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Xavier Vives

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Xavier Vives, IESE, Barcelona-Madrid and CEPR

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ABSTRACT

Endogenous Public Information and Welfare*

This paper performs a welfare analysis of economies with private information when public information is endogenously generated and agents can condition on noisy public statistics in the rational expectations tradition. We find that equilibrium is not (restricted) efficient even when feasible allocations share similar properties to the market context (e.g., linear in information). The reason is that the market in general does not internalize the informational externality when public statistics (e.g., prices) convey information. Under strategic substitutability, equilibrium prices will tend to convey too little information when the "informational" role of prices prevails over its "index of scarcity" role and too much information in the opposite case. Under strategic complementarity, prices always convey too little information. These results extend to the internal efficiency benchmark (accounting only for the collective welfare of the active players). However, received results--on the relative weights placed by agents on private and public information, when the latter is exogenous--may be overturned.

Xavier Vives
IESE Business School
Avda Pearson 21
08034 Barcelona
SPAIN

Email: xvives@iese.edu

1. <u>Introduction</u>

There has been a recent surge of interest in the welfare analysis of economies with private information and in particular on the role of public information in such economies (see, e.g., Morris and Shin 2002; Angeletos and Pavan 2007). Agents may fail to place welfare-optimal weights on private and public information owing to payoff and information externalities. In this paper we examine the issue in a context where public information is endogenously generated and agents can condition on public statistics when making their choices. In the rational expectations tradition, agents learn from prices and from public statistics in general, which are themselves the aggregate outcome of individual decisions.

Endogenous public information is relevant for a broad array of markets and situations. In financial markets, prices are noisy statistics that arise from the decisions of traders. In goods markets, prices aggregate information on the preferences of consumers and the quality of the products. In the overall economy, the release of GDP data is a noisy public signal that is the outcome of actions taken by economic agents.[1]

Any welfare analysis of rational expectations equilibria faces several difficulties. First of all, it must employ a model capable of dealing in a tractable way with the dual role of prices as conveyors of information and determinants of traders' budget constraints. Grossman and Stiglitz (1980) were pioneers in this respect with their CARA-normal model. Second, we require a welfare benchmark against which to test market equilibria in a world with asymmetric information. The appropriate benchmark for measuring inefficiency at the market equilibrium is the team solution in which agents internalize collective welfare but must still rely on private information when making their own decisions (Radner 1979; Vives 1988; Angeletos and Pavan 2007). This is in the spirit of Hayek (1945), where the private signals of agents cannot be communicated to a center. The team-efficient solution internalizes the information externalities associated with the actions of agents in the market. Collective welfare may refer to the surplus of all market participants, active or passive, or may be restricted to the internal welfare of the active agents. The third challenge for such welfare analysis is dealing with the interaction of payoff and informational

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[1]   See, for example, Rodríguez-Mora and Schulstad (2007).

externalities. If we take as a benchmark a pure prediction model with no payoff externalities, then agents will typically rely too much on public information. The reason is that agents do not take into account that their reaction to private information affects the informativeness of public statistics and general welfare. In other words, agents do not internalize an information externality. Pure information externalities will make agents insufficiently responsive to their private information (Vives 1993, 1997; Amador and Weill 2009). We will see that payoff externalities complicate welfare analysis and may rebalance weightings in the opposite direction.

In this paper we consider a tractable linear-quadratic-Gaussian model that allows us to address the three challenges just described when public information is endogenously generated and influenced by the actions of agents. There is uncertainty about a common valuation parameter about which agents have private information, and the endogenous public statistic or "price" is noisy. We use a model with a rational expectations flavor but in the context of a well-specified game, where agents compete in schedules, and allow actions to be strategic substitutes or complements. We focus our attention on linear Bayesian equilibria. The model is flexible and admits several interpretations in terms of firms competing in a homogenous product market, investment complementarities, monopolistic competition, traders (both rational and "behavioral") in a financial market, and asset auctions.

We show that agents correct the slope of their strategy according to what they learn from the public statistic and the character of competition. Under strategic substitutes competition the price's informational and index-of-scarcity roles conflict. With strategic substitutes and private information, a high price is bad news and the equilibrium schedule is steeper than with full information. In fact, in equilibrium schedules may slope the "wrong" way (e.g., downward for a supply schedule) when the informational role of prices dominates their index-of-scarcity role. This will occur when there is little noise in the public statistic. With strategic complements there is no conflict: a high price is good news, and the equilibrium schedule is flatter than with full information. An increase in the degree of the game's complementarity will increase (decrease) the weight placed on private information under strategic substitutes (complements). It is interesting that the impact on the slope of the equilibrium schedule of a change in the exogenous (prior) precision of public

information is opposite to the change in the precision of the noise in the endogenous public signal; consequently, market depth is increasing in the former and decreasing in the latter. The reason is that an increase in the *exogenous* precision of public information decreases the informational component of the public statistic whereas an increase in the *endogenous* precision increases it.

Market equilibria will not be team-efficient even when the allowed allocations have properties (e.g., being linear in information) similar to those of the market equilibrium. This is because the market in general does not internalize the informational externality that results from public statistics (e.g., prices) conveying information. Indeed, a competitive agent is an information taker while the precision of the public statistic is endogenous. The market equilibrium is characterized by the *privately* efficient use of private information and the efficient use of public information. Team efficiency makes efficient use also of public information but instead makes *socially* efficient use of private information. Market equilibria will be team-efficient only in exceptional circumstances (as when the information externality vanishes). This may occur, for example, when public information is exogenous. We find that, under strategic substitutability, equilibrium prices will tend to convey too little information when the informational role of prices prevails and too much information when its index-of-scarcity role prevails. Under strategic complementarity, prices always convey too little information. In a business cycle model with private information about an underlying productivity shock, Angeletos and La'O (2010) find that endogenous public signals introduce inefficiency and that there is always too little learning, whereas if the signals were exogenous then the market outcome (as in our case) would be efficient. In short, the equilibrium use of information over the business cycle is not optimal from the social point of view.

These results can be extended to the internal team profit benchmark (where only the collective welfare of the players is taken into account, for example, ignoring passive consumers). In this case, endogenous public information may overturn conclusions reached using exogenous information models (e.g., Angeletos and Pavan 2007). We find in particular that, in the presence of information externalities and strategic substitutability in payoffs, over-reliance of agents on private information may render prices too informative. With strategic complementarity in payoffs, agents always rely

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too little on private information. This latter result is in stark contrast to the case of exogenous information, where agents under strategic complementarity rely too much on private information (Angeletos and Pavan 2007).

The plan of the paper is as follows. Section 2 presents the model and possible interpretations. Section 3 characterizes the equilibrium and comparative statics properties. Section 4 performs a welfare analysis of the case of a homogenous product, and Section 5 studies the internal team welfare benchmark. Concluding remarks are given in Section 6, and some proofs are gathered in the Appendix.

2. The model and its interpretations

Consider a quadratic payoff game with a continuum of players indexed within the interval $[0,1]$. Player i has the payoff function

$$\pi(x_i, \tilde{x}) = (\alpha - \theta + u)x_i - \beta\tilde{x}x_i - \frac{\lambda}{2}x_i^2,$$

where x_i is the individual action of the player, $\tilde{x} = \int_0^1 x_i \, di$ is the aggregate action, θ and u are parameters that, for the moment, are simply given, and α, λ are positive parameters. Then $\partial^2 \pi / (\partial x_i)^2 = -\lambda < 0$ and $\partial^2 \pi / \partial x_i \partial \tilde{x} = -\beta$, and the slope of the best reply of a player is $m \equiv (\partial^2 \pi / \partial x_i \partial \tilde{x}) / (-\partial^2 \pi / (\partial x_i)^2) = -\beta / \lambda$. Thus we have strategic substitutability (complementarity) for $\beta > 0$ (for $\beta < 0$), and m can be understood as the degree of complementarity in the payoffs. (In the rest of this paper, when discussing strategic substitutability or complementarity we refer to this meaning in the context of this certainty game). We assume that $m < 1/2$ or $2\beta + \lambda > 0$, limiting the extent of strategic complementarity. The condition $2\beta + \lambda > 0$ guarantees that $\pi(x, x)$ is strictly concave in x $(\partial^2 \pi / (\partial x)^2 = -(2\beta + \lambda) < 0)$. Observe that there are no payoff externalities among players when $\beta = 0$.

Consider now a game with uncertainty and in which θ and u are random. The parameter θ is uncertain; it has prior Gaussian distribution with mean $\bar{\theta}$ and

variance σ_θ^2 (we write $\theta \sim N\left(\bar\theta, \sigma_\theta^2\right)$ and, to ease notation, set $\bar\theta = 0$). Player i receives a signal $s_i = \theta + \varepsilon_i$ with $\varepsilon_i \sim N\left(0, \sigma_\varepsilon^2\right)$. Error terms are uncorrelated across players, and the random variables $\{\theta, \varepsilon_i, u\}$ are mutually independent. We establish the convention that error terms cancel in the aggregate: $\int_0^1 \varepsilon_i \, di = 0$ almost surely (a.s.). Then the aggregation of all individual signals will reveal the underlying uncertainty: $\int_0^1 s_i \, di = \theta + \int_0^1 \varepsilon_i = \theta$.[2]

Players have access to the (endogenous) public statistic $p = \alpha + u - \beta \tilde{x}$, where $u \sim N\left(0, \sigma_u^2\right)$; this can be interpreted as the marginal benefit of taking action level x_i, which has cost $\theta x_i + (\lambda/2) x_i^2$.[3] When $\beta = 0$, there are no informational externalities among players.

The timing of the game is as follows. At $t = 0$, the random variables θ and u are drawn but not observed. At $t = 1$, each player observes his own private signal s_i and submits a schedule $X_i(s_i, \cdot)$ with $x_i = X_i(s_i, p)$, where p is the public statistic. The strategy of a player is a map from the signal space to the space of schedules. Finally, the public statistic is formed (the "market clears") by finding a p that solves

$$p = \alpha + u - \beta\left(\int_0^1 X_j(s_j, p) \, dj\right),$$ and payoffs are collected at $t = 1$.

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[2] That is, I assume that the strong law of large numbers (SLLN) holds for a continuum of independent random variables with uniformly bounded variances. Suppose that $\left(q_i\right)_{i \in [0,1]}$ is a process of independent random variables with means $E[q_i]$ and uniformly bounded variances $\text{var}[q_i]$. Then we let $\int_0^1 q_i \, di = \int_0^1 E[q_i] \, di$ a.s. This convention will be used while taking as given the usual linearity property of integrals. Equality of random variables must be assumed to hold almost surely. It can be checked that the results obtained in the continuum economy are the limit of finite economies under the usual SLLN.

[3] Normality of random variables means that prices and quantities can be negative with positive probability. The probability of this event can be controlled, if necessary, by an appropriate choice of means and variances. Furthermore, for this analysis the key property of Gaussian distributions is that conditional expectations are linear. Other prior-likelihood conjugate pairs (e.g., beta-binomial and gamma-Poisson) share this linearity property and can display bounded supports.

Let us assume that there is a unique public statistic $\hat{p}\left(\left(X_j\left(s_j,\cdot\right)\right)_{j\in[0,1]}\right)$ for any realization of the signals.[4] Then, for a given profile $\left(X_j\left(s_j,\cdot\right)\right)_{j\in[0,1]}$ of players' schedules and realization of the signals, the profits for player i are given by

$$\pi_i = px_i - \beta\tilde{x}x_i - \frac{\lambda}{2}x_i^2,$$

where $x_i = X_i\left(s_i,p\right)$, $\tilde{x} = \int_0^1 X_j\left(s_j,p\right)dj$, and $p = \hat{p}\left(\left(X_j\left(s_j,\cdot\right)\right)_{j\in[0,1]}\right)$. This formulation has a rational expectations flavor but in the context of a well-specified schedule game. We will restrict our attention to linear Bayesian equilibria of the schedule game. The model admits several interpretations, as follows.

Firms competing in a homogenous product market with quadratic production costs. In this case, $p = \alpha + u - \beta\tilde{x}$ is the inverse demand for the homogenous product, x_i is the output of firm i, and the cost function of firm i is given by $C(x_i) = \theta x_i + (\lambda/2)x_i^2$. Firms use supply functions as strategies, and markets clear: $p = \alpha + u - \beta\left(\int_0^1 X_i\left(s_i,p\right)di\right)$.

If $\beta > 0$, then demand is downward sloping and we have strategic substitutability in the usual partial equilibrium market. If $\beta < 0$, we have strategic complementarity and demand is upward sloping. The latter situation may arise in the case of a network good with compatibility.

Investment complementarities. In this case, $\beta < 0$ and we have strategic complementarity among investment decisions of the agents. The marginal benefit of investing is $p = \alpha + u - \beta\tilde{x}$, and the cost is $C(x_i) = \theta x_i + (\lambda/2)x_i^2$. The shock to the marginal benefit (u) can be understood as a shock to demand, while the shock to costs (θ) can be viewed as a productivity shock. Agents condition their decisions on the marginal benefit of investment p, derived, for example, from the public signals on macroeconomic data released by the government (which in turn depend on the

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4   We assign zero payoffs to the players if there is no p that solves the fixed point problem. If there are multiple solutions, then the one that maximizes volume is chosen.

aggregate activity level). This description need not be taken literally and is simply meant to capture the reduced form of a dynamic process. For example, consider competitive firms deciding about investment in the presence of macroeconomic uncertainty as represented by the random variable θ, which affects profitability. In predicting θ, each firm has access to a private signal as well as to public information, consisting of aggregate past investment figures compiled by a government agency. Data on aggregate investment incorporates measurement error and, at each period, a noisy measure of the previous period's aggregate investment is made public.[5]

Monopolistic competition. The model applies also to a monopolistically competitive market with quantity-setting firms; in this case, either $\beta > 0$ (goods are substitutes) or $\beta < 0$ (goods are complements). Firm i faces the inverse demand for its product, $p_i = \alpha + u - \beta \tilde{x} - (\lambda/2) x_i$, and has costs θx_i. Each firm uses a supply function that is contingent on its own price: $X(s_i, p_i)$ for firm i. It follows then that observing the price p_i is informationally equivalent (for firm i) to observing $p \equiv \alpha + u - \beta \tilde{x}$.

Our setup encompasses *demand schedule competition* as well. Let a buyer of a homogenous good with unknown ex post value θ face an inverse supply $p = \alpha + u + \beta \tilde{y}$, where $\tilde{y} = \int_0^1 y_i \, di$ and y_i is the demand of buyer i. The buyer's net benefit is given by $\pi_i = (\theta - p) y_i - (\lambda/2) y_i^2$, where λy_i^2 is a transaction or opportunity cost (or an adjustment for risk aversion). The model fits this setup if we let $y_i \equiv -x_i$. Some examples follow.

Firms purchasing labor. A firm purchases labor whose productivity θ is unknown—say, because of technological uncertainty—and faces an inverse linear labor supply (with $\beta > 0$) and quadratic adjustment costs in the labor stock. The firm has a private assessment of the productivity of labor, and inverse supply is subject to a shock.

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[5]   For example, quarterly data on national accounts are subject to measurement error. Rodríguez-Mora and Schulstad (2007) show how government announcements regarding GNP growth affect growth via aggregate investment.

Traders in a financial market. Traders compete in demand schedules for a risky asset with liquidation value θ and face a quadratic adjustment cost in their position (alternatively, the parameter λ proxies for risk aversion). Each trader receives a private signal about the liquidation value of the asset. There are also behavioral traders: those who trade according to the elastic aggregate demand $(\alpha + u - p)/\beta$, where u is random. When $\beta > 0$, the behavioral agents are "value" traders who buy (sell) when the price is low (high). When $\beta < 0$, the behavioral agents are "momentum" traders who buy (sell) when the price is high (low).[6] Our inverse supply follows from the market-clearing equation. It is worth noting that behavioral value (momentum) traders induce strategic substitutability (complementarity) in the actions of informed traders.

Asset auctions. Consider the auction of a financial asset for which (inverse) supply is price elastic: $p = \alpha + \beta \hat{y}$ with $\beta > 0$, where $\hat{y}$ is the total quantity bid. The liquidation value θ of the asset may be its value in the secondary market (say, for a central bank liquidity or Treasury auction). The marginal valuation of a bidder is decreasing in the amount bid.[7] Each bidder receives a private signal about θ, and there are noncompetitive bidders who bid according to u/β. As before, this setup yields an effective inverse supply for the informed bidders: $p = \alpha + u + \beta \tilde{y}$.

Double auction with noise traders. The model can also accommodate, as a limit case of the example just given, a double auction with noise traders demanding a random amount u. Suppose that noise traders bid $(\alpha + \hat{u} - p)/\beta$ with $\hat{u} = \beta u$. Then $\beta^{-1}(\alpha + \beta u - p) \to u$ as $\beta \to \infty$, and market clearing yields $u + \tilde{y} = 0$.

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6 Gennotte and Leland (1990) interpret the case $\beta < 0$ as program traders following a portfolio insurance strategy. Asness, Moskowitz, and Pedersen (2009) study empirical returns of value and momentum strategies. Hendershott and Seasholes (2009) find that program trading accounts for almost 14% of the average daily market volume at the NYSE in 1999-2005 and that program traders lose money on average. See Barberis and Thaler (2003) for a survey of behavioral biases.

7 A justification for the case of liquidity auctions is given in Ewerhart, Cassola, and Valla (2009).

We will maintain a supply interpretation of the model. We let $p \equiv \alpha + u - \beta \tilde{x}$ be the marginal benefit or "price" of taking an action and let $\text{MC}(x_i) \equiv \theta + \lambda x_i$ be the marginal cost.

3. Equilibrium

We are interested in linear (Bayesian) equilibria—LE, for short—of the schedule game for which the public statistic functional is of type $P(\theta,\,u)$. Since the payoffs and the information structure are symmetrical and since payoffs are strictly concave, there is no loss of generality in restricting our attention to symmetric equilibria. Indeed, the solution to the problem of player i,

$$\max_{x_i} E\left[\left(p - \theta - \frac{\lambda}{2}x_i\right)x_i \,\middle|\, s_i,\, p\right],$$

is both unique (given strict concavity of profits) and symmetric across firms (since the cost function and signal structure are symmetric across firms):

$$X(s_i,\,p) = \lambda^{-1}\left[p - E\left[\theta \,\middle|\, s_i,\, p\right]\right],$$

where $p = P(\theta,\,u)$. A strategy for player i may be written as

$$x_i = \hat{b} + \hat{c}p - as_i,$$

in which case the aggregate action is given by

$$\tilde{x} = \int_0^1 x_i\, di = \hat{b} + \hat{c}p - a\theta.$$

It then follows from $p = \alpha + u - \beta\tilde{x}$ that, provided $\hat{c} \neq \beta^{-1}$,

$$p = P(\theta,\,u) = \left(1 + \beta\hat{c}\right)^{-1}\left(\alpha - \beta\hat{b} + z\right);$$

here the random variable $z = u + \beta a\theta$ is informationally equivalent to the "price" or public statistic p. Because u is random, z (and the public statistic) will typically generate a noisy signal of the unknown parameter θ.

Market depth—that is, the inverse of how much the price moves to accommodate a unit increase in u—is given by $\left(\partial P / \partial u\right)^{-1} = 1 + \beta\hat{c}$.[8] Excess demand is given by

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[8]   See, for example, Kyle (1985).

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$$\Xi(p) \equiv \beta^{-1}(\alpha + u - p) - \hat{b} + a\theta - \hat{c}p.$$

The information available to player i is $\{s_i, p\}$ or, equivalently, $\{s_i, z\}$. Since $E[\theta|s_i, p] = E[\theta|s_i, z]$, we can posit strategies of the form

$$X(s_i, z) = b - as_i + cz$$

and obtain that $p = \alpha - \beta b + (1 - \beta c)z$. If $1 + \beta \hat{c} > 0$ then $1 - \beta c > 0$ (since $1 + \beta \hat{c} = (1 - \beta c)^{-1}$) and so p and z will move together. The strategy of player i is then given by

$$X(s_i, z) = \lambda^{-1}[\alpha - \beta b + (1 - \beta c)z - E[\theta|s_i, z]].$$

We can solve for the LE in the usual way: identifying coefficients with the candidate linear strategy $x_i = b - as_i + cz$ by calculating $E[\theta|s_i, z]$. An alternative way of characterizing the LE—and one that is more instructive from the welfare perspective—is to observe that players at a LE make privately efficient use of their signals and efficient use of public information.

The first-order condition (FOC) for player i is $E[p - \mathrm{MC}(x_i)|s_i, z] = 0$. It follows that $E[p - \mathrm{MC}(x_i)] = 0$, from which we can pin down $b = \alpha/(\beta + \lambda)$. Furthermore, the FOC must hold on average, given the private signal of the player. That is,

$$E[E[p - \mathrm{MC}(x_i)|s_i, z]|s_i] = E[p - \mathrm{MC}(x_i)|s_i] = 0.$$

If we assume normality and recall that $E[s_i] = \bar{\theta} = 0$ (by assumption), it follows immediately that

$$E[p - \mathrm{MC}(x_i)|s_i] = E[p - \mathrm{MC}(x_i)] + \frac{\mathrm{cov}[p - \mathrm{MC}(x_i), s_i]}{\mathrm{var}[s_i]}s_i = 0.$$

Because $E[p - \mathrm{MC}(x_i)] = 0$, the equation must hold for all possible signals s_i and with $\mathrm{var}[s_i] > 0$; therefore, $\mathrm{cov}[p - \mathrm{MC}(x_i), s_i] = 0$. Hence firms at a LE make efficient private use of the signals, which yields in particular the elimination of covariance between signals and the margin. We can use the properties of Normal distributions together with some algebra to obtain

$$E\left[\left(p - \mathrm{MC}(x_i)\right)s_i\right] = 0 \quad \text{or} \quad c = \frac{a(\lambda(\tau_\varepsilon + \tau_\theta) + \beta\tau_\varepsilon) - \tau_\varepsilon}{a\beta\tau_\varepsilon(\lambda + \beta)}.$$

Note that neither equation involves the (exogenous) precision of the public statistic τ_u. The efficient use of private information yields c as an increasing (decreasing) function of a if $\beta > 0$ (if $\beta < 0$). See Figure 1 and Figure 2, respectively. In sum: with strategic substitutes, an increase in the weight given to private information must be matched by an *increase* in the weight given to public information; with strategic complements, however, an increase in the weight given to private information must be matched by a *decrease* in the weight given to public information.

Figure 1: Determination of the LE parameters (a, c) as the intersection of the efficient use of private information (Private) and the efficient use of public information (Public) when $\beta > 0$, illustrated for cases of τ_u high, intermediate, and low; T marks the team solution(s).
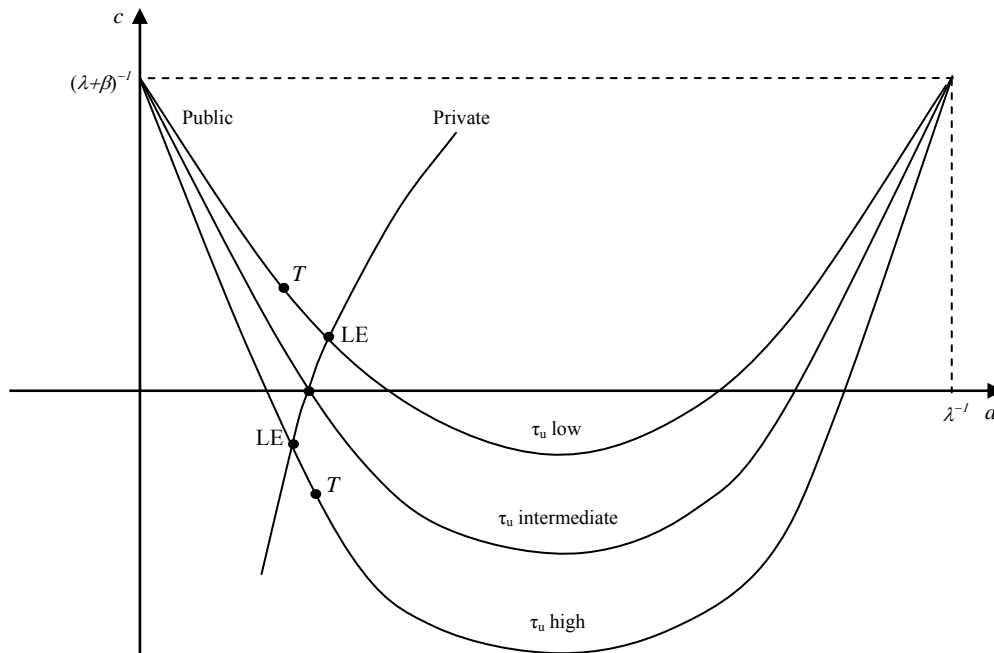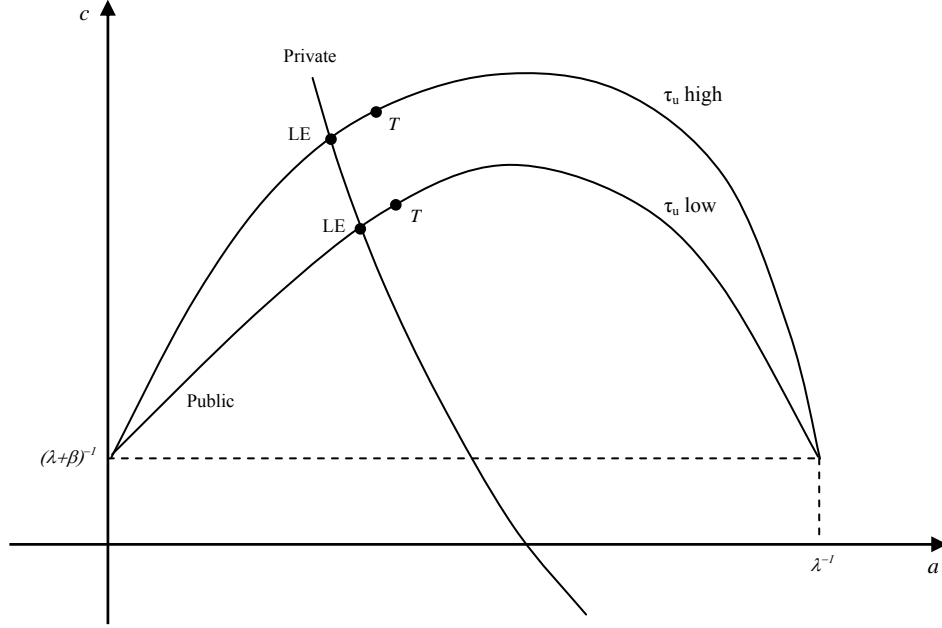
Figure 2: Determination of the LE parameters (a, c) as the intersection of the efficient use of private information (Private) and the efficient use of public information (Public) when $\beta < 0$, illustrated for cases of τ_u high and low; T marks the team solution(s).

With similar reasoning (i.e., using the properties of Gaussian distributions) we obtain that players at a LE make efficient use also of public information, which eliminates the covariation between the margin and public information:

$$E\left[\left(p - \mathrm{MC}(x_i)\right)z\right] = 0 \quad \text{or} \quad c = c(a) \quad \text{with} \quad c(a) \equiv \frac{1}{\beta + \lambda} - \frac{\beta a \tau_u (1 - \lambda a)}{\tau(\beta + \lambda)};$$

here $\tau \equiv \left(\mathrm{var}\left[\theta | p\right]\right)^{-1} = \tau_\theta + \beta^2 a^2 \tau_u$ is the precision of public information about θ. Note that these expressions do not involve the precision of the private signal τ_ε. The equation displayed above yields c as first a decreasing (increasing) and then an increasing (decreasing) function of a if $\beta > 0$ ($\beta < 0$). Therefore, an increase in the weight given to private information may be met by either a decrease or an increase in the weight given to public information. As τ_u increases, the curve shifts downward (upward) in the range $1 - \lambda a > 0$ if $\beta > 0$ ($\beta < 0$), and c tends to $(\lambda + \beta)^{-1}$ as $a \to \lambda^{-1}$ or $a \to 0$. Again, see Figures 1 and 2, respectively.

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In equilibrium, the parameters a and c are determined by the intersection of the (privately) efficient use of private information and the efficient use of public information. The following proposition, whose proof is given in the Appendix, characterizes the linear equilibrium.

Proposition 1. *Let $\tau_\varepsilon \geq 0$ and $\tau_u \geq 0$. Then there is a unique (and symmetric) LE*

$$X\left(s_i, p\right) = \lambda^{-1}\left[\, p - E\left[\theta | s_i, p\right]\right] = \hat{b} - a s_i + \hat{c} p\,,$$

where a is the unique (real) solution of the equation $a = \tau_\varepsilon \lambda^{-1}\left(\tau_\varepsilon + \tau_\theta + \tau_u \beta^2 a^2\right)^{-1}$,

$\hat{c} = \left(\left(\beta + \lambda\right)\left(1 - \beta\lambda\tau_u a^2 \tau_\varepsilon^{-1}\right)^{-1} - \beta\right)^{-1}$, *and $\hat{b} = \alpha\left(1 - \lambda\hat{c}\right)/\left(\beta + \lambda\right)$. In equilibrium,*

$a \in \left(0, \tau_\varepsilon\lambda^{-1}\left(\tau_\theta + \tau_\varepsilon\right)^{-1}\right)$ *and $1 + \beta\hat{c} > 0$.*

Remark 1. We have examined linear equilibria of the schedule game for which the public statistic function is of type $P\left(\theta, u\right)$. In fact, these are the equilibria in strategies with bounded means and with uniformly (across players) bounded variances. (See Claim 1 in the Appendix.)

Remark 2. We can show that the equilibrium in the continuum economy is the limit of equilibria in replica economies that approach the limit economy. Take the homogenous market interpretation with a finite number of firms n and inverse demand $p_n = \alpha + u - \beta\tilde{x}_n$, where $\tilde{x}_n$ is the average output per firm, and with the same informational assumptions. In this case, given the results in Section 5.2 of Vives (2011), the supply function equilibrium of the finite n-replica market converges to the equilibrium in Proposition 1.

The public statistic or price serves a dual role as index of scarcity and conveyor of information. Indeed, a high price has the direct effect of increasing an agent's competitive supply, but it also conveys news about costs—namely, that costs are high (low) if $\beta > 0$ ($\beta < 0$). In equilibrium, the "price impact" is always positive, $\partial P / \partial u = \left(1 + \beta\hat{c}\right)^{-1} > 0$, and excess demand is downward or upward sloping depending on β: $\varXi' = -\left(\beta^{-1} + \hat{c}\right)$ or $\mathrm{sgn}\{\varXi'\} = \mathrm{sgn}\{-\beta\}$. That is, the slope's

direction depends on whether the competition is in strategic substitutes or in strategic complements.

In equilibrium, agents take public information z (or p) as given and use it to form probabilistic beliefs about the underlying uncertain parameter θ. This parameter, in turn, determines the coefficients a and c for private and public information, respectively. At the same time, the informativeness of public information z depends on the sensitivity of strategies to private information a. Agents in the LE behave as information takers and so, from the perspective of an individual agent, public information is exogenous. This fact is at the root of the LE's *informational externality*. That is, agents fail to account for the impact of their own actions on public information and hence on other agents.

Consider as a benchmark the full information case with perfectly informative signals ($\tau_\varepsilon = \infty$). This puts us in a full information competitive equilibrium and we have $c = (\beta + \lambda)^{-1}$, $a = \hat{c} = \lambda^{-1}$, and $X(s_i, p) = \lambda^{-1}(p - \theta)$. In this case, agents have nothing to learn from the price. If signals become noisy ($\tau_\varepsilon < \infty$) then $a < \lambda^{-1}$ and $\hat{c} < \lambda^{-1}$ for $\beta > 0$, with supply functions becoming steeper (lower $\hat{c}$) as agents protect themselves from adverse selection. The opposite happens ($\hat{c} > \lambda^{-1}$ and flatter supply functions) when $\beta < 0$, since then a high price is good news (entailing lower costs). [9]

Two other cases in which $\hat{c} = \lambda^{-1}$ and there is no learning from the price are when signals are uninformative about the common parameter θ $(\tau_\varepsilon = 0)$ and when the public statistic is extremely noisy ($\tau_u = 0$). In the first case, the price has no information to convey; $a = 0$ and $X(s_i, p) = \lambda^{-1}(p - \overline{\theta})$. In the second case, public

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[9]   This follows because, with upward-sloping demand, we assume that $2\beta + \lambda > 0$ and therefore $\lambda > -\beta$.

information is pure noise, $a = \tau_\varepsilon / \lambda (\tau_\varepsilon + \tau_\theta)$, with $X(s_i, p) = \lambda^{-1} (p - E[\theta | s_i])$.[10] In all three cases, there is no information externality via the public statistic.

As τ_u tends to ∞, the precision of prices τ also tends to ∞, the weight given to private information a tends to 0, and the equilibrium collapses (with $1 + \beta \hat{c} \to 0$). Indeed, the equilibrium becomes fully revealing and is not implementable.

The following proposition (proved in the Appendix) presents our results on comparative statics.

<u>Proposition 2</u>. *Let $\tau_\varepsilon > 0$ and $\tau_u > 0$. In equilibrium, the following statements hold.*

(i)   *Responsiveness to private information $a > 0$ decreases from $\lambda^{-1} \tau_\varepsilon (\tau_\theta + \tau_\varepsilon)^{-1}$ to 0 as τ_u ranges from 0 to ∞, decreases with τ_θ and λ, and increases with τ_ε; also, $\mathrm{sgn}\{\partial a / \partial \beta\} = \mathrm{sgn}\{-\beta\}$. Price informativeness τ is increasing in τ_u and τ_ε.*

(ii)   *Responsiveness to the public statistic $\hat{c}$ goes from λ^{-1} to $-\beta^{-1}$ as τ_u ranges from 0 to ∞. Furthermore, $\mathrm{sgn}\{\partial \hat{c} / \partial \tau_u\} = \mathrm{sgn}\{-\partial \hat{c} / \partial \tau_\theta\} = \mathrm{sgn}\{-\beta\}$ and $\mathrm{sgn}\{\partial \hat{c} / \partial \tau_\varepsilon\} = \mathrm{sgn}\{\beta (\beta^2 \tau_u \tau_\varepsilon^2 + 4\lambda^2 \tau_\theta^2 (\tau_\varepsilon - \tau_\theta))\}$. Market depth $1 + \beta \hat{c}$ is decreasing in τ_u and increasing in τ_θ.*

(iii)   *Let $m \equiv -\beta / \lambda$. Then $\mathrm{sgn}\{\partial a / \partial m\} = \mathrm{sgn}\{\beta\}$, $\mathrm{sgn}\{\partial c / \partial m\} = \mathrm{sgn}\{-\partial c / \partial \beta\} > 0$, and $\mathrm{sgn}\{\partial \tau / \partial m\} = \mathrm{sgn}\{-\beta\}$.*

In order to gain further intuition from these results, we first consider the case $\beta > 0$. As τ_u increases from 0, $\hat{c}$ decreases from λ^{-1} (and the slope of supply increases) because of the price's increased informational component. Agents are more cautious when seeing a high price because it may mean higher costs. As τ_u increases more, $\hat{c}$ becomes zero at some point and then turns negative; as τ_u tends to ∞, $\hat{c}$ tends to

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$-\beta^{-1}$.[11] At the point where the scarcity and informational effects balance, agents place zero weight ($\hat{c}=0$) on the public statistic. In this case, agents do not condition on the price and the model reduces to a quantity-setting model à la Cournot (however, not reacting to the price is optimal). See Figure 1. If τ_θ increases then the informational component of the price diminishes, since the agents are now endowed with better prior information, and induces a higher $\hat{c}$ (and a more elastic supply). An increase in the precision of private information τ_ε always increases responsiveness to the private signal but has an ambiguous effect on the slope of supply. The parameter $\hat{c}$ is U-shaped with respect to τ_ε. Observe that $\hat{c}=\lambda^{-1}$ not only when $\tau_\varepsilon=\infty$ but also when $\tau_\varepsilon=0$ and that $\hat{c}<\lambda^{-1}$ for $\tau_\varepsilon\in(0,\infty)$. If τ_ε is high, then a further increase in τ_ε (less noise in the signals) lowers adverse selection and increases $\hat{c}$. If τ_ε is low then the price is relatively uninformative, and an increase in τ_ε increases adverse selection while lowering $\hat{c}$.

If $\beta<0$ then a high price conveys goods news in terms of both scarcity effects and informational effects, so supply is always upward sloping in this case. Indeed, when $\beta<0$ we have $\hat{c}>\lambda^{-1}$; see Figure 2. A high price conveys the good news that average quantity tends to be high and that costs therefore tend to be low. In this case, increasing τ_u, which reinforces the informational component of the price, increases $\hat{c}$—the opposite of what happens when τ_θ increases. The consequence is that market depth $(\partial P/\partial u)^{-1}=1+\beta\hat{c}$ is decreasing in τ_u and increasing in τ_θ. An increase in the precision of private information τ_ε increases responsiveness to the private signal but, as before, has an ambiguous effect on the slope of supply. Now the parameter $\hat{c}$ is hump-shaped with respect to τ_ε because $\hat{c}>\lambda^{-1}$ for $\tau_\varepsilon\in(0,\infty)$ and $\hat{c}=\lambda^{-1}$ in the extremes of the interval $(0,\infty)$.

An increase in the degree of strategic complementarity makes agents more reliant on private information in the strategic substitutes case ($\beta>0$) or less so in the strategic

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[11] See Wilson (1979) for a model in which adverse selection makes demand schedules upward sloping.

complements case ($\beta < 0$). Increased reliance on public information as complementarity increases has been found by Morris and Shin (2002) and Angeletos and Pavan (2007) when public signals are exogenous. In this case, more complementarity increases the value of public information in forecasting aggregate behavior. When public information is endogenous, however, the precision of the public signal changes with the degree of complementarity m. When $\beta > 0$, less substitutability (increasing m) reduces the public precision and there is more reliance on private information; the opposite holds when $\beta < 0$. Note that the weight c given to the public statistic z has both a scarcity and an informational component and is always increasing in m.

Table 1 summarizes the comparative statics results on the equilibrium strategy.

<u>Table 1:</u> Comparative Statics on the Equilibrium Strategy

sgn	$\partial \tau_u$	$\partial \tau_\theta$	$\partial \tau_\varepsilon$
∂a	$-$	$-$	$+$
$\partial \hat{c}$	$-\beta$	β	$\beta\left(\beta^2 \tau_u \tau_\varepsilon^2 + 4\lambda^2 \tau_\theta \left(\tau_\varepsilon - \tau_\theta\right)\right)$

<u>Remark 2.</u> In the financial market interpretation of the model, if momentum traders predominate ($\beta < 0$) then the slope of excess demand $\Xi' = -\left(\beta^{-1} + \hat{c}\right)$ is positive. Less price-sensitive "momentum" traders (a more negative β) increases the degree of complementarity, decreases the weight given to the private information of rational traders, and increases the informativeness of prices: $\tau = \tau_\theta + \beta^2 a^2 \tau_u$ (increasing $|\beta|$ increases $\beta^2 a^2$). It can also be checked that, when $\beta < 0$, an increase in complementarity decreases market depth $1 + \beta\hat{c}$. Less price-sensitive "momentum" traders would then be associated with shallow markets. If "value" traders predominate ($\beta > 0$) then less price sensitivity (higher β) decreases complementarity and, as

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before, decreases the weight given to the private information of rational traders while increasing the informativeness of prices.

Remark 3. For the case of a double auction, let noise be given by $\hat{u} = \beta u$ and let $\beta \to \infty$. It is then immediate from Proposition 1 that, in the limit, $a = \tau_\varepsilon \lambda^{-1} \left(\tau_\varepsilon + \tau_\theta + \tau_u a^2 \right)^{-1}$, $\hat{b} = 0$, and $\hat{c} = \tau_\varepsilon \lambda^{-1} \left(\tau_\varepsilon + \tau_u a^2 \right)^{-1} > 0$. In this case, equilibrium schedules always have their natural (expected) slope. Given a diffuse prior ($\tau_\theta = 0$), we have $\hat{c} = a$ and the equilibrium strategy is $X(s_i, p) = a(p - s_i)$, with trader i supplying or demanding according as the price is (respectively) larger or smaller than the private signal.

4. Welfare analysis of the homogenous product market

Consider the homogeneous product market with quadratic production costs. The inverse demand $p = \alpha + u - \beta \tilde{x}$ arises from a benefit or surplus function $\left(\alpha + u - (\beta/2) \tilde{x} \right) \tilde{x}$, and the welfare criterion is total surplus:

$$TS = \left(\alpha + u - \beta \frac{\tilde{x}}{2} \right) \tilde{x} - \int_0^1 \left(\theta x_i + \frac{\lambda}{2} x_i^2 \right) di .$$

Under our assumptions, $\beta + \lambda > 0$ and the TS function is strictly concave for symmetric solutions.

The LE is partially revealing (with $0 < \tau_u < \infty$ and $0 < \tau_\varepsilon < \infty$), so expected total surplus should be strictly greater in the first-best allocation (full information) than at the LE. The reason is that suppliers produce under uncertainty and rely on imperfect estimation of the common cost component; hence they end up producing different amounts even though costs are identical and strictly convex.

The welfare benchmark that we use is the team solution maximizing expected total surplus subject to employing linear decentralized strategies (as in Vives 1988; Angeletos and Pavan 2007). This team-efficient solution internalizes the information externalities of the actions of agents, and it is restricted to using the same type of

strategies (decentralized and linear) that the market employs. Indeed, when reacting to information, an agent in the market does not take into account the influence her own actions have on public statistics.

At the team-efficient solution, expected total surplus $E[\text{TS}]$ is maximized under the constraint that firms use decentralized linear production strategies. That is,

$$\max_{a,b,c} E[\text{TS}]$$

subject to $\quad x_i = b - as_i + cz$, $\tilde{x} = b - a\theta + cz$, and $z = u + \beta a\theta$.

The following proposition characterizes the team solution (superscript *T*) and compares it with the LE solution (superscript LE).

<u>Proposition 3</u>. *Let* $\tau_\varepsilon > 0$. *Then the team problem has a unique solution with* $a^T > 0$, *and* $\text{sgn}\{a^{\text{LE}} - a^T\} = \text{sgn}\{\beta c^{\text{LE}}\}$.

<u>Proof</u>: Given that $\partial x_i/\partial a = -s_i + c\beta\theta$, $\partial x_i/\partial b = 1$, and $\partial x_i/\partial c = z$, the team solution is characterized by the following first-order conditions:

$$\frac{\partial E[\text{TS}]}{\partial a} = E\big[\big(p - \text{MC}(x_i)\big)\big(-s_i + c\beta\theta\big)\big] = 0,$$

$$\frac{\partial E[\text{TS}]}{\partial b} = E\big[\big(p - \text{MC}(x_i)\big)\big] = 0,$$

$$\frac{\partial E[\text{TS}]}{\partial c} = E\big[\big(p - \text{MC}(x_i)\big)z\big] = 0,$$

where $p = \alpha + u - \beta\tilde{x}$ and $\text{MC}(x_i) = \theta + \lambda x_i$. We know that the constraint $E[p - \text{MC}(x_i)] = 0$ is equivalent to $b = \alpha/(\beta + \lambda)$ and that $E[(p - \text{MC}(x_i))z] = 0$ is equivalent to $c = c(a)$. Also, note that $\partial^2 E[\text{TS}]/\partial^2 b < 0$ and $\partial^2 E[\text{TS}]/\partial^2 c < 0$ whenever $\beta + \lambda > 0$, once we replace b with $\alpha/(\beta + \lambda)$ and c with $c(a)$ in the expression for $E[\text{TS}]$. From this it follows that $E[\text{TS}]$ has a unique maximum attainable for some $a^T > 0$ and that $E[\text{TS}]$ is single-peaked for $a > 0$ (see Claim 2 in the Appendix). Evaluating $\partial E[\text{TS}]/\partial a$ at the LE, where

$E\left[\left(p - \mathrm{MC}(x_i)\right)s_i\right] = 0$, we obtain that $\partial E[\mathrm{TS}]/\partial a = c\beta E\left[\left(p - \mathrm{MC}(x_i)\right)\theta\right]$. Now, because

$$E\left[\left(p - \mathrm{MC}(x_i)\right)s_i\right] = E\left[\left(p - \mathrm{MC}(x_i)\right)\theta\right] + E\left[\left(p - \mathrm{MC}(x_i)\right)\varepsilon_i\right] = 0,$$

it follows that

$$\begin{aligned}
E\left[\left(p - \mathrm{MC}(x_i)\right)\theta\right] &= -E\left[\left(p - \mathrm{MC}(x_i)\right)\varepsilon_i\right] \\
&= E\left[\mathrm{MC}(x_i)\varepsilon_i\right] = E\left[\left(\theta + \lambda x_i\right)\varepsilon_i\right] = -\lambda a^{\mathrm{LE}}\sigma_\varepsilon^2 < 0
\end{aligned}$$

since ε_i is independent of all the model's other random variables and since $a^{\mathrm{LE}} > 0$ when $\tau_\varepsilon > 0$. Hence

$$\mathrm{sgn}\left\{\left.\frac{\partial E[\mathrm{TS}]}{\partial a}\right|_{a=a^{\mathrm{LE}}}\right\} = \mathrm{sgn}\left\{-\beta c^{\mathrm{LE}}\right\},$$

and this equals $\mathrm{sgn}\left\{a^T - a^{\mathrm{LE}}\right\}$ because $E[\mathrm{TS}]$ is single-peaked for $a > 0$ with a maximum at a^T. ♦

If $\beta = 0$ then there is neither a payoff nor an informational externality, and the team and market solutions coincide. For $\beta \neq 0$, $\tau_\varepsilon > 0$, and $\tau_u > 0$, the solutions coincide only if $c^{\mathrm{LE}} = 0$. This occurs only at the LE $a = \tau_\varepsilon / \left(\lambda\left(\tau_\varepsilon + \tau_\theta\right) + \beta\tau_\varepsilon\right)$ (the intermediate case in Figure 1, where $\beta > 0$). When firms do not respond to the price $\left(c = 0\right)$, the model reduces to a quantity-setting model with private information. This is consistent with Vives (1988), where it is shown that a Cournot market with private information and a continuum of suppliers solves a team problem whose objective function is expected total surplus. If $c^{\mathrm{LE}} < 0$ then a should be increased, and the contrary holds for $c^{\mathrm{LE}} > 0$. The team-optimal solution uses public information efficiently but is not bound by the privately efficient use of information.

At the LE with strategic substitutability, for which $\beta > 0$, there is too much (not enough) weight given to private information whenever τ_u is small (large) and supply functions are increasing (decreasing); see Figure 1. With strategic complementarity ($\beta < 0$) we have that both $c^{\mathrm{LE}} > 0$ and $\mathrm{sgn}\left\{\partial E[\mathrm{TS}]/\partial a\right\} = \mathrm{sgn}\left\{-\beta c^{\mathrm{LE}}\right\} > 0$ always, and agents give insufficient weight to private information; see Figure 2.

There is no information externality when firms have perfect information ($\tau_\varepsilon = \infty$) and the full information, first-best outcome (price equal to marginal cost) is obtained; when the price contains no information ($\tau_u = 0$); or when signals are uninformative ($\tau_\varepsilon = 0$). In each of these cases, the team and the market solution coincide in terms of $E[\text{TS}]$. For both the team and the market solutions, if $\tau_u = 0$ then $E[(p - \text{MC}(x_i))z] = 0$ implies that $c = 1/(\beta + \lambda)$ and that $E[\text{TS}]$ is infinite; if $\tau_\varepsilon = 0$, then $a = 0$ and $c = 1/(\beta + \lambda)$.

The conclusion is that, with strategic substitutability, team efficiency requires a decrease (increase) in c when c^{LE} is negative (positive). When $c^{\text{LE}} < 0$, the informational role of the price dominates and the price reveals too little information. In this case, more weight should be given to private signals so that public information becomes more revealing. Conversely, when the price is mainly an index of scarcity, $c^{\text{LE}} > 0$, it reveals too much information and a should be decreased. Only in the knife-edge (Cournot) case, where $c^{\text{LE}} = 0$, is the LE team-efficient. With strategic complementarity, agents place too little weight on private information. When $\beta < 0$, the informational externality is aligned with the price scarcity effect; in this case, it is always preferable to induce agents to rely more on their private information and thereby increase c.

Remark 4. As mentioned in Section 2, the model can be reinterpreted from the viewpoint of buyers (instead of sellers) by letting $y_i \equiv -x_i$. The same welfare analysis applies in this case.

Remark 5. Under monopolistic competition, the total surplus function (consistent with the differentiated demand system) is slightly different:

$$\text{TS} = (\alpha + u - \theta)\tilde{x} - \left(\beta \tilde{x}^2 + (\lambda/2)\int_0^1 x_i^2 \, di\right)\Big/2 \,.$$

Here the market is not efficient under complete information because price is not equal to marginal cost. Each firm has some residual market power. We could proceed with a similar welfare analysis, but in the next section we instead provide a welfare

benchmark that depends only on the payoffs of the game's players (e.g., welfare in the monopolistic competition case is evaluated from the perspective of firms) and hence applies to any interpretation of the model.

Remark 6. If the signals of agents can be communicated to a center, then questions arise concerning the incentives to reveal information and how welfare allocations may be modified. This issue is analyzed in a related model by Messner and Vives (2006), who use a mechanism design approach along the lines of Laffont (1985).

5. Internal welfare benchmark

At the internal team–efficient solution, expected profit $E[\pi_i]$ (where $\pi_i = (\alpha + u - \beta \tilde{x} - \theta) x_i - (\lambda/2) x_i^2$) is maximized under the constraint that agents use decentralized linear strategies. This is the cooperative solution from the players' perspective. That is,

$$\max_{a,b,c} E[\pi_i]$$

subject to $x_i = b - as_i + cz$, $\tilde{x} = b - a\theta + cz$, and $z = u + a\beta\theta$.

It should now be clear that the LE will not be efficient with respect to the internal team benchmark if $\beta \neq 0$ because that benchmark internalizes payoff externalities at the LE.[12] At the *internal team* (IT) benchmark, joint profits are maximized. The question is whether the LE allocates the correct weights (from the players' collective viewpoint) to private and public information. We show that the answer to this question is qualitatively similar to the one derived when analyzing the *total surplus* team benchmark.

Our next proposition characterizes the solution.

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[12]   Indeed, when $\beta = 0$ there are no externalities (payoff or informational) and the internal team and market solutions coincide.

<u>Proposition 4</u>. *Let* $\tau_\varepsilon > 0$. *Then the internal team problem has a unique solution with*

$$a^{IT} > 0, \ and \ \operatorname{sgn}\left\{a^{LE} - a^{IT}\right\} = \operatorname{sgn}\left\{\beta\left(c^{LE}\lambda\sigma_\varepsilon^2 + \left(c^{LE}\beta - 1\right)^2 \sigma_\theta^2\right)\right\}.$$

<u>Proof</u>: Given that $\partial x_i/\partial a = -s_i + c\beta\theta$, $\partial x_i/\partial b = 1$, and $\partial x_i/\partial c = z$, the internal team solution is characterized by the following FOCs:

$$\frac{\partial E[\pi_i]}{\partial a} = E\left[\left(p - MC(x_i)\right)\left(-s_i + c\beta\theta\right) - \beta\theta(c\beta - 1)x_i\right] = 0,$$

$$\frac{\partial E[\pi_i]}{\partial b} = E\left[\left(p - MC(x_i)\right) - \beta x_i\right] = 0,$$

$$\frac{\partial E[\pi_i]}{\partial c} = E\left[\left(p - MC(x_i)\right)z - \beta x_i z\right] = 0.$$

It is easy to see that the constraint $E\left[\left(p - MC(x_i)\right) - \beta x_i\right] = 0$ is equivalent to $b = \alpha/(2\beta + \lambda)$; we can also check that $E\left[\left(p - MC(x_i)\right)z - \beta x_i z\right] = 0$ is equivalent to $c = c^{IT}(a)$, where

$$c^{IT}(a) = \frac{1}{2\beta + \lambda} - \frac{\beta a\tau_u\left(1 - (\lambda + \beta)a\right)}{\tau(2\beta + \lambda)} \quad and \quad \tau = \tau_\theta + \beta^2\tau_u a^2.$$

Similarly as in the last section, it can be shown—after noting that $\partial^2 E[\pi_i]/\partial^2 b < 0$ and $\partial^2 E[\pi_i]/\partial^2 c < 0$ whenever $2\beta + \lambda > 0$ and putting $c^{IT} = (a)$ and $b = \alpha/(2\beta + \lambda)$ in the expression for $E[\pi_i]$—that $E[\pi_i]$ has a unique maximum attainable for some $a^{IT} > 0$ and that $E[\pi_i]$ is single-peaked for $a > 0$ (see Claim 3 in the Appendix). Evaluating $\partial E[\pi_i]/\partial a$ at the LE, where $E\left[\left(p - MC(x_i)\right)s_i\right] = 0$, we obtain

$$\frac{\partial E[\pi_i]}{\partial a} = \beta E\left[c\left(p - MC(x_i)\right)\theta - (c\beta - 1)\theta x_i\right].$$

As in the last section, we have $E\left[\left(p - MC(x_i)\right)\theta\right] = -\lambda a\sigma_\varepsilon^2 < 0$ and, recalling that $\bar{\theta} = 0$, it is easily checked that $E[\theta x_i] = a\sigma_\theta^2(c\beta - 1)$. At the LE we have $c\beta - 1 < 0$ and therefore

$$\frac{\partial E[\pi_i]}{\partial a} = -\beta a^{LE}\left(c^{LE}\lambda\sigma_\varepsilon^2 + \left(c^{LE}\beta - 1\right)^2 \sigma_\theta^2\right).$$

Since $E[\pi_i]$ is single-peaked for $a > 0$ and has a unique maximum at $a^{IT} > 0$ and $a^{LE} > 0$, it follows that

$$\mathrm{sgn}\left\{a^{IT} - a^{LE}\right\} = \mathrm{sgn}\left\{\left.\frac{\partial E[\pi_i]}{\partial a}\right|_{a=a^{LE}}\right\} = \mathrm{sgn}\left\{-\beta\left(c^{LE}\lambda\sigma_\varepsilon^2 + \left(c^{LE}\beta - 1\right)^2\sigma_\theta^2\right)\right\}. \blacklozenge$$

If $c^{LE} \geq 0$ then $\mathrm{sgn}\left\{a^{LE} - a^{IT}\right\} = \mathrm{sgn}\{\beta\}$. This yields the same qualitative result as in the previous section if $c^{LE} > 0$: too much or too little weight given to private information in the presence of (respectively) strategic substitutability or strategic complementarity. In this case, however, if agents use Cournot strategies (i.e., if $c^{LE} = 0$) then the market is not internal team–efficient. This should not be surprising when one considers that, when $c^{LE} = 0$, there is no information externality yet the payoff externality is not internalized, as agents set a quantity that is too large (small) under strategic substitutability (complementarity). If $\beta > 0$ and $c^{LE} < 0$, then $\left(c^{LE}\lambda\sigma_\varepsilon^2 + \left(c^{LE}\beta - 1\right)^2\sigma_\theta^2\right) > 0$ for c^{LE} close to zero or sufficiently negative (τ_u large). For intermediate values of c^{LE} we have $\left(c^{LE}\lambda\sigma_\varepsilon^2 + \left(c^{LE}\beta - 1\right)^2\sigma_\theta^2\right) < 0$ and $a^{IT} - a^{LE} > 0$.

This is the same qualitative result concerning the weight given to private information as derived previously using the total surplus team benchmark—with the following proviso: when $c^{LE} < 0$, it may not be the case that too little weight is given to private information.

It is interesting to note that, if agents cannot use contingent strategies and there is no information externality issue (as in, e.g., cases of Cournot or Bertrand competition), then Angeletos and Pavan (2007) argue that the strategic complementarity case would exhibit over-reliance on private information (the opposite of what occurs with endogenous public information) and that strategic substitutability would exhibit under-reliance on private information (in contrast with the case for endogenous public information, where either under- or over-reliance on private information is possible).

These results have several implications. Consider first the financial market interpretation. If the behavioral traders are momentum traders ($\beta < 0$), then prices always contain too little information (from the collective viewpoint of informed traders). If the behavioral traders are value traders ($\beta > 0$) then the opposite occurs in the usual case of downward-sloping demand schedules for informed traders, which obtain when the volume of *behavioral trading* is large (low τ_u). When the volume generated by behavioral traders is small (high τ_u), demand schedules are upward sloping and prices may contain too little information. This happens for intermediate values of τ_u within its high-value region.

Likewise, in the asset auction interpretation—and from the collective viewpoint of bidders—prices contain too much information in the usual case of downward-sloping demand schedules, which obtain when the volume of *noncompetitive bidding* is large (low τ_u). When the volume generated by noncompetitive bids is small (high τ_u), demand schedules are again upward sloping and prices may contain too little information for intermediate values of τ_u within its high-value region.

6. <u>Concluding remarks</u>.

Rational expectations equilibria (linear Bayesian equilibria) are not total surplus team–efficient even when the allowed allocations share certain properties with the market equilibrium (i.e., both are linear in information). The reason is that, in general, the market does not internalize the informational externality when prices convey information. Only in exceptional circumstances (i.e., when the information externality vanishes) does the market get it right. Under strategic substitutability, prices will tend to convey too little information when the informational role of prices prevails over its index-of-scarcity role, or will convey too much information in the opposite case. Under strategic complementarity, such as in the presence of a network good, prices always convey too little information.

These results extend to the internal team benchmark, in which the players' collective welfare is taken into account, as long as the index-of-scarcity role of prices prevails

over their informational role. When this is not the case, the amount of information in prices may be above or below the welfare benchmark. It follows that received results on the optimal relative weights to be placed on private and public information (when the latter is exogenous) may be overturned.

Several extensions are worth considering. Examples include exploring tax-subsidy schemes to implement team-optimal solutions along the lines of Angeletos and La'O (2008) and Angeletos and Pavan (2009); studying incentives to acquire information (as in Vives 1988; Burguet and Vives 2000; Hellwig and Veldkamp 2009); and examining the circumstances under which more public information actually reduces welfare (as in Burguet and Vives 2000; Morris and Shin 2002; Amador and Weill 2009, 2010).

Appendix

Proof of Proposition 1: From the posited strategy $X(s_i, z) = b - as_i + cz$, where $z = u + \beta a\theta$ and $1 - \beta c \neq 0$, we obtain that $p = \alpha - \beta b + (1 - \beta c) z$. From the first-order condition for player i we have

$$X(s_i, z) = \lambda^{-1} \left[\alpha - \beta b + (1 - \beta c) z - E[\theta | s_i, z] \right].$$

Here $E[\theta / s_i, z] = \gamma s_i + (1 - \gamma) E[\theta / z]$ with $\gamma = \tau_\varepsilon (\tau_\varepsilon + \tau)^{-1}$, $E[\theta / z] = \beta \tau_u a \tau^{-1} z$ (recall that we have normalized $\bar{\theta} = 0$), and $\tau = \tau_\theta + \beta^2 a^2 \tau_u$ from the projection theorem for Gaussian variables. Identifying coefficients with $X(s_i, z) = b - as_i + cz$, we can immediately obtain

$$a = \frac{\tau_\varepsilon}{\lambda(\tau_\varepsilon + \tau)}, \quad c = \frac{1}{(\beta + \lambda)} - \frac{\beta a \tau_u}{(\beta + \lambda)(\tau_\varepsilon + \tau)}, \quad \text{and} \quad b = \frac{\alpha}{\beta + \lambda}.$$

It follows that the equilibrium parameter a is determined as the unique (real), of the following cubic equations, that is positive and lies in the interval $a \in \left(0, \tau_\varepsilon \lambda^{-1} (\tau_\theta + \tau_\varepsilon)^{-1} \right)$:

$$a = \frac{\tau_\varepsilon}{\lambda(\tau_\varepsilon + \tau_\theta + \beta^2 a^2 \tau_u)} \quad \text{or} \quad \beta^2 \tau_u a^3 + (\tau_\varepsilon + \tau_\theta) a - \lambda^{-1} \tau_\varepsilon = 0$$

and

$$c = \frac{1}{(\beta + \lambda)} - \frac{\beta \lambda \tau_u a^2}{(\beta + \lambda) \tau_\varepsilon}. \,^{13}$$

It is immediate from the preceding equality for c that $c < (\beta + \lambda)^{-1}$ (since $a \geq 0$) and that $1 - \beta c > 0$ (since $\beta + \lambda > 0$); therefore,

$$\beta c = \frac{\beta}{\beta + \lambda} - \frac{\beta^2 a \tau_u}{(\beta + \lambda)(\tau_\varepsilon + \tau)} < 1.$$

It follows that

$$X(s_i, p) = \hat{b} - as_i + \hat{c}p,$$

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[13] Indeed, we would obtain the same result (i.e., the efficient use of public and of private information) from the intersection of the two curves in (a, c) space.

where $\hat{b} = b(1 - \lambda\hat{c})$, $b = \alpha/(\beta + \lambda)$, and $\hat{c} = c/(1 - \beta c)$ with $1 + \beta\hat{c} > 0$. From the equilibrium expression for $c = (\beta + \lambda)^{-1}\left(1 - \beta\lambda\tau_u a^2 \tau_\varepsilon^{-1}\right)$ we obtain the expression for

$$\hat{c} = \left(c^{-1} - \beta\right)^{-1}. \blacklozenge$$

<u>Claim 1.</u> *Linear equilibria in strategies with bounded means and with uniformly (across players) bounded variances yield linear equilibria of the schedule game for which the public statistic function is of type* $P(\theta, u)$.

<u>Proof</u>: If for player i we posit the strategy

$$x_i = \hat{b}_i + \hat{c}_i p - a_i s_i$$

then the aggregate action is given by

$$\tilde{x} = \int_0^1 x_i \, di = \hat{b} + \hat{c}p - a\theta - \int_0^1 a_i \varepsilon_i \, di = \hat{b} + \hat{c}p - a\theta,$$

where $\hat{b} = \int_0^1 \hat{b}_i \, di$, $\hat{c} = \int_0^1 \hat{c}_i \, di$, and $a = \int_0^1 a_i \, di$ (assuming that all terms are well-defined). Observe that, according to our convention on the average error terms of the signals, $\int_0^1 a_i \varepsilon_i \, di = 0$ a.s. provided that $\mathrm{var}\left[a_i \varepsilon_i\right]$ is uniformly bounded across agents (since $\mathrm{var}\left[\varepsilon_i\right] = \sigma_\varepsilon^2$, it is enough that a_i be uniformly bounded). In equilibrium, this will be the case. Therefore, if we restrict attention to candidate linear equilibria with parameters a_i uniformly bounded in i and with well-defined average parameters $\hat{b}$ and $\hat{c}$, then $\tilde{x} = \hat{b} + \hat{c}p - a\theta$ and the public statistic function is of the type $P(\theta, u)$. $\blacklozenge$

<u>Proof of Proposition 2</u>: (i) From the equation determining the responsiveness to private information a, $\beta^2 \tau_u a^3 + \left(\tau_\varepsilon + \tau_\theta\right)a - \lambda^{-1}\tau_\varepsilon = 0$, it is immediate that a decreases with τ_u, τ_θ, and λ, that a increases with τ_ε, and that $\mathrm{sgn}\{\partial a/\partial\beta\} = \mathrm{sgn}\{-\beta\}$. As τ_u ranges from 0 to ∞, a decreases from $\lambda^{-1}\tau_\varepsilon\left(\tau_\theta + \tau_\varepsilon\right)^{-1}$ to 0. Price informativeness $\tau = \tau_\theta + \beta^2 a^2 \tau_u$ is increasing in τ_ε (since

a increases with τ_ε) and also in τ_u (since $a = \lambda^{-1}\tau_\varepsilon\left(\tau_\varepsilon + \tau\right)^{-1}$ and a decreases with τ_u).

(ii) As τ_u ranges from 0 to ∞, the responsiveness to public information c goes from $\left(\beta + \lambda\right)^{-1}$ to $-\infty$ (resp. $+\infty$) if $\beta > 0$ (resp. $\beta < 0$). The result follows since, in equilibrium,

$$c = \frac{1}{\beta + \lambda} - \frac{\beta\lambda\tau_u a^2}{\left(\beta + \lambda\right)\tau_\varepsilon} = \frac{1}{\beta + \lambda} - \frac{1}{\left(\beta + \lambda\right)\beta}\left(\frac{1}{a} - \lambda\left(1 + \frac{\tau_\theta}{\tau_\varepsilon}\right)\right)$$

and $a \to 0$ as $\tau_u \to \infty$. It follows that $\mathrm{sgn}\left\{\partial c/\partial \tau_u\right\} = \mathrm{sgn}\left\{-\beta\right\}$ because $\partial a/\partial \tau_u < 0$. Similarly, from the first part of the expression for c we have $\mathrm{sgn}\left\{\partial c/\partial \tau_\theta\right\} = \mathrm{sgn}\left\{\beta\right\}$ since $\partial a/\partial \tau_\theta < 0$. Furthermore, with some work it is possible to show that, in equilibrium,

$$\frac{\partial c}{\partial \tau_\varepsilon} = \left(\beta + \lambda\right)^{-1}\lambda\,\beta\tau_u\tau_\varepsilon^{-1}a\left(2\frac{a\lambda - 1}{\lambda\left(\tau_\theta + \tau_\varepsilon + 3a^2\beta^2\tau_u\right)} + a\tau_\varepsilon^{-1}\right) \quad \text{and}$$

$$\mathrm{sgn}\left\{2\frac{a\lambda - 1}{\lambda\left(\tau_\theta + \tau_\varepsilon + 3a^2\beta^2\tau_u\right)} + a\tau_\varepsilon^{-1}\right\} = \mathrm{sgn}\left\{a\lambda\tau_\theta - 2\tau_\varepsilon + 3a\lambda\tau_\varepsilon + 3a^3\beta^2\lambda\tau_u\right\}$$
$$= \mathrm{sgn}\left\{-2a\lambda\tau_\theta + \tau_\varepsilon\right\}$$
$$= \mathrm{sgn}\left\{\beta^2\tau_u\tau_\varepsilon^2 + 4\lambda^2\tau_\theta^2\left(\tau_\varepsilon - \tau_\theta\right)\right\}.$$

Hence we conclude that $\mathrm{sgn}\left\{\partial c/\partial \tau_\varepsilon\right\} = \mathrm{sgn}\left\{\beta\left(\beta^2\tau_u\tau_\varepsilon^2 + 4\lambda^2\tau_\theta^2\left(\tau_\varepsilon - \tau_\theta\right)\right)\right\}$. Since $\hat{c} = \left(c^{-1} - \beta\right)^{-1}$, it follows that $\hat{c}$ goes from λ^{-1} to $-\beta^{-1}$ as τ_u ranges from 0 to ∞,[14] $\mathrm{sgn}\left\{\partial\hat{c}/\partial \tau_u\right\} = \mathrm{sgn}\left\{-\partial\hat{c}/\partial \tau_\theta\right\} = \mathrm{sgn}\left\{-\beta\right\}$, and $\mathrm{sgn}\left\{\partial\hat{c}/\partial \tau_\varepsilon\right\} = \mathrm{sgn}\left\{\partial c/\partial \tau_\varepsilon\right\}$. It is then immediate that $1 + \beta\hat{c}$ is decreasing in τ_u and increasing in τ_θ.

(iii) Using the change of variables $m \equiv -\beta/\lambda$ and $n \equiv \lambda$ (with $\beta = -mn$) yields

$$\frac{\partial a}{\partial m} = \frac{\partial a}{\partial \beta}\frac{\partial \beta}{\partial m} + \frac{\partial a}{\partial \lambda}\frac{\partial \lambda}{\partial m} \quad \text{and} \quad \frac{\partial \beta}{\partial m} = -\lambda.$$

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[14] Note that if $\beta < 0$ and $\beta + \lambda > 0$ then $\lambda^{-1} < -\beta^{-1}$.

It follows that $\text{sgn}\{\partial a/\partial m\} = \text{sgn}\{-\partial a/\partial \beta\} = \text{sgn}\{\beta\}$. Similarly, $\partial c/\partial m = -\lambda\,(\partial c/\partial \beta)$ and $\text{sgn}\{\partial c/\partial m\} = \text{sgn}\{-\partial c/\partial \beta\} > 0$ since it can be shown that

$$\frac{\partial c}{\partial \beta} = -\frac{1}{(\lambda+\beta)\tau_\varepsilon}\left[\frac{\tau_\varepsilon + \lambda^2\tau_u a^2}{(\lambda+\beta)} + \frac{4a^5\lambda^3\tau_u^2\beta^2}{\left(1+2a^3\tau_u\lambda\beta^2\right)}\right] < 0.$$

Furthermore, $\partial\tau/\partial m = -\lambda\,(\partial\tau/\partial\beta)$ and

$$\frac{\partial\tau}{\partial m} = -\lambda\tau_u\left(2\beta a^2 + 2\beta^2 a\frac{\partial a}{\partial\beta}\right) = -2\lambda\beta a\tau_u\left(a - \frac{2a^4\tau_u\lambda\beta^2}{1+2a^3\tau_u\lambda\beta^2}\right) = -\frac{2\lambda\beta a^2\tau_u}{1+2a^3\tau_u\lambda\beta^2}.\quad\blacklozenge$$

<u>Claim 2.</u> *Let $\tau_\varepsilon > 0$, and let $\text{ETS}(a)$ denote $E[\text{TS}]$ as a function of a in the team problem. Then $\text{ETS}(a)$ has a unique maximum, attained for some $a^T > 0$, and is single-peaked for $a > 0$.*

<u>Proof</u>: Using the expression for TS from Section 4 together with $x_i = b - as_i + cz$, $\tilde{x} = b - a\theta + cz$, and $z = u + \beta a\theta$, some tedious manipulations yield

$$E[\text{TS}] = \alpha b - \frac{\beta+\lambda}{2}b^2 + \left(1 - \frac{\beta+\lambda}{2}c\right)c\sigma_u^2$$
$$+ (1-\beta c)a\left(1 - \frac{\beta+\lambda}{2}(1-\beta c)a\right)\sigma_\theta^2 - \frac{\lambda}{2}a^2\sigma_\varepsilon^2.$$

Now $\partial E[\text{TS}]/\partial a = 0$ immediately yields $a = \dfrac{\tau_\varepsilon(1-\beta c)}{(\beta+\lambda)\tau_\varepsilon(1-\beta c)^2 + \lambda\tau_\theta}$. Setting $b = \alpha/(\beta+\lambda)$ and $c = c(a) = \dfrac{1}{\beta+\lambda} - \dfrac{\beta a\tau_u(1-\lambda a)}{\tau(\beta+\lambda)}$ in the expression for $E[\text{TS}]$, we obtain

$ETS(a) =$
$$\frac{-\tau_\theta\tau_u^2\lambda\beta^2(\beta+\lambda)a^4 + \left(\tau_u\left(\tau_\varepsilon\beta^2(\tau_u+\tau_\theta+\alpha^2\tau_u\tau_\theta) - \tau_\theta\lambda(\tau_\theta(\beta+\lambda)) + \lambda\tau_\varepsilon\right)\right)a^2 + 2\lambda\tau_u\tau_\theta\tau_\varepsilon a + \tau_\varepsilon\tau_\theta^2(\alpha^2\tau_u+1)}{2(\tau_\theta+\beta^2\tau_u a^2)(\beta+\lambda)\tau_u\tau_\theta\tau_\varepsilon}.$$

This function tends to $-\infty$ as a approaches $-\infty$ or $+\infty$, and its denominator is never zero. Hence the function has a global maximum, and

$ETS'(a) =$
$$-\frac{\tau_u^2\lambda\beta^4(\beta+\lambda)a^5 + 2\tau_\theta\tau_u\lambda\beta^2(\beta+\lambda)a^3 + \beta^2\lambda\tau_u\tau_\varepsilon a^2 + \left(\tau_\theta^2\lambda(\beta+\lambda) - \beta^2\tau_u\tau_\varepsilon + \lambda^2\tau_\varepsilon\tau_\theta\right)a - \lambda\tau_\theta\tau_\varepsilon}{(\beta+\lambda)\left(\tau_\theta + a^2\beta^2\tau_u\right)^2\tau_\varepsilon}.$$

The denominator of $\text{ETS}'(a)$ is always positive. Applying Descartes' rule of signs[15] to the numerator of $\text{ETS}'(a)$, we find that there exists a unique positive a such that $\text{ETS}'(a) = 0$. Combining this fact with $\lim_{a \to \infty} \text{ETS}(a) = -\infty$, we conclude that there is a unique extremal value of $\text{ETS}(a)$ when $a > 0$ and that this value is a local maximum. In addition, it is easy to show that $\text{ETS}(a) - \text{ETS}(-a) > 0$ for all $a > 0$. This implies that the positive value of a that is a local maximum is also the global maximum of $\text{ETS}(a)$. $\blacklozenge$

<u>Claim 3.</u> *Let $\tau_\varepsilon > 0$, and let $\pi(a)$ denote $E[\pi_i]$ as a function of a in the internal team problem. Then $\pi(a)$ has a unique maximum, attained for some $a^{\text{IT}} > 0$, and is single-peaked for $a > 0$.*

<u>Proof</u>: Much as in the proof for Claim 2, from the expression $\pi_i = (\alpha + u - \beta \tilde{x} - \theta) x_i - (\lambda/2) x_i^2$ together with $x_i = b - a s_i + c z$, $\tilde{x} = b - a\theta + cz$, and $z = u + \beta a \theta$ we obtain

$$E[\pi_i] = \alpha b - \left(\beta + \frac{\lambda}{2}\right) b^2 + \left(1 - \frac{(2\beta + \lambda)}{2} c\right) c\sigma_u^2$$
$$- (1 - \beta c) a \left[1 - \frac{(2\beta + \lambda)}{2} (1 - \beta c) a\right] \sigma_\theta^2 - \frac{\lambda}{2} a^2 \sigma_\varepsilon^2.$$

Note that now the optimality conditions are

$$a = \frac{\tau_\varepsilon (1 - \beta c)}{(2\beta + \lambda) \tau_\varepsilon (1 - \beta c)^2 + \lambda \tau_\theta}, \quad b = \frac{\alpha}{2\beta + \lambda}, \quad c = \frac{1}{(2\beta + \lambda)} - \frac{a\beta \tau_u (1 - (\beta + \lambda) a)}{(2\beta + \lambda) \tau}.$$

Substituting into $E[\pi_i]$ the expressions for b and c, we obtain

$\pi(a) =$
$$\frac{-\tau_\theta \tau_u^2 \lambda \beta^2 (2\beta + \lambda) a^4 + \left(\tau_u \left(\tau_\varepsilon \beta^2 \left(\tau_u + \tau_\theta + \alpha^2 \tau_u \tau_\theta\right) - \tau_\theta \lambda \left(\tau_\theta (2\beta + \lambda)\right) + \lambda \tau_\varepsilon\right)\right) a^2 + 2\lambda \tau_u \tau_\theta \tau_\varepsilon a + \tau_\varepsilon \tau_\theta^2 \left(\alpha^2 \tau_u + 1\right)}{2 \left(\tau_\theta + \beta^2 \tau_u a^2\right) (2\beta + \lambda) \tau_u \tau_\theta \tau_\varepsilon}$$

and

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[15] The rule states that if the terms of a polynomial of one variable with real coefficients are ordered by descending exponent of the variable, then the number of positive roots of the polynomial is *either* equal to the number of sign differences between consecutive nonzero coefficients *or* less than that number by a multiple of 2. Multiple roots of the same value are counted separately.

$$\pi'(a) =$$

$$-\frac{\tau_u^2 \lambda \beta^4 (2\beta+\lambda)a^5 + 2\tau_\theta \tau_u \lambda \beta^2 (2\beta+\lambda)a^3 + \beta^2(\beta+\lambda)\tau_u \tau_\varepsilon a^2 + \left(\tau_\theta^2 \lambda(2\beta+\lambda) - \beta^2 \tau_u \tau_\varepsilon + (\beta+\lambda)^2 \tau_\varepsilon \tau_\theta\right)a - (\beta+\lambda)\tau_\theta \tau_\varepsilon}{(2\beta+\lambda)\left(\tau_\theta + a^2 \beta^2 \tau_u\right)^2 \tau_\varepsilon};$$

the proof then proceeds similarly to the proof of Claim 2. ♦

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