DISCUSSION PAPER SERIES

No. 8439

ANIMAL SPIRITS, RATIONAL BUBBLES AND UNEMPLOYMENT IN AN OLD-KEYNESIAN MODEL

Roger E A Farmer

INTERNATIONAL MACROECONOMICS

Centre for Economic Policy Research

www.cepr.org

Available online at: www.cepr.org/pubs/dps/DP8439.asp

ISSN 0265-8003

ANIMAL SPIRITS, RATIONAL BUBBLES AND UNEMPLOYMENT IN AN OLD-KEYNESIAN MODEL

Roger E A Farmer, UCLA and CEPR

Discussion Paper No. 8439
June 2011

Centre for Economic Policy Research
77 Bastwick Street, London EC1V 3PZ, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

CEPR Discussion Paper No. 8439

June 2011

ABSTRACT

Animal Spirits, Rational Bubbles and Unemployment in an Old-Keynesian Model*

This paper presents a model of the macroeconomy in which any unemployment rate may be a steady-state equilibrium and every equilibrium unemployment rate is associated with a different value for the price of assets. To select an equilibrium, I construct a theory in which asset price bubbles are caused by the self-fulfilling animal spirits of market participants, selected by a belief function. In contrast to my earlier work on this topic, asset prices may be unbounded. All of the actors in my model have rational expectations and the asset price bubbles that occur are individually rational, even though the equilibria of the model are socially inefficient. My work opens the door for a new class of theories in which market psychology, captured by the belief function, plays an independent role in helping us to understand economic crises.

JEL Classification: E00, E12 and E24
Keywords: Animal Spirits, Old-Keynesian, rational bubbles and unemployment

Roger E A Farmer
Department of Economics
UCLA
Bunche Hall 8283
Box 951477
Los Angeles, CA 90095-1477
USA
Email: rfarmer@econ.ucla.edu

* I wish to thank Pietro Peretto of Duke University for his insightful comments and suggestions on an earlier paper. Thanks also to Dmitry Plotnikov of UCLA for his valuable feedback and comments on this paper.

Submitted 06 June 2011

1 Introduction

The Great Recession of 2008 has presented economists with data that are difficult to explain using the equilibrium macroeconomic models that dominated the profession for the past three decades. It is common to read accounts in the financial press that attribute the 2008 financial crisis to the bursting of an asset price bubble. Although many economists find that to be a plausible description of what happened, there is no universally accepted definition of a bubble and little or no understanding of how a collapse in asset prices might be the cause of a large increase in the unemployment rate.

In a series of recent books and papers, Farmer (2008a,b, 2009, 2010a,b,c,d); Farmer and Plotnikov (2010), I have developed a theory of unemployment that preserves two important insights from Keynes' 1936 book *The General Theory of Employment Interest and Money.* Using the language of general equilibrium theory, the first insight is that any unemployment rate may prevail as a steady-state equilibrium; the second is that 'animal spirits' select which equilibrium is chosen.

In my previous work (Farmer, 2009, 2010b) I modeled animal spirits by adding an equation to the economic model to represent market psychology. I call this equation the *belief function.* Here, I model the belief function as a three state Markov chain and I show how bubbles and crashes, driven by self-fulfilling waves of optimism and pessimism, may generate booms and crashes in economic activity. In contrast to models in which agents behave irrationally, all of the actors in my models are individually rational and have rational expectations of the future.

But although the agents in my model are individually rational, there are search frictions in the labor market that cause equilibria to be socially inefficient. As a consequence, there are many possible unemployment rates that are consistent with market clearing and rational choice by households and firms. I resolve this indeterminacy by introducing the animal spirits of market participants to select an economic equilibrium and I formalize this idea by introducing the belief function as a new fundamental.[1] My work opens the door to a new class of theories in which market psychology plays an independent role in helping us to understand economic crises.

2 Bubbles and Crashes

2.1 Bubbles and Crashes in the Data

At the end of the 1990s, the repeal of the Glass-Steagal Act was followed by a period of financial deregulation and the creation of new financial instruments that allowed speculators to place bets on the housing market that were previously unavailable. Mortgages that had traditionally been held by the banks that originated them were packaged and sold as mortgage backed securities in organized financial markets.

The creation of mortgage backed securities and other forms of collateralized debt obligations was accompanied by a new instrument; the credit default swap, that allowed investors to speculate on the direction of house price movements. At the same time these new securities were created there was an unprecedented increase in house prices in the United States that was not accompanied by increases in rents or by changes in any of the fundamentals one might normally associate with the housing market.[2] Popular accounts of the 2008 financial crisis attribute the increase and subsequent collapse of house prices to the bursting of an asset price bubble.

Figure 1 illustrates the history of house prices, the stock market and unemployment in the U.S. since 1990. From 1990 through 1995 the Case-Shiller home price index increased at a rate of 1% per year, from 76 in the first quarter of 1990 to 80 in the fourth quarter of 1995.[3] Beginning in

[1] In related work, Farmer (2010a), I estimate a three equation monetary model closed with a belief function and I show that it fits the data better than a new-Keynesian model, closed with a Phillips curve.

[2] Kashiwagi (2010) documents these facts in his Ph.D. thesis.

[3] Data is from Robert Shiller, http://www.econ.yale.edu/~shiller/data.htm.

the first quarter of 1996, house price appreciation began to accelerate and the Case-Shiller index reached a peak of 190 in the second quarter of 2006. Between 1995 and 2006 the index grew at an annualized rate of 8%. In the third quarter of 2006, U.S. house prices began to fall for the first time in a century of data and by 2009Q1 the Case Shiller index had fallen by 42% to a value of 129, a value it last attained in 2003.

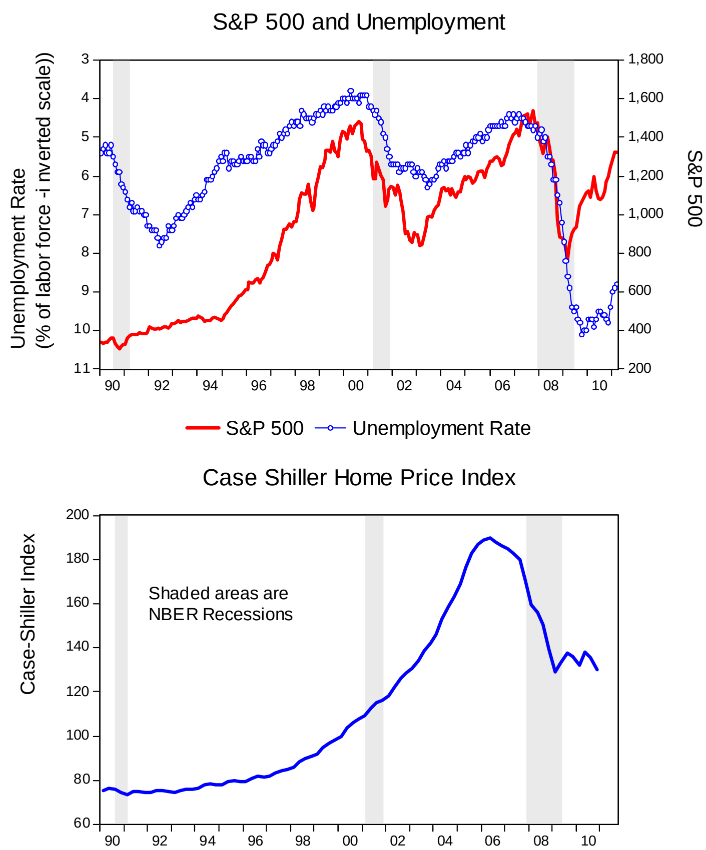

Figure 1: Stock Prices, House Prices and Unemployment Since 1990

The fall in house prices was quickly followed by a stock market crash as the S&P 500 fell from a peak of 1540 in October of 2007 to a trough of 757 in March of 2009, a drop of 56%. The collapse in the value of U.S. financial assets was accompanied by a doubling of the unemployment rate which went from 4.8% in October of 2007 to 10.1% in November of 2009.

Macroeconomic theory, of the kind that has dominated the profession for the past thirty years, cannot easily account for either the huge fall in asset values or the increase in unemployment that followed. This paper can explain both.

2.2 Bubbles and Crashes in Economic Theory

There is no widely accepted definition of a bubble. Some authors say that a bubble occurs if an asset trades at a value other than its 'fundamental value'. That definition allows for a bubble to exist, even if the price of an asset is constant; for example, the value of money in an overlapping generations model is sometimes referred to as a bubble.

In the popular imagination, a bubble is associated with large run-ups of asset prices that eventually burst and lead to considerable economic disruption. Examples include the tulip mania of the seventeenth century, the South Sea Bubble of the early eighteenth century and most recently the financial crisis of 2008.[4]

The academic economics literature on bubbles is vast. Tirole (1985) studied asset bubbles in overlapping generations models and Diba and Grossman (1988) study bubbles in infinite horizon economies. Santos and Woodford (1997) argue that rational bubbles can be ruled out in a wide variety of competitive environments. Abreu and Brunnermeier (2003) explain the existence of bubbles by introducing a subset of irrational agents and Martin and Ventura (2011a; 2011b) study rational bubbles in an overlapping generations model with credit frictions. None of these papers have convincing explanations of why the crash of an asset price bubble should lead to a large increase in the unemployment rate.

Standard labor search models (see the survey by Rogerson, Shimer, and Wright (2005)) are closed in a variety of ways. One way to proceed is to as-

[4] For a description of early examples of bubbles see the 1841 book by Charles Mackay, *Extraordinary Popular Delusions and the Madness of Crowds.*

sume that, when a firm meets a worker, the worker and the firm bargain over the real wage. This approach introduces an additional equation, the Nash bargaining equation, and an additional parameter, the bargaining weight, to close the model. An alternative literature on *competitive search*, for example, Moen (1997), assumes that a set of market makers compete to attract firms and workers by posting wages.

In Farmer (2008a,b, 2009, 2010b) I close a standard search model in a new way.[5] My work drops the wage bargaining equation from a labor contracting model and replaces it with the assumption that animal spirits are an independent driving force of business cycles. I model this by introducing a belief function as a fundamental to describe how beliefs of the future depend on observations of the past. The belief function shifts aggregate demand and aggregate demand determines the unemployment rate in a steady state equilibrium. This assumption is a significant departure from conventional economic theory.

My approach has recently been followed by Kashiwagi (2010), and Kocherlakota (2011). Kashiwagi studies bubbles in the housing market, but in his work there is an upper bound on asset prices and no mechanism to show how the bursting of a bubble in the housing market can impact the labor market. Kocherlakota (2011) combines an overlapping generations model with infinite horizon risk-neutral workers. He shows that bubbles in the asset markets may spillover to the labor market and generate increased unemployment.

By dropping the bargaining equation from a standard search model, I provide an economic environment in which market psychology, modeled by the belief function, exerts an independent influence on economic outcomes, not just in the short run as in the literature on sunspots described in my

[5] My work is closest to work by Hall (2005) which takes the real wage as an exogenous process, determined by a social norm. Unlike Hall, I assume instead, that firms produce as much as is demanded and that variations in stock market values, captured by the belief function, drive aggregate demand. In my work, the real wage adjusts to generate a zero profit equilibrium.

(1993) book and surveyed in Benhabib and Farmer (1999), but also in the *steady state*.

2.3 What's New in this Paper? A Model of Bubbles, Crashes and Depressions

This paper makes two alterations to the environment studied in Farmer (2009). There, I made two important assumptions. First, I assumed that workers are fired and rehired in every period. Second, I assumed that preferences were logarithmic and technology was Cobb-Douglas. Using those assumptions, I was able to show that there exists an equilibrium for any employment rate in the interval $[0, 1]$. For every value of the unemployment rate there was an associated asset price. But in Farmer (2009), the assumptions I made about preferences and technology implied that asset prices were bounded.

In this paper I generalize my previous work to the case of constant relative risk aversion preferences and CES technologies and I study a dynamic version in which labor is a state variable. In this more general model I show that there is a steady state equilibrium for every value of employment in an interval $[0, \mu)$ where $\mu < 1$, and further, if the technology has an elasticity of substitution between 0 and 1 then *asset prices are unbounded.*

Using these more general assumptions about preferences and technology, I provide a simple example in which there may exist asset price bubbles that are associated with significant movements in the unemployment rate. These bubbles are rational, in the sense that all agents have rational expectations and all markets clear, but the bursting of a bubble is associated with high unemployment that is socially inefficient.[6]

[6] As a benchmark against which to compare different equilibria, Appendix A solves the problem of a social planner and shows that the planning problem has a unique efficient unemployment rate in the steady state.

3 A Decentralized Equilibrium

In this section I will describe the environment and the behavior of households and firms in a decentralized search equilibrium.

3.1 Households

There is a continuum of identical households, each of whom derives utility from consumption of a unique commodity, C_t, Households maximize utility,

$$J = E_s \left\{ \sum_{t=s}^{\infty} \beta^{t-s} \frac{C_t^{1-\eta}}{1-\eta} \right\}, \tag{1}$$

subject to the constraints

$$p_{k,t} K_{t+1} + p_t C_t \leq \left(p_{k,t} + rr_t \right) K_t + w_t L_t, \tag{2}$$

$$H_t \leq 1 - L_t, \tag{3}$$

$$L_{t+1} = L_t \left(1 - \delta \right) + \tilde{q}_t H_t. \tag{4}$$

Here, K_t is capital, L_t is employment, w_t is the money wage, p_t is the money price of commodities, $p_{k,t}$ is the money price of capital and and rr_t is the rental rate. Equation (4) represents the assumption that if H_t workers search, $\tilde{q}_t H_t$ of them will find a job where the fraction $\tilde{q}_t$ is determined in equilibrium by the aggregate search technology.

Since we will need to value streams of payments I will assume that there exists a complete set of Arrow securities, one for each realization of S_t. The price at date t of a dollar delivered for sure at date τ in history $S^\tau \equiv \{S_t, S_{t+1}, ...S_\tau\}$ is given by the expression

$$Q_t^\tau = \frac{\beta p_t}{p_\tau} \left(\frac{C_\tau}{C_t} \right)^{-\eta}, \tag{5}$$

where I have suppressed the dependence of Q_t^τ on the history S^τ.

Using this definition, the transversality condition can be written as

$$\lim_{T\to\infty} Q_t^T p_{k,T} K_{T+1} = 0, \quad \text{for all histories } S^T. \tag{6}$$

Since leisure does not yield disutility, households will choose,

$$H_t = 1 - L_t, \tag{7}$$

which implies that all unemployed workers search for a job. Substituting this expression into (4) gives

$$L_{t+1} = L_t\left(1-\delta\right) + \tilde{q}_t\left(1-L_t\right). \tag{8}$$

In addition, the household will allocate resources through time optimally. That assumption leads to the following consumption Euler equation,

$$C_t^{-\eta} = E_t\left\{\beta C_{t+1}^{-\eta}\frac{p_t}{p_{t+1}}\left(\frac{p_{k,t+1}+rr_{t+1}}{p_{k,t}}\right)\right\}. \tag{9}$$

3.2 The Production Technology

The consumption commodity is produced by the technology

$$C_t = \left[b\left(S_t X_t\right)^\rho + aK_t^\rho\right]^{\frac{1}{\rho}}, \qquad a+b=1 \tag{10}$$

where X_t is labor used in production, K_t is capital and S_t is a labor augmenting technology shock.

I further assume that

$$1 > \rho > 0. \tag{11}$$

This assumption places the technology on the linear side of the CES class and I will show in Proposition 2 that the assumption that inequality (11) holds is sufficient to guarantee the existence of equilibria in which the relative price of capital is unbounded. This is important since it permits the existence of rational stochastic bubbles in which asset prices follow explosive trajectories that burst with some probability. In contrast, in the model described in Farmer (2009), the price of assets is bounded.

3.3 Firms in a Search Model

Each firm solves the following problem,

$$\max_{\{K_t,V_t,X_t,L_t\}} E_s \left\{ \sum_{t=s}^{\infty} Q_s^t \left(\left[b\left(S_t X_t\right)^{\rho} + a K_t^{\rho} \right]^{\frac{1}{\rho}} - \frac{w_t}{p_t} L_t - \frac{rr_t}{p_t} K_t, \right) \right\} \tag{12}$$

subject to the constraints,

$$L_t = X_t + V_t, \tag{13}$$

$$L_{t+1} = L_t \left(1 - \delta\right) + q_t V_t. \tag{14}$$

Constraints (13) and (14) hold for all $t = s, \dots$. The sequences of money prices $\{p_t\}$, money wage $\{w_t\}$, money rental rates $\{rr_t\}$ and the present value prices $\{Q_s^t\}$, are taken as given where all variables are contingent on the histories of shocks. In addition, the firm takes the sequence of search efficiencies of a recruiter, $\{q_t\}$ as given.

Using equations (12) – (14) we may write the following Lagrangian for problem (12).

$$\max E_s \sum_{t=s}^{\infty} \left\{ Q_s^t \left(\left[b S_t^{\rho} \left(L_t - V_t\right)^{\rho} + a K_t^{\rho} \right]^{\frac{1}{\rho}} - \frac{w_t}{p_t} L_t - \frac{rr_t}{p_t} K_t \right.\right.$$
$$\left.\left. + \ \psi_t \left[\left(1 - \delta\right) L_t + q_t V_t - L_{t+1} \right] \right) \right\}.$$

This expression is maximized when

$$a \left(\frac{C_t}{K_t} \right)^{1-\rho} = \frac{rr_t}{p_t}, \tag{15}$$

$$bS_t^{\rho}\left(\frac{C_t}{L_t - V_t}\right)^{1-\rho} = \psi_t q_t, \tag{16}$$

and

$$\psi_t = E_s\left\{Q_t^{t+1}\left(b\left(\frac{C_{t+1}}{(L_{t+1} - V_{t+1})}\right)^{1-\rho} - \frac{w_{t+1}}{p_{t+1}} + \psi_{t+1}(1-\delta)\right)\right\}. \tag{17}$$

The equations

$$C_t = \left[bS_t^{\rho}(L_t - V_t)^{\rho} + aK_t^{\rho}\right]^{\frac{1}{\rho}}, \tag{18}$$

and

$$L_{t+1} = L_t(1-\delta) + q_t V_t, \tag{19}$$

must also hold. In addition, any optimal path must satisfy the transversality condition

$$\lim_{T\to\infty} Q_t^T \psi_T = 0 \text{ for all histories } S^T. \tag{20}$$

3.4 How q and $\tilde{q}$ are Determined in a Search Model

The variables $\tilde{q}_t$ and q_t, are determined in equilibrium by market clearing in the markets for search inputs. Let a variable with a bar denote an economy-wide average. Using this notation, $\bar{L}_t$ is the measure of aggregate employment and L_t is the measure of workers hired by the representative firm. These variables are conceptually distinct although they turn out to be equal in equilibrium.

Each period I assume that in aggregate, a measure

$$\bar{m}_t = \left(\Gamma\bar{V}_t\right)^{\theta}\left(1 - \bar{L}_t\right)^{1-\theta}, \tag{21}$$

of workers is hired, where Γ measures the efficiency of the match process and θ measures the elasticity of the recruiting effort by firms. This parameter can be identified in data from estimates of the Beveridge curve. Using U.S. data, Blanchard and Diamond (1990) found estimates of θ to be between 0.3

and 0.5. Since setting $\theta = 0.5$ will simplify some of the algebra of the model, I will make that assumption from this point on. In addition, I assume that a measure δL_t of workers lose their jobs for exogenous reasons.

Together, these assumptions imply that the labor force in period $t+1$ will be given by the expression

$$\bar{L}_{t+1} = \bar{L}_t (1-\delta) + \left(\Gamma \bar{V}_t\right)^{\frac{1}{2}} \left(1 - \bar{L}_t\right)^{\frac{1}{2}}. \tag{22}$$

Since (8) and (19) must also hold in a symmetric equilibrium it follows that

$$q_t = \Gamma^{\frac{1}{2}} \left(\frac{1-\bar{L}_t}{\bar{V}_t}\right)^{\frac{1}{2}}, \tag{23}$$

and

$$\tilde{q}_t = \Gamma^{\frac{1}{2}} \left(\frac{\bar{V}_t}{1-\bar{L}_t}\right)^{\frac{1}{2}}. \tag{24}$$

4 Characterizing Equilibrium

In this section I will lay out the equations that characterize behavior in a symmetric equilibrium of the model and I will study the behavior of a special class of steady state equilibria. I will show that this model possesses a continuum of steady state equilibria for any level of employment in an open interval $L \in [0, \mu)$ where $\mu < 1$.

4.1 The Equations of the Model

The following eight equations characterize the competitive equilibrium conditions. Equations (25) and (26) represent the Euler equation and the pricing kernel.

$$C_t^{-\eta} = E_t \left\{ \beta C_{t+1}^{-\eta} \frac{p_t}{p_{t+1}} \left(\frac{p_{k,t+1} + rr_{t+1}}{p_{k,t}} \right) \right\}, \tag{25}$$

$$Q_t^{t+1} = \frac{\beta p_t}{p_{t+1}} \left(\frac{C_{t+1}}{C_t} \right)^{-\eta}. \tag{26}$$

The next four equations combine optimizing behavior by firms with the search equilibrium condition (23),

$$\psi_t = E_t \left\{ Q_t^{t+1} \left(\psi_{t+1} q_{t+1} - \frac{w_{t+1}}{p_{t+1}} + \psi_{t+1} (1-\delta) \right) \right\}, \tag{27}$$

$$\frac{rr_{t+1}}{p_{t+1}} = a \left(C_{t+1} \right)^{1-\rho}, \tag{28}$$

$$bS_t^{\rho} \left(\frac{C_t}{L_t - V_t} \right)^{1-\rho} = \psi_t q_t, \tag{29}$$

$$L_{t+1} = L_t (1-\delta) + (\Gamma V_t)^{\frac{1}{2}} (1 - L_t)^{\frac{1}{2}}. \tag{30}$$

Here, ψ_t is the shadow price of labor and q_t is given by the labor market search technology as

$$q_t = \Gamma^{\frac{1}{2}} \left(\frac{1 - L_t}{V_t} \right)^{\frac{1}{2}}. \tag{31}$$

Finally, since I assume that $K_t = 1$, the production function,

$$C_t = \left[bS_t^{\rho} (L_t - V_t)^{\rho} + a \right]^{\frac{1}{\rho}}, \tag{32}$$

must hold in aggregate.

These eight equations must determine the nine unknowns,

$$y_t \equiv \left\{ C_t, \;\; L_t, \;\; V_t, \;\; \frac{rr_t}{p_t}, \;\; \frac{w_t}{p_t}, \;\; \frac{p_{k,t}}{p_t}, \;\; Q_t, \;\; q_t, \;\; \psi_t \right\}. \tag{33}$$

The fact that there is one less equation than unknown arises from the absence of markets to allocate search intensity between the time of searching workers and the recruiting activities of firms, a point first made by Greenwald and Stiglitz (1988).

Farmer (2009) proposed closing the model with the function

$$E_t\left[\frac{p_{k,t+1}}{p_{t+1}}\right] = x_t, \tag{34}$$

where x_t is a process that represents how beliefs are influenced by economic events. In this paper I modify that approach and I assume instead that

$$E_t\left[\frac{p_{k,t+1}}{w_{t+1}}\right] = x_t. \tag{35}$$

I will show that this model admits equilibria in which there may exist self-fulfilling asset market bubbles and crashes, where bubbles are in real asset prices using the money wage as the numeraire.

4.2 Steady State Equilibria

In Farmer (2009) I showed, in a static version of this model with a Cobb-Douglas technology and logarithmic preferences, that there is a steady state equilibrium for any value of L in the interval $[0, 1]$. In that model, for each equilibrium value of L, there is a different asset price $p_{k,t}$, but asset prices are bounded above.

In this paper I have built a dynamic model in which labor is a state variable. The purpose of introducing labor as a state variable is to show that there is nothing special about the timing assumption in my earlier work. In the more general framework, any employment rate in an interval $[0, \mu)$ can exist as a steady state equilibrium. The purpose of relaxing the assumption of Cobb-Douglas technology and logarithmic preferences is to show that there will be an equilibrium in which the price of capital is unbounded.

The following definitions and propositions extend my previous work to a dynamic model with more general preferences and technologies and show that, in equilibrium, asset prices are unbounded. I begin by defining a steady state equilibrium.

Definition 1 *A Non-Stochastic Steady State Equilibrium is a vector* $\left\{C, L, V, \frac{rr}{p}, \frac{w}{p}, \frac{p_k}{p}, Q, q, \psi\right\}$ *that solves the equations*

$$\frac{p_k}{w} = \frac{x}{w}, \tag{36}$$

$$\frac{C^{1-\rho}p}{p_k} = \frac{1-\beta}{a\beta}, \tag{37}$$

$$Q = \beta, \tag{38}$$

$$\psi\left(1-\beta\left(1-\delta\right)\right) = \beta q\psi - \beta\frac{w}{p}, \tag{39}$$

$$\frac{rr}{p} = aC^{1-\rho}, \tag{40}$$

$$b\left(\frac{C}{L-V}\right)^{1-\rho} = \psi q, \tag{41}$$

$$\delta^2 L^2 = \Gamma V\left(1-L\right), \tag{42}$$

$$q = \Gamma^{\frac{1}{2}}\left(\frac{1-L}{V}\right)^{\frac{1}{2}}, \tag{43}$$

$$C = \left[b\left(L-V\right)^{\rho} + a\right]^{\frac{1}{\rho}}. \tag{44}$$

These equations are derived from Equations (25) – (32) by assuming that $S_t = 1$ for all t and solving the resulting non-stochastic equations for a steady state.

Proposition 1 *Define the constants* λ, μ *and* Ω *as follows*

$$\lambda = \frac{\Gamma}{\Gamma+\delta^2}, \quad \mu = \frac{\beta\Gamma}{\beta\Gamma+\delta\left(1-\beta\left(1-\delta\right)\right)},$$
$$\Omega = \left(\frac{a\beta}{1-\beta}\right)\frac{\Gamma^{\rho}\left(\Gamma+\delta^2\right)^{1-\rho}}{\beta\Gamma+\delta\left(1-\beta\left(1-\delta\right)\right)}\left(\frac{\beta}{b}\right). \tag{45}$$

For all $L \in [0, \mu)$, *there exists a steady state equilibrium. The values of the endogenous variables* Q, C, V *and* q, *for each value of* L *are given by the* *expressions*

$$
\begin{aligned}
Q &= \beta, \qquad\qquad C = \left(bL^{\rho} \left(1 - \frac{\delta^2 L}{\Gamma (1-L)} \right)^{\rho} + a \right)^{\frac{1}{\rho}}, \\
V &= \frac{\delta^2 L^2}{\Gamma (1-L)}, \qquad q = \Gamma^{\frac{1}{2}} \left(\frac{1-L}{V} \right)^{\frac{1}{2}}, \qquad (46)
\end{aligned}
$$

and the values of the variable $\frac{rr}{p}, \psi$ *and* $\frac{w}{p}$ *are computed from (28), (29) and (27). The price of capital, measured in wage units is described by a continuous function:* $g(L) : [0, \mu) \to \tilde{P} \subset R_+$ *where*

$$
\frac{p_k}{w} = g(L) \equiv \frac{\Omega L^{1-\rho} (1-L)^{\rho} [\lambda - L]^{1-\rho}}{\mu - L}. \qquad (47)
$$

Proposition 2 *If* $0 < \rho < 1$, $\tilde{P} \equiv R_{+,}$ *and the function* g *is strictly increasing with*

$$
g(0) = 0, \quad g(\mu) = \infty. \qquad (48)
$$

By the inverse function theorem there exists a function $h(x) = R_+ \to [0, \mu)$ *such that for all* $x \in R_+$ *there exists a steady state equilibrium where*

$$
L = h(x). \qquad (49)
$$

The vector of endogenous variables y_t *defined in (33) is determined as in Proposition 1.*

Proposition 1 establishes that the equations that define a steady state equilibrium have a solution for a set of values of L less than or equal to some maximum value μ.[7]

Proposition 2 goes further. It shows that L and p_k are related by a monotonically increasing function and that when $L = 0$, $p_k = 0$ and that p_k becomes infinite as L attains it's upper bound.

[7] The parameter Γ measures the efficiency of the match process. As Γ approaches ∞, the set of sustainable equilibrium employment rates approaches the interval $[0, 1]$.

4.3 The Role of Animal Spirits and the Belief Function

Proposition 2 is important since if g is invertible, we may select equilibria by defining a new fundamental that represents the market psychology of participants.

Let

$$E_t\left[p_{k,t+1}\right] = x_t, \tag{50}$$

and let x_t be described by a time series process such that $x_t \in R_+$. To represent this process, I propose to supplement the model by adding a *belief function* to represent the way market psychology influences beliefs about asset values. The belief function is not an alternative to the rational expectations assumption: *it is in addition to it.*

For example, the belief function might be determined entirely by non-economic variables, as in Equation (51),

$$x_t = F\left(x_{t-1}\right). \tag{51}$$

But there is no reason to suppose that market psychology is independent of economic fundamentals. One could assume that

$$x_t = F\left(x_{t-1}, L_{t-1}\right), \tag{52}$$

and indeed, the belief function F might depend on additional economic variables in arbitrarily complicated ways. The form of F in practice is an empirical question. Different assumptions about human psychology will imply different forms for this function and each of them will have different implications for the movements of unemployment, asset prices and all of the other economic variables that we observe in data.

5 A Simplified Model

I have shown that this model may contain many different equilibria and I have argued that there is a role for market psychology to determine outcomes. This section studies a special case of the model and constructs an example of an equilibrium in which rational bubbles and crashes in the asset markets are associated with expansions and contractions of employment.

5.1 A Simplifying Assumption: Allowing Workers to Recruit Themselves

Consider a version of this model in which the labor force is fired and rehired in every period. This is the same timing assumption studied in Farmer (2009) but with more general preferences and technology. This timing assumption leads to the following expression to determine aggregate employment,

$$\bar{L}_t = \left(\Gamma \bar{V}_t\right)^{\frac{1}{2}} \left(H_t\right)^{\frac{1}{2}}, \tag{53}$$

where

$$H_t = 1. \tag{54}$$

In this version of the model, firms decide each period on a plan $\{V_t, X_t\}$ which specifies how many workers will be allocated to recruiting and how many to production. Notice that since firms begin each period with no workers; I am allowing workers to recruit themselves. This fiction is well worth adopting since it allows me to describe equilibria in closed form.

5.2 The Theory of Aggregate Supply

In Farmer (2009) I studied a special case of this model where $\rho = 0$ and $\eta = 1$. In that case, by choosing $w_t = 1$ as the numeraire, I showed that one can derive an 'aggregate supply' equation that describes how the value of the output of the economy, measured in wage units, is related to aggregate employment.[8] That equation took the form,

$$Z_t = \frac{1}{b} L_t, \tag{55}$$

where, in Farmer (2009), I defined

$$Z_t = p_t C_t, \tag{56}$$

to be equal to GDP, measured in wage units. The generalization of Equation (56) to an economy with more general preferences and technology (derived in Appendix D), is the expression,

$$Z_t \equiv p_t C_t^{1-\rho} = g\left(L_t\right), \tag{57}$$

where

$$g\left(L_t\right) \equiv \frac{\Gamma^{\rho} L_t^{1-\rho}}{b S_t^{\rho} \left(\Gamma - L_t\right)^{\rho}}. \tag{58}$$

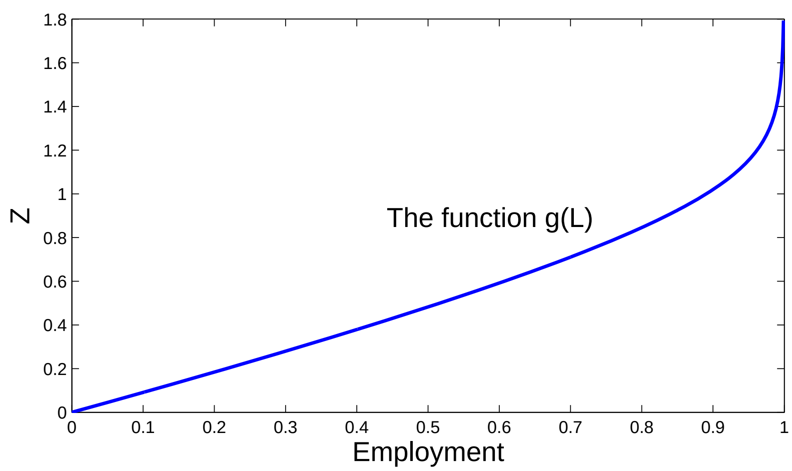

Figure 2: The Function $g\left(L\right)$

[8]See Farmer (2008a) for a discussion of this concept of aggregate supply and it's relationship to Keynes' definition in *The General Theory*.

The function $g(L)$ is graphed in Figure 2 for the case $\Gamma = 1$. This function is approximately linear for most of its domain, but asymptotes to infinity as L_t approaches Γ, where Γ is the maximum feasible value of employment.

The fact that Z_t may be unbounded will be important for a theory of asset market bubbles since I will show that Z_t can be described by a sequence of self-fulfilling beliefs about asset prices. The fact that there exists a steady state equilibrium for all non-negative values of p_k means that it is perfectly rational for investors to keep bidding up the price of an asset since for any value of p_k there is a higher value that is consistent with rational behavior and market clearing.

5.3 The Theory of Aggregate Demand

To close this model we will need a theory of what determines aggregate demand, a term that I associate with the variable Z_t. This section provides that theory by solving the agent's Euler equation to find a relationship between asset prices and aggregate demand.

Consider the Euler equation,

$$C_t^{-\eta} = E_t \left\{ \beta C_{t+1}^{-\eta} \frac{p_t}{p_{t+1}} \left(\frac{p_{k,t+1} + rr_{t+1}}{p_{k,t}} \right) \right\}. \tag{59}$$

We may combine this expression with the first order condition for profit maximization,

$$\frac{rr_t}{p_t} = a C_t^{1-\rho}, \tag{60}$$

to generate the expression,

$$\frac{p_{k,t}}{C_t^{\eta} p_t} = E_t \left\{ \frac{\beta p_{k,t+1}}{p_{t+1} C_{t+1}^{\eta}} + a\beta C_{t+1}^{1-\rho-\eta} \right\}. \tag{61}$$

Recall that we defined

$$Z_t = p_t C_t^{1-\rho}, \tag{62}$$

and consider the relationship between $p_{k,t}$ and Z_t that holds in a non-stochastic steady state equilibrium. In that case, we may cancel terms in $C_t^{-\eta}$, $p_{k,t}$ and p_t from both sides of Equation (62) to derive the following expression,

$$Z = C^{1-\rho} p = p_k \frac{1-\beta}{a\beta}. \tag{63}$$

If we are willing to impose the parametric assumption

$$\eta = 1 - \rho, \tag{64}$$

we can make a much stronger statement. In that case we may iterate Equation (61) forwards and by imposing a transversality condition we arrive at the expression

$$Z_t = p_{k,t} \frac{1-\beta}{a\beta}. \tag{65}$$

In other words, for this parametric restriction, Equation (63) holds *at all points in time.*[9] I will impose the restriction implied by Equation (64) in the remainder of the paper, since it will allow me to construct a very simple example of rational bubbles.

[9] This follows from noting that if $1-\rho = \eta$ then Equation (61) takes the form

$$x_t = \beta E_t \{x_{t+1} + \alpha\}$$

where

$$x_t = \frac{p_{k,t}}{C_t^{\eta} p_t} \equiv \frac{p_{k,t}}{C_t^{1-\rho} p_t}.$$

For $|\beta| < 1$ this equation has the unique solution

$$x_t = \frac{\alpha\beta}{1-\beta}.$$

6 Modeling Rational Bubbles with the Belief Function

To model asset price bubbles, I will assume that asset prices are determined by animal spirits that are themselves generated by market psychology. In my work, an asset price bubble results from a particular belief function that describes how the asset price evolves over time.

6.1 The Wage as Numeraire

In my previous work, Farmer (2009), I used p_t as the numeraire. Here, I will choose w_t. This allows me to describe asset price bubbles as self-fulfilling sequences of asset market prices *relative to the price of labor*. The important point is that the bubbles are in *real* asset prices. An economy experiencing an asset price bubble, in this sense, would also experience a bubble in goods prices relative to the money wage.

To capture the dependence of beliefs on market psychology, I will assume that

$$E_t\left[\frac{p_{k,t+1}}{w_{t+1}}\right] = x_t, \tag{66}$$

and I will represent a bubble as a sequence of the form

$$x_t = \alpha_{s_t} + \gamma_{s_t} x_{t-1} + \varepsilon_t \tag{67}$$

where $\left\{\alpha_{s_t}, \gamma_{s,t}\right\}$ are parameters that are governed by a Markov chain s_t. In this example, market psychology is independent of economic fundamentals and the belief function is described by the parameters of the Markov chain.

6.2 Rational Bubbles and Business Cycles

Figure 3 plots the value of asset prices and unemployment in the U.S. for the period from 1990 through 2011. The asset price series is a weighted average of the Case-Shiller house price index and the S&P 500 with a weight of 2/5 on the house price index and 3/5 on the stock market. These weights are consistent with the fractions of assets held as houses, as opposed to factories and machines, in the Flow of Funds accounts reported by the Federal Reserve Board.

This figure illustrates two things. First, not all movements of the unemployment rate can be explained by movements in wealth. The 1991 recession, for example, shows no contemporaneous movement in asset values. Second, *some* recessions *are* associated with movements in wealth. The 2000 and 2008 recessions were both accompanied by collapses in asset price bubbles.

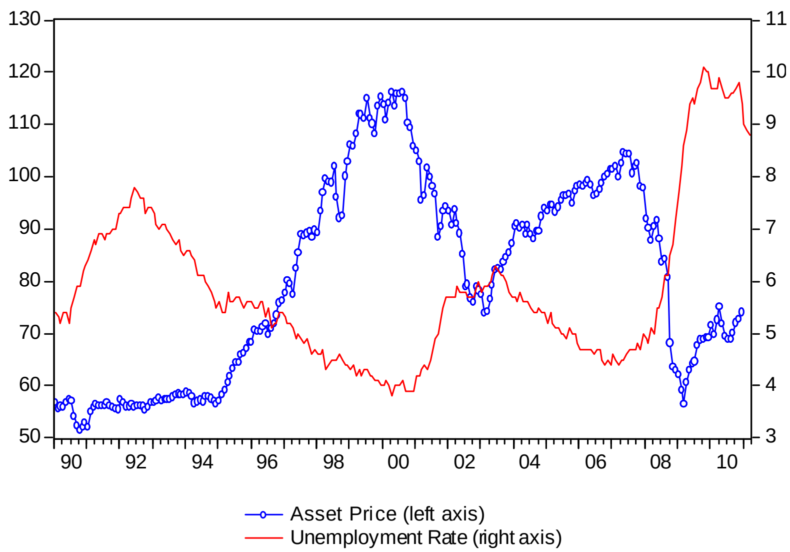

Figure 3: Wealth and Unemployment in the US Since 1990

It should come as no surprise that not all business cycles have the same cause. In his 1929 book *Industrial Fluctuations*, Pigou cited at least six different causes of business cycles including productivity shocks, monetary disturbances, agricultural disturbances, strikes, industrial disputes and shocks to confidence. Business cycle theory since 1970 has focused mainly on productivity disturbances as a cause of cycles and for much of the post-war period, measured total factor productivity appeared to be pro-cyclical, a stylized fact that is consistent with that explanation.

The most recent three business cycles are different. Productivity increased during the 2008 financial crisis, as it did in the 2000 recession, the 1990 recession and previously during the initial phase of the Great Depression.

How can we explain business cycles in which productivity may be procyclical during some recessions, but countercyclical in others? One explanation is that some fluctuations are caused by supply factors of the kind captured by the productivity shock S_t in the model of this paper. Others are caused by demand side factors. The model developed in this paper can explain both.

6.3 Simulating Rational Bubbles

To demonstrate the effects of demand side shocks, I simulated the model of this paper with the belief function described in Equation (68),[10]

$$x_t = \alpha_{s_t} + \gamma_{s_t} x_{t-1}, \tag{68}$$

where s_t follows a three regime Markov chain with transition probabilities $\Pr\{s_t = j | s_{t-1} = i\} = p_{i,j}$. In regime 1, I assume that

[10] For the purposes of this simulation I chose

$$x = F = \left[\left(\frac{L}{1-L}\right)^{\frac{1}{2}}\right]^2.$$

This is equivalent to setting $\rho = 0.5$,

$$x = \frac{L^{1-\rho}}{(1-L)^{\rho}}$$

and

$$x = F(x) = x^2.$$

$$0 < \gamma_1 < 1. \tag{69}$$

This is a stable regime in which x_t converges towards the value

$$\bar{x}_1 = \frac{\alpha_{s_1}}{1 - \gamma_1}. \tag{70}$$

With probability $p_{1,2}$ the economy transits to state 2 in which there is an explosive bubble. In this regime, I assume that

$$\gamma_2 = 1. \tag{71}$$

A bubble crashes with probability $p_{2,3}$ and in this case the economy moves into the crash state which, by assumption, lasts for only one period. In the crash state, x_t is drawn from a truncated normal distribution with mean $\bar{x}_3 < \bar{x}_1$ and variance σ^2.[11] By assumption, $p_{1,3} = p_{2,1} = 0$ which implies that the economy cannot enter the crash state without passing through a bubble. I also assume that $p_{3,1} = 1$; which implies that the crash lasts for only one period.

Table 1	$s_t = 1$ "normal"	$s_t = 2$ "bubble"	$s_t = 3$ "crash"
α_{s_t}	0.38	0.5	10
γ_{s_t}	0.98	1	0
$\sigma^2_{s_t}$	0	0	2
$\Pr(s_{t+1} = 1 \vert s_t)$	0.99	0	1
$\Pr(s_{t+1} = 2 \vert s_t)$	.01	0.975	0
$\Pr(s_{t+1} = 3 \vert s_t)$	0	0.025	0
Steady State	19	–	–
Expected Duration	100	40	1

[11] It is truncated, since x_t is non-negative. In practice, I calibrate the distribution to have a mean of 10 and a standard deviation of 2 and for this distribution the probability of drawing a negative value is essentially zero.

Table 1 describes the parametric assumptions that govern the Markov process in a simulated example of this process. In regime 1, asset prices are explained by the equation,

$$x_t = 0.38 + 0.98 x_{t-1}. \tag{72}$$

In this regime the the asset price returns to a mean of 19, closing half the gap to it's steady state every 35 months. The expected duration of regime 1 is 100 months.

In regime 2 the asset price follows the explosive process,

$$x_t = 0.5 + x_{t-1}. \tag{73}$$

This regime, which represents an asset price bubble, has an expected duration of 40 months. Regime 2 is followed by a crash which lasts for one period.

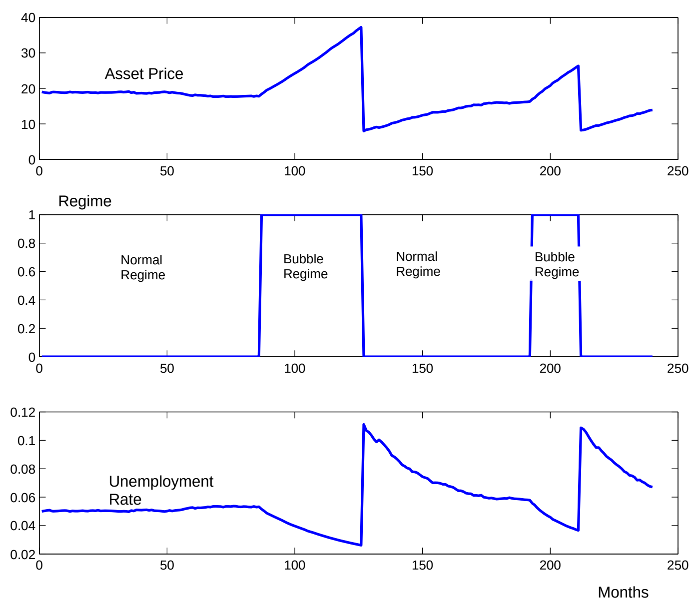

Figure 4: Simulated Asset Market Bubbles

Figure 4 shows a single draw from the process described in Table 1. Since the unemployment rate is a function of the asset price, when the bubble bursts there is a big increase in the unemployment rate, as depicted in artificial data in Figure 3. Immediately following the collapse of the bubble, the asset price reverts to the stable process described by the first column of Table 1.

7 Psychology and Economics

A critic of this paper might observe that I have not provided a deep theory of asset price movements. I have simply grafted an exogenous theory of asset price movements onto an economic model and used the theory to explain movements in the unemployment rate. That is a fair criticism. But I do not claim that the theory in this paper is the last word on asset price bubbles. Instead, I see it as the beginning of a new class of theories that opens the door to understanding how market psychology and economics interact with one another to generate the correlated movements in asset prices and unemployment that we have observed in U.S. time series data.

During the 1960s, economists developed the efficient markets hypothesis. At one level, this hypothesis is the statement that we should not expect to observe profit opportunities in financial markets. At a deeper level, the efficient markets hypothesis asserts that free trade in financial assets will replicate the first best allocation of capital that would be achieved by an omniscient social planner. This stronger form of the hypothesis is an implication of an equilibrium model in which the first and second theorems of welfare economics are both true: Every equilibrium is Pareto efficient and every Pareto efficient allocation can be decentralized by free trade in a complete set of financial markets.

The model developed in this paper accepts the weak form of the efficient markets hypothesis. But it denies that every equilibrium is Pareto efficient. By constructing an *old-Keynesian model* in which *any* unemployment rate can be an equilibrium, I have opened the door for theories that combine self-fulfilling animal spirits with market equilibrium. In my approach, animal spirits are a separate fundamental feature of an economic model that have the same methodological status as preferences, endowments and technology.

Psychology matters for economics. But *how* does it matter? Shiller (2000) and Akerlof and Shiller (2009) have made important observations by pointing to the role of social transmission mechanisms in propagating confidence and Burnside, Eichenbaum, and Rebelo (2011) have applied a model first used to explain the spread of disease to the spread of ideas in a model of house prices. In a more ambitious research program, Frydman and Goldberg (2007, 2012, 2011) propose to replace rational expectations with a model of an 'imperfect knowledge economy'. These are all examples of the application of psychological mechanisms to economic models.

My work demonstrates that we do not need to *replace* the rational expectations assumption. Rational expectations is consistent with many possible alternative theories of what governs human psychology. In a world where any unemployment rate can be a steady state equilibrium, there are many possible dynamic paths for asset prices and many possible ways of modeling beliefs all of which are consistent with rational expectations. Our job as social scientists is to understand which theory of market psychology best explains the data.

8 Concluding Comments

How does my work fit into other recent theories of the crisis? There is a large literature that tries to understand the role of credit in a financial collapse. The work of Hyman Minsky (1975) has been widely cited as an explanation for the end of a bubble and there is a large and expanding literature, with early contributions from Bernanke, Gertler, and Gilchrest (1996) and Kiyotaki and Moore (1997), that draws on the importance of collateral in amplifying credit cycles. That work is important and I do not want to deny the role of leverage in the business cycle. But a theory of credit cycles is incomplete without an explanation of why we should care about booms and busts in asset markets.

The two most recent recessions look a lot more like the 1929 contraction than any of the other post-war recessions. Each of them was accompanied by a boom and subsequent bust in asset prices, a feature that was not present in the other nine post-war recessions. In my view, the deregulation of financial markets in the 1990s had a lot to do with that. But why was a large collapse in asset prices accompanied by a big increase in the unemployment rate? Why was the recovery so slow in the 1930s and why was unemployment still at 9% in April of 2011, 22 months after the NBER declared an end to the recession? That requires a theory that can explain persistent unemployment and that is what I have contributed in this paper by constructing an old-Keynesian model in which any unemployment rate can persist as a steady state equilibrium.

Appendix A

This appendix describes the problem that would be faced by a social planner whose goal was to maximize the utility of the representative household.

The economy satisfies all of the assumptions of standard general equilibrium theory. There are two convex technologies and preferences are assumed to be concave, hence the programming problem, defined as

$$\max_{\{V_t, L_{t+1}\}} E_s \left\{ \sum_{t=s}^{\infty} \beta^{t-s} \left(\frac{\left[bS_t^{\rho}\left(L_t - V_t\right)^{\rho} + a\right]^{\frac{1-\eta}{\rho}}}{1-\eta} + \psi_t \left[L_t \left(1-\delta\right) + \left(\Gamma V_t\right)^{\frac{1}{2}} \left(1 - L_t\right)^{\frac{1}{2}} - L_{t+1} \right] \right) \right\} \quad \text{(A1)}$$

has a unique solution.

Proposition 3 *Define the constants A, B, and C as follows,*

$$A = \frac{\beta \Gamma^{\frac{1}{2}}}{2}, \quad B = 1 - \beta \left(1 - \delta\right), \quad C = \frac{\beta \Gamma^{\frac{1}{2}}}{2}. \quad \text{(A2)}$$

Let $\bar{X}$ be the unique positive root of the quadratic

$$AX^2 + BX - C = 0, \quad \text{(A3)}$$

where $\bar{X}$ is given by the expression

$$\bar{X} = \frac{-\left[1 - \beta\left(1-\delta\right)\right] + \sqrt{\left[1 - \beta\left(1-\delta\right)\right]^2 + \Gamma \beta^2}}{\beta \Gamma^{\frac{1}{2}}}. \quad \text{(A4)}$$

For values of β close to 1, the optimal sequences $\{V_s, L_s\}_{s=t}^{\infty}$ that solve (A1) converge asymptotically to a pair of numbers $\{L, V\}$ where

$$L = \frac{\Gamma^{\frac{1}{2}} \bar{X}}{\delta + \Gamma^{\frac{1}{2}} \bar{X}}, \qquad V = \left(\frac{\delta}{\delta + \Gamma^{\frac{1}{2}} \bar{X}} \right) \bar{X}^2. \quad \text{(A5)}$$

Proof of Proposition 3

Proof. A solution to Problem (A1) must satisfy the following first order conditions,

$$b\frac{\left[bS_t^{\rho}\left(L_t-V_t\right)^{\rho}+a\right]^{\frac{1-\eta-\rho}{\rho}}S_t^{\rho}}{\left(L_t-V_t\right)^{1-\rho}}=\frac{1}{2}\psi_t\Gamma^{\frac{1}{2}}\left(\frac{1-L_t}{V_t}\right)^{\frac{1}{2}}, \tag{A6}$$

$$\psi_t=E_t\left\{\beta\left(b\frac{\left[bS_{t+1}^{\rho}\left(L_{t+1}-V_{t+1}\right)^{\rho}+a\right]^{\frac{1-\eta-\rho}{\rho}}S_{t+1}^{\rho}}{\left(L_{t+1}-V_{t+1}\right)^{1-\rho}}\right.\right.$$
$$\left.\left.+\psi_{t+1}\left[\left(1-\delta\right)-\frac{1}{2}\Gamma^{\frac{1}{2}}\left(\frac{V_{t+1}}{1-L_{t+1}}\right)^{\frac{1}{2}}\right]\right)\right\}, \tag{A7}$$

$$L_{t+1}=L_t\left(1-\delta\right)+\left(\Gamma V_t\right)^{\frac{1}{2}}\left(1-L_t\right)^{\frac{1}{2}}. \tag{A8}$$

These equations must be obeyed by the optimal path $\{L_{s+1},V_s,\psi_s\}_{s=t}^{\infty}$ where L_t is given by an initial condition. Since the problem is concave, the solution is unique.

Let $\{L,V,\psi\}$ be a non-stochastic steady state solution of (A1), defined as a solution to the equations,

$$b\frac{\left[b\left(L-V\right)^{\rho}+a\right]^{\frac{1-\eta-\rho}{\rho}}}{\left(L-V\right)^{1-\rho}}=\frac{\psi}{2}\Gamma^{\frac{1}{2}}\left(\frac{1-L}{V}\right)^{\frac{1}{2}}, \tag{A9}$$

$$\psi=\beta b\frac{\left[b\left(L-V\right)^{\rho}+a\right]^{\frac{1-\eta-\rho}{\rho}}}{\left(L-V\right)^{1-\rho}}+\beta\psi\left(1-\delta\right)-\beta\psi\frac{1}{2}\Gamma^{\frac{1}{2}}\left(\frac{V}{1-L}\right)^{\frac{1}{2}}. \tag{A10}$$

Rearranging these expressions, defining

$$X=\left(\frac{V}{1-L}\right)^{\frac{1}{2}}, \tag{A11}$$

gives

$$AX^2 + BX - C = 0, \tag{A12}$$

where,

$$A = \frac{\beta\Gamma^{\frac{1}{2}}}{2}, \quad B = 1 - \beta(1-\delta), \quad C = \frac{\beta\Gamma^{\frac{1}{2}}}{2}. \tag{A13}$$

This establishes the quadratic defined in the proposition. The values of L and V are found by combining (A11) with the steady state value of (30), given by,

$$\delta L = (\Gamma V)^{\frac{1}{2}} (1-L)^{\frac{1}{2}}. \tag{A14}$$

The local existence and convergence of dynamic paths, when β is 'close enough' to 1, is a consequence of the turnpike property of optimal growth models. See, for example, Cass (1966). ■

Appendix B

Proof of Proposition 1

Proof. Since only real variables are determined in equilibrium we are free to choose the normalization $w = 1$. In a steady state equilibrium it follows from (36) that,

$$p_k = \frac{a\beta}{1-\beta} Z, \tag{B1}$$

where

$$Z \equiv pC^{1-\rho}. \tag{B2}$$

We now seek an expression for z as a function of L.

Combining (39) with (41), using the normalization $w = 1$, we have,

$$\frac{(1-\beta(1-\delta))}{q} b \left(\frac{C}{L-V}\right)^{1-\rho} = \beta b \left(\frac{C}{L-V}\right)^{1-\rho} - \beta\frac{1}{p}. \tag{B3}$$

Combining (42) and (43) gives

$$q = \frac{\Gamma(1-L)}{\delta L}, \tag{B4}$$

and substituting for q from (B4) in (B3) gives

$$\frac{1}{p} = b\left(\frac{C}{L-V}\right)^{1-\rho}\left(\frac{\beta\Gamma(1-L) - \delta(1-\beta(1-\delta))L}{\beta\Gamma(1-L)}\right). \tag{B5}$$

We next seek an expression for V as a function of L. Substituting from (B4) into (43) gives

$$V = \frac{\delta^2 L^2}{\Gamma(1-L)}, \tag{B6}$$

and hence

$$L - V = L\left(1 - \frac{\delta^2 L}{\Gamma(1-L)}\right). \tag{B7}$$

Substituting from (B7) into (B5) and rearranging terms gives

$$\begin{aligned} p_k &\equiv \frac{a\beta}{1-\beta} p C^{1-\rho} \\ &= \left(\frac{a\beta}{1-\beta}\right) \frac{\beta L^{1-\rho}\Gamma^{p}(1-L)^{\rho}\left[\Gamma(1-L) - \delta^2 L\right]^{1-\rho}}{b(\beta\Gamma(1-L) - \delta(1-\beta(1-\delta))L)} \equiv g(L). \end{aligned} \tag{B8}$$

Finally, using the definitions of Ω, μ and λ from (45), we have

$$g(L) = \frac{\Omega L^{1-\rho}(1-L)^{\rho}[\lambda - L]^{1-\rho}}{\mu - L},$$

which establishes the form of the function g. ■

Appendix C

Proof of Proposition 2

Proof. We must show that, for $\rho > 0$, g is strictly increasing. First notice that from (45) that, since $0 < \beta < 1$,

$$\mu < \lambda < 1. \tag{C1}$$

Taking the logarithmic derivative of g gives

$$\left.\frac{L}{g}\frac{\partial g'}{\partial L}\right|_L = (1-\rho) - \rho\frac{L}{1-L} - (1-\rho)\frac{L}{\lambda - L} + \frac{L}{\mu - L}. \tag{C2}$$

Rearranging terms

$$\overbrace{(1-\rho)}^{A_1} + \rho L \overbrace{\left(\frac{1}{\lambda - L} - \frac{1}{1-L}\right)}^{A_2} + L\overbrace{\left(\frac{1}{\mu - L} - \frac{1}{\lambda - L}\right)}^{A_3} > 0 \tag{C3}$$

where

$$A_1 > 0, \;\; A_2 > 0 \text{ and } \;\; A_3 > 0 \;\; \text{for all} \;\; L \leq \mu. \tag{C4}$$

The first inequality follows since $0 \leq \rho \leq 1$, and the second two inequalities follow from the additional facts that $\mu < \lambda < 1$ and $L < \mu$. ■

Appendix D

Aggregate Supply in the Simplified Model

This Appendix derives the aggregate supply curve, Equation (58). By combining Equation (53) with Equation (D1),

$$L_t = q_t V_t, \tag{D1}$$

and imposing the symmetric equilibrium conditions, $\bar{L}_t = L_t$ and $\bar{V}_t = V_t$ we arrive at the following expression for q_t

$$q_t = \frac{\Gamma}{\bar{L}_t}. \tag{D2}$$

Each firm solves the static problem,

$$\max \left[bS_t^{\rho} \left(L_t - V_t\right)^{\rho} + aK_t^{\rho}\right]^{\frac{1}{\rho}} - \frac{w_t}{p_t} L_t - \frac{rr_t}{p_t} K_t \tag{D3}$$

where

$$L_t = X_t + V_t, \tag{D4}$$

and

$$L_t = q_t V_t. \tag{D5}$$

Substituting from (D4) and (D5) into (D3) and using (D2) leads to the reduced form problem,

$$\max \left[bS_t^{\rho} L_t^{\rho} \left(1 - \frac{\bar{L}_t}{\Gamma}\right)^{\rho} + aK_t^{\rho}\right]^{\frac{1}{\rho}} - \frac{w_t}{p_t} L_t - \frac{rr_t}{p_t} K_t, \tag{D6}$$

which is maximized when

$$bS_t^{\rho} \left(1 - \frac{\bar{L}_t}{\Gamma}\right)^{\rho} \left(\frac{C_t}{L_t}\right)^{1-\rho} = \frac{w_t}{p_t}, \tag{D7}$$

and

$$a \left(C_t\right)^{1-\rho} = \frac{rr_t}{p_t}, \tag{D8}$$

where I have made use of the equilibrium assumption to set $K_t = 1$. The aggregate supply curve, Equation (58) is derived by setting $w_t = 1$, imposing $L_t = \bar{L}_t$, and solving (D7) for $Z_t \equiv p_t C_t^{1-\rho}$,

$$Z_t = \frac{\Gamma^{\rho} L_t^{1-\rho}}{bS_t^{\rho} \left(\Gamma - L_t\right)^{\rho}}. \tag{D9}$$

References

ABREU, D., AND M. K. BRUNNERMEIER (2003): "Bubbles and Crashes," *Econometrica*, 71(1), 173–204.

AKERLOF, G. A., AND R. J. SHILLER (2009): *Animal Spirits*. Princeton University Press, Princeton and Oxford.

BENHABIB, J., AND R. E. A. FARMER (1999): "Indeterminacy and Sunspots in Macroeconomics," in *Handbook of Macroeconomics*, ed. by J. B. Taylor, and M. Woodford. North-Holland.

BERNANKE, B., M. GERTLER, AND S. GILCHREST (1996): "The Financial Accelerator and the Flight to Quality," *The Review of Economics and Statistics*, 78(1), 1–15.

BLANCHARD, O., AND P. A. DIAMOND (1990): "The aggregate matching function," in *Growth, Productivity and Unemployment*, ed. by P. A. Diamond. MIT Press.

BURNSIDE, C., M. EICHENBAUM, AND S. REBELO (2011): "Understanding Booms and Busts in Housing Markets," *Northwestern mimeo, Stanford.*

CASS, D. (1966): "Optimum Growth in an Aggregative Model of Capital Accumulation: A Turnpike Theorem," *Econometrica*, 34(4), 833–850.

DIBA, B. T., AND H. I. GROSSMAN (1988): "The Theory of Rational Bubbles in Stock Prices," *The Economic Journal*, 98, 746–754.

FARMER, R. E. A. (1993): *The Macroeconomics of Self-Fulfilling Prophecies.* (1999) MIT Press, Cambridge, MA, second edn.

——— (2008a): "Aggregate Demand and Supply," *International Journal of Economic Theory*, 4(1), 77–94.

——— (2008b): "Old Keynesian Economics," in *Macroeconomics in the Small and the Large*, ed. by R. E. A. Farmer, chap. 2. Edward Elgar, Cheltenham, UK.

——— (2009): "Confidence, Crashes and Animal Spirits," *NBER WP no. 14846*.

——— (2010a): "Animal Spirits, Persistent Unemployment and the Belief Function," *NBER Working Paper 16522*.

——— (2010b): *Expectations, Employment and Prices*. Oxford University Press, New York.

——— (2010c): *How the Economy Works: Confidence, Crashes and Self-fulfilling Prophecies*. Oxford University Press, New York.

——— (2010d): "How to Reduce Unemployment: A New Policy Proposal,," *Journal of Monetary Economics: Carnegie Rochester Conference Issue*, 57(5), 557–572.

Farmer, R. E. A., and D. Plotnikov (2010): "Does Fiscal Policy Matter? Blinder and Solow Revisited," *NBER Working Paper number 16644*.

Frydman, R., and M. D. Goldberg (2007): *Imperfect Knowledge Economics: Exchange Rates and Risk*. Princeton University Press.

——— (2011): *Beyond Mechanical Markets*. Princeton University Press.

——— (2012): "The Imperfect Knowledge Imperative in Modern Macroeconomics and Finance Theory," in *Micro-Micro: Back to the Foundations*, ed. by R. Frydman, and E. S. Phelps. Princeton University Press, forthcoming.

Greenwald, B., and J. E. Stiglitz (1988): "Pareto Inefficiency of Market Economies: Search and Efficiency Wage Models," *American Economic Review*, 78(2), 352–355.

Hall, R. E. (2005): "Employment Fluctuations with Equilibrium Wage Stickiness," *American Economic Review*, 95(1), 50–65.

Kashiwagi, M. (2010): "Search Theory and the Housing Market," Ph.D. thesis, UCLA.

Keynes, J. M. (1936): *The General Theory of Employment, Interest and Money*. MacMillan and Co.

Kiyotaki, N., and J. H. Moore (1997): "Credit Cycles," *Journal of Political Economy*, 105(3), 739–764.

Kocherlakota, N. (2011): "Bubbles and Unemployment," *Federal Reserve Bank of Minneapolis, mimeo.*

Mackay, C. (1841): *Extraordinary Popular Delusions and the Madness of Crowds; reprinted 1980*. Harmony Books, New York.

Martin, A., and J. Ventura (2011a): "Economic Growth with Bubbles," CREI working paper.

——— (2011b): "Theoretical Notes on Bubbles and the Current Crisis," *IMF Review (forthcoming).*

Minsky, H. P. (1975): *John Maynard Keynes*. Columbia University Press.

Moen, E. (1997): "Competitive Search Equilibrium," *Journal of Political Economy*, 105(2), 385–411.

Pigou, A. C. (1929): *Industrial Fluctuations*. McMillan.

Rogerson, R., R. Shimer, and R. Wright (2005): "Search-Theoretic Models of the Labor Market: A Survey," *Journal of Economic Literature*, 43, 959–988.

Santos, M. S., and M. Woodford (1997): "Rational Asset Pricing Bubbles," *Econometrica*, 65(6), 19–58.

Shiller, R. J. (2000): *Irrational Exuberance*. Princeton University Press.

Tirole, J. (1985): "Asset Bubbles and Overlaping Generations," *Econometrica*, 53(6), 1499–1528.