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ABSTRACT

Strategic Asset Allocation in Money Management*

Money managers behave strategically when competing for fund flows within relatively small groups. We study strategic interaction between two risk-averse managers in continuous time, characterizing analytically their unique equilibrium dynamic investments. Driven by chasing and contrarian mechanisms when one is well ahead, they gamble in the opposite direction when their performances are close. We also discuss multiple and mixed-strategy equilibria. Equilibrium policy of each crucially depends on the opponent's risk attitude. Hence, client investors, concerned about how a strategic manager may trade on their behalf, should also learn competitors' characteristics--as against non-strategic settings, where knowing a manager's own characteristics suffices to determine behavior.

JEL Classification: C61, C73, D81, G11 and G20
Keywords: fund flows, incentives, money managers, portfolio choice, relative performance, risk shifting, strategic interactions and tournaments

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1. Introduction

This paper analyzes the dynamic portfolio strategies of money managers in the presence of strategic interactions, arising from each manager's desire to perform well relative to the other managers.[1] Given the prevalent finding in money management that the money flows to relative performance relationship is increasing, a fund manager has incentives to outperform the peers so as to increase her assets under management, and hence her compensation.[2] A common explanation for this relationship between flows and relative performance is that many client investors choose a fund based primarily on widely published fund rankings (*MorningStar*, *Forbes*, etc). Money managers may also care about their relative standing due to psychological aspects, such as envy or crave for higher social status.

When discussing the interaction of fund managers in the presence of relative performance concerns, Brown, Harlow, and Starks (1996) appeal to the notion of a *tournament* in which managers are the competitors and money flows are the prizes awarded based on relative ranking. Since then, a large empirical literature addresses the tournament hypothesis by looking at how risk taking behavior responds to relative performance. While several recent theoretical studies attempt to analyze mutual fund tournaments, their results are obtained under fairly specialized economic settings (as discussed below). To our best knowledge, ours is the first comprehensive analysis of the portfolio choice effects of strategic interactions within a workhorse dynamic asset allocation framework, allowing us to derive a rich set of implications. The effects of strategic considerations are likely to be the strongest when there is a small number of funds competing against other. Despite there being thousands of mutual and hedge funds, there are various cases when the competition occurs within relatively small groups, such as when several top-performing funds compete for leadership or when competition happens within a fund family comprised of a small number of funds (as elaborated in Section 2.2).

We consider two risk averse money managers, and adopt a familiar continuous-time economy for investment opportunities with multiple correlated risky stocks. We assume constant relative risk aversion (CRRA) preferences for a normal manager with no relative performance concerns. We model relative performance concerns by postulating that a manager experiences money flows which depend (positively) on the ratio of her horizon investment return over the other manager's return. We consider a convex option-like flow-performance

[1] The term "money manager" refers to a mutual or hedge fund manager investing in a wide range of assets, and not only in money market instruments as the word "money" might suggest.

[2] There is a large literature documenting that higher relative performance leads to more money flows, including Chevalier and Ellison (1997), Sirri and Tufano (1998) for individual mutual funds, Gallaher, Kaniel, and Starks (2006) for mutual fund families, Agarwal, Daniel, and Naik (2004), Ding, Getmansky, Liang and Wermers (2010) for hedge funds. Relative concerns may also arise within fund families as funds with high relative performance are likely to be advertised more (Jain and Wu (2000)) and also to be cross-subsidized at the expense of other family members (Gaspar, Massa, and Matos (2006)).

specification, consistent with the empirical evidence on fund flows. If the manager's relative performance is above a certain threshold, she receives money flows increasing with performance, and hence displays relative performance concerns. Otherwise if she performs relatively poorly, she receives no (or constant) flows and her objectives are as for a normal manager with no relative concerns. In characterizing managers' behavior, we appeal to the pure-strategy Nash equilibrium concept, in which each manager strategically accounts for the dynamic investment policies of the other manager, and the equilibrium policies of the two managers are mutually consistent.

Solving for the managers' best responses reveals that a manager only chooses two outcomes at the horizon: "winning" by outperforming the other manager and attracting flows, or "losing" by underperforming and getting no flows, and never opting for a "draw." This is due to the local convexity around the performance threshold, inducing the manager to gamble so as to avoid a draw. Moreover, we show that an important feature of a manager when outperforming, and hence displaying relative performance concerns, is that she either "chases" the opponent, increasing her investment policy in response to the opponent's increasing hers, or acts as a "contrarian," decreasing her investments in response to the opponent's increasing hers. In our formulation, a manager is a chaser if her risk aversion coefficient is greater than unity (more empirically plausible), and is a contrarian otherwise.

Moving to equilibrium, we demonstrate that the gambling behavior caused by local convexities leads to the potential non-existence of a pure-strategy Nash equilibrium, since when both managers' performances are close to their thresholds (in the convex region), they cannot agree on who the winner is. Such a situation occurs when the managers' attitude towards risk are considerably different, where one manager may want to outperform the other by just a little to become a winner, while the other manager wants to underperform by a lot to be content with losing. If on the other hand the managers' risk aversions are sufficiently similar, an equilibrium obtains, with one manager emerging as a winner and the other as a loser. However, since the managers are very similar, the opposite outcome may also occur, where the earlier winner and loser positions are switched, giving rise to multiplicity. We show that both of these non-existence and multiplicity issues are resolved when the performance threshold is sufficiently high. This is because with a high threshold, the two managers cannot both be close to their thresholds and gamble at the same time, which is the main cause for the non-existence and multiplicity. This unique equilibrium is more likely to exist for higher values of risk aversions or lower money flow elasticities as this dampens the gambling incentives.

We investigate multiplicity and non-existence of pure equilibrium in detail because certain aspects of the managers' behavior in these two cases may well be of interest to the client investors and regulators. Regarding non-existence, the managers are likely to resort to mixed strategies in which case the risk-taking implications are notably different from those obtained when a pure equilibrium exists, as we explain below. Under multiplicity, investors cannot reliably predict their returns for a given realization of stock prices, indicating a potential role for regulation.[3] We believe that our analysis is the first to identify an important economic role played by the performance threshold and risk aversion heterogeneity in determining whether a pure or mixed equilibrium occurs. In related non-strategic works where the gambling behavior is present (Carpenter (2000), Basak, Pavlova, and Shapiro (2007)), performance thresholds are also present, however in these studies optimal portfolios are unique, and moreover are not much affected qualitatively by the threshold. We uncover that the threshold has a much greater impact on the managers' behavior once the setting is strategic.

When equilibrium is unique, we provide a full characterization of the equilibrium investment policies. The two managers' investments at any interim point in time depend on their performance relative to each other. If a manager significantly outperforms the opponent, both managers are far from the convex region of their objectives and so the chasing and contrarian behaviors dominate the gambling incentives. In particular, when outperforming a chaser moves her investment policy *towards* the opponent's normal policy, while a contrarian tilts her policy *away* from the opponent's normal policy. For the underperforming manager, the relative concerns are weak since the likelihood of her attracting money flows at year-end is low, and hence her equilibrium policy is close to the normal policy. When both managers' performance is close to the threshold, the gambling incentives dominate the chasing and contrarian behaviors, inducing the two managers to gamble in the opposite direction from each other in each individual stock. The exact direction of a manager's gambling in each stock is determined by the relation between the managers' risk aversions, the stock Sharpe ratios and correlations. Considering a setting with two risky stocks and looking at the effect of stock correlation, we uncover rich patterns of the managers' behavior, arising due to the interaction of a diversification effect, the extent of benefits to diversification, and a substitution effect, the extent of the stocks acting as a substitute to each other. A novel result here is that due to the presence of competition the managers' account for each others' actions when responding to a given change in stock correlation. For example, if, having observed that the correlation has changed, one manager decides to change the direction of gambling in one stock then the opponent reacts accordingly by also changing the direction of gambling in that stock.

Since the above results are concerned with individual stocks, they may not readily be testable in the data given that mutual and hedge funds do not typically disclose their dynamic portfolio holdings. To provide testable implications of our model, we express our results in

[3]When suggesting that multiple equilibria may be associated with regulation, we rely on several branches of the literature where the link between multiplicity and role for regulation is well established. Examples include work on bank runs (Diamond and Dybvig (1983)), currency attacks (Obstfeld (1986, 1996)), debt crises (Calvo (1988)), among others. It remains to be investigated how problematic, from the viewpoint of regulators, the multiplicity is in the context of fund competition.

terms of portfolio volatilities, consistent with extant empirical works on fund tournaments which rely on volatilities. We show that an outperforming manager does not choose a normal portfolio volatility but rather, if a chaser, moves her volatility from normal towards the opponent's normal volatility, or if a contrarian, away from the opponent's normal volatility. When underperforming, a manager chooses close to normal portfolio volatility. When the managers' performances are similar, the gambling incentives induce the more risk averse manager to reduce her risk by choosing a lower than normal volatility while inducing the more risk tolerant manager to increase her risk by choosing a higher than normal portfolio volatility.

Above equilibrium investments determine whether a manager ends up as a winner or as a loser at the investment horizon depending on economic conditions. In good states, the more risk averse manager performs worse (consistent with normal behavior) and hence is a loser, while the less risk averse manager with the better performance is a winner, getting the money flows. In bad states, the opposite holds, with the more risk averse manager emerging as the winner, and the less risk averse as the loser. In intermediate states, around their performance thresholds, both managers are losers, not getting any flows. Multiple equilibria obtains when the performance threshold is low, which rules out the unique equilibrium outcome of both managers being losers in intermediate states. Each manager now wants to differentiate herself from the opponent, and so both outcomes, with either manager being a winner and the other a loser, constitute a (multiple) equilibrium. In good and bad states the multiple equilibrium outcomes are as in the unique equilibrium.

To shed some light on the managers' behavior when a pure strategy equilibrium does not exist, we characterize a mixed strategy equilibrium in a binomial version of our model. Focusing on the situation where both managers are driven by gambling incentives (i.e., their performances are similar), we find that in a mixed equilibrium the more risk averse manager does not randomize and increases her risk exposure relative to normal. The more risk tolerant manager randomizes between two risk exposures, one higher and one lower than normal. Comparing these results with the empirical implications described above for the case when a pure equilibrium exists, we see that the managers' (observable) behavior in the mixed equilibrium is notably different from that in the pure equilibrium, implying that the issue of existence of a pure equilibrium is relevant not only from a theoretical but also from a practical perspective.

Our paper is related to several strands of literature. Motivated by the empirical works on fund tournaments (Brown, Harlow, and Stark (1996), Busse (2001), Qiu (2003), Goriaev, Nijman, and Werker (2005), Reed and Wu (2005)), there is a theoretical literature on the effects of strategic considerations on portfolio managers' choices. Most of these works adopt single- or two-period settings and often assume risk neutrality (Goriaev, Palomino and Prat (2003), Taylor (2003), Palomino (2005), Li and Tiwari (2005), Chiang (1999), Loranth and

Sciubba (2006)). Our goal is to characterize optimal portfolios in a standard dynamic asset allocation setting with risk averse managers. We find risk aversion to be the critical driving factor in much of our analysis, including chasing/contrarian behavior, risk shifting, existence of equilibrium. In a dynamic setting like ours, Browne (2000) investigates a portfolio game between two managers. He primarily focuses on the case when the managers face different financial investment opportunities and have practical objectives (e.g., maximizing the probability of beating the other manager). None of these objective functions display local convexities, which we find to have important implications for equilibrium investment policies as well as for the number of equilibria.

If a peer group consists of a large number of competing funds, strategic interactions are likely to be less pronounced. In this case, the behavior of each fund manager is better described by assuming that she seeks to perform well relative to an exogenous benchmark. The manager's behavior in this case has been recently investigated in Basak, Shapiro, and Tepla (2006), van Binsbergen, Brandt, and Koijen (2007), Cuoco and Kaniel (2010). Several works, including Carpenter (2000), Basak, Pavlova, and Shapiro (2007), have demonstrated that convexities in managers' objective functions have significant implications for the optimal portfolios, leading to risk shifting behavior. We contribute to this non-strategic literature by recovering the following novel implications. First, strategic interactions can lead to multiple equilibria (potentially calling for regulation) or to a mixed-strategy equilibrium in which case the managers behave quite differently relative to the pure equilibrium case. Second, when considerably ahead a manager does not opt for a normal portfolio volatility, as might be expected, but rather either increases or decreases her portfolio volatility relative to normal. Third, when their performances are close the managers gamble by moving their portfolio volatilities in the opposite direction from each other. Fourth, changing the correlation between risky stock returns leads to rich patterns of behavior as the managers strategically account for each others' actions when rebalancing their portfolios, e.g., when one manager changes the direction of gambling in one stock the other does as well. In all above points, how exactly a manager adjusts her investments relative to normal crucially depends on her risk aversion *relative* to that of the opponent, which is unlike non-strategic works where it typically suffices to know a manager's own characteristics to determine behavior. This has important implications for client investors concerned about how a strategic manager may trade on their behalf – the client needs to learn the characteristics not only of the manager but also of her competitors.

Finally, our paper is related to the literature that examines the role of relative wealth concerns in finance. DeMarzo, Kaniel, and Kremer (2007, 2008) show that relative wealth concerns may play a role in explaining financial bubbles and excessive real investments. These papers are close in spirit to our work since they also demonstrate how relative wealth concerns may arise endogenously. Abel (1990), Gomez, Priestley, and Zapatero (2009), among many others, demonstrate that models with the "catching-up-with-the-Joneses" feature can explain various empirically observed asset pricing phenomena. Goel and Thakor (2005) investigate how envy leads to corporate investment distortions.

Remainder of the paper is organized as follows. Section 2 presents the economic set-up and provides the money flows justification for relative performance concerns. Section 3 describes the managers' objective functions and characterizes their best responses. Section 4 analyzes the issues of non-existence, uniqueness, and multiplicity of equilibrium. Section 5 characterizes the unique equilibrium and investigates the properties of the equilibrium investment policies. Section 6 describes the managers' behavior in multiple and mixed-strategy equilibrium. Section 7 concludes. Proofs are in the Appendix.

2. Economy with Strategic Asset Allocation

2.1. Economic Set-Up

We adopt a familiar dynamic asset allocation framework along the lines of the standard Merton (1969) set-up. We consider a continuous-time, finite horizon $[0, T]$ economy, in which the uncertainty is driven by an N-dimensional standard Brownian motion $\omega = (\omega_{1}, \dots, \omega_N)^\top$. Financial investment opportunities are given by a riskless bond and N correlated risky stocks. The bond provides a constant interest rate r. Each stock price, S_j, follows a geometric Brownian motion

$$dS_{jt} = S_{jt}\mu_j dt + S_{jt}\sum_{k=1}^{N}\sigma_{jk}d\omega_{kt}, \qquad j = 1, \dots, N,$$

where the stock mean returns $\mu \equiv (\mu_1, \dots, \mu_N)^\top$ and the nondegenerate volatility matrix $\sigma \equiv \{\sigma_{jk},\ j, k = 1, \dots N\}$ are constant,[4] and the (instantaneous) correlation between stock j and ℓ returns is given by $\rho_{j\ell} = \sum_{k=1}^{N}\sigma_{jk}\sigma_{\ell k}/\sqrt{\sum_k \sigma_{jk}^2 \sum_k \sigma_{\ell k}^2}$.

Each money manager i in this economy dynamically chooses an investment policy ϕ_i, where $\phi_{it} \equiv (\phi_{i1}, \dots, \phi_{iN})^\top$ denotes the vector of fractions of fund assets invested in each stock, or the *risk exposure*, given initial assets of W_{i0}. The investment wealth process of manager i, W_i, follows

$$dW_{it} = W_{it}[r + \phi_{it}^\top(\mu - r\bar{1})]dt + W_{it}\phi_{it}^\top\sigma d\omega_t, \tag{1}$$

[4]While it may be of interest to consider a more general setting with a stochastic investment opportunity set, the analytical tractability in the current framework would be lost (as would also be the case for non-strategic models). To characterize the dynamic equilibrium portfolio policies in such a setting, one would need to resort to numerical methods, such as those proposed by Detemple, Garcia, and Rindisbacher (2003) and Cvitanic, Goukasian, and Zapatero (2003). We leave this for future work.

where $\bar{1} = (1, \ldots, 1)^\top$. Dynamic market completeness (under no-arbitrage) implies the existence of a unique state price density process, ξ, with dynamics $d\xi_t = -\xi_t r dt - \xi_t \kappa^\top d\omega_t$, where $\kappa \equiv \sigma^{-1}(\mu - r\bar{1})$ is the constant N-dimensional market price of risk in the economy. The state-price density serves as the driving economic state variable in a manager's dynamic investment problem absent any market imperfections. The quantity $\xi_t(\omega)$ is interpreted as the Arrow-Debreu price per unit probability $\mathcal{P}$ of one unit of wealth in state $\omega \in \Omega$ at time t. In particular, each manager's dynamic budget constraint (1) can be restated as (e.g., Karatzas and Shreve (1998))

$$E[\xi_T W_{iT}] = W_{i0}. \tag{2}$$

This allows us to equivalently define the set of possible investment policies of managers as being the managers' horizon wealth, W_{iT}, subject to the static budget constraint (2).

2.2. Modeling Strategic Interaction

Motivated by empirical findings, we consider the situation where the presence of relative concerns within a group of money managers leads to their strategic interaction, which is likely to be most pronounced when the peer group is comprised of a small number of managers. Looking at US data, Chevalier and Ellison (1997) document strong gambling incentives among top-performing mutual funds which, they suggest, indicates that funds compete for the year-end leadership. For UK funds, Jans and Otten (2008) present evidence of strategic behavior after 1996 by finding that fund managers take actions of their peers into account rather than treating them as exogenous benchmarks (strategic interactions appeared to be absent before 1996). In the context of fund families, Kempf and Ruenzi (2008) document that mutual fund managers belonging to families with a small number of funds behave notably different from managers belonging to large families. They argue that this result is driven by strategic interactions, which are strong in small fund families but less pronounced in large families. Del Guercio and Tkac (2008) investigate how money flows respond to Morningstar rating changes and find "a more complicated and dynamic strategic environment for fund managers than assumed in current tests of managerial incentives and behavior." They conclude that "a theoretical model of managerial incentives in such a [dynamic] environment could point the way toward new empirical tests of strategic managerial behavior." To our knowledge, our paper is the first to provide such a model.

Before a formal description, we briefly discuss how strategic interaction emerges in our setting. We envision a group of money managers, interpreted as mutual or hedge fund managers, where each manager seeks to increase the terminal value of her portfolio. This is consistent with maximizing her own compensation given the widespread use of the linear fee structure in the money management industry. The key feature of our setting is that the manager experiences money flows which depend on her relative performance within a peer group, implying that the managers *rationally* care about their relative performance. Strategic interactions arise as the managers attempt to outperform each other so as to attract a larger share of the flows. We now present a formal description of the above environment.

In the presence of relative concerns, the objective function of manager i has the general form

$$v_i(W_{iT}, R_{iT}), \tag{3}$$

where v_i is increasing in horizon wealth, W_{iT}, and horizon relative return, R_{iT}. We consider a framework with two fund managers, indexed by $i = 1, 2$. The relative returns of managers 1 and 2, R_{1T} and R_{2T}, are defined as the ratio of the two managers' time-T investment returns:

$$R_{1T} = \frac{W_{1T}/W_{10}}{W_{2T}/W_{20}}, \quad R_{2T} = \frac{W_{2T}/W_{20}}{W_{1T}/W_{10}}. \tag{4}$$

We normalize both managers' initial assets to be equal, $W_{10} = W_{20}$, without loss of generality.

We now rationalize the particular form of the objective function (3) to be used in our analysis. Towards this, we take the empirically observed fund-flows to relative performance relation and show that the objective (3) arises endogenously under this relation in a setting where managers care directly only about their own wealth (as established in Lemma 1). Chevalier and Ellison (1997) document that for top-performing mutual funds the flow-performance relation is roughly flat until a certain threshold and then increases sharply. Similarly, Ding, Getmansky, Liang, and Wermers (2010) find that the flow-performance relation for defunct hedge funds exhibits a convex shape as the flows shoot up in the region of high past performance. This evidence is relevant for our model because the top-performing funds, which is our focus, are likely to be defunct. Indeed, as discussed in Ding et al., top performers no longer need to advertise themselves to potential investors, and so may choose to become defunct by not reporting the information to data vendors. Complementing the evidence of convexity in money flows, Koijen (2008) argues that fund managers' objective functions also feature convexities as he finds that, among several objective functions, the only function leading to plausible estimates of risk aversion is the one featuring local convexities.

A manager at time T experiences money flows at a rate f_T depending on her relative performance over the period $[0, T]$, where T can be thought of as year-end. In assuming that f_T depends only on relative performance at the horizon T, and not at interim dates, we rely on the prevailing view that "the most critical rankings are based on annual performance" (Brown, Harlow, and Starks (1996)). Based on the above evidence, we consider a flow-performance relation f_T that resembles the convex payoff profile of a call option – it is flat until a certain *performance threshold* η and then increases with relative performance – given by $f_T = k\mathbb{1}_{\{R_{iT}<\eta\}} + k(R_{iT}/\eta)^\alpha \mathbb{1}_{\{R_{iT}\geq\eta\}}$, with $f_T > 1$ denoting an inflow and $f_T < 1$ an outflow. In this specification, α denotes the *flow elasticity*, the elasticity of money flows with respect to relative performance when it is above the threshold η. The parameter k reflects the idea that a manager, being a top-performer, may experience money inflows even when her relative performance is below η, which can be captured by setting $k > 1$. The presence of the performance threshold η reflects inertia on the part of client investors who award a fund with money flows only after this fund outperforms the opponent by a certain margin.[5] Huang, Wei, and Yan (2007) formally show that inertia is consistent with investor rationality as they find that inertia endogenously arises when rational investors face information acquisition or transaction costs. The absence of inertia corresponds to $\eta = 1$, and the stronger the inertia the higher the performance threshold η. For completeness, we comment on the case of $\eta < 1$ in Remark 1.

The manager, after receiving flows f_T at time T, continues to invest beyond date T until an investment horizon T', $T' > T$, interpreted as expected tenure or compensation date. Manager i, $i = 1, 2$, is assumed to have CRRA preferences defined over the overall value of assets under management at time T':

$$u_i\left(W_{T'}\right) = \frac{\left(W_{T'}\right)^{1-\bar{\gamma}_i}}{1-\bar{\gamma}_i}, \quad \bar{\gamma}_i > 0, \bar{\gamma}_i \neq 1. \tag{5}$$

Manager i then maximizes the expected value of (5) which is equivalent to maximizing the time-T indirect utility function v_{iT}, defined as

$$v_{iT} \quad \equiv \quad \max_{\phi_i} E_T\left[u_i\left(W_{iT'}\right)\right]$$

subject to the dynamic budget constraint (1) for $t \in [T, T']$, given the time-T assets value augmented by money flows, $W_{iT} f_T$. Lemma 1 presents the time-T indirect utility function for our flow-performance specification.

Lemma 1. *For the flow-performance function $f_T = k\mathbb{1}_{\{R_{iT}<\eta\}} + k(R_{iT}/\eta)^{\alpha}\mathbb{1}_{\{R_{iT}\geq\eta\}}$, $\alpha > 0$, the time-T indirect utility function is*

$$v_{iT} = \begin{cases} \frac{k^{1-\bar{\gamma}_i}}{1-\gamma_i} W_{iT}^{1-\bar{\gamma}_i} & R_{iT} < \eta \\ \frac{k^{1-\bar{\gamma}_i}\eta^{-\theta(1-\gamma_i)}}{1-\gamma_i}\left(W_{iT}^{1-\theta} R_{iT}^{\theta}\right)^{1-\gamma_i} & R_{iT} \geq \eta, \end{cases} \tag{6}$$

where $\theta = \alpha/(1+\alpha), \gamma_i = \bar{\gamma}_i + \alpha(\bar{\gamma}_i - 1)$, with the properties that $\theta \in [0, 1)$, $\gamma_i > \bar{\gamma}_i$ if and only if $\bar{\gamma}_i > 1$.

Lemma 1 quantifies how the manager's risk attitude interacts with the shape of the flow-

[5] A real-life example of the use of thresholds is provided by the following description of the investment strategy of DAL Investment Company with 2 billion under management: "We rank them [mutual funds] based on the average performance ... Then we review the rankings to select the funds that are in the top 10% of their risk category for the portfolio. We have specific sell thresholds, and when they reach the threshold, we replace them with the current leaders."

performance relation to determine her objective function v_{iT}. When the manager performs relatively poorly, she gets performance-insensitive money flows, and so inherits the objectives of a *normal manager* with no relative performance concerns, given by the standard CRRA utility. When the manager's relative return is above the performance threshold however, she gets money flows increasing with performance, and hence displays relative concerns, with the parameter θ capturing the manager's *relative performance bias*, the extent to which she biases her objectives towards relative performance concerns. The special case of $\theta = 0$ (or equivalently $\alpha = 0$) corresponds to a normal manager. From (6), we also observe that the manager's attitude towards risk changes in the presence of money flows. Indeed, while $\bar{\gamma}_i$ represents the manager's *intrinsic risk aversion* (over terminal wealth $W_{iT'}$), the parameter γ_i captures her *effective risk aversion* (over the composite $W_{iT}^{1-\theta} R_{iT}^{\theta}$) in the presence of relative performance concerns.[6] For our subsequent maximization problems to be well-defined, we assume that γ_i is positive. This is true if and only if $\bar{\gamma}_i > 1 - 1/(1+\alpha)$, which always holds for a relatively risk averse manager $\bar{\gamma}_i > 1$. Moreover, the manager's attitude towards risk is increased ($\gamma_i > \bar{\gamma}_i$) by the presence of money flows ($\alpha > 0$) for intrinsic risk aversions greater than unity ($\bar{\gamma}_i > 1$), and is decreased for intrinsic risk aversions less than unity.

In what follows, we assume that each manager knows the opponent's objective function. This appears reasonable given that we focus on the very top managers whose portfolio strategies are often in the spotlight of the analysts and financial media. Moreover, parameters of the managers' objective functions can be estimated from the past performance data, as demonstrated by Koijen (2008).

2.3. Nash Equilibrium Policies

In this paper, we appeal to the Nash equilibrium notion to characterize managers' behavior in their strategic interaction via relative performance concerns. Below, we define the structure of the game between managers at time 0, where each manager draws up a plan of how she is going to invest throughout the whole time period $[0, T]$. Outlining the game initially is for expositional ease, since as we establish later (Remark 2), neither manager would want to deviate from a policy chosen initially at any subsequent date t, implying that the equilibrium investment policies are time-consistent.

Information sets. We consider a complete information game at time 0 where each manager knows all primitives and parameters of the model described in Sections 2.1–2.2, namely, the processes for the security prices, own initial wealth and that of the opponent, own risk aversion and relative performance bias and those of the opponent.

[6] The difference between the two risk aversion parameters stems from the fact that manager i assesses a gamble ($W_{iT}+\epsilon$, $W_{iT}-\epsilon$) differently in cases with and without money inflows. In the latter case, ϵ represents an actual change in wealth . In the former case, changing wealth by ϵ leads to money in- or outflows and so effectively the manager faces a different gamble.

Strategy sets. A strategy of manager 1 is a function $\phi_1(t, W_{1t}, W_{2t})$ defined over the space $[0, T] \times (0, +\infty) \times (0, +\infty)$, where $\phi_1(t, W_{1t}, W_{2t})$ is manager 1's investment policy at time t for given values of her own time-t wealth, W_{1t}, and that of the opponent, W_{2t}. The strategy set of manager 1 is the set of all such functions $\phi_1(t, W_{1t}, W_{2t})$. Similarly, manager 2's strategy set is the set of functions $\phi_2(t, W_{1t}, W_{2t})$ defined over $[0, T] \times (0, +\infty) \times (0, +\infty)$. For convenience, we use ϕ_{it} as shorthand notation for manager i's time-t investment policy and drop its arguments.

Managers' payoffs. The managers' payoffs for a policy pair (ϕ_1, ϕ_2) are given as follows. First, the horizon wealth profiles are obtained by substituting ϕ_1 and ϕ_2 into the dynamic wealth processes (1) of manager 1 and 2, respectively. Given these wealth profiles, the managers' expected objective functions (6) are computed, yielding their payoffs for (ϕ_1, ϕ_2).

In order to define a Nash equilibrium, we first introduce the best response policies. Throughout the paper, a symbol with a hat $\hat{}$ denotes an optimal best response quantity, while one with an asterisk * denotes an equilibrium quantity.

Definition 1. *For a given manager 2's dynamic policy ϕ_2, manager 1's best response $\hat{\phi}_1$ is the solution to the following maximization problem:*

$$\max_{\phi_1} \quad E\left[v_1(W_{1T}, R_{1T})\right] \tag{7}$$
$$\text{subject to} \quad dW_{1t} = W_{1t}[r + \phi_{1t}^\top(\mu - r\bar{1})]dt + W_{1t}\phi_{1t}^\top\sigma d\omega_t.$$

Manager 2's best response $\hat{\phi}_2$ follows analogously by switching managers 1 and 2 in the above.

Definition 2. *A pure-strategy Nash equilibrium is a pair of investment policies (ϕ_{1t}^*, ϕ_{2t}^*, $t \in [0, T]$) such that ϕ_1^* is manager 1's best response to ϕ_2^*, and ϕ_2^* is manager 2's best response to ϕ_1^*.*

In a Nash equilibrium, each manager strategically accounts for the actions of the other manager, and the pure-strategy equilibrium policies of the two managers are mutually consistent. A pure-strategy Nash equilibrium is unique when there is a single pair of investment policies satisfying Definition 2. When there is more than one such pair of mutually consistent policies then there are multiple equilibria, and when there is no such pair then a pure-strategy equilibrium does not exist. As discussed previously, in our set-up, for a given horizon wealth W_{iT} satisfying the budget constraint (2) there exists a unique portfolio policy ϕ_{it}, $t \in [0, T]$, replicating it. Hence, for an equilibrium outcome in investment policies (ϕ_{1t}^*, ϕ_{2t}^*, $t \in [0, T]$), there is always an equivalent outcome in terms of horizon wealth policies (W_{1T}^*,W_{2T}^*). We make use of this duality by solving for the equilibrium horizon wealth, and then deriving the corresponding equilibrium investment policies, and moreover for clarity we sometimes discuss the intuition behind our results in terms of horizon wealth rather than investment policies.

3. Managers' Objectives and Best Responses

In our analysis, we investigate a setting in which manager 1 is guided by the objective function

$$v_1(W_{1T}, W_{2T}) = \begin{cases} \frac{k^{1-\bar{\gamma}_1}}{1-\gamma_1} W_{1T}^{1-\bar{\gamma}_1} & W_{1T} < \eta W_{2T} \\ \frac{k^{1-\bar{\gamma}_1}}{1-\gamma_1} \left(W_{1T}^{1-\theta} (\frac{W_{1T}}{\eta W_{2T}})^{\theta} \right)^{1-\gamma_1} & W_{1T} \geq \eta W_{2T}, \end{cases} \tag{8}$$

as given by Lemma 1, where $\eta \geq 1$ is the performance threshold, and manager 2's objective function is as in (8) with subscripts 1 and 2 switched. Here, the convexity of flow-performance relation leads to an asymmetric perception of outperformance and underperformance by the manager, whereby only the former affects her normal objectives.[7] For ease of discussion, we henceforth refer to the manager with a below-threshold performance as not getting any flows (instead of getting constant flows), since the magnitude of constant flows k does not affect our subsequent results (Propositions 1–4, mixed strategy equilibrium in Section 6.2).

Before proceeding with the formal analysis, we provide some basic intuition regarding how the optimal horizon wealth of manager 1 may be affected by manager 2's choice of horizon wealth in the presence of relative performance concerns when outperforming ($W_{1T} \geq \eta W_{2T}$). Suppose that manager 2's horizon wealth increases. This has the following two effects on manager 1. First, higher manager 2's wealth implies lower money flows for manager 1. As a result, manager 1 wants to increase her wealth so as to restore the previous level of flows. Second, higher W_{2T} reduces the incremental effect of a unit change in manager 1's wealth, making it costlier for manager 1 to increase her wealth. In the pivotal logarithmic case ($\gamma_1 = 1$), the two effects exactly offset each other. For a relatively risk averse manager, $\gamma_1 > 1$, the first effect dominates and manager 1 increases her wealth W_{1T} when manager 2 increases hers – manager 1 is *chasing* manager 2. For a relatively risk tolerant manager, $\gamma_1 < 1$, the second effect dominates and manager 1 decreases her wealth – manager 1 is a *contrarian* to manager 2. Chetty (2006), in different economic settings, and Koijen (2008), specifically for fund managers, document a substantial heterogeneity in the estimates of relative risk aversion, which suggests that both types of behavior – chasing and contrarian – are likely to be present among money managers (Section 5.1 discusses how to distinguish empirically these behaviors). Koijen finds the average risk aversion of fund managers to be above unity, implying that the average real-life manager is likely to be a chaser.

To highlight further features of the objective function (8), we plot in Figure 1 its typical shape. From Figure 1, there are three distinct regions of the objective function, depending on manager 1's relative performance at the horizon T, or equivalently on the relation between

[7] Alternatively, the objective (8) could be interpreted as capturing the well-known psychological feature that people care about their relative standing in their profession. Under this interpretation, the asymmetry in perception of out- and underperformance can be due to the fact that people tend to attribute their success to skill and failure to bad luck (Zuckerman (1979)).

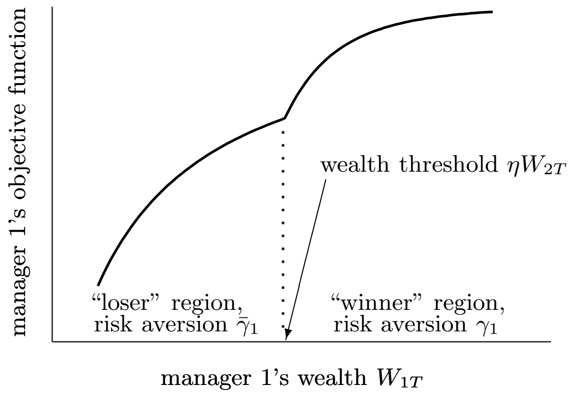


Figure 1: Manager 1's objective function $v_1(W_{1T}, W_{2T})$.

manager 1's wealth and the threshold level ηW_{2T}. When her wealth is above the threshold, the manager gets money flows, and hence we label her as the *winner*. In this case, she is in the region of objectives augmented by relative performance concerns, driven by her effective risk aversion γ_1. When manager 1's wealth is below the threshold, she does not get money flows, and so is labelled as the *loser*. In this case, the manager finds herself in the region of normal objectives with no relative performance concerns, driven by her intrinsic risk aversion $\bar{\gamma}_1$. Finally, when the performance is around the threshold level, the manager is in the region of local convexity. Consequently, there are two main differences from the conventional CRRA objective function. The first is that now both intrinsic and effective risk aversions directly enter the objective specification, and thus have distinct effects on the optimal policies. The second, and a major, difference is the presence of the local convexity in (8). As revealed in the subsequent analysis, it is the interaction of these two features with strategic behavior that underlies the novel predictions of our model.

We now determine the managers' best responses. The managers' optimization problems are non-standard since their objective functions are not globally concave. Nevertheless, interior solutions turn out to exist since the managers' risk aversions limit the sizes of their gambles over the locally convex regions. In Proposition 1 we report the managers' best responses explicitly in closed form.

Proposition 1. *The best response of manager 1 is given by*

$$\hat{W}_{1T} = \begin{cases} (y_1 \xi_T)^{-1/\bar{\gamma}_1} & y_1 \xi_T > b_1(\eta W_{2T}) \quad (loser) \qquad (9) \\ (1+\alpha)^{1/\gamma_1} (y_1 \xi_T)^{-1/\gamma_1} (\eta W_{2T})^{\theta(\gamma_1 - 1)/\gamma_1} & y_1 \xi_T \le b_1(\eta W_{2T}) \quad (winner), \qquad (10) \end{cases}$$

where the boundary function $b_1(\cdot)$ is given by

$$b_1(W) = (1+\alpha)^{\bar{\gamma}_1/\theta} \left(\bar{\gamma}_1/\gamma_1\right)^{\bar{\gamma}_1 \gamma_1/(\gamma_1 - \bar{\gamma}_1)} W^{-\bar{\gamma}_1} \tag{11}$$

and $y_1 > 0$ solves $E[\xi_T \hat{W}_{1T}] = W_{10}$. Switching subscripts 1 and 2 in the above yields *manager 2's best response $\hat{W}_{2T}$. Moreover, when manager i is a winner, her associated relative performance $\hat{R}_{iT}$ is bounded from below by the minimum outperformance margin, $\bar{\eta}_i$, given by $\bar{\eta}_i = (1+\alpha)^{-1/\alpha}(\bar{\gamma}_i/\gamma_i)^{-\bar{\gamma}_i/(\gamma_i-\bar{\gamma}_i)}\eta > \eta$. When manager i is a loser, her relative performance $\hat{R}_{iT}$ is bounded from above by the maximum underperformance margin, $\underline{\eta}_i$, given by $\underline{\eta}_i = (1+\alpha)^{-(1+\alpha)/\alpha}(\bar{\gamma}_i/\gamma_i)^{-\gamma_i/(\gamma_i-\bar{\gamma}_i)}\eta < \eta$.*

Focusing on manager 1, she chooses whether to be a winner or a loser depending on the level of the threshold wealth, ηW_{2T}, relative to the cost of wealth in that state, ξ_T, where the threshold wealth affects manager 1 through the decreasing boundary function $b_1(\cdot)$. The reason a manager may choose to lose is that picking a relatively low wealth and losing in one state allows the manager, because of the budget constraint, to choose a relatively high wealth and win in another state. For a relatively low threshold wealth ηW_{2T}, manager 1 optimally becomes a winner, outperforming the threshold ηW_{2T}, in which case her best response (10) is given by a normal policy, $(y_1\xi_T)^{-1/\gamma_1}$, augmented by the component $W_{2T}^{\theta_1(\gamma_1-1)/\gamma_1}$, accounting for relative performance concerns. The additional component formalizes the earlier intuition that manager 1 increases or decreases her optimal wealth in response to manager 2's increasing hers depending on whether she is a chaser ($\gamma_1 > 1$) or a contrarian ($\gamma_1 < 1$), respectively. For a relatively high threshold wealth, manager 1 opts to be a loser, in which case her best response (9) follows the normal policy.

An important feature here is that a manager only considers two outcomes: winning or losing, never opting for a "draw" by choosing her relative performance $\hat{R}_{iT}$ to be equal or close to the threshold η. This is due to the convexity of her the objective function around the threshold, inducing her to gamble so as to avoid a draw. As presented in Proposition 1, formally, there exists a manager-specific *minimum outperformance margin* $\bar{\eta}_i$, greater than η, so that a winner's relative performance can never be below this margin. Similarly, there is a *maximum underperformance margin* $\underline{\eta}_i$, less than η, so that a loser's relative performance can never be above this margin.

4. Existence and Uniqueness of Pure-Strategy Equilibrium

As we demonstrate below, the managers' risk-shifting motives, together with their strategic interaction, may lead to three possibilities: unique equilibrium, multiple equilibria, or no pure-strategy equilibrium. In this Section, we establish the conditions for each of the possibilities and investigate their underlying economic mechanisms. To facilitate some subsequent discussions, we now describe these three equilibrium types by focusing on the managers' horizon outcomes (winner or loser), complementing the corresponding definitions in terms of investment policies provided in Section 2.3.

For a given state of nature, represented by a realization of ξ_T, a manager is either a winner or a loser, and so there are four possibilities for an equilibrium with two managers. However, both managers cannot be winners as the performance threshold η is greater or equal to 1, and so at most one manager can get money flows. Hence, for each ξ_T there are three possible Nash equilibrium outcomes, denoted by (manager 1 outcome, manager 2 outcome): (winner, loser), (loser, winner), or (loser, loser). Note that the condition for the (loser, loser) outcome is $1/\eta < W_{1T}/W_{2T} < \eta$, which is only possible if $\eta > 1$. A unique Nash equilibrium obtains if for each ξ_T there is one and only one outcome on which both managers agree. Multiple equilibria occur if for each ξ_T the managers agree on at least one outcome and for some ξ_T the managers agree on more than one outcome (e.g., (winner, loser) and (loser, winner)). A pure equilibrium does not exist if for some ξ_T the managers cannot agree on any of the three outcomes. Proposition 2 provides conditions for the uniqueness, multiplicity and non-existence of a pure-strategy equilibrium.

Proposition 2.
(i) A unique pure-strategy Nash equilibrium occurs when

$$\eta > \max\left[(B/A)^{1/(2\bar{\gamma}_1\bar{\gamma}_2)}, (C/A)^{1/(2\gamma_1\bar{\gamma}_2)}, (B/D)^{1/(2\bar{\gamma}_1\gamma_2)}\right]; \tag{12}$$

(ii) multiple pure-strategy Nash equilibria occur when

$$\max\left[A\eta^{\bar{\gamma}_1\bar{\gamma}_2}, C\eta^{-\bar{\gamma}_2(\gamma_1+\theta(\gamma_1-1))}\right] \leq \min\left[B\eta^{-\bar{\gamma}_1\bar{\gamma}_2}, D\eta^{\bar{\gamma}_1(\gamma_2+\theta(\gamma_2-1))}\right]; \tag{13}$$

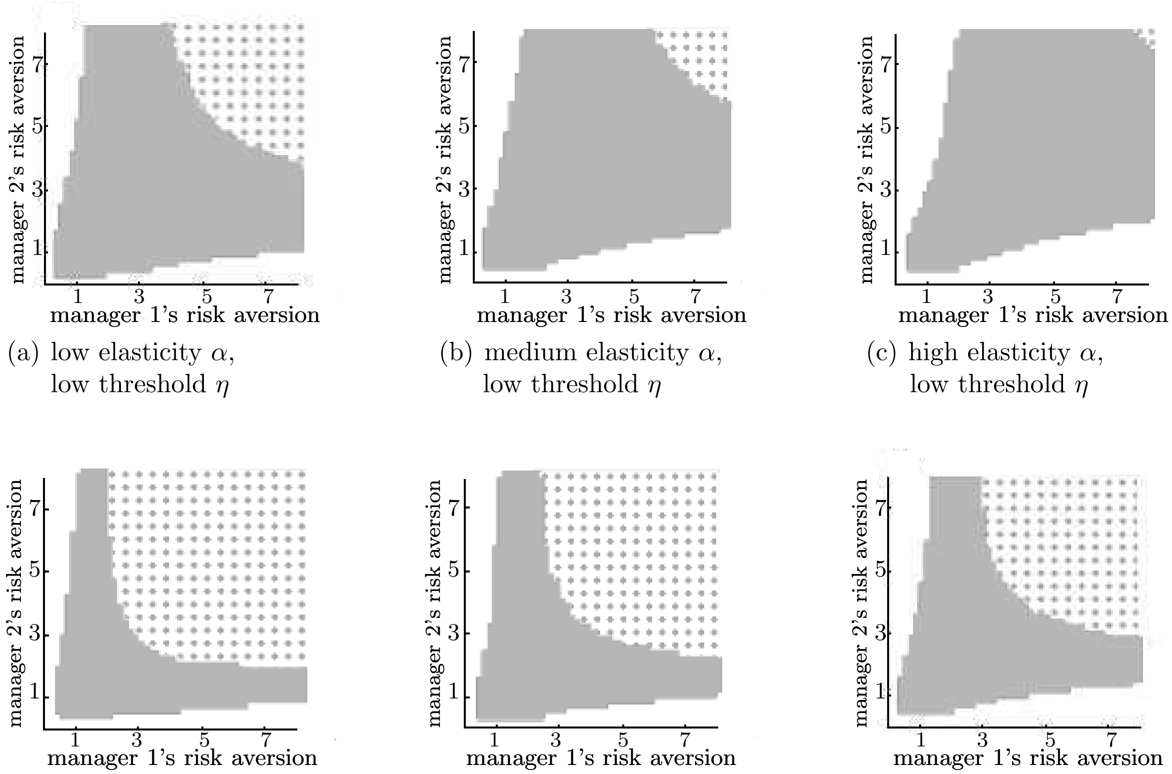
(iii) if neither (12) nor (13) is satisfied, there is no pure-strategy Nash equilibrium.
In the above, the constants A, B, C, D are given by

$$A = (1+\alpha)^{-\bar{\gamma}_1\bar{\gamma}_2/\theta}\left(\bar{\gamma}_1/\gamma_1\right)^{-\bar{\gamma}_1\gamma_1\bar{\gamma}_2/(\gamma_1-\bar{\gamma}_1)}, B = (1+\alpha)^{\bar{\gamma}_1\bar{\gamma}_2/\theta}\left(\bar{\gamma}_2/\gamma_2\right)^{\bar{\gamma}_1\gamma_2\bar{\gamma}_2/(\gamma_2-\bar{\gamma}_2)}, \tag{14}$$

$$C = (1+\alpha)^{\gamma_1\bar{\gamma}_2/\theta-\bar{\gamma}_2}\left(\bar{\gamma}_2/\gamma_2\right)^{\gamma_1\gamma_2\bar{\gamma}_2/(\gamma_2-\bar{\gamma}_2)}, D = (1+\alpha)^{\bar{\gamma}_1-\gamma_2\bar{\gamma}_1/\theta}\left(\bar{\gamma}_1/\gamma_1\right)^{-\bar{\gamma}_1\gamma_1\gamma_2/(\gamma_1-\bar{\gamma}_1)}. \tag{15}$$

The results of Proposition 2 are illustrated in Figure 2 depicting which type of equilibrium occurs for given values of the managers' intrinsic risk aversions, $\bar{\gamma}_1$ and $\bar{\gamma}_2$. In all panels, multiple equilibria occur in solid-filled areas, unique equilibrium occurs in dot-filled areas, and non-existence of pure equilibrium occurs in unfilled areas. The possibility of multiple and no equilibrium is a novel result in the money management literature, and so we start with discussing these two possibilities.

Inspecting the unfilled regions in Figure 2, we see that the non-existence of equilibrium arises when the managers' risk aversions are considerably different in which case their minimum outperformance and maximum underperformance margins are not compatible with



(a) low elasticity α, low threshold η
(b) medium elasticity α, low threshold η
(c) high elasticity α, low threshold η
(d) low elasticity α, high threshold η
(e) medium elasticity α, high threshold η
(f) high elasticity α, high threshold η

Figure 2: Unique, multiple, or no Nash equilibrium. The solid-filled area corresponds to the pairs of managers' intrinsic risk aversions $(\bar{\gamma}_1, \bar{\gamma}_2)$ for which multiple Nash equilibria obtain, the dot-filled area corresponds to the pairs $(\bar{\gamma}_1, \bar{\gamma}_2)$ for which a unique Nash equilibrium obtains, the unfilled area corresponds to the pairs $(\bar{\gamma}_1, \bar{\gamma}_2)$ for which a Nash equilibrium does not exist. In all panels, low and high threshold corresponds to $\eta = 1.05$ and $\eta = 1.1$, respectively; low, medium, and high elasticity corresponds to $\alpha = 0.7$, $\alpha = 1$, and $\alpha = 1.3$, respectively. As we go down the plots η increases, as we go from the left to the right α increases.

each other (see Proposition 1 for the definition of the margins). For example, if manager 1 is sufficiently risk averse her minimum outperformance margin $\bar{\eta}_1$ is only slightly higher than the performance threshold η, implying that in some states she wants her relative performance R_{1T} to be only slightly higher than η. Given that manager 2's relative performance R_{2T} is the inverse of R_{1T}, for an equilibrium to obtain manager 2 needs to agree that her relative performance R_{2T} is slightly lower than $1/\eta$. However, if manager 2 is sufficiently risk tolerant, her maximum underperformance margin $\underline{\eta}_2$ is much lower than $1/\eta$, meaning that she agrees to be a loser only if she underperforms by a lot. Hence, the managers cannot agree on the winning/losing margin when their risk aversions are considerably different, resulting in the non-existence of equilibrium.

Looking at the solid-filled areas in Figure 2, multiple equilibria occur when the managers have (sufficiently) similar risk aversions. Equilibrium now exists because with similar risk aversions the winner's outperformance margin is compatible with the loser's underperformance margin. Multiplicity arises because in some states of the world each of the two outcomes, (winner, loser) and (loser, winner), constitutes an equilibrium. Intuitively, each manager is indifferent between being a winner and a loser in certain states since losing (having a low wealth) in one state will allow her, from the budget constraint, to win (have a high wealth) in another state.

A unique equilibrium is likely to occur when the threshold η is sufficiently high, as evident from the dot-filled areas becoming larger as we move from the top to bottom panels in Figure 2, but is less likely when the flow elasticity α increases, reflected by the shrinking dot-filled areas as we move from the left to right. To understand these findings, recall that non-existence and multiplicity emerge due to a loser's desire to become a winner in certain states, with non-existence occurring if the winner is not willing to become the loser and multiplicity happening if the winner agrees to the switch. Hence, when the loser's desire to win is mitigated, both non-existence and multiplicity are less likely, and this is why the unique equilibrium is more likely when η increases or α decreases. Similarly, when the managers are sufficiently risk averse the loser is less eager to win, and so the unique equilibrium is more likely when the risk aversions are relatively high, as seen in all panels of Figure 2.

Given the novelty of our results pertaining to multiplicity and non-existence of managers' investment policies, it is natural to address their robustness. Basak and Makarov (2011) investigate the strategic interaction of money managers in a similar setting but with no convexities and show that a pure-strategy equilibrium always obtains and is unique. Hence, the key assumption behind multiplicity and non-existence is the convexity of money flows. As discussed earlier, there is extensive empirical literature supporting this assumption.

Remark 1. *Low performance threshold,* $\eta < 1$. While in our setting the performance threshold η is greater or equal to one (capturing investor inertia, as discussed in Section 2.2), in other strategic settings the threshold may be below one. For example, Murphy (1999) documents that the prevalent executive compensation contract in the U.S. is the so called 80/120 plan – a manager receives a fixed salary plus a bonus if her performance exceeds 80% of a certain performance standard. If CEOs of the companies in some industry have compensation of this form, strategic interaction may arise, particularly in oligopolistic industries. Our framework could then be applied by setting $\eta = 0.8$. It turns out the condition for multiplicity (13) equally applies to this case. However, unique equilibrium is not possible for $\eta < 1$, and so when (13) is not satisfied the equilibrium does not exist. To understand why, note that for $\eta < 1$ there emerges an outcome (not present for $\eta > 1$) when both managers are winners. In this (winner, winner) outcome, each manager's actions affect the other's marginal utility which imposes an additional restriction on the managers' behavior. This restriction leads to the non-existence of equilibrium.

5. Unique Equilibrium Investment Policies

The unique Nash equilibrium obtains when condition (12) is satisfied. In this Section, we describe the ensuing equilibrium policies and investigate their properties.

5.1. Characterization of Unique Equilibrium

Proposition 3 reports the investment policies and horizon wealth in the unique equilibrium.

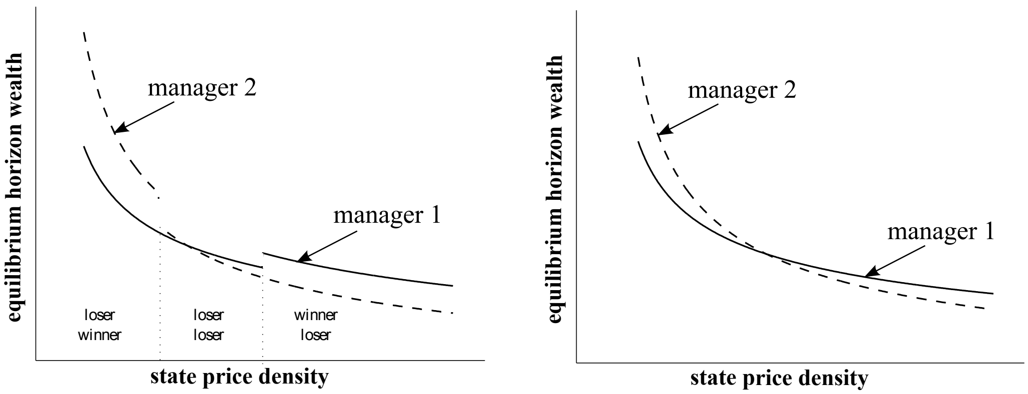
Proposition 3. *Assume the condition for the existence and uniqueness of a Nash equilibrium (12) is satisfied. The equilibrium investment policy of manager 1, ϕ_1^*, is given by*

$$\begin{aligned}\phi_{1t}^* &= (\sigma^\top)^{-1}\kappa \Big\{(1-N(d(\tilde{\gamma},\beta)))\tilde{y}_1 Z(\tilde{\gamma},t)\xi_t^{-1/\tilde{\gamma}}/\tilde{\gamma} \\ &- n(d(\tilde{\gamma},\beta))\tilde{y}_1 Z(\tilde{\gamma},t)\xi_t^{-1/\tilde{\gamma}}/(\|\kappa\|\sqrt{T-t}) + N(d(\bar{\gamma}_1,\beta))y_1^{-1/\bar{\gamma}_1} Z(\bar{\gamma}_1,t)\xi_t^{-1/\bar{\gamma}_1}/\bar{\gamma}_1 \\ &+ n(d(\bar{\gamma}_1,\beta))y_1^{-1/\bar{\gamma}_1} Z(\bar{\gamma}_1,t)\xi_t^{-1/\bar{\gamma}_1}/(\|\kappa\|\sqrt{T-t})\Big\}/W_{1t}^*, \qquad (16)\end{aligned}$$

where $N(\cdot)$ is the standard-normal cumulative distribution function, $n(\cdot)$ is the corresponding probability density function, $\|\cdot\|$ denotes the norm, $y_1 > 0$ solves $E[\xi_T W_{1T}^*] = W_{10}$, and $W_{1t}^*, \tilde{y}_1, \tilde{\gamma}, Z(\cdot), d(\cdot), \beta$ are given in the Appendix. The equilibrium portfolio policy of manager 2, ϕ_{2t}^*, is as in (16) with subscripts 1 and 2 switched.

The associated equilibrium outcomes at the horizon are as follows: The managers are in (winner, loser) when $\xi_T^{\bar{\gamma}_1-\bar{\gamma}_2} \geq yA\eta^{\bar{\gamma}_1\bar{\gamma}_2}$, in (loser, loser) when $yB\eta^{-\bar{\gamma}_1\bar{\gamma}_2} \leq \xi_T^{\bar{\gamma}_1-\bar{\gamma}_2} < yA\eta^{\bar{\gamma}_1\bar{\gamma}_2}$, and in (loser, winner) when $\xi_T^{\bar{\gamma}_1-\bar{\gamma}_2} < yB\eta^{-\bar{\gamma}_1\bar{\gamma}_2}$. Manager 1's equilibrium horizon wealth is $W_{1T}^* = \tilde{y}_1\xi_T^{-(\bar{\gamma}_2+\theta(\gamma_1-1))/(\gamma_1\bar{\gamma}_2)}$ when she is a winner, or $W_{1T}^* = (y_1\xi_T)^{-1/\bar{\gamma}_1}$ when she is a loser. Manager 2's equilibrium wealth W_{2T}^* follows by switching subscripts 1 and 2 in W_{1T}^*. In the above, A and B are as given in (14) and y, $\tilde{y}_1$ are given in the Appendix.

We first look at the managers' equilibrium horizon wealth. Figure 3 plots the equilibrium (latter part of Proposition 3), as well as the normal wealth profiles, as a function of the state price density ξ_T. From Figure 3(a), in good states (low ξ_T), the less risk averse manager 2 has a higher equilibrium wealth than manager 1, in line with the normal wealth profiles as depicted in Figure 3(b). In these states, manager 1 is a loser and manager 2 is a winner, getting the money flows. As we move into intermediate states (middle-ξ_T region), manager 2's relative performance decreases, and after hitting her minimum outperformance margin $\bar{\eta}_2$, jumps down as manager 2 optimally becomes a loser, no longer getting any flows. Finally, as economic conditions deteriorate (high ξ_T), manager 1's relative performance increases, and after reaching the maximum underperformance margin $\underline{\eta}_1$, it jumps upwards as manager 1 becomes a winner and receives money flows.[8] From the viewpoint of potential fund investors, Figure 3 illustrates the importance of accounting for the managers' relative performance concerns, especially in good and bad states where the presence of strategic interactions strongly amplifies the difference between the returns on the managers' portfolios.



(a) Equilibrium horizon wealth profiles W^*_{iT} (b) Normal horizon wealth profiles W^N_{iT}

Figure 3: Managers' unique equilibrium horizon wealth. Equilibrium and normal horizon wealth profiles, W^*_{iT} and W^N_{iT}, for the more risk averse manager 1 (solid plots) and the more risk tolerant manager 2 (dashed plots). The plots are typical.

Turning to investment policies, we first describe the managers' behavior in the deep out- and underperformance regions at an interim point in time $t < T$. Corollary 1 characterizes the limiting equilibrium policies in the interim bad (high ξ_t) and good (low ξ_t) states, henceforth referred to as the interim (winner, loser) and (loser, winner) regions, respectively.

Corollary 1. *The managers are in the interim (winner, loser) region when $\xi_t \to \infty$, and their limiting equilibrium investment policies are $\phi^*_1(\infty) = (\sigma^\top)^{-1}\kappa/\gamma_1 + \theta(\gamma_1 - 1)(\sigma^\top)^{-1}\kappa/(\gamma_1\bar{\gamma}_2)$, $\phi^*_2(\infty) = (\sigma^\top)^{-1}\kappa/\bar{\gamma}_2$. The managers are in the interim (loser, winner) region when $\xi_t \to 0$, and their limiting equilibrium investment policies are $\phi^*_1(0) = (\sigma^\top)^{-1}\kappa/\bar{\gamma}_1$, $\phi^*_2(0) = (\sigma^\top)^{-1}\kappa/\gamma_2 + \theta(\gamma_2 - 1)(\sigma^\top)^{-1}\kappa/(\gamma_2\bar{\gamma}_1)$.*
Consequently, the directions in which the managers' limiting equilibrium policies deviate from their normal policies are given by

$$\operatorname{sgn}(\phi^*_1(\infty) - \phi^N_1) = \operatorname{sgn}(\bar{\gamma}_1 - 1)\operatorname{sgn}(\phi^N_2 - \phi^N_1), \quad (17)$$

$$\operatorname{sgn}(\phi^*_2(0) - \phi^N_2) = \operatorname{sgn}(\bar{\gamma}_2 - 1)\operatorname{sgn}(\phi^N_1 - \phi^N_2), \quad (18)$$

where manager i's normal policy is $\phi^N_i = (\sigma^\top)^{-1}\kappa/\bar{\gamma}_i$.

[8] As established in the proof of Proposition 3 in the Appendix, the unique equilibrium obtains when (loser, loser) region is non-empty, i.e., when the condition $yB\eta^{-\bar{\gamma}_1\bar{\gamma}_2} \le \xi_T^{\bar{\gamma}_1-\bar{\gamma}_2} < yA\eta^{\bar{\gamma}_1\bar{\gamma}_2}$ is satisfied for some ξ_T. In the knife-edge case of identical managers, $\bar{\gamma}_1 = \bar{\gamma}_2$, we see that if (loser, loser) region is non-empty then this region covers the whole interval $\xi_T \in (0, +\infty)$ because the middle term in the above inequality equals one for all values of ξ_T. Hence, when $\bar{\gamma}_1 = \bar{\gamma}_2$, and only in this special case, both managers are losers across all states ξ_T in the unique equilibrium, and so they follow their normal policies.

Equations (17)–(18) reveal that when outperforming, the direction in which manager i strategically adjusts her policy from the normal is determined by whether she is a chaser ($\bar{\gamma}_i > 1$) or a contrarian ($\bar{\gamma}_i < 1$), and whether her normal policy exceeds the opponent's normal policy or not. In particular, if manager i is a chaser, she moves her investment policy from the normal *towards* the normal policy of the opponent. On the other hand, if manager i is a contrarian, she tilts her investment policy *away* from the opponent's normal. These implications can be understood by recalling our earlier discussion of the chasing and contrarian behavior in the presence of relative performance concerns. For example, consider the case where manager 1 is a chaser ($\bar{\gamma}_1 > 1$) while manager 2 is a contrarian ($\bar{\gamma}_2 < 1$). Here, the normal policy of the more risk averse manager 1 is less risky than that of the more risk tolerant manager 2, $\phi_1^N < \phi_2^N$. Being a chaser, manager 1 moves her policy towards the normal policy of manager 2, leading to a higher than normal risk exposure, as predicted by (17). Manager 2 is a contrarian and so moves her policy away from the normal policy of manager 1, leading to a higher than normal risk exposure, as implied by (18).

To best highlight the managers' overall asset allocation behavior, Figure 4 plots managers' equilibrium investments (equation (16)) as a function of manager 1's relative performance at time t, $R_{1t} = W_{1t}/W_{2t}$, with a relatively high R_{1t} ($> \eta$) corresponding to the interim (winner, loser) region and a relatively low R_{1t} ($< 1/\eta$) to the interim (loser, winner) region. For expositional clarity, we specialize the economy to feature two risky stocks, 1 and 2, and without loss of generality let the stock return volatility matrix to be

$$\sigma = \begin{pmatrix} \sigma_1 & 0 \\ \rho\sigma_2 & \sqrt{1-\rho^2}\sigma_2 \end{pmatrix}$$

where ρ is the correlation between stock 1 and 2 returns. Here, the market prices of risk are $\kappa_1 = (\mu_1 - r)/\sigma_1$ and $\kappa_2 = [(\mu_2 - r)/\sigma_2 - \rho(\mu_1 - r)/\sigma_1]/\sqrt{1-\rho^2}$. For the parameter values in Figure 4, stock 1 is the more favorable of the two stocks in the sense that the Sharpe ratio of the stock 1 ($(\mu_1 - r)/\sigma_1$) is relatively high as compared to the Sharpe ratio of stock 2 ($(\mu_2 - r)/\sigma_2$). As a result, both managers tend to invest a larger share of wealth in stock 1 than in stock 2. Apart from this, the profiles of the equilibrium investments in stocks 1 and 2 are similar, as seen by comparing the left panels of Figure 4, (a) and (c), with the right panels, (b) and (d). Hence, in our discussion we focus only on the equilibrium investments in stock 1 (Figure 4(a), (c)).

Figure 4(a) corresponds to the case when both managers are chasers. In the interim (winner, loser) region manager 1 chases manager 2, which from Corollary 1 implies that manager 1 increases her risk exposure as compared to normal and moves her policy towards manager 2's normal policy. For manager 2, the relative performance concerns are weak in the interim (winner, loser) region since the likelihood of attracting money flows at year-end is low. As a result, the equilibrium policy of manager 2 is close to her normal policy. In the

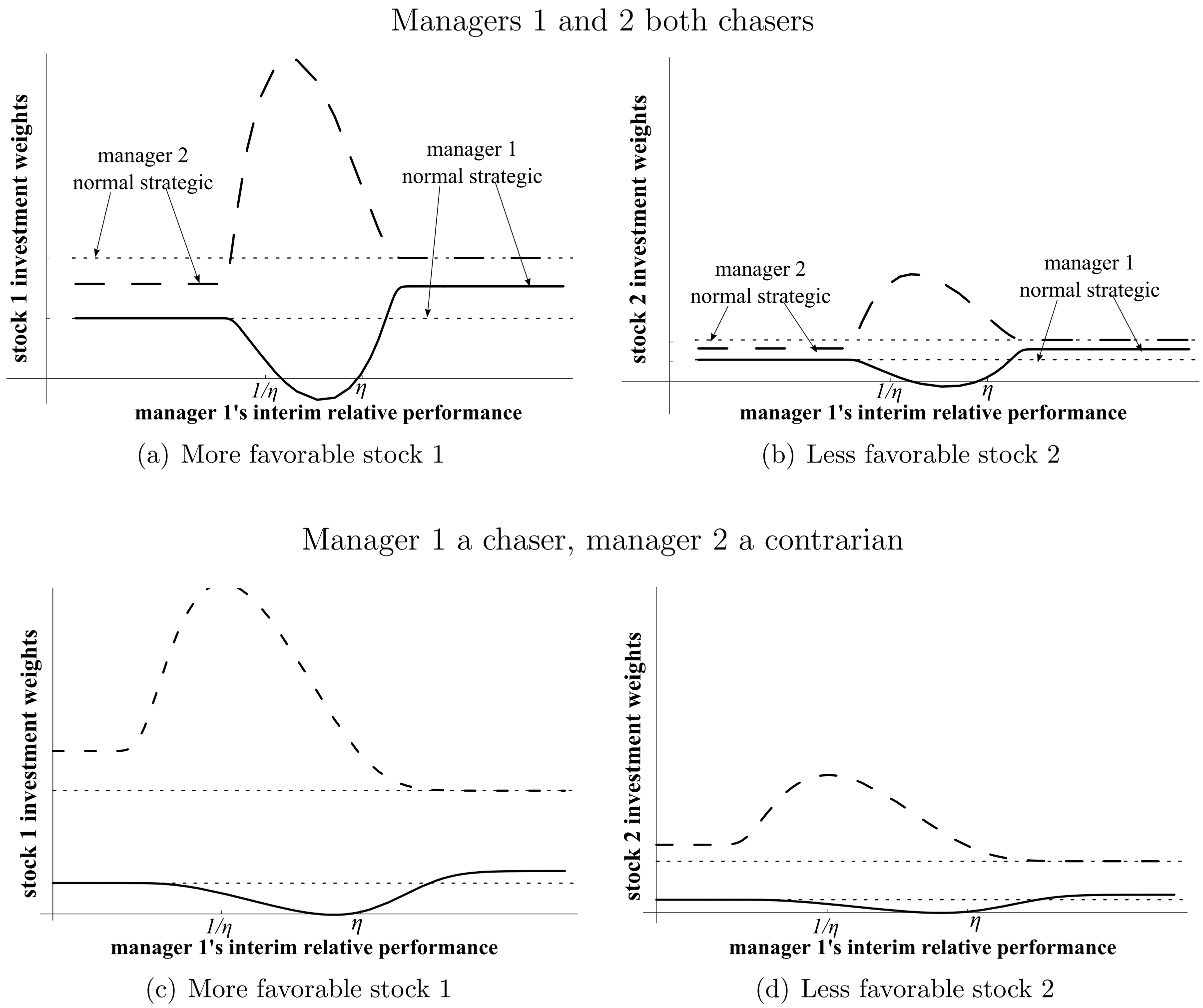


Figure 4: The managers' unique equilibrium investment policies. The time-t investment policies of the more risk averse manager 1 in stock 1 ϕ^*_{11t} (solid lines in panels (a) and (c)) and in stock 2 ϕ^*_{12t} (solid lines in panels (b) and (d)), and the more risk tolerant manager 2 in stock 1 ϕ^*_{21t} (dashed lines in panels (a) and (c)) and in stock 2 ϕ^*_{22t} (dashed lines in panels (b) and (d)). In each panel, the lower dotted line is the normal policy of manager 1 $\phi_1^N = (\sigma^\top)^{-1}\kappa/\bar{\gamma}_1$, the upper dotted line is the normal policy of manager 2 $\phi_2^N = (\sigma^\top)^{-1}\kappa/\bar{\gamma}_2$. Stock 1 is more favorable than stock 2 as it has a relatively higher Sharpe ratio. In panels (a) and (b), the two managers are chasers, and the parameter values are: $r = 0.05, \mu_1 = 0.1, \mu_2 = 0.12, \sigma_1 = 0.15, \sigma_2 = 0.3, \rho = 0.3, \bar{\gamma}_1 = 4, \bar{\gamma}_2 = 2, \alpha = 1.5, \eta = 1.2, t = 0.8, T = 1$, and hence $\theta = 0.6, \gamma_1 = 8.5, \gamma_2 = 3.5$. In panels (c) and (d), manager 1 is a chaser and manager 2 is a contrarian, and the parameter values are: $\bar{\gamma}_1 = 2, \bar{\gamma}_2 = 0.5, \alpha = 0.3, \eta = 1.35, t = 0.95$, and hence $\theta = 0.23, \gamma_1 = 2.3, \gamma_2 = 0.35$, and the remaining parameters are as in panels (a) and (b).

interim (loser, winner) region, manager 2, a chaser, decreases her risk exposure relative to normal as she tilts her investment policy towards manager 1's normal policy. The equilibrium policy of manager 1 is close to normal since the effect of relative performance concerns is small. In Figure 4(c), manager 1 is a chaser but manager 2 is a contrarian. In the interim (winner, loser) region, the outcome of the strategic interaction is as in Figure 4(a): manager

1 increases her risk exposure while manager 2 chooses a policy close to her normal. In the interim (loser, winner) region, Corollary 1 implies that manager 2, a contrarian, increases her risk exposure relative to the normal as she moves her policy away from the manager 1's normal policy. For manager 1, the effect of relative considerations is weak and so she chooses a policy close to the normal.

In the interim (loser, loser) region, the economic mechanism underlying the chasing and contrarian behavior is dominated by the gambling incentives as each manager is in the convex region of her (conditional) objective function. Hence, the behavior of managers in this region is similar in Figures 4(a) and (c). Each manager gambles in order to avoid a situation where her relative performance is close to the threshold level, which is achieved by following a policy that is sufficiently different from that of the opponent. As a result, the managers always gamble in the opposite directions from each other in each individual stock.[9] The more risk averse manager 1 optimally decreases her stock holding when gambling while the more risk tolerant manager 2 gambles by increasing hers, as seen in Figures 4(a) and (c).[10]

While the above findings concerning individual stocks enable us to zoom in on the portfolios of strategic managers, these predictions cannot readily be tested in the data since most mutual and hedge funds do not disclose their dynamic trading patterns. What is observable (or can be estimated), and what the majority of empirical tournament studies focus on, is portfolio volatility, and so to formulate our testable predictions we describe the managers' portfolio volatilities. We have verified numerically that the effect of manager 1's relative performance R_{1t} on the managers' portfolio volatilities is very similar to the effect on their stock investments, as depicted in Figure 4. In particular, when considerably outperforming, a manager does not opt for a normal portfolio volatility but rather, if a chaser, moves her volatility from normal towards the opponent's normal volatility, or if a contrarian, she tilts her volatility away from the opponent's normal volatility. When underperforming, a manager's portfolio volatility is close to normal. When the managers' performances are close and gambling incentives prevail, the more risk averse manager 1 decreases her volatility relative to normal whereas the more risk tolerant manager 2 increases her volatility relative to normal.

[9] Another way to understand this result is to recall a well-known option pricing intuition that a holder of a call option has incentives to increase the volatility of the underlying so as to reduce the chance of the option expiring at-the-money, thereby increasing the option value. In our case, each manager can be thought of as holding a call option (due to convex fund flows) whose strike price is not constant but rather depends on the opponent's horizon return. Hence, gambling in the opposite direction in each stock allows each manager to minimize the chance of ending up at-the-money.

[10] We note that mutual fund managers are often not allowed to go short, while according to Figure 4 it may be optimal for a manager to short risky assets in a certain range of interim relative performance. For tractability, we do not explicitly introduce a no-short-sale constraint. We believe that incorporating such a constraint is not going to significantly change our main insights, because if the constraint were present, it would likely lead to less pronounced gambling by the more risk averse manager in the relevant range of the interim relative performance.

Given these results, we see that a strategic manager may behave rather differently depending on who her opponent is. For example, the manager gambles by substantially *increasing* her portfolio volatility if her opponent happens to be more risk averse than her, but *decreases* her volatility when gambling if the opponent is more risk tolerant. Hence, existing or potential client investors concerned with how a strategic manager may invest on their behalf should learn the characteristics of not only the manager herself but also of her competitors. This insight also holds in a mixed-strategy equilibrium (analyzed later in Section 6.2) where a manager may follow a pure or mixed strategy depending on the opponent's risk attitude. In contrast, in related non-strategic works (Carpenter (2000), Basak, Pavlova, and Shapiro (2007)) knowing a manager's own characteristics typically suffices to determine the behavior.

Remark 2. *Time consistency.* Inspecting the proof of Proposition 3, it is straightforward to observe that the analysis, particularly the equilibrium horizon wealth profiles, does not depend on the time at which the game between managers occurs. Hence, the managers' equilibrium wealth profiles, and consequently equilibrium policies presented in Proposition 3 are time consistent in that neither manager would want to deviate from a policy chosen initially if the managers were to replay the game at any subsequent time $t > 0$. By the same argument, the conditions for existence and multiplicity reported in Proposition 2 do not depend on the time when the game is played. For the above to hold, the structure of the time-t game needs to be the same as that at time 0 (as described in Section 2.3), with the date of the game being the only difference. Concerning the complete information assumption, this means that each manager needs to observe the opponent's time-t wealth, or equivalently, the opponent's return from time 0 to time t. Whether a manager observes the opponent's portfolio prior to t or not does not affect the analysis since the managers know the characteristics of each other's objective functions, and so there is no scope for learning from past portfolio decisions.

5.2. Effects of Stock Correlation on Unique Equilibrium

Figure 5 illustrates the impact of the correlation between the two stocks, ρ, on the managers' equilibrium investment policies. Several noteworthy features arise. First, changing the correlation can affect the equilibrium policies in a non-monotonic way, as in Figures 5(a)-(c), as well as monotonically, as in Figure 5(d). Second, Figures 5(a) and (b) reveal that the equilibrium investment profiles may cross for different values of correlation. Third, as correlation increases the direction of humps in the interim (loser, loser) region can invert, as depicted in Figures 5(b) and (d). Overall, Figure 5 demonstrates that the managers strategically account for each others' actions when responding to a change in correlation. For example, manager 1 inverts the direction of gambling in the interim (loser, loser) region if and only if manager 2 inverts hers (Figures 5(b) and (d)).

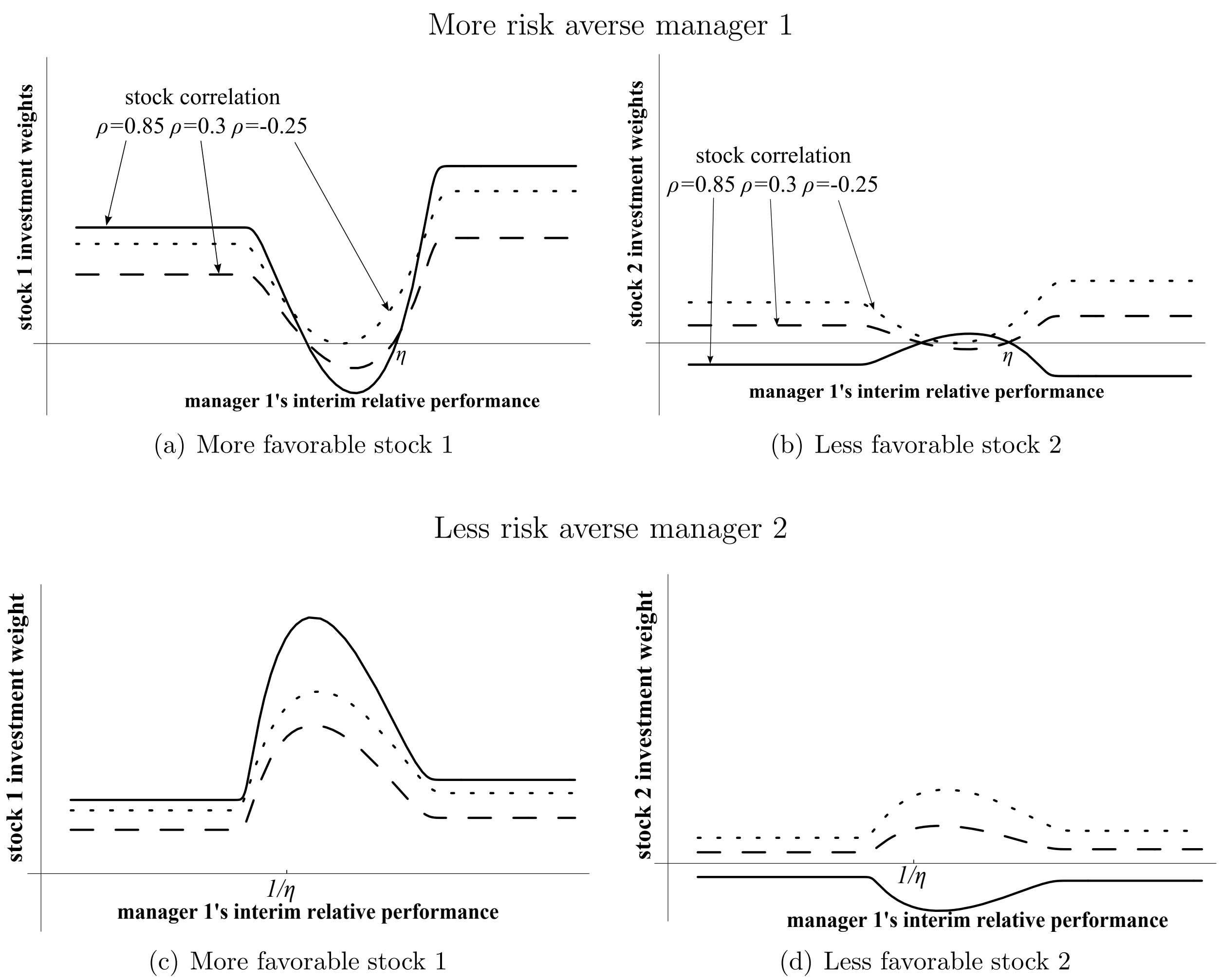


Figure 5: Effect of stock correlation on the managers' unique equilibrium investment policies. The time-t equilibrium policies of manager 1, ϕ^*_{11t} and ϕ^*_{12t}, and manager 2, ϕ^*_{21t} and ϕ^*_{22t}, for varying levels of the correlation, ρ, between stock 1 and 2 returns. In all panels, dotted line corresponds to $\rho = -0.25$, dashed line to $\rho = 0.3$, solid line to $\rho = 0.85$. The other parameter values are as in Figure 4(a) and (b).

Above rich patterns arise due to the interaction of a *diversification* effect, the extent of benefits to diversification, and a *substitution* effect, the extent of the two stocks acting as a substitute to each other. The diversification effect dominates for low correlation values, while the substitution effect dominates for high values of correlation. When the correlation is low, both sources of stock return uncertainty matter considerably and so the two stocks compliment each other by providing a hedge against a specific source of risk. Increasing the correlation reduces the diversification benefits from holding a portfolio of the two stocks, leading the managers to reduce their investments in both stocks. When the correlation is high, the two stocks become close substitutes, in which case the more favorable stock 1 (with a relatively high Sharpe ratio) becomes the primary security through which the managers achieve their desired risk exposures. As the correlation increases further, the managers substitute away from the less favorable stock 2 (with a relatively low Sharpe ratio) into the more favorable stock 1.

Going back to Figure 5, when the correlation increases from -0.25 to 0.3 (moving from the dotted to dashed lines in all panels of Figure 5), the diversification effect dominates and both managers reduce their investments in each stock across all three regions of interim relative performance.As the correlation rises further from $\rho = 0.3$ to $\rho = 0.85$ (moving from dashed to solid lines in Figure 5), in the interim (winner, loser) and (loser, winner) regions the substitution effect induces the managers to increase their investments in the more favorable stock 1 and to finance this by decreasing their holdings in the less favorable stock 2. Hence, the substitution effect works in the opposite direction from the diversification effect for stock 1, leading the equilibrium policies to being non-monotonic in the correlation (Figures 5(a) and (c)). In the interim (loser, loser) region, the more risk averse manager 1 gambles by decreasing her risk exposure. Since the more favorable stock 1 is the primary means to changing her risk exposure, the downward hump becomes more pronounced, as seen in Figure 5(a). The substitution effect implies that this decrease in stock 1 holdings is mirrored by an increase in the less favorable stock 2 holdings. This leads to the inverted shape of the equilibrium policy for stock 2, as depicted in Figure 5(b). The more risk tolerant manager 2 follows the opposite strategy to that of manager 1 and gambles by increasing her risk exposure. Consequently, the size of the upward hump in the primary stock 1 holdings increases (Figure 5(c)). This larger position in stock 1 is financed by a decrease in the less favorable stock 2 holdings, leading to the inverted hump in Figure 5(d).

Remark 3. *Interim Performance.* Starting from Brown, Harlow, and Starks (1996), a number of theoretical and empirical works investigate how the managers behavior is affected by the (exogenously given) difference in their interim performances. We can address this question in our model by introducing an interim performance parameter, R_{10}, reflecting manager 1's advantage ($R_{10} > 1$) or disadvantage ($R_{10} < 1$) at time 0. To obtain the ensuing unique equilibrium policies, in Figures 4–5 we need to scale the graphs along the x-axis by a factor of R_{10}, i.e., when $R_{10} > 1$ ($R_{10} < 1$) the graphs are shrunk (stretched) horizontally. Given this change, the managers' investment policies, and hence portfolio volatilities, are sensitive to interim performance, consistent with empirical evidence. However, all our results on equilibrium existence and multiplicity (Figure 2) remain unchanged, which is in contrast to a static risk-neutral setting where interim performance is a key factor behind the existence of a pure equilibrium (Taylor (2003)). In the knife-edge case when both managers are identical, we can have an equilibrium in which the manager who is far ahead at time 0 is a winner at the horizon T with certainty, across all states. We do not describe this result in detail since it does not hold when, realistically, the managers are even slightly different.

6. Multiple and Mixed-Strategy Equilibrium

Having investigated the managers' behavior in the unique equilibrium, in this Section we look at the other two cases: multiple equilibrium occurring when condition (13) is satisfied, and non-existence of pure equilibrium occurring when both conditions (12) and (13) are not satisfied.

6.1. Characterization of Multiple Equilibria

Proposition 4 reports all horizon wealth profiles that can occur in multiple equilibrium.[11]

Proposition 4. *In multiple Nash equilibrium occuring when condition (13) is satisfied, the equilibrium outcomes are as follows: The managers are in (winner, loser) when* $\xi_T^{\bar{\gamma}_1 - \bar{\gamma}_2} > y \min\left[B\eta^{-\bar{\gamma}_1\bar{\gamma}_2}, D\eta^{\bar{\gamma}_1(\gamma_2+\theta(\gamma_2-1))}\right]$*, in (loser, winner) when* $\xi_T^{\bar{\gamma}_1 - \bar{\gamma}_2} < y \max\left[A\eta^{\bar{\gamma}_1\bar{\gamma}_2}, C\eta^{-\bar{\gamma}_2(\gamma_1+\theta(\gamma_1-1))}\right]$*, and both outcomes, (winner, loser) and (loser, winner), can occur in equilibrium when*

$$y \max\left[A\eta^{\bar{\gamma}_1\bar{\gamma}_2}, C\eta^{-\bar{\gamma}_2(\gamma_1+\theta(\gamma_1-1))}\right] < \xi_T^{\bar{\gamma}_1 - \bar{\gamma}_2} < y \min\left[B\eta^{-\bar{\gamma}_1\bar{\gamma}_2}, D\eta^{\bar{\gamma}_1(\gamma_2+\theta(\gamma_2-1))}\right].$$

Here, A, B, C, D *are as in Proposition 2 and* y *as in Proposition 3. The equilibrium horizon wealth* $W_{iT}^*, i = 1, 2$*, depending on winning or losing, are as given in Proposition 3.*

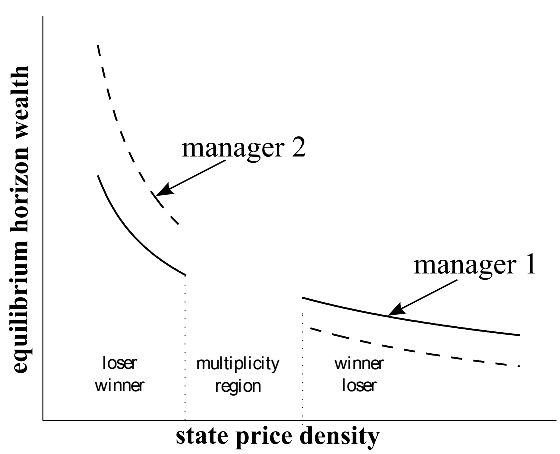


Figure 6: Managers' multiple equilibrium horizon wealth. Multiple equilibrium horizon wealth profiles W_{iT}^* of the more risk averse manager 1 (solid plot) and the more risk tolerant manager 2 (dashed plot). The plots are typical.

Figure 6 plots the multiple equilibrium wealth profiles. Looking at Figures 3 and 6, we observe that the multiple and unique equilibrium wealth profiles differ only in the middle-ξ_T

[11] Due to the nature of multiplicity, describing the corresponding investment policies is not straightforward. Loosely, the number of terms in the expressions for equilibrium policies depend on the number of discontinuities in the horizon wealth profiles, which can be infinite.

region. When the equilibrium is unique, both managers are losers in the middle region, choosing similar horizon wealth profiles (Figure 3). Now, since the performance threshold is relatively low, such wealth profiles are no longer optimal as both managers have incentives to gamble. So, one of the managers must be a winner, implying that her wealth will be considerably higher than the other manager's wealth, preventing the latter from gambling. As discussed in Section 4, in some states the managers are indifferent to switching their rankings. Since now the rankings are different in all states, with neither (winner, winner) nor (loser, loser) outcomes being possible, switching rankings leads to multiple equilibria. When ξ_T is either relatively low or high, the difference between the managers' wealth levels is substantial in the unique equilibrium (Figure 3). So, even with a low performance threshold η neither of the two managers has incentives to gamble in these regions, leading to the multiple and unique equilibrium wealth profiles being the same, as seen by comparing Figures 3 and 6.

Given the multiplicity, one may be interested in whether all equilibria are equally "likely" to occur in reality, or whether a certain equilibrium is more plausible as it survives some reasonable refinement criterion. While an in-depth investigation of this issue is beyond the scope of this paper, we may put forward the following simple refinement criterion. Since real-life money managers incur trading costs, which we have abstracted away from for tractability, the managers would favor the equilibrium with the minimal swings in investments so as to minimize the transaction costs. Since the most pronounced swings are associated with discontinuities in horizon wealth profiles, a natural selection criterion is to pick an equilibrium with the minimum number of the discontinuities, which in our setting amounts to the equilibrium with just one discontinuity. Indeed, inspecting Figure 6 we observe that such an equilibrium is obtained by dividing the middle-ξ_T multiplicity region into two parts, with (loser, winner) outcome occurring in the left part and (winner, loser) outcome occurring in the right part. One can easily see that the structure of the resulting equilibrium wealth profiles is similar to that obtained in the unique equilibrium (Proposition 3), and so the investment policies underlying this selected equilibrium have the same profile as the unique equilibrium investment policies depicted in Figure 3.

6.2. Mixed-Strategy Equilibrium in a Binomial Setting

When a pure-strategy equilibrium does not exist, the managers are likely to resort to mixed strategies, and so we now investigate a mixed-strategy equilibrium.[12] However, it does not appear to be possible to characterize a mixed equilibrium analytically in our continuous-time framework given that the strategy space is infinite-dimensional, and even numerically

[12] An alternative way to deal with non-existence is to perturb an original game so that the perturbed game admits a pure equilibrium (e.g., Harsanyi's (1973) purification approach). Since we are not aware of evidence motivating some specific perturbation, we do not follow this route.

this task seems daunting. To circumvent this difficulty, we turn to a simpler binomial setting,[13] allowing us to not only explore the managers' behavior in a mixed equilibrium but to also examine how the structure of the mixed equilibrium is related to the managers' best responses.

The modified economic setting and parameter values are as follows. We consider a one-period economy where the managers trade at time 0 in a single risky stock and a riskless bond. The stock price can either go up by 15% or down by 10% with equal probabilities and the bond return is $r = 0$, implying that the risk premium is positive. The intrinsic risk aversions of managers 1 and 2 are $\bar{\gamma}_1 = 4$ and $\bar{\gamma}_2 = 2$, the flow elasticity is $\alpha = 5$ and the performance threshold is $\eta = 1$. Both managers' initial wealth is normalized to one. The managers are guided by the same objectives as described before (equation (8)), where the horizon T is now end of the single period. Here, the managers' normal policies are $\phi_1^N = 0.43$ and $\phi_2^N = 0.83$. It is worth noting that the ensuing qualitative results are typical, and not driven by a particular choice of model parameters.

In our example, neither manager has an advantage in performance over the opponent initially, implying that the behavior of each manager is driven by gambling incentives as each is in the convex region of her objective function. Hence, when comparing the below results with those obtained when a pure strategy exists, we should look at the middle region in Figure 4 where gambling behavior is prevalent. We do not present the analysis of a binomial model when one of the managers is sufficiently ahead since the results are very similar to those obtained earlier (see left and right regions in Figure 4). In particular, we obtain a pure equilibrium in which the manager who is behind follows a normal policy, while the manager who is ahead, if a chaser, moves her policy from normal towards the opponent's normal policy, or if a contrarian, away from the opponent's policy.[14]

Towards describing a mixed strategy equilibrium in this setting, we start by providing a brief informal discussion behind this notion of equilibrium. A mixed equilibrium is somewhat similar to a pure equilibrium in that in both cases we look for mutually consistent managers' best responses. What is different now is that a manager's equilibrium strategy may consist of multiple investment policies over which she randomizes. This happens because each of these policies maximizes the manager's objective function, leading to randomization as the manager is indifferent between these policies. Accordingly, a strategy of manager i, $i = 1, 2$, is now given by a pair (ϕ_i, π_i), where the vector $\phi_i \equiv [\phi_{i1}, \phi_{i2}, ..., \phi_{iM_i}]$ (with slightly modified notation) denotes the set of investment policies she randomizes over, $\pi_i \equiv [\pi_{i1}, \pi_{i2}, ..., \pi_{iM_i}]$

[13] A binomial setting is commonly used as a discrete-time alternative of a continuous-time formulation, motivated by studies which establish that continuous-time results can also be obtained by taking the corresponding result in a binomial setting and letting the time step tend to zero (e.g., Cox-Ross-Rubinstein binomial option price formula converging to the Black-Scholes formula).

[14] This provides additional assurance that considering the simpler binomial setting is not likely to affect our main insights. Moreover, we have verified that in the binomial case a pure equilibrium exists (does not exist) when the managers' risk aversions are similar (different) – as in the continuous-time case (Figure 2).

is the vector of corresponding probabilities, and M_i is the number of policies in manager i's set. We now provide a formal definition of the best response strategies and a mixed strategy equilibrium in this finite strategy set, binomial framework.

Definition 3. *For a given manager 2's strategy (ϕ_2, π_2), manager 1's set of best response policies $\hat{\phi}_1 \equiv [\hat{\phi}_{11}, \hat{\phi}_{12}, ..., \hat{\phi}_{1M_1}]$ is such that for any j, $j = 1, 2, ...M_1$, $\hat{\phi}_{1j}$ yields the maximum to the objective function $E\left[v_1(W_{1T}, R_{1T})\right]$. Manager 2's best response set $\hat{\phi}_2$ follows analogously by switching subscripts 1 and 2 above.*

A mixed strategy Nash equilibrium is a quadruple $(\phi_1^*, \pi_1^*, \phi_2^*, \pi_2^*)$ such that ϕ_1^* is manager 1's best response set to (ϕ_2^*, π_2^*) and ϕ_2^* is manager 2's best response set to (ϕ_1^*, π_1^*).

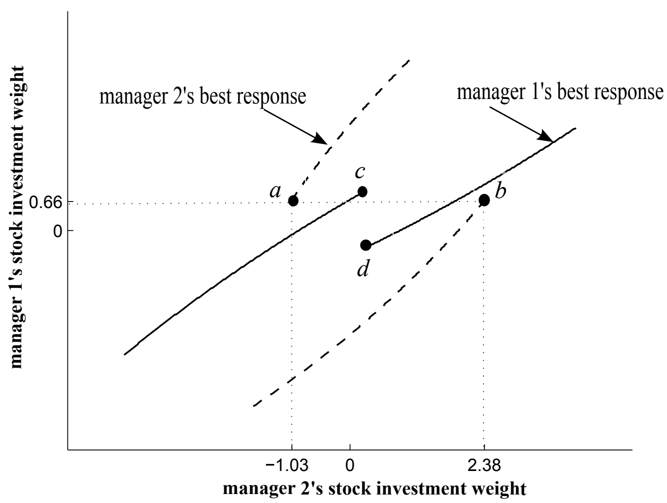


Figure 7: Managers' best responses in a mixed-strategy equilibrium. Best responses of the more risk averse manager 1 $\hat{\phi}_1(\phi_2)$ (solid line) and the more risk tolerant manager 2 $\hat{\phi}_2(\phi_1)$ (dashed line), where the opponent follows a single investment policy, $\hat{\phi}_1(\phi_2)$ is placed along y-axis and $\hat{\phi}_2(\phi_1)$ is placed along x-axis. The parameter values are: the stock return is 15% or -10% with equal probabilities, $r = 0$, $\bar{\gamma}_1 = 4$, $\bar{\gamma}_2 = 2$, $\alpha = 5$, $\eta = 1$.

From Definition 3, to find a mixed equilibrium one may potentially need to look at best responses when the opponent randomizes over several policies. However, it turns out that in our setting much of the insight can be lucidly explained by inspecting "simple" best responses when the opponent follows a single investment policy. Figure 7 plots such best responses of manager 1 $\hat{\phi}_1(\phi_2)$ (solid line) and of manager 2 $\hat{\phi}_2(\phi_1)$ (dashed line), where ϕ_1 and ϕ_2 are numbers representing the opponent's investments. The figure illustrates that there is no equilibrium in pure strategies as the managers' best responses do not cross, justifying the need to look for a mixed equilibrium. Looking at the best responses, we see that manager 2's best response to $\phi_1 = 0.66$ consists of two policies, $\hat{\phi}_{21} = -1.03$ and $\hat{\phi}_{22} = 2.38$ (points a and b in Figure 7). Computing manager 1's best response to (ϕ_2, π_2) for $\phi_2 = [-1.03, 2.38]$, we find that if $\pi_2 = [0.65, 0.35]$ then $\hat{\phi}_1 = 0.66$. Hence, we have the following mixed strategy equilibrium: The more risk averse manager 1 follows a pure strategy $\phi_1^* = 0.66$ and the less risk averse manager 2 follows a mixed strategy given by $\phi_2^* = [-1.03, 2.38]$ and $\pi_2^* = [0.65, 0.35]$.[15]

From the above, we see that a key mechanism behind our results in the pure equilibrium – the managers' desire to avoid a "draw" (their performances being similar) – is also at work in the mixed equilibrium. This is evident from the fact that in the mixed equilibrium manager 1's policy, $\phi_1^* = 0.66$, is close to the average of the two policies of manager 2, $\phi_2^* = [-1.03, 2.38]$. If this were not the case and manager 1's policy were close to, say $\phi_{22}^* = 2.38$, then there would be a draw if manager 2, after randomizing, actually played $\phi_{22}^* = 2.38$. Moreover, another important message from the pure equilibrium analysis, that a manager's behavior crucially depends on her risk aversion *relative* to that of the opponent, remains valid in the mixed equilibrium since it is the manager's *relative* attitude towards risk which determines whether she follows a pure or a mixed strategy.

Comparing the gambling behavior in the pure equilibrium (Section 5.1) to above results, however, we see a notable difference. In particular, while in the mixed equilibrium the more risk averse manager 1 chooses a higher than normal portfolio volatility ($\phi_1^* = 0.66 > 0.43 = \phi_1^N$), she chooses a lower than normal volatility in the unique equilibrium. As for the more risk tolerant manager 2, in the mixed equilibrium she randomizes between two risk exposures, one lower ($\phi_{21}^* = -1.03$) and one higher ($\phi_{22}^* = -2.38$) than normal ($\phi_2^N = 0.83$), whereas in the pure equilibrium she always increases her risk by opting for a higher than normal portfolio volatility. This highlights that the difference between the existence and non-existence of pure equilibrium has important economic implications.

7. Conclusion

In this paper, we analyze the equilibrium portfolios of money managers in presence of strategic interactions driven by relative performance concerns with local convexities. We discover the possibility of three distinct results: multiple equilibria, unique equilibrium, or no equilibrium at all. When the equilibrium is unique, we analyze the properties of the equilibrium investment policies. In the other two cases, we elaborate on the underlying economic mecha-

[15]Switching managers 1 and 2 in the above discussion (looking at points c and d in Figure 7) gives another candidate for a mixed equilibrium in which it is now manager 1 who randomizes over two policies while manager 2 chooses a single policy. However, our analysis reveals that there is no mixed equilibrium of this type.This does not necessarily mean that a mixed equilibrium is unique since we cannot rule out the possibility that there exists a mixed strategy equilibrium of a different structure than the one analyzed in this Section, e.g., an equilibrium where both managers randomize over closed intervals (of investment policies). For several structures, we have checked that a mixed equilibrium of these structures does not exist, however a complete analysis of uniqueness is left for future work.

nisms that lead to non-existence or multiplicity, most of which are driven by the risk shifting incentives combined with the strategic interaction of the managers. While we have focused on money managers, our analysis can potentially be applied to study the behavior of traders working in the same investment bank. Indeed, while it may not be explicitly written in a contract, it is commonly known that promotion of traders depends on their relative (to peers) success, and so strategic interactions may arise as traders compete for promotion.

Given the novelty of our analysis, we believe there are various promising directions for future research. While we assume that money managers have a perfect knowledge of each other's attitude towards risk, it would be valuable to investigate a perfect Bayesian equilibrium in a more realistic framework where the managers do not have such knowledge but can learn about each other's traits by observing the investment policies. It would also be of interest to extend our framework to investigate the possible strategic interactions among CEOs, whose contracts often include a bonus part for high relative performance (Murphy (1999)). Considering a setting where managers compete against each other and at the same time against an exogenous peer-group benchmark could be an interesting generalization. Another natural extension of our framework would be to incorporate flow-performance relations where money flows depend on discrete rankings, leading to discontinuities of the managers' objective functions. Finally, analyzing the case when the investment opportunities of the managers are not perfectly correlated would be worthwhile.

Appendix: Proofs

Proof of Lemma 1. Employing martingale methods, given the CRRA preferences (5), manager i's optimal time-T' wealth profile $\hat{W}_{iT'}$ is given by the first order condition

$$\hat{W}_{iT'} = (y_{iT}\xi_{T'})^{-1/\bar{\gamma}_i}, \tag{A1}$$

where $y_{iT} > 0$ is the Lagrange multiplier attached to her time-T static budget constraint $E_T[\xi_{T'}\hat{W}_{iT'}] = \xi_T W_{iT} f_T$. The Lagrange multiplier is found by substituting (A1) into the budget constraint, which yields

$$y_{iT}^{-1/\bar{\gamma}_i} = \frac{\xi_T W_{iT} f_T}{E_T[\xi_{T'}^{1-1/\bar{\gamma}_i}]}. \tag{A2}$$

Plugging (A2) into (A1), we find manager i's optimal time-T' wealth profile:

$$\hat{W}_{iT'} = \frac{\xi_T W_{iT} f_T}{E_T[\xi_{T'}^{1-1/\bar{\gamma}_i}]}\xi_{T'}^{-1/\bar{\gamma}_i} \tag{A3}$$

Combining (6) and (A3) yields the time-T indirect utility function

$$v_{iT} \equiv E_T[u_i(\hat{W}_{iT'})] = \frac{1}{1-\bar{\gamma}_i}(\xi_T W_{iT} f_T)^{1-\bar{\gamma}_i}(E_T[\xi_{T'}^{1-1/\bar{\gamma}_i}])^{\bar{\gamma}_i}. \tag{A4}$$

Since ξ_t follows a geometric Brownian motion with constant drift and volatility, we have that $E_T[\xi_{T'}^{1-1/\bar{\gamma}_i}] = a\xi_T^{1-1/\bar{\gamma}_i}$ where a is some constant depending on r, κ, and $T' - T$, and we drop it without loss of generality since it does not affect the optimal behavior. Finally, substituting the expectation into (A4), we get

$$v_{iT} = \frac{1}{1-\bar{\gamma}_i}(W_{iT} f_T)^{1-\bar{\gamma}_i}. \tag{A5}$$

The indirect utility function (6) follows by plugging the flow-performance function into (A5) after some manipulation. The stated properties of θ and γ_i are immediate.

Q.E.D.

Proof of Proposition 1. Before presenting the proof, it is worth commenting on how we approach the problem of determining the managers' best responses and equilibrium policies. Given the game structure (Section 2.3), manager i can choose policies of the form $\phi_i(t, W_{1t}, W_{2t})$, however we conjecture that in equilibrium each manager's policy depends only on her own wealth, i.e, that manager i's equilibrium policy has the form $\phi_1^*(t, W_{it})$. Intuitively, despite the fact that the uncertainty is driven by an N-dimensional Brownian motion, a key driving variable in our setting, the state price density ξ, is one-dimensional, and so it is natural to expect that a manager's equilibrium policy should also be driven by just one wealth process. This conjecture enables us, when looking for a manager's best response, to differentiate her objective function with respect to own horizon wealth without taking into account the chain rule effect whereby own wealth affects the objective function through changing the opponent's wealth. After we characterize the equilibrium policies under the above conjecture, we verify that these policies constitute an equilibrium when each manager i can choose policies of the form $\phi_i(t, W_{1t}, W_{2t})$, as discussed at the end of the proof of Proposition 3.

We consider only manager 1; for manager 2 the analysis is analogous. From the above [discussion], we compute manager 1's best response only for the [set; type] of policies that may occur in equilibrium – policies of the form $\phi_2^*(t, W_{2t})$ for which manager 2's horizon wealth profile, $W_{2T}(\xi_T)$, is not affected by manager 1's actions. Hence, we fix $W_{2T}(\xi_T)$ and look for the optimal horizon wealth profile manager 1's $\hat{W}_{1T}$. Although manager 1's objective function has a region of local convexity, we can still use standard optimization techniques once we concavify the objective function (see Basak, Pavlova, and Shapiro (2007) for a more formal proof in a similar setting). Concavification involves finding the range $[\underline{\mathrm{W}}, \overline{W}]$ and the coefficients a and b_1 such that replacing $v_1(\cdot)$ within the range $[\underline{\mathrm{W}}, \overline{W}]$ with a chord $a + b_1 W_{1T}$ will result in a globally concave objective function. Noting that the chord must be tangent to $v_1(\cdot)$ at $\underline{\mathrm{W}}$ and $\overline{W}$, we have the following system of equations to solve for:

$$a + b_1\underline{\mathrm{W}} = \frac{\underline{\mathrm{W}}^{1-\bar{\gamma}_1}}{1-\bar{\gamma}_1} \tag{A6}$$

$$a + b_1\overline{W} = \frac{1}{1-\bar{\gamma}_1}\left(\overline{W}^{1-\theta}\left(\frac{\overline{W}}{\eta W_2}\right)^{\theta}\right)^{1-\gamma_1} \tag{A7}$$

$$b_1 = \underline{\mathrm{W}}^{-\bar{\gamma}_1} = (1+\alpha)\overline{W}^{-\gamma_1} W_2^{\theta(\gamma_1-1)}, \tag{A8}$$

where we have dropped $k^{1-\bar{\gamma}_1}$ from the objective function since it does not affect the optimization problem. Subtracting (A6) from (A7) yields

$$b_1(\overline{W} - \underline{\mathrm{W}}) = \frac{1}{1-\bar{\gamma}_1}\left(\overline{W}^{1-\gamma_1}(\eta W_2)^{\theta(\gamma_1-1)} - \underline{\mathrm{W}}^{1-\bar{\gamma}_1}\right). \tag{A9}$$

Expressing $\underline{\mathrm{W}}$ and $\overline{W}$ in terms of b_1 and W_2 (from (A8)) and plugging into (A9) gives

$$\begin{aligned}&b_1(b_1^{-1/\gamma_1}(1+\alpha)^{1/\gamma_1}W_2^{\theta(\gamma_1-1)/\gamma_1} - b_1^{-1/\bar{\gamma}_1})\\ =\ &\frac{1}{1-\bar{\gamma}_1}\left(b_1^{(\gamma_1-1)/\gamma_1}(1+\alpha)^{(\gamma_1-1)/\gamma_1}W_2^{-\theta(\gamma_1-1)^2/\gamma_1}(\eta W_2)^{\theta(\gamma_1-1)} - b_1^{(\bar{\gamma}_1-1)/\bar{\gamma}_1}\right),\end{aligned}$$

which after some algebra yields the boundary function (11). If $y_1\xi_T$ is higher that the slope of the concavification line b_1, the optimal wealth is to the left from $\underline{\mathrm{W}}$, i.e., manager 1 chooses to be a loser and her normal-type policy (9) obtains. Otherwise, she becomes a winner and (10) obtains, accounting for relative concerns.

Manager 1's relative performance is closest to the threshold ηW_{2T} when she is indifferent between being a winner and a loser, i.e., when

$$y_1\xi_T = b_1(\eta W_{2T}) = (1+\alpha)^{\bar{\gamma}_1/\theta}\left(\bar{\gamma}_1/\gamma_1\right)^{\bar{\gamma}_1\gamma_1/(\gamma_1-\bar{\gamma}_1)}\left(\eta W_{2T}\right)^{-\bar{\gamma}_1}. \tag{A10}$$

If manager 1 chooses to be a winner, her "minimum outperformance margin" $\bar{\eta}_1$ is obtained by plugging (A10) into (10) and dividing the resulting wealth by W_{2T}. This yields

$$\begin{aligned}\bar{\eta}_1 &= (1+\alpha)^{1/\gamma_1}\left((1+\alpha)^{\bar{\gamma}_1/\theta}\left(\bar{\gamma}_1/\gamma_1\right)^{\bar{\gamma}_1\gamma_1/(\gamma_1-\bar{\gamma}_1)}\left(\eta W_{2T}\right)^{-\bar{\gamma}_1}\right)^{-1/\gamma_1}\left(\eta W_{2T}\right)^{\theta(\gamma_1-1)/\gamma_1}/W_{2T}\\ &= (1+\alpha)^{(1-\bar{\gamma}_1/\theta)/\gamma_1}\left(\bar{\gamma}_1/\gamma_1\right)^{-\bar{\gamma}_1/(\gamma_1-\bar{\gamma}_1)}\left(\eta W_{2T}\right)^{(\bar{\gamma}_1+\theta(\gamma_1-1))/\gamma_1}/W_{2T}\\ &= (1+\alpha)^{-1/\alpha}\left(\bar{\gamma}_1/\gamma_1\right)^{-\bar{\gamma}_1/(\gamma_1-\bar{\gamma}_1)}\eta,\end{aligned}$$

where in the last equality we use the expressions for θ and γ_i provided in Lemma 1. If manager 1 chooses to be a loser, her "maximum underperformance margin" $\underline{\eta}_1$ is obtained by substituting (A10) into (9) and dividing the result by W_{2T}, which yields

$$\underline{\eta}_1 = \left((1+\alpha)^{\bar{\gamma}_1/\theta}\left(\bar{\gamma}_1/\gamma_1\right)^{\bar{\gamma}_1\gamma_1/(\gamma_1-\bar{\gamma}_1)}\left(\eta W_{2T}\right)^{-\bar{\gamma}_1}\right)^{-1/\bar{\gamma}_1}/W_{2T} = (1+\alpha)^{-1/\theta}\left(\bar{\gamma}_1/\gamma_1\right)^{-\bar{\gamma}_1/(\gamma_1-\bar{\gamma}_1)}\eta.$$

Q.E.D.

Proof of Proposition 2. Given the complexity of our framework, it does not appear possible to obtain the conditions for existence or multiplicity from the corresponding generic results, such as Baye, Tian, and Zhou (1993) for existence or Cooper and John (1988) for multiplicity. Hence, the below detailed analysis is indeed required to derive these conditions.

For a given realization of ξ_T, we can have one of the three outcomes: (winner, loser), (loser, winner), or (loser, loser). From the best response (9), we can determine the regions of ξ_T for which these outcomes can occur.

(winner, loser). From (10), manager 1 chooses to be a winner if $y_1\xi_T \le b_1(\eta W_{2T})$. Plugging manager 2's wealth when a loser, given by (9) with subscript 1 replaced by 2, and using the definition of $b_1(\cdot)$ (11), yields

$$y_1\xi_T \le (1+\alpha)^{\bar{\gamma}_1/\theta}\left(\bar{\gamma}_1/\gamma_1\right)^{\bar{\gamma}_1\gamma_1/(\gamma_1-\bar{\gamma}_1)}\left(\xi_T y_2\right)^{\bar{\gamma}_1/\bar{\gamma}_2}\eta^{-\bar{\gamma}_1}.$$

Rearranging, we get

$$\xi_T^{\bar{\gamma}_1-\bar{\gamma}_2} \ge y_1^{\bar{\gamma}_2}y_2^{-\bar{\gamma}_1}\left((1+\alpha)^{\bar{\gamma}_1/\theta}\left(\bar{\gamma}_1/\gamma_1\right)^{\bar{\gamma}_1\gamma_1/(\gamma_1-\bar{\gamma}_1)}\right)^{-\bar{\gamma}_2}\eta^{\bar{\gamma}_1\bar{\gamma}_2} \equiv yA\eta^{\bar{\gamma}_1\bar{\gamma}_2}, \tag{A11}$$

where $y \equiv y_1^{\bar{\gamma}_2}y_2^{-\bar{\gamma}_1}$ and A is as given in (14).

From (10), manager 2 chooses to be a loser if $y_2\xi_T > b_2(\eta W_{1T})$. Plugging manager 1's wealth W_{1T}, given in (10), and expanding b_2 yields

$$y_2\xi_T > (1+\alpha)^{\bar{\gamma}_2/\theta}\left(\bar{\gamma}_2/\gamma_2\right)^{\bar{\gamma}_2\gamma_2/(\gamma_2-\bar{\gamma}_2)}\left(\eta(1+\alpha)^{1/\gamma_1}(y_1\xi_T)^{-1/\gamma_1}(\eta W_{2T})^{\theta(\gamma_1-1)/\gamma_1}\right)^{-\bar{\gamma}_2}.$$

After some algebra, we get

$$\xi_T^{\bar{\gamma}_1-\bar{\gamma}_2} > y_1^{\bar{\gamma}_2}y_2^{-\bar{\gamma}_1}(1+\alpha)^{\gamma_1\bar{\gamma}_2/\theta-\bar{\gamma}_2}\left(\bar{\gamma}_2/\gamma_2\right)^{\gamma_1\gamma_2\bar{\gamma}_2/(\gamma_2-\bar{\gamma}_2)}\eta^{-\bar{\gamma}_2(\gamma_1+\theta(\gamma_1-1))} \equiv yC\eta^{-\bar{\gamma}_2(\gamma_1+\theta(\gamma_1-1))}, \tag{A12}$$

where C is as given in (15). The outcome (winner, loser) can occur provided that both (A11) and (A12) are satisfied, which means ξ_T satisfies:

$$\xi_T^{\bar{\gamma}_1-\bar{\gamma}_2} > y\max\left[A\eta^{\bar{\gamma}_1\bar{\gamma}_2}, C\eta^{-\bar{\gamma}_2(\gamma_1+\theta(\gamma_1-1))}\right]. \tag{A13}$$

(loser, winner). The expressions are obtained from (A11) and (A12) by switching subscripts 1 and 2, leading to the conditions on $\xi_T^{\bar{\gamma}_2-\bar{\gamma}_1}$. For ease of comparison with (winner, loser) case, we then invert the obtained inequalities to get the conditions on $\xi_T^{\bar{\gamma}_1-\bar{\gamma}_2}$:

$$\begin{aligned} \xi_T^{\bar{\gamma}_1-\bar{\gamma}_2} &\leq y_1^{\bar{\gamma}_2}y_2^{-\bar{\gamma}_1}\left((1+\alpha)^{\bar{\gamma}_2/\theta}\left(\bar{\gamma}_2/\gamma_2\right)^{\bar{\gamma}_2\gamma_2/(\gamma_2-\bar{\gamma}_2)}\right)^{\bar{\gamma}_1}\eta^{-\bar{\gamma}_1\bar{\gamma}_2} \equiv yB\eta^{-\bar{\gamma}_1\bar{\gamma}_2}, && \text{(A14)}\\ \xi_T^{\bar{\gamma}_1-\bar{\gamma}_2} &< y_1^{\bar{\gamma}_2}y_2^{-\bar{\gamma}_1}(1+\alpha)^{\bar{\gamma}_1-\gamma_2\bar{\gamma}_1/\theta}\left(\bar{\gamma}_1/\gamma_1\right)^{-\gamma_2\gamma_1\bar{\gamma}_1/(\gamma_1-\bar{\gamma}_1)}\eta^{\bar{\gamma}_1(\gamma_2+\theta(\gamma_2-1))} \equiv yD\eta^{\bar{\gamma}_1(\gamma_2+\theta(\gamma_2-1))}, \end{aligned}$$

where B and D are given by (14) and (15), respectively. Combining the two conditions, (loser, winner) can occur for ξ_T satisfying

$$\xi_T^{\bar{\gamma}_1-\bar{\gamma}_2} < y\min\left[B\eta^{-\bar{\gamma}_1\bar{\gamma}_2}, D\eta^{\bar{\gamma}_2(\gamma_2+\theta(\gamma_2-1))}\right]. \tag{A15}$$

(loser, loser). The conditions for this outcome follow from the observation that manager i wants to be a loser in those states in which she does not want to be a winner. Hence, manager 1 wants to be a loser for ξ_T such that (A11) is not satisfied. Similarly, manager 2 chooses to be a loser when (A14) does not hold. So, (loser, loser) can occur for ξ_T given by

$$yB\eta^{-\bar{\gamma}_1\bar{\gamma}_2} < \xi_T^{\bar{\gamma}_1-\bar{\gamma}_2} < yA\eta^{\bar{\gamma}_1\bar{\gamma}_2}. \tag{A16}$$

Inspection of (A13), (A15), and (A16) reveals that if (loser, loser) region is not empty, i.e., if

$$B\eta^{-\bar{\gamma}_1\bar{\gamma}_2} < A\eta^{\bar{\gamma}_1\bar{\gamma}_2}, \tag{A17}$$

the three regions can never overlap, meaning that multiple equilibria are not possible. For the unique equilibrium to exist, i.e., for the three regions to fully cover the interval $(0, +\infty)$, it must be the case that

$$A\eta^{\bar{\gamma}_1\bar{\gamma}_2} \geq C\eta^{-\bar{\gamma}_2(\gamma_1+\theta(\gamma_1-1))}, B\eta^{-\bar{\gamma}_1\bar{\gamma}_2} \leq D\eta^{\bar{\gamma}_2(\gamma_2+\theta(\gamma_2-1))}, \tag{A18}$$

in which case the unique equilibrium has the following structure. (winner, loser) occurs for $\xi_T^{\bar{\gamma}_1-\bar{\gamma}_2} > yA\eta^{\bar{\gamma}_1\bar{\gamma}_2}$, (loser, loser) for $yB\eta^{-\bar{\gamma}_1\bar{\gamma}_2} \leq \xi_T^{\bar{\gamma}_1-\bar{\gamma}_2} \leq yA\eta^{\bar{\gamma}_1\bar{\gamma}_2}$, and (loser, winner) for $\xi_T^{\bar{\gamma}_1-\bar{\gamma}_2} < yB\eta^{-\bar{\gamma}_1\bar{\gamma}_2}$. Combining (A17) and (A18) yields the condition for the existence and uniqueness of equilibrium (12).

If (loser, loser) region is empty, i.e., if (A17) is not satisfied, then for an equilibrium to exist it must be the case that the remaining two outcomes fully cover $(0, +\infty)$. Hence, from (A13) and (A15), we get the multiple equilibria condition (13). In a knife-edge case when (13) holds as an equality, the equilibrium is unique. In this case, (winner, loser) occurs for ξ_T satisfying (A13), (loser, winner) occurs for the other ξ_T. In all other cases, when (13) holds as a strict inequality, multiple equilibria obtains. The structure of the equilibria is as follows. (winner, loser) occurs for $\xi_T^{\bar{\gamma}_1-\bar{\gamma}_2} > y\min\left[B\eta^{-\bar{\gamma}_1\bar{\gamma}_2}, D\eta^{\bar{\gamma}_1(\gamma_2+\theta(\gamma_2-1))}\right]$, (loser, winner) occurs for $\xi_T^{\bar{\gamma}_1-\bar{\gamma}_2} < y\max\left[A\eta^{\bar{\gamma}_1\bar{\gamma}_2}, C\eta^{-\bar{\gamma}_2(\gamma_1+\theta(\gamma_1-1))}\right]$. The region

$$y\max\left[A\eta^{\bar{\gamma}_1\bar{\gamma}_2}, C\eta^{-\bar{\gamma}_2(\gamma_1+\theta(\gamma_1-1))}\right] < \xi_T^{\bar{\gamma}_1-\bar{\gamma}_2} < y\min\left[B\eta^{-\bar{\gamma}_1\bar{\gamma}_2}, D\eta^{\bar{\gamma}_1(\gamma_2+\theta(\gamma_2-1))}\right]$$

is consistent with both (winner, loser) and (loser, winner) outcomes, hence for such ξ_T we get the multiplicity of equilibria with these two outcomes. Other structures of equilibrium are not possible, hence there is no pure-strategy Nash equilibrium if neither (12) nor (13) is satisfied.

As a consistency check, we now verify that the conditions for uniqueness and multiplicity, (12) and (13), respectively, cannot hold simultaneously. Condition (12) can equivalently be stated as the threshold η being higher than each of the three terms inside the maximum operator on the left-hand side of (12), and so η is higher than the first term $(B/A)^{1/(2\bar{\gamma}_1\bar{\gamma}_2)}$. Rearranging yields $A\eta^{\bar{\gamma}_1\bar{\gamma}_2} > B\eta^{-\bar{\gamma}_1\bar{\gamma}_2}$. The multiplicity condition (13) is equivalent to the statement that each of the two terms inside the maximum operator on the left-hand side is lower than each of the two terms inside the minimum operator on the right-hand side. Hence, $A\eta^{\bar{\gamma}_1\bar{\gamma}_2} < B\eta^{-\bar{\gamma}_1\bar{\gamma}_2}$, contradicting the above (rearranged) uniqueness condition.

Let us summarize the above analysis. When the performance threshold η is equal (or close) to 1, the outcome (loser, loser) cannot happen in equilibrium. Hence, for an equilibrium to exist, the conditions for the two remaining outcomes have to cover the interval $(0, +\infty)$. As shown above, the outcome (winner, loser) is an equilibrium when ξ_T is higher than the left-hand side in the multiple-equilibria condition (13), while (loser, winner) is an equilibrium when ξ_T is lower than the right-hand side in (13). Hence, if (13) is satisfied, for every $\xi_T \in (0, +\infty)$ at least one of the two outcomes is an equilibrium. In the region where

ξ_T is between the left- and right-hand side of (13), any of the two outcomes can occur in equilibrium, giving rise to multiple equilibria. The condition for uniqueness where the performance threshold η is above a certain critical value (12) admits the outcome (loser, loser) for some states ξ_T. In this case the regions corresponding to the three equilibrium outcomes cover all possible states of the world and do not overlap. Hence, a unique equilibrium occurs. If neither (12) nor (13) holds, a pure equilibrium does not exist.

Q.E.D.

Proof of Proposition 3. First, let us define the constants $\tilde{y}_1, \tilde{\gamma}, \beta$ and functions $Z(\cdot), d(\cdot)$ to be used in the analysis below:

$$
\begin{aligned}
\tilde{y}_1 &= y_1^{-1/\gamma_1} y_2^{-\alpha(\bar{\gamma}_1-1)/(\gamma_1\bar{\gamma}_2)} \left((1+\alpha)^{-1}\eta^{-\alpha(\bar{\gamma}_1-1)}\right)^{-1/\gamma_1},\ \tilde{\gamma} = \frac{\gamma_1\bar{\gamma}_2}{\bar{\gamma}_2+\alpha(\bar{\gamma}_1-1)},\\
\beta &= \left[\eta^{\bar{\gamma}_1\bar{\gamma}_2} y_2^{-\bar{\gamma}_1} y_1^{\bar{\gamma}_2}(1+\alpha)^{-\bar{\gamma}_1\bar{\gamma}_2/\theta}\left(\bar{\gamma}_1/\gamma_1\right)^{-\bar{\gamma}_1\gamma_1\bar{\gamma}_2/(\gamma_1-\bar{\gamma}_1)}\right]^{1/(\bar{\gamma}_1-\bar{\gamma}_2)},\\
Z(z,t) &= e^{((1-z)/z)\left(r+\|\kappa\|^2/(2z)\right)(T-t)},\ d(z,x) = \frac{\ln(x/\xi_t) + \left(r+(2-z)\|\kappa\|^2/(2z)\right)(T-t)}{\|\kappa\|\sqrt{T-t}}.
\end{aligned}
$$

From Proposition 2, an equilibrium exists and is unique if the uniqueness condition (12) is satisfied. In the proof of Proposition 2, we have established the structure of the managers' equilibrium wealth profiles – for each realization of ξ_T we identified whether manager i, $i=1,2$, is a winner or a loser. We now determine the associated equilibrium horizon wealth profiles corresponding to these two outcomes. Focusing on manager 1, from (9) her optimal wealth is $(y_1\xi_T)^{-1/\bar{\gamma}_1}$ when she is a loser, i.e., when $\xi_T^{\bar{\gamma}_1-\bar{\gamma}_2} \leq yA\eta^{\bar{\gamma}_1\bar{\gamma}_2}$. Otherwise, when $\xi_T^{\bar{\gamma}_1-\bar{\gamma}_2} > yA\eta^{\bar{\gamma}_1\bar{\gamma}_2}$, manager 1 is a winner, and her best response wealth is given in (10). As the performance threshold η is greater than one, manager 2 is a loser whenever manager 1 is a winner, and so in equilibrium chooses $W^*_{2T} = (y_2\xi_T)^{-1/\bar{\gamma}_2}$. Plugging this into (10) yields the equilibrium manager 1's wealth when she is a winner:

$$
W^*_{1T} = y_1^{-1/\gamma_1} y_2^{-\theta(\gamma_1-1)/(\gamma_1\bar{\gamma}_2)}(1+\alpha)^{1/\gamma_1}\eta^{\theta(\gamma_1-1)/\gamma_1}\xi_T^{-(\bar{\gamma}_2+\theta(\gamma_1-1))/(\gamma_1\bar{\gamma}_2)}.
$$

To derive the associated equilibrium investment policy of manager 1, ϕ^*_{1t}, we first determine the time-t value of the equilibrium wealth W^*_{1t}. Since $\xi_t W^*_{1t}$ is a martingale, we have

$$
\xi_t W^*_{1t} = E_t[\xi_T W^*_{1T}] = y_1^{-1/\bar{\gamma}_1} E_t[\xi_T^{(\bar{\gamma}_1-1)/\bar{\gamma}_1}\mathbb{1}_{\{\xi_T<\beta\}}] + \tilde{y}_1 E_t[\xi_T^{(\tilde{\gamma}-1)/\tilde{\gamma}}\mathbb{1}_{\{\xi_T\geq\beta\}}]. \tag{A19}
$$

To evaluate the two conditional expectations, we use the following property of the truncated log-normal distribution. If x is a log-normally distributed random variable such that $\ln x$ is normal with mean m and variance v^2, then

$$E[x^n \mathbb{1}_{x<a}] = e^{nm+n^2v^2/2}\, N\left((\ln a - m - nv^2)/v\right). \tag{A20}$$

Given that as of time t $\ln \xi_T$ has mean $\ln \xi_t - (r + \|\kappa\|^2/2)(T-t)$ and variance $\|\kappa\|^2(T-t)$, applying (A20) to (A19) yields

$$W_{1t}^* = N(d(\bar{\gamma}_1, \beta)) y_1^{-1/\bar{\gamma}_1} Z(\bar{\gamma}_1, t) \xi_t^{-1/\bar{\gamma}_1} + (1 - N(d(\tilde{\gamma}, \beta))) \tilde{y}_1 Z(\tilde{\gamma}, t) \xi_t^{-1/\tilde{\gamma}}. \tag{A21}$$

From Itô's Lemma, the coefficient of the Brownian motion ω in the dynamic process for W_{1t}^* is equal to

$$-\kappa_j \xi_t \frac{\partial W_{1t}^*}{\partial \xi_t},$$

where κ_j is the j-th component of the market price of risk vector κ. Equating each of these coefficients to the corresponding diffusion term in (1), we obtain the following system of linear equations:

$$\xi_t \frac{\partial W_{1t}^*}{\partial \xi_t} \kappa = W_{1t}^* \sigma^\top \phi_{1t}^*.$$

Solving the system and substituting the derivative of (A21) with respect to ξ_t into the solution yields manager 1's equilibrium investment policy. For manager 2, the analysis is analogous.

As explained in the proof of Proposition 1, in the above analysis we conjecture that manager 1's equilibrium policy is of the form $\phi_1(t, W_{1t})$, and similarly for manager 2. It is straightforward to observe that the policies derived under this conjecture constitute an equilibrium in the original game described in Section 2.3 where manager i is allowed to choose strategies of the form $\phi_i(t, W_{1t}, W_{2t})$. Indeed, when computing manager 1's best response we do not restrict her policy to have the conjectured form, the conjecture was applied only to manager 2's policy. That is, manager 1, while able to choose a policy of the form $\phi_1(t, W_{1t}, W_{2t})$, optimally chooses the policy of the form $\phi_1(t, W_{1t})$ whenever the opponent's policy has the form $\phi_2(t, W_{2t})$, and similarly for manager 2. Investigating whether there exists another equilibrium in which each of the two policies depends non-trivially on the opponent's wealth is rather complex, and so is beyond the scope of the current paper.

Q.E.D.

Proof of Corollary 1. The limiting equilibrium policies of manager 1 is straightforwardly obtained by letting ξ_t tend to 0 or ∞ in (16), and similarly for manager 2.

Substituting the equilibrium limiting policies into (17) and multiplying both parts by the inverse of $(\sigma^\top)^{-1}\kappa$, we obtain that (17) is equivalent to

$$\operatorname{sgn}\left(\frac{1}{\gamma_1} + \frac{\theta(\gamma_1 - 1)}{\gamma_1 \bar{\gamma}_2} - \frac{1}{\bar{\gamma}_1}\right) = \operatorname{sgn}(\bar{\gamma}_1 - 1)\, \operatorname{sgn}\left(\frac{1}{\bar{\gamma}_2} - \frac{1}{\bar{\gamma}_1}\right). \tag{A22}$$

Rearranging the argument of $sgn(\cdot)$ on the left-hand side of (A22) yields

$$\begin{aligned}\frac{\bar{\gamma}_1\bar{\gamma}_2 + \bar{\gamma}_1\alpha(\bar{\gamma}_1 - 1) - \bar{\gamma}_2(\bar{\gamma}_1 + \alpha(\bar{\gamma}_1 - 1))}{\gamma_1\bar{\gamma}_1\bar{\gamma}_2} &= \frac{\bar{\gamma}_1\alpha(\bar{\gamma}_1 - 1) - \bar{\gamma}_2\alpha(\bar{\gamma}_1 - 1)}{\gamma_1\bar{\gamma}_1\bar{\gamma}_2} \\ &= \frac{\alpha}{\gamma_1}(\bar{\gamma}_1 - 1)\left(\frac{1}{\bar{\gamma}_2} - \frac{1}{\bar{\gamma}_1}\right).\end{aligned}$$

Since α and γ_1 are positive, (A22) obtains. Switching subscripts 1 and 2 in (17) gives (18).

Q.E.D.

Proof of Proposition 4. In Proposition 2 we show that multiple equilibria occur if the multiplicity condition (13) is satisfied. In the proof of Proposition 2, we describe the structure of the multiple equilibria, i.e., the states in which outcome (winner, loser) or outcome (loser, winner) occurs in equilibrium, and the states in which either of the two outcomes is possible. In Proposition 3, we describe what horizon wealth manager i, $i = 1, 2$, chooses in equilibrium when she is a winner and when she is a loser. Combining the results of these two Propositions, we obtain the horizon wealth profiles that can occur in the case of multiple equilibria.

Q.E.D.

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