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A LARGE-MARKET RATIONAL EXPECTATIONS EQUILIBRIUM MODEL

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ABSTRACT

A Large-Market Rational Expectations Equilibrium Model*

This paper presents a market with asymmetric information where a privately revealing equilibrium obtains in a competitive framework and where incentives to acquire information are preserved. The equilibrium is efficient, and the paradoxes associated with fully revealing rational expectations equilibria are precluded without resorting to noise traders. The rate at which equilibria in finite replica markets with n traders approach the equilibrium in the continuum economy is $1/\sqrt{n}$, slower than the rate of convergence to price-taking behavior ($1/n$); and the per capita welfare loss is dissipated at the rate $1/n$, slower than the rate at which inefficiency due to market power vanishes ($1/n^2$). The model admits a reinterpretation in which behavioral traders coexist with rational traders, and it allows us to characterize the amount of induced mispricing.

JEL Classification: D82, D84 and G14

Keywords: adverse selection, behavioral traders, complementarities, double auction, information acquisition, multi-unit auctions and rate of convergence

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1. Introduction

Rational expectations models have proved to be a workhorse for the analysis of situations involving uncertainty and private information. An important aim has been to provide a workable model of Hayek's (1945) idea that prices aggregate the dispersed information of agents in the economy, given agents' dual role as index of scarcity and conveyors of information. However, the concept of a rational expectations equilibrium (REE) is not without problems—and this is especially true of fully revealing REE in competitive markets. The concept has two main difficulties. First, the equilibrium need not be implementable; that is, it may not be possible to find a trading mechanism (in a well-specified game) that delivers the fully revealing REE. Second, if information is costly to acquire, then agents at a fully revealing REE will have no incentive to purchase information and so the equilibrium breaks down (this is the “paradox of informationally efficient markets”; see Grossman and Stiglitz 1980). An added problem arises if the competitive REE is defined in a finite-agent economy, since then traders realize that prices convey information but do not realize the impact of their actions on the price (this is the “schizophrenia” problem of Hellwig 1980). These problems are typically overcome by considering noisy REE in large economies. Indeed, noise traders in competitive models have prevented trade from collapsing.¹

This paper presents a simple, competitive, large-market model *without* noise traders and in which the valuation of each trader has a common and a private value component. It shows how to obtain a privately revealing equilibrium in a well-specified game where each trader submits a demand schedule and has incentives to rely on his private signal and on the price. In a privately revealing equilibrium the price *and* the private signal of a trader are sufficient statistics for the trader in the market. The equilibrium is efficient, preserves incentives to acquire information, and overcomes the problems of fully revealing REE without reliance on noise trading. Furthermore, the Bayesian equilibrium in demand schedules obtained in the large market is not an artifact of the continuum specification for traders. We verify that the large limit market well approximates large finite markets in which traders are strategic and have incentives to influence prices, thus

¹ See, for example, Diamond and Verrecchia (1981) and Admati (1985).

providing a foundation for REE with rational traders in the context of the model presented. In addition, the model admits a reinterpretation in which behavioral traders coexist with rational traders (under an approach similar to that of Serrano-Padial 2010).

The model is of the linear-normal variety, as in Grossman and Stiglitz (1980), and it assumes declining marginal valuations. It is quite tractable and allows us to address the case of a good with exogenous supply as well as the case of a double auction; in addition, it enables us to characterize explicitly not only information acquisition but also rates of convergence of finite markets to the continuum limit. The model admits interpretation in terms of both financial markets and markets for goods.

We find that there is a unique linear equilibrium that is privately revealing and efficient. In equilibrium, a high price indicates a high valuation, and this reduces responsiveness to price when there is private information. Indeed, demand schedules in this case are steeper and there is a greater extent of adverse selection in the market (which increases with the correlation of valuations and the noise in the signals). If the information effect is large enough, demand schedules may be upward sloping. Demand becomes steeper also as the slope of marginal valuation is steeper and as the slope of exogenous supply is flatter. The second of these observations can be reinterpreted as an inverse measure of the mass of “value” behavioral traders in a double auction market. Then we find that the demand of rational traders is upward sloping for a large enough mass of behavioral traders and also that the amount of mispricing increases with this mass. The case of a downward-sloping exogenous supply of the good allows us to capture complementarities among the agents in the market, makes aggregate excess demand upward sloping, and can be interpreted in terms of the presence of “momentum” or “positive-feedback” behavioral traders. Mispricing is then more severe than with value behavioral traders, and it increases as rational traders increase relative to momentum traders. Rational traders benefit more from the presence of momentum traders than from that of value traders.

If the signals are costly to acquire and if traders face a convex cost of acquiring precision, then there is an upper bound on the correlation of valuation parameters below which there

are incentives to purchase some precision. This upper bound is decreasing in the precision of the prior and in the marginal cost of acquiring precision; this bound is 1—that is, perfect correlation—when the marginal cost (at zero precision) of acquiring precision is zero or when the prior is diffuse. A more diffuse prior or less correlation among valuations induces more efforts to acquire information. The rate at which equilibria in finite replica markets (with n traders and corresponding exogenous supply) approach the equilibrium in the continuum economy is $1/\sqrt{n}$, the same rate at which the average signal of the traders tends to the limit average valuation parameter. Convergence accelerates as we approach a common value environment with better signals or with less prior uncertainty. The corresponding (per capita) welfare loss in the finite market with respect to the limit market is of the order of $1/n$, and again convergence is faster when closer to the common value case or when there is less prior uncertainty. However, the effect of noise in the signals is ambiguous here because it has opposing effects on allocative and distributive efficiency. The rate of convergence of prices, $1/\sqrt{n}$, is slower than the rate of convergence to price-taking behavior, $1/n$, with a corresponding dissipation rate of the (per capita) welfare loss due to market power, $1/n^2$.

The model developed here can be applied to explaining how banks bid for liquidity in central bank auctions (or how bidders behave in Treasury auctions) and to assessing the effect of pollution damages on market outcomes. In particular, the model can be used to simulate the impact of a financial crisis on central bank liquidity auctions.

This paper is related to at least four strands of the literature. First, it is related to work on information aggregation, and on the foundations of REE in auction games, that developed from the pioneering studies of Wilson (1977) and Milgrom (1981) and have more recently been extended by Pesendorfer and Swinkels (1997). The convergence to price taking and to efficiency as double auction markets grow large has been analyzed in Wilson (1985), Satterthwaite and Williams (1989), and Rustichini, Satterthwaite, and Williams (1994). Along these lines, Cripps and Swinkels (2006) also allow private and common value components of uncertainty. Our results on the model's double auction

version are more closely related to Reny and Perry (2006), who present a double auction model with a unique and privately revealing REE that is implementable as a Bayesian equilibrium in symmetric increasing bidding strategies; they also offer an approximation in a finite large market. Given the nature of our own model, the results presented here deal with multi-unit demands and enable characterizations of an equilibrium's comparative static properties and of information acquisition. Furthermore, the model allows us to study convergence rates and to analyze the effect of an exogenous supply of the good.

Second, a parallel literature on information aggregation has developed in the context of Cournot markets (Palfrey 1985; Vives 1988). Third, the literature on strategic competition in terms of schedules in uniform price auctions has developed from the seminal work of Wilson (1979) and Kyle (1989) (see also Wang and Zender 2002). Vives (2011a,b) considers strategic supply competition and provides a finite-trader counterpart to the model in this paper. Consistently with the analysis in double auction settings, we find that price taking obtains at a rate of $1/n$ whereas efficiency is achieved at the rate $1/n^2$ (where n is the number of traders).²

Finally, the fourth strand to which this paper is related is the growing literature on behavioral models (e.g., De Long et al. 1990; Daniel, Hirshleifer, and Subrahmanyam 1998; Hong and Stein 1999; Serrano-Padial 2010). This connection will be emphasized throughout our discussion.

The balance of the paper is organized as follows. Section 2 presents the model. Section 3 summarizes the problems with the concept of a fully revealing REE and introduces our approach. Section 4 characterizes the equilibrium and its properties; Section 5 presents some extensions of the model. Section 6 deals with information acquisition, and Section 7 considers large but finite markets. Finally, the Appendix gathers some of the proofs.

² See Rostek and Weretka (2010) for an asymmetric correlation structure for valuation parameters.

2. The model

A continuum of traders—indexed in the unit interval $i \in [0,1]$, which is endowed with the Lebesgue measure—face a linear, downward-sloping inverse supply for a homogenous good $p = \alpha + \beta \tilde{x}$. Here $\alpha, \beta > 0$ and \tilde{x} denotes aggregate quantity in our continuum economy (and also per capita quantity, since we have normalized the measure of traders to 1). We have $\tilde{x} = \int_0^1 x_i di$, where x_i is the individual quantity demanded by trader i . We interpret $x_i < 0$ to mean that the trader is a (net) supplier.

Traders are assumed to be risk neutral. The profits of trader i when the price is p are

$$\pi_i = (\theta_i - p)x_i - \frac{\lambda}{2}x_i^2,$$

where θ_i is a value private to the trader and λx_i is a marginal transaction or opportunity cost (it could also be interpreted as a proxy for risk aversion).

We assume that θ_i is normally distributed (with mean $\bar{\theta} > \alpha$ and variance σ_θ^2). The parameters θ_i and θ_j , $j \neq i$, are correlated with correlation coefficient $\rho \in [0,1]$. We therefore have $\text{cov}[\theta_i, \theta_j] = \rho\sigma_\theta^2$ for $j \neq i$. Trader i receives a signal $s_i = \theta_i + \varepsilon_i$; all signals are of the same precision, and ε_i is normally distributed with $E[\varepsilon_i] = 0$ and $\text{var}[\varepsilon_i] = \sigma_\varepsilon^2$. Error terms in the signals are correlated neither with themselves nor with the θ_i parameters.

Our information structure encompasses the case of a common value and also that of private values. If $\rho = 1$, the valuation parameters are perfectly correlated and we are in a *common value* model. When $0 < \rho < 1$, we are in a *private values* model if signals are perfect and $\sigma_\varepsilon^2 = 0$ for all i ; traders receive idiosyncratic, imperfectly correlated shocks, and each trader observes her shock with no measurement error. If $\rho = 0$, then the

parameters are independent and we are in an *independent values* model. Under our assumption of normality, conditional expectations are affine.³

Let the average valuation parameter be $\tilde{\theta} \equiv \int \theta_j dj$, normally distributed with mean $\bar{\theta}$ and $\text{cov}[\tilde{\theta}, \theta_i] = \text{var}[\tilde{\theta}] = \rho \sigma_\theta^2$.⁴ An equivalent formulation that highlights the aggregate and idiosyncratic components of uncertainty is to let $\eta_i \equiv \theta_i - \tilde{\theta}$ and observe that $\theta_i = \tilde{\theta} + \eta_i$, where $\text{cov}[\eta_i, \tilde{\theta}] = 0$ and $\text{cov}[\eta_i, \eta_j] = 0$ for $i \neq j$. We adopt the convention that the average of independent and identically distributed random variables with mean zero is zero.⁵ We then have $\tilde{s} \equiv \int s_i di = \int \theta_i di + \int \varepsilon_i di = \tilde{\theta}$ almost surely, since $\int \varepsilon_i di = 0$ according to our convention. Note that if $\rho = 0$ then $\tilde{\theta} = \bar{\theta}$ (a.s.).

3. Rational expectations equilibria and the schedule game

In this section we begin by defining REE and expounding on its problems. We then move on to our game-theoretic approach and interpretations of the model.

3.1. Rational expectations equilibrium

A (competitive) rational expectations equilibrium is a (measurable) price function mapping the average valuation (state of the world) $\tilde{\theta}$ into prices $P(\tilde{\theta})$ and a set of trades $x_i, i \in [0, 1]$, such that the following two statements hold.

³ With normal distributions there is positive probability that prices and quantities are negative in equilibrium. We can control for this if necessary by restricting the variances of the distributions and of the parameters α , β , λ , and $\bar{\theta}$.

⁴ This can be justified as the continuum analogue of the finite case with n traders. Then, under our assumptions, the average parameter $\tilde{\theta}_n$ is normally distributed with mean $\bar{\theta}$, $\text{var}[\tilde{\theta}_n] = (1 + (n-1)\rho)\sigma_\theta^2 n^{-1}$, and $\text{cov}[\tilde{\theta}_n, \theta_i] = \text{var}[\tilde{\theta}_n]$. The result is obtained by letting n tend to infinity.

⁵ See Vives (1988) for a justification of this convention. In any event, we will see that the equilibrium in the continuum economy is the limit of equilibria in the appropriate finite economies under the standard laws of large numbers.

- (1) Trader i maximizes its expected profit, $E[\pi_i | s_i, p]$, conditional on knowing the functional relationship $P(\tilde{\theta})$ as well as the underlying distributions of the random variables.
- (2) Markets clear: $Z(p) \equiv \int_0^1 x_i di - \beta^{-1}(p - \alpha) = 0$.

Thus each trader optimizes while taking prices as given, as in the usual competitive equilibrium, but infers from prices the relevant information.

This equilibrium concept is problematic. Consider the common value case ($\rho = 1$); we shall present a fully revealing REE that is not, however, implementable. Suppose there is a competitive equilibrium of a full information market in which the traders know θ . At this equilibrium, price equals marginal benefit, $p = \theta - \lambda x_i$; therefore, individual demand is $x_i = \lambda^{-1}(\theta - p)$. The equilibrium price is given by the market-clearing condition $Z(p) = 0$ and is equal to $p = (\lambda\alpha + \beta\theta)/(\lambda + \beta)$. This allocation is also a fully revealing REE of our economy (Grossman 1981). Indeed, looking at the price allows each trader to learn θ , which is the only relevant uncertainty, and the allocation is a REE equilibrium because firms optimize and markets clear. However, this REE has a strange property: the price is fully revealing even though a trader's demand is independent of the signals received. How has the information been incorporated into the price? In other words, what is the game and the market microstructure that yields such a result? In this case we cannot find a game that delivers as an equilibrium the fully revealing REE.⁶

3.2. The schedule game

We will restrict our attention to REE that are the outcome of a well-specified game—that is, implementable REE. The natural way to implement competitive REE in our context is

⁶ If we were to insist that prices be measurable in excess demand functions, then the fully revealing REE would not exist (see Beja 1977; Anderson and Sonnenschein 1982). However, fully revealing REE are implementable if each agent is informationally “small” or irrelevant in the sense that his private information can be predicted from the joint information of other agents (Palfrey and Srivastava 1986; Postlewaite and Schmeidler 1986; Blume and Easley 1990).

to consider competition among demand functions (see Wilson 1979; Kyle 1989) in a market where each trader is negligible.⁷

We assume that traders compete in terms of their demand functions for the exogenous supply of the good. The game's timing is as follows. At $t = 0$, random variables $(\theta_i)_{i \in [0,1]}$ are drawn but not observed. At $t = 1$, traders observe their own private signals, $(s_i)_{i \in [0,1]}$, and submit demand functions $X_i(s_i, \cdot)$ with $x_i = X_i(s_i, p)$, where p is the market price. The strategy of a trader is therefore a map from the signal space to the space of demand functions. At $t = 2$ the market clears, demands are aggregated and crossed with supply to obtain an equilibrium price,⁸ and payoffs are collected. An implementable REE is associated with a Bayesian Nash equilibrium of the game in demand functions. Hereafter we discuss only the linear Bayesian demand function equilibrium (DFE).

3.3. Interpretations of the model

The model and game admit several interpretations in terms of financial markets and markets for goods as long as there are enough participants to justify the use of the continuum model assumption (this issue is dealt formally with in Section 7).

The good may be a financial asset such as central bank funds or Treasury notes, and the traders are the bidders (banks and other intermediaries) in the auction who use demand functions. In the open-market operation of central bank funds, the average valuation $\tilde{\theta}$ is related to the price (interest rate) in the secondary interbank market and λ may reflect the structure of a counterparty's pool of collateral. A bidder bank must offer the central bank collateral in exchange for funds, and the bidder's first preference is to offer the least liquid one. Given an increased allotment of funds, the bank must offer more liquid types of collateral at a higher opportunity cost; this implies a declining marginal valuation for

⁷ See Gul and Postlewaite (1992) and Mas-Colell and Vives (1993) for results on the implementation of efficient allocations in large economies.

⁸ If there is no market-clearing price then assume that the market closes; if there are many market-clearing prices, choose the one that maximizes volume.

the bidder.⁹ The marginal value for funds of bank i has the idiosyncratic component $\eta_i \equiv \theta_i - \tilde{\theta}$, which is uncorrelated with the like component of other banks. Bank i receives an imperfect signal about its overall valuation θ_i . In a Treasury auction, bidders will have private information related to different expectations about the future resale value $\tilde{\theta}$ of the securities (e.g., different beliefs concerning how future inflation will affect securities denominated in nominal terms) and to the idiosyncratic liquidity needs of traders.¹⁰ We should expect that the common value component is more significant in Treasury auctions than in central bank auctions, since the main dealers buy Treasury bills primarily for resale.¹¹

The good could also be an input (such as labor of uncertain productivity) whose traders are the firms that want to purchase it. Our model also accommodates the case where firms compete in supply functions to fill an exogenous demand, as in procurement auctions. In this case we assume that $\bar{\theta} < \alpha$, since θ_i is now a cost parameter and typically $x_i < 0$. For example, θ_i could be a unit ex post pollution or emission penalty to be levied on the firm and about which the producer has some private information.

4. Bayesian demand function equilibrium

In this section we use Proposition 1 to characterize the symmetric¹² equilibrium of the demand schedule game before discussing its properties. We then extend the range of the model to double auctions and inelastic supply, market structures with complementarities, and behavioral traders.

⁹ See Ewerhart, Cassola, and Valla (2010).

¹⁰ For example, Hortaçsu and Kastl (2008) cannot reject the hypothesis that bidders in Canadian 3-month T-bill auctions have private values.

¹¹ See Bindseil, Nyborg, and Strebulaev (2005).

¹² The symmetry requirement could be relaxed. Then the (linear and symmetric) equilibrium would be unique in the class of (linear) equilibria with uniformly bounded second moments (equivalently, in the class of equilibria with linear price functional of the type $P(\tilde{\theta})$).

Proposition 1. Let $\rho \in (0,1)$ and $\sigma_\varepsilon^2/\sigma_\theta^2 < \infty$. Then there is a unique symmetric DFE given by

$$X(s_i, p) = (E[\theta_i | s_i, p] - p)\lambda^{-1} = b + as_i - cp.$$

Here

$$a = \frac{1}{\lambda(1+M)}, \quad b = -\frac{\alpha}{\beta}(1-\lambda a), \quad c = \frac{1}{\beta}(a(\beta + \lambda) - 1),$$

and $M = \sigma_\varepsilon^2 / ((1-\rho)\sigma_\theta^2)$. Moreover, $a > 0$ and $-\beta^{-1} < c \leq a \leq \lambda^{-1}$. Also, c is decreasing in M and λ and is increasing in β ; $Z' = -(c + \beta^{-1}) < 0$; and the equilibrium price is given by

$$p = \frac{\lambda\alpha + \beta\tilde{\theta}}{\lambda + \beta}.$$

Proof: See the Appendix.

It is worthwhile to highlight some properties of this equilibrium.

The equilibrium is, first of all, *privately revealing*.¹³ The price p reveals the average parameter $\tilde{\theta}$ and, for trader i , either pair (s_i, p) or $(s_i, \tilde{\theta})$ is a sufficient statistic for the joint information in the market. In particular, at equilibrium we have $E[\theta_i | s_i, p] = E[\theta_i | s_i, \tilde{\theta}]$. The privately revealing character of the equilibrium implies that the incentives to acquire information are preserved.

Second, the equilibrium is *efficient*: it is a price-taking equilibrium, the price reveals $\tilde{\theta}$, and firms act with a sufficient statistic for the shared information in the economy.¹⁴ Indeed, at equilibrium we have that price equals marginal benefit with full (shared)

¹³ See Allen (1981).

¹⁴ A fully revealing REE must be ex post Pareto optimal. The reason is that it can be viewed as the competitive equilibrium of an economy with fully informed agents and so, according to the first welfare theorem, it cannot be improved on by a social planner with access to the pooled information of agents (Grossman 1981).

information: $p = E[\theta_i | s_i, \tilde{\theta}] - \lambda x_i$. This would not be the case if traders had market power, since then a wedge would be introduced between price and marginal benefit (see Vives 2011a). Neither would the equilibrium be efficient if price were noisy, since then a trader would not take into account the information externality that her trade has on other traders through the effect on the informativeness of the price (see, e.g., Amador and Weill 2010; Vives 2011c).

In the common value case, the equilibrium breaks down. If $\rho = 1$ (and $0 < \sigma_\varepsilon^2 < \infty$) then, as stated in Section 3, there is a fully revealing REE but it is not implementable (indeed, a linear equilibrium does not exist).

When signals are perfect ($\sigma_\varepsilon^2 = 0$ and $M \equiv \sigma_\varepsilon^2 \left((1 - \rho) \sigma_\theta^2 \right)^{-1} = 0$), we have that $a = c = \lambda^{-1}$, $b = 0$, and $x_i = \lambda^{-1}(\theta_i - p)$. Bidders have nothing to learn from prices, and the equilibrium is just the usual complete information competitive equilibrium (which, we remark, is independent of ρ). When $M > 0$, bidders learn from prices and the demand functions are steeper: $c < \lambda^{-1}$. Indeed, the larger is M (which is increasing in ρ and in $\sigma_\varepsilon^2 / \sigma_\theta^2$ and can be viewed as an index of adverse selection), the more the price serves to inform about the common value component and the steeper are the demand functions (lower c). The response to a price increase is to reduce the amount demanded according to the usual scarcity effect, but this impulse is moderated (or even reversed) by an information effect because a high price conveys the good news that the average valuation is high. Indeed, if $\beta \lambda^{-1} = M$ then $c = 0$; for larger values of M , we have $c < 0$ and demand is upward sloping.¹⁵ As $M \rightarrow \infty$ we have that $a \rightarrow 0$, $b \rightarrow -\alpha/\beta$, and $c \rightarrow -1/\beta$. Then the linear equilibrium collapses because, in the limit, traders put no weight on their private signals. We have already seen that when $\rho = 1$ (and $0 < \sigma_\varepsilon^2 < \infty$) there is a fully revealing REE that is not implementable. When signals are pure noise

¹⁵ C. Wilson (1980) finds an upward-sloping demand schedule in a market with asymmetric information whose quality is known only to the sellers.

($\sigma_\varepsilon^2 = \infty$), the equilibrium is $X(p) = \lambda^{-1}(\bar{\theta} - p)$ because $E[\theta_i | s_i, p] = \bar{\theta}$ (even if $\rho = 1$). However, this equilibrium is not the limit of DFE as $\sigma_\varepsilon^2 \rightarrow \infty$ ($M \rightarrow \infty$).

If $\rho = 0$ then the price conveys no information on values, $c = \lambda^{-1}$, and $X(s_i, p) = \lambda^{-1}(E[\theta_i | s_i] - p)$. Again this is not the limit of DFE as $\rho \rightarrow 0$. However, it can be checked that there is no discontinuity in outcomes.

5. Extensions

In this section we extend the model to some boundary cases and new interpretations: inelastic supply and double auctions, complementarities, and behavioral traders.

5.1. Inelastic supply and double auctions

The case in which an auctioneer supplies q units of the good is easily accommodated by letting $\beta \rightarrow \infty$ and $\alpha/\beta \rightarrow -q$. From the inverse supply function we obtain the average quantity $y = (p - \alpha)/\beta \rightarrow q$; then $c \rightarrow a$ and $b \rightarrow (1 - \lambda a)q$. Here demand is always downward sloping, and the strategy of a trader is $X(s_i, p) = b + a(s_i - p)$ and $p = \tilde{\theta} - \lambda q$. The good can be in zero net supply ($q = 0$) as in a double auction, in which case $b = 0$ and $p = \tilde{\theta}$.¹⁶ A trader is a buyer or a seller depending on whether her private signal is larger or smaller than the price.

Reny and Perry (2006) obtain a related result in a double auction with a unit mass of traders, each of whom desires at most one unit of the good. Each trader receives a conditionally independent signal about the random value of the good. The value of the good and the signals of the agents are assumed to be strictly affiliated (and the densities of the random variables are smooth and positive on the unit interval). It is assumed also that the valuation of a trader is (strictly) increasing in his signal and (weakly) increasing in the good's value. In contrast to our model, in Reny and Perry's model there is a

¹⁶ In this case there is also a no-trade equilibrium.

common value for the good but the payoff of a trader depends directly on the signal he receives. This signal provides a private value component to the trader's valuation.

Once traders have received their signals, they submit bids to the auctioneer. A buyer (resp., seller) indicates the maximum (minimum) price she is willing to pay for (resp., for which he is willing to sell) the desired unit. The auctioneer then uses the bids to form supply and demand schedules and finds a market-clearing price. Buyers whose bids are above the market-clearing price obtain one unit, and those with bids below the market-clearing price come away with nothing.

Unlike the case for our DFE, traders in a double action cannot condition on the market price because they submit a single bid that is contingent only on private information. Nonetheless, there is a unique (and privately revealing) REE that is implementable as a Bayesian equilibrium of the double auction in symmetric increasing bidding strategies. The equilibrium is *privately revealing* because the price reveals the value of the good and this, together with the signal received by a trader, determines his payoff. The equilibrium is *efficient* because the privately revealing REE is just the competitive equilibrium when the state is known. This REE is implementable as a double auction even in a pure common value case (when the valuation of a trader is independent of his signal), in contrast to the demand competition model, owing to the double auction mechanism whereby bids are for a single unit. At the REE both buyers and sellers are indifferent between using (or not) their private signal, so they might as well use it.

Reny and Perry (2006) also provide a strategic foundation for the rational expectations equilibrium in a finite-market counterpart of the double auction continuum model. This issue is addressed in Section 7.2 to follow.

5.2. Complementarities

Letting $\beta < 0$ allows for complementarities. For example, if traders are suppliers ($x_i < 0$) then $\beta < 0$ means that increasing the aggregate quantity leads to price increases, a dynamic typical of network goods; conversely, if traders are demanders ($x_i > 0$) then

$\beta < 0$ means that increasing the aggregate quantity lowers the price, as may occur with labor supply when the income effect dominates. We can allow negative values of β with $0 > \beta > -\lambda\alpha/\bar{\theta}$. The last inequality ensures that $E[p] > 0$ in equilibrium and implies that $\beta + \lambda > 0$ (since $\bar{\theta} - \alpha > 0$). When $\beta < 0$ we have that $-\beta^{-1} > c \geq a$ and that c increases in M ; in other words, the exogenous supply is downward sloping and an increase in M makes demand flatter. Furthermore, excess demand is upward sloping: $Z' = -(c + \beta^{-1}) > 0$. Now the information and the scarcity effect work in the same direction, and a high price conveys the unequivocal bad news that the average valuation is low.

5.3. Behavioral traders in a double auction

Consider a double auction—say, with zero net supply ($q = 0$), equilibrium $X(s_i, p) = a(s_i - p)$, and price $p = \tilde{\theta}$. Introduce a mass of $(|\beta|)^{-1}$ boundedly rational traders who supply the good according to the fixed schedule $y = (p - \alpha)/\beta$. We shall interpret $\beta > 0$ as meaning that “value” traders predominate and $\beta < 0$ as meaning that “momentum” or “positive-feedback” traders dominate. Value traders sell when the price is high, whereas momentum traders sell when it is low.¹⁷ We are in the context of our model (provided that $\beta + \lambda > 0$), Proposition 1 applies, and rational traders adjust by using the strategy $X(s_i, p) = b + as_i - cp$ with $c \leq a$ if $\beta > 0$ and $c \geq a$ if $\beta < 0$. That is, rational traders use a steeper demand schedule with value traders and a flatter one with momentum traders. Note that demand will be upward sloping ($c < 0$) for a large enough mass of value behavioral traders (i.e., when $\beta < \lambda M$).

The presence of behavioral traders makes total excess demand flatter (i.e., $|Z'| = |c + \beta^{-1}| = |\lambda^{-1} + \beta^{-1}|(1 + M)^{-1}$ is increasing in $(|\beta|)^{-1}$), and with momentum traders

¹⁷ See Hirshleifer (2001) for a survey of behavioral biases in asset pricing; see Asness, Moskowitz, and Pedersen (2009) for an empirical analysis of the returns to value and momentum trading.

($\beta < 0$) the excess demand slopes upward ($Z' > 0$). Since $\text{var}[p] = (1 + \lambda\beta^{-1})^{-2} \text{var}[\tilde{\theta}]$ and since $\beta + \lambda > 0$, it follows that price volatility increases with $|\beta|$ and therefore decreases with the mass of behavioral traders. It can be checked that expected profits of rational traders are always larger with behavioral traders and that they decrease with β .¹⁸ These profits are minimal when $\beta = \infty$ and there are no behavioral traders, and they are maximal when $\beta \rightarrow -\lambda$. This means that expected profits increase with the mass of behavioral traders, $(|\beta|)^{-1}$, when value traders predominate and decrease with the mass of behavioral traders when momentum traders predominate. Expected profits are higher when $\beta < 0$, so rational traders would prefer to have momentum traders around.

The amount of mispricing due to the presence of behavioral traders is easily seen to be

$$|p - \tilde{\theta}| = \frac{\lambda}{\lambda + \beta} |\alpha - \tilde{\theta}|,$$

which is decreasing in β/λ . In words: when value trading predominates ($\beta > 0$), the mispricing is increasing in its mass, β^{-1} ; when momentum trading predominates ($\beta < 0$), the level of mispricing is higher but the level decreases with the mass of momentum traders, $(|\beta|)^{-1}$. De Long et al. (1990) also find that increasing the proportion of rational traders relative to positive-feedback traders may increase the amount of mispricing. An alternative measure of mispricing is $\text{var}[p - \tilde{\theta}] = (1 + \beta\lambda^{-1})^{-2} \text{var}[\tilde{\theta}]$, from which we conclude that mispricing is increasing in ρ and in σ_{θ}^2 . This finding is consistent with behavioral models in which greater uncertainty and more adverse selection (in our model, increasing in ρ) increase misvaluation (see, e.g., Daniel, Hirshleifer, and Subrahmanyam 1998; Hong and Stein 1999).

Serrano-Padial (2010) studies a pure common value double auction with a continuum of traders, a fraction of whom are naïve (in the sense that they use a fixed bidding strategy

¹⁸ See the expression for expected profits in the proof of Proposition 2.

independent of what other traders do). In his model, nonnegligible amounts of naïve trade are compatible with fully revealing prices.

6. Information acquisition

Now suppose that, in a first stage of the game, private signals must be purchased at a cost, which is increasing and convex in the precision $\tau_\varepsilon \equiv 1/\sigma_\varepsilon^2$ of the signal,¹⁹ according to a smooth function $H(\cdot)$ that satisfies $H(0) = 0$ with $H' > 0$ for $\tau_\varepsilon > 0$, and $H'' \geq 0$. Hence there are nonincreasing returns to information acquisition. At a second stage, traders receive signals according to the precision purchased and compete in demand functions. The following result summarizes our analysis of symmetric equilibria of the two-stage game.

Proposition 2. *Let $\rho \in [0,1]$. There is a unique symmetric equilibrium in the two-stage game with costly information acquisition where $\tau_\varepsilon^* = 0$ if $(2\lambda\tau_\theta^2)^{-1} \leq H'(0)$ and $\tau_\varepsilon^* > 0$ if $\rho < \bar{\rho} \equiv 1 - \tau_\theta \sqrt{2\lambda H'(0)}$, in which case τ_ε^* is decreasing in λ , τ_θ , and ρ . Otherwise there is no equilibrium.*

Proof: Suppose that, at the first stage, all traders but i have chosen a precision $\tau_\varepsilon > 0$. Then the market equilibrium (which is unaffected by the actions of a single trader) exhibits, according to Proposition 1, the price $p = (\lambda\alpha + \beta\tilde{\theta})/(\lambda + \beta)$, a price that reveals $\tilde{\theta}$. Trader i receives a signal with precision τ_{ε_i} and chooses x_i to maximize

$$\left(E[\theta_i | s_i, \tilde{\theta}] - p\right)x_i - \frac{\lambda}{2}x_i^2,$$

which yields the first-order condition $E[\theta_i | s_i, \tilde{\theta}] - p = \lambda x_i$. Expected profits are given by

$E[\pi_i] = \frac{\lambda}{2}E[x_i^2]$, where $x_i = (E[\theta_i | s_i, \tilde{\theta}] - p)/\lambda$ and $p = (\lambda\alpha + \beta\tilde{\theta})/(\lambda + \beta)$. Note that

¹⁹ For a random variable η , we use τ_η to denote $1/\sigma_\eta^2$.

$E[\pi_i]$ does not depend on τ_ε because the equilibrium reveals $\tilde{\theta}$. We remark that

$$E[\theta_i | s_i, \tilde{\theta}] = \varsigma_i s_i + (1 - \varsigma_i) \tilde{\theta}, \text{ where}$$

$$\varsigma_i = \frac{(1 - \rho) \tau_{\varepsilon_i}}{(1 - \rho) \tau_{\varepsilon_i} + \tau_\theta} \quad \text{and} \quad E[x_i^2] = (E[x_i])^2 + \text{var}[x_i]$$

with $E[x_i] = (\alpha - \bar{\theta}) / (\beta + \lambda)$ and

$$\begin{aligned} \text{var}[x_i] &= \left(\frac{1}{\beta + \lambda} \frac{1}{\lambda} \right)^2 \text{var}[\varsigma_i (\lambda + \beta) s_i + (\lambda (1 - \varsigma_i) - \varsigma_i \beta) \tilde{\theta}] \\ &= \left(\frac{1}{\lambda + \beta} \frac{1}{\lambda} \right)^2 \left(\varsigma_i^2 (\lambda + \beta)^2 (\tau_{\varepsilon_i}^{-1} + (1 - \rho) \tau_\theta^{-1}) + \lambda^2 \rho \tau_\theta^{-1} \right). \end{aligned}$$

We can use this fact to obtain

$$E[\pi_i] = \frac{\lambda}{2} \left(\left(\frac{\alpha - \bar{\theta}}{\beta + \lambda} \right)^2 + \frac{\tau_{\varepsilon_i} \tau_\theta^{-1} (1 - \rho) \left((\beta + \lambda)^2 - \beta \rho (\beta + 2\lambda) \right) + \rho \lambda^2}{(\tau_\theta + (1 - \rho) \tau_{\varepsilon_i}) (\beta + \lambda)^2 \lambda^2} \right).$$

It follows that the marginal benefit of increasing the precision of information is

$$\frac{\partial E[\pi_i]}{\partial \tau_{\varepsilon_i}} = \frac{(1 - \rho)^2}{2\lambda (\tau_\theta + (1 - \rho) \tau_{\varepsilon_i})^2}.$$

Observe that this marginal benefit is decreasing in τ_{ε_i} provided that $\rho < 1$ (and thus

$E[\pi_i]$ is strictly concave in τ_{ε_i}). Let

$$\phi(\tau_\varepsilon) \equiv \frac{\partial E[\pi_i]}{\partial \tau_\varepsilon} - H'(\tau_\varepsilon).$$

Then $\phi(\infty) < 0$ and $\phi' < 0$. We have that $\phi(0) = (1 - \rho)^2 (2\lambda \tau_\theta^2)^{-1} - H'(0) > 0$ if and only if $\rho < \bar{\rho} \equiv 1 - \tau_\theta \sqrt{2\lambda H'(0)}$, in which case there is a unique interior solution τ_ε^* to the equation $\phi(\tau_\varepsilon) = 0$. Note that τ_ε^* is decreasing in λ , τ_θ , and ρ because ϕ is.

For $\rho \geq \bar{\rho}$ we have that $\tau_\varepsilon = 0$ at a candidate equilibrium. Since

$$\frac{\partial E[\pi_i]}{\partial \tau_{\varepsilon_i}} = \frac{1}{2\lambda (\tau_\theta + \tau_{\varepsilon_i})^2}$$

when no trader other than i purchases information (the price then is $p = (\lambda\alpha + \beta\bar{\theta})/(\lambda + \beta)$ and the expression for $\partial E[\pi_i]/\partial \tau_{\varepsilon_i}$ is the same as when $\rho = 0$), it follows that $\tau_{\varepsilon} = 0$ is an equilibrium only if $\phi(0) = (2\lambda\tau_{\theta}^2)^{-1} - H'(0) \leq 0$. Otherwise (i.e., if $(2\lambda\tau_{\theta}^2)^{-1} > H'(0)$), it will benefit a single trader to purchase information and there is no symmetric equilibrium in the game. (In fact, we can show that neither is there an asymmetric equilibrium in the class of trading strategies with bounded second moments.) ♦

A more diffuse prior or less correlation of valuations induces more acquisition of information. In particular, when $\rho < 1$ an equilibrium exists if $H'(0) = 0$ or if the prior is diffuse (τ_{θ} small). As $\rho \rightarrow \bar{\rho}$ we have $\tau_{\varepsilon}^* \rightarrow 0$, and the demand function equilibrium collapses.

It is worth remarking that the same equilibrium would obtain in a one-shot game where traders choose simultaneously the demand function and the precision of the signal. This corresponds to the case where information acquisition is covert (nonobservable). The equivalence of the games follows from the existence of a continuum of traders.

Hence we see that the incentives to acquire information are preserved because the equilibrium is privately revealing—as long as we are not too close to the common value case, or otherwise the marginal cost of acquiring information at zero precision is zero (and $\rho < 1$). Jackson (1991) shows the possibility of fully revealing prices in a common value environment with costly information acquisition (and under some specific parametric assumptions) when there is a finite number of agents.

Application. Consider the example of banks bidding for liquidity and the impact of a crisis. In this scenario we may expect that the correlation ρ of the values of the banks increases (equivalently, that the volatility of the price $\tilde{\theta}$ in the secondary market for liquidity increases) and that λ also increases as it becomes more costly to supply more

liquid collateral. The direct effect of an increase in ρ or λ is to make the demand schedules of the banks steeper (Proposition 1), and this effect is reinforced by the induced decrease in information precision (τ_ε^* goes down, according to Proposition 2). The effect of the crisis is thus that demand schedules are steeper and the signals noisier. These effects are consistent with the empirical evidence gathered by Cassola, Hortaçsu, and Kastl (2009) when studying European Central Bank auctions. These authors find that the aggregate bid curve became steeper after the subprime crisis in August 2007.²⁰ When our model is interpreted as representing behavioral traders, we find that such a crisis would increase mispricing because it is increasing in ρ and in λ for $\beta > 0$, when the value traders predominate.

7. Finite markets and convergence to the limit equilibrium

The question arises of whether the results obtained in the large market are simply an artifact of the continuum specification. In this section, we answer this question in the negative. We show that the equilibria in finite markets tend to the equilibrium of the continuum economy as the market grows large, which justifies our use of a continuum model to approximate the large market with demand function competition. We illustrate the argument with the double auction case.

7.1. The convergence result

Consider the following replica economy. Suppose that inverse supply is given by $P_n(y) = \alpha + \beta y n^{-1}$; here y is total quantity and n is the number of traders (buyers), each with same benefit function as before. Increasing n will increase the number of buyers and increase the supply at the same rate. Denote with subscript n the magnitudes in the n -replica market. The information structure is the finite-trader counterpart of the structure described in Section 2. We have that

²⁰ Market power leading to bid shading may reinforce the steepness of the bid curve (see Vives (2011a)).

$\tilde{\theta}_n \equiv \left(\sum_{i=1}^n \theta_i \right) / n \sim N\left(\bar{\theta}, (1+(n-1)\rho)n^{-1}\sigma_\theta^2\right)$ and $\text{cov}[\tilde{\theta}_n, \theta_i] = \text{var}[\tilde{\theta}_n]$. Note also that $\tilde{\theta}_n \rightarrow \tilde{\theta} \sim N\left(\bar{\theta}, \rho\sigma_\theta^2\right)$ in mean square and that $\text{cov}[\tilde{\theta}_n, \tilde{\theta}] = \text{var}[\tilde{\theta}]$.²¹

It follows from Proposition 1 in Vives (2011a) that, for $\rho \in [0,1)$, there is a unique (symmetric) DFE of the form $X_n(s_i, p) = b_n + a_n s_i - c_n p$ for any n . The equilibrium is privately revealing, and the price reveals the average signal of the traders, \tilde{s}_n . Furthermore, by Section S.4 in Vives (2011b), for $\rho \in [0,1)$ there is a symmetric equilibrium of the game with covert information acquisition in the n -replica market—provided that the cost of information acquisition at zero precision is not too high.

The following proposition establishes that, as n grows large, the equilibria of finite markets converge to the limit equilibrium. Denote by ETS (resp., $n^{-1}\text{ETS}_n$) the per capita expected total surplus in the continuum (resp., in the n -replica markets).

Proposition 3. *Consider the n -replica market.*

(i) *Let $\rho \in [0,1)$. For given $\sigma_\varepsilon^2 \geq 0$, the symmetric DFE of the n -replica market converge to the limit equilibrium as n tends to infinity:*

(a) $a_n \xrightarrow{n} a$, $c_n \xrightarrow{n} c$, and $b_n \xrightarrow{n} b$;

(b) $p_n - p \xrightarrow{n} 0$ in mean square at rate $1/\sqrt{n}$ with

$$nE\left[(p_n - p)^2\right] \xrightarrow{n} \left(\frac{\beta}{\beta + \lambda}\right)^2 \text{AV},$$

where $\text{AV} = (1 - \rho)\sigma_\theta^2 + \sigma_\varepsilon^2$ if $\rho > 0$ and $\text{AV} = \sigma_\theta^4 (\sigma_\theta^2 + \sigma_\varepsilon^2)^{-1}$ if $\rho = 0$;

(c) *the per capita welfare loss $\text{WL}_n \equiv \text{ETS} - n^{-1}\text{ETS}_n \xrightarrow{n} 0$ at the rate $1/n$, and the total welfare loss*

²¹ See the Appendix for definitions of “in mean square” and of convergence (and rates of convergence) for random variables.

$$n\text{WL}_n \xrightarrow{n} \frac{AV}{2(\beta + \lambda)} + \frac{(1-\rho)^2 \sigma_\theta^4}{2\lambda((1-\rho)\sigma_\theta^2 + \sigma_\varepsilon^2)}.$$

(ii) Let $\rho \in [0, \bar{\rho})$, where $\bar{\rho} \equiv 1 - \tau_\theta \sqrt{2\lambda H'(0)} > 0$. Then the equilibrium $\tau_\varepsilon^* > 0$ of the continuum economy with endogenous information acquisition is the limit of the unique equilibrium $\tau_\varepsilon^*(n)$ of the covert information acquisition game with replica markets for n large.

Proof: See the Appendix.

7.2. Illustration of the argument in the double auction case

Consider the double auction case (with inelastic per capita supply of q) to illustrate the argument for (a) and (b) in Proposition 3(i).

Suppose that traders $j \neq i$ employ linear strategies, $X(s_j, p) = b + as_j - cp$. Then the market-clearing condition, $\sum_{j \neq i} X(s_j, p) + x_i = nq$, $c \neq 0$, implies that trader i faces a residual inverse supply:

$$p = I_i + dx_i, \quad \text{where } d = ((n-1)c)^{-1} \text{ and } I_i = d \left((n-1)b + a \sum_{j \neq i} s_j - qn \right).$$

The (endogenous) parameter d is the slope of inverse residual supply and the wedge introduced by market power. All the information that the price provides to trader i about the signals of others is contained in the intercept I_i . The information available to trader i is $\{s_i, p\}$ or, equivalently, $\{s_i, I_i\}$. Trader i chooses x_i to maximize

$$E[\pi_i | s_i, p] = x_i \left(E[\theta_i | s_i, p] - p \right) - \frac{\lambda}{2} x_i^2 = x_i \left(E[\theta_i | s_i, p] - I_i - dx_i \right) - \frac{\lambda}{2} x_i^2.$$

The first-order condition (FOC) is $E[\theta_i | s_i, p] - p = (d + \lambda)x_i$. An equilibrium requires that $d > 0$.²²

²² The second-order sufficient condition is fulfilled when $d > 0$.

A trader bids according to $p = E[\theta_i | s_i, p] - (d + \lambda)x_i$, and competitive bidding obtains when $d = 0$. A buyer ($x_i > 0$) underbids, $p < E[\theta_i | s_i, p] - \lambda x_i$; since $d > 0$, a seller ($x_i < 0$) overbids, $p > E[\theta_i | s_i, p] - \lambda x_i$.

From the FOC and the normal updating formulas for $E[\theta_i | s_i, p]$, we immediately obtain the coefficients of the linear equilibrium strategy:

$$X_n(s_i, p) = b_n + a_n s_i - c_n p, \quad c_n = \frac{n-2-M_n}{\lambda(n-1)(1+M_n)},$$

where

$$M_n \equiv \frac{\rho \sigma_\varepsilon^2 n}{(1-\rho)(\sigma_\varepsilon^2 + (1+(n-1)\rho)\sigma_\theta^2)} \quad \text{and} \quad a_n = \frac{(1-\rho)\sigma_\theta^2}{(\sigma_\varepsilon^2 + (1-\rho)\sigma_\theta^2)} (d_n + \lambda)^{-1}$$

for $d_n = ((n-1)c_n)^{-1}$. Now, in the finite economy (unlike the elastic exogenous supply case), we require $n-2-M_n > 0$ in order to guarantee the existence of an equilibrium (i.e., to obtain $d_n > 0$ and $c_n > 0$). (Observe that the inequality is always fulfilled for n large because M_n is bounded.) The reason for this requirement is that, if the inequality does not hold, then traders will seek to exploit their market power by submitting vertical schedules, and that is incompatible with the existence of equilibrium when there is no elastic exogenous supply.

The equilibrium price p_n reveals the average signal \tilde{s}_n ; therefore, $E[\theta_i | s_i, p_n] = E[\theta_i | s_i, \tilde{s}_n]$ and $n^{-1} \sum_{i=1}^n E[\theta_i | s_i, \tilde{s}_n] = E[\tilde{\theta}_n | \tilde{s}_n]$. Averaging the FOCs, we obtain that $E[\tilde{\theta}_n | \tilde{s}_n] - p_n = (d_n + \lambda)\tilde{x}_n = (d_n + \lambda)q$ and hence $p_n = E[\tilde{\theta}_n | \tilde{s}_n] - (d_n + \lambda)q$.

We have that $X_n(s_i, p) \xrightarrow{n} (1 - \lambda a)q + a(s_i - p)$, the trading strategy in the double auction in the limit economy.²³ Furthermore, $p_n \xrightarrow{n} p = \tilde{\theta} - \lambda q$ in mean square at the rate $1/\sqrt{n}$. This follows because $d_n \xrightarrow{n} 0$ and $E[\tilde{\theta}_n | \tilde{s}_n] \xrightarrow{n} \tilde{\theta}$ in mean square (given that $\tilde{\theta}_n \xrightarrow{n} \tilde{\theta}$ and $(\sum_i \varepsilon_i)/n \xrightarrow{n} 0$ in mean square, both at rate $1/\sqrt{n}$). In fact, we have $nE\left[\left(\tilde{\theta} - E[\tilde{\theta}_n | \tilde{s}_n]\right)^2\right] \xrightarrow{n} AV$, where $AV = (1 - \rho)\sigma_\theta^2 + \sigma_\varepsilon^2$ if $\rho > 0$ and $AV = \sigma_\theta^4 (\sigma_\theta^2 + \sigma_\varepsilon^2)^{-1}$ if $\rho = 0$. This means that the convergence is faster (in terms of asymptotic variance) the closer we are to the common value case, the less prior uncertainty there is, and the less noisy are the signals (if $\rho > 0$).²⁴ The market power distortion $d_n = ((n-1)c_n)^{-1}$ (i.e., the amount of over- or underbidding) is of the order $1/n$.

Reny and Perry (2006) provide a strategic foundation for the Bayesian equilibrium/REE of the double auction in their continuum model. In a finite-market counterpart of their double auction continuum model, the authors use a symmetry-preserving rationing rule²⁵ to prove that, with enough buyers and sellers and with a sufficiently fine grid of prices, the following statement holds: generically in the valuation functions of the traders and the fineness of the grid, there is a Bayesian equilibrium in monotonically increasing bid functions that is very close to the unique REE of the continuum economy. The strategy of their involved proof is to show an appropriate continuity property for the equilibrium in the limit market. The main obstacle in the proof is that, with a finite number of traders,

²³ This statement is proved as follows: $c_n \xrightarrow{n} a \equiv \lambda^{-1}(M+1)^{-1}$ if $\rho > 0$ (since then $M_n \xrightarrow{n} M$), and $c_n \xrightarrow{n} \lambda^{-1}$ if $\rho = 0$ (since then $M_n = 0$); furthermore, $a_n \xrightarrow{n} a$ because $d_n = ((n-1)c_n)^{-1} \xrightarrow{n} 0$. It can be checked similarly that $b_n \xrightarrow{n} (1 - \lambda a)q$.

²⁴ If $\rho = 0$ then $\tilde{\theta} = \bar{\theta}$; in this case, more noise in the signals makes $E[\tilde{\theta}_n | \tilde{s}_n]$ closer to $\bar{\theta}$, which speeds up convergence. See the Appendix for the definition of the asymptotic variance of convergence.

²⁵ In the double auction with a finite number of buyers and sellers, a rationing rule must be established for traders who bid exactly the market-clearing price.

in the double auction the strategies of buyers and sellers are not symmetric.²⁶ The incentives of buyers to underbid and of sellers to overbid in order to affect the price disappear as the market grows large and price-taking behavior obtains. In our DFE, the strategy of a trader is symmetric and the trader perceives that her influence on the price is given by $d_n > 0$. A buyer underbids and a seller overbids, and the incentives to manipulate the market also disappear as n grows and $d_n \rightarrow 0$. We can in addition characterize the *rate* at which this happens (and at which convergence to the limit equilibrium obtains) and distinguish between the dissipation of market power and the averaging of noise terms.

7.3. Summary and interpretation of results in the elastic supply case

In a finite n -replica market, traders have the capacity to influence prices; the price reveals the average signal of the traders \tilde{s}_n , which is a noisy estimate of $\tilde{\theta}_n$. We find that, for n large, such an equilibrium is close to the equilibrium in the limit economy where traders have no market power and where the price reveals the average parameter $\tilde{\theta}$. Convergence to the equilibrium of the continuum economy occurs as $1/\sqrt{n}$, the rate at which the average signal \tilde{s}_n of the traders (or the average estimate $E[\tilde{\theta}_n | \tilde{s}_n]$) tends to the average parameter $\tilde{\theta}$ in the continuum economy. Convergence to price-taking behavior is faster (at the rate $1/n$, since d_n is of the order $1/n$; see Proposition 7 in Vives 2011a), but convergence to the limit is delayed by the slower convergence of the agents' average signal. However, this latter convergence is faster (in terms of asymptotic variance) as we approach a common value environment (i.e., as $\rho \rightarrow 1$), when there are better signals (low σ_ε^2 for $\rho > 0$), and/or with less prior uncertainty (low σ_θ^2).

In the finite market, the per capita welfare loss (with respect to that in the limit market) is of the order of $1/n$; see part (c) of Proposition 3(i). Here again, convergence is faster (in

²⁶ The consequence is that the signal of each agent need not be affiliated with the order statistics of the bids of other agents. This failure of “single crossing” implies that standard proofs from auction theory, which rely on relationships between affiliation and order statistics with symmetric strategies, do not apply here.

terms of asymptotic variance) when closer to the common value case and slower if there is more prior uncertainty. The effect of noise in the signals is ambiguous if $\rho > 0$ since an increase in σ_ε^2 will tend to raise allocative inefficiency while diminish distributive inefficiency. The reason is that the expression for total expected welfare loss in the finite market,

$$n\text{WL}_n = n \left((\beta + \lambda) E \left[(\tilde{x}_n - \tilde{x})^2 \right] + \lambda E \left[(u_{in} - u_i)^2 \right] \right) / 2$$

(where $u_{in} \equiv x_{in} - \tilde{x}_n$ and $u_i \equiv x_i - \tilde{x}$), has two components on the right-hand side; the first component reflects allocative inefficiency (is the average quantity at the right level?), and the second reflects distributive inefficiency (is a given average quantity efficiently distributed among market participants?). The first term converges to $((1 - \rho)\sigma_\theta^2 + \sigma_\varepsilon^2) / (2(\beta + \lambda))$ if $\rho > 0$, and the second term converges to $(1 - \rho)^2 \sigma_\theta^4 / (2\lambda((1 - \rho)\sigma_\theta^2 + \sigma_\varepsilon^2))$ as $n \rightarrow \infty$. Increases in the correlation of parameters ρ or in the precision of the prior $\tau_\theta \equiv (\sigma_\theta^2)^{-1}$ will decrease both terms; however, the first term increases with σ_ε^2 whereas the second term decreases with σ_ε^2 (since more noise in the signals aligns more individual and average quantities).²⁷

The overall convergence result is again driven by the rate of information aggregation and not by the faster rate of convergence to price-taking behavior, which implies a welfare loss of the order of $1/n^2$ (cf. Proposition 7 in Vives 2011a). This latter result is consistent with results on the asymptotic dissipation of inefficiency that have been obtained in the double auction literature, which culminated in the work of Cripps and Swinkels (2006); these authors employed a generalized private value setting in which bidders can be asymmetric and can demand or supply multiple units.

²⁷ When $\rho = 0$, an increase in the noise of the signals reduces both terms.

Appendix

Proof of Proposition 1: Trader i chooses x_i to maximize

$$E[\pi_i | s_i, p] = x_i \left(E[\theta_i | s_i, p] - p \right) - \frac{\lambda}{2} x_i^2,$$

which yields the FOC $p = E[\theta_i | s_i, p] - \lambda x_i$. Positing linear strategies $X(s_i, p) = b + as_i - cp$ while using the inverse supply function $p = \alpha + \beta \tilde{x}$ and our convention $\int s_i di = \tilde{\theta}$, we obtain (provided that $1 + \beta c \neq 0$) an expression for the price $p = (1 + \beta c)^{-1} (\alpha + \beta b + \beta a \tilde{\theta})$. The vector $(\theta_i, s_i, \tilde{\theta})$ is normally distributed with $E[\theta_i] = E[\tilde{\theta}] = E[s_i] = \bar{\theta}$ and with variance-covariance matrix

$$\sigma_\theta^2 \begin{pmatrix} 1 & 1 & \rho \\ 1 & \xi^{-1} & \rho \\ \rho & \rho & \rho \end{pmatrix},$$

where $\xi = \tau_\varepsilon / (\tau_\theta + \tau_\varepsilon)$. It follows that $E[\theta_i | s_i, \tilde{\theta}] = \zeta s_i + (1 - \zeta) \tilde{\theta}$ for $\zeta = (1 + \sigma_\varepsilon^2 / (1 - \rho) \sigma_\theta^2)^{-1}$. Given joint normality of the stochastic variables $(\theta_i, s_i, \tilde{\theta})$, we obtain

$$\begin{pmatrix} \theta_i \\ s_i \\ p \end{pmatrix} \sim N \left(\begin{pmatrix} \bar{\theta} \\ \bar{\theta} \\ C + D\bar{\theta} \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \sigma_\theta^2 & D\rho\sigma_\theta^2 \\ \sigma_\theta^2 & \sigma_\theta^2 + \sigma_\varepsilon^2 & D\rho\sigma_\theta^2 \\ D\rho\sigma_\theta^2 & D\rho\sigma_\theta^2 & D^2\rho\sigma_\theta^2 \end{pmatrix} \right).$$

Here $C = (1 + \beta c)^{-1} (\alpha + \beta b)$ and $D = (1 + \beta c)^{-1} (\beta a)$. If we use the projection theorem for normal random variables and assume that $\beta a \neq 0$, then

$$E[\theta_i | s_i, p] = -\frac{C\sigma_\varepsilon^2}{D((1 - \rho)\sigma_\theta^2 + \sigma_\varepsilon^2)} + \frac{(1 - \rho)\sigma_\theta^2}{(1 - \rho)\sigma_\theta^2 + \sigma_\varepsilon^2} s_i + \frac{\sigma_\varepsilon^2}{D((1 - \rho)\sigma_\theta^2 + \sigma_\varepsilon^2)} p.$$

Using the first-order condition, we obtain

$$\begin{aligned} \frac{\sigma_\varepsilon^2}{(\sigma_\varepsilon^2 + (1 - \rho)\sigma_\theta^2)} \frac{\alpha + \beta b}{\beta a} - \frac{(1 - \rho)\sigma_\theta^2}{(\sigma_\varepsilon^2 + (1 - \rho)\sigma_\theta^2)} s_i + \left(1 - \frac{\sigma_\varepsilon^2}{(\sigma_\varepsilon^2 + (1 - \rho)\sigma_\theta^2)} \frac{1 + \beta c}{\beta a} \right) p \\ = -\lambda(b + as_i - cp); \end{aligned}$$

then, using the method of undetermined coefficients, we obtain the following system of equations:

$$\left. \begin{aligned} \frac{(1-\rho)\sigma_\theta^2}{\lambda(\sigma_\varepsilon^2 + (1-\rho)\sigma_\theta^2)} &= a \\ -\frac{\sigma_\varepsilon^2}{(\sigma_\varepsilon^2 + (1-\rho)\sigma_\theta^2)} \frac{\alpha + \beta b}{\beta a} &= \lambda b \\ 1 - \frac{\sigma_\varepsilon^2}{(\sigma_\varepsilon^2 + (1-\rho)\sigma_\theta^2)} \frac{(1 + \beta c)}{\beta a} &= \lambda c \end{aligned} \right\}$$

The solution to this system gives the result because for $\rho \in (0,1)$, $\sigma_\theta^2 > 0$, and $\sigma_\varepsilon^2 < \infty$ we have that $1 + \beta c = a(\beta + \lambda) > 0$, and

$$\begin{aligned} a &= \frac{1}{\lambda \left(1 + \left(\sigma_\varepsilon^2 / ((1-\rho)\sigma_\theta^2) \right) \right)} = \frac{1}{\lambda(1+M)}, \\ b &= -\frac{\alpha}{\beta} \frac{\sigma_\varepsilon^2}{(\sigma_\varepsilon^2 + (1-\rho)\sigma_\theta^2)} = -\frac{\alpha}{\beta}(1-\lambda a), \\ c &= \frac{\lambda^{-1}(1-\rho)\sigma_\theta^2 - \beta^{-1}\sigma_\varepsilon^2}{(\sigma_\varepsilon^2 + (1-\rho)\sigma_\theta^2)} = \frac{1}{\beta}(a(\beta + \lambda) - 1). \end{aligned}$$

It is immediate that $a > 0$, that $-\beta^{-1} < c \leq a \leq \lambda^{-1}$ for $\beta > 0$, and that c decreases in M and λ but increases in β . Finally, we can use $p = (1 + \beta c)^{-1}(\alpha + \beta b + \beta a \tilde{\theta})$ together with the expressions for the equilibrium coefficients to show that $p = (\lambda\alpha + \beta\tilde{\theta})/(\lambda + \beta)$ and $\tilde{x} = (\tilde{\theta} - \alpha)/(\beta + \lambda)$. ♦

Measures of speed of convergence. We say that the sequence (of real numbers) b_n is of the order n^ν (ν a real number) whenever $n^{-\nu}b_n \xrightarrow[n]{\rightarrow} k$ for some nonzero constant k .²⁸ The constant of convergence k is a refined measure of the speed of convergence. We say that the sequence of random variables $\{y_n\}$ converges *in mean square* to zero at the rate $1/\sqrt{n^r}$ (or that y_n is of the order $1/\sqrt{n^r}$) if $E[(y_n)^2]$ converges to zero at the rate $1/n^r$

²⁸ This definition is stronger than necessary but it will suffice for our purposes.

(i.e., if $E[(y_n)^2]$ is of the order $1/n^r$). Given that $E[(y_n)^2] = (E[y_n])^2 + \text{var}[y_n]$, a sequence $\{y_n\}$ such that $E[y_n] = 0$ and $\text{var}[y_n]$ is of order $1/n$ and converges to zero at the rate $1/\sqrt{n}$. A more refined measure of the speed of convergence for a given convergence rate is provided by the *asymptotic variance*. Suppose that $E[(y_n)^2] \xrightarrow{n} 0$ at the rate $1/n^r$ and $E[y_n] = 0$. Then the asymptotic variance of convergence is given by the constant $\lim_{n \rightarrow \infty} n^r E[(y_n)^2]$. A higher asymptotic variance means that the speed of convergence is slower.

Proof of Proposition 3:

(i)(a) From the proof of Proposition 7 in Vives (2011b) we have that

$$c_n \xrightarrow{n} c \equiv \frac{\lambda^{-1} - \beta^{-1}M}{M+1}, \text{ where } M = \frac{\sigma_\varepsilon^2}{(1-\rho)\sigma_\theta^2} \text{ if } \rho > 0 \text{ and } c_n \xrightarrow{n} \lambda^{-1} \text{ if } \rho = 0.$$

Furthermore,
$$a_n = \frac{(1-\rho)\sigma_\theta^2}{(\sigma_\varepsilon^2 + (1-\rho)\sigma_\theta^2)} (d_n + \lambda)^{-1} \xrightarrow{n} a \quad \text{because}$$

$d_n = (\beta^{-1}n + (n-1)c_n)^{-1} \xrightarrow{n} 0$. Note that $nd_n \xrightarrow{n} (\beta^{-1} + c_\infty)^{-1}$, which is equal to $(\beta^{-1} + \lambda^1)^{-1}(1+M)$ if $\rho > 0$ or to $(\beta^{-1} + \lambda^1)^{-1}$ if $\rho = 0$. Convergence for b_n follows similarly.

(i)(b) From Proposition 1 in Vives (2011a) we have that

$$\tilde{x}_n = \left(E[\tilde{\theta}_n | \tilde{s}_n] - \alpha \right) / (\beta + \lambda + d_n), \quad \text{where } \tilde{s}_n = n^{-1} \left(\sum_i s_i \right) = \tilde{\theta}_n + n^{-1} \left(\sum_i \varepsilon_i \right),$$

$$E[\tilde{\theta}_n | \tilde{s}_n] = \zeta_n \tilde{s}_n + (1 - \zeta_n) \bar{\theta}, \quad \text{and } \zeta_n \equiv \text{var}[\tilde{\theta}_n] / \left(\text{var}[\tilde{\theta}_n] + \sigma_\varepsilon^2 n^{-1} \right).$$

It follows that
$$\text{var} \left[E[\tilde{\theta}_n | \tilde{s}_n] \right] = \zeta_n \text{var}[\tilde{\theta}_n] = \frac{\left((1 + (n-1)\rho)\sigma_\theta^2 \right)^2}{\left((1 + (n-1)\rho)\sigma_\theta^2 + \sigma_\varepsilon^2 \right) n}.$$
 Observe that $d_n \xrightarrow{n} 0$ and

$E[\tilde{\theta}_n | \tilde{s}_n] \xrightarrow{n} \tilde{\theta}$ in mean square (since $\tilde{\theta}_n \xrightarrow{n} \tilde{\theta}$ and $(\sum_i \varepsilon_i)/n \xrightarrow{n} 0$ in mean square, both at rate $1/\sqrt{n}$). It is immediate that

$$E\left[\left(\tilde{\theta} - E\left[\tilde{\theta}_n \mid \tilde{s}_n\right]\right)^2\right] = \frac{\sigma_\theta^2(1-\rho)(\rho(n-1)+1) + n\rho\sigma_\varepsilon^2}{n(\sigma_\theta^2 + \sigma_\varepsilon^2 + (n-1)\rho\sigma_\theta^2)} \sigma_\theta^2 \quad \text{and}$$

$$nE\left[\left(\tilde{\theta} - E\left[\tilde{\theta}_n \mid \tilde{s}_n\right]\right)^2\right] \xrightarrow{n} AV \quad , \quad \text{where} \quad AV = (1-\rho)\sigma_\theta^2 + \sigma_\varepsilon^2 \quad \text{if} \quad \rho > 0 \quad \text{and}$$

$AV = \sigma_\theta^4(\sigma_\theta^2 + \sigma_\varepsilon^2)^{-1}$ if $\rho = 0$. We have that $\tilde{x} = (\tilde{\theta} - \alpha)/(\beta + \lambda)$, that

$$E\left[\left(\tilde{x} - \tilde{x}_n\right)^2\right] = E\left[\left(\frac{(\beta + \lambda)(\tilde{\theta} - E[\tilde{\theta}_n \mid \tilde{s}_n]) + d_n(\tilde{\theta} - \alpha)}{(\beta + \lambda)(\beta + \lambda + d_n)}\right)^2\right] \quad , \quad \text{and} \quad \text{that} \quad \text{both} \quad d_n \quad \text{and}$$

$E\left[\left(\tilde{\theta} - E\left[\tilde{\theta}_n \mid \tilde{s}_n\right]\right)^2\right]$ are of order $1/n$; hence we obtain

$$nE\left[\left(\tilde{x} - \tilde{x}_n\right)^2\right] \xrightarrow{n} AV/(\beta + \lambda)^2. \quad \text{Therefore,} \quad \tilde{x} - \tilde{x}_n \xrightarrow{n} 0 \quad \text{in mean square. The results}$$

follow since $p_n - p = \beta(\tilde{x}_n - \tilde{x})$.

(i)(c) Total surplus (per capita) in the continuum and in the n -replica markets are given, respectively, by

$$TS = \int_0^1 \left(\theta_i x_i - \frac{\lambda}{2} x_i^2 \right) di - \left(\alpha + \beta \frac{\tilde{x}}{2} \right) \tilde{x} \quad \text{and} \quad n^{-1}TS_n = n^{-1} \sum_{i=1}^n \left(\theta_i x_i - \frac{\lambda}{2} x_i^2 \right) - \left(\alpha + \beta \frac{\tilde{x}_n}{2} \right) \tilde{x}_n.$$

We can then write the expected welfare loss as

$$WL_n \equiv E[TS] - n^{-1}E[TS_n] = \left((\beta + \lambda)E\left[\left(\tilde{x}_n - \tilde{x}\right)^2\right] + \lambda E\left[\left(u_m - u_i\right)^2\right] \right) / 2,$$

where $u_m \equiv x_m - \tilde{x}_n$ and $u_i \equiv x_i - \tilde{x}$ (this follows as in the proof of Proposition 3 in Vives 2011a). We already know from the proof of part (i)(b) that

$$nE\left[\left(\tilde{x} - \tilde{x}_n\right)^2\right] \xrightarrow{n} AV/(\beta + \lambda)^2.$$

We also know that $u_m = (\tilde{t}_n - t_m)/(\lambda + d_n)$ and $u_i = (\tilde{\theta} - t_i)/\lambda$, where $\tilde{t}_n \equiv E\left[\tilde{\theta}_n \mid \tilde{s}_n\right]$,

$$t_m \equiv E\left[\theta_i \mid s_i, \tilde{s}_n\right] = \bar{\theta} + \frac{(1-\rho)\sigma_\theta^2}{(\sigma_\theta^2(1-\rho) + \sigma_\varepsilon^2)}(s_i - \bar{\theta}) + \frac{\sigma_\varepsilon^2\sigma_\theta^2\rho n}{(((n-1)\rho + 1)\sigma_\theta^2 + \sigma_\varepsilon^2)(\sigma_\theta^2(1-\rho) + \sigma_\varepsilon^2)}(\tilde{s}_n - \bar{\theta}),$$

and $t_i \equiv E\left[\theta_i \mid s_i, \tilde{\theta}\right] = \varsigma s_i + (1-\varsigma)\tilde{\theta}$ for $\varsigma = \left(1 + \frac{\sigma_\varepsilon^2}{(1-\rho)\sigma_\theta^2}\right)^{-1}$. As a result,

$$E\left[\left(u_m - u_i\right)^2\right] = \frac{1}{(\lambda + d_n)^2 \lambda^2} \left(\frac{(1-\rho)\sigma_\theta^2}{(\sigma_\theta^2(1-\rho) + \sigma_\varepsilon^2)} \right)^2 E\left[\left((\tilde{s}_n - \tilde{\theta})\lambda - (\tilde{\theta} - s_i)d_n\right)^2\right].$$

Further computations yield

$$E\left[\left((\tilde{s}_n - \tilde{\theta})\lambda - (\tilde{\theta} - s_i)d_n\right)^2\right] = n^{-1}\left(\sigma_\theta^2(1-\rho) + \sigma_\varepsilon^2\right)\left(2\lambda d_n + \lambda^2 + nd_n^2\right).$$

Therefore, $E\left[(u_m - u_i)^2\right] = \frac{(2\lambda d_n + \lambda^2 + nd_n^2)(1-\rho)^2 \sigma_\theta^4}{(\sigma_\theta^2(1-\rho) + \sigma_\varepsilon^2)(\lambda + d_n)^2 \lambda^2 n}$; since d_n is of order $1/n$, we

have $\lim_{n \rightarrow \infty} n \frac{\lambda E\left[(u_m - u_i)^2\right]}{2} = \frac{(1-\rho)^2 \sigma_\theta^4}{2\lambda(\sigma_\theta^2(1-\rho) + \sigma_\varepsilon^2)}$. It follows that

$$n\text{WL}_n \rightarrow \frac{\text{AV}}{2(\beta + \lambda)} + \frac{(1-\rho)^2 \sigma_\theta^4}{2\lambda(\sigma_\theta^2(1-\rho) + \sigma_\varepsilon^2)}.$$

(ii) Let $E[\pi_i]$ be the expected profits of trader i when the other traders $j \neq i$ have information precision τ_ε and use identical strategies based on linear demand schedules with coefficients (b, a, c) . Suppose that trader i has precision τ_{ε_i} and optimizes his demand schedule. If we put $d_n = (\beta^{-1}n + (n-1)c)^{-1}$, then it follows from Section S.4 in Vives (2011b) that

$$\psi_n(\tau_\varepsilon) \equiv \left. \frac{\partial E[\pi_i]}{\partial \tau_{\varepsilon_i}} \right|_{\tau_{\varepsilon_i} = \tau_\varepsilon} = \frac{1}{2(2d_n + \lambda)} \frac{(\tau_\varepsilon(1-\rho)(1 + \rho(n-1)) + \tau_\theta)^2}{(\tau_\varepsilon(1-\rho) + \tau_\theta)^2 (\tau_\varepsilon(1 + \rho(n-1)) + \tau_\theta)^2}.$$

Interior symmetric equilibria for information precision are characterized by the solution of $\psi_n(\tau_\varepsilon) - H'(\tau_\varepsilon) = 0$, where d_n is given by Proposition 1 in Vives (2011a) for any particular τ_ε . Let $\psi_n(0) \equiv \lim_{\tau_\varepsilon \rightarrow 0} \psi_n(\tau_\varepsilon) = (2(2d_n(0+) + \lambda)\tau_\theta^2)^{-1}$ and $d_n(0+) \equiv \lim_{\tau_\varepsilon \rightarrow 0} d_n(\tau_\varepsilon)$. The existence condition for an interior symmetric equilibrium is $H'(0) < \psi_n(0)$. Note that $\psi_n(0) \xrightarrow{n} (2\lambda\tau_\theta^2)^{-1}$ because $d_n(0+) \xrightarrow{n} 0$. Therefore, if $\phi(0) = (1-\rho)^2(2\lambda\tau_\theta^2)^{-1} - H'(0) > 0$ (given that $\rho \in [0, 1 - \tau_\theta\sqrt{2\lambda H'(0)}]$), then $(2\lambda\tau_\theta^2)^{-1} - H'(0) > 0$ and the condition $H'(0) < \psi_n(0)$ is fulfilled for n large. Observe

that, since $d_n \xrightarrow{n} 0$, we have $\psi_n(\tau_\varepsilon) \xrightarrow{n} \frac{(1-\rho)^2}{2\lambda(\tau_\theta + (1-\rho)\tau_\varepsilon)^2}$. For n large there is a unique symmetric equilibrium of the information acquisition game $\tau_\varepsilon^*(n)$, the solution to $\phi_n(\tau_\varepsilon) \equiv \psi_n(\tau_\varepsilon) - H'(\tau_\varepsilon) = 0$. This is easily checked because, for n large, $\psi_n(\tau_\varepsilon)$ is strictly decreasing in τ_ε . It follows that $\tau_\varepsilon^*(n) \rightarrow \tau_\varepsilon^* > 0$ (where $\phi(\tau_\varepsilon^*) = 0$) as $n \rightarrow \infty$. ♦

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