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FINANCIAL-FRICTION MACROECONOMICS WITH HIGHLY LEVERAGED FINANCIAL INSTITUTIONS

Sheung Kan Luk, Economics Department and Lady Margaret Hall, Oxford University
David Vines, Economics Department, Institute for New Economic Thinking, Oxford Martin School, and Balliol College, Oxford University; Centre for Applied Macroeconomic Analysis, Research School of Economics, Australian National University; and CEPR

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Centre for Economic Policy Research
77 Bastwick Street, London EC1V 3PZ, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

Financial-Friction Macroeconomics with Highly Leveraged Financial Institutions*

This paper adds a highly-leveraged financial sector to the Ramsey model of economic growth and shows that this causes the economy to behave in a highly volatile manner: doing this strongly augments the macroeconomic effects of aggregate productivity shocks. Our model is built on the financial accelerator approach of Bernanke, Gertler and Gilchrist (BGG), in which leveraged goods-producers, subject to idiosyncratic productivity shocks, borrow from a competitive financial sector. In the present paper, by contrast, it is the financial institutions which are leveraged and subject to idiosyncratic productivity shocks. Financial institutions can only obtain their funds by paying an interest rate above the risk-free rate, and this risk premium is anti-cyclical, and so augments the effects of shocks. Our parameterisation, based on US data, is one in which the leverage of the financial sector is two and a half times that of the goods-producers in the BGG model. This causes a much more significant augmentation of aggregate productivity shocks than that which is found in the BGG model.

JEL Classification: E22, E32 and E44
Keywords: financial accelerator, highly leveraged financial institutions, leverage and volatility

Sheung Kan Luk
Department of Economics
University of Oxford
Wellington Square
Oxford OX1 2JD
UK

Email:
sheung.luk@economics.ox.ac.uk

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David Vines
Department of Economics
University of Oxford
Manor Road Building
Manor Road
OXFORD OX1 3UQ

Email:
david.vines@economics.ox.ac.uk

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1 Introduction

1.1 Financial-Friction Macroeconomics

Until three years ago we were living in the 'Great Moderation'. Macroeconomic outcomes were unusually good. And microeconomic policy seemed good too. In particular, light-touch regulation of the financial sector seemed just what was needed.

Then things fell apart.

There are now a number of valuable narrative accounts of the global financial crisis[1], and a clear microeconomic analysis of what happened is beginning to emerge.[2] But we still do not understand enough about the macroeconomic amplification of the crisis. The crisis begain in the sub-prime mortgage market in the US, which is rather small, valued at around $1.3 trillion.[3] and the sub-prime-related losses are estimated to have been the rather small sum of about $300-400 billion (Blundell-Wignall, 2008; Roubini, 2008). By contrast, according to the IMF's Global Financial Stability Report (IMF, 2010), the banking system write-downs totalled about $2.3 trillion world-wide. This suggests an amplification of the shock within the financial system of about 6-8 times. We lack an understanding of why collapse in a small market nearly brought down the entire global financial system. This paper provides one response to that challenge: it shows how leverage can lead to such propagation.

Our analysis of how a financial shock can be amplified builds on the analysis of Bernanke Gertler and Gilchrist (1999) – a paper which referred to from now on as BGG. In that paper the final-goods producers are leveraged, but subject to idiosyncratic productivity shocks, so that loans to them are subject to a risk premium. Ampification happens because this risk premium is anti-cyclical. But in BGG the financial sector obtains its money from depositors without any financial frictions. This assumption makes that model poorly suited to analysis of the present financial crisis.

By contrast, in the present paper the final-goods producers raise finance from financial institutions by issuing shares. Finanical insitutions can diversify away the risk coming from idiosyncratic shocks affecting the final-goods producers which do not cause any risk premium in the model. By contrast financial institutions are leveraged, and – because they are also subject to idiosyncratic productivity shocks – risky. Consumers are unable to diversify away this risk. Financial institutions can therefore only obtain their funds by paying an interest rate to consumers which is above the risk-free rate; we show that this risk premium is anti-cyclical, and so augments the effects of shocks. The financial-accelerator effect in our model operates between depositors and financial institutions,

[1] Tett (2009) and Lewis (2010) offer fascinating pictures of what went wrong in the US financial system; Garnaut and Llewellyn Smith (2009) offer a useful account of the ensuing global events.

[2] See Dewatripont, Rochet, and Tirole (2010) and the Review article of that book by Vines (2011).

[3] "Will subprime mess ripple through economy?" (3/13/2007), Associated Press. Available at http://www.msnbc.msn.com/id/17584725.

rather than, as in BGG, between financial institutions and the final-goods producers. By assumption consumers are unable to invest directly in final-goods producers because it is assumed that they lack the ability to diversify away the risk attached to each final-goods producer.

Our model of a leveraged financial system is based on the analysis of Hirakata, Sudo, and Ueda (2009), a paper henceforth referred to as HSU, and our calibration is based on theirs. That paper supposes that financial institutions are leveraged but also that entrepreneurs are leveraged so that financial institutions lend money to entrepreneurs, rather than assuming like us that the financial instituions buy shares issued by the entrepreneurs. In effect the HSU setup contains two BGG-like contracts, joined together, giving rise to what they call 'chained credit contracts'. Our analysis is simpler than theirs, and also, we think, more realistic. In the run-up to the crisis a large proportion of the financial system consisted of investment banks, which provided credit to final-goods producers by purchasing asset-backed securities, rather than by providing the kind of fixed-interest loans that are provided by commercial banks.

Our setup provides three new insights into what caused the amplification which happened during the financial crisis. First, our setup produces a stronger amplification of shocks than BGG. This is because of the way the model is calibrated, but that choice of calibration is deliberate and reflects reality. In BGG the firms in the leveraged sector of the model, the final-goods producers, have a leverage of 2, a number which the authors based on US data. By contrast in our model, the firms in the leveraged sector of the model, the financial sector, have a leverage of 5, a number which we take from HSU who obtained it from US data.[4] Second, we show that our setup leads to a greater amplification of shocks than HSU. That is because their assumption that financial institutions lend money to entrepreneurs, rather than buying their shares, dampens the amplification in their system in a way which does not happen in our model.[5] Third, we show that the extent of amplification depends on the extent to which goods-producers are externally financed, through the issue of shares, rather than self-financed. This is the case, even although goods-producers sell shares and are not leveraged. It happens simply because when the productive sector becomes less self-financing, that will increase,

[4]The leverage of investment banks before the crisis was approximately 30. Adrian and Shin (2008) examine the balance sheets of what were the five largest US investment banks, namely Bear Stearns, Goldman Sachs, Lehman Brothers, Merrill Lynch and Morgan Stanley, from data provided in the quarterly 10-K and 10-Q filings with the Securities and Exchange Commission. The leverage refers to the ratio of total assets to book equity. The aggregate sample from 1992Q1 to 2008Q1 shows that the mean leverage is 27.45 times, with a standard deviation of 3.85.

The effective leverage before the crisis may have been even higher than this figure suggests, since the balance sheets do not include off-balance sheet items such as asset-backed commercial paper (ABCP) conduits and structured investment vehicles (SIVs), the sizes of which are harder to estimate. (See Acharya and Schnabl, 2009.)

Nevertheless we take the much lower number of 5 to avoid exaggerating our results. This number was obtained by HSU as the leverage for the whole of the the financial system, much of which is much less higly leveragted than the insitutions just described.

[5]We will explain below why this is the case.

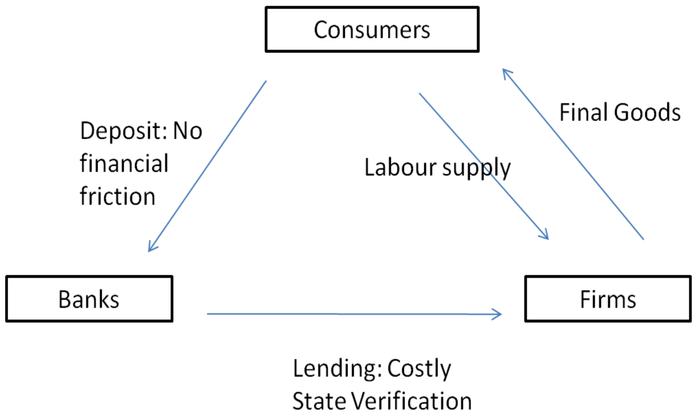

Figure 1: The BGG model

ceteris paribus, the size of the financial system and so cause the financial sector to became more leveraged unless, at the same time, the financial system becomes more self-financing in a way which exactly compensates for the reduced self-financing in the productive sector.

We believe that the crisis turned out to be so bad because all three of these things were at work in the run-up to the crisis.

1.2 A Diagrammatic Representation of Financial Friction Possibilities

Figure 1 shows the economic interactions in the BGG model. In that model, goods in the economy are produced by imperfectly competitive firms which are leveraged: they borrow from a competitive financial sector. The productivity of the goods producing firms is randomly distributed, leading to the possibility of bankruptcy for each individual firm, and so to a risk premium on the borrowings of these firms. The higher the leverage, the greater the probability of bankruptcy and so the larger the risk premium. As BGG show, this risk premium is anti-cyclical, that is to say the premium narrows in response to a positive shock, and thus is disturbance-amplifying, to an extent which depends on the leverage of the borrowing firms.

Figure 2 depicts the setup in the HSU model. In that model, financial institutions are leveraged: they borrow from a competitive set of 'investors', who collect, and invest, the savings of consumers. HSU show that if the financial sector is also risky, and each financial institution faces the possibility of bankruptcy, then the interest rate which consumers require from the financial institutions to which they lend is subject to a risk premium, and that this risk premium is also anti-cyclical, and therefore also disturbance-amplifying, to an extent which depends on the leverage of the financial institutions. But, as we will discuss in the final section of the paper, the contract which financial institutions make

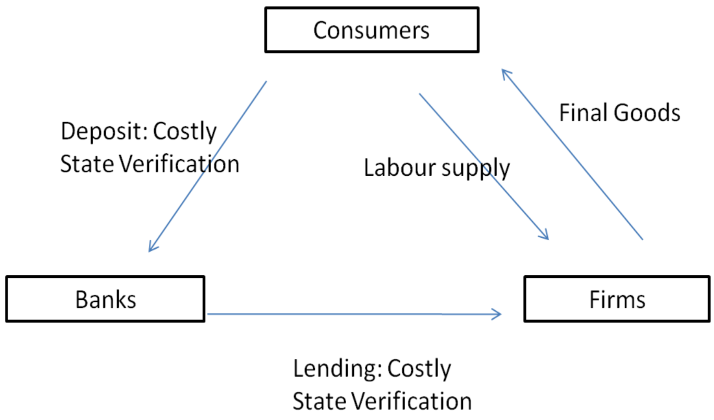

Figure 2: The HSU model

with the goods-producing firms makes the risk premium attached to firm-borrowing from financial institutions pro-cyclical, and so helps to damp disturbances. We will see that this means that in the HSU setup, all of the volatility of the macro-economy, compared with that of a Ramsey model without financial frictions, comes from the leveraged nature of the financial sector, emphasised by HSU, rather than from the leveraged nature of the goods producing firms, emphasised by BGG.

Figure 3 depicts the setup of the model developed in this paper – a model henceforth referred to as the Luk-Vines model, or LV. In this model, as in HSU, financial institutions are leveraged: they borrow from a competitive set of 'investors', who collect, and invest, the savings of consumers. As in HSU, the interest rate which consumers require from the financial institutions to which they lend is subject to a risk premium, and this risk premium is anti-cyclical, and therefore also disturbance-amplifying, to an extent which depends on the leverage of the financial institutions. But the financial contract which financial institutions make with the goods producing firms is a share contract, rather than a loan contract, which means that the contract which financial institutions make with the goods producing firms does not help to damp disturbances, in the way which happens in the HSU model.

1.3 The Financial Accelerator and Macroeconomic Outcomes in this Model

The model which we will present below is built on the Ramsey model with adjustment costs in capital formation. In this model, the financial sector costlessly allocates the savings of consumers for use as investment by final-goods producers. Investment is driven by Tobin's Q. When it is greater than unity, this leads to an increase in investment (and vice versa), by an amount which depends on the size of adjustment costs. For obvious

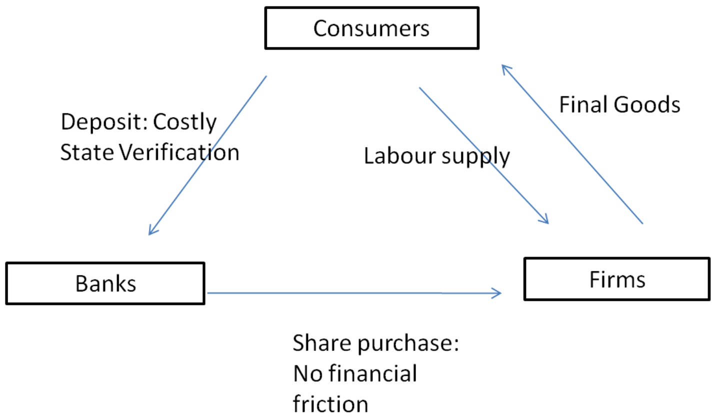

Figure 3: The Luk-Vines model

reasons, we call this the Q-Ramsey model.

When, in such a model, there is a sustained negative shock to the aggregate level of productivity, the long-run level of the capital stock will fall, leading to a reduction in investment. Furthermore, the sustainable level of current consumption will fall, and, in the interests of consumption smoothing, consumers will wish to cut their level of consumption immediately. That is, although aggregate supply is reduced, aggregate demand will fall, and if adjustment costs in capital investment are sufficiently small, the fall in demand will be larger than the fall in supply. This means that the rate of interest will fall. The fall in the interest rate ensures that aggregate demand remains exactly equal to the lower level of aggregate supply. In this frictionless Ramsey model, the interest rate received by consumers is equal to the cost of capital to investors.

In a BGG model with financial friction, a fall in aggregate productivity will mean that a larger proportion of the goods-producing firms are likely to go bankrupt. Financial institutions understand this fact, and so increase the risk premium which they charge to goods-producing firms on the loans which they make to these firms. This risk premium behaves in an anti-cyclical manner, and so augments the effects of shocks. In particular it will magnify the reduction in investment caused by an aggregate productivity shock – this is the 'financial-accelerator' effect of BGG. But the financial system is competitive and can so investors can diversify away the effects of the increased risk. As a result, in that model, consumers continue to lend any amount to financial institutions at a risk-free rate.

In the model presented below, consumers do not directly invest in the goods-producing firms, but instead deposit their money with 'investors'. These investors allocate their money across financial institutions, and offer consumers interest on their deposits. Financial institutions purchase shares in goods-producing firms, can diversify their investments in these firms, and so do not need to earn a risk premium since all risk is diversified away. Financial institutions also suffer from idiosyncratic shocks and so the investments by investors in these institutions are risky. As a result, investors demand a risk premium, over and above the risk-free rate, to compensate them for the probability of default by the financial institutions to which they lend; financial institutions thus need to earn a return on their investments sufficient to cover what these investors demand. In our model, this risk premium behaves in an anti-cyclical manner, and so augments the effects of shocks, just as in the BGG model. As in BGG, it will magnify the reduction in investment caused by the shock, with a 'financial-accelerator' effect.

When, in our model, there is a sustained negative shock to the aggregate level of productivity, the long-run level of the capital stock will fall, leading to a reduction in investment, and, as above, the sustainable level of current consumption will fall, so that, in the interests of consumption smoothing, consumers cut their level of consumption immediately. As a result – if adjustment costs in capital investment are sufficiently small – the risk-free rate of interest will fall. As noted above the risk premium will rise in these circumstances. We shall also see that, in these circumstances, the leverage of financial institutions rises, since the net worth of the financial institutions falls by more than the fall in the value of the capital of the goods-producing firms in which they are invested. As a result, the model produces a positively-sloped 'leverage supply function'; increases in leverage are associated with increases in the risk premium.

This underpinnings of this reduced-form leverage supply function are easy to understand. Financial institutions suffer from a known distribution of idiosyncratic shocks. Given this distribution, when leverage rises, a financial institution has an increases risk of bankruptcy. As a result the interest rate which an investor will demand will rise, even if the investor is risk neutral.

In our model, consumers are assumed not to buy the shares of goods producing firms directly. It may be that individual consumers lack sufficient knowledge of goods-producing firms. In addition consumers may be too small to diversify risk individually, and so need to buy funds managed by financial institutions. Although our model oversimplifies, a significant proportion of financial intermediation is in reality achieved by means of consumers depositing money in financial institutions which purchase shares on their behalf.

1.4 Alternative Formulations of the Leverage Supply Function

In the literature, there are some other approaches to the creation of macroeconomic models with financial market frictions which offer different ways of microfounding the leverage supply function. We now briefly review these opther approaches, none of which seems satisfactory.

First, Kiyotaki and Moore (1997) develop a collateral framework which supposes that borrowers of funds face credit limits which are binding, and that they have to keep their credit under the desired, and optimal, level. The credit limit comes from an assumption that lenders cannot force borrowers to pay for their debts unless the contracts are fully secured by durable assets such as land, housing or capital. But such a costly enforcement framework implies that leverage is fixed at unity. It is therefore not helpful in thinking about highly leveraged financial institutions.

Second, the costly-financial-intermediation approach assumes that there is a representative bank operating in a perfectly competitive environment, which provides loans and deposits to the agents in the economy. The financial intermediation is costly. For example, Gerali et. al. (2009) assume that the bank incurs a quadratic cost when its leverage deviates from an exogenous target level, something which gives rise to a positive premium on the lending rate whenever this happens, and so leads to a leverage supply function. However, the way in which this intermediation cost varies with leverage is not microfounded.

The third approach to this problem analyses the moral hazard problem in the banking sector. Gertler and Karadi (2009) and Gertler and Kiyotaki (2010) microfound the leverage supply function considering the following problem: A bank takes deposits and lends to firms. It has an option to divert a fraction of available funds. If it does, the bank will be bankrupt and depositors suffer a loss. The banker chooses to cheat when the quantity of divertable funds is larger than the present discounted value of the bank if it continues to exist. In equilibrium, depositors limit their deposits so that cheating never occurs. This setup means that , for any given bank net worth, a higher lending rate relative to the cost of funds increases the banks continuing value, and so means that depositors are willing to lend more, leading to an upward-sloping leverage supply function. It is unclear, however, whether the need to defend asset-holdings against theft, upon which this setup depends, is important in the way in which it supposes.

By contrast, the approach which we will use is the costly-state-verification approach due to BGG, but adopted for an analysis of financial institutions. Our aim is to show that this approach enables us to properly microfound the leverage supply function, and so to provide a persuasive approach to financial-frictions macroeconomics in a world with highly-leveraged financial institutions.

1.5 Plan of this Paper

Section 2 sets out our model of the financial system. We outline the nature of the credit contracts in the economy. We set out the nature of the loan contract between investors and financial institutions, and the nature of the share contract between financial institutions and final-goods producers . These analyses are *micro*economic analyses, in that they take the price of shares, and the risk-free rate of interest, as given.

Section 3 presents the *macro*economic model in which the financial system is embedded. It is a basic Ramsey model with adjustment costs attached to capital investment, in which there are financial frictions between investors and financial institutions, and in which financial institutions purchase shares of goods-producing firms rather than lending to them. These features give rise to financial accelerator properties in the full macro system.

Section 4 describes the calibration of the model and then discusses simulation results. We simulate the effects of a permanent negative productivity shock in the goods producing sector, and compare the results provided by our model with those provided by the BGG and HSU models. This shock causes investment to fall and the risk free interest rate to fall in all models. However, in our model, the risk premium also rises a great deal. This means that investment falls by more in our model and that there will be a larger fall in the risk free interest rate, and so a smaller fall of consumption in order that resources remain fully utilised. At the end of this section we also show that the larger the proportion of investment in goods producing firms that is financed externally, the larger the amplification of the shock.

Section 5 presents brief conclusions.

There are three Appendices. In the first of these we write down our model in log-linear form. The second and third appendices describe the HSU model and the BGG model.

2 Credit Contracts, and the Evolution of the Net Worth of Financial Institutions and Entrepreneurs

This section discusses the credit contracts between investors, financial institutions and entrepreneurs in a partial equilibrium framework. The main purpose of the section is to discuss the effects of financial frictions in the financial intermediation process. We take as given the net worths, the price of shares, the return on capital and the risk free interest rate. Each firm will maximise profits by choosing its optimal stock of capital, but for the economy as a whole, these choices will need to be consistent with the stock of capital in existence. In the subsequent section we endogenise these variables within a full general equilibrium macroeconomic system.

Three types of agents, namely investors, financial institutions and entrepreneurs (i.e the producers of goods), participate in the financial market. The financial institutions and entrepreneurs earn positive profits, accumulating net worth. Denote $N_{i,t}^F$ the net worth of the financial institution indexed i, where $i \in \{1, 2, ..., \infty\}$. The aggregate net worth of the financial institutions is denoted as

$$N_t^F \equiv \sum_i N_{i,t}^F$$

Each financial institution i buys the shares of entrepreneurs indexed $j_i \in \{1, 2, ..., \infty\}$. The net worth of the entrepreneur indexed j_i is $N_{j_i,t}^E$. Denote $N_{i,t}^E$ as the sum of the net worths of the entrepreneurs whose shares are bought by financial institution i.

$$N_{i,t}^E \equiv \sum_{j_i} N_{j_i,t}^E, \qquad \text{for each } i.$$

The aggregate net worth of the entrepreneurs is denoted as

$$N_t^E \equiv \sum_i N_{i,t}^E$$

The net worths of the financial institutions alone are not enough to finance the entrepreneurs' purchase of capital, so the financial institutions take loans from investors. The investors are risk neutral agents who collect the deposits from consumers, paying them the risk-free rate R_{t+1}, and play no interesting part in the model.

In any period of time, entrepreneurs purchase capital for use in the following period, after which the depreciated capital is sold. This means that at time t, entrepreneur j_i purchases capital for use at $t+1$. The quantity of capital purchased is denoted $K_{j_i,t+1}$, with the subscript denoting the period in which the capital is actually used, and the superscript j_i denoting the entrepreneur. The price paid per unit of capital in period t is Q_t. Denote $K_{i,t+1}$ The sum of capital purchased by the entrepreneurs whose shares are bought by financial institution i. Then

$$K_{i,t+1} \equiv \sum_{j_i} K_{j_i,t+1}, \qquad \text{for each } i.$$

Denote the aggregate capital stock in period $t+1$ as K_{t+1}, then

$$K_{t+1} \equiv \sum_i K_{i,t+1}$$

Both the entrepreneurs and financial institutions face idiosyncratic shocks to their return to capital. In each period, each entrepreneur draws from a distribution of ω^E, where ω^E follows a log-normal distribution with mean one and variance $(\sigma^E)^2$. The realised gross return on capital from entrepreneur i is $\omega_{j_i}^E R_{t+1}^E$, where $\omega_{j_i}^E$ is an idiosyncratic disturbance to entrepreneur j_i's return and R_{t+1}^E is the realised aggregate return to capital of the entrepreneurs. Similarly, in each period, each financial institution draws from a distribution of ω^F, where ω^F follows a log-normal distribution with mean one and variance $(\sigma^F)^2$. The realised gross return on capital from financial institution i is $\omega_i^F R_{t+1}^F$, where ω_i^F is an idiosyncratic disturbance to financial institution i's return and R_{t+1}^F is the realised aggregate return to capital of the financial institutions. We further assume that ω^E and ω^F are independent across time and across entrepreneurs and financial institutions. Lenders can diversify away the idiosyncratic risks of the borowers by investing in a lot of borrowers.

2.1 The Loan Contract for Each Financial Institution

We first describe the contract between the consumers and the financial institutions, which is a loan contract.

We follow the financial frictions approach pioneered by BGG, which assumes costly state verification as a way of motivating financial frictions. The return of each financial institution is subject to an idiosyncratic shock ω^F of the kind described above. If the investors want to observe the shock for a specific financial institution, they need to pay a monitoring cost, which is assumed to be a fixed fraction μ^F of the financial institution's total capital. Lenders will only need to pay such a cost if a borrower becomes bankrupt; the existence of such a cost is what drives the risk premium on lending to financial insitutions.

What follows is a more detailed derivation of a BGG-like contact than can be found elsewhere. We show how such a contract can be represented diagrammatically. Such a diagram helps to make clear the connection between the microfounded financial system and the overall workings of the macroeconomy.

Since each financial institution receives a different realisation of the random shock every period, the financial institutions will have different net worths. We now investigate the optimal contracting problem facing the financial institution indexed i with a net worth of $N^F_{i,t}$.

The solution to this costly state verification problem is the standard debt contract due to Townsend (1979). Here is why. Due to the monitoring cost, it would be too costly for the investors to monitor every contract. Then there is a cutoff value of the shock $\bar{\omega}^F_i$ for each financial institution i above which the financial institution will make positive profits and below which it is bankrupt. Above the cutoff value, the financial institution i gets a return sufficient to pay the investors a fixed rate of interest Z^F_i and keep what is left for itself; the investors have no incentive to monitor the financial institution's true return. Below the cutoff value, the financial institution is bankrupt, and the investor pays the monitoring cost and take the financial institution's entire wealth. (Note that, in some sense, the wealth of the financial institution is acting as an (incomplete) collateral for the loan.)

In our model, each financial institution purchases shares from the entrepreneurs. These entrepreneurs wish to hold a capital stock of $K_{i,t+1}$, for reasons discussed in the macroeconomic part of the model. They finance $N^E_{i,t+1}$ of such holdings of capital from their own net worth. If Q_t is the price of shares in capital then such entrepreneurs will need to sell $(Q_t K_{i,t+1} - N^E_{i,t+1})$ shares.

Consider financial institution i which purchases $(Q_t K_{i,t+1} - N^E_{i,t+1})$ of these shares. Let the net worth of this financial institution be $N^F_{i,t+1}$. We assume that financial intermediation is costless. Then this financial institution will need to borrow $(Q_t K_{i,t+1} - N^E_{i,t+1} -$

$N^F_{i,t+1}$) from investors.

Now suppose that financial institution i is one whose idiosyncratic shock is exactly $\bar{\omega}^F_i$. The returns to this financial institution will be exactly equal to the contractual rate of interest $Z^F_{i,t+1}$, in the contract between the financial institution and its investors, times what has been borrowed from its investors $(Q_tK_{i,t+1}-N^E_{i,t+1}-N^F_{i,t+1})$. It is useful for expositional reasons to describe $(Q_tK_{i,t+1}-N^E_{i,t+1})$ as the expenditure on shares by financial institution i and to describe $(Q_tK_{i,t+1}-N^E_{i,t+1}-N^F_{i,t+1})$ as this institution's loans from investors. Since the returns to this financial institution are exactly $\bar{\omega}^F_{i,t+1}R^F_{t+1}(Q_tK_{i,t+1}-N^E_{i,t+1})$, this means that

$$\bar{\omega}^F_{i,t+1}R^F_{t+1}(Q_tK_{i,t+1}-N^E_{i,t+1}) = Z^F_{i,t+1}(Q_tK_{i,t+1}-N^E_{i,t+1}-N^F_{i,t+1}). \tag{1}$$

It is clear from this equation that there is a one-to-one mapping between $Z^F_{i,t+1}$, and $\bar{\omega}^F_{i,t+1}$; a higher contractual rate of interest will cause more firms to go bankrupt and so cause $\bar{\omega}^F_{i,t+1}$ to rise.

To be perfectly insured against the idiosyncratic shocks of the financial institutions, the investors sign contracts with an infinite number of financial institutions. The solution of the loan contract defines how the investment revenue is split between the lenders (investors) and the borrowers (financial institutions) under all possible realisations of future state-spaces. Suppose the idiosyncratic shock has a cumulative distribution function $F^F(\omega^F)$. Let $\Gamma^F(\bar{\omega}^F_i)$ be the share that goes to the lender , and $G^F(\bar{\omega}^F_i)$ is defined as

$$G^F(\bar{\omega}^F_i) = \int_0^{\bar{\omega}^F_i} \omega^F dF^F(\omega^F)$$

then[6]

$$\begin{aligned}
\Gamma^F(\bar{\omega}^F_i)R^F(QK_i-N^E_i) &= \int_0^{\bar{\omega}^F_i} \omega^F dF^F(\omega^F) \times R^F(QK_i-N^E_i) \\
&\quad +[1-F^F(\bar{\omega}^F_i)]Z^F_i(QK_i-N^E_i-N^F_i) \\
&= G^F(\bar{\omega}^F_i)R^F(QK_i-N^E_i)+[1-F^F(\bar{\omega}^F_i)]\bar{\omega}^F_iR^F(QK_i-N^E_i) \\
\therefore \qquad \Gamma^F(\bar{\omega}^F_i) &= G^F(\bar{\omega}^F_i)+[1-F^F(\bar{\omega}^F_i)]\bar{\omega}^F_i. \qquad (2)
\end{aligned}$$

The first term on the right hand side of the last equation accounts for what the investors get in expectation (before monitoring) from financial institution i if it defaults, and the second term refers to the fixed interest payment the investors get (expressed in the share of profits) from the financial institution if it does not default, with probability $1-F^F(\bar{\omega}^F_i)$.

[6]Given the log-normal assumption, it can be shown that

$$F^F(\bar{\omega}^F) = \Phi\left(\frac{\ln\bar{\omega}^F + \frac{1}{2}(\sigma^F)^2}{\sigma^F}\right), \text{ and } \qquad G^F(\bar{\omega}^F) = \Phi\left(\frac{\ln\bar{\omega}^F - \frac{1}{2}(\sigma^F)^2}{\sigma^F}\right),$$

where Φ is the cumulative distribution function of the standard normal distribution. Γ^F and Ψ^F are determined using (2) and (3).

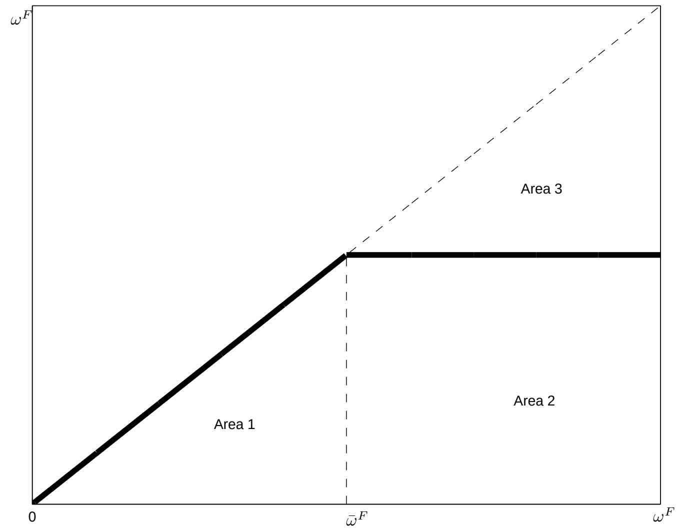

Figure 4: The Γ^F function. Area 1 $= G^F(\bar{\omega}^F) = \int_0^{\bar{\omega}^F} \omega^F dF^F$. Area 2 $= (1 - F^F(\bar{\omega}^F))\bar{\omega}^F$. Area 3 = 1- Area 1 - Area 2. Here, the share of revenue retained by the investors $= \Gamma^F(\bar{\omega^F})$ = Area under solid line = Area 1 + Area 2. The share of revenue retained by the financial institutions = Area 3. It is easy to see that $\Gamma^F(\bar{\omega}^F)$ is increasing in the cutoff value $\bar{\omega}^F$.

Figure 4 provides a graphical representation: the $\Gamma^F(\bar{\omega}_i^F)$ function is the area below the solid line integrated by the density function. One can easily show that $\Gamma_\omega^F(\bar{\omega}_i^F) = 1 - F^F(\bar{\omega}_i^F) > 0$, that is the higher the cutoff value, the larger is the share that goes to the lender. The borrower gets the share $(1 - \Gamma^F(\bar{\omega}_i^F))$. After taking into account the monitoring cost, the net share that goes to the lender is

$$\Psi^F(\bar{\omega}_i^F) = \Gamma^F(\bar{\omega}_i^F) - \mu^F G^F(\bar{\omega}_i^F), \tag{3}$$

where the last term on the right hand side accounts for the resources wasted in monitoring. The sign of Ψ_ω^F is ambiguous. However, in the optimal contract, it remains true that the optimal $\bar{\omega}_i^F$ will be such that $\Psi_\omega^F(\bar{\omega}_i^F) > 0$.[7]

[7]The proof is the following: for the log-normal distribution, $\bar{\omega}^F h^F(\bar{\omega}^F)$ is increasing in $\bar{\omega}^F$, where $h^F(\bar{\omega}^F) = f^F(\bar{\omega}^F)/(1 - F^F(\bar{\omega}^F))$ is the hazard rate. Therefore,

$$\begin{aligned} G_\omega^F(\bar{\omega}^F) &= \bar{\omega}^F f^F(\bar{\omega}^F) = \bar{\omega}^F h^F(\bar{\omega}^F)(1 - F^F(\bar{\omega}^F)) > 0 \\ \Gamma_\omega^F(\bar{\omega}^F) &= G_\omega^F(\bar{\omega}^F) + (1 - F^F(\bar{\omega}^F)) - \bar{\omega}^F f^F(\bar{\omega}^F) = 1 - F^F(\bar{\omega}^F) > 0 \end{aligned}$$

There exists an $\bar{\omega}^{F*}$ such that $\Psi_\omega^F(\bar{\omega}^F) = \Gamma_\omega^F(\bar{\omega}^F) - \mu^F G_\omega^F(\bar{\omega}^F) = (1 - F^F(\bar{\omega}^F))(1 - \mu^F \bar{\omega}^F h^F(\bar{\omega}^F)) > 0$ for $\bar{\omega}^F < \bar{\omega}^{F*}$ and for $\bar{\omega}^F > \bar{\omega}^{F*}, \Psi_\omega^F(\bar{\omega}^F) < 0$. For any $\bar{\omega}_1^F$ such that $\bar{\omega}_1^F > \bar{\omega}^{F*}$, there exist a $\bar{\omega}_2^F$ such that $\bar{\omega}_2^F < \bar{\omega}^{F*} < \bar{\omega}_1^F$ and $\Psi^F(\bar{\omega}_2^F) = \Psi^F(\bar{\omega}_1^F)$. Since the smaller $\bar{\omega}_2^F$ implies a smaller bankruptcy rate for the borrower than $\bar{\omega}_1^F$ while keeping the lender's share of profit unchanged, the borrower will never choose any $\bar{\omega}_1^F > \bar{\omega}^{F*}$. Hence, $\bar{\omega}^F$ has an interior solution and in the optimal contract $\Psi_\omega^F(\bar{\omega}^F) > 0$.

We assume that the investors are perfectly competitive, making zero profit. As the idiosyncratic risk is totally diversifiable for the investors, the required return to the investors is the risk-free rate R_{t+1}. The participation constraint for these investors requires that these investors' expected earnings on their loans $E_t[[\Gamma^F(\bar{\omega}^F_{i,t+1})-\mu^F G^F(\bar{\omega}^F_{i,t+1})]R^F_{t+1}](Q_t K_{i,t+1}-N^E_{i,t+1})$ are exactly equal to the risk free rate, R_{t+1}, (which these investors pay to consumers) times what they lend to this financial institution which is equal to $(Q_t K_{i,t+1} - N^E_{i,t+1} - N^F_{i,t+1})$. Therefore we can write the 'expected participation constraint' as

$$E_t[[\Gamma^F(\bar{\omega}^F_{i,t+1}) - \mu^F G^F(\bar{\omega}^F_{i,t+1})]R^F_{t+1}](Q_t K_{i,t+1} - N^E_{i,t+1}) = R_{t+1}(Q_t K_{i,t+1} - N^E_{i,t+1} - N^F_{i,t+1}), \tag{4}$$

The variables Q, R are determined macroeconomically and are given to each financial institution, the net worths of the financial institution and of entrepreneurs are predetermined, and the expected value of the return on share-holdings, R^F, is also given to the individual financial institution.[8] The demand for shares will be chosen by each financial institution in the light of this.[9]

Since financial institutions are risk-neutral, financial institution i will maximise its expected profit, $E_t[(1-\Gamma^F(\bar{\omega}^F_{i,t+1}))R^F_{t+1}(Q_t K_{i,t+1} - N^E_{i,t+1})]$, by choosing a desired expenditure on shares, $Q_t K_{i,t+1} - N^E_{i,t+1}$, in period t, and choosing the value of $E_t\bar{\omega}^F_{t+1}$ in period $t+1$, given $E_t R^F_{t+1}$, and subject to the participation constraint in expectation.

The Lagrangian for this problem is

$$\begin{aligned} L &= E_t\left[(1-\Gamma^F(\bar{\omega}^F_{i,t+1}))R^F_{t+1}(Q_t K_{i,t+1} - N^E_{i,t+1})\right. \\ &\quad \left. +\lambda_{i,t+1}\left[(\Gamma^F(\bar{\omega}^F_{i,t+1}) - \mu^F G^F(\bar{\omega}^F_{i,t+1}))R^F_{t+1}(Q_t K_{i,t+1} - N^E_{i,t+1}) - R_{t+1}(Q_t K_{i,t+1} - N^E_{i,t+1} - N^F_{i,t+1})\right]\right] \end{aligned}$$

[8] We will discuss in detail how it is determined macroeconomically in the next section.

[9] As will be discussed in the next section, macroeconomically these demands must add up to the outstanding stock of shares issued by the entrepreneurs.

and the first order conditions are[10]

$$\bar{\omega}^F_{i,t+1} \quad : \qquad \lambda(\bar{\omega}^F_{i,t+1}) = \frac{\Gamma^F_\omega(\bar{\omega}^F_{i,t+1})}{\Gamma^F_\omega(\bar{\omega}^F_{i,t+1}) - \mu^F G^F_\omega(\bar{\omega}^F_{i,t+1})}, \qquad \lambda_\omega(\bar{\omega}^F_{i,t+1}) > 0 \tag{5}$$

$$Q_t K_{i,t+1} - N^E_{i,t+1} \quad : \qquad \frac{E_t(R^F_{t+1})}{R_{t+1}} = E_t\left(\frac{\lambda(\bar{\omega}^F_{i,t+1})}{\lambda(\bar{\omega}^F_{i,t+1})(\Gamma^F(\bar{\omega}^F_{i,t+1}) - \mu^F G^F(\bar{\omega}^F_{i,t+1})) + (1 - \Gamma^F(\bar{\omega}^F_{i,t+1}))}\right) \tag{6}$$

These first order conditions are similar to the contract in BGG and are easy to understand intutitively. The Lagrange multiplier to the participation constraint $\lambda_{i,t+1}$, derived in Equation (5), measures the shadow value of an additional unit of net worth (internal funds) to the financial institution. Equation (6) shows that $\lambda_{i,t+1}$ will be larger than unity precisely because $E_t R^F_{t+1}$ is bigger than R_{t+1}, by an amount which depends on the size of the monitoring cost parameter μ^F. And Equation (5) shows that the size of $\lambda_{i,t+1}$ determines the size of the cutoff value of the shock $\bar{\omega}^F_{i,t+1}$. We call $E_t(R^F_{t+1})/R_{t+1}$ the external finance premium.

The two first order conditions solve for an optimal expected cutoff value $E_t\bar{\omega}^F_{i,t+1}$. We use Equations (5) and (6) to eliminate the Lagrange multiplier $\lambda_{i,t+1}$ and get the 'combined FOC':

$$E_t\left(\frac{\frac{\Gamma^F_\omega(\bar{\omega}^F_{i,t+1})}{\Gamma^F_\omega(\bar{\omega}^F_{i,t+1}) - \mu^F G^F_\omega(\bar{\omega}^F_{i,t+1})}}{\frac{\Gamma^F_\omega(\bar{\omega}^F_{i,t+1})}{\Gamma^F_\omega(\bar{\omega}^F_{i,t+1}) - \mu^F G^F_\omega(\bar{\omega}^F_{i,t+1})}(\Gamma^F(\bar{\omega}^F_{i,t+1}) - \mu^F G^F(\bar{\omega}^F_{i,t+1})) + (1 - \Gamma^F(\bar{\omega}^F_{i,t+1}))}\right) = \frac{E_t(R^F_{t+1})}{R_{t+1}} \tag{7}$$

This combined FOC equates the slope of the constant expected profits line and the slope of the participation constraint. This is shown diagrammatically in Figure 5 in $(Q_t K_{i,t+1} - N^E_{i,t+1}, E_t\bar{\omega}^F_{i,t+1})$ space.[11] Suppose the financial institution i chooses a point on the left of the tangency point on the expected participation constraint. This point is

[10]To derive the first derivative of λ with respect to $\bar{\omega}^F$, note that

$$\begin{aligned} G^F_{\omega\omega}(\bar{\omega}^F) &= \frac{d}{d\bar{\omega}^F}[\bar{\omega}^F h(\bar{\omega}^F)(1 - F^F(\bar{\omega}^F))] = (1 - F^F(\bar{\omega}^F))\frac{d(\bar{\omega}^F h^F(\bar{\omega}^F))}{d\bar{\omega}^F} - \bar{\omega}^F h^F(\bar{\omega}^F) f^F(\bar{\omega}^F), \\ \Gamma^F_{\omega\omega}(\bar{\omega}^F) &= -f^F(\bar{\omega}^F). \end{aligned}$$

Therefore,

$$\begin{aligned} & \Gamma^F_\omega(\bar{\omega}^F) G^F_{\omega\omega}(\bar{\omega}^F) - \Gamma^F_{\omega\omega}(\bar{\omega}^F) G^F_\omega(\bar{\omega}^F) \\ = \; & (1 - F^F(\bar{\omega}^F))^2 \frac{d(\bar{\omega}^F h^F(\bar{\omega}^F))}{d\bar{\omega}^F} - \bar{\omega}^F h^F(\bar{\omega}^F) f^F(\bar{\omega}^F)(1 - F^F(\bar{\omega}^F)) - (-f^F(\bar{\omega}^F))\bar{\omega}^F h^F(\bar{\omega}^F)(1 - F^F(\bar{\omega}^F)) \\ = \; & (1 - F^F(\bar{\omega}^F))^2 \frac{d(\bar{\omega}^F h^F(\bar{\omega}^F))}{d\bar{\omega}^F} > 0 \end{aligned}$$

And $\lambda_\omega(\bar{\omega}^F) = \frac{\mu^F}{(\Gamma^F_\omega(\bar{\omega}^F) - \mu^F G^F_\omega(\bar{\omega}^F))^2} \times [\Gamma^F_\omega(\bar{\omega}^F) G^F_{\omega\omega}(\bar{\omega}^F) - \Gamma^F_{\omega\omega}(\bar{\omega}^F) G^F_\omega(\bar{\omega}^F)] > 0$.

[11]The constant expected profits line is concave because the a rise in the cutoff value increases the Γ function in a decreasing rate. Therefore, $(1 - \Gamma)$ is increasing more than linearly, which means that the desired expenditure on shares has to rise in an increasing rate as the expected cutoff value rises linearly. The reasoning for concavity of the participation constraint is similar.

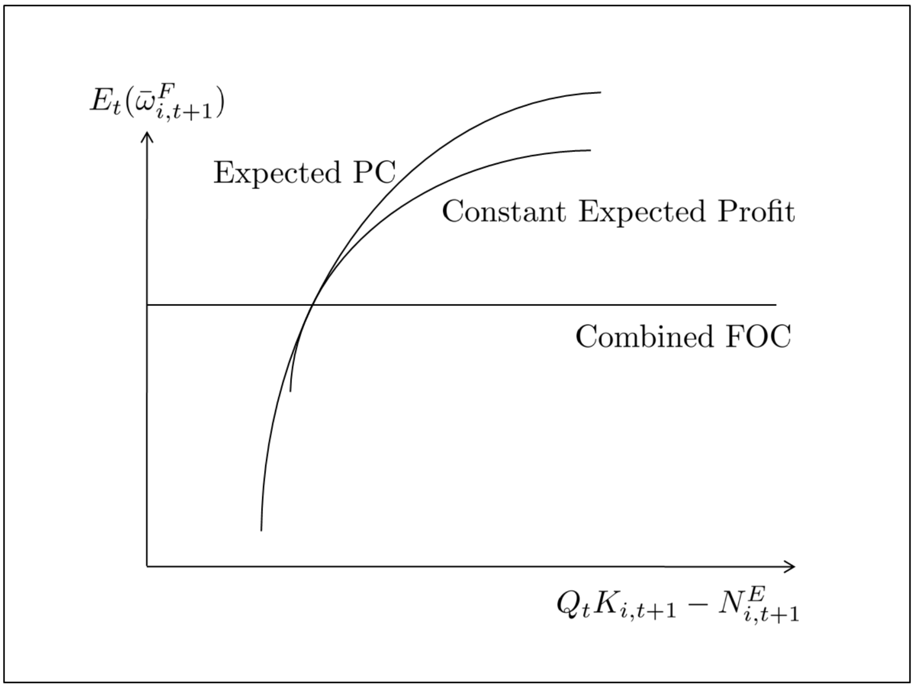

Figure 5: The constant expected profits line, the expected participation constraint and the combined first order condition of financial institution i's profit maximisation problem in the loan contract.

not optimal because by demanding an additional unit of share purchase, and incurring a higher expected cutoff value, i.e. by paying a higher non-default contractual interest rate $Z^F_{i,t+1}$ according to Equation (1), the financial institution can get higher expected profits. This is because, an additional unit of share purchase requires a rise in the expected cutoff by $\frac{1}{QK-N^E}\left(\frac{R/R^F-\Psi}{\Psi'}\right)$ unit; on the othe hand, an additional unit of share purchase will increase the expected profits unless the expected cutoff value rises by $\frac{1}{QK-N^E}\left(\frac{1-\Gamma}{\Gamma'}\right)$ unit. Below the optimal solution, the expected returns on shares $E_t R^F_{t+1}$ relative to the risk free rate R_{t+1} is more than enough to cover the costs associated with the marginal increase in monitoring costs paid by the investors.

This combined FOC is scale independent, which comes from the fact that monitoring costs are a constant fraction μ^F of the financial institution's desired expenditure on shares. As a result, both the marginal rate of substitution in the constant expected profits line and the slope of the participation constraint are linear in the desired expenditure on shares. A rise in the desired expenditure on shares increase the slopes in an offseting way, so the expected cutoff value is independent of the desired expenditure on shares. This 'combined FOC' is represented in the bottom-right panel of Figure 6 as a horizontal line in $(K_i, E(\bar{\omega}^F_i))$ space.

Taken together, the participation constraint and the combined FOC fully specify the

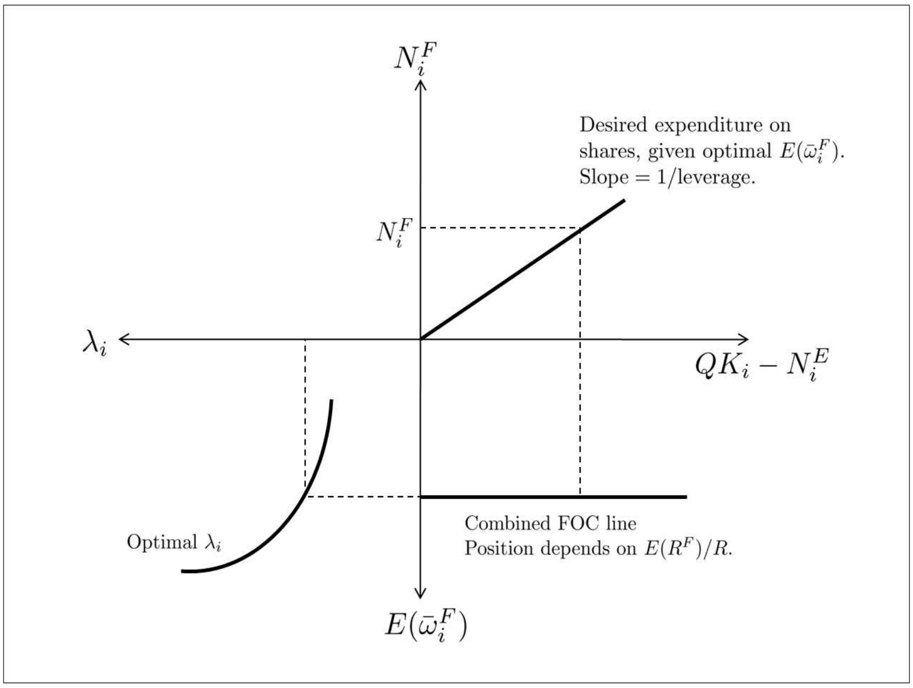

Figure 6: A graphical solution to individual financial institution's profit maximisation problem. Equations (5), (7) and (8) given the optimal expected cutoff value are sketched in the bottom-left, bottom-right and top-right hand corners respectively.

credit contract: they fully determine the purchase of shares by a financial institution and the cutoff value. It is helpful to show that the outcome is one in which, for a given external finance premium, the leverage is constant and a financial institution's desired expenditure on shares depends on its net worth.

We use the expected participation constraint and the combined FOC, Equation (7), to solve for the desired expenditure on shares by financial institution i:

$$Q_t K_{i,t+1} - N^E_{i,t+1} = \left[1 + E_t\left(\frac{\Gamma^F_\omega(\bar{\omega}^F_{i,t+1})}{1-\Gamma^F(\bar{\omega}^F_{i,t+1})} \times \frac{\Gamma^F(\bar{\omega}^F_{i,t+1}) - \mu^F G^F(\bar{\omega}^F_{i,t+1})}{\Gamma^F_\omega(\bar{\omega}^F_{i,t+1}) - \mu^F G^F_\omega(\bar{\omega}^F_{i,t+1})}\right)\right] N^F_{i,t+1} \quad (8)$$

Given the external finance premium, Equation (7) solves for the optimal expected cutoff value $E_t\bar{\omega}^F_{i,t+1}$, and Equation (8) for the desired expenditure on shares in terms of the expected cutoff value $E_t\bar{\omega}^F_{i,t+1}$. The meaning of this is as follows. According to the participation constraint, if the financial institution is to expand the amount of shares which it purchases, it must pay a higher contractual interest rate to its investors, and so must incur a higher value of $\bar{\omega}^F_{i,t+1}$. This means that, given the optimal $\bar{\omega}^F_{i,t+1}$, chosen in the way described above, the optimal amount of shares for the financial institution to purchase is pinned down for any given level of net worth of the financial institution. This equation is represented by the upward-sloping linear line in the top right hand corner in Figure 6, showing how share purchase depends on net worth. The line is flatter than 45 degrees because the leverage of the financial institution, $(Q_t K_{i,t+1} - N^E_{i,t+1})/N^F_{i,t+1}$, is bigger than unity.

Thus, Figure 6 graphically describes the solution of the financial institution i's profit maximisation problem.[12] Conditional on the optimal expected cutoff value and for a given N^E_i, the financial institution's desired expenditure on shares depends on the financial institution's net worth in such a way that leverage is constant.

2.2 Aggregation of the Loan Contract

We aggregate across financial institutions in this subsection. Consider again the combined first order condition Equation (7). The external finance premium on the right hand side is determined macroeconomically. This equation implies that every financial institution will choose the same expected cutoff value, $E_t\bar{\omega}^F_{i,t+1}$, regardless of its net worth, so that $E_t\bar{\omega}^F_{i,t+1} = E_t\bar{\omega}^F_{t+1}$ for all i. The combined FOC, Equation (7), can thus be replaced by the following equation for the economy as a whole as follows:

$$E_t\left(\frac{\frac{\Gamma^F_\omega(\bar{\omega}^F_{t+1})}{\Gamma^F_\omega(\bar{\omega}^F_{t+1}) - \mu^F G^F_\omega(\bar{\omega}^F_{t+1})}}{\frac{\Gamma^F_\omega(\bar{\omega}^F_{t+1})}{\Gamma^F_\omega(\bar{\omega}^F_{t+1}) - \mu^F G^F_\omega(\bar{\omega}^F_{t+1})}(\Gamma^F(\bar{\omega}^F_{t+1}) - \mu^F G^F(\bar{\omega}^F_{t+1})) + (1-\Gamma^F(\bar{\omega}^F_{t+1}))}\right) = \frac{E_t(R^F_{t+1})}{R_{t+1}} \quad (9)$$

[12] The external finance premium solves the optimal expected cutoff value which pins down the Lagrange multiplier in the manner depicted in the bottom left hand corner of Figure 6.

and the expected participation constraint, Equation (4), can be aggregated to obtain

$$E_t[[\Gamma^F(\bar{\omega}^F_{t+1}) - \mu^F G^F(\bar{\omega}^F_{t+1})]R^F_{t+1}](Q_t K_{t+1} - N^E_{t+1}) = R_{t+1}(Q_t K_{t+1} - N^E_{t+1} - N^F_{t+1}) \quad (10)$$

These two equations will hold at any point in time in which there had not been any unanticipated shocks.

However, after a shock, $Q_t K_{t+1} - N^E_{t+1}$ has already been fixed, consumers must continue to receive the fisk free rate R_{t+1}. Taken together, these two things determines what the realised cutoff value $\bar{\omega}^F_{t+1}$ must be. It is this *realised* participation constraint,

$$[\Gamma^F(\bar{\omega}^F_t) - \mu^F G^F(\bar{\omega}^F_t)]R^F_t(Q_{t-1}K_t - N^E_t) = R_t(Q_{t-1}K_t - N^E_t - N^F_t), \quad (11)$$

which is implemented in the model at any time t, along with the combined FOC, which contains expectations of what will happen in period $t+1$. See Equations (44) and (45).

It is helpful to show how these equations can be combined to obtained a 'leverage supply function', since we will use this to discuss our simulation results. To obtain an analytic solution would require us to substitute out the expected cutoff value, $E_t\bar{\omega}^F_{t+1}$, which is impossible because of the complexity of the distribution functions. Instead, we approach the problem implicitly. We first take the first derivative of Equation (9):

$$\frac{dE_t\bar{\omega}^F_{t+1}}{d(E_t(R^F_{t+1})/R_{t+1})} = E_t\left(\frac{R_{t+1}}{R^F_{t+1}}\frac{\lambda(\bar{\omega}^F_{t+1})}{\lambda_\omega(\bar{\omega}^F_{t+1})}\left(\frac{(1-\Gamma^F(\bar{\omega}^F_{t+1})) + \lambda(\bar{\omega}^F_{t+1})(\Gamma^F(\bar{\omega}^F_{t+1}) - \mu^F G^F(\bar{\omega}^F_{t+1}))}{1-\Gamma^F(\bar{\omega}^F_{t+1})}\right)\right) > 0 \quad (12)$$

This equation shows that a rise in the external finance premium raises the expected cutoff value in the optimal contract. A higher expected cutoff value leads to more monitoring which is required to 'use up' the funds made available because $E_t R^F_{t+1} > R_{t+1}$.

We now determine the effect of $E_t\bar{\omega}^F_{t+1}$ on leverage, starting with Equation (8). We can substitute $\bar{\omega}^F_{i,t+1} = \bar{\omega}^F_{t+1}$ into the equation showing the desired expenditure on shares by financial institution i to obtain:

$$Q_t K_{i,t+1} - N^E_{i,t+1} = \left[1 + E_t\left(\frac{\Gamma^F_\omega(\bar{\omega}^F_{t+1})}{1-\Gamma^F(\bar{\omega}^F_{t+1})} \times \frac{\Gamma^F(\bar{\omega}^F_{t+1}) - \mu^F G^F(\bar{\omega}^F_{t+1})}{\Gamma^F_\omega(\bar{\omega}^F_{t+1}) - \mu^F G^F_\omega(\bar{\omega}^F_{t+1})}\right)\right] N^F_{i,t+1} \quad (13)$$

This means that all financial institutions will choose the same leverage whatever their net worth. Summing across all the financial institutions, and dividing by N^F_{t+1}, we obtain an equation showing the leverage of the financial system as a whole.

$$\frac{Q_t K_{t+1} - N^E_{t+1}}{N^F_{t+1}} = 1 + E_t\left(\frac{\Gamma^F_\omega(\bar{\omega}^F_{t+1})}{1-\Gamma^F(\bar{\omega}^F_{t+1})} \times \frac{\Gamma^F(\bar{\omega}^F_{t+1}) - \mu^F G^F(\bar{\omega}^F_{t+1})}{\Gamma^F_\omega(\bar{\omega}^F_{t+1}) - \mu^F G^F_\omega(\bar{\omega}^F_{t+1})}\right) \quad (14)$$

Figure 6 enables us to picture how the aggregate desired expenditure on shares for the economy as a whole is determined as a function of the net worth of financial institutions, N^F_{t+1}. In Figure 6 the slope of the desired expenditure on shares line by financial institution

i is equal to the inverse of the leverage. The linearity of Equation (13) means that the slope of the aggregate desired expenditure on shares line is equal to the slope of the desired expenditure on shares line for financial institution i. We next take the first derivative of Equation (14):

$$\frac{d\left(\frac{Q_t K_{t+1} - N^E_{t+1}}{N^F_{t+1}}\right)}{dE_t\bar{\omega}^F_{t+1}} = E_t\left(\frac{\lambda_\omega(\bar{\omega}^F_{t+1})}{\lambda(\bar{\omega}^F_{t+1})}\left(\frac{Q_t K_{t+1} - N^E_{t+1}}{N^F_{t+1}} - 1\right) + \frac{\Gamma^F_\omega(\bar{\omega}^F_{t+1})}{1 - \Gamma^F(\bar{\omega}^F_{t+1})}\frac{Q_t K_{t+1} - N^E_{t+1}}{N^F_{t+1}}\right) > 0, \tag{15}$$

This equation shows that the leverage is a positive function of $E_t\bar{\omega}^F_{t+1}$.

Taking Equations (12) and (15) toghether, we obtain a solution for the loan contract. This shows that leverage is an increasing function of the external finance premium:

$$\frac{Q_t K_{t+1} - N^E_{t+1}}{N^F_{t+1}} = s\left(\frac{E_t R^F_{t+1}}{R_{t+1}}\right), \quad s' > 0 \tag{16}$$

We will call Equation (16) a leverage supply function.

We use this leverage supply function to show the way in which $E_t R^F_{t+1}$ must be determined, macroeconomically. For the economy as a whole, the supply of capital is fixed at any point in time. Since the market for shares has to clear, this means that financial institutions need to be sufficiently leveraged to hold whatever capital is in existence. This means that, all terms on the left hand side of Equation (16) are exogenous to the financial system taken as a whole. As a result it must be the case that Equation (16) ends up determining $E_t R^F_{t+1}/R_{t+1}$. In other words, at any point in time the return on holding shares R^F_{t+1} must be sufficiently large to induce financial institutions to hold them. How this actually happens dpends on the actual workings of the whole economy, and in particular on how Q is determined.

2.3 The Share Contract

The contract between the financial institutions and the entrepreneurs is a share contract. As equity holders of the entrepreneurs, the financial institutions absorb the profits and losses of the entrepreneurs. There is no problem of asymmetric information and no monitoring takes place. The financial institutions and the entrepreneurs split the profits according to the shares of their investmsents, regardless of the idiosyncratic shock to the entrepreneurs ω^E. To diversify away from the entrepreneur-specific idiosyncratic risk, each financial institution will invest in an infinite number of entrepreneurs.

Consider financial institution i again. It finances a worth of $(QK_{i,t} - N^E_{i,t})$ to the entrepreneurs. Denote the aggregate return to capital as R^E_{t+1} and financial institution i's average share of profits from its share contracts (with an infinite number of enterpreneurs

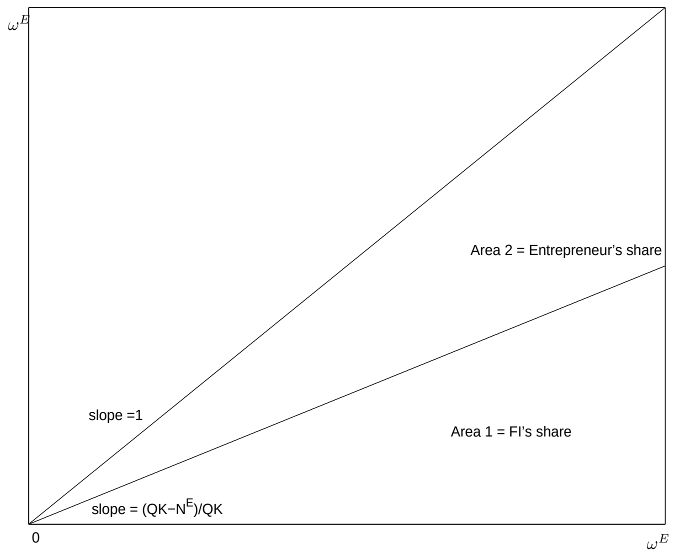

Figure 7: The Γ^E function. Area 1 $= \Gamma^E = \frac{QK-N^E}{QK}$. Area 2 $= 1 - \Gamma^E = \frac{N^E}{QK}$.

$j_i = \{1, 2, ..., \infty\})$ as $\Gamma^E_{i,t+1}$, then

$$\begin{aligned} \Gamma^E_{i,t+1} R^E_{t+1} Q_t K_{i,t+1} &= \frac{Q_t K_{i,t+1} - N^E_{i,t+1}}{Q_t K_{i,t+1}} \int_0^\infty \omega^E R^E_{t+1} Q_t K_{i,t+1} dF^E(\omega^E) \\ \therefore \qquad \Gamma^E_{i,t+1} &= \frac{Q_t K_{i,t+1} - N^E_{i,t+1}}{Q_t K_{i,t+1}} \end{aligned} \tag{17}$$

The share Γ^E_i is independent of the idiosyncratic shock ω^E because there is no asymmetric information. Since the entrepreneurs in the entire economy draw the idiosyncratic shocks from the same distribution, every financial institution should receive the same average share of profits. Denote Γ^E the aggregate share of profits in the share contracts that goes to the financial institutions, then $\Gamma^E_{i,t} = \Gamma^E_t$. Hence,

$$\Gamma^E_t = \frac{Q_t K_{t+1} - N^E_{t+1}}{Q_t K_{t+1}} \tag{18}$$

Figure 7 illustrates the share contract. The steeper line is the 45-degree line; the flatter line shows the financial institutions' share of profit in the share contracts. The split of the profits does not depend on the realisation of the idiosyncratic shock.

Finally, the realised return to financial institutions' investment equals the realised return on entrepreneurs' investment:

$$R^E_t = R^F_t \tag{19}$$

In Appendix B, we describe the 'chained loan contract' in the Hirakata, Sudo and Ueda (2009) model. HSU assume the contract between the financial institutions and the entrepreneurs is also a loan contract. In the simulations we will compare our model with the HSU model.

2.4 Dynamic Behaviour of Net Worth

Next, we consider the evolution of the aggregate net worths of the financial institutions and the entrepreneurs. As discussed in the previous section, the nature of the loan contract is such that the leverage of the financial institutions is independent of the net worth of the institutions N_t^F. In the share contract, the distribution of idiosyncratic shocks to entrepreneurs is symmetric and the Γ^E function depends only on aggregate variables. We only need to keep track of the aggregate levels of net worths in each period. Let V_t^E and V_t^F be the earnings from credit contracts of the entrepreneurs and the financial institutions, then

$$V_t^E = (1 - \Gamma_t^E) R_t^E Q_{t-1} K_t = R_t^E N_t^E \tag{20}$$

$$V_t^F = \Gamma_t^E (1 - \Gamma^F(\bar{\omega}_t^F)) R_t^E Q_{t-1} K_t = (1 - \Gamma^F(\bar{\omega}_t^F)) R_t^E (Q_{t-1} K_t - N_t^E) \tag{21}$$

There are two technical matters. First, it is necessary to start the financial institutions and the entrepreneurs off with some net worth in order to allow them to begin their operations. Following BGG and Calstrom and Fuerst (1996), we assume that in addition to operating firms, the entrepreneurs and the financial institutions supplement their income by working in the general labour market. Both inelastically supply one unit of labour. The net worths come from earnings from the credit contracts and labour income. As in BGG, in the calibrations we keep the share of labour income going to financial institutions and entrepreneurs small, so that the modification has no significant direct effects on the results.

Second, in order to prevent the financial institutions and the entrepreneurs from growing out of the financial constraints, we assume that at the end of every period, constant fractions $(1 - \gamma^E)$ and $(1 - \gamma^F)$ of entrepreneurs and financial institutions respectively quit their businesses. When they quit, they consume their net worth C_t^E and C_t^F.

So, the law of motions of net worths for the entrepreneurs and the financial institutions are

$$N_{t+1}^E = \gamma^E V_t^E + W_{Et} \tag{22}$$

$$N_{t+1}^F = \gamma^F V_t^F + W_{Ft} \tag{23}$$

where W_{Et} and W_{Ft} denote the labour incomes of the entrepreneurs and the financial institutions. Consumption of the quitting entrepreneurs and financial institutions are

$$C_t^E = (1 - \gamma^E) V_t^E \tag{24}$$

$$C_t^F = (1 - \gamma^F) V_t^F \tag{25}$$

3 The Full Macro Model

Our model adopts the same basic real business cycle framework with capital adjustment costs and adds in the credit contracts. By embeding the partial equilibrium in the general equilibrium framework, we endogenise the risk-free rate R, gross return on capital R^E and the price of capital Q.

The economy consists of five types of agents, namely consumers, investors, financial institutions, entrepreneurs and capital producers. In the following we describe the aggregate behaviour for the consumers, entrepreneurs and the capital producers.

3.1 Consumers

Our consumers are reasonably conventional. Homogeneous consumers consume, save and supply labour. They save by lending funds to investors. For simplicity, we assume log utility in consumption and separability between consumption utility and labour disutility. The consumer's maximisation problem is

$$\max_{C_t, L_{Ct}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \chi \frac{L_{Ct}^{1+\varphi}}{1+\varphi} \right) \tag{26}$$

subject to the budget constraint

$$C_t + B_{t+1} = W_t L_{Ct} + R_t B_t + \Pi_t \tag{27}$$

where C_t is consumption, L_{Ct} is labour supply by the consumers, W_t is the real wage and B_{t+1} is the real amount of lending. Π_t is the profits received from the ownership of the capital producing firms. R_t is the real return from lending from period $t-1$ to t. β is the discount factor, χ the utility weight on leisure and φ the inverse of Frisch elasticity of labour supply.

The first order conditions are standard:

$$1 = \beta E_t \left(\frac{C_t}{C_{t+1}} R_{t+1} \right) \tag{28}$$

$$W_t = \chi C_t L_{Ct}^{\varphi} \tag{29}$$

The first equation is the Euler equation that governs intertemporal consumption decision, meaning that consumption is rising when the interest rate is high. The second equation is the condition of intratemporal substitution between consumption and labour, meaning that, *ceteris paribus,* the labour supply rises when the real wage is high, and that the labour supply falls when consumption rises.

3.2 The Entrepreneurial Sector

The entrepreneurs produce goods using capital and labour. We assume the following Cobb-Douglas production function:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{30}$$

where A_t is the level of technology of goods production. L_t is a composite of the labour supplied by the consumers L_{Ct}, financial institutions L_{Ft} and entrepreneurs L_{Et} which has the following form:

$$L_t = L_{Ct}^{(1-\Omega_E-\Omega_F)} L_{Et}^{\Omega_E} L_{Ft}^{\Omega_F} \tag{31}$$

Entrepreneurs have to buy capital for production one period in advance. So, they choose K_{t+1} at the end of period t. In order to do so, these entrepreneurs issue shares in the equity market at the prevailing price of capital Q_t. Capital is depreciated at a rate δ during production. The remaining capital $(1-\delta)K_{t+1}$ is sold to capital producing firms at the prevailing price of capital Q_{t+1}. Entrepreneurs choose their demand for labour within the period to maximise profits. The entrepreneurs' maximisation problem is the following:

$$\max_{K_{t+1}, L_{Ct+1}, L_{Ft+1}, L_{Et+1}} E_t[Y_{t+1} + (1-\delta)K_{t+1}Q_{t+1} - R_{t+1}^E Q_t K_{t+1} - W_{t+1}L_{Ct+1} - W_{Et+1}L_{Et+1} - W_{Ft+1}L_{Ft+1}] \tag{32}$$

The entrepreneurs' first order conditions are:

$$\begin{aligned} E_t(R_{t+1}^E) &= E_t\left(\frac{\frac{\alpha Y_{t+1}}{K_{t+1}} + (1-\delta)Q_{t+1}}{Q_t}\right) && (33) \\ (1-\alpha)(1-\Omega_F-\Omega_E)\frac{Y_t}{L_{Ct}} &= W_t && (34) \\ (1-\alpha)\Omega_F Y_t &= W_{Ft} && (35) \\ (1-\alpha)\Omega_E Y_t &= W_{Et} && (36) \end{aligned}$$

The realised return for the entrepreneurs is

$$R_t^E = \frac{\frac{\alpha Y_t}{K_t} + (1-\delta)Q_t}{Q_{t-1}} \tag{37}$$

Note that R^E depends on the prices of capital at which the capital is bought and sold. This is in contrast with the expected return

$$\text{Expected return}_t = E_t(R_{t+1}^E) = E_t\left(\frac{\frac{\alpha Y_{t+1}}{K_{t+1}} + (1-\delta)Q_{t+1}}{Q_t}\right)$$

which depends on the expected future prices of capital but not on past prices.

3.3 The Capital Producers

Capital producing firms own technology that converts goods into capital. They purchase depreciated capital. We assume also that actual goods need to be invested to create new capital. Capital is produced within the period. Then they sell the capital back to the firms. However, there are adjustment costs J to capital investment. Capital accumulation is defined as follows:

$$K_{t+1} = (1-\delta)K_t + J\left(\frac{I_t}{K_t}\right)K_t \tag{38}$$

The adjustment costs J is such that $J' > 0$ and $J'' < 0$. The function is normalised such that $J(\delta) = \delta$. New capital is produced within the period and sold to goods producing firms at the same price Q_t. Their profit maximisation problem is:

$$\max_{I_t} \Pi_t = Q_t K_{t+1} - (1-\delta)Q_t K_t - I_t = Q_t J\left(\frac{I_t}{K_t}\right)K_t - I_t \tag{39}$$

The first order condition is

$$Q_t J'\left(\frac{I_t}{K_t}\right) = 1 \tag{40}$$

The profits are transfered to the consumers every period.

3.4 Aggregate Demand

To close the model, we need the goods market equilibrium:

$$Y_t = C_t + C_t^E + C_t^F + I_t + \mu^F G^F(\bar{\omega}_t^F) R_t^F (Q_{t-1}K_t - N_t^E) \tag{41}$$

The final term corresponds to the cost associated with the investors' monitoring.

3.5 The Full Macroeconomic Model

To summerise, the model equations are:

$$
\begin{aligned}
E_t(C_{t+1}) &= \beta C_t R_{t+1} && (42)\\
W_t &= \chi C_t L_{Ct}^{\varphi} && (43)\\
1 &= \frac{[\Gamma^F(\bar{\omega}_t^F) - \mu^F G^F(\bar{\omega}_t^F)] R_t^F (Q_{t-1}K_t - N_t^E)}{R_t(Q_{t-1}K_t - N_t^E - N_t^F)} && (44)\\
\frac{E_t(R_{t+1}^F)}{R_{t+1}} &= E_t\left(\frac{\lambda(\bar{\omega}_{t+1}^F)}{\lambda(\bar{\omega}_{t+1}^F)[\Gamma^F(\bar{\omega}_{t+1}^F) - \mu^F G^F(\bar{\omega}_{t+1}^F)] + (1 - \Gamma^F(\bar{\omega}_{t+1}^F))}\right) && (45)\\
R_t^F &= R_t^E && (46)\\
N_{t+1}^F &= \gamma^F(1 - \Gamma^F(\bar{\omega}_t^F)) R_t^F (Q_{t-1}K_t - N_t^E) + (1-\alpha)\Omega_F Y_t && (47)\\
N_{t+1}^E &= \gamma^E R_t^E N_t^E + (1-\alpha)\Omega_E Y_t && (48)\\
C_t^F &= (1-\gamma^F)(1 - \Gamma^F(\bar{\omega}_t^F)) R_t^F (Q_{t-1}K_t - N_t^E) && (49)\\
C_t^E &= (1-\gamma^E) R_t^E N_t^E && (50)\\
Y_t &= A_t K_t^{\alpha} L_{Ct}^{(1-\alpha)(1-\Omega_E-\Omega_F)} && (51)\\
W_t &= (1-\alpha)(1-\Omega_E-\Omega_F)\frac{Y_t}{L_{Ct}} && (52)\\
R_t^E &= \frac{\alpha \frac{Y_t}{K_t} + (1-\delta) Q_t}{Q_{t-1}} && (53)\\
K_{t+1} &= (1-\delta)K_t + J\left(\frac{I_t}{K_t}\right) K_t && (54)\\
1 &= Q_t J'\left(\frac{I_t}{K_t}\right) && (55)\\
Y_t &= C_t + C_t^E + C_t^F + I_t + \mu^F G^F(\bar{\omega}_t^F) R_t^F (Q_{t-1}K_t - N_t^E) && (56)
\end{aligned}
$$

where $\lambda(\bar{\omega}_{t+1}^F)$ is the shadow price of internal funds of the financial institutions, defined in Equation (5).

The 15 equations above ensure that (i) consumers maximise utility given the relative prices; (ii) financial institutions, entrepreneurs and capital producers maximise their respective profits given the relative prices; (iii) the resource constraints hold; (iv) and markets clear.

Given the initial conditions for $\{K_0, N_0^E, N_0^F\}$ and realisation of productivity shocks $\{A_t\}_{t=0}^{\infty}$, the system solves $\{C_t, R_t, W_t, L_{Ct}, Y_t, K_{t+1}, N_{t+1}^F, N_{t+1}^E, C_t^F, C_t^E, R_t^E, Q_t, I_t, R_t^F, \bar{\omega}_t^F\}_{t=0}^{\infty}$.

The log-linearised equations which we actually simulate are given in Appendix A.

Table 1: Parameter Values

Parameter	α	β	Ω_E	Ω_F	δ	χ	φ	J''	A
Numerical Value	0.35	0.99	0.01	0.01	0.025	0.3	0.33	-10	5.3958

Table 2: Credit contract Parameters and Steady State Values

Parameter	μ^F	γ^E	γ^F	$\bar{\omega}^F$	σ^F	G^F	Ψ^F	Γ^F	λ
Value	0.0452	0.9837	0.9622	0.7974	0.1074	0.0153	0.7961	0.7968	1.0212

4 Calibration and Simulation

4.1 Calibration

We discuss the calibration of our model here. We also discuss the calibrations for the models that we use to compare with our model.

The standard parameters of our model are set following HSU and BGG based on US data, given in Table 1.[13]

In our model, the credit contract parameter values are set to satisfy the following steady state conditions:

1. the steady state external finance premium $R^E - R = 0.02/4$ (same as BGG and HSU);
2. the steady state entrepreneurial leverage K/N^E is 2 times (same as BGG and HSU);
3. the steady state financial institution capital to net worth ratio K/N^F is 10 times (same as HSU);
4. the financial institution failure rate is 2%, i.e. $F^F(\bar{\omega}^F) = 2\%$ (same as HSU).

We use the combined FOC, the participation constraint and the cumulative distribution function $F^F(.)$ in the steady state to solve for the parameters μ^F and σ^F and the steady state cutoff value $\bar{\omega}^F$. The credit contract parameters and steady state values are given in Table 2. The steady state values of the macroeconomic variables are given in Table 3.

[13]All parameters except the inverse of Frisch elasticity of labour supply are the same as BGG (1999). We use a different φ because we want to be consistent with the HSU model, in which the formulation of labour disutility is slightly different from BGG. BGG's period t utility of leisure is $\chi \ln(1 - L_{Ct})$. In fact, the parameter φ has no significant impact to the impulse responses.

Table 3: Steady State Macroeconomic variables

Parameter	Y	C	C^E	C^F	I	W	L_C	K
Value	100	67.2264	7.2401	3.3462	21.8199	26.8863	2.3692	872.796
Parameter	N^E	N^F	R	R^E	R^F	Z^F		
Value	436.398	87.2796	1.0101	1.0151	1.0151	1.0118		

We wish to compare our model with the HSU model. Since this model is not well-known, it is fully set out in Appendix B. In the HSU model, the contract between the financial institutions and entrepreneurs is also a loan contract. In this comparison, our objective is to compare the financial accelerator effects which arise when financial institutions buy shares from the entrepreneurs with what happens when financial institutions make loans to entrepreneurs. A fair comparison between the two models is possible since our model is identical to the HSU model except for the way for the specification of credit contracts between finanicial insitutions and goods-producing firms. In the calibration of the credit contract parameters in the HSU model, we require two additional parameters which come from the steady state conditions in that model. These steady state conditions are as follows: the failure rate of entrepreneurs is 2% per period; and the equilibrium spread between the contractual rate which financial institutions lend to the entrepreneurs and the rate which they borrow from the investors in the model is assumed to be 230 basis points annually. These additional steady state conditions are necessary for the identification of the monitoring cost, the variance of the idiosyncratic shock and the steady state cutoff value in the loan contract between financial intermediaries and entrepreneurs. Details are given in Appendix B.

We will also compare our model with the BGG model. In that setup there is no friction between investors and financial institutions; the only financial friction comes from the loan contract between investors and entrepreneurs. To make a fair comparison between the models possible, we have constructed a version of the BGG model which keeps the macroeconomic setup of our model. This setup differs from the published BGG setup in four ways. First, we assume all goods prices are flexible. Second, we follow the formulation of labour disutility used in our model. Third, we set the macroeconomic parameter values equal to those in our model, presented in Table 1. Finally, we treat $\bar{\omega}^E$ as an endogenous variable, where $\bar{\omega}^E$ is the level of the idiosyncratic productivity shock ω^E below which entrepreneurs are bankrupt.[14] We calibrate the credit contract parameters using

[14]In the published version of the BGG model, the authors treat the value of $\bar{\omega}^E$ as a constant, equal to its steady-state value. But in the BGG model, $\bar{\omega}^E$ should be endogenous, in exacly the way that it is in our model, for reasons which we have identified in the body of our paper. In fact, BGG introduce a rather clever adjustment to the budget constraint of entrepreneurs which almost exactly offsets the effects of their erroneous simplification. But, in order to assist with our comparison between different approaches to financial frictions, it is appropriate for us to use a version of the BGG model in which $\bar{\omega}^E$ is properly endogenous.

the first two steady state relations in our baseline calibration, that is (i) a steady state external finance premium of 2% per annum; (ii) a steady state entrepreurial leverage of 2; and together with (iii) an entrepreneurial failure rate of 2%. By doing so, we make the slightest change to the macroeconomic environment (in particular, the steady state values are about the same as the values in our baseline model) so that all the dissimilarities in the simulation results come from the nature of the credit contracts in the two setups. This version of the BGG model, and the calibration details, are reported in Appendix C.

We will also compare our model with the Q-Ramsey model. We construct the Q-Ramsey model by removing the financial imperfection in our model. This is achieved by (1) fixing the external finance premium; and (2) fixing the cutoff value of the idiosyncratic shock at their steady state levels instead of allowing them to respond to changes in the leverage supply function, Equation (16). This comparison is fair because the Q-Ramsey model has the same steady state as our model, but the financial accelerator has been turned off.

4.2 Simulated Outcomes

In this section, we simulate the impulse response of credit spreads and other macroeconomic variables to a 10% permanent negative shock to productivity. The results are presented in Figure 8 and 9.

In response to a negative productivity shock, the demand for capital falls. The share price Q jumps down so that along the adjustment path Q is rising, leading to capital gains and a higher expected return to entrepreneurs' capital $E(R^E_{+1})$. (The extent to which the expected return to capital has to rise and the share price to fall depends on the cost of funds of the entrepreneurs, as will be discussed.) This unanticipated fall in asset prices reduces the realised return on capital to the entrerpeneurs R^E_t in the period in which the unanticipated shock occurs. Since the financial institutions hold the shares of the entrepreneurs, their return R^F_t falls by the same amount. This shifts the realised participation constraint to the left in $(K_{t+1}, \bar{\omega}^F_{t+1})$ space relative to the expected participation constraint, raising the cutoff value for the financial institutions relative to the expected cutoff value, as shown in Figure 10.

The aggregate net worth of the financial institutions falls not only because the realised return is low relative to the expected return evaluated one period before, but also because a higher cutoff value increases bankruptcy and reduces the share of profit that goes to the financial institutions in the loan contract $(1 - \Gamma^F(\bar{\omega}^F_t))$. With these two effects, the fall in net worth of the financial institutions is proportionally bigger than the fall in share price, so their leverage increases. The investors are now exposed to bigger risks of default. To compensate for the rise in the monitoring costs, the investors require a higher return from the financial institutions, forcing up the external finance premium R^F/R. The transmission mechanism discussed above comes from the loan contract and is the same as BGG.

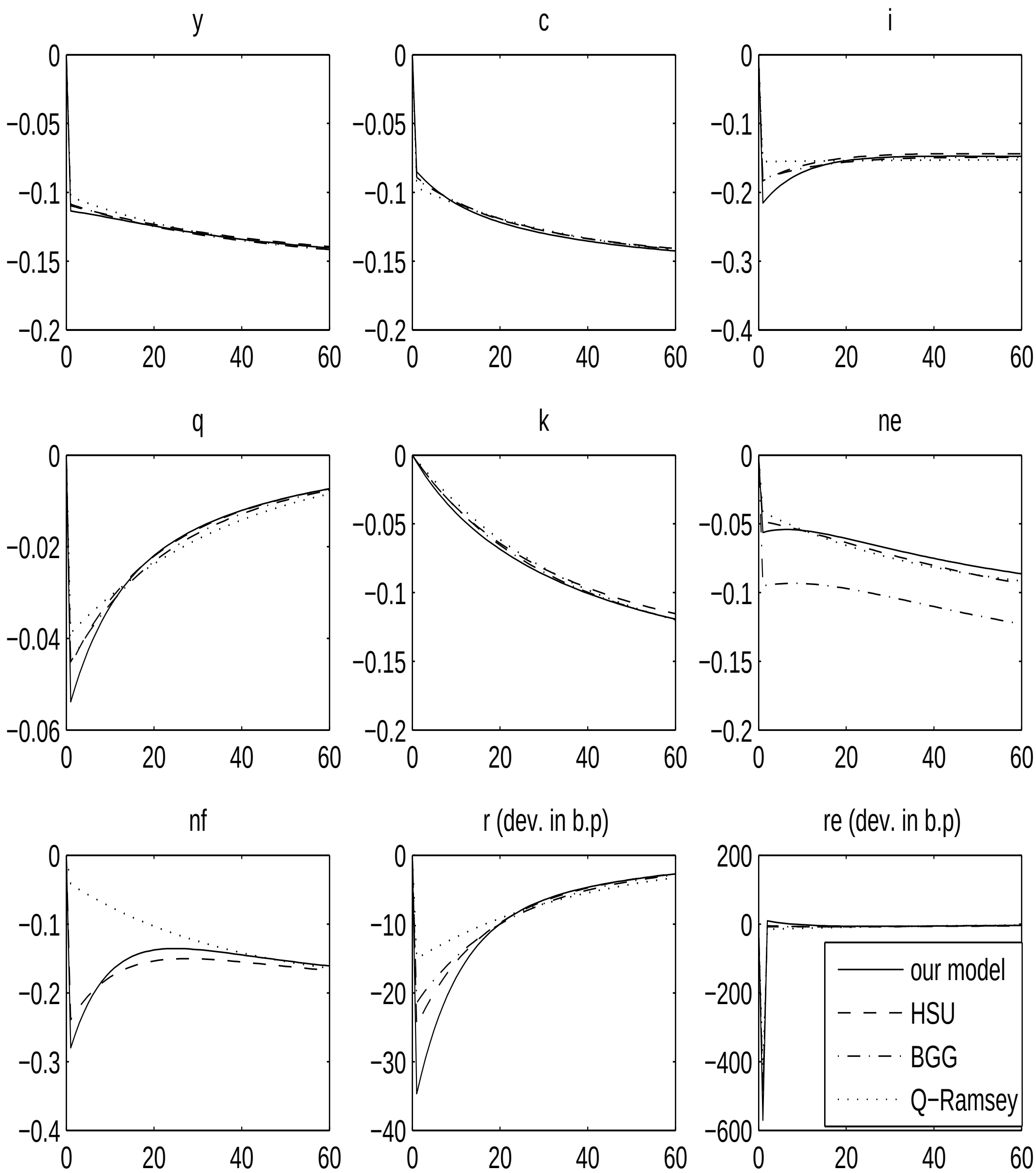

Figure 8: IRF of Luk-Vines, HSU and BGG models: permanent productivity shock 1.

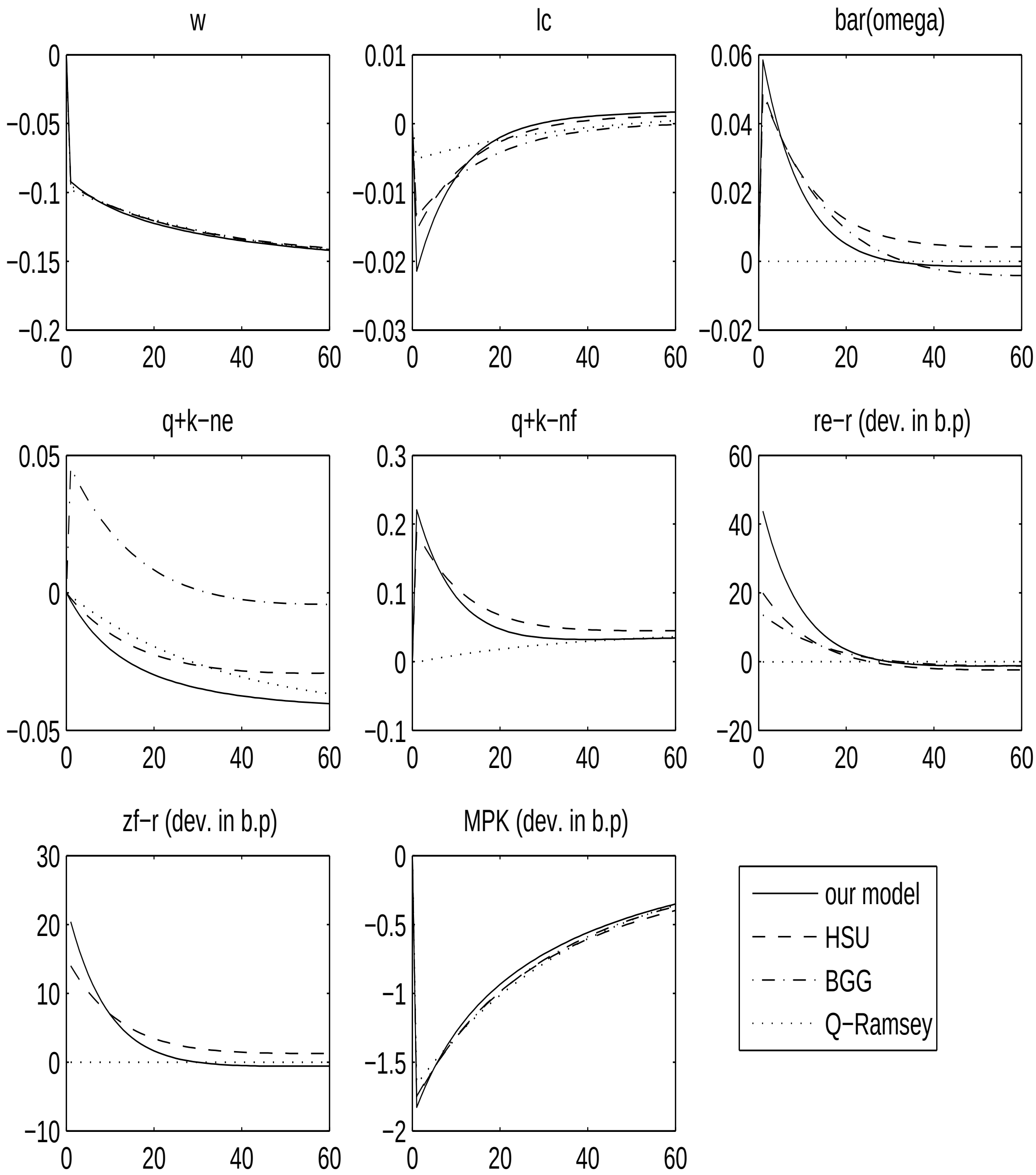

Figure 9: IRF of Luk-Vines, HSU and BGG models: permanent productivity shock 2.

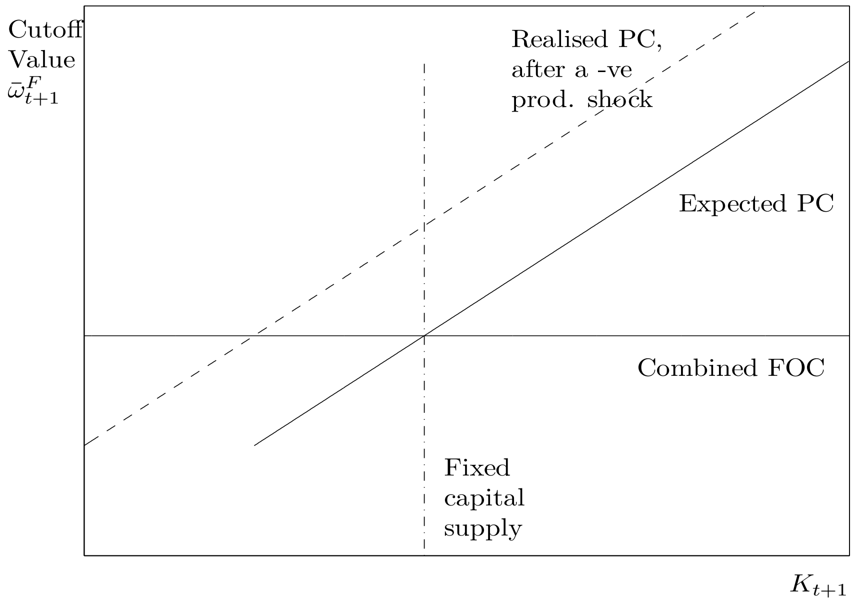

Figure 10: Luk-Vines Partial Equilibrium

In the share contract between the financial institutions and the entrepreneurs, as the cost of funds of the financial institutions rises, they require a higher return from the entrepreneurs. Owing to the counter-cyclical external finance premium, the price of capital Q has to fall further initially relative to a model without the financial accelerator to allow for bigger capital gains along the adjustment path. This leads to a larger fall in the demand for investment relative to a model without the financial accelerator. Q here has an interpretation of share price because the financial contract concerned is a share contract.

We compare the impulse repsonses of our model and the HSU model. The impacts on the aggregate variables are qualitatively similar, but our amplifications are quanitatively larger than HSU. In particular, the rise in external finance premium in our model is significantly larger than in the HSU model, leading to a larger financial accelerator effect and a more pronounced fall in investment. Our setup is more volatile for the following reason: in the HSU model, financial institutions maximise profits subject to the participation constraints of the entrepreneurs. Subsequent to a negative shock hitting the economy, the financial institutions are forced to set a lower cutoff value $\bar{\omega}^E$ in order not to violate the entrepreneurs' participation constraint, otherwise the entrepreneurs will be better off using their own funds only and not participate in the credit contract.[15] The fall in the cutoff value $\bar{\omega}^E$, *ceteris paribus*, dampens the amplification of the shock. In our setup, the contract between financial institutions and entrepreneurs is a share contract which does not involve a participation constraint for the entrepreneurs, so the dampening effect

[15]See Appendix B for a more detailed analysis.

described above is absent, leading to a larger amplification than the loans economy.

We compare the impulse repsonses of our model and the BGG model. From Figure 8 and 9, a negative productivity shock causes the entrepreneurial net worth in the BGG setup to jump down by more than in our model and HSU, leading to an immediate rise in leverage in the entrepreneurial sector. This is due to the fact that the financial accelerator in BGG operates in the contract between the investors and entrepreneurs, whereas in our model as well as HSU, the amplification comes from the financial friction between investors and financial institutions. The simulation results show that, the external finance premium in BGG is anti-cyclical, but its increase in response to a negative productivity shock is smaller than in our baseline setup and in HSU. This is simply a result of the calibration of the models which reflects the fact that, in the real world, the financial sector is much more highly leveraged than the entrepreneurial sector. Higher leverage makes the economy more volatile.

4.3 Varying the Size of Entrepreneurial Net Worth

We investigate how the size of the entrepreneurial sector affects the volatility of the economy. We proceed by reducing the steady state entrepreneurial net worth N^E. The entrepreneurs will need less net worth of their own in order to get funding from the financial sector. We simulate the model with entrepreneurial sizes 50%, 30%, 10% and 0% of the capital stock, keeping the size of the financial sector constant at 10% of capital stock.

In the case in which entrepreneurial net worth falls to zero, the entrepreneurs do not need any funds of their own in order to obtain fundings from the banks. In other words the financial institutions own the entire goods producing sector. Therefore, the banks and the entrepreneurial sector can be thought of as one sector. In the full system, we collapse entrepreneurial net worth N^E and the entrepreneurial share of labour Ω_E to 0. It turns out that the system in this special case has an identical structure with the BGG model. The only distinction is in parameterisation. The steady state leverage of the financial-entrepreneurial sector is 10 times in our setup, which is larger than 2 times in BGG's calibration.

We re-parameterise the steady state for each entrepreneurial size and simulate a 10% permanent negative productivity shock. The impulse responses of a permanent productivity shock are presented in Figure 11 and 12.

Compared with the other models, the model with entrepreneurial net worth equal to zero (i.e. the re-parameterised BGG model) is the most volatile system. As we increase the steady state entrepeneurial net worth N^E and recalibrate the model, the volatility of the system falls. The reason is the following. When the size of entrepreneurial net worth is smaller in steady state (and so when the leverage of the production sector is bigger), a larger share of revenue goes to the financial institutions. When the economy is hit by a negative productivity shock, financial institutions suffer a larger loss in revenue, and so,

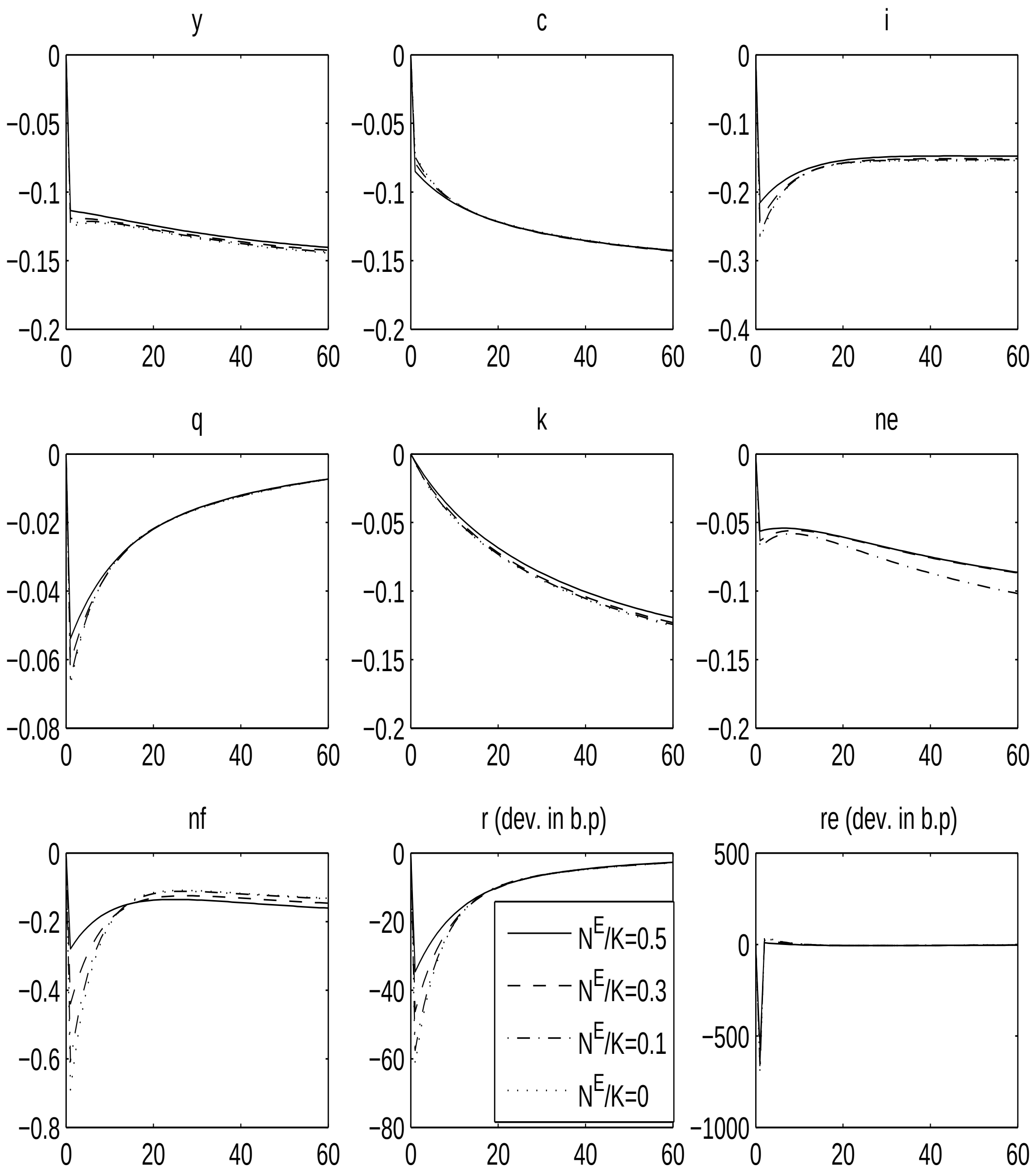

Figure 11: IRF of revised HSU model with different entrepreneurial net worths N^E: permanent productivity shock 1.

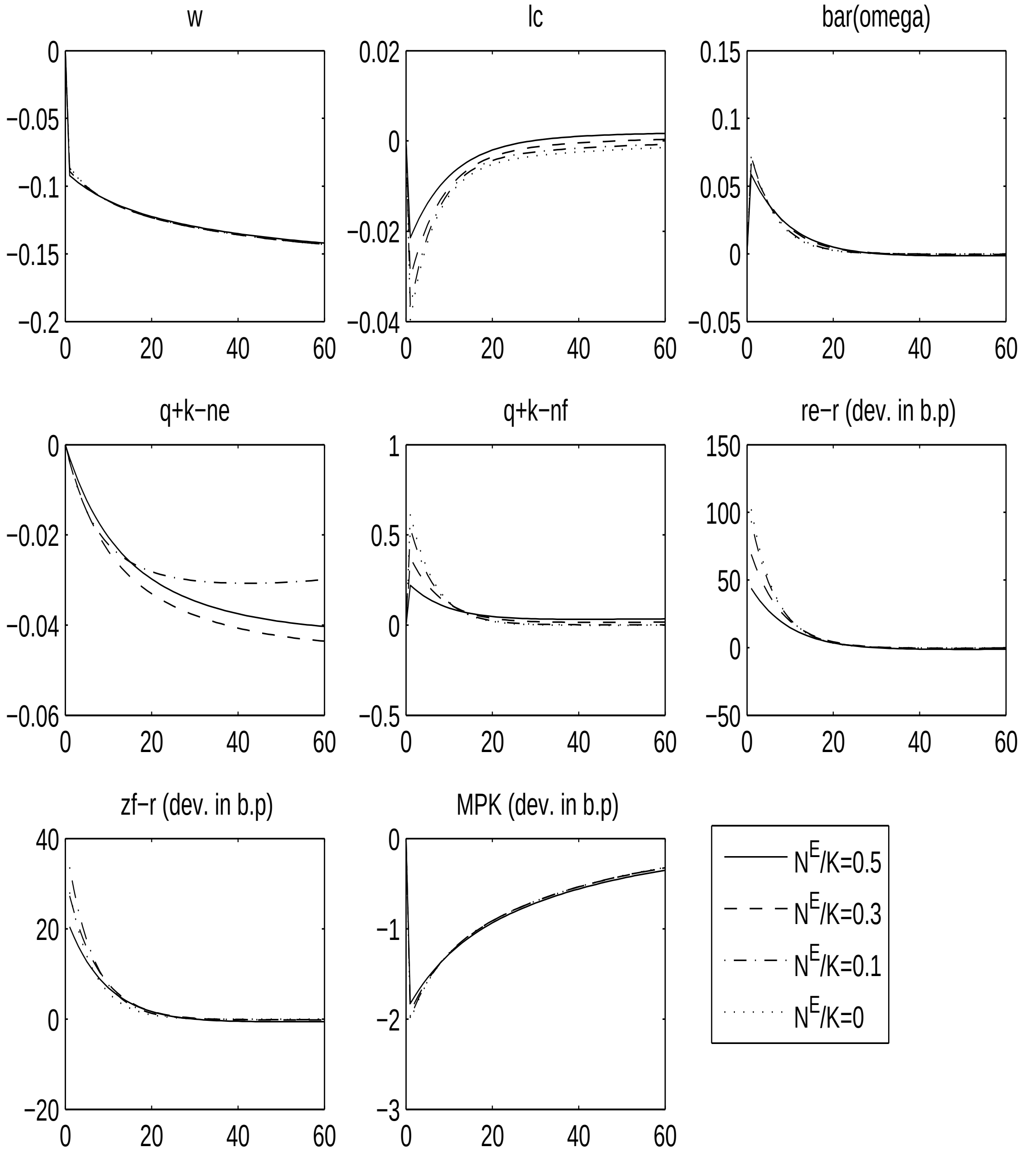

Figure 12: IRF of revised HSU model with different entrepreneurial net worths N^E: permanent productivity shock 2.

keeping the size of financial institutions fixed, a larger proportional fall in revenue, and so a larger proportional decrease in net worth. Therefore, the leverage of the financial sector $(QK - N^E)/N^F$ initially rises by more, which - for reasons analysed above - causes a bigger rise in the external finance premium in the contract between the financial institutions and the investors, and thus a bigger fall in investment. On the other hand, in the limit, if N^E approaches the value of the entire capital stock, the size of the financial sector shrinks to zero. The system then reduces to a simple Q-Ramsey economy in which there is no influence of financial frictions.

We have seen that a reduction in entrepreneurial net worth N^E relative to the stock of capital K reduces macroeconomic volatility. This effect could be counteracted by increasing the net worth of the financial sector. This is because a smaller N^E increases the leverage of the financial sector, $(QK - N^E)/N^F$, and a higher N^F reduces it. When a negative shock occurs, the larger self-financing of the financial sector acts as a buffer in the loan contract to the increase in volatility caused by the reduction in N^E. In particular, it can be shown that, if N^E falls, and at the same time N^F rises by an amount which keeps the steady state leverage of the financial sector identical to that in the baseline model, then macroeconomic volatility will remain unchanged.

5 Conclusion

The work of John Maynard Keynes has taught us that microeconomic policy reforms will only work well when the macroeconomic environment is supportive. In his biography of Keynes, first published in 1951, Roy Harrod describes a speech which he heard Keynes gave in 1924, about how to fix the Post-World-War-I recession in the UK. Harrod makes clear how unpersuasive Keynes' proposals were. They focused on microeconomic ideas, like, for example, a plan to subsidise particular forms of investment. But the main difficulties which Keynes were macroecnomic: caused by a shortage of aggregate demand resulting from the fact that the British economy was no longer competitive internationally. By contrast, says Harrod 'when we have Keynes's General Theory... in our hands [which was not finished until 12 years later], it is easy in retrospect to give a theoretical defence of the practical policy which [Keynes] outlined in 1924.' (Harrod, 1972, pp. 413-14)

At present a raft of microeconomic reforms to financial regulation are being discussed – reforms which we all want to be successful. But Keynes's experience suggests these reforms might not succeed until we know how to carry out macroprudential regulation. To do that will require us develop a macroeconomic theory which properly describes the effects on the macroeconomy of a financial sector which is both highly leveraged and risky. The present paper has responded to that challenge.

Our paper has explored how leverage within the financial system can make the economy more volatile in response to shocks. In our model, each financial institution faces the possibility of bankruptcy. This means that the interest rate which consumers require from the financial institutions to which they lend is subject to a risk premium. This risk premium is anti-cyclical, and therefore disturbance-amplifying, to an extent which depends on the degree of leverage of the financial institutions. In a downturn, caused, say by a reduction in productivity, the share price of firms will fall, increasing the likelihood of bankruptcy of the financial institutions which hold their shares, and causing the risk premium to rise, which is disturbance-amplifying. We have compared our findings with those of both BGG and HSU, and have shown that our setup is more disturbance-amplifying than either of those other setups.

In conclusion it is worth mentioning another important approach to this problem, coming from a study of the way in which financial institutions are connected to each other. When the crisis hit, another thing that financial institutions did, when seeking to re-build their balance sheets, was to seek to rebuild their position by withdrawing loans to other financial institutions, rather than merely by attempting to sell the shares of the firms in which they had invested, as is assumed in this paper. It has become apparent that 'network inter-connectedness' can mean that such a withdrawal of loans to other financial institutions can lead to a domino-effect operating within the financial system. This effect can operate as in the case of Northern Rock in which a withdrawal of funding led to the collapse of the institution. Or it can operate as in the case of Lehman Bros, in which the collapse of one institution can lead to the collapse of other institutions. Recent studies (e.g. by Haldane, 2009), focus on the fact that financial systems which appear to be in a satisfactory equilibrium state can collapse after what seems to be a small shock, as a result of such domino effects. Moore (2011) has produced similar results. Work by May et. al. (2007) and Haldane and May (2010) has shown how epidemiological models may be used to study the spread of financial contagion in this way. Such work is also important in showing how a small shock can lead to a big crisis within the financial system.

Such features could be included in the model which we have presented in this paper, but we have not yet done this.

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A Log-linearised Model Equations

This appendix reports the log-linearised form of the equations of the model. The equations in levels are reported as Equations (42) to Equation (56) in the main text. We denote X the steady state value of the variable X_t and $\hat{X}_t$ the percentage deviation of the variable from its steady state such that $\hat{X}_t = (X_t - X)/X$. The log-linearised equations are the following:

The consumption Euler equation:

$$\hat{C}_t - E_t\hat{C}_{t+1} + \hat{R}_{t+1} = 0 \tag{57}$$

The labour supply:

$$\hat{W}_t = \hat{C}_t + \varphi \hat{L}_{Ct} \tag{58}$$

Realised participation constraint for investors:

$$\begin{aligned} \hat{R}^F_t - \hat{R}_t &= \left(\frac{1}{K - N^E - N^F} - \frac{1}{K - N^E}\right)[K(\hat{Q}_{t-1} + \hat{K}_t) - N^E\hat{N}^E_t] \\ &\quad - \frac{N^F}{K - N^E - N^F}\hat{N}^F_t - \frac{\Psi^F_\omega}{\Psi^F}\bar{\omega}^F\hat{\bar{\omega}}^F_t \end{aligned} \tag{59}$$

The first order condition of financial institutions' profit maximisation:

$$0 = \frac{\Gamma^F_\omega}{\Gamma^F_\omega - \mu^F G^F_\omega}(E_t(\hat{R}^F_{t+1}) - \hat{R}_{t+1}) - (1 - \Gamma^F)\frac{R^F}{R}\left(\frac{\Gamma^F_{\omega\omega}}{\Gamma^F_\omega} - \frac{\Gamma^F_{\omega\omega} - \mu^F G^F_{\omega\omega}}{\Gamma^F_\omega - \mu^F G^F_\omega}\right)E_t\bar{\omega}\hat{\bar{\omega}}^F_{t+1} \tag{60}$$

Returns on holding shares:

$$\hat{R}^E_t = \hat{R}^F_t \tag{61}$$

The law of motion of financial instiuttions' net worth:

$$\begin{aligned} N^F\hat{N}^F_{t+1} &= \gamma^F(1 - \Gamma^F)R^F[(K - N^E)\hat{R}^F_t + K(\hat{Q}_{t-1} + \hat{K}_t) - N^E\hat{N}^E_t] \\ &\quad - \gamma^F R^F(K - N^E)\Gamma^F_\omega\bar{\omega}^F\hat{\bar{\omega}}^F_t + (1 - \alpha)\Omega_F Y\hat{Y}_t \end{aligned} \tag{62}$$

The law of motion of entrepreneurs' net worth:

$$\hat{N}^E_{t+1} = \gamma^E R^E(\hat{R}^E_t + \hat{N}^E_t) + (1 - \gamma^E R^E)\hat{Y}_t \tag{63}$$

Consumption by quitting financial institutions:

$$\hat{C}^F_t = \hat{R}^F_t + \frac{K}{K - N^E}(\hat{Q}_{t-1} + \hat{K}_t) - \frac{N^E}{K - N^E}\hat{N}^E_t - \frac{\Gamma^F_\omega}{1 - \Gamma^F}\bar{\omega}^F\hat{\bar{\omega}}^F_t \tag{64}$$

Consumption by quitting entrepreneurs:

$$\hat{C}^E_t = \hat{R}^E_t + \hat{N}^E_t \tag{65}$$

The production function:

$$\hat{Y}_t = \hat{A}_t + \alpha \hat{K}_t + (1-\alpha)(1-\Omega_E-\Omega_F)\hat{L}_{Ct} \tag{66}$$

Labour demand:

$$\hat{W}_t = \hat{Y}_t - \hat{L}_{Ct} \tag{67}$$

Capital demand:

$$R^E(\hat{Q}_{t-1} + E_{t-1}\hat{R}_t^E) = (R^E - (1-\delta))(\hat{Y}_t - \hat{K}_t) + (1-\delta)\hat{Q}_t \tag{68}$$

Capital accumulation:

$$\hat{K}_{t+1} = (1-\delta)\hat{K}_t + \delta \hat{I}_t \tag{69}$$

Investment decision:

$$\hat{Q}_t = -J''\delta(\hat{I}_t - \hat{K}_t) \tag{70}$$

Aggregate demand:

$$\begin{aligned} Y\hat{Y}_t &= C\hat{C}_t + C^E\hat{C}_t^E + C^F\hat{C}_t^F + \delta K\hat{I}_t + \mu^F R^F(K-N^E)G_\omega^F\bar{\omega}^F\hat{\bar{\omega}}_t^F \\ &\quad + \mu^F G^F R^F[(K-N^E)\hat{R}_t^F + K(\hat{Q}_{t-1} + \hat{K}_t) - N^E\hat{N}_t^E] \end{aligned} \tag{71}$$

The derivatives of the distribution functions are defined as follows:

$$F_\omega^F = \frac{1}{\sigma^F\bar{\omega}^F}\phi\left(\frac{\ln\bar{\omega}^F + \frac{1}{2}(\sigma^F)^2}{\sigma^F}\right) > 0 \tag{72}$$

$$G_\omega^F = \frac{1}{\sigma^F\bar{\omega}^F}\phi\left(\frac{\ln\bar{\omega}^F - \frac{1}{2}(\sigma^F)^2}{\sigma^F}\right) > 0 \tag{73}$$

$$\Gamma_\omega^F = 1 - F^F(\bar{\omega}^F) = 1 - \Phi\left(\frac{\ln\bar{\omega}^F + \frac{1}{2}(\sigma^F)^2}{\sigma^F}\right) > 0 \tag{74}$$

$$\Gamma_{\omega\omega}^F = -F_\omega^F = -\frac{1}{\sigma^F\bar{\omega}^F}\phi\left(\frac{\ln\bar{\omega}^F + \frac{1}{2}(\sigma^F)^2}{\sigma^F}\right) < 0, \tag{75}$$

where ϕ is the probability density function of the standard normal distribution. For $G_{\omega\omega}^F$, we make use of the fact that $\phi'(x) = -x\phi(x)$. Therefore,

$$\begin{aligned} G_{\omega\omega}^F &= -\frac{1}{\sigma^F(\bar{\omega}^F)^2}\phi\left(\frac{\ln\bar{\omega}^F - \frac{1}{2}(\sigma^F)^2}{\sigma^F}\right) + \frac{1}{\sigma^F\bar{\omega}^F}\phi'\left(\frac{\ln\bar{\omega}^F - \frac{1}{2}(\sigma^F)^2}{\sigma^F}\right)\frac{1}{\sigma^F\bar{\omega}^F} \\ &= -\frac{1}{\sigma^F(\bar{\omega}^F)^2}\phi\left(\frac{\ln\bar{\omega}^F - \frac{1}{2}(\sigma^F)^2}{\sigma^F}\right) - \frac{1}{\sigma^2(\bar{\omega}^F)^2}\phi\left(\frac{\ln\bar{\omega}^F - \frac{1}{2}(\sigma^F)^2}{\sigma^F}\right)\left(\frac{\ln\bar{\omega}^F - \frac{1}{2}(\sigma^F)^2}{\sigma^F}\right) \end{aligned} \tag{76}$$

The numerical values of these steady state parameters are reported in Table 4.

Table 4: Distributions regarding ω^F

Parameter	G^F	Γ^F	Ψ^F	F^F_ω	G^F_ω	Γ^F_ω
Value	0.0153	0.7968	0.7961	0.5651	0.4506	0.98
Parameter	$\Gamma^F_{\omega\omega}$	$G^F_{\omega\omega}$	Ψ^F_ω	$\Psi^F_{\omega\omega}$		
Value	-0.5651	10.802	0.9596	-1.0538		

B The HSU model

This appendix describes the credit contract and the general equilibrium of the HSU model which we use to compare with our baseline model. The HSU model features the 'chained financial contract'. It also involves the investors, financial institutions and the entrepreneurs. Financial institutions mediate funds between the entrepreneurs and the investors. However, in the HSU model, both contracts are loan contracts. Financial institutions need to pay a linear monitoring cost μ^E when an entrepreneur goes bankrupt. The financial institutions choose the contractual rates and loan amounts for both contracts subject to the participation constraints of the investors and the entrepreneurs.

The financial institutions' maximisation problem is the following:

$$\max_{\bar{\omega}^F,\bar{\omega}^E,K}(1-\Gamma^F(\bar{\omega}^F))R^F(QK-N^E) \tag{77}$$

where $(1-\Gamma^F(\bar{\omega}^F))$ is the borrower's (expected) share of profit in the contract, and $R^F(QK-N^E)$ is the gross return of the contract, defined by

$$R^F(QK-N^E)=\Psi^E(\bar{\omega}^E)R^EQK. \tag{78}$$

On the right hand side, $\Psi^E(\bar{\omega}^E)$ is the lender's share of profit in the contract after monitoring, and R^EQK is the gross return of the entrepreneurs' production.

To ensure the investors participate, the zero profit condition for the perfectly competitive investor must be satisfied:

$$\Psi^F(\bar{\omega}^F)R^F(QK-N^E)=R(QK-N^E-N^F) \tag{79}$$

where $\Psi^F(\bar{\omega}^F)$ is the lender's share of profit in the contract after monitoring, while the right hand side is the opportunity cost of the investors' lending. Investors sign contracts with a lot of banks, the idiosyncratic risks of the banks are diversifiable.

To ensure the entrepreneurs participate, there is a participation constraint for the entrepreneurs:

$$(1-\Gamma^E(\bar{\omega}^E))R^EQK\geq R^EN^E \tag{80}$$

The condition states that in the contract between the financial institutions and the entrepreneurs, the entrepreneurs' expected return is set to equal to the return from their alternative option. This condition is always binding.

The banks problem is therefore maximisation of Equation (77) subject to Equation (78), (79) and (80). The problem can be simplified to

$$\max_{\bar{\omega}^F, \bar{\omega}^E, K} (1 - \Gamma^F(\bar{\omega}^F))\Psi^E(\bar{\omega}^E)R^E QK \tag{81}$$

such that

$$\begin{aligned} \Psi^F(\bar{\omega}^F)\Psi^E(\bar{\omega}^E)R^E QK &= R(QK - N^E - N^F) && (82)\\ (1 - \Gamma^E(\bar{\omega}^E))QK &= N^E && (83) \end{aligned}$$

We write down the Lagrangian with λ_1 the Lagrangian multiplier for the investors' zero profit constraint and λ_2 the Lagrangian multiplier for the entrepreneurs' participation constraint:

$$\begin{aligned} L = &(1 - \Gamma^F(\bar{\omega}^F))\Psi^E(\bar{\omega}^E)R^E QK + \lambda_1[\Psi^F(\bar{\omega}^F)\Psi^E(\bar{\omega}^E)R^E QK - R(QK - N^E - N^F)] \\ &+\lambda_2[(1 - \Gamma^E(\bar{\omega}^E))QK - N^E] && (84) \end{aligned}$$

The first order conditions are

$$\begin{aligned} K &: (1 - \Gamma^F(\bar{\omega}^F))\Psi^E(\bar{\omega}^E)R^E + \lambda_1[R - \Psi^F(\bar{\omega}^F)\Psi^E(\bar{\omega}^E)R^E] = \lambda_2(1 - \Gamma^E(\bar{\omega}^E)) && (85)\\ \bar{\omega}^F &: -\Gamma^F_\omega(\bar{\omega}^F) + \lambda_1\Psi^F_\omega(\bar{\omega}^F) = 0 && (86)\\ \bar{\omega}^E &: (1 - \Gamma^F(\bar{\omega}^F))\Psi^E_\omega(\bar{\omega}^E)R^E + \lambda_1\Psi^F(\bar{\omega}^F)\Psi^E_\omega(\bar{\omega}^E)R^E - \lambda_2\Gamma^E_\omega(\bar{\omega}^E) = 0 && (87) \end{aligned}$$

Rearranging the last two equations to solve for the Lagrange multipliers,

$$\begin{aligned} \lambda_1 &= \frac{\Gamma^F_\omega(\bar{\omega}^F)}{\Psi^F_\omega(\bar{\omega}^F)} && (88)\\ \lambda_2 &= \left(\frac{(1 - \Gamma^F(\bar{\omega}^F))\Psi^E_\omega(\bar{\omega}^E)}{\Gamma^E_\omega(\bar{\omega}^E)} + \frac{\Gamma^F_\omega(\bar{\omega}^F)\Psi^F(\bar{\omega}^F)\Psi^E_\omega(\bar{\omega}^E)}{\Gamma^E_\omega(\bar{\omega}^E)\Psi^F_\omega(\bar{\omega}^F)}\right) R^E && (89) \end{aligned}$$

where λ_1 is the marginal value of the internal funds of the financial institution, and λ_2 is the marginal increase in the profits of the financial institutions per unit increase in entrepreneurial net worth. Substitute these back into the first order condition for K and rearrange:

$$\begin{aligned} \frac{R^E}{R} &= \frac{\lambda^E(\bar{\omega}^E)}{(1 - \Gamma(\bar{\omega}^E)) + \lambda^E(\bar{\omega}^E)(\Gamma(\bar{\omega}^E) - \mu^E G(\bar{\omega}^E))} \\ &\quad \times \frac{\lambda^F(\bar{\omega}^F)}{(1 - \Gamma(\bar{\omega}^F)) + \lambda^F(\bar{\omega}^F)(\Gamma(\bar{\omega}^F) - \mu^F G(\bar{\omega}^F))} && (90) \end{aligned}$$

where $\lambda^F(\bar{\omega}^F)$ and $\lambda^E(\bar{\omega}^E)$ are given by

$$\lambda^E(\bar{\omega}^E) = \frac{\Gamma^E_\omega(\bar{\omega}^E)}{\Gamma^E_\omega(\bar{\omega}^E) - \mu^E G^E_\omega(\bar{\omega}^E)} > 1, \qquad \lambda^E_\omega(\bar{\omega}^E) > 0 \tag{91}$$

$$\lambda^F(\bar{\omega}^F) = \frac{\Gamma^F_\omega(\bar{\omega}^F)}{\Gamma^F_\omega(\bar{\omega}^F) - \mu^F G^F_\omega(\bar{\omega}^F)} > 1, \qquad \lambda^F_\omega(\bar{\omega}^F) > 0 \tag{92}$$

The external finance premium can be written as the product of functions of $\bar{\omega}^E$ and $\bar{\omega}^F$. Notice that the structure of this first order condition is identical to that in the BGG contract except for the fact that Equation (90) contains one more shock that affects the external finance premium. Hence, HSU refer to this contract as the 'chained financial contract'.

The loan contract between financial institutions and entrepreneurs is shock-dampening. To see this, refer to the simulation of a permanent negative shock to productivity in Figure 9. The shock reduces entrepreneurs' leverage QK/N^E in the HSU model along the adjustment path. From the participation constraint of the entrepreneurs (80), a fall in entrepreneurs' leverage requires a rise in the share of profit that goes to the entrepreneurs, so the cutoff value for the entrepreneurs $\bar{\omega}^E$ has to fall. Since the $\bar{\omega}^E$ component on the right hand side of Equation (90) is increasing in $\bar{\omega}^E$, a fall in $\bar{\omega}^E$, *ceteris paribus*, reduces the external finance premium, which dampens a positive shock.

The net worth accumulation equations are

$$N^E_{t+1} = \gamma^E(1 - \Gamma^E(\bar{\omega}^E_t))R^E_t Q_{t-1}K_t + (1-\alpha)\Omega_E Y_t \tag{93}$$

$$N^F_{t+1} = \gamma^F \Psi^E(\bar{\omega}^E_t)(1 - \Gamma^F(\bar{\omega}^F_t))R^E_t Q_{t-1}K_t + (1-\alpha)\Omega_F Y_t \tag{94}$$

The aggregate demand has to include the resources used in the monitoring of entrepreneurs:

$$Y_t = C_t + C^E_t + C^F_t + I_t + \mu^E G^E(\bar{\omega}^E_t)R^E_t Q_{t-1}K_t \tag{95}$$

$$+\mu^F G^F(\bar{\omega}^F_t)R^F_t(Q_{t-1}K_t - N^E_t) \tag{96}$$

The other equations in the general equilibrium are unchanged.

To make a fair comparision, the general equilibrium parameter values used in the HSU model are the same as ours and are reported in Table 1. We calibrate the credit contract parameter values to satisfy the same set of steady state relations in our baseline model described in Section 4.1. Two more steady state relations are required to calibrate the HSU model: (1) The annualised failure rate of entrepreneurs is 2%; and (2) the spread between the financial institutions' loan rate (Z^E) and the borrowing rate (Z^F) is 230 basis points annually. The credit contract parameteres and steady state values used in our calibration of the HSU model are reported in Table 5 and Table 6.

Table 5: Credit contract Parameters and Steady State Values in HSU

Parameter	$\bar{\omega}^E$	$\bar{\omega}^F$	σ^E	σ^F	μ^E	μ^F	γ^E	γ^F
Value	0.5010	0.7974	0.3127	0.1074	0.0263	0.0241	0.9837	0.9629
Parameter	G^E	G^F	Ψ^E	Ψ^F	Γ^E	Γ^F	Ψ'^E	Ψ'^F
Value	0.009	0.0153	0.4998	0.7964	0.5	0.7968	0.9759	0.9692

Table 6: Steady State variables in HSU

Parameter	Y	C	C^E	C^F	I	W	L_C	K
Value	100	67.2264	7.2401	3.3407	21.8199	26.8863	2.3692	872.796
Parameter	N^E	N^F	R	R^E	R^F	Z^E	Z^F	
Value	436.398	87.2796	1.0101	1.0151	1.0146	1.0172	1.0113	

C The BGG model

We describe the BGG model in this appendix. It is necessary to do this, even although this model is well known, because the BGG setup adopted here differs from the published BGG setup in four ways, all of which are necessary to make possible a fair comparison with our model. First, we assume all goods prices are flexible, since nothing in BGG is intrinsically dependent on the existence of nominal rigidities. Second, we follow the formulation of labour disutility used in our baseline model. Third, we set the macroeconomic parameter values equal to those in our baseline model. Finally, as explained, we change the way in which $\bar{\omega}^E$ is treated, where $\bar{\omega}^E$ is the level of the idiosyncratic productivity shock below which entrepreneurs are bankrupt. See footnote 14.

As explained in the main text, in the BGG model, there is no financial friction between investors and financial institutions. We may think of entrepreneurs as financed directly by loans from the consumers. The external finance premium reflects only entrepreneurial riskiness, and the financial accelerator comes only from this effect.

In the BGG credit contract, entrepreneurs maximise their expected revenue

$$\max_{\bar{\omega}^E, K}(1 - \Gamma^E(\bar{\omega}^E))R^E QK \tag{97}$$

subject to the participation constraint to the investors:

$$(\Gamma^E(\bar{\omega}^E) - \mu^E G(\bar{\omega}^E))R^E QK = R(QK - N^E) \tag{98}$$

The first order conditions are

$$\bar{\omega}^E \quad : \quad \lambda(\bar{\omega}^E) = \frac{\Gamma^E_\omega(\bar{\omega}^E)}{\Gamma^E_\omega(\bar{\omega}^E) - \mu^E G^E_\omega(\bar{\omega}^E)}, \qquad \lambda_\omega(\bar{\omega}^E) > 0 \tag{99}$$

$$K \quad : \quad \frac{R^E}{R} = \frac{\lambda(\bar{\omega}^E)}{\lambda(\bar{\omega}^E)(\Gamma^E(\bar{\omega}^E) - \mu^E G^E(\bar{\omega}^E)) + (1 - \Gamma^E(\bar{\omega}^E))} \tag{100}$$

The nature of the BGG contract is identical to the loan contract presented in the main text, and the intuitions are the same. The only difference is that in our baseline model the loan contract occurs between the investors and the financial institutions whereas in the BGG model, the loan contract occurs between investors and entrepreneurs.

The net worth accumulation of the entrepreneurs is

$$N^E_{t+1} = \gamma^E(1 - \Gamma^E(\bar{\omega}^E_{t+1}))R^E_t Q_{t-1}K_t + (1-\alpha)\Omega_E Y_t \tag{101}$$

where the second term is the 'enterpeneurial wage'. The consumption by quiting entrepreneurs is given by:

$$C^E_t = (1 - \gamma^E)(1 - \Gamma^E(\bar{\omega}^E_{t+1}))R^E_t Q_{t-1}K_t \tag{102}$$

The aggregate demand equation now has to account for the consumption by quiting entrepreneurs and the resource wasted by monitoring in the BGG loan contract:

$$Y_t = C_t + C^E_t + I_t + \mu^E G^E(\bar{\omega}^E_t)R^E_t Q_{t-1}K_t \tag{103}$$

Since there is not any separate financial institutinos, we no longer need any supply of labour by the financial institutions. We set the parameter Ω_F to be zero. Therefore, the production function and the demand of labour are given by

$$Y_t = A_t K^\alpha_t L^{(1-\alpha)(1-\Omega_E)}_{Ct} \tag{104}$$

$$W_t = (1-\alpha)(1-\Omega_E)\frac{Y_t}{L_{Ct}} \tag{105}$$

The other equations in the general equilibrium are unchanged.

We calibrate the credit contract parameters using the first two steady state relations in our baseline calibration, that is (i) a steady state external finance premium of 2% per annum; (ii) a steady state entrepreurial leverage of 2; and together with (iii) an entrepreneurial failure rate of 2%. The credit contract parameters and steady state values used in our calibration of the BGG model are reported in Table 7 and 8.

Table 7: Credit contract Parameters and Steady State Values in BGG

Parameter	$\bar{\omega}^E$	σ^E	μ^E	γ^E	G^E	Γ^E	G^E_ω	Γ^E_ω
Value	0.4991	0.3143	0.0561	0.9798	0.0089	0.4980	0.1540	0.98

Table 8: Steady State variables in BGG

Parameter	Y	C	C^E	I	W
Value	100	68.7581	8.9774	21.82	27.4139
Parameter	L_C	K	N^E	R	R^E
Value	2.3473	872.80	436.40	1.0101	1.0151