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Veronica Guerrieri and Péter Kondor

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Veronica Guerrieri, University of Chicago and NBER
Péter Kondor, Central European University and CEPR

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ABSTRACT

Fund Managers, Career Concerns, and Asset Price Volatility*

We propose a model of delegated portfolio management with career concerns. Investors hire fund managers to invest their capital either in risky bonds or in riskless assets. Some managers have superior information on the default risk. Looking at the past performance, investors update beliefs on their managers and make firing decisions. This leads to career concerns which a¤ect investment decisions, generating a counter-cyclical 'reputational premium.' When default risk is high, the bond's return is high to compensate uninformed managers for the high risk of being fired. As default risk changes over time, the reputational premium amplifies price volatility.

Veronica Guerrieri
University of Chicago, GSB
5807 South Woodlawn Avenue
Chicago, IL 60637
USA

Péter Kondor
Department of Economics
Central European University
Nador u. 9
Budapest, H-1051
HUNGARY

Email:
Veronica.Guerrieri@chicagogsb.edu

Email:
kondorp@ceu.hu

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1 Introduction

The financial turmoil of 2007-2008 has fueled a lively debate on asset price volatility and the role of financial intermediaries. In the years before the crisis, a number of market observers were concerned about a growing overenthusiasm for risky investments in debt instruments, including high-yield corporate bonds, mortgage-backed assets and emerging market bonds. Financial intermediaries and their incentives have received growing attention to explain these types of episodes. One observer comments on the bond-financed leveraged buy-out boom in 2005:

> The head of one of the biggest commercial lenders in the US describes the amount of leverage on some buy-out deals as "nutty". Much of the wildest lending is being done by hedge funds awash with cash, he says. "Some funds believe they have to invest the money even if it's not a smart investment. They think the people that gave them the money expect them to invest it. But it's madness." (March 14, 2005, Financial Times)

In this paper, we develop a general equilibrium model of portfolio management where fund managers have career concerns. We show that managers' career concerns distort their investment decisions and magnify asset price volatility.

Figure 1 shows the pattern of the yield spreads of AAA and B-graded corporate bonds, BB-graded commercial mortgage-backed assets, and a sample of emerging market bonds between October 1994 and May 2011. The figure shows two periods in which all spreads shrunk to very low levels, close to the AAA corporate spreads: in 1996-1997 and then again from 2005 to the summer of 2007. These periods have been often referred to as periods of overenthusiasm preceding the emergence of a crisis (e.g. Kamin and von Kleist, 1999, Duffie et al., 2003). The figure also shows four episodes of high turbulence in which the spreads of many high-risk bonds jump up and capital tends to flow out of these markets, a phenomenon dubbed as flight-to-quality. Our model explores the role of career concerns in explaining both episodes of apparent overenthusiasm and episodes of flight-to-quality.

We consider a model where investors delegate their portfolio decision to risk-neutral fund managers. Fund managers can invest either in a risky asset or in a riskless one. For concreteness, we assume the risky asset is a bond subject to default. Some managers –the "informed managers"– have superior information about the realization of the default state. Investors would prefer to delegate their portfolio decision to informed managers, but the managers' type is private information. Every period, each investor has a manager working for him. At the end of the period, the investor updates his belief about the manager's type
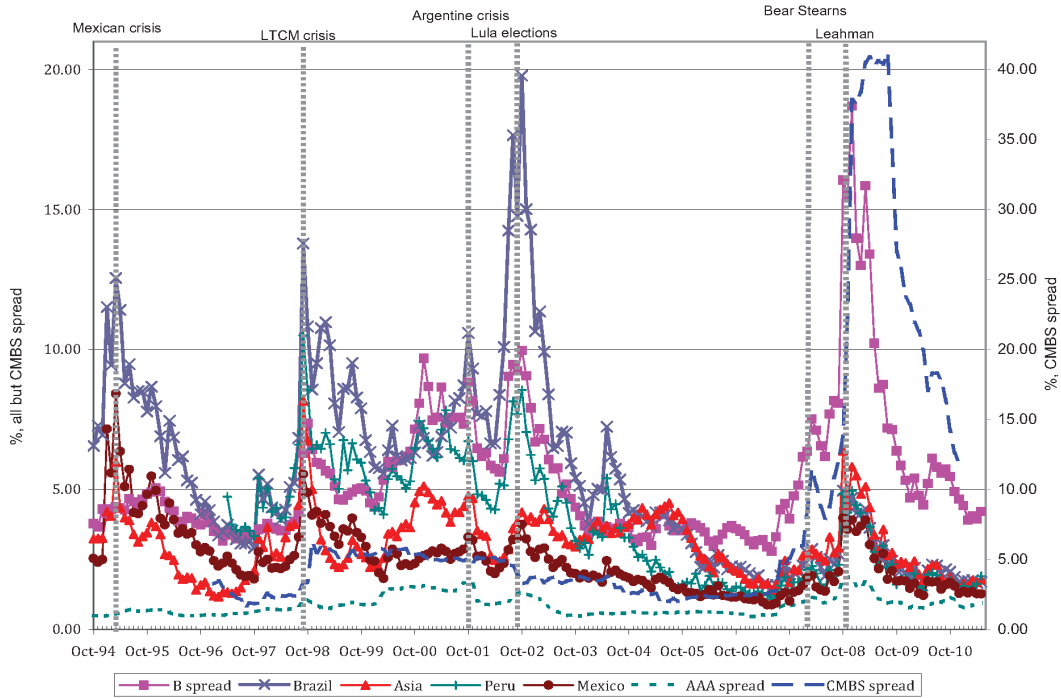
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Figure 1: The JPMorgan EMBI+ spread for Asia, Brazil, Bulgaria, Mexico, Peru, the yield spread of AAA corporate bonds and an index of the yield spread of B-graded corporate bonds and BB-graded comercial mortgage-backed assets (with scale on the right side of the figure) between October 1994 and April 2010. Source: Datastream, St. Louis Fed.

based on his performance, and decides whether to retain him or to hire a new one. The investors' firing decision distorts the investment decision of uninformed managers who would like to be perceived as informed.

The main result of the paper is that managers' career concerns generate a premium on the return of risky bonds, which may be positive or negative depending on the default probability. Uninformed fund managers are concerned that the realized returns of their investment hurt their reputation. A default event hurts the reputation of uninformed managers who invested in the risky bond, and a no default event hurts the reputation of uninformed managers who invested in the riskless asset. Thus, when the default risk is high, the premium for investing in the risky bond is positive to compensate for the risk of being fired. When instead the default risk is low, the risky bond trades at a negative premium. As the default risk changes stochastically over time, the reputational premium amplifies the bond price volatility, relative to an economy with no career concerns.

We also explore a more general version of the model, where we allow for persistent default risk. In this case, career concerns have additional effects on asset price volatility. First, the

reputational premium is magnified when it is positive, that is, when default risk is high, and dampened when it is negative, that is, when default risk is low. Moreover, there is an additional source of volatility in asset price dynamics, driven by the labor market. The more employed managers are informed, the higher is the informational content of future prices, which increase the expected utility of employed uninformed managers. This makes reputation more valuable and increases the distorting effect of career concerns. This also implies that asset price movements may not driven only by changes in fundamentals.

It is well known that the premium on risky assets (the difference between their expected return and the riskless return) is time-varying and, in particular, lower in good times than in bad times. Common explanations appeal to time-varying marginal utility of consumption due to habit formation, time-varying probability of disasters or slow-moving component in consumption risk.[1] In our model the premium changes, because of the time-varying risk for a manager of being fired. In particular, because of career concerns, fund managers rationally undervalue cash-flows connected to small probability states and bid up, above fundamental, the price of assets with left skewed distribution, that is, assets with a "crash risk".[2] This is in contrast to consumption-based explanations where the premium is always positive and is broadly consistent with several empirical observations in different markets.

Perhaps the best example is the observed mispricing of senior trenches of collateralized debt obligations (CDOs) in the period before the recent financial crisis. Coval, Jurek and Stafford (2008) argue that between 2004 and September of 2007 these assets provided too little compensation for risk compared to portfolios of securities of the same pay-off structure. Senior CDO trenches are subject to a crash risk by construction as these assets deliver higher returns than Treasuries in most states at the expense of significant losses in economic crises. As CDOs are traded only by institutional investors, our mechanism is a good candidate to explain this phenomenon.

On a similar line, Brunnermeier and Nagel (2004) argue that hedge funds were investing heavily in technological stocks during the dotcom bubble, although they seemed aware of the mispricing. It is common to believe that during the bubble the short-term return of technological stocks were subject to crash risk. The probability that the bubble was going to collapse during that next day or month was perceived to be small, even if fund managers were sure it would eventually collapse. Our model suggests that hedge funds were willing to buy technological stocks at highly inflated prices because of their fear of losing reputation (and hence funds) if they missed the high returns generated by the bubble. This is consistent with

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[1]See Cochrane (2006) for a detailed review.

[2]The price distortion in our model is similar to the behavioral bias explored in Gennaioli, Shleifer and Vishny (2010), where investors neglect small risks. However, our mechanism relies on rational expectations.

the additional fact, reported in Brunnermeier and Nagel (2004), that the largest hedge fund, Tiger Fund, which refused to invest in technology stocks, experienced severe fund outflows in 1999 compared to its main competitor who did invest in technology stocks, Quantum Fund.

Literature review. Our paper is related to models with career concerns, such as Scharfstein and Stein (1990), Zwiebel (1995), and Ottaviani and Sorensen (2006). These papers are close to our work because career concerns distort managers' decisions to convince their clients that they are informed. However, in contrast to our paper, their main mechanism is based on herding behavior. In these papers, each agent herds on others' decisions because going against the average action is a bad signal about his ability. In our model, in equilibrium, fund managers make their investment decisions regardless of other managers' decision. That is, there are no strategic complementarities. Moreover, this literature typically concentrates on partial equilibrium models while our focus is on the interaction of career concerns and asset prices. A notable exception is Rajan (1994), who shows that herding may motivate bank executives to overextend credit in good times, hence amplifying the effect of real shocks. However, in bad times banks provide the right amount of credit and there is no amplification effect. In contrast, in our paper career concerns always generate an amplification effect, given that when the default risk is high (bad times) managers are worried of being fired and underinvest in the risky bonds.

There is also a growing literature which analyzes the effect of delegated portfolio management on asset prices, such as Allen and Gorton (1993), Shleifer and Vishny (1997), Vayanos (2003), Cuoco and Kaniel (2010), Kaniel and Kondor (2011).[3] However, all these papers take managers' distorted incentives as given. Two papers more related to ours are Dasgupta and Prat (2008) and Dasgupta, Prat and Verardo (2011) who introduce reputational concerns into a Glosten and Milgrom (1985) type of sequential trading model.[4] They show that reputational concerns can lead to excessive trading, slow revelation of information and (if the market maker has market power) biased prices. They are the first to use the term reputational premium and to point out that reputation may lead managers to choose bets with negative net present value. However, both these papers take the reputational concerns as exogenously given and the mechanism behind the reputational premium is based on herding behavior. Also, they do not explore the effect of reputation on asset price volatility, which is the focus of our paper. Dasgupta and Prat (2006) is the first paper to microfound reputational concerns of fund managers who are afraid to be fired. However, the mechanism is

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[3]See Bhattacharya et al. (2008) for a survey on fund managers and incentives.

[4]More recently, Vayanos and Woolley (2008) show that learning about managerial ability can explain momentum and reversal, while Malliaris and Yan (2010) connect reputational concerns of hedge fund managers to the skewness of their returns and slow moving capital.

again based on herding and the paper does not explore asset price volatility. To the best of our knowledge, He and Krishnamurthy (2008, 2011) are the only papers that connect portfolio delegation and asset price volatility. In contrast to our work, the distortion arising here magnifies asset prices reaction only to bad shocks. The mechanism in these papers is based on the design of optimal contracts to face moral hazard issues in the relationship between investors and managers. We view this mechanism as complementary to career concerns.

Our paper is also related to a large literature on the propagation and amplification of fundamental shocks due to the interaction between asset values and collateralized lending. Seminal papers in this area are Bernanke and Gertler (1989), Kiyotaki and Moore (1997), and Gromb and Vayanos (2002)[5]. The main difference with our mechanism is again that these papers have typically an asymmetric distortion, given that collateral constraints build into the model an external finance premium, usually generating underinvestment. In our paper, instead, we microfound the financial distortion, generating a premium that can be either positive or negative.

The rest of the paper is organized as follows. In Section 2, we introduce an example to illustrate the main mechanism of our model. In Section 3, we describe the environment. In Section 4, we define and characterize a stationary equilibrium. We also show our main amplification result. In Section 5, we explore a more general version of the model with persistent default risk and show some numerical exercises. Finally, Section 6 concludes. The Appendix includes all the proofs which are not in the text.

2   An Example

In this section, we introduce a simple example to present the main mechanism of the model, that is, how prices may be distorted by the career concerns of fund managers.

There is a large group of risk-neutral fund managers who have to decide whether to invest a unit of capital in a riskless asset or in a risky bond. The risky bond has price p, and pays 1 if there is no default and 0 if there is default.[6] The probability of default is equal to q. The riskless asset pays the safe return $R < 1/p$. The riskless asset is in infinite supply, while the supply of the risky bonds is fixed and smaller than the total capital invested by the managers. Assume that a manager gets a fraction γ of his investment returns and obtains

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[5]See also Aghion, Banerjee and Piketty (1999), Rampini (2004), Krishnamurthy (2003), Danielsson, Shin and Zigrand (2004), Morris and Shin (2004), Bernardo and Welch (2004), Gai, Kondor and Vause (2005), and Guerrieri and Lorenzoni (2009).

[6]All the arguments would go through if we had a more general risky asset. In that case, the event of default would be the analog of a bad state when the asset's return is below its expected value, and the no-default event would be the analog of a good state when the asset's return is above its expected value.

a constant reward W if and only if his investment is successful, that is, if he invests in the risky bond when there is no default and in the riskless asset when there is default. This reward scheme can be interpreted as the reduced form of our baseline model (presented in the next section), where unsuccessful managers are fired and get zero continuation utility, while successful managers are retained and W is their continuation utility.

The bond market clears if and only if managers are indifferent between investing in the risky bond and in the riskless asset. Hence, the equilibrium price of the risky bond has to satisfy the following indifference condition:[7]

$$(1 - q)(\gamma/p + W) = \gamma R + qW. \tag{1}$$

The left-hand side of equation (1) represents the expected payoff of a manager who invests in the risky bond. With probability $1 - q$ there is no default and the manager gets a return γ/p and the reward W. If instead there is default, the manager gets zero revenues and no reward. Similarly, the right-hand side of equation (1) represents the expected payoff of a manager who invests in the riskless asset. He gets always a return γR, but he obtains the reward only if there is default.[8]

We are now ready to characterize the price distortion generated by the reward W on the bond price. Define the *premium* Π as the difference between the expected return on the risky bond and the risk free rate

$$\Pi \equiv \frac{1 - q}{p} - R.$$

Condition (1) immediately implies that $\Pi = 0$ when the reward is zero, that is, $W = 0$. In this case, fund managers care only about the expected returns of the bond and the premium is zero. When instead $W > 0$, the premium can be negative or positive. In particular, if $q > 1/2$, the payoff of the risky bond is skewed to the left as the probability of default is larger than the probability of no default. In this case, investing in the riskless asset has an advantage over the risky bond as it ensures the reward payment with larger probability. If the expected return of the two assets were equal, all managers would prefer the riskless one, because of this advantage. Thus, in equilibrium there must be a positive premium on risky bonds to induce managers to hold them. Similarly, if $q < 1/2$ the payoff of the risky bond is skewed to the right. In this case, the risky bond has an advantage and the premium is negative.

In the rest of the paper, we build a general equilibrium model of delegated portfolio management where the reward scheme proposed above is endogenous and generates career

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[7]This indifference condition is the analog of condition (8) for the baseline model.
[8]The equlibrium price is consistent with the assumption that $1/p > R$ if $R > W(1 - 2q)/q$.

concerns. To this end, we need both a dynamic environment and some form of heterogeneous information. Investors need fund managers to manage their capital, but managers may be more or less informed about the bond's default risk and their type is private information. Based on their investment performance, investors learn about their type and make their firing decision. This model generates an incentive scheme similar to the example, where W is an equilibrium object equal to the discounted expected utility of an uninformed manager who retains his job. In particular, uninformed managers' career concerns generate a reputational premium analogous to the one described above. We show that the presence of such a reputational premium magnifies the volatility of asset prices, when the default risk is time-varying.

Once we move to a fully fledged model, we need to take care of a number of additional steps. The most important is to make sure that the bond price is not always fully revealing, otherwise the investor would attach no value to having an informed manager and the reputational premium would vanish.

3 Model

Consider an infinite-horizon economy, in discrete time, populated by three groups of agents: investors, fund managers, and borrowers. Each period t is divided in morning and afternoon. There is a continuum of mass one of infinitely lived, risk-neutral investors who discount future payoffs at the rate β. Investors are endowed with 1 unit of consumption goods in the morning which can be invested in two ways. It can either be invested in a safe asset that pays a rate of return $R > 1$ in the afternoon, or it can be used to buy bonds issued by the borrowers at the price p_t. In the afternoon, the borrowers either repay their debt or default according to an aggregate shock χ_t: if $\chi_t = 1$ all borrowers repay, if $\chi_t = 0$ all borrowers default.[9] The probability of default $q_t = E_t[\chi_t]$ is a random variable drawn independently at the beginning of each period from the cumulative distribution function $F(q)$ with support $[\underline{q}, \overline{q}]$.

Investors cannot invest their endowment on their own, but need to employ fund managers. Fund managers are also infinitely lived, risk neutral and have discount factor β. Each investor can only employ one fund manager and each fund manager can only work for a single investor. For simplicity, we fix the contract between investors and fund managers and assume that fund managers keep a share γ of the returns and leave the rest to the investors. We assume that investors and fund managers can only consume in the afternoon and that there is no technology to transfer consumption across periods. This drastically simplifies the investors' and fund managers' behavior, given that there is no saving decision: in the morning the

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[9] In the working paper version, Guerrieri and Kondor (2009), the default decision is modeled endogenously.

investors' endowment is fully invested, in the afternoon the returns are fully consumed.

There are two types of fund managers: informed (I) and uninformed (U). There is a mass M^I of informed managers and a large continuum of uninformed managers. The manager's type is private information. Informed managers receive in the morning a perfect signal about default, that is, they observe χ_t.[10] All other agents, in the morning, can only observe the probability of default q_t. In the afternoon, after the default event is publicly revealed and investment returns are realized, the investor decides whether to retain his manager or to fire him and hire a new one.

Each period, there is a mass b_t of one-period-lived borrowers. The borrowers' behavior is mechanical. In the morning, they supply bonds inelastically to finance 1 unit of consumption. In the afternoon they repay if and only if $\chi_t = 1$. The nominal bond supply b_t is a random variable, drawn independently each period from a uniform distribution on $[\underline{b}, \overline{b}]$. The realization of b_t is not observed by investors and fund managers. Its role is to ensure that the bond market price is not always fully revealing.

To complete the description of the environment we need to specify the structure of the bond market and of the labor market.

Bond Market. In the morning of period t, each manager submits a demand schedule for bonds to an auctioneer. For simplicity, we restrict managers to three choices: invest their whole endowment in bonds, $d = 1$, invest zero in bonds, $d = 0$, or declare indifference between the two, $d = \{0, 1\}$. A demand schedule is a map $d : \mathcal{R}_+ \to \{0, 1, \{0, 1\}\}$ which for any price $p \geq 0$ gives the manager's demand $d(p)$. The auctioneer collects all the demand schedules, selects the equilibrium price and assigns the bonds to the managers. In particular, if p is the equilibrium price, managers with demand $d(p) = 1$ receive $1/p$ bonds, managers with $d(p) = 0$ receive no bonds, and managers with demand $d(p) = \{0, 1\}$ are selected randomly to receive 0 or $1/p$ bonds so as to clear the market. Let $\theta_{i,t} \in \{0, 1\}$ denote the realized investment in the risky bond for manager i at time t, that is, $\theta_{i,t} = 1$ if manager i gets $1/p$ bonds and $\theta_{i,t} = 0$ if he gets zero bonds.

Labor Market. In the afternoon, an investor who employs manager i, observes his realized investment $\theta_{i,t}$ and the default realization χ_t. At the same time, the investor receives an

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[10]The extreme assumption that informed managers have a perfect signal is not crucial for our mechanism. However, it simplifies the analysis of investors' beliefs, making the model more tractable. Also, our mechanism would go through if both types of investors receive some information, with some better informed than the others. One simple extension of our information structure that would keep the analysis close to ours is the following: some managers have a perfect signal, while others may either have a perfect signal or a completely uninformed signal. In this case, the less informed managers who obtain the perfect signal would behave as the better informed ones, while the marginal traders would be the less informed managers with uninformative signal.

additional exogenous signal of the manager's type. This signal is denoted by $\sigma_{i,t}$, for manager i at time t, and can take two values, 0 or 1. If the manager is informed the signal is always $\sigma_{i,t} = 0$. If the manager is uninformed, the signal is $\sigma_{i,t} = 0$ with probability ω and $\sigma_{i,t} = 1$ with probability $1 - \omega$. Therefore, with probability $1 - \omega$ the type of the uninformed manager is perfectly revealed. In the equilibrium analysis, we will clarify that the introduction of this exogenous signal is useful to ensure the existence of a stationary equilibrium and hence makes the analysis more tractable. Finally, given all the information available, the investor updates his beliefs about the manager's type and chooses whether to retain him or fire him and hire a new one. Also, in the afternoon any investor-manager match is exogenously terminated with probability $1 - \delta$. This ensures that the pool of unemployed managers always contains informed managers.

At the end of the afternoon, each investors who is not matched to a manager—either because he fired him or because of exogenous termination—searches for one. At the same time, unemployed managers choose either to pay a cost κ to look for a job or to stay inactive. Then matching takes place. The matching technology is Leontief: given A searching investors and Z unemployed managers looking for a job, the number of matches created is $\min\{A, Z\}$. Therefore, the probability of being matched is $\min\{A, Z\}/Z$ for investors and $\min\{A, Z\}/A$ for managers. Our assumptions ensure that, in equilibrium, investors are always on the short side of the market, that is, $A < Z$, so that investors are always matched with probability 1.

Given that there is a continuum of managers, an investor will never meet the same manager twice. Moreover, we assume that an investor can only observe the trading history of the manager he employs. Therefore, from the point of view of an investor, all newly employed managers are equivalent and the probability that a newly employed manager is informed is equal to the fraction of informed managers in the unemployed pool.

The timeline below summarizes the timing of the model.
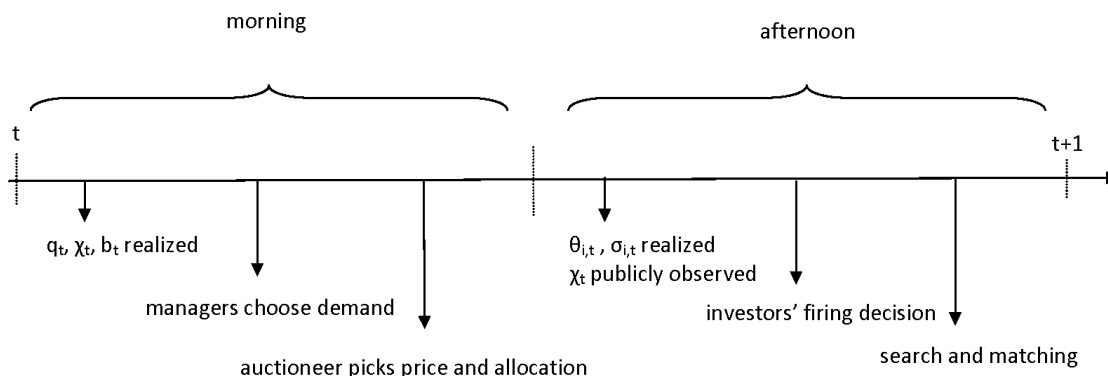
In specifying preferences, contracts and market structure, we have made a number of

simplifying assumptions. The role of these assumptions is to allow us to focus the analysis on two key decision variables: the fund managers' decision whether to invest in the riskless or in the risky asset and the investors' decision to retain or fire their managers at the end of each period. Investors acquire information on whether their fund manager is informed or uninformed by observing their investment decisions and their realized returns. They fire the fund manager whenever their belief about the quality of the manager is lower than the average quality of a newly hired manager. This firing decision is the source of career concerns for fund managers and, hence, of price distortions.

4   Equilibrium

In this section, we define and construct a stationary symmetric equilibrium. Throughout the analysis we make four assumptions.

Assumption 1 *No fully revealing prices:* $M^I < \min\{\underline{b}, \overline{b} - \underline{b}, 1 - \overline{b}\}$.

This assumption ensures that there are states of the world in which prices do not fully reveal the default shock. This implies that informed managers make higher expected returns than uninformed ones and investors strictly prefer to hire informed managers. It also ensures that uninformed managers are the marginal traders when information is not revealed.

Assumption 2 *Informative exogenous signal:* $\omega < 1/(1 + \delta)$.

This assumption is needed for the existence of a stationary equilibrium. It ensures that, even though the proportion of informed managers in the unemployed pool fluctuates over time, it is always better to keep a manager who never made a mistake.

Assumption 3 *High default risk:* $\underline{q} > (1 + 1/(\delta\omega\beta))^{-1}$.

This assumption ensures that the default probability is sufficiently high that the equilibrium price of risky bonds is always lower or equal than $1/R$.[11]

Assumption 4 *Informed managers' entry:* $\kappa \leq \gamma R$.

This last assumption is sufficient to ensure that it is profitable for an informed manager to search for a job, rather than staying inactive.

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[11] As we will see below, if the default probability is too low, the presence of career concerns can make the expected return on risky bonds lower than the risk-free return.

4.1 Definition

We now introduce the main equilibrium objects and define a *stationary* equilibrium as an equilibrium where prices and allocations are stationary. As we will explain below, there is one non-stationary equilibrium object: the distribution of investors' beliefs about the employed managers' types. However, thanks to our simplifying assumptions, this does not affect the stationarity of prices and allocations. We say that a variable is stationary if it depends only on the current realization of the aggregate and idiosyncratic shocks. At each time t, the economy is hit by three aggregate shocks: the default probability q_t, the default shock χ_t, and the supply shock b_t. Let $s_t \equiv (q_t, \chi_t, b_t)$. Also, at time t each manager i is hit by two idiosyncratic shocks: the realized investment $\theta_{i,t}$ and the exogenous signal $\sigma_{i,t}$.

Bond market. In a stationary symmetric equilibrium, all informed managers submit the same demand schedule contingent on the realization of the default shock χ_t. We denote this demand schedule by $d^I(p; \chi_t)$. Uninformed managers do not observe χ_t, so they submit a demand schedule contingent only on the default probability q_t, which we denote by $d^U(p; q_t)$. The auctioneer picks a price $P(s_t)$ and a bond allocation consistent with the demand schedules submitted by the managers and market clearing.

The equilibrium bond allocation is described by the function $X(d; s_t)$ which gives the equilibrium probability of investing in risky bonds for a manager demanding $d \in \{0, 1, \{0, 1\}\}$.[12] This means that $X(d^I(P(s_t); \chi_t); s_t)$ and $X(d^U(P(s_t); q_t); s_t)$ represent the equilibrium probability that $\theta_{i,t} = 1$ if manager i is, respectively, informed or uninformed. By the law of large numbers, they are also equal, respectively, to the fraction of informed and uninformed managers investing in risky bonds. Also, in a stationary equilibrium the measures of informed and uninformed employed managers are constant. Let us denote them respectively by N^I and N^U, so that $N^I + N^U = 1$. Market clearing on the bond market requires

$$N^I X(d^I(P(s_t); \chi_t); s_t) + N^U X(d^U(P(s_t); q_t); s_t) \leq b_t, \tag{2}$$

with equality when the equilibrium price is strictly positive.

Labor market. At the beginning of each period t, there is a distribution of existing investor-manager matches. The investor matched with manager i believes that with probability $\eta_{i,t}$ the manager is informed. Then, after observing the realized investment $\theta_{i,t}$, the exogenous

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[12]To be consistent with the managers' demand, the function X must satisfy $X(0; s_t) = 0$, $X(1; s_t) = 1$, and $X(\{0, 1\}; s_t) \in [0, 1]$.

signal $\sigma_{i,t}$, and the aggregate shock s_t, the investor updates his belief to

$$\eta_{i,t+1} = H\left(\eta_{i,t}, \theta_{i,t}, \sigma_{i,t}, s_t\right).$$

Since $\eta_{i,t}$ appears in this expression, beliefs depend in general on the whole past history of the investor-manager match. Therefore, the belief distribution is non-stationary, that is, in general, it depends both on current and past shocks. In spite of this non-stationarity, our information structure and Assumption 2 allow us to construct an equilibrium in which the firing strategy is stationary and, in particular, such that investors always retain managers who never made a mistake. This is a key step in constructing our stationary equilibrium. A detailed explanation is in the proof of Proposition 1. The firing strategy is described by the function $\phi_{i,t} = \Phi\left(\theta_{i,t}, \sigma_{i,t}, s_t\right)$, where $\phi_{i,t} = 1$ corresponds to firing and $\phi_{i,t} = 0$ to retention.

In a stationary equilibrium, the measures of employed managers, N^I and N^U, have to be consistent with job market flows. Let μ denote the ratio of searching investors to searching managers and $Z^j\left(s_t\right)$ denote the measure of managers of type j looking for a job, with $j = I, U$.[13] Also, let $\xi^j\left(s_t\right)$ be the equilibrium probability that a manager of type j is fired.[14] By the law of large numbers, $\xi^j\left(s_t\right)$ is also the fraction of employed managers of type j fired at the end of a period with shock s_t. Then, a fraction $1 - \delta$ of the remaining managers is exogenously separated. Since $\mu Z^j\left(s_t\right)$ managers of type j are matched at the end of each period, the following condition ensures the stationarity of N^j:

$$\left[\xi^j\left(s_t\right) + \left(1 - \xi^j\left(s_t\right)\right)\left(1 - \delta\right)\right] N^j = \mu Z^j\left(s_t\right). \tag{3}$$

This gives us our last equilibrium object: the fraction of informed managers in the unemployment pool, $\varepsilon\left(s_t\right) = Z^I\left(s_t\right)/\left(Z^I\left(s_t\right) + Z^U\left(s_t\right)\right)$, which is also only a function of s_t.

We are now ready to write down the optimization problem for the managers, which is key to understand the equilibrium price dynamics.[15] From now on we drop time subscripts. The uninformed managers' behavior is characterized by the Bellman equation

$$\begin{aligned} W &= E[\max_{d \in \{0,1,\{0,1\}\}} E[X\left(d; s\right)\left(\left(1 - \chi\right)\gamma/p + \left[1 - \Phi\left(1, \sigma; s\right)\right]\delta\beta W\right) + \\ &\quad \left(1 - X\left(d; s\right)\right)\left(\gamma R + \left[1 - \Phi\left(0, \sigma; s\right)\right]\delta\beta W\right) \mid p, q]], \end{aligned} \tag{4}$$

where W denotes the expected utility of an employed uninformed manager at the end of a

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[13] The total measure of uninformed managers actively looking for a job is then $Z\left(s\right) = Z^I\left(s\right) + Z^U\left(s\right)$.

[14] That is, $\xi^j\left(s_t\right) \equiv E\left[\Phi\left(\theta_{i,t}, \sigma_{i,t}, s_t\right) \mid j, s_t\right]$, where the expectation is taken with respect to $\theta_{i,t}$ and $\sigma_{i,t}$.

[15] In the appendix, we also write down the optimization problem for the investors.

period.[16] The maximization problem can be interpreted as follows. Given his choice of d, the manager receives risky bonds with probability $X(d;s)$. In this case, if there is no default he receives the current return γ/p, while if there is default his current return is zero. If he is not fired—with probability $1 - \Phi(1,\sigma;s)$—and the match is not exogenously terminated—with probability δ—he keeps his job and receives the continuation utility βW. If he loses his job, he gets zero continuation utility, given free entry in the managers' labor market. With probability $1 - X(d;s)$ the manager receives the riskless bond. He then always receives the safe current return γR. His continuation utility is computed as above, except that his firing probability is now $\Phi(0,\sigma;s)$.

The Bellman equation that characterizes the informed managers' behavior is the same as (4), except that the expectation inside the maximization operator is conditioned also on χ.

Finally, since we assumed a large continuum of uninformed managers and a search cost κ, the free entry condition is

$$\mu W - \kappa = 0. \tag{5}$$

Given that informed managers have more information than uninformed ones, they can always mimic their behavior and their expected utility when employed is larger than W. Together with Assumption 4, this implies that unemployed informed managers get positive expected utility when searching for a job, so that they all search and

$$Z^I(s) = M^I - (1 - \xi^I(s))\delta N^I \text{ for all } t. \tag{6}$$

We are now ready to define an equilibrium.

Definition 1 *A stationary symmetric equilibrium is given by demand schedules $d^I(p;\chi)$ and $d^U(p;q)$, a price function $P(s)$, a bond allocation $X(d;s)$, a firing strategy $\Phi(\theta,\sigma;s)$, a law of motion for the investor beliefs H, a measure of employed informed managers N^I and a matching probability for unemployed managers μ, such that:*

1. *fund managers' demand schedules are optimal, given the equilibrium price and investors' firing strategy;*

2. *investors' firing strategy is optimal, given their beliefs, the equilibrium price, and managers' demand schedules;*

3. *the bond allocation is consistent with managers' demand schedules;*

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[16]For consistency of notation, we adopt the convention that when $p = 0$ and $\chi = 1$ the rate of return $(1-\chi)/p$ is zero.

4. the bond market clears;

5. investors' beliefs H are consistent with Bayes' law on the equilibrium path;

6. N^I and μ are consistent with stationary labor market flows and with free entry of uninformed managers.

4.2 Characterization

We are now ready to construct a symmetric stationary equilibrium. We focus on an equilibrium where informed managers signal their type by making the "right" investment decisions, i.e., by choosing investment so as to maximize expected returns conditional on their information.[17]

In equilibrium, three possible regimes of information revelation can arise. The more interesting regime is when prices do not reveal any information. In this case, only the informed managers know the default realization and demand the risky bond if and only if there is no default. The uninformed managers, who only know the expected default probability, are the marginal traders and the price makes them indifferent between the risky bond and the riskless asset. The auctioneer allocates the risky bonds to the informed managers if they demand any. The residual bonds are allocated randomly to a fraction of uninformed managers to clear the market. Informed managers are never fired while uninformed managers are fired whenever it is revealed that they are not informed.

Moreover, there are two regimes with full information. First, if the bond price is equal to zero, default is revealed, no manager invests in risky bonds and hence no manager is fired, except if the exogenous signal reveals that he is uninformed. Second, if the bond price is equal to $1/R$ no default is revealed and the two assets have the same safe return. In this case, all managers are indifferent between the two assets and are never fired, again, except for the exogenous signal.

The next proposition claims that an equilibrium of this type exists under our four assumptions. The information regime depends on the realization of the variable $z \equiv b - (1 - \chi) N^I$. The variable z can be interpreted as the excess supply of risky bonds relative to the demand of informed managers, who are willing to demand risky bonds when $\chi = 0$. When z is small enough, uninformed managers learn that informed managers demand risky bonds and that there is no default, while when z is high enough, they learn that no informed managers demand risky bonds and hence that there is default. When z is in an intermediate range, prices do not reveal any information.

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[17]In Section 4.5 we discuss the possibile existence of other equilibria.

Proposition 1 *Under Assumptions 1-4, there exists a stationary symmetric equilibrium with three possible regimes: if $z \in [\underline{b} - N^I, \underline{b})$ no default is revealed and the bond price is equal to $1/R$, if $z \in (\overline{b} - N^I, \overline{b}]$ default is revealed and the bond price is equal to zero, if $z \in [\underline{b}, \overline{b} - N^I]$ there is no information revelation and the bond price is equal to*

$$P(q) = \frac{1-q}{R} \left[\frac{1 - [\alpha + (1-\alpha) E(q)] \delta\omega\beta}{1 - [\alpha + (1-\alpha) E(q)] \delta\omega\beta - (1-2q) \delta\omega\beta} \right] \; \text{for all } q \in [\underline{q}, \overline{q}], \qquad (7)$$

where $\alpha \equiv N^I/(\overline{b} - \underline{b})$.

The key feature of our equilibrium is that the optimal investment strategy of uninformed managers is affected by reputational concerns. When prices reveal no information, the uninformed managers are the marginal traders and the equilibrium price $P(q)$ makes them indifferent between investing in the risky bond and in the risk-less asset, that is,

$$(1-q)(\gamma/P(q) + \omega\delta\beta W) = \gamma R + q\omega\delta\beta W. \qquad (8)$$

This condition is analogous to condition (1) in the example of Section 2. The left-hand side represents the expected payoff of investing in risky bonds. When investing in risky bonds, if there is no default, the manager gets a return $\gamma/P(q)$ and he is not fired, as long as the exogenous signal does not reveal that he is uninformed (which happens with probability ω). Conditional on not being fired, a manager gets discounted continuation utility βW only if he is not exogenously separated (which happens with probability δ). The right-hand side represents the expected payoff of investing in the riskless asset. When investing in the riskless asset, he always gets a return γR but he is not fired only if default occurs, with probability q, and the exogenous signal does not reveal that he is uninformed.

Rearranging condition (8), we obtain an expression for the equilibrium price under no information revelation, for given W:

$$P(q) = \frac{\gamma(1-q)}{\gamma R - (1-2q)\delta\omega\beta W} \; \text{for all } q \in [\underline{q}, \overline{q}]. \qquad (9)$$

Reputational concerns come from the fact that uninformed managers are fired whenever their type is revealed. This distorts their investment strategy, given that they internalize the fact that when the default probability is low, there is a low probability to be fired if they invest in the risky bond and a high probability to be fired if they invest in the riskless asset.[18]

To complete the characterization of equilibrium prices, it remains to solve for W, the expected utility of employed uninformed managers. Using the Bellman equation (4), after

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[18]Racall that Assumption 3 is sufficient to ensure that $P(q) \in (0, 1/R)$.

some algebra, we obtain

$$W = \gamma R + [\alpha + (1 - \alpha) E(q)] \delta \omega \beta W, \tag{10}$$

where $\alpha \equiv N^I/(\bar{b} - \underline{b})$ is the probability that the price is fully revealing.[19] To interpret this expression, notice that uninformed managers are indifferent between their equilibrium strategy and always buying the riskless asset.[20] Under this strategy, uninformed managers always receive current returns equal to γR. Then, if prices are fully revealing they are fired only for exogenous reasons, while if prices are non-revealing they are also fired when no default occurs (which happens with expected probability $1 - E(q)$). Notice that W is increasing in the measure of employed informed managers N^I. This is because the more employed managers are informed, the higher is the probability that prices are fully revealing. This, in turns, increases the probability that uninformed manager are not fired and hence increases their value of being employed W and the career concerns' distortion.[21]

4.3 Amplification

We now compare the behavior of our model with a benchmark model with no career concerns. This allows us to derive our main result: managers' career concerns magnify the price volatility of risky bonds, generating a counter-cyclical premium.

As a benchmark model with no career concerns, consider our model with $M^I = 0$. In this case, all managers are uninformed, so investors are indifferent between keeping the manager working for them and hiring a new one. Then, there exists an equilibrium where managers are never fired and maximize their expected returns in each period. We call this equilibrium the *benchmark equilibrium*. The bond price in the benchmark equilibrium is determined by the standard no-arbitrage condition

$$P^B(q) = \frac{1 - q}{R} \text{ for all } q \in [\underline{q}, \bar{q}]. \tag{11}$$

Similarly to Section 2, when there is no information revelation, let $\Pi(q)$ be the difference

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[19]Given the equilibrium price schedule (19), this probability can be easily derived as the probability that $z \in (\underline{b} - N^I, \underline{b}] \cup (\bar{b} - N^I, \bar{b}]$.

[20]Always buying the riskless asset is optimal when prices are fully revealing, since either risky bonds always default or they are equivalent to riskless bonds. It is also optimal when prices are non-revealing, because then, by construction, the price makes uninformed managers indifferent between the two assets.

[21]Although in our model the amount of information in prices switches between extremes, the externality would survive in more general frameworks with regimes of partial revelation. The classic Grossman-Stiglitz logic applies: more informed agents increase the information content of prices, which improves the relative profitability of uninformed agents' trades, making the distortion bigger.

between the expected repayment on bonds and the risk free rate R, that is,

$$\Pi(q) \equiv \frac{1-q}{p} - R. \tag{12}$$

We call $\Pi(q)$ the *reputational premium* because it characterizes the price distortion generated by the reputational concerns of uninformed managers. Indeed, condition (11) immediately shows that the reputational premium in the benchmark equilibrium with no career concerns is equal to zero for all $q \in [\underline{q}, \overline{q}]$.

When instead there is a positive measure of informed managers, $M^I > 0$, the reputational premium can be negative or positive. In particular, when $p = 1/R$ and no default is revealed, the premium is equal to zero, while when $p = 0$ and default is revealed, no bonds are traded and the premium is not well defined. When instead there is no revelation and $p = P(q)$, it is easy to check that $\Pi(q)$ is negative if and only if $q < 1/2$. When $\underline{q}$ and $\overline{q}$ are such that $\underline{q} < 1/2 < \overline{q}$ (which is consistent with Assumptions 1-4), the equilibrium premium switches sign depending on the realization of q. When the default probability is particularly low, investing in the risky bond is a relatively safe bet because there is a higher chance to mimic the investment of informed managers. Hence, uninformed managers have a high probability of not being fired and this compensates them for a negative premium (discount) on the bond. When instead the default probability is high, uninformed managers investing in the risky bond have a large probability of being fired and hence they demand a positive premium. In short, the equilibrium price reflects this preference for large probability events. It follows that in equilibrium, the reputational premium varies with q, magnifying the volatility of prices.

Figure 2 represents graphically the price schedule $P(q)$ defined in (9) and the price schedule for the benchmark equilibrium $P^B(q)$ defined in (11). The intersection of these two functions at the realized default probability q, gives the prices in our equilibrium and in the benchmark one respectively. The figure shows that both the price schedules are monotonically decreasing in q and they intersect at $q = 1/2$. Moreover, $P(q)$ is steeper than $P^B(q)$ at $q = 1/2$ and for q not too close to 1, so that $\Pi(q) > 0$ if and only if $q > 1/2$. This also immediately implies that the price of the risky bond reacts more to a change in q in our model in comparison to the benchmark as long as q is not too high.

Notice that career concerns have effects not only on volatility but also on the average price level (except when q is near $1/2$). To focus on the volatility in prices generated by career concerns controlling for level effects, we look at the volatility of log prices. Next proposition states our main amplification result: the presence of career concerns increases price volatility.
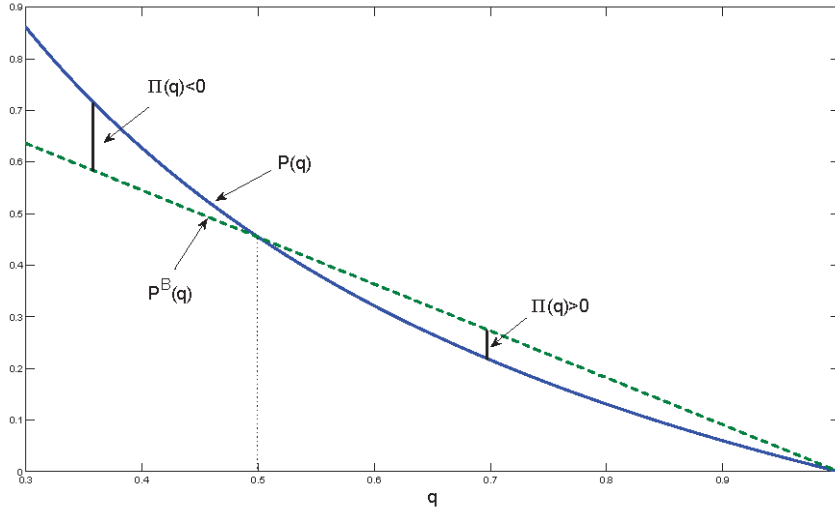
Figure 2: The figure plots the price schedule in the model with career concerns $P(q)$ together with the price schedule for the benchmark equilibrium $P^B(q)$. The parameters used are: $M^I = .25$, $\beta = .99$, $\delta = .85$, $\omega = .5$, $\kappa = .1$, $\bar{b} - \underline{b} = .355$, $\underline{q} = .3$, $\bar{q} = 1$, $E(q) = .6$.

Proposition 2 *In equilibrium, the reputational premium* $\Pi(q)$ *is negative whenever* $q < 1/2$, *and positive otherwise. Equilibrium prices are more volatile than in the benchmark equilibrium, that is,* $Var\left(\log P(q, \chi, b)\right) \geq Var\left(\log P^B(q)\right)$.

Proposition 2 shows that managers' career concerns amplify the price reaction of risky bonds to changes in their default risk. In particular, when the default risk increases, the economy can switch from regimes with high bond prices (low spreads) to regimes with low bond prices (high spreads). The first type of regimes are frequently described as regimes of *abundant liquidity*. To describe phenomena where the economy switches to the second type of regime, common terms are *flight-to-quality* and *flight-to-liquidity*. In our model, phenomena of this type can arise even if fund managers are risk-neutral and their aggregate funds are constant. In good times, when the default probability of credit instruments is low, it is very attractive for uninformed managers to invest in these instruments, because they are likely to gain high returns and improve their reputation. If the default probability increases, investing in the risky bond becomes less appealing because their reputation starts deteriorating. Hence, prices increase not only because of the higher default probability, but also because of an additional premium coming from career concerns.

It is well established in the literature that the premium on risky assets is time-varying and, in particular, that in good times it is lower than in bad times. Standard explanations are based on time-varying marginal utility of consumption, on time-varying probability of disasters or on slow-moving component in consumption risk. A common element of these

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different mechanisms is that the premium is always positive. Our model generates a time-varying component of the premium on risky assets that can be negative. In good times some managers are willing to take risky bets without the sufficient compensation in returns. This unique implication of our model and the presence of managers' career concerns seem consistent with a number of empirical observations that we have described in the introduction (Brunnermeier and Nagel, 2004 and Coval, Jurek and Stafford, 2008).

Finally, next proposition shows that our amplification effect is stronger when there are more informed managers, M^I is larger, when the entry cost κ is higher, and when the managers' returns share γ is lower.

Proposition 3 *The amplification effect is stronger the larger is the measure of informed managers M^I, the higher is the entry cost κ, and the lower is γ.*

The first comparative static result is the most intuitive. The larger is the measure of informed managers in the population, the larger is the measure of informed managers who are employed.[22] This generates a positive externality for the uninformed managers. The more employed managers are informed, the higher is the probability that prices will reveal information about the default state, hence increasing the expected utility of employed managers who are uninformed. This makes their reputation more valuable and amplifies the distortion generated by career concerns.

The reason why we should expect larger price volatility in markets that are more costly for managers, either because of setting up costs or because of worse contracting terms, is more subtle. Given that in equilibrium there is free entry of uninformed managers, the higher is the entry cost or the smaller are the expected returns from working, the smaller is the measure of uninformed managers looking for a job. This increases the hiring probability for all managers and hence increases the measure of informed managers who are employed, again making career concerns more important.[23]

In our equilibrium, investors have to delegate their investment decision. One could generalize the model to the case where the delegation decision is endogenous and investors have

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[22] There is an indirect effect that goes in the opposite direction. As more informed managers are employed, the value of being employed increases, inducing more uninformed managers to look for a job. This makes the job finding probability smaller also for the informed managers. In the appendix, we show that this effect is dominated by the direct one.

[23] The effect on amplification of β and ω is in general ambiguous. On the one hand, for given N^I, when β is higher, managers care more about their future and hence about their reputation. Also, career concerns are stronger when there is a lower chance that managers lose their job for exogenous reasons and hence when ω is higher. However, on the other hand, an increase in either one of these parameters also increases the expected utility of being employed. More uninformed managers search for a job, reducing the hiring probability and the measure of informed employed managers. This reduces the importance of career concerns.

the same information of uninformed managers. Under some additional parameter restrictions, our equilibrium would survive this generalization. First of all, delegation is costly because the investor has to give up a share γ of his returns to the manager, so γ has to be low enough to induce voluntarily delegation. The benefit of delegation is the possibility of hiring an informed manager able to get higher returns. This implies that the more informed managers are employed in equilibrium, the higher is the expected return to delegation. However, if an investor knew that his manager was uninformed, he would strictly prefer to invest himself. For example, when the reputational premium is negative, uninformed managers are indifferent between the two assets because they are compensated by a reputational gain, but the investor would strictly prefer to invest in the riskless asset. Hence, to make sure that the expected benefit of delegation is high enough relative to the expected cost, we would need some additional parameter restriction, such as M^I large enough or $\bar{q} - \underline{q}$ small enough.

4.4 Limit Equilibrium

We now show that the reputational distortion in this economy survives even when the measure of informed managers becomes infinitesimal, that is, in the limit case with $M^I \to 0$. As long as there is a positive measure of informed managers in the population, there is an expected gain for any investor to fire a manager who revealed to be uninformed and hire a new random one. This implies that reputation is valuable and affects the investment decision of uninformed managers, who are the marginal traders.

In particular, as $M^I \to 0$, the sequence of stationary equilibria constructed so far converges to a *limit equilibrium* which is essentially the same as the equilibrium described in Proposition 1 in the regime of no information revelation. As the fraction of informed managers becomes infinitesimal, the uninformed managers demand essentially all the bonds supplied and hence don't learn any information from the equilibrium price.

Proposition 4 *Under Assumption 1-4, when $M^I \to 0$, there exists a limit equilibrium where no information is revealed and*

$$P(q) = \frac{1-q}{R} \left[\frac{1 - \delta\omega\beta E(q)}{1 - \delta\omega\beta (E(q) + 1 - 2q)} \right] \text{ for all } q \in [\underline{q}, \bar{q}].$$

The proof of this proposition is an obvious generalization of the proof of Proposition 1 and hence omitted. In a limit equilibrium, prices never reveal any information and uninformed managers are always the marginal traders. Informed managers demand the risky bond if and only if there is no default, while uninformed managers cannot follow the same strategy. The equilibrium price makes them indifferent between demanding the risky bond and the riskless

asset, so that bonds can be allocated to clear the market.[24] At the end of the period, investors fire managers who failed to mimic the informed managers' strategy and hence revealed to be uninformed. Clearly, informed managers are never fired.

For a given default probability q, the equilibrium price $P(q)$ is determined by the same indifference condition (9), where the expected continuation utility of an employed uninformed manager W satisfies[25]

$$W = \gamma R + E(q)\,\delta\omega\beta W. \tag{13}$$

Combining (9) and (13) we obtain the equilibrium price $P(q)$ given in Proposition 4 and the following reputational premium:

$$\Pi(q) \equiv \frac{R\delta\omega\beta(2q-1)}{1 - \delta\omega\beta E(q)} \text{ for all } q \in [\underline{q}, \overline{q}].$$

This immediately shows that, also in the limit case, the reputational premium varies with q and can be positive or negative depending on q being, respectively, above or below $1/2$. Hence, the reputational premium does not disappear when the informational asymmetry becomes infinitesimal, that is, as $M^I \to 0$. This implies that there is a form of discontinuity at $M^I = 0$: the limit equilibrium as $M^I \to 0$ is different from our benchmark equilibrium at $M^I = 0$.[26]

This limiting result is useful to highlight that in our equilibrium the source of the distortion is really the fact that reputation matters. As long as there is a positive probability of hiring an informed manager, investors fire any manager who reveal to be uninformed, irrespective on how few informed managers are around. However, the discontinuity at zero is not particularly robust. For example, things change if we introduce a positive firing cost. When $M^I \to 0$, the advantage for an investor of firing a manager who revealed to be uninformed is infinitesimally small, because the proportion of informed managers in the pool of unemployed goes to zero. This implies that if there is a positive firing cost, investors decide to keep their managers, regardless of their performance history. This implies that our benchmark equilibrium exists even for M^I sufficiently close to zero.

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[24] Assumption 1 guarantees that $b/Z \in (0,1)$ for any b, so that there are always some uninformed managers investing in the risky bond and some investing in the risk-free asset.

[25] This condition is the limit of condition (10) for $N^I \to 0$.

[26] When $M^I = 0$ there is a continuum of equilibria, including the limit equilibrium for $M^I \to 0$ and our benchmark equilibrium.

4.5 Multiple equilibria

In this paper we focus on a stationary equilibrium where informed managers signal their type by making the "right" investment decision, that is, the one that maximizes expected returns conditional on their information. This means that informed managers always weakly prefer to invest in risky bonds if there is no default. Given the signalling nature of the game, there may be multiple equilibria, as informed managers can take different actions to signal their type.

To illustrate the possibility of multiplicity, one could think at an equilibrium where informed managers signal their type by choosing different investment strategy depending on their job tenure. Let *junior* managers be managers in their first period of job and *senior* managers be managers who have been employed for at least one period. Consider an equilibrium where junior informed managers make the "wrong" investment decision, that is, invest in the risky bond if there is default and in the riskless asset if there is no default, while senior informed managers behave as in our equilibrium. In such an equilibrium, the investors' firing strategy depends on the manager's tenure. In particular, investors still fire managers whenever they reveal to be uninformed, but now this means firing junior managers if they make the "right" decision. One can make additional parameter restrictions to make sure that informed managers actually find it optimal to make losses in their first period of work and that investors strictly prefer informed to uninformed managers. This can be done by picking $\beta\delta$ high enough, that is, assuming that investors and managers are sufficiently patient.

This equilibrium is similar to our equilibrium, as the behavior of the senior managers is analogous to the behavior of all the managers in our equilibrium. In particular, the equilibrium price is going to be the price that makes the senior uninformed managers indifferent between demanding risky bonds and riskless assets. Given this price, the junior uninformed managers strictly prefer to invest in the risky bond if $q > 1/2$ and in the riskless asset otherwise. This is because career concerns of junior and senior managers work in the opposite direction. For example, when $q > 1/2$ the risky bond pay a positive premium to compensate the reputational disadvantage for the senior uninformed managers, while it bears a reputational advantage for the junior. This clearly makes the equilibrium characterization a bit more complicated, given that market clearing, and hence all the equilibrium object, will depend on an extra state variable: the fraction of newly hired uninformed managers.

5 Persistent Default Risk

In this section, we generalize the model to allow for persistent default probability. In particular, we allow q_t to be distributed according to a first-order Markov process with cumulative density function $F(q_t|q_{t-1})$ with support $[\underline{q}, \overline{q}]$. The environment is a natural generalization of the baseline one with iid shocks.

When the default risk is persistent, the expected value of being an employed uninformed manager varies over time, affecting the reputational premium. This amplifies the price response to changes in the default risk when the reputational premium is positive and it dampens it when it is negative. Moreover, the flow of employed informed managers now varies over time. This also provides an independent source of price volatility.

5.1 Equilibrium with persistent shocks

Once we allow the default probability to follow a first-order Markov process, a stationary equilibrium does not exist anymore. We then focus on Markov equilibria. The definition of a Markov symmetric equilibrium is a natural generalization of Definition 1, where q and N^I become state variables. The managers' demand schedules, $d^I(p; \chi)$ and $d^U(p; q)$, and the investors' firing rule $\Phi(\theta, \sigma; s)$ are similar to the ones defined in the iid environment. However, the equilibrium price $P(s, N^I)$ and the bond allocation $X(d; s, N^I)$ are now also functions of N^I. Moreover, the equilibrium measure of employed informed managers follows the law of motion $G(q, N^I)$ and the matching probability for unemployed managers $\mu(q, N^I)$ is also a function of the states q and N^I.

We can characterize a Markov equilibrium with similar features to the equilibrium in the baseline model. This equilibrium is very similar to the one constructed in Section 4.2, with the exception that the price, the bond allocation, and the labor market flows now depend on the states q and N^I. There are three revelation regimes: no information is revealed, default is revealed, no default is revealed. Informed managers always maximize expected returns conditional on their information and hence demand the bond if and only if there is default whenever $p < 1/R$ and are indifferent when $p = 1/R$. Uninformed managers mimic the informed whenever the default state is revealed, while when there is no revelation, they are indifferent between risky bonds and riskless assets. The auctioneer picks the price and the bond allocation which are consistent with the managers' demand and ensure market clearing. Finally, each investor fires his manager whenever either his investment or the exogenous signal reveal that he is uninformed.

Let us now focus on the main differences with the iid case. First of all, as mentioned above, the labor market flows are no longer constant in equilibrium and N^I becomes a state

variable. As in the baseline model, there is free entry in the managers' labor market, that is, the hiring probability $\mu(q, N^I)$ needs to satisfy

$$\mu(q, N^I)W(q, N^I) = \kappa, \tag{14}$$

where $W(q, N^I)$ denotes the expected value of being employed for uninformed managers at the end of the period. As in the baseline, given that the expected value of being employed is always higher for an informed manager than for an uninformed manager, all unemployed informed managers at time t search for a job. In equilibrium, no informed manager is fired. Hence, the measure of employed informed managers at the beginning of time $t + 1$ must be equal to the measure of informed managers employed at time t whose job was not exogenously terminated, plus the measure of unemployed informed managers who found a job at the end of time t, that is, the law of motion for N_t^I is given by

$$N_{t+1}^I = G(q_t, N_t^I) \equiv \delta N_t^I + \mu(q_t, N_t^I) \left(M^I - \delta N_t^I \right). \tag{15}$$

The fact that the measure of informed employed managers evolves over time is a novel feature of the equilibrium. In particular, as shown by numerical examples below, this measure tends to be persistent.

Turning to the equilibrium price, as in the baseline, if no information is revealed, uninformed managers are the marginal traders. The indifference condition for the uninformed managers yields

$$P(q, N^I) = \frac{\gamma (1 - q)}{\gamma R - (1 - 2q) \, \delta \omega \beta W(q, N^I)}, \tag{16}$$

with

$$W(q, N^I) = \gamma R + [\alpha(N^I) + \left(1 - \alpha(N^I)\right) E[q'|q]] \delta \omega \beta E[W(q', G(q, N_I))|q], \tag{17}$$

where $\alpha(N^I) = N^I/(\bar{b} - \underline{b})$ is the probability that the price is fully revealing. The only difference with expression (9) is that W now depends on the states q and N^I.

Finally, to ensure that successful uninformed managers are never fired in equilibrium, we replace Assumption 2, with the following one (which is tighter).

Assumption 5 *Informative exogenous signal with persistent q:*

$$\omega < (1 - M^I) \left[1/ (1 - \delta) (1 - \delta \omega \beta) + \delta\right]^{-1}.$$

The next proposition gives conditions for the existence of a Markov equilibrium, where again $z \equiv b - (1 - \chi) N^I$.

Proposition 5 *If there exist three functions $W(q, N^I)$, $\mu(q, N^I)$, and $G(q, N^I)$ that satisfy equations (15)-(17) and Assumptions 1 and 3-5 hold, there exists a Markov equilibrium with three possible regimes: if $z \in [\underline{b} - N^I, \underline{b})$ no default is revealed and the bond price is equal to $1/R$, if $z \in (\overline{b} - N^I, \overline{b}]$ default is revealed and the bond price is equal to zero, if $z \in [\underline{b}, \overline{b} - N^I]$ there is no information revelation and the bond price is given by expression (16).*

In the limit case with $M^I \to 0$, the measure of employed informed managers N^I also converges to 0, and hence the only state variable is q. This allows us to show analytically the existence of the functions $W(q)$ and $\mu(q)$. However, in the general case, we need numerical methods to show existence of an equilibrium. For the parameters we tried, we find that our equilibrium exists and has similar qualitative properties, as the ones illustrated below.

5.2 Limit Case: Amplification

Let us first focus on the limit case with $M^I \to 0$. As just mentioned, this case is particularly tractable because N^I is no longer a state variable and we can derive some analytical results.

As in the baseline case, when $M^I \to 0$, prices do not reveal any information. The value of being an employed uninformed managers then reduces to

$$W(q) = \gamma R + E[q'|q]\delta\omega\beta E[W(q')|q].$$

It is easy to see that the right-hand-side of the previous equation is a contraction and hence that there exists an equilibrium function $W(q)$. In particular, W is increasing in the default risk q. Recall that in equilibrium the price makes uninformed managers indifferent between investing in the two assets, so that their expected utility can be calculated as the value of always investing in the safe assets. The higher is q, the higher is the expected default probability tomorrow and hence the lower is the chance to be fired when investing in the riskless asset. Moreover, given that W is increasing in q there is a reinforcing effect due to the fact that the higher is q, the higher is the expected value of being employed in the future.

The expression for the reputational premium is the same as in the baseline except that now W depends on q, that is,

$$\Pi(q) = -\frac{(1 - 2q)\delta\omega\beta W(q)}{\gamma}. \tag{18}$$

As in the baseline, the premium is positive when $q > 1/2$, but becomes negative when $q < 1/2$. Also, the absolute value of the premium is increasing in the expected utility of employed uninformed managers W that represents the reward to good reputation. Relative

to the baseline, the fact that W increases with q magnifies the premium when positive, but dampens it when negative. This amplifies the price response to changes in q in bad states (when q is relatively high), but it dampens it in good states (when it is relatively low).

5.3   A Numerical Example

Let us now illustrate the equilibrium for the general case with $M^I > 0$, using a numerical example. We will see that, as in the limit case, the amplification effect is magnified relative to the baseline when the reputational premium is positive and dampened when it is negative. Moreover, in the general case W moves over time not only because of movements in the default risk, but also because of movements in N^I. This leads to an additional result: the bond price can move independently from changes in fundamentals, which in this model are simply equal to the current default risk.

In the general case, when no information is revealed, the expression for the reputational premium is the same as (18), except that W now depends both on the default risk q and the measure of employed manager N^I according to equation (17). The numerical exercise shows that W is increasing not only in q but also in N^I. As more employed managers are informed, the higher is the probability that prices are fully revealing. When there is full revelation, uninformed managers are better off because they are never fired and their expected utility W is higher.

The fact that W is increasing in q immediately confirms that the same result for the limit case goes through: the reputational premium is magnified when positive and dampened when negative. The new result is that now W, and hence the reputational premium, increases with N^I. Moreover, the law of motion for N^I, given by expression (15), shows that the current q_t and N_t^I affect the future measure of informed employed managers N_{t+1}^I. In particular, the numerical example shows that N_t^I is persistent, that is, N_{t+1}^I increases with N_t^I, regardless of q_t. This implies that even if q_t does not change, N_t^I can increase just because it was high in the past. In turns, this can increase $W\left(N_t^I, q_t\right)$, even if q_t does not change.

Figure 3 shows a sample simulation for a specific realization of sequence of shocks $\{q_t\}$ represented in Panel A. Panel B plots the simulated path for N^I. It is easy to see that N^I gradually increases whenever $q = q^L$ and gradually decreases whenever $q = q^H$. Also, N^I fluctuates around its iid counterpart, N^{I*}. Panel C compares the simulated pattern of the reputational premium for the baseline model, Π^*, and the persistent model, Π. Finally, Panel D plots the ratio Π/Π^* so that the comparison is more evident. Panel C and D show our two insights.

First, as in the limit case, the persistence of q magnifies the reputational premium when
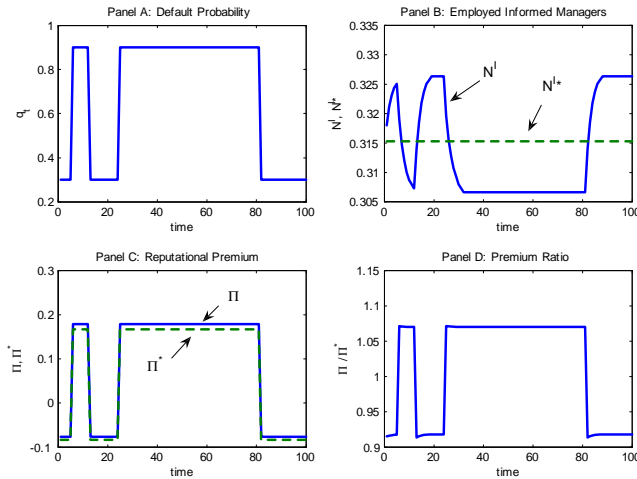
Figure 3: Figure 2: Panel A shows a specific realization of sequence of shocks q_t. Panel B compares the simulated behavior of Γ^I and Γ^{I*} under that specific sequence of shocks. Panel C compares the evolution of Π and Π^* for the same simulation. Panel D plots the ratio of Π/Π^*. The parameters used are: $M^I = .25$, $\beta = .99$, $\delta = .85$, $\omega = .5$, $\kappa = .1$, $\bar{b} - \underline{b} = .355$, $\underline{q} = .3$, $\bar{q} = .9$, $\pi^L = .1$, and $\pi^H = .9$.

positive $(q = q^H)$ and dampens it when negative $(q = q^L)$.

Second, the premium varies over time even for a sequence of realizations where the default risk does not change. For example, if the default risk stays equal to q^L for a sequence of periods, N^I keeps increasing and, hence, so does W. This implies that the reputational discount increases in absolute value, even though the fundamentals do not change. In contrast, if the economy experiences a sequence of high realizations of default risk q^H, N^I decreases, hence reducing W and dampening the reputational premium. This shows that when the default risk is persistent, there is an additional source of volatility in asset price dynamics, driven by the labor market. The measure of employed informed agents changes the future informational content of prices, which changes career prospects of uninformed managers, affecting current prices. However, the figure shows that in our example this effect is small. We leave to future research a more quantitative evaluation of these effects.

6   Conclusion

In this paper, we have proposed a general equilibrium model of delegated portfolio management with time-varying default risk, where career concerns distort asset prices. In particular, risky bonds trade with a reputational premium, which may be positive or negative depend-

ing on the default risk. For example, when the default probability is high, the return on the risky bond has to be high to compensate the uninformed managers for the high risk of being fired. As the default risk changes over time, the countercyclical reputational premium amplifies the volatility of the risky bond price.

We believe a promising direction for future research is the introduction of alternative risky assets in the managers' portfolio choice. In this case, our mechanism would generate contagion. Imagine that there are two risky bonds and a riskless asset. The reputational cost of investing in the riskless asset depends on the default probability of both the risky bonds. If none of them defaults, the manager who invests in the riskless asset loses his reputation. Thus, if the probability of default of any of the risky bonds decreases, the riskless asset will be less attractive, and the prices of both bonds will have to increase in order to make uninformed managers indifferent between different investment opportunities.

Finally, it would be interesting to develop the supply side of the model in the context of sovereign debt.[27] A large literature on business cycle in emerging markets highlights that emerging market bond spreads are very volatile.[28] In particular, the magnitude of volatility of interest rates is hard to reconcile with models where bond prices are determined by the standard no-arbitrage condition. Our model provides an appealing framework to think about this excess volatility.

Appendix

Proof of Proposition 1

Here we prove that, under Assumptions 1-4, there exists a symmetric stationary equilibrium as claimed in proposition 1. First, let us fully describe the equilibrium objects. In equilibrium, the price schedule is

$$
p(s) = \begin{cases} \frac{1}{R} & \text{if } z \in [\underline{b} - N^I, \underline{b}) \\ P(q) & \text{if } z \in [\underline{b}, \overline{b} - N^I] \\ 0 & \text{if } z \in (\overline{b} - N^I, \overline{b}] \end{cases} , \tag{19}
$$

where $P(q)$ is given in equation (7); the managers' demand schedules are

$$
d^I(p; \chi) = \begin{cases} \{0, 1\} & \text{if } p = 1/R \\ 1 - \chi & \text{otherwise} \end{cases} \quad \text{and} \quad d^U(p; q) = \begin{cases} 0 & \text{if } p = 0 \\ \{0, 1\} & \text{otherwise} \end{cases} ;
$$

---

[27] In the working paper version, we propose a first attempt in this direction.

[28] See Neumeyer and Perri (2005), Uribe and Yue (2006), Aguiar and Gopinath (2006), and Arellano (2008).

the bond allocation is

$$X(d;s) = \begin{cases} d & \text{if } d \in \{0,1\} \\ \frac{b}{N} & \text{if } d = \{0,1\} \text{ and } z \in [\underline{b} - N^I, \underline{b}) \\ \frac{z(\chi,b)}{N^U} & \text{if } d = \{0,1\}, \ z \in [\underline{b}, \overline{b} - N^I] \\ 0 & \text{if } d = \{0,1\} \text{ and } z \in (\overline{b} - N^I, \overline{b}] \end{cases} \quad ;$$

and the investors' firing rule is

$$\Phi(\theta, \sigma; s) = \begin{cases} 0 & \text{if } \sigma = 0 \text{ and either } p(s) = 1/R \text{ or } \theta = 1 - \chi \\ 1 & \text{otherwise} \end{cases} \quad .$$

The rest of the proof proceeds in five additional steps: first, we show how equilibrium prices reveal information conditional on different shocks; second, we derive the equilibrium values for N^I, N^U, and μ that are consistent with stationary labor market flows and managers' free entry; third, we show that managers' demand schedules are optimal given investors' firing rule and equilibrium price schedule and allocation; fourth, we show that the equilibrium allocation is consistent with demand schedule and bond market clearing; fifth, we show that investors' firing strategy is optimal, given managers' demand schedules, equilibrium price schedule and allocation.

Step 1. First, we want to describe how equilibrium prices reveal information. If $p = 1/R$, then $z \in [\underline{b} - N^I, \underline{b})$. In this case, uninformed managers learn that $\chi = 0$ because z can be smaller than $\underline{b}$ only if a positive mass of informed managers is demanding risky bonds, which only happens if $\chi = 0$. If $p = 0$, then $z \in (\overline{b} - N^I, \overline{b}]$. In this case, uninformed managers learn that $\chi = 1$ because z can be greater than $\overline{b} - N^I$ only if no informed managers are demanding risky bonds, which only happens if $\chi = 1$. Finally, when $p = P(q)$, then $z \in [\underline{b}, \overline{b} - N^I]$ and the uninformed managers' updated beliefs are:

$$\Pr(\chi = 1 | p = P(q)) = \frac{\Pr(\chi = 1, z \in [\underline{b}, \overline{b} - N^I])}{\Pr(\chi = 1, z \in [\underline{b}, \overline{b} - N^I]) + \Pr(\chi = 0, z \in [\underline{b}, \overline{b} - N^I])}. \tag{20}$$

Since b is independent of χ and uniformly distributed on $[\underline{b}, \overline{b}]$, we have

$$\Pr(\chi = 1, z \in [\underline{b}, \overline{b} - N^I]) = q \Pr(b \in [\underline{b}, \overline{b} - N^I]) = q \frac{\overline{b} - \underline{b} - N^I}{\overline{b} - \underline{b}}$$

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and

$$\Pr(\chi = 0, z \in [\underline{b}, \bar{b} - N^I]) = (1 - q) \Pr(b \in [\underline{b} + N^I, \bar{b}]) = (1 - q) \frac{\bar{b} - \underline{b} - N^I}{\bar{b} - \underline{b}},$$

where Assumption A1 guarantees that $\bar{b} - \underline{b} > N^I$, so that these are strictly positive probabilities. Substituting in (20), it follows that $\Pr(\chi = 1 | p = P(q)) = q$ and the price $P(q)$ is completely uninformative.

Step 2. The firing probabilities consistent with the equilibrium firing strategy according to $\xi_t^s = E\left[\Phi\left(\theta_{i,t}, \sigma_{i,t}, \chi_t, p_t\right) | x_t^s\right]$ can be reduced to $\xi_t^I = 0$ for all q_t, χ_t, and b_t and

$$\xi_t^U = \begin{cases} 1 - \omega \left[x_t^U (1 - \chi_t) + \left(1 - x_t^U\right) \chi_t\right] & \text{if } z_t \in [\underline{b}, \bar{b} - N^I] \\ 1 - \omega & \text{if } z_t \notin [\underline{b}, \bar{b} - N^I] \end{cases}. \tag{21}$$

Using $\xi_t^I = 0$, condition (3) with $s = I$, and (6), we obtain

$$N^I = \frac{\mu M^I}{1 - \delta (1 - \mu)}. \tag{22}$$

Also, equation (5) can be rewritten as

$$W = \frac{\kappa}{\mu}. \tag{23}$$

From condition (10), we obtain

$$W = \frac{\gamma R}{1 - \delta \omega \beta \left[\frac{N^I}{\bar{b} - \underline{b}} + \left(1 - \frac{N^I}{\bar{b} - \underline{b}}\right) E(q)\right]} \tag{24}$$

and by combining (22)-(24) we obtain an equation in μ only $g(\mu) = 0$, where

$$g(\mu) = \frac{\kappa}{\mu} - \gamma R \left\{1 - \delta \omega \beta \left[E(q) + \frac{M^I (1 - E(q))}{(\bar{b} - \underline{b}) \left(\delta + \frac{1 - \delta}{\mu}\right)}\right]\right\}^{-1}. \tag{25}$$

Notice that $\lim_{\mu \to 0} g(\mu) = \infty$, $\lim_{\mu \to 1} g(\mu) < 0$ thanks to the assumption that $\kappa < \gamma R$, and $g'(\mu) < 0$ by inspection. It immediately follows that there exists a unique $\mu \in (0, 1)$ such that $g(\mu) = 0$. Given μ, one can use equation (22) to solve for a unique $N^I < M^I$, and hence a unique $N^U = N - N^I$, and equation (23) to solve for a unique W.

Step 3. Here we verify that the managers' demand schedules are optimal, given the

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investors' firing rule and the equilibrium price schedule and allocation. For informed managers, it is easy to see that their demand is optimal both because it maximizes their current expected returns and because, given the investors' firing rule, it maximizes their continuation utility by ensuring them to be never fired. Turning to uninformed managers, their behavior is characterized by the Bellman equation (4). When prices are fully revealing, it is easy to check that the uninformed managers' strategy is optimal, as it perfectly mimics the informed managers' behavior. Let us then focus on the case of non-revealing prices, when $p = P(q)$. In this case, substituting the investors' firing rule, the maximization problem in (4) becomes

$$\max_{d \in \{0,1,\{0,1\}\}} E \left[X(d; q, \chi, b)(1 - \chi) \left(\gamma \frac{1}{p} + \omega \delta \beta W \right) + (1 - X(d; q, \chi, b))(\gamma R + \chi \omega \delta \beta W) \mid q, p = P(q) \right].$$

We need to check that when $p = P(q)$ it is optimal for the uninformed manager to demand $d = \{0, 1\}$. Next, we show that the allocation probability $X(\{0, 1\}; q, \chi, b)$ is independent of χ, conditional on q and $p = P(q)$, that is,

$$E[X(\{0, 1\}; q, \chi, b) | q, \chi = 1, p = P(q)] = E[X(\{0, 1\}; q, \chi, b) | q, \chi = 0, p = P(q)].$$

From the equilibrium price schedule (19), we know that $p = P(q)$ if $z \in [\underline{b}, \bar{b} - N^I]$. Assumption A1 ensures that this happens with positive probability. Recall that $z = b - (1 - \chi) N^I$. Hence, when $p = P(q)$ and $\chi = 1$ it must be that $b \in [\underline{b}, \bar{b} - N^I]$, while when $p = P(q)$ and $\chi = 0$ it must be that $b \in [\underline{b} + N^I, \bar{b}]$. One can then derive

$$E[X(\{0, 1\}; q, \chi, b) | q, \chi = 1, p = P(q)] = \int_{\underline{b}}^{\bar{b} - N^I} \frac{b}{N^U} dF(b) = \frac{1}{2N^U}[(\bar{b} - N^I)^2 - \underline{b}^2],$$

$$E[X(\{0, 1\}; q, \chi, b) | q, \chi = 0, p = P(q)] = \int_{\underline{b} + N^I}^{\bar{b}} \frac{b - N^I}{N^U} dF(b) = \frac{1}{2N^U}[(\bar{b} - N^I)^2 - \underline{b}^2].$$

It follows that these two expressions are the same, which implies that $d = \{0, 1\}$ is optimal for uninformed managers whenever condition (8) is satisfied. This is guaranteed by the construction of $P(q)$ in equation (19).

Step 4. It is easy to check that the bond allocation $X(d; q, \chi, b)$ is consistent with the managers' demand and that the bond market always clears. In particular, when $z(\chi, b) \in (\bar{b} - N^I, \bar{b}]$, default is fully revealed and the price is 0. In this case, there is an excess supply and the market clearing condition (2) holds with inequality. When $z \in [\underline{b} - N^I, \underline{b})$, no default is revealed and all managers are indifferent between risky bonds and the riskless asset. In this case, risky bonds are randomly allocated to all managers, informed and uninformed, and the probability of investing in risky bonds is equal to b/N. When $z(\chi, b) \in [\underline{b}, \bar{b} - N^I]$,

informed managers invest in the bond if and only if there is default and only uninformed managers are indifferent. Hence, to clear the market, the probability of investing in the bond for a manager who is indifferent must be equal to $(b - N^I)/N^U$ if $\chi = 0$ and to b/N^U if $\chi = 1$, or, more compactly, to $z(\chi, b)/N^U$. Assumption A1 ensures that in all these cases we have $X(d; q, \chi, b) \in (0, 1)$.

Step 5. Here we show that the investors' firing rule $\Phi(\theta, \sigma; s)$ is optimal. Let us first write down the investors' optimization problem. At the end of time t, an investor with posterior belief η' that his manager is informed, chooses to fire him only if the value of hiring a random new manager, informed with probability ε, is higher than the value of keeping him. Given that an investor searching for a new manager has probability 1 of matching, his firing decision ϕ solves

$$J(\eta', \varepsilon) = \max_{\phi \in \{0,1\}} (1 - \phi)V(\eta') + \phi V(\varepsilon), \tag{26}$$

where $V(\eta)$ denotes the value of being matched with a manager informed with probability η, that is,

$$V(\eta) = E\left[(1 - \gamma)(\theta(1 - \chi)/P(s) + (1 - \theta)R) + \beta J(H(\eta, \theta, \sigma, s), \varepsilon) | \eta\right].$$

Notice that the prior belief η affects the right-hand-side of the above expression in two ways: directly through the Bayes rule H and through the distribution of θ. A manager informed with probability η invests in the risky bond, $\theta = 1$, with probability $\eta x^I(s) + (1 - \eta) x^U(s)$. If the manager invests in the risky bond, the investor gets $(1 - \gamma)/P(s)$ only if there is no default, while if the manager invests in the riskless asset, the investor gets $(1 - \gamma)R$ for sure.

As problem (26) shows, each period investors' current payoffs are given by a share $1 - \gamma$ of the return on their current investment. The expected return made by informed and uninformed managers is the same when $p = 1/R$ or $p = 0$, and there is full revelation. However, when $p = P(q)$, the expected return of an informed manager is higher. Therefore, $V(\eta)$ is increasing in η and investors prefer to have informed managers investing their capital. Given the updated belief η' and the fraction of informed managers in the unemployment pool ε, problem (26) implies that an investor will fire his manager if and only if $\eta' < \varepsilon$. Therefore, to check that the firing rule is optimal we need to show that, for any belief η that can arise in equilibrium, the updated belief η' is greater than ε whenever $\sigma = 0$ and either $p = 1/R$ or $\theta = 1 - \chi$, and is smaller than ε otherwise. The second part of this statement is easy to check, because $\sigma = 1$ or $\theta \neq 1 - \chi$ and $p < 1/R$ can only happen when the manager is uninformed. Therefore, in this case $\eta = 0$ which is always smaller than $\varepsilon > 0$. That is,

when the manager is exogenously revealed to be uninformed or when he makes a mistake at a non-revealing price, he is immediately identified and fired. The first part of the statement above is harder to check because η' depends on the history of the match and ε depends on the current shocks. Next, we show that Assumption A2 is sufficient to ensure that this is the case.

By definition ε satisfies

$$\varepsilon = \frac{Z^I}{Z^I + Z^U} > 0, \tag{27}$$

that is, the probability that a newly hired manager is informed is equal to the ratio of unemployed informed managers relative to all the unemployed managers. When manager i realizes $\theta_{i,t} = 1 - \chi_t$ and/or $p_t \in \{0, 1/R\}$, the investor's belief is updated according to $\eta_{i,t+1} = \eta_{i,t}/[\eta_{i,t} + (1 - \xi_t^U)(1 - \eta_{i,t})]$, where ξ_t^U defined in equation (21) denotes the proportion of uninformed managers who are fired. Next, we show that assumption A2 is sufficient to make sure that in equilibrium $\eta_{i,t+1} \geq \varepsilon_t$ for any ξ_t^U and $\eta_{i,t+1} > 0$.

First, consider an investor who has just hired manager i at the end of $t-1$ and hence, by definition, has prior belief $\eta_{i,t} = \varepsilon_{t-1}$. In this case, if $\theta_{i,t} = 1 - \chi_t$ and/or $p_t \in \{0, 1/R\}$, then $\eta_{i,t+1} = \varepsilon_{t-1}/[\varepsilon_{t-1} + (1 - \xi_t^U)(1 - \varepsilon_{t-1})]$. Next, we want to show that $\eta_{i,t+1} \geq \varepsilon_t$. This condition can be rewritten as

$$\frac{1 - \varepsilon_t}{\varepsilon_t} \geq \left(\frac{1 - \varepsilon_{t-1}}{\varepsilon_{t-1}}\right)(1 - \xi_t^U). \tag{28}$$

Using expression (27) for ε_t with $Z_t^I = M^I - \delta N^I$ from condition (6), we have that $(1 - \varepsilon_t)/\varepsilon_t = Z_t^U/(M^I - \delta N^I)$, and, hence, condition (28) can be rewritten as $Z_t^U/Z_{t-1}^U \geq 1 - \xi_t^U$, where $Z_t^U = (1 - \delta(1 - \xi_t^U))N^U/\mu$. Hence, in order for (28) to be satisfied it must be that $1 - \delta(1 - \xi_t^U) > (1 - \delta(1 - \xi_{t-1}^U))(1 - \xi_t^U)$, which is ensured by assumption A2, given that $\xi_t^U \in [1 - \omega, 1]$ for all t.

Let us now consider managers who were working for an investor for longer than 1 period. First, notice that the investors' beliefs about any manager who is still working at time t but was hired at time $t' < t$ must be higher than the initial belief $\varepsilon_{t'-1}$, given that if he was not fired he never made any mistake, that is, $\eta_{i,t} \geq \varepsilon_{t'-1}$. Hence, the posterior belief about a manager who was hired at time t' and did not make a mistake at time t is $\eta_{i,t+1} = \eta_{i,t}/[\eta_{i,t} + (1 - \xi_t^U)(1 - \eta_{i,t})] \geq \varepsilon_{t'-1}/[\varepsilon_{t'-1} + (1 - \xi_{t'}^U)(1 - \varepsilon_{t'-1})]$. It follows that a sufficient condition for this manager not being fired is $(1 - \varepsilon_t)/\varepsilon_t \geq (1 - \xi_{t'}^U)(1 - \varepsilon_{t'})/\varepsilon_{t'}$, which, by the same argument, is satisfied when assumption A2 holds, completing the proof of this step.

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Proof of Proposition 2

Taking logs of expression (19), one obtains

$$
\log p(s) = \begin{cases} -\log R & \text{if } z(\chi,b) \in [\underline{b}-N^I, \underline{b}) \\ \log P(q) & \text{if } z(\chi,b) \in [\underline{b}, \bar{b}-N^I] \\ \log 0 & \text{if } z(\chi,b) \in (\bar{b}-N^I, \bar{b}] \end{cases},
$$

where

$$
P(q) = \frac{1-q}{R} \left[\frac{1-[\alpha+(1-\alpha)E(q)]\delta\omega\beta}{1-[\alpha+(1-\alpha)E(q)]\delta\omega\beta-(1-2q)\delta\omega\beta} \right],
$$

with $\alpha = N^I/(\bar{b}-\underline{b})$. Recall that Assumption 3 ensures that $P(q) \in (0,1/R)$ for any $q \in [\underline{q}, \bar{q}]$. Taking logs of this expression and of equation (11) we can define

$$
\begin{aligned}
h(q) &\equiv \log P^B(q) = \log(1-q) - \log R, \\
g(q) &\equiv \log P(q) = \log(1-q) - \log R - \log\left[\frac{1-[\alpha+(1-\alpha)E(q)]\delta\omega\beta}{1-[\alpha+(1-\alpha)E(q)]\delta\omega\beta-(1-2q)\delta\omega\beta} \right].
\end{aligned}
$$

Next, differentiate the last two expressions with respect to q and obtain $h'(q) = -1/(1-q)$ and $g'(q) = -1/(1-q) + 2\beta\delta\omega/[1-\beta\delta\omega(1-2q+\alpha+(1-\alpha)E(q))]$. Assumptions 1 and 3 guarantee that $|g'(q)| > |h'(q)|$. Define $\bar{h} \equiv \int h(q)\,dF(q)$, $\bar{g} \equiv \int g(q)\,dF(q)$, and q_0 such that $g(q_0) = \int g(q)\,dF(q)$. Then

$$
Var(g(q)) = \int (g(q)-\bar{g})^2 \, dF(q) = \int (g(q)-g(q_0))^2 \, dF(q) > \int (h(q)-h(q_0))^2 \, dF(q),
$$

where the last inequality follows from $|g'(q)| > |h'(q)|$ and the monotonicity of both h and g. Moreover, from a standard property of the second moment, we can write

$$
\int (h(q)-h(q_0))^2 \, dF(q) = \int (h(q)-\bar{h})^2 \, dF(q) + \int (\bar{h}-h(q_0))^2 \, dF(q).
$$

Combining the last two expressions we then obtain $Var(g(q)) > \int (h(q)-\bar{h})^2 \, dF(q) + \int (\bar{h}-h(q_0))^2 \, dF(q) \geq Var(h(q))$. This implies that $Var(\log P(q,\chi,b)) > Var(\log P^B(q))$ whenever $z(\chi,b) \in [\underline{b}, \bar{b}-N^I]$. For any other $z(\chi,b) \notin [\underline{b}, \bar{b}-N^I]$, $\log P(q,\chi,b) = \log P^B(q)$, completing the proof that $Var(\log P(q,\chi,b)) \geq Var(\log P^B(q))$.

Proof of Proposition 3

From the proof of Proposition 2, it is straightforward that the amplification effect is going to be stronger, the higher is the absolute value of $d\log P(q)/dq$, or $|g'(q)|$ in the notation

of the proof. Such an object is larger, the larger is $y(q;x)$, where

$$y(q;x) \equiv 2\beta\delta\omega \left[1 - \beta\delta\omega \left(1 - 2q + \frac{N^I(x)}{\bar{b} - \underline{b}}(1 - E(q)) + E(q)\right)\right]^{-1},$$

where with some slight abuse of notation I define by $N^I(x)$ the equilibrium measure of informed employed managers as a function of the parameter x, which can be equal to κ, γ, or M^I. Recall that Assumption 3 is sufficient to ensure that $y(q;x) > 0$ for all $q \in [\underline{q}, \bar{q}]$. We can then differentiate this expression and obtain that for any given $q \in [\underline{q}, \bar{q}]$ we have $dy(q;x)/d\kappa = \left(dy/dN^I\right)\left(dN^I/dx\right)$, where

$$\frac{dy}{dN^I} = 2(\beta\delta\omega)^2 \left(\frac{1 - E(q)}{\bar{b} - \underline{b}}\right) \left[1 - \beta\delta\omega \left(1 - 2q + \frac{N^I(\kappa,\gamma)}{\bar{b} - \underline{b}}(1 - E(q)) + E(q)\right)\right]^{-2} > 0.$$

Next, note from (22) that N^I depends on γ and κ only through μ, so that $dN^I/d\kappa = \left(dN^I/d\mu\right)(d\mu/d\kappa)$ and $dN^I/d\gamma = \left(dN^I/d\mu\right)(d\mu/d\gamma)$, where $dN^I/d\mu = (1 - \delta)M^I/(1 - \delta(1 - \mu))^2 > 0$. Instead M^I affects N^I both directly and through μ, so that $dN^I/dM^I = \mu/[1 - \delta(1 - \mu)] + \left(dN^I/d\mu\right)(d\mu/dM^I)$. First, we can rewrite the implicit function for μ as $v(\mu;x) = 0$ with

$$v(\mu;x) \equiv \frac{x}{\mu} - R\left\{1 - \delta\omega\beta\left[E(q) + \frac{M^I(1 - E(q))}{(\bar{b} - \underline{b})\left(\delta + \frac{1-\delta}{\mu}\right)}\right]\right\}^{-1}, \tag{29}$$

where $x \equiv \kappa/\gamma$. Applying the implicit function theorem, we obtain $d\mu/d\kappa = -v_x/(\gamma v_\mu)$ and $d\mu/d\gamma = \kappa v_x/(\gamma^2 v_\mu)$. We can derive $v_x = 1/\mu$ and

$$v_\mu = -\frac{1}{\mu^2}\left[x + R\delta\omega\beta(1 - \delta)\frac{M^I(1 - E(q))}{(\bar{b} - \underline{b})\left(\delta + \frac{1-\delta}{\mu}\right)^2}\left\{1 - \delta\omega\beta\left[E(q) + \frac{M^I(1 - E(q))}{(\bar{b} - \underline{b})\left(\delta + \frac{1-\delta}{\mu}\right)}\right]\right\}^{-2}\right].$$

It is immediate that $v_x > 0$ and $v_\mu < 0$, so that $d\mu/d\kappa > 0$ and $d\mu/d\gamma < 0$. Combining these results with $dy/dN^I > 0$ and $dN^I/d\mu > 0$, we obtain $dy(q;\kappa)/d\kappa > 0$ and $dy(q;\gamma)/d\gamma < 0$. Next, we can rewrite the implicit function for μ as a function $v\left(\mu;M^I\right) = 0$, where $v\left(\mu;M^I\right)$ is equal to the right-hand-side of expression (29). Applying the implicit function theorem we now obtain $d\mu/dM^I = -v_{M^I}/v_\mu$, where

$$v_{M^I} = -\delta\omega\beta R\frac{(1 - E(q))}{(\bar{b} - \underline{b})\left(\delta + \frac{1-\delta}{\mu}\right)}\left\{1 - \delta\omega\beta\left[E(q) + \frac{M^I(1 - E(q))}{(\bar{b} - \underline{b})\left(\delta + \frac{1-\delta}{\mu}\right)}\right]\right\}^{-2}.$$

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After some algebra we can show that $dN^I/dM^I > 0$, so that $dy\left(q; M^I\right)/dM^I > 0$, completing the proof.

Proof of Proposition 5

Given that we assumed that there exist three functions $W\left(q, N^I\right)$, $\mu\left(q, N^I\right)$, and $G\left(q, N_I\right)$ satisfying equations (14), (15), and (17), the proof follows closely the proof of proposition 1. The only slightly different step is to prove that the investors' firing strategy is optimal, which we analyze next.

Here we show that Assumption 5 is sufficient to ensure that the belief that an employed manager is informed if he did not reveal to be uninformed is always higher than the probability that a newly hired manager is informed. That is, the posterior probability, $\eta_{i,t+1}$, that manager i is informed if $\sigma_{i,t} = 0$ and either $p_t = 1/R$ or $\theta_{i,t} = 1 - \chi_t$ is larger than the probability that an unemployed manager at time t is informed, ε_t. The proof follows closely the one for the iid case, except that now the job flows are not constant over time. First, consider an investor who has just hired manager i so that his prior belief $\eta_{i,t} = \varepsilon_t$. In this case, if $\sigma_{i,t} = 0$ and either $p_t = 1/R$ or $\theta_{i,t} = 1 - \chi_t$, then $\eta_{i,t+1} = \varepsilon_{t-1}/\left[\varepsilon_{t-1} + \left(1 - \xi_t^U\right)\left(1 - \varepsilon_{t-1}\right)\right]$. Next, we want to show that $\eta_{i,t+1} \geq \varepsilon_t$. This condition can be rewritten as (28) and, substituting for ε_t using expression (27), we obtain

$$1 - \xi_t^U \leq \frac{Z_t^U / \left(M^I - \delta N_t^I\right)}{Z_{t-1}^U / \left(M^I - \delta N_{t-1}^I\right)}, \tag{30}$$

where $N_{t+1}^U = \delta\left(1 - \xi_t\right)N_t^U + \mu_t Z_t^U$ and $N_t^U = 1 - N_t^I$. Hence, we can rewrite condition (30) as follows:

$$1 - \xi_t^U \leq \frac{1 - N_{t+1}^I - \delta\left(1 - \xi_t\right)\left(1 - N_t^I\right)}{1 - N_t^I - \delta\left(1 - \xi_{t-1}\right)\left(1 - N_{t-1}^I\right)} \frac{\mu_t\left(M^I - \delta N_{t-1}^I\right)}{\mu_{t+1}\left(M^I - \delta N_t^I\right)}.$$

Given that $\xi_t^U \in [1 - \omega, 1]$ and $\mu_t = \kappa/W_t$, a sufficient condition is then

$$\omega \leq \left[\frac{1 - N_{t+1}^I}{1 - N_t^I} - \delta\omega\right] \frac{W_{t+1}}{W_t} \frac{M^I - \delta N_{t-1}^I}{M^I - \delta N_t^I}.$$

where $N_t^I \in \left[0, M^I\right]$. From expression (17), it is straightforward that $W_t \in \left[\gamma R, \gamma R/\left(1 - \delta\omega\beta\right)\right]$ and hence a stricter condition is $\omega \leq \left[1 - M^I - \delta\omega\right]\left(1 - \delta\right)\left(1 - \delta\omega\beta\right)$, which ensures that Assumption 5 is sufficient for condition (30) to be satisfied. A similar argument to the iid case applies when managers have been employed for more than 1 period, completing the proof.

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