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**THE MORE WE KNOW ON THE
FUNDAMENTAL, THE LESS WE
AGREE ON THE PRICE**

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ABSTRACT

The more we know on the fundamental, the less we agree on the price*

I allow heterogeneity in trading horizons across groups in a standard differential information model of a financial market. This can explain the empirical facts that after public announcements trading volume increases, more private information is incorporated into prices and volatility increases. Public information, in such environments, has the important secondary role of helping agents to learn about the information of other agents. As a consequence, whenever the correlation between private information across groups is sufficiently low, a public announcement increases disagreement among short horizon traders on the expected selling price, even if it decreases disagreement about the fundamental value of the asset. Additional testable implications are also suggested.

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* This paper builds on Chapter 2 of my PhD thesis at the LSE. I am grateful for Hyun Song Shin and Dimitri Vayanos for the guidance. Previous versions were circulated under the titles: "The more we know the less we agree: Public announcements and higher-order expectations" and "The more we know, the less we agree: Public announcements, higher-order expectations, and rational inattention". I am also grateful for the help received from Amil Dasgupta, Gabrielle Demange, Bernardo Guimaraes, Christian Hellwig, Gergely Ujhelyi, Jakub Steiner, Stijn Van Nieuwerburgh, Laura Veldkamp and seminar participants at Cambridge, CEU, Chicago, LSE, NYU and the SED 2009 meeting in Istanbul. I also thank John Harbord for editorial assistance. All remaining errors are my own. I gratefully acknowledge the EU grant "Archimedes Prize" (HPAW-CT-2002-80054), and the financial support of the Central Bank of Hungary.

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The more we know on the fundamental, the less we agree on the price

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May 2011

Abstract

I allow heterogeneity in trading horizons across groups in a standard differential information model of a financial market. This can explain the empirical facts that after public announcements trading volume increases, more private information is incorporated into prices and volatility increases. Public information, in such environments, has the important secondary role of helping agents to learn about the information of other agents. As a consequence, whenever the correlation between private information across groups is sufficiently low, a public announcement increases disagreement among short horizon traders on the expected selling price, even if it decreases disagreement about the fundamental value of the asset. Additional testable implications are also suggested.

1 Introduction

Why do announcements of public information set off a frenzy of trading? Intuition suggests that public information brings beliefs closer to each other. With less disagreement, there should be less reason to trade.

*E-mail address: kondorp@ceu.hu. This paper builds on Chapter 2 of my PhD thesis at the LSE. I am grateful for Hyun Song Shin and Dimitri Vayanos for the guidance. Previous versions were circulated under the titles: "The more we know the less we agree: Public announcements and higher-order expectations" and "The more we know, the less we agree: Public announcements, higher-order expectations, and rational inattention". I am also grateful for the help received from Amil Dasgupta, Gabrielle Demange, Bernardo Guimaraes, Christian Hellwig, Gergely Ujhelyi, Jakub Steiner, Stijn Van Nieuwerburgh, Laura Veldkamp and seminar participants at Cambridge, CEU, Chicago, LSE, NYU and the SED 2009 meeting in Istanbul. I also thank John Harbord for editorial assistance. All remaining errors are my own. I gratefully acknowledge the EU grant "Archimedes Prize" (HPAW-CT-2002-80054), and the financial support of the Central Bank of Hungary.

For a fresh look on this puzzle, my starting point in this paper is that the trading horizon differ across market participants. That is, some groups of traders buy assets knowing that later they will have to resell it to others. Public information, in such an environment, aside from providing information about the uncertain fundamentals, has an important secondary role in helping each agent to guess the private information of other agents. I show that this observation provides a novel explanation of the well established stylized facts that after public announcements trading volume increases, more private information is incorporated into prices and volatility increases. In particular, I show that these facts arise naturally in a generalized Grossman-Stiglitz type model where agents' trading horizon vary and their is sufficient heterogeneity in their information sets. I also suggest additional testable implications.

The main result is based on a simple observation. Agents' opinion about the opinion of others (higher-order expectations) respond differently to public information than agents' opinion on the fundamentals of an economic object (fundamental expectations). In particular, a public announcement might increase disagreement among agents in higher-order expectations, even if it decreases disagreement in fundamental expectations. A typical case of this when agents collect private information on different dimensions of the fundamental. For an extreme example, consider two groups: A -agents and B -agents. Suppose that while B -agents form their expectation about the fundamental, each A -agent has to guess B -agents' average fundamental expectation. That is, A -agents form second-order expectations. This might be the case in financial markets if A -agents expect to resell their assets to B -agents. Suppose that the fundamental is the sum of two independent factors, $\theta = \theta_A + \theta_B$. While each A -agent observes a different noisy signal on θ_A , each B -agent observes a different noisy signal on θ_B . The public announcement, observed by all, is a noisy version of the fundamental, $y = \theta + \eta$. Without a public announcement, A -agents agree in their guess, because their private signals do not reveal any information on B -agents' signal. However, there is disagreement with a public announcement. For example, an A -agent with a high private signal on the first factor relative to the announcement concludes that most probably, the other factor is low. Therefore, the average signal of B -agents and their average fundamental expectation must also be low. An A -agent with a low private signal relative to the announcement reaches the opposite conclusion. Thus, the announcement *polarizes* second-order expectations. Interestingly, first-order expectations are not polarized as disagreement among members of any of the groups fundamental expectation decreases after the public announcement.

I incorporate this intuition into an economic context by analyzing a generalized, differential information model of financial markets in the tradition of Grossman-Stiglitz (1980) and Hellwig (1980). Importantly, I allow agents to have heterogeneous trading horizons and

to observe private signals with weak unconditional correlation. In particular, I assume two groups of traders; *A*-traders and *B*-traders. There are three periods. Agents trade in the first and second periods and the fundamental value of the asset realizes in the third. While *B*-traders consume their financial wealth in the third period, *A*-traders have to liquidate their assets and consume the proceeds in the second period. Consequently, in the first period *A*-traders know that their consumption depends on the second period equilibrium price they receive from *B*-traders for their assets as opposed to the fundamental value of the asset. The information structure is general in the sense that the unconditional correlation of private signals across groups can range from 0 to 1, depending on the parameters. Consistently with the example above, whenever this correlation is sufficiently low, a public announcement in the first period increases the dispersion in *A*-traders' forecast of the second period price. I refer to an information structure which satisfies this condition on the correlation structure as a weakly correlated information structure.

I consider two versions of the model. In the first case, *B*-traders are not present in the market in the first period. I interpret this case as a model of an asset being traded in geographically distinct locations. Examples include currencies and cross-listed stocks. In the second case, I present a general model of trading with heterogeneous horizon where *B*-traders and *A*-traders are both present in the first period.

Consider the simpler case where only *A*-traders are present in the first period. As a main result, under a weakly correlated information structure, the announcement induces an upward shift in trading and in the amount of private information incorporated in prices in both periods. Moreover, the volatility of first period price can also increase. As I discuss below, these implications are consistent with a vast body of empirical work. Polarization creates trading volume in the first period because the increased disagreement of *A*-traders translates to active speculative trading after the announcement. Interestingly, it induces more trade also among *B*-traders in the second period, because *A*-traders' more active trading makes first period prices more informative. This reduces the uncertainty for *B*-traders making them more aggressive in trading on their private information.

With the help of the second case, I analyze the effect of an increase in the share of *A*-traders in the population. I interpret the increase of this share as a proxy for increased market segmentation and/or an increased fraction of short-horizon traders in the economy. The drawback of the additional complexity of this version is that the availability of analytical results becomes limited. Relying partially on numerical simulations, I show that under a weakly correlated information structure the elasticity of volume with respect to public information is positive in the first period. Moreover, increasing the share of *A*-traders increases the elasticity. When this share is sufficiently high, this elasticity is also positive in the second

period. In some weakly correlated information structures, the elasticity of first period price volatility with respect to public information is also positive .

The analysis of the second case helps to find new ways to test this theory. First, recent work has constructed empirical proxies for the investment horizon of the investor base of financial assets.¹ Thus, a simple testable implication is that trading volume of assets with a larger share of short-horizon investors in their investor base should respond more strongly to public announcements. Second, the event of cross-listing of stocks might be used as a natural experiment where the degree of market segmentation of the given asset increases. Thus, the volume and information content of prices of cross-listed stocks should respond more strongly to a release of public information after the event than before. Although interpreting this work as a direct test of our theory has caveats, Bailey, Karolyi and Salva (2005) document that volume and volatility of cross-listed stocks indeed follow this pattern.

This paper is the first to highlight that public announcements can polarize market participants' valuation of an asset without polarizing their fundamental expectations. It is also the first to point out the potential of this observation to explain empirical patterns around public announcements in financial markets.

There is a large previous literature focusing on the trade and price pattern around public announcements. The stylized fact that the trading volume of stocks increases around earnings announcements has been known for decades.² Recent studies based on high-frequency data sets give a more detailed picture.³ First, this stylized fact is true across various markets and various type of public information releases. Second, within the day, trading volume drops for a period before the expected announcement and increases only afterwards. Third, the private information incorporated into prices through trading increases significantly after announcements.⁴ Finally, public announcements also increase return volatility.

As neither in representative agent models, nor in standard differential information models should the price adjustment caused by the new public information be accompanied by abnormal trading volume or volatility, even the basic stylized facts are puzzling from the view point of the most standard models. Motivated by this fact, Kim and Verrecchia (1991) introduce heterogeneous risk-aversion and information precision, while He and Wang (1995) introduce residual uncertainty for the final pay-off into a dynamic version of the standard

¹See Wahal and McConnell (2000) and Gaspar et al. (2005).

²See Beaver (1968), Bamber (1987), Ziebart (1990), Kandel and Pearson (1995).

³Fleming and Remolona (1999) and Green (2005) focus on the market for US treasuries, while Evans and Lyons (2008) and Love and Payne (2008) focus on currency markets, and Chae (2005) and Krinsky and Lee (1996) focus on equity markets.

⁴Krinsky and Lee (1996) and Fleming and Remolona (1999) decompose the bid-ask spread around announcements, while Evans and Lyons (2008) analyze the joint distribution of the orderflow and prices to arrive to this conclusion.

model. These papers highlight that small modifications to the standard framework do lead to some trading volume around announcements. However, in these models trading volume increases because agents build up speculative positions before the announcement which they liquidate after observing the announcement. Informative trading does not increase after the announcement, because disagreement decreases. This is hard to reconcile with the stylized facts above.

In line with the empirical results, the majority of the literature is settled on the conclusion that a viable explanation for these patterns requires models where public announcements increase the disagreement among agents on the valuation of the asset. Observing that public signals in common prior environments generally decrease disagreement on fundamentals, this literature developed in two directions. The first group (e.g. Kim and Verrecchia (1997), Evans and Lyons (2001)) relaxes the assumption that public announcements are modeled by public signals. Instead, public announcements are modelled as combinations of public and private signals.⁵ Thus, the announcement can increase disagreement. The disadvantage of this line of work is that its generality is limited. As the new assumption is on the nature of information content of announcements, its potential to explain empirical patterns unrelated to announcements is naturally small. This is in contrast to the second group starting with Varian (1989), Harris and Raviv (1993) and Kandel and Pearson (1995) assuming heterogeneous priors. That is, agents with differing priors process the same public signal but reach a different posterior valuation. The assumption of heterogeneous priors proved to be useful for addressing various other empirical puzzles⁶.

Like the aforementioned two groups of papers, this paper also argues that trading volume increases around public announcement because traders' disagreement increases on the valuation of the asset. Importantly, I point out that this is consistent with the combination of common priors and public signals as long as a fraction of agents have a short trading horizon. As short horizon agents focus on the future price instead of the fundamental, in this case public announcement can polarize market participants' valuation of the asset without polarizing their fundamental expectation. This model shares the advantage with heterogeneous priors that its main assumption, the presence of short-horizon investors, also proved to be a fruitful approach in a wide range of economic contexts.⁷

⁵Rabin and Shrag (1999) uses the same modelling strategy to explain confirmation bias.

⁶For example, heterogeneous prior models were shown to explain puzzles related to bubbles (Harrison and Kreps (1979), Morris (1996), Sheinkman and Xiong (2003, Biais and Bossaerts (1998)), IPO overpricing (Morris(1996)), momentum and post-announcement drift (Banerjee, Kaniel and Kramer (2009)). See also Dixit and Weibull (2007), Acemoglu, Chernozhukov and Yildiz (2009) for the discussion of polarization due to the relaxation of the common prior assumption in other contexts.

⁷For example, Tirole (1985) and Woodford and Santos (1997) analyze the role of short-horizons (OLG models) in rational bubbles, Froot, Scharfstein and Stein (1992) connect short-horizon of traders to herding,

In the given context there is a fundamental trade-off between heterogeneous prior models and common priors-differential information models (including the one in this paper). On one hand, unlike models with heterogeneous priors, common prior models are constrained by No Trade Theorems (e.g., Milgrom and Stokey (1982)). That is, differential information does not generate trade in itself without some type of noise in price determination. Thus, these models analyze changes in trading volume for a given amount of noise. Because of this constraint, the assumption of heterogeneous priors looks as a natural candidate to explain patterns related to the enormous trading volume of financial markets. On the other hand, this assumption implies a lack of learning from other agents actions. Thus, these models tend to be inconsistent with the evidence that after public announcement a large flow of private information is incorporated into the price. Because of this trade-off, the two class of assumptions have a complimentary role in explaining patterns of trade and prices around announcements.⁸

More broadly, this paper fits into the recent flow of papers analyzing the effect of higher-order expectations in various contexts. The most closely related part of this literature⁹ analyzes environments where various groups of agents act sequentially and the pay-off of early actors depends on the actions of groups acting later (e.g. Allen, Morris and Shin (2006), Makarov and Rytchkov (2007) Banerjee, Kaniel and Kremer (2009), Goldstein, Ozdenoren and Yuan (2008), Angeletos, Lorenzoni and Pavan (2007)). Thus, early actors have to guess the information of agents acting later. Applications include financial markets, currency attacks and the interaction between stock prices and real investment. None of these papers consider information structures with the possibility of polarized higher-order expectations.

The structure of this paper is as follows. In the next section, I illustrate with an example how public signals can polarize higher-order expectations in Gaussian information structures. In section 3, I present the financial application, characterize the equilibrium and discuss additional empirical implications. Finally I conclude.

and Vives (1995) relates short-horizon traders to the informational efficiency of prices. See Chapter 8.3 in Vives (2008) for more related literature on short-horizon traders in rational financial models. Also, starting with Holmstrom and Ricart i Costa (1986) and Shleifer and Vishny (1990), there is a series of models providing microfoundations for the existence of investors with short-horizon.

⁸See Banerjee and Kremer (2010) as a notable example of mixing these two sets of assumptions.

⁹Other papers concentrate on whether imperfect information of decision makers leads to a unique equilibrium in coordination games (e.g. Morris and Shin (1998), Angeletos and Werning (2006), Hellwig, Mukherji and Tsyvinski (2006)), and on "Beauty contest" environments where the pay-off of agents is a weighted sum of the deviation of their actions from an optimal level and of the deviation of their actions from the average action of others (e.g. Morris and Shin (2002), Woodford (2002), Hellwig (2002), Angeletos and Pavan (2007)).

2 Polarized second-order expectations: an example

Before introducing a model of a financial market, I illustrate the driving force of the results by a simple example. In this example, a public announcement increases disagreement in second-order expectations without increasing disagreement in first-order expectations.

Consider groups A and B with a unit mass of agents in each group indexed by i and j respectively. B -agents forms expectations about a fundamental, θ . A -agents forms expectations about the average expectation of B -agents.¹⁰ These are second-order expectations on θ . The fundamental value is the sum of two independent factors, $\theta = \theta_A + \theta_B$. Each A -agent i observes a private signal about the first factor, $x^i = \theta_A + \varepsilon^i$, while each B -agent j observes a private signal on the second factor, $z^j = \theta_B + \varepsilon^j$. The difference across groups' information sets represents an unmodelled specialization in information acquisition. I am interested in the change of dispersion in first and second order expectations after the release of a public signal, $y = \theta + \eta$. I assume that each factor and noise term is drawn from independent distributions

$$\theta_A, \theta_B \sim N\left(0, \frac{1}{\kappa}\right), \varepsilon^i, \varepsilon^j \sim N\left(0, \frac{1}{\alpha}\right), \eta \sim N\left(0, \frac{1}{\beta}\right).$$

Consider first the case before the public announcement. The fundamental expectation of each B -agent is a linear function of the private signal

$$E(\theta|z^j) = b^n z^j$$

where $b^n > 0$ and the n superscript stands for no-announcement. The average expectation in group B is

$$\bar{E}(\theta|z^j) \equiv \int_0^1 E(\theta|z^j) dj = b^n \theta_B.$$

Then a measure of dispersion of fundamental expectations is

$$\int_0^1 |E(\theta|z^j) - \bar{E}(\theta|z^j)| dj = \frac{|b^n|}{\sqrt{\alpha}} \sqrt{\frac{2}{\pi}}.$$

Each agent i in group A forms the second order expectation

$$E(\bar{E}(\theta|z^j)|x^i) = a^n E(\theta_B|x^i) = 0.$$

The second-order expectation is 0 independently of the private signal of agent i , because

¹⁰In the main model, each agent in group A eventually wants to sell her asset to someone in group B , this is why she is interested in the expectations of group B . However, to keep our example simple, in this section we do not model the motivations of agents.

A -agents have private information about the A factor only, while B -agents have private information about the B factor only and the two factors are independent. As the fundamental expectations of B -agents depend on their private signal only, A -agents' private information is useless in forming expectations about the fundamental expectation of the average B -agents. Consequently, the dispersion in the second order expectations of A -agents is also 0.

Consider now the case with public announcement. Any B -agent's fundamental expectation is linear and can be written as

$$E(\theta|z^j) = bz^j + cy$$

where $b, c > 0$ are constants. As above, the dispersion of fundamental expectations is

$$\int_0^1 |E(\theta|z^j, y) - \bar{E}(\theta|z^j, y)| dj = \frac{|b|}{\sqrt{\alpha}} \sqrt{\frac{2}{\pi}}.$$

By calculating the coefficients¹¹ it is easy to show that the dispersion decreases after the announcement as

$$b = \frac{\alpha\kappa}{\kappa^2 + \alpha\kappa + \beta(\alpha + 2\kappa)} > \frac{\alpha\kappa}{\kappa^2 + \alpha\kappa} = b^n.$$

This is intuitive. Each B -agent has a more precise knowledge about the fundamental after observing the public signal, thus, the disagreement among B -agents decreases.

Note that A -agents second-order expectations are not independent of private signals anymore as

$$E(\bar{E}(\theta|z^j, y) | x^i, y) = bE(\theta_B | x^i, y) + cy = b(ax^i + c'y) + cy$$

where $a, c' \neq 0$ are the coefficients in $E(\theta_B | x^i, y)$. Consequently, the dispersion in second-order expectations increases from zero to

$$\int_0^1 |E(\bar{E}(\theta|z^j, y) | x^i, y) - \bar{E}(\bar{E}(\theta|z^j, y) | x^i, y)| di = \frac{|ba|}{\sqrt{\alpha}} \sqrt{\frac{2}{\pi}} > 0.$$

¹¹Whenever I calculate the coefficients of conditional expectations of normal variables throughout the paper, I use the Projection Theorem. This states that if \mathbf{v}_θ and \mathbf{v}_s are vectors of variables which are jointly normally distributed with the vector of expected values μ_θ, μ_s , respectively and the covariance matrix of the vector $[\mathbf{v}_\theta, \mathbf{v}_s]$ is

$$\begin{bmatrix} \Sigma_\theta & \Sigma_{\theta,s} \\ \Sigma_{s,\theta} & \Sigma_{s,\theta} \end{bmatrix},$$

where $\Sigma_\theta, \Sigma_{\theta,s}, \Sigma_{s,\theta}, \Sigma_{s,\theta}$ are the appropriate submatrices, then

$$(\mathbf{v}_\theta | \mathbf{v}_s) \sim N(\mu_\theta + \Sigma_{\theta,s} \Sigma_s^{-1} (\mathbf{v}_s - \mu_s), \Sigma_\theta - \Sigma_{\theta,s} \Sigma_s^{-1} \Sigma_{s,\theta}).$$

Thus, second-order expectations are *polarized* by the public announcement. The idea is that as public signal gives information about the sum of the two factors, together with this information, a private signal on one factor is informative about the likely value of the other factor. For example, an A -agent with a high private signal on θ_A relative to the announcement concludes that most probably, the other factor is low. Consequently, the average signal of a B -agent and her fundamental expectation must also be low. In contrast, an A -agent with low private signal on θ_A reaches the opposite conclusion. Thus, there will be dispersion among A -agents about the expectation of the average B -agent.

Observe that polarization in higher-order expectations differs from polarization in first-order expectations, because of the critical role of the strength of connection across private information sets. When an A -agent forms expectations on the fundamental expectations of the average B -agent, she has to forecast the private signal of that agent. When the A -agent's private signal, x^i , is informative about the private signal z^j , then the dispersion of A -agents' second-order expectations is high. Polarization occurs when conditional on the public signal this informativeness increases.

One might wonder why this property has not received any attention in the previous literature. There are two likely reasons. The first is that interest in models where higher-order expectations play an important role is relatively recent. The second is that even in such models the information structure is virtually always assumed to be of the form where both private and public signals are noisy observations of the fundamental: $x^i = \theta + \varepsilon^i$, $z^j = \theta + \varepsilon^j$, $y = \theta + \eta$. Virtually all CARA-Normal models of financial markets impose this information structure. This is why I refer to this structure as the *standard information structure*. To highlight the effect of moving from the standard information structure towards the extreme specification in the above example, in the rest of the paper I use a more general information structure than the one in the example. In particular, I assume that the fundamental is the sum of three factors

$$\theta = \theta_A + \theta_B + \theta_C \tag{1}$$

and the private signals of A and B -agents and the public signal are

$$x^i = \theta_A + \theta_C + \varepsilon^i, \tag{2}$$

$$z^j = \theta_B + \theta_C + \varepsilon^j \tag{3}$$

and

$$y = \theta + \eta. \tag{4}$$

All factors and noise terms are drawn from independent normal distributions

$$\theta_A, \theta_B \sim \mathbf{N}\left(0, \frac{1}{\kappa}\right), \theta_C \sim \mathbf{N}\left(0, \frac{1}{\omega}\right), \varepsilon^i, \varepsilon^j \sim \mathbf{N}\left(0, \frac{1}{\alpha}\right), \eta \sim \mathbf{N}\left(0, \frac{1}{\beta}\right). \quad (5)$$

Note that apart from the group specific factors θ_A, θ_B there is also a common factor θ_C which all agents learn about. The advantage of this structure is that by choosing $\kappa \rightarrow \infty$, it nests the standard, single factor information structure, while by choosing $\omega \rightarrow \infty$ it results in the structure of the presented example where the private information sets are independent. Nevertheless, this structure is simple enough to give tractable expressions. I refer to the information structure given by (1)-(5) as the *general information structure*.

Throughout the paper, instead of comparing equilibrium objects with and without announcement, I consider only the situation when the public signal is observed, and think about the announcement as an increase in the precision of the public information, β . I also refer to β as the amount of public information. The following proposition gives a necessary and sufficient condition for polarization in the informational environment given by (1)-(5).

Proposition 1 *Given the information structure (1)-(5), a public announcement always decreases disagreement among agents' fundamental expectations in each group. That is,*

$$\frac{\partial \int_0^1 |E(\theta|z^j, y) - \bar{E}(\theta|z^j, y)| dj}{\partial \beta}, \frac{\partial \int_0^1 |E(\theta|x^i, y) - \bar{E}(\theta|x^i, y)| di}{\partial \beta} < 0.$$

Furthermore, a public announcement increases disagreement among A -agents about the average fundamental expectation of B -agents, that is

$$\frac{\partial \int_0^1 |E[\bar{E}(\theta|z^j, y) | x^i, y] - \bar{E}[\bar{E}(\theta|z^j, y) | x^i, y]| di}{\partial \beta} > 0,$$

if and only if

$$\beta > \frac{\kappa^2}{\omega} \quad (6)$$

holds.

To see the intuition behind condition (6), note that in our information structure it is equivalent to the condition

$$\text{corr}(z^i, x^j) < \text{corr}(z^i, y) \text{corr}(x^j, y) \quad (7)$$

where $\text{corr}(\cdot, \cdot)$ is the correlation between the variables. Thus, the proposition states that more public information polarizes A -agents' second-order expectations if and only if the

correlation in private information across groups is small relatively to the product of the correlation between private and public information in the two groups. This condition trivially holds in our example, where the correlation between private signals of agents in different groups is zero. In contrast, the standard information structure imposes a rigid structure on the correlation structure of signals and θ . In particular,

$$\text{cov}(x^i, z^j) = \text{cov}(z^j, z^n) = \text{cov}(x^i, x^m) = \text{cov}(x^i, y) = \text{cov}(z^j, y) = \text{var}(\theta) \quad (8)$$

for any agents i, j, n, m . It is easy to check that this structure violates (7) and (6).

Throughout the paper, I refer to the combination of (1)-(5) and assumption (6) as the *weakly correlated information structure*.

In the next section, I argue that the statistical property highlighted in this section has important economic consequences by modifying a standard workhorse model of financial markets with differential information.

3 Trading with heterogeneous horizon and dispersed information

In this section, I consider the effect of public information on strategies and prices in modified versions of a standard, three-period REE, Grossman-Stiglitz type model.¹² I deviate from the basic model along two main dimensions. First, I consider the general information structure instead of the standard information structure, and second, I allow for the interaction of two groups who consume at different time points.

The effect of public announcement to trading positions can differ from its effect on expectations derived in the example of the previous section. First, in the model endogenously determined prices serve as public signals and pay-off relevant variables for early consumers. Second, as traders are risk averse, their position does not depend only on their expectation of the pay-off of their portfolio, but also the uncertainty about the pay-off. The main purpose of this section is to analyze how these channels affect the mapping between the example and observables in a financial market.

The main finding of this section is that the combination of heterogeneous trading horizon and weakly correlated information structure implies polarization in higher-order expectations. This leads to increasing trading volume, increased information content of prices

¹²See Grossmann and Stiglitz (1980), Hellwig (1980), Diamond and Verrecchia (1981), Brown and Jennings (1989), He and Wang (1995) and Allen, Morris and Shin (2006).

and potentially increased volatility of prices around announcements in otherwise standard Grossman-Stiglitz type models. This is consistent with a vast body of empirical work cited in the introduction.

3.1 General set-up

Just as in the example, I consider two groups of traders, A and B , trading the same risky asset and a riskless bond. The return on the bond is normalized to 1. There are three periods, $t = 1, 2, 3$. The uncertain fundamental value θ is realized in period 3. Each agent has CARA utility over final wealth with the identical risk-aversion parameter γ . The total measure of agents is 1 in the first period and $\mu \in (0, 1]$ in the second period. The total supply of assets, u_1 , in period 1 and $\mu u_2 \equiv u_1 + \Delta u_2$ in period 2 are normally and independently distributed.¹³ Each active trader in period t forms her demand d_t^i or d_t^j conditional on her information set \mathcal{I}_t^i or \mathcal{I}_t^j and the price p_t . In equilibrium, the price p_t has to clear the market.

I will consider two main versions of the model. In both cases, A -traders trade in period 1 and sell their portfolio to B traders in period 2 and consume the proceeds, while B traders liquidate their portfolio in period 3 for the fundamental value θ and consume the proceeds. However, in Case 1, only A -traders trade in period 1. B -traders arrive and trade in period 2 only. Thus the problems of traders are determined as follows.

Case 1 *The demand of each A -trader for given price p_1 solves*

$$\begin{aligned} \max_{d_1^i} E \left[-e^{-\gamma W_A^i} | p_1, \mathcal{I}_1^i \right] \\ W_A^i = d_1^i (p_2 - p_1) \end{aligned} \tag{9}$$

and the demand of each B -trader for given price p_2 solves

$$\begin{aligned} \max_{d_2^j} E \left[-e^{-\gamma W_B^j} | p_2, \mathcal{I}_2^j \right] \\ W_B^j = d_2^j (\theta - p_2). \end{aligned} \tag{10}$$

The measure of the two groups are equal and normalized to 1. Components of the random endowment, u_1 and u_2 are drawn independently from the distributions

$$u_1 \sim \mathbf{N} \left(0, \frac{1}{\delta_1^2} \right), u_2 \sim \mathbf{N} \left(0, \frac{1}{\delta_2^2} \right),$$

¹³The independence of u_1 and u_2 implies that $\frac{\text{cov}(u_1, \Delta u_2)}{\text{var}(u_1)} = -1$. This is clearly a stark assumption, but leads to the simplest analysis. The model can be generalized to include any correlation structure across the noise terms. The main results are robust to this treatment.

There are various potential interpretations of this case. For example, one can see this case as a (part of the) 24-hour day in the market of global currencies. In reality, the main direct participants of these markets are dealers. Dealers receive orders from their customers¹⁴, but also trade on their own account. They trade with each other either directly or through interdealer electronic brokerage services. The structure of Case 1 emphasizes two stylized facts of global currency markets. First, a large part of trading volume is generated by dealers operating in distinct geographical locations and during their local daylight hours. Second, dealers tend not to hold positions overnight.¹⁵ That is, they do not pass on positions at the end of the day, even to other dealers of the same financial firm but located in an other geographical location. For example, consider the U.K. pound/US dollar market. A large share of the trading goes through dealers located in either in London or New York. Thus, in terms of the model, A -traders are dealers located in London, B -traders are dealers located in New York. Then period 1 represents trading hours in London and period 2 represents trading hours in New York. Because dealers do not want to hold positions overnight, they maximize their end of day utility. Because the trading hours in London end shortly after trading hours start in New York, Londoners will sell their excess positions to New Yorkers at the end of their trading day.¹⁶ Cross-listed stocks is an other example where geographical segmentation seems to play an important role.¹⁷ In general, I will refer to this set-up as a model of local traders in global markets. This structure is especially useful for my purposes because of its analytical tractability.

In the second case, I allow B -traders to trade in both periods. Thus, in the first period the two types of agents coexist. Thus, the utility of traders is determined as follows.

Case 2 *Each A -trader solves problem (9) while a B -trader solves problem (10) in the second*

¹⁴Starting from Diamond and Verrecchia (1981), it is common to interpret the random supply u_1, u_2 as the sum of initial endowments of traders. In our context, this random endowment could be interpreted as the customer orders of dealers. There is a set-up which is formally equivalent to our version, but uses this alternative interpretation of the random supply.

¹⁵See Lyons (2006), page 46.

¹⁶As understanding this particular market is not the main purpose of this model, I keep the framework close to the standard models of trading with differential information. Thus, I abstract from many other institutional features of this market, e.g. the interdealer trade, the structure of price quotations and market orders, the interday dynamics etc.

¹⁷Although in theory local markets could work as one global market, several studies find significant segmentation in trading activity in these markets. For example, Pulatkonak and Sofianos (1999) finds that 40% of the variation of the US-market share of trading volume of cross-listed stocks can be explained by the hours of overlap in trading between the NYSE and the home market for the stock. See also Rosenthal and Young (1990) and Froot and Dabora (1999).

period and

$$\begin{aligned} & \max_{d_1^i} E \left[-e^{-\gamma W_B^i} | p_1, \mathcal{I}_1^j \right] \\ W_B^j &= d_1^j (p_2 - p_1) + d_2^j (\theta - p_2) \end{aligned} \tag{11}$$

in the first period, where d_2^j is the optimal strategy in period 2.

The measure of A -traders is $(1 - \mu)$ and the measure of B -traders is μ . The random supply, u_1 and u_2 are drawn independently from the distributions

$$u_1 \sim \mathbf{N} \left(0, \frac{1}{\delta_1^2} \right), \mu u_2 \sim \mathbf{N} \left(0, \frac{\mu^2}{\delta_2^2} \right)$$

thus, the supply per capita of the asset is independent of μ .

This case might fit more to equity markets where individuals and institutions with different investment horizon coexist. While some individuals trade very frequently with the explicit purpose of opening and closing positions within a day ("day traders"), others are saving for retirement. It is also an empirically documented fact that the investment horizon of financial institutions varies, perhaps in line with the dispersion in their managers' incentive schemes and the duration of their liabilities.¹⁸ Depending on the interpretation of the group of long-term and short-term traders, the interpretation for the length of each interval should also vary. Again, the context of cross-listed stocks might be useful in this case. In this context, one can think of $(1 - \mu)$, the fraction of B -traders, as the degree of market integration of the two markets for the particular asset. When this fraction is high, traders can optimize over when to trade. When this fraction is low a large group of traders (A -traders) have to expect future prices to incorporate information of another group of traders (B -traders) who are not present on their local market. Before the asset is cross-listed, the local market for the asset is integrated and μ is close to one. After the cross-listing, the two markets for the assets are less integrated and μ is low. I include Case 2 for two reasons. First, to show that the main results are robust to the additional complexity that heterogeneous groups coexist, and second, to present testable implications on the effect of market integration on trading activity. I will refer to this set up as a model of heterogeneous trading horizon. The drawback of this case is that I have to rely partially on numerical simulations for its analysis.

Note that Case 2 is set up in such a way that as $\mu \rightarrow 0$, the structure converges to case 1 in the following sense: while the model is undefined for $\mu = 0$, because there are no traders

¹⁸See Derrien, Kecskes and Thesmar (2009), Hotchkiss and Strickland (2003), Ke and Petroni (2004), Ke, Petroni, and Yu (2008), Yan and Zhang (2009)).

to trade in period 2, it is well defined for any $\mu > 0$. As $\mu \rightarrow 0$, the structure converges to a set up where there are only B -traders in period 2 and there are only A -traders in period 1. This results in a very similar set up to Case 1. In contrast, when $\mu = 1$, it is a 2 period model with homogeneous traders. This extreme case differs from the standard set-up of Brown and Jennings (1989) only to the extent that the information structure is more general.

The information structure is described in (1)-(5). The information sets of agents are

$$\begin{aligned}\mathcal{I}_1^i &= \{x^i, y\} \\ \mathcal{I}_1^j &= \{z^j, y\} \\ \mathcal{I}_2^j &= \{z^j, y, p_1\}.\end{aligned}$$

I look for a linear Rational Expectation Equilibrium defined as follows.

Definition 1 *A linear Rational Expectation Equilibrium is given by the linear price functions p_1, p_2 , mapping the aggregate random variables to prices and individual demands, d_1^i, d_2^j in Case 1 and d_1^i, d_1^j, d_2^j in Case 2 such that*

1. *in Case 1, d_1^i and d_2^j solve problems (9)-(10), respectively and p_t clear the market in period $t = 1, 2$,*
2. *in Case 2, d_1^i, d_1^j, d_2^j solve problems (9)-(11), respectively and p_t clear the market in period $t = 1, 2$.*

Before proceeding to the analysis of the model, it is useful to sum up how our structure nests the usual assumptions made in the literature.

1. If $\kappa \rightarrow \infty$ the information structure becomes the standard informational structure.
2. If $\delta_1 \rightarrow 0$, the second period environment is the same as a static environment, as first period prices become uninformative.
3. If $\beta \rightarrow 0$, the environment converge to an environment with no public announcement.

Thus, combining different subsets of this limits, this model nests many models in the literature. For example, with $\delta_1 \rightarrow 0, \kappa \rightarrow \infty$, the second period is close to Hellwig (1981). Case 1 with $\kappa \rightarrow \infty$ is the two period version of Allen, Morris, Shin (2006), while case 2 with $\kappa \rightarrow \infty$ is close to Brown and Jennings (1989) or to the two period version of He and Wang (1995). In this sense, the presented framework is general.

3.2 Case 1: local traders in a global market

In this section, I characterize the equilibrium of Case 1 of the model. First, I show that a RE equilibrium always exists in this model. Second, I analyze trading volume. I show that both A -traders in the first period and B -traders in the second period trade more after a public announcement whenever $\frac{\kappa^2}{\omega} < \beta$, that is, whenever A -traders expectation about the second period prize is polarized by the public announcement. Finally, I analyze the volatility of prices.

3.2.1 Equilibrium

The derivation of the equilibrium is standard. First, I conjecture the price functions

$$p_2 = \frac{\mathbf{b}_2(\theta_B + \theta_C) + \mathbf{c}_2y + \mathbf{g}_2q_1 - u_2}{\mathbf{e}_2} \quad (12)$$

$$p_1 = \frac{\mathbf{a}_1(\theta_A + \theta_B) + \mathbf{c}_1y - u_1}{\mathbf{e}_1} \quad (13)$$

where $\mathbf{a}_1, \mathbf{b}_2, \mathbf{c}_1, \mathbf{c}_2, \mathbf{e}_1, \mathbf{e}_2$, and \mathbf{g}_2 are undetermined coefficients. Second, I derive the optimal demand given these price functions. For this, observe that p_1 and y are informationally equivalent to y and the "price signal" q_1 of the first period defined as

$$q_1 \equiv \frac{\mathbf{e}_1p_1 - \mathbf{c}_1y}{\mathbf{a}_1} = (\theta_A + \theta_B) - \frac{u_1}{\mathbf{a}_1}. \quad (14)$$

The conditional precision of q_1 is

$$\tau_1^2 \equiv \frac{1}{\text{var}(q_1|\theta_A + \theta_B)} = \delta_1^2 \mathbf{a}_1^2.$$

Similarly, p_2, y , and q_1 are informationally equivalent to y, q_1 and the price signal q_2 of the second period defined as

$$q_2 \equiv \frac{\mathbf{e}_2p_2 - \mathbf{c}_2y - \mathbf{g}_2q_1}{\mathbf{b}_2} = (\theta_B + \theta_C) - \frac{u_2}{\mathbf{b}_2} \quad (15)$$

with a conditional precision of

$$\tau_2^2 \equiv \frac{1}{\text{var}(q_2|(\theta_B + \theta_C))} = \mathbf{b}_2^2 \delta_2^2. \quad (16)$$

I also define b_2, c_2, e_2, g_2 and b_1, c_1, e_1 as the linear coefficients of the conditional expecta-

tions

$$E(\theta|z^j, y, q_1, q_2) = b_2 z^j + c_2 y + e_2 q_2 + g_2 q_1 \quad (17)$$

$$E(q_2|x^i, y, q_1) = a_1 x^i + c_1 y + e_1 q_1 \quad (18)$$

and

$$\tau_\theta^2 \equiv \frac{1}{\text{var}(\theta|z^j, y, q_1, q_2)}$$

$$\tau_q^2 \equiv \frac{1}{\text{var}(q_2|x^i, y, q_1)}$$

as the corresponding precision. Note that all the expectational coefficients and precisions are functions of the primitive parameters and the equilibrium values of τ_1, τ_2 . Then, the first order condition of the problem of B traders, (10), gives

$$d_2^j = \frac{\tau_\theta^2}{\gamma} (E(\theta|z^j, y, q_2, q_1) - p_2) \quad (19)$$

and the problem (9) gives

$$d_1^i = \frac{1}{\gamma} \left(\frac{\mathbf{e}_2}{\mathbf{b}_2} \right)^2 \tau_q^2 (E(p_2|x^i, y, q_1) - p_1). \quad (20)$$

Note that the form of (19) and (20) differ because A traders are interested in the next period price, p_2 , as opposed to the fundamental value. The term $\left(\frac{\mathbf{e}_2}{\mathbf{b}_2} \right)^2 \tau_q^2$ is the precision of p_2 conditional on the information set of A traders.

Imposing market clearing and using expressions (17)-(18) and definitions (14)-(15) give expressions for p_2 and p_1 as linear functions of the random variables with coefficients which depend on the primitives and $\mathbf{b}_2, \mathbf{b}_1, \mathbf{c}_2, \mathbf{c}_1, \mathbf{e}_2, \mathbf{e}_1$ and \mathbf{g}_2 . For an equilibrium, I have to find $\mathbf{b}_2, \mathbf{b}_1, \mathbf{c}_2, \mathbf{c}_1, \mathbf{e}_2, \mathbf{e}_1$ and \mathbf{g}_2 which ensure that these price functions are identical to conjectures (12)-(13). The next proposition follows.

Proposition 2 1. *For all parameters there exists a linear RE equilibrium. In this equilibrium demand functions and prices are given as follows:*

$$d_2^j = \mathbf{b}_2 z^j + \mathbf{c}_2 y + \mathbf{g}_2 q_1 - \mathbf{e}_2 p_2 \quad (21)$$

$$d_1^i = \mathbf{a}_1 x^i + \mathbf{c}_1 y - \mathbf{e}_1 p_1 \quad (22)$$

$$p_2 = \frac{\mathbf{b}_2 (\theta_B + \theta_C) + \mathbf{c}_2 y + \mathbf{g}_2 q_1 - u_2}{\mathbf{e}_2} \quad (23)$$

$$p_1 = \frac{\mathbf{a}_1 \bar{x} + \mathbf{c}_1 y - u_1}{\mathbf{e}_1}$$

where

$$\mathbf{b}_2 = \tau_\theta^2 \frac{b_2}{\gamma} \quad (24)$$

$$\mathbf{c}_2 = \tau_\theta^2 \frac{b_2}{\gamma} \frac{c_2}{b_2 + e_2} \quad (25)$$

$$\mathbf{e}_2 = \tau_\theta^2 \frac{b_2}{\gamma (b_2 + e_2)}. \quad (26)$$

$$\mathbf{g}_2 = \tau_\theta^2 \frac{b_2}{\gamma} \frac{g_2}{e_2 + b_2} \quad (27)$$

and

$$\mathbf{a}_1 = \frac{\tau_q^2}{\gamma} \frac{a_1}{e_2 + b_2} \quad (28)$$

$$\mathbf{c}_1 = \frac{\tau_q^2}{\gamma} \frac{((b_2 + e_2) c_1 + c_2) a_1}{(e_2 + b_2) ((e_2 + b_2) (a_1 + e_1) + g_2)}$$

$$\mathbf{e}_1 = \frac{\tau_q^2}{\gamma} \frac{a_1}{(e_2 + b_2) ((e_2 + b_2) (a_1 + e_1) + g_2)}.$$

Furthermore, all coefficients and equilibrium constants are calculated at $\tau_1 = \tau_1^*$ and $\tau_2 = \tau_2^*$ where $[\tau_1^*, \tau_2^*]$ is the fixed point of the system

$$\delta_2 \tau_\theta^2 \frac{b_2}{\gamma} = \tau_2 \quad (29)$$

$$\delta_1 \frac{\tau_q^2}{\gamma} \frac{b_1}{(e_2 + b_2)} = \tau_1. \quad (30)$$

2. When $\kappa \rightarrow \infty$, there is a unique linear RE equilibrium where

$$\tau_2^* = \alpha \frac{\delta_2}{\gamma}$$

$$\tau_1^* = \alpha^2 \delta_1 \frac{\delta_2^2}{\gamma (\gamma^2 + \alpha \delta_2^2)}.$$

3. When $\frac{\kappa^2}{\omega} = \beta$, there exists a unique linear RE equilibrium where $\tau_1^* = 0$ and τ_2^* is the unique solution of

$$\alpha \delta_2 (\kappa + \omega) \frac{\kappa}{\gamma} = \tau_2 (\kappa^2 + (\kappa + \omega) (\alpha + \tau_2^2) + 2\kappa\omega).$$

Note that the Proposition states existence in general, and uniqueness at two particular points of the parameter space. Given previous work, it is not surprising that there is a unique equilibrium in the limit where our information structure converges to the standard information structure. There is also a unique equilibrium at $\beta = \frac{\kappa^2}{\omega}$. Recall from Proposition 1 that this is an important point in our parameter space, as second-order expectations are polarized by the public signal if and only if $\beta > \frac{\kappa^2}{\omega}$.

3.2.2 Trading volume and information content in trades

I start with a general analysis on traders' equilibrium demand. Note first that rearranging (15) for p_2 and substituting in to (21) gives the first equation in the chain

$$d_2^j = \mathbf{b}_2 (z^j - q_2) = \frac{\tau_\theta^2}{\gamma} [b_2 (z^j - q_2)] = \mathbf{b}_2 \varepsilon^j + u_2. \quad (31)$$

The second equation comes from (24), while the last one is a consequence of the definition of (15). This chain of equations is intuitive. The first expression states that each agent j forms her price contingent demand as follows. She considers the difference between z^j and q_2 ; her private signal and a noisy measure of the average private signal of other agents as it is aggregated in the given market price. If agent j has a higher private signal than this noisy signal of average private information, she buys the asset; otherwise she sells the asset. However, the amount she buys or sells also depends on \mathbf{b}_2 , which I refer to as the agent's trading intensity. The larger the trading intensity, the more aggressively the agent bets on this difference. The second expression decomposes trading intensity. Intuitively, the term in the squared bracket shows how difference in information translates to differences in estimated fundamental value. The larger this term, the larger the agent's perceived difference between her estimate and that of the market. The term $\frac{\tau_\theta^2}{\gamma}$ shows how the difference in opinion is translated into positions. The smaller the risk aversion of the agent, γ and the larger the precision of her fundamental estimation, τ_θ^2 , the larger bet she wants to take for every unit of differences in opinion. Importantly, all b_2 , \mathbf{b}_2 and τ_θ^2 are functions of the deep parameters and τ_2 , the precision of the price signal.

The last expression in (31) shows that at the equilibrium prices, agents end up with a position which is a composite of two parts. The second part is just the per-capita supply. I refer to this part as the risk-sharing position. The first one is the trading intensity weighted private noise. I refer to this part as the speculative position. Importantly, agents cannot distinguish these two parts of their own positions as they know neither the supply nor the noise term in their private signals.¹⁹ Still this decomposition helps us to understand how

¹⁹In fact, as explained and clarified in Biais, Bossaerts and Spatt (2010), the fact that the demand of

and why trading volume and other equilibrium objects react to public information.

In the same way that I derived (31), I also derive the analogous expressions for A traders

$$d_1^i = \mathbf{a}_1 (x^i - q_1) = \frac{\tau_q^2}{\gamma} \frac{a_1}{e_2 + b_2} (x^i - q_1) = \mathbf{a}_1 \varepsilon^i + u_2. \quad (32)$$

The same interpretation holds.

For the purposes of this paper, it is also useful to point out how first period demand is related to higher-order expectations. The market clearing condition in period 2 gives

$$p_2 = \int_0^1 E(\theta | z^j, y, q_2, q_1) dj - \frac{\gamma}{\tau_\theta^2} u_2.$$

Thus, I rewrite first period demand as

$$\frac{1}{\gamma} \left(\frac{\mathbf{e}_2}{\mathbf{b}_2} \right)^2 \tau_q^2 \left[E \left(\int_0^1 E(\theta | z^j, y, q_2, q_1) dj - \frac{\gamma}{\tau_\theta^2} u_2 | x^i, y, q_1 \right) - p_1 \right]. \quad (33)$$

Note that the term in the squared bracket is the difference between the A -trader's expectation of the expectation of the average B -trader (a second order expectation) and the first period price. As I will argue, this second-order expectation carries all the intuition built in Section 2. The term $\left(\frac{\mathbf{e}_2}{\mathbf{b}_2} \right)^2 \tau_q^2$ is the precision of A -traders' estimate. This part is endogenously determined in this model and could modify the basic intuition of Section 2. Importantly, in a RE equilibrium formally agents do not form expectations about the expectations of others. Still, the logic of the example in Section 2 can be applied in two ways. First, one can interpret expression (33) in an *as if* sense. Traders in the first period form their demand as if they were forecasting the expectation of the average B trader. Second, as I show in Appendix C, our model is a specific large number limit of a strategic model where the agents do form expectations about the strategies of others.²⁰ Thus, the intuition of the presented example also applies in this sense.

Similar to the decomposition of demands in (31), I also define and decompose trading trader i positively depends on the error term in her private signal, ε^i , is a form of winner's curse. If the trader could distinguish between ε^i and u_1 , she would avoid this curse.

²⁰Appendix C is available from the author's website.

volume as one of the key objects of interest. The expected volume²¹ in each period is

$$\begin{aligned} V_2 &\equiv E(|d_2^j|) = \sqrt{\frac{2}{\pi} \left(\frac{1}{\delta_2^2} + \frac{\mathbf{b}_2^2}{\alpha} \right)} \\ V_1 &\equiv E(|d_1^i|) = \sqrt{\frac{2}{\pi} \left(\frac{1}{\delta_2^2} + \frac{\mathbf{a}_1^2}{\alpha} \right)}. \end{aligned} \quad (34)$$

I refer to the first term in the brackets on the left hand side as the risk-sharing part of volume and the second part as speculative volume. While the risk-sharing part is exogenously given by the variance of the random supply, the speculative part depends on the equilibrium trading intensities, \mathbf{b}_2 , \mathbf{a}_1 . Note that volume is not influenced by the realization of fundamental factors or the public announcement. As our main interest is the change in that part of volume which is driven by dispersion in private information, I also define speculative volume as the realized volume when aggregate random variables are at their expected value.

$$\begin{aligned} V_2^S &\equiv \frac{1}{2} \int |d_2^j| |_{u_2=0} = \frac{|\mathbf{b}_2|}{\sqrt{2\alpha\pi}} \\ V_1^S &\equiv \frac{1}{2} \int |d_2^j| |_{u_1=0} = \frac{|\mathbf{a}_1|}{\sqrt{2\alpha\pi}} \end{aligned}$$

It is apparent that in Case 1, changes in the amount of public information affects expected volume and speculative volume in very similar ways. However, this second measure will turn out to be of independent interest in Case 2.

It is important to point out that neither in this part nor in the rest of the paper I present arguments against the classic No Trade Theorems. Just as in any other common prior set-up, differential information does not generate trade in itself in this model. To induce trade, prices must be non-fully revealing. Indeed, both the risk sharing and the speculative components of volumes and holdings in (31) and (34) go to zero as the noise in supply diminishes. However, the decomposition of holdings and volume in (31) and (34) also

²¹By the properties of folded normal distributions, the total realized volume in each period is

$$\begin{aligned} V_2 &\equiv \frac{1}{2} \int |d_2^j| = \frac{1}{2} \left(\frac{|\mathbf{b}_2|}{\sqrt{\alpha}} \sqrt{\frac{2}{\pi}} \exp\left(-\alpha \frac{u_2^2}{2\mathbf{b}_2^2}\right) + u_2 \left(1 - 2\Phi\left(-\sqrt{\alpha} \frac{u_2}{|\mathbf{b}_2|}\right)\right) \right) \\ V_1 &\equiv \frac{1}{2} \int |d_1^i| = \frac{1}{2} \left(\frac{|\mathbf{a}_1|}{\sqrt{\alpha}} \sqrt{\frac{2}{\pi}} \exp\left(-\alpha \frac{u_1^2}{2\mathbf{a}_1^2}\right) + u_1 \left(1 - 2\Phi\left(-\sqrt{\alpha} \frac{u_1}{|\mathbf{a}_1|}\right)\right) \right). \end{aligned}$$

Observe that just as the measures of volume in the main text, these expressions are also increasing in trading intensity and affected by the amount of public information only through the change in trading intensity. Thus, considering this measure would not change our qualitative results.

illustrate that dispersion in private information adds to trading volume in a market where prices are not fully revealing. To see this, consider the limit $\alpha \rightarrow \infty$. This coincides with the symmetric information benchmark. In this limit, the speculative part of equilibrium demand diminishes and only the risk-sharing part remains. Thus, the additional effect of differential information to trade for a given amount of noise is measured by the speculative component in each object. Given that this component depends on the equilibrium objects $\mathbf{b}_2, \mathbf{a}_1$, the way the combination of traders' heterogeneous trading horizon and the weakly correlated information structure influences the speculative component is non-trivial. The analysis of this is the main focus of this paper.

I am also interested in the information content of prices. I define a measure for this as

$$C_1 \equiv \frac{1}{\text{var}(q_1|\theta_A + \theta_C)} = \tau_1^2 = \delta_1^2 \mathbf{a}_1^2 \quad (35)$$

$$C_2 \equiv \frac{1}{\text{var}(q_2|\theta_B + \theta_C)} = \tau_2^2 = \delta_2^2 \mathbf{b}_2^2 \quad (36)$$

where I used (28) and (24) for the last equation in each expression, respectively. When this measure is zero, the price do not aggregate any private information. When it is infinity, it aggregates private information perfectly. Note from (34)-(36) that to study the effect of public information on trading volume and information content of prices, it is sufficient if I study its effect on the absolute value of trading intensities $|\mathbf{b}_2|, |\mathbf{a}_1|$. When the trading intensity increases in absolute value, so do our measures of volume and information content of prices.

I start the analysis with the limit where the importance of group specific information diminishes, i.e., $\kappa \rightarrow \infty$. As pointed out above, this limit corresponds to the standard information structure. The following proposition shows that public information does not affect trading volume and information content of prices in this case.

Proposition 3 *When $\kappa \rightarrow \infty$, trading volume and information content of trades are both unaffected by the amount of public information β . That is*

$$\frac{\partial \mathbf{b}_2}{\partial \beta} = \frac{\partial \mathbf{a}_1}{\partial \beta} = \frac{\partial V_t}{\partial \beta} = \frac{\partial C_t}{\partial \beta} = 0.$$

for $t = 1, 2$.

Consider the result on information content and volume. Note that the effect of public

information on trading intensity in each period can be decomposed as

$$\frac{\partial |\mathbf{b}_2|}{\partial \beta} = \frac{1}{\gamma} \frac{\partial \tau_\theta^2 b_2}{\partial \beta} = \frac{1}{\gamma} \left(\tau_\theta^2 \frac{\partial b_2}{\partial \beta} + b_2 \frac{\partial \tau_\theta^2}{\partial \beta} \right) \quad (37)$$

$$\frac{\partial |\mathbf{a}_1|}{\partial \beta} = \frac{1}{\gamma} \frac{\partial \frac{\tau_q^2}{e_2 + b_2} b_1}{\partial \beta} = \frac{1}{\gamma} \left(\tau_\theta^2 \frac{\partial a_1}{\partial \beta} + a_1 \frac{\partial \frac{\tau_q^2}{e_2 + b_2}}{\partial \beta} \right). \quad (38)$$

The first term in the bracket is the effect of public information on the weight of private signals in each agent's conditional expectation, while the second term is the effect on the precision of their expectations. It is easy to check that in the limit $\kappa \rightarrow \infty$ the first term is always negative while the second term is always positive and their absolute size is the same. Intuitively, more public information decreases disagreement among agents. If an agent knows more from public sources, she will rely less on her private signal. Less disagreement decreases trading intensity. On the other hand, more information makes agents more certain in their estimation of the fundamental value. This increases trading intensity. Proposition 3 states that these two effects exactly cancel out in the standard information structure. As already been pointed out in previous work (e.g. Kim and Verrecchia (1991) and Wang (1995)), this result is not robust. Still the existence of the two opposing forces is a general feature of previous CARA-Normal RE models.

In contrast, an important result in this paper is that in our set up the effect of an announcement on precision and conditional expectation not only do not cancel out, but they have the same positive sign, leading to large increase in trading volume as a response to more public information.

Let us turn to the general case when κ is finite so the common factor does not fully dominate the fundamental value. I start the characterization with the following Lemma.

Lemma 1 *In every point where $\frac{\partial V_t}{\partial \beta}$ exists for both $t = 1, 2$*

$$\text{sgn} \left(\frac{\partial |\mathbf{a}_1|}{\partial \beta} \right) = \text{sgn} \left(\frac{\partial |\mathbf{b}_2|}{\partial \beta} \right).$$

The Lemma states that for any combinations of the parameters, public information affects absolute trading intensity, and consequently trading volume and information content of prices, in the same way across the two periods. The underlying intuition is that if A -traders trade more aggressively, the price in the first period becomes sufficiently more informative. Hence the precision of B -traders' estimated pay-off increases. Consequently, the Lemma states that even if decreasing disagreement among B -traders decreases trading intensity, the effect on precision will dominate.

Now I turn to the main result of this section. Recall from Proposition 1 that in our example the public signal polarizes A -traders expectations on the expectation of the average B -traders in a weakly correlated information structure, that is, if and only if $\beta > \frac{\kappa^2}{\omega}$. Furthermore, expression (33) shows that second period price is closely related to the average expectation of B -traders. Thus, if polarization is indeed the main determinant of increase in trading volume, A -traders' volume should increase if and only if $\beta > \frac{\kappa^2}{\omega}$. By Lemma 1, B -traders' trading volume should also increase under the same condition. That is, polarization among A -traders increases trading volume among both group of traders, even if disagreement about the pay-off among B -traders decreases after the announcement. By previous arguments, trading intensities of A and B traders, $\mathbf{a}_1, \mathbf{b}_2$ and information content of prices should change similarly. This is indeed the case as illustrated in Figure 1 and Figure 2. Figure 1 shows trading intensities, and the precision of traders estimates, τ_θ^2, τ_q^2 on the top and bottom panels, respectively. The bottom panel on Figure 2 shows speculative trading volume in period 1 and 2. In the next proposition, I show that these results are general as long as trading intensities $\mathbf{a}_1, \mathbf{b}_2$ are continuously differentiable in the amount of public information, β .²²

Proposition 4 *There are $\omega_{\min} \in [0, \frac{\kappa^2}{\beta}), \omega_{\max} \in (\frac{\kappa^2}{\beta}, \infty]$ that as long as $\omega \in (\omega_{\min}, \omega_{\max})$ there are corresponding τ_1^*, τ_2^* which are continuous in ω and continuously differentiable in β and ω . Furthermore, when $(\omega_{\min}, \omega_{\max})$ is the largest such set,, as long as $\omega \in (\frac{\kappa^2}{\beta}, \omega_{\max})$*

$$\frac{\partial |\mathbf{a}_1|}{\partial \beta}, \frac{\partial |\mathbf{b}_2|}{\partial \beta} > 0.$$

That is, in weakly correlated information structures, the absolute value of trading intensities, volume and information content of prices all increase in both periods.

3.2.3 Volatility of prices

Turning to prices, by definition, the coefficients $\frac{\mathbf{b}_2}{\mathbf{e}_2}, \frac{\mathbf{a}_1}{\mathbf{e}_1}$ show the price effects of the part of fundamentals which agents have private information on, the coefficients $\frac{1}{\mathbf{e}_2}, \frac{1}{\mathbf{e}_1}$ show the price effect of supply shocks, while $\frac{\mathbf{c}_2}{\mathbf{e}_2}, \frac{\mathbf{c}_1}{\mathbf{e}_1}$ show price effect of public information. The first two sets of coefficients are particularly important, because they determine the relevant measure

²²While experiments with a wide range of parameters suggest that this neighborhood is the whole parameter space, i.e. $\omega_{\min} = 0, \omega_{\max} = \infty$, a general proof for this was not found. Hence the weaker statement.

of price volatility as

$$\Sigma_1 \equiv \text{var}(p_1|y) = \left(\frac{\mathbf{a}_1}{\mathbf{e}_1}\right)^2 \left(\frac{1}{\kappa} + \frac{1}{\omega}\right) + \frac{1}{(\mathbf{e}_1)^2 \delta_1} \quad (39)$$

$$\Sigma_2 \equiv \text{var}(p_2|y, p_1) = \left(\frac{\mathbf{b}_2}{\mathbf{e}_2}\right)^2 \left(\frac{1}{\kappa} + \frac{1}{\omega}\right) + \frac{1}{(\mathbf{e}_2)^2 \delta_1}. \quad (40)$$

Both measures are conditioned by publicly observed variables in the given period.

Starting again with the standard information structure, the following proposition holds.

Proposition 5 *When $\kappa \rightarrow \infty$, in each period,*

1. *prices are positively affected by the average information of traders, and this effect decreases in the precision of public information. That is,*

$$\frac{\mathbf{b}_2}{\mathbf{e}_2}, \frac{\mathbf{a}_1}{\mathbf{e}_1} > 0, \frac{\partial \frac{\mathbf{b}_2}{\mathbf{e}_2}}{\partial \beta}, \frac{\partial \frac{\mathbf{a}_1}{\mathbf{e}_1}}{\partial \beta} < 0.$$

2. *Prices are positively affected by public information and this effect increases in the precision of public information,*

$$\frac{\mathbf{c}_2}{\mathbf{e}_2}, \frac{\mathbf{c}_1}{\mathbf{e}_1} > 0, \frac{\partial \frac{\mathbf{c}_2}{\mathbf{e}_2}}{\partial \beta}, \frac{\partial \frac{\mathbf{c}_1}{\mathbf{e}_1}}{\partial \beta} > 0.$$

3. *Prices are negatively affected by supply shocks, and the effect decreases in the precision of public information*

$$\frac{\mathbf{1}}{\mathbf{e}_2}, \frac{\mathbf{1}}{\mathbf{e}_1} > 0, \frac{\partial \frac{\mathbf{1}}{\mathbf{e}_2}}{\partial \beta}, \frac{\partial \frac{\mathbf{1}}{\mathbf{e}_1}}{\partial \beta} < 0.$$

4. *Price volatility decreases in the precision of public information*

$$\frac{\partial \Sigma_1}{\partial \beta}, \frac{\partial \Sigma_2}{\partial \beta} < 0.$$

As the last statement in the proposition shows, under the standard information structure, more precise public information decreases price volatility in each period. The result is intuitive. If public information is more precise, agents rely more on public pieces of information and less on every other piece of information. Thus, the price will be more sensitive to public information, but less sensitive to every other shock. As only sensitivities to private information and supply shocks, $\frac{\mathbf{b}_2}{\mathbf{e}_2}$, $\frac{\mathbf{a}_1}{\mathbf{e}_1}$, and $\frac{\mathbf{1}}{\mathbf{e}_2}$, $\frac{\mathbf{1}}{\mathbf{e}_1}$ affect our volatility measure, more precise public information decreases price volatility.

As I show in the following proposition, this monotonic pattern generally disappears in a weakly correlated structure.

Proposition 6 *For any set of other parameters,*

1. *there is an interval $\mathcal{B}_1 \subseteq (\frac{\omega}{\kappa^2}, \infty)$ that if $\beta \in \mathcal{B}_1$*

$$\frac{\partial \left| \frac{\mathbf{a}_1}{\mathbf{e}_1} \right|}{\partial \beta} \Big|_{\beta \in \mathcal{B}_1 \subseteq (\frac{\omega}{\kappa^2}, \infty)} > 0,$$

that is, in certain weakly correlated structures the absolute value of the price effect of the average information of traders in period 1 is increasing in the precision of public information.

2. *If the variance of the supply shock, $\frac{1}{\delta_1}$ is sufficiently small and τ_1^*, τ_2^* are continuous in β in $(\frac{\omega}{\kappa^2}, \infty)$, then there is an interval $\mathcal{B}_2 \subseteq (\frac{\omega}{\kappa^2}, \infty)$ that if $\beta \in \mathcal{B}_2$,*

$$\frac{\partial \left| \frac{1}{\mathbf{e}_1} \right|}{\partial \beta} \Big|_{\beta \in \mathcal{B}_2 \subseteq (\frac{\omega}{\kappa^2}, \infty)} > 0,$$

that is, in certain weakly correlated structures the absolute price effect of the supply shock in period 1 is increasing in the precision of public information.

The result states that in weakly correlated structures, the price can become more sensitive both to shocks in the average private information and to supply shocks. Especially when the variance of the supply shock is small, typically there is a set of parameters where both sensitivities increase in precision. Figure 2 shows the equilibrium price coefficients, while Figure 3 depicts our volatility measure as a function of the precision of the public information in a typical case. It is apparent that there is a range where more public information increases the volatility of price in period 1. This range is indeed within the interval $(\frac{\omega}{\kappa^2}, \infty)$, that is, it corresponds to weakly correlated information structures.

3.3 Case 2: Heterogeneous trading horizon

In this part, I analyze Case 2 where B traders enter in the first period. Thus, traders with different horizons coexist. I focus on the effect of the changing measure of the two groups on trading activity. I argue that in a weakly correlated information structure, the lower the μ , (the larger the share of short-horizon traders or, equivalently, the smaller the level of integration of the market of a given asset), the larger the response to announcements of volume, information content of prices and, potentially, volatility.

3.3.1 Equilibrium

The structure of the second period is the same as in case 1, so conjecture (12) and definitions of q_2 , τ_2 , and τ_θ , b_2 , c_2 , e_2 , g_2 are used as before. However, for the first period I conjecture

$$p_1 = \frac{\mathbf{a}_1(\theta_A + \theta_B) + \mathbf{b}_1(\theta_B + \theta_C) + \mathbf{c}_1 y - u_1}{\mathbf{e}_1} \quad (41)$$

and define q_1 as the price signal corresponding to period 1

$$q_1 \equiv \frac{\mathbf{e}_1 p_1 - \mathbf{c}_1 y}{\mathbf{a}_1 + \mathbf{b}_1} = (1 - \phi)\theta_A + \phi\theta_B + \theta_C - \frac{1}{\mathbf{a}_1 + \mathbf{b}_1} u_1 \quad (42)$$

where ϕ is the share of B -traders' private information in the total private information content of the first period price. It is defined as

$$\phi \equiv \frac{\mathbf{b}_1}{\mathbf{a}_1 + \mathbf{b}_1}. \quad (43)$$

The conditional precision of q_1 is

$$\tau_1^2 \equiv \frac{1}{\text{var}(q_1 | \theta_A + \theta_B)} = \delta_1^2 (\mathbf{a}_1 + \mathbf{b}_1)^2. \quad (44)$$

Finally, I define b_A, c_A, e_A and b_B, c_B, e_B as the linear coefficients of the conditional expectations

$$E(q_2 | x^i, y, q_1) = a_A x^i + c_A y + e_A q_1 \quad (45)$$

$$E(q_2 | z^j, y, q_1) = b_B z^j + c_B y + e_B q_1 \quad (46)$$

and

$$\tau_A^2 \equiv \frac{1}{\text{var}(q_2 | x^i, y, q_1)}$$

$$\tau_B^2 \equiv \frac{1}{\text{var}(q_2 | z^j, y, q_1)}$$

as the corresponding precision.

The problem of each B -trader in the second period and that of each A -trader in the first period are very similar to their respective problems in case 1. The optimal demand of these traders leads to the same formulations of (19) and (20), respectively.

However, B -traders have to solve a two period problem in period 1. I show in the

appendix that their demand function takes the form of

$$d_1^j = \frac{(\tau_\theta^2 b_2^2 + \tau_B^2) (E(p_2|z^j, y, q_1) - p_1)}{(b_2 + e_2)^2 \gamma} + \frac{\tau_\theta^2 b_2^2 (z^j - E(q_2|z^j, y, q_1))}{(b_2 + e_2) \gamma}, \quad (47)$$

a weighted sum of the trader's expected price change between period 1 and 2 and her expected demand in period 2. I refer to the first term as the myopic component and the second term as the hedging component of demand.

Just as in Case 1, I have to find $\mathbf{b}_2, \mathbf{b}_1, \mathbf{c}_2, \mathbf{c}_1, \mathbf{e}_2, \mathbf{e}_1$ and \mathbf{g}_2 , which ensure that the price functions coincide with their respective conjectures. The next proposition follows.

Proposition 7 *Suppose that the system*

$$\delta_2 \tau_\theta^2 \frac{b_2}{\gamma} = \tau_2 \quad (48)$$

$$\delta_1 \frac{\mu (\tau_B^2 b_B + \tau_\theta^2 b_2^2) + (1 - \mu) \tau_A^2 a_A}{\gamma (e_2 + b_2)} = \tau_1, \quad (49)$$

$$\frac{\mu (\tau_B^2 b_B + \tau_\theta^2 b_2^2)}{\mu (\tau_B^2 b_B + \tau_\theta^2 b_2^2) + (1 - \mu) \tau_A^2 a_A} = \phi, \quad (50)$$

has a fixed point $\tau_1^*, \tau_2^*, \phi^*$. Then there is a linear RE equilibrium. In this equilibrium price and demand in period 2 is given by (21), (23) and (24)-(27) and

$$p_1 = \frac{\mathbf{a}_1 (\theta_A + \theta_C) + \mathbf{b}_1 (\theta_B + \theta_C) + \mathbf{c}_1 y - u_1}{\mathbf{e}_1}$$

and

$$d_1^i = \mathbf{a}_A x^i + \mathbf{c}_A y - \mathbf{e}_A p_1 \quad (51)$$

$$d_1^j = \mathbf{b}_B x^j + \mathbf{c}_B y - \mathbf{e}_B p_1 \quad (52)$$

where

$$(1 - \mu) \mathbf{a}_A = \mathbf{a}_1 \quad (53)$$

$$\mu \mathbf{b}_B = \mathbf{b}_1$$

$$(1 - \mu) \mathbf{c}_A + \mu \mathbf{c}_B = \mathbf{c}_1$$

$$(1 - \mu) \mathbf{e}_A + \mu \mathbf{e}_B = \mathbf{e}_1 \quad (54)$$

and

$$\mathbf{b}_B = \frac{\tau_B^2 b_B + \tau_\theta^2 b_2^2}{\gamma(e_2 + b_2)} \quad (55)$$

$$\mathbf{a}_A = \frac{\tau_A^2 a_A}{\gamma(e_2 + b_2)} \quad (56)$$

and $\mathbf{c}_A, \mathbf{c}_B, \mathbf{e}_A, \mathbf{e}_B$ can also be written as analytical functions of the parameters and τ_1, τ_2, ϕ only. Furthermore, all coefficients are calculated at $\tau_1 = \tau_1^*$ and $\tau_2 = \tau_2^*$ and $\phi = \phi^*$

3.3.2 Trading volume

In the second period, by the same analysis as in case 1, the equilibrium demand of each trader is described by (31) and the volume in the second period is described by (37). For demand and expected volume in the first period, observe that from (51)-(52) and the definition of q_1

$$\begin{aligned} d_1^i &= \mathbf{a}_A x^i + y \left(\mathbf{c}_A - \mathbf{e}_A \frac{\mathbf{c}_1}{\mathbf{e}_1} \right) - \mathbf{e}_A \frac{q_1 (\mathbf{a}_1 + \mathbf{b}_1)}{\mathbf{e}_1} = \\ &= \mathbf{a}_A \varepsilon^i + \left[\mathbf{a}_1 \left(\frac{1}{(1-\mu)} - \frac{\mathbf{e}_A}{\mathbf{e}_1} \right) (\theta_A + \theta_C) - \mathbf{e}_A \frac{\mathbf{b}_1}{\mathbf{e}_1} (\theta_B + \theta_C) + y \left(\mathbf{c}_A - \mathbf{e}_A \frac{\mathbf{c}_1}{\mathbf{e}_1} \right) \right] + \frac{\mathbf{e}_A}{\mathbf{e}_1} u_1, \end{aligned}$$

$$d_1^j = \mathbf{b}_B x^j + y \left(\mathbf{c}_B - \mathbf{e}_B \frac{\mathbf{c}_1}{\mathbf{e}_1} \right) - \mathbf{e}_B \frac{q_1 (\mathbf{a}_1 + \mathbf{b}_1)}{\mathbf{e}_1} = \quad (57)$$

$$= \mathbf{b}_B \varepsilon^j + \left[\mathbf{b}_1 \left(\frac{1}{\mu} - \frac{\mathbf{e}_B}{\mathbf{e}_1} \right) (\theta_B + \theta_C) - \mathbf{e}_B \frac{\mathbf{a}_1}{\mathbf{e}_1} (\theta_A + \theta_C) + y \left(\mathbf{c}_B - \mathbf{e}_B \frac{\mathbf{c}_1}{\mathbf{e}_1} \right) \right] + \frac{\mathbf{e}_B}{\mathbf{e}_1} u_1 \quad (58)$$

and, consequently,

$$\begin{aligned} V_1 &= \sqrt{\frac{1}{2\pi}} (1-\mu) \left(\frac{(\mathbf{a}_A)^2}{\alpha} + \left[\frac{\left(\frac{\mathbf{a}_1}{(1-\mu)} + \mathbf{c}_A - \frac{\mathbf{e}_A}{\mathbf{e}_1} (\mathbf{a}_1 + \mathbf{c}_1) \right)^2 + \left(\mathbf{c}_A - \mathbf{e}_A \frac{\mathbf{c}_1 + \mathbf{b}_1}{\mathbf{e}_1} \right)^2}{\kappa} + \frac{\left(\mathbf{c}_A - \mathbf{e}_A \frac{\mathbf{c}_1}{\mathbf{e}_1} \right)^2}{\beta} \right] + \frac{\left(\frac{\mathbf{e}_A}{\mathbf{e}_1} \right)^2}{\delta_1} \right)^{\frac{1}{2}} + \\ &+ \sqrt{\frac{1}{2\pi}} \mu \left(\frac{(\mathbf{b}_B)^2}{\alpha} + \left[\frac{\left(\frac{\mathbf{b}_1}{\mu} + \mathbf{c}_B - \frac{\mathbf{e}_B}{\mathbf{e}_1} (\mathbf{b}_1 + \mathbf{c}_1) \right)^2 + \left(\mathbf{c}_B - \mathbf{e}_B \frac{\mathbf{c}_1 + \mathbf{b}_1}{\mathbf{e}_1} \right)^2}{\kappa} + \frac{\left(\mathbf{c}_B - \mathbf{e}_B \frac{\mathbf{c}_1}{\mathbf{e}_1} \right)^2}{\beta} \right] + \frac{\left(\frac{\mathbf{e}_B}{\mathbf{e}_1} \right)^2}{\delta_1} \right)^{\frac{1}{2}}. \end{aligned}$$

Unlike in Case 1, equilibrium demand does depend on the realization of aggregate random variables. The reason is that in period 1, A -traders' and B -traders' demand react differently to each piece of information. This is so because both the joint distribution of signals in each trader's information set and the trading horizon differ across groups. As a consequence, apart

from the risk-sharing and speculative parts of trades defined in Case 1, there is also trade across groups. This latter part of equilibrium demand and expected volume is in squared brackets in each expression. Note that the population weighted average of the terms in the squared brackets is 0.

Speculative realized volume separates the within-group part of trade:

$$V_1^S = \frac{1}{2} \left(\int |d_1^i| di + \int |d_1^j| di \right) |_{\theta_A=\theta_B=\theta_C=\beta=u_1=0} = \sqrt{\frac{1}{2\pi} \frac{1}{\sqrt{\alpha}}} ((1-\mu) |\mathbf{a}_A| + \mu |\mathbf{b}_B|).$$

Our measures for the information content of prices remain similar to case 1.

$$C_1 \equiv \frac{1}{\text{var}(q_1|\theta_A + \theta_C)} = \tau_1^2 = \delta_1^2 (\mathbf{a}_1 + \mathbf{b}_1)^2 \quad (59)$$

$$C_2 \equiv \frac{1}{\text{var}(q_2|\theta_B + \theta_C)} = \tau_2^2 = \delta_2^2 \mathbf{b}_2^2. \quad (60)$$

Just as before, it is useful to establish the following result.

Proposition 8 *In the limit $\kappa \rightarrow \infty$, there is a unique equilibrium where*

$$\begin{aligned} \mathbf{b}_2 &= \alpha \frac{\delta_2}{\gamma} \\ \mathbf{a}_1 &= (1-\mu) \frac{\alpha}{\gamma} \delta_1 \frac{\alpha \delta_2^2}{\gamma^2 + \alpha \delta_2^2} \\ \mathbf{b}_1 &= \mu \frac{\alpha}{\gamma} \delta_1. \end{aligned}$$

Thus

$$\frac{\partial \mathbf{b}_t}{\partial \beta} = \frac{\partial \mathbf{a}_1}{\partial \beta} = \frac{\partial \mathbf{C}_t}{\partial \beta} = \frac{\partial \mathbf{V}_t^S}{\partial \beta} = 0$$

for $t = 1, 2$.

The proposition shows that under the standard information structure, even if traders with heterogeneous horizon coexist, more public information has no effect on trading intensities, the information content of trade or speculative volume. Numerical simulations (not shown) show that the effect on total expected volume also diminishes as $\kappa \rightarrow \infty$.

I analyze the equilibrium in the general case with the help of Figures 4-9. In each panel of each figure, the x -axis shows the amount of public information measured by β and four curves correspond to different fractions of long-horizon traders in period 1, μ . The thicker the line, the larger the fraction of long-horizon traders.²³

²³The discontinuity on each curve corresponding to $\mu = 0.01$ shows the only identified segment of the

The top panels on Figure 5 show the trading intensities of A -traders and B -traders in the first period, and the top and middle panels on Figure 4 show the total trading intensity in period 1 and B -traders trading intensity in the second period respectively. First I turn to the question how the increasing share of long-horizon traders in period 1 affect trading volume. Trading intensity of A -traders is shown on the North-West panel of Figure 5. It changes with public information the same way as in Case 1. It decreases in β in absolute value as long as $\beta < \frac{\kappa^2}{\omega}$ and increases in absolute value when $\beta > \frac{\kappa^2}{\omega}$. B -traders' trading intensity in the second period is shown on the top panel of Figure 4. It decreases in public information as long as their fraction in the economy is large. However, when μ is small, as in Case 1, B -traders' trading intensity decreases in β if $\beta < \frac{\kappa^2}{\omega}$, and increases otherwise. This is consistent with the observation that Case 2 is close to Case 1 if μ is small. Trading intensity of B -traders in the first period is shown on the North-East panel of Figure 5. Interestingly, it increases in public information for any β . This is surprising because the intuition shown in Section 2 do not apply for B -traders. If B -traders in period 1 were to forecast the forecast of the average B -trader in period 2, the dispersion in their forecasts would decrease in the amount of public information, because (6) would not hold. The correlation between the private information set of B -traders in period 1 and the average B -trader in period 2 is high. To understand the intuition, I decompose the trading intensity \mathbf{b}_B on Figure 5 as follows. The term $\frac{\tau_\theta^2 b_2^2}{\gamma(e_2 + b_2)}$ is the hedging component of the trading intensity. The bottom left panel on Figure 5 shows that this component is decreasing in β . The term $\frac{\tau_B^2 b_B}{\gamma}$ is the numerator in the myopic component of the trading intensity in (47). Comparing this term to equilibrium value of \mathbf{b}_2 shows that this term would be the trading intensity, if the true value were to realize in period 2 instead of period 3. The bottom right panel on Figure 5 shows that this term is also decreasing in β . Thus, trading intensity \mathbf{b}_B increases in β solely because of the remaining term, $\frac{1}{b_2 + e_2}$, the numerator of the myopic component. Note that this term is the inverse of the sensitivity of p_2 to the fundamental. Intuitively, as public information increases, the second period price is more correlated to the fundamental, so in the first period all traders can estimate p_2 with more certainty. While, this effect is not sufficient to influence the sign of the derivative of \mathbf{a}_A with respect to public information, it switches that of \mathbf{b}_B .

The analysis of measures of volume is illustrated by Figure 6. As my focus is the effect of changing share of long-horizon traders, μ , on the effect of public announcements on volume, I report the public information elasticity of the two volume measures. Just as before, in

parameter space where problem (48)-(50) does not have a fixed point. Consequently the equilibrium does not exist. Figure 4 illustrates the reason for this. As it apperent on the top and bottom panels, this segment corresponds to a zero-measure set of parameters such that sufficiently close to this set $\mu \mathbf{b}_1 + (1 - \mu) \mathbf{a}_1 \rightarrow 0$ so $\phi \rightarrow \pm\infty$.

each panel the amount of public information is on the x -axis and each of the four curves correspond to a different fraction of long-horizon traders in period 1, μ . The top and bottom panels show the elasticity of speculative and expected volume respectively.

Consider speculative volume first. It is apparent that in a weakly correlated information structure, the larger the fraction of short-term traders, the larger the response in speculative volume. That is, when public announcement polarize A -traders price forecast, then their percentage response is much larger than that of B -traders. While I find this result numerically robust to any change in parameters, analytically I prove the weaker statement that it holds for the direct effect when τ_1, τ_2, ϕ are held constant.

Lemma 2 *Holding τ_1, τ_2, ϕ fixed, the public information elasticity of speculative volume is decreasing in the fraction of long-horizon traders (B -traders) in period 1:*

$$\frac{\partial \frac{\partial V_1^S}{\partial \beta} \frac{\beta}{V_1^S}}{\partial \mu} \Big|_{\tau_1=\bar{\tau}_1, \tau_2=\bar{\tau}_2, \phi=\bar{\phi}} < 0.$$

Turning to expected total volume, the bottom panel on Figure 6 shows that the qualitative results are the same as with our first measure. The public information elasticity of expected volume decreases in the fraction of long-term traders, if β is sufficiently large.

3.3.3 Volatility of prices

Similar to case 1, the coefficients $\frac{\mathbf{b}_2}{\mathbf{e}_2}, \frac{\mathbf{a}_1}{\mathbf{e}_1}, \frac{\mathbf{b}_1}{\mathbf{e}_1}$ show the price effects of the part of fundamentals which agents have private information on, the coefficients $\frac{1}{\mathbf{e}_2}, \frac{1}{\mathbf{e}_1}$ show the price effect of supply shocks while $\frac{\mathbf{c}_2}{\mathbf{e}_2}, \frac{\mathbf{c}_1}{\mathbf{e}_1}$ show price effect of public information. The definition of price volatility in the second period is still given by (40). The definition in the first period changes to

$$\Sigma_1 \equiv \text{var}(p_1|y) = \left[\left(\frac{\mathbf{a}_1}{\mathbf{e}_1} \right)^2 + \left(\frac{\mathbf{b}_1}{\mathbf{e}_1} \right)^2 \right] \frac{1}{\kappa} + \left(\frac{\mathbf{a}_1}{\mathbf{e}_1} + \frac{\mathbf{b}_1}{\mathbf{e}_1} \right)^2 \frac{1}{\omega} + \frac{1}{(\mathbf{e}_1)^2 \delta_1}.$$

As in case 1, it is useful to start with the standard information structure. The next Proposition shows that the coexistence of A and B traders does not change the conclusion that the standard information structure is inconsistent with volatility-generating public announcements.

Proposition 9 *In Case 2, in the limit $\kappa \rightarrow \infty$, price coefficients $\frac{\mathbf{b}_2}{\mathbf{e}_2}, \frac{\mathbf{b}_1}{\mathbf{e}_1}, \frac{\mathbf{c}_2}{\mathbf{e}_2}, \frac{\mathbf{c}_1}{\mathbf{e}_1}, \frac{1}{\mathbf{e}_2}, \frac{1}{\mathbf{e}_1}$ inherit all the properties of Case 1 described in Proposition 5. Also, price in period 1 is positively affected by the average information of A traders and this effect decreases in the precision of*

public information. That is,

$$\frac{\mathbf{a}_1}{\mathbf{e}_1} > 0, \frac{\partial \mathbf{a}_1}{\partial \beta} < 0.$$

Thus, just as in Case 1,

$$\frac{\partial \Sigma_1}{\partial \beta}, \frac{\partial \Sigma_2}{\partial \beta} < 0.$$

I proceed with the numerical analysis of the general case. In Figures 8-9 I plot the relevant equilibrium coefficients, and Figure 7 depicts our volatility measures. It is apparent that only when the share of long-horizon traders is sufficiently low, that is, the structure is sufficiently close to Case 1, does volatility in period 1 increase with the amount of public information in any range of the parameter space. This illustrates that the assumption of heterogeneous trading horizons is essential for this result.

3.3.4 New empirical predictions

The analysis of Case 2 of our model provides additional empirical predictions to test the presented theory.

First, there are widely used empirical proxies to measure the heterogeneity in trading horizon in the investor base of different stocks (e.g. Wahal and McConnell (2000) and Gaspar et al. (2005)).²⁴ If a larger share of short-horizon traders in the investor base were found to be connected to larger volume and inflow of information responses to announcements, this would be an evidence consistent with the predictions shown in the previous section. I am not aware of any existing studies on such connection.

An alternative empirical strategy is to rely on natural experiments when the characteristics of the investor base of a given asset changes abruptly and significantly. Cross-listings of stocks can potentially provide such a natural experiment. As an example, Bailey, Karolyi and Salva (2005) focus on the trading volume and return volatility of stocks around earning announcements before and after these stocks were cross-listed on NYSE. They find that both volatility and volume response increases after the announcement. They find a larger effect for those stocks which were originally listed in the exchange of a developed economy as opposed to an emerging economy exchange. Using a large number of controls, they conclude that this effect must be implied by the change in the informational environment due to the cross-listing. However, they cannot explain their findings with the existing theoretical models and

²⁴Recent empirical work has found that more short-term investors affect the quality of accounting disclosure, R&D spending, mergers and acquisitions and trade-off between, dividends and repurchases, less equity issue and less investment. (see Bushee (1998), Gaspar, Massa, and Matos (2005), and Derrien, Kecskés and Thesmar (2009)).

call it a puzzle. Although this work is not a direct test of our model, I argue that their finding is consistent with the proposed theory. Consider case 2 of the model and Figure 6. Although cross-listing changes a range of characteristics of the trading environment of firms, for our purposes think of cross-listing as an increase in the heterogeneity of the investors base or, equivalently, a drop in the level of integration of the market of the asset. That is, μ jumps. Figure 6 shows that this jump should increase the volume response to public announcements regardless of the volatility measure as long as the prior public information, β is sufficiently high. Figure 7 shows that this jump might result in an increase in the price volatility response to earning announcements. Regarding the difference between emerging market firms and developed market firms, a reasonable assumption is that while cross-listing increases the amount of disclosed public information prior to the announcement for both firms, emerging market firms will be less transparent both before and after cross-listing. While our model does not provide a clear prediction on this comparative static, it is easy to see that there are scenarios under which it would provide the same results as the empirical evidence.

4 Conclusion

In this paper, I have shown that in Gaussian information structures where the connection between private signals is sufficiently weak, a public announcement leads to polarized higher-order expectations regardless of the content of the announcement. I illustrated the economic relevance of this properties by a noisy rational expectations model of financial markets. I have shown that these properties can explain stylized facts of trading patterns around announcements like high trading volume, more informative and more volatile prices.

I believe that the observation that public information might polarize higher-order expectations without polarization in first-order expectations has further economic implications in a wide range of contexts. As another example, in a companion paper, Kondor (2009), I analyze a version of the speculative currency attack model of Morris and Shin (1998) where the central bank has imperfect knowledge of the state of the economy. To assess the probability of a devaluation, speculators have to second guess the expectation of the central bank. I show that the fact that a public announcement can polarize higher-order expectations implies that generating and disclosing more public information can destabilize the exchange rate system.

As regards further research, empirical analyzes on the relative effect of announcements on trading patterns and price informativeness across assets and markets with different characteristics (e.g. degree of heterogeneity in the investor base, frequency of announcements, the

importance of private information) could help to establish the importance of the presented mechanism relative to others.

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Appendix: Proofs

A.1 Proof of Proposition 1

Note that

$$\begin{aligned}\bar{E}(\theta|z^j, y) &= \begin{bmatrix} \frac{1}{\omega} + \frac{1}{\kappa} & \frac{2}{\kappa} + \frac{1}{\omega} \end{bmatrix} \begin{bmatrix} \frac{1}{\omega} + \frac{1}{\kappa} + \frac{1}{\alpha} & \frac{1}{\omega} + \frac{1}{\kappa} \\ \frac{1}{\omega} + \frac{1}{\kappa} & \frac{2}{\kappa} + \frac{1}{\omega} + \frac{1}{\beta} \end{bmatrix}^{-1} \begin{bmatrix} \theta_B + \theta_C \\ y \end{bmatrix} \\ &= \frac{\beta(\kappa^2 + \alpha\kappa + \alpha\omega + 2\kappa\omega)y + (\theta_B + \theta_C)\alpha\kappa(\kappa + \omega)}{\alpha\kappa^2 + \kappa^2\beta + \kappa^2\omega + \alpha\kappa\beta + \alpha\kappa\omega + \alpha\beta\omega + 2\kappa\beta\omega}\end{aligned}$$

and

$$\begin{aligned}E((\theta_B + \theta_C)|x^i, y) &= \begin{bmatrix} \frac{1}{\omega} & \frac{1}{\kappa} + \frac{1}{\omega} \end{bmatrix} \begin{bmatrix} \frac{1}{\omega} + \frac{1}{\kappa} + \frac{1}{\alpha} & \frac{1}{\omega} + \frac{1}{\kappa} \\ \frac{1}{\omega} + \frac{1}{\kappa} & \frac{2}{\kappa} + \frac{1}{\omega} + \frac{1}{\beta} \end{bmatrix}^{-1} \begin{bmatrix} x^i \\ y \end{bmatrix} = \\ &= \frac{\beta(\kappa + \omega)(\alpha + \kappa)y + x^i\alpha(\kappa^2 - \beta\omega)}{\alpha\kappa^2 + \kappa^2\beta + \kappa^2\omega + \alpha\kappa\beta + \alpha\kappa\omega + \alpha\beta\omega + 2\kappa\beta\omega},\end{aligned}$$

$$\begin{aligned}E(\bar{E}(\theta|z^j, y)|x^i, y) - \bar{E}(\bar{E}(\theta|z^j, y)|x^i, y) &= \\ &= \alpha^2\kappa(\kappa + \omega)(\kappa^2 - \beta\omega) \frac{\varepsilon^i}{(\alpha\kappa^2 + \kappa^2\beta + \kappa^2\omega + \alpha\kappa\beta + \alpha\kappa\omega + \alpha\beta\omega + 2\kappa\beta\omega)^2}.\end{aligned}$$

By the property of folded normal distributions,

$$\begin{aligned}\frac{\partial \int_j |E(\bar{E}(\theta|z^j, y)|x^i, y) - \bar{E}(\bar{E}(\theta|z^j, y)|x^i, y)| dj}{\partial \beta} &= \\ &= \frac{\partial \left| \frac{\alpha^2\kappa(\kappa + \omega)(\kappa^2 - \beta\omega)}{(\alpha\kappa^2 + \kappa^2\beta + \kappa^2\omega + \alpha\kappa\beta + \alpha\kappa\omega + \alpha\beta\omega + 2\kappa\beta\omega)} \right| \sqrt{\frac{2}{\alpha\pi}}}{\partial \beta} = \\ &= \sqrt{\frac{2}{\alpha\pi}} \frac{-\alpha^2\kappa^2(\alpha + \kappa)(\kappa + \omega)^3}{(\alpha\kappa^2 + \kappa^2\beta + \kappa^2\omega + \alpha\kappa\beta + \alpha\kappa\omega + \alpha\beta\omega + 2\kappa\beta\omega)^2} \text{sgn}(\kappa^2 - \beta\omega)\end{aligned}$$

which proves the statement.

A.2 Proof of Proposition 2

From (19) and (17), market clearing implies

$$p_2 = b_2(\theta_B + \theta_C) + c_2y + e_2q_2 + g_2q_1 - \frac{\gamma}{\tau_\theta}u_2.$$

From (15), this is equivalent to

$$\frac{q_2 \mathbf{b}_2 + \mathbf{c}_2 y + \mathbf{g}_2 q_1}{\mathbf{e}_2} = b_2 (\theta_B + \theta_C) + c_2 y + e_2 q_2 + g_2 q_1 - \frac{\gamma}{\tau_\theta^2} u_2$$

or

$$\frac{\mathbf{b}_2 (\theta_B + \theta_C) - u_2 + \mathbf{c}_2 y + \mathbf{g}_2 q_1}{\mathbf{e}_2} = b_2 (\theta_B + \theta_C) + c_2 y + e_2 \left((\theta_B + \theta_C) - \frac{u_2}{\mathbf{b}_2} \right) + g_2 q_1 - \frac{\gamma}{\tau_\theta^2} u_2.$$

This expression has to hold for any realizations of each random variable. This holds for any η , u_1 , $\theta_B + \theta_C$, u_2 if and only if

$$\begin{aligned} \frac{\mathbf{c}_2}{\mathbf{e}_2} &= c_2, \\ \frac{\mathbf{g}_2}{\mathbf{e}_2} &= g_2 \\ \frac{\mathbf{b}_2}{\mathbf{e}_2} &= b_2 + e_2 \\ \frac{1}{\mathbf{e}_2} &= \frac{\gamma}{\tau_\theta^2} + \frac{e_2}{\mathbf{b}_2}, \end{aligned}$$

respectively. Combining these equations give expressions (24)-(27), which in turn imply (23). Using the same expressions I also get (21) as

$$\begin{aligned} d_2^j &= \frac{\tau_\theta^2}{\gamma} (b_2 z^j + c_2 y + e_2 q_2 + g_2 q_1 - p_2) = \frac{\tau_\theta^2}{\gamma} \left(b_2 z^j + c_2 y + e_2 q_2 + g_2 q_1 - \frac{q_2 \mathbf{b}_2 + \mathbf{c}_2 y + \mathbf{g}_2 q_1}{\mathbf{e}_2} \right) = \\ &= \frac{\tau_\theta^2}{\gamma} \left(b_2 z^j + e_2 q_2 - \frac{q_2 \mathbf{b}_2}{\mathbf{e}_2} \right) = \frac{\tau_\theta^2}{\gamma} \left(b_2 z^j + \left(e_2 - \frac{\mathbf{b}_2}{\mathbf{e}_2} \right) q_2 \right) = \mathbf{b}_2 (z^j - q_2). \end{aligned}$$

Expression (87) is implied by the definition of τ_1 and (24). The same steps give all the corresponding expressions for period 1.

Note that the proposition gives all the equilibrium objects in terms of $b_2, c_2, e_2, g_2, a_1, c_1, e_1, \tau_\theta^2, \tau_q^2$. These coefficients are determined by the Projection Theorem using the observations that that covariance matrix of $[z^j, y, q_1, q_2]$, and its covariance with the fundamental value, θ are

$$\begin{bmatrix} \frac{1}{\kappa} + \frac{1}{\omega} + \frac{1}{\alpha} & \frac{1}{\kappa} + \frac{1}{\omega} & \frac{1}{\omega} & \frac{1}{\kappa} + \frac{1}{\omega} \\ \frac{1}{\kappa} + \frac{1}{\omega} & \frac{2}{\kappa} + \frac{1}{\omega} + \frac{1}{\beta} & \frac{1}{\omega} + \frac{1}{\kappa} & \frac{1}{\omega} + \frac{1}{\kappa} \\ \frac{1}{\omega} & \frac{1}{\omega} + \frac{1}{\kappa} & \frac{1}{\omega} + \frac{1}{\kappa} + \frac{1}{\tau_1^2} & \frac{1}{\omega} \\ \frac{1}{\omega} + \frac{1}{\kappa} & \frac{1}{\omega} + \frac{1}{\kappa} & \frac{1}{\omega} & \frac{1}{\kappa} + \frac{1}{\omega} + \frac{1}{\tau_2^2} \end{bmatrix}, \begin{bmatrix} \frac{1}{\kappa} + \frac{1}{\omega} \\ \frac{2}{\kappa} + \frac{1}{\omega} \\ \frac{1}{\kappa} + \frac{1}{\omega} \\ \frac{1}{\kappa} + \frac{1}{\omega} \end{bmatrix}$$

, respectively, while the covariance matrix of $[x^i, y, q_1]$ and its covariance with the second

period price signal q_2 are

$$\begin{bmatrix} \frac{1}{\omega} + \frac{1}{\kappa} + \frac{1}{\alpha} & \frac{1}{\omega} + \frac{1}{\kappa} & \frac{1}{\omega} + \frac{1}{\kappa} \\ \frac{1}{\omega} + \frac{1}{\kappa} & \frac{2}{\kappa} + \frac{1}{\omega} + \frac{1}{\beta} & \frac{1}{\omega} + \frac{1}{\kappa} \\ \frac{1}{\omega} + \frac{1}{\kappa} & \frac{1}{\omega} + \frac{1}{\kappa} & \frac{1}{\omega} + \frac{1}{\kappa} + \frac{1}{\tau_1^2} \end{bmatrix}, \begin{bmatrix} \frac{1}{\omega} \\ \frac{1}{\kappa} + \frac{1}{\omega} \\ \frac{1}{\omega} \end{bmatrix}$$

,respectively.. Thus, all the equilibrium are functions of the primitives and τ_1 and τ_2 only. Consequently, for existence I only have to prove that equilibrium values for τ_1 and τ_2 exist. Using the explicit expressions for $b_2, e_2, a_1, \tau_\theta^2, \tau_q^2$, (87)-(30) define this as a fixed point problem

$$\tau_2 = \delta_2 \alpha (\kappa + \tau_1^2) \frac{\kappa + \omega}{\gamma (\kappa^2 + \tau_1^2 \tau_2^2 + \alpha \kappa + \alpha \omega + 2\kappa \omega + \alpha \tau_1^2 + \kappa \tau_1^2 + \kappa \tau_2^2 + \omega \tau_1^2 + \omega \tau_2^2)}$$

$$\tau_1 = \delta_1 \frac{\alpha}{\gamma} \frac{\tau_2^2}{\kappa + \omega} \frac{\kappa^2 - \beta \omega}{(\tau_1^2 + \kappa)(\tau_2^2 + \alpha)} \frac{(\tau_1^2 + \tau_2^2 + \alpha + \beta + \omega) \kappa^2 + (2\tau_1^2 \tau_2^2 + \alpha \beta + \alpha \omega + 2\beta \omega + 2\alpha \tau_1^2 + \beta \tau_1^2 + \beta \tau_2^2 + \omega \tau_1^2 + \omega \tau_2^2) \kappa + (\alpha \beta \tau_1^2 + \alpha \omega \tau_1^2 + \beta \omega \tau_1^2 + \beta \omega \tau_2^2 + \beta \tau_1^2 \tau_2^2 + \omega \tau_1^2 \tau_2^2 + \alpha \beta \omega)}{(\tau_1^2 + \tau_2^2 + \alpha + \beta + \omega) \kappa^2 + (2\tau_1^2 \tau_2^2 + \alpha \beta + \alpha \omega + 2\beta \omega + 2\alpha \tau_1^2 + \beta \tau_1^2 + \beta \tau_2^2 + \omega \tau_1^2 + \omega \tau_2^2) \kappa + (\alpha \beta \tau_2^2 + \alpha \omega \tau_2^2 + \beta \omega \tau_1^2 + \beta \omega \tau_2^2 + \beta \tau_1^2 \tau_2^2 + \omega \tau_1^2 \tau_2^2 + \alpha \beta \omega)}$$

The problem simplifies, if I rewrite this as a fixed point problem in the space of $[\tau_1, \tau_2, Y]$ where

$$\tau_2 = \delta_2 \alpha (\kappa + \tau_1^2) \frac{\kappa + \omega}{\gamma (\kappa^2 + \tau_1^2 \tau_2^2 + \alpha \kappa + \alpha \omega + 2\kappa \omega + \alpha \tau_1^2 + \kappa \tau_1^2 + \kappa \tau_2^2 + \omega \tau_1^2 + \omega \tau_2^2)} \quad (61)$$

$$\tau_1 = \delta_1 \frac{(\kappa^2 - \beta \omega) \alpha \tau_2^2}{\gamma (\kappa + \tau_1^2) (\kappa + \omega) (\alpha + \tau_2^2)} Y \quad (62)$$

$$Y = \frac{(\tau_1^2 + \tau_2^2 + \alpha + \beta + \omega) \kappa^2 + (2\tau_1^2 \tau_2^2 + \alpha \beta + \alpha \omega + 2\beta \omega + 2\alpha \tau_1^2 + \beta \tau_1^2 + \beta \tau_2^2 + \omega \tau_1^2 + \omega \tau_2^2) \kappa + (\alpha \beta \tau_1^2 + \alpha \omega \tau_1^2 + \beta \omega \tau_1^2 + \beta \omega \tau_2^2 + \beta \tau_1^2 \tau_2^2 + \omega \tau_1^2 \tau_2^2 + \alpha \beta \omega)}{(\tau_1^2 + \tau_2^2 + \alpha + \beta + \omega) \kappa^2 + (2\tau_1^2 \tau_2^2 + \alpha \beta + \alpha \omega + 2\beta \omega + 2\alpha \tau_1^2 + \beta \tau_1^2 + \beta \tau_2^2 + \omega \tau_1^2 + \omega \tau_2^2) \kappa + (\alpha \beta \tau_2^2 + \alpha \omega \tau_2^2 + \beta \omega \tau_1^2 + \beta \omega \tau_2^2 + \beta \tau_1^2 \tau_2^2 + \omega \tau_1^2 \tau_2^2 + \alpha \beta \omega)} \quad (63)$$

define the fixed point. Let the left hand side of each of these equations be called $F_2(\tau_2, \tau_1)$, $F_1(\tau_2, \tau_1, Y)$ and $F_Y(\tau_2, \tau_1)$, respectively. Also, let $\hat{Y} \equiv F_Y(\tau_2, \tau_1)$ and $\hat{\tau}_2(\tau_1)$, $\hat{\tau}_1(\tau_2, Y)$ implicitly defined as

$$\hat{\tau}_2 \equiv F_2(\hat{\tau}_2, \tau_1)$$

$$\hat{\tau}_1 \equiv F_1(\tau_2, \hat{\tau}_1, Y)$$

By the Implicit Function Theorem, it is easy to check that $\frac{\partial \hat{\tau}_2}{\partial \tau_1} > 0$. Thus, for any τ_1, Y , $\hat{\tau}_2 \leq \tau_2^{\max}$ where τ_2^{\max} is defined as

$$\tau_2^{\max} = \lim_{\tau_1 \rightarrow \infty} \hat{\tau}_2(\tau_1, Y).$$

By simple derivation, $\frac{\partial \hat{Y}}{\partial \tau_2^2} > 0$ and $\frac{\partial \hat{Y}}{\partial \tau_1^2} < 0$. Thus, for any τ_2, τ_1 , $Y^{\min} \leq \hat{Y} \leq Y^{\max}$ where

$$Y^{\min} = \lim_{\tau_2 \rightarrow 0} \lim_{\tau_1 \rightarrow \infty} F_Y(\tau_2, \tau_1) = \frac{\kappa^2 + 2\alpha\kappa + \alpha\beta + \kappa\beta + \alpha\omega + \kappa\omega + \beta\omega}{(\kappa + \omega)(\kappa + \beta)}$$

$$Y^{\max} = \lim_{\tau_1 \rightarrow 0} \lim_{\tau_2 \rightarrow \infty} F_Y(\tau_2, \tau_1) = (\kappa + \omega) \frac{\kappa + \beta}{\kappa^2 + 2\alpha\kappa + \alpha\beta + \kappa\beta + \alpha\omega + \kappa\omega + \beta\omega}$$

Finally, by the Implicit Function Theorem, whenever $\kappa^2 > \beta\omega$, $\frac{\partial \hat{\tau}_1}{\partial \tau_2}, \frac{\partial \hat{\tau}_1}{\partial Y} > 0$, while whenever $\kappa^2 < \beta\omega$, $\frac{\partial \hat{\tau}_1}{\partial \tau_2}, \frac{\partial \hat{\tau}_1}{\partial Y} < 0$. Thus, if τ_1^{\max} is defined as the unique solution of

$$Y^{\max} \alpha \frac{\kappa^2 - \beta\omega}{\gamma(\kappa + \omega)(\kappa + (\tau_1^{\max})^2)} = \tau_1^{\max}$$

then for any τ_2 and Y , $\hat{\tau}_1 \in [0, \tau_1^{\max}]$ when $\kappa^2 > \beta\omega$ and $\hat{\tau}_1 \in [\tau_1^{\max}, 0]$ when $\kappa^2 < \omega\beta$.

Consequently, if the space S is defined as

$$[0, \tau_1^{\max}] \times [0, \tau_2^{\max}] \times [Y^{\min}, Y^{\max}]$$

,and as

$$[\tau_1^{\max}, 0] \times [0, \tau_2^{\max}] \times [Y^{\min}, Y^{\max}]$$

in the case of $\kappa^2 > \omega\beta$ and $\kappa^2 < \omega\beta$ respectively, then the system (61)-(63) maps S to itself and S is a closed convex set. Thus, there is there exist a fixed point, $[\tau_1^*, \tau_2^*, Y^*]$ of the system (61)-(63), by the Brouwer Fixed Point Theorem. Then, I conclude that as long as the denominator of the equilibrium objects described in the proposition are not-zero the equilibrium exists. It is easy to check that this criteria excludes at most a zero measured set of the parameter space.

A.3 Proof of Proposition 3

The result is a consequence of Proposition 2 the fact that

$$\tau_2^* = \alpha \frac{\delta_2}{\gamma}$$

$$\tau_1^* = \alpha^2 \delta_1 \frac{\delta_2^2}{\gamma(\gamma^2 + \alpha\delta_2^2)}$$

is the fixed point of the system

$$\begin{aligned}\lim_{\kappa \rightarrow \infty} F_2(\tau_2, \tau_1) &= \tau_2 \\ \lim_{\kappa \rightarrow \infty} F_1(\tau_2, \tau_1, F_Y(\tau_2, \tau_1)) &= \tau_1.\end{aligned}$$

A.4 Proof of Lemma 1 and Proposition 4

Substituting in F_Y and reorganizing $F_1(F_Y(\tau_1), \tau_2, \tau_1) = \tau_1$ and $F_2(\tau_2, \tau_1) = \tau_2$ as polynoms in τ_1 and τ_2 respectively, check that in any equilibrium (τ_1^*, τ_2^*) has to solve

$$\begin{aligned}0 &\equiv G_1 \\ 0 &\equiv G_2\end{aligned}$$

where

$$G_1 = \gamma\tau_1(\kappa + \omega)(\tau_1^2 + \kappa)(\tau_2^2 + \alpha)Z_2 + \alpha\tau_2^2(\beta\omega - \kappa^2)Z_1$$

with

$$\begin{aligned}Z_1 &= \kappa^2\tau_1^2 + \kappa^2\tau_2^2 + \alpha\kappa^2 + \kappa^2\beta + \kappa^2\omega + 2\alpha\kappa\tau_1^2 + \alpha\beta\tau_1^2 + \kappa\beta\tau_1^2 + \kappa\beta\tau_2^2 + \alpha\omega\tau_1^2 + \kappa\omega\tau_1^2 + \\ &\quad + \kappa\omega\tau_2^2 + \beta\omega\tau_1^2 + \beta\omega\tau_2^2 + 2\kappa\tau_1^2\tau_2^2 + \beta\tau_1^2\tau_2^2 + \omega\tau_1^2\tau_2^2 + \alpha\kappa\beta + \alpha\kappa\omega + \alpha\beta\omega + 2\kappa\beta\omega \\ Z_2 &= \kappa^2\tau_1^2 + \kappa^2\tau_2^2 + \alpha\kappa^2 + \kappa^2\beta + \kappa^2\omega + 2\alpha\kappa\tau_2^2 + \alpha\beta\tau_2^2 + \kappa\beta\tau_1^2 + \kappa\beta\tau_2^2 + \kappa\omega\tau_1^2 + \alpha\omega\tau_2^2 + \\ &\quad + \kappa\omega\tau_2^2 + \beta\omega\tau_1^2 + \beta\omega\tau_2^2 + 2\kappa\tau_1^2\tau_2^2 + \beta\tau_1^2\tau_2^2 + \omega\tau_1^2\tau_2^2 + \alpha\kappa\beta + \alpha\kappa\omega + \alpha\beta\omega + 2\kappa\beta\omega\end{aligned}$$

and

$$G_2 = \gamma(\tau_1^2 + \kappa + \omega)\tau_2^3 + \gamma(\kappa^2 + \alpha\kappa + \alpha\omega + 2\kappa\omega + \alpha\tau_1^2 + \kappa\tau_1^2 + \omega\tau_1^2)\tau_2 - (\omega + \alpha\kappa\delta_2(\tau_1^2 + \kappa)).$$

Note that for any fixed τ_1 G_2 is a monotonically increasing function with a single root. Also

$$\begin{aligned}\frac{\partial G_2(\tau_1, \tau_2)}{\partial \tau_1^2} \Big|_{\tau_2 = \hat{\tau}_2(\tau_1)} &= (\tau_2\gamma(\tau_2^2 + \gamma(\alpha + \kappa + \omega)) - \alpha\kappa\delta_2) = \\ &= (F_2(\tau_2, \tau_1)\gamma(\tau_2^2 + \gamma(\alpha + \kappa + \omega)) - \alpha\kappa\delta_2) = \\ &= \left(\delta_2\alpha(\kappa + \tau_1^2) \frac{(\kappa + \omega)\gamma(\tau_2^2 + (\alpha + \kappa + \omega))}{\gamma(\kappa^2 + \tau_1^2\tau_2^2 + \alpha\kappa + \alpha\omega + 2\kappa\omega + \alpha\tau_1^2 + \kappa\tau_1^2 + \kappa\tau_2^2 + \omega\tau_1^2 + \omega\tau_2^2)} - \alpha\kappa\delta_2 \right) = \\ &= \alpha\delta_2(\kappa + \omega)(\kappa + \tau_1^2) \frac{\alpha + \kappa + \omega + \tau_2^2}{\kappa^2 + \tau_1^2\tau_2^2 + \alpha\kappa + \alpha\omega + 2\kappa\omega + \alpha\tau_1^2 + \kappa\tau_1^2 + \kappa\tau_2^2 + \omega\tau_1^2 + \omega\tau_2^2} > 0\end{aligned}$$

where $\hat{\tau}_2(\tau_1)$ is defined as in the Proof of Proposition 2.

Also, by the implicit function theorem

$$\frac{\partial \tau_1^*}{\partial \beta} = - \frac{\det \begin{pmatrix} \frac{\partial G_1}{\partial \beta} & \frac{\partial G_1}{\partial \tau_2} \\ \frac{\partial G_2}{\partial \beta} & \frac{\partial G_2}{\partial \tau_2} \end{pmatrix}}{\det \begin{pmatrix} \frac{\partial G_1}{\partial \tau_1} & \frac{\partial G_1}{\partial \tau_2} \\ \frac{\partial G_2}{\partial \tau_1} & \frac{\partial G_2}{\partial \tau_2} \end{pmatrix}} = - \frac{\frac{\partial G_1}{\partial \beta} \frac{\partial G_2}{\partial \tau_2}}{\frac{\partial G_1}{\partial \tau_1} \frac{\partial G_2}{\partial \tau_2} - \frac{\partial G_1}{\partial \tau_2} \frac{\partial G_2}{\partial \tau_1}}. \quad (64)$$

$$\frac{\partial \tau_2^*}{\partial \beta} = - \frac{\det \begin{pmatrix} \frac{\partial G_1}{\partial \beta} & \frac{\partial G_1}{\partial \tau_1} \\ 0 & \frac{\partial G_2}{\partial \tau_1} \end{pmatrix}}{\det \begin{pmatrix} \frac{\partial G_1}{\partial \tau_1} & \frac{\partial G_1}{\partial \tau_2} \\ \frac{\partial G_2}{\partial \tau_1} & \frac{\partial G_2}{\partial \tau_2} \end{pmatrix}} = - \frac{\frac{\partial G_1}{\partial \beta} \frac{\partial G_2}{\partial \tau_1}}{\frac{\partial G_1}{\partial \tau_1} \frac{\partial G_2}{\partial \tau_2} - \frac{\partial G_1}{\partial \tau_2} \frac{\partial G_2}{\partial \tau_1}}. \quad (65)$$

Suppose that at a given point $\frac{\partial \tau_1^*}{\partial \beta}$ and $\frac{\partial \tau_2^*}{\partial \beta}$ exists. Clearly, $\frac{\partial \tau_1^*}{\partial \beta} = 0$ is possible only if $\frac{\partial G_1}{\partial \beta} = 0$, but then $\frac{\partial \tau_2^*}{\partial \beta} = 0$ also. If $\frac{\partial \tau_1^*}{\partial \beta} \neq 0$, then

$$\frac{\frac{\partial \tau_2^*}{\partial \beta}}{\frac{\partial \tau_1^*}{\partial \beta}} = \frac{\frac{\partial G_2}{\partial \tau_1}}{\frac{\partial G_2}{\partial \tau_2}} > 0.$$

This proves Lemma 1.

For Proposition 4, consider the next Lemma first.

Lemma 3 For any $\beta \geq \frac{\kappa^2}{\omega}$, $\frac{\partial G_1}{\partial \beta} |_{\tau_1 = \tau_1^*} > 0$

Proof. Consider G_1 . It is clear that $Z_1, Z_2, \frac{\partial Z_2}{\partial \beta}, \frac{\partial Z_1}{\partial \beta} > 0$ and

$$\begin{aligned} \frac{\partial G_1}{\partial \beta} &= \gamma \tau_1 (\kappa + \omega) (\tau_1^2 + \kappa) (\tau_2^2 + \alpha) \frac{\partial Z_2}{\partial \beta} + \alpha \tau_2^2 \omega Z_1 + \alpha \tau_2^2 (\beta \omega - \kappa^2) \frac{\partial Z_1}{\partial \beta} = \\ &= \alpha \tau_2^2 (\kappa^2 - \beta \omega) \frac{Z_1}{Z_2} \frac{\partial Z_2}{\partial \beta} + \alpha \tau_2^2 \omega Z_1 + \alpha \tau_2^2 (\beta \omega - \kappa^2) \frac{\partial Z_1}{\partial \beta} = \\ &= \left(\frac{\alpha \tau_2^2 Z_1 (\kappa + \tau_2^2) (\kappa + \omega)^2 (\alpha + \kappa + \tau_1^2)}{(\tau_1^2 + \tau_2^2 + \alpha + \beta + \omega) \kappa^2 + (2\tau_1^2 \tau_2^2 + \alpha \beta + \alpha \omega + 2\beta \omega + 2\alpha \tau_2^2 + \beta \tau_1^2 + \beta \tau_2^2 + \omega \tau_1^2 + \omega \tau_2^2) \kappa + (\beta (\tau_1^2 \tau_2^2 + \alpha \omega + \alpha \tau_2^2 + \omega \tau_1^2 + \omega \tau_2^2) + \omega \tau_2^2 (\alpha + \tau_1^2))} \right) + \\ &\quad + \alpha \tau_2^2 (\beta \omega - \kappa^2) \frac{\partial Z_1}{\partial \beta} > 0 \end{aligned}$$

where I used the equilibrium condition $\tau_1 = F_1$. ■

Note that $0 \equiv G_2$ has a single solution $\hat{\tau}_2$ for any τ_1^2 , and $0 \equiv G_2$ will have at least one solution $\hat{\tau}_1$ for given τ_2 , but might have more than one. However, when $\beta \omega = \kappa^2$, then $\hat{\tau}_1 = 0$ is the only solution of $0 \equiv G_1$. Thus, the system has a unique fixed point where $\tau_1^* = 0, \tau_2^* > 0$.

Note that $\hat{\tau}_2$ is continuous in ω and τ_1 . Thus, τ_2^* is also continuous in ω as long as $\hat{\tau}_1$ is continuous in ω . Also, as G_2 is a 5-th order polynomial in τ_1 a necessary condition for $\hat{\tau}_1$ to be discontinuous at a given point is that $\frac{\partial G_1}{\partial \tau_1} = 0$ at that point.

Consider the point $\omega = \frac{\kappa^2}{\beta}$ where $\tau_1^* = 0$ and $\tau_2^* > 0$. It is simple to check that at that point $\frac{\partial G_1}{\partial \tau_1} > 0$, $\frac{\partial G_2}{\partial \tau_1} = 2\tau_1 \frac{\partial G_2}{\partial \tau_1^2} = 0$, $\frac{\partial G_2}{\partial \tau_2} > 0$, $\frac{\partial G_1}{\partial \tau_2} = 0$. As from Lemma 3, $\frac{\partial G_1}{\partial \beta} > 0$, at this point $\frac{\partial \tau_1^*}{\partial \beta} < 0$ and $\frac{\partial \tau_2^*}{\partial \beta} = 0$. Also, the fact that $\frac{\partial G_1}{\partial \tau_1} > 0$, $\frac{\partial G_1}{\partial \tau_1} \frac{\partial G_2}{\partial \tau_2} - \frac{\partial G_1}{\partial \tau_2} \frac{\partial G_2}{\partial \tau_1} > 0$ and both are continuous in ω at that point, imply that there is an open set around $\omega = \frac{\kappa^2}{\beta}$ that within this set τ_1^*, τ_2^* are continuous functions of ω and continuously differentiable in β . Define $\omega^{\min}, \omega^{\max}$ in a way that the set $(\omega^{\min}, \omega^{\max})$ is the largest such open set around $\omega = \frac{\kappa^2}{\beta}$. Then by definition $\frac{\partial G_1}{\partial \tau_1} \frac{\partial G_2}{\partial \tau_2} - \frac{\partial G_1}{\partial \tau_2} \frac{\partial G_2}{\partial \tau_1}$ cannot change sign within this set. Also, as $\beta > \frac{\kappa^2}{\omega}$ implies $\tau_1 < 0$, $\frac{\partial G_2}{\partial \tau_1} = 2\tau_1 \frac{\partial G_2}{\partial \tau_1^2} < 0$ in this region, while $\frac{\partial G_2}{\partial \tau_2} > 0$, from Lemma 3, the second statement holds.

A.5 Proof of Proposition 5

The result comes from a series of mechanical calculations. In particular,

$$\begin{aligned} \lim_{\kappa \rightarrow \infty} \frac{\mathbf{b}_2}{\mathbf{e}_2} &= \frac{\alpha + \tau_2^2}{\alpha + \beta + \omega + \tau_1^2 + \tau_2^2} \\ \lim_{\kappa \rightarrow \infty} \frac{\mathbf{a}_1}{\mathbf{e}_1} &= \frac{\alpha^2 + \tau_1^4 + \tau_1^2 \tau_2^2 + 2\alpha \tau_1^2 + \alpha \tau_2^2 + \beta \tau_1^2 + \omega \tau_1^2}{(\alpha + \beta + \omega + \tau_1^2)(\alpha + \beta + \omega + \tau_1^2 + \tau_2^2)} \\ \lim_{\kappa \rightarrow \infty} \mathbf{e}_1 &= \tau_2^2 (\alpha + \beta + \omega + \tau_1^2 + \tau_2^2) \alpha \frac{\alpha + \beta + \omega + \tau_1^2}{(\alpha + \tau_2^2)(\alpha^2 + \tau_1^4 + \tau_1^2 \tau_2^2 + 2\alpha \tau_1^2 + \alpha \tau_2^2 + \beta \tau_1^2 + \omega \tau_1^2)} \\ \lim_{\kappa \rightarrow \infty} \mathbf{e}_2 &= \alpha \frac{\alpha + \beta + \omega + \tau_1^2 + \tau_2^2}{\alpha + \tau_2^2} \\ \lim_{\kappa \rightarrow \infty} \frac{\mathbf{c}_2}{\mathbf{e}_2} &= \frac{\beta}{\alpha + \beta + \omega + \tau_1^2 + \tau_2^2} \\ \lim_{\kappa \rightarrow \infty} \frac{\mathbf{c}_1}{\mathbf{e}_1} &= \beta \frac{2\alpha + \beta + \omega + \tau_1^2 + \tau_2^2}{(\alpha + \beta + \omega + \tau_1^2)(\alpha + \beta + \omega + \tau_1^2 + \tau_2^2)}. \end{aligned}$$

As I already showed that τ_1 and τ_2 are insensitive to β in this limit, the partial derivatives of these expressions with respect to β give all the results.

A.6 Proof of Proposition 6

I start with the analysis of the relevant equilibrium objects when $\beta = \frac{\omega}{\kappa^2}$. At this point, $\tau_1^* = 0$, $\tau_2^* > 0$ and given as the unique root of

$$\delta_2 \alpha \kappa \frac{\kappa + \omega}{\gamma (\kappa^2 + \alpha \kappa + \alpha \omega + 2\kappa \omega + \kappa \tau_2^2 + \omega \tau_2^2)} = \tau_2.$$

Also, as I showed in the proof of Proposition 4, at this point τ_1^* and τ_2^* are continuously differentiable in β at this point and $\frac{\partial(\tau_1^*)^2}{\partial\beta} = \frac{\partial(\tau_2^*)^2}{\partial\beta} = 0$. Therefore, at this point, as β changes each equilibrium objects changes only by the direct effect of β . I am interested in the properties of \mathbf{e}_1 and $\frac{\mathbf{a}_1}{\mathbf{e}_1}$ near this point. As

$$\begin{aligned} \lim_{\tau_1 \rightarrow 0} \frac{\mathbf{a}_1}{\mathbf{e}_1} &= \lim_{\tau_1 \rightarrow 0} ((e_2 + b_2)(a_1 + e_1) + g_2) = \\ &= \frac{\alpha(\kappa^2 - \beta\omega)\kappa(\kappa + \omega)(\alpha + \tau_2^2)}{(\kappa^2\tau_2^2 + \alpha\kappa^2 + \kappa^2\beta + \kappa^2\omega + \kappa\beta\tau_2^2 + \kappa\omega\tau_2^2 + \beta\omega\tau_2^2 + \alpha\kappa\beta + \alpha\kappa\omega + \alpha\beta\omega + 2\kappa\beta\omega)(\alpha\kappa^2 + \kappa^2\beta + \kappa^2\omega + \alpha\kappa\beta + \alpha\kappa\omega + \alpha\beta\omega + 2\kappa\beta\omega)} \end{aligned}$$

and

$$\begin{aligned} &\frac{\partial}{\partial\beta} \frac{(\kappa^2 - \beta\omega)\alpha\kappa(\kappa + \omega)(\alpha + \tau_2^2)}{(\kappa^2\tau_2^2 + \alpha\kappa^2 + \kappa^2\beta + \kappa^2\omega + \kappa\beta\tau_2^2 + \kappa\omega\tau_2^2 + \beta\omega\tau_2^2 + \alpha\kappa\beta + \alpha\kappa\omega + \alpha\beta\omega + 2\kappa\beta\omega)(\alpha\kappa^2 + \kappa^2\beta + \kappa^2\omega + \alpha\kappa\beta + \alpha\kappa\omega + \alpha\beta\omega + 2\kappa\beta\omega)} \Big|_{\frac{\kappa^2}{\omega} = \beta} = \\ &= -\omega \frac{\alpha\kappa(\kappa + \omega)(\alpha + \tau_2^2)}{(\kappa^2\tau_2^2 + \alpha\kappa^2 + \kappa^2\beta + \kappa^2\omega + \kappa\beta\tau_2^2 + \kappa\omega\tau_2^2 + \beta\omega\tau_2^2 + \alpha\kappa\beta + \alpha\kappa\omega + \alpha\beta\omega + 2\kappa\beta\omega)(\alpha\kappa^2 + \kappa^2\beta + \kappa^2\omega + \alpha\kappa\beta + \alpha\kappa\omega + \alpha\beta\omega + 2\kappa\beta\omega)} < 0 \end{aligned}$$

I conclude that

$$\frac{\partial \left| \frac{\mathbf{a}_1}{\mathbf{e}_1} \right|}{\partial\beta} \Big|_{\kappa^2 = \beta} > 0.$$

This implies the first part of the statement. Also, using the expression for \mathbf{e}_1 and the observation that

$$\begin{aligned} \frac{e_1 + a_1}{a_1} &= \frac{\alpha + \tau_1^2}{\alpha} \\ \frac{g_2}{e_2 + b_2} &= \tau_1^2 \frac{\alpha + \kappa + \tau_2^2}{(\kappa + \tau_1^2)(\alpha + \tau_2^2)} \end{aligned}$$

I rewrite it as

$$\begin{aligned} \mathbf{e}_1 &= \frac{\tau_q^2}{\gamma} \frac{1}{(e_2 + b_2)} \frac{1}{(e_2 + b_2) \frac{e_1 + a_1}{a_1} + \frac{g_2}{a_1}} = \frac{\tau_q^2}{\gamma} \frac{1}{(e_2 + b_2)^2} \frac{1}{\frac{\alpha + \tau_1^2}{\alpha} + \frac{g_2}{(e_2 + b_2)a_1}} = \\ &= \frac{\tau_q^2}{\gamma (e_2 + b_2)^2} \frac{1}{\frac{\alpha + \tau_1^2}{\alpha} + \frac{\tau_1^2}{a_1} \frac{\alpha + \kappa + \tau_2^2}{(\kappa + \tau_1^2)(\alpha + \tau_2^2)}}. \end{aligned}$$

As

$$\frac{\tau_1^2}{a_1} = \frac{\left(\delta_1 \frac{\tau_q^2}{\gamma} \frac{a_1}{(e_2 + b_2)} \right)^2}{a_1} = \left(\delta_1 \frac{\tau_q^2}{\gamma} \right)^2 \frac{a_1}{(e_2 + b_2)^2},$$

$$\mathbf{e}_1|_{\kappa^2=\beta} = \frac{\tau_q^2}{\gamma} \frac{1}{(e_2 + b_2)^2} > 0.$$

Also,

$$\begin{aligned} \lim_{\beta \rightarrow \infty} \left(\frac{\alpha + \tau_1^2}{\alpha} + \frac{\tau_1^2}{a_1} \frac{\alpha + \kappa + \tau_2^2}{(\kappa + \tau_1^2)(\alpha + \tau_2^2)} \right) &= \\ &= \kappa \frac{(-\tau_2^2 - \alpha - \kappa - \omega)\tau_1^4 + (-\alpha^2 - 2\alpha\kappa - \alpha\tau_2^2 - 2\omega\alpha - \kappa^2 - \kappa\tau_2^2 - 2\omega\kappa - \omega\tau_2^2)\tau_1^2 + (\omega\alpha^2 + \omega\alpha\tau_2^2)}{\alpha\omega(\kappa + \tau_1^2)(\alpha + \tau_2^2)}, \end{aligned}$$

,where, for any fixed τ_2 , the numerator is a monotonically decreasing function in τ_1^2 . As τ_2^* is finite for any τ_1 , and $\lim_{\delta_1 \rightarrow \infty} \tau_1^2 = \infty$,

$$\lim_{\delta_1 \rightarrow \infty} \lim_{\beta \rightarrow \infty} \left(\frac{\alpha + \tau_1^2}{\alpha} + \frac{\tau_1^2}{a_1} \frac{\alpha + \kappa + \tau_2^2}{(\kappa + \tau_1^2)(\alpha + \tau_2^2)} \right) = -\infty$$

in equilibrium. As $\frac{\tau_q^2}{\gamma(e_2 + b_2)^2} > 0$, there must be a sufficiently large δ_1 and $\beta \in \left(\frac{\kappa^2}{\omega}, \infty\right)$ that $\frac{1}{\mathbf{e}_1}$ is negative. As $\mathbf{e}_1|_{\kappa^2=\beta} > 0$, this implies the second part of the Lemma.

A.7 Proof of Proposition 7

Period 2 is equivalent to case 1. For period 1 objects, first, I derive expression (47). In period 1, B traders maximize the expected utility

$$\begin{aligned} \max_{d_1^i} E_1 \left(-\exp \left(-\gamma (p_2 - p_1) d_1^j - \gamma \frac{E(\theta|q_2, q_1, y, z^j) - p_2}{\gamma \text{var}(\theta|q_2, q_1, y, \bar{z}^j)} (\theta - p_2) \right) | z^j, q_1, y \right) &= \\ E_1 \left(E_2 \left(-\exp \left(-\gamma (p_2 - p_1) d_1^j - \frac{E(\theta|q_2, q_1, y, z^j) - p_2}{\text{var}(\theta|q_2, q_1, y, \bar{z}^j)} (\theta - p_2) \right) | q_2, q_1, y, z^j \right) | z^j, q_1, y \right) \end{aligned}$$

as $E(\exp(\theta)) = \exp(E(\theta) + \frac{1}{2}\text{var}(\theta))$

$$\begin{aligned} E_2 \left(-\exp \left(-\gamma (p_2 - p_1) d_1^j - \gamma \frac{E(\theta|q_2, q_1, y, z^j) - p_2}{\gamma \text{var}(\theta|q_2, q_1, y, \bar{z}^j)} (\theta - p_2) \right) | q_2, q_1, y, z^j \right) &= \\ = \exp(-\gamma (p_2 - p_1) d_1^j) \exp \left(-\frac{(E(\theta|q_2, q_1, y, z^j) - p_2)^2}{\text{var}(\theta|q_2, q_1, y, \bar{z}^j)} + \frac{1}{2} \frac{(E(\theta|q_2, q_1, y, z^j) - p_2)^2}{\text{var}(\theta|q_2, q_1, y, \bar{z}^j)} \right) \end{aligned}$$

thus, the trader maximizes

$$\begin{aligned}
& E_1 \left(-\exp \left(-\gamma (p_2 - p_1) d_1^j - \frac{(E(\theta|q_2, q_1, y, z^j) - p_2)^2}{\text{var}(\theta|q_2, q_1, y, z^j)} \frac{1}{2} \right) \middle| z^j, q_1, y \right) = \\
& = E_1 \left(-\exp \left(-\gamma (p_2 - p_1) d_1^j - \text{var}(\theta|q_2, q_1, y, z^{jj}) (\mathbf{b}_2)^2 (z^j - q_2)^2 \frac{1}{2} \right) \middle| z^j, q_1, y \right) = \\
& = E_1 \left(-\exp \left(-\gamma \left(\frac{\mathbf{b}_2 q_2 + \mathbf{g}_2 q_1 - \mathbf{c}_2 y}{\mathbf{e}_2} - p_1 \right) d_1^j - \frac{(\mathbf{b}_2)^2 \left((z^j)^2 - 2q_2 z^j + q_2^2 \right)}{\tau_\theta^2} \frac{1}{2} \right) \middle| z^j, q_1, y \right)
\end{aligned}$$

where I used that

$$\frac{E(\theta|q_2, q_1, y, z^j) - p_2}{\gamma \text{var}(\theta|q_2, q_1, y, z^{jj})} = \mathbf{b}_2 (z^j - q_2) = \tau_\theta^2 \frac{b_2}{\gamma} (z^j - q_2).$$

A property of normal distributions is that if C is constant scalar, L is a $nx1$ constant vector, N is an nxn constant matrix and M is an $nx1$ stochastic matrix and \mathcal{I} is an information set, then

$$\begin{aligned}
& E(-\exp(C + L'M - M'NM') | \mathcal{I}) = \\
& - |W|^{-1/2} |2N + W^{-1}|^{-1/2} \exp \left(C + L'Q - Q'NQ + \frac{1}{2} (L' - 2Q'N) (2N + W^{-1})^{-1} (L - 2NQ) \right)
\end{aligned} \tag{66}$$

where $Q = E(M|I)$ and $W = \text{var}(M|I)$. Let $q_2 = M$ and

$$\begin{aligned}
C &= -\gamma \left(\frac{\mathbf{g}_2 q_1 - \mathbf{c}_2 y}{\mathbf{e}_2} - p_1 \right) d_1^j - \tau_\theta^2 b_2^2 (z^j)^2 \frac{1}{2} \\
L &= -\gamma \frac{a_2'}{e_2'} d_1^j + \tau_\theta^2 b_2^2 z^j \\
N &= \tau_\theta^2 b_2^2 \frac{1}{2}
\end{aligned}$$

then the term in the brackets in (66) is

$$\begin{aligned}
& -\gamma \left(\frac{\mathbf{g}_2 q_1 - \mathbf{c}_2 y}{\mathbf{e}_2} - p_1 \right) d_1^j - \tau_\theta^2 b_2^2 (z^j)^2 \frac{1}{2} + \\
& \left(-\gamma \frac{\mathbf{b}_2}{\mathbf{e}_2} d_1^j + \tau_\theta^2 b_2^2 z^j \right) E(q_2 | z^j, q_1, y) \\
& - E^2(q_2 | z^j, q_1, y) \frac{1}{2} \tau_\theta^2 b_2^2 \\
& + \frac{1}{2} \frac{\left(-\gamma \frac{\mathbf{b}_2}{\mathbf{e}_2} d_1^j + \tau_\theta^2 b_2^2 z^j - E(q_2 | z^j, q_1, y) \tau_\theta^2 b_2^2 \right)^2}{\tau_\theta^2 b_2^2 + \tau_B^2}.
\end{aligned}$$

Thus, the trader maximizes

$$\gamma \left(\frac{\mathbf{g}_2 q_1 - \mathbf{c}_2 y}{\mathbf{e}_2} - p_1 \right) d_1^j + \gamma \left(d_1^j \frac{\mathbf{b}_2}{\mathbf{e}_2} \right) E(q_2 | z^j, q_1, y) - \frac{1}{2} \frac{\left(-\gamma \frac{\mathbf{b}_2}{\mathbf{e}_2} d_1^j + \tau_\theta^2 b_2^2 z^j - E(q_2 | z^j, q_1, y) \tau_\theta^2 b_2^2 \right)^2}{\tau_\theta^2 b_2^2 + \tau_B^2}$$

taking the first order condition gives

$$\frac{\left(\frac{\mathbf{g}_2 q_1 - \mathbf{c}_2 y}{\mathbf{e}_2} + \frac{\mathbf{b}_2}{\mathbf{e}_2} E(q_2 | z^j, q_1, y) - p_1 \right) (\tau_\theta^2 b_2^2 + \tau_B^2) + \tau_\theta^2 b_2^2 \frac{\mathbf{b}_2}{\mathbf{e}_2} (z^j - E(q_2 | z^j, q_1, y))}{\gamma \left(\frac{\mathbf{b}_2}{\mathbf{e}_2} \right)^2} = d_1^j \quad (67)$$

which is equivalent to (47). Collecting coefficients of z^j and using that $\frac{\mathbf{b}_2}{\mathbf{e}_2} = b_2 + e_2$, gives the expression for \mathbf{b}_B as

$$\frac{\frac{\mathbf{b}_2}{\mathbf{e}_2} b_B (\tau_\theta^2 b_2^2 + \tau_B^2) + \tau_\theta^2 b_2^2 \frac{\mathbf{b}_2}{\mathbf{e}_2} (1 - b_B)}{\gamma \left(\frac{\mathbf{b}_2}{\mathbf{e}_2} \right)^2} = \frac{b_B (\tau_\theta^2 b_2^2 + \tau_B^2) + \tau_\theta^2 b_2^2 (1 - b_B)}{\gamma (e_2 + b_2)} = \frac{\tau_B^2 b_B + \tau_\theta^2 b_2^2}{\gamma (e_2 + b_2)}.$$

The demand of A traders in the first period is

$$d_1^i = \frac{E(p_2 | x^i, q_1, y) - p_1}{\gamma \text{var}(p_2 | x^i, q_1, y)} = \tau_A^2 \frac{\frac{\mathbf{b}_2}{\mathbf{e}_2} E(q_2 | x^i, q_1, y) + \frac{\mathbf{g}_2 q_1 - \mathbf{c}_2 y}{\mathbf{e}_2} - p_1}{\gamma \left(\frac{\mathbf{b}_2}{\mathbf{e}_2} \right)^2}. \quad (68)$$

Collecting terms multiplying x^i gives

$$\mathbf{a}_A = \frac{\tau_A^2 a_A}{\gamma (e_2 + b_2)}.$$

where (51)-(52) define coefficients $\mathbf{a}_A, \mathbf{c}_A, \mathbf{e}_A$ and $\mathbf{b}_B, \mathbf{c}_B, \mathbf{e}_B$. As, by market clearing, (53)-(54) have to hold, the system (48)-(50) comes from plugging (55)-(56) into $\mathbf{a}_1, \mathbf{b}_1$ and the

resulting formulas into (43),(44) and (16).

Finally, I derive expressions for $\mathbf{c}_A, \mathbf{e}_A$ and $\mathbf{c}_B, \mathbf{e}_B$ and $\mathbf{c}_1, \mathbf{e}_1$ as functions of the primitives and τ_1, τ_2 . First, collecting the terms multiplying y in (67) and (68), plugging in the definition of q_1 gives

$$\mathbf{c}_B = \frac{\mathbf{e}_2}{\gamma(\mathbf{b}_2)^2} \left(\tau_B^2 \mathbf{b}_2 c_B + (\tau_\theta^2 b_2^2 + \tau_B^2) \left(\mathbf{c}_2 - \mathbf{g}_2 \frac{\mathbf{c}_1}{\mathbf{a}_1 + \mathbf{b}_1} \right) \right) \quad (69)$$

$$\mathbf{c}_A = \frac{\mathbf{e}_2}{\gamma(\mathbf{b}_2)^2} \tau_A^2 \left(\mathbf{c}_2 + \mathbf{b}_2 c_A - (\mathbf{g}_2 + \mathbf{b}_2 e_A) \frac{\mathbf{c}_1}{\mathbf{a}_1 + \mathbf{b}_1} \right). \quad (70)$$

Using $\mathbf{c}_1 = \mu \mathbf{c}_B + (1 - \mu) \mathbf{c}_A$ and $\mathbf{a}_1 + \mathbf{b}_1 = \mu \mathbf{a}_A + (1 - \mu) \mathbf{b}_B$ This gives

$$\frac{\mathbf{c}_1}{\mathbf{a}_1 + \mathbf{b}_1} = \frac{(1 - \mu) \tau_A^2 (\mathbf{c}_2 + \mathbf{b}_2 c_A) + \mu (\tau_B^2 \mathbf{b}_2 c_B + (\tau_\theta^2 b_2^2 + \tau_B^2) \mathbf{c}_2)}{\mathbf{b}_2 (\tau_A^2 (1 - \mu) (a_A + e_A) + \mu (\tau_B^2 a_B + \tau_\theta^2 b_2^2)) + \mathbf{g}_2 ((1 - \mu) \tau_A^2 + \mu (\tau_\theta^2 b_2^2 + \tau_B^2))}.$$

Plugging back this and the expressions for $\mathbf{e}_2, \mathbf{b}_2, \mathbf{g}_2$ into expressions (69)-(70) gives the result. By analogous steps I get

$$\begin{aligned} \frac{\mathbf{e}_1}{\mathbf{a}_1 + \mathbf{b}_1} &= \frac{\frac{\mathbf{e}_2}{\mathbf{b}_2} (\mu (\tau_B^2 + \tau_\theta^2 b_2^2) + (1 - \mu) \tau_A^2)}{\tau_A^2 (1 - \mu) \left(a_A + e_A + \frac{\mathbf{g}_2}{\mathbf{b}_2} \right) + \tau_B^2 \mu \left(a_B + \frac{\mathbf{g}_2}{\mathbf{b}_2} \right) + \mu \tau_\theta^2 b_2^2 \left(1 + \frac{\mathbf{g}_2}{\mathbf{b}_2} \right)} \\ \mathbf{e}_A &= \left(\frac{\mathbf{e}_2}{\mathbf{b}_2} \right)^2 \frac{\tau_A^2}{\gamma} \left(1 - \frac{\mathbf{e}_1}{\mathbf{a}_1 + \mathbf{b}_1} (\mathbf{g}_2 + \mathbf{b}_2 e_A) \right) \\ \mathbf{e}_B &= \frac{1}{\gamma} \left(\frac{\mathbf{e}_2}{\mathbf{b}_2} \right)^2 \left(1 - \frac{\mathbf{e}_1}{\mathbf{a}_1 + \mathbf{b}_1} \frac{\mathbf{g}_2}{\mathbf{b}_2} \right) (\tau_B^2 + \tau_\theta^2 b_2^2). \end{aligned}$$

A.8 Proof of Lemma 2

By definition

$$\begin{aligned} &\frac{\partial V_1^S}{\partial \beta} \frac{\beta}{V_1^S} \Big|_{\tau_1=\bar{\tau}_1, \tau_2=\bar{\tau}_2, \phi=\bar{\phi}} = \\ &= (1 - \mu) \frac{\frac{\partial |a_A \tau_A^2|}{\partial \beta} \Big|_{\tau_1=\bar{\tau}_1, \tau_2=\bar{\tau}_2, \phi=\bar{\phi}}}{(1 - \mu) |a_A \tau_A^2| + \mu (b_B \tau_B^2 + b_2^2 \tau_\theta^2)} + \mu \frac{\frac{\partial b_B \tau_B^2 + b_2^2 \tau_\theta^2}{\partial \beta} \Big|_{\tau_1=\bar{\tau}_1, \tau_2=\bar{\tau}_2, \phi=\bar{\phi}}}{(1 - \mu) |b_A A| + \mu (b_B \tau_B^2 + b_2^2 \tau_\theta^2)} + \frac{\partial \frac{1}{b_2 + e_2}}{\partial \beta} \Big|_{\tau_1=\bar{\tau}_1, \tau_2=\bar{\tau}_2, \phi=\bar{\phi}}. \end{aligned}$$

By simple substitution, I can show that $\frac{\partial |a_A \tau_A^2|}{\partial \beta} \Big|_{\tau_1=\bar{\tau}_1, \tau_2=\bar{\tau}_2, \phi=\bar{\phi}} > 0$ and $\frac{\partial b_B \tau_B^2 + b_2^2 \tau_\theta^2}{\partial \beta} \Big|_{\tau_1=\bar{\tau}_1, \tau_2=\bar{\tau}_2, \phi=\bar{\phi}} < 0$.

A.9 Proof of Proposition 9

The results for coefficients in period 2 are implied by the same steps as in Proposition 5. Below I provide the main steps for the rest of the results. Just as in Case 1, I use the fact that $(\tau_1^*)^2, (\tau_2^*)^2$ do not change in this limit with β , so I have to only consider the direct effects.

1.

$$= \frac{(\alpha + \tau_2^2)\mu}{\alpha + \beta + \omega + \tau_1^2 + \tau_2^2} \frac{\alpha^2 \tau_2^2 + \alpha^3 \mu + \alpha \tau_2^4 + \alpha^2 \mu \tau_1^2 + \alpha^2 \mu \tau_2^2 + \alpha \tau_1^2 \tau_2^2 (2 - \mu) + \beta \tau_1^2 \tau_2^2 + \tau_1^4 \tau_2^2 + \tau_1^2 \tau_2^4 (1 - \mu) + \omega \tau_1^2 \tau_2^2}{(\tau_1^2 \tau_2^2 + \alpha^2 \mu + \alpha \tau_2^2 + \beta \tau_2^2 + \omega \tau_2^2)(\tau_2^2 + \alpha \mu)} > 0$$

and

$$\begin{aligned} & \frac{\partial \left(\lim_{\kappa \rightarrow \infty} \frac{\mathbf{b}_1}{\mathbf{e}_1} \right)}{\partial \beta} = \\ & = \frac{\partial \left(\frac{(\alpha + \tau_2^2)\mu}{\alpha + \beta + \omega + \tau_1^2 + \tau_2^2} \right)}{\partial \beta} \frac{\alpha^2 \tau_2^2 + \alpha^3 \mu + \alpha \tau_2^4 + \alpha^2 \mu \tau_1^2 + \alpha^2 \mu \tau_2^2 + \alpha \tau_1^2 \tau_2^2 (2 - \mu) + \beta \tau_1^2 \tau_2^2 + \tau_1^4 \tau_2^2 + \tau_1^2 \tau_2^4 (1 - \mu) + \omega \tau_1^2 \tau_2^2}{(\tau_1^2 \tau_2^2 + \alpha^2 \mu + \alpha \tau_2^2 + \beta \tau_2^2 + \omega \tau_2^2)(\tau_2^2 + \alpha \mu)} + \\ & + \frac{(\alpha + \tau_2^2)\mu}{\alpha + \beta + \omega + \tau_1^2 + \tau_2^2} \frac{\partial \frac{\alpha^2 \tau_2^2 + \alpha^3 \mu + \alpha \tau_2^4 + \alpha^2 \mu \tau_1^2 + \alpha^2 \mu \tau_2^2 + \alpha \tau_1^2 \tau_2^2 (2 - \mu) + \beta \tau_1^2 \tau_2^2 + \tau_1^4 \tau_2^2 + \tau_1^2 \tau_2^4 (1 - \mu) + \omega \tau_1^2 \tau_2^2}{(\tau_1^2 \tau_2^2 + \alpha^2 \mu + \alpha \tau_2^2 + \beta \tau_2^2 + \omega \tau_2^2)(\tau_2^2 + \alpha \mu)}}{\partial \beta} \end{aligned}$$

where

$$\frac{\partial \left(\frac{(\alpha + \tau_2^2)\mu}{\alpha + \beta + \omega + \tau_1^2 + \tau_2^2} \right)}{\partial \beta} = -\mu \frac{\alpha + \tau_2^2}{(\alpha + \beta + \omega + \tau_1^2 + \tau_2^2)^2} < 0$$

and

$$\begin{aligned} & \frac{\partial \frac{\alpha^2 \tau_2^2 + \alpha^3 \mu + \alpha \tau_2^4 + \alpha^2 \mu \tau_1^2 + \alpha^2 \mu \tau_2^2 + \alpha \tau_1^2 \tau_2^2 (2 - \mu) + \beta \tau_1^2 \tau_2^2 + \tau_1^4 \tau_2^2 + \tau_1^2 \tau_2^4 (1 - \mu) + \omega \tau_1^2 \tau_2^2}{(\tau_1^2 \tau_2^2 + \alpha^2 \mu + \alpha \tau_2^2 + \beta \tau_2^2 + \omega \tau_2^2)(\tau_2^2 + \alpha \mu)}}{\partial \beta} = \\ & = -\tau_2^2 (\alpha + \tau_2^2) \frac{\tau_1^2 \tau_2^2 + \alpha^2 \mu + \alpha \tau_2^2 - \mu \tau_1^2 \tau_2^2}{(\tau_2^2 + \alpha \mu) (\tau_1^2 \tau_2^2 + \alpha^2 \mu + \alpha \tau_2^2 + \beta \tau_2^2 + \omega \tau_2^2)^2} < 0. \end{aligned}$$

2.

$$\lim_{\kappa \rightarrow \infty} \frac{\mathbf{a}_1}{\mathbf{e}_1} = \tau_2^2 \frac{1 - \mu}{\alpha + \beta + \omega + \tau_1^2 + \tau_2^2} \frac{\alpha^2 \tau_2^2 + \tau_1^2 \tau_2^4 (1 - \mu) + \tau_1^4 \tau_2^2 + \alpha^3 \mu + \alpha \tau_2^4 + \alpha^2 \mu \tau_1^2 + \alpha^2 \mu \tau_2^2 + \alpha \tau_1^2 \tau_2^2 (2 - \mu) + \beta \tau_1^2 \tau_2^2 + \omega \tau_1^2 \tau_2^2}{(\tau_1^2 \tau_2^2 + \alpha^2 \mu + \alpha \tau_2^2 + \beta \tau_2^2 + \omega \tau_2^2)(\tau_2^2 + \alpha \mu)} > 0$$

and

$$\frac{\partial \lim_{\kappa \rightarrow \infty} \frac{\mathbf{a}_1}{\mathbf{e}_1}}{\partial \beta} = \frac{\partial \tau_2^2 \frac{1-\mu}{\alpha+\beta+\omega+\tau_1^2+\tau_2^2} \frac{\alpha^2 \tau_2^2 + \tau_1^2 \tau_2^4 (1-\mu) + \tau_1^4 \tau_2^2 + \alpha^3 \mu + \alpha \tau_2^4 + \alpha^2 \mu \tau_1^2 + \alpha^2 \mu \tau_2^2 + \alpha \tau_1^2 \tau_2^2 (2-\mu) + \beta \tau_1^2 \tau_2^2 + \omega \tau_1^2 \tau_2^2}{(\tau_1^2 \tau_2^2 + \alpha^2 \mu + \alpha \tau_2^2 + \beta \tau_2^2 + \omega \tau_2^2)(\tau_2^2 + \alpha \mu)}}{\partial \beta} +$$

$$+ \tau_2^2 \frac{1-\mu}{\alpha+\beta+\omega+\tau_1^2+\tau_2^2} \frac{\partial \frac{\alpha^2 \tau_2^2 + \tau_1^2 \tau_2^4 (1-\mu) + \tau_1^4 \tau_2^2 + \alpha^3 \mu + \alpha \tau_2^4 + \alpha^2 \mu \tau_1^2 + \alpha^2 \mu \tau_2^2 + \alpha \tau_1^2 \tau_2^2 (2-\mu) + \beta \tau_1^2 \tau_2^2 + \omega \tau_1^2 \tau_2^2}{(\tau_1^2 \tau_2^2 + \alpha^2 \mu + \alpha \tau_2^2 + \beta \tau_2^2 + \omega \tau_2^2)(\tau_2^2 + \alpha \mu)}}{\partial \beta} < 0$$

3.

$$\lim_{\kappa \rightarrow \infty} \frac{\mathbf{c}_1}{\mathbf{e}_1} = \beta \frac{\tau_2^4 + \tau_1^2 \tau_2^2 + \alpha^2 \mu + 2\alpha \tau_2^2 + \beta \tau_2^2 + \omega \tau_2^2}{(\alpha + \beta + \omega + \tau_1^2 + \tau_2^2)(\tau_1^2 \tau_2^2 + \alpha^2 \mu + \alpha \tau_2^2 + \beta \tau_2^2 + \omega \tau_2^2)}$$

and

$$\frac{\partial \beta \frac{\tau_2^4 + \tau_1^2 \tau_2^2 + \alpha^2 \mu + 2\alpha \tau_2^2 + \beta \tau_2^2 + \omega \tau_2^2}{(\alpha + \beta + \omega + \tau_1^2 + \tau_2^2)(\tau_1^2 \tau_2^2 + \alpha^2 \mu + \alpha \tau_2^2 + \beta \tau_2^2 + \omega \tau_2^2)}}{\partial \beta} > 0.$$

$$\lim_{\kappa \rightarrow \infty} \frac{\mathbf{1}}{\mathbf{e}_1} = \frac{\alpha \gamma (\alpha + \tau_2^2)}{(\alpha + \tau_2^2 - \alpha \mu)(\alpha + \beta + \omega + \tau_1^2 + \tau_2^2)} \frac{\alpha^2 \tau_2^2 + \tau_1^2 \tau_2^4 (1-\mu) + \tau_1^4 \tau_2^2 + \alpha^3 \mu + \alpha \tau_2^4 + \alpha^2 \mu \tau_1^2 + \alpha^2 \mu \tau_2^2 + \alpha \tau_1^2 \tau_2^2 (2-\mu) + \beta \tau_1^2 \tau_2^2 + \omega \tau_1^2 \tau_2^2}{(\tau_1^2 \tau_2^2 + \alpha^2 \mu + \alpha \tau_2^2 + \beta \tau_2^2 + \omega \tau_2^2)}$$

$$\frac{\partial \left(\frac{\alpha^2 \tau_2^2 + \tau_1^2 \tau_2^4 + \tau_1^4 \tau_2^2 + \alpha^3 \mu + \alpha \tau_2^4 + \alpha^2 \mu \tau_1^2 + \alpha^2 \mu \tau_2^2 + 2\alpha \tau_1^2 \tau_2^2 + \beta \tau_1^2 \tau_2^2 - \mu \tau_1^2 \tau_2^4 + \omega \tau_1^2 \tau_2^2 - \alpha \mu \tau_1^2 \tau_2^2}{(\tau_1^2 \tau_2^2 + \alpha^2 \mu + \alpha \tau_2^2 + \beta \tau_2^2 + \omega \tau_2^2)} \right)}{\partial \beta} =$$

$$- \tau_2^2 (\alpha + \tau_2^2) \frac{\alpha^2 \mu + \alpha \tau_2^2 + (1-\mu) \tau_1^2 \tau_2^2}{(\tau_1^2 \tau_2^2 + \alpha^2 \mu + \alpha \tau_2^2 + \beta \tau_2^2 + \omega \tau_2^2)^2} < 0$$

$$\frac{\partial \left(\frac{\alpha \gamma (\alpha + \tau_2^2)}{(\alpha + \tau_2^2 - \alpha \mu)(\alpha + \beta + \omega + \tau_1^2 + \tau_2^2)} \right)}{\partial \beta} =$$

$$- \alpha \gamma \frac{\alpha + \tau_2^2}{(\alpha + \tau_2^2 - \alpha \mu)(\alpha + \beta + \omega + \tau_1^2 + \tau_2^2)^2} < 0.$$

4. it is a direct consequence of statements 1,2 and 4.

B Appendix: Figures

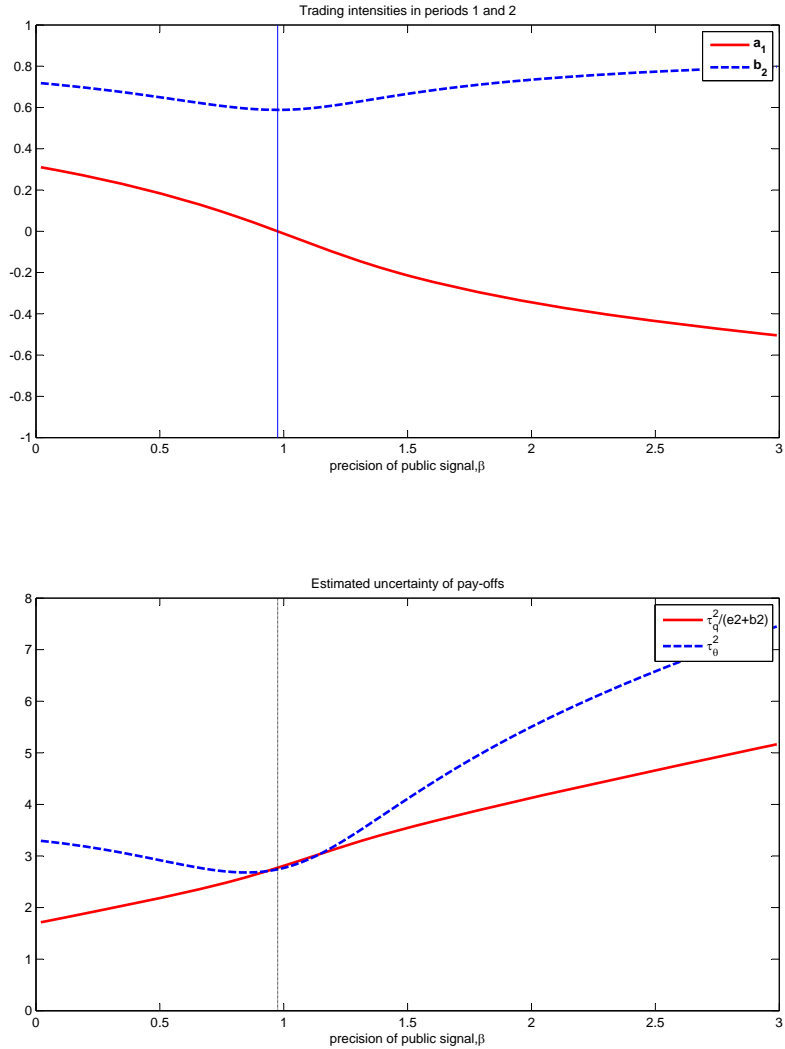


Figure 1: Trading intensities and estimated pay-off uncertainty in Case 1. The top panel shows trading intensities $\mathbf{a}_1, \mathbf{b}_2$, in period 1 and 2. The bottom panel shows A-traders' estimated precision of second period price, $\frac{\tau_q^2}{e_2+b_2}$, and B-traders' estimated precision of the fundamental, τ_θ^2 . The x-axis is the precision of the public signal, β . The vertical line depicts $\beta = \frac{\kappa^2}{\omega}$, the threshold above which second-order expectations are polarized by more public information.

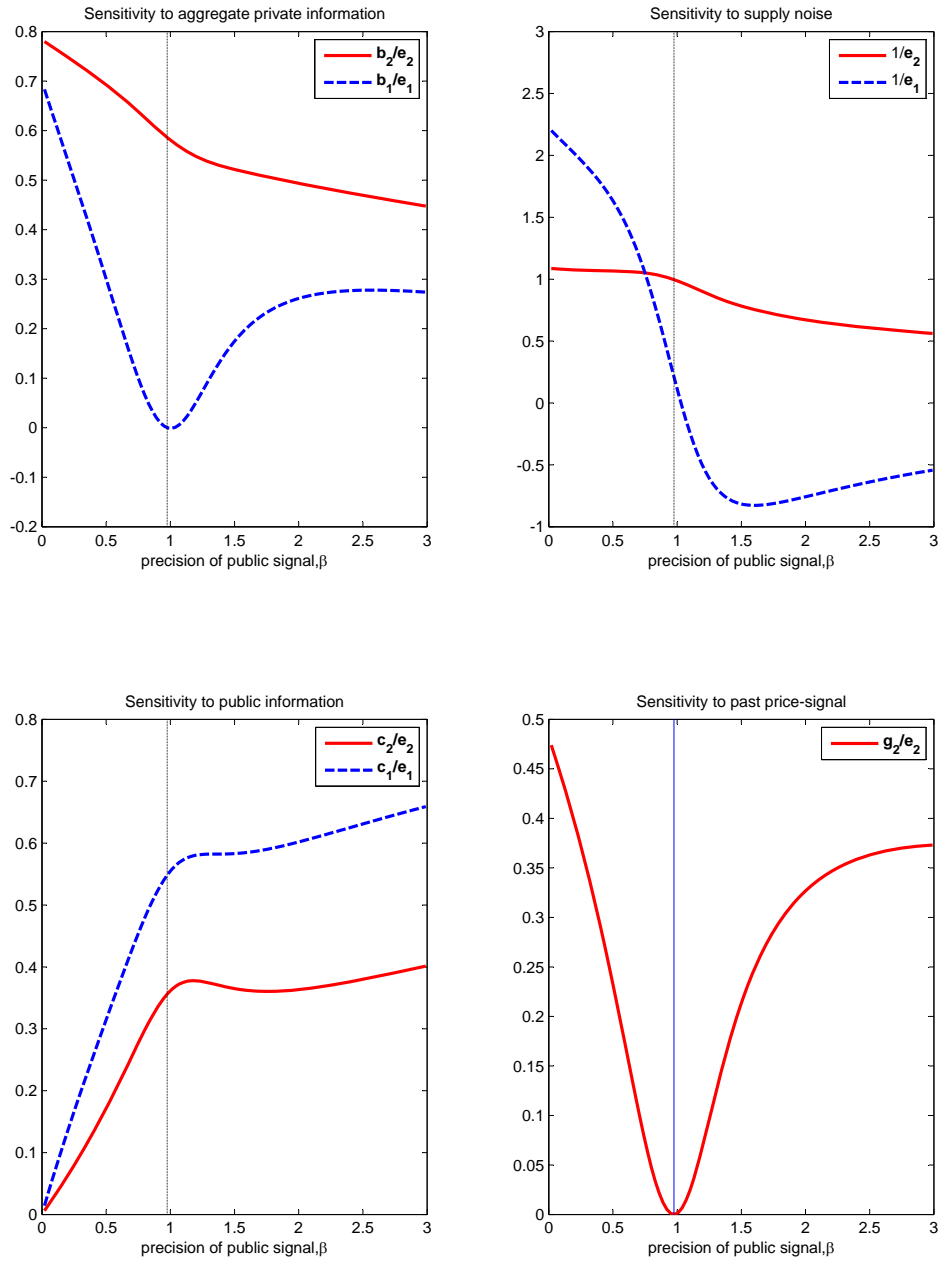


Figure 2: Coefficients of the price function in Case 1. Each panel shows a given coefficient of the price function in periods 1 and 2. The x-axis is the precision of the public signal, β . The vertical line depicts $\beta = \frac{\kappa^2}{\omega}$, the threshold above which second-order expectations are polarized by more public information.

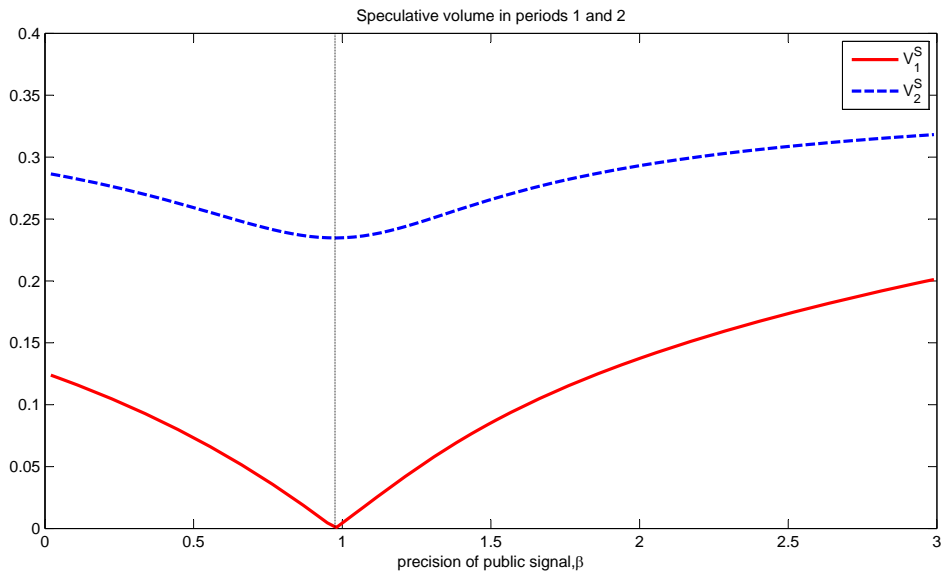
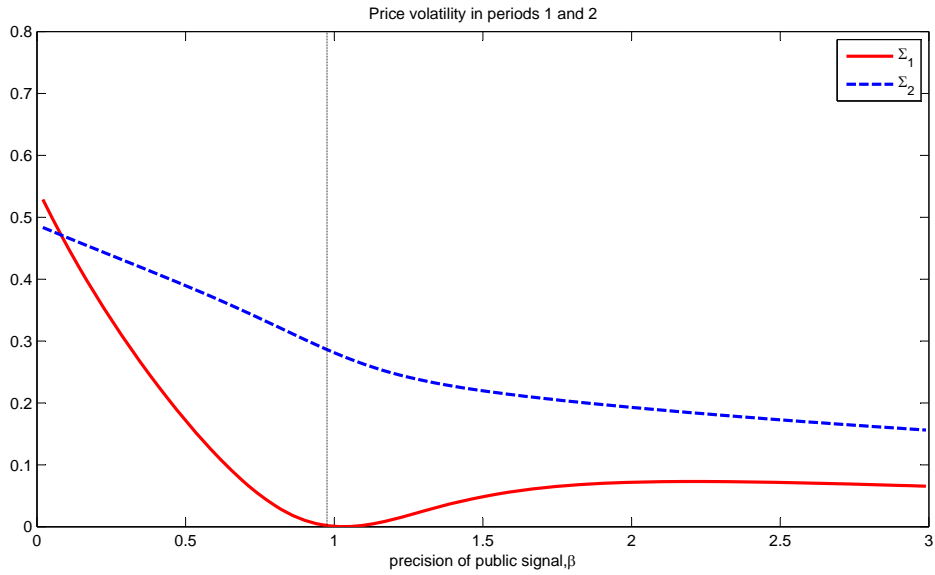


Figure 3: Price volatility and speculative volume in Case 1. The top panel shows the volatility of prices in period 1 and 2. The bottom panel shows speculative volume (for each standard deviation unit of private signal noise $\frac{1}{\sqrt{\alpha}}$) in period 1 and 2. The x-axis is the precision of the public signal, β . The vertical line depicts $\beta = \frac{\kappa^2}{\omega}$, the threshold above which second-order expectations are polarized by more public information.

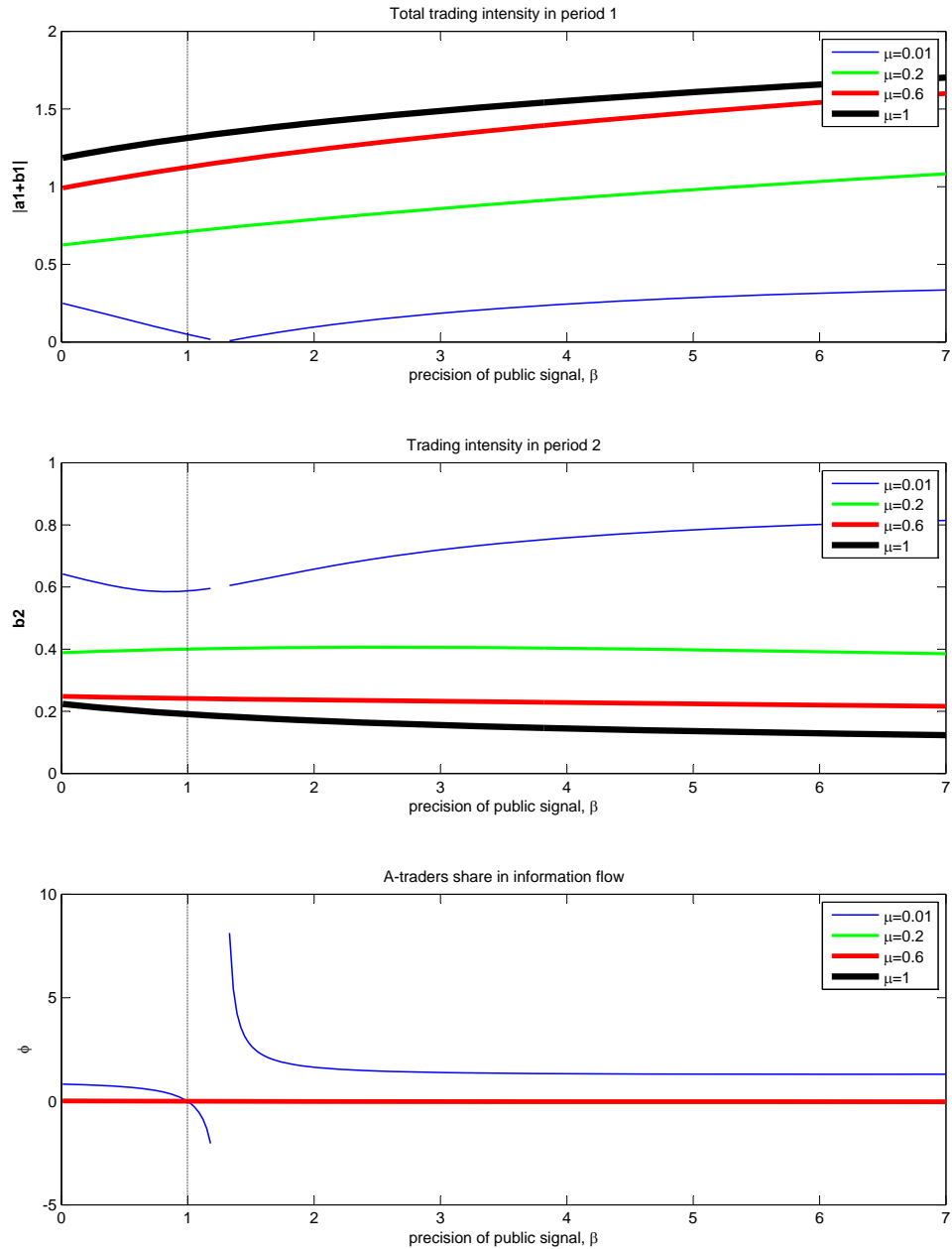


Figure 4: Trading intensities in Case 2. From top to bottom, panels show the total trading intensity in period 1 and 2, $|a_1 + b_1|, b_2$ and the share of A-traders private information in all the private information incorporated in first period prices, ϕ . In each plot, different curves correspond to different fraction of B-traders on the market, μ . The thicker the curve, the larger the fraction. The x-axis is the precision of the public signal, β . The vertical line depicts $\beta = \frac{\kappa^2}{\omega}$, the threshold above which second-order expectations are polarized by more public information.

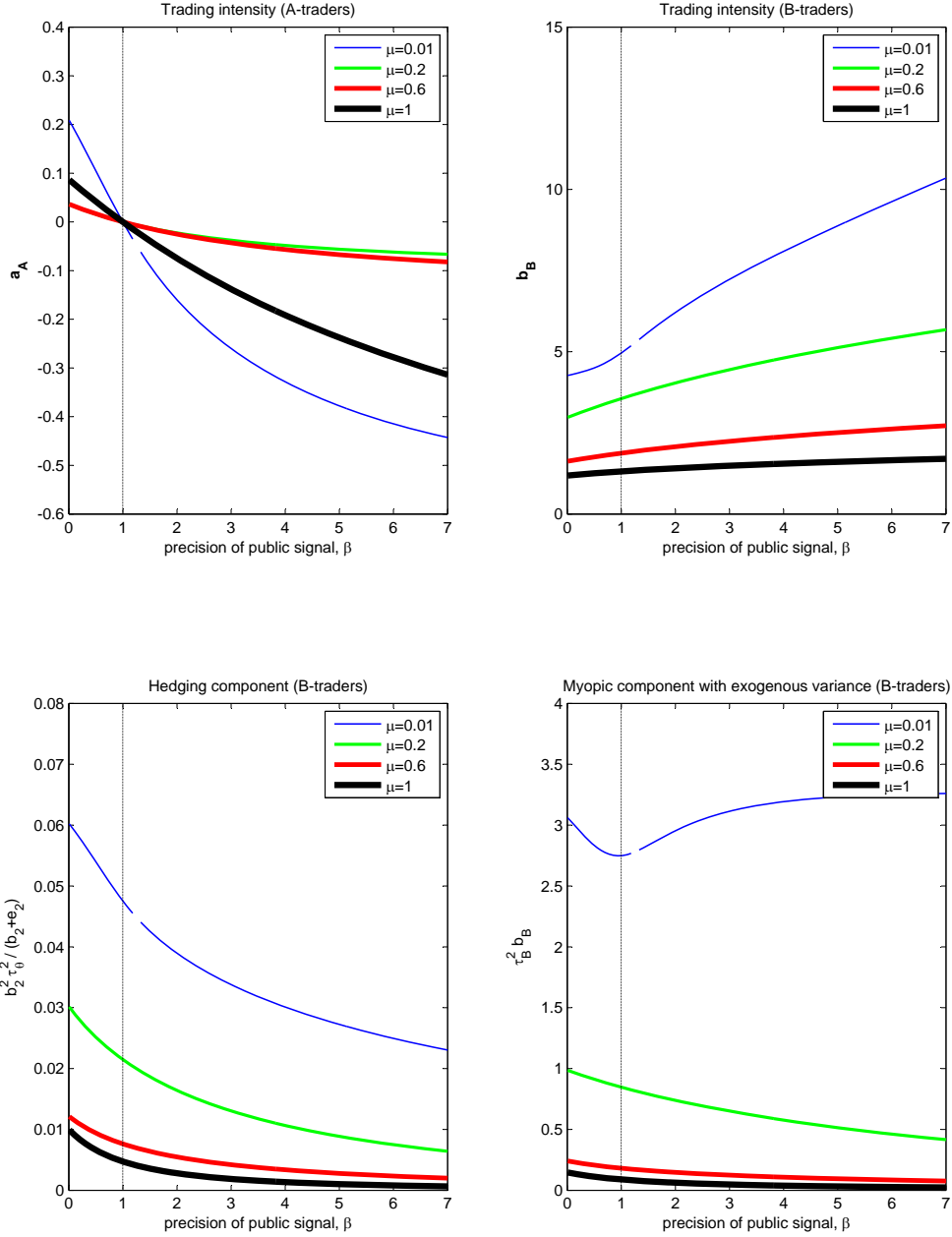


Figure 5: Trading intensity of different groups in period 1 in Case 2. The top panels show the trading intensities of A-traders and B-traders in period 1, \mathbf{a}_A , \mathbf{b}_B . The bottom panel shows the decomposition of \mathbf{b}_B . The left and right panel show the intensity corresponding to the hedging component and myopic component respectively. In each plot, different curves correspond to different fraction of B-traders on the market, μ . The thicker the curve, the larger the fraction. The x-axis is the precision of the public signal, β . The vertical line depicts $\beta = \frac{\kappa^2}{\omega}$, the threshold above which second-order expectations are polarized by more public information.

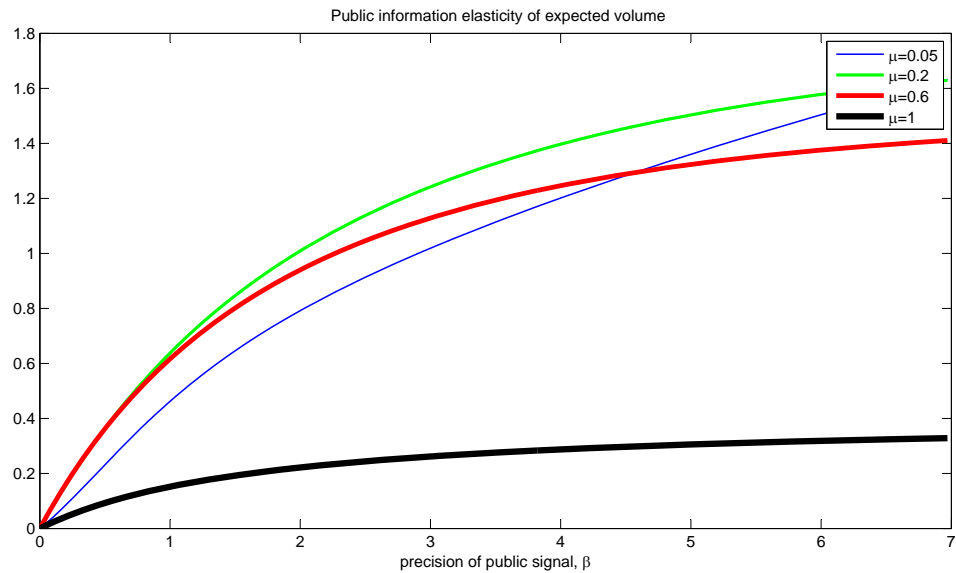
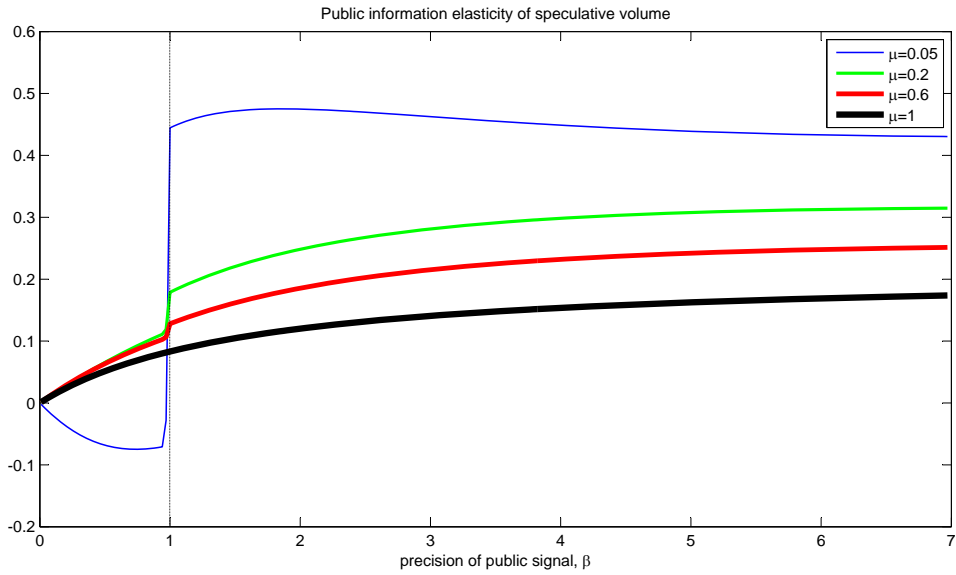


Figure 6: Elasticity of volume in period 1 in Case 2. The top and bottom panels show the elasticity of speculative and expected volume with respect to the precision of public information, respectively. In each plot, different curves correspond to different fractions of B-traders on the market, μ . The thicker the curve, the larger the fraction. The x-axis is the precision of the public signal, β . The vertical line depicts $\beta = \frac{\kappa^2}{\omega}$, the threshold above which second-order expectations are polarized by more public information.

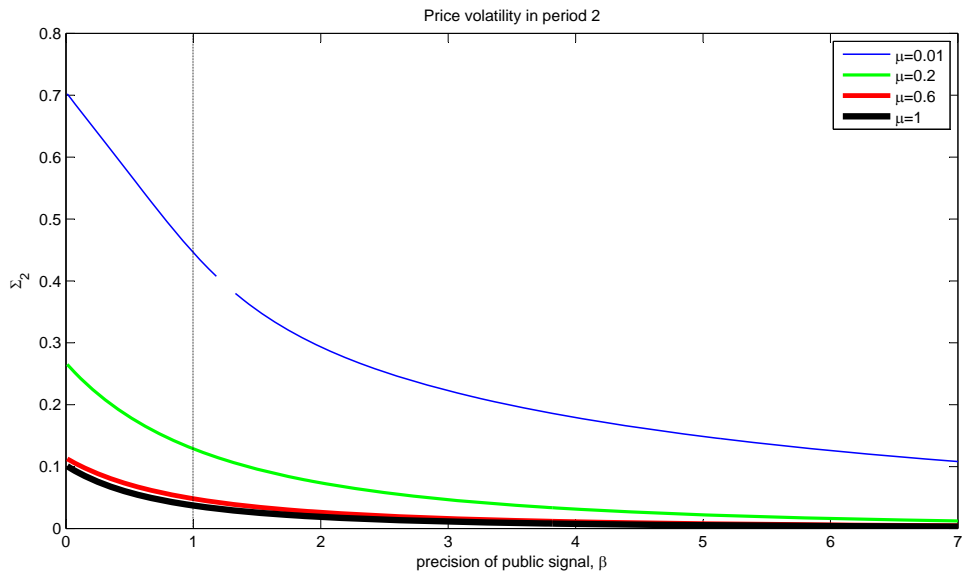
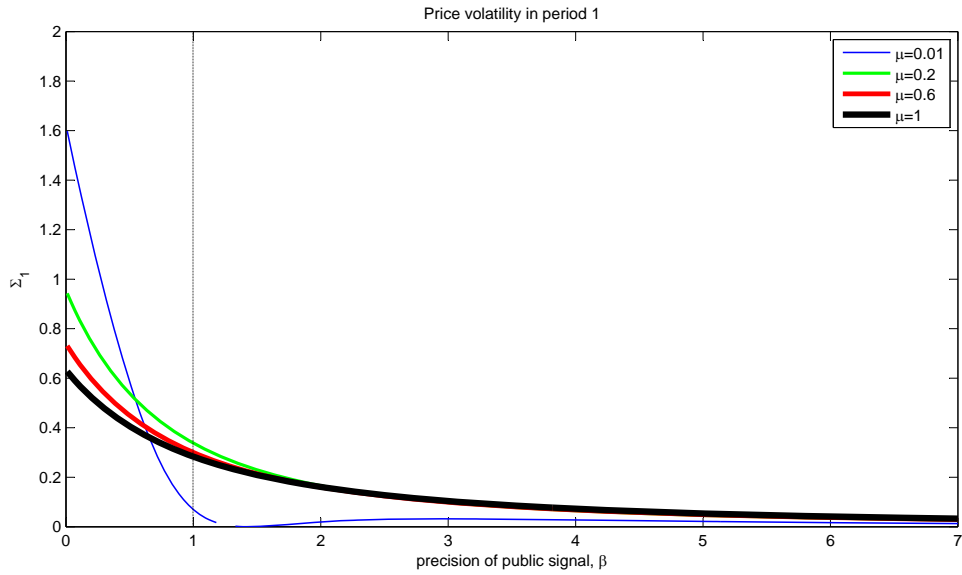


Figure 7: Price volatility in period 1 and 2 in Case 2. In each plot, different curves correspond to different fraction of B-traders on the market, μ . The thicker the curve, the larger the fraction. The x-axis is the precision of the public signal, β . The vertical line depicts $\beta = \frac{\kappa^2}{\omega}$, the threshold above which second-order expectations are polarized by more public information.

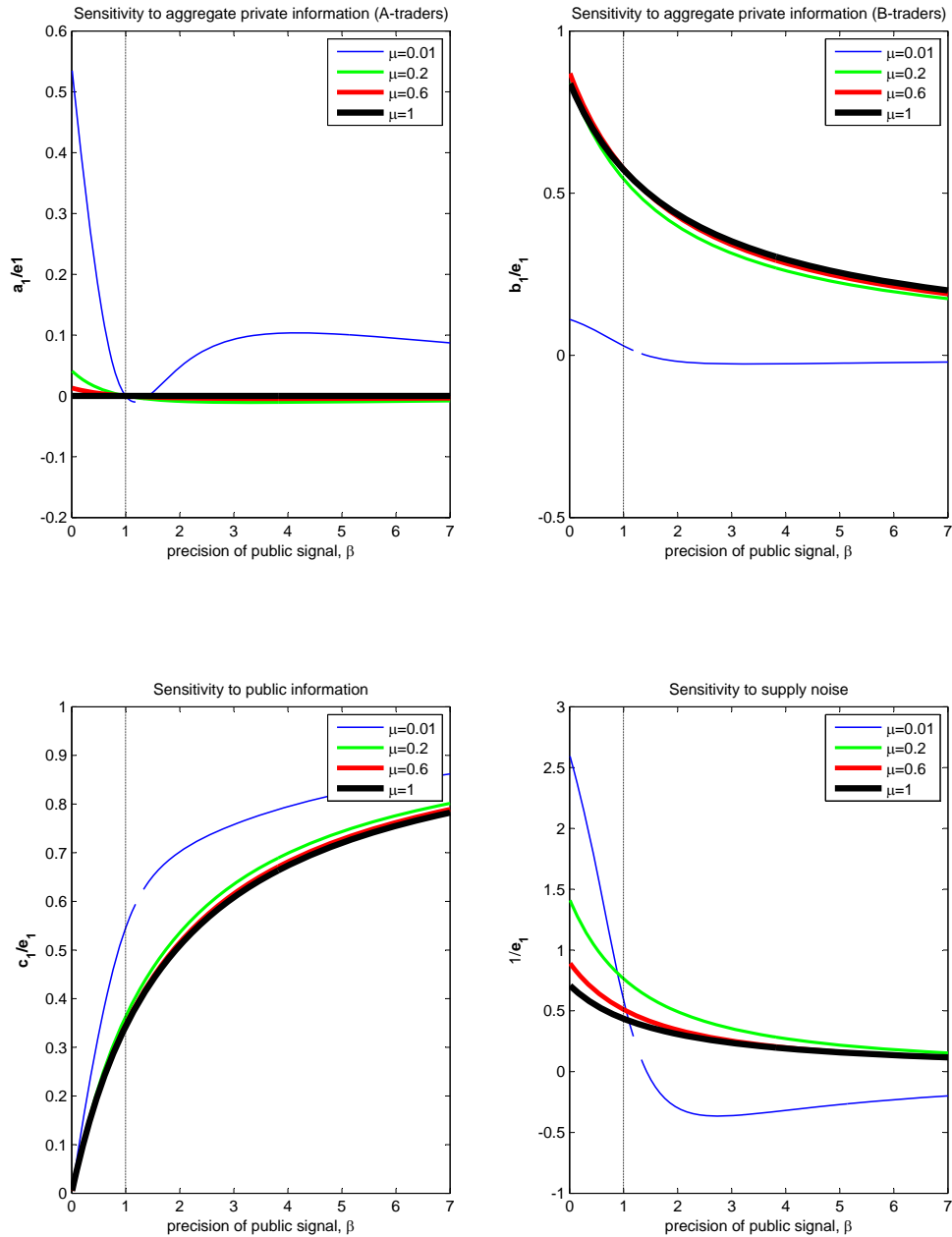


Figure 8: Coefficients of the price function in period 1 in Case 2. Each panel shows a given coefficient of the price function. In each plot, different curves correspond to different fraction of B-traders on the market, μ . The thicker the curve, the larger the fraction. The x-axis is the precision of the public signal, β . The vertical line depicts $\beta = \frac{\kappa^2}{\omega}$, the threshold above which second-order expectations are polarized by more public information.

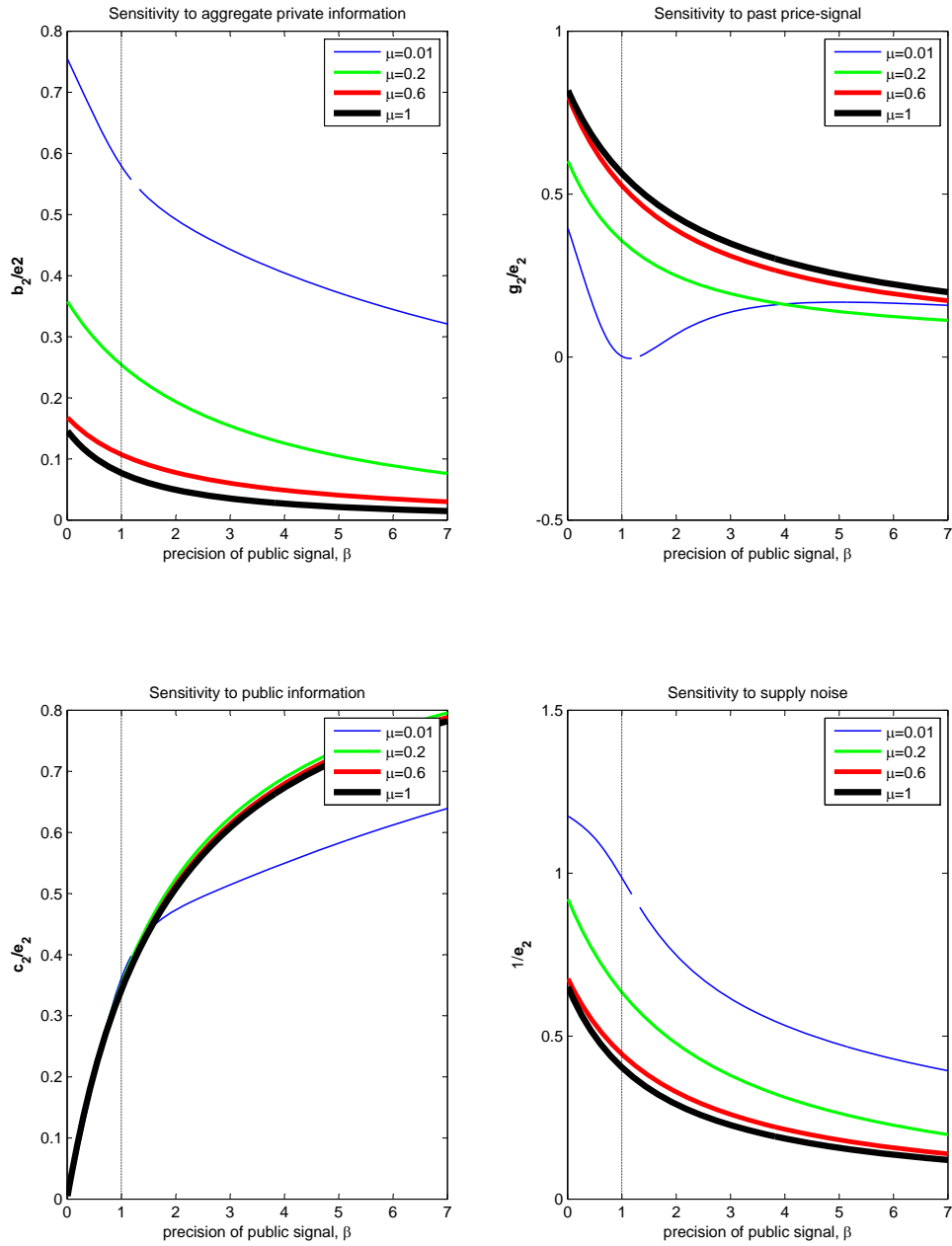


Figure 9: Coefficients of the price function in period 2 in Case 2. Each panel shows a given coefficient of the price function. In each plot, different curves correspond to different fraction of B-traders on the market, μ . The thicker the curve, the larger the fraction. The x-axis is the precision of the public signal, β . The vertical line depicts $\beta = \frac{\kappa^2}{\omega}$, the threshold above which second-order expectations are polarized by more public information.