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No. 8336

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*FINANCIAL ECONOMICS*



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Discussion Paper No. 8336  
April 2011

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## ABSTRACT

### Security Design: Signaling versus Speculative Markets\*

We determine optimal security design and retention of asset-backed securities by a privately informed issuer with positive NPV uses for immediate cash. In canonical models, investors revert to prior beliefs if issuers pool at zero-retentions (originate-to-distribute), and separating equilibria are welfare-dominated since separation entails signaling via asset-retention and underinvestment. However, we show speculative markets arise if and only if issuers pool, creating previously overlooked costs. Pooling induces socially costly information acquisition by speculators. Further, in pooling equilibria, issuers never sell safe claims, leaving uninformed investors exposed to adverse selection and distorting risk sharing. In such equilibria, issuers retain zero interest in the asset, and speculator effort is maximized by splitting cash flow into a risky senior ("debt") tranche and residual junior ("equity") claim. Optimal leverage trades off per-unit speculator gains against endogenous declines in uninformed debt trading. Issuer incentives to implement the pooling equilibrium, with distorted risk sharing, are strong precisely when efficient risk sharing, achieved through separation, has high social value. In such cases, a tax on issuer proceeds can raise welfare by encouraging issuer retentions. Taxation dominates mandatory skin-in-the-game as a policy response, since the latter creates gratuitous underinvestment.

JEL Classification: G21, G32

Keywords: security design, issuer, type, pooling, separating, speculator

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\* We thank seminar participants at Toulouse School of Economics, the Bachelier Seminar, University of Bath, London Business School, London School of Economics, the 2010 Rising Stars in Finance Conference, Inova Lisbon, Bocconi and Princeton University for valuable feedback. We owe special thanks to Franklin Allen, James Dow, Sebastien Pouget and Conrad Raff for helpful discussions.

Submitted 25 March 2011

The canonical model of financing and security design under asymmetric information (see e.g. Tirole (2005) Chapter 6) considers the firm to have two choices: send credible signals or accept underpricing as uninformed investors set prices based upon prior beliefs. A potential weakness of this standard framework is that outside investors are assumed to be passive, doing nothing other than providing funding on an actuarially fair basis. Thus, in the canonical framework there is no role for demand-side factors to affect the choice between separation and pooling. This prediction is at odds with the empirical observation that the buy-side appears to play a critical role in security design.<sup>1</sup>

In this paper, we develop a theory of optimal financing which assigns a leading role to speculative markets in influencing security design, as well as issuer choice between signaling and pooling. In contrast to the canonical model, the proposed theory recognizes the fact that speculative markets can produce information. Consequently, a privately informed issuer need not choose between signaling and reversion to uninformed prices. Rather, in reality issuers must choose between signaling and reliance on speculative markets for information production. As discussed below, this alternative perspective radically changes predictions regarding optimal security design, securitization levels, and social welfare.

The setting for our model is as follows. A privately informed owner is contemplating the sale of optimally-designed asset-backed securities in order to fund a positive NPV project having constant returns. One may think of the issuer as being a bank facing capital requirements that lead to a preference for receiving immediate cash rather than holding risky assets. Alternatively, one can think of the issuer of the asset-backed securities as being a non-financial firm. Under this second interpretation, the security is a claim on a fixed asset already owned by the firm and not on assets purchased with the proceeds raised. See DeMarzo and Duffie (1999) for additional motivation for this setup.<sup>2</sup>

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<sup>1</sup>See Allen and Gale (1994) for a comprehensive discussion of clientele-driven contract innovation.

<sup>2</sup>We follow DeMarzo and Duffie (1999) and take the underlying asset as given. We do not analyze which assets are optimal for asset-backing. See DeMarzo (2005) for an analysis of that issue.

The underlying asset value is either  $L$  or  $H$ . The owner faces a continuum of risk averse uninformed investors who may have an insurance demand for buying claims due to endowment shocks. In addition, there is a speculator who can exert costly effort in order to acquire a signal regarding true asset value. The presence of the informed speculator discourages uninformed investors from buying securities, since they face adverse selection in competitive markets.

One possibility is for the owner to signal positive information by selling a senior debt claim secured by the asset having face value  $L$ . The original owner then retains the residual junior claim. This retention is a costly signal in that it leads to underinvestment in the scalable growth option. Meanwhile, the low type sells the entire asset in the form of a pass-through security and invests at the first-best level. In this separating equilibrium, positive private information is credibly signaled by issuer retentions, implying there is no role for costly information acquisition and that the previously uninformed investors are insulated from adverse selection. Although a similar separating contract has been described in earlier papers, e.g. Myers and Majluf (1984), we are the first to recognize that the separating equilibrium has two attractive features from a social welfare perspective. First, there is no expenditure on socially wasteful information acquisition. Second, risk sharing is efficient.

The other possibility for the high type to consider is a *pooling equilibrium* in which both types sell the entire asset and adopt identical security designs. The key difference between our pooling equilibrium and that considered in the canonical model is that the two owner types do not expect the same total revenues. This is because speculative markets can produce information and bring prices closer to fundamentals. If he opts to pool, the high type will promote speculator effort by choosing a securitization structure that maximizes the speculator's gains. This is accomplished by bifurcating the underlying asset's cash flow into a risky senior (debt) claim and a residual (equity) claim. The rational uninformed investors only trade the senior debt claim since this claim has lower adverse selection costs. The speculator hides behind uninformed demand in the risky debt market, and this is her only source of trading profits. The optimal debt face value trades off higher per-unit speculator profit against endogenous declines in uninformed demand.

The pooling equilibrium has an attractive feature in that expected investment is first-best since both types sell the entire underlying asset. However, in this equilibrium, socially wasteful information acquisition occurs. Further, risk sharing is distorted due to the adverse selection faced by uninformed investors. It follows that from a social welfare perspective the separating equilibrium dominates the pooling equilibrium whenever risk sharing is sufficiently important, e.g. when uninformed investors face larger endowment shocks or are more risk averse. Our finding that separating equilibria may, in fact, dominate pooling equilibria is interesting inasmuch as the standard signaling model, e.g. Myers and Majluf (1984), implies it is socially preferable for issuers to pool by selling a pass-through (equity) security and investing at first-best, with security mispricing being of no social concern.

Another interesting prediction generated by the model is that privately-informed owners are less likely to implement the separating equilibrium, which entails efficient risk sharing, precisely when risk sharing has high social value. The argument is as follows. Suppose the high type is considering pooling. He knows that when the uninformed investors face large endowment shocks, or are more risk averse, they are more willing to trade despite facing adverse selection. The higher level of uninformed demand provides the speculator with increased cover for her trades, allowing her to earn greater profits. This promotes speculator effort, causing prices to move closer to fundamentals in the pooling equilibrium. It follows that the high type is more likely to “choose” the pooling equilibrium when risk sharing is socially important. Formally, we show that the equilibrium set always includes the separating equilibrium, with pooling equilibria being feasible only if endowment shocks are sufficiently large, or risk aversion is sufficiently high.

An important conflict highlighted by the model is that privately informed owners fail to internalize the positive externality they confer to previously uninformed investors when they signal private information via retention of claims on future cash flow.<sup>3</sup> This creates the potential for socially excessive levels of securitization, especially when risk sharing matters most.<sup>4</sup> However, a tax

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<sup>3</sup>Pagano and Volpin (2010) identify externalities arising from another choice, coarse information disclosure.

<sup>4</sup>Policymakers have begun to question the risk sharing benefits of securitization. Adair Turner, Chairman of the

on the gross proceeds from securitized asset sales can serve to move markets back to the separating equilibrium when this equilibrium is socially optimal. Mandating that all securitizers maintain skin-in-the-game is an inefficient policy response to the problem identified here since this results in gratuitous underinvestment (by low types).

Our paper is most closely related to that of Gorton and Pennacchi (1990). In their economy, uninformed investors are also rational, being reluctant to trade informationally-sensitive securities. They show uninformed investors have an incentive to set up an intermediary that will pool endowments and bifurcate underlying cash flows into *riskless* debt and residual equity, with the former being used by uninformed investors as a store of value. Essentially, one can think of the uninformed investors as choosing financial structure in their model. In contrast, we determine the securities that will be chosen by privately-informed owners. This leads to very different optimal security designs. For example, we show that in pooling equilibria safe debt will never be issued, since the high type wants to promote information production by allowing the speculator to profit from trades in *risky* debt markets.

In a recent paper, Dang, Gorton and Holmström (2010) also analyze the interplay between security design and risk sharing. They derive an “ignorance is bliss result” predicting that optimal security design actually minimizes incentives for information acquisition. In their economy information is problematic for three reasons: it is costly to produce; it creates adverse selection; and it reduces welfare due to an asymmetric benefit to positive news. In contrast, we show that privately-informed owners may have an incentive to promote information acquisition since this drives prices closer to fundamentals. Thus, in our model, the privately optimal security design does not minimize informational-sensitivity.

Boot and Thakor (1993) assume the asset must be fully securitized, so their model is silent on the issuer’s choice between pooling and separating via retentions, the main choice margin we analyze from a welfare perspective. The two models share the implication that tranching can be 

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U.K. Financial Services Authority, recently contended, “the argument that they created great allocative efficiency benefits via market completion was hugely overstated.”



used to promote information acquisition. However, the causal mechanisms are very different. In their model, the role of tranching is to relax speculator wealth constraints, as he buys the risky claim in a market with exogenous uninformed demand. We focus on the opposite lever. Specifically, in our model the role of tranching is to *endogenously* shift uninformed demand in order to stimulate information acquisition by an unconstrained speculator. The optimal structuring also differs. In their model, it is optimal to carve out a riskless senior claim. In the pooling equilibrium of our model, it is never optimal to issue a riskless claim since uninformed investors would then boycott the risky security market, resulting in zero information production.

Fulghieri and Lukin (2001) also analyze the role of security design in information production. Their model is silent on the question of pooling versus separation since the assumed fixed investment scale precludes separation. They also model uninformed demand exogenously, precluding analysis of risk sharing and social welfare.

DeMarzo and Duffie (1999) analyze optimal security design from the perspective of an issuer who places intrinsic value on immediate liquidity. In their model, the issuer chooses security design before observing asset value. After the structure is locked-in, the issuer observes asset value and decides how much to sell. Under technical conditions, e.g. monotonicity, debt is an optimal security since its low information-sensitivity results in low price impact. In contrast, we consider a setting where the issuer knows the asset's value before choosing the security design. Further, we allow for a speculator to acquire information. Finally, the model of DeMarzo and Duffie is silent on risk sharing since they assume universal risk-neutrality.

Nachman and Noe (1994) analyze a setting, like ours, where the issuer is privately informed at the time the security is designed. In their setting, the scale of investment is fixed, and there is no possibility for separation or informed speculation. Under technical conditions, e.g. monotonicity, they show firms will pool at a debt contract, since debt minimizes the cross-subsidy from high to low types.<sup>5</sup>

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<sup>5</sup>Axelson (2007) considers a setting where the issuer is uninformed, showing equity can be optimal as it aligns funding with investment value.

Allen and Gale (1988) evaluate optimal security design in a setting with endogenously incomplete markets, with the firm having a monopoly on issuing securities due to the need for asset-backing. Our model is predicated upon a very different friction. Allen and Gale assume symmetric information, but firms incur a cost when introducing a security. Our model features asymmetric information, but no direct cost of introducing a security.

The remainder of the paper is as follows. Section I describes the economic setting. Section II abstracts from the security design problem in order to describe the market-making process. Section III analyzes optimal structuring in a setting where the entire asset must be securitized. Section IV considers the general setting in which issuers optimize both the security design and retentions.

## I. Economic Setting

This section describes preferences, endowments, and the market-making process. To provide a benchmark, we finish this section by describing equilibrium if information were symmetric.

### A. Preferences and Endowments

There are two periods, 1 and 2, with a single nonstorable consumption good available in each period. This consumption good is the numeraire. There is an underlying asset with type ( $\tau$ ) that is either high ( $H$ ) or low ( $L$ ), with Owner being the only agent endowed with perfect knowledge of the type. The asset delivers  $\tau$  units of the good in period 2 with perfect certainty, with  $L \in (0, H)$ . The uninformed prior probability of the asset being type  $H$  is  $q \in (0, 1)$ .

Owner possesses the only tangible real asset in the economy. Owner has no endowment other than this real asset. The tangibility of the asset allows courts to verify its value in period 2. In contrast, the endowments of the various agents are not verifiable by courts. Consequently, other agents cannot issue securities and cannot short-sell.<sup>6</sup> Further, there can be no endowment-contingent contracts. Rather, courts can only enforce asset-backed payments contingent upon the observed

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<sup>6</sup>We could allow the marketmakers to supply a limited amount of risk-free assets without changing the qualitative results.

value in  $\{L, H\}$ . Since endowments are not verifiable by courts, Arrow securities are not possible and risk sharing may be inefficient. Allen and Gale (1988) also consider an incomplete markets setting in which security design influences risk sharing.

There is a continuum of uninformed investors having measure one. Our uninformed investors are analogous to the “liquidity traders” in the model of Gorton and Pennacchi (1990). To fix ideas, one can think of the uninformed investors as the analog of defined benefit pension funds. Such pensions want to transfer funds across periods and do not particularly value high upside potential. Rather, they are very averse to being unable to meet their obligations to pensioners. Given such preferences, and lacking the resources to gather inside information in securities markets, they naturally seek relatively safe stores of value.

In the model, uninformed investors may have an insurance motive for purchasing securities delivering consumption in period 2. The uninformed investors are sufficiently wealthy in aggregate to buy the entire underlying asset since each has a first period endowment  $y_1 \geq H$ . Uninformed investors face a common endowment shock, with their period 2 endowment  $y_2$  being either 0 or  $-\phi$ . Just prior to securities market trading in period 1, uninformed investors privately observe a noisy signal regarding the size of their period 2 endowment. In particular, with probability one-half they find that they are “invulnerable” to a negative shock and will have endowment  $y_2 = 0$  with probability one. With probability one-half they are “vulnerable” to a negative shock. Conditional upon being vulnerable, they face a probability  $\eta \in (0, 1]$  of  $y_2 = -\phi$ .

Uninformed investors are risk-neutral over first period consumption  $c_1$  and risk-averse over second period consumption  $c_2$ . They are indexed by the intensity of their risk-aversion as captured by a preference parameter  $\theta$ . The utility of uninformed investor- $\theta$  takes the form:

$$U(c_1, c_2; \theta) \equiv c_1 + \min(0, \theta c_2). \tag{1}$$

The preference parameters have compact support  $\Theta \equiv [0, \theta^{\max}]$ . Throughout,  $\theta^{\max}$  is assumed to be sufficiently high such that there is always strictly positive uninformed demand for at least one

security.<sup>7</sup> The  $\theta$  parameters have density  $f$  with cumulative density  $F$ . This distribution has no atoms, with  $f$  being strictly positive and continuously differentiable. We have here followed the tractable specification of risk-aversion employed by Dow (1998) in that second period utility is piecewise linear, and has a concave kink. Other smooth utility functions could be assumed at the cost of more complex aggregate demand functions. The essential assumption is that uninformed investors are averse to negative consumption in period 2, so they are potentially willing to buy stores of value even when facing adverse selection in securities markets.

There is a single speculator  $S$  who is risk-neutral and indifferent regarding the timing of consumption having utility equal to  $c_1 + c_2$ . In the first period, she is endowed  $y_1^S \geq H$  units of the numeraire, so she can afford to buy the entire asset. Her second period endowment is irrelevant and normalized at zero.

The speculator is unique in that she receives a noisy signal of asset type and can exert costly effort to increase signal precision. Letting  $s \in \{s_L, s_H\}$  denote the signal and  $\tau$  the true asset type,  $S$  chooses  $\sigma \equiv \Pr(s = s_\tau)$  from the feasible set  $[1/2, 1]$ . Her non-pecuniary effort cost function  $e$  is strictly positive, strictly increasing, strictly convex, twice continuously differentiable, and satisfies

$$\begin{aligned} \lim_{\sigma \downarrow \frac{1}{2}} e(\sigma) &= 0 \\ \lim_{\sigma \downarrow \frac{1}{2}} e'(\sigma) &= 0 \\ \lim_{\sigma \uparrow 1} e'(\sigma) &= \infty. \end{aligned}$$

If  $S$  puts in any effort, the signal becomes informative since

$$\sigma > \frac{1}{2} \Rightarrow \Pr[\tau = H | s = s_H] = \frac{\Pr[\tau = H \cap s = s_H]}{\Pr[s = s_H]} = \frac{q\sigma}{q\sigma + (1-q)(1-\sigma)} > q. \quad (2)$$

The final set of agents in the economy is a continuum of market-makers having measure one. They are risk-neutral and indifferent regarding the timing of consumption having utility equal to  $c_1 + c_2$ . In the first period each market-maker is endowed with  $y_1^{MM} \geq H$  units of the numeraire,

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<sup>7</sup>This avoids the need to continually check upper limits of integration when computing their demand.

so they too can afford to buy the entire asset. Their second period endowment is irrelevant and normalized at zero.

## B. The Market-Making Game

We characterize perfect Bayesian equilibria (PBE) of signaling games, requiring: all agents have a belief at each information set; strategies must be sequentially rational given beliefs; and beliefs are determined using Bayes' rule and the equilibrium strategies for all information sets on the equilibrium path.

Section IV determines the nature of equilibrium in the full security design game—a game for which there can be pooling and separating equilibria supported by signals in the form of asset retention. A necessary precursor to that analysis is determining the nature of equilibria in the event that the types pool at the same security design. If the two types pool at the same security design, play passes to the market-making game. The *market-making game* is a proper signaling game played between the informed speculator and market-makers.

The market-makers and uninformed investors enter the market-making game holding their prior belief that the asset has high quality with probability  $q$ . The market-making game starts with  $S$  choosing  $\sigma$  at personal cost  $e(\sigma)$ . Her choice of  $\sigma$  is not observable, but is correctly inferred by other agents in equilibrium. Then  $S$  privately observes the noisy signal  $s \in \{s_L, s_H\}$  regarding the asset type. Next, uninformed investors privately observe whether or not they are vulnerable to a negative endowment shock in period 2. Market orders are then submitted, with market-makers setting prices competitively. To do so, in the market-making game market-makers form beliefs regarding the signal received by  $S$ .

The market-making process is in the spirit of Kyle (1985) and Glosten and Milgrom (1985). The speculator and uninformed investors simultaneously submit non-negative market orders. Market-makers then set prices based upon observed aggregate demands in all markets. There is no market segmentation. At this information set, market-makers must have a belief about the signal  $s$  for any aggregate demand configuration. Market-makers clear all markets, buying all securities not

purchased by uninformed investors or the speculator.

Since Owner is the only agent capable of issuing claims delivering goods in period 2, market-makers cannot be called upon to take short positions. To this end, we impose the following technical assumption.

$$A1 : \phi \leq \frac{L}{2}.$$

The role of Assumption 1 is as follows. The aggregate demand of uninformed investors is weakly increasing in  $\phi$ . Therefore, to avoid the possibility of aggregate demand exceeding supply for any security, the endowment shock must be sufficiently small. Sufficiency of Assumption 1 for ensuring no shorting by the market-makers is established below.

To summarize, the sequence of events in period 1 is as follows: Owner observes the asset type and chooses a structuring; the speculator exerts effort cost  $e(\sigma)$  and then observes her signal; uninformed investors observe whether they are vulnerable to negative  $y_2$ ; the speculator and uninformed investors simultaneously submit market orders; and finally market-makers observe aggregate demands in each market and set prices competitively. In period 2 endowments ( $y_2$ ) are realized, the asset value is verified by the courts and the various claimants are paid.

### C. Symmetric Information Benchmark

If information were symmetric Owner would sell the entire asset as a pass-through security (an equity claim to the entire asset cash flow). The speculator would not put in any effort and would simply consume her first period endowment of  $y_1^S$ . The uninformed investors would not buy any claims to second period consumption if invulnerable. If vulnerable, a subset of uninformed investors would submit buy orders for the equity which would be correctly priced at  $\tau$ . Specifically, all uninformed investors with  $\theta > \eta^{-1}$  would buy  $\phi/\tau$  shares of equity ensuring  $c_2 = 0$  in the event of a negative endowment shock and  $c_2 = \phi$  if not. The remaining vulnerable uninformed investors would not buy any of the equity, implying  $c_2 = -\phi$  in the event of a negative endowment shock and  $c_2 = 0$  if not.

## II. Market-Making with a Single Security

We set the stage for subsequent analysis by initially ignoring the security design problem altogether, focusing on how prices would be set by market-makers if both types were to sell the entire underlying asset in the form of a pass-through security.

Many of the results derived in this section are relevant for cases where the owner bifurcates the asset into two securities. To handle bifurcation into two claims  $A$  and  $B$ , let  $(A_L, A_H)$  and  $(B_L, B_H)$  denote their respective period 2 payoffs as a function of the verified value in  $\{L, H\}$ . Security  $B$  is treated as the default in the case of only one security being issued.

Since she cannot short-sell, the optimal strategy for the speculator is to place a buy order if and only if she receives a positive signal. She attempts hiding her buy orders behind those of uninformed investors. The optimal size of her buy order is equal to the size of the aggregate buy order placed by uninformed investors when they are vulnerable to negative endowment shocks. This latter quantity is denoted  $X$ .

An uninformed investor will not place a buy order if invulnerable to negative  $y_2$  since the marginal utility of any increase in  $c_2$  is then zero. An individual uninformed investor may place a buy order if vulnerable since there is an insurance motive to avoiding negative consumption. However, uninformed investors are rational, weighing adverse selection costs against insurance motives when determining optimal demand. Each uninformed investor conditions demand on his idiosyncratic preference parameter  $\theta$ .

Table 1 lists the possible aggregate demand configurations confronting market-makers. After observing aggregate demand, market-makers form beliefs regarding the signal received by the speculator based upon the observed aggregate demand  $D$ , with:

$$\begin{aligned}\Pr[s = s_H | D = 2X] &= 1 \\ \Pr[s = s_H | D = X] &= 1 - q - \sigma + 2q\sigma \\ \Pr[s = s_H | D = 0] &= 0.\end{aligned}\tag{3}$$

Beliefs over  $s$  can be mapped to beliefs over the asset type, with

$$\Pr[\tau = H | D] = \Pr[\tau = H | s = s_H] \Pr[s = s_H | D] + \Pr[\tau = H | s = s_L] \Pr[s = s_L | D]\tag{4}$$

where

$$\begin{aligned}\Pr[\tau = H|s = s_H] &= \frac{q\sigma}{1 - q - \sigma + 2q\sigma} \\ \Pr[\tau = H|s = s_L] &= \frac{q(1 - \sigma)}{q + \sigma - 2q\sigma}.\end{aligned}\tag{5}$$

Substituting (5) into (4) one obtains:

$$\begin{aligned}\Pr[\tau = H|D = 2X] &= \frac{q\sigma}{1 - q - \sigma + 2q\sigma} \\ \Pr[\tau = H|D = X] &= q \\ \Pr[\tau = H|D = 0] &= \frac{q(1 - \sigma)}{q + \sigma - 2q\sigma}.\end{aligned}\tag{6}$$

Market-maker beliefs and equilibrium prices ( $P$ ) increase monotonically in aggregate demand with:

$$\begin{aligned}P(D) &= L + (H - L)\Pr[\tau = H|D] \quad \forall D \in \{0, X, 2X\} \\ \Rightarrow P(2X) &> P(X) > P(0).\end{aligned}\tag{7}$$

To support the PBE conjectured in Table 1 it is sufficient to verify the speculator has no incentive to deviate regardless of the signal she receives. To that end, off the equilibrium path market-makers form adverse beliefs from the perspective of the speculator, setting prices based upon:

$$\Pr[s = s_H|D] = 1 \quad \forall D \notin \{0, X, 2X\}.$$

It is readily verified that the speculator has no incentive to change her signal-contingent trading strategy when confronted with such beliefs. While such beliefs off the equilibrium path are sufficient to support the conjectured PBE of the market-making game, it is worthwhile to briefly discuss their plausibility. Note that any  $D \notin \{0, X, 2X\}$  must be due to the speculator placing a strictly positive order. The chosen specification of beliefs off the equilibrium path is predicated on the intuitive notion that market-makers should view any such (positive) order as being placed by  $S$  after having observed  $s_H$ . After all, if a negative signal is received, the speculator stands to incur a loss from buying securities unless the market-makers form the most favorable beliefs from her perspective,



which would entail  $\Pr[s = s_H|D] = 0$ . Conversely, if a positive signal is received, the speculator stands to make a strictly positive trading gain provided  $\Pr[s = s_H|D] < 1$ .

### A. Expected Revenue

The expected revenue of the owner, conditional upon  $\tau = H$ , is given by:

$$E[R|\tau = H] \equiv \bar{R}_H = L + (H - L) \left[ \frac{\sigma \Pr[\tau = H|D = 2X]}{2} + \frac{(1 - \sigma) \Pr[\tau = H|D = 0]}{2} + \frac{\Pr[\tau = H|D = X]}{2} \right]. \quad (8)$$

Equation (8) can be rewritten as:

$$\begin{aligned} \bar{R}_H(\sigma) &= HZ(\sigma) + L[1 - Z(\sigma)] \\ Z(\sigma) &\equiv \frac{1}{2} \left[ \frac{q\sigma^2}{1 - q - \sigma + 2q\sigma} + \frac{q(1 - \sigma)^2}{q + \sigma - 2q\sigma} + q \right]. \end{aligned} \quad (9)$$

Anticipating, the variable  $Z$  plays a critical role in the model. It measures the high type's expectation of the market-makers' updated belief. Some intuition regarding the interpretation of the variable  $Z$  is provided by the following equation for expected underpricing for the high type:

$$H - E[R|\tau = H] = (H - L)[1 - Z(\sigma)]. \quad (10)$$

The preceding equation shows that  $1 - Z$  is a proxy for expected underpricing from the perspective of the high type. To take a limiting example, if it were possible to achieve  $Z = 1$ , then each type would receive the correct type-specific asset valuation.

From Bayes' rule one may relate the expected revenue of the low type, denoted  $\bar{R}_L(\sigma)$ , to that of the high type as follows

$$E(\tau) = qH + (1 - q)L = q\bar{R}_H(\sigma) + (1 - q)\bar{R}_L(\sigma), \quad (11)$$

which implies

$$\begin{aligned} \bar{R}_L(\sigma) &= Hz(\sigma) + L[1 - z(\sigma)] \\ z(\sigma) &\equiv \left( \frac{q}{1 - q} \right) [1 - Z(\sigma)]. \end{aligned} \quad (12)$$

Lemma 1 shows that the owner of the high value asset benefits from higher speculator signal precision, since increased signal precision drives prices closer to fundamental value. All but the most important proofs are presented in the appendix.

**Lemma 1** *The expected revenue of the owner of a high value asset is increasing in the precision of the signal received by the speculator.*

From Lemma 1 it follows that  $Z$  is increasing in  $\sigma$ , with

$$\begin{aligned} Z\left(\frac{1}{2}\right) &= q \\ Z(1) &= \frac{1+q}{2}. \end{aligned} \tag{13}$$

It is worth noting that if the speculator fails to put in effort ( $\sigma = 1/2$ ), then  $Z = q$  and the expected revenue of each type reverts to the unconditional expected revenue  $E(R)$  as in a standard analysis of pooling equilibria.

## B. Incentive Compatible Information Acquisition

Consider next the incentives of the speculator. From Table 1 it follows that her expected gross trading gain is

$$\begin{aligned} G(\sigma, X) &= X \cdot \left[ \begin{aligned} &\left(\frac{q\sigma}{2}\right) [H - P(2X)] + \left(\frac{q\sigma}{2}\right) [H - P(X)] \\ &+ \left(\frac{(1-q)(1-\sigma)}{2}\right) [L - P(2X)] + \left(\frac{(1-q)(1-\sigma)}{2}\right) [L - P(X)] \end{aligned} \right] \\ &= \frac{q(1-q)(2\sigma-1)(B_H - B_L)X}{2} \end{aligned} \tag{14}$$

It is readily verified that the speculator's trading gain increases linearly in each of its arguments, and that the marginal benefit of signal precision is increasing in  $X$ , with

$$\begin{aligned} G_1(\sigma, X) &= q(1-q)(B_H - B_L)X > 0 \\ G_2(\sigma, X) &= \frac{q(1-q)(2\sigma-1)(B_H - B_L)}{2} > 0 \\ G_{11}(\sigma, X) &= G_{22}(\sigma, X) = 0 \\ G_{12}(\sigma, X) &= q(1-q)(B_H - B_L) > 0. \end{aligned} \tag{15}$$

The incentive compatible signal precision, denoted  $\sigma_{ic}$  satisfies:

$$e'(\sigma_{ic}) = q(1 - q)(B_H - B_L)X. \quad (16)$$

Define the inverse function of  $e'$  as follows

$$\psi \equiv [e']^{-1}.$$

We may rewrite the incentive compatible signal precision as

$$\sigma_{ic} = \psi[q(1 - q)(B_H - B_L)X]. \quad (17)$$

From the implicit function theorem and the convexity of the effort cost function  $e$  it follows that:

$$\begin{aligned} \frac{\partial \sigma_{ic}}{\partial X} &= \frac{q(1 - q)(B_H - B_L)}{e''(\sigma_{ic})} \geq 0 \\ \frac{\partial \sigma_{ic}}{\partial (B_H - B_L)} &= \frac{Xq(1 - q)}{e''(\sigma_{ic})} > 0 \\ \frac{\partial \sigma_{ic}}{\partial q} &= \frac{X(B_H - B_L)(1 - 2q)}{e''(\sigma_{ic})}. \end{aligned} \quad (18)$$

Since the incentive compatible signal precision plays a critical role, we summarize these findings in the following lemma.

**Lemma 2** *The incentive compatible signal precision of the speculator is increasing in the aggregate demand of the uninformed investors when vulnerable ( $X$ ); increasing in the wedge between the value of claim  $B$  under high and low types ( $B_H - B_L$ ); increasing in  $q$  for  $q < 1/2$ ; and decreasing in  $q$  for  $q > 1/2$ .*

### C. Aggregate Uninformed Demand: Pass-Through Security

The next step is to determine aggregate uninformed demand ( $X$ ) for security  $B$  in response to their being vulnerable to a negative endowment shock. Before conducting this analysis it is worth recalling that if there were symmetric information regarding the asset's value, uninformed investors with  $\theta > \eta^{-1}$  would fully insure against negative consumption in the second period. In particular,

whenever vulnerable to negative endowment shocks they would submit demand for  $\phi/\tau$  units of equity for an asset of type  $\tau \in \{L, H\}$ . Such a demand would result in  $c_2 = 0$  if  $y_2 = -\phi$ .

Letting  $x^*(\theta)$  denote the optimal  $\theta$ -contingent demand, aggregate uninformed demand is

$$X \equiv \int_0^{\theta^{\max}} x^*(\theta) f(\theta) d\theta. \quad (19)$$

Each uninformed investor has measure zero and acts as a price-taker. If vulnerable, an individual uninformed investor expects the security to be overpriced since he knows a subset of uninformed investors will submit positive demands, pushing prices higher as market-makers revise upward their assessment of the probability of the asset being of high value. Despite facing adverse selection, an individual uninformed investor is willing to submit a buy order if  $\theta$  is sufficiently high.

In order to characterize uninformed demand, it is useful to compute the expected price of the asset conditional upon uninformed investors being vulnerable to a negative endowment shock. With this in mind, for the remainder of paper let  $\chi$  be an indicator function for uninformed investors being vulnerable to a negative endowment shock.

$$\begin{aligned} E[P|\chi = 1] &\equiv \bar{P}^- & (20) \\ \bar{P}^- &= [q\sigma + (1-q)(1-\sigma)]P(2X) + [q(1-\sigma) + \sigma(1-q)]P(X) \\ &= qH + (1-q)L + q(1-q)(2\sigma - 1)(H - L). \end{aligned}$$

Equation (20) is consistent with the intuition that uninformed investors face adverse selection when submitting buy orders, since the asset is overpriced relative to its unconditional expected value. Additionally, the equation reveals there is no adverse selection if the speculator does not put in any effort ( $\sigma = 1/2$ ). Finally, it can be seen that adverse selection is increasing in the precision of the speculator's signal. This latter finding is at the heart of the trade-off in the model. Structures that encourage information production by the speculator simultaneously worsen adverse selection as perceived by uninformed investors, biasing them away from first-best insurance against negative endowment shocks.

Recalling the functional form for uninformed investor utility in equation (1), consider the change in expected utility experienced by uninformed investors for various demand perturbations:

$$\begin{aligned}
x \in \left(0, \frac{\phi}{H}\right) &\implies \frac{\partial E(U|\chi = 1)}{\partial x} = \eta\theta[qH + (1 - q)L] - \bar{P}^- & (21) \\
x \in \left(\frac{\phi}{H}, \frac{\phi}{L}\right) &\implies \frac{\partial E(U|\chi = 1)}{\partial x} = \eta\theta(1 - q)L - \bar{P}^- \\
x > \frac{\phi}{L} &\implies \frac{\partial E(U|\chi = 1)}{\partial x} = -\bar{P}^-.
\end{aligned}$$

From the last line in (21) it is apparent that no uninformed investor ever buys more than  $\phi/L$  units of equity since this results in  $c_2 > 0$  with probability one and zero marginal consumption utility in the second period. In order to further characterize optimal uninformed demand, we define two cutoff values for the preference parameter:

$$\begin{aligned}
\theta_1 &\equiv \frac{\bar{P}^-}{\eta[qH + (1 - q)L]} & (22) \\
\theta_2 &\equiv \frac{\bar{P}^-}{\eta(1 - q)L}.
\end{aligned}$$

From the demand perturbation equations (21) it follows that:

$$\begin{aligned}
\theta \in [0, \theta_1] &\implies x^*(\theta) = 0 & (23) \\
\theta \in (\theta_1, \theta_2) &\implies x^*(\theta) = \frac{\phi}{H} \\
\theta \in [\theta_2, \theta^{\max}] &\implies x^*(\theta) = \frac{\phi}{L}.
\end{aligned}$$

Adverse selection distorts trading decisions and the level of insurance against negative endowment shocks. Uninformed investors with relatively low risk-aversion (low  $\theta$ ) do not buy any insurance against negative consumption. Such investors have  $c_2 = -\phi$  if there is a negative endowment shock. Information acquisition by the speculator results in an increase in the number of uninformed investors foregoing insurance. To see this, note that  $\sigma > 1/2$  implies  $\theta_1 > \eta^{-1}$ , with the former representing the cutoff for the purchase of equity when there is asymmetric information and latter representing the cutoff for the purchase of equity under symmetric information.

Uninformed investors with intermediate  $\theta$  parameters underinsure relative to the symmetric information case. Specifically, in the event of a negative endowment shock, at their optimal demand

$c_2 = 0$  if  $\tau = H$  but  $c_2 = -\phi(H - L)/H$  if  $\tau = L$ . Uninformed investors with high  $\theta$  parameters actually overinsure in that at their optimal demand  $c_2 = 0$  if  $\tau = L$  but  $c_2 = \phi(H - L)/L$  if  $\tau = H$ .

Lemma 3 summarizes the properties of aggregate uninformed demand.

**Lemma 3** *If types pool by selling the underlying asset in the form of a pass-through security, aggregate uninformed demand when vulnerable to a negative endowment shock is*

$$\begin{aligned} X &= \phi \cdot \left[ \frac{F(\theta_2) - F(\theta_1)}{H} + \frac{1 - F(\theta_2)}{L} \right] \\ \theta_1 &= \eta^{-1} \left[ 1 + \frac{q(1 - q)(H - L)(2\sigma - 1)}{qH + (1 - q)L} \right] \\ \theta_2 &= \eta^{-1} \left[ 1 + \frac{q(1 - q)(H - L)(2\sigma - 1) + qH}{(1 - q)L} \right]. \end{aligned} \tag{24}$$

Finally, we may pull together the incentive compatible signal precision from Lemma 2 and the uninformed demand from Lemma 3 to verify existence of equilibrium in the market-making game. We have the following proposition.

**Proposition 1** *In the market-making game, for any  $B_H > B_L$ , there exists a unique equilibrium pair  $(\sigma^{eq}, X^{eq})$  satisfying*

$$\begin{aligned} \sigma_{ic}(X^{eq}) &= \sigma^{eq} \in \left( \frac{1}{2}, 1 \right) \\ X(\sigma^{eq}) &= X^{eq} \in \left( 0, \frac{\phi}{B_L} \right). \end{aligned}$$

### III. Optimal Structuring under Full Securitization

Section IV analyzes the full security design problem, including the question of optimal claim retentions and the potential for separating equilibria. In order to analyze the full problem, this

section considers the following instructive thought experiment. Suppose, for this section only, that the owner *must* sell the entire asset and is not allowed to retain any interest in it. What type of security designs would emerge in equilibrium? We note that this is precisely the question analyzed by Boot and Thakor (1993). However, they consider a setting with a wealth-constrained speculator and pure noise-trading. This precludes welfare analysis in their model. Further, we derive different optimal security designs.

If the owner must sell the entire asset, there is no possibility of a separating equilibrium. This is because the owner of a low type asset would find it optimal to mimic whatever structuring would be chosen by the owner of a high type asset, since any other structure would fully reveal the negative private information, leaving him to collect the minimum expected revenue  $L$ . It follows that with full securitization, one may confine attention to the securitization structure that gives the high type the highest pooling payoff.

An important simplifying result is that attention can be confined to two publicly traded securities without loss of generality. This result is stated as Lemma 4.

**Lemma 4** *Any outcome attainable with three or more publicly traded securities is attainable with two publicly traded securities.*

Following Nachman and Noe (1994) and DeMarzo and Duffie (1999), attention is confined to securities with nonnegative payoffs that are weakly increasing in the asset value in period 2. Monotonicity is assumed for two reasons. First, monotone securities are commonplace. Second, if one of the securities, say security  $A$ , was decreasing, the owner of security  $B$  could benefit at the expense of the owner of  $A$  by making a clandestine contribution of additional funds to the asset pool. As argued by DeMarzo and Duffie (1999), only securities with monotone payoffs will be observed if such hidden contributions are feasible, and demanding monotonicity is then without loss of generality.

The major difference between the analysis of one and two securities is that uninformed investors insure themselves in the most efficient way possible. In particular, they buy the security that exposes

them to the lowest degree of adverse selection per unit payoff. Therefore, we begin the analysis of multiple securities with a focus on uninformed demand.

### A. Uninformed Demand: Multiple Securities

Uninformed investors have positive demands only if vulnerable to a negative endowment shock. By the same reasoning applied in Section II, uninformed investors are concerned about overpricing of securities if vulnerable to a negative endowment shock since they correctly anticipate high aggregate demand.

Before proceeding, we first argue that if the original owner must sell off all cash flow rights, it is never optimal to issue a safe claim. To demonstrate this, suppose to the contrary Owner carves out a safe claim with  $B_H = B_L \leq L$ . Then all uninformed investors with  $\theta > \eta^{-1}$  would use security  $B$ , and only  $B$ , to insure if vulnerable to a negative endowment shock, setting demand at  $x_B = \phi/B_L$ . Such a financial structure would achieve first-best risk sharing. However, in the present setting, the goal of the owner (high type) is to encourage information production since this increases his expected revenue (Lemma 1). With safe debt available there will be no information production since there is no market in which the speculator can make trading gains. We state this result as Lemma 5.

**Lemma 5** *If the owner were to issue a safe claim, uninformed demand would be confined to that claim, resulting in zero speculator effort ( $\sigma^{eq} = 1/2$ ). Market-makers would then revert to their prior probability  $q$  of the asset being type  $H$ . If the owner must securitize the entire asset, he will never issue a safe claim.*

Lemma 5 highlights an inherent conflict between private and public incentives in choosing security design. In particular, the sale of a riskless claim would here achieve first-best risk sharing. However, if the owner must sell the entire asset, he necessarily finds it optimal to sacrifice risk sharing in order to encourage information acquisition. This prediction stands in stark contrast with those derived by Gorton and Pennacchi (1990) and Dang, Gorton and Holmström (2010), who predict that issuers will attempt to minimize informational-sensitivity.



In light of Lemma 5, the remainder of this section focuses on structures in which both claims are risky. To compute the optimal demand for each uninformed investor it is useful compute price expectations conditional upon vulnerability to a negative endowment shock. To this end, let:

$$\begin{aligned}\bar{P}_A^- &\equiv E[P_A|\chi = 1] \\ \bar{P}_B^- &\equiv E[P_B|\chi = 1].\end{aligned}$$

Under this section's working conjecture, to be verified, that all uninformed demand is concentrated in a single security, say security  $B$ , there is zero aggregate demand for  $A$  since the speculator will also refrain from trading in that market given the lack of any cover provided by uninformed investors. Therefore, Table 1 continues to be the relevant table depicting aggregate demand (for security  $B$ ). Since there is no market segmentation, the aggregate demand for security  $B$  is also used by the market-makers in setting prices for security  $A$ . That is, the market-makers will set prices as a function of aggregate demand for  $B$  as follows:

$$\begin{aligned}P_A(D) &= A_L + (A_H - A_L) \Pr[\tau = H|D] \quad \forall D \in \{0, X, 2X\} \\ P_B(D) &= B_L + (B_H - B_L) \Pr[\tau = H|D] \quad \forall D \in \{0, X, 2X\}.\end{aligned}\tag{25}$$

Using Table 1 we arrive at the following expressions for the expected prices computed by uninformed investors when vulnerable to negative endowment shocks:

$$\begin{aligned}\bar{P}_A^- &= [q\sigma + (1-q)(1-\sigma)]P_A(2X) + [q(1-\sigma) + \sigma(1-q)]P_A(X) \\ &= qA_H + (1-q)A_L + q(1-q)(2\sigma - 1)(A_H - A_L)\end{aligned}\tag{26}$$

and

$$\begin{aligned}\bar{P}_B^- &= [q\sigma + (1-q)(1-\sigma)]P_B(2X) + [q(1-\sigma) + \sigma(1-q)]P_B(X) \\ &= qB_H + (1-q)B_L + q(1-q)(2\sigma - 1)(B_H - B_L).\end{aligned}\tag{27}$$

Equations (26) and (27) are consistent with the intuition that uninformed investors perceive overpricing for any security unless it is riskless. Further, the degree of perceived adverse selection is increasing in the precision of the speculator's signal.

Assume without loss of generality that security  $A$  is more informationally sensitive in the sense of taking a larger percentage claim in the event that  $\tau = H$ :

$$\begin{aligned} \frac{A_H}{H} &\geq \frac{A_L}{L} & (28) \\ \Rightarrow \frac{\bar{P}_A^-}{qA_H + (1-q)A_L} &\geq \frac{\bar{P}_B^-}{qB_H + (1-q)B_L} \\ \Rightarrow \frac{\bar{P}_A^-}{(1-q)A_L} &\geq \frac{\bar{P}_B^-}{(1-q)B_L}. \end{aligned}$$

The two inequalities presented in (28) imply that security  $A$  is viewed by uninformed investors as having higher adverse selection costs per unit of  $c_2$  provided.

The intuition behind optimal uninformed demand is simple. For uninformed investors with  $\theta$  sufficiently low, demand is zero for both securities, with adverse selection dominating insurance motives. For intermediate values of  $\theta$ , the agent partially insures, buying enough units of period 2 cash flow such that his consumption is zero if the actual asset type is  $H$ , which implies that consumption is negative if the actual asset type is  $L$ . Finally, if  $\theta$  is sufficiently high, the uninformed investor completely insures in the sense of purchasing enough units of the security such that  $c_2$  reaches zero even if the actual asset type is  $L$ , which implies  $c_2 > 0$  if the asset type is  $H$ . Of course, for any given level of insurance, uninformed investors seek the least costly security combination.

Let  $\Phi$  be an indicator for  $y_2 = -\phi$ . Second period consumption can then be expressed as a function of the actual asset type with

$$c_2(x_A, x_B, \Phi, \tau) = x_A A_\tau + x_B B_\tau - \Phi \phi \quad \forall \quad \tau \in \{L, H\}.$$

The optimal portfolio is determined using perturbation arguments. Attention is confined to portfolios satisfying  $c_2(x_A, x_B, 1, L) \leq 0$ . Otherwise,  $c_2 > 0$  regardless of the actual asset type, despite the fact that the marginal utility of second period consumption is then equal to zero.

Consider an arbitrary portfolio such that  $c_2(x_A, x_B, 1, H) < 0$  and evaluate a local perturbation.

We have:

$$\begin{aligned}\frac{\partial E[U|\chi = 1]}{\partial x_A} &= \eta\theta [qA_H + (1 - q)A_L] - \bar{P}_A \\ \frac{\partial E[U|\chi = 1]}{\partial x_B} &= \eta\theta [qB_H + (1 - q)B_L] - \bar{P}_B.\end{aligned}\tag{29}$$

If  $\theta$  is sufficiently low, both perturbation gains listed in (29) are negative and optimal uninformed demand is zero. Specifically:

$$\theta \leq \frac{\bar{P}_B}{\eta[qB_H + (1 - q)B_L]} \equiv \theta_1^B \Leftrightarrow (x_A^*, x_B^*) = (0, 0).\tag{30}$$

Next, consider an arbitrary portfolio  $(x_A, x_B)$  such that  $c_2(x_A, x_B, 1, H) = \varepsilon$  where  $\varepsilon$  is arbitrarily small. That is, we are considering points just above the kink that arises at portfolios such that  $c_2(x_A, x_B, 1, H) = 0$ . Performing a perturbation one finds:

$$\begin{aligned}\frac{\partial E[U|\chi = 1]}{\partial x_A} &= \eta\theta(1 - q)A_L - \bar{P}_A \\ \frac{\partial E[U|\chi = 1]}{\partial x_B} &= \eta\theta(1 - q)B_L - \bar{P}_B.\end{aligned}\tag{31}$$

If  $\theta$  is sufficiently high, such a perturbation increases the maximand. Further, since the maximand is piece-wise linear, it would then be optimal to fully insure against negative consumption, achieving  $c_2(x_A^*, x_B^*, 1, L) = 0$ . Finally, from the inequality in (28), the minimal cost means of achieving this full insurance is to purchase only security  $B$ . Formally, we have:

$$\theta \geq \frac{\bar{P}_B}{\eta(1 - q)B_L} \equiv \theta_2^B \Leftrightarrow (x_A^*, x_B^*) = \left(0, \frac{\phi}{B_L}\right).\tag{32}$$

The final case to consider is  $\theta \in (\theta_1^B, \theta_2^B)$ . From the perturbation arguments given above, we know such uninformed investors partially insure, with  $c_2(x_A^*, x_B^*, 1, H) = 0$ . Now, consider the marginal utility ( $MU$ ) per unit of period 1 numeraire allocated to the purchase of each security (on the relevant region where the uninformed investor is partially insuring). From the inequality in (28) we know:

$$MU_B = \frac{\eta[qB_H + (1 - q)B_L]}{\bar{P}_B} \geq \frac{\eta[qA_H + (1 - q)A_L]}{\bar{P}_A} = MU_A.\tag{33}$$

It follows that security  $B$  yields the highest marginal utility on the region of partial insurance, so that

$$\theta \in (\theta_1^B, \theta_2^B) \Rightarrow (x_A^*, x_B^*) = \left(0, \frac{\phi}{B_H}\right). \quad (34)$$

This establishes Proposition 2.

**Proposition 2** (*Full Securitization*) *There is zero uninformed demand for the riskier claim  $A$ . When vulnerable to a negative endowment shock, aggregate uninformed demand for security  $B$  is*

$$\begin{aligned} X(\phi, B_L, B_H) &= \phi \cdot \left[ \frac{F(\theta_2^B) - F(\theta_1^B)}{B_H} + \frac{1 - F(\theta_2^B)}{B_L} \right] \\ \theta_1^B &= \eta^{-1} \left[ 1 + \frac{q(1-q)(B_H - B_L)(2\sigma - 1)}{qB_H + (1-q)B_L} \right] \\ \theta_2^B &= \eta^{-1} \left[ 1 + \frac{q(1-q)(B_H - B_L)(2\sigma - 1) + qB_H}{(1-q)B_L} \right]. \end{aligned} \quad (35)$$

Proposition 2 can be contrasted with a result obtained by Boot and Thakor (1993). In their model, speculators make trading gains in the riskier levered equity claim. This results from their particular specification of noise trading. In our model, uninformed investors optimally insure themselves using only the least information-sensitive claim. Consequently, in our model the speculator is unable to make trading gains in the market for the riskier claim. A similar effect is operative in the model of Gorton and Pennacchi (1990), since they too predict that uninformed investors will buy only the safest claim, which is riskless debt in their model.

## B. Optimal Structuring

Lemma 1 shows that the expected revenue of a high type owner is increasing in  $\sigma$ . In this section's setting in which all cash flow rights must be sold, the objective of such an owner is to maximize the incentive compatible speculator signal precision  $\sigma_{ic}$  defined implicitly by equation (16). From

convexity of the effort cost function  $e$  it follows that the optimal security design solves:

$$PROGRAM \ 1 \tag{36}$$

$$(B_L^*, B_H^*) \in \arg \max_{B_L, B_H} (B_H - B_L)X(\phi, B_L, B_H) \tag{37}$$

*s.t.*

$$(LIS) \quad \frac{B_L}{L} \geq \frac{B_H}{H}$$

$$(Monotonicity) \quad B_H \geq B_L \tag{38}$$

$$(Limited Liability) \quad B_L \leq L.$$

Intuitively, the optimal security design under full securitization maximizes the product of the speculator's per-unit profit and endogenous uninformed trading volume. This creates a natural trade-off given that uninformed demand decreases with informational sensitivity. The constraint labeled *LIS* ensures that security  $B$  is, in fact, the low information-sensitivity security in which uninformed demand is concentrated. The three listed constraints ensure all other limited liability and monotonicity constraints are respected since they imply:

$$A_\tau(0, \tau) \in [0, \tau] \quad \forall \tau \in \{L, H\}$$

$$B_L > 0$$

$$B_H \in (0, H]$$

$$A_H \geq \frac{HA_L}{L}.$$

Conveniently, Program 1 is independent of the choice of  $B_L$  provided  $B_L \in (0, L]$ . This is because uninformed demand is homogeneous degree negative one in  $(B_L, B_H)$ . For example, if Owner were to cut both state contingent payoffs in half, each uninformed investor would simply double his demands. Thus, examining the objective function in Program 1, the optimal policy is unique up to a scalar, since

$$(B_H - B_L)X(\phi, B_L, B_H) = (\zeta B_H - \zeta B_L)X(\phi, \zeta B_L, \zeta B_H) \quad \forall \zeta \in (0, 1]. \tag{39}$$

Given this finding, let

$$B_H \equiv \kappa B_L.$$

In this case, the aggregate demand defined in Proposition 2 simplifies as follows (with slight abuse of notation):

$$\begin{aligned} X &= X(\phi, B_L, \kappa) = \frac{\phi}{B_L} \cdot \left[ 1 - \frac{F(\theta_1(\kappa))}{\kappa} - \frac{(\kappa - 1)F(\theta_2(\kappa))}{\kappa} \right] \\ \theta_1(\kappa) &\equiv \eta^{-1} \left[ 1 + \frac{q(1-q)(2\sigma - 1)(\kappa - 1)}{1 + q(\kappa - 1)} \right] \\ \theta_2(\kappa) &\equiv \eta^{-1} \left[ 1 + \frac{q(1-q)(2\sigma - 1)(\kappa - 1) + q\kappa}{(1 - q)} \right]. \end{aligned} \quad (40)$$

Increases in  $\kappa$  reduce uninformed demand since both cutoffs are increasing in  $\kappa$  with:

$$\begin{aligned} \theta'_1(\kappa) &= \frac{q(1-q)(2\sigma - 1)}{\eta[1 + q(\kappa - 1)]^2} > 0 \\ \theta'_2(\kappa) &= \eta^{-1} \left[ q(2\sigma - 1) + \frac{q}{(1 - q)} \right] > 0. \end{aligned} \quad (41)$$

Making the substitution  $B_H = \kappa B_L$  throughout Program 1 allows us to simplify the optimal structuring problem as follows.

**Lemma 6** *Suppose total securitized cash flow is worth  $l$  if the asset is low quality and  $h \geq l$  if the asset is high quality. Total expected revenue received for the securitized claims, conditional upon the asset being of high quality, is maximized with any  $B_L^* \in (0, l]$  and  $B_H^* = \kappa^* B_L^*$  where  $\kappa^*$  solves*

$$\begin{aligned} & \text{PROGRAM 2} \\ \kappa^* &\in \arg \max_{\kappa} M(\kappa) \equiv \phi(\kappa - 1) \left[ 1 - \frac{F(\theta_1(\kappa))}{\kappa} - \frac{(\kappa - 1)F(\theta_2(\kappa))}{\kappa} \right] \\ & \text{s.t.} \\ \kappa &\leq \frac{h}{l}. \end{aligned}$$

It is worth noting that Program 2 is relevant for arbitrary levels of securitization, including cases where the owner retains some interest in the asset. Full securitization of the asset pertains to the special case where one sets  $(l, h) = (L, H)$  in Program 2. The generality of Lemma 6 will prove useful in Section IV which considers partial securitization.

Lemma 7 establishes a sufficient condition under which the objective function in Program 2 is strictly concave.

**Lemma 7** *If the cumulative distribution function ( $F$ ) for uninformed investors' preference parameter  $\theta$  is weakly convex, then the maximand in Program 2 ( $M$ ) is strictly concave.*

The intuition behind Lemma 7 is as follows. If  $F$  is convex then marginal increases in  $\kappa$  result in ever larger reductions in aggregate uninformed demand. Further, the benefit to the speculator of the increase in per-unit profits stemming from an increase in  $\kappa$  is spread over a progressively smaller trading base. Consequently, the maximand is strictly concave. The remainder of the paper assumes

$$A2 : F \text{ is weakly convex.}$$

The Lagrangian for Program 2 can be written as:

$$\mathcal{L}(\kappa) \equiv M(\kappa) + \lambda \left( \frac{h}{l} - \kappa \right). \quad (42)$$

In Program 2, the optimal policy is characterized by a unique pair  $(\kappa^*, \lambda^*)$  satisfying the following first-order condition

$$M'(\kappa^*) = \lambda^* \quad (43)$$

and the complementary slackness conditions:

$$\begin{aligned} \left( \frac{h}{l} - \kappa^* \right) \lambda^* &= 0 \\ \lambda^* &\geq 0. \end{aligned} \quad (44)$$

For the remainder of the analysis, we shall assume that  $H/L$  is sufficiently high such that the *LIS* constraint does not bind if the asset is fully securitized. To this end, define  $\kappa^{**}$  to be the unconstrained maximizer of the objective function  $M$ :

$$\kappa^{**} \equiv (M')^{-1}(0).$$

And we then adopt the technical assumption:

$$A3 : \frac{H}{L} > \kappa^{**} \Rightarrow \kappa^*(L, H) = \kappa^{**}, \quad \lambda^*(L, H) = 0.$$

One can understand the role of Assumption 3 as follows. Think of Owner as progressively raising  $\kappa$ , bringing the low-information-sensitivity claim  $B$  closer and closer to a linear claim. Doing so raises the per-unit profit of the speculator, but also diminishes demand for  $B$ , with Assumption 2 implying the demand cost rises with  $\kappa$ . Assumption 3 is predicated on the notion that the demand cost dominates before  $B$  becomes linear. That is, Assumption 3 ensures that when Owner fully securitizes the underlying real asset, it is never optimal to package it as a pass-through security (equity).

Differentiating the maximand yields:

$$M'(\kappa) = \phi \left[ 1 - \frac{F(\theta_1)}{\kappa} - \frac{(\kappa - 1)F(\theta_2)}{\kappa} \right] - \phi \left[ \frac{\kappa - 1}{\kappa} \right] \left[ \frac{F(\theta_2) - F(\theta_1)}{\kappa} + f(\theta_1)\theta'_1(\kappa) + (\kappa - 1)f(\theta_2)\theta'_2(\kappa) \right]. \quad (45)$$

The first term in (45) captures the gain from increasing informational sensitivity (via  $\kappa$ ), as it increases the speculator's per-unit trading gain. The negative term captures the cost of increasing informational sensitivity in terms of reducing equilibrium uninformed demand, behind which the speculator hopes to hide her trading. Canceling terms one obtains:

$$M'(\kappa) = \phi \left[ 1 - F(\theta_2) \left( 1 - \frac{1}{\kappa^2} \right) - \frac{F(\theta_1)}{\kappa^2} - \left( \frac{\kappa - 1}{\kappa} \right) [f(\theta_1)\theta'_1(\kappa) + (\kappa - 1)f(\theta_2)\theta'_2(\kappa)] \right]. \quad (46)$$

We have then established the following proposition which characterizes equilibrium security design in a pooling equilibrium with full securitization.

**Proposition 3** *If the entire asset is securitized, the equilibrium structuring consists of a claim with low-information-sensitivity with  $B_L^* \in (0, L]$  and  $B_H^* = \kappa^{**}B_L^*$ , where  $\kappa^{**} < H/L$  is the unique solution to*

$$1 - \frac{F(\theta_1)}{\kappa^{**}} - \frac{(\kappa^{**} - 1)F(\theta_2)}{\kappa^{**}} = \left[ \frac{\kappa^{**} - 1}{\kappa^{**}} \right] \left[ \frac{F(\theta_2) - F(\theta_1)}{\kappa^{**}} + f(\theta_1)\theta'_1(\kappa^{**}) + (\kappa^{**} - 1)f(\theta_2)\theta'_2(\kappa^{**}) \right]. \quad (47)$$

*The second residual claim attracts zero aggregate uninformed demand. All informed trading gains are derived in the market for the low information-sensitivity claim.*



The following corollary shows that under full securitization optimal structuring can be achieved by combining standard securities.

**Corollary** (*Full Securitization*) *One optimal securitization structure consists of a liquid risky senior debt tranche with face value  $\kappa^{**}L$  and a residual illiquid junior (equity) claim. Another optimal securitization structure consists of liquid equity and an illiquid call option on the underlying asset with strike price  $\kappa^{**}L$ .*

Since we have an analytical solution for uninformed demand, numerical illustrations are simple. Figure 1 depicts aggregate uninformed demand, conditional upon vulnerability, assuming that the distribution of  $\theta$  parameters ( $F$ ) is the uniform distribution with support  $[0, 8]$ ,  $q = 1/2$ , and  $\eta = 3/4$ . As shown in the figure uninformed demand declines monotonically in informational sensitivity, as measured by  $\kappa$ , since increases in  $\kappa$  induce marginal investors to either forego purchase of the security ( $\theta_1$  increasing) or to purchase less units ( $\theta_2$  increasing). Consistent with the fact that aggregate demand increases linearly in  $\phi$ , Figure 1 also shows that increases in the magnitude of endowment shocks induce outward shifts in the uninformed demand curves. For these same parameter values, Figure 2 plots the objective function  $M$  for Program 2, which pins down the optimal informational sensitivity  $\kappa^{**}$ . As shown in the figure, the optimal value of  $\kappa$  is actually independent of  $\phi$ , reflecting the fact that the maximand is linear in  $\phi$ . However, it is also apparent that higher values of  $\phi$  result in correspondingly higher values of  $M$ , which implies higher incentive compatible signal precision.

At this point it is worth recalling the working assumption that the only party capable of issuing securities is Owner. That is, other agents cannot issue securities or short-sell. Now recall that the market-makers clear markets for all securities, buying one minus the combined aggregate demand of the uninformed investors and the speculator. But is it possible for aggregate demand to exceed supply? To address this question, notice that the maximum aggregate demand coming from the uninformed investors and speculator is  $2X$ . We can write uninformed demand as:

$$X(\phi, B_L, \kappa) = \left( \frac{\phi}{B_L} \right) \left[ \frac{F(\theta_2) - F(\theta_1)}{\kappa} + 1 - F(\theta_2) \right].$$

Therefore, to avoid the possibility of market-makers being called upon to short-sell,  $\phi$  must be

sufficiently small in relation to  $B_L$ . The no-shorting constraint is clearly easiest to satisfy at  $B_L = L$ , where the choice of  $B_L$  was otherwise arbitrary when we ignored the no-shorting constraint. From Assumption 1 it follows that the market-makers are never called upon to short-sell (even at suboptimal  $\kappa$ ) since

$$\phi \leq L/2 \Rightarrow 2X(\phi, B_L^* = L, \kappa) < 1. \quad (48)$$

#### IV. Optimal Retentions and Security Design

Section III assumed the original asset owner must sell the entire asset. Such a setting is relevant when there is a forced asset sale due to antitrust enforcement, bankruptcy liquidation, or unbounded liquidity needs. This section considers a more general setting in which the owner chooses both the degree and design for securitization of the original asset.

The assumptions for the remainder of the paper are as follows. Owner is risk-neutral and values consumption equally in both periods, having utility of the form  $c_1 + c_2$ . Further, Owner has access to a linear production technology allowing him to convert each unit of numeraire received from investors in period 1 into  $\beta > 1$  units of numeraire in that same period. In contrast to the original real asset, the value of this short-term production technology is not verifiable by courts, so this stream of cash flow cannot be securitized.

We consider this particular setup for three reasons. First, our interest is in analyzing optimal asset-backed securities, where a firm raises funds by selling claims against a specified asset. Second, this setup approximates a number of real-world settings. For example, one may think of a distressed bank as placing high, yet bounded, value on the immediate receipt of cash coming from securitization of an underlying asset. Finally, this setup allows us to retain our focus on the optimal securitization of a single real asset, here the original real asset with values in  $\{L, H\}$ . This allows us to address how the option to retain some cash flow rights affects the optimal structure.

If there were no intrinsic benefit to receiving funds immediately ( $\beta = 1$ ), the owner of the high quality asset would not sell any claims on the real asset given asymmetric information. He would then obtain his first-best payoff  $H$  by holding onto the entire asset. Conversely, if there were symmetric information and if  $\beta$  were greater than one, then an owner of either asset type would sell all cash flow rights in the form of a pass-through security.

### A. The Security Design Game

Maskin and Tirole (1992) show the equilibrium set of signaling games can be narrowed and Pareto-improved (from the perspective of the privately informed party) by expanding the set of feasible initial actions. Tirole (2005) describes an application of the formulation of Maskin and Tirole (1992) to security issuance by a privately informed party. We adapt the game of Tirole (2005) to our setting.

The sequencing of events is as follows. The entire *security design game* actually consists of two connected signaling games: an offer game and the market-making game. The latter game was already described in Section I. The *offer game* is a signaling game played between Owner and all outside investors. This game begins with Owner privately observing asset value. He then approaches the market-makers (e.g. investment banks) and publicly proposes a *menu* of two securitization structures, say  $\Sigma \in \{\Sigma^1, \Sigma^2\}$ , that he would like the option to choose from subsequently. This step resembles a shelf-registration in that Owner is locking in a pair of optional future financial configurations. Each structure stipulates all payoffs for claimants as a function of the verified asset value in period 2. The market-makers then agree to clear markets competitively for whatever structure  $\Sigma$  the owner subsequently chooses from his menu. All agents in the economy must have a belief regarding the asset type in response to any menu offer, including those off the equilibrium path. Beliefs at this stage are labeled *offer beliefs*. To support candidate PBE, menu offers off the equilibrium path are punished with outside investors inferring  $\tau = L$  with probability one.

The sole difference between this formulation and the game described by Tirole (2005) is that in our model market-makers cannot agree to providing cross-subsidies. Rather, they simply agree to

compete and clear markets for the securitization structure subsequently chosen by Owner. Using the terminology of Tirole (2005), competitive market-making implies that all investors in securities find them profitable type-by-type. This stands in contrast to a setting in which investors can pre-commit to subsequently buying some set of securities at a loss, a possibility allowed in the formulation of Tirole (2005).

In the next stage of the offer game, Owner selects a securitization structure  $\Sigma$  from the menu he initially proposed, with the choice being incentive compatible. After observing the selection of Owner, all other agents revise beliefs using Bayes' rule where possible. The beliefs formed at this stage are labeled *selection beliefs*. It is worth stressing that both types can offer the same menu, but they do not necessarily select the same securitization structure from that menu. Indeed, in a *separating equilibrium* of the offer game, the initial securitization proposal is such that the  $\Sigma$  subsequently selected from the menu reveals the true asset type  $\tau$ . In any separating equilibrium securities are correctly priced and all agents trade in full knowledge of the true type. There is no incentive for the speculator to put in effort in a separating equilibrium of the offer game.

In a *pooling equilibrium* of the offer game, both owner types propose the same trivial menu with  $\Sigma^1 = \Sigma^2$ . In such cases, no information is revealed about the asset type after the selection stage of the offer game. If and only if a pooling equilibrium occurs in the offer game, play then passes to the market-making game described in Section I. Recall, all relevant players enter the market-making game holding their prior belief that  $\Pr[\tau = H] = q$ , as is appropriate when the offer game reveals no information regarding  $\tau$ . Then a signaling game ensues between the speculator and market-makers, where market-makers use aggregate demand to form beliefs regarding the signal  $s$  received by the speculator and set prices accordingly.

## **B. The Least-Cost Separating Equilibrium**

The owner can credibly signal positive private information by retaining sufficient rights. To this end, assume Owner designs a third security  $C$  with value-contingent payoffs  $(C_L, C_H)$ . Owner holds security  $C$  and sells the other two securities  $A$  and  $B$  to public investors in a competitive market.

From Lemma 4 it follows that confining attention to no more than two publicly traded securities is without loss of generality.

We begin by evaluating the least-cost separating equilibrium (LCSE) from the perspective of the high type. Note that in the LCSE the speculator has no incentive to acquire information since the original owner's private information is fully revealed by his financing choice.

The LCSE minimizes the low type's incentive to mimic by giving him his first-best allocation in which he sells the entire asset in the form of a pass-through security. The LCSE makes the high type as well off as possible subject to the constraint that the low type does not want to mimic. In the LCSE, there is no need for the high type to sell more than one public security, call it security  $B$ . The LCSE is then the solution to:

$$\begin{aligned} & \max_{(B_L, B_H, C_L, C_H)} C_H + \beta B_H \\ & s.t. \end{aligned}$$

$$\text{No Mimic} : \beta L \geq C_L + \beta B_H$$

*Limited Liability*

*Monotonicity.*

To determine the LCSE, we first ignore the monotonicity constraint and then verify it is slack. Clearly, in this relaxed program the optimal policy is to loosen the no-mimic constraint to the maximum extent by setting  $C_L^* = 0$ , implying  $B_L^* = L$ . Further, the no-mimic constraint must bind at the optimum, implying  $B_H^* = L$  and  $C_H^* = H - L$ . Since the neglected monotonicity constraint is satisfied we have established the following proposition.

**Proposition 4** *In the least-cost separating equilibrium, a low type asset is sold in its entirety in the form of a pass-through security. The owner of a high type asset sells only a safe senior (debt) claim with face value  $L$ , retaining the residual (levered equity) claim. Uninformed investors then achieve first-best insurance against negative endowment shocks by purchasing correctly priced claims. The speculator does not exert effort.*

The intuition behind Proposition 4 is simple. In the LCSE, the low type would always mimic if the high type were to sell any risky claim since he would then benefit from security overvaluation. Therefore, the best the high type can do is to get the maximum liquidity possible subject to zero informational-sensitivity. Debt with face value  $L$  achieves this objective.

In the LCSE, the high type experiences a loss relative to symmetric information equal to  $(\beta - 1)(H - L)$ . This deadweight loss reflects that fact that first-best entails him selling off the entire asset instead of just the claim to  $L$ . As in the model of Myers and Majluf (1984), in the LCSE asymmetric information results in the high type cutting back the scale of his investment to below first-best.

The socially attractive feature of the LCSE is that it achieves first-best risk sharing, regardless of the actual asset type. To see this, note that all marketed claims (the equity of the low type and the debt of the high type) are correctly priced since the equilibrium is fully-revealing. Therefore, the uninformed investors will insure themselves just as they did under perfect information. Specifically, all uninformed investors with  $\theta > \eta^{-1}$  will purchase enough insurance such that they will achieve  $c_2 = 0$  in the event of a negative endowment shock.

Proposition 4 is interesting in that it shows that the results in Gorton and Pennachi (1990) are overly restrictive in that their model relies upon safe debt to achieve perfect risk sharing. However, perfect risk sharing is achieved in any *separating* equilibrium, even if the separation is predicated upon the issuance of risky claims.

### C. The Equilibrium Set

This subsection maps some of the results of Maskin and Tirole (1992) and Tirole (2005) to our setting, relying on somewhat different proofs due to differences in the economic settings considered.

The next lemma places a lower bound on what each type must receive in any equilibrium.

**Lemma 8** *In any equilibrium of the security design game, each owner type must receive a payoff weakly greater than his least-cost separating payoff.*

**Proof.** Suppose to the contrary that some type received less than his LCSE payoff. He could then

profitably deviate by issuing safe debt with face value  $L$  and retaining residual cash flow rights. ■

The following lemma characterizes the equilibrium set.

**Lemma 9** *The equilibrium set of the security design game always includes the least-cost separating equilibrium. It also includes pooling equilibria with a single contract on the offered menu provided that contract weakly Pareto dominates the least-cost separating equilibrium (from the perspective of both owner types).*

**Proof.** Consider first supporting the LCSE. If beliefs were set to  $\Pr[\tau = H] = 0$  in response to any deviating menu, then no such deviation is profitable. Suppose next there is a pooling contract weakly Pareto dominating the LCSE. If beliefs were set to  $\Pr[\tau = H] = 0$  in response to any deviating menu, the deviator would get weakly less than his LCSE payoff and the deviation is not profitable. ■

### C. The Pooling Equilibrium

Consider next the nature of pooling equilibrium—an equilibrium in which both types offer a trivial menu such that  $\Sigma^1 = \Sigma^2$ . Confining attention to pooling equilibria, the best such equilibrium from the perspective of the high type maximizes

$$C_H + E[P_A + P_B | \tau = H]. \quad (49)$$

Any pair  $(C_L, C_H)$  held by the original owner leaves a total residual stream  $(l, h)$  of payments that will be packaged and sold to outside investors:

$$(C_L, C_H) \Rightarrow (l, h) \equiv (L - C_L, H - C_H).$$

We characterize the optimal nature and scope of securitization using a two step procedure. First, Lemma 6 can be used to characterize the optimal structuring for the sale of residual cash flows after netting out the retained claim. Then  $(C_L, C_H)$  are optimized in light of their effect on the value attainable in this residual structuring problem.

Before proceeding with the formal solution, it is useful to sketch the intuition. For the owner of a high quality asset, the benefit of increasing  $C_H$  is that he marginally reduces his exposure to underpricing. However, this retention of cash flow rights from the long-term tangible real asset reduces the amount he can invest in the profitable short-term project. Further, an increase in  $C_H$  reduces  $h/l$ . If  $h/l \leq \kappa^{**}$ , where  $\kappa^{**}$  is defined in Proposition 3, the *LIS* constraint in Program 2 is binding and the incentive compatible signal precision falls below that attainable under full securitization.

Let  $M^*(l, h)$  denote the maximum value obtained in Program 2 given that the total value of publicly traded claims on the real asset is in  $\{l, h\}$ :

$$M^*(l, h) \equiv M[\kappa^*(l, h)]. \quad (50)$$

From (16) and the definition of  $M^*$  it follows that the maximized incentive compatible signal precision is:

$$\sigma^*(l, h) \equiv \psi[q(1 - q)M^*(l, h)]. \quad (51)$$

From the Envelope Theorem we know:

$$\begin{aligned} M_1^*(l, h) &= \frac{\partial \mathcal{L}}{\partial l} = \frac{-h\lambda^*(l, h)}{l^2} \Rightarrow \sigma_1^*(l, h) = \frac{-q(1 - q)h\lambda^*(l, h)}{l^2 e'(\sigma^*)} \leq 0 \\ M_2^*(l, h) &= \frac{\partial \mathcal{L}}{\partial h} = \frac{\lambda^*(l, h)}{l} \Rightarrow \sigma_2^*(l, h) = \frac{q(1 - q)\lambda^*(l, h)}{l e'(\sigma^*)} \geq 0. \end{aligned} \quad (52)$$

The inequalities in (52) convey an important trade-off. Specifically, when the *LIS* constraint is binding, increases in  $l$  reduce the value obtained in Program 2 and with it the incentive compatible signal precision  $\sigma^*$ . Conversely, increases in  $h$  loosen the *LIS* constraint, potentially leading to higher  $\sigma^*$ . Thus, consistent with the intuition provided above, a high value of  $C_H$  imposes a cost in terms of the power of incentives that can be provided to the speculator. Lower incentives then lead to more severe mispricing of the public claims.

With this in mind, we turn to characterizing the preferred pooling equilibrium for the high type. The following lemma simplifies the analysis greatly in showing that the low payoff is always fully securitized in the high type's preferred pooling equilibrium.



**Lemma 10** *The optimal pooling contract for the high type entails a zero payoff to the owner if asset value is low ( $C_L^* = 0$  and  $l^* = L$ ).*

The intuition for Lemma 10 is that the high type has no desire to retain rights to cash flow in the event of a low realization since he knows this is a probability zero event for him. For this same reason he also knows that the market will overvalue a claim to low-type payoffs. Further, he places high value on immediate liquidity.

Lemma 10 allows us to write the high type's program as a one dimensional optimization:

*PROGRAM 3*

$$\max_h \mu(L, h) \equiv H - h + \beta[L + (h - L)Z(\sigma^*(L, h))]$$

*s.t.*

$$\text{Incentive Compatability} : \sigma^*(L, h) = \psi[q(1 - q)M^*(L, h)]$$

$$\text{LL\&Mono} : h \in [L, H].$$

It is readily verified that if the owner of the high quality asset opts to pool at a structuring in which only safe debt is securitized he gets the same payoff as what he attains under the LCSE contractual structure, with

$$\mu(L, L) = H - L + \beta L. \tag{53}$$

Further, if  $\beta Z[\sigma^*(L, H)] \leq 1$  the optimal pooling contract for the high type is to pool at a structuring in which only safe debt is securitized. To see this, note that

$$\beta Z[\sigma^*(L, H)] \leq 1 \Rightarrow \mu(L, h) \leq \mu(L, L) \quad \forall \quad h \in (L, H]. \tag{54}$$

This leads directly to Proposition 5.

**Proposition 5** *If  $\beta Z[\sigma^*(L, H)] \leq 1$  the payoffs and outcomes under the least-cost separating contract are the unique payoffs and outcomes of the security design game.*

The intuition for Proposition 5 is straightforward. If the speculator cannot be incentivized to produce sufficiently precise signals, even when her incentives are maximized under full asset securitization, then the costs of underpriced securities exceed the value of immediate liquidity and the owner of a high quality asset eschews issuing any risky security. Rather, he gets the maximal liquidity possible selling only safe debt.

Recall, the objective in Program 3 is to find the pooling contract preferred by the high type. Apparently, if  $\beta Z[\sigma^*(L, H)] \leq 1$  it is impossible to find a pooling contract that makes him better off. And since we ignored the welfare of the low type in that program, it follows that there is no Pareto-improving contract across the owner types when  $\beta Z \leq 1$ . Also, the actual outcome for all agents under the pooling contract described in Proposition 5 is identical to that under the LCSE.

Consider next the optimal pooling contract for the high type when the speculator can be incentivized to produce more precise signals, in the sense that  $\beta Z[\sigma^*(L, H)] > 1$ . To analyze this case, note first that the maximand in Program 3 is linear once  $h$  is sufficiently high. In particular,

$$\forall h \in (L\kappa^{**}, H), \quad \mu_2(L, h) = \beta Z(\sigma^*(L, H)) - 1 \quad (55)$$

which is a strictly positive constant in the posited scenario. It follows that:

$$\beta Z[\sigma^*(L, H)] > 1 \Rightarrow \mu(L, H) > \mu(L, h) \quad \forall h \in [L\kappa^{**}, H]. \quad (56)$$

And further, using the fact that the maximand is linear for high values of  $h$  we know:

$$\beta Z[\sigma^*(L, H)] > 1 \quad (57)$$

$\Downarrow$

$$\begin{aligned} \mu(L, L\kappa^{**}) &= \mu(L, H) - [H - L\kappa^{**}][\beta Z(\sigma^*(L, H)) - 1] \\ &= H - L + \beta L + (L\kappa^{**} - L)[\beta Z(\sigma^*(L, H)) - 1] \geq \mu(L, h) \quad \forall h \in [L, L\kappa^{**}]. \end{aligned}$$

Proposition 6 follows immediately from the inequalities in (56) and (57).

**Proposition 6** *If  $\beta Z[\sigma^*(L, H)] > 1$ , the preferred pooling contract for the high type entails full securitization of the asset, with optimal bifurcation of the asset following Proposition 3. Under*

*this structuring, the speculator exerts effort and uninformed investors are imperfectly insured. Both owner types are strictly better off under this pooling contract than under the least-cost separating equilibrium, so the latter is not a unique equilibrium of the security design game.*

Propositions 5 and 6 taken together offer a complete characterization of the equilibrium set. The former predicts that when the information maximizing structuring described in Proposition 3 is insufficient for inducing high speculator effort ( $\sigma$ ), the high type finds it impossible to improve upon the LCSE. In such cases, the high type will underinvest, securitizing only safe debt and keeping all risk on his own books. In contrast, when the structuring described in Proposition 3 does induce sufficiently high speculator effort, both types may respond by pooling at that same structuring, with the entire asset sold off to outside investors.

Note that our model nests the canonical signaling model (e.g. Tirole (2005)) as a special case. In the canonical model, there is no possibility for meaningful information production by speculators ( $\sigma = 1/2$ ), so market beliefs are equal to prior beliefs ( $Z = q$ ) if the types pool. Thus, in the canonical model pooling occurs only if  $\beta q > 1$ . In contrast, our model predicts that information production by speculators makes the pooling equilibrium inherently more attractive to the high type. This manifests itself in a weaker condition for pooling ( $\beta Z > 1$ ).

Another interesting feature of our model is that it predicts that buy-side factors will play a critical role in determining optimal security design and the nature of equilibrium. To see this, recall that the high type's expectation for market maker beliefs ( $Z$ ) is determined by speculator effort. In turn, the willingness of the speculator to put in effort is determined by her ability to make gains in trading with uninformed investors. In turn, the volume of uninformed demand depends upon the size of their endowment shocks and degree of risk aversion.

For example, consider the effect of increasing the size of endowment shocks ( $\phi$ ) from a very low level to a very high level (or a large first-order stochastic dominant shift in the risk aversion parameters). This would increase  $Z$ , moving the asset-backed securities market from the separating to the pooling equilibrium. The total volume of securitization would increase, as issuers no longer

retain any claim on the underlying assets (e.g. originate-to-distribute). Further, the volume of riskless debt would fall from a positive level to zero.

#### D. Private versus Public Incentives in Securitization

The model also allows us to determine whether the privately optimal “choice” between separating and pooling equilibria is socially optimal. We here consider a utilitarian social planner placing equal weight on all agents, although the arguments carry over for any weighting with strictly positive weights on all agents.

To set a benchmark, consider first social welfare under symmetric information. Here, the owner would sell the entire asset in the first period, regardless of type, converting each unit of cash raised into  $\beta > 1$  units of consumption. The speculator and market-makers would consume their endowments. If vulnerable to a negative endowment shock, all uninformed investors with  $\theta > \eta^{-1}$  would spend  $\phi$  units of first period numeraire in order to insure against negative consumption. The remaining uninformed investors would go without insurance and incur utility losses associated with negative consumption. The implied ex ante social welfare is:

$$W^{FB} = \beta[qH + (1 - q)L] + y_1^S + y_1^{MM} + y_1 - \frac{\phi}{2}[1 - F(\eta^{-1})] - \frac{\eta\phi}{2} \int_0^{\eta^{-1}} \theta f(\theta) d\theta. \quad (58)$$

Ex ante, the planner computes the following expectation (over types) of social welfare loss in the LCSE relative to first-best:

$$DWL^{SEP} = q(\beta - 1)(H - L). \quad (59)$$

The only deadweight loss in the LCSE is the loss in NPV resulting from the high type operating the new short-term project below optimal scale. From a risk sharing perspective the LCSE is attractive, since credible signaling of private information eliminates asymmetric information across investors. So the speculator does not exert effort and there is efficient risk sharing.

Consider next the pooling equilibrium in which the asset is fully securitized under the corresponding optimal structuring described in Proposition 3. Note first that the equalities in equation

(11) imply that the expected level of investment in the pooling equilibrium is equal to the socially optimal level. However, this equilibrium also entails costly effort on the part of the speculator and results in inefficient risk sharing.

The calculation of social welfare in the pooling equilibrium is a bit more involved. As a first step it can be computed as:

$$\begin{aligned}
W^{POOL} &= \beta[qH + (1-q)L] + y_1^S + y_1^{MM} + y_1 + [G - e] \\
&\quad - \frac{\eta\phi}{2} \left[ \int_0^{\theta_1} \theta f(\theta) d\theta + (1-q)(1-\kappa^{-1}) \int_{\theta_1}^{\theta_2} \theta f(\theta) d\theta \right] \\
&\quad - \frac{1}{2}[1 - F(\theta_2)] \left( \frac{\phi}{B_L} P_B^- \right) - \frac{1}{2}[F(\theta_2) - F(\theta_1)] \left( \frac{\phi}{B_H} P_B^- \right).
\end{aligned} \tag{60}$$

The first line in equation (60) measures the value of aggregate investment, plus the first period endowments plus net trading gains to the speculator. The second line measures the costs incurred by the uninformed investors when they have negative  $c_2$ . The third line measures the expected units of  $c_1$  that are spent by the uninformed investors when they purchase their optimal portfolios. This equation simplifies as follows:

$$\begin{aligned}
W^{POOL} &= \beta[qH + (1-q)L] + y_1^S + y_1^{MM} + y_1 - e - \frac{\phi}{2}[1 - F(\theta_1)] - \frac{\eta\phi}{2} \int_0^{\theta_1} \theta f(\theta) d\theta \\
&\quad - \frac{\eta\phi}{2}(1-q)(1-\kappa^{-1}) \int_{\theta_1}^{\theta_2} (\theta - 1)f(\theta) d\theta - \frac{\phi}{2}q[\kappa - 1][1 - F(\theta_2)] \\
&\quad + \frac{(1-\eta)\phi}{2}(1-q)(1-\kappa^{-1})[F(\theta_2) - F(\theta_1)].
\end{aligned} \tag{61}$$

Subtracting (61) from (58) one obtains the following expression for the deadweight loss in the pooling equilibrium:

$$DWL^{POOL} = e[\sigma^*(\phi)] + \frac{\phi}{2} \left[ \int_{\bar{v}^{-1}}^{\theta_1} (\eta\theta - 1)f(\theta) d\theta + (1-q)(1-\kappa^{-1}) \int_{\theta_1}^{\theta_2} (\eta\theta - 1)f(\theta) d\theta + q(\kappa - 1)[1 - F(\theta_2)] \right]. \tag{62}$$

Equation (62) has the following intuition. The first term reflects the fact that speculator effort is socially costly. The term in large square brackets reflects the fact that the existence of asymmetric

information in the pooling equilibrium leads to distortions in the portfolios of the uninformed investors relative to first-best. The first term in the large brackets captures the fact that a socially inefficient number of uninformed investors forego insurance altogether. The second term in the large brackets reflects the fact that adverse selection induces a socially inefficient number of uninformed investors to only partially insure against negative consumption. And the final term represents the social cost associated with overinsurance ( $c_2 > 0$ ) by extremely risk averse uninformed investors.

From equation (62) it is readily verified that the deadweight loss associated with the pooling equilibrium is increasing in the size of endowment shocks as follows:

$$\frac{\partial DWL}{\partial \phi} = e' \frac{\partial \sigma^*}{\partial \phi} + \frac{1}{2} \left[ \int_{\eta^{-1}}^{\theta_1} (\eta\theta - 1) f(\theta) d\theta + (1 - q) (1 - \kappa^{-1}) \int_{\theta_1}^{\theta_2} (\eta\theta - 1) f(\theta) d\theta + q(\kappa - 1)[1 - F(\theta_2)] \right].$$

Note that the deadweight loss in the separating equilibrium is independent of  $\phi$ , but increasing in  $\beta$ . Conversely, the deadweight loss under pooling is independent of  $\beta$ , but increasing in  $\phi$ . It follows that by equating the deadweight losses across the two types of equilibria we may pin down a critical value of  $\beta$ , call it  $\beta_{public}$ , at which the social planner would be just indifferent between the two equilibria. Specifically:

$$\beta_{public}(\phi) = \frac{e[\sigma^*(\phi)] + \frac{\phi}{2} \left[ \int_{\eta^{-1}}^{\theta_1} (\eta\theta - 1) f(\theta) d\theta + (1 - q) (1 - \kappa^{-1}) \int_{\theta_1}^{\theta_2} (\eta\theta - 1) f(\theta) d\theta + q(\kappa - 1)[1 - F(\theta_2)] \right]}{q(H - L)}. \quad (63)$$

It is readily verified that  $\beta_{public}$  is increasing in  $\phi$ . Intuitively, an increase in  $\phi$  raises the risk sharing cost associated with the pooling equilibrium, so the only way to maintain social planner indifference across the equilibria is to have a compensating increase in  $\beta$ , which raises the deadweight cost of the underinvestment associated with the separating equilibrium.

Similarly, we may pin down a critical value of  $\beta$ , call it  $\beta_{private}$  at which the high type would be just indifferent between the two equilibria. From Proposition 6 we know that the indifference region is determined by:

$$\beta_{private}(\phi) = [Z(\sigma(\phi))]^{-1} \Rightarrow \frac{d\beta_{private}}{d\phi} = -[Z(\sigma(\phi))]^{-2} \left[ \frac{\partial \sigma}{\partial \phi} \right] < 0.$$

In contrast to the social planner, the high type is more attracted to the pooling equilibrium for higher values of  $\phi$  since large endowment shocks stimulate uninformed demand and speculator effort, resulting in less underpricing in the pooling equilibrium. Hence, to maintain indifference for the high type, a compensating decrease in  $\beta$  is required in response to a marginal increase in  $\phi$ .

Figure 3 pulls the entire analysis together in a convenient way. The solid upward sloping line depicts  $\beta_{public}$ . The social planner prefers the *separating* equilibrium at points to the right of  $\beta_{public}$  reflecting the fact that high values of  $\phi$  are associated with large losses due to inefficient risk sharing. The dotted downward sloping line depicts  $\beta_{private}$ . The high type prefers the *pooling* equilibrium at points to the right of  $\beta_{private}$ , reflecting the fact that large endowment shocks stimulate speculator effort and mitigate the extent of underpricing he faces in the pooling equilibrium.

Private and public preferences conflict on Regions 1 and 4. On Region 1, new investment is very valuable (high  $\beta$ ) and efficient risk sharing is less important since endowment shocks ( $\phi$ ) are small. The social planner prefers the pooling equilibrium on this region, being willing to sacrifice efficient risk sharing in exchange for higher investment. However, the high type here prefers the separating equilibrium, recognizing that he will face severe underpricing if he pools with the low type, given that low  $\phi$  values induce low speculator effort. That is, the high type signals because he wants to avoid conferring relatively large transfers to the low type. However, such transfers from high to low types are irrelevant for the (neutral) social planner.

Conversely, on Region 4 the planner prefers the pooling equilibrium, with the high  $\phi$  values raising the risk sharing and speculator effort costs inherent in the pooling equilibrium. However, on this same region the high type prefers pooling, since he recognizes that high  $\phi$  values stimulate speculator effort and mitigate the extent of underpricing. Similarly, an increase in risk-aversion via a first-order stochastic dominant shift in  $\theta$  would also increase the social welfare loss associated with pooling, while simultaneously making that equilibrium more attractive to the owner of a high quality asset. Taken together, these results indicate that the private sector will often prefer the pooling equilibrium, with high volumes of securitized asset sales and inefficient risk sharing, precisely when

this risk sharing has the highest social value. The next section discusses potential policy responses.

### E. Policy Implications

The preceding subsection shows that an unregulated private sector may implement a different equilibrium than that preferred by a social planner. We now consider corrective policy responses. We assume the planner knows the model parameters  $(\phi, \beta)$ , which were assumed to be common knowledge across all agents in the model.

Suppose first the planner knows the unregulated market is at a point  $(\phi, \beta)$  in Region 4 of Figure 3. The planner would like to push the private sector to implement the separating equilibrium since achieving efficient risk sharing is of paramount importance, due to the exposure of uninformed investors to large endowment shocks. Proposition 5 shows the separating equilibrium is the unique equilibrium if  $\beta Z \leq 1$ . However, if the unregulated market is in Region 4, as currently posited, it must be the case that  $\beta Z > 1$ . Therefore, the planner simply needs to lower the net value issuers obtain per dollar of gross funding raised via securitization. This can be accomplished by taxing the gross proceeds raised via securitization. The minimum tax rate required solves

$$(1 - t^{\min})\beta Z[\sigma^*(L, H)] = 1. \quad (64)$$

This leads to the following proposition.

**Proposition 7** *In order to induce the market to move from a pooling equilibrium to a socially preferred separating equilibrium, the minimum required tax rate on securitization proceeds is*

$$t^{\min} = 1 - \frac{1}{\beta Z[\sigma^*(L, H)]}.$$

From this proposition it can be seen that the minimum tax rate is increasing in  $\beta$  and the size of the endowment shock  $\phi$ , since an increase in  $\phi$  results in an increase in  $Z$ . That is, policy must lean harder against the wind of securitization when market pressures are pushing banks toward socially excessive levels of securitization. One could also argue that such a securitization tax would pay a double dividend from a public finance perspective since the tax revenue could be used to finance cuts in other marginal tax rates.



To the extent that policymakers are confident that the unregulated private market is indeed in Region 4, the natural policy prescription is to discourage banks from selling off claims on assets for which they hold private information. However, during the securitization boom, bank capital requirements worked in the opposite direction, discouraging banks from holding risky assets. Moreover, a recent paper by Han, Park and Pennacchi (2009) finds that tax incentives discouraged asset retention by banks. In particular, assets held on bank balance sheets are hit with corporate income tax, while assets sold to SPV's often escape corporate tax altogether.

On Region 4 the model prescribes encouraging signaling via the retention of claims on cash flow. However, that is not the same as prescribing that all issuers keep a mandated level of skin-in-the-game. Mandated retentions are problematic for two reasons. First, even if this policy had the ability to move markets to the separating equilibrium, it does so in an inefficient way since it causes the low type to underinvest for no reason. Of course, this is just a policy corollary of the standard result in signaling theory that no distortion should be imposed on low types in the least-costly separating equilibrium. Second, it is not at all clear that such proposals would actually induce separation. To see this, suppose a 10% stake would be sufficient to credibly signal private information in an unregulated market in which low types sell the entire asset. Now suppose the government imposes a mandatory stake of 5% on all securitizers. In this case, the high type might need to raise his stake to 15% to send a credible signal. But he may perceive this as too costly and instead opt for pooling with the other banks at the 5% stake.

It would be incorrect to interpret the model as prescribing that policymakers should always discourage securitized asset sales.<sup>8</sup> In fact, there are times when policymakers should try to encourage securitization, and discourage signaling via retentions. To see this, suppose the policymaker knows the unregulated private market will fall into Region 1. Region 1 may describe an economy such as Japan during the “lost decades”, where new investment is very valuable (high  $\beta$ ) while risk sharing is of secondary importance due to low levels of uncertainty (small  $\phi$ ). In such an economy, the policymaker wants to see higher levels of investment funded with securitization proceeds.

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<sup>8</sup>Plantin (2010) makes a similar argument in the context of a moral hazard model.

How can a policymaker jump-start the securitization process, as is appropriate in Region 1? First, capital requirements could be increased, discouraging banks from sitting on existing assets. Second, investment tax credits could be put in place, raising  $\beta$ , but this policy is probably not sufficiently targeted. Third, generous tax treatment could be provided for immediate realization of capital gains and losses on asset-backed securities. Interestingly, Lewis (1989) argues that the Tax Reform Act of 1981 had precisely this effect on thrifts, encouraging them to realize tax losses via the sale of mortgage bonds. This revitalized an otherwise dormant market.

### Conclusions

This paper analyzes security design when the issuer has an intrinsic motive for raising funds immediately, but is concerned about mispricing given that he is privately informed regarding asset value. Prices are set by competitive market-makers, with an endogenously informed speculator trading against uninformed investors placing rational orders. The high-type issuer may choose to retain a portion of the asset in order to signal his private information. In this separating equilibrium, a low type sells the entire asset in equity form while a high type only sells safe debt, retaining levered equity on his own books. Since the type is fully revealed, there is no motive for costly information acquisition by the speculator and perfect risk sharing is achieved since uninformed investors do not fear adverse selection. However, there is a social cost to the separating equilibrium since the high type invests less than first-best.

Instead of signaling, the issuer can securitize the entire asset in a pooling equilibrium. In contrast to canonical signaling models, we find that separating equilibria may actually be socially preferred over pooling equilibria. This is because the failure of issuers to credibly signal results in the formation of speculative markets. In turn, speculative markets give rise to costly information acquisition by speculators and expose uninformed investors to adverse selection.

In pooling equilibria, high types rely on speculators to acquire information, driving prices closer to fundamentals. Here, all speculator gains are derived in the market for the less information-

sensitive claim, with the optimal structuring maximizing the product of the speculator's per-unit gain and uninformed trading volume. Uninformed investors imperfectly insure in pooling equilibria, as adverse selection distorts their trading decisions.

The model provides a unique perspective on the policy debate regarding the extent to which securitization increases or decreases social welfare. The model highlights the following fundamental conflict between private and social incentives in choosing securitization structures: Private incentives to implement the pooling equilibrium with full asset sales (originate-to-distribute) are strongest precisely when the gains to efficient risk sharing are highest. Specifically, increases in the size of endowment shocks and/or risk-aversion encourage owners to rely upon speculative activity, rather than signaling, since higher uninformed trading volume subsidizes information acquisition by speculators, which reduces the extent of mispricing in pooling equilibria. Thus, the private sector will engage in socially excessive securitization and distort risk sharing when society most highly values the risk sharing benefits that advocates commonly attribute to such structures.

The essential public policy problem highlighted by the model is that securitizers fail to internalize the negative externality they impose on uninformed investors when they sell informationally-sensitive securities in speculative markets. In fact, our analysis shows that even when privately-informed owners have the ability to fully insulate uninformed investors from adverse selection, they have an incentive to issue securities with nonzero informational-sensitivity in order to promote information production by speculators. Further, our analysis shows that when endowment shocks are large or risk aversion is high, private owners will not engage in enough signaling via retention of risk on their own balance sheets (i.e. will engage in excessive securitization), because they fail to internalize the social benefits signaling provides in terms of improved risk sharing. We show that a tax on securitization proceeds is a valid policy response to this problem.

It is incorrect to interpret our model as implying that securitization and financial innovation are bad things. In our model, the bifurcation of cash flows into streams with varying risk characteristics can serve vital social purposes. For example, bifurcation is also necessary to support separating

equilibria with efficient risk sharing. Further, it would be incorrect to interpret this model as supporting proposals forcing banks to keep more skin in the game since compulsory retentions for all securitizers creates gratuitous underinvestment and potentially discourages socially valuable signaling.

A unique feature of our model is that it gives an explicit role to demand-side factors in influencing both the nature and extent of securitization, without invoking behavioral biases or irrational noise trading. For example, the equilibrium shifts from partial to full securitization of assets, with concomitant increases in risky debt, when uninformed investors face larger shocks and/or when they are more risk averse. Further, the model yields some unique predictions relative to most of the existing security design literature. For example, in pooling equilibria issuers do not attempt to minimize informational-sensitivity. Rather, they deliberately create a degree of adverse selection in order to subsidize informed speculation. Further, in contrast to models where the uninformed parties choose the security design, our model predicts that issuers fail to issue riskless debt in the pooling equilibrium.

## Appendix: Proofs

*Lemma 1*

Substituting beliefs from equation (6) into the expected revenue (8), we obtain:

$$\bar{R}_H(\sigma) = L + \left( \frac{q(H-L)}{2} \right) \left[ \frac{\sigma^2}{1-q-\sigma+2\sigma q} + q + \frac{(1-\sigma)^2}{q+\sigma-2\sigma q} \right]. \quad (65)$$

We need only verify the square bracketed term is increasing. Let

$$\begin{aligned} a(\sigma) &\equiv q + \sigma - 2\sigma q \\ \Omega(\sigma) &\equiv 1 + \frac{\sigma^2}{(1-a)} + \frac{(1-\sigma)^2}{a}. \end{aligned}$$

We need only verify  $\Omega$  is increasing. Differentiating we obtain:

$$\begin{aligned} \Omega'(\sigma) &= \frac{2(1-a)\sigma + (1-2q)\sigma^2}{(1-a)^2} - \frac{2a(1-\sigma) + (1-2q)(1-\sigma)^2}{a^2} \\ &= \frac{[2(1-a) + (1-2q)\sigma]\sigma a^2 - (1-a)^2(1-\sigma)[2a + (1-2q)(1-\sigma)]}{(1-a)^2 a^2} \end{aligned} \quad (66)$$

This is strictly positive if and only if.

$$\begin{aligned} [2(1-a) + \sigma(1-2q)]\sigma a^2 &> (1-a)^2(1-\sigma)[2a + (1-2q) - \sigma(1-2q)] \\ &\Downarrow \\ [(1-a) + (1-q)]\sigma a^2 &> (1-a)^2(1-\sigma)[a + (1-q)] \\ &\Downarrow \\ (1-q)\sigma a^2 &> (1-a)[(1-\sigma)(1-a)a + (1-\sigma)(1-q)(1-a) - \sigma a^2] \\ &\Downarrow \\ (1-q)\sigma a^2 &> (1-a)[a(1-a) - a\sigma + (1-\sigma)(1-q)(1-a)] \\ &\Downarrow \\ [(1-q)a + 1-a]\sigma a &> (1-a)^2[a + (1-\sigma)(1-q)] \\ &\Downarrow \end{aligned}$$

$$\begin{aligned}
[1 - qa] \sigma a &> (1 - a)^2(1 - q\sigma) \\
&\Downarrow \\
\sigma a - \sigma qa^2 &> (1 - a)^2 - q\sigma(1 - a)^2 \\
&\Downarrow \\
q\sigma [(1 - a)^2 - a^2] + \sigma a &> (1 - a)^2 \\
&\Downarrow \\
q\sigma + \sigma a(1 - 2q) &> (1 - a)^2 \\
&\Downarrow \\
a^2 + q(\sigma - a) &> (1 - a)^2 \\
&\Downarrow \\
q^2(2\sigma - 1) + 2[\sigma - q(2\sigma - 1)] &> 1 \\
&\Downarrow \\
(q - 1)^2(2\sigma - 1) - (2\sigma - 1) + 2\sigma &> 1 \\
&\Downarrow \\
(q - 1)^2(2\sigma - 1) &> 0. \blacksquare
\end{aligned}$$

*Proposition 1*

Consider a graph with  $X$  on the vertical axis and  $\sigma$  on the horizontal axis. Plotting aggregate uninformed demand, we know  $X(1/2) > 0$  and that  $X$  is strictly decreasing in  $\sigma$  on  $[1/2, 1]$ . Plotting the incentive compatible signal precision, we know  $\sigma_{ic}$  is strictly increasing in  $X$  with  $\sigma_{ic}^{-1}(1/2) = 0$  and the limit as  $\sigma_1$  converges to one of  $\sigma_{ic}^{-1}(\sigma_1) = \infty$ . Thus, the two curves intersect once, and only once, implying a unique equilibrium exists. ■

*Lemma 4*

Suppose Owner sells  $N \geq 3$  securities. Rank these securities in descending order in terms of the

ratio of their payoff if value is low relative to their payoff if value is high. Section III establishes that hedge trading will be concentrated in security 1, and security 1 will be the only source of informed trading gains. Aggregate demand of the uninformed investors and informed trader will then be zero in securities 2 to  $N$ . Therefore, one may roll up these securities into a single security having no effect on  $\sigma$  or expected revenues. ■

*Lemma 7*

Differentiating the maximand one obtains

$$\frac{M'(\kappa)}{\phi} = 1 - F(\theta_2) + \kappa^{-2}[F(\theta_2) - F(\theta_1)] - (1 - \kappa^{-1})[f(\theta_1)\theta'_1 + f(\theta_2)\theta'_2(\kappa - 1)].$$

And

$$\begin{aligned} \frac{M''(\kappa)}{\phi} &= -2\kappa^{-2}f(\theta_1)\theta'_1 - 2\kappa^{-3}[F(\theta_2) - F(\theta_1)] \\ &\quad - (1 - \kappa^{-1})[f'(\theta_1)(\theta'_1)^2 + f'(\theta_2)(\theta'_2)^2] \\ &\quad - (1 - \kappa^{-1})[2(1 + \kappa^{-1})f(\theta_2)\theta'_2 - f(\theta_1)|\theta''_1|]. \end{aligned}$$

Since  $F$  is convex, a sufficient condition for  $M'' < 0$  is  $|\theta''_1| \leq \theta'_2$ , which always holds. ■

*Lemma 10*

The following program characterizes the preferred pooling equilibrium for the high type.

*PROGRAM 3A*

$$\max_{l,h} \quad \mu(l,h) \equiv H - h + \beta[l + (h - l)Z(\sigma^*(l,h))]$$

*s.t.*

$$\textit{Incentive Compatability} \quad : \quad \sigma^*(l,h) = \psi[q(1 - q)M^*(l,h)]$$

$$\textit{Monotonicity} \quad : \quad h \geq l$$

$$\textit{Limited Liability} \quad : \quad h \in [0, H] \text{ and } l \in [0, L].$$

Program 3A is not necessarily concave. Therefore, instead of relying on first-order conditions, we pin down the optimal policy via perturbation and dominance arguments. The lemma is proved in a series of steps. First, we claim

$$h^* = H \Rightarrow l^* = L.$$

and

$$l^* < L \Rightarrow h^* < H.$$

To demonstrate this, note

$$h^* = H \Rightarrow \forall l \in (0, L), \quad \lambda^*(l, h^*) = 0 \Rightarrow \mu_1(l, h^*) > 0.$$

Next we claim

$$\lambda^*(l^*, h^*) = 0 \Rightarrow l^* = L.$$

To demonstrate this suppose to the contrary that  $(l_0, h_0)$  are optimal with  $\lambda^*(l_0, h_0) = 0$  but  $l_0 < L$ . Then consider increasing  $l$  by  $\varepsilon$  arbitrarily small, noting that such an increase meets all constraints including monotonicity since  $\lambda^*(l_0, h_0) = 0$  implies  $h_0 > l_0$ . The gain is  $\varepsilon\beta(1 - Z) > 0$ , contradicting the initial conjecture.

Next we claim

$$\lambda^*(l^*, h^*) > 0 \Rightarrow l^* = L.$$

To demonstrate this claim, suppose to the contrary that  $(l_0, h_0)$  are optimal with  $\lambda^*(l_0, h_0) > 0$  but  $l_0 < L$ . Then let  $\kappa_0 \equiv h_0/l_0$  and consider all pairs  $(l, \kappa_0 l)$ . By construction, all such pairs keep  $Z$  fixed at  $Z[\sigma^*(l_0, h_0)] \equiv Z_0$ . Then consider

$$\frac{d}{dl} \mu[l, \kappa_0 l] = \beta(1 - Z_0) + \kappa_0[\beta Z_0 - 1] \quad \forall l \in (0, L).$$

Note that the value of this derivative is constant by construction. We next claim the derivative must be weakly positive. For if it is not, the optimal policy is to decrease both  $l$  and  $h$  to zero leaving the owner to collect  $\mu = H$  which is strictly dominated by  $l = h = L$ . Finally, since the derivative is weakly positive  $l^* = L$ . ■



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Table 1: Aggregate Demand Outcomes

Type	Signal	Vulnerable	Informed Demand	Hedge Demand	Aggregate Demand	Probability
$H$	$s_H$	$Y$	$X$	$X$	$2X$	$\frac{q\sigma}{2}$
$H$	$s_H$	$N$	$X$	$0$	$X$	$\frac{q\sigma}{2}$
$H$	$s_L$	$Y$	$0$	$X$	$X$	$\frac{q(1-\sigma)}{2}$
$H$	$s_L$	$N$	$0$	$0$	$0$	$\frac{q(1-\sigma)}{2}$
$L$	$s_L$	$Y$	$0$	$X$	$X$	$\frac{(1-q)\sigma}{2}$
$L$	$s_L$	$N$	$0$	$0$	$0$	$\frac{(1-q)\sigma}{2}$
$L$	$s_H$	$Y$	$X$	$X$	$2X$	$\frac{(1-q)(1-\sigma)}{2}$
$L$	$s_H$	$N$	$X$	$0$	$X$	$\frac{(1-q)(1-\sigma)}{2}$

