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ABSTRACT

Learning from Prices, Liquidity Spillovers, and Market Segmentation*

We describe a new mechanism that explains the transmission of liquidity shocks from one security to another ("liquidity spillovers"). Dealers use prices of other securities as a source of information. As prices of less liquid securities convey less precise information,a drop in liquidity for one security raises the uncertainty for dealers in other securities, thereby affecting their liquidity. The direction of liquidity spillovers is positive if the fraction of dealers with price information on other securities is high enough. Otherwise liquidity spillovers can be negative. For some parameters, the value of price information increases with the number of dealers obtaining this information. In this case, related securities can appear segmented, even if the cost of price information is small.

JEL Classification: G10, G12 and G14
Keywords: colocation, contagion, liquidity risk, liquidity spillovers, transparency and value of price information

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1 Introduction

The "flash crash" of May 6, 2010 provides a striking illustration of how a drop in the liquidity of one security can quickly propagate to other securities. As shown in the CFTC-SEC report on the flash crash, buy limit orders for the E-mini futures contract on the S&P 500 index vanished in a few minutes after 2:30 p.m. on May 6, 2010.[1] This evaporation of liquidity in the E-mini futures was soon followed by a similar phenomenon in the SPY Exchange Traded Fund (another derivative security on the S&P 500 index) and in the S&P 500 index component stocks (see Figure 1.12 in the joint CFTC-SEC report), resulting in a very high volatility in transaction prices (with some stocks trading as low as a penny or as high as $100,000).

Why do such liquidity spillovers arise? Addressing this question is of broad interest. It can shed light on sudden and short systematic liquidity crises such as the flash crash. More generally, it can explain why liquidity co-varies across securities.[2] Co-movements in liquidity have important implications for asset pricing since they are a source of systematic risk (see for instance Acharya and Pedersen (2005), Korajczyk and Sadka (2008) and Amihud et al. (2005) for a survey). Yet, their cause(s) is not well understood. Co-variations in liquidity may be driven by systematic variations in the demand for liquidity (see Hendershott and Seasholes (2009) or Koch, Ruenzi and Starks (2010)) or systematic variations in the supply of liquidity. One possibility is that financing constraints constitute a systematic liquidity factor because they bind liquidity providers in different securities at the same time. This mechanism is formalized by Gromb and Vayanos (2002) and Brunnemeier and Pedersen (2007) and has received empirical support from analysis of NYSE stocks (see for instance, Coughenour and Saad (2004) or Comerton-Forde et al. (2010)). Another related explanation is that a drop in the capital available to financial intermediaries active in multiple securities can trigger an increase in risk aversion, impairing the supply of liquidity in these securities (as in Kyle and Xiong (2001)).

In this paper we analyze a new mechanism that generates co-movements in the supply of liquidity in different securities, even when dealers active in these securities are *distinct* and *not* simultaneously hit by a market wide shock. Dealers in a security often rely on the prices of other securities to set their quotes. For instance, dealers in a stock learn information from the prices of other stocks in their industry or stock index futures. We show that cross-security learning by dealers causes *liquidity spillovers* and thereby co-movements in liquidity.

To see this intuitively, consider a dealer in security X who uses the price of security Y as a source of information. Movements in the price of security Y are informative because they reflect news about fundamentals known to dealers in security Y. However, this signal is noisy since price movements in security Y also reflect transient price pressures due to uninformed trades. These transient price pressures account for a larger fraction of price volatility when the cost of

[1] See "Findings regarding the market events of May 6, 2010," CFTC-SEC joint report available at http://www.sec.gov/news/studies/2010/marketevents-report.pdf

[2] Evidence of co-variations in liquidity are provided in Chordia et al. (2000), Hasbrouck and Seppi (2001), Huberman and Halka (2001), Korajczyk and Sadka (2008), Corwin and Lipson (2011) for stocks and Chordia et al. (2005) for bonds and stocks.

liquidity provision for dealers in security Y is higher.[3] For this reason, the informativeness of the price of security Y for dealers in security X is smaller when security Y is less liquid.[4] Now suppose that a shock *specific* to security Y decreases the cost of liquidity provision for dealers in this security (e.g., dealers in this security face less stringent limits on their positions). Thus, security Y becomes more liquid and, for this reason, the price of security Y becomes more informative for dealers in security X (transient price pressures in security Y contribute less to its volatility relative to news about fundamentals). As a result, inventory risk for dealers in security X is lower and the cost of liquidity provision for these dealers declines as well. In this way, the improvement in liquidity for security Y spreads to security X, as shown in Figure 1.

[**Insert Figure 1 about here**]

To formalize this intuition, we consider a model with distinct pools of risk averse dealers operating in two securities, X and Y, with a two-factor structure. Dealers in a given market have identical information on one of the risk factors. However, dealers operating in different markets are informed on different risk factors. For this reason, dealers in one market can learn information about the risk factor on which they have no information by watching the price of the other security. We explore two cases: the case in which learning is *two-sided* (dealers in each security learn from each other's price) and the case in which learning is *one-sided* (the price of one security is informative for dealers in another security but not vice versa).[5] We refer to dealers who engage in cross-security price monitoring as being "pricewatchers." The fraction of pricewatchers associated with a security sets *the dealers' level of attention* to the other security.

The model generates the spillover mechanism portrayed in Figure 1 and a rich set of implications. First, when learning is two-sided, an exogenous shock to the cost of liquidity provision in one security (say Y) is amplified by the propagation of this shock to the cost of liquidity provision in the other security (say X). Indeed, as learning is two-sided, the change in the liquidity of security X feeds back on the liquidity of security Y, which sparks a chain reaction amplifying the initial shock. Hence, liquidity is fragile in our model: a small exogenous drop in the liquidity of one market can ultimately result in a disproportionately large drop in the liquidity of this market and other related markets.

[3] For stocks listed on the NYSE, Hendershott, Li, Menkveld and Seasholes (2010) show that 25% of the monthly return variance is due to transitory price changes. Interestingly, they also find that transient price pressures are stronger when market-makers' inventories are relatively large. This finding implies that price movements are less informative when dealers' cost of liquidity provision is higher, in line with our model.

[4] In this paper, we measure liquidity by the sensitivity of prices to market order imbalances, as in Kyle (1985). The market is more liquid when this sensitivity is low. Empirically, this sensitivity can be measured by regressing price changes on order imbalances (see for instance Glosten and Harris (1988) or Korajczyk and Sadka (2008)).

[5] For instance, consider dealers in a stock and dealers in stock index futures. The stock return is determined both by a systematic factor and an idiosyncratic factor whereas the stock index futures return is only driven by the systematic factor. Suppose that dealers in the stock index futures are well informed on the systematic factor. In this case, dealers in the stock can learn information about the systematic factor from the price of the stock index futures whereas dealers in the stock index futures have nothing to learn from the price of individual stocks. In this case learning is one sided.

Second, when learning is two-sided, the model can feature multiple equilibria with differing levels of liquidity. The reason is as follows. Suppose that dealers in security X *expect* a drop in the liquidity of security Y. Then, dealers in security X expect the price of security Y to be noisier, which makes the market for security X less liquid. But as a consequence, the price of security X becomes less informative for dealers in security Y and the liquidity of security Y drops, which validates the expectation of dealers in security X. Hence, dealers' expectations about the liquidity of the other security can be *self-fulfilling*. For this reason, there exist cases in which, for the same parameter values, the liquidity of securities X and Y can be either relatively high or relatively low.[6] A sudden switch from a high to a low liquidity equilibrium is an extreme form of co-variation in liquidity and fragility since it corresponds to a situation in which the liquidity of several related securities dries up without an apparent reason.

Third, an increase in the fraction of pricewatchers in a security has an ambiguous impact on the liquidity of this security. On the one hand, this increase improves liquidity because pricewatchers require a smaller compensation for inventory risk (as they have more information). On the other hand, entry of new pricewatchers impairs liquidity because it exposes inattentive dealers (i.e., dealers without price information) to adverse selection. Indeed, pricewatchers bid relatively conservatively for the security when they receive bad signals and relatively aggressively when they receive good signals. As a result, inattentive dealers are more likely to end up with relatively large (small) holdings when the value of the security is low (large). In reaction to this winner's curse, inattentive dealers shade their bids, which reduces market liquidity. The net effect on liquidity is always positive when dealers' risk bearing capacity (i.e., dealers' risk tolerance divided by the variance of dealers' aggregate dollar inventory) is low enough. Otherwise, an increase in the fraction of pricewatchers can impair market liquidity when the fraction of pricewatchers is small.

Fourth, the exposure of inattentive dealers to adverse selection implies that liquidity spillovers can be *negative*. To see why, suppose that the liquidity of security Y improves. This improvement implies that the price of security Y conveys more precise information to pricewatchers in security X. Thus, the informational disadvantage of inattentive dealers increases and, as a result, the liquidity of security X may drop. For this to happen, we show that the fraction of pricewatchers must be small enough and dealers' risk bearing capacity must be large.

In a last step, we endogenize the fraction of pricewatchers by introducing a cost of attention to prices. There are several possible interpretations for this cost. It may simply reflect the fact that monitoring the price of other securities requires attention (it is time consuming) and human dealers have limited attention.[7] More importantly maybe, real-time data on prices are costly to acquire. Data vendors (Reuters, Bloomberg, etc. . .) or trading platforms charge a fee for real time datafeed.[8] In particular, some market-makers can choose to pay a "co-location"

[6] There also exist cases in which the equilibrium is unique, even if learning is two-sided.

[7] Recent empirical papers (Corwin and Coughenour (2008), Boulatov et al. (2010) and Chakrabarty and Moulton (2009)) find that attention constraints for NYSE specialists have an effect on market liquidity. Thus, modelling dealer attention is important to understand liquidity.

[8] Market participants often complain about these data fees.For instance, the fee charged by Nasdaq for the dissemination of corporate bond prices has been very controversial. For accounts of these debates, see, for in-

fee to trading platforms in order to obtain the right to place their computers close to platforms' matching engines. In this way, they possess a split second advantage in accessing and reacting to changes in prices. Last, in the absence of real time price reporting (as for instance in some OTC markets), real time price information is available only to a few privileged dealers and very costly to collect for other participants.[9]

When learning is one-sided, the value of price information declines with the fraction of pricewatchers. Thus, the equilibrium fraction of pricewatchers is unique and inversely related to the cost of price information. When dealers' risk bearing capacity is low, a decrease in the cost of price information leads to an improvement in liquidity. Otherwise, liquidity is a U-shaped function of this cost. Indeed, for relatively high values of the cost of price information, a decrease in this cost triggers entry of a few pricewatchers, which is a source of adverse selection risk and impairs liquidity, as explained previously.

In contrast, when learning is two-sided, the value of monitoring the price of, say, security X for dealers in security Y can *increase* with the fraction of pricewatchers in either security (for some parameter values). The reason is as follows. As explained previously, if dealers' risk bearing capacity is low enough, an increase in the fraction of pricewatchers in security Y makes this security more liquid. This improvement in liquidity spreads to security X, which makes the price of this security more informative. Thus, information on the price of security X becomes more valuable for dealers in security Y. Furthermore, the value of information on the price of security X for dealers in security Y also increases in the fraction of pricewatchers in security X. Indeed, as the number of pricewatchers in security X increases, the price of this security becomes more informative, which strengthens its informational value for dealers in security Y.

This finding is surprising since usually the value of financial information declines with the number of investors buying information (Grossman and Stiglitz (1980) or Admati and Pfleiderer (1986)). This principle does not necessarily apply to price information because the precision of price information increases in the number of dealers buying this information.

One consequence is that dealers' decisions to acquire price information on other securities are self-reinforcing both within and across markets. As a result, there can be multiple levels of attention in equilibrium for a fixed value of the cost of attention to prices. In particular, *for identical parameter values*, the markets for the two securities can appear well integrated (the fraction of pricewatchers is high) or segmented (the fraction of pricewatchers is low). As an illustration we construct an example in which, for a fixed correlation in the payoffs of both securities, the markets for securities X and Y are either fully integrated (all dealers are pricewatchers) or segmented (no dealer is a pricewatcher). For dealers in security X, monitoring the price of the other security does not have much value if there are no pricewatchers in security Y and vice versa. Thus, the situation in which the two markets are segmented is self-sustaining

stance, "Latest Market Data Dispute Over NYSE's Plan to Charge for Depth-of-Book Data Pits NSX Against Other U.S. Exchanges," Wall Street Technology, May 21, 2007; the letter to the SEC of the Securities Industry and Financial Markets Association (SIFMA) available at `http://www.sifma.org/regulatory/comment_letters/41907041.pdf`, and "TRACE Market Data Fees go to SEC," Securities Industry News, 6/3/2002.

[9]For instance, a bond dealer may be an employee of a trading firm also active in credit default swaps (CDS). In this way, the dealer may be privy of information on trades in CDSs written on the bond.

and can persist even if the cost of attention declines.

The mechanism that leads to liquidity spillovers in our model generates predictions distinct from the mechanisms based on funding constraints or systematic shifts in risk aversion described in Brunnemeier and Pedersen (2008), Gromb and Vayanos (2002) or Kyle and Xiong (2001). In our model, funding restrictions or an increase in risk aversion for dealers in one asset class (e.g., stocks) can initially spark a drop in the liquidity of this class of assets. However, in contrast to other theories of co-variations in liquidity supply, our model predicts that this shock can spread to other asset classes (e.g., bonds) even if there is no tightening of funding constraints for dealers in other asset classes. The only requirement is that the prices of assets in the first class are used as a source of information to value assets in other classes. Furthermore, as explained previously, in our model liquidity spillovers can be negative while theories based on funding constraints imply positive liquidity spillovers.

Isolating the role of cross-asset learning in liquidity spillovers is challenging empirically because this mechanism can operate simultaneously with other sources of systematic variations in liquidity. One way to address this difficulty consists in studying the effects of changes in trading technologies that affect dealers' ability to learn from the prices of other assets. One strategy is to consider cases in which a security switches from an opaque trading system (e.g., an OTC market) to a more transparent trading system (a case in point is the implementation of post trade transparency in the U.S. bond market in 2002). In this case, dealers in related securities can more easily use the information conveyed by the price of the previously opaque security. This is similar to a decrease in the cost of price information in our model. Another approach is to study the effect of changes in co-location fees. Indeed, dealers who co-locate can be seen as pricewatchers in our model (they have very quick access to prices of other securities and can thereby make their strategies contingent on these prices). Hence, variations in co-location fees should also affect the fraction of pricewatchers. We develop predictions about the effects of such changes in trading technologies in the last part of the paper.

Our model is related to models of contagion (King and Wadhwani (1990), Kodres and Pritsker (2002), or Pasquariello (2007)) and cross-asset price pressures (Andrade, Chang and Seasholes (2008), Bernhardt and Taub (2008), Pasquariello and Vega (2009), Boulatov, Hendershott and Livdan (2010). These models describe various mechanisms through which a shock on investors' information or liquidity traders' demand in one security can affect the *prices* of other securities.[10] None of these models however studies the role of cross-asset learning in the transmission of a *liquidity shock* (i.e., a change in the sensitivity of price to order imbalances) in one security to other securities, as we do here. Our paper is also linked to the literature on the value of financial information (e.g., Grossman and Stiglitz (1980), Admati and Pfleiderer (1986)). We contribute to this literature by studying the value of securities price information. As explained previously, we show that price information is special in the sense that its value can increase with the number of investors buying this information, an effect which does not arise in standard models of information acquisition. In this respect, our paper adds to the few

[10]Most of these models build upon the multi-asset pricing models of Admati (1985) and Caballe and Krishnan (1994).

papers identifying conditions under which the value of financial information may increase with the number of informed investors (Barlevy and Veronesi (2000), Veldkamp (2006), Chamley (2007), and Ganguli and Yang (2009)).

The rest of the paper is organized as follows. Section 2 describes the model. In Section 3, we consider the case in which the fraction of pricewatchers is fixed and we show how liquidity spillovers and multiple equilibria arise in this set-up. In Section 4, we study how the value of price information depends on the fraction of pricewatchers and we endogenize this fraction. Section 5 discusses testable implications of the model and Section 6 concludes. Proofs are collected in the Appendix or the Internet Appendix.

2 The model

We consider two securities, denoted D and F. These securities pay-off at date 2 and their payoffs, v_D and v_F, are given by a factor model with two risk factors δ_D and δ_F, i.e.,

$$v_D = \delta_D + d_D \times \delta_F + \eta, \tag{1}$$

$$v_F = d_F \times \delta_D + \delta_F + \nu. \tag{2}$$

The random variables δ_D, δ_F, η and ν are independent and have a normal distribution, with mean zero. The variance of η is denoted σ_η^2. We make additional parametric assumptions that simplify the exposition without affecting our conclusions. First, there is no idiosyncratic risk for security F (i.e., $\nu = 0$). Second, the variance of the factors is normalized to one. Third, we assume that $d_F = 1$ and $d_D \in [0, 1]$, so that the payoffs of the two securities are positively correlated. To simplify notations, we therefore denote d_D by d. When $d = 0$, the payoff of security D does not depend on factor δ_F. Thus, the price of security F cannot convey new information to dealers in security D. In this case, we say that learning is one-sided.

Trades in securities D and F take place at date 1. In each market, there are two types of traders: *(i)* a continuum of risk-averse speculators and *(ii)* liquidity traders. The aggregate demand of liquidity traders in market j is $u_j \sim N(0, \sigma_{u_j}^2)$. Liquidity traders' demands in both markets are independent and are absorbed by speculators. Hence, in the rest of the paper, we refer to speculators as *dealers* and to u_j as the size of the *demand shock* in market j.

Dealers are specialized: they are active in only one security. In this way, we rule out co-movements in liquidity which arise simply because the same dealers are active in multiple securities.[11] Dealers specialized in security j have perfect information on factor δ_j and no information on factor δ_{-j}. However, they can follow the price of the other security to obtain information on this factor. We denote by μ_j the fraction of dealers specialized in security j who monitor the price of security $-j$ and we refer to μ_j, as the *level of attention* to security $-j$.

[11] In reality, dealer firms are active in multiple securities. However, these firms delegate trade-related decisions to individuals who operate on specialized trading desks. Naik and Yadav (2003) show empirically that the decision-making of these trading desks is largely decentralized (e.g., dealers' trading decisions within a firm are mainly driven by their own inventory exposure rather than the aggregate inventory exposure of the dealer firm to which they belong). Their results suggest that there is no direct centralized information sharing between dealers within these firms.

We refer to these dealers as being *pricewatchers*. Other dealers are called *inattentive dealers*. We use W to index the decisions made by pricewatchers and I to index the decisions made by inattentive dealers. The polar cases, in which there are either no pricewatchers in either market ($\mu_D = \mu_F = 0$) or all dealers are pricewatchers ($\mu_D = \mu_F = 1$) are called the "no attention case" and the "full attention case," respectively. Table 1 summarizes the various possible cases that will be considered in the paper.

Attention/Learning	One-Sided: $d = 0$	Two-Sided: $d > 0$
No Attention	$\mu_D = \mu_F = 0$	$\mu_D = \mu_F = 0$
Limited Attention	$\mu_j > 0$ and $\mu_{-j} < 1$	$\mu_j > 0$ and $\mu_{-j} < 1$
Full Attention	$\mu_D = \mu_F = 1$	$\mu_D = \mu_F = 1$

Table 1: Various Cases

Each dealer in market j has a CARA utility function with risk tolerance γ_j. Thus, if dealer i in market j holds x_{ij} shares of the risky security, her expected utility is

$$E\left[U\left(\pi_{ij}\right)|\delta_j, \mathcal{P}_j^k\right] = E\left[-\exp\left\{-\gamma_j^{-1}\pi_{ij}\right\}|\delta_j, \mathcal{P}_j^k\right], \tag{3}$$

where $\pi_{ij} = (v_j - p_j)x_{ij}$ and $\mathcal{P}_j^k$ is the *price information* available to a dealer with type $k \in \{W, I\}$ operating in security j.

As dealers submit price contingent demand functions, they all act as if they were observing the clearing price in their market. Thus, we have $\mathcal{P}_j^W = \{p_j, p_{-j}\}$ and $\mathcal{P}_j^I = \{p_j\}$. We denote the demand function of a pricewatcher by $x_j^W(\delta_j, p_j, p_{-j})$ and that of an inattentive dealer by $x_j^I(\delta_j, p_j)$.[12] In each period, the clearing price in security j, p_j, is such that the demand for this security is equal to its supply, i.e.,

$$\mu_j x_j^W(\delta_j, p_j, p_{-j})di + (1-\mu_j)x_j^I(\delta_j, p_j)di + u_j = 0, \quad \text{for } j \in \{D, F\}. \tag{4}$$

As in many other papers (e.g., Kyle (1985) or Vives (1995)), we will measure the level of illiquity in security j by the sensitivity of the clearing price to the demand shock (i.e., $\partial p_j/\partial u_j$). In equilibrium, the aggregate inventory position of dealers in security j after trading at date 1 is $-u_j$ and the total dollar value of this position at date 1 is $-u_j \times v_j$. The risk associated with this position for dealers in security j can be measured by its variance conditional on information on risk factor δ_j, i.e., $\sigma_{u_j}^2 \text{Var}[v_j|\delta_j]$. Thus, the ratio of dealers' risk tolerance to this variance (the total amount of risk taken by the dealers) is a measure of the risk bearing capacity of the

[12] As pricewatchers observe the price in security $-j$, they can make their trading strategy in security j contingent on this price. Alternatively, one can assume that pricewatchers do not observe directly the price of security $-j$ but are allowed to place limit orders (a demand function) in security j contingent on the price of other securities. Such indexed limit orders have been proposed by Black (1995) but are typically not offered by exchanges. See Cespa (2004) for an analysis of trading mechanisms that allow multi-price contingent orders.

market. We denote this ratio by $\mathcal{R}_j$:

$$\mathcal{R}_j = \frac{\gamma_j^2}{\sigma_{u_j}^2 \mathrm{Var}[v_j|\delta_j]}. \tag{5}$$

The higher is $\mathcal{R}_j$, the higher is the risk bearing capacity of the dealers in security j. As we shall see this ratio plays an important role for some of our findings.

There are several ways to interpret the two securities in our model. For instance, as in King and Wadhwani (1990), securities D and F could be two stock market indexes for two different countries. Alternatively, they could represent a derivative and its underlying security. For instance, security D could be a credit default swap (CDS) and security F the stock of the firm on which the CDS is written. When $d = f = 1$ and $\sigma_\eta^2 = 0$, the payoff of the two securities is identical, as in Chowdry and Nanda (1991). In this case, the two securities can be viewed as the stock of a cross-listed firm and its American Depository Receipt (ADR) in the U.S. for instance. Factor δ_F can then be viewed as the component of the firm's cash-flows that comes from its sales in the U.S. In each of these cases, it is natural to assume that dealers have specialized information. For instance, dealers in country j will be well informed on local fundamental news but not on foreign fundamental news as in King and Wadhwani (1990).[13]

3 Attention and liquidity spillovers

3.1 Benchmark: No attention

We first analyze the equilibrium in the no attention case ($\mu_D = \mu_F = 0$). For instance, the markets for securities D and F may be opaque so that dealers in each security can obtain information on the price of the other security only after some delay. Alternatively, the prices of each security are available in real time but accessing this information is so costly that no dealer chooses to be informed on the price of the other security (see Section 4).

Lemma 1. *(Benchmark) When $\mu_F = \mu_D = 0$, the equilibrium price in market j is:*

$$p_j = \delta_j + B_{j0} u_j, \tag{6}$$

with $B_{D0} = \gamma_D^{-1}(\sigma_\eta^2 + d^2)$ and $B_{F0} = \gamma_F^{-1}$.

The sensitivity of the equilibrium price for security j to the aggregate demand shock in this market, the illiquidity of security j, is given by B_{j0} (we use index "0" to refer to the case in which $\mu_F = \mu_D = 0$). In the no attention case, the illiquidity of security D is determined by parameters σ_η^2, d, and γ_D. We refer to these parameters as being the "liquidity fundamentals" of security D. Similarly, we refer to γ_F as a liquidity fundamental of security F since it only affects the illiquidity of security F. Illiquidity increases with dealers' risk aversion (γ_j decreases) and uncertainty on the securities' payoffs (σ_η^2 increases).

[13]In the case of the CDS market, dealers in CDS are often affiliated with lenders and therefore better informed on the likelihood of defaults (and size of associated losses) than dealers in the stock market (see Acharya and Johnson (2007))

Importantly, in the benchmark case, there are no liquidity spillovers: a change in the illiquidity fundamental of one market does not affect the illiquidity of the other market. For instance, an increase in the risk tolerance of dealers in security D makes this security more liquid but it has no effect on the illiquidity of the other security.[14] In contrast, with limited or full attention, a change in the illiquidity fundamental of one security will affect the illiquidity of the other security, as shown in the next sections.

3.2 Liquidity spillovers with full attention

In this section, we consider the case in which *all* dealers are pricewatchers, that is the full attention case ($\mu_D = \mu_F = 1$). The analysis is more complex than in the benchmark case as dealers in one security extract information about the factor that is unknown to them from the price of the other security. To solve this signal extraction problem, dealers must form beliefs on the relationship between clearing prices and risk factors. We will focus on equilibria in which these beliefs are correct, i.e., the rational expectations equilibria of the model. We first show that, in contrast to the benchmark case, the levels of illiquidity of both markets are interdependent and this interdependence leads to multiple equilibria (Section 3.2.1). We then provide an explanation for this finding and we show that that the interdependence in the illiquidity of securities D and F leads to liquidity spillovers: a shock to the illiquidity fundamental of one security propagates to the other security (Section 3.2.2). Finally, we show that when learning is two-sided, the total effect of a small shock on the illiquidity fundamental of one security can be much larger than the initial effect of such a shock (Section 3.2.3).

3.2.1 Equilibria with full attention

In our model, a linear rational expectations equilibrium is a set of prices $\{p^*_{j1}\}_{j\in\{D,F\}}$ such that

$$p^*_{j1} = R_{j1}\delta_j + B_{j1}u_j + A_{j1}\delta_{-j} + C_{j1}u_{-j}, \tag{7}$$

and p^*_{j1} clears the market of asset j for each realization of $\{u_j,\ \delta_j,\ u_{-j}, \delta_{-j}\}$ when dealers anticipate that clearing prices satisfy equation (7) and choose their trading strategy to maximize their expected utility (given in equation (3)). We say that the equilibrium is non-fully revealing if pricewatchers in security j cannot infer perfectly the realization of risk factor δ_{-j} from observing the price of security $-j$. The sensitivity of the price in market j to the demand shock in this market, i.e., the "illiquidity of market j," is measured by B_{j1} in the full attention case. Index "1" is used to refer to the equilibrium when $\mu_D = \mu_F = 1$.

Proposition 1. *With full attention and $\sigma^2_\eta > 0$, there always exists a non-fully revealing linear rational expectations equilibrium. At any non-fully revealing equilibrium, $B_{j1} > 0$, $R_{j1} = 1$ and*

[14] In our model, a variation in risk tolerance of dealers in one security is just one way to vary the cost of liquidity provision for dealers in one asset class. In reality variations in this cost may be due to variations in risk tolerance, inventory limits or financing constraints for dealers in this asset class. The important point is that they do not directly affect dealers in other asset classes.

the coefficients, A_{j1} and C_{j1} can be expressed as functions of B_{j1} and B_{-j1}. Moreover

$$B_{D1} = f_1(B_{F1}; \gamma_D, \sigma_\eta^2, d, \sigma_{u_F}^2) = \frac{\sigma_\eta^2}{\gamma_D} + \frac{d^2 B_{F1}^2 \sigma_{u_F}^2}{\gamma_D(1 + B_{F1}^2 \sigma_{u_F}^2)}, \quad (8)$$

$$B_{F1} = g_1(B_{D1}; \gamma_F, \sigma_{u_D}^2) = \frac{B_{D1}^2 \sigma_{u_D}^2}{\gamma_F(1 + B_{D1}^2 \sigma_{u_D}^2)}. \quad (9)$$

Proposition 1 shows that the illiquidities of securities D and F are interdependent since B_{D1} is a function of B_{F1} and vice versa. Moreover, all coefficients in the equilibrium price function can be expressed as functions of the illiquidity of securities D and F. Thus, the number of non-fully revealing linear rational expectations equilibria is equal to the number of pairs $\{B_{D1}^*, B_{F1}^*\}$ solving the system of equations (8) and (9). In general, we cannot characterize these solutions analytically and therefore cannot solve for the equilibria in closed-form. However, we can find these solutions numerically. In Figure 2 we illustrate the determination of the equilibrium levels of illiquidity by plotting the functions $f_1(\cdot)$ and $g_1(\cdot)$ for specific values of the parameters.

[**Insert Figure 2 about here**]

The equilibria are the values of B_{D1} and B_{F1} at which the curves representing the functions $f_1(\cdot)$ and $g_1(\cdot)$ intersect. In panel (a) we set $\gamma_j = d = 1$, $\sigma_{u_j} = 2$, and $\sigma_\eta = 0.2$. In this case, we obtain three equilibria: one with a low level of illiquidity, one with a medium level of illiquidity and one with a relatively high level of illiquidity. In panels (b) and (c), we pick values of σ_η or d such that the correlation between the payoffs of securities D and F is smaller ($\sigma_\eta = 1$ in panel (b) while $d = 0.9$ in panel (c)). In this case, we obtain a unique equilibrium. More generally, when d is low relative to σ_η^2, the model has a unique rational expectations equilibrium, as shown in the next corollary.

Corollary 1. *If $4d^2 < \sigma_\eta^2$ and $\mu_D = \mu_F = 1$ then there is a unique non-fully revealing rational expectations equilibrium.*

In particular, when learning is one sided ($d = 0$), there exists a unique non-fully revealing linear rational expectations equilibrium. Furthermore, in this case, we can characterize the equilibrium in closed-form (see Corollary 6 below).[15]

The case in which $\sigma_\eta^2 = 0$ requires a separate analysis. In this case, it is still true that if there exists a non-fully revealing equilibrium then B_{D1} and B_{F1} solve the system of equations (8) and (9). However, in this case, the unique solution to this system of equations can be $B_{D1} = B_{F1} = 0$ so that a non-fully revealing equilibrium does not exist. As an example, consider the case in which the two securities are identical: $d = 1$, $\sigma_\eta^2 = 0$, $\gamma_F = \gamma_D = \gamma$, $\sigma_{u_j}^2 = \sigma_u^2$. We refer to this case as the *symmetric case.*

[15] The condition given in Corollary 1 is sufficient to guarantee the existence of a unique rational expectations equilibrium when all dealers are pricewatchers, but it is not necessary. Numerical simulations show that there exist multiple equilibria when d is high relative to σ_η^2. Moreover it can be shown formally that the model has either one or three non-fully revealing rational expectations equilibria.

Lemma 2. *In the symmetric case with full attention, if $\sigma_u^2 > 4\gamma^2$, there are two non fully revealing linear rational expectations equilibria: a "High" illiquidity equilibrium and a "Low" illiquidity equilibrium. The levels of illiquidity in each of these equilibria are*

$$\textit{High}: \qquad B^{H*} = \frac{\sigma_u + (\sigma_u^2 - 4\gamma^2)^{1/2}}{2\gamma\sigma_u}, \tag{10}$$

$$\textit{Low}: \qquad B^{L*} = \frac{\sigma_u - (\sigma_u^2 - 4\gamma^2)^{1/2}}{2\gamma\sigma_u}, \tag{11}$$

with $B^{H*} > B^{L*}$. If $\sigma_u^2 < 4\gamma^2$, a non-fully revealing equilibrium does not exist.

3.2.2 Cross-asset learning and liquidity spillovers

We now explain why cross-asset learning is naturally conducive to multiple equilibria and liquidity spillovers. To this end, it is useful to analyze in detail how dealers in one security extract information from the price of the other security. Our starting point is the following lemma.

Lemma 3. *With full attention, in any non-fully revealing linear rational expectations equilibrium,*

$$p_j^* = (1 - A_{j1}A_{-j1})\omega_j + A_{j1}p_{-j}^*, \ \textit{for } j \in \{F, D\}. \tag{12}$$

where $\omega_j \equiv \delta_j + B_{j1}u_j$ for $j \in \{D, F\}$. Hence, ω_{-j} is a sufficient statistic for the price information, $\mathcal{P}_j^W = \{p_j^*, p_{-j}^*\}$, available to pricewatchers operating in security j.

In other words, ω_{-j} is the signal about the risk factor δ_{-j} that pricewatchers operating in security j extract from the price of security $-j$. In the absence of information on the price of security $-j$, the precision of the forecast formed by dealers in security j about the payoff of security j is $(\text{Var}[v_j|\delta_j])^{-1}$. In contrast, with access to price information, the precision of this forecast is[16]

$$\text{Var}[v_j|\delta_j, \omega_{-j}]^{-1} = (\text{Var}[v_j|\delta_j]\left(1 - \rho_{j1}^2\right))^{-1}, \tag{13}$$

where

$$\rho_{j1}^2 \stackrel{def}{=} \frac{E[v_j\omega_{-j}\,|\delta_j\,]^2}{\text{Var}[v_j\,|\delta_j\,]\text{Var}[\omega_{-j}]}. \tag{14}$$

Hence, the higher ρ_{j1}^2 is, the greater the informativeness of the signal conveyed by the price of security $-j$ to dealers in security j. For this reason, we refer to ρ_{j1}^2 as the informativeness of the price of security $-j$ about the payoff of security j for dealers operating in security j. Using the definition of ω_j, we obtain

$$\rho_{D1}^2 = \frac{d^2}{(\sigma_\eta^2 + d^2)(1 + B_{F1}^2\sigma_{u_F}^2)}, \tag{15}$$

$$\rho_{F1}^2 = \frac{1}{1 + B_{D1}^2\sigma_{u_D}^2}. \tag{16}$$

[16] This result follows from the fact that if X and Y are two random variables with normal distribution then $\text{Var}[X|Y] = \text{Var}[X] - \text{Cov}^2[X, Y]/\text{Var}[Y]$ and the fact that $E[\omega_{-j}\,|\delta_j\,] = 0$.

When $d = 0$, the price of security F does not convey information to dealers in security D ($\rho^2_{D1} = 0$) since the payoff of security D does not depend on the risk factor known to dealers in security F. Using the expressions for B_{j1} given in Proposition 1, we obtain that

$$B_{j1} = B_{j0}(1 - \rho^2_{j1}). \tag{17}$$

This observation yields the following result.

Corollary 2. *The markets for securities D and F are less illiquid with full attention than with no attention, i.e.,* $B_{j1} \leq B_{j0}$. *Moreover, with full attention, an increase in the informativeness of the price of security* $-j$ *for dealers in security* j *makes security* j *more liquid, i.e.,*

$$\frac{\partial B_{j1}}{\partial \rho^2_{j1}} \leq 0. \tag{18}$$

The intuition for this result is straightforward. By watching the price of another security, dealers learn information. Hence, they face less uncertainty about the payoff of the security in which they are active. For this reason, with full attention, dealers require a smaller premium than with no attention to absorb a given demand shock (first part of the corollary) and this premium decreases with the informativeness of prices (last part of the corollary).

Price movements in security j are driven both by news about factor δ_j and demand shocks specific to this security. The contribution of demand shocks to price variations becomes relatively higher when security j becomes more illiquid. As a consequence the price of security j becomes less informative for dealers in other markets when security j becomes more illiquid. To see this, remember that the signal about factor δ_j conveyed by the price of security j to dealers in security $-j$ is $\omega_j = \delta_j + B_{j1}u_j$. Clearly, this signal is noisier when B_{j1} is higher, which yields the following result.

Corollary 3. *With full attention, an increase in the illiquidity of security* j *makes its price less informative for dealers in security* $-j$*:*

$$\frac{\partial \rho^2_{-j1}}{\partial B_{j1}} \leq 0. \tag{19}$$

Corollaries 2 and 3 explain why the illiquidity of security D and F are interdependent when dealers in the two securities learn from each other's prices. Indeed, the illiquidity of security $-j$ determines the informativeness of the price of this security for dealers in security j (Corollary 3) and as a result the illiquidity of security j (Corollary 2).

This observation helps us to understand how multiple equilibria can arise when dealers learn from each other's prices. Consider dealers in security F. They do not directly observe the sensitivity of the price to demand shocks in security D, i.e., the illiquidity of security D. Hence, ultimately, the informativeness of the price of security D for dealers in security F depends on their belief regarding the illiquidity of security D. Similarly, the informativeness of the price of security F for dealers in security D depends on their belief regarding the illiquidity of security F. In sum, the illiquidity of security j depends on the beliefs of the dealers active in this security about the illiquidity of security $-j$, which itself depends on the beliefs of its dealers about the illiquidity of security j. This loop leads to multiplicity as, for the same values of the exogenous parameters, various systems of beliefs can be self-sustaining.[17]

This circularity breaks down when dealers in security D do not use the information contained in the price of security F (either because $\mu_D = 0$ or because $d = 0$). In this case, the illiquidity of security D is uniquely pinned down by its "fundamentals" (γ_D and σ_η^2) and, as a result, the beliefs of dealers in security F regarding the liquidity of security D are uniquely defined as well (since dealers' expectations about the illiquidity of the other security must be correct in equilibrium). More generally, when d is low relative to σ_η^2, security D is not much exposed to factor δ_F. Thus, the beliefs of dealers in security D about the liquidity of security F play a relatively minor role in the determination of the liquidity of security D and, for this reason, the equilibrium is unique, as shown in Corollary 1.

The interdependence in the illiquidity of securities D and F has another implication. In contrast to the benchmark case, an exogenous change in the illiquidity of one market (due for instance to an increase in dealers' risk tolerance in this market) affects the illiquidity of the other market. We call this effect a *liquidity spillover*. To see this point, consider the effect of an increase in the risk tolerance of dealers in security D. The immediate effect of this increase is to make security D more liquid as in the benchmark case. Hence, its price becomes more informative for dealers in security F (Corollary 3), which then becomes more liquid (Corollary 2) because inventory risk for dealers in security F is smaller when they are all better informed. Thus, the improvement in the liquidity of security D spreads to liquidity F, although security F experiences no change in its liquidity fundamentals.

More formally, consider the system of equations (8) and (9). Other things equal, an increase in the risk tolerance of dealers in security D makes this security more liquid since $\partial f_1/\partial\gamma_D < 0$. In turn this improvement spreads to security F because $\partial g_1/\partial B_{D1} \neq 0$. More generally, any exogenous change in the illiquidity of security D will spill over to security F because $\partial g_1/\partial B_{D1} \neq 0$. Similarly, an exogenous change in the illiquidity of security F will spill over to security D when $\partial f_1/\partial B_{F1} \neq 0$. The direction (positive/negative) of these liquidity spillovers is determined by the signs of $\partial g_1/\partial B_{D1}$ and $\partial f_1/\partial B_{F1}$.

Corollary 4. *With full attention, liquidity spillovers are always positive, i.e.,* $\partial f_1/\partial B_{F1} \geq 0$ *and* $\partial g_1/\partial B_{D1} > 0$. *Moreover when learning is one sided* ($d = 0$)*, there is no spillover from security* F *to security* D *because the price of security* F *conveys no information to dealers in security* D. *In contrast, when learning is two-sided* ($d > 0$)*, liquidity spillovers operate in both directions.*

Intuitively, positive liquidity spillovers generate positive co-movements in illiquidity across-securities. In our model, illiquidity is not stochastic (it is a deterministic function of the parameters). However, we can create variations in illiquidity by picking randomly the exogenous

[17]Ganguli and Yang (2009) consider a single security model of price formation similar to Grossman and Stiglitz (1980). They show that multiple non-fully revealing linear rational expectations equilibria arise when investors have private information both on the asset payoff and the aggregate supply of the security. The source of multiplicity here is different since dealers have no supply information in our model.

parameters of the model (e.g., the risk tolerance of dealers in security D) and compute the resulting covariance for illiquidity of securities F and D. Figure 5 in Section 3.3 provides an example that shows how positive liquidity spillovers result in positive covariation in liquidity.

3.2.3 Amplification: the illiquidity multiplier

With two-sided learning, liquidity spillovers operate in both directions. As a consequence, the total effect of a small change in the illiquidity fundamentals of one security is higher than the direct effect of these changes.

To see this consider the chain of effects that follows a small *reduction*, denoted by $\Delta\gamma_D < 0$, in the risk tolerance of dealers in security D. The *direct effect* of this reduction is to increase the illiquidity of security D by $(\partial f_1/\partial\gamma_D)\Delta\gamma_D > 0$. As a consequence, the price of this security becomes less informative. Hence, dealers in security F face more uncertainty and security F becomes less liquid as well, although its liquidity fundamental (γ_F) is *unchanged*. The immediate increase in the illiquidity of security F is equal to $(\partial g_1/\partial B_{D1})(\partial f_1/\partial\gamma_D)\Delta\gamma_D > 0$. When learning is two sided ($d > 0$), this increase in illiquidity for security F leads to an even larger increase in the illiquidity of security D, starting a new vicious loop (as the increase in illiquidity for security D leads to a further increase in illiquidity for security F etc,...). As a result, the total effect of the initial decrease in the risk tolerance of dealers in security D is an order of magnitude larger than its direct effect on the illiquidity of both securities. The next corollary formalizes this discussion.

Corollary 5. *Let*

$$\kappa \equiv \frac{1}{(1 - (\partial g_1/\partial B_{D1})(\partial f_1/\partial B_{F1}))}, \tag{20}$$

and assume that $d > 0$. With full attention, the total effects of a change in the risk tolerance of dealers in security D is given by

$$\underbrace{\frac{dB_{D1}}{d\gamma_D}}_{\text{Total Effect}} = \kappa \underbrace{\frac{\partial f_1}{\partial\gamma_D}}_{\text{Direct Effect}} < 0,$$

$$\underbrace{\frac{dB_{F1}}{d\gamma_D}}_{\text{Total Effect}} = \kappa \underbrace{\frac{\partial g_1}{\partial B_{D1}}\frac{\partial f_1}{\partial\gamma_D}}_{\text{Direct Effect}} < 0.$$

and there always exists at least one equilibrium in which $\kappa > 1$.

Thus, the initial effects of a small change in the risk tolerance of dealers in security D are amplified by a factor κ. We call κ the "illiquidity multiplier." This illiquidity multiplier can be relatively large when the illiquidity of each market is very sensitive to the illiquidity of the other market ($(\partial g_1/\partial B_{D1})(\partial f_1/\partial B_{F1})$ is high). In this sense, cross-asset learning is a source of fragility for financial markets.[18]

[18] Allen and Gale (2004) define a financial market as being fragile if "*small shocks have disproportionately large effects*." (Allen and Gale (2004), page 1015).

Figure 3 illustrates this point for specific values of the parameters (in all our numerical examples we choose the parameter values such that there is a unique rational expectations equilibrium, except otherwise stated). It shows the value of κ for various values of the idiosyncratic risk of security D (σ_η) and the resulting values for the direct and total effects of a change in this risk tolerance on the illiquidity of securities D and F, as a function of σ_η. In this example, the total drop in illiquidity of each security after a decrease in risk tolerance for dealers in security D can be up to ten times bigger than the direct effect of this drop.

Table 2 provides another perspective on the illiquidity multiplier by showing the elasticity, denoted $\mathcal{E}_{B_{j1},\gamma_D}$, of illiquidity in each security to the risk tolerance of dealers in security D, i.e., the percentage change in illiquidity in each security for a one percent increase in the risk tolerance of dealers in security D. The table also shows the elasticity that would be obtained ($\hat{\mathcal{E}}_{B_{j1},\gamma_D}$) in the absence of the illiquidity multiplier (e.g., $\kappa = 1$ if $\mu_D = 0$). For instance, when $\gamma_D = 1.8$, a drop of 1% in the risk tolerance of dealers in security D triggers an increase of 9% in the illiquidity of security D and 14.9% in the illiquidity of security F. This is much larger than what would be obtained in the absence of bi-directional spillovers (e.g., if $\mu_D = 0$) since in this case the illiquidity of securities D and F would increase by only 1% and 1.5% respectively.

γ_D	κ	B_{D1}	B_{F1}	Elasticities			
				$\mathcal{E}_{B_{D1},\gamma_D}$	$\hat{\mathcal{E}}_{B_{D1},\gamma_D}$	$\mathcal{E}_{B_{F1},\gamma_D}$	$\hat{\mathcal{E}}_{B_{F1},\gamma_D}$
2.2	1.54	0.19	2.11	−1.54	−1.00	−2.80	−1.81
2	2.16	0.23	2.87	−2.16	−1.00	−3.80	−1.76
1.8	9.94	0.36	5.94	−9.49	−1.00	−14.95	−1.50
1.62	2.35	0.57	11.01	−2.35	−1.00	−2.54	−1.08
1.46	1.65	0.70	13.41	−1.65	−1.00	−1.45	−0.88
1.31	1.39	0.82	15.29	−1.39	−1.00	−1.00	−0.72

Table 2: The table shows the impact of the illiquidity multiplier for different shocks to the risk aversion of dealers in market D. Other parameter values are $d = 1$, $\sigma_\eta = .62$, $\sigma_{u_F} = .1$, $\sigma_{u_D} = 1.6$, $\gamma_D = 1.8$, and $\gamma_F = 1/24$.

The corollary focuses on the effect of an increase in the risk tolerance of dealers in security D but the effects of a change in the other exogenous parameters of the model (γ_F and σ_η^2) are also magnified for the same reasons.

Last, we note that when the equilibrium is unique, it is necessarily such that $\kappa > 1$ (an implication of the last part of Corollary 5). When there are multiple equilibria, there is in general one equilibrium for which $\kappa < 0$. This equilibrium delivers "unintuitive"comparative statics.[19] For instance, in this equilibrium, a reduction in the risk tolerance of dealers in, say, security D *increases* the liquidity of both securities. Such an equilibrium may exist because,

[19]It is possible to show that the model has three equilibria when it admits multiple equilibria. The equilibrium with $\kappa < 0$ is unstable whereas the two other equilibria (for which $\kappa > 1$) are stable.

as explained previously, the illiquidity of each security is in part determined by dealers' beliefs about the illiquidity of the other market. These beliefs may be disconnected from the illiquidity fundamentals of each security and yet be self-fulfilling.

3.3 Limited attention, adverse selection, and negative liquidity spillovers

We now turn our attention to the more general case in which $0 < \mu_D \leq 1$ and $0 < \mu_F \leq 1$. That is, we allow for limited attention by dealers in either security. In this case, the pricewatchers (dealers who monitor the price of the other security) have an informational advantage over inattentive dealers (dealers who do not monitor this price). This advantage is a source of adverse selection for inattentive dealers. This effect yields two new results: (a) liquidity spillovers can be negative and (b) an increase in the fraction of pricewatchers in one security can reduce the liquidity of this security when the fraction of pricewatchers is small. We now explain the intuition for these two results in more details. We proceed as follows. We first generalize Proposition 1 when attention is limited (Section 3.3.1). We then show that liquidity spillovers can be negative with limited attention and we provide a sufficient condition on the parameters for liquidity spillovers to be always positive (Section 3.3.2). Finally, we study the effect of a change in the fraction of pricewatchers in a security on the liquidity of this security (Section 3.3.3).

3.3.1 Equilibria with limited attention

As with full attention, a linear rational expectations equilibrium is a set of prices $\{p_j^*\}_{j\in\{D,F\}}$ such that

$$p_j^* = R_j\delta_j + B_j u_j + A_j\delta_{-j} + C_j u_{-j}, \tag{21}$$

and p_j^* clears the market of asset j for each realizations of $\{u_j, \delta_j, u_{-j}, \delta_{-j}\}$ when dealers anticipate that clearing prices satisfy equation (21) and choose their trading strategies to maximize their expected utility. The next proposition generalizes Proposition 1 when $0 < \mu_D \leq 1$ and $0 < \mu_F \leq 1$.

Proposition 2. *Suppose $\sigma_\eta^2 > 0$. With limited attention (i.e., $0 < \mu_D \leq 1$ and $0 < \mu_F \leq 1$), there always exists a non fully revealing linear rational expectations equilibrium. At any non-fully revealing equilibrium, $B_j > 0$, $R_j = 1$ and the coefficients A_j and C_j can be expressed as functions of B_j and B_{-j}. Moreover*

$$B_j = B_{j0}(1-\rho_j^2) \times \frac{\gamma_j^2\mu_j\rho_j^2 + \sigma_{u_j}^2 \mathrm{Var}[v_j|\delta_j](1-\rho_j^2)}{\gamma_j^2\mu_j^2\rho_j^2 + \sigma_{u_j}^2 \mathrm{Var}[v_j|\delta_j](1-\rho_j^2)(1-\rho_j^2(1-\mu_j))}, \tag{22}$$

where $\rho_D^2 \equiv d^2/((\sigma_\eta^2 + d^2)(1 + B_F^2\sigma_{u_F}^2))$ and $\rho_F^2 \equiv 1/(1 + B_D^2\sigma_{u_D}^2)$.

Proposition 2 generalizes Proposition 1 when attention is limited. As in the full attention case, it can be shown that (i) pricewatchers in security j extract a signal $\omega_{-j} = \delta_{-j} + B_{-j}u_{-j}$ from the price of security $-j$ and that (ii) variable ρ_j^2 is the informativeness of this signal. As the pricewatchers' trading strategy depends on the information they obtain from watching the price of security $-j$ (i.e., ω_{-j}), the price of security j partially reveals pricewatchers' private information.[20] Equation (21) implies that observing the price of security j and risk factor δ_j is informationally equivalent to observing $\hat{\omega}_j \equiv A_j\delta_{-j} + B_j u_j + C_j u_{-j}$. Thus, in equilibrium, the information set of inattentive dealers, $\{\delta_j, p_j\}$, is informationally equivalent to $\{\delta_j, \hat{\omega}_j\}$. In what follows, we refer to $\hat{\omega}_j$ as inattentive dealers' price signal. Using the expressions for A_j and C_j (given in the proof of Proposition 2), we obtain that $\hat{\omega}_j = A_j\omega_{-j} + B_j u_j$. Hence, when $B_j > 0$, inattentive dealers' price signal is less precise than pricewatchers' price signal, which means that inattentive dealers in security j are at an informational disadvantage compared to pricewatchers.

This disadvantage creates an adverse selection problem for the inattentive dealers. Indeed, relative to inattentive dealers, pricewatchers will bid aggressively when the price of security $-j$ indicates that the realization of the risk factor δ_{-j} is high and conservatively when the price of security $-j$ indicates that the realization of the risk factor δ_{-j} is low. As a consequence, inattentive dealers in one security will tend to have relatively large holdings of the security when its value is low and relatively small holdings of the security when its value is large. This bias in inattentive dealers' portfolio holdings is a source of adverse selection, which is absent when all dealers are pricewatchers. This new effect is key to understanding why liquidity spillovers may be negative in the limited attention case (see below).

Substituting ρ_D^2 and ρ_F^2 by their expressions in equation (22), we can express B_j as a function of B_{-j}. Formally, we obtain:

$$B_D = f(B_F; \mu_D, \gamma_D, \sigma_\eta^2, d, \sigma_{u_F}^2), \tag{23}$$

$$B_F = g(B_D; \mu_F, \gamma_F, \sigma_{u_D}^2), \tag{24}$$

where the expressions for the functions $f(\cdot)$ and $g(\cdot)$ are given in the Appendix (see equations (A.26) and (A.28)). The linear rational expectations equilibria are completely characterized by the solution(s) of this system of equations. As in the full attention case and for the same reason, there might be multiple equilibria and we cannot in general provide an analytical characterization of these equilibria. Of course, when $\mu_D = \mu_F = 1$, the solutions to the previous system of equations are those obtained in the full attention case since this case is nested in the limited attention case.

3.3.2 When are liquidity spillovers positive?

As mentioned previously, liquidity spillovers from one security to the other can be negative when the fraction of pricewatchers in the latter security is relatively small. The intuition for negative spillovers is more easily seen when learning is one sided ($d = 0$) or when no dealers

[20] Pricewatchers' trading strategy (demand function) can be written as

$$x_j^W(p_j, \omega_{-j}) = a_j^W\left(E[v_j \mid \delta_j, p_{-j}] - p_j\right) = a_j^W(\delta_j - p_j) + b_j^W\omega_{-j},$$

where expressions for coefficients a_j^W and b_j^W are provided in the proof of Proposition 2.

in security D are pricewatchers ($\mu_D = 0$). Indeed, in these cases, the price of security F conveys no information to dealers in security D. Thus, the level of illiquidity in security D is as in the benchmark case ($B_D = B_{D0}$) and the level of illiquidity in security F is readily obtained by substituting this expression for B_D in equation (22). Hence, there is a unique rational expectations equilibrium and we can characterize the equilibrium in closed form, which considerably simplifies the analysis. Remember that $\mathcal{R}_F$ is a measure of dealers' risk bearing capacity in security F (see equation (5)). We obtain the following result.

Corollary 6. *With one-sided learning ($d = 0$) or no pricewacthers in security D ($\mu_D = 0$), there is a unique linear rational expectations equilibrium where the levels of illiquidity of securities D and F are*

$$B_D = B_{D0}, \tag{25}$$

$$B_F = \frac{B_D^2\sigma_{u_D}^2(B_D^2\sigma_{u_F}^2\sigma_{u_D}^2 + \mu_F\gamma_F^2)}{\gamma_F(\mu_F^2\gamma_F^2(1 + B_D^2\sigma_{u_D}^2) + B_D^2\sigma_{u_D}^2\sigma_{u_F}^2(\mu_F + B_D^2\sigma_{u_D}^2))}. \tag{26}$$

In this equilibrium, liquidity spillovers from security D to security F are positive for all values of μ_F if $\mathcal{R}_F \leq 1$. In contrast, if $\mathcal{R}_F > 1$, liquidity spillovers from security D to security F are negative when $\mu_F < \hat{\mu}_F$ and positive when $\mu_F \geq \hat{\mu}_F$, where $\hat{\mu}_F$ is strictly smaller than one and defined in the proof of the corollary.

When $\mu_D = \mu_F = 1$, the corollary describes the equilibrium obtained with full attention and one sided learning. In this case, as explained previously, liquidity spillovers from security D to security F are always positive. In contrast, when μ_F is small enough and $\mathcal{R}_F > 1$, liquidity spillovers from security D to security F can be negative.

To see why, consider a decrease in the risk tolerance of the dealers operating in security D (γ_D decreases). This decrease makes security D less liquid and therefore less informative for pricewatchers in security F. Thus, uncertainty about the payoff of security F increases. As with full attention, this "uncertainty effect" increases the illiquidity of security F. However, with limited attention, there is a countervailing effect that we call the "adverse selection effect." Indeed, as pricewatchers' private information is less precise, their informational advantage is smaller. As a consequence, inattentive dealers are less exposed to adverse selection. This effect reduces the illiquidity of security F. Intuitively the reduction in uncertainty has a small effect on illiquidity when (i) few dealers receive price information ($\mu_F < \hat{\mu}_F$) and (ii) when dealers' risk bearing capacity is high (i.e., $\mathcal{R}_F > 1$) since in this case uncertainty is not a big driver of illiquidity. When these conditions are met, the adverse selection effect prevails over the uncertainty effect. As a result the increase in the illiquidity of security D *reduces* the illiquidity of security F. Otherwise, the uncertainty effect dominates and liquidity spillovers from security D to F are positive.

We now consider the more general case in which learning is two-sided ($d > 0$). The next corollary shows that liquidity spillovers in this case are positive if the fraction of pricewatchers in securities D and F is high enough.

Corollary 7. *Let*

$$\overline{\mu}_j = \max\left\{0, \frac{\mathcal{R}_j - 1}{\mathcal{R}_j}\right\}, \text{ for } j \in \{D, F\}. \tag{27}$$

If $\mu_D \in [\overline{\mu}_D, 1]$ and $\mu_F \in [\overline{\mu}_F, 1]$ then liquidity spillovers from security D to security F and vice versa are positive for all values of d.

Thus, the model will feature positive liquidity spillovers if the level of attention is higher than $\overline{\mu}_j$ for $j \in \{D, F\}$. This threshold is always less than one and can be as low as zero if dealers' risk bearing capacity is small enough in both markets, i.e., if $\mathcal{R}_j \leq 1$ for $j \in \{D, F\}$. In contrast, when the fraction of pricewatchers in security j is less than $\overline{\mu}_j$, liquidity spillovers from security $-j$ to security j can be negative for the reasons explained previously.

As an example, suppose that the parameter values are as follows: $\sigma_{u_F} = 0.1$, $\sigma_{u_D} = 1$, $\gamma_F = 1$, $d = 1$, $\mu_D = \mu_F = 0.1$, and $\sigma_\eta = 1$. In this case, $\overline{\mu}_D = 0$ while $\overline{\mu}_F = 0.9$. Thus, liquidity spillovers from security F to security D are positive while liquidity spillovers from security D to security F can be negative since $\mu_F < \overline{\mu}_F$ (Corollary 7). For instance Figure 4 considers the effect of an increase in the risk tolerance of dealers in security D. This increase reduces the illiquidity of security D but it *increases* the illiquidity of security F because liquidity spillovers from security D to security F are negative in this case.

[Insert Figure 4 about here]

Our model predicts the existence of positive or negative liquidity spillovers between securities. Empirically, these spillovers should translate into positive or negative co-movement in liquidity. We illustrate this point with the following experiment. For a given value of μ_F, we compute the illiquidity of securities F and D assuming that γ_D is uniformly distributed in $[0.5, 1]$ and setting $\sigma_{u_F} = \sigma_{u_D} = 1/2$, $\sigma_\eta = 2$, $\gamma_F = 1/2$. For these values of the parameters $\overline{\mu}_j = 0$ and liquidity spillovers are therefore positive. We then compute the covariance between the resulting equilibrium values for B_D and B_F. Figure 5, Panel (a) and Panel (b) show this covariance as a function of μ_F when $d = 0$ and $d = 0.9$, respectively (for $\mu_D = 0.1$ and $\mu_D = 0.9$). In both cases, the covariance between the illiquidity of securities D and F is positive because illiquidity spillovers are positive. In panel (c) we set $\sigma_{u_F} = 0.1$, $d = 0.9$ and $\mu_D = 0.9$ while other parameters are unchanged. In this case liquidity spillovers from security D to security F can be negative when μ_F is smalle enough. As a result the covariance between the illiquidity of security D and the illiquidity of security F is negative for relative low values of μ_F and positive otherwise.

[Insert Figure 5 about here]

3.3.3 Is attention good for market liquidity?

We now study the relationship between the illiquidity of a security and the fraction of pricewatchers in this security. We already know that the illiquidity of security j is always smaller with full attention than with no attention (see Corollary 2). However, as shown below, for small values of the fraction of pricewatchers, the illiquidity of a security may be strictly higher than with no attention. Hence, the relationship between illiquidity and attention is non monotonic. Again it is easier to establish this result when learning is one sided ($d = 0$) or when $\mu_D = 0$ since in these cases the equilibrium is unique and we can characterize it in closed-form. We obtain the following result.

Corollary 8. *Consider the cases in which learning is one sided ($d = 0$) or in which there are no pricewacthers in security D ($\mu_D = 0$).*

1. *If $\mathcal{R}_F \leq 1$, an increase in attention by dealers in security F reduces the illiquidity of this security.*

2. *If $\mathcal{R}_F > 1$, an increase in attention by dealers in security F reduces the illiquidity of this security if $\mu_F \geq \mu_F^\bigstar$ and increases its illiquidity if $\mu_F < \mu_F^\bigstar$ where $0 < \mu_F^\bigstar < 1$ (see the appendix for the analytical expression of $\mu_F^\bigstar$).*

The impact of a change in the fraction of pricewatchers in security F on the liquidity of this market is determined by both the adverse selection effect and the uncertainty effect, which play in opposite directions. On the one hand, an increase in the fraction of pricewatchers in security F raises the exposure to adverse selection for inattentive dealers in security F. On the other hand, more dealers have relatively low inventory holding costs because more dealers are better informed about the payoff of security j. The first effect raises illiquidity while the second effect decreases illiquidity. As shown in Corollary 8, the second effect always prevails when the risk bearing capacity of dealers in security F is less than one. When this condition is not satisfied, the adverse selection effect dominates when the fraction of pricewatchers is small ($\mu_F < \mu_F^\bigstar$). Hence, the relationship between the liquidity of security F and the fraction of pricewatchers is non monotonic: it increases in the fraction of pricewatchers when this fraction is less than $\mu_F^\bigstar$, reaches a maximum when this fraction is equal to $\mu_F^\bigstar$ and then decreases.

When learning is two-sided, i.e., $d > 0$, the analysis of the impact of a change in attention in one market is more complex because liquidity spillovers operate in both directions. Hence, as explained in Section 3.2.3, the total impact of a change in the fraction of pricewatchers in one security on the illiquidity of this security is determined both by the *direct* impact of this change on illiquidity (measured by $(\partial f / \partial \mu_D)$ or $(\partial g / \partial \mu_F)$) and the *indirect* impact which accounts for the spillover effects described in the previous section. This indirect impact can be positive or negative depending on the direction of liquidity spillovers between the two markets. Signing the total impact of an increase in attention in one market on the level of illiquidity in both markets is therefore challenging. However, the next corollary shows that if $\mathcal{R}_j \leq 1$ then more attention leads to a more liquid market for both securities in at least one of the possible rational expectations equilibria of the model. When the equilibrium is unique, it must therefore have this property if $\mathcal{R}_j \leq 1$.

Corollary 9. *If $\mathcal{R}_j \leq 1$ for $j \in \{D, F\}$ then, other things equal, an increase in attention by dealers in security j reduces the illiquidity of this security ($(\partial f / \partial \mu_D) < 0$ and $(\partial g / \partial \mu_F) < 0$).*

Furthermore, there is always an equilibrium in which an increase in attention by dealers in security j reduces the illiquidity of both securities in equilibrium.

To save space, we provide the proof of this result in the Internet Appendix. We illustrate this corollary with a numerical example. We set $\sigma_\eta = 0.77$, $\sigma_{u_j} = 1$, $\gamma_j = 1$ and $d = 1$, so that learning is two-sided. In Figure 6, we plot the relationship between the illiquidity of security D and the fraction of pricewatchers in this security for $\mu_D \in \{0.001, 0.002, \ldots, 1\}$ when $\mu_F = 0.5$ (panel (a)) and $\mu_F = 0.9$ (panel (b)) when B_F is fixed at its equilibrium value for $\mu_D = 0.001$ (bold curve) and when B_F adjusts to its equilibrium value for each value of μ_D (dotted curve). Thus, the bold curve represents the direct effect of a change in the fraction of pricewatchers in security D (i.e., the effect holding constant the liquidity of security F) while the dotted curve represents the evolution of the equilibrium value of the illiquidity of security D, after accounting for spillover effects. The difference between the two curves shows the amount by which spillover effects magnify the direct effect of a change in attention on illiquidity.

[Insert Figure 6 about here]

Table 3 provides a summary of our main results when the level of attention in each market is exogenous.

Panel A – One-sided learning $d = 0$				
Attention	Risk bearing capacity	Spillovers from from D to F	$\uparrow \mu_F$ on B_D	$\uparrow \mu_F$ on B_F
No		No spillovers	No effect	No effect
Limited	$\mathcal{R}_F \leq 1$	$+$	No Effect	$-$
	$\mathcal{R}_F > 1$	$+$ iff $\mu_F \geq \widehat{\mu}_F$	No Effect	$-$ iff $\mu_F \geq \mu_F^\star$
Full		$+$	N.A.	N.A.
Panel B – Two-sided learning $d > 0$				
Attention	Risk bearing capacity	Spillovers from from j to $-j$	$\uparrow \mu_j$ on B_j	$\uparrow \mu_j$ on B_{-j}
No		No spillovers	No effect	No effect
Limited	$\mathcal{R}_j \leq 1$	$+$	$-$	$-$
	$\mathcal{R}_j > 1$	Ambiguous/can be negative	Ambiguous	Ambiguous
Full		$+$	N.A.	N.A.

Table 3: Summary of the main findings with exogenous attention.

4 Endogenous attention

We now endogenize the level of dealers' attention to the prices of other securities, i.e., μ_j. To this end we introduce a *cost of attention*, C (see the introduction of the paper for interpretations of this cost).[21] We assume that dealers simultaneously make their decision to be a pricewatcher at date 0, before trades take place at date 1. Dealers who become pricewatchers pay the cost C. Other dealers do not pay this cost and cannot make their strategy contingent on the price in the other market. Once these decisions have been made, trades take place as described in the previous section.

Dealers' decisions to be a pricewatcher hinges on a comparison between the cost of attention and the value of attention, i.e., the informational value of the price of the other security. Let ϕ_j be the value of the information contained in the price of security $-j$ for dealers in security j when a fraction μ_j of dealers in security j are informed about the price of security $-j$. This value is the maximum fee that a dealer in security j is willing to pay in order to observe the price of security $-j$, p_{-j}. This fee solves:

$$E\left[U\left(\left(v_j - p_j\right) x_j^W - \phi_j\right)\right] = E\left[U\left(\left(v_j - p_j\right) x_j^I\right)\right]. \tag{28}$$

In general, the solution to this equation depends on the level of illiquidity in security $-j$ since this level determines the informational content of the price of security $-j$. We stress this feature by explicitly writing ϕ_j as a function of the illiquidity of security $-j$: $\phi_j = \phi_j(\mu_j, B_{-j})$. In making their monitoring decisions, dealers take the fraction of pricewatchers as given. Hence, for a given fraction of pricewatchers in each market, a dealer in security j chooses to monitor the price of security $-j$ if $\phi_j(\mu_j, B_{-j}) > C$ and abstains from monitoring this price if $\phi_j(\mu_j, B_{-j}) < C$. When $\phi_j(\mu_j, B_{-j}) = C$, the dealer is indifferent between monitoring the price of security $-j$ or not.

The fraction of pricewatchers in each market results from this cost-benefit analysis and is ultimately determined by the cost of attention. In the rest of this section, we study the effect of varying the cost of attention on the equilibrium fraction of pricewatchers and market illiquidity. This analysis yields two new insights. First, a decrease in the cost of attention can impair market liquidity. Second, when learning is two-sided, the value of attention for dealers in one security can increase both in the level of attention by dealers in the *same* security and dealers in the other security. As a consequence, dealers' attention decisions reinforce each other *and* multiple equilibria with differing levels of attention can arise for the same level of the cost of attention.

4.1 Attention decisions with one-sided learning

When $d = 0$, learning is one-sided: dealers in security D learn no information from the price of security F. In this case, monitoring the price of security F for dealers in security D is worthless ($\phi_D(\mu_D, B_F) = 0$) and as a result all dealers in security D optimally abstain from paying the cost of attention, i.e., $\mu_D = 0$. Thus, the level of illiquidity in security D is as given in the

[21] In our analysis we take the cost of attention as being exogenous. In reality, part of this cost is determined by pricing decisions of data vendors (Bloomberg, Reuters, exchanges, etc...). An interesting extension of our paper would be to endogenize this cost by studying the optimal pricing policy of sellers of price information in our set-up. Cespa and Foucault (2009) study the optimal pricing policy for a monopolist seller of price information. But they restrict their attention to the case with a single security.

benchmark case, i.e., $B_D = \sigma_\eta^2/\gamma_D$ for all possible values of μ_F. Hence, in this section we write $\phi_F(\mu_F, B_D)$ as $\phi_F(\mu_F)$ to simplify notation.

Using the specification of dealers' utility functions and the fact that all variables have a normal distribution, we obtain that[22]

$$\phi_F(\mu_F) = \frac{\gamma_F}{2} \ln\left(\frac{\mathrm{Var}[v_F|\delta_F, \hat{\omega}_F]}{\mathrm{Var}[v_F|\delta_F, \omega_D]}\right) > 0. \tag{29}$$

As explained in Section 3.3, pricewatchers in security F obtain a signal ω_D about factor δ_D from monitoring the price of security D. The price information privately observed by pricewatchers leaks partially through the price of security F as pricewatchers trade on this information, which conveys a signal $\hat{\omega}_F$ to inattentive dealers. However, this signal is less informative than the signal obtained by pricewatchers since price movements in security F are also affected by the demand shock in this security. For this reason, pricewatchers can form a more precise forecast of the payoff of security F than inattentive dealers, that is $\mathrm{Var}[v_F|\delta_F, \hat{\omega}_F] > \mathrm{Var}[v_F|\delta_F, \omega_D]$ and the value of being a pricewatcher is always strictly positive. Intuitively, the value of monitoring the price of security D for dealers in security F decreases in the fraction of pricewatchers in security F because the leakage effect is stronger when the fraction of pricewatchers in security F is higher. We establish this result in the next corollary.

Proposition 3. *If $d = 0$,*

$$\phi_F(\mu_F) = \frac{\gamma_F}{2} \ln\left(1 + \frac{\sigma_{u_F}^2 \sigma_{u_D}^2 B_D^2}{\gamma_F^2 \mu_F^2 (1 + B_D^2 \sigma_{u_D}^2) + \sigma_{u_F}^2 \sigma_{u_D}^4 B_D^4}\right). \tag{30}$$

with $B_D = \sigma_\eta^2/\gamma_D$. Thus, the value of monitoring the price of security D for dealers in security F decreases in the fraction of pricewatchers in security F.

Hence, with one sided learning, the value of acquiring price information declines with the fraction of dealers buying this information, as usual in models of information acquisition (e.g., Grossman and Stiglitz (1980) or Admati and Pfleiderer (1986)). Let $\mu_F^*(C)$ be the fraction of dealers in security F who decide to pay the cost of attention. As $\phi_F(\mu_F)$ decreases in μ_F, there are three possible cases:

1. If $\phi_F(1) > C$, then the value of monitoring the price of security D for dealers in security F exceeds the cost of monitoring even when all dealers pay the cost of monitoring. Thus, $\mu_F^*(C) = 1$.

2. If $\phi_F(0) < C$, then the value of monitoring the price of security D for dealers in security F is always lower than the cost of monitoring. Thus, $\mu_F^*(C) = 0$.

3. Otherwise, the equilibrium fraction of pricewatchers is such that dealers in security F are just indifferent between monitoring the price of security D or not. That is, $\mu_F^*(C)$ is the unique solution of $\phi_F(\mu_F) = C$.

[22]Our expression for the value of information is standard in models of information acquisition with normally distributed variables and CARA utility functions (see for instance Admati and Pfleiderer (1986)). Thus, for brevity we omit the derivation of this result, which can be obtained upon request.

We obtain the following result.

Proposition 4. *With one sided learning ($d = 0$), the fraction $\mu_F^*(C)$ of dealers in security F who monitor the price of security D in equilibrium decreases in the cost of attention. This fraction is:*

1. $\mu_F^*(C) = 0$, *if* $C > \overline{C}$.
2. $\mu_F^*(C) = \sqrt{\frac{\sigma_{u_F}^2 \sigma_{u_D}^2 B_D^2 (1 - B_D^2 \sigma_{u_D}^2 (e^{2C/\gamma_F} - 1))}{\gamma_F^2 (1 + B_D^2 \sigma_{u_D}^2)(e^{2C/\gamma_F} - 1)}}$, *if* $\underline{C} \leq C \leq \overline{C}$.
3. $\mu_F^*(C) = 1$, *if* $C < \underline{C}$,

where closed-form solutions for the thresholds $\underline{C}$ and $\overline{C}$ are given in the proof of the proposition and $B_D = \sigma_\eta^2 / \gamma_D$.

The illiquidity of security F is in part determined by the fraction of pricewatchers in this security (see Section 3.3). As this fraction is itself determined by the cost of attention, the illiquidity of security F is ultimately determined by the cost of attention. The next corollary describes the effect of a change in the cost of attention on the illiquidity of security F.

Corollary 10. *With one sided learning ($d = 0$):*

1. *If $\mathcal{R}_F \leq 1$ then the illiquidity of security F increases in the cost of attention for dealers active in this security.*
2. *If $\mathcal{R}_F > 1$, there exists a value of $C^* \in (\underline{C}, \overline{C})$ such that the illiquidity of security F increases in the cost of attention for dealers active in this security when $C \leq C^*$ and decreases in the cost of attention otherwise (the closed-form solution for C^* is given in the proof of the corollary).*

A decrease in the cost of attention leads to an increase in the fraction of pricewatchers in security F when learning is one-sided. As explained in Section 3.3, this evolution has an ambiguous effect on the illiquidity of security F. On the one hand, more attention reduces the uncertainty on the payoff of security F. On the other hand, inattentive dealers are more exposed to adverse selection if the attention of their competitors increases. As shown in Corollary 6, the uncertainty effect always dominates when $\mathcal{R}_F \leq 1$. Thus, in this case, a reduction in the cost of monitoring for dealers in security F always improves the liquidity of this security. When $\mathcal{R}_F > 1$, the adverse selection effect dominates as long as the fraction of pricewatchers remains small, i.e., when C is greater than C^*. Indeed, for this range of values for the cost of attention, only a few dealers choose to be pricewatchers. As a result, a small decline in the cost of attention reinforces the adverse selection risk for inattentive dealers and market liquidity deteriorates. Figure 7 illustrates the impact that a change in the cost of attention has on the fraction of pricewatchers, illiquidity, and the value of information with one-sided learning.

[Insert Figure 7 about here]

4.2 Attention decisions with two sided learning

We now consider the case in which $d > 0$, so that dealers in each security can learn information from the price of the other security. In this case, our main finding is that the value of price information for dealers in a given market can be increasing in the fraction of pricewatchers in both markets. This finding is counterintuitive since usually the value of financial information declines with the fraction of investors acquiring this information (see Grossman and Stiglitz (1980) or Admati and Pfleiderer (1986)). The value of price information has this property when learning is one-sided, as we have just shown in Proposition 4. In contrast, when learning is two-sided, price information is *special*: its value can increase in the number of investors who buy this information. As we shall see the main reason for this counter-intuitive result is that the value of price information tends to be higher for securities that are more liquid and securities tend to be more liquid when the fraction of pricewatchers is large enough.

Using again the dealers' utility functions specification and the fact that all variables are normally distributed, we obtain that the value of monitoring the price of security $-j$ for dealers in security j is

$$\phi_j(\mu_j, B_{-j}(\mu_j, \mu_{-j})) = \frac{\gamma_j}{2} \ln \left(\frac{\mathrm{Var}[v_j|\delta_j, \hat{\omega}_j]}{\mathrm{Var}[v_j|\delta_j, \omega_{-j}]} \right), \tag{31}$$

where we stress the fact that the illiquidity of each market in equilibrium is a function of the fraction of pricewatchers in either market. To save space we provide the explicit expression for $\phi_j(\mu_j, B_{-j}(\mu_j, \mu_{-j}))$ in the Internet Appendix. For a fixed fraction of pricewatchers in market $-j$, we have

$$\frac{d\phi_j}{d\mu_j} = \underbrace{L_j}_{\text{Leakage effect}} + \underbrace{\Lambda_j}_{\text{Feedback effect}}. \tag{32}$$

with $L_j \equiv (\partial\phi_j/\partial\mu_j)$ and $\Lambda_j \equiv (\partial\phi_j/\partial B_{-j})(\partial B_{-j}/\partial\mu_j)$. Thus, the total effect of an increase in the fraction of pricewatchers in security j on the value of being a pricewatcher is the sum of two effects: the *leakage effect* (that we described in the previous section) and the *feedback effect*, which is new as it arises only when learning is two-sided. To understand this feedback effect, consider an increase in the fraction of pricewatchers in security D (the reasoning is symmetric for an increase in μ_F). When $d > 0$, this increase affects the liquidity of security D and thereby the liquidity of security F. In turn, the change in the liquidity of security F feeds back on the value of monitoring this security since, as explained before, it affects the informativeness of the price of security F for dealers in security D if $d > 0$. The change in the value of information due to this feedback effect is measured by Λ_D. It is zero when learning is one-sided because in this case dealers in security D learn no information from the price of security F (hence $\partial\phi_D/\partial B_F = 0$).[23]

The total effect of an increase in the fraction of pricewatchers in security j on the value of information in this market is positive if and only if the feedback effect outweighs the leakage effect

$$\Lambda_j > -L_j > 0. \tag{33}$$

[23]Moreover, $\partial B_D/\partial\mu_F = 0$ when $d = 0$ since the illiquidity of security D is independent of μ_F in this case ($B_D = (\sigma_\eta^2/\gamma_D)$). Thus, $\Lambda_F = 0$ as well when learning is one-sided.

If the feedback effect dominates (i.e., condition (33) holds true), the value of being a pricewatcher in security j increases in the fraction of pricewatchers in this security. Obviously, a necessary condition for this to happen is that the feedback effect is positive, which is a possibility when $\mathcal{R}_j \leq 1$. To see this, consider again the value of monitoring security F for dealers in security D. When $\mathcal{R}_D \leq 1$, as shown in Corollary 9, an increase in the fraction of pricewatchers in security D reduces the illiquidity of security F ($\partial B_F/\partial \mu_D < 0$). As a consequence, the price of security F becomes more informative for dealers in security D and the value of monitoring this price is higher ($\partial \phi_D/\partial B_F < 0$), at least for some parameter values. In this case, the feedback effect for security D is positive: $\Lambda_D > 0$.

We have not been able to delineate the set of parameters under which the feedback effect dominates the leakage effect. However, numerical simulations show that this set is not empty. To see this, consider Figure 8. Panel (a) on this figure plots the value of monitoring security F for pricewatchers in security D (i.e., $\phi_D(\mu_D, B_F)$) for two values of μ_F. In both cases the value of observing the price of security F increases with the fraction of pricewatchers in security D, which means that the feedback effect dominates the leakage effect.

[Insert Figure 8 about here]

Now consider the effect of a change in the fraction of pricewatchers located in market $-j$ on the value of monitoring this market for dealers in asset j. This cross-market monitoring effect is measured by

$$\frac{d\phi_j}{d\mu_{-j}} = \left(\frac{\partial \phi_j}{\partial B_{-j}}\frac{\partial B_{-j}}{\partial \mu_{-j}}\right). \tag{34}$$

As shown in Corollary 9, an increase in the fraction of pricewatchers in, say, security D reduces the illiquidity of this security ($(\partial B_D/\partial \mu_D) < 0$) if $\mathcal{R}_D \leq 1$. In turn this effect makes the price of security D more informative for dealers in security F and increases the value of monitoring this price for dealers in security F ($(\partial \phi_F/\partial B_D) < 0$). In this case, $(d\phi_F/d\mu_D) > 0$. That is, an increase in the fraction of pricewatchers in security D makes the value of monitoring the price of security D higher for dealers in security F.

Figure 8 illustrates the cross-market monitoring effect as well. First, consider panel (a) again. It shows that the value of monitoring the price of security F for dealers in security D is higher, all else being equal when $\mu_F = 0.9$ than when $\mu_F = 0.1$. Moreover, panel (b) shows that an increase in the fraction of pricewatchers in security D makes the value of monitoring security D higher for dealers in security F.

Thus, price information is special because the decision of each dealer to buy this information can reinforce each other both in the *same* market and across *different* markets.[24] The model shows that this happens in two distinct ways: (i) the value of being informed about the price of another security can increase in the fraction of dealers who follow this security ("within market

[24] The leakage effect implies that dealers' decisions to buy price information are "strategic substitutes": the acquisition of price information by one dealer reduces the value of being a pricewatcher for other dealers. In contrast, when positive, the feedback effect works to make dealers' decisions to buy price information "strategic complements": the acquisition of price information by one dealer strengthens the value of being a pricewatcher for other dealers operating in the same market.

complementarity") and (ii) the value of being informed about the price of another security can increase in the fraction of pricewatchers in this security ("cross market complementarity"). Both types of complementarity in dealers' monitoring decisions are absent when $d = 0$ and they do not necessarily both operate when $d > 0$ (in particular the leakage effect may prevail over the feedback effect even though the cross-market complementarity operates).

Now consider whether a dealer in market j should become a pricewatcher. In making this decision, the dealer takes the fraction of pricewatchers in both markets as given. If $\phi_j(\mu_j, B_{-j}(\mu_D, \mu_F)) > C$, it is optimal for the dealer to be a pricewatcher since the value of monitoring the price in the other market is higher than the cost. If $\phi_j(\mu_j, B_{-j}(\mu_D, \mu_F)) < C$, it is optimal for the dealer not to monitor the price in the other market and finally, if $\phi_j(\mu_j, B_{-j}(\mu_D, \mu_F)) = C$, the dealer is just indifferent. Given these observations, the equilibrium fractions of pricewatchers in each market, (μ_D^*, μ_F^*), are displayed in Table 4.

μ_j^*, μ_{-j}^*	When
$\mu_j^* = \mu_{-j}^* = 1$	$\phi_j(1, B_{-j}(1,1)) > C$ for $j \in \{D, F\}$
$\mu_j^* = 1$, $\mu_{-j}^* \in (0,1)$	$\phi_j(1, B_{-j}(1, \mu_{-j}^*)) > C$ and $\phi_{-j}(\mu_{-j}^*, B_{-j}(1, \mu_{-j}^*)) = C$
$\mu_j^*, \mu_{-j}^* \in (0,1)$	$\phi_j(\mu_j^*, B_{-j}(\mu_j^*, \mu_{-j}^*)) = C$ for $j \in \{D, F\}$
$\mu_j^* = 0, \mu_{-j}^* \in (0,1)$	$\phi_j(0, B_{-j}(0, \mu_{-j}^*)) < C$ and $\phi_{-j}(\mu_{-j}^*, B_{-j}(1, \mu_{-j}^*)) = C$
$\mu_j^*, \mu_{-j}^* = 0$	$\phi_j(0, B_{-j}(0,0)) < C$ for $j \in \{D, F\}$.

Table 4: The equilibrium fraction of pricewatchers in markets j and $-j$.

Complementarities in attention decisions among dealers located in different markets lead to multiple equilibria for the levels of attention. Indeed, these complementarities imply that the value of cross-market monitoring will be relatively high when the fraction of pricewatchers in both markets is high and relatively low when the fraction of pricewatchers in both markets is low. Thus, for intermediate values of the cost of monitoring, there is room for multiple equilibria with various levels of market integration for the same values of the parameters (in particular the correlation of the payoffs of the two securities being fixed).

It is worth stressing that the multiplicity of possible attention levels in equilibrium is a phenomenon distinct from the multiplicity of rational expectations equilibria. Indeed, one may have a single linear rational expectations equilibrium for each possible level of attention in each security and yet multiple equilibrium levels of attention. As an example, consider the parameter values of Figure 8 again and suppose $C = 0.06$. For the parameter values in Figure 8, there is a unique non-fully rational expectations equilibrium for each value of μ_D and μ_F. However, there are three possible pairs of equilibrium values for the levels of attention in each market: (i) $\mu_D^* = \mu_F^* = 1$, (ii) $\mu_D^* = 0$, $\mu_F^* = 1$ and (iii) $\mu_D^* \simeq 0.3$, $\mu_F^* = 1$. In all these equilibria, all dealers in security F pay attention to the price of security D. In contrast, *for the same parameter values*, we can have an equilibrium in which dealers in security D do not follow security F

($\mu_D^* = 0$), an equilibrium in which all dealers in security D follow security F ($\mu_D^* = 1$) or an equilibrium in which only a fraction of dealers in security D buy price information on security F ($\mu_D^* \simeq 0.3$). Thus, for the same fundamentals, dealers in security D can appear to neglect the information contained in the price of security F or to be very sensitive to this information.

We may also have situations in which, for the *same parameter values*, the markets for the two securities appear fully segmented because dealers in either market pay no attention to the other market ($\mu_D^* = \mu_F^* = 0$) or fully integrated because all dealers are pricewatchers ($\mu_D^* = \mu_F^* = 1$). To see this, consider the case in which the two markets are perfectly symmetric: $\gamma_F = \gamma_D = \gamma$, $d = 1$, $\sigma_\eta = 0$ and $\sigma_{u_F} = \sigma_{u_D} = \sigma_u$. In this case, we obtain (see the Internet Appendix for a derivation):

$$\phi_j(\mu_j, B_{-j}) = \frac{\gamma}{2} \ln\left(1 + \frac{B_{-j}^2 \sigma_u^4}{\gamma^2 \mu_j^2 (1 + B_{-j}^2 \sigma_u^2) + B_{-j}^4 \sigma_u^6}\right). \tag{35}$$

In this symmetric case, there are two non-fully revealing rational expectations equilibria if $\mu_D = \mu_F = 1$ (see Section 3.2). For the discussion, we focus on the high illiquidity equilibrium in which the level of illiquidity in markets D and F is B^{H*} (given in equation (10)). In the symmetric case, parameters are identical in the two markets. Hence, by symmetry, we have $\phi_F(1, B^{H*}) = \phi_D(1, B^{H*})$ and $\phi_F(0, B_{F0}) = \phi_D(0, B_{D0})$. That is, the value of price information is identical in each market in the full attention case and in the no attention case, respectively. Let ϕ_0 be the value of price information in the no attention case and let ϕ_1 be the value of price information in the full attention case. Using equation (35), we obtain the following result.

Proposition 5. *In the symmetric case (i.e., $\gamma_F = \gamma_D = \gamma$, $d = 1$, $\sigma_\eta = 0$ and $\sigma_{u_F} = \sigma_{u_D} = \sigma_u$):*

1. *The value of monitoring prices in market $-j$ for dealers in market j is strictly higher when $\mu_D = \mu_F = 1$ than when $\mu_D = \mu_F = 0$, that is, $\phi_1 > \phi_0$ for $j \in \{H, L\}$.*

2. *Moreover if the cost of attention is such that $\phi_0 < C < \phi_1$, then the cases in which all dealers are pricewatchers ($\mu_D^* = \mu_F^* = 1$) and no dealers are pricewatchers ($\mu_D^* = \mu_F^* = 0$) are possible equilibria.*

The first part of the proposition shows that in the symmetric case the value of monitoring is always higher when all dealers are pricewatchers than when no dealers are pricewatchers. For this reason, for the *same* parameters value, the markets for securities F and D can be either fully integrated (all dealers in each market account for the price information available in the other market) or fully segmented, as claimed in the second part of the proposition. As an illustration, suppose that $\sigma_\delta = \sigma_u = 1$, $\gamma = 1/2$. In this case, we have

$$\phi_0 = \frac{\gamma}{2} \ln\left(1 + \frac{\gamma^2}{\sigma_u^2}\right) \approx 0.055, \;\; \phi_1 = \frac{\gamma}{2} \ln\left(1 + \frac{(B^{H*})^2 \sigma_u^4}{\gamma^2 (1 + (B^{H*})^2 \sigma_u^2) + (B^{H*})^4 \sigma_u^6}\right) \approx 0.127.$$

Thus, for any value of $C \in [0.055, 0.127]$, the markets for securities F and D can be either fully segmented or fully integrated, depending on whether dealers in both markets coordinate on the high or the low attention equilibrium. The liquidity of both markets and the informativeness of prices are higher if dealers coordinate on the high attention equilibrium. Interestingly, in this case, the markets can remain segmented even if the cost of attention decreases, unless it falls below $C = 0.055$.

In summary, when learning is two sided, the value of price information can increase in the fraction of pricewatchers. This property means that dealers' decisions to monitor the price of another security are complements both within and across markets. That is, they reinforce each other. As a consequence, multiple equilibria with differing levels of attention are sustainable and two securities may appear segmented even though the correlation of their payoffs is high and the cost of monitoring is relatively low.

This result has interesting implications. First, it implies that fads, traditions, or other coordination devices may determine the degree of integration between two securities, independently of the correlation in the payoffs of these securities. Second, a decrease in the cost of attention (due for instance to better information linkages between markets) does not necessarily entail greater market integration, unless the cost is very low. Third, dealers operating in related but opaque segments may undervalue the benefit of greater market integration. Indeed, in the low attention equilibrium, the value of getting price information is low. Thus, data vendors will perceive a weak demand and will therefore lack incentives to collect and disseminate price information. In this case, regulatory intervention is needed. A case in point is the U.S. corporate bond market where real time dissemination of bond prices took off only under regulatory pressure (see Bessembinder et al. (2006)).

5 Testable implications

One way to test whether cross-security learning is a source of liquidity spillovers is to consider changes in trading technologies that affect dealers' ability to learn from the prices of other securities. According to our model, these changes should affect the extent of liquidity spillovers across securities and the levels of liquidity for these securities. In contrast, theories of liquidity co-movements based on market wide changes in dealers' risk bearing capacity (e.g., Brunnermeier and Pedersen (2009)) make no predictions on such changes in trading technologies. In the rest of the paper, we illustrate this approach with two thought experiments.

5.1 From opaque to transparent markets

Suppose that the markets for securities D and F are opaque so that the cost of obtaining information on the prices of securities D and F is high. In this case, the fraction of pricewatchers in both securities is low. Let us denote the fraction of pricewatchers in this environment by μ_j^b for $j \in \{D, F\}$. Now suppose that the market for security D becomes transparent while the market for security F remains opaque. After this switch, the fraction of pricewatchers in security D remains unchanged whereas the fraction of pricewatchers in security F is higher since transparency reduces the cost of acquiring information on the price of security D. That is $\mu_D^b = \mu_D^a$ but $\mu_F^a > \mu_F^b$ where μ_j^a is the fraction of pricewatchers in security j *after* the switch to a new trading system for security F. To simplify the discussion, let us assume that

$\mu_D^b = \mu_D^a = 0$. In this case the model has a unique rational expectations equilibrium for all values of μ_F and we can use Corollaries 6 and 8 to develop predictions about the effects of this change in market design.

In this case, if dealers' risk bearing capacity in security F is relatively low ($\mathcal{R}_F \leq 1$), the liquidity of security F should increase after the market for security D becomes transparent (see Corollary 8), even though the market structure for security F is identical before and after the change affecting the other security. Moreover, co-variation in liquidity between securities D and F should be positive and greater than before the change in market design as explained in Section 3.3 (Corollary 6).

If instead, dealers' risk bearing capacity in security F is relatively large ($\mathcal{R}_F > 1$) and the fraction of pricewatchers in security F remains small ($\mu_F^a < \mu_F^\star$) then the liquidity of security F should decrease after the market for security D becomes transparent (Corollary 8). The reason is that the transparency of security D exposes inattentive dealers active in security F to adverse selection by giving an informational advantage to pricewatchers (see Section 3.3.3). Moreover, in this case, liquidity spillovers from security D to security F will be negative (Corollary 6).

The implementation of the TRACE system in the U.S. corporate bond market is a field experiment close to the thought experiment we just described. Until 2002, the U.S. corporate bond market was very opaque: the price of each transaction was known only to the parties involved in the transaction. This situation changed when the SEC required dissemination of transaction prices for a subset of bonds through a reporting system called TRACE. This requirement initially applied to 498 bonds and was implemented in July 2002. Bessembinder et al. (2006) study the effects of this reform of the bond market on the liquidity of TRACE eligible bonds (security D in our thought experiment) and non-TRACE-eligible bonds (security F).[25] Interestingly, Bessembinder et al. (2006) find a significant increase in liquidity for non-TRACE eligible bonds, as predicted by our model (see Table 3, page 272 in Bessembinder et al.(2006)).

The model makes the additional prediction, which to our knowledge has yet to be tested, that the liquidity of non-TRACE bonds should become more sensitive to changes in the liquidity of TRACE bonds after the implementation of TRACE. This prediction can be tested by analyzing the lead-lag relationships between measures of liquidity for TRACE-eligible bonds and non-TRACE bonds. The model implies that a shock to the liquidity of TRACE bonds should have a greater effect in absolute value on the liquidity of non-TRACE bonds after the implementation of TRACE and that the direction of this effect might be negative if few dealers in non-TRACE bonds watch the prices of TRACE bonds.

Bessembinder et al.(2010) also finds that the liquidity of the TRACE eligible bonds increases. This finding is consistent with the model as well. To see this suppose now that both the markets for securities D and F become transparent. If $\mathcal{R}_j \leq 1$ for both securities or μ_j is high enough then the liquidity of both securities is higher in the transparent system, for all values of the fraction of pricewatchers (see Corollary 9).

[25]Edward et al. (2005) and Goldstein et al. (2007) also consider the effects of greater transparency in the U.S. bond markets. However, they do not analyze the effects of greater transparency on non-eligible bonds.

5.2 Co-location fees

The recent years have witnessed a growth of so called "high frequency market-makers" (e.g., GETCO, Optiver, etc. . .), who use highly automated strategies. These market-makers often use price information available about one security to take positions in other securities. For instance, they monitor quote updates in stock index futures and use this information to set their quotes in other securities.

The case in which $d = 0$ can be used to analyze this type of trading strategy. Indeed, in this case we can interpret security D as providing information on a market wide risk factor (δ_D) and security F as a security that loads on this factor and another factor (δ_F). We interpret pricewatchers in security F as high frequency market-makers: they watch in real-time the price of security D and use this information to determine their position in security F.

As explained in the introduction, high frequency market-makers obtain price information faster than other market participants by co-locating their computers close to trading platforms' matching engines, at a cost equal to the co-location fee charged by the platform.[26] Thus, the co-location fee is one component of the cost of price information.

Now suppose that the co-location fee declines. In this case, Proposition 4 implies that the number of high frequency market-makers should increase since the cost of price information declines. If the risk bearing capacity of high-frequency market-makers is low ($\mathcal{R}_F \leq 1$), entry of new pricewatchers should improve the liquidity of security F. Moreover, liquidity spillovers from security D to security F should be positive and stronger after the reduction in the co-location fee (see Corollary 10 and Figure 5).

If instead the risk bearing capacity of high-frequency market-makers is high ($\mathcal{R}_F > 1$), the scenario is more complex. If $C > C^*$, entry of new high frequency market-makers increases the exposure to adverse selection for other dealers in security F. Thus, the liquidity of security F should drop after the reduction in the co-location fee (see Corollary 10). Moreover, liquidity spillovers from security D to security F can be negative in this case. Indeed, an improvement in liquidity for security D allows pricewatchers in security F to obtain more precise information. Thus, if the fraction of pricewatchers remains small, the risk of adverse selection for inattentive dealers increases and the liquidity of security F drops following an increase in liquidity for security D.

Jovanovic and Menkveld (2010) study entry of a high frequency market-maker in Dutch stocks traded on Chi-X (a European trading platform). They show empirically that following this entry, quotes in Chi-X become relatively more informative on price innovations in the Dutch index futures.[27] Moreover, the liquidity of the stocks in which the high frequency market-maker is active improves. This is consistent with the model when $\mathcal{R}_F \leq 1$. In this case the model makes the additional prediction that the liquidity of Dutch shares should become more sensitive

[26]This fee can be significant. For instance, the monthly fee for this service for stocks listed on NYSE Amex is as high as \$61,000 per month in 2011. See NYSE Amex equities price list 2011 at `http://www.nyse.com/pdfs/amex-equities-prices.pdf`.

[27]Hendershott and Riordan (2010) also find empirically that high frequency traders make the market more informationally efficient.

to changes in the liquidity of the Dutch index futures after entry of the high-frequency market-maker.

6 Conclusions

In this paper we analyze a new mechanism that explains the transmission of liquidity shocks in one market to another market ("liquidity spillovers"). Central to this mechanism is the fact that dealers in one security often use the price of other securities as a source of information to set their quotes. The price of a security conveys a noisier signal about fundamentals when the market for this security is less liquid. As a result, a drop in the liquidity of one security propagates to other securities because it increases the level of uncertainty for dealers in all other securities. This propagation of the initial liquidity shock makes all prices less informative, which amplifies the initial drop in liquidity. For this reason, even small initial shocks on market liquidity in one asset class can ultimately result in large market wide changes in liquidity.[28]

The model provides several additional insights:

1. Liquidity spillovers are not necessarily positive. The direction of these spillovers depends on the fraction of dealers with price information on other securities. When this fraction is large, liquidity spillovers are positive. In contrast, liquidity spillovers can be negative when price information is only available to a relatively small number of dealers and dealers' risk bearing capacity is large.

2. A decrease in the cost of price information can increase market illiquidity if it triggers too small an increase in the fraction of dealers who acquire information on the price of other securities.

3. The value of price information can increase, for some parameter values, with the fraction of dealers buying this information. For this reason, for the same parameter values, multiple levels of segmentation (high, medium or low) between securities can be sustained in equilibrium.

Future work should study the implications of our model for asset pricing. The model implies that the extent of liquidity co-movements between assets is in part determined by the cost of acquiring price information. Hence, liquidity risk and therefore risk premia should be sensitive to changes in trading technologies that affect this cost, as explained in the last part of our paper. Moreover the model implies that the liquidity of some securities could covary negatively

[28]In line with this transmission mechanism, the CFTC-SEC report on the Flash crash emphasizes the role that uncertainty on the cause (transient price pressures or changes in fundamentals) of the large price movements in the E-mini futures on the S&P500 played in the evaporation of liquidity during the Flash crash. The authors of this report write (on page 39): "*market makers that track the prices of securities that are underlying components of an ETF are more likely to pause their trading if there are price-driven or data-driven integrity questions about those prices. Moreover extreme volatility in component stocks makes it very difficult to accurately value an ETF in real-time. When this happens, market participants who would otherwise provide liquidity for such ETFs may widen their quotes or stop providing liquidity [...].*" This is consistent with our model in which the liquidity of a security drops when prices of other securities become less reliable as a source of information.

with the liquidity of other securities. These securities should therefore provide a good hedge against market wide variations in liquidity and offer negative risk premia for this risk. Do such securities exist in reality? Do they have the characteristics that our model predicts (relatively few well informed dealers with high risk bearing capacity)? We leave these questions for future research.

A Appendix

Proof of Lemma 1

If $\mu_D = 0$ then all dealers in security D only observe factor δ_D when they choose their demand function. As dealers have a CARA utility function, it is immediate that their demand function in this case is

$$x_D^I(\delta_D) = \gamma_D \frac{E[v_D|\delta_D] - p_D}{\text{Var}[v_D|\delta_D]} = \gamma_D \frac{\delta_D - p_D}{\sigma_\eta^2 + d^2}. \tag{A.1}$$

Using the clearing condition, we deduce that the clearing price is such that:

$$p_D = \delta_D + \left(\frac{\sigma_\eta^2 + d^2}{\gamma_D}\right) u_D = \delta_D + B_{D0} u_D.$$

A similar reasoning yields the expression of the clearing price for security F. □

Proof of Proposition 1

This proposition is a special case of Proposition 2, which considers the more general case in which μ_j can take any value. This proposition is proved below. □

Proof of Lemma 2

In the symmetric case, we can proceed as in the proof of Proposition 2 to show that a non-fully revealing linear rational expectations equilibrium exists if and only if the system of equations (8) and (9) has at least one strictly positive solution. Solving this sytem shows that this is the case if and only if $\sigma_u^2 \geq 4\gamma^2$ and that in this case the system of equations (8) and (9) has two solutions: $B_D^* = B_F^* = B^{H*}$ and $B_D^* = B_F^* = B^{L*}$. Otherwise, the unique solution of this system is $B_{D1}^* = 0$ and $B_{F1} = 0$. Hence, there is no non-fully revealing linear rational expectations equilibria when $\sigma_u^2 < 4\gamma^2$. □

Proof of Lemma 3

See Step 1 in the proof of Proposition 2. □

Proof of Corollary 1

From Step 3 in the proof of Proposition 2, we deduce that when $\mu_D = \mu_F = 1$, there is a unique non-fully revealing equilibrium if and only if $\Psi_1'(B_{D1}) < 0$, $\forall B_{D1}$. Using the expression for $\Psi_1(\cdot)$ (equation (A.32)), we obtain

$$\Psi_1'(B_{D1}) = -\gamma_D \gamma_F^2 (1 + B_{D1}^2 \sigma_{u_D}^2)^2 + \\ 4B_{D1}\sigma_{u_D}^2(\sigma_\eta^2 - \gamma_D B_{D1}) \left(\gamma_F^2(1 + B_{D1}^2\sigma_{u_D}^2) + B_{D1}^2\sigma_{u_D}^2\sigma_{u_F}^2\right) + B_{D1}^3\sigma_{u_D}^4\sigma_{u_F}^2\gamma_D(4\gamma_D^{-1}d^2 - B_{D1}).$$

Remember that when $\mu_D = \mu_F = 1$, $B_{D1} > \sigma_\eta^2/\gamma_D$ (see Step 3 in the proof of Proposition 2). Hence, if $4d^2/\gamma_D \leq \sigma_\eta^2/\gamma_D$ then $\Psi_1'(B_{D1}) < 0$.

□

Proof of Corollary 2

The result follows immediately from equation (17) □

Proof of Corollary 3

The result follows immediately from equations (15) and (16). □

Proof of Corollary 4

The result follows immediately from the definition of functions $f_1(\cdot)$ and $g_1(\cdot)$ in Proposition 1. □

Proof of Proposition 2

Step 1. We show below (Step 2) that if $p_j^* = R_j\delta_j + B_ju_j + A_j\delta_{-j} + C_ju_{-j}$ is a rational expectations equilibrium then $R_j = 1$ and $C_j = A_jB_{-j}$. Hence, in a rational expectations equilibrium, the price in market j can be written $p_j^* = \omega_j + A_j\omega_{-j}$, where $\omega_j = \delta_j + B_ju_j$. Thus, $\{\delta_j,\ \omega_{-j}\}$ is a sufficient statistic for $\{\delta_j, p_{-j}, p_j\}$. Clearly, the equilibrium is non-fully revealing if and only if $B_j > 0$. Moreover, $\{\delta_j, \hat{\omega}_j\}$ is a sufficient statistic for $\{\delta_j, p_j\}$, where $\hat{\omega}_j = B_ju_j + A_j\omega_{-j}$ and since $\omega_{-j} = p_{-j}^* - A_{-j}\omega_j$, we can also write the equilibrium price in market j as

$$p_j^* = \omega_j + A_j(p_{-j}^* - A_{-j}\omega_j) = (1 - A_jA_{-j})\omega_j + A_jp_{-j}^*.$$

These observations prove Lemma 3.

Step 2. Equilibrium in market j.
Pricewatchers' demand function. A pricewatcher's demand function in market j, $x_j^W(\delta_j, p_j, p_{-j})$, maximizes

$$E\left[-\exp\left\{-\left((v_j - p_j)x_j^W\right)/\gamma_j\right\}|\delta_j, p_j, p_{-j}\right].$$

We deduce that

$$\begin{aligned} x_j^W(\delta_j, p_j, p_{-j}) &= \gamma_j\left(\frac{E[v_j|\delta_j, p_{-j}, p_j] - p_j}{\mathrm{Var}[v_j|\delta_j, p_{-j}]}\right) \\ &= a_j^W(E[v_j|\delta_j, p_{-j}, p_j] - p_j), \end{aligned} \tag{A.2}$$

with $a_j^W = \gamma_j\mathrm{Var}[v_j|\delta_j, p_{-j}]^{-1}$.

As $\{\delta_D,\ \omega_F\}$ is a sufficient statistic for $\{\delta_D, p_F, p_D\}$, we deduce (using well-known properties of normal random variables) that

$$\begin{aligned} E[v_D|\delta_D, p_F, p_D] &= E[v_D|\delta_D, \omega_F] \\ &= \delta_D + \frac{d}{(1 + B_F^2\sigma_{u_F}^2)}\omega_F, \end{aligned} \tag{A.3}$$

and

$$a_D^W = \frac{\gamma_D}{\mathrm{Var}[v_D \mid \delta_D, \omega_F]} \tag{A.4}$$

$$= \gamma_D\left(\frac{1 + B_F^2\sigma_{u_F}^2}{d^2B_F^2\sigma_{u_F}^2 + \sigma_\eta^2(1 + B_F^2\sigma_{u_F}^2)}\right) \tag{A.5}$$

$$= \frac{\gamma_D}{\mathrm{Var}[v_D|\delta_D](1 - \rho_D^2)}, \tag{A.6}$$

where $\rho_D^2 \equiv d^2/((\sigma_\eta^2 + d^2)(1 + B_F^2\sigma_{u_F}^2))$. Thus,

$$x_D^W(\delta_D, \omega_F) = a_D^W(\delta_D - p_D) + b_D^W\omega_F,$$

where

$$\begin{aligned} b_D^W &= \frac{\gamma_D}{\text{Var}[v_D|\delta_D, \omega_F]} \frac{\text{Cov}[v_D, \omega_F]}{\text{Var}[\omega_F]} \\ &= d a_D^W \left(\frac{1}{1 + B_F^2\sigma_{u_F}^2} \right). \end{aligned} \tag{A.7}$$

Similarly, for pricewatchers in security F we obtain

$$x_F^W(\delta_F, \omega_D) = a_F^W(\delta_F - p_F) + b_F^W\omega_D, \tag{A.8}$$

where $\omega_D = \delta_D + B_D u_D$, and

$$a_F^W = \gamma_F \left(\frac{1 + B_D^2\sigma_{u_D}^2}{B_D^2\sigma_{u_D}^2} \right) = \frac{\gamma_F}{\text{Var}[v_F|\delta_F](1 - \rho_F^2)}, \quad b_F^W = a_F^W \frac{1}{1 + B_D^2\sigma_{u_D}^2}, \tag{A.9}$$

where $\rho_F^2 \equiv (1 + B_D^2\sigma_{u_D}^2)^{-1}$.

Inattentive Dealers. An inattentive dealers' demand function in market j, $x_j^I(\delta_j, p_j)$, maximizes:

$$E\left[-\exp\left\{-\left((v_j - p_j)\, x_j^I\right)/\gamma_j\right\} |\delta_j, p_j\right].$$

We deduce that

$$\begin{aligned} x_j^I(\delta_j, p_j) &= \gamma_j \left(\frac{E[v_j|\delta_j, p_j] - p_j}{\text{Var}[v_j|\delta_j, p_{-j}]} \right) \\ &= a_j^I(E[v_j|\delta_j, p_j] - p_j), \end{aligned} \tag{A.10}$$

with $a_j^I = \gamma_j \text{Var}[v_j|\delta_j, p_{-j}]^{-1}$.

As $\{\delta_D, \hat{\omega}_D\}$ is a sufficient statistic for $\{\delta_D, p_D\}$, we deduce (using well-known properties of normal random variables) that

$$\begin{aligned} E[v_D|\delta_D, p_D] &= E[v_D|\delta_D, \hat{\omega}_D] \\ &= \delta_D + \frac{d A_D}{A_D^2(1 + B_F^2\sigma_{u_F}^2) + B_D^2\sigma_{u_D}^2}\hat{\omega}_D, \end{aligned} \tag{A.11}$$

and

$$a_D^I = \frac{\gamma_D}{\text{Var}[v_D \mid \delta_D, \hat{\omega}_D]} \tag{A.12}$$

$$= \gamma_D \frac{A_D^2(1 + B_F^2\sigma_{u_F}^2) + B_D^2\sigma_{u_D}^2}{d^2(A_D^2 B_F^2\sigma_{u_F}^2 + B_D^2\sigma_{u_D}^2) + \sigma_\eta^2(A_D^2(1 + B_F^2\sigma_{u_F}^2) + B_D^2\sigma_{u_D}^2)}. \tag{A.13}$$

Thus,

$$x_D^I(\delta_D, \hat{\omega}_D) = a_D^I(\delta_D - p_D) + b_D^I\hat{\omega}_D,$$

where

$$b_D^I = \frac{\gamma_D}{\text{Var}[v_D|\delta_D, \hat{\omega}_D]} \frac{\text{Cov}[v_D, \hat{\omega}_D]}{\text{Var}[\hat{\omega}_D]}$$
$$= a_D^I \frac{dA_D}{A_D^2(1 + B_F^2\sigma_{u_F}^2) + B_D^2\sigma_{u_D}^2}. \tag{A.14}$$

Similarly, for market F we obtain:

$$x_F^I(\delta_F, \hat{\omega}_F) = a_F^I(\delta_F - p_F) + b_F^I\hat{\omega}_F, \tag{A.15}$$

where

$$a_F^I = \gamma_F \frac{A_F^2(1 + B_D^2\sigma_{u_D}^2) + B_F^2\sigma_{u_F}^2}{A_F^2 B_D^2\sigma_{u_D}^2 + B_F^2\sigma_{u_F}^2}, \quad b_F^I = a_F^I \frac{A_F}{A_F^2(1 + B_D^2\sigma_{u_D}^2) + B_F^2\sigma_{u_F}^2}. \tag{A.16}$$

Clearing price in market j. The clearing condition in market $j \in \{D, F\}$ imposes

$$\mu_j x_j^W(\delta_j, p_j, p_{-j}) + (1 - \mu_j)x_j^I(\delta_j, p_j) + u_j = 0.$$

Let $a_j = \mu_j a_j^W + (1 - \mu_j)a_j^I$. Using equations (A.2) and (A.10), we solve for the clearing price and we obtain

$$p_j^* = \delta_j + \left(\frac{\mu b_j^W + (1 - \mu_j)b_j^I A_j}{a_j}\right)\omega_{-j} + \left(\frac{(1 - \mu_j)b_j^I B_j + 1}{a_j}\right)u_j, \tag{A.17}$$

Remember that we are searching for an equilibrium such that $p_j^* = R_j\delta_j + B_j u_j + A_j\delta_{-j} + C_j u_{-j}$. We deduce from equation (A.17) that in equilibrium, we must have $R_j = 1$,

$$B_j = \left(\frac{(1 - \mu_j)b_j^I B_j + 1}{a_j}\right), \; A_j = \left(\frac{\mu b_j^W + (1 - \mu_j)b_j^I A_j}{a_j}\right), \text{ and } C_j = A_j B_{-j}.$$

Thus

$$B_j = \frac{1}{a_j - (1 - \mu_j)b_j^I}, \quad \text{for } j \in \{D, F\}, \tag{A.18}$$
$$A_j = \mu_j B_j b_j^W, \quad \text{for } j \in \{D, F\}. \tag{A.19}$$

Coefficients A_j and C_j ultimately depend on the coefficients $\{B_j, B_{-j}\}$. Hence, the equilibrium is fully characterized once coefficients B_j and B_{-j} are known as claimed in the proposition. Substituting (A.6) in (A.7) and rearranging we obtain

$$b_D^W = d\gamma_D \frac{1}{d^2 B_F^2\sigma_{u_F}^2 + \sigma_\eta^2(1 + B_F^2\sigma_{u_F}^2)}. \tag{A.20}$$

Using (A.19) (for $j = D$) and (A.20), we can rewrite (A.14) as

$$b_D^I = a_D^I \frac{d^2\mu_D\gamma_D(d^2 B_F^2\sigma_{u_F}^2 + \sigma_\eta^2(1 + B_F^2\sigma_{u_F}^2))}{B_D(\mu_D^2 d^2\gamma_D^2(1 + B_F^2\sigma_{u_F}^2) + \sigma_{u_D}^2(d^2 B_F^2\sigma_{u_F}^2 + \sigma_\eta^2(1 + B_F^2\sigma_{u_F}^2))^2)}. \tag{A.21}$$

Similarly, using (A.19) (for $j = D$) and (A.20), we can rewrite (A.13) as

$$a_D^I = \frac{\gamma_D \left(\mu_D^2 d^2 \gamma_D^2 (1 + B_F^2 \sigma_{u_F}^2) + \sigma_{u_D}^2 (d^2 B_F^2 \sigma_{u_F}^2 + \sigma_\eta^2 (1 + B_F^2 \sigma_{u_F}^2))^2\right)}{(d^2 B_F^2 \sigma_{u_F}^2 + \sigma_\eta^2 (1 + B_F^2 \sigma_{u_F}^2))(\mu_D^2 d^2 \gamma_D^2 + \sigma_{u_D}^2 (\sigma_\eta^2 + d^2)(\sigma_\eta^2 (1 + B_F^2 \sigma_{u_F}^2) + d^2 B_F^2 \sigma_{u_F}^2))} \tag{A.22}$$

$$= \frac{\gamma_D \left(\mu_D^2 \gamma_D^2 \rho_D^2 + \sigma_{u_D}^2 (d^2 + \sigma_\eta^2)(1 - \rho_D^2)^2\right)}{(d^2 + \sigma_\eta^2)(1 - \rho_D^2)(\mu_D^2 \gamma_D^2 \rho_D^2 + \sigma_{u_D}^2 (d^2 + \sigma_\eta^2)(1 - \rho_D^2))}. \tag{A.23}$$

Inserting (A.23) in (A.21) yields after some algebra

$$b_D^I = \gamma_D^2 \frac{d^2 \mu_D}{B_D (\mu_D^2 d^2 \gamma_D^2 + \sigma_{u_D}^2 (\sigma_\eta^2 + d^2)(\sigma_\eta^2 (1 + B_F^2 \sigma_{u_F}^2) + d^2 B_F^2 \sigma_{u_F}^2))}. \tag{A.24}$$

We can now replace (A.6), (A.23) and (A.24) in (A.18) and, after some tedious algebra, we obtain

$$B_D = f(B_F; \mu_D, \gamma_D, \sigma_\eta^2, d, \sigma_{u_F}^2), \tag{A.25}$$

where

$$f(B_F; \mu_D, \gamma_D, \sigma_\eta^2, d, \sigma_{u_F}^2) = \frac{B_{D0}(1 - \rho_D^2)\left(\mu_D \gamma_D^2 \rho_D^2 + (\sigma_\eta^2 + d^2)\sigma_{u_D}^2 (1 - \rho_D^2)\right)}{\rho_D^2 \mu_D^2 \gamma_D^2 + \sigma_{u_D}^2 (\sigma_\eta^2 + d^2)(1 - \rho_D^2)(1 - \rho_D^2 (1 - \mu_D))}, \tag{A.26}$$

with $\rho_D^2 = d^2/((\sigma_\eta^2 + d^2)(1 + B_F^2 \sigma_{u_F}^2))$ and $B_{D0} = (\sigma_\eta^2 + d^2)/\gamma_D$. In a similar way we obtain

$$B_F = g(B_D; \mu_F, \gamma_F, \sigma_{u_D}^2), \tag{A.27}$$

where

$$g(B_{D1}; \mu_F, \gamma_F, \sigma_{u_D}^2) = \frac{B_{F0}(1 - \rho_F^2)\left(\mu_F \gamma_F^2 \rho_F^2 + \sigma_{u_F}^2 (1 - \rho_F^2)\right)}{\rho_F^2 \mu_F^2 \gamma_F^2 + \sigma_{u_F}^2 (1 - \rho_F^2)(1 - \rho_F^2 (1 - \mu_F))}, \tag{A.28}$$

with $\rho_F^2 = (1 + B_D^2 \sigma_{u_D}^2)^{-1}$ and $B_{F0} = \gamma_F^{-1}$. Last, as $\text{Var}[v_D|\delta_D] = \sigma_\eta^2 + d^2$ and $\text{Var}[v_F|\delta_F] = 1$, we obtain that

$$B_j = B_{j0}(1 - \rho_j^2) \times \frac{\gamma_j^2 \mu_j \rho_j^2 + \sigma_{u_j}^2 \text{Var}[v_j|\delta_j](1 - \rho_j^2)}{\gamma_j^2 \mu_j^2 \rho_j^2 + \sigma_{u_j}^2 \text{Var}[v_j|\delta_j](1 - \rho_j^2)(1 - \rho_j^2 (1 - \mu_j))}, \tag{A.29}$$

as claimed in the proposition.

Step 3. Existence of a non-fully revealing equilibrium with full attention ($\mu_j = 1$).
Let $f_1(B_{F1}; \gamma_D, \sigma_\eta^2, d, \sigma_{u_F}^2) \equiv f(B_F; 1, \gamma_D, \sigma_\eta^2, d, \sigma_{u_F}^2)$ and $g_1(B_{D1}; \gamma_F, \sigma_{u_D}^2) \equiv g(B_{D1}; 1, \gamma_F, \sigma_{u_D}^2)$. When $\mu_D = \mu_F = 1$, we deduce from equations (A.25) and (A.27) that a non-fully rational expectations equilibrium exists if and only if the following system of equations has a strictly positive solution

$$B_{D1} = f_1(B_{F1}; \gamma_D, \sigma_\eta^2, d, \sigma_{u_F}^2) = \frac{\sigma_\eta^2}{\gamma_D} + \frac{d^2 B_{F1}^2 \sigma_{u_F}^2}{\gamma_D (1 + B_{F1}^2 \sigma_{u_F}^2)}, \tag{A.30}$$

$$B_{F1} = g_1(B_{D1}; \gamma_F, \sigma_{u_D}^2) = \frac{B_{D1}^2 \sigma_{u_D}^2}{\gamma_F (1 + B_{D1}^2 \sigma_{u_D}^2)}. \tag{A.31}$$

Note that $B_{F1} > 0$ if and only if $B_{D1} > 0$. Let

$$\Psi_1(B_{D1}) \equiv f_1(g_1(B_{D1}; \gamma_F, \sigma_{u_D}^2); \gamma_D, \sigma_\eta^2, d, \sigma_{u_F}^2) - B_{D1}.$$

Substituting the expression for B_{F1} in equation (A.30), we deduce that the equilibrium levels for the illiquidity of security B_{D1} solve $\Psi_1(B_{D1}) = 0$. Thus, a non-fully revealing equilibrium exists if and only if $\Psi_1(B_{D1}) = 0$ has at least one strictly positive root. Using the expression for $g_1(B_{D1}; \gamma_F, \sigma^2_{u_D})$, we obtain

$$\Psi_1(B_{D1}) = \left(\sigma^2_\eta - \gamma_D B_{D1}\right)\left(\gamma_F^2(1 + B^2_{D1}\sigma^2_{u_D})^2 + B^4_{D1}\sigma^4_{u_D}\sigma^2_{u_F}\right) + d^2 B^4_{D1}\sigma^4_{u_D}\sigma^2_{u_F}, \quad \text{(A.32)}$$

which is a polynomial of degree 5 in B_{D1}. Observe that $\Psi_1(\cdot)$ is continuous and

$$\Psi_1\left(\frac{\sigma^2_\eta}{\gamma_D}\right) \geq 0, \quad \Psi_1\left(\frac{\sigma^2_\eta + d^2}{\gamma_D}\right) < 0.$$

Thus, (A.32) has at least one solution B^*_{D1} in the interval $[\sigma^2_\eta/\gamma_D, (\sigma^2_\eta + d^2)/\gamma_D]$. As $\sigma^2_\eta > 0$, this proves existence of a non-fully revealing equilibrium when $\mu_D = \mu_F = 1$.

Step 4. Existence of a non fully revealing equilibrium with limited attention ($\mu_j <$ 1).

With limited attention, we deduce from equations (A.25) and (A.27) that a non-fully revealing equilibrium exists if and only if the following equation has one strictly positive solution

$$\Psi(B_D) \equiv f(g(B_D; \mu_F, \gamma_F, \sigma^2_{u_D}); \mu_D, \gamma_D, \sigma^2_\eta, d, \sigma^2_{u_F}) - B_D = 0.$$

Calculations show that $\Psi(\cdot)$ is an odd-degree polynomial in B_D with negative leading coefficient. Hence,

$$\lim_{B_D \to \infty} \Psi(B_D) = -\infty,$$

while, for $\sigma^2_\eta > 0$,

$$\Psi(0) = \gamma_F^{12}\mu_F^8\sigma^2_\eta(d^2\gamma_D^2\mu_D + \sigma^2_\eta\sigma^2_{u_D}(d^2 + \sigma^2_\eta)) > 0.$$

Thus, there always exists a strictly positive value B^*_D, such that $\Psi(B^*_D) = 0$ when $\sigma^2_\eta > 0$. □

Proof of Corollary 5

Step 1: The total effect of a change in γ_D on the illiquidity of security D is given by

$$\frac{dB_{D1}}{d\gamma_D} = \frac{\partial f_1}{\partial \gamma_D} + \frac{\partial f_1}{\partial B_F}\frac{dB_F}{d\gamma_D}.$$

As

$$\frac{dB_{F1}}{d\gamma_D} = \frac{\partial g_1}{\partial B_D}\frac{dB_{D1}}{d\gamma_D},$$

and $(\partial g_1/\partial B_{D1})(\partial f_1/\partial B_{F1}) > 0$ (since $d > 0$), we deduce that:

$$\frac{dB_{D1}}{d\gamma_D} = \kappa\frac{\partial f_1}{\partial \gamma_D},$$
$$\frac{dB_{F1}}{d\gamma_D} = \kappa\left(\frac{\partial g_1}{\partial B_{D1}}\frac{\partial f_1}{\partial \gamma_D}\right),$$

with $\kappa = 1 - ((\partial g_1/\partial B_{D1})(\partial f_1/\partial B_{F1}))$.

Step 2: Now we prove that there alwauys exists at least one non fully revealing rational expectations equilibrium for which $\kappa > 1$. Let

$$h_1(B_{D1}) \equiv f_1(g_1(B_{D1}; \gamma_F, \sigma^2_{u_D}); \gamma_D, \sigma^2_\eta, d, \sigma^2_{u_F}).$$

Note that

$$\frac{\partial h_1}{\partial B_{D1}} = \frac{\partial f_1}{\partial B_{F1}} \frac{\partial g_1}{\partial B_{D1}}.$$

Hence, if $h'_1(B_{D1}) < 1$ at an equilibrium value for B_{D1} then there exists at least one equilibrium in which $\kappa > 1$. Remember that the equilibrium values for B_{D1} solve (see Step 3 in the proof of Proposition 2)

$$\Psi_1(B_{D1}) \equiv h_1(B_{D1}) - B_{D1} = 0.$$

Hence, the roots of the polynomial $\Psi_1(B_{D1})$ are the possible equilibrium values for the illiquidity of security D. Using equation (A.32), we obtain

$$\Psi_1(B_{D1}) = -B^5_{D1}\gamma_D\sigma^4_{u_D}(\gamma^2_F + \sigma^2_{u_F}) + B^4_{D1}\sigma^4_{u_D}(\gamma^2_F\sigma^2_\eta + (d^2 + \sigma^2_\eta)\sigma^2_{u_F}) \\ - 2B^3_{D1}\gamma_D\gamma^2_F\sigma^2_{u_D} + 2B_{D1}\gamma^2_F\sigma^2_\eta\sigma^2_{u_D} - B_{D1}\gamma_D\gamma^2_F + \gamma^2_F\sigma^2_\eta.$$

Using Descartes' rule of signs, we obtain that $\Psi_1(\cdot)$ has five, three or one positive root. These roots correspond to the intersections of the function $h_1(B_{D1})$ with the 45-degree line. As $h_1(0) = \sigma^2_\eta/\gamma_D > 0$ and,

$$h'_1(B_{D1}) = \frac{4B^3_D d^2\gamma^2_F\sigma^4_{u_D}\sigma^2_{u_F}(1 + B^2_D\sigma^2_{u_D})}{\gamma_D(\gamma^2_F(1 + B^2_D\sigma^2_{u_D})^2 + B^4_D\sigma^4_{u_D}\sigma^2_{u_F})^2} > 0,$$

the function $h_1(B_{D1})$ cuts for the first time the 45-degree line from above. Hence, at this intersection point, we must have $h'_1(B_{D1}) < 1$. Let B^{L*}_{D1} be this intersection point. When the equilibrium is unique, the equilibrium level of illiquidity must be B^{L*}_{D1} as otherwise $h_1(\cdot)$ would never cut the 45-degree line and therefore an equilibrium would not exist. When there are multiple equilibria, B^{L*}_{D1} is the lowest level of illiquidity for security D among all non-fully revealing equilibria since this is the lowest positive root of $\Psi_1(B_{D1})$. Thus, there always exists an equilibrium in which $h'_1(B_{D1}) < 1$ at the equilibrium value for B_{D1}.

□

Proof of Corollary 6

Step 1: For the expressions for the illiquidity levels in securities D and F, see the paragraph that precedes the corollary.

Step 2: For the second part, we differentiate B_F with respect to B_D and we obtain that

$$\frac{\partial B_F}{\partial B_D} = \frac{2B_{D\mu_F\sigma^2_{u_D}}(\gamma^4_F\mu^2_F + B^2_D\gamma^2_F\sigma^2_{u_D}\sigma^2_{u_F}(2\mu_F - B^2_D(1 - \mu_F)\sigma^2_{u_D}) + B^4_D\sigma^4_{u_D}\sigma^4_{u_F})}{\gamma_F((\gamma_F\mu_F)^2(1 + B^2_D\sigma^2_{u_D}) + B^2_D\sigma^2_{u_D}\sigma^2_{u_F}(\mu_F + B^2_D\sigma^2_{u_D}))^2}. \quad \text{(A.33)}$$

The numerator of this expression contains a quadratic polynomial in μ_F with two real roots. Let $\mathcal{P}(\mu_F)$ be this polynomial. One root of $\mathcal{P}(\mu_F)$ is always negative. The other root is

$$\hat{\mu}_F = \frac{B_D^2\sigma_{u_D}^2\sigma_{u_F}\left(-(2+B_D^2\sigma_{u_D}^2)\sigma_{u_F} + \sqrt{4\gamma_F^2 + B_D^2\sigma_{u_D}^2\sigma_{u_F}^2(4+B_D^2\sigma_{u_D}^2)}\right)}{2\gamma_F^2}.$$

As the leading coefficient on $\mathcal{P}(\mu_F)$ (i.e., the coefficient on μ_F^4) is positive, we deduce that $(\partial B_F/\partial B_D)$ is positive if and only if $\mu_F > \hat{\mu}_F$. Direct calculations show that $\hat{\mu}_F \leq 0$, if $\mathcal{R}_F \leq 1$. Thus, in this case, $(\partial B_F/\partial B_D)$ is positive for all values of μ_F. Otherwise $\hat{\mu}_F > 0$ and $(\partial B_F/\partial B_D) < 0$ if and only if $\mu_F < \hat{\mu}_F$. This implies that $\hat{\mu}_F < 1$, as otherwise liquidity spillovers would be negative even when $\mu_F = 1$ (which we know is impossible from Corollary 4). □

Proof of Corollary 7

First observe that a change in B_{-j} only affects the illiquidity of security j through its effect on ρ_j^2. As ρ_j^2 decline in B_{-j}, we deduce that liquidity spillovers from security j to security $-j$ are positive if and only if $(\partial B_j/\partial \rho_j^2) < 0$. Now we show that $\mu_j \geq \overline{\mu}_j$ is a sufficient condition for this to be the case. Observe that $B_j = B_{j0}(1-\rho_j^2)G(\mu_j, \rho_j^2)$ with

$$G(\mu_j, \rho_j^2) \equiv \frac{\gamma_j^2\mu_j\rho_j^2 + \sigma_{u_j}^2\mathrm{Var}[v_j|\delta_j](1-\rho_j^2)}{\gamma_j^2\mu_j^2\rho_j^2 + \sigma_{u_j}^2\mathrm{Var}[v_j|\delta_j](1-\rho_j^2)(1-\rho_j^2(1-\mu_j))}. \tag{A.34}$$

Therefore, we have:

$$\frac{\partial B_j}{\partial \rho_j^2} = -B_{0j}G(\mu_j, \rho_j^2) + B_{0j}(1-\rho_j^2)\frac{\partial G}{\partial \rho_j^2}, \tag{A.35}$$

Now observe that:

$$\frac{\partial G(\mu_D, \rho_D^2)}{\partial \rho_D^2} = \frac{(\sigma_\eta^2+d^2)(1-\mu_D)(1-\rho_D^2)\sigma_{u_D}^2(\gamma_D^2\mu_D(1+\rho_D^2) + (\sigma_\eta^2+d^2)(1-\rho_D^2)\sigma_{u_D}^2)}{(\gamma_D^2\mu_D^2\rho_D^2 + \sigma_{u_D}^2\mathrm{Var}[v_D|\delta_D](1-\rho_D^2)(1-\rho_D^2(1-\mu_D)))^2} > 0.$$

Inserting this expression and the expression for $G(\mu_D, \rho_D^2)$ in equation (A.35), we obtain after some algebra

$$\frac{\partial B_D}{\partial \rho_D^2} = -\frac{\mathrm{Var}[v_D|\delta_D]\mu_D}{\gamma_D(\gamma_D^2\mu_D^2\rho_D^2 + \sigma_{u_D}^2\mathrm{Var}[v_D|\delta_D](1-\rho_D^2)(1-\rho_D^2(1-\mu_D)))^2}\times$$
$$(\gamma^4\mu_D^2\rho_D^4 + \sigma_{u_D}^2\mathrm{Var}[v_D|\delta_D](1-\rho_D^2)(\mathrm{Var}[v_D|\delta_D](1-\rho_D^2)\sigma_{u_D}^2 - \gamma_D^2(1-\mu_D-\rho_D^2(1+\mu_D)))).$$

As $\rho_D^2 < 1$, we deduce that the sign of $(\partial B_D/\partial \rho_D^2)$ is the opposite of the sign of

$$\mu_D - \left(\frac{\mathcal{R}_D - 1}{\mathcal{R}_D}\right)\left(\frac{1-\rho_D^2}{1+\rho_D^2}\right),$$

which is positive if $\mu_D \geq \overline{\mu}_D$. We deduce that $(\partial f/\partial B_F) > 0$ if $\mu_D > \overline{\mu}_D$. A similar reasoning shows that $(\partial g/\partial B_D) > 0$ if $\mu_F > \overline{\mu}_F$. □

Proof of Corollary 8

Using the expression for B_F in the one sided case (see equation (25)), we obtain

$$\frac{\partial B_F}{\partial \mu_F} = -\frac{B_D^2\sigma_{u_D}^2(\gamma_F^4\mu_F^2 - B_D^4\sigma_{u_D}^4\sigma_{u_F}^2(\gamma_F^2(1-2\mu_F) - \sigma_{u_F}^2) + B_D^2\gamma_F^2\mu_F\sigma_{u_D}^2(\gamma_F^2\mu_F + 2\sigma_{u_F}^2))}{\gamma_F((\gamma_F\mu_F)^2(1+B_D^2\sigma_{u_D}^2) + B_D^2\sigma_{u_D}^2\sigma_{u_F}^2(\mu_F + B_D^2\sigma_{u_D}^2))^2}. \tag{A.36}$$

The sign of this derivative is the same as the sign of its numerator, which is a quadratic polynomial in μ_F with a positive leading coefficient. Hence, its sign is positive for all values of μ_F that are larger, in absolute value, than the two real roots of this polynomial. Upon inspection, the first of these roots is always negative, whereas the other root is

$$\mu_F^\bigstar = \frac{-B_D^2\sigma_{u_D}^2\sigma_{u_F}\left(\sigma_{u_F}(1+B_D^2\sigma_{u_D}^2) - ((1+B_D^2\sigma_{u_D}^2)(\gamma_F^2 + B_D^2\sigma_{u_D}^2\sigma_{u_F}^2))^{1/2}\right)}{\gamma_F^2(1+B_D^2\sigma_{u_D}^2)}.$$

We observe that $\mu_F^\bigstar \leq 0$ if and only if $\mathcal{R}_F \leq 1$. Thus, in this case, $(\partial B_F/\partial\mu_F) < 0$, as claimed in Part 1 of the corollary. When $\mathcal{R}_F > 1$, we have $\mu_F^\bigstar > 0$ and $(\partial B_F/\partial\mu_F) > 0$ if and only if $\mu_F < \mu_F^\bigstar$, as claimed in the second part of the corollary. Last we observe that $\mu_F^\bigstar < 1$ as otherwise the illiquidity of security F would be smaller with full attention than with no attention, which is never true (see Corollary 4). □

Proof of Proposition 3

Using the notations introduced in the proof of Proposition 2, we have

$$\text{Var}[v_F|\delta_F, \hat{\omega}_F] = \gamma_F(a_F^I)^{-1},$$
$$\text{Var}[v_F|\delta_F, \omega_F] = \gamma_F(a_F^W)^{-1},$$

where

$$a_F^W = \gamma_F\left(\frac{1+B_D^2\sigma_{u_D}^2}{B_D^2\sigma_{u_D}^2}\right), \quad a_F^I = \gamma_F\left(\frac{\mu_F^2\gamma_F^2(1+B_D^2\sigma_{u_D}^2) + B_D^4\sigma_{u_D}^4\sigma_{u_F}^2}{B_D^2\sigma_{u_D}^2(\mu_F^2\gamma_F^2 + B_D^2\sigma_{u_D}^2\sigma_{u_F}^2)}\right).$$

We deduce that

$$\phi_F(\mu_F, B_D) = \frac{\gamma_F}{2}\ln\left(\frac{a_F^W}{a_F^I}\right),$$

and the expression for $\phi_F(\mu_F, B_D)$ given in the corollary follows. It is then immediate that $\partial\phi_F(\mu_F)/\partial\mu_F < 0$. □

Proof of Proposition 4

As explained in the text, the fraction of pricewatchers in equilibrium is zero iff $\phi_F(0) < C$. Using equation (30), we deduce that this condition is satisfied iff $C > \overline{C}$ where

$$\overline{C} = \frac{\gamma_F}{2}\ln\left(1 + \frac{1}{\sigma_{u_D}^2 B_D^2}\right).$$

Similarly, the fraction of pricewatchers in equilibrium is one iff $\phi_F(1) > C$. Using equation (30), we deduce that this condition is satisfied iff $C < \underline{C}$ where:

$$\underline{C} = \frac{\gamma_F}{2}\ln\left(1 + \frac{\sigma_{u_F}^2\sigma_{u_D}^2 B_D^2}{\gamma_F^2(1+B_D^2\sigma_{u_D}^2) + \sigma_{u_F}^2\sigma_{u_D}^4 B_D^4}\right).$$

Otherwise the fraction of pricewatchers in equilibrium solves $\phi_F(\mu_F, B_D) = C$ and we obtain the expression for $\mu_F^*(C)$ by inverting $\phi_F(\mu_F)$ given in equation (30). □

Proof of Corollary 10

For a given value of C, the level of illiquidity of security F is given by $B_F(\mu_F^*(C))$ where $B_F(\cdot)$ is given in equation (26) when $d = 0$. Thus:

$$\frac{\partial B_F}{\partial C} = \left.\frac{\partial B_F}{\partial \mu_F}\right|_{\mu_F=\mu_F^*(C)} \left(\frac{\partial \mu_F^*(C)}{\partial C}\right).$$

We know that $(\partial\mu_F^*(C)/\partial C) \leq 0$ (Proposition 4). Moreover, using equation (26), we deduce that when $d = 0$, $(\partial B_F/\partial\mu_F) < 0$ if and only if $\mu_F > \widehat{\mu}_F$ where

$$\widehat{\mu}_F = \left(\frac{\sigma_\eta^4 \sigma_{u_D}^2 \sigma_{u_F}}{\gamma_F}\right)\sqrt{\frac{\max\{\gamma_F^2 - \sigma_{u_F}^2 \mathrm{Var}[v_F|\delta_F], 0\}}{\gamma_D^2 + \sigma_\eta^4\sigma_{u_D}^2}}.$$

Thus, when $\gamma_F^2 \leq \sigma_{u_F}^2 \mathrm{Var}[v_F|\delta_F]$, $\widehat{\mu}_F = 0$ and $(\partial B_F/\partial\mu_F)|_{\mu_F=\mu_F^*(C)} < 0$. It follows that $(\partial B_F/\partial C) > 0$. When $\gamma_F^2 > \sigma_{u_F}^2\mathrm{Var}[v_F|\delta_F]$ then $\widehat{\mu}_F > 0$. As $\mu_F^*(C)$ decreases with C from one to zero over $[\underline{C}, \overline{C}]$, there exists a value $C^* \in (\underline{C}, \overline{C})$ such $\mu_F^*(C) = \widehat{\mu}_F$ and $\mu_F^*(C) < \widehat{\mu}_F$ iff $C > C^*$. Thus, in this case, $(\partial B_F/\partial\mu_F) < 0$ iff $C < C^*$. The second part of the corollary follows. □

Proof of Proposition 5

We have

$$\phi_j(1, B^{H*}) = \frac{\gamma}{2}\ln\left(1 + \frac{(B^{H*})^2\sigma_u^4}{\gamma^2(1 + (B^{H*})^2\sigma_u^2) + (B^{H*})^4\sigma_u^6}\right), \tag{A.37}$$

and

$$\phi_j(0, B_{j0}) = \frac{\gamma}{2}\ln\left(1 + \frac{\sigma_\delta^2}{B_{j0}^2\sigma_u^2}\right) = \frac{\gamma}{2}\ln\left(1 + \frac{\gamma^2}{\sigma_u^2}\right)$$

Thus,

$$\phi_j(1, B^{H*}) > \phi_j(0, B_{j0}) \Leftrightarrow \frac{(B^{H*})^2\sigma_u^4}{\gamma^2(1 + (B^{H*})^2\sigma_u^2) + (B^{H*})^4\sigma_u^6} > \frac{\gamma^2}{\sigma_u^2}. \tag{A.38}$$

We deduce that $\phi_j(1, B^{H*}) > \phi_j(0, B^*(0))$ if and only if

$$-\gamma^2\sigma_u^6(B^{H*})^4 + (\sigma_u^4 - \gamma^4)\sigma_u^2(B^{H*})^2 - \gamma^4 > 0. \tag{A.39}$$

Using the expression for B^{H*} given in equation (10), we obtain that

$$(B^{H*})^2 = \frac{(B^{H*}\sigma_u^2 - \gamma)}{\gamma\sigma_u^2}. \tag{A.40}$$

Thus, we can rewrite condition (A.39) as

$$-\gamma\sigma_u^2(B^{H*}\sigma_u^2 - \gamma)^2 + (\sigma_u^4 - \gamma^4)(B^{H*}\sigma_u^2 - \gamma) - \gamma^5 > 0.$$

It can be checked that this inequality holds true if B^{H*} belongs to

$$\left(\frac{\gamma}{\sigma_u^2} + \frac{\sigma_u^4 - \gamma^4 - ((\sigma_u^4 - \gamma^4)^2 - 4\gamma^6\sigma_u^2)^{1/2}}{2\gamma\sigma_u^4}, \frac{\gamma}{\sigma_u^2} + \frac{\sigma_u^4 - \gamma^4 + ((\sigma_u^4 - \gamma^4)^2 - 4\gamma^6\sigma_u^2)^{1/2}}{2\gamma\sigma_u^4}\right).$$

Straighforward calculations show that this is the case when $\sigma_u^2 > 4\gamma^2$, which is required for the existence of a symmetric equilibrium.

Part 2: Suppose that $\mu_D^* = \mu_F^* = 1$. Then in this case, the value of monitoring market j for a dealer in security $-j$, given the actions of other dealers, is ϕ_1. As this value is higher than C, monitoring is optimal. Hence $\mu_D^* = \mu_F^* = 1$ is an equilibrium. Now suppose that $\mu_D^* = \mu_F^* = 0$. Then in this case, the value of monitoring market j for a market-maker in market $-j$, given the actions of other dealers, is ϕ_0. As this value is lower than C, not monitoring is optimal. Hence $\mu_D^* = \mu_F^* = 0$ is an equilibrium. □

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Figures

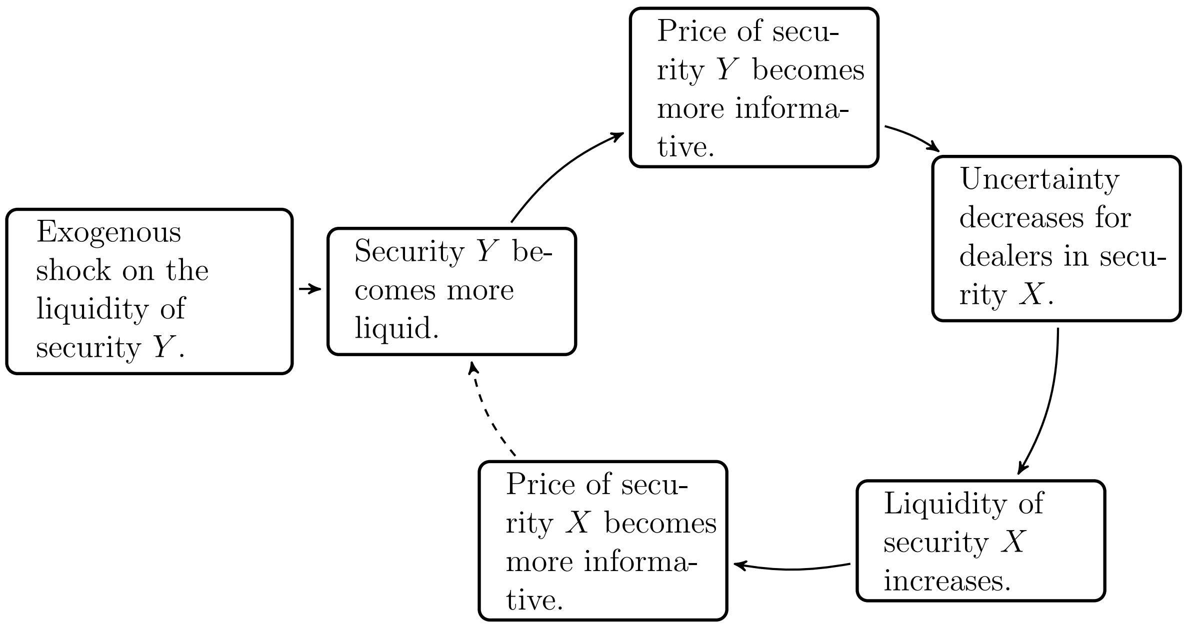

Figure 1: Cross-asset learning and liquidity spillovers.

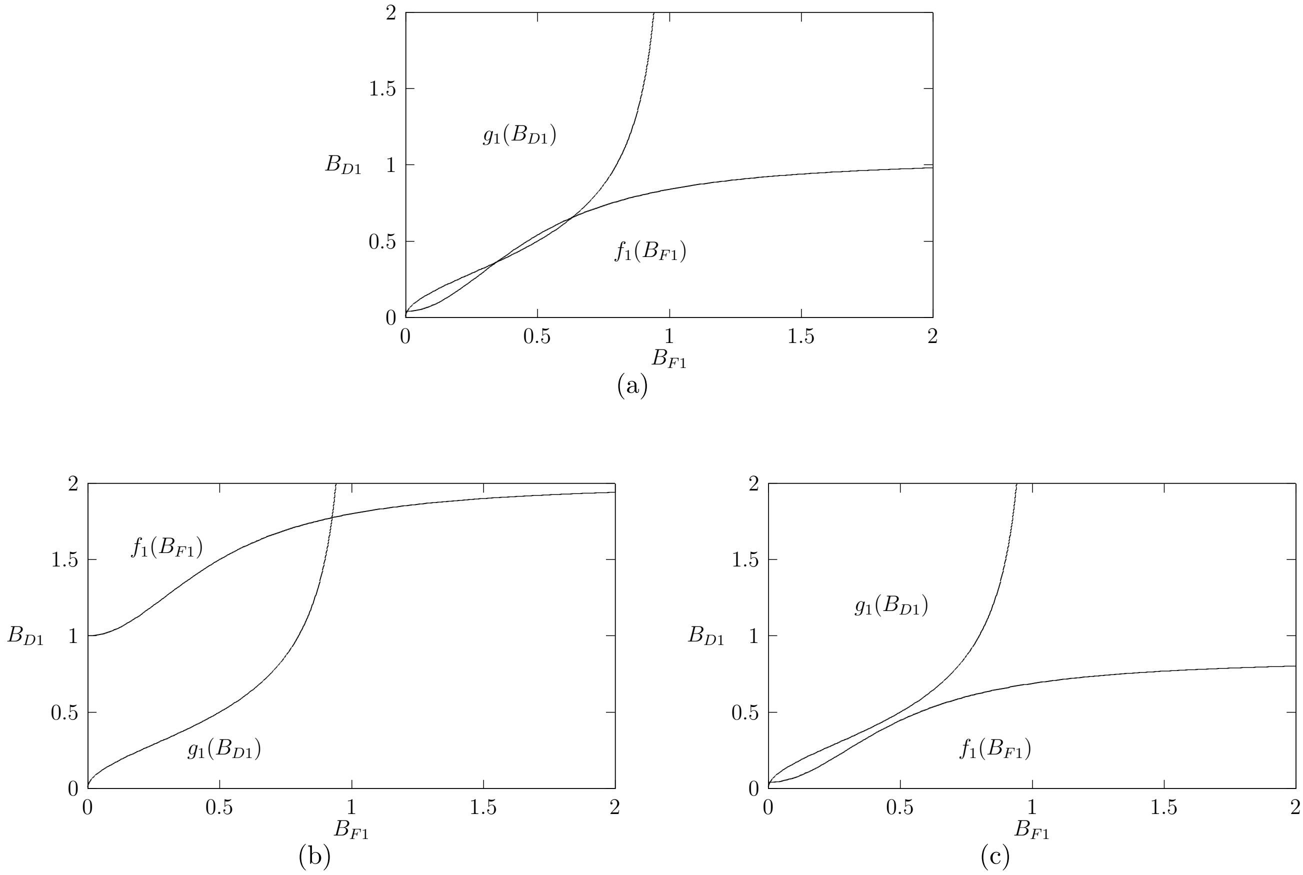

Figure 2: Equilibrium determination with full attention: multiplicity (panel (a)) and uniqueness (panel (b) and (c)). Parameters' values are as follows: $\gamma_j = d = 1$, $\sigma_{u_j} = 2$, and $\sigma_\eta = .2$ (panel (a)), while in panel (b) we set $\sigma_\eta = 1$ and in panel (c) we set $d = 0.9$.

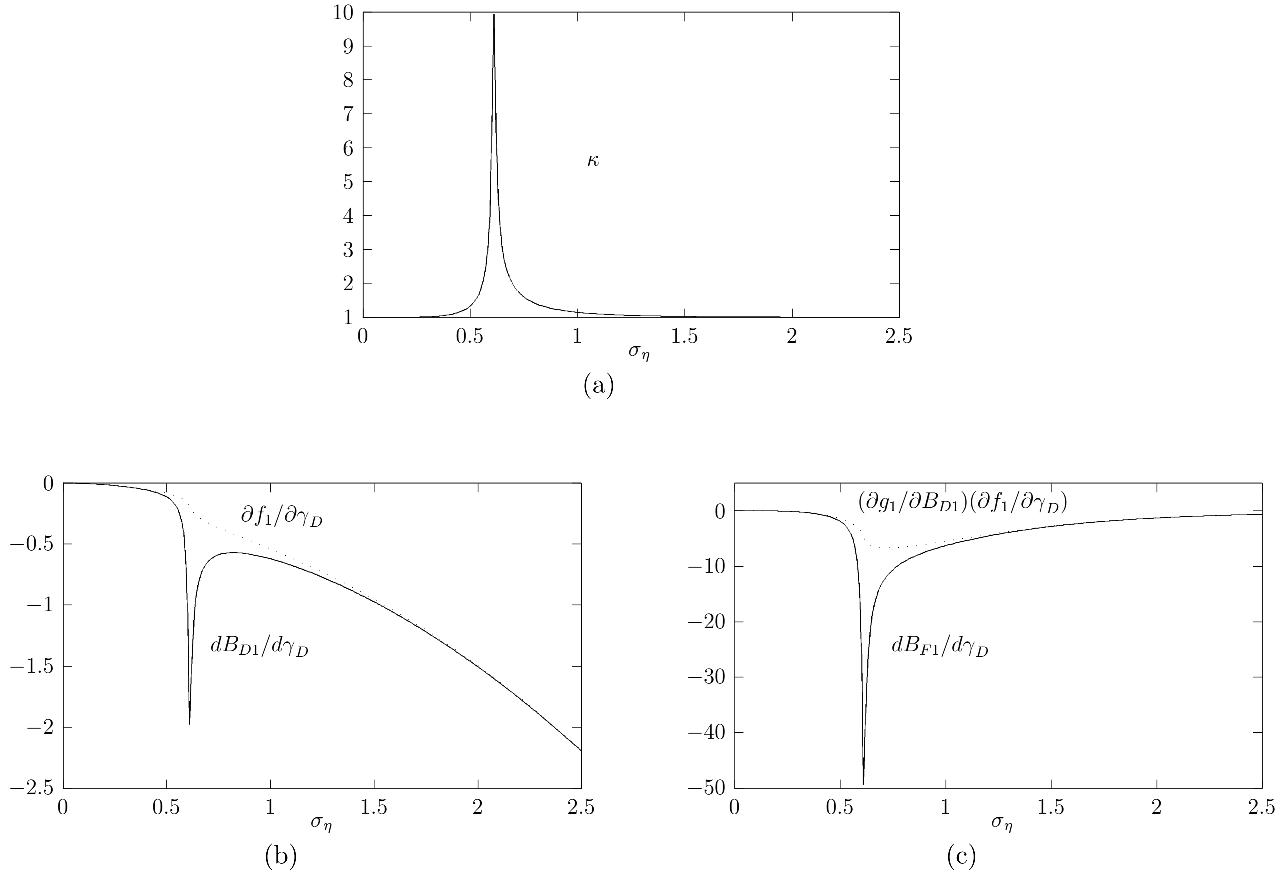

Figure 3: Illiquidity multiplier. In panel (a) we plot κ as a function of σ_η. Panels (b) and (c) show the direct effect (dotted line) and total effect (plain line) of a change in the risk tolerance of the dealers in security D on the illiquidity of securities D and F, respectively as a function of σ_η. Other parameter values are $\sigma_{u_F} = .1$, $\sigma_{u_D} = 1.6$, $\gamma_D = 1.8$, and $\gamma_F = 1/24$.

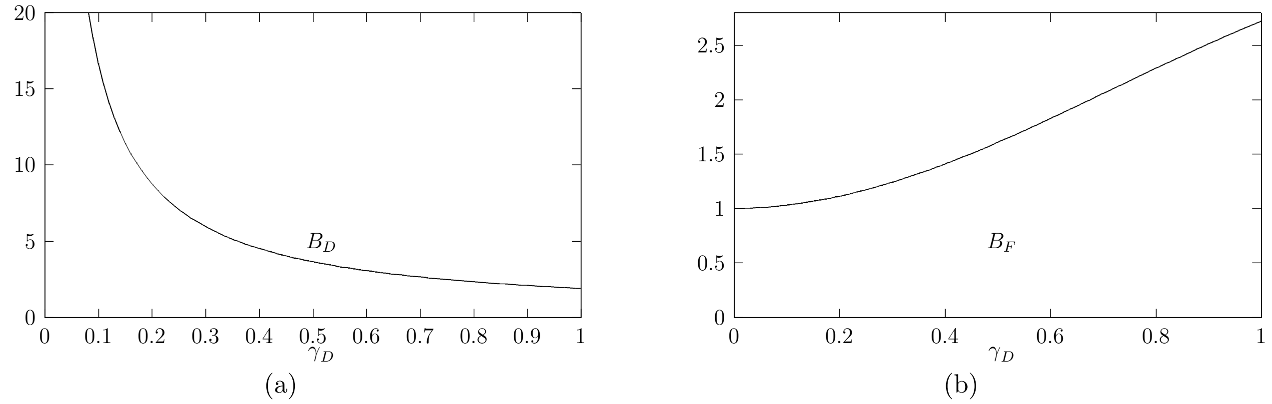

Figure 4: Negative liquidity spillovers. Parameters' values are as follows: $\sigma_{u_F} = .1$, $\sigma_{u_D} = 1$, $\gamma_F = 1$, $d = 1$, $\mu_F = \mu_D = .1$, $\sigma_\eta = 1$, and $\gamma_D \in \{.01, .02, \ldots, 1\}$.

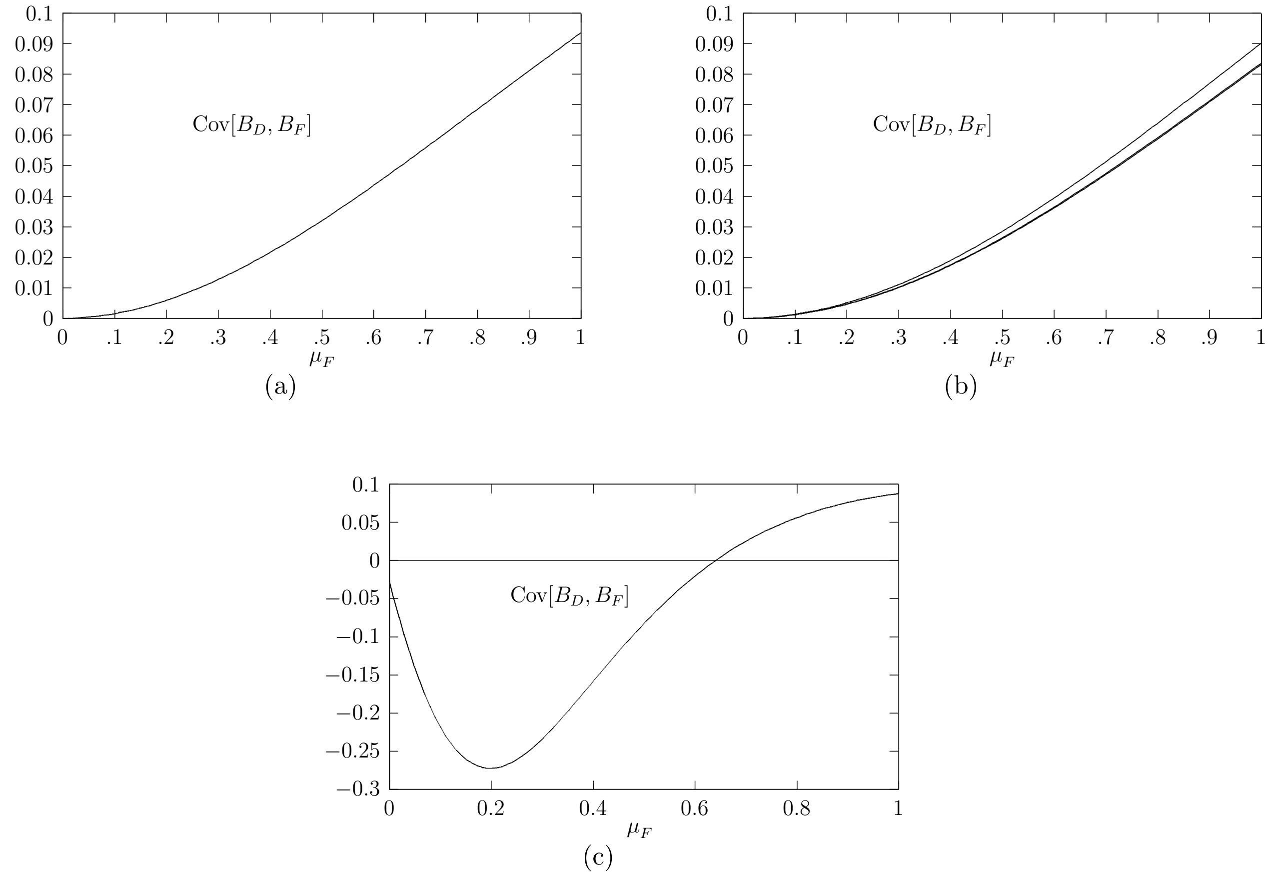

Figure 5: Comovement in illiquidity. The figure displays the covariance between the illiquidity of security F and the illiquidity of security D as a function of μ_F when $d = 0$ (panel (a)) and $d = 0.9$ (panels (b) and (c)). In panel (b) the covariance between the illiquidity of the two securities is higher when $\mu_D = 0.9$ (light curve) than when $\mu_D = 0.1$ (bold curve), for all values of $\mu_F > 0$. Other parameter values are $\sigma_{u_F} = \sigma_{u_D} = 1/2$, $\sigma_\eta = 2$, $\gamma_F = 1/2$, and $\mu_D \in \{0.1, 0.9\}$ for panels (a) and (b), while in panel (c) we set $\sigma_{u_F} = 0.1$, $d = \mu_D = 0.9$ and keep the other parameters' values fixed.

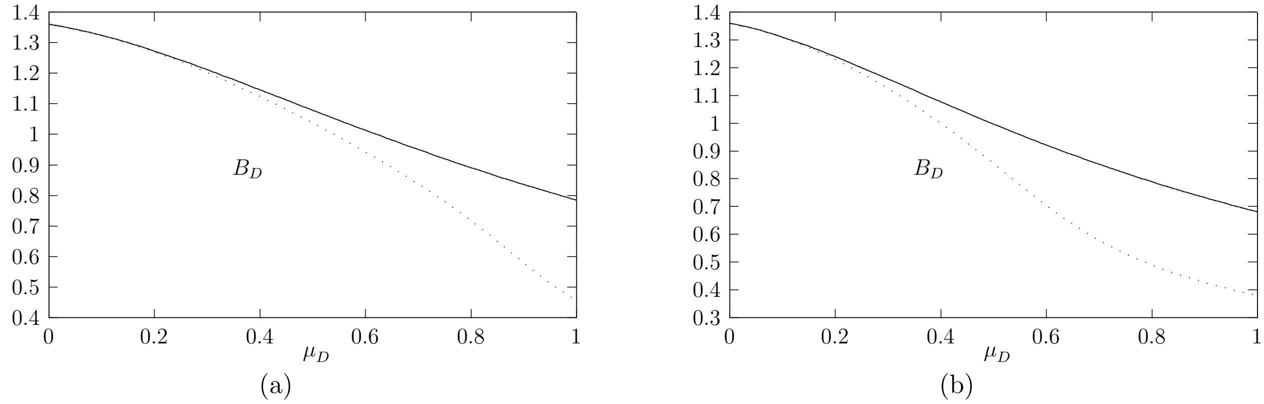

Figure 6: The figure displays the illiquidity of security D as a function of μ_D when $\mu_F = 0.5$ (in panel (a)) and when $\mu_F = 0.9$ (panel (b)) when B_F is fixed at its equilibrium value for $\mu_D = 0.001$ (bold curve) and when instead it adjusts to its equilibrium value for each value of μ_D (dotted curve). The difference between the two curves shows the amount by which spillover effects magnify the direct effect of a change in attention on illiquidity. Parameters' values are as follows: $\sigma_{u_D} = \sigma_{u_F} = 1$, $\sigma_\eta = 0.77$ and $d = \gamma_D = \gamma_F = 1$.

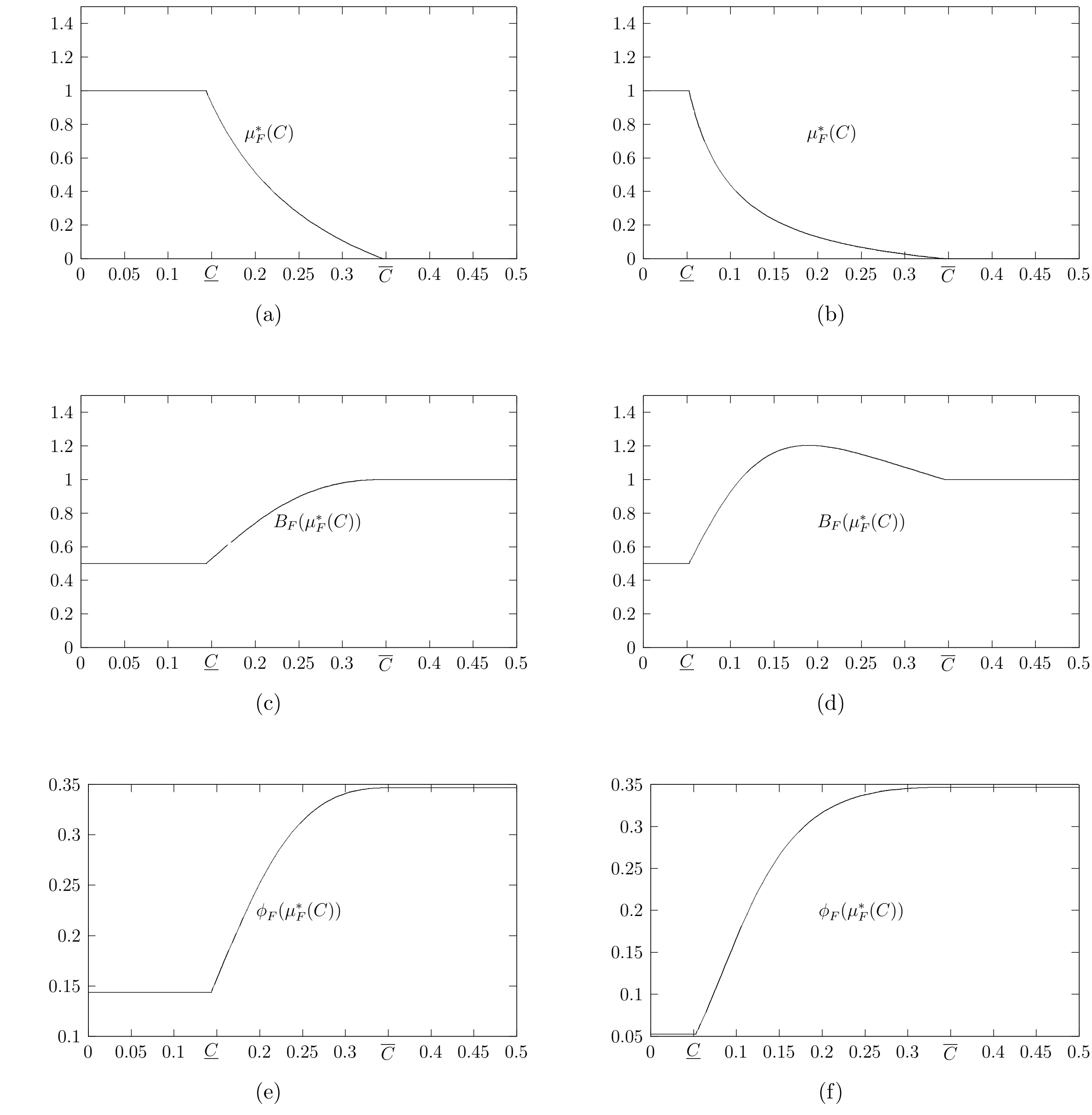

Figure 7: Impact of a change in the cost of attention on the fraction of pricewatchers, illiquidity, and the value of information with one-sided learning. Case with $\mathcal{R}_F \leq 1$ (panels (a), (c), and (e)), and case with $\mathcal{R}_F > 1$ (panels (b), (d), and (f)). Parameters' values are as follows: $\sigma_{u_D} = 1$, $\gamma_F = \gamma_D = 1$, $d = 0$, and $\sigma_\eta = 1$, with $\sigma_{u_F} = 1$ in panels (a), (c), and (e) whereas $\sigma_{u_F} = 0.5$ in panels (b), (d), and (f).

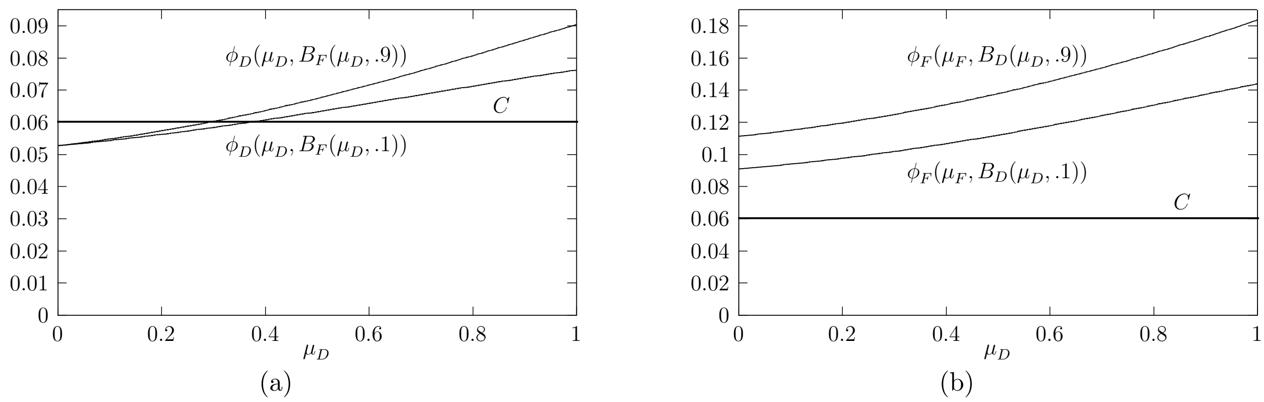

Figure 8: Positive feedback effect and cross-market monitoring effect. In panel (a) we plot ϕ_D as a function of μ_D, for $\mu_F \in \{0.1, 0.9\}$. In panel (b) we plot ϕ_F as a function of μ_D, for $\mu_F \in \{0.1, 0.9\}$. Other parameter values are as follows: $\sigma_\eta = 1$, $\sigma_{u_F} = \sigma_{u_D} = 1$, $\gamma_F = \gamma_D = 1$, and $d = 1$.