment to unemployment and thus the unemployment rate increases.

Denote the equilibrium employment rate in the segregation equilibrium as e_j^{SEG} (Section 5.1) and the integration equilibrium as e_j^{INT} (Section 5.2). We have:

Proposition 8 *Integration can be beneficial to black workers but detrimental to white workers if*

$$e_B^{SEG} < e_B^{INT} + \frac{N_W}{N}\left(e_W^{INT}\frac{\lambda_W}{\lambda_B} - e_B^{INT}\right) < e_W^{SEG}\frac{\lambda_W}{\lambda_B} \tag{40}$$

This proposition shows that the main difference between the segregation and the integration equilibrium is the meeting process with weak ties, one of the main sources of employment as shown in Proposition 4. In the segregation equilibrium, type$-j$ workers only meet weak ties of the same race j. In particular, they meet an employed person informed about a job at rate $e_j\lambda_j$. So if black workers are strongly discriminated against such that λ_B is quite low, they get stuck in a d_0 dyad and stay there for a long time. In the integration equilibrium, black and white workers meet workers of different races and the rate at which they meet an employed person informed about a job is $M_{BW} = \frac{N_B}{N}e_B\lambda_B + \frac{N_W}{N}e_W\lambda_W$. As a result, if the ratio λ_W/λ_B is quite high, then it will always be beneficial for black workers to interact with white workers. However, integration would be detrimental to whites only if this ratio is not too large. Also, the fraction of white workers in the population and the time spent interacting with weak ties are important determinants of how integration affects blacks' workers since they strongly affect the meeting process with weak ties. We will investigate in more detail these issues in our numerical simulation exercises below.

5.2.3 Choosing social interactions

Instead of assuming that $\omega_W > \omega_B$, let us derive this result. As in Section 4.2, the expected utility of a type$-j$ worker is:

$$EV_j(\omega_j) = e_j^*(\omega_j)y + \left[1 - e_j^*(\omega_j)\right]b - TC_j(\omega_j)$$

where $e_j^*(\omega)$ are defined in Proposition 7 for $j = B, W$ and $TC_j(\omega_j)$ is the total cost of meeting a weak tie. We assume that it is more costly to meet a weak tie from the other race than from own race. Normalize the cost of meeting a weak tie of the same race to zero. Then, we have:

$$TC_B(\omega_B) = \frac{N_W}{N}\omega_B\, c_B \text{ and } TC_W(\omega_W) = \frac{N_B}{N}\omega_W\, c_W$$

As a result, we have

$$EV_B(\omega_B) = e_B^*\,(y-b) + b - \frac{N_W}{N}c_B$$

$$EV_W(\omega_W) = e_B^*\,(y-b) + b - \frac{N_B}{N}c_W$$

Each individual j optimally chooses ω_j that maximizes $EV_j(\omega_j)$. The first-order conditions yield:

$$\frac{\partial e_B^*(\omega_B^*)}{\partial \omega_B}(y-b) = \frac{N_W}{N} c_B \tag{41}$$

$$\frac{\partial e_W^*(\omega_W^*)}{\partial \omega_W}(y-b) = \frac{N_B}{N} c_W \tag{42}$$

Since we have assumed that $\lambda_W > \lambda_B$, i.e. whites are more likely to hear from a job than blacks, then, other things being equal, whites will tend to interact more with their weak ties than blacks. Also, even without assuming anything about the marginal cost c_j, since $N_W > N_B$, the total costs of interacting with weak ties, i.e. $TC_j(\omega_j)$, will tend to be higher for blacks than for whites. This will lead to $\omega_B^* < \omega_W^*$. This cannot be proved analytically but, with c_B sufficiently high compared to c_W, this will be true since $\frac{\partial e_j^*(\omega_j^*)}{\partial \omega_j}$ does not depend on c_j.

5.2.4 Numerical simulations

Let us now run some numerical simulations. Take our model and interpret the time period as one quarter of a year. The job destruction rate is equal to $\delta = 0.05$, that is, workers keep on average their job for 5 years. The job information rate is equal to $\lambda_W = 1$, which means that an employed white worker is aware about a job every 3 months, while $\lambda_B = 0.2$, which means that an employed black worker is aware about a job every 15 months. We fix the total population to $N = 1,000$ with 20 percent blacks and 80 percent whites, i.e. $N_B = 200$ and $N_W = 800$. Finally, we assume that $\omega_B = 0.2$ and $\omega_W = 0.8$, meaning that blacks mainly interact with their strong ties (80 percent of their time) while whites interact mostly with their weak ties (80 percent of their time), which are most likely to be white since 80 percent of workers are white in the population. To summarize, the two main differences between blacks and whites are difference in λs (capturing discrimination) and ωs (capturing isolation).

The results of the simulations are displayed in Tables 2a and 2b where we compare the cases when blacks and whites are segregated (Section 5.1) and when they are not (Section 5.2). Confirming Proposition 8, integration is beneficial to black workers since their unemployment rate decreases from nearly 40 percent (segregation) to a little more than 15 percent (integration) while it has only a small negative effect on whites' outcomes. When integration occurs, even though they meet weak ties only 20 percent of their time ($\omega_B = 0.2$), they are much more likely to meet white workers since they compose 80 percent of the population. Indeed, when meeting a weak tie, the probability of obtaining a job is $\frac{N_B}{N} e_B^* \lambda_B + \frac{N_W}{N} e_W^* \lambda_W$, so that the chance of meeting a white is four times more likely than of meeting a black worker.

This implies, in particular, that black workers do not stay that much in a d_0 dyad (only 3.69 percent of their time)[19] because they are likely to meet a white weak tie who is employed and has heard about a job. On the contrary, in the segregation case, the situation is catastrophic because they only meet black weak and strong ties who, because of discrimination, have a low chance of being employed. As a result, the unemployment rate is nearly 40 percent and they now spend more than 26 percent of their time in a d_0 dyad. Whites are not very much affected when they are segregated from black workers, because they still obtain a job at a fast rate due to their "old boy networks".

To sum-up, when black workers are isolated from good-quality networks emanating from white workers and are discriminated against, they get stuck in a state where they friends are unemployed and, as a result, have little chance to escape unemployment. On the contrary, when they can interact with whites workers and partly benefit from their old boy networks, they leave unemployment at a much faster rate.

Table 2a: Segregation / **Table 2b: Integration**

	Black workers	White workers		Black workers	White workers
e_j^* (%)	60.41	95.00	e_j^* (%)	84.80	94.22
u_j^* (%)	39.59	5.00	u_j^* (%)	15.20	5.78
d_{0j}^*	26.69	1.24	d_{0j}^*	3.69	1.70
$2d_{0j}^*/N_j$ (%)	26.69	0.31	$2d_{0j}^*/N_j$ (%)	3.69	0.43
d_{1j}^*	25.80	37.62	d_{1j}^*	23.25	42.86
$2d_{1j}^*/N_j$ (%)	25.80	9.41	$2d_{1j}^*/N_j$ (%)	23.25	10.71
d_{2j}^*	47.51	361.14	d_{2j}^*	73.06	355.44
$2d_{2j}^*/N_j$ (%)	47.51	90.28	$2d_{2j}^*/N_j$ (%)	73.06	88.86

Let us now investigate in more detail the integration equilibrium and determine the role of the meeting bias with white weak ties (i.e. the fraction of whites in the population N_W/N), the time spent with weak ties, the job-destruction rate and labor-market discrimination on the labor-market outcomes of black and white workers.

Figures 2a and 2b display the simulation results of the effect of N_W on the employment rates and time spent in a d_0 dyad.[20] One can see that increasing N_W has a big impact on the labor-market outcomes of black workers. When there are extremely few whites around, then we are back to the segregation equilibrium where blacks experience low employment rate and spend most of their time in a d_0 dyad. When the number of whites in the population

[19] For the interpretation of the results, it is better to use $2d_{0j}^*/N_j$ than d_{0j}^* since the former is normalized and gives the time spent in a d_0 dyad. The same applies for d_{1j}^* and d_{2j}^*.

[20] In all figures, the solid curve corresponds to the outcomes of black workers while the dashed curve describes the outcomes of white workers.

increases, black workers get more exposed to the white' social network and, as a result, both their unemployment rate and the time spent in a d_0 sharply decrease. For example, when blacks are mostly surrounded by blacks so that $N_W = 100$ (i.e. blacks constitute 90 percent of the population), they spend more than 80 percent of their time in a d_0 dyad and their unemployment rate is more than 30 percent. In 1979, the average black lived in a neighborhood that was 63.6 percent black, even though blacks constituted only 14.9 percent of the population (Borjas, 1998, Table 1). In our model, if we take $N_B = 636$ (or $N_W = 364$) so that $N_B/N = 63.6$ percent, then blacks will spend 35 percent of their time in a d_0 dyad and their unemployment rate will be equal to 22 percent. As it can be seen in Figures 2a and 2b, white workers are much less affected by the increase in N_W mainly because they are less discriminated against.

[*Insert Figures* 2a *and* 2b *here*]

Let us now consider the effect of the time spent with weak ties, ω_j, on workers' outcomes, i.e. we look at the effect of ω_B on e_B^* and d_{0B}^* and of ω_W on e_W^* and d_{0W}^* (Figures 3a and 3b). Even though the effects look globally similar between black and white workers, the qualitative effects are quite different. For black workers, the effect of ω_B is very strong, confirming Proposition 4. The more time they spend with their weak ties, i.e. the more they are exposed to the white's social network, the higher is their employment rate and the less time they spend in a d_0-dyad. For example, switching from $\omega_B = 0.1$ to $\omega_B = 0.9$ changes the black employment rate from 80 to 94 percent and the time spend in a d_0-dyad from 30 to 2 percent. For black workers, this really shows the *role* and *strength of weak ties* in leaving unemployment (Granovetter, 1973, 1974, 1983) and the idea that weak ties are superior to strong ties for providing support in getting a job. For white workers, the effects are much smaller. For example, the effect of weak tie on employment, though positive, are quite small. Increasing the interaction with weak ties from 3 to 95 percent (meaning spending no time to spending the whole time with weak ties) "only" increases the employment rate of whites by 8.5 percent (from 86.6 to 94 percent).

[*Insert Figures* 3a *and* 3b *here*]

We continue our numerical simulations by investigating in Figures 4a and 4b the role of the job-destruction rate δ. Because of less discrimination and better-quality social networks, white workers are much less affected by a downturn economy. Their employment rate and time spent in a d_0-dyad slightly decrease following a large increase in δ. On the contrary, black workers are much more sensitive to a negative shock in the economy. For example, varying δ from 0.06 (where the average duration of employment is a little more than 4 years) to 0.2 (where the average duration of employment is 1 year and 3 months) leads to a decrease in employment from 82 to 48 percent for black workers and "only" from 93 to 76 percent for white workers.

[*Insert Figures* 4*a and* 4*b here*]

Finally, we study the effect of labor-market discrimination on workers' outcomes (Figures 5a and 5b). Interestingly, one can see that the main difference between black and white workers is due to the λs, which captures both labor-market discrimination and social networks. Indeed, λ_j, is the rate at which employed workers of type j hear about a job. As documented in Section 5.1, there is an important bias in hiring by ethnicity. White employers tend to hire white workers while black employers tend to hire black employees. Since white employers are much more numerous than black employers, there is a gap between λ_B and λ_W. Labor (tasted-based and statistically) discrimination is also well documented (see, for example, the recent survey by Charles and Guryan, 2011) and can also explain the difference in λs between black and white workers. In Figures 5a and 5b, we provide a counterfactual study by setting the values of λ_B and λ_W so that the ratio λ_B/λ_W is 90 percent (i.e. $\lambda_B = 0.9$ and $\lambda_W = 1$) while it was before 20 percent (i.e. $\lambda_B = 0.2$ and $\lambda_W = 1$). In other words, black and white workers have nearly the same λs[21] and we look at the effect of weak ties on employment and on the time spent in a d_0-dyad. Compared to Figures 3a and 3b, we see that the curves for blacks and whites are getting really close. Interestingly, these figures also show that if λ_B and λ_W are very close but black workers use less their weak ties than whites, then outcome differences become large again. For example, if we keep $\lambda_B = 0.9$ and $\lambda_W = 1$ but assume that $\omega_B = 0.01$ and $\omega_W = 0.2$, then the black unemployment rate u_B^* will be equal to 25 percent while the white unemployment rate u_W^* will be equal to 6 percent. Even if we set $\lambda_B = \lambda_W = 1$ (no labor discrimination at all) with $\omega_B = 0.01$ and $\omega_W = 0.2$, then the black unemployment rate u_B^* is equal to 23 percent and it is only equal to 6 percent for white workers. This again confirms the strength of weak ties in getting a job.

[*Insert Figures* 5*a and* 5*b here*]

Our result is related to that of Calvó-Armengol and Jackson (2004). Contrary to the present model where only a very specific network structure (i.e. the dyad) is assumed, they explicitly model a social network (which can have any possible structure) where information flows between individuals having a link with each other. They show that an equilibrium with a clustering of workers with the same status is likely to emerge since, in the long run (i.e. steady state), employed workers tend to be friends with employed workers. The main difference with our approach is that individuals exchange job information only with their strong ties (as defined by their direct friends). In their model, weak ties (as defined by friends

[21] If $\lambda_B = \lambda_W$, then the two curves would be exactly the same.

of friends) will indirectly help individuals because, by providing job information to their strong ties, they help them to become employed. The two approaches are complementary. In Calvó-Armengol and Jackson (2004), if because of some initial condition some black workers are unemployed, then in steady-state they will still be unemployed because both their strong and weak ties will also be unemployed. In our framework, it is discrimination and segregation that make black workers only interacting with strong ties, who are themselves likely to be unemployed.

6 Discussion and policy implications

Though there is a considerable body of evidence examining ethnic disadvantage in the labor market, most of these studies tend to focus on individual characteristics such as education. This paper tries to gauge the importance of connections that black workers have with others and endeavours to ascertain whether such connections hinder labor market achievement. Because of labor discrimination, we show that the fact that black workers tend to be isolated from the social network of white workers can have dramatic consequences in the labor market. This may lead them to get stuck in a situation where they meet mostly their strong ties, who tend to be unemployed, and cannot therefore not help them leave unemployment. They are missing the connection to weak ties, especially whites, who have contacts with networks outside ego's network and therefore offer new sources of information on job opportunities.

Our analysis offers interesting policy implications. We have shown that discrimination and segregation (or separation) are crucial in understanding labor-market outcomes of ethnic minorities. Policies that promote social integration and thus increase the interracial interactions between weak ties would have positive effects on the labor-market outcomes of minority workers. Such policies, like the Moving to Opportunity (MTO) programs (Katz et al., 2001; Rosenbaum and Harris, 2001; Kling et al., 2005), have been implemented in the United States. By giving housing assistance to low-income families, the MTO programs help them relocate to better and richer neighborhoods.[22] In light of our results, our model predicts that, relative to the 'control' group, displaced workers (from low- to high-rental-housing areas) should improve their social network and therefore their labor outcomes. If labor market participation is a good 'proxy' for labor outcomes, then the findings of Rosenbaum and Harris (2001) confirm some of the predictions of our model. Indeed, using the

[22] See also Beaman (2011) and Edin et al. (2003) who both exploit natural experiments, consisting of refugee resettlement programs in the U.S. and Sweden, respectively, to try to disentangle social network referral effects from sorting or correlation in unobservable attributes. Beaman (2011) finds that a one standard deviation increase in the number of network members in a given year lowers the employment probability of someone arriving one year later by 4.9 percentage points. Edin et al. (2003) find similar positive results.

survey data from the MTO program in Chicago, the findings of these authors, based on interviews an average of 18 months after families moved from public housing to higher rental housing areas, show an increase in labor force participation and employment. More precisely, Rosenbaum and Harris (2001) show that: "After moving to their new neighborhoods, the Section 8 respondents were far more likely to be actively participating in the labor force (i.e. working or looking for a job), while for MTO respondents, a statistically significant increase is evident only for employment per se."

Another way of reducing the unemployment rate of minorities in the context of our model is to observe that *institutional connections* can be engineered to create connections between job seekers and employers in ways that parallel social network processes. For example, scholars like Granovetter (1979) and Wilson (1996) have called for poverty reduction programs to "create connections" between employers and poor and disadvantaged job seekers. While labor market intermediaries of all types aim to place workers with employers, especially with respect to poor populations, there is some disagreement about how these linkages work. Although strengthening connections being poor job seekers and employers is often seen as desirable, past research has questioned whether labor market intermediaries actually perform this function for those most in need. Recently, Autor and Houseman (2010) have argued that, in the low-wage sector, temporary services can help workers in the short term, but is not helpful in the longer-term because temporary employment weakens workers' search efforts for direct hire jobs. On the employer's side, a number of studies have shown that employers often stigmatize low wage workers who are sent to them by public and private labor market intermediaries (e.g., Laufer and Winship, 2004). In general, employers are concerned that since intermediaries targeting poor populations specialize in hard-to-employ populations, candidates referred by these organizations will be adversely selected, constituting the labor market "left-overs" who could not find a job through other means (Autor, 2009; Burtless, 1985; Van Ours, 1994). While low-wage employers generally stigmatize job-seekers sent to them from labor market intermediary organizations, Fernandez (2010) shows how it is that such biases can be overcome. To the degree that intermediary organizations can help the firm address its recruitment problems, "created connections" can serve as functional substitutes for social network processes in matching people to jobs. Actors will choose to work with brokers to the extent that brokers provide goods or services that are of greater value than those available through alternative means.

To conclude, we believe that weak ties generate 'bridging' social capital. Bridging social capital refers to ties across networks that may make the resources exist in one network accessible to a member of another. These social relationships enable members to 'get ahead'. These are needed to extend beyond family to connect to a broader range of resources and opportunities that exist in networks to which they are otherwise not connected. If black workers do not have access to weak ties (especially whites), in particular because they are segregated and separated from them, then their main source of information about jobs will be provided by their strong ties. But if the latter are themselves unemployed, the chance of escaping unemployment will be very low.

References

[1] Addison J.T. and Portugal, P. (2002) "Job search methods and outcomes," *Oxford Economic Papers* 54, 505-533.

[2] Akerlof, G. (1997), "Social distance and social decisions," *Econometrica* 65, 1005-1027.

[3] Alesina, A. and E. La Ferrara (2002), "Who trust others?" *Journal of Public Economics* 85, 207-234.

[4] Altonji, J.G. and R.M. Blank (1999), "Race and gender in the labor market," *Handbook of Labor Economics*, In: O. Ashenfelter and D. Card (Eds.), *Handbook of Labor Economics Vol. 3*, Amsterdam: Elsevier, pp. 3143-3259.

[5] Autor, D.H. and S.N. Houseman (2010), "Do temporary-help jobs improve labor market outcomes for low-skilled workers? Evidence from "Work First"," *American Economic Journal: Applied Economics* 2, 96-128.

[6] Autor, D.H. (2009), "The economics of labor market intermediation: An analytic framework," In: D. H. Autor (Ed.), *Studies of Labor Market Intermediation*, Chicago: University of Chicago Press, pp. 1-23.

[7] Ballester, C., Calvó-Armengol, A. and Y. Zenou (2006), "Who's who in networks. Wanted: the key player," *Econometrica* 74, 1403-1417.

[8] Barber, A. E., M. J. Wesson, Q. M. Roberson and M. S. Taylor (1999), "A tale of two job markets: Organizational size and its effects on hiring practices and job search behavior," *Personnel Psychology* 52, 841-867.

[9] Bartram, D., P. A. Lindley, L. Marshall and J. Foster (1995), "The recruitment and selection of young-people by small businesses," *Journal of Occupational Organizational Psychology* 68, 339-358.

[10] Bates, T. (1994), "Utilization of minority employees in small business: A comparison of nonminority and black-owned enterprises," *Review of Black Political Economy* 23, 113-121.

[11] Battu, H. Seaman,P. and Y. Zenou (2011), "Job contact networks and the ethnic minorities," *Labour Economics* 18, 48-56.

[12] Bayer, P., Ross, S.L. and G. Topa (2008), "Place of work and place of residence: Informal hiring networks and labor market outcomes," *Journal of Political Economy* 116, 1150-1196.

[13] Beaman, L.A. (2011), "Social networks and the dynamics of labor market outcomes: Evidence from refugees resettled in the U.S.," *Review of Economic Studies*, forthcoming.

[14] Bentolila, S., Michelacci, C. and J. Suarez (2010), "Social contacts and occupational choice," *Economica* 77, 20-45.

[15] Berthiaume, J. and W. Parsons (2006), "Referral bonuses: A popular way to bring in new talent." SHRM, http://moss07.shrm.org/hrdisciplines/compensation/Articles/Pages/CMS_016894.aspx.

[16] Bisin, A. and T. Verdier (2000), "Beyond the Melting Pot: Cultural Transmission, Marriage, and the Evolution of Ethnic and Religious Traits," *Quarterly Journal of Economics* 115, 955-988.

[17] Bisin, A. and T. Verdier (2001), "The Economics of Cultural Transmission and the Dynamics of Preferences," *Journal of Economic Theory* 97, 298-319.

[18] Blau, D.M. and P.K. Robins (1990) "Job search outcomes for the employed and unemployed," *Journal of Political Economy* 98, 637-655.

[19] Borjas, G.J. (1998), "To ghetto or not to ghetto: Ethnicity and residential segregation," *Journal of Urban Economics* 44, 228-253.

[20] Bridges, W.P. and W.J. Villemez (1986), "Informal hiring and income in the labor market," *American Sociological Review* 51, 574-582.

[21] Burtless, G. (1985), "Are targeted wage subsidies harmful? Evidence for a wage voucher experiment," *Industrial and Labor Relations Review* 39, 105-114.

[22] Cahuc, P. and F. Fontaine (2009), "On the efficiency of job search with social networks," *Journal of Public Economic Theory* 11, 411-439.

[23] Calvó-Armengol, A. (2004), "Job contact networks," *Journal of Economic Theory* 115, 191-206.

[24] Calvó-Armengol, A. and M. Jackson (2004), "The effects of social networks on employment and inequality," *American Economic Review* 94, 426-454.

[25] Calvó-Armengol, A. and M. Jackson (2007), "Networks in labor markets: Wage and employment dynamics and inequality," *Journal of Economic Theory* 132, 27-46.

[26] Calvó-Armengol, A., Patacchini, E., and Y. Zenou (2009), "Peer effects and social networks in education," *Review of Economic Studies* 76, 1239-1267.

[27] Calvó-Armengol, A., Verdier, T. and Y. Zenou (2007), "Strong and weak ties in employment and crime," *Journal of Public Economics* 91, 203-233.

[28] Calvó-Armengol, A. and Y. Zenou (2005), "Job matching, social network and word-of-mouth communication," *Journal of Urban Economics* 57, 500-522.

[29] Cappelli, P. and K. Chauvin (1991), "An interplant test of the efficiency wage hypothesis," *Quarterly Journal of Economics* 106, 769-787.

[30] Carrington, W.J. and K.R. Troske (1998), "Interfirm segregation and the black/white wage gap," *Journal of Labor Economics* 16, 231-260.

[31] Charles, K.K. and J. Guryan (2011), "Studying discrimination: Fundamental challenges and recent progress," *Annual Review of Economics* 3, 479-511.

[32] Costa, D.L. and M.E. Kahn (2003), "Civic engagement and community heterogeneity: An economist's perspective," *Perspectives on Politics* 1, 103-111.

[33] Currarini, S., Jackson, M.O., and P. Pin (2009), "An economic model of friendship: Homophily, minorities, and segregation," *Econometrica* 77, 1003-1045.

[34] Cutler, D., Glaeser, E.L. and J.L. Vigdor (1999), "The rise and decline of the American ghetto," *Journal of Political Economy* 107, 455-506.

[35] Darity, Jr., W.A. and P.L. Mason (1998), "Evidence on discrimination in employment: Codes of color, codes of gender," *Journal of Economic Perspectives* 12, 63-90.

[36] Davis, S.J. and J.C. Haltiwanger (1992), "Gross job creation, gross job destruction, and employment reallocation," *Quarterly Journal of Economics* 107, 819-63.

[37] Diamond, P. (1981), "Mobility costs, frictional unemployment, and efficiency," *Journal of Political Economy* 89, 798-812.

[38] Falcón, L.M. (1995), "Social networks and employment for Latinos, Blacks, and Whites," *New England Journal of Public Policy* 11, 17-28.

[39] Falcón, L.M. and E. Melendez (2001), "The role of social networks in the labor market outcomes of Latinos, Blacks and Non-Hispanic Whites," Paper presented at the Russell Sage Foundation Conference on Residential Segregation Social Capital and Labor Markets, New York.

[40] Fehr, E., Kirchsteiger, R. and A. Riedl (1996), "Involuntary unemployment and non-compensating wage differentials in an experimental labour market," *Economic Journal* 106, 106-121.

[41] Fehr, E. and L. Goette (2007), "Do workers work more if wages are high? Evidence from a randomized field experiment," *American Economic Review* 97, 298-317.

[42] Fernandez, R.M. (2010), "Creating connections for the disadvantaged: Networks and labor market intermediaries at the hiring interface," MIT Sloan School Working Paper No. 4778-10.

[43] Fong, E. and W. Isajiw (2000), "Determinants of friendship choices in multiethnic society," *Sociology Forum* 15, 249-271.

[44] Frijters, P. Shields, M.A. and S. Wheatley-Price (2005), "Immigrant job search in the UK: Evidence from panel data," *Economic Journal* 115, F359-F376.

[45] Giuliano, L. and M. Ransom (2011), "Manager ethnicity and employment segregation," IZA Discussion Paper No. 5437.

[46] Glaeser, E.L., Sacerdote, B., and J.A. Scheinkman (1996), "Crime and social interactions," *Quarterly Journal of Economics* 111, 508-548.

[47] Glaeser, E.L., Laibson, D.I., Scheinkman, J.A., and C. L. Soutter (2000), "Measuring trust," *Quarterly Journal of Economics* 115, 811-846.

[48] Goldsmith, A.H., Veum, J.R. and W. Jr. Darity (2000), "Working hard for the money? Efficiency wages and worker effort," *Journal of Economic Psychology* 21, 351-385.

[49] Goyal, S. (2007), *Connections: An Introduction to the Economics of Networks*, Princeton: Princeton University Press.

[50] Granovetter, M.S. (1973), "The strength of weak ties," *American Journal of Sociology* 78, 1360-1380.

[51] Granovetter, M.S. (1974), *Getting a Job: A Study of Contacts and Careers*, Cambridge, MA: Harvard University Press.

[52] Granovetter, M.S. (1979), "Placement as brokerage: Information problems in the labor market for rehabilitated workers," In: D. Vandergoot and J.D. Worrall (Eds.), *Placement in Rehabilitation: A Career Development Perspective*, Baltimore, MD: University Park Press, pp. 83-101.

[53] Granovetter, M.S. (1983), "The strength of weak ties: A network theory revisited," *Sociological Theory* 1, 201-233.

[54] Green, P.G., L.M. Tigges and I. Browne (1995), "Social resources, job search, and poverty in Atlanta", *Research in Community Sociology* 5, 161-182.

[55] Green, G.P, Tigges, L.M. and D. Diaz (1999), "Racial and ethnic differences in job search strategies in Atlanta, Boston and Los Angeles," *Social Science Quarterly* 80, 263-278.

[56] Green, P.Gary, L.M. Tigges and D. Diaz (1999), "Racial and ethnic differences in job-search strategies in Atlanta, Boston and Los Angeles," *Social Science Quarterly* 80, 263-278.

[57] Heckman, J.J. (2011), "The American family in black and white: A post-racial strategy for improving skills to promote equality," *Daedalus* 140, 70-89.

[58] Hellerstein, J.K. and D. Neumark (2008), "Workplace segregation in the United States: Race, ethnicity, and skill," *Review of Economics and Statistics* 90, 459-477.

[59] Hellerstein, J.K. and D. Neumark (2011),"Employment in black urban labor markets: Problems and solutions," NBER Working Paper No. 16986.

[60] Holzer, H.J. (1987), "Informal job search and black youth unemployment," *American Economic Review* 77, 446-452.

[61] Holzer, H. (1988), "Search method use by unemployed youth," *Journal of Labor Economics* 6, 1-20.

[62] Ihlanfeldt, K.R. and Sjoquist, D.L. (1998), "The spatial mismatch hypothesis: A review of recent studies and their implications for welfare reform," *Housing Policy Debate* 9, 849-892.

[63] Iceland, J. and D.H. Weinberg (2002), "Racial and ethnic segregation in the United States: 1980-2000," U.S. Census Bureau, Census 2000 Special Reports. Available at http://www.census.gov/hhes/www/housing/housing_patterns/pdf/censr-3.pdf.

[64] Ioannides, Y.M. and D.L. Loury (2004), "Job information networks, neighborhood effects, and inequality," *Journal of Economic Literature* 42, 1056-1093.

[65] Ioannides, Y.M. (2011), *From Neighborhoods to Nations: The Economics of Social Interactions*, forthcoming.

[66] Jackson, M.O. (2008), *Social and Economic Networks*, Princeton: Princeton University Press.

[67] Kain, J.F. (1968), "Housing segregation, negro employment, and Metropolitan decentralization," *Quarterly Journal of Economics* 82, 175-197.

[68] Katz, L.F., Kling, J.R. and J.B. Liebman (2001), "Moving to opportunity in Boston: Early results of a randomized mobility experiment," *Quarterly Journal of Economics* 116, 607-654.

[69] Kling, J.R., Ludwig, J. and L.F. Katz (2005), "Neighborhood effects on crime for female and male youth: Evidence from a randomized housing voucher experiment," *Quarterly Journal of Economics* 120, 87-130.

[70] Lai, G., N. Lin and S.-Y. Leung (1998), "Network resources, contact resources, and status attainment," *Social Networks* 20, 159-178.

[71] Laufer, J.K. and S. Winship (2004), "Perception vs. reality: Employer attitudes and the rebranding of workforce intermediaries," In: R.P. Giloth (Ed.), *Workforce Intermediaries for the Twenty-First Century*, Philadelphia: Temple University Press, pp. 216-240.

[72] Lin, N., W.M. Ensel and J.C. Vaughn (1981), "Social resources and strength of ties: Structural factors in occupational status attainment," *American Sociological Review* 46, 393-405.

[73] Marsden, P.V. and J.S. Hurlbert (1988), "Social resources and mobility outcomes: A replication and extension," *Social Forces* 66, 1038-1059.

[74] Massey, D.S. and Denton, N.A. (1993), *American Apartheid: Segregation and the Making of the Underclass*, Cambridge: Harvard University Press.

[75] Montgomery, J.D. (1991), "Social networks and labor-market outcomes: Toward an economic analysis," *American Economic Review* 81, 1408-1418.

[76] Montgomery, J.D. (1994), "Weak ties, employment, and inequality: An equilibrium analysis," *American Journal of Sociology* 99, 1212-1236.

[77] Mortensen, D. and T. Vishwanath (1994), "Personal contacts and earnings: It is who you know!," *Labour Economics* 1, 187-201.

[78] Mouw, T. (2002), "Racial differences in the effects of job contacts: Conflicting evidence from cross-sectional and longitudinal data," *Social Science Quarterly* 31, 511-538.

[79] Nagin, D.S., J.B. Rebitzer, S. Sanders and L.J. Taylor (2002), "Monitoring, motivation, and management: The determinants of opportunistic behavior in a field experiment," *American Economic Review* 92, 850-873.

[80] Pager, D. (2007), "The use of field experiments for studies of employment discrimination: Contributions, critiques, and directions for the future," *Annals of the American Academy of Political and Social Science* 609, 104-133.

[81] Patacchini, E. and Y. Zenou (2008), "The strength of weak ties in crime," *European Economic Review* 52, 209-236.

[82] Pellizzari, M. (2010), "Do friends and relatives really help in getting a good job?" *Industrial and Labor Relations Review* 63, 494-510.

[83] Putnam R. (2007), "E Pluribus Unum: Diversity and community in the twenty first century; The 2006 Johan Skytte Prize Lecture," *Scandinavian Political Studies*, 30, 137-174.

[84] Quillian, L. and M.E. Campbell (2003), "Beyond black and white: The present and future of multiracial friendship segregation," *American Sociology Review* 68, 540-566.

[85] Rebitzer, J.B. (1995), "Is there a trade-off between supervision and wages? An empirical test of efficiency wage theory," *Journal of Economic Behavior and Organization* 28, 107-129.

[86] Rees, A. (1966), "Information networks in labor markets," *American Economic Review* 56, 559-566.

[87] Reuter, E.B. (1945), "Racial Theory," *American Journal of Sociology* 50, 452-461.

[88] Rosenbaum, E. and L.E. Harris (2001), "Residential mobility and opportunities: Early impacts of the Moving to Opportunity demonstration program in Chicago," *Housing Policy Debate* 12, 321-346.

[89] Rushton, J.P. and A. Jensen (2005), "Thirty years of research on race differences in cognitive ability," *Psychology, Public Policy, and Law* 11, 235-294.

[90] Saloner, G. (1985), "Old boy networks as screening nechanism," *Journal of Labor Economics* 3, 255-267.

[91] Sigelman, L., Bledsoe, T., Welch, S., and M.W. Combs (1996), "Making contact? Black-white social interaction in an urban setting," *American Journal of Sociology* 101, 1306-1332.

[92] Shapiro, C., Stiglitz, J.E. (1984), "Equilibrium unemployment as a worker discipline device," *American Economic Review* 74, 433-444.

[93] Stoll, M.A., Raphael, S. and H.J. Holzer (2004), "Black job applicants and the hiring officer's race," *Industrial and Labor Relations Review* 57, 267-287.

[94] Strobl, E. and F. Walsh (2007), "Estimating the shirking model with variable effort," *Labour Economics* 14, 623-637.

[95] Topa, G. (2001), "Social interactions, local spillovers and unemployment," *Review of Economic Studies* 68, 261-295.

[96] Topa, G. (2011), "Labor markets and referrals," In: J. Benhabib, A. Bisin and M.O. Jackson (Eds.), *Handbook of Social Economics*, Amsterdam: Elsevier Science, pp. 1193-1221.

[97] Tuch, S.A., Sigelman, L. and J.A. Macdonald (1999), "Trends: Race relations and America Youth, 1976-1995," *Public Opinion Quarterly* 63, 109-148.

[98] Turner, S. (1997), "Barriers to a better break: Employers discrimination and spatial mismatch in Metropolitan Detroit," *Journal of Urban Affairs* 19, 123-141.

[99] Van Ours, J.C. (1994), "Matching unemployed and vacancies at the public employment office," *Empirical Economics* 19, 37-54.

[100] Wahba, J. and Y. Zenou (2005), "Density, social networks and job search methods: Theory and application to Egypt," *Journal of Development Economics* 78, 443-473.

[101] Wial, H. (1991), "Getting a good job: Mobility in a segmented labor market," *Industrial Relations* 30, 396-416.

[102] Wilson, W.J. (1996), *When Work Disappears: The World of the New Urban Poor*, New York: Alfred A. Knopf.

[103] Wilson, W.J. (2009), "The Moynihan report and research on the black community," Annals of the American Academy of Political and Social Science 621, 34-46,

[104] Yakubovich, V. (2005), "Weak ties, information, and influence: How workers find jobs in a local Russian labor market," *American Sociological Review* 70, 3, 408-421.

Appendix (For Online Publication)

Proof of Proposition 1

We establish the proof in two steps. First, Lemma 1 characterizes all steady-state dyad flows. Lemma 2 then provides conditions for their existence.

Lemma 1 *There exists at most two different steady-state equilibria:* (i) *a full-unemployment equilibrium* $\mathcal{U}$ *such that* $e^* = 0$ *and* $u^* = 1$, (ii) *an interior equilibrium* $\mathcal{I}$ *such that* $0 < e^* < 1$ *and* $0 < u^* < 1$.

Proof.

By combining (5) to (8), we easily obtain:

$$e^* = [(1 - \omega + \omega e^*)\lambda + \delta] \frac{2\omega e^* \lambda}{\delta^2} d_0^* \tag{43}$$

We consider two different cases.

(i) If $e^* = 0$, then equation (43) is satisfied. Furthermore, using (5) and (6), this implies that $d_1^* = d_2^* = 0$ and, using (7) and (9), we have $d_0^* = 1/2$ and $u^* = 1$. This is referred to as steady-state $\mathcal{U}$ (full unemployment).

(ii) If $e^* > 0$, then solving equation (43) yields:

$$e^* = \frac{1}{\lambda\omega}\left[\frac{\delta^2}{2\omega\lambda d_0^*} - \delta\right] - \frac{(1-\omega)}{\omega}$$

Define $Z = (1-\omega)/\omega$, $B = \delta/(\lambda\omega)$. This equation can now be written as:

$$e^* = \frac{B^2}{2d_0^*} - B - Z \tag{44}$$

Moreover, by combining (5) and (6), we obtain:

$$d_1^* = \frac{2e^*}{B} d_0^* \ , \qquad d_2^* = \frac{(Z + e^*)\, e^*}{B^2} d_0^* \tag{45}$$

• Let us first focus on the case where $e^* = 1$. In that case, it has to be that only d_2-dyads exist and thus $d_0^* = d_1^* = 0$, which, using (45) implies that: $d_2^* = 0$. So this case is not possible.

• Let us now thus focus on the case: $0 < e^* < 1$ (which implies that $0 < u^* < 1$)

By plugging (44) and (45) in (7) and after some algebra, we obtain that d_0^* solves $\Phi(d_0^*) = 0$ where $\Phi(x)$ is the following second-order polynomial:

$$\Phi(d_0^*) = -\frac{Z}{B}x^2 - \frac{(1+Z)}{2}x + \left(\frac{B}{2}\right)^2 \tag{46}$$

∎

Lemma 2

(*i*) *The steady-state equilibrium* $\mathcal{U}$ *always exists.*

(*iv*) *The steady-state equilibrium* $\mathcal{I}$ *exists when* $\delta < \lambda[\omega + \sqrt{\omega(4-3\omega)}]/2$.

Proof.

(*i*) In this equilibrium $e^* = 0$, which implies that $h(e) = (1-\omega)\,\lambda$ and $q(e) = 0$. There are only d_0-dyads so all workers are unemployed and will never receive a job offer since $q(e) = 0$. So when a d_0-dyad is formed it is never destroyed and thus this equilibrium is always sustainable.

(*ii*) We know from Lemma 1 that a steady-state $\mathcal{I}$ exists and that $e^* \neq 1$. We now have to check that $e^* > 0$ and $0 < d_0^* < 1/2$. Let us thus verify whether there exists some $0 < d_0^* < 1/2$ such that $\Phi(d_0^*) = 0$, where $\Phi(\cdot)$ is given by (46). We have $\Phi(0) = (B/2)^2 > 0$ and $\Phi'(0) = -(1+Z)/2 < 0$. Therefore, (46) has a unique positive root smaller than 1/2 if and only if

$$\Phi(1/2) = \frac{1}{4}\left[B^2 - (1+Z) - \frac{Z}{B}\right] = \frac{1}{4}(1+\frac{1}{B})(B^2 - B - Z) < 0.$$

The unique positive solution to $x^2 - x - Z = 0$ is $\left[1 + \sqrt{(4-3\omega)/\omega}\right]/2$. Then, $d_0^* < 1/2$ if and only if $B < \left[1 + \sqrt{(4-3\omega)/\omega}\right]/2$, equivalent to:

$$\frac{\delta}{\lambda} < \frac{\omega + \sqrt{\omega(4-3\omega)}}{2}$$

Observe that $d_0^* < 1/2$ guarantees that $e^* > 0$. ∎

Proof of Proposition 2

By differentiating (17), we obtain:

$$\frac{\partial e^*}{\partial \delta} = \frac{1-\omega}{\omega\sqrt{\lambda\left[\lambda + 4\delta\left(1-\omega\right)\right]}} - \frac{1}{\lambda\omega} \tag{47}$$

We have:

$$\frac{\partial e^*}{\partial \delta} < 0 \Leftrightarrow \frac{1-\omega}{\omega\sqrt{\lambda\left[\lambda + 4\delta\left(1-\omega\right)\right]}} < \frac{1}{\lambda\omega}$$

which is equivalent to:

$$\left(1-\omega\right)^2 \lambda < \lambda + 4\delta\left(1-\omega\right)$$

Simplifying further this inequality leads to:

$$\lambda\omega\left(\omega - 2\right) < 4\delta\left(1-\omega\right)$$

which is always true since $\omega < 1$.

By differentiating (12), we get:

$$\frac{\partial d_0^*}{\partial \delta} = \frac{1}{\lambda\omega B}\left[\frac{2Zd_0^{*2} + B^3}{4d_0^* Z + \left(1+Z\right)B}\right] > 0 \tag{48}$$

By differentiating (13), we get:

$$\frac{\partial d_1^*}{\partial \delta} = \frac{2\lambda\omega}{\delta^2}\left[\frac{\partial e^*}{\partial \delta} d_0^* \delta + e^* \frac{\partial d_0^*}{\partial \delta}\delta - e^* d_0^*\right] \tag{49}$$

Using (47), (48) and the fact that $B \equiv \delta/(\lambda\omega)$, we obtain:

$$\frac{\partial d_1^*}{\partial \delta} = \frac{2\lambda\omega}{\delta^2}\left[\frac{\left(1-\omega\right)d_0^*\delta}{\omega\sqrt{\lambda\left[\lambda + 4\delta\left(1-\omega\right)\right]}} - \frac{d_0^*\delta}{\lambda\omega} + \frac{2Zd_0^{*2}e^* + B^3 e^*}{4d_0^* Z + \left(1+Z\right)B} - e^* d_0^*\right]$$

This is clearly ambiguous. However, if we go back to (49), observe that if $\frac{\partial e^*}{\partial \delta} d_0^* \delta + e^* \frac{\partial d_0^*}{\partial \delta}\delta < 0$, then $\frac{\partial d_1^*}{\partial \delta} < 0$. This is equivalent to:

$$\frac{\partial e^*}{\partial \delta}\frac{\delta}{e^*} + \frac{\partial d_0^*}{\partial \delta}\frac{\delta}{d_0^*} < 0$$

$$\Leftrightarrow \frac{\partial d_0^*}{\partial \delta}\frac{\delta}{d_0^*} < -\frac{\partial e^*}{\partial \delta}\frac{\delta}{e^*}$$

$$\Leftrightarrow \left|\frac{\partial d_0^*}{\partial \delta}\frac{\delta}{d_0^*}\right| < \left|\frac{\partial e^*}{\partial \delta}\frac{\delta}{e^*}\right|$$

Finally, by differentiating (14), we get:

$$\frac{\partial d_2^*}{\partial \delta} = \frac{(\lambda\omega)^2}{\delta^3}\left\{(Z+2e^*)\frac{\partial e^*}{\partial \delta}d_0^*\delta + \left(Ze^* + e^{*2}\right)\frac{\partial d_0^*}{\partial \delta}\delta - 2\left(Ze^* + e^{*2}\right)d_0^*\right\}$$

Since $(Z+2e^*)\frac{\partial e^*}{\partial \delta}d_0^*\delta < 0$, let us show that $(Ze^* + e^{*2})\frac{\partial d_0^*}{\partial \delta}\delta - 2(Ze^* + e^{*2})d_0^* < 0$. Using (48) and the fact that $B \equiv \delta/(\lambda\omega)$, this last inequality is equivalent to:

$$\frac{2Zd_0^{*2} + B^3}{4d_0^* Z + (1+Z)B} - 2d_0^* < 0$$

which is equivalent to:

$$-\frac{3}{2}\frac{Z}{B}d_0^{*2} - \frac{(1+Z)}{2}d_0^* + \left(\frac{B}{2}\right)^2 < 0$$

Since d_0^* is defined as (see (12)):

$$-\frac{Z}{B}d_0^{*2} - \frac{(1+Z)}{2}d_0^* + \left(\frac{B}{2}\right)^2 = 0$$

the inequality above is thus equivalent to:

$$-\frac{1}{2}\frac{Z}{B}d_0^{*2} < 0$$

which is always true. ∎

Proof of Proposition 3

Differentiate first (12). We obtain:

$$\frac{\partial d_0^*}{\partial \lambda} = -\frac{\frac{Z\lambda\omega}{\delta}d_0^{*2} + \frac{\delta^2}{2\lambda^3\omega^2}}{2\frac{Z}{B}d_0^* + \frac{1+Z}{2}} < 0 \tag{50}$$

Now, by differentiating (11), we have:

$$\frac{\partial e^*}{\partial \lambda} = \frac{\delta}{\lambda^2\omega} - \frac{\delta^2}{\lambda^2\omega^2 4d_0^*}\left(\frac{2d_0^*}{\lambda} + \frac{\partial d_0^*}{\partial \lambda}\right)$$

Thus, using (50), we have:

$$\frac{\partial e^*}{\partial \lambda} = \frac{\delta}{\lambda^2\omega}\left[1 - \frac{\delta}{4\omega d_0^*}\left(\frac{2d_0^*}{\lambda} - \frac{2Zd_0^{*2} + \frac{\delta^3}{\lambda^4\omega^3}}{4Zd_0^* + (1+Z)B}\right)\right] \tag{51}$$

As a result,

$$\frac{\partial e^*}{\partial \lambda} > 0 \Leftrightarrow 4\omega + \frac{2Zd_0^*\delta + \frac{\delta^4}{\lambda^4\omega^3 d_0^*}}{4Zd_0^* + (1+Z)B} > \frac{2\delta}{\lambda}$$

$$\Leftrightarrow 2\omega + \frac{Zd_0^*\delta + \frac{\delta^4}{2\lambda^4\omega^3 d_0^*}}{4Zd_0^* + (1+Z)B} > \frac{\delta}{\lambda}$$

$$\Leftrightarrow 2\omega\left[4Zd_0^* + (1+Z)B\right] + Zd_0^{*2}\delta\lambda + \frac{\delta^4}{2\lambda^3\omega^3} > 4Z\delta d_0^{*2} + (1+Z)B\delta d_0^*$$

$$\Leftrightarrow 4\lambda^3\omega^4\left[4Zd_0^* + (1+Z)B\right] + 2\lambda^4\omega^3 Z\delta d_0^{*2} + \delta^4 > 8\lambda^3\omega^3 Z\delta d_0^{*2} + 2\lambda^3\omega^3(1+Z)B\delta d_0^*$$

$$\Leftrightarrow \frac{2\omega}{\delta}\left[4\frac{Z}{B}d_0^* + (1+Z)\right] - \frac{Z}{B}d_0^{*2}(4-\lambda) - (1+Z)d_0^* + \frac{\delta^3}{2B\lambda^3\omega^3} > 0$$

$$\Leftrightarrow \frac{8\omega}{\delta}\frac{Z}{B}d_0^* - \frac{Z}{B}d_0^{*2} - \frac{(1+Z)}{2}d_0^* - \frac{Z}{B}d_0^{*2} - \frac{(1+Z)}{2}d_0^* - 2\frac{Z}{B}d_0^{*2} + \frac{Z}{B}d_0^{*2}\lambda + \frac{\delta^3}{2B\lambda^3\omega^3} + \frac{2\omega}{\delta}(1+Z) > 0$$

We know from (12) that

$$-\frac{Z}{B}d_0^{*2} - \frac{(1+Z)}{2}d_0^* + \left(\frac{B}{2}\right)^2 = 0$$

thus

$$\Leftrightarrow \frac{8\omega}{\delta}\frac{Z}{B}d_0^* - \frac{B^2}{2} - 2\frac{Z}{B}d_0^{*2} + \frac{Z}{B}d_0^{*2}\lambda + \frac{\delta^3}{2B\lambda^3\omega^3} + \frac{2\omega}{\delta}(1+Z) > 0$$

$$\Leftrightarrow \frac{Z}{B}d_0^{*2}(\lambda-2) + \frac{8\omega}{\delta}\frac{Z}{B}d_0^* + \frac{\delta^3}{2B\lambda^3\omega^3} + \frac{2\omega}{\delta}(1+Z) - \frac{B^2}{2} > 0$$

Use the values of $Z = \frac{1-\omega}{\omega}$ and $B = \frac{\delta}{\lambda\omega}$. Then this inequality is equivalent to:

$$(1-\omega)\lambda(\lambda-2)\delta d_0^{*2} + 8\omega(1-\omega)\lambda d_0^* + 2\delta > 0$$

If $\lambda \geq 2$, this inequality is always true. Combining (10) and $\lambda \geq 2$ leads to

$$\lambda > \max\left\{\frac{2\delta}{\omega + \sqrt{\omega(4-3\omega)}}, 2\right\}$$

which is condition (20).

By differentiating (14), we obtain:

$$\begin{aligned} B^3\frac{\partial d_2^*}{\partial\lambda} &= \frac{\partial e^*}{\partial\lambda}(Z+2e^*)Bd_0^* + \left(Ze^* + e^{*2}\right)\frac{\partial d_0^*}{\partial\lambda}B + 2\left(Ze^* + e^{*2}\right)\frac{\delta}{\lambda^2\omega}d_0^* \\ &= \frac{\partial e^*}{\partial\lambda}(Z+2e^*)\frac{\delta}{\lambda\omega}d_0^* + 2\left(Ze^* + e^{*2}\right)\frac{\delta}{\lambda^2\omega}d_0^* + \left(Ze^* + e^{*2}\right)\frac{\delta}{\lambda\omega}\frac{\partial d_0^*}{\partial\lambda} \end{aligned}$$

Using (50), we have:

$$\begin{aligned}
B^3\frac{\partial d_2^*}{\partial\lambda} &= \frac{\partial e^*}{\partial\lambda}\left(Z+2e^*\right)\frac{\delta}{\lambda\omega}d_0^* + 2\left(Ze^*+e^{*2}\right)\frac{\delta}{\lambda^2\omega}d_0^* - \frac{\left(1-\omega\right)\lambda d_0^{*2}+\frac{\delta^3}{2\lambda^3\omega^2}}{2\frac{\lambda^2\omega(1-\omega)}{\delta}d_0^*+\frac{\lambda}{2}}\left(Ze^*+e^{*2}\right)\\
&= \frac{\partial e^*}{\partial\lambda}\left(Z+2e^*\right)\frac{\delta}{\lambda\omega}d_0^* + \left(Ze^*+e^{*2}\right)\left[\frac{2\delta d_0^*}{\lambda^2\omega} - \frac{2\left(1-\omega\right)\lambda\delta d_0^{*2}+\frac{\delta^4}{\lambda^3\omega^2}}{4\lambda^2\omega\left(1-\omega\right)d_0^*+\lambda\delta}\right]\\
&= \frac{\partial e^*}{\partial\lambda}\left(Z+2e^*\right)\frac{\delta}{\lambda\omega}d_0^* + \left(Ze^*+e^{*2}\right)\left[\frac{2\lambda^2\delta\omega\left(1-\omega\right)d_0^{*2}\left(4-\lambda\right)+2\delta^2\lambda d_0^*-\frac{\delta^4}{\lambda\omega}}{\lambda^2\omega\left[4\lambda^2\omega\left(1-\omega\right)d_0^*+\lambda\delta\right]}\right]
\end{aligned}$$

We know from (12) that

$$-\frac{\left(1-\omega\right)\lambda}{\delta}d_0^{*2} - \frac{1}{2\omega}d_0^* + \frac{\delta^2}{4\lambda^2\omega^2} = 0$$

and thus we have:

$$B^3\frac{\partial d_2^*}{\partial\lambda} = \frac{\partial e^*}{\partial\lambda}\left(Z+2e^*\right)\frac{\delta}{\lambda\omega}d_0^* - \frac{2\delta\left(Ze^*+e^{*2}\right)\left(1-\omega\right)\lambda\left(\lambda-2\right)d_0^{*2}}{\lambda\left[4\lambda^2\omega\left(1-\omega\right)d_0^*+\lambda\delta\right]}$$

Using (51), we obtain:

$$\begin{aligned}
B^3\frac{\partial d_2^*}{\partial\lambda} &= \frac{\delta}{\lambda^2\omega} - \delta^3\left(Z+2e^*\right)\left(\frac{2\lambda\omega\left(1-\omega\right)d_0^{*2}\left(4-\lambda\right)+2d_0^*\delta-\frac{\delta^3}{\lambda^2\omega}}{4\lambda^4\omega^3\left[4\lambda\omega\left(1-\omega\right)d_0^*+\delta\right]}\right)\\
&\quad - \frac{2\delta\left(Ze^*+e^{*2}\right)\left(1-\omega\right)\lambda\left(\lambda-2\right)d_0^{*2}}{\lambda\left[4\lambda^2\omega\left(1-\omega\right)d_0^*+\lambda\delta\right]}
\end{aligned}$$

Using (12), we get:

$$B^3\frac{\partial d_2^*}{\partial\lambda} = \frac{\delta}{\lambda^2\omega} + \frac{\delta^4\left(Z+2e^*\right)\left(\frac{(1-\omega)\lambda d_0^{*2}(\lambda-2)}{\delta}-\frac{d_0^*}{\omega}\right) - 4\lambda^3\delta\omega^2\left(Ze^*+e^{*2}\right)\left(1-\omega\right)\lambda\left(\lambda-2\right)d_0^{*2}}{2\lambda^4\omega^2\left[4\lambda\omega\left(1-\omega\right)d_0^*+\delta\right]}$$

$$\Leftrightarrow B^3\frac{\partial d_2^*}{\partial\lambda} = \frac{\delta}{\lambda^2\omega} + \frac{\left(1-\omega\right)\omega\lambda\left(\lambda-2\right)\left[\delta^3\left(Z+2e^*\right)-4\lambda^2\delta\omega^2\left(Z+e\right)e^*\right]d_0^{*2} - \delta^4\left(Z+2e^*\right)d_0^*}{2\lambda^4\omega^3\left[4\lambda\omega\left(1-\omega\right)d_0^*+\delta\right]}$$

A sufficient condition for $\frac{\partial d_2^*}{\partial\lambda} > 0$ is:

$$\left(1-\omega\right)\omega\lambda\left(\lambda-2\right)\left[\delta^3\left(Z+2e^*\right)-4\lambda^2\delta\omega^2\left(Z+e\right)e^*\right]d_0^{*2} - \delta^4\left(Z+2e^*\right)d_0^* > 0$$

which is equivalent to:

$$d_0^* > \frac{\delta^3}{\left(1-\omega\right)\omega\lambda\left(\lambda-2\right)\left[\delta^2-4\lambda^2\omega^2\left(\frac{Z+e}{Z+2e^*}\right)e^*\right]}$$

An upper bound for $\left(\frac{Z+e}{Z+2e^*}\right) e^*$ is 1 and thus this condition can be written as:

$$d_0^* > \frac{\delta^3}{(1-\omega)\,\omega\lambda\,(\lambda-2)\,(\delta^2-4\lambda^2\omega^2)}$$

which is condition (21).

Finally, by differentiating (13), we get:

$$\frac{\partial d_1^*}{\partial \lambda} = \frac{2\omega}{\delta}\left[\frac{\partial e^*}{\partial \lambda} d_0^* \lambda + e^* \frac{\partial d_0^*}{\partial \lambda}\lambda + e^* d_0^*\right] \tag{52}$$

This is clearly ambiguous. However, since $e^* d_0^* > 0$, a sufficient condition for $\frac{\partial d_1^*}{\partial \lambda} > 0$ is:

$$\frac{\partial e^*}{\partial \lambda} d_0^* \lambda + e^* \frac{\partial d_0^*}{\partial \lambda}\lambda > 0$$

$$\Leftrightarrow \frac{\partial e^*}{\partial \lambda}\frac{\lambda}{e^*} + \frac{\partial d_0^*}{\partial \lambda}\frac{\lambda}{d_0^*} > 0$$

$$\Leftrightarrow \frac{\partial e^*}{\partial \lambda}\frac{\lambda}{e^*} > -\frac{\partial d_0^*}{\partial \lambda}\frac{\lambda}{d_0^*}$$

$$\Leftrightarrow \left|\frac{\partial e^*}{\partial \lambda}\frac{\lambda}{e^*}\right| > \left|\frac{\partial d_0^*}{\partial \lambda}\frac{\lambda}{d_0^*}\right|$$

Proof of Proposition 4

(i) By totally differentiating (12), we obtain:

$$\frac{\partial d_0^*}{\partial \omega} = \frac{\frac{\lambda}{\delta} d_0^2 + \frac{1}{2\omega^2} d_0 - \frac{\delta^2}{2\lambda^2\omega^3}}{2\frac{\lambda(1-\omega)}{\delta} d_0 + \frac{1}{2\omega}}$$

and thus

$$\operatorname{sgn} \frac{\partial d_0^*}{\partial \omega} = \operatorname{sgn} \left[\frac{\lambda}{\delta} d_0^2 + \frac{1}{2\omega^2} d_0 - \frac{\delta^2}{2\lambda^2\omega^3}\right]$$

Let us study

$$\Phi(d_0) \equiv \frac{\lambda}{\delta} d_0^2 + \frac{1}{2\omega^2} d_0 - \frac{\delta^2}{2\lambda^2\omega^3}$$

$$\Phi(0) = -\frac{\delta^2}{2\lambda^2\omega^3} < 0$$

$$\Phi'(d_0) = 2\frac{\lambda}{\delta} d_0 + \frac{1}{2\omega^2} > 0 \text{ when } d_0 \geq 0$$

$$\Phi''(d_0) = 2\frac{\lambda}{\delta} > 0$$

We have a quadratic function that crosses only once the positive orthant. Let us calculate $\widehat{d_0} > 0$ the value for which $\Phi(d_0)$ crosses the d_0-axis. For that, we have to solve: $\Phi(\widehat{d_0}) = 0$. It is easy to verify that:

$$\widehat{d_0} = \frac{\delta}{4\lambda\omega^2}\left(\sqrt{1+\frac{8\delta\omega}{\lambda}} - 1\right) > 0$$

It should be clear that if $\widehat{d_0} < 1/2$, then $\Phi(d_0) < 0$ for $0 < d_0 < 1/2$ and thus $\frac{\partial d_0^*}{\partial \omega} < 0$. Let us thus check that $\widehat{d_0} < 1/2$, which is equivalent to:

$$\Omega\left(\frac{\delta}{\lambda}\right) \equiv 2\left(\frac{\delta}{\lambda}\right)^3 - \omega\frac{\delta}{\lambda} - \omega^3 < 0$$

We have:

$$\Omega(0) = -\omega^3 < 0$$

$$\Omega'\left(\frac{\delta}{\lambda}\right) = 6\left(\frac{\delta}{\lambda}\right)^2 - \omega$$

with

$$\Omega'\left(\frac{\delta}{\lambda}\right) < 0 \Leftrightarrow \frac{\delta}{\lambda} < \sqrt{\frac{\omega}{6}}$$

As a result, when $\frac{\delta}{\lambda} < \sqrt{\frac{\omega}{6}}$, $\widehat{d_0} < 1/2$ and thus $\frac{\partial d_0^*}{\partial \omega} < 0$. Since we are in equilibrium $\mathcal{I}$, condition (10) has to hold, i.e.

$$\frac{\delta}{\lambda} < \frac{\omega + \sqrt{\omega(4-3\omega)}}{2}$$

Let us show that

$$\sqrt{\frac{\omega}{6}} < \frac{\omega + \sqrt{\omega(4-3\omega)}}{2}$$

This inequality is equivalent to:

$$4 + 2\sqrt{\omega(4-3\omega)} > \frac{2}{3} + 2\omega$$

which is always true since $\omega < 1$ and thus $4 > \frac{2}{3} + 2\omega$. Consequently, when condition (22) holds, i.e. $\frac{\delta}{\lambda} < \sqrt{\frac{\omega}{6}}$, $\frac{\partial d_0^*}{\partial \omega} < 0$, then condition (10) is always satisfied.

(*ii*) By totally differentiating (11), we obtain:

$$
\begin{aligned}
\frac{\partial e^*}{\partial \omega} &= \frac{\partial B}{\partial \omega}\left(\frac{B}{2}-1\right)-\frac{B^2}{4}\frac{1}{d_0}\frac{\partial d_0^*}{\partial \omega}-\frac{\partial Z}{\partial \omega} \\
&= \frac{-\delta}{\lambda\omega^2}\left(\frac{\delta}{2\lambda\omega}-1\right)-\frac{\delta^2}{4\lambda^2\omega^2}\frac{1}{d_0}\frac{\partial d_0^*}{\partial \omega}+\frac{1}{\omega^2} \\
&= \frac{\delta}{\lambda\omega^2}-\frac{\delta^2}{4\lambda^2\omega^2}\frac{1}{d_0}\frac{\partial d_0^*}{\partial \omega}+\frac{1}{\omega^2}-\frac{\delta^2}{2\lambda^2\omega^3} \\
&= \frac{1}{\omega^2}\left[\frac{\delta}{\lambda}-\frac{\delta^2}{4\lambda^2}\frac{1}{d_0}\frac{\partial d_0^*}{\partial \omega}+1-\frac{\delta^2}{2\lambda^2\omega}\right]
\end{aligned}
$$

Thus, we have:

$$\frac{\partial e^*}{\partial \omega}>0 \Leftrightarrow \frac{\delta}{\lambda}-\frac{\delta^2}{4\lambda^2}\frac{1}{d_0}\frac{\partial d_0^*}{\partial \omega}+1>\frac{\delta^2}{2\lambda^2\omega}$$

Since $\frac{\partial d_0^*}{\partial \omega}<0$ when (22) holds, then it suffices to show that:

$$\frac{\delta}{\lambda}+1>\frac{\delta^2}{2\lambda^2\omega}$$

which is always true if

$$\frac{\delta^2}{2\lambda^2\omega}<1$$

This is equivalent to:

$$\frac{\delta}{\lambda}<\sqrt{2\omega}$$

But since

$$\sqrt{\frac{\omega}{6}}<\sqrt{2\omega}$$

is always true, then condition (22) guarantees that both

$$\frac{\partial d_0^*}{\partial \omega}<0 \text{ and } \frac{\partial e^*}{\partial \omega}>0$$

Since $e^*=1-u^*$, $\frac{\partial e^*}{\partial \omega}>0 \Leftrightarrow \frac{\partial u^*}{\partial \omega}<0$. To summarize, when condition (22) holds, i.e. $\frac{\delta}{\lambda}<\sqrt{\frac{\omega}{6}}$, we have: $\frac{\partial d_0^*}{\partial \omega}<0$, $\frac{\partial e^*}{\partial \omega}>0$, $\frac{\partial u^*}{\partial \omega}<0$, and condition (10) is always satisfied.

Finally, from (13) and (14), it is easy to see that $\frac{\partial d_1^*}{\partial \omega}$ and $\frac{\partial d_2^*}{\partial \omega}$ cannot be signed. ▮

Proof of Proposition 6

First, it is easily verified that, using (17),

$$\frac{\partial e^2}{\partial^2\omega}=\frac{\sqrt{\lambda}}{2\lambda\omega^3}\left[\frac{2\delta\sqrt{\lambda+4\delta(1-\omega)}-\frac{(\lambda+4\delta-2\delta\omega)}{2\sqrt{\lambda+4\delta(1-\omega)}}4\delta}{\lambda+4\delta(1-\omega)}\right]<0 \tag{53}$$

so that second order condition is always satisfied. As a result, ω^*, the solution to (27), is unique.

Second, by differentiating (27), it is straightforward to show that

$$\frac{\partial \omega^*}{\partial y} > 0 \qquad \frac{\partial \omega^*}{\partial b} < 0 \qquad \frac{\partial \omega^*}{\partial c} < 0$$

Third, by differentiating (27), we have

$$\frac{\partial \omega^*}{\partial \delta} = -\frac{\frac{\partial^2 e^*(\omega)}{\partial \omega \partial \delta}}{\frac{\partial e^{*2}}{\partial^2 \omega}} \text{ and } \frac{\partial \omega^*}{\partial \lambda} = -\frac{\frac{\partial^2 e^*(\omega)}{\partial \omega \partial \lambda}}{\frac{\partial e^{*2}}{\partial^2 \omega}}$$

Since $\frac{\partial e^{*2}}{\partial^2 \omega} < 0$ (see (53)), the sign of $\frac{\partial \omega^*}{\partial \delta}$ is the same as of $\frac{\partial^2 e^*(\omega)}{\partial \omega \partial \delta}$ and the sign of $\frac{\partial \omega^*}{\partial \lambda}$ is the same as of $\frac{\partial^2 e^*(\omega)}{\partial \omega \partial \lambda}$. Let us now calculate these cross-derivatives.

By differentiating (17), we have:

$$2\lambda\omega^2 \frac{\partial e^*}{\partial \omega} = 2\delta + \lambda - \lambda \left[\frac{\lambda + 2\delta(2-\omega)}{\sqrt{\lambda[\lambda + 4\delta(1-\omega)]}} \right] \tag{54}$$

Thus, by differentiating again this equation with respect to δ, we obtain:

$$2\lambda\omega^2 \frac{\partial^2 e^*}{\partial \omega \partial \delta} = 2 - \frac{2\lambda^2 + 4\lambda\delta(2-\omega)(1-\omega)}{[\lambda + 4\delta(1-\omega)]\sqrt{\lambda[\lambda + 4\delta(1-\omega)]}}$$

$$\Leftrightarrow \lambda\omega^2 \frac{\partial^2 e^*}{\partial \omega \partial \delta} = \frac{[\lambda + 4\delta(1-\omega)]\sqrt{\lambda[\lambda + 4\delta(1-\omega)]} + 2\lambda\delta(2-\omega)(1-\omega) - \lambda^2}{[\lambda + 4\delta(1-\omega)]\sqrt{\lambda[\lambda + 4\delta(1-\omega)]}}$$

Let us show that

$$\lambda\omega^2 \frac{\partial^2 e^*}{\partial \delta^2} > 0$$

This is equivalent to

$$[\lambda + 4\delta(1-\omega)]\sqrt{\lambda[\lambda + 4\delta(1-\omega)]} + 2\lambda\delta(2-\omega)(1-\omega) > \lambda^2$$

$$\Leftrightarrow \lambda\sqrt{\frac{\lambda + 4\delta(1-\omega)}{\lambda}} + 4\delta(1-\omega)\sqrt{\frac{\lambda + 4\delta(1-\omega)}{\lambda}} + 2\delta(2-\omega)(1-\omega) > \lambda$$

Since

$$\lambda\sqrt{\frac{\lambda + 4\delta(1-\omega)}{\lambda}} > \lambda \Leftrightarrow \sqrt{1 + \frac{4\delta(1-\omega)}{\lambda}} > 1$$

is always true, then $\lambda\omega^2 \frac{\partial^2 e^*}{\partial \omega \partial \delta} > 0$ and thus $\frac{\partial^2 e^*}{\partial \omega \partial \delta} > 0$. As a result, $\frac{\partial \omega^*}{\partial \delta} > 0$.

Now, by differentiating (54) with respect to λ, we obtain:

$$2\omega^2 \frac{\partial^2 e^*}{\partial \omega \partial \lambda} = -\frac{\delta}{\lambda^3} + \frac{4\delta\lambda + 8\delta^2(2-\omega)(1-\omega)}{[\lambda + 4\delta(1-\omega)]\sqrt{\lambda[\lambda + 4\delta(1-\omega)]}}$$

$$\Leftrightarrow 2\omega^2 \frac{\partial^2 e^*}{\partial\omega\partial\lambda} = \frac{4\delta\lambda^4 + 8\delta^2\lambda^3 (2-\omega)(1-\omega) - \delta[\lambda + 4\delta(1-\omega)]\sqrt{\lambda[\lambda + 4\delta(1-\omega)]}}{\lambda^3[\lambda + 4\delta(1-\omega)]\sqrt{\lambda[\lambda + 4\delta(1-\omega)]}}$$

Let us show that:

$$4\delta\lambda^4 + 8\delta^2\lambda^3 (2-\omega)(1-\omega) > \delta[\lambda + 4\delta(1-\omega)]\sqrt{\lambda[\lambda + 4\delta(1-\omega)]}$$

This is equivalent to:

$$16\lambda^7 + 64\delta^2\lambda^5 (2-\omega)^2 (1-\omega)^2 + 64\lambda^6\delta(2-\omega)(1-\omega) > [\lambda + 4\delta(1-\omega)]^3$$

$$\begin{aligned} &\Leftrightarrow 16\lambda^7 + 64\lambda^5\delta^2 (2-\omega)^2 (1-\omega)^2 + 64\lambda^6\delta(2-\omega)(1-\omega) \\ > \ &\lambda^3 + 12\lambda^2\delta(1-\omega) + 48\lambda\delta^2(1-\omega)^2 + 64\delta^3(1-\omega)^3 \end{aligned}$$

$$\begin{aligned} \Leftrightarrow \ &\lambda^3\left(\lambda^4 - 1\right) + 12\lambda\delta(1-\omega)\left[(2-\omega)\lambda^4 - 1\right][\lambda + 4\delta(1-\omega)] \\ &+15\lambda^7 + 52\delta(1-\omega)\lambda^6(2-\omega) + 16\delta(1-\omega)\lambda^5\delta(2-\omega)^2(1-\omega) - 64\delta^3(1-\omega)^3 > 0 \end{aligned}$$

Observe that (10), i.e.

$$\delta < \lambda\left[\frac{\omega + \sqrt{\omega(4-3\omega)}}{2}\right]$$

implies that (by taking the upper bound of ω on the right-hand side):

$$\delta < \lambda\left[\frac{1+\sqrt{4}}{2}\right] = \frac{3}{2}\lambda$$

As a result,

$$64\delta^3(1-\omega)^3 < 64\left(\frac{3}{2}\right)^3\lambda^3 = 216\lambda^3$$

Since we are considered the case for which $\lambda \geq 2$,

$$15\lambda^7 > 15\lambda^4\lambda^3 = 15 \times 2^4\lambda^3 = 240\lambda^3$$

and thus the inequality above is always true. As a result, $2\omega^2 \frac{\partial^2 e^*}{\partial\omega\partial\lambda} > 0$, thus $\frac{\partial^2 e^*}{\partial\omega\partial\lambda} > 0$ and $\frac{\partial\omega^*}{\partial\lambda} > 0$. ∎

Proof of Proposition 7

Lemma 3 *There exist two types of steady-state equilibria: (i) a full-unemployment equilibrium $\mathcal{U}$ such that $e_j^* = 0$ and $u_j^* = 1$, (ii) an interior equilibrium $\mathcal{I}$ such that $0 < e_j^* < 1$ and $0 < u_j^* < 1$, $\forall j = B, W$.*

Proof. By combining (30) to (34), we easily obtain:

$$e_j^* = \left[(1-\omega_j)\lambda_j + \omega_j \left(\frac{N_B}{N} e_B^* \lambda_B + \frac{N_W}{N} e_W^* \lambda_W \right) + \delta \right] \frac{2\omega_j \left(\frac{N_B}{N} e_B^* \lambda_B + \frac{N_W}{N} e_W^* \lambda_W \right)}{\delta^2 N_j} d_{0j}^* \tag{55}$$

We consider the following different cases.

(ia) If $e_B^* = e_W^* = 0$, then equation (55) is satisfied. We have that $d_{1j}^* = d_{2j}^* = 0$ and $d_0^* = N_j/2$ and $u_j^* = 1$. This is referred to as steady-state $\mathcal{U}$ (full unemployment).

(ib) If $e_B^* = 0$ and $e_W^* > 0$, then solving equation (43) yields for blacks:

$$0 = \left[(1-\omega_B)\lambda_B + \omega_B \frac{N_W}{N} e_W^* \lambda_W + \delta \right] \frac{2\omega_B \lambda \frac{N_W}{N} e_W^* \lambda_W}{\delta^2 N_B} d_{0B}^*$$

The only way this equation can hold is that $e_W^* = 0$ (indeed d_{0B}^* cannot be equal to zero since this implies that $d_{2B}^* = d_{1B}^* = 0$ and thus $d_{0B}^* = \frac{N_W}{2} - d_{2B}^* - d_{1B}^*$ cannot hold) and we are back in case (ia) where $e_B^* = e_W^* = 0$ and steady-state $\mathcal{U}$ prevails.

(ic) If $e_W^* = 0$ and $e_B^* > 0$, then by a similar reasoning as in case (ib), we end up with $e_B^* = e_W^* = 0$ and steady-state $\mathcal{U}$ prevails.

(ii) Let us assume that $e_B^* > 0$ and $e_W^* > 0$. Let us see if it is possible to have either $e_B^* = 1$ or $e_W^* = 1$ or both. If either $e_B^* = 1$ or $e_W^* = 1$ or both $e_B^* = e_W^* = 1$, then it is easily verified that $d_{0j}^* = d_{1j}^*/2 - d_{1j}^* = -d_{1j}^*/2 < 0$, which is impossible. As a result, if $e_B^* > 0$ and $e_W^* > 0$, then it has to be that $e_B^* < 1$ and $e_W^* < 1$. We call this steady-state equilibrium $\mathcal{I}$ because it is an interior equilibrium for which $0 < e_B^* < 1$ and $0 < e_W^* < 1$. ∎

Let us now focus on the case $0 < e_B^* < 1$ and $0 < e_W^* < 1$ (which implies that $0 < u_B^* < 1$ and $0 < u_W^* < 1$) and prove Proposition 7. Using (30) to (34), we have:

$$d_{2j}^* = \frac{\omega_j \left[(1-\omega_j)\lambda_j + \omega_j \left(\frac{N_B}{N} e_B^* \lambda_B + \frac{N_W}{N} e_W^* \lambda_W \right) \right] \left(\frac{N_B}{N} e_B^* \lambda_B + \frac{N_W}{N} e_W^* \lambda_W \right)}{\delta^2} d_{0j}^*$$

$$d_{1j}^* = \frac{2\omega_j \left(\frac{N_B}{N} e_B^* \lambda_B + \frac{N_W}{N} e_W^* \lambda_W \right)}{\delta} d_{0j}^*$$

$$d_{0j}^* = \frac{N_j}{2} - d_{2j}^* - d_{1j}^*$$

First, we plug the values of d_{2j}^* and d_{1j}^* from (30) and (31) into $N_j e_j^* = 2d_{2j}^* + d_{1j}^*$ to obtain:

$$N_j e_j^* = \left[(1-\omega_j)\lambda_j + \omega_j \left(\frac{N_B}{N} e_B^* \lambda_B + \frac{N_W}{N} e_W^* \lambda_W \right) + \delta \right] \left(\frac{N_B}{N} e_B^* \lambda_B + \frac{N_W}{N} e_W^* \lambda_W \right) \frac{2\omega_j}{\delta^2} d_{0j}^*$$

which is (35)

Second, we plug the values of d_{2j}^* and d_{1j}^* from (30) and (31) into $d_{0j}^* = \frac{N_j}{2} - d_{2j}^* - d_{1j}^*$ to obtain:

$$d_{0j}^* + \left[(1-\omega_j)\lambda_j + \omega_j\left(\frac{N_B}{N}e_B^*\lambda_B + \frac{N_W}{N}e_W^*\lambda_W\right) + 2\delta\right]\left(\frac{N_B}{N}e_B^*\lambda_B + \frac{N_W}{N}e_W^*\lambda_W\right)\frac{\omega_j}{\delta^2}d_{0j}^* = \frac{N_j}{2}$$

which is (36)By plugging the value of d_{0j}^* into (35), we obtain () and (). By dividing (35) for $j = B$ and $j = W$, we obtain:

$$\frac{N_B e_B^*}{N_W e_W^*} = \frac{\left[(1-\omega_B)\lambda_B + \omega_B\left(\frac{N_B}{N}e_B^*\lambda_B + \frac{N_W}{N}e_W^*\lambda_W\right) + \delta\right]}{\left[(1-\omega_W)\lambda_W + \omega_W\left(\frac{N_B}{N}e_B^*\lambda_B + \frac{N_W}{N}e_W^*\lambda_W\right) + \delta\right]}\frac{\omega_B}{\omega_W}\frac{d_{0B}^*}{d_{0W}^*}$$

As a result, $e_B^* < e_W^*$ is equivalent to:

$$\frac{(1-\omega_B)\lambda_B + \omega_B\left(\frac{N_B}{N}e_B^*\lambda_B + \frac{N_W}{N}e_W^*\lambda_W\right) + \delta}{(1-\omega_W)\lambda_W + \omega_W\left(\frac{N_B}{N}e_B^*\lambda_B + \frac{N_W}{N}e_W^*\lambda_W\right) + \delta} < \frac{\omega_W}{\omega_B}\frac{d_{0W}^*}{d_{0B}^*}\frac{N_B}{N_W}$$

This is equivalent to:

$$\left(\frac{N_B}{N}e_B^*\lambda_B + \frac{N_W}{N}e_W^*\lambda_W\right)\left(\omega_W^2 d_{0W}^* N_B - \omega_B^2 d_{0B}^* N_W\right) + \delta\left(\omega_W d_{0W}^* N_B - \omega_B d_{0B}^* N_W\right)$$
$$+ (1-\omega_W)\lambda_W\omega_W d_{0W}^* N_B - (1-\omega_B)\lambda_B\omega_B d_{0B}^* N_W > 0$$

Thus, if the following inequalities are satisfied

$$\begin{cases} \omega_W^2 d_{0W}^* N_B > \omega_B^2 d_{0B}^* N_W \\ \omega_W d_{0W}^* N_B > \omega_B d_{0B}^* N_W \\ (1-\omega_W)\lambda_W\omega_W d_{0W}^* N_B > (1-\omega_B)\lambda_B\omega_B d_{0B}^* N_W \end{cases}$$

then $e_B^* < e_W^*$. If $\omega_W > \omega_B$, then these inequalities are equivalent to

$$\begin{cases} \frac{\omega_W}{\omega_B}\frac{d_{0W}^*}{d_{0B}^*}\frac{N_B}{N_W} > 1 \\ \frac{\omega_W}{\omega_B}\frac{d_{0W}^*}{d_{0B}^*}\frac{N_B}{N_W} > \left(\frac{1-\omega_B}{1-\omega_W}\right)\frac{\lambda_B}{\lambda_W} \end{cases}$$

that is

$$\frac{\omega_W}{\omega_B}\frac{d_{0W}^*}{d_{0B}^*}\frac{N_B}{N_W} > \max\left\{1, \left(\frac{1-\omega_B}{1-\omega_W}\right)\frac{\lambda_B}{\lambda_W}\right\}$$

which is condition (39). ∎

Proof of Proposition 8

Integration would be beneficial for blacks iff

$$e_B^{\mathrm{SEG}}\lambda_B < \frac{N_B}{N}e_B^{\mathrm{INT}}\lambda_B + \frac{N_W}{N}e_W^{\mathrm{INT}}\lambda_W$$

which is equivalent to:

$$N\, e_B^{\text{SEG}} < N\, e_B^{\text{INT}} + N_W \left(e_W^{\text{INT}} \frac{\lambda_W}{\lambda_B} - e_B^{\text{INT}} \right)$$

Integration would be detrimental for whites iff

$$e_W^{\text{SEG}} \lambda_W > \frac{N_B}{N} e_B^{\text{INT}} \lambda_B + \frac{N_W}{N} e_W^{\text{INT}} \lambda_W$$

$$\Leftrightarrow N\, e_W^{\text{SEG}} \frac{\lambda_W}{\lambda_B} > N e_B^{\text{INT}} + N_W \left(e_W^{\text{INT}} \frac{\lambda_W}{\lambda_B} - e_B^{\text{INT}} \right)$$

Combining these two inequalities leads to

$$N\, e_B^{\text{SEG}} < N\, e_B^{\text{INT}} + N_W \left(e_W^{\text{INT}} \frac{\lambda_W}{\lambda_B} - e_B^{\text{INT}} \right) < N\, e_W^{\text{SEG}} \frac{\lambda_W}{\lambda_B}$$

$$\Leftrightarrow e_B^{\text{SEG}} < e_B^{\text{INT}} + \frac{N_W}{N} \left(e_W^{\text{INT}} \frac{\lambda_W}{\lambda_B} - e_B^{\text{INT}} \right) < e_W^{\text{SEG}} \frac{\lambda_W}{\lambda_B}$$

which is (40). ∎

Figure 2a: The effect of the number of white workers on employment rates

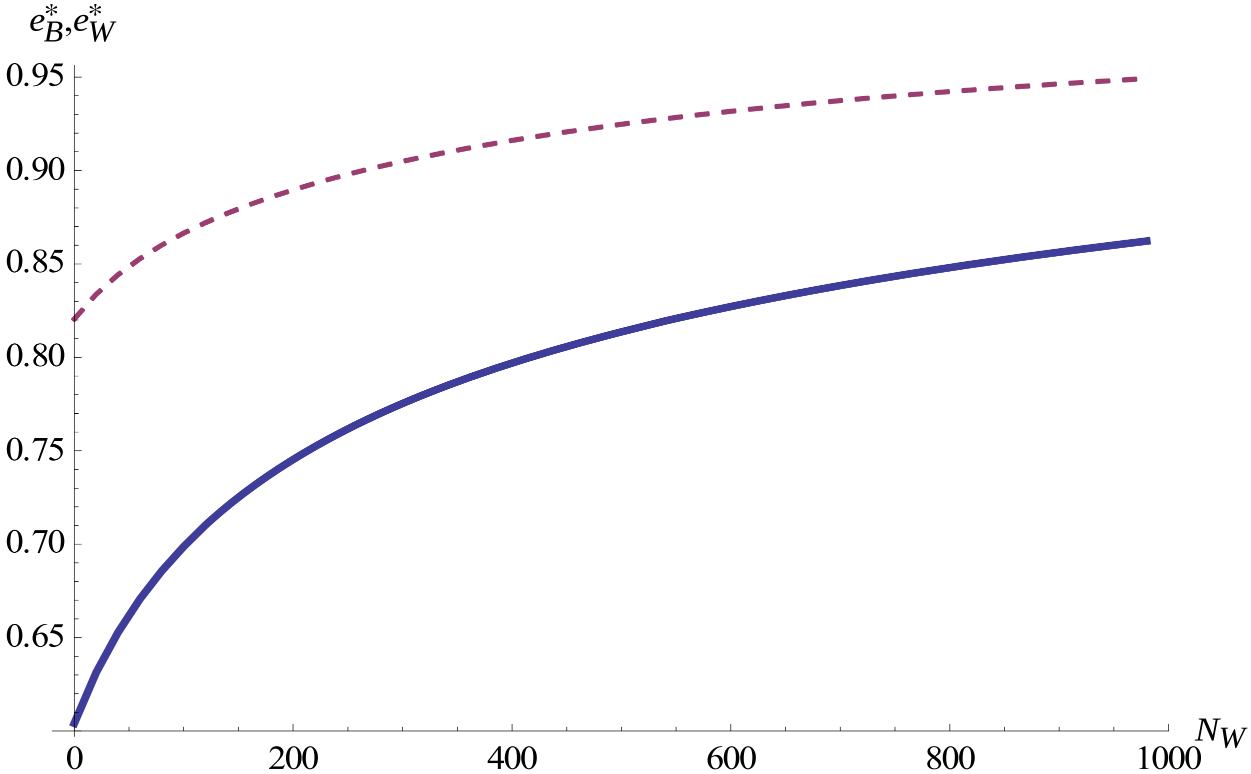

Figure 2b: The effect of the number of white workers on the time spent in a d_0 dyad

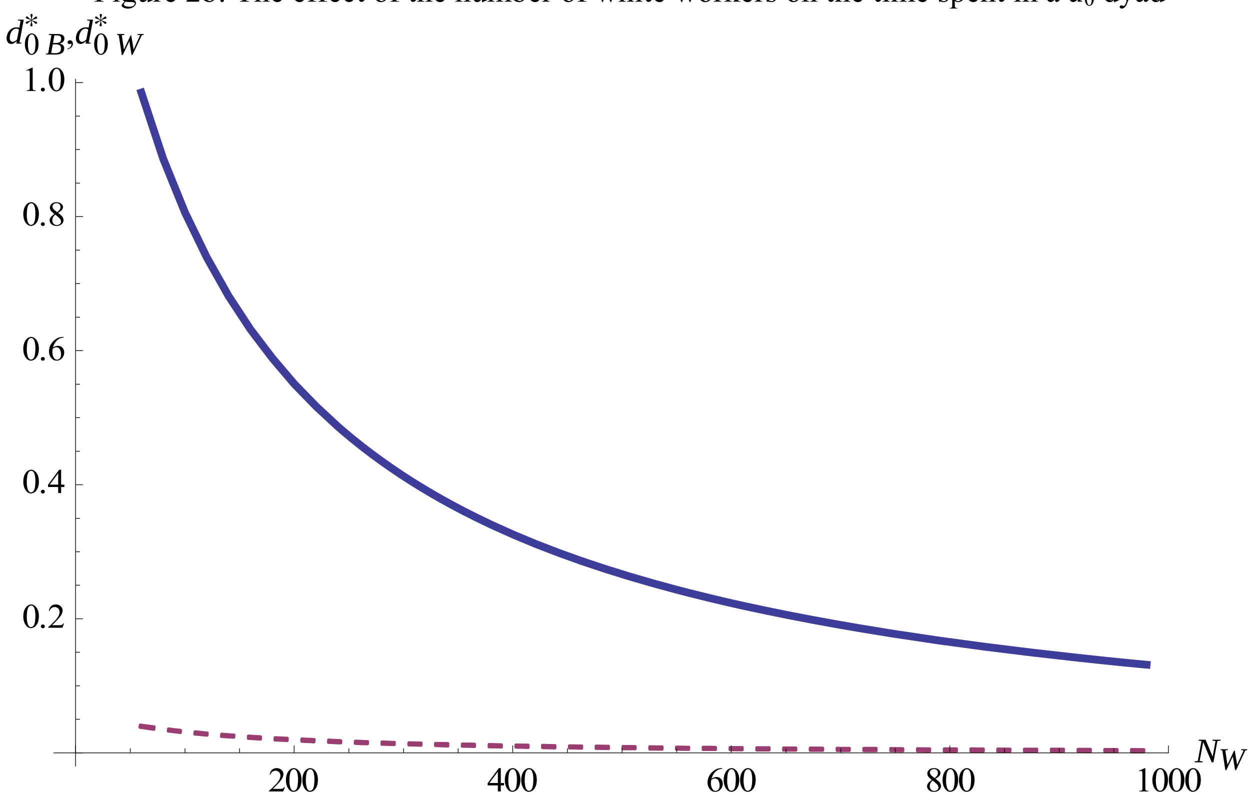

Figure 3a: The effect of weak ties on employment rates

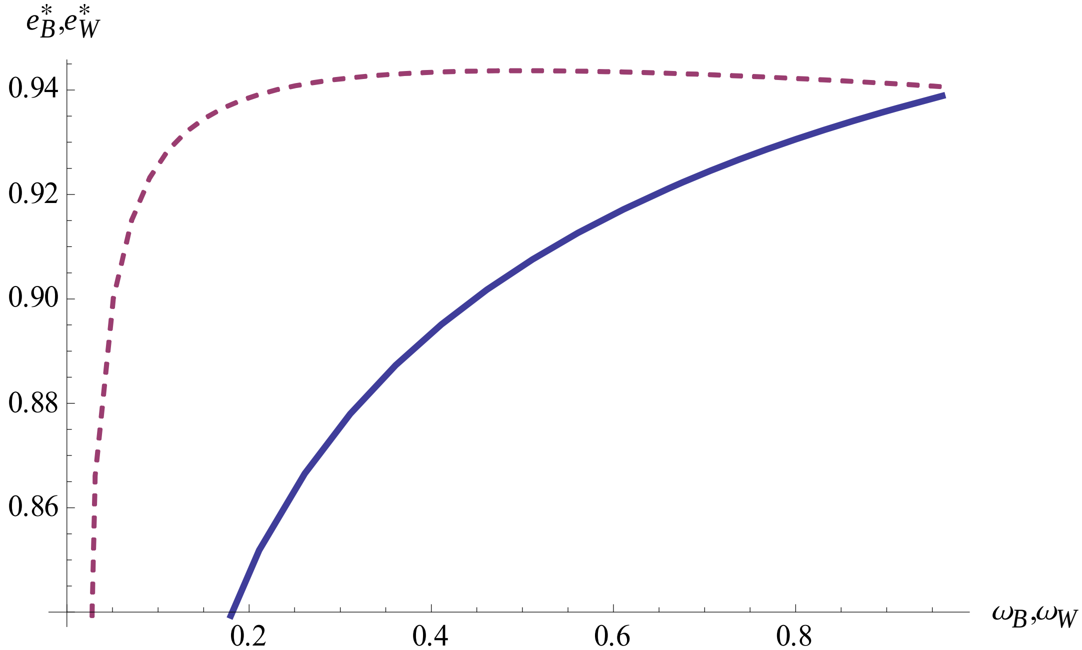

Figure 3b: The effect of weak ties on the time spent in a d_0 dyad

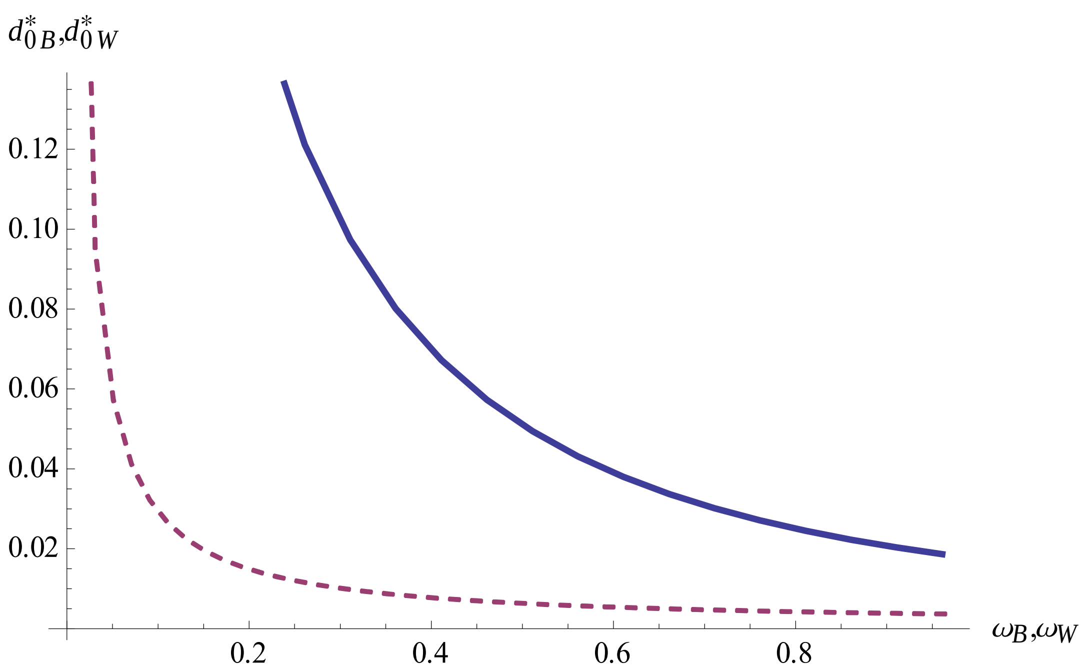

Figure 4a: The effect of the job-destruction rate on employment rates

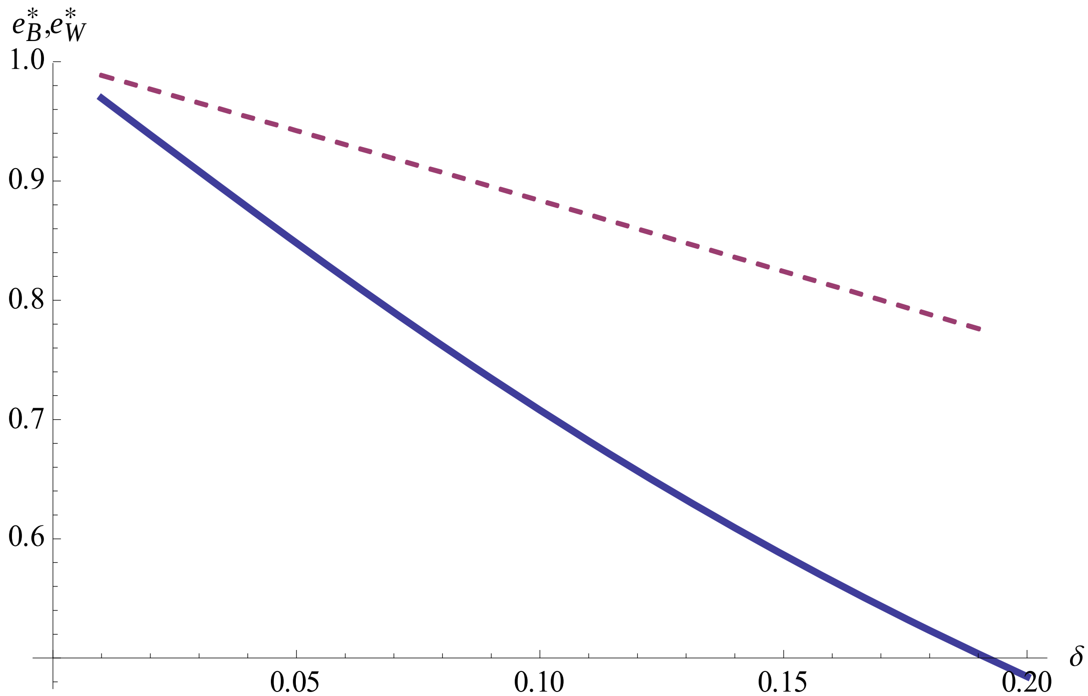

Figure 4b: The effect of the job-destruction rate on the time spent in a d_0 dyad

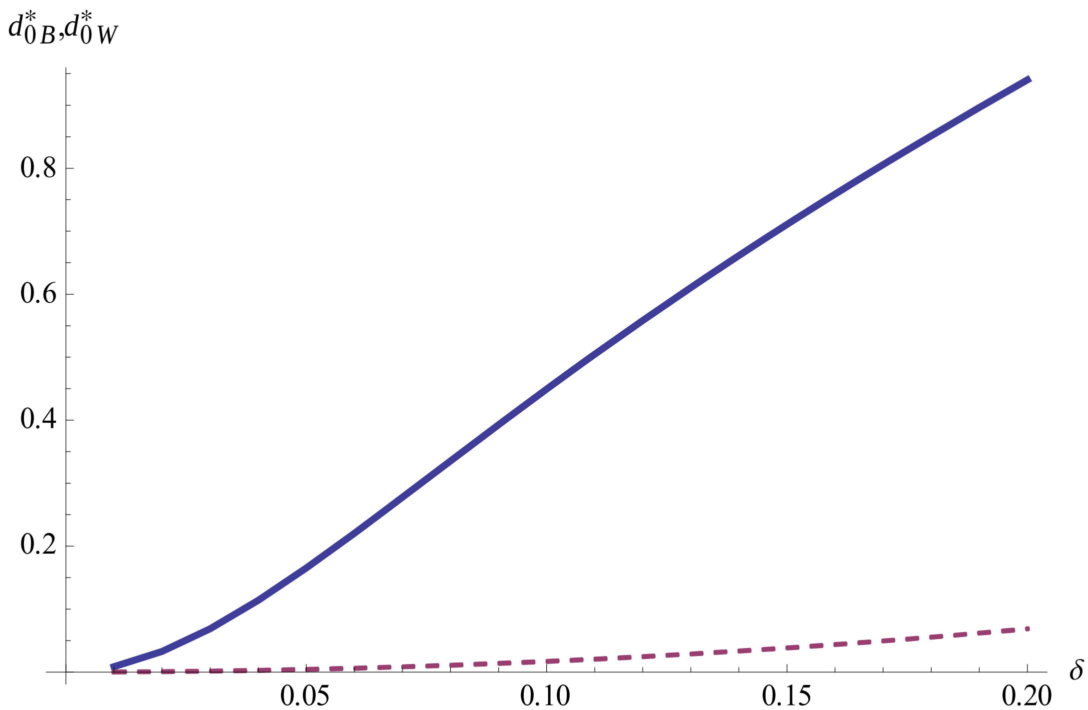

Figure 5a: The effect of the job-information rate on employment rates (λ_B=0.9, λ_W=1)

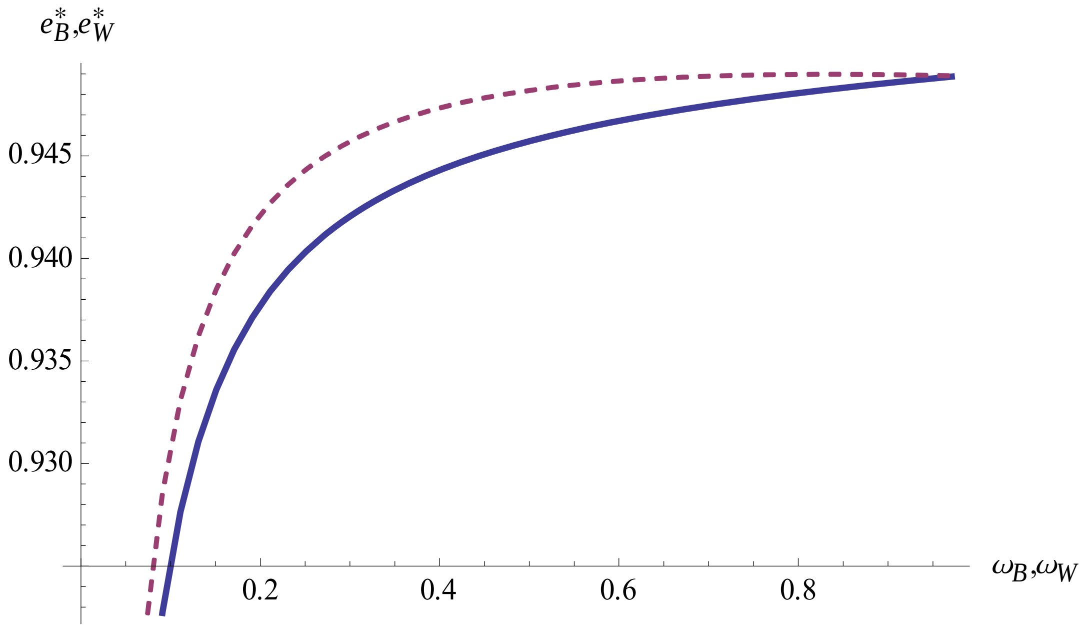

Figure 5b: The effect of the job-information rate on employment rates (λ_B=0.9, λ_W=1)