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ABSTRACT

Resource Allocation in the Brain*

When an individual performs several tasks simultaneously, resources must be allocated to different brain systems to produce energy for neurons to fire. Following the evidence from neuroscience, we model the brain as an organization in which a coordinator allocates limited resources to the brain systems responsible for the different tasks. Systems are privately informed about the amount of resources necessary to perform their task and compete to obtain the resources. The coordinator arbitrates the demands while satisfying the resource constraint. We show that the optimal mechanism is to impose to each system with privately known needs a cap in resources that depends negatively on the amount of resources requested by the other system. This allocation can be implemented using a physiologically plausible mechanism. Finally, we provide some implications of our theory: (i) performance is inversely related to the difficulty of the task and can be flawless for sufficiently simple tasks, (ii) the dynamic allocation rule exhibits inertia (current allocations are increasing in past needs), and (iii) different cognitive tasks are performed by different systems only if the tasks are sufficiently important.

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1 Introduction

Our ability to handle tasks depends on the coordination of multiple brain mechanisms. Research in the brain sciences has established that decision making requires the allocation of scarce resources (most notably oxygen and glucose) to the brain systems involved in understanding tasks, planning responses, and implementing actions. The objective of this paper is to study the relationship between the mechanisms for allocating resources in the brain and the quality of the resulting decisions.

To this purpose, we develop a *parsimonious theory of constrained optimal behavior* based on resource allocation under neurophysiological limitations. This approach affords a new perspective on decision-making which is different from traditional bounded rationality models, as it provides foundations for "mistakes" and "biases" in decision-making that do not rely on the ad-hoc imposition of imperfections. It also sets a benchmark for determining the environments in which second-best choices are most likely to arise. The fundamental features of brain processes that will constitute the building blocks of our theory are briefly introduced here and more thoroughly reviewed in section 1.1.[1] First, there is brain specialization. Different brain systems are recruited to perform different tasks and neurons in a given system respond exclusively to features of that particular task. These neurons remain active as long as they receive resources and the task is not completed. The behavior of neurons in a system is therefore consistent with the maximization of task performance. Second, there is communication of needs. The consumption of resources in a brain system triggers a signal which results in more resources being allocated to that system. Third, the resource allocation process is centralized. Some areas of the lateral prefrontal cortex (LPFC) play an active role when attention is divided, for instance when two tasks have to be completed at the same time. This points to the existence of what has been called a 'Central Executive System' (CES) whose role is to coordinate the systems involved in the different tasks. Fourth, resources are scarce. The brain is endowed with a fixed and often insufficient amount of oxygen and glucose to perform the tasks.

In section 2, we build an agency model based on these four fundamental principles of the brain architecture. In our model, CES (the principal) is responsible for allocating resources to systems with privately known needs (the agents) given a resource constraint. More precisely, we consider the case of an individual who must perform three tasks $(0, 1, 2)$

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[1] Notice that the paper takes the brain architecture as given. It does not address important questions related to its evolutionary rationale. We refer the reader to Robson (2001b) and Robson and Samuelson (2009) for formal models of the biological basis of economic behavior and to Robson (2001a) for a survey of the literature. These papers, however, consider fitness of the individual rather than fitness of neuronal groups as the evolutionary selection criterion.

at the same time, each carried out by a different system $(0, 1, 2)$. The amount of resources necessary to perform a task is a function of its complexity, and performance decreases with the difference between resources needed and resources obtained. Total resources are available in a fixed amount. CES seeks to maximize the sum of performances in the three tasks; it knows the complexity of task 0 and extracts information from systems 1 and 2 about the complexity of tasks 1 and 2 vía a communication mechanism.

We first conduct a normative analysis where we assume that CES can resort to any communication mechanism. This allows us to restrict attention to incentive compatible direct revelation mechanisms. We characterize the optimal mechanism and show that the allocation is such that each system is guaranteed a minimum level of resources. A system can obtain resources above that minimum if and only if at least one of the other systems chooses not to exhaust its allocation (Proposition 1). We then perform some comparative statics and find that a natural resource monotonicity principle holds under fairly general conditions: (i) if one system becomes less important from the viewpoint of CES then it receives fewer resources whereas all other systems receive (weakly) more resources, and (ii) if the total amount of resources available increases then all systems benefit to some extent (Proposition 2).

The normative analysis is important in that it sets an upper bound on the attainable performance. Our second result is to show that the optimal mechanism can be implemented using a simple and physiologically plausible autoregulation process: systems receive resources at different rates, they choose whether to deplete them and, if they do, CES decides whether to provide more resources (Proposition 3). This finding is critical because it establishes that observing a simple allocation rule does not necessarily imply that individuals are subject to ad-hoc limitations. Instead, it reflects that for our problem the constrained optimal choice can be implemented with a simple procedure.

Next, we derive behavioral implications of our mechanism and confront them with the experimental results obtained in neuroscience studies. Most notably, our theory predicts that performance will be inversely related to the difficulty of the cognitive task and flawless for sufficiently simple ones (Implications 1 and 2). It also predicts performance improvements over time and task-inertia: if needs are drawn independently over time, the allocation of resources at a certain date will depend positively on the needs experienced in the past (Proposition 4 and Implications 3 and 4). These results match the experimental neuroscience evidence and arise in our framework only under private information of needs.[2]

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[2]In other words, if CES knew the needs of all three systems, performance would never be flawless, it would be identical for easy and difficult tasks and the allocation would not exhibit dynamic improvements or inertia.

Also, we show that in a biologically plausible mechanism, the time required to complete an easy task is shorter the more difficult that same task was in the past (Implication 5). Finally, we propose a novel testable implication regarding the architecture of the brain: from an informational viewpoint it is efficient to concentrate cognitive tasks in one system whenever the importance of the tasks is relatively low and to separate them into different systems otherwise (Proposition 5 and Implication 6).

1.1 Evidence from neuroscience

This section briefly reviews the neuroscience evidence underlying our theory.[3] We will refer to it when we introduce the formal elements of our model.

Principles of neural activation, resource allocation and behavioral performance. The literature studying cerebral blood flow has established the existence of a functional coupling between neural activity and brain metabolism. Cerebral activation processes are accompanied by a dynamic adjustment of cerebral blood flow. Blood flow is correlated with oxygen delivery to the brain. The increase in blood flow following the presentation of a task is positively related to the performance in that task (Duschek and Schandry (2004, 2006)). Therefore, task performance, neural activation and resource allocation of energy inputs are highly correlated. These correlations result from the mechanisms used by brain units, or neurons, to complete their function.

Neurons are information messengers that use electrical impulses and chemical signals to transmit information between different areas of the nervous system. This communication process is used by the brain to detect features of the environment (in the sensory system), make choices (in the decision system) and send orders to act accordingly (in the motor system). Typically, brain areas are either unconnected or unidirectionally connected to other areas. This feature of the brain anatomy is the result of evolution, which optimizes the number and location of the highly scarce and energetically demanding neural connections. Limited neural connectivity immediately implies a restricted flow of information or, in the economics language, asymmetric information. Communication between neurons requires energy which is delivered by the oxidation of glucose extracted from arterial blood. This energy is used for propagating signals and returning the membrane to its resting potential after firing (Attwell and Laughlin, 2001).

Studies that use scanning techniques (PET and fMRI) to observe brain activity are indirectly informing us about how resources are allocated in the brain. Activity is detected

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[3]It can be skipped by readers who either have a background in neuroscience or are not interested in the details of the brain architecture. Readers interested in a yet more detailed introduction to these physiological processes are referred to Brocas (2009).

vía methods that observe differentials in consumption of resources (Fox et al. (1988), Hyder et al. (2002)). The correlation between neuronal activity and those differentials results from the existence of a variety of internal adaptive mechanisms that work to adjust the response to stimuli. This process, called autoregulation, helps diverting blood where it is most needed. Since resources (glucose and oxygen) are carried by blood, blood flow differentials are also considered a reliable measure for brain activity. In particular, PET monitors detect changes in blood flow, glucose usage or oxygen consumption. fMRI signals reflect the degree of blood oxygenation and flow, and measure the blood-oxygen-level dependent (BOLD) response.[4]

Tasks and systems. When a decision-maker is facing a task, populations of neurons specialized in different features relevant for that task are recruited. These constitute a system. To understand the 'objective' of a system, it is useful to look at its components: the neurons. Neurons fire in response to certain inputs. For instance, the spiking activity of a neuron in the visual system represents a small part of the visual environment, as the neuron is sensitive to the presence of a few specific features only.[5] As such, neurons are only concerned about transmitting information regarding the features they are tuned to detect. Given such a construct, a system only transmits information detected by its components, that is, information relevant for that task. Moreover, neurons keep firing as long as they detect relevant information. Combining these premises, a system can be represented as an entity that cares exclusively about transmitting information to perform its own task.[6] In terms of resources, this means that glucose and oxygen must be brought in to help the system complete the desired firing. Overall, a system seeks the ideal amount of resources to optimize performance, and poor performance occurs when the actual amount allocated differs from the ideal amount.[7]

Demand and supply of resources. As neurons do not have any reserve of energy, they rely on an on-demand mechanism to obtain the resources required to perform their activity. When neurons fire, they not only transmit information but also request more glucose and oxygen to be brought in quickly. This can be construed as a message requesting

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[4]The *joint* observation that one system receives more oxygen when a certain task is performed *and* that subjects with a lesion in that system are unable to perform the task provides yet another indirect support for the idea that the system utilizes resources to perform the task.

[5]However, neurons respond not only to the presence or absence of features but also to their values by producing graded responses. They do so by controlling the number of spikes they fire.

[6]A system is related to a task. That is, neurons active in one task are part of the system performing that task, but they can also be active in a different task involving a different system. In other words, two systems do not need to be two physically different areas of the brain.

[7]Some medical conditions are characterized by the inability to regulate the amount of resources in the brain. For instance, too much or too little glucose in insulin-dependent diabetes patients have detrimental effects on cognitive functions (Cox et al., 2005).

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additional resources. In a very schematic way, we can represent the process as a dynamic communication mechanism. Neurons initiate firing based on the available resources in the bloodstream at the moment. Resource depletion constitutes a signal that more resources are needed. Once that message is correctly interpreted, they receive (or not) additional resources.

Centralization of the resource allocation process. A number of fMRI studies have found that certain regions of the LPFC are active when two tasks are performed simultaneously. These regions are not active (i) when only one task is presented to the subject, (ii) when both tasks are presented but the subject is instructed to selectively focus on only one of them, or (iii) when both tasks are presented and performed sequentially (D'Esposito et al. (1995), Herath et al. (2001), Szameitat et al. (2002), Jiang (2004)). The same phenomenon is observed for branching, that is, when subjects must keep in mind a main goal while performing concurrent subgoals (Koechlin et al. (1999)). In parallel, the literature on task switching has found that several regions of the PFC are activated when a switch occurs (Monsell (2003)). The results point to the existence of a *Central Executive System* (CES) whose role is to "coordinate the concurrent processing of the different streams of information" (Szameitat et al. (2002, p. 1184)). Studies using other techniques also support this idea: patients with brain lesions in the left DLPFC have problems switching between the attributes they are instructed to attend to (Rogers et al. (1998), Keele and Rafal (2000)) and subjects in a TMS study whose DLPFC has been disrupted exhibit an impaired ability to divide attention between tasks (Johnson et al. (2007)).

Scarcity of resources. The brain utilizes 20% of the total resting energy consumption. Most of the energy used in the brain is for the propagation of action potentials (47%) and for restoring postsynaptic ion fluxes after receptors have been stimulated by the neurotransmitter (34%) (Attwell and Laughlin, 2001). In contrast, maintenance of the resting potential in brain cells accounts for only 13% of the total energy consumption. Glucose is the only fuel normally used by brain cells. Given that neurons cannot store it, they depend on the bloodstream for its supply. Cognitive tasks are associated with glucose consumption and glucose is not always present in sufficient amounts to support them (McNay and Gold, 2001). More generally, the metabolic cost of brain activity is high which may be the limiting factor for both the number of neurons that can be active at any given point in time as well as the maximum frequency of firing of individual neurons (Lennie (2003), Attwell and Gibb (2005)). This evidence suggests that resources are scarce and their allocation is inevitably constrained.

Behavioral interferences and neural activity patterns. Single- vs. dual-task experiments have established some interesting results on neural activation and behavioral patterns.

Studies have shown that the volume of activation is smaller in the dual-task condition than in the sum of the two related single-task conditions (Just et al. (2001), Loose et al. (2003), Johnson and Zatorre (2006), Newman et al. (2007)).[8] Sub-additivity suggests the existence of "biological mechanisms that place an upper bound on the amount of cortical tissue that can be activated at any given time" (Just et al. (2001, p. 424)).[9] Some other studies highlight a significant behavioral interference when subjects perform the dual-task. In particular, reaction times (Jiang, 2004)) and error rates (Szameitat et al., 2002) increase, suggesting that the two tasks compete for attention. Behaviorally, performance in the dual-task is lower than in the sum of the two single tasks (Just et al., 2001), which is consistent with the above mentioned activation patterns.

1.2 Related literature in economics

From a theoretical viewpoint, the problem is related to two strands of the information economics literature. First, the research on *mechanism design without transfers* (Holmström (1977), Melumad and Shibano (1991), Alonso and Matouschek (2007, 2008)).[10] While the absence of transfers typically requires justification in the literature on organizations, in our setting the neurobiology evidence indicates that CES can choose how to allocate resources between systems but does not possess any other means of 'compensation'. Recent papers on this literature have studied problems with multiple agents (Martimort and Semenov (2008), Carrasco and Fuchs (2009)) or multiple actions (Koessler and Martimort, 2009). The present paper combines aspects of multiple agents and multiple actions in a novel setting with capacity constraints. Second the *axiomatic social choice* literature that studies rationing problems (Sprumont (1991), Barbera, Jackson and Neme (1997) and Moulin (2000)). This literature has provided characterizations of rationing mechanisms that satisfy efficiency, strategy-proofness and certain additional properties. For instance, Sprumont (1991) shows that the uniform rule is the unique mechanism that guarantees efficiency, strategy-proofness and symmetry, while Barbera, Jackson and Neme (1997) allow for asymmetric rules and characterize those that additionally satisfy replacement-monotonicity. In contrast to this literature, our focus is on the mechanisms that are

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[8]These studies measure activity in the sensory and association areas that are active in one (and only one) of the tasks. They are designed to minimize overlapping areas by choosing tasks that are known to recruit different brain systems (e.g., mental rotation of visually depicted objects and auditory comprehension).

[9]A puzzling result in Just et al. (2001) is that, contrary to the other papers reviewed above, LPFC activation does not change between the single- and dual-task treatments. A possible explanation is that subjects are requested to perform high-level cognitive tasks so that the single-task treatment may already be producing significant activation in the LPFC.

[10]Mechanism design problems *with* transfers and quasi-linear utilities are generally simpler to solve, since it is possible to use standard Lagrangian techniques (see e.g. Fudenberg and Tirole (1991, ch. 7)).

efficient, strategy-proof and that *maximize the expected performance of systems.*

From a conceptual viewpoint, the paper is related to the behavioral economics literature that studies decision-making when individuals have *imperfect self-knowledge*. Recent papers have emphasized the gains from ignorance under decreasing impatience (Carrillo and Mariotti (2000), Bénabou and Tirole (2002, 2004), Brocas and Carrillo (2005), Ali (2009)) and the strategic choice of actions as self-signaling devices (Bodner and Prelec (2003), Bénabou and Tirole (2006), Dal Bo and Terviö (2007)). Our model focuses on a different set of issues, namely performance in a multi-tasking environment. It also departs significantly in the methodological approach: rather than building a model of boundedly rational behavior based either on introspection or on empirical / experimental data, we take the neuroscience findings about the brain architecture as inputs for modeling the constraints in the optimization problem.[11]

2   The model

2.1   Systems and objectives

Based on the evidence described in section 1.1, we build the following resource allocation model. First, there is a set of systems. Each system is responsible for a task, which can be as simple as a basic motor skill action or as complex as a high order reasoning. Systems are composed of neurons. They demand resources (glucose, oxygen) in an amount that depends on the complexity of the task to be performed. Resource deficits imply a decrease in performance. Second, there is a Central Executive System (CES) which is responsible for the optimal allocation of the scarce resources between systems and whose objective is to maximize overall performance in the tasks.

Formally, we assume there are three tasks, and system l ($\in L = \{0, 1, 2\}$) is responsible for task l. As reviewed in section 1.1, system l can be formally represented as a selfish entity focused exclusively on the performance in its own task. Let $\Theta_l = [0, \bar{\theta}_l]$ be the set of possible resources task l may require. If, in a given instance, $\theta_l \in \Theta_l$ is the actual amount of resources necessary to carry out task l flawlessly and x_l are the resources allocated to system l, the system seeks $x_l = \theta_l$ resources. Without loss of generality, a system can be endowed with a performance function $\Pi_l(x_l; \theta_l)$ that is maximized at $x_l = \theta_l$. There is a loss whenever $x_l < \theta_l$ because the task cannot be completed with perfect accuracy. The effect of too many resources is ambiguous. On the one hand, the system may be able to costlessly discard resources above θ_l (they are present in the bloodstream but

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[11]In that respect, the paper is closer to Brocas and Carrillo (2008) which studies the dynamic choices of an individual when brain systems have different mental representations of current vs. distant prospects.

neurons do not consume them), which would formally mean that $\Pi_l(x_l; \theta_l) = \Pi_l(\theta_l; \theta_l)$ for all $x_l \geq \theta_l$. On the other hand, evidence suggests that when autoregulation fails, too many resources can impair cognitive functions or, formally, that $\Pi_l(x_l; \theta_l) < \Pi_l(x'_l; \theta_l)$ for all $x_l > x'_l \geq \theta_l$. In any case, the performance function should be increasing in x_l up to θ_l and non-increasing above it. To simplify the exposition, we assume a simple quadratic expression:[12]

$$\Pi_l(x_l; \theta_l) = -\frac{1}{\beta_l} (x_l - \theta_l)^2, \quad \beta_l > 0 \tag{1}$$

As the absolute difference between needs and resources granted $|\theta_l - x_l|$ increases, performance deteriorates. Also, tasks that are more complex require more resources. Finally, a decrease in β_l raises the marginal cost of unfulfilled needs for any level of needs and resources.

2.2 Optimization under full information

The optimization problem of CES consists in distributing a fixed amount of resources k among the three systems so as to maximize the sum of performances on the tasks given their respective needs. We can formally represent it as:

$$\max_{\{x_0, x_1, x_2\}} \quad \Pi_0(x_0; \theta_0) + \Pi_1(x_1; \theta_1) + \Pi_2(x_2; \theta_2)$$
$$\text{s.t.} \quad x_0 + x_1 + x_2 \leq k \tag{R}$$
$$x_0 \geq 0, \ x_1 \geq 0, \ x_2 \geq 0 \tag{F}$$

The *resource constraint* (R) reflects the maximum resources k available to perform the three tasks.[13] The *feasibility constraint* (F) captures the minimum resources that can be allocated to each system (the analysis can be trivially extended to a positive minimum amount of resources necessary for a system to operate). Notice that CES is 'utilitarian' in the sense that each system carries equal weight.[14]

The problem is interesting when $\theta_0 + \theta_1 + \theta_2 > k$, as it implies that (R) binds at the optimum and systems are rationed. Given (1) and assuming an interior solution for

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[12]Because we will assume scarcity of resources it will always be the case that, in equilibrium, $x_l \leq \theta_l$. Hence, identical results would be obtained if we assumed instead $\Pi_l(x_l; \theta_l) = \Pi_l(\theta_l; \theta_l)$ for all $x_l \geq \theta_l$.

[13]As discussed in section 1.1 we refer to "metabolic resources", that is, the (scarce) energy required by neurons to fire. One could alternatively interpret the model in terms of "computational resources" or "attentional resources", although this concept is less precisely defined in neurobiology. All terminologies fit equally well our model at our level of abstraction (except perhaps in section 3.5).

[14]A formally equivalent interpretation would be to say that performance of system l is given by $\tilde{\Pi}_l(x_l; \theta_l) = -(x_l - \theta_l)^2$ and that CES maximizes a weighted sum of performances $\sum_l \tilde{\Pi}_l(x_l; \theta_l)/\beta_l$, where $1/\beta_l$ measures the relative importance of system l. These weights could be related to a system of rewards imposed externally to complete these tasks.

all systems ($x_l^F > 0$), the first-best outcome to this problem is (corner solutions are also straightforward to analyze):

$$x_l^F = \theta_l - \frac{\beta_l}{\displaystyle\sum_{l' \in L} \beta_{l'}} \left(\sum_{l' \in L} \theta_{l'} - k \right) \tag{2}$$

Equation (2) simply means that, at equilibrium, no system receives its needs. More precisely, system l obtains its desired amount minus a fraction of the excess resource demand. This fraction depends on the cost for system l to receive less than θ_l relative to the cost for the other systems (or on the relative importance of system l for CES in the alternative interpretation). For instance, if $\beta_1 = \beta_2$, the shortage is the same for systems 1 and 2: $x_1^F - \theta_1 = x_2^F - \theta_2$. Under first-best rationing, the performance of system l is:

$$\Pi_l(x_l^F; \theta_l) = -\frac{\beta_l}{\left(\sum \beta_{l'} \right)^2} \left(\sum \theta_{l'} - k \right)^2 \tag{3}$$

From this equation, notice that the performance of a system depends on the aggregate needs ($\sum \theta_{l'}$) but not on the relative needs of each of them. For instance, if $\beta_1 = \beta_2$, performance is the same $\Pi_1(x_1^F; \theta_1) = \Pi_2(x_2^F; \theta_2)$ independently of whether $\theta_1 \gtrless \theta_2$. More importantly, (3) implies that if the actual resources required to complete tasks are known, the individual should under-perform in *all* tasks whenever the resource constraint is hit ($x_l < \theta_l$ for all l).

3   Imperfect knowledge of needs

The more realistic and interesting situation arises when CES does not know how many resources are required by some of the systems. As motivated in section 1.1, information asymmetry matches the physiological evidence on brain connectivity. It introduces an *endogenous cost* of resource allocation and information processing.

In the rest of the paper, we will consider two classes of systems. System 0 is responsible for a *basic motor skill* task 0 which corresponds, for example, to lifting an object or looking in a certain direction. The needs to perform this task, θ_0, are known. Systems 1 and 2 are responsible for *higher order cognitive* tasks. These include vision, audition, abstract projection and language, among others. We use subscripts i and j for systems 1 and 2 with $i \neq j$. The needs of system i, θ_i, are unknown to CES, and depend crucially on the type and difficulty of the cognitive task to be performed (face identification, auditory comprehension, mental representation of shapes, word recognition, etc.). CES only knows that θ_1 and θ_2 are independently drawn from continuous distributions with c.d.f. $F^1(\theta_1)$

and $F^2(\theta_2)$ and densities $f^1(\theta_1)$ and $f^2(\theta_2)$.[15] Let $h_i(\theta_i) = \frac{f^i(\theta_i)}{1-F^i(\theta_i)}$ be the hazard rate of θ_i. We assume that the distribution of needs of system i has an increasing hazard rate (IHR). This condition has no meaningful interpretation in our context. It is imposed only to facilitate the technical resolution of the problem and ensure some regularity properties of the solution.

Assumption 1 (IHR) $h_i(\theta_i') \geq h_i(\theta_i)$ *for all* $\theta_i' \geq \theta_i$.

Our objective is to find the resource allocation mechanism which is optimal from the viewpoint of CES given its imperfect knowledge of needs.

The first step of our analysis consists in adopting a *normative approach* and determine the optimal allocation when CES can use any conceivable communication mechanism: each system sends a message requesting resources and CES responds with an allocation as a function of the messages received. The obvious advantage of this approach is that we can apply the revelation principle. However, is it realistic to think in these terms? The answer is yes and no. On the one hand, our entire research rests on the fact that the brain has some specific, well-documented physiological limitations in the availability, transmission and processing of information, so putting no restrictions on the type of communication allowed contradicts that view. On the other hand, we show in Appendix A1 that a pseudo-dynamic mechanism where CES allocates some initial resources, systems choose whether to consume them and, as a function of their choice, CES decides whether to grant more resources is formally equivalent to a static incentive compatible mechanism where systems (truthfully) report their needs. Either way, a crucial advantage of the normative analysis is that it provides an upper bound on the attainable performance of CES. A contribution of the paper will rest on the subsequent *positive approach* (section 3.5), where we do investigate if the optimal allocation described in the normative analysis can indeed be implemented using a physiologically plausible mechanism. Lastly, we restrict attention to mechanisms that are ex-post efficient and can be implemented in dominant strategies. From a neuroeconomic viewpoint, implementation in dominant strategies seems most natural as it ensures that a system does not have to 'form beliefs' about the objectives, needs, demands or even the 'existence' of other systems.[16] Ex-post efficiency rules out the possibility that once the needs are reported both the CES and the systems agree to a change in the allocation.

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[15]The results can be trivially extended to more than one system (and therefore more than one task) with known needs. By contrast, extensions to three or more systems with unknown needs are challenging.

[16]From a theory viewpoint it would be interesting (but also difficult) to determine the optimal mechanism in Bayesian strategies, as Carrasco and Fuchs (2009) do in a somewhat related theoretical setting. However, we would have a hard time interpreting this type of mechanisms in our context.

Applying the revelation principle, we can without loss of generality restrict attention to direct revelation mechanisms where each system i 'announces' its needs $\tilde{\theta}_i \in \Theta_i$. Based on the announcements, CES 'commits' to a resource allocation rule:

$$\mathcal{X}(\tilde{\theta}_1, \tilde{\theta}_2) = \left\{ x_0(\tilde{\theta}_1, \tilde{\theta}_2), x_1(\tilde{\theta}_1, \tilde{\theta}_2), x_2(\tilde{\theta}_1, \tilde{\theta}_2) \right\}_{(\tilde{\theta}_1, \tilde{\theta}_2) \in \Theta_1 \times \Theta_2}$$

The allocation rule is constructed in a way that announcing $\tilde{\theta}_i = \theta_i$ is a dominant strategy for system i, that is:

$$\Pi_i(x_i(\theta_i, \theta_j); \theta_i) \geq \Pi_i(x_i(\tilde{\theta}_i, \theta_j); \theta_i) \qquad \forall\, i,\, \theta_i,\, \tilde{\theta}_i,\, \theta_j \qquad \text{(DSIC)}$$

Notice that the assumptions imposed on systems are minimal. Following the evidence reviewed above, the only concern of a system is to obtain the resources necessary to complete its task. Each system realizes that resources are scarce (simply by observing that its needs are not always fulfilled) and that their availability may depend on external factors. However, awareness of the necessities or even the existence of other systems and other tasks is not required.

3.1 The optimization problem

Given the imperfect knowledge of needs, we will assume that CES maximizes the *expected* performance in the tasks. From the work by Sprumont (1991) and Barbera, Jackson and Neme (1997) on social choice rules it is known that all the direct revelation mechanisms which are ex-post efficient and incentive compatible in dominant strategies have a resource cap structure: each system with private needs is granted the resources it requests up to a cap, and the cap imposed on one system depends on the report of the other system. In our setup, it means that the set of ex-post efficient, direct revelation mechanisms $D(\bar{x}_1(\theta_2), \bar{x}_2(\theta_1)) \in \mathcal{X}(\theta_1, \theta_2)$ are of the form:

$$D(\bar{x}_1(\theta_2), \bar{x}_2(\theta_1)) = \begin{cases} x_1(\theta_1, \theta_2) & = & \min\{\theta_1, \bar{x}_1(\theta_2)\} \\ x_2(\theta_1, \theta_2) & = & \min\{\theta_2, \bar{x}_2(\theta_1)\} \\ x_0(\theta_1, \theta_2) & = & k - x_1(\theta_1, \theta_2) - x_2(\theta_1, \theta_2) \end{cases} \qquad (4)$$

The problem under asymmetric information reduces to:

$$\max_{D(\bar{x}_1(\theta_2), \bar{x}_2(\theta_1))} \int\!\!\int \left[\Pi_0(x_0(\theta_1, \theta_2); \theta_0) + \Pi_1(x_1(\theta_1, \theta_2); \theta_1) + \Pi_2(x_2(\theta_1, \theta_2); \theta_2) \right] dF^1(\theta_1)\, dF^2(\theta_2)$$

$$\text{s.t.} \quad x_0(\theta_1, \theta_2) + x_1(\theta_1, \theta_2) + x_2(\theta_1, \theta_2) \leq k \qquad \forall\, \theta_1, \theta_2 \quad \text{(R)}$$

$$x_0(\theta_1, \theta_2) \geq 0,\ x_1(\theta_1, \theta_2) \geq 0,\ x_2(\theta_1, \theta_2) \geq 0 \qquad \forall\, \theta_1, \theta_2 \quad \text{(F)}$$

where the *dominant strategy incentive compatibility constraint* (DSIC) is automatically satisfied by the mechanism $D(\bar{x}_1(\theta_2), \bar{x}_2(\theta_1))$ and therefore ignored, and (R) and (F) are

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the resource and feasibility constraints introduced previously. The problem presupposes that CES does not necessitate resources to coordinate the needs of systems. This goes largely against the evidence presented in section 1.1 but it is imposed only for simplicity. Indeed, one could trivially extend the model and assume that CES requires $\tilde{k}$ resources for coordinating activities and that only $k - \tilde{k}$ resources are available for the three systems.[17]

Next, we assume that $\theta_0 > k$, which is sufficient to ensure that (R) always binds at the optimum. Intuitively, it means that the resources are always exhausted, even when the individual is not performing any cognitive task.[18] The assumption is not necessary, but it makes the optimization slightly simpler by removing one variable (x_0) and one constraint (R) from the problem. It also makes the comparative statics cleaner by preventing corner solutions to depend on the realizations of the private information parameters θ_1 and θ_2. Finally, notice that the marginal cost of failing short from the needs of the motor skill system depends on β_0. We can then use this parameter to scale down (or even make arbitrarily small) the importance of task 0 in the problem. Using (R) to express x_0 as a function of x_1 and x_2, inserting this expression in $\Pi_0(\cdot)$ and using the quadratic formulation for $\Pi_l(\cdot)$, we can rewrite the problem as:

$$\mathcal{P}: \max_{D(\bar{x}_1(\theta_2), \bar{x}_2(\theta_1))} \int\int -\left[\frac{1}{\beta_1}\left(x_1(\theta_1, \theta_2) - \theta_1\right)^2 + \frac{1}{\beta_2}\left(x_2(\theta_1, \theta_2) - \theta_2\right)^2\right.$$

$$\left. + \frac{1}{\beta_0}\left(k - x_1(\theta_1, \theta_2) - x_2(\theta_1, \theta_2) - \theta_0\right)^2\right] dF^1(\theta_1) dF^2(\theta_2)$$

$$\text{s.t.} \quad x_1(\theta_1, \theta_2) \geq 0, \; x_2(\theta_1, \theta_2) \geq 0, \; x_1(\theta_1, \theta_2) + x_2(\theta_1, \theta_2) \leq k \quad \forall \theta_1, \theta_2 \qquad \text{(F)}$$

Notice that our initial problem with three systems (two with private needs, one with public needs) and free resources up to a fixed capacity constraint k can be formally rewritten as $\mathcal{P}$, a problem with only two systems (both with private needs) and where the sum of resources is increasingly costly and bounded above.[19] In the next section, we determine the optimal caps $\bar{x}_1(\theta_2)$ and $\bar{x}_2(\theta_1)$.

3.2 Priority mechanisms

In order to derive the $D(\bar{x}_1(\theta_2), \bar{x}_2(\theta_1))$ allocation mechanism that solves problem $\mathcal{P}$, it is instructive to study first a simpler 'priority mechanism.' Following Moulin (2000), let

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[17]We can even assume that more difficult tasks require more resources for coordination (formally, $\tilde{k}(\theta_1, \theta_2)$ with $\partial\tilde{k}/\partial\theta_i > 0$). All that matters is that different allocation *mechanisms* do not require different amounts of resources (otherwise, the revelation principle may not hold).

[18]Recall that system 0 can represent several systems with known needs.

[19]In other words, our problem would be solved in an identical manner under the (biologically less probable) assumption that total resources k are unbounded but there is a convex cost $c(k)$ to obtain them.

us call priority mechanism $\mathbf{P}_i$ a rule that first allocates to system i its desired needs θ_i (provided that $\theta_i \leq k$), and then divides the remaining resources $k - \theta_i$ optimally between systems 0 and j.[20] Assume the parameters of the model are such that all systems receive a positive amount of resources.[21] We summarize the characteristics of the optimal priority mechanism $\mathbf{P}_i$ in the following lemma.

Lemma 1 *The optimal priority mechanism $\mathbf{P}_i$ is given by $\bar{x}_i^{P_i}(\theta_j) = k$ for all θ_j and $\bar{x}_j^{P_i}(\theta_i) = y_j(\theta_i)$, where $y_j(\theta_i)$ is continuous, non-increasing and such that*

$$\frac{1}{\beta_j}\Big(E[\theta_j \,|\, \theta_j \geq y_j(\theta_i)] - y_j(\theta_i)\Big) = \frac{1}{\beta_0}\Big(\theta_0 - [k - \theta_i - y_j(\theta_i)]\Big) \tag{5}$$

Furthermore, under Assumption 1, $y_j'(\theta_i) \in (-1, 0)$ whenever $0 < y_j(\theta_i) < k - \theta_i$.

Proof: See Appendix A2. □

When CES chooses to satisfy all the needs of system i, the only issue left is to decide on the split of the remaining resources $k - \theta_i$ between 0 and j. Given its restricted set of tools (and, in particular, its inability to provide some 'compensation' in exchange of truthful revelation of needs), the optimum takes the form of a resource cap $y_j(\theta_i)$ granted to system j. Needs below the cap are fully satisfied, whereas needs above the cap are constrained. The remaining resources $k - \theta_i - \min\{\theta_j, y_j(\theta_i)\}$ are assigned to system 0. As reflected by (5), the cap is uniquely determined using an intuitive marginal analysis argument. Conditional on system j being effectively constrained ($\theta_j > y_j$), the marginal benefit of allocating resources to system 0 (right-hand side of the equation, where needs are θ_0 and resources granted are $k - \theta_i - y_j$) must be equal to the *expected* marginal benefit of allocating resources to system j (left-hand side of the equation, where expected needs are $E[\theta_j \,|\, \theta_j \geq y_j]$ and resources granted are y_j). Last, a simple resource cap is actually the optimal priority mechanism even if we consider priority mechanisms that are not necessarily ex-post efficient.[22]

Under mild technical conditions (Assumption 1), if the needs of system i increase, the amount of resources allocated *both* to system 0 and to system j decrease (formally, $y_j'(\theta_i) \in (-1, 0)$). This property means, in particular, that both non-priority systems 0 and j are always better-off if the priority system i requests fewer resources. Finally, notice

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[20]Therefore, a $\mathbf{P}_i$-priority mechanism is equivalent to $\succsim$-priority mechanism in Moulin (2000) where systems j and 0 are in the same priority class and system i has priority over both of them.

[21]For all the mechanisms described in the paper, the appendix provides necessary and sufficient conditions for interior solutions to exist. It also characterizes the mechanisms when those conditions are not satisfied.

[22]The formal argument can be found in the proof of the Lemma 1.

that the priority mechanism $\mathbf{P}_i$ will be particularly inefficient from the viewpoint of CES when θ_i is large (since fewer resources will be available for the other systems independently of their needs) and when β_i is large (since it would be relatively inexpensive to constrain system i to an allocation below its needs).

3.3 Optimal resource allocation with unknown needs

With these premises in mind, we can now characterize the optimal allocation mechanism. Let us denote by $\mathbf{M}$ the $D(\bar{x}_1(\theta_2), \bar{x}_2(\theta_1))$ mechanism that solves problem $\mathcal{P}$. Proposition 1 provides a characterization of $\mathbf{M}$.

Proposition 1 (Characterization) *The optimal mechanism* $\mathbf{M}$ *is:*

$$\bar{x}_1^*(\theta_2) = \begin{cases} y_1(\theta_2) & if \quad \theta_2 < k_2 \\ k_1 & if \quad \theta_2 \geq k_2 \end{cases} \quad and \quad \bar{x}_2^*(\theta_1) = \begin{cases} y_2(\theta_1) & if \quad \theta_1 < k_1 \\ k_2 & if \quad \theta_1 \geq k_1 \end{cases}$$

where $y_j(\theta_i)$ *is given by (5), and* k_1 *and* k_2 *are such that:*

$$\frac{1}{\beta_1}\Big(E[\theta_1 \,|\, \theta_1 \geq k_1] - k_1\Big) = \frac{1}{\beta_2}\Big(E[\theta_2 \,|\, \theta_2 \geq k_2] - k_2\Big) = \frac{1}{\beta_0}\Big(\theta_0 - k_0\Big) \qquad (6)$$

with $k_0 \equiv k - k_1 - k_2$. *In other words,* $\mathbf{M}$ *follows* $\mathbf{P}_1$ *when* $\theta_1 < k_1$ *and* $\mathbf{P}_2$ *when* $\theta_2 < k_2$. *It allocates fixed amounts* (k_0, k_1, k_2) *to systems 0, 1 and 2 when* $\theta_1 \geq k_1$ *and* $\theta_2 \geq k_2$.

Proof: See Appendix A3. □

In what follows, we denote the equilibrium allocation by $x_1^*(\theta_1, \theta_2)$ and $x_2^*(\theta_1, \theta_2)$. The optimal mechanism $\mathbf{M}$ divides the space into four regions determined by the constants k_1 and k_2. If both cognitive systems request 'few' resources ($\theta_1 < k_1$ and $\theta_2 < k_2$) they each receive their desired amounts ($x_1^*(\theta_1, \theta_2) = \theta_1$ and $x_2^*(\theta_1, \theta_2) = \theta_2$). If system i requests 'few' resources and system j requests 'many' resources ($\theta_i < k_i$ and $\theta_j \geq k_j$), system i receives full priority and system j is constrained according to $\mathbf{P}_i$ (see (5)) ($x_i^*(\theta_i, \theta_j) = \theta_i$ and $x_j^*(\theta_i, \theta_j) = \min\{\theta_j, y_j(\theta_i)\}$). If both systems require 'many' resources ($\theta_1 \geq k_1$ and $\theta_2 \geq k_2$), they both receive fixed amounts ($x_1^*(\theta_1, \theta_2) = k_1$ and $x_2^*(\theta_1, \theta_2) = k_2$). In all cases, the remaining resources (if any) are allocated to system 0. The values k_1 and k_2 are given by the intersection of the cap functions $y_2(\theta_1)$ and $y_1(\theta_2)$. As reflected by (6), they correspond to the amounts where the marginal benefit of allocating resources to system 0 equals the expected marginal benefit of allocating them to either of the constrained systems 1 and 2. Naturally, $k_1 = k_2$ when the cognitive tasks are symmetric from the CES viewpoint (that is, when $F^1(\cdot) = F^2(\cdot)$ and $\beta_1 = \beta_2$). Figure 1 provides a graphical representation of the resources $(x_1^*(\theta_1, \theta_2), x_2^*(\theta_1, \theta_2))$ allocated to systems 1 and 2 under

M for every pair of needs (θ_1, θ_2). Since $k_1 + k_2 < k$, the resources $x_0^*(\theta_1, \theta_2) = k - x_1^*(\theta_1, \theta_2) - x_2^*(\theta_1, \theta_2) > 0$ are going to system 0.
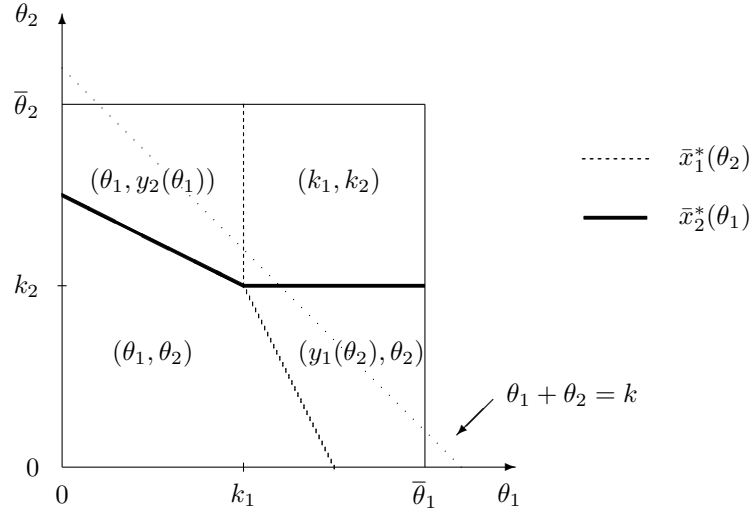

Figure 1. Allocation (x_1^*, x_2^*) as a function of needs (θ_1, θ_2) in the optimal mechanism **M**.

The intuition for the optimality of **M** is as follows. By the very definition of a "priority mechanism", the cap on the resources granted to system j in the optimal mechanism **M** has to be *at least* as high as the cap in the mechanism $\mathbf{P}_i$ that gives priority to the other system (formally, $\bar{x}_j^*(\theta_i) \geq \bar{x}_j^{P_i}(\theta_i) = y_j(\theta_i)$ for all θ_i). Suppose now that, following $\mathbf{P}_2$, $\bar{x}_1^*(\theta_2) = y_1(\theta_2)$ for all $\theta_2 < k_2$. It means that system 2 can be unconstrained in that interval or, equivalently, that $\bar{x}_2^*(\theta_1) = k_2 \; (> y_2(\theta_1))$ is achievable for all $\theta_2 < k_2$. Using a symmetric reasoning for $\bar{x}_2^*(\theta_1)$ when $\theta_1 < k_1$, we conclude that **M** is a feasible mechanism. However, it does not prove its optimality. One could think that for some $\theta_2 < k_2$ improvements could be made by setting a cap $\tilde{x}_1(\theta_2) > y_1(\theta_2)$ at the expense of $\tilde{x}_2(\theta_1) < k_2$. This, however, is not possible. Indeed, whenever the constraint of system 1 is binding $(\tilde{x}_1(\theta_2) < \theta_1)$, its allocation $x_1^*(\theta_1, \theta_2)$ cannot depend on its needs θ_1. But then, using the first order condition of system 2, it implies that the allocation of system 2 cannot depend on θ_1 either, which contradicts the possibility of a cap $\tilde{x}_2(\theta_1) < k_2$.

Notice that under mechanism **M**, system $l \in \{0, 1, 2\}$ has a minimum amount of guaranteed resources k_l. If the needs of a system exceed the amount allocated, extra resources can be obtained, but only if one of the other systems does not exhaust its quota. The optimal mechanism **M** has two important properties that we describe below.

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Implication 1 *If needs are public, the individual will always under-perform in the cognitive tasks. If needs are private, the individual will perform flawlessly in simple cognitive tasks and severely under-perform in difficult cognitive tasks.*

Recall that under complete information, each system always receives fewer resources than its actual needs (see (2)). This implies that the individual will always under-perform in tasks 1 and 2. Under asymmetric information, on the contrary, each system has a chance to be granted all its needs. Therefore, the individual will perform cognitive tasks flawlessly as long as they are simple enough. The result is illustrated in Figure 2. It represents the allocation of resources to system i under complete information (dotted line) and incomplete information (full line) as a function of its needs θ_i and for a given announcement θ_j of the needs by system j. The result implies in particular that flawless decision-making is *not* consistent with the complete information model (except in the uninteresting case where total resources exceed maximum needs). By contrast, the incomplete information model captures the possibility that an individual may make correct decisions even under multi-tasking.
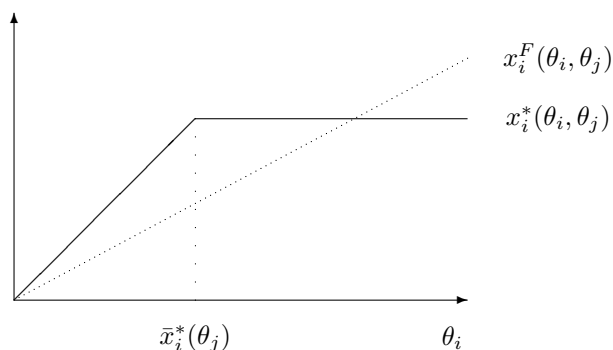
Figure 2. Allocations x_i^F and x_i^* as a function of the needs θ_i.

We can also see from Figure 2 that, as the needs of system i increase, the allocation obtained by that system under $\mathbf{M}$ is fully responsive up to a level $(dx_i^*/d\theta_i = 1)$, and non-responsive afterwards $(dx_i^*/d\theta_i = 0)$. This is to be contrasted with the full information case in which the allocation increases always. Thus, there exists a cutoff value such that, compared to the full information case, a system with private information will obtain more resources and therefore perform better when its needs are below the cutoff, that is, when the task is easy. Conversely, it will obtain less resources and therefore perform worse when its needs are above the cutoff, that is, when the task is difficult. The testable implication

of the asymmetric information model is that, fixing the difficulty of one task and varying the difficulty of the other, the individual should perform flawlessly in the latter up to a point and have a decreasing performance afterwards.

According to **M**, the relative performance across cognitive tasks is also qualitatively different under incomplete information. To make this point clear, we consider the case where both cognitive tasks are of equal importance ($\beta_1 = \beta_2$) and we fix the total amount of needs $\theta_0 + \theta_1 + \theta_2$.

Implication 2 *Consider a multi-task setting where both cognitive tasks are of equal importance ($\beta_1 = \beta_2$). If needs are public, the individual will perform equally in simple and in difficult tasks. If needs are private, the individual will perform better in simple tasks than in difficult tasks.*

When $\beta_1 = \beta_2$, under full information, the performance of a system depends on total needs but not on how these needs are distributed across systems. In other words, fixing a total amount of needs $(\theta_0 + \theta_1 + \theta_2)$, we have $\theta_1 - x_1^F = \theta_2 - x_2^F$ and the relative performance of systems 1 and 2 is the same (see (3)). Under private information and again for a given total amount of realized needs, the system with smallest needs (doing the easy task) will obtain relatively more resources than the system with the highest needs (doing the difficult task). That is, if $\theta_1 < \theta_2$, $\theta_1 - x_1^*(\theta_1, \theta_2) < \theta_2 - x_2^*(\theta_1, \theta_2)$. Therefore, the system performing the easy task will perform better than the system performing the difficult task. This asymmetry between simple and difficult tasks also constitutes a testable implication of the theory. If the individual must perform two tasks of different levels of difficulty, the model predicts that the individual will perform relatively better the easier task. This prediction matches the neuroscience evidence (D'Esposito et al. (1995)).[23]

3.4 Comparative statics

In this section we study how the allocation mechanism is affected by changes in the resources available and the relative importance of the systems. Consider the optimal mechanism **M** given the parameters $(\beta_0, \beta_1, \beta_2, \theta_0, k)$. We have the following result.

Proposition 2 (Comparative statics) *The resources $x_l^*(\theta_1, \theta_2)$ allocated to system l ($\in \{0, 1, 2\}$) (weakly) increase if β_l decreases or if β_{-l} or k increases. Also, $x_i^*(\theta_1, \theta_2)$ decreases and $x_0^*(\theta_1, \theta_2)$ increases if θ_0 increases.*

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[23]Under more general functional forms of $\Pi(\cdot)$ it may not be the case that performance under full information is identical in the easy and difficult tasks. However, it will still be true that asymmetric information exacerbates the difference in performance.

<u>Proof</u>: See Appendix A4. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ □

Under fairly general conditions, the comparative statics follow a general resource mono-tonicity principle which can be summarized as *"abundance is shared and relative impor-tance is compensated."* If a system becomes more valuable for CES (due, for example, to an increase in the marginal cost of under-performance), it receives more resources at the expense of *both* the other systems. Conversely, if new resources become available, then *all* systems benefit from this surplus.[24]

Changes in β_0 are particularly illuminating. If shortages in the amount granted to the motor skill task become less and less costly (β_0 increases), systems 1 and 2 receive more resources ($y_i(\theta_j)$ increases for all θ_j). They also become less sensitive to each other's demands ($|y_i'(\theta_j)|$ decreases for all θ_j), because higher needs of system j come more at the expense of system 0 and less at the expense of system i. Eventually, x_0^* hits the non-negativity constraint. Once this occurs, system 0 receives no resources and the problem reduces to allocating a fixed amount k between systems 1 and 2.

The optimal mechanism $\mathbf{M}$ and the comparative statics can be illustrated with the following stylized analytical example.[25]

Example 1 (Uniform) Suppose that $\theta_i \sim \mathrm{U}[0, \overline{\theta}_i]$. Let $\theta_0 = k$. Using (5)-(6), we get:

$$y_i(\theta_j) = \frac{\beta_0 \overline{\theta}_i}{\beta_0 + 2\beta_i} - \frac{2\beta_i}{\beta_0 + 2\beta_i}\theta_j \quad \text{and} \quad k_i = \frac{\beta_0 \overline{\theta}_i + 2\beta_j \overline{\theta}_i - 2\beta_i \overline{\theta}_j}{\beta_0 + 2\beta_i + 2\beta_j}$$

where the slope of the cap function is constant: $y_i'(\theta_j) = -\mu_i \equiv -\frac{2\beta_i}{\beta_0 + 2\beta_i} \in (-1, 0)$.

Naturally, one may wonder how much efficiency is lost under the optimal mechanism $\mathbf{M}$ relative to the full information case, and also how much is gained under $\mathbf{M}$ relatively to some simpler priority mechanisms. We address this question with the following analytical example.

Example 2 (Utility loss) Suppose that $\theta_l \sim \mathrm{U}[0, 1]$, $\beta_l = 1$ for all l and $\theta_0 - k \equiv r \in [0, 1/2]$. The expected utility under first best (Full Information), second-best (Mechanism $\mathbf{M}$), a mechanism that gives priority to i, then j, then 0 (Priority i-j-0) and a mechanism

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[24]As demonstrated in Appendix A2 and A3, Assumption 1 is a sufficient condition so that the cap in resources for the non-priority system decreases at a rate slower than the increase in the needs of the priority system (formally, $y_j'(\theta_i) \in (-1, 0)$). This immediately implies the resource monotonicity property. Moreover, it also ensures that $y_2(\theta_1)$ and $y_1(\theta_2)$ intersect at most once, and therefore that an equilibrium where every system receives a positive amount of resources is necessarily unique.

[25]The algebraic details in Examples 1 and 2 are omitted for brevity but are available from the authors.
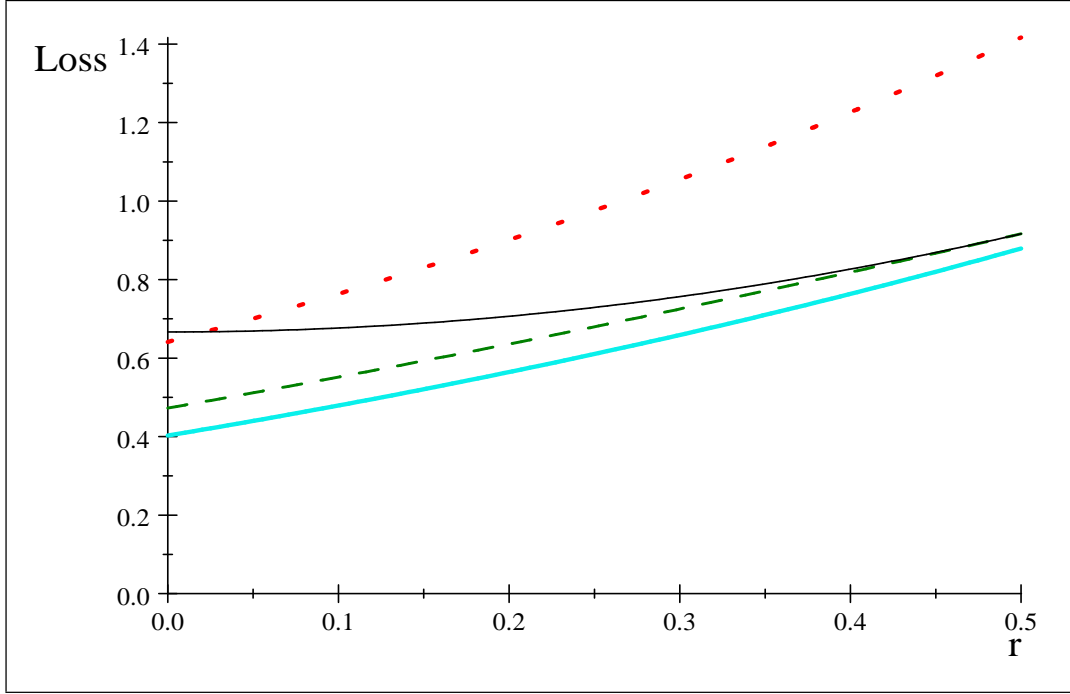
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Figure 3. Utility loss under full information (light thick line), mechanism $\mathbf{M}$ (dashed line), Priority i-j-0 (dotted line) and Priority 0-i-j (dark thin line).

that gives priority to 0, then i, then j (Priority 0-i-j) as a function of r are given by:[26]

Full information	Mechanism $\mathbf{M}$	Priority i-j-0	Priority 0-i-j
$-\frac{52r+30r^2+4r^3+5r^4+29}{72}$	$-\frac{5132r+1974r^2-592r^3+176r^4+3191}{6750}$	$-\frac{496r+312r^2+64r^3+16r^4+277}{432}$	$-\frac{3r^2+2}{3}$

These utility losses are graphically represented in Figure 3.

When $r = 0$, $\mathbf{M}$ performs substantially better than other simple mechanisms: the utility loss under $\mathbf{M}$ is 20% bigger than under first-best (and it decreases with r), whereas the utility loss under either priority mechanisms is 60% bigger than under first-best. As r increases, the inefficiency of Priority i-j-0 increases whereas the inefficiency of Priority 0-i-j decreases simply because, other things being equal, higher needs of system 0 relative

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[26]We restrict attention to $r \geq 0$ to ensure that resources are scarce and to $r \leq 1/2$ so that system 0 is not overwhelmingly important (for instance when $r = 1$ Full information, Mechanism $\mathbf{M}$ and Priority 0-i-j perform identically because all three mechanisms allocate all the resources to system 0, the system with greatest needs).

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to systems 1 and 2 (i.e., higher r) implies that more resources should be granted to that system.

3.5   Implementation

The solution described in Proposition 1 represents a normative upper bound on the efficiency of the resource allocation problem. A direct revelation mechanism where each system 'communicates' its needs truthfully given the 'commitment' by CES to split resources following a pre-specified rule is nothing but an abstract formalization of the problem. Indeed, although systems are unable to literally send messages (truthful or not) to CES, they can still signal their needs through consumption. As discussed in Appendix A1, the direct revelation and signal-through-consumption mechanisms can be formally equivalent. However, that approach is still fairly abstract. The purpose of this section is to determine whether the efficient allocation rule can be reached using a simple and biologically plausible process. Assume that the tasks must be completed between time 0 and time k, and only one unit of resources is delivered at each instant.

Proposition 3 (Implementation) M *can be implemented with the following procedure.*

(i) CES sends resources to system l at a rate $r_l = k_l/k$.

(ii) As long as i and j consume resources, the flow rates (r_0, r_1, r_2) are maintained. If i stops consuming at time $\tilde{t}$ (where $r_i \tilde{t} = \theta_i$), then resources are redirected to j and 0 at revised rates ν_j and $(1 - \nu_j)$, such that $r_j \tilde{t} + \nu_j(k - \tilde{t}) = y_j(r_i \tilde{t})$.

(iii) If both i and j stop consuming, all the remaining resources are redirected to 0.

<u>Proof</u>: Trivial to check, thus omitted.   □

The mechanism follows the biological principles highlighted in the introduction. CES sends resources simultaneously to the systems in charge of performing tasks. The systems deplete the resources, which allow neurons to fire. Depletion is (correctly) interpreted by CES as a signal that more resources are needed. The process is dynamic but extremely fast. If one system stops consuming (i.e., neurons stop firing), no further resources are sent to it. This autoregulation mechanism is extremely simple for systems: it just requires them to grab any incoming resources until they are satiated (i.e, until the task is performed at full satisfaction). It means in particular that, for the optimal mechanism to work, each system's knowledge about the existence and needs of other systems is virtually nil. In fact, systems do not even need to know their own needs at any point in time, only whether an extra resource is valuable or not. On the other hand, the mechanism requires a certain degree of sophistication by CES, which must be able to select different flow rates for different systems and be ready to redirect resources when some needs are satiated. We

hypothesize that the activity measured in the LPFC in the dual-task experiments reviewed in section 1.1 (Szameitat et al. (2002) and others) captures precisely this extra top-down involvement of CES in the coordination and allocation of resources. This also suggests the existence of a mechanism linking CES to the vascular delivery system. It would be interesting to design experiments to test that hypothesis, as the existing literature remains vague as to how CES is involved. [27]

4   Task inertia and performance improvements

In this section we study the sequential allocation of resources. To this purpose consider the following extension of the basic model. Suppose that CES has imperfect knowledge of the distribution $F^i(\cdot)$ from which the needs of system i ($\in \{1,2\}$) are drawn. More precisely, there is an underlying state $s_i \in S_i = [\underline{s}_i, \overline{s}_i]$ that determines the distribution of needs for system i. For example, suppose the individual performs an auditory comprehension task which is often (though not always) complex. Then, the auditory system will often (though, again, not always) require substantial resources. This is formally captured by an underlying state s_i that places high probability on auditory needs being large.

We order the states from highest likelihood of small needs to highest likelihood of large needs, and assume that a (strict) Monotone Likelihood Ratio Property (MLRP) holds.[28]

Assumption 2 (MLRP) $\dfrac{d}{d\theta_i}\left(\dfrac{f^i_{s_i}(\theta_i|s_i)}{f^i(\theta_i|s_i)}\right) > 0 \quad \forall\ i,\ \theta_i,\ s_i.$

According to this assumption, needs increase (in a stochastic sense) as we move towards a higher state. Stated differently, the state s_i is a parameter that captures how complex the task is, and therefore how important the needs are likely to be. When the individual performs tasks only once, the problem is identical to the one studied previously, as CES is not interested in states *per se* but only as a way to identify more accurately the needs of systems. To see this, suppose the state s_i is drawn from a known distribution $P^i(s_i)$ with density $p^i(s_i)$, and that states are independent across systems ($p^i(s_i|s_j) = p^i(s_i)$ for

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[27]Evidence suggests that CES is involved in the allocation and coordination of "attentional resources" (see D'Esposito et al. (1995)). Our working hypothesis in this section is that attentional resources are closely related to metabolic resources. As such our mechanism requires CES to have a direct or indirect relationship to the hemoglobin delivery system.

[28]Subscripts in c.d.f. or density functions denote partial derivatives with respect to that argument.

all s_j).[29] The probability that system i has needs θ_i is:

$$g^i(\theta_i) = \int_{\underline{s}_i}^{\overline{s}_i} f^i(\theta_i|s_i)p^i(s_i)ds_i \tag{7}$$

We can then perform the very same analysis as before where $f^i(\cdot)$ is replaced by $g^i(\cdot)$.

The problem becomes more interesting when the individual performs the same set of tasks in consecutive periods. We assume that s_i remains constant over time to obtain clear-cut solutions. At each date t, the needs of system i are, conditional on the state s_i, drawn independently according to $F^i(\theta_i|s_i)$. The past realization of needs then conveys information about the state, which itself informs about the distribution of present needs. Formally and applying Bayes rule, the probability that the needs of system i at date t are θ_i^t given that its needs at date $t-1$ were θ_i^{t-1} is:

$$g^i(\theta_i^t|\theta_i^{t-1}) = \int_{\underline{s}_i}^{\overline{s}_i} f^i(\theta_i^t|s_i)p^i(s_i|\theta_i^{t-1})ds_i = \frac{\displaystyle\int_{\underline{s}_i}^{\overline{s}_i} f^i(\theta_i^t|s_i)f^i(\theta_i^{t-1}|s_i)p^i(s_i)ds_i}{\displaystyle\int_{\underline{s}_i}^{\overline{s}_i} f^i(\theta_i^{t-1}|s_i)p^i(s_i)ds_i} \tag{8}$$

The following lemma is a key step for our subsequent analysis.

Lemma 2 *Under Assumption 2,* $\dfrac{d}{d\theta_i^t}\left(\dfrac{g^i_{\theta_i^{t-1}}(\theta_i^t\,|\,\theta_i^{t-1})}{g^i(\theta_i^t|\theta_i^{t-1})}\right) > 0 \quad \forall\ i,\ \theta_i^t,\ \theta_i^{t-1}.$

Proof: See Appendix A5. □

According to Lemma 2, *MLRP begets MLRP*: if the individual experiences high needs at some date, it means that the state is likely to be high (in an MLRP sense), and therefore that needs are likely to be high also in the future (again, in an MLRP sense).

The dynamic allocation of needs in the framework developed above has some new features. Suppose that, at the end of the period, CES learns what the needs of each system were at that date. This occurs for example if the performance $\Pi_i(\cdot)$ of system i is observed after system i has performed the task.[30] Then, the needs reported by systems at some date affect current allocations but not future allocations. Hence, independently of whether system i is myopic (most likely) or forward-looking (least likely), it will 'communicate' its needs in order to optimize exclusively its present allocation. The mechanism **M** developed

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[29] If states are not independent, then θ_i and θ_j are correlated. The optimal mechanism must then exploit this correlation, as it is well known in the mechanism design literature.

[30] For instance the individual receives feedback about its performance which is interpreted by CES.

in Proposition 1 as well as the implementation procedure described in Proposition 3 remain optimal at each date t, where $f^i(\theta_i)$ is simply replaced by $g^i(\theta_i^t \mid \theta_i^{t-1})$ updated using the posterior $p^i(s_i \mid \theta_i^{t-1})$. This mechanism, however, has new interesting properties.

Proposition 4 (Task inertia) *The resources* $x_{it}^*(\theta_1^t, \theta_2^t)$ *allocated to system* i *at date* t *(weakly) increase if* θ_i^{t-1} *increases or if* θ_j^{t-1} *decreases.*

<u>Proof</u>: See Appendix A6. □

The idea is simple. If CES realizes that the needs of system i in the previous period were high, it concludes that state s_i is likely to be high which, other things being equal, shifts the updated distribution of system i's future needs towards high values. As a result, it becomes optimal to grant more resources to system i in the current period, that is, to set a higher cap. Given our resource monotonicity principle, the more generous allocation to system i comes at the expense of both systems 0 and j.

Using Proposition 1, we can then compare two models, depending on whether at the beginning of date t the needs (θ_1^t, θ_2^t) of systems 1 and 2 are known by CES (the 'public information' case, as in section 2) or unknown by CES (the 'private information' case, as in section 3). In both models, the underlying state is unknown, so there is learning over time about s_i, and therefore about the distribution $f^i(\theta_i|s_i)$ from which needs are drawn.[31] These two models yield two different implications for observed behavior that are summarized below.

Implication 3 *In a dynamic multi-task setting, the expected performance is constant under public information and improves over time under private information.*

With public information, the resource allocation of CES does not depend on his assessment of the state. Nevertheless, higher states lead to higher needs and therefore lower expected performance whereas lower states leads to lower needs and therefore higher expected performance (see equation (3)). From the perspective of date 0, however, learning about the state may go in either direction. With private information, a new effect is in place. Over time, CES learns about s_i through the realization of θ_i. This reduces the information asymmetry between CES and system i, which results in an improved expected performance.[32]

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[31]Public information refers to the fact that, at the beginning of date t, the needs (θ_1^t, θ_2^t) are known whereas the needs at future dates are not.

[32]To be precise, improved expected performance follows from two observations. First, the requirement of dominant strategy incentive compatibility does not depend on the common beliefs on the distribution of needs. Thus, every mechanism that is feasible and DSIC remains so after the CES updates his belief

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This result is in line with experimental evidence. Subjects tend to adapt their behavior and improve their performance in the presence of feedback about performance even when there is nothing to 'learn' about the characteristics of the task. This observation combined with our analysis suggests again that the asymmetric information model better captures the observed behavior.

Implication 4 *In a dynamic multi-task setting, the allocation rule exhibits path-independence under public information and path-dependence under private information.*

With public information, present needs are a sufficient statistic to determine the optimal allocation. With private information, the allocation mechanism depends on the distribution from which needs are drawn: higher past needs of system i reflects a higher likelihood of present needs inducing a more favorable treatment by CES through a higher consumption cap. This more favorable treatment translates into a higher performance of system i at the expense of systems j and 0.

Task inertia and a path-dependent performance is a particularly interesting result in the light of the recent neuroscience research. Indeed, suppose that for the first few periods the task performed by system i is more complex than the task performed by system j. Not surprisingly, resources are primarily directed to system i. Suppose now that, at some point, there is a reversion in complexity. There is substantial fMRI evidence of residual activity right after the change in the previously crucial but now unimportant system i. Conversely too few resources are allocated to the previously unimportant but now crucial system j following the reversion in complexity. This misallocation vanishes after a few periods. Behaviorally, it translates into a short-term lowered performance (slower response and more mistakes) in the task for which system j is responsible (Wylie and Allport (2000), Monsell (2003), Yeung et al. (2006)). Neuroscientists argue that this phenomenon is due to a "task inertia" or a "task switching cost." However, the evidence on the existence of such cost is not accompanied by an understanding of where it comes from and why it vanishes rapidly. In order to generate this effect in a model with public information, we would need to impose some ad-hoc cost of adaptation. Perhaps more satisfactorily, inertia and a path-dependent allocation arise very naturally when critical resources need to be allocated to competing tasks under private information.

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over the state s_i. Second, since for any belief the CES has available the same direct mechanisms, it cannot do worse when it has additional information on the state and systems act myopically. Formally, letting D be the set of feasible and DSIC mechanisms:

$$E\left[\max_D E\left[\Pi_{CES} \mid I\right]\right] \geq \max_D E\left[\Pi_{CES}\right].$$

The result is also related to 'priming', an effect widely documented in psychology. According to this literature, making some identity features salient has a short-lived but significant impact on the behavior of individuals (see e.g. Wegner and Bargh (1998)). A reinterpretation of our model may provide some foundations for this effect. Indeed, information regarding identity traits triggers activity in certain systems which remain operational for a short period of time when the individual subsequently performs other unrelated tasks, hence generating the inertia.

Last, the biologically plausible mechanism discussed in Proposition 3 has a natural implication in the multi-period framework.

Implication 5 *In a dynamic multi-task setting, the biologically plausible mechanism predicts that the initial rate of resources sent to system i at date t increases when the needs of system i at $t-1$ increase. Furthermore, if in equilibrium needs of system i are satiated $(\theta_i^t < \bar{x}_{it}^*(\theta_j))$, the individual completes that task quicker the higher the past needs.*

It suffices to apply Proposition 3 to each period. Interestingly, the rates r_i and ν_i will change over time as a function of learning. It comes immediately from Proposition 4 that the initial rates allocated to task i at date t increase if the needs θ_i^{t-1} to complete task i at date $t-1$ increase and if the needs θ_j^{t-1} to complete task j at date $t-1$ decrease. More resources are sent if the task is expected to be more difficult given the feedback obtained. This also implies that the task should be completed quicker at date t as more resources are available more rapidly, providing another testable implication of the theory.

5 The architecture of brain systems: integration vs. specialization

So far, we have assumed that each system performs exactly one task. In reality, systems are responsible for multiple tasks and tasks require the coordination of multiple systems. There are numerous reasons for such an organization of the brain. In this section, we focus on a specific aspect that builds on the core premise of our theory: restricted communication channels. More precisely, we study from an informational viewpoint the trade-off between *integrating* the two cognitive tasks into one system vs. *specializing* systems into performing one cognitive task each.

Specialization corresponds to the case already analyzed in section 3. To study integration, we assume that one system, denoted by I, performs tasks 1 and 2. More precisely, system I has access to the same information and performs the same tasks as the previous systems 1 and 2. For transparency, we assume that this system puts the same relative

weight on each task as the CES. Formally, I is endowed with the following performance function:

$$\Pi_I(x_1, x_2; \theta_1, \theta_2) \equiv \Pi_1(x_1; \theta_1) + \Pi_2(x_2; \theta_2) = -\frac{1}{\beta_1}(x_1 - \theta_1)^2 - \frac{1}{\beta_2}(x_2 - \theta_2)^2$$

Under integration, CES can only choose the total resources that are allocated to tasks 1 and 2. System I knows the relative needs in tasks 1 and 2 and decides how to split the resources between the tasks. Following (2), system I with needs (θ_1, θ_2) who is granted resources k' ($< \theta_1 + \theta_2$) splits them according to:[33]

$$
\begin{aligned}
\hat{x}_1(k'; \theta_1, \theta_2) &= \theta_1 - \frac{\beta_1}{\beta_1 + \beta_2}(\theta_1 + \theta_2 - k') \\
\hat{x}_2(k'; \theta_1, \theta_2) &= \theta_2 - \frac{\beta_2}{\beta_1 + \beta_2}(\theta_1 + \theta_2 - k')
\end{aligned}
\tag{9}
$$

His equilibrium performance given resources k' is then:

$$\Pi_I(\hat{x}_1, \hat{x}_2; \theta_1, \theta_2, k') = -\frac{1}{\beta_1 + \beta_2}\left(\theta_1 + \theta_2 - k'\right)^2$$

Notice that the performance of the integrated system, $\Pi_I(\cdot)$, depends only on the sum of needs $\theta_1 + \theta_2$. It follows that any mechanism in which CES allocates total resources to system I can only separate the needs in tasks 1 and 2 based on the one dimensional statistic $\theta_1 + \theta_2$. The overall performance from the viewpoint of CES if system I receives k' resources and splits them according to (9) and system 0 receives the remaining $k - k'$ resources is:

$$\Pi_{CES} = -\frac{1}{\beta_0}\left(\theta_0 - (k - k')\right)^2 - \frac{1}{\beta_1 + \beta_2}\left(\theta_1 + \theta_2 - k'\right)^2 \tag{10}$$

Our next lemma provides a characterization of the optimal mechanism $\mathbf{I}$ under integration of tasks 1 and 2 by system I.

Lemma 3 *The optimal integration mechanism* $\mathbf{I}$ *is such that CES sets a fixed cap* $\hat{k}$ *on the total resources allocated to system I. The optimal cap* $\hat{k}$ *satisfies:*

$$\frac{1}{\beta_1 + \beta_2}\left(E[\theta_1 + \theta_2 \,|\, \theta_1 + \theta_2 \geq \hat{k}] - \hat{k}\right) = \frac{1}{\beta_0}\left(\theta_0 - [k - \hat{k}]\right) \tag{11}$$

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[33]More precisely, system I allocates resources according to (9) as long as $k' \geq \max\{\theta_2 - \beta_2\theta_1/\beta_1, \theta_1 - \beta_1\theta_2/\beta_2\}$. This condition ensures that it is optimal for system I to allocate a positive amount of resources to both tasks. If the condition is violated, however, system I will assign all resources to one task: to task 1 if $k' < \theta_2 - \beta_2\theta_1/\beta_1$ and to task 2 if $k' < \theta_1 - \beta_1\theta_2/\beta_2$.

<u>Proof</u>: See Appendix A7. □

The optimal resource cap in the integration mechanism $\mathbf{I}$ is determined in a similar way as the optimal caps in the priority mechanism $\mathbf{P}_i$ and the specialization mechanism $\mathbf{M}$ (see (5), (6) and (11)). Indeed, from the overall performance function Π_{CES} described in (10), we notice that conditional on the required resources exceeding the cap, (11) equalizes the expected marginal benefit for CES of allocating one extra unit of resources to system I with the marginal benefit of allocating that extra unit to system 0.

We can then compare the performance from CES viewpoint of the specialization mechanism $\mathbf{M}$ and the integration mechanism $\mathbf{I}$ as a function of the importance attached by CES to each of the tasks. For notational convenience, we perform the following change of variables: $\frac{1}{\beta_0} \equiv \frac{1-\gamma}{\beta_0'}$ and $\frac{1}{\beta_i} \equiv \frac{\gamma}{\beta_i'}$ with $\gamma \in [0,1]$. The new parameter γ captures the importance of the two cognitive tasks relative to the motor skill task in the overall performance function. We also focus on the case where $\bar{\theta}_1 + \bar{\theta}_2 = k$. We impose this assumption to be in the interesting situation where integration and specialization yield identical performance if either $\gamma = 0$ or $\gamma = 1$. In the former case, only the motor skill task matters and optimality requires $x_0 = k$. In the latter case, only the cognitive tasks matter and first best can be achieved by setting $x_1 = \theta_1$ and $x_2 = \theta_2$.[34] Which brain architecture dominates when $\gamma \in (0,1)$ depends on the informational trade-off. The result is summarized below.

Proposition 5 (Brain architecture) *Assume that $f_1(\bar{\theta}_1)f_2(\bar{\theta}_2) > 0$. There exist $\underline{\gamma}$ and $\overline{\gamma}$ with $0 < \underline{\gamma} \leq \overline{\gamma} < 1$ such that, from the viewpoint of CES, integration dominates specialization for all $\gamma \in (0, \underline{\gamma})$ and specialization dominates integration for all $\gamma \in (\overline{\gamma}, 1)$.*

<u>Proof</u>: See Appendix A8. □

From a pure informational perspective, integration has both benefits and costs for CES. On the one hand, given a fixed amount of resources allocated to the cognitive tasks, the ability of the integrated system to split them efficiently between tasks 1 and 2 is beneficial for CES. This occurs because, for a given amount of total resources granted to tasks 1 and 2, both CES and system I would follow the same splitting rule. Moreover, this splitting rule is undertaken by system I under full information of needs. On the other hand, under integration CES is unable to make the resources granted to task i contingent on the needs in task j. In other words, the direct revelation mechanism under integration allows CES

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[34]If we assume $\bar{\theta}_1 + \bar{\theta}_2 > k$, then integration dominates specialization when $\gamma = 1$ (and therefore also when $\gamma \to 1$) but for ad-hoc reasons. Indeed, when $\gamma = 1$, *all* the resources are directed to tasks 1 and 2. Since resources are sometimes scarce ($\theta_1 + \theta_2 > k$ for some (θ_1, θ_2)), it is efficient to have system I (which by assumption knows the relative needs in those tasks) deciding how to split k between the two.

to learn only about the sum of needs in the cognitive tasks whereas under specialization CES learns about each of the needs separately. This information loss is costly.

According to Proposition 5, the benefits of specialization offset those of integration when the importance of the cognitive tasks is high relative to the motor skill task ($\gamma > \overline{\gamma}$). The benefits of integration offset those of specialization when the importance of the cognitive tasks is low relative to the motor skill task ($\gamma < \underline{\gamma}$).[35] Intuitively, when the cognitive tasks are important, the majority of resources are allocated to tasks 1 and 2. It then becomes relatively more valuable to get extra information about the needs in each of the tasks in order to determine how much to grant to task 0. This is obtained through specialization. Conversely, when the motor skill task is very valuable, most of the resources are allocated to system 0. At the margin, it is then important to optimize the (few) resources granted to the cognitive tasks, and this is achieved through integration.

Figure 4 depicts the equilibrium allocation as a function of needs under integration (bold line – mechanism $\mathbf{I}$). The allocation under specialization is the same as in Figure 1 and is superimposed in the graph (dotted lines – mechanism $\mathbf{M}$). Two differences between the mechanisms deserve emphasis. First, under mechanism $\mathbf{I}$ the amount of resources consumed in task i ($\in \{1,2\}$) are $x_i^{**}(\theta_1, \theta_2) = \theta_i$ if $\theta_1 + \theta_2 \leq \hat{k}$ and $x_i^{**}(\theta_1, \theta_2) = \hat{x}_i(\hat{k}; \theta_1, \theta_2)$ if $\theta_1 + \theta_2 > \hat{k}$. It means that, in equilibrium, the needs in either none or both cognitive tasks are constrained. This contrasts with mechanism $\mathbf{M}$ which has four regions so that, for some parameters, the needs of one and only one task are constrained. Second and by construction, under mechanism $\mathbf{I}$ the cap is set on total resources for the cognitive tasks, so lower needs in task i do not result in spillovers for task 0 when $\theta_1 + \theta_2 > \hat{k}$. Again, this contrasts with mechanism $\mathbf{M}$, where lower needs by system i result in more resources for both system j and system 0.

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[35]We show for a parametric example with $\theta_i \sim U[0,1]$ and $\beta_1 = \beta_2$ that $\underline{\gamma} = \overline{\gamma}$. Unfortunately, the uniqueness of the cutoff does not generalize (the example involves substantial algebra and is therefore omitted for brevity but it is available upon request), so, in general, we cannot determine what happens when $\gamma \in [\underline{\gamma}, \overline{\gamma}]$.
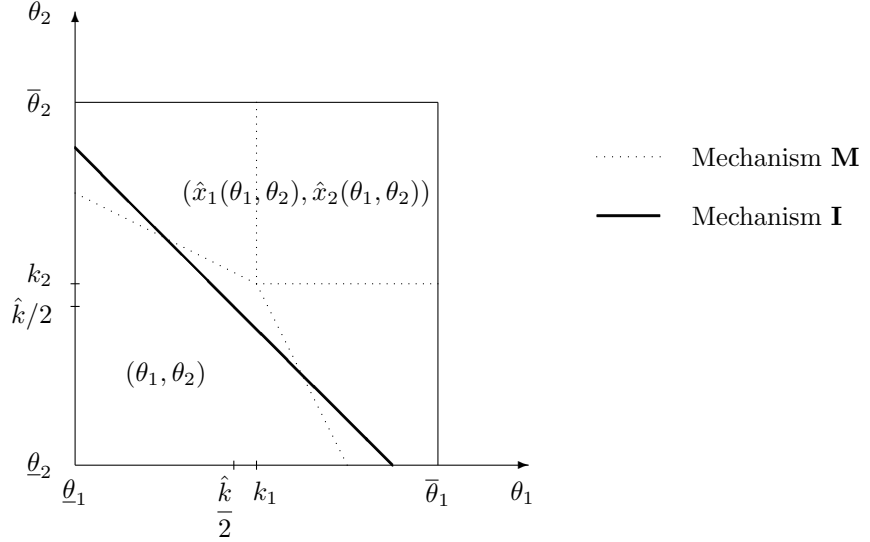
Figure 4. Optimal allocation under integration (**I**) and specialization (**M**).

An immediate implication, or rather an intuitive reformulation of Proposition 5 is summarized below.

Implication 6 *Different cognitive tasks should be performed by different systems if the tasks are crucial for CES and by the same system if they are not.*

Finally, note that integration is necessarily associated with a tendency to under-perform in all cognitive tasks if the amount of resources allocated to the integrated system is insufficient. This is the case because the allocation mechanism is done as if information were public. Observing flawless decision-making in simple tasks and asymmetric performance in easy versus difficult tasks indicates tasks are likely to be separated. This property could a priori be tested.

6   Discussion and conclusions

Incorporating choice imperfections in the decisions of agents has become central to behavioral economics. Observed behaviors or "outputs" such as empirical evidence, experimental data or sometimes mere introspection have been the main source of inspiration for models of bounded rationality. The premise of the present research is that "inputs" such as physiological constraints in our ability to perceive events, process information, and select

between options should also be used as building blocks for new theories of individual decision making. This paper follows this alternative route. It determines the constrained optimal allocation of resources to brain systems when multiple tasks are performed simultaneously. It shows that the optimal mechanism takes a resource cap structure and that it can be implemented using a physiologically plausible process. Some implications of the theory are discussed, most notably the inverse relation between task difficulty and performance, the endogenous emergence of task inertia and the conditions for the optimality of task integration.

Although our theory is strictly motivated by the neurobiology of the brain, the model can also be applied to more traditional areas of economics. For example, it can be straightforwardly reinterpreted as a manager in a firm whose objective is to allocate scarce funds between self-interested units (research, production, marketing, etc.) given private information of needs.[36] It can also capture the decision problem of colluding firms who decide how to split the market without using side transfers that would provide compromising evidence of their illegal activities. Other natural applications include provision of private goods to a group of individuals and lobbying activities in political contexts.

The paper also makes an observation (though, admittedly, tangential to its core objective) on the relationship between the literatures on mechanism design without transfers and axiomatic social choice. We demonstrate that, under IHR, resource monotonicity is a characteristic of the allocation rule that maximizes total welfare. This is related to but different from Barbera, Jackson and Neme (1997) who show that any allocation mechanism must necessarily have a cap structure (where the cap is non-increasing in the requests of other systems) if we impose resource monotonicity on top of strategy proofness and ex-post efficiency.

As a final point, economists often express reservations about the idea that brain systems may have competing goals. In particular, wouldn't it be more efficient if every subpart pursued the common good? There are evolutionary, physiological and empirical arguments against this common interest approach. First, the well-known neural Darwinism (Edelman, 1987) and neuronal selectionism (Changeux, 1985) theories provide evidence and models where neuronal groups within the brain compete with each other for stimulus and reward resources.[37] Second, and paradoxically, a cooperative approach would require a greater

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[36]As mentioned in section 1.2, the literature on organizations has studied related questions. However, to our knowledge this problem with two actions and two agents with private information has not been treated. The implications for inertia in organizations and the trade-off integration vs. specialization of units within a firm are also potentially important in this application.

[37]Under this approach, biological evolution encourages fitness of the neuronal system, rather than fitness at a higher level (the individual) or a lower level (the gene). See also Tooby and Cosmides (1992) for an

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degree of brain connectivity and/or sophistication. Indeed, each system would have to be able either to 'communicate' its needs back to the central decision-maker or to perform a non-trivial marginal analysis and give up worthy resources whenever these are more valuable to other systems. Instead, the physiological evidence reviewed in section 1.1 points towards a lack of information flowing from systems to CES (possibly due to the scarcity of the energetically costly neural connections) and a simplistic 'deplete-until-satiation' behavior of neurons in the decision systems. These two features are consistent with the mechanism described in Proposition 3. Third, some of the empirical regularities discussed in the paper (the possibility of flawless behavior, the inverse relation between task complexity and performance, the prevalence of task inertia) arise naturally in our non-cooperative model with private information but would not be present in a model with common objectives.

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evolutionary theory of internal conflicts in changing environments and Livnat and Pippenger (2006) for a computational model showing the advantages of having modules with opposing objectives.

Appendix

A1. Interpreting the direct revelation mechanism.

Suppose there are two stages. In stage $\tau \in \{1, 2\}$, CES allocates a local budget to each system (y_1^τ, y_2^τ). A budget is a function of past "messages", which in our case corresponds to past "consumptions". We denote by m_i^τ the consumption of system i in stage τ.

In stage 2, the resources needed by system i are r_i^2 and its allocation is $y_i^2(m_1^1, m_2^1)$. It is optimal to consume exactly what is needed or, if this is not possible, to deplete the local budget:

$$m_i^2\Big(y_i^2(m_1^1, m_2^1), r_i^2\Big) = \min\Big\{y_i^2(m_1^1, m_2^1), r_i^2\Big\}$$

In stage 1, the resources needed by system i are θ_i and its allocation is y_i^1. Consumption cannot exceed the allocation $(m_i^1 \leqslant y_i^1)$. If $\theta_i < y_i^1$, then it is (weakly) optimal to consume $m_i^1 = \theta_i$. If $\theta_i > y_i^1$, system i chooses m_i^1 and the ex-post utility is:

$$-\Big(m_i^1 + \min\Big\{y_i^2(m_1^1, m_1^2), \theta_i - m_i^1\Big\} - \theta_i\Big)^2$$

where stage 2 needs are replaced by total needs minus stage 1 consumption. We look for a solution in dominant strategies, that is:

$$-\Big(m_i^1 + \min\Big\{y_i^2(m_i^1, m_j^1), \theta_i - m_i^1\Big\} - \theta_i\Big)^2 \geqslant -\Big(\tilde{m}_i^1 + \min\Big\{y_i^2(\tilde{m}_i^1, m_j^1), \theta_i - \tilde{m}_i^1\Big\} - \theta_i\Big)^2$$

for all $m_i^1 \leqslant y_i^1$ and $\tilde{m}_i^1 \leqslant y_i^1$, yielding a solution $m_i^{1*}(\theta_i, y_i^1)$. At equilibrium, system i consumes $m_i^{1*}(\theta_i, y_i^1)$ in stage 1. He receives $y_i^2(m_i^{1*}(\theta_i, y_i^1), m_j^{1*}(\theta_j, y_j^1))$ in stage 2 and consumes $m_i^{2*}(\theta_i, \theta_j, y_i^1, y_j^1) = \min\big\{y_i^2(m_i^{1*}(\theta_i, y_i^1), m_j^{1*}(\theta_j, y_j^1)), \theta_i - y_i^1\big\}$. Total consumption is $m_i^{1*}(\theta_i, y_i^1) + m_i^{2*}(\theta_i, \theta_j, y_i^1, y_j^1)$ which, by construction, is less than $y_i^1 + y_i^2(m_i^{1*}(\theta_i, y_i^1), m_j^{1*}(\theta_j, y_j^1))$.

Let $x_i(\theta_i, \theta_j, y_i^1, y_j^1) = m_i^{1*}(\theta_i, y_i^1) + m_i^{2*}(\theta_i, \theta_j, y_i^1, y_j^1)$. Again by construction, for all θ_j and for all y_i^1, y_j^1, we have:

$$-\Big(x_i(\theta_i, \theta_j, y_i^1, y_j^1) - \theta_i\Big)^2 \geqslant -\Big(x_i(\theta_i', \theta_j, y_i^1, y_j^1) - \theta_i\Big)^2$$

which means that the two-stage mechanism where stage 2 budget depends on stage 1 consumption is formally equivalent to a direct mechanism where, for any initial local budgets (y_1^1, y_2^1), each system i is asked to report its total needs θ_i and receives a final allocation x_i that is divided among the two stages. The mechanism is direct and incentive compatible in dominant strategies. Moreover, for any such mechanisms with initial budgets (y_1^1, y_2^1), there exists an equivalent mechanism with no budget in stage 1. That mechanism

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is itself equivalent to a static mechanism in which all resources are allocated in stage 2. We can thus restrict to such mechanisms.

A2. Proof of Lemma 1.

Under a priority mechanism $\mathbf{P}_i$ the center assigns resources θ_i to system i. We first restrict attention to priority mechanisms that set an upper bound on the resources allocated to system j, i.e. $x_j^{\mathbf{P}_i}(\theta_1, \theta_2) = \min\{\theta_j, y_j(\theta_i)\}$, and characterize the optimal upper bound. We then show that this mechanism is optimal in the general class of priority mechanisms.

(a) **Optimal cap $y_j(\theta_i)$**.

The expected performance when total resources assigned to systems 0 and j are $k' = k - \theta_i$ and a resource cap $y_j(\theta_i)$ is imposed on system j is given by (omitting the dependence of $y_j(\theta_i)$ on θ_i to avoid clutter)

$$J_j(y_j) = -\int_{y_j}^{\bar{\theta}_j} \frac{1}{\beta_j} (y_j - \theta_j)^2 \, dF^j(\theta_j) - \int_{\Theta_j} \frac{1}{\beta_0} (x_0(\theta_j) - \theta_0)^2 \, dF^j(\theta_j)$$

where $x_0(\theta_j) = k' - \theta_j$ if $\theta_j \le y_j$ and $x_0(\theta_j) = k' - y_j$ if $\theta_j > y_j$. The optimal threshold y_j solves

$$\max J_j(y_j) \quad \text{s.t.} \quad 0 \le y_j \le k' \tag{12}$$

We now show that the function $J_j(y_j)$ is quasiconcave in $[0, k']$. By differentiating $J_j(y_j)$ we have

$$J_j'(y_j) = 2\left(1 - F^j(y_j)\right)\left[\frac{1}{\beta_j}\left(E\left[\theta_i \mid \theta_i \ge y_j\right] - y_j\right) - \frac{1}{\beta_0}\left(y_j + \theta_0 - k'\right)\right] \tag{13}$$

Assumption 1 implies that $d\left(E[\theta_i \mid \theta_i \ge y_j] - y_j\right)/dy_j \le 0$. Therefore the term in brackets in (13) is strictly decreasing in y_j and there exists at most one point $\hat{y}_j \in [0, k']$ such that $J_j'(\hat{y}_j) = 0$. Moreover, whenever $J_j'(\hat{y}_j) = 0$, we have $J_j''(\hat{y}_j) \le 0$. Therefore $J_j(y_j)$ is either monotonic or, if $\hat{y}_j$ exists such that $J_j'(\hat{y}_j) = 0$, increasing for $y_j \le \hat{y}_j$ and decreasing for $y_j \ge \hat{y}_j$. This establishes that $J_j(y_j)$ is quasiconcave. With these insights we can now solve (12).

First, for $y_j = 0$ to be a solution of (12) it is necessary and sufficient that $J_j'(0) \le 0$ which is equivalent to $\beta_j(\theta_0 - k') \ge \beta_0 E[\theta_j]$. For $\hat{y}_j = k'$ to be a solution of (12) it is necessary and sufficient that $J_j'(k') \ge 0$ which is equivalent to $\beta_0\left(E[\theta_j \mid \theta_j \ge k'] - k'\right) \ge \beta_j \theta_0$. In all other cases the maximizer of (12) is the unique solution to $J_j'(y_j) = 0$ thus satisfying:

$$\frac{1}{\beta_j}\left(E[\theta_j \mid \theta_j \ge y_j] - y_j\right) = \frac{1}{\beta_0}\left(\theta_0 - (k' - y_j)\right)$$

In particular, the uniqueness of maximizer and the Maximum Theorem establish that $y_j(k')$ is continuous. Summarizing, the optimal threshold $y_j(\theta_i)$ under $\mathbf{P}_i$ is:

$$
\begin{cases}
y_j(\theta_i) = 0 & \text{if } \beta_j(\theta_0 + \theta_i - k) \geq \beta_0 E[\theta_j] \\
y_j(\theta_i) = k - \theta_i & \text{if } \beta_0(E[\theta_j | \theta_j \geq k - \theta_i] - (k - \theta_i)) \geq \beta_j \theta_0 \\
\frac{1}{\beta_j}(E[\theta_j - y_j(\theta_i) | \theta_j \geq y_j(\theta_i)]) = \frac{1}{\beta_0}(\theta_0 - (k - \theta_i - y_j(\theta_i))) & \text{otherwise}
\end{cases}
\tag{14}
$$

At any point θ_i at which $y_j(\theta_i') = k - \theta_i'$ for θ_i' in a neighborhood of θ_i we have $y_j'(\theta_i) = -1$. We can implicitly differentiate (14) for any interior solution to find

$$
y_j'(\theta_i) = -\frac{\beta_j/\beta_0}{1 - \frac{d}{dy_j(\theta_i)} E[\theta_j | \theta_j \geq y_j(\theta_i)] + \beta_j/\beta_0} \in [-1, 0)
$$

since Assumption 1 implies $dE[\theta_j | \theta_j \geq y_j(\theta_i)]/dy_j \leq 1$. In particular, if $h_j'(\theta_j) > 0$ then $y_j'(\theta_j) \in (-1, 0)$.

(b) **Optimality of resource-cap priority mechanisms.**

The overall performance under a priority mechanism $\mathbf{P}_i$ given (θ_i, θ_j) and resources $x_j \in [0, k - \theta_i]$ to system j is

$$
\begin{aligned}
J(x_j, \theta_i, \theta_j) &= -\frac{1}{\beta_j}(x_j - \theta_j)^2 - \frac{1}{\beta_0}(k - \theta_i - x_j - \theta_0)^2 \\
&= -\left(\frac{1}{\beta_j} + \frac{1}{\beta_0}\right)(x_j - y_{CES}(\theta_j))^2 - \frac{(k - \theta_0 - \theta_i - \theta_j)^2}{\beta_0 + \beta_j}
\end{aligned}
$$

where
$$
y_{CES}(\theta_j) = \max\left\{\frac{\beta_0}{\beta_0 + \beta_j}\theta_j - \frac{\beta_j}{\beta_0 + \beta_j}(\theta_i + \theta_0 - k), 0\right\}
$$

is the amount of resources optimally allocated to system j if the CES knew θ_j. The search for the optimal $\mathbf{P}_i$ is equivalent to a delegation problem where system j is offered a set of "resources" $D_j(\theta_i) \subset [0, k - \theta_i]$. This problem has been studied in Alonso and Matouschek (2008) who provide conditions for the optimal $D_j(\theta_i)$ to simply involve an upper resource cap. Proposition 3 in Alonso and Matouschek (2008) states that, defining the forward bias

$$
S_j(\theta_j) = \left(1 - F^j(\theta_j)\right)\left(\theta_j - E\left[y_{CES}(z) \mid z \geq \theta_j\right]\right),
\tag{15}
$$

the optimal mechanism involves an upper resource cap if and only if there exists $\hat{y}_j$ such that (i) $S_j(\hat{y}_j) = 0$ and $S_j(\theta_j) > 0$ for $\theta_j > \hat{y}_j$, and (ii) $S_j(\theta_j)$ is concave in $\theta_j \in [0, \hat{y}_j]$. First, we note that

$$
S_j(\theta_j) = -\frac{1}{2}\left(\frac{\beta_0 \beta_j}{\beta_j + \beta_0}\right) J_j'(\theta_j)
$$

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From (13) we have that $J_j'(\hat{y}_j) = 0$ and $J_j'(\theta_j) < 0$ for $\theta_j > \hat{y}_j$ thus satisfying condition (i). Differentiating (15) yields

$$S_j'(\theta_j) = \left(1 - F^j(\theta_j)\right)\left[1 - h_j(\theta_j)(\theta_j - y_{CES}(\theta_j))\right]$$

The difference between system j's preferred allocation and the CES preferred allocation for system j, $\theta_j - y_{CES}(\theta_j)$, is positive and increasing in θ_j. Assumption 1 then implies that $h_j(\theta_j)(\theta_j - y_{CES}(\theta_j))$ is increasing in θ_j. From the second order conditions on $\hat{y}_j$ we have $J_j''(\hat{y}_j) \le 0$ implying $S_j'(\hat{y}_j) \ge 0$ and $h_j(\hat{y}_j)(\hat{y}_j - y_{CES}(\hat{y}_j)) \le 1$. Therefore $1 - h_j(\theta_j)(\theta_j - y_{CES}(\theta_j))$ is positive and decreasing in $[0, \hat{y}_j]$. It follows that $S_j'(\theta_j)$ as the product of two decreasing and positive functions must be decreasing. Therefore $S_j(\theta_j)$ is concave in $\theta_j \in [0, \hat{y}_j]$ thus satisfying condition (ii).

A3. Proof of Proposition 1.

The proof of Proposition 1 will be based on the following two lemmas.

Lemma A *The thresholds $y_2(\theta_1)$ and $y_1(\theta_2)$ in the priority mechanisms $\mathbf{P}_1$ and $\mathbf{P}_2$ intersect if and only if the following two conditions are satisfied:*

$$\begin{aligned}
E[\theta_1 \,|\, \theta_1 \ge k + \beta_0/\beta_2 E[\theta_2] - \theta_0] - (k - \theta_0 + \beta_0/\beta_2 E[\theta_2]) &< \beta_1/\beta_2 E[\theta_2] \\
E[\theta_2 \,|\, \theta_2 \ge k + \beta_0/\beta_1 E[\theta_1] - \theta_0] - (k - \theta_0 + \beta_0/\beta_1 E[\theta_1]) &< \beta_2/\beta_1 E[\theta_1]
\end{aligned} \qquad (16)$$

Furthermore, if $h_1(\theta_1)$ and $h_2(\theta_2)$ are strictly increasing and $y_i(\theta_j) < k - \theta_j$ then $y_2(\theta_1)$ and $y_1(\theta_2)$ intersect at a single point.

Proof: We first show that if at least one of the conditions (16) is not satisfied then $y_1(\theta_2)$ and $y_2(\theta_1)$ never intersect. The functions $y_1(\theta_2) - y_2^{-1}(\theta_2)$ and $y_2(\theta_1) - y_1^{-1}(\theta_1)$ are weakly increasing for $\theta_i \in [y_i(0), y_i(\bar{\theta}_j)] \cap [0, \bar{\theta}_i]$, since at any point of differentiability $d(y_i(\theta_j) - y_j^{-1}(\theta_j))/d\theta_j = y_i'(\theta_j) - (1/y_j'(y_j^{-1}(\theta_j))) \ge 0$.[38] Therefore, a necessary and sufficient condition for $y_1(\theta_2)$ and $y_2(\theta_1)$ to never intersect is that either $y_1(0) - y_2^{-1}(0) > 0$ or $y_2(0) - y_1^{-1}(0) > 0$.

Consider first the case $y_1(0) - y_2^{-1}(0) > 0$. In other words, under a priority mechanism $\mathbf{P}_1$ there is a θ_1 such that whenever system 1 requests at least θ_1 system 2 obtains zero resources $(y_2(\theta_1) = 0)$ and $\theta_1 < y_1(0) \le k$. The minimum value $\tilde{\theta}_1$ at which $y_2(\tilde{\theta}_1) = 0$ is given by $\tilde{\theta}_1 = k - \theta_0 + \beta_0/\beta_2 E[\theta_2]$ according to (14). The condition $y_1(0) > \tilde{\theta}_1$ can be restated as requiring that the marginal effect on overall performance of increasing the threshold to system 1 at $\tilde{\theta}_1$ must be positive, which from Lemma 1 implies:

$$\frac{1}{\beta_1}\left(E\left[\theta_1 \,\Big|\, \theta_1 \ge \tilde{\theta}_1\right] - \tilde{\theta}_1\right) - \frac{1}{\beta_0}\left(\tilde{\theta}_1 + \theta_0 - k\right) \ge 0$$

---

[38]It can be readily shown that any other point must entail a binding constraint $y_i(\theta_j) = k - \theta_j$ in which case the functions $y_i(\theta_j) - y_j^{-1}(\theta_j)$ are continuous and weakly increasing.

Substituting the value of $\tilde{\theta}_1$ this requires:

$$E\left[\theta_1 \mid \theta_1 \geq k + \beta_0/\beta_2 E\left[\theta_2\right] - \theta_0\right] - (k - \theta_0 + \beta_0/\beta_2 E\left[\theta_2\right]) \geq \beta_1/\beta_2 E\left[\theta_2\right]$$

Following a similar analysis, $y_2(0) - y_1^{-1}(0) > 0$ if and only if:

$$E\left[\theta_2 \mid \theta_2 \geq k + \beta_0/\beta_1 E\left[\theta_1\right] - \theta_0\right] - (k - \theta_0 + \beta_0/\beta_1 E\left[\theta_1\right]) \geq \beta_2/\beta_1 E\left[\theta_1\right].$$

Second, suppose that $y_1(\theta_2) < k - \theta_2$ and $y_2(\theta_1) < k - \theta_1$, i.e. both $y_i(\theta_j)$ are interior solutions of (14) and that the hazard rates are strictly increasing. We now show that, provided (16), $y_2(\theta_1) - y_1^{-1}(\theta_1) = 0$ has a unique solution. From $h_2'(\theta_2) > 0$, $h_1'(\theta_1) > 0$ and Lemma 1 it follows that $0 > y_2'(\theta_1) > -1$ and $d(y_1^{-1}(\theta_1))/d\theta_1 < -1$. Taking both implications together we have that the difference $y_2(\theta_1) - y_1^{-1}(\theta_1)$ is *strictly* increasing in θ_1 and thus changes sign at most once. Therefore if the curves $y_1(\theta_2)$ and $y_2(\theta_1)$ intersect at an interior point, then they intersect only once.

Lemma B *Let* $\Theta_i^+ = \left\{\theta_i : \theta_j > \bar{x}_j^*(\theta_i) \Rightarrow \theta_i > \bar{x}_i^*(\theta_j)\right\}$ *be the set of values* θ_i *such that under an optimal mechanism* **M** *and for any* (θ_i, θ_j) *in which system j receives less than* θ_j, *system i receives less than* θ_i. *If θ_1 and θ_2 are independent, then $\bar{x}_j^*(\theta_i)$ is constant in* Θ_i^+.

Proof: Let $\theta_i \in \Theta_i^+$. Then the optimal cap on system j, $\bar{x}_j^*(\theta_i)$, must satisfy the first order condition:

$$\int_{\bar{x}_j^*}^{\bar{\theta}_j} \left(\frac{1}{\beta_j}\left(\bar{x}_j^* - \theta_j\right) - \frac{1}{\beta_0}\left(k - \theta_0 - \left(\bar{x}_j^* + x_i\left(\theta_i, \theta_j\right)\right)\right)\right) dF^j(\theta_j) = 0$$

By definition, system i receives less than its needs if $\theta_j \geq \bar{x}_j^*$ when $\theta_i \in \Theta_i^+$. Therefore its allocation $x_i(\theta_i, \theta_j)$ is independent of θ_i, $x_i(\theta_i, \theta_j) = \bar{x}_i^*(\theta_j)$. It follows that for all $\theta_i \in \Theta_i^+$ the optimum $\bar{x}_j^*$ is independent of θ_i.

Using Lemmas A and B, we can now prove the characterization of the mechanism **M**.

Proof of Proposition 1: The first order condition for $\bar{x}_i^*$ at θ_j can be written as

$$\frac{1}{\beta_i}\left(E\left[\theta_i \mid \theta_i \geq \bar{x}_i^*\right] - \bar{x}_i^*\right) - \frac{1}{\beta_0}\left(\bar{x}_i^* + \theta_0 - k\right) = \frac{1}{\beta_0}\left(E\left[x_j\left(\theta_i, \theta_j\right) \mid \theta_i \geq \bar{x}_i^*\right]\right) \qquad (17)$$

From the proof of Lemma 1 the cap $\bar{x}_i^*$ must (weakly) decrease for any $x_j'(\theta_i, \theta_j)$ such that $E[x_j'(\theta_i, \theta_j) \mid \theta_i \geq \bar{x}_i^*] \geq E[x_j(\theta_i, \theta_j) \mid \theta_i \geq \bar{x}_i^*]$. In words, the optimal cap on a system must be lower if the other system receives more resources. This observation allows us to establish the following lower and upper bounds on the resources allocated to each system under an optimal mechanism **M** : (i) system i obtains at least the same resources than

under the priority mechanism $\mathbf{P}_j$, and (ii) system i's resources never exceed those obtained under $\mathbf{P}_j$ when system j demands zero resources. Indeed, since

$$0 \leq E[x_j(\theta_1, \theta_2) \,|\, \theta_j \geq y] = E[\min\{\theta_j, \bar{x}_j(\theta_i)\} \,|\, \theta_j \geq y] \leq \theta_j$$

we then have:

$$y_i(\theta_j) \leq \bar{x}_i^*(\theta_j) \leq y_i(0) \tag{18}$$

We consider first the case in which $y_1(\theta_2)$ and $y_2(\theta_1)$ intersect at an interior point (Lemma A). To characterize $\mathbf{M}$ we proceed in several steps:

(i) *The sets Θ_i^+ are non-empty.*
Suppose that $\bar{\theta}_i > y_i(0)$. From Lemma A we have that $y_2(0) < y_1^{-1}(0)$, that is the maximum resources granted to system 2 in a priority mechanism $\mathbf{P}_1$ (which occurs when system 1 demands zero) are less than the needs of system 2 that under $\mathbf{P}_2$ would lead system 1 to obtain zero resources. From (18), $y_2(0)$ represents an upper bound on the resources that system 2 would obtain under $\mathbf{M}$. Therefore, for $\theta_2 \geq y_2(0)$ we have $\theta_2 \in \Theta_2^+$ as system 2 is necessarily constrained. Lemma A also implies that $y_1(0) < y_2^{-1}(0)$. Therefore, for $\theta_1 \geq y_1(0)$ we have $\theta_1 \in \Theta_1^+$.

(ii) *Mechanism M behaves like a priority mechanism P_i for $\theta_i \leq k_i$*
From Lemma B let $k_i = \bar{x}_i^*(\theta_j)$ be the constant cap for $\theta_j \in \Theta_j^+$. Then k_i represents the minimum resources guaranteed to system in the sense that $k_i \leq \bar{x}_i^*(\theta_j)$ for all θ_j. To see this, suppose by way of contradiction that there exists a θ_j' such that $\bar{x}_i^*(\theta_j') < k_i$. Then (17) implies that under $\mathbf{M}$

$$E[x_j(\theta_i, \theta_j') \,|\, \theta_i \geq k_i] > E[x_j(\theta_i, \theta_j) \,|\, \theta_i \geq k_i] \tag{19}$$

for all $\theta_j \in \Theta_j^+$. Given that $\mathbf{M}$ is incentive compatible and ex-post efficient then $x_j(\theta_i, \theta_j') > x_j(\theta_i, \theta_j)$ only if $\theta_j' > \theta_j$. But this leads to a contradiction since then (19) implies $\theta_j' > \max \Theta_j^+ = \bar{\theta}_j$. Therefore, $k_i \leq \bar{x}_i^*(\theta_j)$ for all θ_j which implies that for $\theta_i \leq k_i$ system i always obtains its resource needs $x_i(\theta_i, \theta_j) = \theta_i$. Therefore, for $\theta_i \leq k_i$ (17) is satisfied by $\bar{x}_j^*(\theta_i) = y_j(\theta_i)$.

(iii) *Optimal guaranteed resources satisfy $k_j = y_j(k_i)$.*
Define k_i^* as the point of intersection of $y_1(\theta_2)$ and $y_2(\theta_1)$, i.e. $k_j^* = y_j(k_i^*)$. We now show that $k_i = k_i^*$.

First, for any $\theta_1 > k_1$ system 1 obtains at least resources k_1. This implies that the resources obtained by system 2 cannot exceed those obtained under a priority mechanism $\mathbf{P}_1$ when system 1 demands resources k_1, i.e. $x_2(\theta_1, \theta_2) \leq \bar{x}_2^*(\theta_1) \leq y_2(k_1)$ for $\theta_1 > k_1$. Therefore the optimal k_1 that satisfies (17) is (weakly) higher than the cap under a priority mechanism $\mathbf{P}_2$ when system 2 demanded resources $y_2(k_1)$, i.e.

$$k_1 \geq y_1(y_2(k_1))$$

or $y_2(k_1) - y_1^{-1}(k_1) \geq 0$. By Lemma A, $y_2(\theta_1) - y_1^{-1}(\theta_1)$ is an increasing function implying that

$$k_1 \geq k_1^* \tag{20}$$

Analogously we obtain that $k_2 \geq k_2^*$.

Second, system 2 always obtains at least resources k_2 when $\theta_2 \geq k_2$. Therefore the optimal k_1 that satisfies (17) cannot exceed the cap under a priority mechanism $\mathbf{P}_2$ when system 2 demands resources k_2 , i.e.

$$k_1 \leq y_1(k_2) \tag{21}$$

Combining (20) and (21) and $k_2 \geq k_2^*$ we have

$$k_1 \leq y_1(k_2) \leq y_1(k_2^*) = k_1^* \leq k_1$$

A similar reasoning yields $k_2 = k_2^*$.

Finally, we also consider the case in which $y_1(\theta_2)$ and $y_2(\theta_1)$ never intersect. If $y_i(0) - y_j^{-1}(0) > 0$ then by Lemma A we have $y_j(\theta_i) < y_i^{-1}(\theta_i)$ for all θ_i which implies that $k_i = y_i(0)$. Therefore $\mathbf{M}$ is a priority mechanism $\mathbf{P}_i$ for $\theta_i \leq y_i(0)$ while it implements the allocation $x_i(\theta_1, \theta_2) = k_i$ and $x_j(\theta_1, \theta_2) = 0$ for $\theta_i > y_i(0)$.

A4. Proof of Proposition 2.

The thresholds $y_i(\theta_j)$ are defined in (14). A decrease in β_1 to $\beta_1' < \beta_1$ relaxes the conditions $(1/\beta_0)(\theta_0 + \theta_2 - k) < (1/\beta_1)E[\theta_1]$ and $(1/\beta_1)(E[\theta_1 \mid \theta_1 \geq k - \theta_2] - (k - \theta_2)) \geq (1/\beta_0)\theta_0$ implying that if $y_1(\theta_2)(\beta_1) \geq 0$ then $y_1(\theta_2)(\beta_1') \geq 0$, and if $y_1(\theta_2)(\beta_1) = k - \theta_2$ then $y_1(\theta_2)(\beta_1') = k - \theta_2$.

If $y_1(\theta_2)(\beta_1)$ satisfies $(1/\beta_1)(E[\theta_1 \mid \theta_1 \geq y_1(\theta_2)] - y_1(\theta_2)) = (1/\beta_0)(y_1(\theta_2) + \theta_2 + \theta_0 - k)$, then implicitly differentiating we have that $\partial y_1(\theta_2)/\partial \beta_1 < 0$. In summary, if $\beta_1' < \beta_1$ then $y_1(\theta_2)(\beta_1') \geq y_1(\theta_2)(\beta_1)$ for all $\theta_2 \in [0, \bar{\theta}_2]$. Since $y_2(\theta_1)$ does not depend on β_1, we have $\bar{x}_1^*(\theta_2)(\beta_1') \geq \bar{x}_1^*(\theta_2)(\beta_1)$, $\bar{x}_2^*(\theta_1)(\beta_1') \leq \bar{x}_2^*(\theta_1)(\beta_1)$, and $k_1(\beta_1') \geq k_1(\beta_1)$, $k_2(\beta_1') \leq k_2(\beta_1)$.

Following a similar argument for a decrease in β_0 or an increase in θ_0 we find that if $\beta_0' < \beta_0$ or $\theta_0' > \theta_0$ then $y_i(\theta_j)(\beta_0') \leq y_i(\theta_j)(\beta_0)$ and $y_i(\theta_j)(\theta_0') \leq y_i(\theta_j)(\theta_0)$ for $\theta_j \in [0, \bar{\theta}_j]$.

Finally, let $H_i(k_i) = E[\theta_i \mid \theta_i \geq k_i] - k_i$, which satisfies $H_i'(k_i) < 0$ whenever $h_i'(\theta_i) > 0$. If $k_i(\beta_0) > 0$ and $k_i(\theta_0) > 0$ from Proposition 1 we have that the guaranteed levels k_i satisfy:

$$(1/\beta_1)H_1(k_1) = (1/\beta_2)H_2(k_2) \qquad \qquad \text{if} \quad k_1 + k_2 = k \tag{22}$$

$$(1/\beta_1)H_1(k_1) = (1/\beta_2)H_2(k_2) = (1/\beta_0)(k_1 + k_2 + \theta_0 - k) \quad \text{if} \quad k_1 + k_2 < k \tag{23}$$

We now show that both guaranteed levels are reduced for $\beta_0' < \beta_0$ or $\theta_0' > \theta_0$. First consider the case (22). Then for any $\beta_0' < \beta_0$ or $\theta_0' > \theta_0$ such that we still have $k_1(\beta_0') +$

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$k_2\left(\beta_0'\right) = k$ or $k_1\left(\theta_0'\right) + k_2\left(\theta_0'\right) = k$ the guaranteed levels do not change as (22) does not depend on β_0 or θ_0. Next suppose that $k_1 + k_2 < k$. Then by the implicit function theorem applied to (23):

$$\frac{\partial k_i}{\partial \beta_0} = -\frac{(1/\beta_j)H_j'(k_j)(1/\beta_0^2)\left(k_1 + k_2 + \theta_0 - k\right)}{\Delta} \qquad \text{and} \qquad \frac{\partial k_i}{\partial \theta_0} = \frac{(1/\beta_j)H_j'(k_j)(1/\beta_0)}{\Delta}$$

where $\Delta = (1/\beta_1)(1/\beta_2)H_1'(k_1)H_2'(k_2) - (1/\beta_0)\sum_{j=1,2}(1/\beta_j)H_j'(k_j) > 0$. Therefore $\partial k_i/\partial \beta_0 > 0$ and $\partial k_i/\partial \theta_0 < 0$.

A5. Proof of Lemma 2.

For simplicity, we ignore subscripts and superscripts in θ. From (7) and (8), we have:

$$\frac{g_\theta^i(\tilde{\theta}|\theta)}{g^i(\tilde{\theta}|\theta)} = \frac{\int_{\underline{s}_i}^{\overline{s}_i} f_\theta^i(\theta|s_i)f^i(\tilde{\theta}|s_i)p^i(s_i)ds_i}{\int_{\underline{s}_i}^{\overline{s}_i} f^i(\theta|s_i)f^i(\tilde{\theta}|s_i)p^i(s_i)ds_i} - \frac{\int_{\underline{s}_i}^{\overline{s}_i} f_\theta^i(\theta|s_i)p^i(s_i)ds_i}{\int_{\underline{s}_i}^{\overline{s}_i} f^i(\theta|s_i)p^i(s_i)ds_i}$$

Therefore, $\dfrac{d}{d\tilde{\theta}}\left(\dfrac{g_\theta^i(\tilde{\theta}|\theta)}{g^i(\tilde{\theta}|\theta)}\right) > 0$

$$\Leftrightarrow \int_{\underline{s}_i}^{\overline{s}_i} f_\theta^i(\theta|s_i)f_{\tilde{\theta}}^i(\tilde{\theta}|s_i)p^i(s_i)ds_i \int_{\underline{s}_i}^{\overline{s}_i} f^i(\theta|s_i)f^i(\tilde{\theta}|s_i)p^i(s_i)ds_i$$

$$> \int_{\underline{s}_i}^{\overline{s}_i} f_\theta^i(\theta|s_i)f^i(\tilde{\theta}|s_i)p^i(s_i)ds_i \int_{\underline{s}_i}^{\overline{s}_i} f^i(\theta|s_i)f_{\tilde{\theta}}^i(\tilde{\theta}|s_i)p^i(s_i)ds_i$$

Suppose that $\displaystyle\int_{\underline{s}_i}^{\overline{s}_i} f^i(\theta|s_i)f_{\tilde{\theta}}^i(\tilde{\theta}|s_i)p^i(s_i)ds_i > 0$. Then:

$$\frac{d}{d\tilde{\theta}}\left(\frac{g_\theta^i(\tilde{\theta}|\theta)}{g^i(\tilde{\theta}|\theta)}\right) > 0 \quad \Leftrightarrow \quad \frac{\int_{\underline{s}_i}^{\overline{s}_i} f_\theta^i(\theta|s_i)f_{\tilde{\theta}}^i(\tilde{\theta}|s_i)p^i(s_i)ds_i}{\int_{\underline{s}_i}^{\overline{s}_i} f^i(\theta|s_i)f_{\tilde{\theta}}^i(\tilde{\theta}|s_i)p^i(s_i)ds_i} > \frac{\int_{\underline{s}_i}^{\overline{s}_i} f_\theta^i(\theta|s_i)f^i(\tilde{\theta}|s_i)p^i(s_i)ds_i}{\int_{\underline{s}_i}^{\overline{s}_i} f^i(\theta|s_i)f^i(\tilde{\theta}|s_i)p^i(s_i)ds_i}$$

$$\Leftrightarrow \frac{\int_{\underline{s}_i}^{\overline{s}_i} \frac{f_\theta^i(\theta|s_i)}{f^i(\theta|s_i)}f_{\tilde{\theta}}^i(\tilde{\theta}|s_i)p^i(s_i|\theta)ds_i}{\int_{\underline{s}_i}^{\overline{s}_i} f_{\tilde{\theta}}^i(\tilde{\theta}|s_i)p^i(s_i|\theta)ds_i} > \frac{\int_{\underline{s}_i}^{\overline{s}_i} \frac{f_\theta^i(\theta|s_i)}{f^i(\theta|s_i)}f^i(\tilde{\theta}|s_i)p^i(s_i|\theta)ds_i}{\int_{\underline{s}_i}^{\overline{s}_i} f^i(\tilde{\theta}|s_i)p^i(s_i|\theta)ds_i}$$

$$\Leftrightarrow \int_{\underline{s}_i}^{\overline{s}_i} l^i(\theta|s_i)q(s_i)ds_i > \int_{\underline{s}_i}^{\overline{s}_i} l^i(\theta|s_i)r(s_i)ds_i$$

where $l^i(\theta|s_i) = \dfrac{f_\theta^i(\theta|s_i)}{f^i(\theta|s_i)}$, $q(s_i) = \dfrac{f_{\tilde{\theta}}^i(\tilde{\theta}|s_i)p^i(s_i|\theta)}{\int_{\underline{s}_i}^{\overline{s}_i} f_{\tilde{\theta}}^i(\tilde{\theta}|s_i)p^i(s_i|\theta)ds_i}$, $r(s_i) = \dfrac{f^i(\tilde{\theta}|s_i)p^i(s_i|\theta)}{\int_{\underline{s}_i}^{\overline{s}_i} f^i(\tilde{\theta}|s_i)p^i(s_i|\theta)ds_i}$.

Integrating by parts and given $Q(\bar{s}_i) = R(\bar{s}_i) = 1$ and $Q(\underline{s}_i) = R(\underline{s}_i) = 0$, we have:

$$\int_{\underline{s}_i}^{\bar{s}_i} l^i(\theta|s_i)\Big(q(s_i) - r(s_i)\Big)ds_i > 0 \;\Leftrightarrow\; \int_{\underline{s}_i}^{\bar{s}_i} \Big(Q(s_i) - R(s_i)\Big)l^i_{s_i}(\theta|s_i)ds_i < 0.$$

Assumption 2 implies $l^i_{s_i}(\theta|s_i) > 0$. Also, given $\displaystyle\int_{\underline{s}_i}^{\bar{s}_i} f^i_{\tilde{\theta}}(\tilde{\theta}|s_i)p^i(s_i|\theta)ds_i > 0$, then

$$Q(s_i) - R(s_i) < 0 \;\Leftrightarrow\; \frac{\int_{\underline{s}_i}^{s_i} f^i_{\tilde{\theta}}(\tilde{\theta}|x)p^i(x|\theta)dx}{\int_{\underline{s}_i}^{s_i} f^i(\tilde{\theta}|x)p^i(x|\theta)dx} < \frac{\int_{\underline{s}_i}^{\bar{s}_i} f^i_{\tilde{\theta}}(\tilde{\theta}|s_i)p^i(s_i|\theta)ds_i}{\int_{\underline{s}_i}^{\bar{s}_i} f^i(\tilde{\theta}|s_i)p^i(s_i|\theta)ds_i} \tag{24}$$

So a sufficient condition for $Q(s_i) - R(s_i) < 0$ is $\dfrac{d}{ds_i}\left[\dfrac{\int_{\underline{s}_i}^{s_i} f^i_{\tilde{\theta}}(\tilde{\theta}|x)p^i(x|\theta)dx}{\int_{\underline{s}_i}^{s_i} f^i(\tilde{\theta}|x)p^i(x|\theta)dx}\right] > 0$.

According to Assumption 2, for all $s_i > s'_i$:

$$\frac{f^i_{\tilde{\theta}}(\tilde{\theta}|s_i)}{f^i(\tilde{\theta}|s_i)} > \frac{f^i_{\tilde{\theta}}(\tilde{\theta}|s'_i)}{f^i(\tilde{\theta}|s'_i)} \;\Leftrightarrow\; f^i_{\tilde{\theta}}(\tilde{\theta}|s_i)f^i(\tilde{\theta}|s'^i_i(\tilde{\theta}|s_i)f^i_{\tilde{\theta}}g(\tilde{\theta}|s'_i)$$

$$\Rightarrow\; f^i_{\tilde{\theta}}(\tilde{\theta}|s_i)\int_{\underline{s}_i}^{s_i} f^i(\tilde{\theta}|x)p^i(x|\theta)dx > f^i(\tilde{\theta}|s_i)\int_{\underline{s}_i}^{s_i} f^i_{\tilde{\theta}}(\tilde{\theta}|x)p^i(x|\theta)dx$$

$$\Rightarrow\; \frac{d}{ds_i}\left[\frac{\int_{\underline{s}_i}^{s_i} f^i_{\tilde{\theta}}(\tilde{\theta}|x)p^i(x|\theta)dx}{\int_{\underline{s}_i}^{s_i} f^i(\tilde{\theta}|x)p^i(x|\theta)dx}\right] > 0,$$

which proves that $\dfrac{d}{d\tilde{\theta}}\left(\dfrac{g^i_{\theta}(\tilde{\theta}|\theta)}{g^i(\tilde{\theta}|\theta)}\right) > 0$ when $\displaystyle\int_{\underline{s}_i}^{\bar{s}_i} f^i(\theta|s_i)f^i_{\tilde{\theta}}(\tilde{\theta}|s_i)p^i(s_i)ds_i > 0$.

Now, suppose that $\displaystyle\int_{\underline{s}_i}^{\bar{s}_i} f^i(\theta|s_i)f^i_{\tilde{\theta}}(\tilde{\theta}|s_i)p^i(s_i)ds_i < 0$. Then:

$$\frac{d}{d\tilde{\theta}}\left(\frac{g^i_{\theta}(\tilde{\theta}|\theta)}{g^i(\tilde{\theta}|\theta)}\right) > 0 \;\Leftrightarrow\; \frac{\int_{\underline{s}_i}^{\bar{s}_i} f^i_{\theta}(\theta|s_i)f^i_{\tilde{\theta}}(\tilde{\theta}|s_i)p^i(s_i)ds_i}{\int_{\underline{s}_i}^{\bar{s}_i} f^i(\theta|s_i)f^i_{\tilde{\theta}}(\tilde{\theta}|s_i)p^i(s_i)ds_i} < \frac{\int_{\underline{s}_i}^{\bar{s}_i} f^i_{\theta}(\theta|s_i)f^i(\tilde{\theta}|s_i)p^i(s_i)ds_i}{\int_{\underline{s}_i}^{\bar{s}_i} f^i(\theta|s_i)f^i(\tilde{\theta}|s_i)p^i(s_i)ds_i}$$

$$\Leftrightarrow\; \int_{\underline{s}_i}^{\bar{s}_i} \Big(Q(s_i) - R(s_i)\Big)l^i_{s_i}(\theta|s_i)ds_i > 0.$$

But given $\displaystyle\int_{\underline{s}_i}^{\bar{s}_i} f^i_{\tilde{\theta}}(\tilde{\theta}|s_i)p^i(s_i|\theta)ds_i < 0$, then

$$Q(s_i) - R(s_i) > 0 \;\Leftrightarrow\; \frac{\int_{\underline{s}_i}^{s_i} f^i_{\tilde{\theta}}(\tilde{\theta}|x)p^i(x|\theta)dx}{\int_{\underline{s}_i}^{s_i} f^i(\tilde{\theta}|x)p^i(x|\theta)dx} < \frac{\int_{\underline{s}_i}^{\bar{s}_i} f^i_{\tilde{\theta}}(\tilde{\theta}|s_i)p^i(s_i|\theta)ds_i}{\int_{\underline{s}_i}^{\bar{s}_i} f^i(\tilde{\theta}|s_i)p^i(s_i|\theta)ds_i}$$

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which is the very same condition as (24).

A6. Proof of Proposition 4.

Given Proposition 1 and Lemma 2 it suffices to show that if $\overline{x}_i^*(\theta_j)(G_1)$ is the optimal resource cap when θ_1 is distributed according to G_1 then for $G_1' \succ_{MLRP} G_1$ we have $\overline{x}_1^*(\theta_2)(G_1') \geq \overline{x}_1^*(\theta_2)(G_1)$ and $\overline{x}_2^*(\theta_1)(G_1') \leq \overline{x}_2^*(\theta_1)(G_1)$.

If $G_1' \succ_{MLRP} G_1$, then the hazard rates satisfy $h_{G_1'}(\theta_1) \leq h_{G_1}(\theta_1)$ and $\frac{1-G_1'(\theta_1)}{1-G_1(\theta_1)}$ increases in θ_1. Since

$$E_{G_1}[\theta_1 \,|\, \theta_1 \geq y_1] - y_1 = \int_{y_1}^{\bar{\theta}_1} \frac{1 - G_1(\theta_1)}{1 - G_1(y_1)} d\theta_1$$

it follows that

$$E_{G_1'}[\theta_1 \,|\, \theta_1 \geq y_1] - y_1 > E_{G_1}[\theta_1 \,|\, \theta_1 \geq y_1] - y_1 \tag{25}$$

From (25) and the definition of $y_1(\theta_2)$ in (14) we have: (i) if $y_1(\theta_2)(G_1) = 0$ then $y_1(\theta_2)(G_1') \geq 0$; (ii) if $y_1(\theta_2)(G_1) = k - \theta_2$ then $y_1(\theta_2)(G_1') = k - \theta_2$; and (iii) if $y_1(\theta_2)(G_1)$ satisfies

$$\frac{1}{\beta_1}\Big(E[\theta_1 \,|\, \theta_1 \geq y_1(\theta_2)] - y_1(\theta_2)\Big) = \frac{1}{\beta_0}\Big(y_1(\theta_2) + \theta_2 + \theta_0 - k\Big)$$

then $y_1(\theta_2)(G_1) < y_1(\theta_2)(G_1')$. In summary, if $G_1' \succ_{MLRP} G_1$ then $y_1(\theta_2)(G_1') \geq y_1(\theta_2)(G_1)$ for all $\theta_2 \in [\underline{\theta}_2, \bar{\theta}_2]$. Since $y_2(\theta_1)(G_1') = y_2(\theta_1)(G_1)$, we can immediately conclude that (i) $k_1(G_1') > k_1(G)$, $k_2(G_1') < k_2(G_1)$ and (ii) $\overline{x}_1^*(\theta_2)(G_1') \geq \overline{x}_1^*(\theta_2)(G_1)$, $\overline{x}_2^*(\theta_1)(G_1') \leq \overline{x}_2^*(\theta_1)(G_1)$.

A7. Proof of Lemma 3.

For fixed resources k' (≥ 0) and needs (θ_1, θ_2) system I will choose to distribute them according to

$$\begin{array}{lll}
\text{1. If } k' \geq \max_i\left\{\theta_i - \frac{\beta_i}{\beta_j}\theta_j\right\} \text{ then} & \left\{\begin{array}{l} x_1(k'; \theta_1, \theta_2) = \theta_1 - \beta_1(\theta_1 + \theta_2 - k')/(\beta_1 + \beta_2) \\ x_2(k'; \theta_1, \theta_2) = \theta_2 - \beta_2(\theta_1 + \theta_2 - k')/(\beta_1 + \beta_2) \end{array}\right. & \\
\text{2. If } k' < \theta_1 - \frac{\beta_1}{\beta_2}\theta_2 & \text{then} \quad \left\{\begin{array}{ll} x_1(k'; \theta_1, \theta_2) & = \quad k' \\ x_2(k'; \theta_1, \theta_2) & = \quad 0 \end{array}\right. & (26) \\
\text{3. If } k' < \theta_2 - \frac{\beta_2}{\beta_1}\theta_1 & \text{then} \quad \left\{\begin{array}{ll} x_1(k'; \theta_1, \theta_2) & = \quad 0 \\ x_2(k'; \theta_1, \theta_2) & = \quad k' \end{array}\right. &
\end{array}$$

and the ex-post performance of system I is

$$\begin{array}{llll}
\Pi_I(\theta_1, \theta_2, k') & = & -\frac{1}{\beta_1 + \beta_2}(\theta_1 + \theta_2 - k')^2 & \text{if} \quad k' \geq \max_i\{\theta_i - \frac{\beta_i}{\beta_j}\theta_j\} \\
\Pi_I(\theta_1, \theta_2, k') & = & -\frac{1}{\beta_1}(\theta_1 - k')^2 - \frac{1}{\beta_2}\theta_2^2 & \text{if} \quad k' < \theta_1 - \frac{\beta_1}{\beta_2}\theta_2 \\
\Pi_I(\theta_1, \theta_2, k') & = & -\frac{1}{\beta_1}\theta_1^2 - \frac{1}{\beta_2}(\theta_2 - k')^2 & \text{if} \quad k' < \theta_2 - \frac{\beta_2}{\beta_1}\theta_1
\end{array}$$

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The fact that resources allocated to each task must be non-negative accounts for the allocation rules whenever $k' < \max_i \left\{ \theta_i - \frac{\beta_i}{\beta_j} \theta_j \right\}$. In fact, non-negativity implies that system I cannot "borrow" negative resources from tasks with lower needs and redirect them to tasks with higher needs. This insight provides an upper bound on the ex-post performance of system I given optimal distribution rules (26):

$$\Pi_I(\theta_1, \theta_2, k') \leq -\frac{1}{\beta_1 + \beta_2} \left(\theta_1 + \theta_2 - k' \right)^2 \tag{27}$$

Clearly, for any k' and needs (θ_1, θ_2), expected overall performance is

$$\Pi_{CES} = \Pi_I(\theta_1, \theta_2, k') - 1/\beta_0 (\theta_0 + k' - k)^2.$$

To study the optimal mechanism under integration we consider a related optimization problem (problem $\mathcal{P}'$) where the performances of CES and system I are given by:

$$\begin{aligned}
\Pi_I'(\theta_1, \theta_2, k') &= -\frac{1}{\beta_1 + \beta_2} \left(\theta_1 + \theta_2 - k' \right)^2 \tag{28} \\
\Pi_{CES}' &= -\frac{1}{\beta_0} \left(\theta_0 - (k - k') \right)^2 + \Pi_I(\theta_1, \theta_2, k')
\end{aligned}$$

We then show that the optimal mechanism for problem $\mathcal{P}'$ is also the optimal mechanism for our original specification.

(i) *Optimal Mechanism for Problem $\mathcal{P}'$.*

The overall performance from the viewpoint of CES in (28) can be written as

$$\Pi_{CES}' = - \left(\frac{1}{\beta_1 + \beta_2} + \frac{1}{\beta_0} \right) \left(k' - k_{CES}'(\theta_1, \theta_2) \right)^2 - \frac{(k - \theta_1 - \theta_2 - \theta_0)^2}{\beta_0 + \beta_1 + \beta_2}$$

where

$$k_{CES}'(\theta_1, \theta_2) = \max \left\{ \frac{\beta_0}{\beta_0 + \beta_1 + \beta_2} (\theta_1 + \theta_2) - \frac{\beta_1 + \beta_2}{\beta_0 + \beta_1 + \beta_2} (\theta_0 - k), 0 \right\}$$

is the optimal assignment to system I if the CES knew (θ_1, θ_2). The search for the optimal mechanism is equivalent to a delegation problem where system I is offered a set of resources $D_I \subset [0, k]$ and selects its preferred level from D_I. We will apply the conditions in Proposition 3 in Alonso and Matouschek (2008) to show that the optimal D_I is an interval where the maximum resources allowed to system I never exceeds its highest needs.

First, with $z = \theta_1 + \theta_2$ we have $G(z) = \int_{\Theta_1} \int_{\theta_2}^{\theta_2 = z - \theta_1} f(\theta_1, \theta_2) \, d\theta_1 d\theta_2$ and $g(z) = \int_{\Theta_1} f_1(\theta_1) f_2(z - \theta_1) \, d\theta_1$ (which follows from independence of θ_1 and θ_2). The distribution function $G(z)$ is the convolution of two distributions with an increasing hazard rate,

and therefore has also an increasing hazard rate (Barlow et al 1963). Second, the difference between system I's preferred choice of resources z and the CES preferred resources $k'_{CES}(z)$ is a non-negative, non-decreasing function of z. Thus the delegation problem satisfies the same conditions as in Lemma 1 implying that the optimal resource allocation rule for problem $\mathcal{P}'$ takes the form $k'(z) = \min\{z, \hat{k}\}$ for some $\hat{k} \leq k$.

(ii) *Optimal Mechanism under integration.*

First we observe that under a mechanism of the form $k(z) = \min\{z, \hat{k}\}$ the allocation for each task whenever $z \leq \hat{k}$ is the same for a system I with preferences as in (28) or in our original setup. Moreover, the fact that the solution to problem $\mathcal{P}'$ is of the form $k'(z) = \min\{z, \hat{k}\}$ implies that it is never optimal to "rule out" intermediate resources, i.e. offer mechanisms of the form

$$k'(z) = \begin{cases} \min\{z, k_1\} & \text{if} \quad z \leq (k_1 + k_2)/2 \\ \min\{\max\{z, k_2\}, \hat{k}\} & \text{if} \quad z > (k_1 + k_2)/2 \end{cases}$$

where resources $k' \in (k_1, k_2)$ are not available to system I. We will now show that the expected performance to the CES in our original setup when ruling out intermediate resources (k_1, k_2) is (weakly) lower than in problem $\mathcal{P}'$ with preferences (28). Since the CES cannot gain under problem $\mathcal{P}'$ by ruling out intermediate resources it therefore cannot be optimal to do so in our original setup.

Suppose that in the original setup the CES rules out any resource $k' \in (k_1, k_2)$. As a result, system I with needs (θ_1, θ_2) such that $k_1 < \theta_1 + \theta_2 < k_2$ and allocating resources according to (26) will decide whether to demand total resources k_1 or k_2. Let A_i be the set of needs (θ_1, θ_2) that will lead system I to demand k_i, that is

$$A_i = \{(\theta_1, \theta_2) : k_1 \leq \theta_1 + \theta_2 \leq k_2, \Pi_I(\theta_1, \theta_2, k_i) \geq \Pi_I(\theta_1, \theta_2, k_j)\}$$

For problem $\mathcal{P}'$ the equivalent set A_i^S is

$$A_i^S = \{(\theta_1, \theta_2) : k_1 \leq \theta_1 + \theta_2 \leq k_2, |\theta_1 + \theta_2 - k_i| \leq |\theta_1 + \theta_2 - k_j|\}$$

We first note that whenever system I chooses k_2 with preferences (28) it will also choose k_2 when it allocates resources according to (26), i.e. $A_2^S \subset A_2$ and $A_1 \subset A_1^S$. Furthermore, as resources granted exceed resource needs in A_2^S system I will choose the same allocation in both cases implying that $\Pi_{CES} = \Pi'_{CES}$ for $(\theta_1, \theta_2) \in A_2^S$. From (27) the performance of system I is dominated by $-1/(\beta_1 + \beta_2)(\theta_1 + \theta_2 - k_1)^2$ for $(\theta_1, \theta_2) \in A_1^S$ implying that $\Pi_{CES} \leq \Pi'_{CES}$ in A_1^S. Therefore expected performance from ruling out intermediate resources is lower when system I behaves as (26) than in problem $\mathcal{P}'$.

(iii) *Optimal cap $\hat{k}$.*

The expected performance of CES with a resource allocation rule $k^{\mathbf{I}}(z) = \min\{z, k'\}$ is

$$J_{CES}^I(k') = \int_0^{z=k'} -\frac{1}{\beta_0}(k-\theta_0-z)^2 \, dG(z) + \int_{z=k'}^{\bar{z}} \left(-\frac{1}{\beta_0}(k-\theta_0-k')^2 - \frac{1}{\beta_1+\beta_2}(z-k')^2\right) dG(z)$$

where $\bar{z} = \bar{\theta}_1 + \bar{\theta}_2$. The optimal cap $\hat{k}$ satisfies the first order condition

$$2\int_{z=\hat{k}}^{\bar{z}} \left(\frac{1}{\beta_0}(k-\theta_0-\hat{k}) + \frac{1}{\beta_1+\beta_2}(z-\hat{k})\right) dG(z) = 0$$

which translates into

$$\frac{1}{\beta_1+\beta_2}\left(E\left[z \,\middle|\, z \geq \hat{k}\right] - \hat{k}\right) = \frac{1}{\beta_0}\left(\theta_0 - \left(k-\hat{k}\right)\right).$$

A8. Proof of Proposition 5.

Denote by $J_{CES}^I(\gamma)$ the maximum expected performance under integration and by $J_{CES}^M(\gamma)$ the maximum expected performance under specialization.

(a) **Relative performance of Integration vs. Specialization as $\gamma \to 1$.**

(a-i) *Performance under integration.*

Given that $\bar{z} = \bar{\theta}_1 + \bar{\theta}_2 \leq k$, Lemma 3 establishes that for γ close to 1 the optimal integration mechanism **I** sets a cap on resources $\hat{k}$ where

$$\hat{k} = \frac{\gamma\beta_0'}{\gamma\beta_0' + (1-\gamma)(\beta_1'+\beta_2')}E\left[z \,\middle|\, z \geq \hat{k}\right] - \frac{(1-\gamma)(\beta_1'+\beta_2')}{\gamma\beta_0' + (1-\gamma)(\beta_1'+\beta_2')}(\theta_0 - k) \qquad (29)$$

Since $\theta_0 > k$ it follows that $\hat{k} < \bar{z}$ whenever $\gamma < 1$ and $\hat{k} \to \bar{z}$ as $\gamma \to 1$. To study the performance of integration as $\gamma \to 1$ first determine the rate at which the cap $\hat{k}$ increases. To this end we make two preliminary observations. First, given the bounded support of θ_1 and θ_2, the p.d.f. of $g(\bar{z})$ satisfies

$$g(\bar{z}) = \int_{\Theta_1} f^1(\theta_1) f^2(\bar{\theta}_1 - \theta_1 + \bar{\theta}_2) \, d\theta_1 = 0$$

as $f^2(\bar{\theta}_1 - \theta_1 + \bar{\theta}_2) = 0$ for $\theta_1 < \bar{\theta}_1$. Second,

$$g_-'(\bar{z}) = -f^1(\bar{\theta}_1) f^2(\bar{\theta}_2) < 0$$

In order to compute $\partial\hat{k}/\partial\gamma|_{\gamma=1^-}$ as an application of the implicit function theorem to (29) we first compute $dE\left[z - \hat{k} \,\middle|\, z \geq \hat{k}\right]/d\hat{k}|_{\gamma=1^-}$

$$\frac{dE\left[z - \hat{k} \,\middle|\, z \geq \hat{k}\right]}{d\hat{k}}\Bigg|_{\gamma=1^-} = -1 + \lim_{\gamma=1^-} \frac{g(\hat{k})}{1 - G(\hat{k})}E\left[z - \hat{k} \,\middle|\, z \geq \hat{k}\right].$$

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Defining the function $N(k) = \int_k^{\bar{z}} (z-k)dG(z) = \int_k^{\bar{z}} (1-G(z))dz$, we have

$$\frac{g(k)}{1-G(k)} E\left[z-k \,|\, z \geq k\right] = \frac{N''(k)N(k)}{(-N'(k))^2}$$

The (leftward) Taylor series expansion of $N(k)$ around $\bar{z}$ is given by $N(k) = \frac{g'(\bar{z})}{3!}(k-\bar{z})^3 + O((k-\bar{z})^4)$ for $k \leq \bar{z}$. With this expression we readily obtain

$$\lim_{\gamma \to 1^-} \frac{g(\hat{k})}{1-G(\hat{k})} E\left[z-\hat{k}\,\Big|\, z \geq \hat{k}\right] = \lim_{\gamma \to 1^-} \frac{N''(\hat{k})N(\hat{k})}{(-N'(\hat{k}))^2} = \frac{\frac{g'(\bar{z})}{3!}\frac{g'(\bar{z})}{1!}}{\left(\frac{g'(\bar{z})}{2!}\right)^2} = \frac{2}{3}$$

Thus

$$\left.\frac{dE\left[z-\hat{k}\,\Big|\, z \geq \hat{k}\right]}{d\hat{k}}\right|_{\gamma=1^-} = -1 + \lim_{\gamma=1^-} \frac{g(\hat{k})}{1-G(\hat{k})} E\left[z-\hat{k}\,\Big|\, z \geq \hat{k}\right] = -\frac{1}{3}$$

and by the implicit function theorem

$$\left.\frac{\partial \hat{k}}{\partial \gamma}\right|_{\gamma=1^-} = -\frac{(\beta_1' + \beta_2')(\bar{z} + \theta_0 - k)}{\beta_0' \left.\frac{dE[z-\hat{k}|z\geq\hat{k}]}{d\hat{k}}\right|_{\gamma=1^-}} = 3\frac{\beta_1' + \beta_2'}{\beta_0'}(\bar{z} + \theta_0 - k) > 0. \tag{30}$$

(a-ii) *Performance under specialization.*

To study the case of specialization we will consider the mechanism $\mathbf{M}_D$ defined by

$$\begin{aligned}
x_1^{\mathbf{M}_D}(\theta_1, \theta_2) &= \min\{\theta_1, y_1(\theta_2)\} \\
x_2^{\mathbf{M}_D}(\theta_1, \theta_1) &= \min\{\theta_2, y_2(\theta_1)\} \\
x_0^{\mathbf{M}_D}(\theta_1, \theta_2) &= k - x_1^{\mathbf{M}_D}(\theta_1, \theta_2) - x_2^{\mathbf{M}_D}(\theta_1, \theta_2)
\end{aligned}$$

where

$$y_i(\theta_j) = \begin{cases} \widetilde{k}_i & \text{if } \theta_j \leq \hat{k} - \bar{\theta}_i \\ \hat{k} - \theta_j & \text{if } \theta_j \geq \hat{k} - \bar{\theta}_i \end{cases}$$

and, with $\delta_i > 0$,

$$\widetilde{k}_i = \bar{\theta}_i - (1-\gamma)\frac{\beta_i'}{\beta_0'\left(\gamma + \frac{\delta_i}{2}\beta_i'\right)}\left(\bar{\theta}_i + \theta_0 - k\right) \tag{31}$$

The mechanism M_D always satisfies the resource constraint (given that $\bar{\theta}_1 + \bar{\theta}_2 \leq k$) and is dominant strategy incentive compatible. Clearly $\mathbf{M}_D$ is not necessarily optimal under specialization, i.e. $J_{CES}^M(\gamma) \geq J_{CES}^{M_D}(\gamma)$. Nevertheless $\mathbf{M}_D$ is simpler to analyze than

the optimal mechanism under specialization and will suffice to show that specialization dominates integration as $\gamma \to 1$.

(a-iii) *Comparison integration-specialization.*

Define the sets

$$
\begin{aligned}
A_i &= \left\{ (\theta_1, \theta_2) : \theta_i \geq \widetilde{k}_i, \theta_j \leq \hat{k} - \bar{\theta}_i \right\}, \quad i \in \{1, 2\} \\
B &= \left\{ (\theta_1, \theta_2) : \theta_1 + \theta_2 \geq \hat{k} \right\}
\end{aligned}
$$

As the allocation rule under the mechanisms $\mathbf{I}$ and $\mathbf{M}_D$ coincides outside the sets A_i and B we only need to estimate the difference $J_{CES}^{M_D}(\gamma) - J_{CES}^{I}(\gamma)$ in each of these three sets.

First, for $(\theta_1, \theta_2) \in A_i$ (i) the mechanism $\mathbf{M}_D$ restricts the allocation to system i to $\widetilde{k}_i$ while it grants its needs to system j and (ii) both tasks obtain their needs under the mechanism $\mathbf{I}$. We thus have

$$
\left(J_{CES}^{M_D}(\gamma) - J_{CES}^{I}(\gamma) \right) \Big|_{(\theta_1, \theta_2) \in A_i}
$$

$$
= \int_{A_i} \left(-\frac{1-\gamma}{\beta_0'} \left(k - \widetilde{k}_i - \theta_j - \theta_0 \right)^2 - \frac{\gamma}{\beta_i'} \left(\widetilde{k}_i - \theta_i \right)^2 + \frac{1-\gamma}{\beta_0'} (k - \theta_i - \theta_j - \theta_0)^2 \right) dF^1 dF^2
$$

$$
= 2 \int_{A_i} \int_{\widetilde{k}_i}^{\theta_i} \left(\frac{1-\gamma}{\beta_0'} (s + \theta_j + \theta_0 - k) - \frac{\gamma}{\beta_i'} \left(s - \widetilde{k}_i \right) \right) ds \, dF^1 dF^2
$$

$$
\geq 2 \int_{A_i} \int_{\widetilde{k}_i}^{\theta_i} \left(\frac{1-\gamma}{\beta_0'} (s + \theta_0 - k) - \frac{\gamma}{\beta_i'} \left(s - \widetilde{k}_i \right) \right) ds \, dF^1 dF^2
$$

which leads to the estimate

$$
2 \int_{A_i} \int_{\widetilde{k}_i}^{\theta_i} \left(\frac{1-\gamma}{\beta_0'} (s + \theta_0 - k) - \frac{\gamma}{\beta_i'} \left(s - \widetilde{k}_i \right) \right) ds \, dF^1 dF^2 \geq \delta_i \left(\bar{\theta}_i - \widetilde{k}_i \right) E \left[\theta_i - \widetilde{k}_i | A_i \right] \Pr \left[A_i \right]
$$

where $\Pr \left[A_i \right] = (1 - F^i(\widetilde{k}_i)) F^j(\hat{k} - \bar{\theta}_i)$ and the last inequality follows from the definition of $\widetilde{k}_i$ and that, for $1 > \gamma > \frac{\beta_i'}{\beta_0' + \beta_i'}$,

$$
\frac{1-\gamma}{\beta_0'} (s + \theta_0 - k) - \frac{\gamma}{\beta_i'} \left(s - \widetilde{k}_i \right) \geq \frac{1-\gamma}{\beta_0'} (\bar{\theta}_i + \theta_0 - k) - \frac{\gamma}{\beta_i'} \left(\bar{\theta}_i - \widetilde{k}_i \right) = \left(\bar{\theta}_i - \widetilde{k}_i \right) \frac{\delta_i}{2} > 0
$$

Second, for $(\theta_1, \theta_2) \in B$ both tasks obtain less than their needs both under $\mathbf{M}_D$ and $\mathbf{I}$.[39] Thus

$$
\left(J_{CES}^{M_D}(\gamma) - J_{CES}^{I}(\gamma) \right) \Big|_{(\theta_1, \theta_2) \in B}
$$

---

[39] This can be seen by observing that mechanism $\mathbf{M}_D$ restricts the resources to both tasks only if mechanism $\mathbf{I}$ restricts total resources to system I. This follows by the observation that $\widetilde{k}_1 + \widetilde{k}_2 > \hat{k}$ for γ

$$
= -\int_B \left(\sum_{i=1}^2 \frac{\gamma}{\beta_i'} \left(\hat{k} - \theta_1 - \theta_2 \right)^2 - \frac{\gamma}{\beta_1 + \beta_2} \left(\theta_1 + \theta_2 - \hat{k} \right)^2 \right) dF^1 dF^2
$$

$$
= -\gamma \left(\bar{\theta}_1 + \bar{\theta}_2 - \hat{k} \right)^2 \left(\frac{1}{\beta_1'} + \frac{1}{\beta_2'} - \frac{1}{\beta_1' + \beta_2'} \right) \Pr\left[B\right]
$$

Using both estimates we obtain the lower bound

$$
J_{CES}^{M_D}(\gamma) - J_{CES}^I(\gamma) \geq \left(\sum_{i=1}^2 (1 - F^i(\widetilde{k}_i)) + \Pr\left[B\right] \right) \left(\bar{\theta}_1 + \bar{\theta}_2 - \hat{k} \right)^2 (C_1(\gamma) - C_2(\gamma)) \quad (32)
$$

where

$$
C_1(\gamma) = \frac{\sum_{i=1}^2 \delta_i \left(\bar{\theta}_i - \widetilde{k}_i \right) E\left[\theta_i - \widetilde{k}_i | A_i \right] \Pr\left[A_i\right]}{\left(\sum_{i=1}^2 (1 - F^i(\widetilde{k}_i)) + \Pr\left[B\right] \right) \left(\bar{\theta}_1 + \bar{\theta}_2 - \hat{k} \right)^2}
$$

$$
C_2(\gamma) = \left(\frac{1}{\beta_1'} + \frac{1}{\beta_2'} - \frac{1}{\beta_1' + \beta_2'} \right) \frac{\gamma \Pr\left[B\right]}{\sum_{i=1}^2 (1 - F^i(\widetilde{k}_i)) + \Pr\left[B\right]}
$$

The following limits follow by application of L'Hôpital's rule

$$
\lim_{\gamma=1^-} \frac{\bar{\theta}_i - \widetilde{k}_i}{\bar{\theta}_1 + \bar{\theta}_2 - \hat{k}} = \frac{-\left. \frac{d\widetilde{k}_i}{d\gamma} \right|_{\gamma=1^-}}{-\left. \frac{d\hat{k}}{d\gamma} \right|_{\gamma=1^-}} > 0, \quad \lim_{\gamma=1^-} \frac{\bar{\theta}_i - E\left[\theta_i | A_i\right]}{\bar{\theta}_1 + \bar{\theta}_2 - \hat{k}} = \frac{-\left. \frac{dE[\theta_i | A_i]}{d\gamma} \right|_{\gamma=1^-}}{-\left. \frac{d\hat{k}}{d\gamma} \right|_{\gamma=1^-}} > 0
$$

$$
\lim_{\gamma=1^-} \frac{\Pr\left[B\right]}{\sum_{i=1}^2 (1 - F^i(\widetilde{k}_i)) + \Pr\left[B\right]} = \frac{g\left(\bar{\theta}_1 + \bar{\theta}_2 \right) \left. \frac{d\hat{k}}{d\gamma} \right|_{\gamma=1^-}}{-\sum_{i=1}^2 f^i(\bar{\theta}_i) \left. \frac{d\widetilde{k}_i}{d\gamma} \right|_{\gamma=1^-} - g\left(\bar{\theta}_1 + \bar{\theta}_2 \right) \left. \frac{d\hat{k}}{d\gamma} \right|_{\gamma=1^-}} = 0
$$

close to 1. Indeed, since

$$
\left. \frac{\partial \widetilde{k}_i}{\partial \gamma} \right|_{\gamma=1} = \beta_i' \frac{\bar{\theta}_i + \theta_0 - k}{\beta_0' \left(1 + \frac{\delta_i}{2} \beta_i' \right)} > 0,
$$

comparing (30) and (31) we have that

$$
\sum_{i=1}^2 \left. \frac{\partial \widetilde{k}_i}{\partial \gamma} \right|_{\gamma=1} = \sum_{i=1}^2 \beta_i' \frac{\bar{\theta}_i + \theta_0 - k}{\beta_0' \left(1 + \frac{\delta_i}{2} \beta_i' \right)}
$$

$$
< \sum_{i=1}^2 \beta_i' \frac{\bar{\theta}_i + \theta_0 - k}{\beta_0'}
$$

$$
< \sum_{i=1}^2 \beta_i' \frac{\bar{z} + \theta_0 - k}{\beta_0'} = \frac{1}{3} \left. \frac{\partial \hat{k}}{\partial \gamma} \right|_{\gamma=1^-}
$$

Therefore $\left. \frac{\partial}{\partial \gamma} \left(\sum_{i=1}^2 \widetilde{k}_i - \hat{k} \right) \right|_{\gamma=1} < 0$ implying that for γ in a neighborhood of $\gamma = 1$, $\widetilde{k}_1 + \widetilde{k}_2 > \hat{k}$.

$$\lim_{\gamma=1^-} \frac{\sum_{i=1}^2 \Pr[A_i]}{\sum_{i=1}^2 (1 - F^i(\widetilde{k}_i)) + \Pr[B]} = 1$$

which imply that

$$\lim_{\gamma \to 1^-} C_1(\gamma) > 0 \quad \text{and} \quad \lim_{\gamma \to 1^-} C_2(\gamma) = 0$$

Therefore there exists a neighborhood M of $\gamma = 1$ where $\gamma > \max_i \left[\frac{\beta_i'}{\beta_0' + \beta_i'} \right]$ and $J_{CES}^M(\gamma) \geq J_{CES}^{M_D}(\gamma) > J_{CES}^I(\gamma)$ for $\gamma \in M, \gamma \neq 1$.

(b) **Relative performance of Integration vs. Specialization as $\gamma \to 0$.**
Given $\theta_0 > k$, when $\gamma = 0$ the CES assigns all resources to system 0 both under aggregation and specialization implying $J_{CES}^I(0) = J_{CES}^M(0)$. By the Milgrom-Segal Envelope Theorem (Milgrom and Segal, 2002) we have

$$\left. \frac{\partial J_{CES}^I(\gamma)}{\partial \gamma} \right|_{\gamma=0^+} = -\frac{1}{\beta_1' + \beta_2'} E\left[(\theta_1 + \theta_2)^2 \right]$$

$$\left. \frac{\partial J_{CES}^M(\gamma)}{\partial \gamma} \right|_{\gamma=0^+} = -\frac{1}{\beta_1'} E\left[\theta_1^2 \right] - \frac{1}{\beta_2'} E\left[\theta_2^2 \right]$$

Then

$$\left. \frac{\partial \left(J_{CES}^I(\gamma) - J_{CES}^M(\gamma) \right)}{\partial \gamma} \right|_{\gamma=0^+} = \frac{\beta_1' \beta_2'}{(\beta_1' + \beta_2')} E\left[\left(\frac{\theta_1}{\beta_1'} - \frac{\theta_2}{\beta_2'} \right)^2 \right] > 0$$

where the strict inequality follows from independence of θ_1 and θ_2. Therefore, there exists a neighborhood N of $\gamma = 0$ where $J_{CES}^I(\gamma) > J_{CES}^M(\gamma)$ for $\gamma \in N, \gamma \neq 0$.

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