DISCUSSION PAPER SERIES

No. 8352

WAR SIGNALS: A THEORY OF TRADE, TRUST AND CONFLICT

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***DEVELOPMENT ECONOMICS and
INTERNATIONAL MACROECONOMICS***

Centre for Economic Policy Research

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ABSTRACT

War Signals: A Theory of Trade, Trust and Conflict*

We construct a dynamic theory of civil conflict hinging on inter-ethnic trust and trade. The model economy is inhabitated by two ethnic groups. Inter-ethnic trade requires imperfectly observed bilateral investments and one group has to form beliefs on the average propensity to trade of the other group. Since conflict disrupts trade, the onset of a conflict signals that the aggressor has a low propensity to trade. Agents observe the history of conflicts and update their beliefs over time, transmitting them to the next generation. The theory bears a set of testable predictions. First, war is a stochastic process whose frequency depends on the state of endogenous beliefs. Second, the probability of future conflicts increases after each conflict episode. Third, "accidental" conflicts that do not reflect economic fundamentals can lead to a permanent breakdown of trust, plunging a society into a vicious cycle of recurrent conflicts (a war trap). The incidence of conflict can be reduced by policies abating cultural barriers, fostering inter-ethnic trade and human capital, and shifting beliefs. Coercive peace policies such as peacekeeping forces or externally imposed regime changes have instead no persistent effects.

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Submitted 08 April 2011

1 Introduction

Over 16 million people are estimated to have died due to civil conflicts in the second half of the 20th century (cf. Fearon and Laitin, 2003). Such events are geographically concentrated and highly persistent: As many as 68 percent of all outbreaks took place in countries where multiple conflicts were recorded.[1] This observation has motivated a large body of research searching for institutional roots of civil conflict. Yet, weak institutions are unlikely to be the sole explanation. For instance, various studies show that democracy has no systematic effect on the risk of civil war after controlling for other factors such as ethnic diversity, GDP per capita and natural resource abundance.[2] Moreover, several developing countries with relatively solid institutions plunge into recurrent conflicts, whereas other countries with weak institutions and high ethnic cleavages never experience civil conflicts.[3]

In this paper, we propose a theory based on asymmetric information and social learning, arguing that cross-community distrust and pessimism about the viability of peaceful trade can make societies fall into vicious spirals of violence and civil conflicts. Consistent with this view, Figure 1 shows that a country-level measure of average trust is negatively correlated with the frequency of civil wars during the period 1981-2008.[4] Clearly, causality can run both ways: whilst distrust between communities increases the probability of civil conflict, war erodes cross-community trust.

In our theory, the dynamic link between trust and conflict operates through trade. On the one hand, since conflict disrupts cross-community business relationships (hereafter, *trade*), thriving trade deters war by raising its opportunity cost. Conversely, scant trade opportunities make conflict more likely. On the other hand, trade hinges on trust. Many business relationships involving members of different communities (e.g., seller-buyer, employer-employee, supplier-producer, lender-borrower) require specific human-capital investments on both sides (e.g., learning the language or the customs of the other group, maintaining a cross-community social network). How much each community is prepared to invest depends then on the belief that the other community will also invest (hereafter, *trust*). Therefore, trust fosters trade, which in turns deters civil conflict.

We formalize our ideas with the aid of a dynamic model, in which two groups can engage in

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[1] This number is based on the sample covered in Collier and Hoeffler (2004). DeRouen and Bercovitch (2008) find that even more than three quarters of all civil wars are due to "enduring internal rivalries" between ethnic groups that repeatedly get into war against each other. See, among others, Collier and Hoeffler (2004), Collier, Hoeffler and Rohner (2009), Quinn, Mason and Gurses (2007), and Walter (2004), who have found that past wars are strong predictors of future wars.

[2] See, e.g., Fearon and Laitin (2003), Montalvo and Reynal-Querol (2005), and Collier and Rohner (2008).

[3] Columbia, India, Turkey, Sri Lanka and the Philippines fare relatively well in terms of democracy and other institutional indicators, conditional on their stage of development. Yet, they are prone to civil conflicts. Interestingly, the average level of trust as measured by the World Values Survey is significantly lower in these countries than in the average non-OECD country (0.16 vs. 0.22). On the opposite front, Bhutan, Cameroon, Gabon, Kazahstan, Togo, China and Vietnam have low scores on democracy and high ethnic fractionalization, but no recent history of civil war. Data on trust are only available for China and Vietnam among these countries. Their average trust is 0.51, even larger than in the average OECD country.

[4] Trust is a dummy variable and takes a value of 1 if "Yes" is replied to question A165 ("Can most people be trusted?") of the World Values Survey (2010). The civil war data is from PRIO (2010). The correlation is robust to conditioning on several covariates, including democracy.
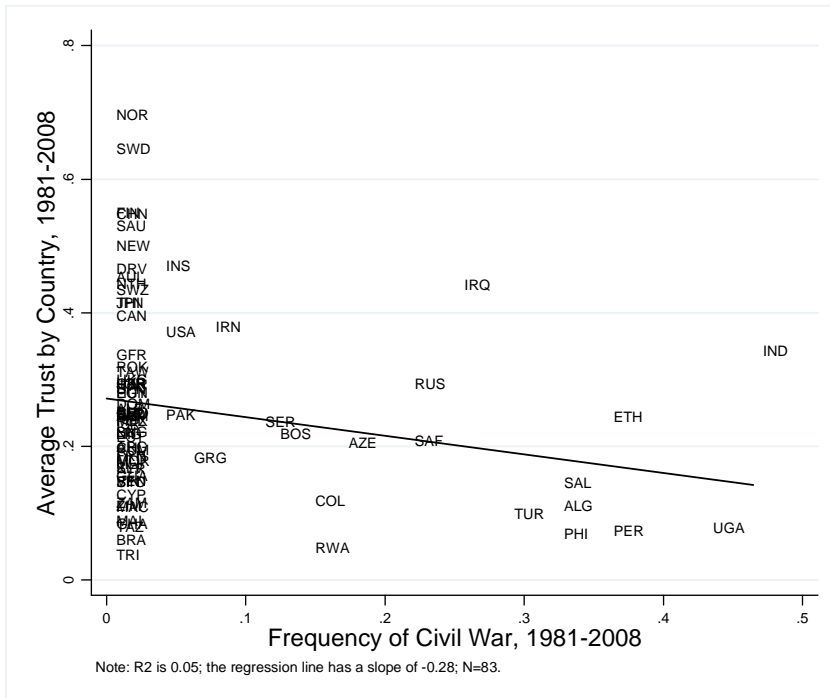
Figure 1: Country-Level Trust and Frequency of Civil War

inter-ethnic trade partnerships, necessitating human-capital investments on both sides in order to be carried out successfully. There are strategic complementarities to such investments: The proportion of investors in a group increases the other group's expected return to investments by increasing their probability to find trading partners. Investment costs are heterogenous across agents. The key parameter is a group-specific fixed effect, which pins down the average investment cost (*propensity to trade*), and about which information is asymmetric: One group ignores the average propensity to trade of the other group, and forms and updates beliefs about it. Finally, one group can stage war against the other, at the cost of foregoing inter-ethnic trade in the current period. Not only does the onset of a conflict destroy current trade but it also undermines future trust and trade by signaling to the victim group that the attacking group regards the opportunity cost of war as low, i.e., has a low propensity to trade. Since beliefs are transmitted across generations, a war today reduces future investments and trade, thereby increasing the probability of future conflicts.

The theory's predictions are consistent with the negative correlation between war and trust in Figure 1. They also imply that wars are persistent: each outbreak increases the probability that a country will fall again into a civil war in the future. This is consistent with the empirical evidence that peace duration reduces significantly the risk of future civil war, even after controlling for country fixed effects (see Martin, Mayer and Thoenig 2008b). Furthermore, "accidental wars", e.g., aggressions initiated by a belligerent minority of a group against the will of the majority of the group itself, may

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lead to the permanent breakdown of peaceful relationships across groups even in spite of good economic fundamentals. More precisely, repeated such episodes can trigger the collapse of trust, and plunge a society into recurrent conflicts (*war traps*) where inter-ethnic trade relationships are weak even in peace times. This result is shown to be robust to different informational assumptions. While in our benchmark model the only informative signal is the peace-and-war history, we relax the informational assumptions in an extension where we allow traders to acquire direct information about the other group's type. Learning traps are found to be robust in such an environment.

The analysis yields a number of policy implications. First, increasing the profitability of inter-ethnic trade reduces the probability of recurrent wars. Thus, policies abating barriers, such as educational policies promoting the knowledge of several national languages, as well as subsidies to trade-enhancing human-capital investments can help sustain peace. Second, policies targeting beliefs may be useful. For instance, credible campaigns documenting and publicizing success stories of inter-ethnic business relationships, joint ventures, etc. can reduce the probability of future conflicts through changing beliefs about trade opportunities. On the contrary, attempts to *impose* peace through coercion – e.g., peacekeeping forces or externally-imposed regime changes – have ultimately no persistent effects. This is consistent with empirical studies in the conflict literature that we discuss in more detail below.

1.1 Motivating evidence

The *war-deterring effect of trade* has been widely documented in the empirical literature (see, e.g., Martin, Mayer and Thoenig 2008a). While most formal evidence is about international trade, a number of case studies document that inter-ethnic trade has a similar effect within countries. In a seminal study, Horowitz (2000) argues that strong economic inter-group complementarities create powerful incentives for inter-ethnic peace. He shows that in various Asian countries like Indonesia, Myanmar, Malaysia and India "middleman minorities" have often been protected from political violence, as they provide very valuable services to the local ethnic majority. According to Horowitz (2000: 117), "by and large, (...) the supposed economic resentments of businessmen by their customers often do not exist. (...) Rather than resenting alien rural traders, peasants often welcome them". A similar conclusion is reached by Jha (2008) who studies Hindu-Muslim interactions using town-level data for India. He finds that during Medieval times in India's trade ports Hindus and Muslims could provide each other with complementary services, and argues that this led to religious tolerance and a lower level of political violence in Medieval trade ports than in other Indian towns. Interestingly, such a situation persists today. In a similar vein, Varshney (2001, 2002) argues that the existence of inter-ethnic civil society involvement and business associations can curtail the potential for riots in India. According to Varshney, for the prevention of ethnic conflict "trust based on *interethnic*, not *intraethnic*, networks is critical" (2001: 392). Bardhan (1997) provides anecdotal evidence of waning inter-ethnic business links due to exogenous factors lowering the opportunity costs of conflict, and resulting in the outbreak

of riots.[5]

Also in other parts of the world one can observe that inter-ethnic trade prevents conflict. Olsson (2010) shows that an exogenous change in climatic conditions (i.e. a severe drought) has led to a collapse of the inter-ethnic trade between farmers and herders in Sudan's Darfur region, and that this has been followed by the outbreak of conflict.[6] Similarly, Porter et al. (2010) carried out in-depth interviews with market traders in Nigeria and have found that in the context of inter-ethnic tensions, the existence of strong inter-ethnic trade links and associations can prevent the outbreak of full-blown riots.

There is also evidence that *war lowers future inter-group trust and trade*. At the country-pair level, Guiso, Sapienza and Zingales (2009) provide causal evidence on the sample of European countries that a more intense history of bilateral warfare over the 1000-1970 period reduces the current level of bilateral trust; this in turn affects negatively the current levels of bilateral trade, foreign direct investments, and portfolio investments. Glick and Taylor (2010) use a larger panel of country pairs and find that inter-state wars have a strong and persistent disruptive effect on future bilateral trade. Within-country evidence is more scattered, since there are no formal statistics of inter-ethnic trade. Yet, there are several telling case studies. In Rwanda throughout the 1980s inter-ethnic trust was high and sustained symbiotic business relationships, cooperation in agricultural production associations and mixed rotating savings groups involving both Hutus and Tutsis (Ingelaere, 2007; Pinchotti and Verwimp, 2007). Survey data indicate that trust plunged as of October 1990, after localized fighting erupted in northern Rwanda between the Rwandan Patriotic Front (RPF), a rebel group formed from Tutsi refugees in Uganda, and the Hutu-dominated government of Habyarimana (Ingelaere, 2007). The collapse of trust was followed by waning trade and business links between the communities, until inter-ethnic cooperation ceased altogether at the onset of the 1994 genocide.[7] Even many years after the conflict the average inter-ethnic trust levels are significantly lower than in the 1980's (Ingelaere, 2007) and also inter-ethnic trade is persistently lower (Colletta and Cullen, 2000).

Similarly, UNICEF (2003) documents that in several of Darfur's conflicts inter-group trust and inter-ethnic business have collapsed in the aftermath of fighting. For example, the civil war has resulted in the breakdown of the traditional economic arrangements between nomads and farmers regulating

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[5]"On the Moradabad riots of 1980: The higher wages in the brass industry and entrepreneurship brought about not only greater prosperity among the Muslims, it also began to lessen the importance of the middlemen, often Hindu, in business transactions. Some of the Muslim entrepreneurs even managed to get direct orders from West Asian countries. The Hindu middlemen thus edged out began to rally round the Jan Sangh (now BJP) which has its base among petty businessmen" (Bardhan, 1997: 1397).

[6]Also UNICEF (2003) finds that exogenous collapses in inter-ethnic markets have resulted in conflicts in Darfur: "The groups confronting each other in the current conflicts have a long history of guarded cooperation and relative peaceful coexistence. In the past, they exchanged goods and services; indeed some of the herds that the Arab nomads reared belonged to wealthy Fur who did not opt to become nomads themselves. The Fur sold most of their herds on the onset of the drought in 1982/83. This was considered a severance of economic relations, which strained the relations between the Fur and the Arabs" (UNICEF, 2003: 53).

[7]Colletta and Cullen, (2000:45) find that while vertical (within-group) social capital remained intact, "conflict deeply penetrated such forms of horizontal social capital as exchange, mutual assistance, collective action, trust and the protection of the vulnerable. [...] The use of credit in exchanges was common in preconflict Rwanda. This practice has diminished over time, in part due to decreased levels of trust as a consequence of warfare".

the use of pastureland and access to water in the Upper Nice region. This collapse of economic cooperation spurred kidnappings and other forms of inter-ethnic violence, triggering an escalation of further local conflicts. The same pattern is also found elsewhere in Africa. Dercon and Gutierrez-Romero (2010) study the 2007 Kenyan electoral violence. Their survey data indicate that violence decreased trust between ethnic groups (while increasing trust within ethnic groups). Furthermore, after episodes of violence, people indicated that they tend to do less business with people from other ethnic groups and that they find violence more justifiable.

There are also case studies documenting the detrimental effect of war on trust and trade in Europe. Blagojevic (2009) finds that after the Bosnian war inter-ethnic trust collapsed and that the economic cooperation between the Serbian population on the one hand and the Bosniak and Croatian population on the other hand broke down, being replaced by intense inter-ethnic competition. Similarly, according to Kaufman (1996) the war in Moldova led to a climate of distrust between the Moldavans and the Russian-speaking minority, which resulted in a substantial decline in inter-ethnic business cooperation.

1.2 Related Literature

Our paper relates to different streams of economic literature. The link between trust, specific investments and business relationships is related to a large body of literature on contractual incompleteness where successful economic relationships hinge on various forms of bilateral investments. The salience of this issue in the context of cross-community trade is emphasized by Dixit (2003). In Hauk and Saez Marti (2002) and Tabellini (2009) a costly investment leads to the adoption of pro-social norms preventing opportunistic behavior; in Greif (1994) and Rauch (1999) it leads to the development of a social network where reputation and retaliation can be enforced; in Dewatripont and Tirole (2005) it leads to the acquisition of communication tools, such as the other group's customs and language.[8]

Learning traps are related to the literature on herding, social learning, and informational cascades. This includes Banerjee (1992); Bikhchandani, Hirshleifer and Welch (1992); Ely and Valimaki (2003); Fernandez (2007) and Piketty (1995). The theory is also related to the theoretical literature on supermodular games with strategic complementarities (Baliga and Sjostrom, 2004; Chamley, 1999; Chassang and Padro i Miquel, 2010 and Cooper and John, 1988). While most of these papers emphasize the possibility of static multiplicity, in our paper we constrain parameters to yield a unique equilibrium under perfect information.[9] The dynamic nature of the model of conflict is related to Yared (2010). The importance of luck and the persistent effects of negative shocks link our contribution with Acemoglu and Zilibotti (1997). Also related to our research are the recent papers by Aghion et al. (2010) and Aghion, Algan and Cahuc (2011) focusing on the relation between public policy, on

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[8]See also the recent advertising campaign of HSBC branded "Never underestimate the importance of local knowledge", emphasizing the importance and market value of a good knowledge of the system of customs, norms and social conventions for inter-cultural business relationships.

[9]Among these papers, Chamley (1999) is the closest to us as he also studies coordination in a dynamic setting with learning and strategic complementarities. However, in his model the dynamics are driven by exogenous changes in the unobservable fundamentals and the possibility of persistence and absorbing states with learning traps is absent.

the one hand, and beliefs and norms of cooperation in the labor market, on the other hand.[10]

This paper is also related more generally to the economic literature studying conflict. Some existing theories focus on institutional failures, such as weak state capacity and weak institutions (Besley and Persson, 2010, 2011; Fearon, 2005). In Besley and Persson (2011) the lack of checks and balances implies that rent-sharing strongly depends on who is in power, thereby strengthening incentives to fight.[11] Poverty and natural resource abundance have also been found to fuel conflict, as the former reduces the opportunity cost of fighting, while the latter results in a larger "pie" that can be appropriated (cf. Torvik, 2002, and Collier and Hoeffler, 2004). Esteban and Ray (2008) emphasize the role of ethnic diversity, and argue that ethnic polarization can favor the collective action needed for appropriation by generating the right mix of capital and labor for the groups. In Rohner (2011) ethnic diversity increases the risk of conflict by reducing the reputational cost of non-cooperative behavior. While explaining why some countries are more prone to conflicts than others, most of such theories do not explain why a civil war today makes future conflict more likely. An exception is Acemoglu, Ticchi and Vindigni (2010) who argue that in weakly-institutionalized states civilian governments have incentives to select small and weak armies to prevent coups. This has the undesired consequence of making it harder for the state to end insurgency and rebellion. Collier and Hoeffler (2004) argue that current conflict makes conflict recurrence more likely due to the existence of conflict-specific capital, like cheap military equipment.

The plan of the paper is the following. Section 2 presents the benchmark model of inter-ethnic trade and conflict. Section 3 extends it to a dynamic environment where beliefs are transmitted across generations, and derives the main results. Section 4 presents a major extension where agents can learn from the observation of trade history together with warfare history. Section 5 discusses some policy implications. Section 6 concludes and discusses avenues for future research. The proofs of all Lemmas, Propositions and Corollaries are in the Appendix (proofs of Lemma 2 and Propositions 6–10 in a webpage Appendix).

2 The Static Model

2.1 Setup

The model economy is populated by a continuum of risk-neutral individuals belonging to two "ethnic groups" (A and B), each of unit mass. The interactions between the two groups are described by a two-stage game. First, group A decides whether or not to stage war against group B. Next, inter-ethnic trade may occur. No economically interesting decisions are made under the shadow of war. In case of

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[10] Aghion, Algan and Cahuc (2011) document a negative empirical correlation between the quality of labor relations and state regulation of the minimum wage. They explain this evidence with the aid of a model in which agents learn about the quality of labor relations, and where state regulation prevents workers from learning through experimentation. Their model features multiple equilibria: one characterized by good labor relations, and another characterized by low trust and strong minimum wage regulation.

[11] Another stream of literature views civil wars as failure of bargaining processes due to private information (Fearon, 1995), commitment problems (Powell, 2006), issue indivisibilities or political bias of leaders (Jackson and Morelli, 2007).

peace, each agent in group A is randomly matched with an agent in group B, and trade only occurs if both agents have made a human capital investment prior to the match.[12] In this case, each trading partner receives a return z, where we assume that $0 < z \leq 1$.

Investment decisions are based on an *ex-ante* comparison between costs and benefits. We define ι to be the investment cost net of the return that accrues to the investor irrespective of trade (note that ι can be negative). ι is heterogenous across agents – reflecting individual shocks to ability and investment opportunities. It is assumed to be *i.i.d.* across agents, and to be drawn from a probability density function (p.d.f.), $f^J : \mathbb{R} \to \mathbb{R}^+$, where $J \in \{A, B\}$. We denote by $F^J : \mathbb{R} \to [0,1]$ the corresponding cumulative distribution function (c.d.f.). Group A can be of two types: $f^A \in \{f^+, f^-\}$, and accordingly $F^A \in \{F^+, F^-\}$, where F^- first-order stochastically dominates F^+. Since investment costs are a barrier to trade, we say that group A has a high propensity to trade (A is of the *high* type) when $F^A = F^+$, and has a low propensity to trade (A is of the *low* type) when $F^A = F^-$. Instead, we assume that F^B has a unique realization. This assumption is for tractability, as it avoids the difficulty of a two-dimensional learning process.

We introduce a technical assumption that is maintained throughout the rest of the paper.

Assumption 1 *There exists $\varepsilon > 0$ such that the p.d.f.'s $f^B(\iota)$, $f^+(\iota)$ and $f^-(i)$ are non-decreasing in the subrange $\iota \in [0, z + \varepsilon]$.*

Assumption 1 ensures that, at least in the interval $[0, z + \varepsilon]$, there are fewer people with a low (or negative) than with a high net investment cost. The implications of this assumption will be explained in the next section.

The net benefit of war to group A is assumed to be stochastic and is denoted by $\tilde{V} \in \{V_L, V, V_H\}$. $\tilde{V}$ is interpreted as the value of the resources grabbed through war minus the military or psychological costs associated with war. Group A's gains from trade that are foregone by staging war are denoted by $S^k \in [\hat{S}^{\min}, \hat{S}^{\max}]$, where $k \in \{+, -\}$. S^k is the (endogenous) opportunity cost of war. Group A decide whether or not to stage war by a unanimity rule. When war is decided upon, all agents in group A know the realization of $\tilde{V}$, but ignore the realization of their individual cost ι.

Assumption 2 $V_L < \hat{S}^{\min} < V < \hat{S}^{\max} < V_H.$

The intermediate realization, V, is the most frequent one, and is referred to as *business as usual* (BAU). Under BAU, staging war is profitable if $V > S^k$, and unprofitable otherwise. V_H corresponds to a situation in which the military cost of making war is exceptionally low, implying that the benefit of war exceeds its opportunity cost. Conversely, V_L corresponds to an unusually high cost to stage war, due, e.g., to a failure in the collective action.[13] As $S^k \geq \hat{S}^{\min}$, peace necessarily occurs when $\tilde{V} = V_L$;

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[12]We interpret this investment as the familiarization with the customs of the other community, such as learning a foreign language, becoming aware of informal rules and traditions, getting in touch with external networks, etc.

[13]The stochastic process can alternatively be driven by shocks to the political process or psychological costs of conflict. When $\tilde{V} = V_H$, the perceived cost of staging war is low, due to an explosion of hatred (Gurr, 1970) or due to the capture

likewise, as $S^k \leq \hat{S}^{\max}$, war erupts when $\tilde{V} = V_H$. We refer to the infrequent realizations V_H and V_L as a *war shock* and a *peace shock*, with probabilities $\lambda_W < 1/3$ and $\lambda_P < 1/3$, respectively. Hence, the probability of BAU is $1 - \lambda_W - \lambda_P > 1/3$. These shocks echo the recent literature that views the onset of war as "stochastic" (Gartzke, 1999), in particular due to stochastic shocks to coordination costs of rebellion (Collier and Hoeffler, 1998), or to rebel capability (Gates, 2002; Buhaug, Gates and Lujala, 2009).

2.2   Perfect information

To establish a benchmark, we first consider the case in which group A's type is public knowledge. In this case, war spoils trade but conveys no information. Consider the investment problem during peace. Due to random matching, the expected gain from trade for an investor in group A is $z \cdot n_B$, whereas the expected gain from trade for an investor in group B is $z \cdot n_A$. Thus, all agents with $\iota \leq zn_B$ (resp. $\iota \leq zn_A$) in group A (resp. group B) invest. The Nash equilibrium conditional on group A type, $k \in \{+, -\}$, is given by the fixed point

$$\left\{ n_A^k, n_B^k \right\} = \left\{ F^k(zn_B^k), F^B(zn_A^k) \right\} \tag{1}$$

Although the strategic complementarity in investments could yield multiple Nash equilibrium, Assumption 1 is sufficient to ensure that the Nash equilibrium is unique under perfect information. This restriction allows us to focus more sharply on the dynamic interaction between belief formation and warfare.

The trade surplus accruing to group A if peace is maintained is given by

$$S^k \equiv z \cdot n_A^k \cdot n_B^k - \int^{zn_B^k} \iota dF^k(\iota) \tag{2}$$

where $n_A^k \cdot n_B^k$ represents the number of successful trade relationships and $\int^{zn_B^k} \iota dF^k(\iota)$ is the aggregate investment cost. Note that the previous equation implies that necessarily $S^k \in [\hat{S}^{\min}, \hat{S}^{\max}]$ with $\hat{S}^{\min} = -\int^z \iota dF^+(\iota)$ and $\hat{S}^{\max} = z$.

Proposition 1 *Under Assumption 1 and perfect information, the Nash Equilibrium of the invest-ment/trade continuation game conditional on $k \in \{+, -\}$ exists and is unique.*
The equilibrium investments are given by $\{n_A^-, n_A^+, n_B^-, n_B^+\}$ consistent with equation (1), where $n_A^- \leq n_A^+$ and $n_B^- \leq n_B^+$.
The equilibrium trade surplus accruing to group A is given by S^k – as described by (2). Moreover, $S^- \leq S^+$.

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of the political process by a biased political elite (Jackson and Morelli, 2007). On the contrary, a temporary political moderation or a high reluctance to start a conflict would lead to $\tilde{V} = V_L$. Yet another interpretation is that there are shocks to the beliefs of group A about the net benefits of war which are driven by the acquisition of private information. This interpretation is related to Fearon (1995).

Since $S^- < V < S^+$, the low type always stages war while the high type retains peace under BAU. Thus, war is more frequent when $k = -$ (probability is $1 - \lambda_P$) than when $k = +$ (probability is λ_W).

2.3 Asymmetric Information

In the rest of the paper we assume that group B can observe neither group A's type nor the realization of $\tilde{V}$. In this environment, war has a signaling component: staging war signals a low propensity to trade, although the signal is not perfectly revealing.[14] For instance, if $S^- < V < S^+$ and war is staged, group B cannot be sure that A is the low type, since war may have erupted due to a war shock. We denote by π_{-1} the prior belief held by group B that $k = +$. Beliefs are common knowledge, and are updated using Bayes' rule after the observation of war or peace. We denote by (π_W, π_P) the posterior belief conditional on war and peace, respectively.

The timing of the game is the following.

1. The *war stage*: all agents in group B receive the prior belief π_{-1}, all agents in group A observe the state $\tilde{V}$, and group A decides whether to stage war or to keep peace.

2. The *investment/trade stage*: agents in group B update their beliefs. If there is war, there are no further choices and all agents receive their payoffs. If there is peace, all agents in both groups draw ι from the distribution of net costs, and each of them decides in a decentralized way whether or not to invest. Finally, the two groups are randomly matched to trade partners, gains from trade are realized, and consumption occurs.

The equilibrium concept is Perfect Bayesian Equilibria (PBE).

Definition 1 *A strategy for an agent in population A specifies for each of her possible types, $k \in \{+, -\}$ and for each state $\tilde{V} \in \{V_L, V, V_H\}$, a war action ("stage war" or "keep peace"), and, for each possible realization of the investment cost, ι, an investment action ("invest" or "not invest"). A strategy for an agent in population B specifies an "investment action" ("invest" or "not invest") for each of the possible realizations of the investment cost, ι. A PBE is a strategy profile, a belief system and a triplet $\left(n_A^-, n_A^+, n_B\right) \in [0,1]^3$ such that: (i) in the investment/trade continuation game all agents choose their investment so as to maximize the expected pay-off given the posterior beliefs after peace (π_P) and the realization of the net investment cost (ι); $\left(n_A^-, n_A^+, n_B\right)$ yields the associated measure of agents who optimally invest in group A for each type, $k \in \{+, -\}$, and for group B, respectively. (ii) all agents in group A choose unanimously the probability of staging war on group B so as to maximize their expected utility, given group A's type (k), the state $\tilde{V} \in \{V_L, V, V_H\}$ and beliefs (π_{-1}), (iii) beliefs are updated using Bayes' rule.*

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[14]Note that $\tilde{V}$ is neither observable ex-ante nor verifiable ex-post to group B. Otherwise, the process of belief updating would be more complicated. Also, the fact that each generation plays only once and that there is no intergenerational altruism rules out the standard complications associated with signalling games.

2.3.1 Investment/Trade Continuation Game

We solve the PBE backwards, starting from the Nash equilibrium of the investment/trade continuation game under peace. Since the investments of agents in group A are subject to no uncertainty, group A's reaction function continues to be given by $F^k(zn_B)$, with $k \in \{+, -\}$. However, since n_A depends on the unknown type k, group B faces some uncertainty, and its reaction function becomes $F^B(zE_B(n_A \mid \pi_P)) = F^B(z[\pi_P n_A^+ + (1 - \pi_P) n_A^-])$. The equilibrium proportions of investors in the two groups satisfy the following fixed point:

$$\{n_A^-, n_A^+, n_B\} = \{F^-(zn_B), F^+(zn_B), F^B(z[\pi_P F^+(zn_B) + (1 - \pi_P) F^-(zn_B)])\} \qquad (3)$$

Proposition 2 characterizes the set of Nash equilibria of the investment/trade game.

Proposition 2 *Under Assumption 1 and given a belief $\pi_P \in (0, 1)$, the Nash Equilibrium of the investment/trade continuation game conditional on $k \in \{+, -\}$ exists and is unique.*
The equilibrium investments are given by $\{n_A^-(\pi_P), n_A^+(\pi_P), n_B(\pi_P)\}$ implicitly defined by equation (3). $n_A^-(\pi_P)$, $n_A^+(\pi_P)$ and $n_B(\pi_P)$ are continuous and weakly increasing. Moreover, $n_A^-(\pi_P) \leq n_A^+(\pi_P)$.
The equilibrium trade surplus accruing to group A, $S^k(\pi_P)$, is given by (2) and is weakly increasing in π_P. Moreover, $S^-(\pi_P) \leq S^+(\pi_P)$.

Note that trust affects the investments of both groups, due to the strategic complementarity. Pessimistic beliefs (i.e., low π_P), induce agents in group B to expect that only few agents in group A will invest, determining a low n_B. In turn, a low n_B reduces the proportion of investors in group A, whatever its true type $k \in \{-, +\}$. As a result both S^+ and S^- are increasing in trust. Figure 2 plots the functions $S^+(\pi_P)$ and $S^-(\pi_P)$ in the case of a uniform distribution of investment costs.[15]

2.3.2 War Decision and PBE

In this subsection, we analyze the decision of group A of whether or not to stage war. As discussed above, such decision is based on a comparison between the opportunity cost of war, given by (2), and the stochastic realization of its benefit, $\tilde{V}$. Since S^+ and S^- depend on posterior beliefs, we must first characterize the belief-updating process. To this aim it is useful to rescale beliefs in term of likelihood ratio and to introduce some new notation.

Notation 1 *(i) $r_W(r_{-1})$ and $r_P(r_{-1})$ denote the mapping from prior to posterior likelihood ratios conditional on war and peace, respectively, where $r_{-1} \equiv \pi_{-1}/(1 - \pi_{-1})$ and $r_s \equiv \pi_s/(1 - \pi_s)$ for $s \in \{W, P\}$.*
(ii) $\sigma^+(r_{-1})$ and $\sigma^-(r_{-1})$ denote the probability that peace is maintained under BAU by the high and low type, respectively.

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[15]In particular, we set $z = 0.9$ and assume a uniform distribution of investment costs on the following supports: $F^B \sim [0, 1]$, $F^+ \sim [-0.25, 1]$, $F^- \sim [0, 1.25]$.

Proposition 2 and all ensuing results in the previous section can be expressed in terms of this new notation by replacing π_P by $r_P/(1+r_P)$ in each expression. Bayes' rule implies that[16]

$$\ln r_P(r_{-1}) = \ln r_{-1} + \ln \frac{\lambda_P + (1-\lambda_W - \lambda_P)\,\sigma^+(r_{-1})}{\lambda_P + (1-\lambda_W - \lambda_P)\,\sigma^-(r_{-1})}, \tag{4}$$

$$\ln r_W(r_{-1}) = \ln r_{-1} - \ln \frac{1-\lambda_P - (1-\lambda_W - \lambda_P)\,\sigma^-(r_{-1})}{1-\lambda_P - (1-\lambda_W - \lambda_P)\,\sigma^+(r_{-1})}, \tag{5}$$

where $\sigma^k(r_{-1})$ is the key choice variable:[17]

$$\sigma^k(r_{-1}) = \begin{cases} 0 & \text{if} \quad S^k\left(\frac{r_P(r_{-1})}{1+r_P(r_{-1})}\right) < V \\ \in[0,1] & \text{if} \quad S^k\left(\frac{r_P(r_{-1})}{1+r_P(r_{-1})}\right) = V \\ 1 & \text{if} \quad S^k\left(\frac{r_P(r_{-1})}{1+r_P(r_{-1})}\right) > V \end{cases}. \tag{6}$$

Intuitively, peace (war) is chosen with probability one under BAU whenever $S^k > V$ ($S^k < V$). If $S^k = V$ agents are indifferent, and the Nash equilibrium may involve mixed strategies. Proposition 3 establishes the existence of the PBE (the proof follows immediately from Proposition 2 and is omitted).

Proposition 3 *A PBE exists and is fully characterized by the set of equations (2), (3), (4), (5), (6), given a prior belief π_{-1} and the definitions in Notation 1.*

To see how beliefs are updated along the equilibrium path, note that when either $V < S^-(\pi_P)$ or $V > S^+(\pi_P)$ the probability of war is independent of group A's type: if $V < S^-(\pi_P)$, then both types retain peace under BAU ($\sigma^+(r_{-1}) = \sigma^-(r_{-1}) = 1$), while if $V > S^+(\pi_P)$, then both types stage war under BAU ($\sigma^+(r_{-1}) = \sigma^-(r_{-1}) = 0$). Therefore, in either of these two cases the observation of war or peace does not affect beliefs. On the contrary, war/peace is informative whenever $S^-(\pi_P) \leq V \leq S^+(\pi_P)$ – where one inequality is necessarily strict. In this case, the low type would stage war whereas the high type would preserve peace under BAU ($\sigma^+(r_{-1}) = 1$ and $\sigma^-(r_{-1}) = 0$). Thus, peace strengthens the trust of group B towards group A, while war undermines it. More formally, $S^-(\pi_P) \leq V \leq S^+(\pi_P) \Leftrightarrow \pi_P > \pi_{-1} > \pi_W$. We refer to this situation as an informative PBE.

Definition 2 *Given π_{-1} (and, hence, r_{-1}), a PBE is "informative" iff $\sigma^+(r_{-1}) > \sigma^-(r_P)$, or identically, $r_P(r_{-1}) > r_{-1} > r_W(r_{-1})$. A PBE is "uninformative" (or a "learning trap") iff $\sigma^+(r_{-1}) = \sigma^-(r_P)$, or identically $r_P(r_{-1}) = r_{-1} = r_W(r_{-1})$.*

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[16] After peace, the posterior is given by $\ln r_P = \ln r_{-1} + \ln \Lambda_P^+/\Lambda_P^-$ where Λ_P^k represents the probability of observing peace if the true type is $k \in \{+,-\}$. Peace signal is observed with certainty under a peace shock (an event of probability λ_P) or with probability $\sigma^k(r_{-1})$ under BAU (an event of probability $1-\lambda_P - \lambda_W$). Hence, $\Lambda_P^k \equiv \lambda_P + (1-\lambda_W - \lambda_P)\,\sigma^k(r_{-1})$. Similarly, the after-war posterior is $\ln r_W = \ln r_{-1} + \ln \Lambda_W^+/\Lambda_W^-$ where $\Lambda_W^k \equiv \lambda_W + (1-\lambda_W - \lambda_P)\left(1 - \sigma^k(r_{-1})\right)$.

[17] For instance, if under BAU the high type finds it optimal to keep peace ($\sigma^+(r_{-1}) = 1$) while the low type finds it optimal to stage war ($\sigma^-(r_{-1}) = 0$), then $r_P = ((1-\lambda_W)/\lambda_P)\cdot r_{-1}$, where the updating factor after peace is given by the probability of no war shock divided by the probability of a peace shock. Conversely, $r_W = (\lambda_W/(1-\lambda_P))\cdot r_{-1}$, where the updating factor after war is given by the probability of a war shock divided by the probability of no peace shock.
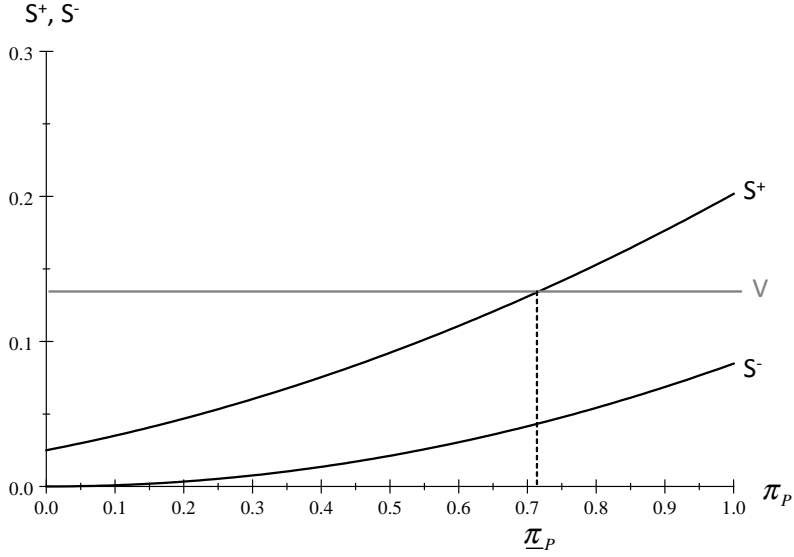
Figure 2: Surplus from trade and gains from war as function of the posterior belief

Figure 2 plots the functions $S^+(\pi_P)$ and $S^-(\pi_P)$ for the particular example mentioned before, and a particular value of the parameter V. Note that war/peace is informative if and only if $\pi_P \geq \underline{\pi}_P$: if $\pi_P < \underline{\pi}_P$, pessimistic beliefs are not updated and peace is viewed as an accident due to a shock. Note also that S^+ and S^- are functions of the posterior π_P, which is endogenous. We now discuss the equilibrium mapping from prior to posterior.

Notation 2 *Let*

$$\underline{r}^*(V) \equiv \begin{cases} \frac{(S^+)^{-1}(V)}{1-(S^+)^{-1}(V)} & if \quad V \geq S^+(0) \\ 0 & if \quad V < S^+(0) \end{cases} , \quad \underline{r}(V) \equiv \frac{\lambda_P}{1-\lambda_W}\underline{r}^*(V) \tag{7}$$

$$\bar{r}(V) \equiv \begin{cases} \frac{(S^-)^{-1}(V)}{1-(S^-)^{-1}(V)} & if \quad V \leq S^-(1) \\ \infty & if \quad V > S^-(1) \end{cases} , \quad \bar{r}^*(V) \equiv \frac{\lambda_P}{1-\lambda_W}\bar{r}(V), \tag{8}$$

where $\underline{r}^*(V) < \underline{r}(V) < \bar{r}(V) < \bar{r}^*(V)$.

Intuitively, $\underline{r}^*(V)$ is the threshold posterior belief such that both types stage war under BAU if $r_P \leq \underline{r}^*(V)$. As long as $r_{-1} \geq \frac{\lambda_P}{1-\lambda_W}\underline{r}^*(V)$, the posterior can be larger or equal to $\underline{r}^*(V)$. Likewise, $\bar{r}(V)$ is the threshold posterior belief such that both types retain peace under BAU if $r_P \geq \bar{r}(V)$. As long as $r_{-1} \geq \frac{\lambda_P}{1-\lambda_W}\bar{r}(V)$, the posterior can be larger or equal to $\bar{r}(V)$. Given these definitions, the following Lemma can be established.

Lemma 1 *An uninformative PBE exists if and only if either $r_{-1} \leq \underline{r}^*(V)$ or $r_{-1} \geq \bar{r}(V)$. Informative PBE exist if and only if $r_{-1} \in [\underline{r}(V), \bar{r}(V)]$. If $r_{-1} \in [\underline{r}(V), \underline{r}^*(V)]$, then there are multiple PBE. Otherwise, the PBE is unique.*

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Uninformative PBE are associated to either very pessimistic or very optimistic priors. Intuitively, when trust is very low (high), trade opportunities are scant (abundant) and both the high and the low type stage war (keep peace) under BAU. Informative PBE arise in an intermediate range of beliefs, although the range may be open to the right as in the case in Figure 2. Two ranges of priors have special properties: $r_{-1} \in [\underline{r}(V), \underline{r}^*(V)]$ and $r_{-1} \in [\bar{r}(V), \bar{r}^*(V)]$. When $r_{-1} \in [\underline{r}(V), \underline{r}^*(V)]$, the mapping from priors to posteriors yields multiple PBE, of which one is uninformative and two are informative. When $r_{-1} \in [\bar{r}(V), \bar{r}^*(V)]$, the mapping from priors to posteriors yields a unique PBE, but this involves randomization of the low type ($\sigma^-(r_{-1}) \in (0,1)$). See the proof for details. The webpage Appendix also provides an intuitive discussion of the set of PBE in these two ranges. For the sake of the dynamic analysis, the (small) region of multiple PBE is the source of uninteresting technical complications. While none of our results depends on a specific selection criterion, we make the following convenient assumption.

Assumption 3 *In the range of prior beliefs such that multiple PBE exists, the most informative equilibrium is selected.*

Since the rest of our analysis emphasizes the possibility for economies to fall into uninformative equilibria, this is a conservative selection criterion.

3   The Dynamic Model

In this section, we extend the analysis to a dynamic economy populated by overlapping generations of two-period lived agents. In the first period of their lives (*childhood*) agents make no economic choice, and receive the common belief from their parents' generation. In the second period (*adulthood*) agents make all economic decisions. After group A decide whether or not to stage war, adult agents in group B update their beliefs, make investment decisions and (if no war has erupted) trade, and transmit their updated beliefs to their children. We assume that all agents in the first generation are endowed with identical prior beliefs. This is a natural assumption, since all agents have access to the same past warfare history. The dynamics of beliefs is the driving force of the stochastic process of war and peace and trade in this economy.

Definition 3 *A Dynamic Stochastic Equilibrium (DSE) is a sequence of PBE with an associated sequence of beliefs such that, given an initial likelihood ratio r_0 the posterior likelihood ratio at t is the prior likelihood ratio at $t + 1$, for all $t \geq 0$.*

It is useful to distinguish between two parametric cases. In the first case, the value of war is high ($V > S^-(1)$), and the DSE can converge to an uninformative PBE with pessimistic beliefs, but not to an uninformative PBE with optimistic beliefs. In the second case, the value of war is lower ($V \in [S^+(0), S^-(1)]$), and the DSE can converge with positive probability to both an uninformative PBE with pessimistic beliefs, and an uninformative PBE with optimistic beliefs.

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3.1 High Value of War

The following Proposition characterizes the dynamic equilibrium when the value of war is high (the proof follows immediately from Lemma 1 and its proof, and is omitted).

Proposition 4 *Assume $V > S^-(1)$ and the selection criterion of Assumption 3. Let $\underline{r}(V)$ be defined as in (7). The DSE is characterized as follows:*
The PBE at time t is unique and given by Proposition 3, after setting $r_{-1} = r_{t-1}$. In particular, if $r_{t-1} < \underline{r}(V)$, then both types choose war under BAU ($\sigma^+(r_{t-1}) = \sigma^-(r_{t-1}) = 0$), and the PBE is uninformative. If $r_{t-1} \geq \underline{r}(V)$, then the low type chooses war while the high type chooses peace under BAU ($\sigma^+(r_{t-1}) = 1$ and $\sigma^-(r_{t-1}) = 0$), and the PBE is informative.
The equilibrium law of motion of beliefs is given by the following stochastic process:

$$\ln r_t = \begin{cases} \ln r_{t-1} & \text{if } r_{t-1} \in [0, \underline{r}(V)] \\[2mm] \ln r_{t-1} + (1 - I_{WAR,t}) \ln\left(\frac{1-\lambda_W}{\lambda_P}\right) - I_{WAR,t} \ln\left(\frac{1-\lambda_P}{\lambda_W}\right) & \text{if } r_{t-1} > \underline{r}(V) \end{cases} \tag{9}$$

where $I_{WAR} \in \{0,1\}$ is an indicator function of war, with the following conditional probability

$$\Pr(I_{WAR,t} = 1 | r_{t-1}) = \begin{cases} 1 - \lambda_P & \text{if } r_{t-1} \in [0, \underline{r}(V)] \\[2mm] I^- \cdot (1 - \lambda_P) + (1 - I^-) \cdot \lambda_W & \text{if } r_{t-1} > \underline{r}(V) \end{cases}. \tag{10}$$

where $I^- \in \{0,1\}$ is an indicator functions of $\{k = -\}$.

The stochastic process (9) is represented in Figure 3. Note that, conditional on r_{t-1}, the realizations of r_t are independent of k. However, the probability of peace and war do depend on k, as in equation (10).

Suppose, first, that the true state of nature is $k = -$. In this case, the probability of war is high for all levels of r_{t-1} – see equation (10). Interestingly, group B never learns with certainty that A has a low propensity to trade, as learning comes to a halt as soon as r falls below the barrier $\underline{r}(V)$. On the contrary, a low-probability long sequence of peace episodes could make group B converge almost surely to the false belief that $k = +$. However, we will see that when $k = -$ the probability that such incorrect learning occurs is zero.

Consider, next, the case in which $k = +$. In this case, if the economy starts with $r_0 > \underline{r}(V)$, the probability of war is low. Yet, an unlucky sequence of war shocks can spoil trust. As r falls below the barrier $\underline{r}(V)$, the probability of war increases discretely from λ_W to $1 - \lambda_P$. Moreover, agents rationally stop updating their beliefs, and even a long sequence of peace episodes fails to restore trust.

We introduce now a formal definition of a war-dominated learning trap.[18]

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[18]Some of the states in the WDLT are non-recurrent, namely, they cannot be reached unless they are chosen as initial conditions. Figure 3 shows the lower bound to the set of recurrent states, $\frac{\lambda_W}{1-\lambda_P}\underline{r}(V)$.
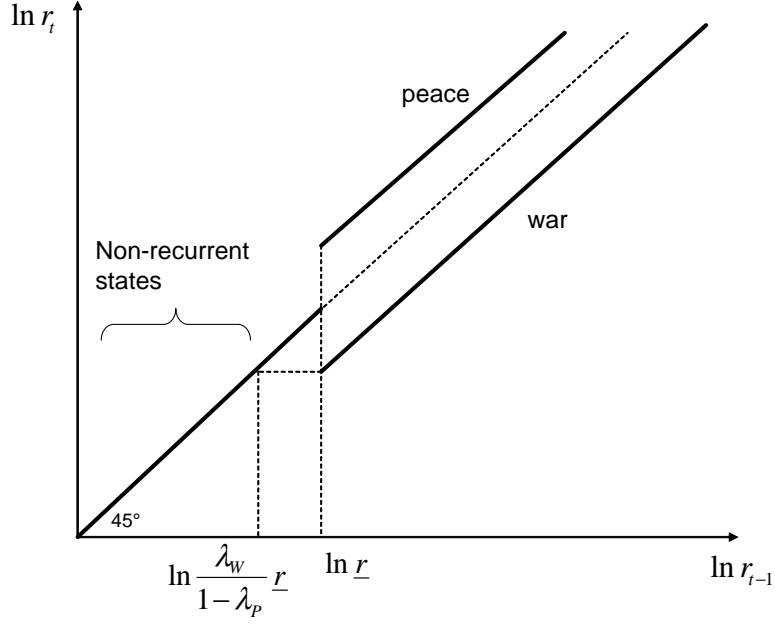
Figure 3: Dynamics of beliefs

Definition 4 *A war-dominated learning trap (WDLT) is a set of states, $\Omega_{WDLT} \subset R^+$, such that if $r_t \in \Omega_{WDLT}$ then $\forall s \geq t, r_s = r_t$, and the incidence of war is high, $\Pr(I_{WAR,s} = 1) = 1 - \lambda_P$, for all continuation paths $[r_s]_{s=t}^{\infty}$.*

It follows immediately from Proposition 4 that $\Omega_{WDLT} = [0, \underline{r}(V)]$.

Since, given any $r_t \notin \Omega_{WDLT}$, there exists a finite number of war episodes leading into Ω_{WDLT}, the economy falls into the WDLT with a positive probability. Does this imply that the DSE necessarily converge in probability to the WDLT? The answer is not straightforward, as Figure 3 suggests. On the one hand, when $r > \underline{r}(V)$, peace is common fare, so there is a high probability that trust increases over time. Positive updating never comes to a halt, as there is no upper barrier and r_t can grow without bound. On the other hand, whatever high level of trust has been achieved, a sufficiently long sequence of war shocks can destroy it and drive the economy into the WDLT. Thankfully, this need not be the case. We show below that the stochastic process of r_t can cross at some point in time the barrier $\underline{r}(V)$, but can also alternatively end in an almost correct learning, $\ln r_t \to +\infty$, without ever falling into the low-trust trap. Both long-run outcomes occur with positive probability.

The stochastic process for $\ln r_t$ is an asymmetric random walk with a drift. Given an initial condition $\ln r_0 \geq \ln \underline{r}(V)$, and an unobserved true state of nature $k \in \{+, -\}$, a key step towards the characterization of the long-run distribution is the determination of the *first passage time* of the random walk below the barrier $\ln \underline{r}(V): T = \min\{t; \ln r_t \leq \ln \underline{r}(V)\}$. If this stopping time T is finite, it means that learning stops in T and priors are trapped in the WDLT. If the stopping time T is infinite, then learning takes place at each period in time and one must study under which condition

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the process converges toward perfect learning. Applying methods from stopping-time theory (see, e.g., Shreve 2004), we establish the following Proposition.

Proposition 5 *Assume that $V > S^-(1)$ and let $r_0 \notin \Omega_{WDLT}$. Then:*

(i) If $k = -$ the DSE enters the WDLT in finite time almost surely: $Pr\{\exists T < \infty | \ r_T \in \Omega_{WDLT}\} = 1.$

(ii) If $k = +$, the DSE enters the WDLT in finite time with probability $Pr\{\exists T < \infty | r_T \in \Omega_{WDLT}\} = P_{WDLT}(r_0) \in (0,1)$. With probability $1 - P_{WDLT}(r_0)$, the DSE converges to perfect learning, i.e., $r_t \to \infty$, and to a low war incidence, $\Pr(I_{WAR,t} = 1) \to \lambda_W$. The probability $P_{WDLT}(r_0)$ has the following bounds:

$$0 < \frac{\lambda_W}{1 - \lambda_P} \frac{r(V)}{r_0} < P_{WDLT}(r_0) \leq \frac{r(V)}{r_0} < 1.$$

The technical intuition of Proposition 5 is the following: When the true state of nature is $k = -$ the stochastic process r cannot stay forever in the region of the informative equilibrium. Were this true, agents would observe an infinite number of realizations of the war/peace process. Then, by the strong law of large numbers, the empirical frequency of war/peace would converge to the underlying probabilities $(1 - \lambda_P, \lambda_P)$. Hence, agents would learn that the state of nature is $-$, namely $\ln r_t \to -\infty$. However, this would imply that at some finite T, r_T falls below $r(V)$ and the economy enters the WDLT. When the true state of nature is $k = +$, a positive-probability set of finite sequences of wars drives r_t below $r(V)$. In this case, group B stops learning and the economy is trapped in a WDLT. However, the probability of falling into a WDLT is less than unity. With the complement probability no such sequence is realized, and r never exits from the region $[r(V), \infty)$. In this case, group B observes an infinite number of realizations of the war/peace process, and the strong law of large numbers ensures then that the empirical frequency of war/peace converges to the underlying probabilities $(\lambda_W, 1 - \lambda_W)$. Thus, group B ultimately learns that the true state of nature is almost surely $k = +$.

For general parameter values, we can only provide bounds to the probability of falling into the WDLT. The expression of the bounds is very simple. Both the lower and upper bound decrease with the distance between the prior and the barrier: the higher the trust, the less likely the barrier will ever be reached. Interestingly, the probability of ever falling into a WDLT decreases after a sequence of peace episodes. Thus, peace fosters trust and decreases the probability of falling into the war trap. Conversely, a few war incidents increase the risk of an irreversible crisis. The lower bound of $P_{WDLT}(r_0)$ also increases with $\lambda_W / (1 - \lambda_P)$. This is intuitive, as this ratio is inversely related to the informational value of the war/peace signal. If this ratio were unity, the two states of nature would be observationally equivalent and there would be no learning. Moreover, as should be expected, since $r(V)$ is non-decreasing in V, the probability for the economy to fall into a WDLT is non-decreasing in the value of war, V.

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A direct consequence of Proposition 5 is that after a war the probability for an economy to enter a trap increases. $I_{WAR,t} = 1$ implies $r_{t+1}/r_t = \frac{\lambda_W}{1-\lambda_P} < 1$. Hence we get

$$1 < \frac{Pr\left\{\exists T < \infty, r_T \in \Omega_{WDLT} \mid I_{WAR,t} = 1, r_t\right\}}{Pr\left\{\exists T < \infty, r_T \in \Omega_{WDLT} \mid r_t\right\}} < \left(\frac{1-\lambda_P}{\lambda_W}\right)^2$$

In the particular case in which $\lambda_W = \lambda_P = \lambda$, we can obtain an exact characterization of $P_{WDLT}(r)$:

Corollary 1 *Assume that $V > S^-(1)$, $\lambda_W = \lambda_P = \lambda < 1/3$ and let $r_0 \notin \Omega_{WDLT}$. Then, $P_{WDLT}(r_0) = \left(\frac{\lambda}{1-\lambda}\right)^{\Delta_0}$ where $P_{WDLT}(r_0)$ is defined as in Proposition 5 and $\Delta_0 \equiv \lceil \ln(r_0/\underline{r}(V))/\ln((1-\lambda)/\lambda)\rceil$. Further, if T denotes the expected first passage time T into the trap, then $E(T \mid T < \infty) = \Delta_0/(1-2\lambda)$.*

The term Δ_0 yields the count of the net number of wars (i.e., number of wars minus number of peace episodes) which are needed to drive the initial prior r_0 below $\underline{r}(V)$. The corollary is consistent with the general discussion of Proposition 5. In particular, $P_{WDLT}(r_0)$ increases with the noise term λ. Moreover, after a war the probability of entering into the trap increases by a factor $(1-\lambda)/\lambda > 1$.

3.2   Low Value of War

In this section we analyze the case in which the value of war is low, $V \in [S^+(0), S^-(1)]$.[19] The main new implication of this case is that there are two learning traps, one with frequent and one with rare wars. A particular example is represented in Figure 4. In the range $\pi_P \leq \bar{\pi}_P$ the implications are qualitatively identical to those in Figure 2. However, in the range $\pi_P \geq \bar{\pi}_P$, the trade surplus is larger than the value of war for both types ($S^+(\pi_P) > S^-(\pi_P) > V$), and thus even the low type chooses peace under BAU. In such a range, the equilibrium is uninformative and peace prevails even if group A has a low propensity to trade.

As before, the process of revision of beliefs is characterized by equations (4)-(5) whereas the mapping of prior beliefs into equilibrium strategies is characterized by (6). There is, however, a new case: for a range of priors in the neighborhood of the threshold $\bar{\pi}_P$ ($r \in [\bar{r}^*(V), \bar{r}(V)]$) the PBE is unique and features the indifference of the low type between war and peace. Then, group A chooses a mixed strategy in the war game under BAU, $\sigma^-(r_t) \in (0,1)$. In such a range, the informativeness of the observation of war or peace decreases as we increase r until we reach $\bar{r}(V)$. As $r_{t-1} \geq \bar{r}(V)$, $\sigma^- \to 1$. The intuition for why the PBE involves randomization is as follows. First, recall that in this region $\sigma^+ = 1$. Then, if $\sigma^- = 0$, peace would be highly informative. Fast updating would increase the trade surplus, making group A regret staging war. Conversely, if $\sigma^- = 1$, peace would be uninformative. The absence of belief updating would keep the trade surplus low, making group A regret retaining peace.

We can now state the analogue of Proposition 4 for the low-V case.

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[19]We do not study the case in which $V \in [0, S^+(0)]$. This case is the mirror image of the high-V case, and features learning traps with frequent peace but no WDLT. The region of parameters that sustain this type of equilibrium is thin for reasons that will become clearer in later sections.
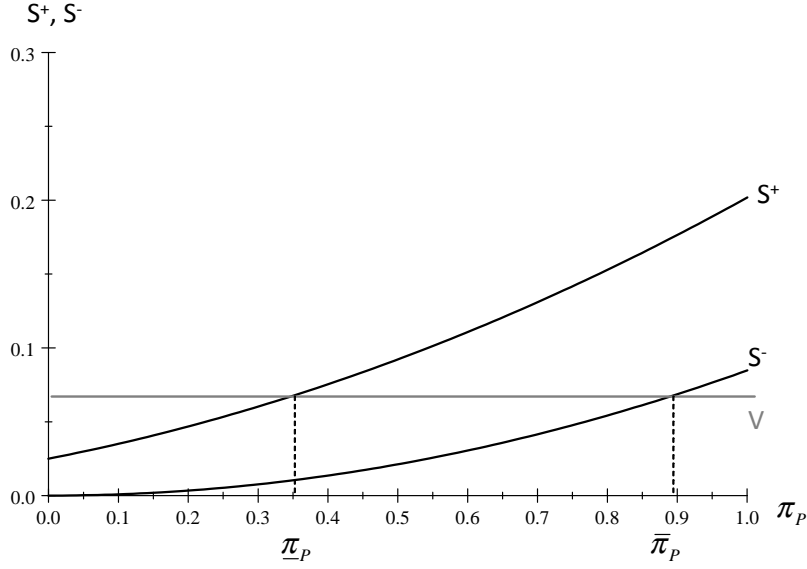
Figure 4: Surplus from trade and gains from war as a function of beliefs in the presence of two traps

Proposition 6 *Assume $V \in [S^+(0), S^-(1)]$ and the selection criterion of Assumption 3. Let $\bar{r}(V) \equiv \frac{(S^-)^{-1}(V)}{1-(S^-)^{-1}(V)}$ and $\bar{r}^*(V) \equiv \frac{\lambda_P}{1-\lambda_W}\bar{r}(V)$. The DSE is characterized as follows:*
The PBE at time t is unique and characterized by Proposition 3 after setting $r_{-1} = r_{t-1}$. In particular, if $r_{t-1} < \bar{r}^*(V)$ the DSE is characterized as in Proposition 4. If $r_{t-1} \in [\bar{r}^*(V), \bar{r}(V)]$, the high type chooses peace while the low type randomizes the war/peace choice under BAU ($\sigma^+(r_{t-1}) = 1$ and $\sigma^-(r_{t-1}) = \hat{\sigma}^-(r_{t-1}) = \frac{(1-\lambda_W)\frac{r_{t-1}}{\bar{r}(V)}-\lambda_P}{1-\lambda_W-\lambda_P} \in [0,1]$), and the PBE is informative. Finally, if $r_{t-1} > \bar{r}(V)$, then both types choose peace under BAU ($\sigma^+(r_{t-1}) = \sigma^-(r_{t-1}) = 1$), and the PBE is uninformative. Given an initial condition r_0, the equilibrium law of motion of beliefs is given by the following stochastic process:

$$
\ln r_t = \begin{cases} \ln r_{t-1} & \text{if} \quad r_{t-1} \in [0, \underline{r}(V)) \cup]\bar{r}(V), \infty) \\[2mm] \ln r_{t-1} + (1 - I_{WAR,t}) \ln\left(\frac{1-\lambda_W}{\lambda_P}\right) - I_{WAR,t} \ln\left(\frac{1-\lambda_P}{\lambda_W}\right) & \text{if} \quad r_{t-1} \in [\underline{r}(V), \bar{r}^*(V)] \\[2mm] (1 - I_{WAR,t}) \ln \bar{r}(V) + I_{WAR,t} \ln \frac{\lambda_W \bar{r}(V) r_{t-1}}{\bar{r}(V) - r_{t-1}(1-\lambda_W)} & \text{if} \quad r_{t-1} \in [\bar{r}^*(V), \bar{r}(V)] \end{cases}
$$

$$(11)$$

where $I_{WAR(t)} \in \{0,1\}$ is an indicator function of War at date t with the following conditional proba-
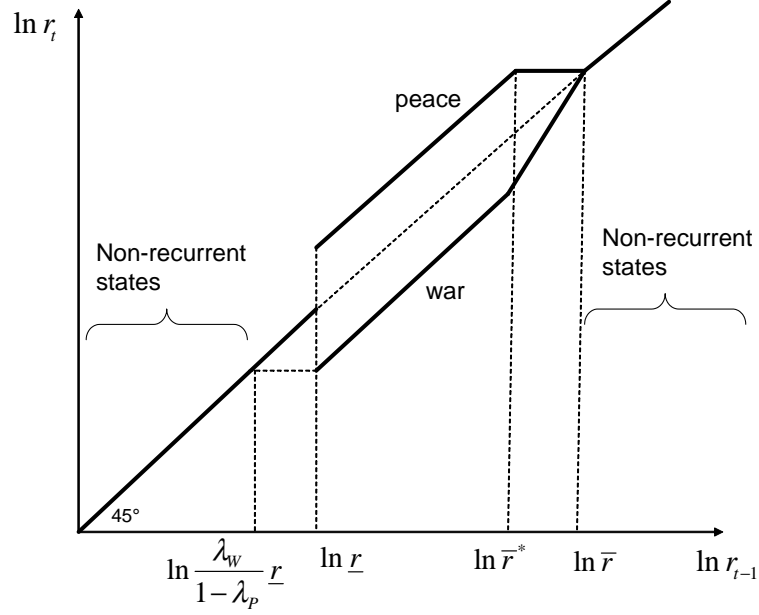
Figure 5: Dynamics of beliefs with two traps

$$
\Pr\left(I_{WAR,t} = 1 \mid r_{t-1}\right) = \begin{cases} 1 - \lambda_P & \text{if} \quad r_{t-1} \in [0, \underline{r}\left(V\right)[\\ \\ I^- \cdot (1 - \lambda_P) + (1 - I^-) \cdot \lambda_W & \text{if} \quad r_{t-1} \in [\underline{r}\left(V\right), \bar{r}^*\left(V\right)] \\ \\ I^- \cdot \left(1 - \lambda_P - (1 - \lambda_P - \lambda_W)\hat{\sigma}^-\left(r_{t-1}\right)\right) + (1 - I^-) \cdot \lambda_W & \text{if} \quad r_{t-1} \in [\bar{r}^*\left(V\right), \bar{r}\left(V\right)] \\ \\ \lambda_W & \text{if} \quad r_{t-1} \in]\bar{r}\left(V\right), \infty] \end{cases} .
$$

where $I^- \in \{0, 1\}$ is an indicator function of $\{k = -\}$.

Figure 5 illustrates the equilibrium dynamics of beliefs, as given by equation (11). The main difference with respect to Figure 3 is that in the high-prior region there is no learning, since peace is preferred by group A even when it is of a low type. Note that if the economy first enters the range $r_{t-1} \in [\bar{r}^*\left(V\right), \bar{r}\left(V\right)]$, and then peace prevails for another period, beliefs get stuck to $r_{t+s} = \bar{r}\left(V\right)$ for all $s \geq 0$. Namely, $\bar{r}\left(V\right)$ is an absorbing state. Larger r are non-recurrent states, which can only be reached if the economy starts there.

Definition 5 *A peace-dominated learning trap (PDLT) is a set of states $\Omega_{PDLT} \subset R^+$ such that, if $r_t \in \Omega_{PDLT}$ then $\forall s \geq t, r_s = r_t$ and the incidence of war is low, $\Pr\left(I_{WAR,s} = 1\right) = \lambda_W$, for all continuation paths $[r_s]_{s=t}^{\infty}$.*

It follows from Proposition 6 and the definition of a PDLT that $\Omega_{PDLT} = [\bar{r}\left(V\right), \infty)$.

Given an initial prior in the informative region ($r_0 \in [\underline{r}\,(V)\,, \bar{r}\,(V)]$), the economy starts in an informative equilibrium and there are learning dynamics. Eventually, the economy gets stuck into either of the two traps.

Proposition 7 *Assume that $V \in [S^+(0), S^-(1)]$ and let $r_0 \in [\underline{r}\,(V)\,, \bar{r}\,(V)]$. Then, in both states of nature, $k = +$ and $k = -$, the DSE exits the informative equilibrium regime almost surely, and learning comes to a halt in finite time. The final belief is such that with probability $P_{WDLT}(r_0) > 0$ the economy is in a WDLT and with probability $1 - P_{WDLT}(r_0) > 0$ it is in a PDLT.*

When $k = +$ the probability has the following bounds

$$\frac{\frac{\bar{r}(V)}{r_0} - \frac{1-\lambda_W}{\lambda_p}}{\frac{1-\lambda_P}{\lambda_W}\frac{\bar{r}(V)}{\underline{r}(V)} - \frac{1-\lambda_W}{\lambda_p}} \leq P_{WDLT}(r_0) \leq \frac{\frac{\underline{r}(V)}{r_0} - \frac{\lambda_P}{1-\lambda_W}\frac{\underline{r}(V)}{\bar{r}(V)}}{1 - \frac{\lambda_P}{1-\lambda_W}\frac{\underline{r}(V)}{\bar{r}(V)}}$$

When $k = -$ the probability has the following bounds

$$\frac{\frac{\lambda_p}{1-\lambda_W} - \frac{r_0}{\bar{r}(V)}}{\frac{\lambda_p}{1-\lambda_W} - \frac{\lambda_W}{1-\lambda_P}\frac{\underline{r}(V)}{\bar{r}(V)}} \leq P_{WDLT}(r_0) \leq \frac{\frac{1-\lambda_W}{\lambda_P}\frac{\bar{r}(V)}{\underline{r}(V)} - \frac{r_0}{\underline{r}(V)}}{\frac{1-\lambda_W}{\lambda_P}\frac{\bar{r}(V)}{\underline{r}(V)} - 1}$$

The intuition behind this Proposition is the same as in the discussion of the high-V case (cf. Proposition 5). In both states of nature, the process of priors cannot stay forever in the informative equilibrium regime. Otherwise agents could observe an infinite number of realizations of the war/peace process. Thus, by virtue of the strong law of large numbers, the empirical frequency of war/peace would converge to the true underlying probability, which is either $(1-\lambda_P, \lambda_P)$, if $k = -$, or $(\lambda_W, 1-\lambda_W)$ if $k = +$. This would enable agents to learn the true state of nature.[20]

4 Learning from Trade

In the analysis so far, the information set of group B was limited to the history of warfare. However, the inference about group A's propensity to trade could be improved if group B observed directly part of the trade history. For instance, if public records existed of the outcome of past inter-ethnic trade, group B could attain a perfect inference. Clearly, this would not be a realistic scenario, since in reality cross-community trade and business links are decentralized and hardly distinguishable from intra-community trade.

In this section, we expand the information set of group B. In particular, we allow agents to retain *some* memory of the information acquired through their individual family trade history. To retain tractability, we make the simplifying assumption that as soon as an agent invests and attempts to

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[20] Contrary to Corollary 1 we cannot provide a closed-form characterization of the probability $P_{WDLT}(r_0)$ when $\lambda_W = \lambda_P$. The reason is that the stochastic process (11) is not a random walk, due to its behavior in the region $r_{t-1} \in [\bar{r}^*(V), \bar{r}(V)]$.

trade, she observes the true k.[21] This ex-post "hard" information is not useful to the trader herself, but can be transmitted to the offspring. Without additional assumptions, all families would learn perfectly k over time. To prevent the informational friction from vanishing in the long run, we make the realistic assumption that the inter-generational transmission of hard information is subject to shocks: with the exogenous probability θ, the child of an informed parent fails to receive the information. In this model, θ is an inverse measure of the efficiency of learning from trade history, and $1/\theta$ is the average number of generations through which the family history is transmitted.

In summary, in every period there is both a hard information inflow (uninformed families that engage in trade learn k) and an exogenous outflow. In war times, nobody trades and the net inflow is negative. In peace times, the net inflow can be either positive or negative. This model captures in a stylized fashion the notion that information depreciates: If trade used to be intense in the far past, but it waned more recently, the information gathered through past trade fades away. This model is tractable, since it reduces the heterogeneity of information sets within group B to a two-point distribution: perfectly informed agents and agents who only observe the warfare history.

As in the benchmark model, we solve the game backwards. The distribution of beliefs in group B is now more complicated. Besides uninformed agents who still hold a public posterior belief conditional on the observation of peace/war, $\{\pi_P, \pi_W\}$, there is now a share of perfectly informed agents. We define by β^- and β^+, respectively, this share of informed agents conditional on group A type being $k = -$ and $k = +$. Recall that all agents in group A know the type. However, agents in group B ignore it, and thus the uninformed in this group cannot tell whether the share of informed agents is $\beta = \beta^-$ or $\beta = \beta^+$. Different from the benchmark model, the aggregate investment of group B is now type-contingent too, as some agents in group B know k. More formally, agents in group A have perfect information and observe n_B^k and their reaction function continues to be given by $n_A^k = F_A^k(zn_B^k)$. In group B a share β^k of the agents take their investment decisions under perfect information while a share $1 - \beta^k$ take a decision based on their common public belief π_P. For $k \in \{-, +\}$ the reaction functions of group B are now given by $n_B^k = \beta^k \cdot zn_A^k + (1 - \beta^k) \cdot zE\left[n_A \mid \pi_P\right]$, where $E\left[n_A \mid \pi_P\right] = \pi_P \cdot n_A^+ + (1 - \pi_P) \cdot n_A^-$.

Proposition 8 *Under assumption 1, for a given $\left(\pi_P, \beta^-, \beta^+\right) \in [0,1]^3$, the Nash Equilibrium of the investment/trade continuation game exists and is the unique 4-tuple $\{n_A^-, n_A^+, n_B^-, n_B^+\} \in [0,1]^4$ such that $n_A^k\left(\pi_P, \beta^-, \beta^+\right) = F_A^k(zn_B^k\left(\pi_P, \beta^-, \beta^+\right))$ and $n_B^k\left(\pi_P, \beta^-, \beta^+\right)$ is the implicit solution of the following fixed-point equation*

$$n_B^k = \beta^k F^B(zF^k(zn_B^k)) + (1 - \beta^k)F^B\left[z\pi_P F^+(zn_B^+) + z\left(1 - \pi_P\right)F^-(zn_B^-)\right]. \qquad (12)$$

The investment decision of agents in group A, $n_A^k\left(\pi_P, \beta^-, \beta^+\right)$, depends on both β^- and β^+, despite the fact that group A knows its type. Indeed, both β^- and β^+ affect the investment of the

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[21]Beyond its simplifying nature, this is an adversary assumption, since our goal is to show that learning traps are a robust outcome even if one increases the extent of information available in societies. Thus, assuming that private learning from trade is very effective plays against our result. Less extreme assumptions would harm tractability, since the entire distribution of private signals would become a state variable.
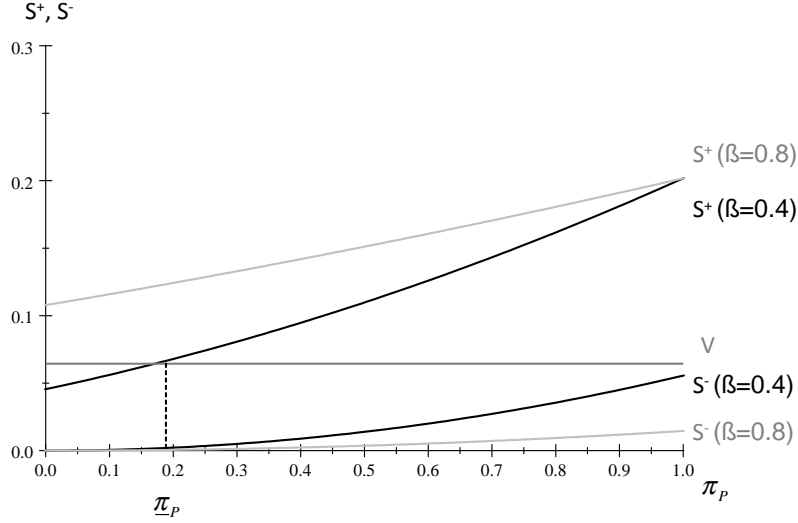
Figure 6: Surplus from trade with exogenous β

uninformed agents in group B who ignore the true type. Due to the strategic complementarity, then, β^- and β^+ also affects the investment of group A and of the informed in group B.

4.1 Exogenous β

To aid intuition, we consider first an economy in which the proportion of informed agents is exogenous, time invariant and common knowledge. Clearly, in this case $\beta^+ = \beta^- = \beta$ in equation (12). For a given after-peace belief π_P, the static equilibrium and the associated trade surplus now depend both on the belief and on the share of informed agents: $S^-(\pi_P; \beta)$ and $S^+(\pi_P; \beta)$.

Lemma 2 *Under assumption 1, for a given $(\pi_p, \beta) \in [0,1]^2$, a unique equilibrium exists and is characterized by a 4-tuple $\left(n_A^-, n_A^+, n_B^-, n_B^+\right) \in [0,1]^4$ which is continuous and non-decreasing in π_P. Moreover, the trade surplus functions $S^-(\pi_P, \beta), S^+(\pi_P, \beta)$ are continuous and non decreasing in π_P. S^- is non increasing in β, whilst S^+ is non decreasing in β. Finally, $S^-(\pi_P,\ \beta\ ) \leq S^+(\pi_P,\ \beta\ ).$*
$$ \underset{(+)\ \ (-)}{} \qquad \underset{(+)\ \ (+)}{}$$

Lemma 2 yields the intuitive result that $\partial S^-/\partial\beta \leq 0$, while $\partial S^+/\partial\beta \geq 0$. Consequently, the wedge between the two surpluses increases in β, $\partial(S^+ - S^-)/\partial\beta \geq 0$. Intuitively, as the share of informed agents increases, the equilibrium outcomes in the two states of nature become more separated, approaching the perfect information equilibrium as $\beta \to 1$. Such a divergence between the two trade surplus functions makes war more and more informative for any given π_P. This in turn makes learning traps harder to sustain. Figure 6 is drawn for the same distribution of investment costs and parameter values as in figure 4. Hence, for the benchmark case of $\beta = 0$ the surpluses S^+ and S^- would be identical to the ones in figure 4, where both a WDLT and a PDLT exist. Initially, increasing β simply reduces the range of posteriors consistent with the existence of two traps. A

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further increase in β rules out the PDLT (as shown by the black lines in figure 6 capturing $\beta = 0.4$), and an even further increase eventually also rules out the WDLT (as shown by the light grey lines in figure 6 capturing $\beta = 0.8$). The result that the range of sustainability of learning traps falls with β is general.

In summary, this subsection has shown that learning traps are robust to the assumption that an exogenous share of the population is informed about the type of group A, as long as the share of informed agents is not too large.

4.2 Endogenous β

In this subsection, we consider economies with an endogenous proportion of informed agents who acquire information through trade and transmit it to their offspring. This extension increases complexity considerably as there are now three state variables to keep track of, $(\pi_t, \beta_t^+, \beta_t^-) \in [0,1]^3$. The PBE Definition 1 is modified in three respects. First, a strategy for an agent in group B specifies an "investment action" for each of her possible types, *informed* or *uninformed,* and for each of the possible realizations of the investment cost. Second, the PBE is defined up to a triplet, $(\pi_{t-1}, \beta_{t-1}^+, \beta_{t-1}^-) \in [0,1]^3$. Third, the triplet $(n_{At}^-, n_{At}^+, n_{Bt})$ is replaced by the 4-tuple $(n_{At}^-, n_{At}^+, n_{Bt}^+, n_{Bt}^-)$, where (n_{Bt}^+, n_{Bt}^-) yields the measure of agents who optimally invest in group B for each type $k \in \{+, -\}$.

The share of informed agents evolves according to the following law of motion:

$$\beta_{t+1}^k = (1 - \theta)[n_{Bt}^k + (1 - n_{Bt}^k)\beta_t^k]. \tag{13}$$

The set of informed agents at t consists of the children either of traders or of informed non-traders which did not experience any memory loss.

Definition 6 *A DSE is a sequence of PBE with an associated sequence of beliefs and a measure of informed agents such that, given an initial condition $(\pi_0, \beta_0^+, \beta_0^-)$, the posterior belief at t is the prior belief at t+1 and the law of motion of β_t^+ and β_t^- is given by (13).*

As in the benchmark model, we define the equilibrium in terms of the likelihood ratio $r \equiv \pi/(1-\pi)$. We first extend the definition of learning trap to the new environment.

Definition 7 *A WDLT (resp. PDLT) is a set of states, $\Omega_{WDLT} \subset \mathbb{R}^+ \times [0,1]^2$ (resp. $\Omega_{PDLT} \subset \mathbb{R}^+ \times [0,1]^2$), such that if $(r_t, \beta_t^+, \beta_t^-) \in \Omega_{WDLT}$ (resp. if $(r_t, \beta_t^+, \beta_t^-) \in \Omega_{PDLT}$) then $\forall s \geq t, r_s = r_t$, and the incidence of war is high (resp. low), $\Pr(I_{WAR,s} = 1) = 1 - \lambda_P$ (resp. $\Pr(I_{WAR,s} = 1) = \lambda_W$), for all continuation paths $[r_s, \beta_s^+, \beta_s^-]_{s=t}^{\infty}$.*

Note that we do not require the stationarity of β_t^+ and β_t^- for an economy to be in a learning trap. Our aim, next, is to characterize the parameter range of θ that is compatible with the existence of the learning traps, given the remaining parameters. This is not straightforward, since the equilibrium path is governed by a three-dimensional stochastic process $(r_t, \beta_t^-, \beta_t^+)$ which admits no closed-form solution. To achieve tractability, we make the following further assumption:

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Assumption 4 ι^B *is uniformly distributed on* $[0,1]$ *and* ι^A *is uniformly distributed on* $[-x_A, 1-x_A]$ *with* $x_A \in \{-x, +x\}$ *and* $x < 1/2$.

Note that this assumption is nested in Assumption 1 when $z < 1 - x$. However, the results of the next two Propositions are valid for any $z \in [0,1]$.

Proposition 9 *Assume* V *such that* $S^+(0) < V < \min\{S^-(1), S^+(1), 1/2\}$. *(i) A WDLT exists if and only if* $\theta \geq \theta_W \equiv zx/(1+zx)$; *(ii) A PDLT exists if and only if* $\theta \geq \theta_P \equiv 1 - (z^2 - x - \sqrt{2V})/(z^3 - z^3\sqrt{2V})$; *(iii) we have* $\theta_P > \theta_W$.

To see the intuition, note first that if families never forget, i.e., $\theta = 0$, then the economy necessarily converges to perfect learning. Intuitively, peace occurs with a positive probabilty, since $\lambda_P > 0$. Moreover, under peace there is some trade, and thus trading families learn. Then, the process converges to the full-information equilibrium. Imposing a lower bound on θ has similar effects to imposing a bound on the exogenous β in section 4.1. In particular, when $\theta > 0$ there exist upper bounds to β^+ and β^- corresponding to the limit of an infinite sequence of peace realizations. This sets upper bounds to the shares of informed agents, denoted by β^+_∞ and β^-_∞. Suppose, next, that $k = +$ and that the state at $t-1$ is $(r_{t-1}, \beta^+_\infty, \beta^-_{t-1})$, being such that both the high and the low type would stage war under BAU so that $r_t = r_{t-1}$ under both peace and war. Then, $(r_{t-1}, \beta^+_\infty, \beta^-_{t-1}) \in \Omega_{WDLT}$. Intuitively, the share of informed agents cannot increase, since it is already at its upper bound. If such a share falls, investments will fall too, strengthening further the case for war. Thus, uninformed agents never learn, and the economy is in a WDLT.

Interestingly, WDLT are more robust to private learning from trade history than PDLT. More formally, $\theta_W < \theta_P$. The key difference is that in a PDLT beliefs are overly optimistic and, hence, many agents invest and trade. The diffusion of private information sets apart the state-contingent equilibrium trade surplus (S^-, S^+), restoring the informativeness of the war/peace process (i.e. $S^- < V < S^+$), and eventually destabilizing a candidate PDLT. While such a mechanism is also present in a WDLT, it is dampened by overly pessimistic beliefs which keep trade low and so limit the accumulation of information.[22]

Proposition 9 yields an existence result for learning traps. The next Proposition establishes that economies starting in an informative equilibrium, $(r_0, \beta^+_0, \beta^-_0) \notin \Omega_{WDLT}$, can actually fall in WDLT with a positive probability as long as the WDLT is non empty (i.e. $\theta > zx/(1+zx)$). To this purpose, we identify a finite time-passage T, corresponding to a non-zero measure subset of continuation paths over the period $0, ..., T$, such that $(r_T, \beta^-_T, \beta^+_T) \in \Omega_{WDLT}$. Basically, these paths include a sequence of war shocks which manages to drive r_T into a range of sufficiently pessimistic beliefs. Moreover,

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[22] The difference can be large, as shown by the following numerical example. Suppose that, on average, the propensity to trade is 10% larger in the good than in the bad state of nature (i.e. $x = 0.05$). Assume, in addition, that in the absence of private learning from trade, there would exist a PDLT and a WDLT "of equal size" (i.e., $z = 1, V = (1-x)^2/2$). With such parameter configuration the thresholds are equal to $\theta_W = 0.047$ and $\theta_P = 0.5$. Hence, family memory should last on average no more than two generations for the PDLT to vanish (i.e., $1/\theta_P = 2$), while the WDLT is sustained as long as memory persists on average for up to twenty one generations (i.e., $1/\theta_W = 21$).

by disrupting trade, such sequence depletes the share of informed agents β_0^+ such that $\beta_0^+/\beta_T^+ = 1/(1-\theta)^T < 1$. When the pace of decrease of the informational externality of trade, $1/(1-\theta)$, is larger than the informativeness of war, $(1-\lambda_P)/\lambda_W$, this sequence of war shocks is able to drive the economy into the WDLT.[23]

Proposition 10 *Assume $\theta > \max\left(1 - \frac{\lambda w}{1-\lambda_P}, \frac{zx}{1+zx}\right)$. Suppose $(r_0, \beta_0^+, \beta_0^-) \notin \Omega_{WDLT}$. Then, the economy falls into a WDLT in finite time with a strictly positive probability, $Pr\{\exists T < \infty, (r_T, \beta_T^+, \beta_T^-) \in \Omega_{WDLT}\} > 0$.*

In conclusion, learning traps are robust to the presence of a positive share of informed agents. However, as the share of informed people increases (i.e., as we lower θ), learning traps with incorrect beliefs become harder to sustain. Eventually, for θ sufficiently small, such learning traps are ruled out. WDLT are more robust than PDLT to private learning from trade. Economies starting in informative equilibria can fall into learning traps even though agents learn from trade.

5   Policy Implications

In this section we outline some comparative statics and discuss policy implications of our theory. Our model implies that *larger individual returns from trade (i.e., larger z)* make human capital investments more attractive, thereby increasing the expected trade surplus (equation (2) shows that $\partial S^+(\pi_P)/\partial z \geq 0$ and $\partial S^-(\pi_P)/\partial z \geq 0$). Thus, policies subsidizing inter-group trade push up the opportunity cost of war, narrowing on the one hand the range of beliefs for which WDLT occur, and enlarging on the other hand the range of beliefs for which PDLT arise (more formally, this corresponds to an upward shift of $S^+(\pi_P)$ and $S^-(\pi_P)$ in the Figures 2 and 4). This prediction is in line with the empirical results of Horowitz (2000) on affirmative action and ethnic conflict. He finds that preferential programs aiming at improving the integration of less advanced ethnic groups in the national economy have reduced the potential for conflict in various countries such as India, Indonesia, Malaysia and Nigeria.[24] Since trade typically thrives in fast-growing economies, our theory is also broadly consistent with the empirical finding that high economic growth reduces the risk of war recurrence (Sambanis, 2008; Walter, 2004). Our theory also provides a rationale to *subsidize human capital investments* which reduce inter-ethnic barriers. Public education initiatives promoting, for example, the knowledge of several national languages can lower the obstacles to inter-group trade. This is in line with the empirical findings that higher education expenditures and enrollment rates decrease the risk of civil wars (Thyne, 2006).

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[23]Proving convergence to a PDLT is harder. We conjecture that convergence may occur under more restrictive conditions. On the one hand, peace must occur to make beliefs more optimistic over time. On the other hand, this would reveal to an increasing share of group B that group A is of the low type.

[24]Horowitz (2000) and Whah (2010) show that the programs since the 1970s of state-induced inter-ethnic joint venture companies in Malaysia have in many instances enhanced trust between the Malay and the Chinese population and resulted in lower social tensions. Similarly, Augenbraun et al. (1999) find that microenterprise lending by donors in Bosnia for inter-ethnic joint ventures has worked well, not only on purely economic grounds, but also in lowering tensions between groups.

Unsurprisingly, *larger windfall gains from war (i.e., larger $\tilde{V}$)* expand the range of beliefs such that the economy can plunge in a WDLT. This is in line with the empirical findings that more abundant natural resources hinder lasting recovery and fuel war recurrence (see, e.g., Doyle and Sambanis, 2000; Fortna, 2004; Sambanis, 2008). International measures such as embargoes on arms exports to, or natural resource imports, from regimes arising from ethnic aggression could limit trust depletion and war recurrence.

Our theory has more subtle implications about the effectiveness of *international peacekeeping.* The model predicts that international peacekeeping efforts that limit themselves to "stopping the shooting" will only have a short-lasting effect on political stability. To reach a permanent effect, e.g. to get a country out of a WDLT, peacekeeping must be complemented first and replaced later by trade- and trust-enhancing measures. In fact, the prolonged insistence on external peacekeeping may be detrimental, as it may undermine the externality of peace on learning and trust. In other words, local groups may attribute peace to the presence of foreign troops, and fail to update their beliefs about the propensity to trade of other communities. These predictions are in line with the conclusion of a study on survival of peace duration by Sambanis (2008: 30): "UN missions have a robust positive effect on peacebuilding outcomes, particularly participatory peace, but the effects occur mainly in the short run and are stronger when peacekeepers remain." Indeed, he finds that the effect becomes insignificant once UN troops have left, and concludes that an enduring peace hinges on economic development and the rebuilding of institutions rather than on past UN peacekeeping.

Similar conclusions are reached by Luttwak (1999: 37) who argues that simple peacekeeping – without trade-promoting or trust-restoring measures– does not lead to lasting peace, but just interrupts hostilities that will recur once the UN troops leave: "(Peacekeeping), perversely, can systematically prevent the transformation of war into peace. The Dayton accords are typical of the genre: they have condemned Bosnia to remain divided into three rival armed camps, with combat suspended momentarily but a state of hostility prolonged indefinitely... Because no path to peace is even visible, the dominant priority is to prepare for future war rather than to reconstruct devastated economies and ravaged societies."

Our theory also suggests that policies *targeting beliefs directly* may be important, especially when there is no fundamental reason for persistent distrust and war. If the state of the world was $k = +$, there may sometimes be ways to credibly communicate this to the population (e.g., by documenting and publicizing successful episodes of inter-ethnic business cooperation). There is empirical evidence that inter-group prejudices can be reduced by targeted media exposure (cf. Paluck, 2009; Paluck and Green, 2009). According to Paluck's (2009) findings the listeners exposed to the "social reconciliation" radio soap opera in Rwanda were significantly more likely to find it "not naive to trust" and to feel empathy for other Rwandans than the control group exposed to a "health" radio soap opera. Similarly, Bardhan (1997) shows that direct targeting of beliefs of the Muslims and Hindus by spreading success stories of cooperation can reduce distrust and the potential for conflicts in India: "Public information on what actually happened, on how a disturbance started, on who tried to take advantage of it, on

instances of intercommunity cooperation in the face of tremendous odds. etc., if effectively transmitted in the early stages, can stop some of the vicious rumors that fuel communal riots and calm group anxieties" (Bardhan, 1997: 1395).

6 Conclusion

The economic theory of civil conflicts is rooted in the rational choice paradigm. In contrast, a number of political scientists emphasize the notion of *grievance* (e.g. Gurr, 1970; Sambanis, 2001). This view is supported by empirical studies showing that wars tend to reoccur more frequently if they are associated with grievances and ethnic identities (Doyle and Sambanis, 2000; Licklider, 1995). In a recent survey article, Blattman and Miguel (2010) argue that incorporating such factors in economic models is one of the big challenges of theory. In this paper, we take a first step in this direction. We provide a theory where asymmetric information and cultural transmission of beliefs explain why societies can plunge into recurrent civil conflicts. In our theory, conflicts are not a mere explosion of irrational grievances, but are associated with the collapse of trust, a notion that is closely connected to that of grievance.[25] The persistent effects of conflict on trust, and the possible emergence of irreversible vicious circles, is explained by a rational belief updating process under imperfect information.

We emphasize the link between trade and war, which has been highlighted in the recent literature as an important factor explaining international conflicts. We believe the link trust-trade-war to be even more salient in the analysis of inter-community conflicts within societies, where business relationships (e.g., seller-buyer, employer-employee, supplier-producer, lender-borrower) are decentralized and do not need the mediation of institutions that can aggregate and diffuse information.

While our current study presents a rational-agent theory, integrating more explicit psychological aspects may cast additional light on the issues at hand. In some work in progress (Rohner, Thoenig and Zilibotti, 2011) we find that children who are exposed to war at a tender age suffer from a permanent deficit of trust, and that the effect is significantly larger than for adults exposed to war. To the extent to which the earlier age is especially "formative" in terms of beliefs and values, this is broadly consistent with the view that war erodes trust. Finally, we abstracted from institutions. As emphasized by Aghion et al. (2011), institutions and beliefs are not independent factors: on the one hand, institutions can matter through their effect on the trust-building process, whilst on the other hand trust can influence institutional developments that can deter conflict. Studying these connections is also left to future research.

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[25] For instance, Downes (2006) writes: "*The key issues concern the adversary's intentions... The process of fighting a war gives both belligerents plentiful evidence of the adversary's malign intentions. Beyond the normal costs of conflict, civil wars are often characterized by depredations against civilians including ethnic cleansing, massacre, rape, bombing, starvation, and forced relocation. These factors produce deep feelings of hostility and hatred, and make it hard for former belligerents to trust each other. Belligerents have little reason to believe their opponent's intentions suddenly have become benign... Moreover, even if the adversary's intentions seem benign now, what guarantee is there that they will not change in future? These issues are of critical importance.*"

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Appendix

A   Proof of Propositions 1 and 2

Under asymmetric information group B agents hold a prior $\pi_P \in [0,1]$ on group A type. Perfect information corresponds to the subcases $\pi_P = 0$ and $\pi_P = 1$. We provide hereafter the proof of Proposition **2;** the proof of Proposition **1** follows directly by setting $\pi_P = 0$ or $\pi_P = 1$.

We start by proving existence. Equation (3) implies that

$$n_B = \tilde{F}^B (n_B; \pi_P) \equiv F^B(z\left[\pi_P F^+ (zn_B) + (1 - \pi_P) F^- (zn_B)\right]), \tag{14}$$

where $\tilde{F}^B$ is a continuous function with the following properties: (i) For all π_P, $\tilde{F}^B (0; \pi_P) \geq 0$ and $\tilde{F}^B (1; \pi_P) < 1$; (ii) $\tilde{F}^B (n_B; \pi_P)$ is increasing and convex in n_B, (iii) $\tilde{F}^B (n_B; \pi_P)$ is increasing in π_P. Property (i) follows from Assumption 1. Property (ii) follows from the fact that (due to the standard properties of p.d.f.) $\tilde{F}^B$ is a continuous, non-decreasing transformation of convex combination of p.d.f. that are themselves continuous, nondecreasing and convex in n_B, where convexity follows from Assumption 1. Property (iii) follows from stochastic dominance. Given property (i) and the continuity of $\tilde{F}^B$, the intermediate value theorem guarantees that, for any $\pi_P \in [0, 1]$, there exists $n_B \in (0, 1)$ such that $n_B = \tilde{F}^B (n_B; \pi_P)$.

Properties (i), (ii) and (iii) guarantee jointly that the mapping $n_B (\pi_B)$ implicitly defined by (14) is unique and is monotonically increasing. To prove uniqueness we proceed by contradiction. Let us assume that there exists a second fixed point $\hat{n}_B = \tilde{F}^B (\hat{n}_B; \pi_P)$. Without loss of generality we assume $n_B < \hat{n}_B$. The fixed point $\hat{n}_B \in [n_B, 1]$ can be written as the following convex combination of the interval bounds: $\hat{n}_B = \frac{1-\hat{n}_B}{1-n_B} \times n_B + \frac{\hat{n}_B - n_B}{1-n_B} \times 1$. Applying to $\hat{n}_B$ the convexity criterion of $\tilde{F}^B$ yields

$$\tilde{F}^B (\hat{n}_B; \pi_P) \leq \frac{1 - \hat{n}_B}{1 - n_B} \tilde{F}^B (n_B; \pi_P) + \frac{\hat{n}_B - n_B}{1 - n_B} \tilde{F}^B (1; \pi_P)$$

From definition of the fixed points $(n_B, \hat{n}_B)$ this inequality yields $\hat{n}_B \leq \frac{1-\hat{n}_B}{1-n_B} n_B + \frac{\hat{n}_B - n_B}{1-n_B} \tilde{F}^B (1; \pi_P)$. This leads to $\tilde{F}^B (1; \pi_P) \geq 1$, which contradicts property (i).

Given the existence of a unique function $n_B (\pi_P)$, the existence and uniqueness of n_A^- and n_A^+ such that $n_A^- = F^- (zn_B (\pi_P)) = n_A^- (\pi_P)$ and $n_A^+ = F^+ (zn_B (\pi_P)) = n_A^+ (\pi_P)$ follow immediately. Thus, equation (3) has a unique fixed point and defines a unique triplet of equilibrium functions. Finally, stochastic dominance implies that $\left(n_A^- (\pi_P), n_B^- (\pi_P)\right) \leq \left(n_A^+ (\pi_P), n_B^+ (\pi_P)\right)$.

Let us now turn to the equilibrium value of the trade surplus S^k for $k \in \{-, +\}$. Integrating by parts (2) yields

$$
\begin{aligned}
S^k (\pi_P) &\equiv zn_A^k (\pi_P) n_B (\pi_P) - \int^{zn_B(\pi_P)} \iota \, dF^k \\
&= zn_A^k (\pi_P) n_B (\pi_P) - \left[\iota \, F^k\right]^{zn_B(\pi_P)} + \int^{zn_B(\pi_P)} F^k (\iota) \, d\iota \\
&= zn_A^k (\pi_P) n_B (\pi_P) - zn_b (\pi_P) F^k (zn_B (\pi_P)) + \int^{zn_B(\pi_P)} F^k (\iota) \, d\iota
\end{aligned}
$$

From (3) we get that at equilibrium $n_A^k = F^k(zn_B)$. Combined with the previous equation this gives

$$S^k(n_B(\pi_P)) = zF^k(zn_B(\pi_P))n_B - \int^{zn_B(\pi_P)} \iota dF^k(\iota) = \int^{zn_B(\pi_P)} F^k(\iota)\ d\iota \qquad (15)$$

Given that F^k is non negative and $n_B(\pi_P)$ is non decreasing in π_P we conclude that $S^k(\pi_P)$ is non decreasing in π_P. Moreover F^- first-order stochastically dominates F^+; $\forall \iota$, $F^+(\iota) \geq F^-(\iota)$. Hence $\int^{zn_B(\pi_P)} F^+(\iota)\ d\iota \geq \int^{zn_B(\pi_P)} F^-(\iota) d\iota$. We conclude that $\forall \pi_P \in [0,1]$, $S^-(\pi_P) \leq S^+(\pi_P)$.

B   Proof of Lemma 1

We first prove that an uninformative PBE exists if and only if the prior is in either the range $r_{-1} \leq \underline{r}^*(V)$ or $r_{-1} \geq \bar{r}(V)$. Guess that a PBE exists. Since $r_P = r_{-1}$, $r_{-1} \leq \underline{r}^*(V) \Rightarrow r_P \leq \underline{r}^*(V)$ and $r_{-1} \geq \bar{r}(V) \Rightarrow r_P \geq \bar{r}(V)$. Then, by the definitions of $\underline{r}^*(V)$ and $\bar{r}(V)$, both types find it optimal to stage war under BAU ($\sigma^+(r_{-1}) = \sigma^-(r_{-1}) = 0$) if $r_{-1} \leq \underline{r}^*(V)$. Likewise, both types retain peace under BAU ($\sigma^+(r_{-1}) = \sigma^-(r_{-1}) = 1$) if $r_P \geq \bar{r}(V)$. The guess is then fulfilled, proving the "if" part. To prove the "only if" part suppose, to draw a contradiction, that an uninformative PBE exists in the range $r_{-1} \in (\underline{r}^*(V), \bar{r}(V))$. Then, $r_P \in (\underline{r}^*(V), \bar{r}(V))$. However, given a posterior in such range, the good type would retain peace ($\sigma^+(r_{-1}) = 1$) whereas the low type would stage war ($\sigma^-(r_{-1}) = 0$) under BAU, contradicting the assumption that peace is uninformative and that $r_P = r_{-1}$.

Next, we prove that informative PBE exist if and only if $r_{-1} \in [\underline{r}(V), \bar{r}(V)]$. We consider first the subrange $r_{-1} \in [\underline{r}(V), \bar{r}^*(V)] \subset [\underline{r}(V), \bar{r}(V)]$, and prove that in this subrange there exists an informative pure-strategy PBE such that $\sigma^+(r_{-1}) = 1$ and $\sigma^-(r_{-1}) = 0$. Guess that such a PBE exists. Since $r_P = \frac{1-\lambda_W}{\lambda_P}r_{-1}$, then $r_{-1} \in [\underline{r}(V), \bar{r}^*(V)] \Rightarrow r_P \in [\underline{r}^*(V), \bar{r}(V)]$. Then, by the definitions of $\underline{r}^*(V)$ and $\bar{r}(V)$, the high type finds it optimal to retain peace ($\sigma^+(r_{-1}) = 1$) while the low type finds it optimal to stage war ($\sigma^-(r_{-1}) = 0$) under BAU. This fulfils the guess, establishing the existence of an informative pure-strategy PBE in the subrange $r_{-1} \in [\underline{r}(V), \bar{r}^*(V)]$. Next, consider the complementary subrange $r_{-1} \in [\bar{r}^*(V), \bar{r}(V)] \subset [\underline{r}(V), \bar{r}(V)]$. In this subrange, an informative pure-strategy PBE such that $\sigma^+(r_{-1}) = 1$ and $\sigma^-(r_{-1}) = 0$ does not exist, since then $r_P = \frac{1-\lambda_W}{\lambda_P}r_{-1} > \bar{r}(V)$ implying that both types would find it optimal to retain peace, contradicting that $\sigma^+(r_{-1}) = 1$ and $\sigma^-(r_{-1}) = 0$. However, there exists a unique mixed-strategy informative PBE, such that the high type chooses peace ($\sigma^+(r_{-1}) = 1$) while the low type is indifferent between war and peace, and chooses war with probability $\hat{\sigma}^-(r_{-1}) = \frac{(1-\lambda_W)\frac{r_{-1}}{\bar{r}(V)}-\lambda_P}{1-\lambda_W-\lambda_P}$. Bayes' rule implies then that $r_P = \bar{r}(V)$, fulfilling the guess that the low type is indifferent between war and peace (consequently, war erupts with probability $\lambda_W < 1/3$ if $k = +$ and with probability $1-\lambda_P-(1-\lambda_P-\lambda_W)\hat{\sigma}^-(r_{-1}) > \lambda_W$ if $k = -$).

The fact that there are multiple PBE if and only if $r_{-1} \in [\underline{r}(V), \underline{r}^*(V)]$ follows immediately from the analysis above [note that in this range there exist three equilibria, since a mixed-strategy informative equilibrium such that $r_P = \underline{r}^*(V)$ also exists. However, if $r_{-1} \in [\underline{r}^*(V), \bar{r}(V)]$ the informative PBE is unique].

C   Proof of Proposition 5

The proof strategy consists of first showing that the stochastic process (9) can be reformulated as an asymmetric random walk with a drift on the real line. Then, applying the properties of Martingale

processes, we characterize the probability of the stopping time $Pr\{\exists T < \infty | \; r_T \in \Omega_{WDLT}\}$. The discrete-time nature of the process introduces some technical complications that would not feature in continuous-time processes. In particular, in discrete time when the random walk has a drift there is a compact set of possible stopping-time values, r_T, and which value in this set is reached depends on the realization of the stochastic process (i.e., r_T is not deterministic). This complication (which would not feature in continuous time) does not arise in the particular case of Corollary 1 in which the random walk has no drift.

The stochastic process (9) can be expressed, after rearranging terms, as

$$Z_t = \delta + Z_{t-1} \pm 1 \text{ with probability } (\rho, 1 - \rho), \tag{16}$$

where $Z_t \equiv \ln r_t/s$, $\delta \equiv d/s < 1$,

$$\rho \equiv 1_{k=+} \times (1 - \lambda_W) + 1_{k=-} \times \lambda_P \tag{17}$$

$$s \equiv \frac{1}{2}\left[\ln(\frac{1-\lambda_W}{\lambda_P}) + \ln(\frac{1-\lambda_P}{\lambda_W})\right] > 0 \tag{18}$$

$$d \equiv \frac{1}{2}\left[\ln(\frac{1-\lambda_W}{\lambda_P}) - \ln(\frac{1-\lambda_P}{\lambda_W})\right] \in]-s, s[. \tag{19}$$

Z_t is a random walk with drift which is defined up to an initial condition $Z_0 \equiv \ln r_0/s$. The process Z_t hits a downward barrier as soon as it falls into the range $]Z^* - 1 + \delta, Z^*]$ where $Z^* \equiv \ln \bar{r}_W(V)/s < Z_0$.

Our next goal is to characterize the first passage time $T \equiv \min\{t; Z^* - 1 + \delta < Z_t \leq Z^* < 0\}$. Our approach generalizes the analysis of Shreve (2004, chap.5) to a random walk with drift. To this aim, we define a family of Martingales $M_t(u)$ which corresponds to a deterministic transformation of Z_t :

$$M_t(u) \equiv e^{u(Z_t - Z^*) - tF(u)} \tag{20}$$

where $u \in \mathbb{R}$ and

$$F(u) \equiv u\delta + \ln(\rho e^u + (1 - \rho)e^{-u}) \tag{21}$$

Using the definitions (17), (18) and (19) we can show that equation $F(u) = 0$ has two roots. One of them is $u = 0$. The other is $u = u^*$, where

$$u^* = -s < 0 \text{ when } k = +, \tag{22}$$
$$u^* = s > 0 \text{ when } k = -.$$

Moreover, $F(u) \underset{u \to 0^-}{>} 0$ when $k = -$ and $F(u) \underset{u \to u^{*-}}{>} 0$ when $k = +$.

The process M_t is a Martingale, since

$$M_{t+1} = e^{u(Z_{t+1} - Z^*) - (t+1)F(u)} = e^{u(Z_{t+1} - Z_t)}e^{-F(u)}M_t,$$

where $E_t[M_{t+1}] = M_t e^{-F(u)}E_t\left[e^{u(Z_{t+1} - Z_t)}\right] = M_t e^{-F(u)}(\rho e^u + (1 - \rho)e^{-u} + u\delta) = M_t$. Next, let $t \wedge T \equiv \min(t, T)$. Since a Martingale stopped at a stopping time is a Martingale, $M_{t \wedge T}$ is a Martingale. Thus, for all $t \in \mathbb{N}$, $M_{0 \wedge T} = E_0[M_{t \wedge T}]$. Hence:

$$e^{u(Z_0 - Z^*)} = E\left[e^{u(Z_{t \wedge T} - Z^*)}e^{-(t \wedge T)F(u)}\right] \tag{23}$$

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We will now show that there exists a range of u, $u < \min(u^*, 0)$, such that the process in (23) is bounded as t goes to infinity. To see why note first that $\forall u < 0$ and $\forall t \in [0, \infty)$, $0 \le e^{u(Z_{t \wedge T} - Z^*)} \le 1$ since $Z_{t \wedge T} \ge Z^*$. Next, recall that $\forall u < \min(0, u^*)$, $F(u) > 0$. Hence, $\forall t \in [0, \infty), 0 < e^{-(t \wedge T)F(u)} < 1$. Since the process is bounded, we can apply the theorem of dominated convergence to (23), implying that, $\forall u < \min(u^*, 0)$,

$$e^{u(Z_0 - Z^*)} = \lim_{t \to \infty} E\left[e^{u(Z_{t \wedge T} - Z^*)}e^{-(t \wedge T)F(u)}\right] = E\left[\lim_{t \to \infty} e^{u(Z_{t \wedge T} - Z^*)}e^{-(t \wedge T)F(u)}\right]$$

$$= \begin{cases} e^{u(Z_T - Z^*)}\lim_{t \to \infty} e^{-TF(u)} & \text{if} \quad T < \infty \\ \lim_{t \to \infty} e^{u(Z_t - Z^*)}e^{-tF(u)} \le \lim_{t \to \infty} e^{-tF(u)} = 0 & \text{if} \quad T \to \infty \end{cases}$$

This yields

$$e^{u(Z_0 - Z^*)} = E\left[e^{-u(Z^* - Z_T)}1_{T < \infty}e^{-TF(u)}\right]. \tag{24}$$

By the definition of the stopping time T we have $Z_T \in]Z^* - 1 + \delta, Z^*]$. This implies

$$1 \le e^{-u(Z^* - Z_T)} < e^{-u(1 - \delta)}. \tag{25}$$

We can at this point prove the following crucial Lemma.

Lemma 3 *For $k = -, \Pr(T < \infty) = 1$. For $k = +$, $0 < e^{-s(1-\delta)}e^{-s(Z_0 - Z^*)} < \Pr(T < \infty) \le e^{-s(Z_0 - Z^*)} < 1$.*

Proof. Suppose $k = -$. From our discussion of (21) we have $\forall u < 0$, $F(u) > 0$. Thus, the process $e^{-u(Z^* - Z_T)}1_{T < \infty}e^{-TF(u)}$ is bounded between 0 and $e^{-u(1-\delta)}$. Applying the theorem of dominated convergence to (24) yields, then,

$$\lim_{u \to 0^-} e^{u(Z_0 - Z^*)} = \lim_{u \to 0^-} E\left[e^{-u(Z^* - Z_T)}1_{T < \infty}e^{-TF(u)}\right] = E\left[\lim_{u \to 0^-} e^{-u(Z^* - Z_T)}1_{T < \infty}e^{-TF(u)}\right]$$

which is equivalent to

$$\begin{aligned} 1 &= E\left[1_{T < \infty}\right] \\ &= \Pr(T < \infty) \end{aligned}$$

Suppose, next, that $k = +$. From our discussion of (21) we have $\forall u < u^* = -s < 0$, $F(u) > 0$. Thus, $\forall u < u^*$, the process $e^{-u(Z^* - Z_T)}1_{T < \infty}e^{-TF(u)}$ is bounded between 0 and $e^{-u(1-\delta)}$. Applying the theorem of dominated convergence to (24) yields:

$$\lim_{u \to u^{*-}} e^{u(Z_0 - Z^*)} = \lim_{u \to u^{*-}} E\left[e^{-u(Z^* - Z_T)}1_{T < \infty}e^{-TF(u)}\right] = E\left[\lim_{u \to u^{*-}} e^{-u(Z^* - Z_T)}1_{T < \infty}e^{-TF(u)}\right]$$

which is equivalent to

$$e^{u^*(Z_0 - Z^*)} = E\left[e^{-u^*(Z^* - Z_T)}1_{T < \infty}e^{-TF(u^*)}\right] = E\left[e^{-u^*(Z^* - Z_T)}1_{T < \infty}\right] \tag{26}$$

Premultiplying inequality (25) by $1_{T < \infty}$ we have $E\left[1_{T < \infty}\right] \le E\left[e^{-u^*(Z^* - Z_T)}1_{T < \infty}\right] < e^{-u^*(1-\delta)}E\left[1_{T < \infty}\right]$. Combined with (22) and (26) this leads to

$$0 < e^{-s(1-\delta)}e^{-s(Z_0 - Z^*)} < \Pr(T < \infty) \le e^{-s(Z_0 - Z^*)} < 1$$

∎

If $k = -$, Lemma (3) implies that $Pr\{\exists T < \infty \mid r_T \in \Omega_{WDLT}\} = 1$, proving the first part of Proposition (5). If $k = +$, using the definitions (16), (17), (18) and (19) we can rewrite the chain of inequalities given by $0 < e^{-s(1-\delta)}e^{-s(Z_0 - Z^*)} < \Pr(T < \infty) \leq e^{-s(Z_0 - Z^*)} < 1$ as $0 < \frac{\lambda_W}{1-\lambda_P}\frac{r(V)}{r_0} < P_{WDLT} \leq \frac{r(V)}{r_0} < 1$. Hence, with probability $0 < 1 - P_{WDLT} < 1$, the process does not enter the trap in finite time and stays in the learning regime. Finally, by the Strong Law of Large Numbers, the process r_t must converge to perfect learning, i.e., $r_t \to \infty$. This proves the second part of Proposition 5.

D Proof of Corollary 1

When $\lambda_W = \lambda_P = \lambda$, the state space of the stochastic process (9) is isomorphic to $\mathbb{Z}$ (e.g., the termination value of the process is the same after the sequence war-war-peace and after the sequence peace-war-war). This implies that the value of the belief at the stopping time T is deterministically characterized by the initial condition: $\pi_T = \bar{\pi}(\pi_0)$ where $\ln(\bar{\pi}(\pi_0)/(1 - \bar{\pi}(\pi_0))) \leq \ln \bar{r}_W(V) < \ln(\bar{\pi}(\pi_0)/(1 - \bar{\pi}(\pi_0))) - \ln\frac{1-\lambda}{\lambda}$. Since the belief $\pi_t \in [0,1]$ is a (bounded) Martingale, then

$$
\begin{aligned}
\forall t, \pi_0 &= E[\pi_t] \\
&= \pi_0 \times E[\pi_t \mid k = +] + (1 - \pi_0) \times E[\pi_t \mid k = -],
\end{aligned}
\tag{27}
$$

The Martingale Convergence Theorem implies that π_t converges almost surely to a random variable π^*. When $k = -$, the Strong Law of Large Numbers implies that $\pi^* = \bar{\pi}(\pi_0)$. When $k = +$, the support of π^* is equal to the two atoms $\{\bar{\pi}(\pi_0), 1\}$ with a probability distribution $(P_{WDLT}, 1 - P_{WDLT})$. Taking the limit of (27) as $t \to +\infty$ yields:

$$
\forall t, \pi_0 = \pi_0 \times [P_{WDLT} \times \bar{\pi} + (1 - P_{WDLT}) \times 1] + (1 - \pi_0) \times \bar{\pi}(\pi_0),
$$

where,

$$
P_{WDLT} = \frac{\bar{\pi}(\pi_0)/(1 - \bar{\pi}(\pi_0))}{\pi_0/(1 - \pi_0)},
$$

proving the first part of the corollary 1.

To prove that $E(T \mid T < \infty) = \Delta_0/(1 - 2\lambda)$, we return to the proof of Proposition 5, and note that when $\lambda_W = \lambda_P = \lambda < 1/3$ the stochastic process Z_t in equation (16) is a random walk without drift, i.e., $\delta = 0$. Moreover, $\rho > 1/2$ iff $k = +$ and $\rho < 1/2$ iff $k = -$. Thus, $Z_t = Z_{t-1} \pm 1$ with probability $(\rho, 1-\rho)$, where $\rho = 1_{k=+} \times (1-\lambda) + 1_{k=-} \times \lambda$. As proven above, Z_T (where T denotes the stopping time) is entirely determined by initial conditions: $Z_T = Z^*(Z_0)$ where $Z^*(Z_0) \leq \ln \bar{r}_W(V)/s < Z^*(Z_0) + 1$ and $(Z^*(Z_0) - Z_0) \in \mathbb{Z}^-$. Moreover, $u^* = \ln\frac{1-\rho}{\rho}$ (where $F(u)$ and u and are defined by (21) in the proof of Proposition 5, and u^* is the non-zero root of F), implying that u^* is negative (positive) if and only if $k = +$ ($k = -$). Equation (24) becomes, then,

$$
\forall u < \min(u^*, 0), e^{u(Z_0 - Z^*)} = E\left[1_{T<\infty}e^{-TF(u)}\right].
\tag{28}
$$

Equation (28) is the Laplace transform of the random variable T when $1_{T<\infty} = 1$:

$$
\forall F > 0, E\left[1_{T<\infty}e^{-T \cdot F}\right] = e^{u(Z_0 - Z^*(Z_0))}.
\tag{29}
$$

Differentiating (29) with respect to F yields:

$$E\left[-1_{T<\infty}Te^{-(T+1)\cdot F}\right] = -(Z_0 - Z^*(Z_0))e^{-u(Z_0-Z^*(Z_0))}\frac{\partial u}{\partial F}$$

Using (21) leads to

$$E\left[-1_{T<\infty}Te^{-(T+1)\cdot F}\right] = -(Z_0 - Z^*(Z_0))e^{-u(Z_0-Z^*(Z_0))}\frac{\rho e^u + (1-\rho)e^{-u}}{\rho e^u - (1-\rho)e^{-u}}$$

Applying the dominated convergence theorem when $u \uparrow \min(0, u^*)$ (and so $F \downarrow 0$) yields:

$$E\left[1_{T<\infty}T\right] = (Z_0 - Z^*(Z_0))e^{-\min(u^*,0)(Z_0-Z^*(Z_0))}\frac{1}{2\rho e^{\min(u^*,0)} - 1} \tag{30}$$

By definition,

$$E\left[1_{T<\infty}T\right] = E\left[T \mid 1_{T<\infty}\right]E\left[1_{T<\infty}\right] = E\left[T \mid T < \infty\right]\Pr\left[T < \infty\right]. \tag{31}$$

Setting $Z_T = Z^*(Z_0)$ equation (26) becomes $\Pr[T < \infty] = e^{-\min(0,u^*)(Z_0-Z^*(Z_0))}$. Together with (31) this leads to

$$E\left[1_{T<\infty}T\right] = E\left[T \mid T < \infty\right]e^{-\min(0,u^*)(Z_0-Z^*(Z_0))}$$

Combining (30) and (31) yields

$$E\left[T \mid T < \infty\right] = \frac{Z_0 - Z^*(Z_0)}{2\rho e^{\min(u^*,0)} - 1} = \frac{Z_0 - Z^*(Z_0)}{|1 - 2\rho|} = \frac{\Delta_0}{1 - 2\lambda},$$

proving the second part of corollary 1.

E Multiple PBE and Mixed-Strategy PBE in Section 2.3.2

Consider Figure 7. The left-hand panel illustrates a case in which $r_{-1} \in (\underline{r}(V), \underline{r}^*(V))$ and the mapping from prior to posterior induces multiple PBE. The figure displays the relationship between two endogenous variables: the war choice for the high type (σ^+) and the posterior conditional on peace (r_P). The black solid step function shows the optimal war choice for a high type according to equation (6) – recall that in this range $\sigma^- = 0$. Note that $S^+(r_P/(1+r_P)) = V$ at $r_P = \underline{r}^*(V)$, implying that the high type is indifferent between war and peace, hence, any randomization between war and peace is optimal. The grey schedule yields the Bayesian updating, corresponding to equation (4). The crossing points pin down three PBE, corresponding to different self-fulfilling posteriors. The intuition for the multiplicity of equilibria is the following. Suppose agents believe peace to be informative (uninformative). Then, $r_P > \underline{r}^*(V)$ $(r_P = r_{-1} < \underline{r}^*(V))$, the trade surplus is larger (smaller) than the expected benefit of war, and peace (war) is strictly the optimal choice. This fulfils the expectation that peace is informative (uninformative). A third equilibrium in mixed strategies exists, corresponding to the point where the grey schedule intersects the horizontal segment of the black schedule. The mixed equilibrium is not stable to small perturbations of beliefs. The multiplicity disappears when $r_{-1} < \underline{r}(V)$, as the grey curve is shifted down and crosses the step function only once, at $\sigma^+ = 0$. Likewise, there is no multiplicity when $r_{-1} > \underline{r}^*(V)$, as the grey curve only crosses the step function at $\sigma^+ = 1$. Therefore, multiple PBE only arise for a small set of the prior belief space.

The right-hand panel illustrates a case in which $r_{-1} \in (\bar{r}^*(V), \bar{r}(V))$. In this case, the mapping from prior to posterior induces a unique PBE involving randomization of the low type between war and peace (the high type chooses peace with unit probability). The black solid step function shows in this case the optimal war and peace choice for a low type according to equation (6) – recall that in this range $\sigma^+ = 1$. In this case, $S^-(r_P/(1+r_P)) = V$ at $r_P = \bar{r}(V)$, implying that any randomization between war and peace is optimal to the low type. In this case, however, only the interior crossing point is a PBE. To see why the corners are not equilibria, suppose agents believe peace to be informative (uninformative). Then, $r_P > \bar{r}(V)$ $(r_P = r_{-1} < \bar{r}(V))$, the trade surplus is larger (smaller) than the expected benefit of war, and peace (war) is strictly the optimal choice. However, this does not fulfil the expectation that peace is informative (uninformative), since $\sigma^+ = \sigma^- = 1$ $(\sigma^+ = 1$ and $\sigma^- = 0)$ Therefore, the mixed-strategy equilibrium is the only PBE. Moreover, this equilibrium is stable to small perturbations of beliefs. Increasing (decreasing) r_{-1} increases (decreases) the probability that the low type retains peace. When $r_{-1} \geq \bar{r}(V)$ $(r_{-1} \leq \bar{r}^*(V))$ the equilibrium features pure strategies, is uninformative (informative) and entails $\sigma^+ = \sigma^- = 1$ $(\sigma^+ = 1$ and $\sigma^- = 0)$.

F Proof of Lemma 2 and Propositions 6–10

F.1 Proof of Proposition 6

The proof of this Proposition follows immediately from Lemma 1 and its proof.
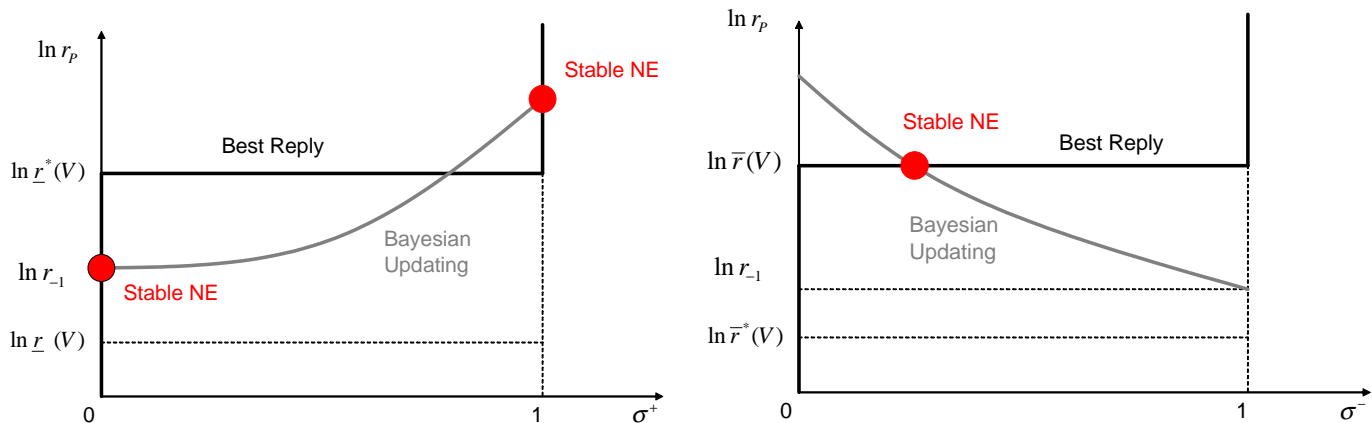
Figure 7: Multiple PBE and Mixed-Strategy PBE

F.2   Proof of Proposition 7

That the DSE exits the informative equilibrium almost surely follows from the proofs of Proposition 5 and of Lemma 3 (see also the discussion in the text).

Next, we must bound the probability of the first passage time of a stochastic process with initial condition r_0. The approach is again similar to the proof of Proposition 5, although now there is both a downward $(\underline{r}(V))$ and an upward $(\bar{r}(V))$ barrier. Moreover, the stochastic process (11) is not a random walk, due to its behavior in the region $r_{t-1} \in [\bar{r}^*(V), \bar{r}(V)]$. We decompose our problem into two parts. First, we study the first passage time of the process (11) out of $(\underline{r}(V), \bar{r}^*(V))$. Second, we study the behavior of (11) in the range $(\bar{r}^*(V), \bar{r}(V))$. The latter turns out to be a one-period problem which is simple to characterize.

Let us first study the probability of a first passage time of the random walk defined by (16), (17), (18) and (19) in presence of a downward (upward) barrier Z_-^* (Z_+^*) and an initial condition Z_0, $Z_-^* < Z_0 < Z_+^*$. The method is very close to the one used for the proof of Proposition 5. Let $T_- = \min\{t; Z_t < Z_-^*\}$ and $T_+ = \min\{t; Z_t > Z_+^*\}$. As discussed in the main text, the strong law of large numbers implies $\Pr(T^- = T^+ = \infty) = 0$ and $\Pr(T^- < T^+ < \infty) = 1 - \Pr(T^+ < T^- < \infty)$. Hence we can focus our analysis on $\Pr(T^- < T^+ < \infty)$. Using the definition (20) we define the Martingale $M_t^- \equiv e^{u(Z_t - Z_-^*) - tF(u)}$. For all $t \in \mathbb{N}$, $M_{0 \wedge T_- \wedge T_+}^- = E_0\left[M_{t \wedge T_- \wedge T_+}^-\right]$. Hence,

$$e^{u(Z_0 - Z_-^*)} = E\left[e^{u(Z_{t \wedge T_- \wedge T_+} - Z_-^*)} e^{-(t \wedge T_- \wedge T_+)F(u)}\right].$$

Following a similar line of argument as the one in equations (23) and (24), we obtain

$$e^{u^*(Z_0 - Z_-^*)} = E\left[e^{u^*(Z_{T_-} - Z_-^*)} 1_{T^- < T^+}\right] + E\left[e^{u^*(Z_{T_+} - Z_-^*)} 1_{T^+ < T^-}\right] \tag{32}$$

Let us consider the subcase $k = -$. From definition (22), we have that $e^{-u^*(1-\delta)} < e^{u^*(Z_{T_-} - Z_-^*)} \leq 1$ and $e^{u^*(Z_+^* - Z_-^*)} \leq e^{u^*(Z_{T_+} - Z_-^*)} < e^{u^*(Z_+^* - Z_-^*)} e^{u^*(1+\delta)}$. Noticing that $E\left[1_{T^- < T^+}\right] = \Pr(T^- < T^+ < \infty) = 1 - E\left[1_{T^+ < T^-}\right]$, we can take the limit $u \to u^* > 0$ into (32) and combine it with the two last

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inequalities to obtain the following bounds

$$\frac{1 - e^{u^*(Z_0 - Z_+^*)}}{1 - e^{-u^*(1 - \delta - Z_+^* + Z_-^*)}} < \Pr(T^- < T^+ < \infty) \le \frac{e^{u^*(Z_+^* - Z_-^* + 1 + \delta)} - e^{u^*(Z_0 - Z_-^*)}}{e^{u^*(Z_+^* - Z_-^* + 1 + \delta)} - 1} \tag{33}$$

In the alternative subcase $k = +$, definition (22) implies $1 \le e^{u^*(Z_{T_-} - Z_-^*)} < e^{-u^*(1 - \delta)}$ and $e^{u^*(Z_+^* - Z_-^*)} e^{u^*(1 + \delta)}$ $< e^{u^*(Z_{T_+} - Z_-^*)} \le e^{u^*(Z_+^* - Z_-^*)}$. Taking the limit $u \to u^* < 0$ into (32) leads to an expression identical to equation (33).

The process (11) restricted to $(\underline{r}(V), \bar{r}^*(V))$ is a random walk. As a consequence, $P_{WDLT}(r_0)$ is bounded below by the lower bound of inequality (33) with the following barriers $Z_-^* \equiv \ln \underline{r}(V) / s$ and $Z_+^* \equiv \ln \bar{r}^*(V) / s$. Now imagine that the process first escapes the interval $(\underline{r}(V), \bar{r}^*(V))$ by crossing the barrier $\bar{r}^*(V)$. By definition, the process (11) restricted to $(\bar{r}^*(V), \bar{r}(V))$ is a one-period process with a probability of not crossing $\bar{r}(V)$ at most equal to λ_W (resp. $1 - \lambda_P$) if $k = +$ (resp. $k = -$). As a consequence, $P_{WDLT}(r_0)$ is bounded from above by the upper bound of inequality (33), with the barriers $Z_-^* \equiv \ln \underline{r}(V) / s$ and $Z_+^* \equiv \ln \bar{r}(V) / s$.

F.3   Proof of Proposition 8

For a given $\left(\pi_P, \beta_t^-, \beta_t^+, z\right) \in [0, 1]^4$ let denote $G^+(n^-, n^+), G^-(n^-, n^+)$ the RHS of (12). Let define the function $\mathbf{G}$ such that $\forall (n^-, n^+) \in [0, 1]^2, \mathbf{G}(n^-, n^+) \equiv [G^-(n^-, n^+), G^+(n^-, n^+)]$. An equilibrium of the investment game corresponds to a fixed point of $\mathbf{G}$. Following its definition we see that $\mathbf{G}$ is a continuous map from $[0, 1]^2$ to $[0, 1]^2$. The Brouwer fixed point theorem implies that $\mathbf{G}$ has at least one fixed point.

To prove uniqueness of the equilibrium we proceed by contradiction. Let us assume that $\mathbf{G}$ admits two fixed points $\mathbf{n}_0 \equiv \left(n_0^-, n_0^+\right)$ and $\mathbf{n}_1 \equiv \left(n_1^-, n_1^+\right)$. We define $\mathbf{a} \equiv (a^-, a^+) \in [0, 1]^2$ and $\mathbf{b} \equiv (b^-, b^+) \in [0, 1]^2$ as the intercepts of the line $n_0 n_1$ with the convex hull of $[0, 1]^2$. By definition, the points $\mathbf{n}_0$ and $\mathbf{n}_1$ are included in the segment $[\mathbf{a}, \mathbf{b}]$. Without loss of generality let us also rank them such that $\mathbf{n}_0 \in [\mathbf{a}, \mathbf{n}_1]$ and $\mathbf{n}_1 \in [\mathbf{n}_0, \mathbf{b}]$. In term of linear combinations we define $(\sigma_0, \sigma_1) \in [0, 1]^2$ such that

$$\mathbf{n}_0 = \sigma_0 \times \mathbf{a} + (1 - \sigma_0) \times \mathbf{n}_1 \tag{34}$$

$$\mathbf{n}_1 = \sigma_1 \times \mathbf{n}_0 + (1 - \sigma_1) \times \mathbf{b} \tag{35}$$

Following assumption 1 we know that $G^-(n^-, n^+)$ and $G^+(n^-, n^+)$ are convex (see the proof of Proposition 2). Applying the convexity criterions of G^- and G^+ to (34) and (35) and using the fact that $\{\mathbf{n}_0, \mathbf{n}_1\}$ are fixed points of $\mathbf{G}$ we get

$$n_0^- \le \sigma_0 \times G^-\left(a^-\right) + (1 - \sigma_0) \times n_1^- \tag{36}$$

$$n_1^- \le \sigma_1 \times n_0^- + (1 - \sigma_1) \times G^-\left(b^-\right) \tag{37}$$

$$n_0^+ \le \sigma_0 \times G^+\left(a^+\right) + (1 - \sigma_0) \times n_1^+ \tag{38}$$

$$n_1^+ \le \sigma_1 \times n_0^+ + (1 - \sigma_0) \times G^+\left(b^+\right) \tag{39}$$

The fact that $\mathbf{a}$ and $\mathbf{b}$ belong to the convex hull of $[0, 1]^2$ implies that the set of equations (34)-(35) and the set of conditions (36)-(39) are not mutually compatible. For example, let us consider the subcase where $a^+ = 1, b^+ = 0$. Following assumption 1 we know that $G^+(1) < 1$ and $G^+(0) \ge 0$. As a consequence equation (38) rewrites as $n_0^+ < \sigma_0 + (1 - \sigma_0) \times n_1^+$ while equation (34) rewrites as

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$n_0^+ = \sigma_0 + (1 - \sigma_0) \times n_1^+$. A contradiction. The same line of argument applies to the five other generic subcases, namely $(a^+, b^-) = (1,0), (a^+, b^-) = (1,1), (a^-, b^-) = (0,1), (a^+, b^-) = (0,1), (a^-, b^+) = (0,0)$. We conclude from this discussion that the equilibrium exists and is unique.

F.4 Proof of Lemma 2

From continuity of the system (12) we get that the equilibrium value of $\left(n_A^-, n_A^+, n_B^-, n_B^+\right)$ is continuous in π_P and β.

Let us first prove that for a given β the equilibrium value $n_B^-(\pi_p, \beta)$ is non decreasing in π_P. We proceed by contradiction. Let us assume that there exists some compact subset of [0,1] such that $n_B^-(\pi_p, \beta)$ is decreasing in π_P. A look at (12) shows that $n_B^+(0, \beta) < n_B^+(1, \beta)$ and $n_B^-(0, \beta) < n_B^-(1, \beta)$. By continuity of the path $\left[n_B^-(\pi_P, \beta), n_B^+(\pi_P, \beta)\right]_{\pi_P \in [0,1]}$ in the space $[0,1]^2$ we conclude that there exists (π_0, π_1) with $\pi_0 < \pi_1$ such that

$$n_B^-(\pi_0, \beta) = n_B^-(\pi_1, \beta) \tag{40}$$

From (12) we know that any equilibrium is such that

$$n_B^- - \beta F^B\left(zF^-\left(zn_B^-\right)\right) = n_B^+ - \beta F^B\left(zF^+\left(zn_B^+\right)\right) \tag{41}$$

Combining (40) and (41) yields

$$n_B^+(\pi_0, \beta) - \beta F^B\left(zF^+\left(zn_B^+(\pi_0, \beta)\right)\right) = n_B^+(\pi_1, \beta) - \beta F^B\left(zF^+\left(zn_B^+(\pi_1, \beta)\right)\right) \tag{42}$$

Following assumption 1 we know that F^B, F^+, F^- are non decreasing and convex. So equality (42) yields

$$n_B^+(\pi_0, \beta) = n_B^+(\pi_1, \beta) \tag{43}$$

The two conditions (40) and (43) imply that $\pi_0 = \pi_1$. A contradiction.

We deduce from the previous discussion that $n_B^-(\pi_P, \beta)$ is non decreasing in π_P. A similar argument can be applied to show that $n_B^-(\pi_P, \beta)$ is also non increasing in β and $n_B^+(\pi_P, \beta)$ is non decreasing in π_P and β.

Finally, it is clear that $\forall (\pi_P, \beta) \in [0,1]^2, n_B^-(\pi_P, \beta) \leq n_B^+(\pi_P, \beta)$. For a given β, it is true for $\pi_P = 0$ and $\pi_P = 1$. As a consequence, if it was not true, there would be a π such that $n_B^-(\pi, \beta) = n_B^+(\pi, \beta)$. Using (41) this would imply $F^B(zF^-(zn_B^-)) = F^B(zF^+(zn_B^-))$, which is not compatible with the fact that F^- $FOSD$ F^+.

The trade surplus $S^k(\pi_P, \beta)$ with $k \in \{-, +\}$ is given by

$$S^k(\pi_P, \beta) = \int^{zn_B^k(\pi_P, \beta)} F^k(\iota) \; d\iota$$

Given that F^- $FOSD$ F^+ and given that $n_B^-(\underset{(+) \; (-)}{\pi_P, \; \beta}) \leq n_B^+(\underset{(+) \; (+)}{\pi_P, \; \beta})$ we get that the trade surplus $S^-(\pi_P, \beta), S^+(\pi_P, \beta)$ are continuous and $S^-(\underset{(+) \; (-)}{\pi_P, \; \beta}) \leq S^+(\underset{(+) \; (+)}{\pi_P, \; \beta})$.

F.5 Proof of Proposition 9

F.5.1 Investment/trade continuation game

We assume that the initial share of informed agents, β_0, is CK. This implies that the initial condition is such that $\beta_0^+ = \beta_0^- = \beta_0$. For a given triplet $(\pi_P, \beta^+, \beta^-)$ the stage game equilibrium is characterized by (12). Given $\beta_0^+ = \beta_0^- = \beta_0$ it is straightforward to show by forward iteration of (13) that for all continuation paths we have $n_B^- \leq n_B^+$ and $\beta^- \leq \beta^+$. As a consequence, for each $(\pi_P, \beta^+, \beta^-)$ the game equilibrium is given by the following equations.

Regime A: for $\beta^- < 1 - \frac{x}{\pi_P z^2}$ and $\beta^+ > \frac{-x+2xz^2(1-\pi_P)+(1-z^2)}{(1-\pi_P)(1-z^2+x)z^2} + \frac{x-(1-z^2)}{x+(1-z^2)}\frac{\pi_P}{1-\pi_P}\beta^-$, the Nash equilibrium is

$$
\begin{cases}
n_B^+ = z\dfrac{-x(1-\beta^+)+(1-z^2)\beta^+ + \pi_P(1+x)(1-\beta^+)+\pi_P z^2(\beta^+-\beta^-)}{1-z^2(1-\pi_P(1-\beta^-))}; \;\; n_B^- = \dfrac{z\pi_P(1+x)(1-\beta^-)-zx}{1-z^2(1-\pi_P(1-\beta^-))} \\[4mm]
n_A^+ = 1; \;\; n_A^- = zn_B^- - x
\end{cases}
$$

Regime B: for $\beta^+ < \min\left\{\frac{1-\pi_P z^2-x}{z^2(1-\pi_P)}, \frac{1-2\pi_P z^2}{z^2(1-\pi_P)} + \frac{\pi_P}{(1-\pi_P)}\beta^-\right\}$, the Nash equilibrium is

$$
\begin{cases}
n_B^+ = \dfrac{zx(\beta^+ + (1-\beta^+)\pi_P)}{1-z^2(\beta^+ + (1-\beta^+)\pi_P)}; \;\; n_B^- = \dfrac{zx\pi_P(1-\beta^-)}{1-z^2(\beta^+ + (1-\beta^+)\pi_P)} \\[4mm]
n_A^+ = zn_B^+ + x; \;\; n_A^- = 0
\end{cases}
$$

Regime C: for $\beta^- > 1 - \frac{x}{\pi_P z^2}$ and $\beta^+ > \frac{1-\pi_P z^2-x}{z^2(1-\pi_P)}$ the Nash equilibrium is

$$
\begin{cases}
n_B^+ = z(1-\beta^+)\pi_P + z\beta^+; \;\; n_B^- = z(1-\beta^-)\pi_P \\[4mm]
n_A^+ = 1; \;\; n_A^- = 0
\end{cases}
$$

Regime D: for $\beta^+ > \frac{1-2\pi_P z^2}{z^2(1-\pi_P)} + \frac{\pi_P}{(1-\pi_P)}\beta^-$ and $\beta^+ < \frac{-x+2xz^2(1-\pi_P)+(1-z^2)}{(1-\pi_P)(1-z^2+x)z^2} + \frac{x-(1-z^2)}{x+(1-z^2)}\frac{\pi_P}{1-\pi_P}\beta^-$ the Nash equilibrium is

$$
\begin{cases}
n_B^+ = xz\dfrac{-1+(2-z^2)\beta^+ + \pi_P(2(1-\beta^+)+z^2(\beta^+-\beta^-))}{(1-z^2)(1-z^2(\beta^+-\pi_P(\beta^+-\beta^-)))}; \;\; n_B^- = xz\dfrac{-1+z^2\beta^+ - \pi_P(z^2(\beta^+-\beta^-)-2(1-\beta^-))}{(1-z^2)(1-z^2(\beta^+-\pi_P(\beta^+-\beta^-)))} \\[4mm]
n_A^+ = zn_B^+ + x; \;\; n_A^- = zn_B^- - x
\end{cases}
$$

As a consequence, when $\pi_P < x/z^2$, the economy is in regime B iff $\beta^+ < \frac{1-\pi_P z^2-x}{z^2(1-\pi_P)}$. Otherwise it is in regime C. When $x/z^2 < \pi_P < 1/2z^2$, the economy is in regime A iff $\beta^- < 1 - \frac{x}{\pi_P z^2}$ and $\beta^+ > \frac{-x+2xz^2(1-\pi_P)+(1-z^2)}{(1-\pi_P)(1-z^2+x)z^2} + \frac{x-(1-z^2)}{x+(1-z^2)}\frac{\pi_P}{1-\pi_P}\beta^-$. It is in regime B iff $\beta^+ < \min\left\{\frac{1-\pi_P z^2-x}{z^2(1-\pi_P)}, \frac{1-2\pi_P z^2}{z^2(1-\pi_P)} + \frac{\pi_P}{(1-\pi_P)}\beta^-\right\}$. It is in regime C iff $\beta^- > 1 - \frac{x}{\pi_P z^2}$ and $\beta^+ > \frac{1-\pi_P z^2-x}{z^2(1-\pi_P)}$. Otherwise it is in regime D. When $\pi_P > 1/2z^2$, the economy is in regime C iff $\beta^- > 1 - \frac{x}{\pi_P z^2}$. It is in regime D when $\beta^+ < \frac{-x+2xz^2(1-\pi_P)+(1-z^2)}{(1-\pi_P)(1-z^2+x)z^2} + \frac{x-(1-z^2)}{x+(1-z^2)}\frac{\pi_P}{1-\pi_P}\beta^-$ and in regime A otherwise. A sufficient condition for regime D to disappear is $\pi_P > \frac{x+(1-z^2)}{2x}$.

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F.5.2 Existence of Ω_{WDLT}

Hereafter we rescale the problem in terms of odds ratio $r \equiv \pi_P/(1-\pi_P)$. We first characterize a subset of the WDLT in the following Lemma.

Lemma 4 *Assume* $V \in (S^+(0), \min(S^+(1), 1/2))$. *For all* $\theta \geq zx/(1+zx)$, *there exists* $\left(\underline{r}_0(\theta), \beta_\infty^+(\theta)\right) \in \mathbb{R}^+ \times (0,1]$ *such that*

$$\left\{\left(r, \beta_0^+, \beta_0^-\right) \in \mathbb{R}^+ \times [0,1]^2 \mid 0 \leq r \leq \underline{r}_0(\theta), 0 \leq \beta_0^+ = \beta_0^- \leq \beta_\infty^+(\theta)\right\} \text{ is a non empty subset of } \Omega_{WDLT}.$$

Proof. We first provide a proof of this Lemma in the limit case $z = 1$. Then we consider the general case $z < 1$.

Let us consider $\theta \in [0,1]$ and $(r, \beta) \in \mathbb{R}^+ \times [0,1]$. We want to provide a sufficient condition for $\left(r_0 = r, \beta_0^+ = \beta, \beta_0^- = \beta\right) \in \Omega_{WDLT}$. From definition (7) $\left(r_0 = r, \beta_0^+ = \beta, \beta_0^- = \beta\right) \in \Omega_{WDLT}$ iff for all continuation paths $\left[r_t, \beta_t^+, \beta_t^-\right]_0^\infty$, $r_t = r$ and

$$S^+\left(r, \beta_t^+, \beta_t^-\right) < V \tag{44}$$

In all generality the stochastic dynamics of S^+ are difficult to characterize except if $(r, \beta_t^+, \beta_t^-)$ are low enough. Hence we also impose the following additional sufficient condition which guaranties that all continuation paths $\left[r, \beta_t^+, \beta_t^-\right]_0^\infty$ evolve within regime B (see Section F.5.1)

$$\begin{cases} \beta_t^+ \leq 1 - (1+r)x \text{ for } r \in \left[0, \frac{x}{1-x}\right] \\ \beta_t^+ \leq 1 - r \text{ for } r \in \left[\frac{x}{1-x}, 1\right] \end{cases} \tag{45}$$

Within regime B the trade surplus is an increasing function of β_t^+

$$S^+\left(r, \beta_t^+, \beta_t^-\right) = \left(n_{Bt}^+ + x\right)^2/2 = \frac{(1+r)^2 x^2/2}{\left(1-\beta_t^+\right)^2} \tag{46}$$

and the lom of β_t^+ is derived from (13)

$$\beta_t^+/(1-\theta) = (1 - I_{WAR,t-1})\left[rx + (1+x)\beta_{t-1}^+\right] + I_{WAR.t-1}\beta_{t-1}^+ \tag{47}$$

We notice that the threshold β_∞^+ defined by

$$\beta_\infty^+(r, \theta) = xr/\left(\frac{\theta}{1-\theta} - x\right) \tag{48}$$

corresponds to the fixed-point of (47) when $I_{WAR,t-1} = 0$ for all t. Moreover it is clear from (47) that $\beta_{t-1}^+ \leq \beta_\infty^+$ implies $\beta_t^+ \leq \beta_\infty^+$. We impose an additional sufficient condition on the initial condition, namely that

$$\beta \leq \beta_\infty^+(r, \theta) \tag{49}$$

This implies $\beta_t^+ \leq \beta_\infty^+(r, \theta)$ for all continuation paths $\left[r, \beta_t^+, \beta_t^-\right]_0^\infty$. As a consequence, for all continuation paths $S^+\left(r, \beta_t^+, \beta_t^-\right) \leq S^+\left(r, \beta_\infty^+, \beta_t^-\right)$. Combining (46) and (48), we get that the sufficient condition (45) becomes

$$\theta \geq \Sigma(r) \equiv \begin{cases} \frac{(1-x)x}{1/(1+r)-x^2} \text{ for } r \in \left[0, \frac{x}{1-x}\right] \\ \\ \frac{1-x/\sqrt{2V}}{1-(1+x)/\sqrt{2V}+1/(1+r)x} \text{ for } r \in \left[\frac{x}{1-x}, 1\right] \end{cases} \tag{50}$$

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and the necessary and sufficient condition (44) becomes

$$\theta \geq \Gamma(r) \equiv \frac{1 - x/\sqrt{2V}}{1 + 1/x\,(1+r) - (1+x)\,/\sqrt{2V}} \tag{51}$$

The two functions $\Sigma(r)$ and $\Gamma(r)$ are upward-sloping with $\Sigma(0) = \Gamma(0) = x/\,(1+x)$. Moreover, $V \in [S^+\,(0)\,, S^-\,(1)]$ implies $V \leq 1/2$, which in turn implies $\Sigma'(0) < \Gamma'(0)$. By L'Hospital rule this implies that there exists an open neighborhood of $r = 0$ such that $\Sigma(r) < \Gamma(r)$. Moreover, for the set $\{(r, \theta) \in [0, 1] \times [0, 1] \mid \theta \geq \Sigma(r) \text{ and } \theta \geq \Gamma(r)\}$ the two conditions (50) and (51) are verified. We define $\Gamma^{-1}(\theta)$ for $\theta \in [0, r^*]$ and $\underline{r}_0(\theta) \equiv \Gamma^{-1}(r^*)$ for $\theta \in [r^*, 1]$ and $\beta_\infty^+(\theta) \equiv \beta_\infty^+(\underline{r}_0(\theta), \theta)$ where β_∞^+ is given by (49). Consequently, for any $\theta \geq x/\,(1+x)$, the set $\{0 \leq r \leq \underline{r}_0(\theta), 0 \leq \beta_0^+ = \beta_0^- \leq \beta_\infty^+(\theta)\}$ is non empty. Moreover, for all continuation paths $[r, \beta_t^+, \beta_t^-]_0^\infty$ the condition (44) is verified; so $(r, \beta_0^+, \beta_0^-) \in \Omega_{WDLT}$.

Let us consider now the general case $z < 1$. We intend to show that the conditions $\Sigma(r)$ and $\Gamma(r)$ still satisfy $\Sigma(0) = \Gamma(0) = x/(r+x)$ and $\Sigma'(0) < \Gamma'(0)$. If correct, by L'Hospital rule, we deduce that there exists an open neighborhood of $r = 0$ such that $\Sigma(r) < \Gamma(r)$. This allows us to conclude the proof in a manner similar to the subcase $z = 1$. For $z < 1$ the sufficient condition (45) becomes

$$\beta_t^+ \leq \frac{1-x}{z^2}\,(1+r) - r \tag{52}$$

The condition (46) becomes

$$S^+\left(r, \beta_t^+, \beta_t^-\right) \equiv \frac{x^2}{2(1 - z^2(\beta_t^+ + (1 - \beta_t^+)r/(1+r))^2} < V \tag{53}$$

The lom of (61) becomes

$$\frac{\beta_t^+}{1-\theta} = (1 - I_{WAR,t-1})\left[\beta_{t-1}^+ + \frac{xz(1 - \beta_{t-1}^+)((1 - \beta_{t-1}^+)r/(1+r) + \beta_{t-1}^+)}{1 - z^2((1 - \beta_{t-1}^+)r/(1+r) + \beta_{t-1}^+)}\right] + I_{WAR,t-1}\beta_{t-1}^+ \tag{54}$$

As a consequence the threshold $\beta_\infty^+(r, \theta)$ is now defined as the root of the second order polynomial $A_1\left(\beta_\infty^+\right)^2 + A_2\beta_\infty^+ + A_3 = 0$ with $A_1(r, \theta) \equiv -z^2\left(\frac{\theta}{1-\theta} - \frac{x}{z}\right)$ and $A_2(r, \theta) \equiv \frac{\theta}{1-\theta}\left(1 + r - rz^2\right) - xz(1 - r)$ and $A_3(r, \theta) \equiv -xzr$. For r close to 0, a first order Taylor expansion leads to $\beta_\infty^+(r, \theta) = \left[-A_2(r, \theta) \pm \sqrt{A_2(r, \theta)^2 - 4A_1(r, \theta)A_3(r, \theta)}\right]/2A_1(r, \theta) \underset{r \sim 0}{\simeq} -A_3(r, \theta)/A_2(r, \theta)$. For $\frac{\theta}{1-\theta} \geq xz$ this yields

$$\beta_\infty^+(r, \theta) \simeq \frac{xzr}{\frac{\theta}{1-\theta} - xz} \tag{55}$$

The conditions $\Sigma(r)$ and $\Gamma(r)$ are obtained by plunging $\beta_\infty^+(r, \theta)$ into (52) and (53). This leads to

$$\theta \geq \Sigma(r) \equiv \frac{(1-x)\,x}{(1-x)\,(x + 1/z) - rz/\,(1+r)} \tag{56}$$

$$\theta \geq \Gamma(r) \equiv \frac{1 - x/\sqrt{2V}}{\left(1 - x/\sqrt{2V}\right)(1 + 1/xz) - rz/\,(1+r)\,x} \tag{57}$$

where condition (57) can be verified if and only if $V > x^2/2 = S^+(0)$.

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We want now to analyze the behavior of $\Sigma(r)$ and $\Gamma(r)$ in the neighborhood of $r = 0$. First we notice that $\Sigma(r) = \Gamma(0) = x/(x+z)$. Secondly we get that $\Sigma'(0) < \Gamma'(0)$ if and only if $V < 1/2$. ∎

From Lemma 4 we see that $\theta \geq x/(z+x)$ implies $\Omega_{WDLT} \neq \emptyset$. This is a sufficient condition for the existence of Ω_{WDLT}. We want to show now that this is also a necessary condition. We proceed by contradiction.

Let us assume that there exists $\hat{\theta} < x/(z+x)$ such that $\Omega_{WDLT} \neq \emptyset$. Hence there exists at least one couple $(\bar{r}, \bar{\beta}) \in \mathbb{R} \times [0,1]$ such that $(r_0 = \bar{r}, \beta_0^+ = \bar{\beta}, \beta_0^- = \bar{\beta}) \in \Omega_{WDLT}$. Let us consider $r \in (0, \bar{r})$. We define $C(r, \bar{\beta})$ as the set of equilibrium paths $[r_t, \beta_t^+, \beta_t^-]_0^\infty$ starting with the initial condition $(r_0 = r, \beta_0^+ = \bar{\beta}, \beta_0^- = \bar{\beta})$. We compare it to $C(\bar{r}, \bar{\beta})$, the set of equilibrium paths $[\bar{r}_t, \bar{\beta}_t^+, \bar{\beta}_t^-]_0^\infty$ starting with the initial condition $(r_0 = \bar{r}, \beta_0^+ = \bar{\beta}, \beta_0^- = \bar{\beta})$. From Section F.5.1 we know that $\left.\frac{\partial n_b^+}{\partial r}\right|_{\beta^+, \beta^-} > 0$ and $\left.\frac{\partial n_b^-}{\partial r}\right|_{\beta^+, \beta^-} > 0$. Given that $r < \bar{r}$ and $\beta_0^+ = \beta_0^- = \bar{\beta}_0^+ = \bar{\beta}_0^- = \bar{\beta}$ a forward iteration on the laws of motion (13) implies that for all path in $C(r, \bar{\beta})$, at each period t, there exists a path in $C(\bar{r}, \bar{\beta})$ such that $n_{bt}^+ \leq \bar{n}_{bt}^+$. As a consequence $\forall t, R^+(n_{bt}^+) \leq R^+(\bar{n}_{bt}^+)$. Following the trade surplus definition (2) this implies $\hat{S}^{\max}(r, \bar{\beta}) < \hat{S}^{\max}(\bar{r}, \bar{\beta})$, where $\hat{S}^{\max}(r, \bar{\beta}) = \arg\max_{C(r, \bar{\beta})} S^+$. From definition (7) we know that $\hat{S}^{\max}(\bar{r}, \bar{\beta}) < V$. This in turn leads to $\hat{S}^{\max}(r, \bar{\beta}) < V$ and so $(r_0 = r, \beta_0^+ = \bar{\beta}, \beta_0^- = \bar{\beta}) \in \Omega_{WDLT}$.

Any continuation path $[r_t, \beta_t^+, \beta_t^-]_0^\infty$ of $(r_0 = r, \beta_0^+ = \bar{\beta}, \beta_0^- = \bar{\beta})$ is almost surely in Ω_{WDLT} : In particular this must be the case for the subset of continuation paths characterized by a non interrupted series of war shocks over the period $\{1, ..., T\}$. Given that this sequence has a positive probability $(\lambda_W)^T$, this implies that $(r_T, \beta_T^+, \beta_T^-) \in \Omega_{WDLT}$: From definition (7) this means $r_T = r_t$. Along such a sequence of wars, trade is fully disrupted and the share of informed agents is depleted as memory loss takes place at a pace θ. We have: $(\beta_T^-, \beta_T^+) = \theta^T(\bar{\beta}, \bar{\beta})$. As a consequence we get that $\forall T < \infty, (r, \theta^T \bar{\beta}, \theta^T \bar{\beta}) \in \Omega_{WDLT}$.

For $\varepsilon, \eta > 0$, let us define the set $\Omega(\varepsilon, \eta) \equiv \{(r, \beta_0^+, \beta_0^-) \in \mathbb{R} \times [0,1]^2 \mid 0 < r < \varepsilon, 0 \leq \beta_0^+ = \beta_0^- < \eta\}$. From the previous discussion we see that $\Omega_{WDLT} \neq \emptyset$ implies $\exists \varepsilon, \eta > 0$ such that $\Omega(\varepsilon, \eta) \subset \Omega_{WDLT}$. The interpretation is clear: if the WDLT is non empty, it must include the cases where beliefs are extremely pessimistic and the initial share of informed agents is very low. Hence there exists at least one $(\hat{r}, \hat{\beta}) \in \Omega(\varepsilon, \eta)$ such that $\Sigma(\hat{r}) < \Gamma(\hat{r})$ and $\hat{\beta} < \beta_\infty^+(\hat{\theta})$. Moreover, the same line of reasoning as above implies that $(r_0 = \hat{r}, \beta_0^+ = \hat{\beta}, \beta_0^- = \hat{\beta}) \in \Omega_{WDLT}$ for $\theta = \hat{\theta} \Rightarrow (r_0 = \hat{r}, \beta_0^+ = \hat{\beta}, \beta_0^- = \hat{\beta}) \in \Omega_{WDLT}$ for $\theta > \hat{\theta}$.

In particular this is the case for any θ such that $\Sigma(\hat{r}) < \theta < \Gamma(\hat{r})$, a non empty range given L'Hospital rule and the fact that $\Sigma(0) = \Gamma(0)$ and $\Sigma'(0) < \Gamma'(0)$. For such a θ we have $S^+(r, \beta_t^+, \beta_t^-) < V$ for all continuation paths $[r_t, \beta_t^+, \beta_t^-]_0^\infty$, starting with the initial condition $(r_0 = \hat{r}, \beta_0^+ = \beta_0^- < \beta_\infty^+(\theta))$. This means that condition (44) is verified; but this condition is equivalent to (51), namely $\theta \geq \Gamma(\hat{r})$. A contradiction.

F.5.3 Existence of Ω_{PDLT}

This proof follows the same line than the previous one: For all $\theta \geq \theta_P$ with $\theta_P \equiv 1 - \frac{z^2 - (x+\sqrt{2V})}{z^3(1-\sqrt{2V})}$ we characterize a non empty subset of Ω_{PDLT}; then for $\theta < \theta_P$ we show by contradiction that Ω_{PDLT} must be empty.

We first start with the specific case $z = 1$. Let us consider $\theta \in [0, 1]$ and $(r, \beta) \in [1, +\infty) \times [0, 1]$. We want to provide a sufficient condition for $(r_0 = r, \beta_0^+ = \beta, \beta_0^- = \beta) \in \Omega_{PDLT}$. From definition (7)

$\left(r_0 = r, \beta_0^+ = \beta, \beta_0^- = \beta\right) \in \Omega_{PDLT}$ iff for all continuation paths $\left[r_t, \beta_t^+, \beta_t^-\right]_0^\infty$, $r_t = r$ and

$$S^- \left(r, \beta_t^+, \beta_t^-\right) > V \tag{58}$$

In all generality the stochastic dynamics of S^- is difficult to characterize except if r is larger than 1 and (β_t^+, β_t^-) are low enough. Hence we also impose the following additional sufficient condition which guaranties that all continuation paths $\left[r, \beta_t^+, \beta_t^-\right]_0^\infty$ evolve within regime A (see Section F.5.1)

$$\beta_t^- \leq 1 - x\left(1 + r\right)/r \text{ and } r \geq 1 \tag{59}$$

Within regime A the trade surplus is an increasing function of β_t^-

$$S^- \left(r, \beta_t^+, \beta_t^-\right) = \left(n_{Bt}^- - x\right)^2/2 = \left(1 - \frac{x\left(1 + r\right)}{\left(1 - \beta_t^-\right)r}\right)^2/2 \tag{60}$$

and the lom of β_t^- is derived from (13)

$$\beta_t^-/(1 - \theta) = (1 - I_{WAR,t-1})\left[1 - x/r - x\beta_{t-1}^-\right] + I_{WAR,t-1}\beta_{t-1}^- \tag{61}$$

This is an oscillating dynamics upper bounded by $\beta_{\max}^-$ as long as $I_{WAR,t-1} = 0$ and condition (59) is satisfied. Hence we get that $\beta \leq \beta_{\max}^-$ implies that for all continuation paths we have $\beta_t^- \leq \beta_{\max}^-$ where

$$\beta_{\max}^- \left(r, \theta\right) = (1 - \theta)(1 - x/r) \tag{62}$$

Combining (59) and (62), we obtain that the sufficient condition (59) becomes

$$\theta \geq \Phi(r) \equiv \frac{x}{1 + x - x\left(1 + r\right)/r} \tag{63}$$

and the necessary and sufficient condition (58) becomes

$$\theta \geq \Delta(r) \equiv \frac{(1 + r)/\left(1 - \sqrt{2V}\right) - 1}{r/x - 1} \tag{64}$$

In the space $(r, \theta) \in [1, +\infty) \times [0, 1]$ the two functions $\Phi(r)$ and $\Delta(r)$ are decreasing in r with $\Phi(+\infty) = x$ and $\Delta(+\infty) = x/(1 - \sqrt{2V})$. Given that $\Phi(+\infty) < \Delta(+\infty)$ we infer that for all $\theta \geq \Delta(+\infty)$ there exists a threshold $\tilde{r}(\theta)$ such that $\forall r > \tilde{r}(\theta)$, the couple (r, θ) verifies conditions (63) and (64); this in turn implies that for $\beta \leq \beta_{\max}^- \left(r, \theta\right)$ we have $(r, \beta, \beta) \in \Omega_{PDLT}$.

Let us consider now the general case $z < 1$. Conditions (59) and (60) become

$$\beta_t^- \leq 1 - \frac{x(1 + r)}{z^2 r} \tag{65}$$

$$S^- \left(r, \beta_t^+, \beta_t^-\right) = \left[\frac{z^2 r(1 + x)(1 - \beta_t^-)/(1 + r) - z^2 x}{1 - z^2(1 - r(1 - \beta_t^-)/(1 + r))} - x\right]^2/2 > V \tag{66}$$

Moreover the law of motion (61) is given by

$$\beta_t^-/(1 - \theta) = (1 - I_{WAR,t-1})\left[\frac{z(\frac{r}{1+r}(1 + x)(1 - \beta_{t-1}^-) - x)(1 - \beta_{t-1}^-)}{1 - z^2(1 - \frac{r}{1+r}(1 - \beta_{t-1}^-))}\right] + \beta_{t-1}^-$$

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As a consequence we obtain

$$\beta_{\max}^{-}(r,\theta) = (1-\theta)z\frac{r-x}{1-z^2+r} \tag{67}$$

Combining (67) with (65) and (66) we get the implicit definitions of $\Phi(r)$ and $\Delta(r)$ respectively

$$(1-\theta)z\frac{r-x}{1-z^2+r} \equiv 1 - \frac{x(1+r)}{z^2 r}$$

$$\left[\frac{z^2 r(1+x)(1-\beta_{\max}^{-}(r,\theta))/(1+r) - z^2 x}{1-z^2(1-r(1-\beta_{\max}^{-}(r,\theta))/(1+r))}\right] = x + \sqrt{2V}$$

Taking the limit $r \to +\infty$ in the two previous equations leads to $\beta_{\max}^{-}(+\infty,\theta) = (1-\theta)z$ and $\Phi(+\infty) = 1 - \frac{1}{z} + \frac{x}{z^3}$ and $\Delta(+\infty) = 1 - \frac{z^2-\left(x+\sqrt{2V}\right)}{z^3\left(1-\sqrt{2V}\right)}$. Hence we have $\Phi(+\infty) < \Delta(+\infty)$ iff $V < \left(x-z^2\right)^2/2 = S^{-}(1)$. Thus, for all $\theta \geq \Delta(+\infty)$ there exists a threshold $\tilde{r}(\theta)$ such that $\forall r > \tilde{r}(\theta)$, the couple (r,θ) verifies conditions (65) and (66); this in turn implies that for $\beta \leq \beta_{\max}^{-}(r,\theta)$ we have $(r,\beta,\beta) \in \Omega_{PDLT}$.

Let us prove now that $\theta < \Delta(+\infty) \equiv \theta_P$ leads to $\Omega_{PDLT} = \emptyset$. We proceed by contradiction. Let us assume that there exists $\hat{\theta} < \theta_P$ such that $\Omega_{WDLT} \neq \emptyset$. Following the same reasoning than in the previous section this implies $\exists \varepsilon, \eta > 0$, such that $\left\{\left(r,\beta_0^+,\beta_0^-\right) \in \mathbb{R} \times [0,1]^2 \mid 1/\varepsilon < r, 0 \leq \beta_0^+ = \beta_0^- < \eta\right\} \subset \Omega_{PDLT}$. But this must imply that $\Phi(r) \geq \Delta(r)$ for all $r > 1/\varepsilon$. A contradiction given that $\Phi(+\infty) < \Delta(+\infty)$. Finally, straightforward computations show that $\theta_P > \theta_W$.

F.6 Proof of Proposition 10

Let us consider a triplet $(r_0,\beta_0^+,\beta_0^-) \notin \Omega_{WDLT}$. Our goal is to show that there is a non-zero measure subset of continuation paths $\left[\beta_t^-,\beta_t^+,r_t\right]_{t=0}^{\infty}$ which enter into Ω_{WDLT} in finite time. To this purpose we aim to exhibit a non-zero measure scenario over the period $0,...,T$ such that $\left(r_T,\beta_T^-,\beta_T^+\right) \in \Omega_{WDLT}$. The proof proceeds in two stages. First we show that with a strictly positive probability the equilibrium path belief can go in finite time below the cutoff $\underline{r}_0(\theta)$ as given by Lemma 4: $\Pr\left\{\exists T_1 < \infty \mid r_{T_1} < \underline{r}_0(\theta)\right\} > 0$. Then we show that just after the threshold $\underline{r}_0(\theta)$ is reached, there is a non zero probability sequence of T_2 consecutive wars which takes place over the time range $[T_1, T_1+T_2]$ such that $\left(r_{T_1+T_2}, \beta_{T_1+T_2}^+\right)$ verifies the sufficient condition of Lemma 4. Setting $T = T_1 + T_2$ we get $\left(r_{T_1+T_2}, \beta_{T_1+T_2}^+, \beta_{T_1+T_2}^-\right) \in \Omega_{WDLT}$.

Stage 1: Given the initial conditions $(r_0,\beta_0^+,\beta_0^-) \notin \Omega_{WDLT}$ there must be a subset S_1, of strictly positive measure, of continuation paths which violate definition (7). This implies that the first passage time $t_1 \equiv \arg\min_{S_1}\{t \mid S^-(t) < V < S^+(t)\}$ is finite. In particular let us consider the subset $\sigma_1 \subset S_1$ consisting of paths such that there is War at date t_1. The subset σ_1 has a strictly positive measure. Moreover we have: $r_{t_1} = r_0$ and $\ln r_{t_1+1} = \ln r_{t_1} - \ln\frac{1-\lambda_P}{\lambda_W}$. There are two possibilities. Either $\left(r_{t_1+1}, \beta_{t_1+1}\right)$ verifies condition (4) and the proof is completed. Or there is a strictly positive measure subset $S_2 \subset \sigma_1$ of continuation paths which violate definition (7). This implies that the first passage time $t_2 \equiv \arg\min_{S_2}\{t \mid S^-(t) < V < S^+(t)\}$ is finite. In particular, let us consider the subset of $\sigma_2 \subset S_2$ consisting of paths such that there is War at date t_2. The subset σ_2 has a positive measure and we have $r_{t_2} = r_{t_1} = r_0$ and $\ln r_{t_2+1} = \ln r_{t_2} - \ln\frac{1-\lambda_P}{\lambda_W} = \ln r_0 - 2\ln\frac{1-\lambda_P}{\lambda_W}$. This line of reasoning is applied

for a finite number of N steps corresponding to the date t_N such that $\ln r_{t_N} = \ln r_0 - N \ln \frac{1-\lambda_P}{\lambda_W}$ and $r_{t_N} < \underline{r}_0(\theta)$. Then we know that the subset σ_{t_N} has a strictly positive measure and we set $T_1 = t_N$.

Stage 2: The continuation paths starting at date T_1 are such that $r_{T_1} < \underline{r}_0(\theta)$ and $\beta_{T_1} \in [0,1]$. Let us consider the subset of continuation paths with a sequence of T war shocks over the period $t = T_1, ..., t = T_1 + T$. This subset has a measure $(\lambda_W)^T > 0$. Moreover, no trade takes place during the sequence of war shocks. Thus, at date $T_1 + T$, and using (13), we get $\beta_{T_1+T}^+ = (1-\theta)^T \beta_{T_1}$ and $\underline{r}_0(\theta) >$ r_{T_1+T} and $\ln r_{T_1+T} \geq \ln \hat{r}_{T_1+T} \equiv \ln r_{T_1} - T \ln \frac{1-\lambda_P}{\lambda_W}$ where $\hat{r}_{T_1+T}$ corresponds to the posterior belief arising when all the stage equilibria are informative during the sequence of war shocks. By definition, the cutoff $\beta_\infty^+(r, \theta)$ is increasing in r; thus $\beta_\infty^+(r_{T_1+T}, \theta) \geq \beta_\infty^+(\hat{r}_{T_1+T}, \theta)$. We now want to characterize a finite time T such that $\left(r_{T_1+T}, \beta_{T_1+T}^+\right)$ verifies condition of Lemma 4: $\beta_{T_1+T}^+ < \beta_\infty^+(r_{T_1+T}, \theta)$. A sufficient condition is $\beta_{T_1+T}^+ < \beta_\infty^+(\hat{r}_{T_1+T}, \theta)$ which is equivalent to $(1-\theta)^T \beta(T_1) < x\hat{r}_{T_1+T} / \left[\frac{\theta}{1-\theta} - x\right]$. Taking the log and using the definition of $\hat{r}_{T_1+T}$ this is equivalent to

$$T \times \log \frac{(1-\theta)(1-\lambda_P)}{\lambda_W} < \ln r_{T_1} - \log \beta(T_1) + \log x - \log\left(\frac{\theta}{1-\theta} - x\right)$$

Clearly, this condition is verified for a sufficiently large (but finite) T if $\log \frac{(1-\theta)(1-\lambda_P)}{\lambda_W} < 0$. Setting $T_2 = T$ we get that $\left(r_{T_1+T_2}, \beta_{T_1+T_2}^+\right)$ verifies condition of Lemma 4 and so belongs to Ω_{WDLT}.