

DISCUSSION PAPER SERIES

No. 8330

**TAX-ADJUSTED DISCOUNT RATES: A
GENERAL FORMULA UNDER
CONSTANT LEVERAGE RATIOS**

Peter Molnár and Kjell G Nyborg

FINANCIAL ECONOMICS



Centre for Economic Policy Research

www.cepr.org

Available online at:

www.cepr.org/pubs/dps/DP8330.asp

TAX-ADJUSTED DISCOUNT RATES: A GENERAL FORMULA UNDER CONSTANT LEVERAGE RATIOS

Peter Molnár, Norwegian School of Economics and Business Administration
Kjell G Nyborg, Universität Zurich and CEPR

Discussion Paper No. 8330
April 2011

Centre for Economic Policy Research
77 Bastwick Street, London EC1V 3PZ, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

This Discussion Paper is issued under the auspices of the Centre's research programme in **FINANCIAL ECONOMICS**. Any opinions expressed here are those of the author(s) and not those of the Centre for Economic Policy Research. Research disseminated by CEPR may include views on policy, but the Centre itself takes no institutional policy positions.

The Centre for Economic Policy Research was established in 1983 as an educational charity, to promote independent analysis and public discussion of open economies and the relations among them. It is pluralist and non-partisan, bringing economic research to bear on the analysis of medium- and long-run policy questions.

These Discussion Papers often represent preliminary or incomplete work, circulated to encourage discussion and comment. Citation and use of such a paper should take account of its provisional character.

Copyright: Peter Molnár and Kjell G Nyborg

ABSTRACT

Tax-Adjusted Discount Rates: A General Formula under Constant Leverage Ratios*

Cooper and Nyborg (2008) derive a tax-adjusted discount rate formula under a constant proportion leverage policy, investor taxes and risky debt. However, their analysis assumes zero recovery in default. We extend their framework to allow for positive recovery rates. We also allow for differences in bankruptcy codes with respect to the order of priority of interest payments versus repayment of principal in default, which may have tax consequences. The general formula we derive differs from that of Cooper and Nyborg when recovery rates in default are anticipated to be positive. However, under continuous rebalancing, the formula collapses to that of Cooper and Nyborg. We provide an explanation for why the effect of the anticipated recovery rate is not directly visible in the general continuous rebalancing formula, even though this formula is derived under the assumption of partial default. The errors from using the continuous approximation formula are sensitive to the anticipated recovery in default, yet small. The "cost of debt" in the tax adjusted discount rate formula is the debt's yield rather than its expected rate of return.

JEL Classification: G31 and G32

Keywords: cost of debt, partial default, personal taxes, risky debt, tax shields and tax-adjusted discount rates

Peter Molnár
Institutt for foretaksøkonomi
Norges Handelshøyskole
Helleveien 30
5045 Bergen
NORWAY

Kjell G Nyborg
Department of Banking and Finance
University of Zurich
Plattenstrasse 14
8032 Zurich
SWITZERLAND

Email: peter.molnar@nhh.no

Email: kjell.nyborg@bf.uzh.ch

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=166325

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=115147

* We would like to thank Ian Cooper for comments and NCCR-FINRISK for financial support.

Submitted 04 April 2011

1 Introduction

In most tax systems, there is a tax advantage to debt arising from the tax deductibility of interest payments. Estimating the value to the resulting debt tax shield is thus an important part of company and project valuation. One approach to incorporating the debt tax shield into valuation is to use tax-adjusted discount rates, whereby unlevered after tax cash flows are discounted at a rate that takes account of the tax shield. The appropriate tax adjusted discount rate depends on the debt policy being pursued (Taggart, 1991). In this paper, we derive a general formula under a constant debt to value policy. Our analysis allows for personal taxes, risky debt, and partial default. Moreover, in default we allow for different regimes with respect to whether interest or principal payments have priority. Allowing for partial default is important since, in practice, complete default is rare.

Our analysis extends the framework developed by Cooper and Nyborg (2008), which itself is an extension of the seminal contribution of Miles and Ezzell (1980). While the Miles-Ezzell (ME) formula for tax-adjusted discount rates does not take into account the effects of personal taxes, as was pointed out by Miller (1977), personal taxes can greatly affect the tax advantage to debt. Cooper and Nyborg's analysis allows for both personal taxes and risky debt.¹ However, their formula is based on an assumption of zero recovery in default. We extend their framework to allow for positive recovery rates. This changes the tax-adjusted discount rate formula when the quantity of debt is rebalanced only infrequently (once a year, for example). However, under continuous rebalancing, the formula collapses to that of Cooper and Nyborg. We provide an explanation for why the effect of the anticipated recovery rate is not directly visible in the general continuous

¹Taggart (1991) extends the ME formula to allow for personal taxes, but allows only for riskfree debt. Sick (1990) shows that the same formula is valid even if the debt is risky, under the assumption that default gives rise to a tax liability. Cooper and Nyborg (2008) show that the tax adjusted discount rate formula is substantially different from that derived by Sick under the Miles and Ezzell assumption that default does not give rise to a tax liability. They also discuss the respective merits of this assumption versus that of Sick. We use the ME assumption.

rebalancing formula, even though this formula is derived under the assumption of partial default.

A notable feature of the results in Cooper and Nyborg (2008) is that the “cost of debt” in the tax adjusted discount rate formula is the debt’s yield rather than its expected rate of return. Intuitively, this reflects that it is the interest payment and not the expected rate of return that is tax deductible, and the interest payment per dollar of debt equals the yield (in their setup). This result also holds in our more general setting. In addition, our general formula when rebalancing is not continuous also contains an adjustment for the anticipated loss in default.

The rest of this paper is organized as follows. Section 2 describes the setup, including the modelling of partial default and its tax implications. Section 3 contains the analysis and Section 4 concludes.

2 The model

The model follows Cooper and Nyborg (2008), except that we allow for partial default.

2.1 Basics

The debt to value ratio, $L \in [0, 1)$, is constant over time. The firm’s expected pre-tax cash flow at time t is C_t and the corporate tax rate is T_C . The tax adjusted (or levered) discount rate, R_L , is fundamentally related to the unlevered discount rate R_U by

$$V_{Ut} = \sum_{i=t+1}^T \frac{C_i(1 - T_C)}{(1 + R_U)^i} \quad (1)$$

and

$$V_{Lt} = \sum_{i=t+1}^T \frac{C_i(1 - T_C)}{(1 + R_L)^i}, \quad (2)$$

where V_{Ut} denotes the value of the unlevered firm at time t and V_{Lt} denotes the value of the levered firm at time t , $t = 1, \dots, T$.

The representative investor has a tax rate T_{PE} on equity income and capital gains and T_{PD} on interest income. The tax saving per dollar of interest, T_S , is given by

$$T_S = (1 - T_{PD}) - (1 - T_C)(1 - T_{PE}). \quad (3)$$

Following Taggart (1991) and Cooper and Nyborg (2008), define

$$T^* = T_S / (1 - T_{PD}) \quad (4)$$

and

$$R_{FE}(1 - T_{PE}) = R_F(1 - T_{PD}). \quad (5)$$

R_F is the rate of return on a riskfree bond. Thus, R_{FE} can be interpreted as a riskfree equity rate. We have

$$1 - T^* = \frac{(1 - T_C)(1 - T_{PE})}{1 - T_{PD}} \quad (6)$$

and

$$R_{FE} = R_F \frac{1 - T_{PD}}{1 - T_{PE}} = R_F \frac{1 - T_C}{1 - T^*}. \quad (7)$$

The tax adjusted discount rate is found by first studying the relationship between the value of the levered and unlevered firm at $T - 1$, where T is the terminal date of the project. At $T - 1$ the unlevered value is given by (1), with $T - 1$ in place of t . The after-tax cash-flow to investors is $C_T(1 - T_C)(1 - T_{PE}) + V_{UT-1}T_{PE}$, where the second term is the tax deduction associated with tax on capital gains (only the difference between the final price and the purchase price is taxed).²

The value of the levered firm is the value of the unlevered firm plus additional tax effects. One effect comes from the tax deductibility of interest payments. A second effect is that the tax saving at the personal level associated with capital gains taxation is now $V_{LT-1}T_{PE}$. Hence, we have

$$V_{LT-1} = V_{UT-1} + PV(\text{tax_saving}) + \frac{(V_{LT-1} - V_{UT-1})T_{PE}}{1 + R_F(1 - T_{PD})}. \quad (8)$$

²Following Cooper and Nyborg (2008), we assume for simplicity that capital gains tax arises every period and that capital losses can be offset by gains elsewhere.

As in Cooper and Nyborg (2008), the term $(V_{LT-1} - V_{UT-1}) T_{PE}$ is discounted by $R_F (1 - T_{PD})$ because it is riskless. The term $PV(\text{tax_saving})$ is the present value of tax savings from the tax deductibility of interest payments. To value this, and subsequently derive the expression for the tax adjusted discount rate, we need to consider the recovery rate in default.

2.2 Partial Default

Let Y_D denote the yield on risky debt. This is constant over time. Cooper and Nyborg (2008) use a binomial model whereby the debt is either paid back in full or defaults completely. That is, the return to \$1 of risky debt at any date is $1 + Y_D$ in case of solvency and 0 in case of default. In contrast, in our model of partial default we assume that in default the payoff to \$1 in the bond is $(1 + \alpha) < 1 + Y_D$. In other words, the recovery rate is $\frac{1+\alpha}{1+Y_D}$.³ Note that if α is negative, there are not sufficient funds to repay the principal in full. Cooper and Nyborg's model is the special case that $\alpha = -1$.

Thus, taking date T as an example: if there is no default, the tax saving is $Y_D LV_{LT-1} T_S$.⁴ If there is default, the tax saving depends on the bankruptcy code, as outlined below.

To calculate taxes and tax savings, we must decompose the payoff to the bond into principal and coupon payments. This is done in Table 1. We allow for different rules with respect to whether the principal or coupons are paid first in bankruptcy

Let us denote by δY_D the part of the bond payment in default considered by the tax code as an interest payment. Thus, the tax saving is $Y_D T_S$ in solvency and $\delta Y_D T_S$ in case of default. Using Table 1 we see that⁵

$$\delta = \begin{cases} \min \left[1, \frac{1+\alpha}{Y_D} \right] & \text{if interest is paid first} \\ \max \left[\frac{\alpha}{Y_D}, 0 \right] & \text{if principal is paid first.} \end{cases} \quad (9)$$

³Note that α can be both positive and negative. $\alpha \in [-1, 0)$ represents the situation that the payment to the bondholders is smaller than the principal. $\alpha \in [0, R_F)$ represents the situation that the payment to the bondholders is larger than or equal to the principal. α cannot be above R_F , since this would imply that the return on risky debt would dominate the risk-free rate in every possible state of the world.

⁴ LV_{LT-1} is value of the debt at debt $T - 1$ and $Y_D LV_{LT-1}$ is the interest payment at date T .

⁵We assume that if $\delta Y_D > 0$, then there are taxable earnings of at least this amount.

Table 1: Bond payoff decomposition.

	Solvent	Default	
		Interest paid first	Principal paid first
Total	$1 + Y_d$	$1 + \alpha$	$1 + \alpha$
Principal	1	$\max [1 + \alpha - Y_d, 0]$	$\min [1, 1 + \alpha]$
Interest (coupon)	Y_d	$\min [Y_d, 1 + \alpha]$	$\max [\alpha, 0]$

Based on the principle that repayment of capital should not be taxed, the case that the principal is repaid first is arguably the most relevant one in practice.

3 Analysis

3.1 The value of the tax shield

To value the payoff $Y_D LV_{LT-1} T_S$ in case of solvency and $\delta Y_D LV_{LT-1} T_S$ in case of bankruptcy we create a portfolio from the riskless asset and the risky bond that replicates these payoffs. The payoffs to the riskless asset and the risky bond are summarized in Table 2.

Table 2: Payoff to the riskless asset and to the risky bond.

	No Personal Taxes		Personal taxes	
	Solvent	Default	Solvent	Default
Riskless Asset	$1 + R_F$	$1 + R_F$	$1 + R_F(1 - T_{PD})$	$1 + R_F(1 - T_{PD})$
Risky Bond	$1 + Y_D$	$1 + \alpha$	$1 + Y_D(1 - T_{PD})$	$1 + \alpha - \alpha T_X$

Note: αT_X is derived below. See equation 10.

The payoffs after investor taxes in solvency are modified by multiplying the interest payment by $(1 - T_{PD})$. To get the post-tax payoff to the risky bond in case of default, we sum the direct payoff $(1 + \alpha)$ and the tax effect αT_X . αT_X depends on the tax rates T_{PE} , T_{PD} and on δ . Table 3 calculates this tax effect.⁶

⁶When $\alpha T_X > 0$, investors are paying taxes, when $\alpha T_X < 0$, investors gets a tax-deductible loss. We assume that investors can utilize this tax loss.

Table 3: Bond in default.

Total payoff	$1 + \alpha$
Interest	δY_D
Principal	$1 + \alpha - \delta Y_D$
Capital loss	$-\alpha + \delta Y_D$
Personal tax effect	$(-\alpha + \delta Y_D) T_{PE} - \delta Y_D T_{PD}$

Therefore

$$-\alpha T_X = (-\alpha + \delta Y_D) T_{PE} - \delta Y_D T_{PD}. \quad (10)$$

We can replicate the tax shield, which has payoff $T_S Y_D$ in case of solvency and $\delta T_S Y_D$ in default, by investing in the riskless asset and the risky bond. Denote the amount invested in the riskless asset by a and the amount invested in the risky bond by b . Thus, (a, b) is the solution to the following system of equations:

$$a [1 + R_F (1 - T_{PD})] + b [1 + Y_D (1 - T_{PD})] = T_S Y_D \quad (11)$$

and

$$a [1 + R_F (1 - T_{PD})] + b [1 + \alpha (1 - T_X)] = \delta T_S Y_D. \quad (12)$$

Since the price of both the riskless asset and the risky bond are normalized to 1, the value of the tax shield is

$$a + b = \frac{-\alpha (1 - T_X) + [\delta Y_D + (1 - \delta) R_F] (1 - T_{PD})}{[1 + R_F (1 - T_{PD})] [Y_D (1 - T_{PD}) - \alpha (1 - T_X)]} T_S Y_D. \quad (13)$$

Therefore

$$PV(\text{tax saving}) = \frac{-\alpha (1 - T_X) + [\delta Y_D + (1 - \delta) R_F] (1 - T_{PD})}{[1 + R_F (1 - T_{PD})] [Y_D (1 - T_{PD}) - \alpha (1 - T_X)]} T_S Y_D LV_{LT-1}. \quad (14)$$

3.2 The tax adjusted discount rate

Combining (8) and (14), we have

$$V_{LT-1} = V_{UT-1} + Y_D LV_{LT-1} T_S \frac{-\alpha (1 - T_X) + [\delta Y_D + (1 - \delta) R_F] (1 - T_{PD})}{[1 + R_F (1 - T_{PD})] [Y_D (1 - T_{PD}) - \alpha (1 - T_X)]} + \frac{(V_{LT-1} - V_{UT-1}) T_{PE}}{1 + R_F (1 - T_{PD})}. \quad (15)$$

This can be rewritten using (3)-(7) as

$$V_{LT-1} = V_{UT-1} + \frac{LV_{LT-1}T^*Y_D(1-T_C)}{(1-T^*)(1+R_{FE})} \frac{[\delta Y_D + (1-\delta)R_F](1-T_{PD}) - \alpha(1-T_X)}{Y_D(1-T_{PD}) - \alpha(1-T_X)}. \quad (16)$$

By (1) and (2),

$$V_{UT-1} = V_{LT-1} \frac{1+R_L}{1+R_U}. \quad (17)$$

Thus, we obtain the relationship between R_L and R_U :

$$R_L = R_U - (1+R_U) \frac{LY_D T^* (1-T_C)}{(1-T^*)(1+R_{FE})} \frac{[\delta Y_D + (1-\delta)R_F](1-T_{PD}) - \alpha(1-T_X)}{Y_D(1-T_{PD}) - \alpha(1-T_X)}. \quad (18)$$

Substituting (10) into (18) yields

$$R_L = R_U - \frac{LY_D T^* (1-T_C)}{1-T^*} \frac{1+R_U}{1+R_{FE}} \frac{(1-\delta)R_F(1-T_{PD}) + \delta Y_D(1-T_{PE}) - \alpha(1-T_{PE})}{Y_D(1-T_{PD} + \delta(T_{PD} - T_{PE})) - \alpha(1-T_{PE})}, \quad (19)$$

where δ is given by (9).

While we have derived R_L by analyzing the model at time $T-1$, the same inductive argument as in Cooper and Nyborg (2008) can be used to establish that R_L as given by (19) holds at any date t . Thus, this is our general tax adjusted discount rate, that takes account of risky debt, personal taxes, and partial default.

The formula for the tax adjusted discount rate derived by Cooper and Nyborg (2008) is:

$$R_L = R_U - \frac{LY_D T^* (1-T_C)}{1-T^*} \frac{1+R_U}{1+R_{FE}} \frac{1+R_F}{1+Y_D}. \quad (\text{C\&N 12})$$

As pointed out above, Cooper and Nyborg's model corresponds to the special case that $\alpha = -1$ and therefore also $\delta = 0$. However, substituting these values into (19) yields:

$$R_L = R_U - \frac{LY_D T^* (1-T_C)}{1-T^*} \frac{1+R_U}{1+R_{FE}} \frac{(1-T_{PE}) + R_F(1-T_{PD})}{(1-T_{PE}) + Y_D(1-T_{PD})}, \quad (20)$$

which is slightly different from (C&N 12). The reason for this difference comes from the tax treatment of capital losses. In the case of complete default, the investor suffers a capital loss of 1 per dollar invested in the bond. This loss is tax deductible and therefore the total payoff to the investor is $1 \times T_{PE}$, assuming a capital gains of T_{PE} . In contrast,

Cooper and Nyborg's formula is derived under the assumption that the total payoff to the investor here is $1 \times T_{PD}$.⁷

3.3 Continuous rebalancing

The rates of return in the analysis above may be interpreted as annual returns, with rebalancing of the debt to the target leverage ratio carried out once a year. In this subsection, we derive the general expression for the tax adjusted discount rate under continuous rebalancing of the debt.

We start by dividing the year into n equal periods, with rebalancing happening at the end of each period. The annually compounded rates R_U and R_F are not affected by the frequency of rebalancing. Let

$$r_{U,n} = (1 + R_u)^{\frac{1}{n}} - 1 \quad (21)$$

be the unlevered discount rate over a period of length $1/n$. Define $r_{F,n}$ and $r_{FE,n}$ analogously.

We assume that the binomial process for the risky debt holds over each period of length $1/n$, with the per period yield being denoted by $y_{D,n} = (1 + Y_D)^{1/n} - 1$.

For simplicity, we assume that the pre-tax payoff to the risky bond in default, $1 + \alpha$, is unaffected by the length of a period. However, as a period becomes arbitrarily small, the per period yield on the bond also becomes small. Thus, to ensure a recovery rate below 1 for arbitrarily short periods, we assume that $1 + \alpha < 1$, i.e., $\alpha < 0$. This also means that when the principal is viewed as being paid first in bankruptcy, $\delta = 0$. We consider this the most relevant case as it is consistent with the principle that repayment of capital is not taxed. In the case that interest is paid first in bankruptcy, δ may depend on n . Clearly, it is 0 if $1 + \alpha = 0$. If $1 + \alpha > 0$, there is n' such that for all $n > n'$, $1 + \alpha > y_{D,n}$, since $y_{D,n}$ is decreasing in n and converges to zero. Thus, for sufficiently large n , δ will be equal

⁷While we differ in this detail, both we and Cooper and Nyborg assume elsewhere that capital gains are taxed at T_{PE} . In particular, Cooper and Nyborg (2008) use a capital gains tax of T_{PE} in their equation (A2), as do we in the corresponding expression in this paper. Thus, (20) is arguably the more correct tax adjusted discount rate formula under complete default in the Cooper and Nyborg model.

to 1 if the tax code treats interest as being paid first in bankruptcy. In short, our model collapses to either having $\delta = 0$ or $\delta = 1$.

Thus, using (19), the tax adjusted discount rate over a period of length $1/n$ is

$$r_{L,n} = r_{U,n} - \frac{Ly_{D,n}T^*(1-T_C)}{1-T^*} \frac{1+r_{U,n}}{1+r_{FE,n}} \frac{(1-\delta)r_{F,n}(1-T_{PD}) + \delta y_{D,n}(1-T_{PE}) - \alpha(1-T_{PE})}{y_{D,n}(1-T_{PD} + \delta(T_{PD} - T_{PE})) - \alpha(1-T_{PE})}. \quad (22)$$

Denote the third fraction on the right hand side by A . Multiplying both sides by n , we have

$$nr_{L,n} = nr_{U,n} - ny_{D,n} \frac{LT^*(1-T_C)}{1-T^*} \left(\frac{1+r_{U,n}}{1+r_{FE,n}} \right) \times A. \quad (23)$$

Now define $R_{U,n} = nr_{U,n}$, $R_{L,n} = nr_{L,n}$, $Y_{D,n} = ny_{D,n}$. These are the annualized rates corresponding to $r_{U,n}$, $r_{L,n}$, and $y_{D,n}$, respectively. By definition, $\hat{R}_U = \lim_{n \rightarrow \infty} R_{U,n}$ is the continuously compounded rate that corresponds to R_U . $\hat{Y}_D = \lim_{n \rightarrow \infty} Y_{D,n}$ is the continuously compounded rate corresponding to Y_D . $\hat{R}_L = \lim_{n \rightarrow \infty} R_{L,n}$ is the tax adjusted discount rate under continuous rebalancing. \hat{R}_U , \hat{Y}_D , and \hat{R}_L are continuously compounded rates stated on a standard per annum basis.

Using these definitions in (23), we have

$$\lim_{n \rightarrow \infty} R_{L,n} = \lim_{n \rightarrow \infty} \left\{ R_{U,n} - Y_{D,n} \frac{LT^*(1-T_C)}{1-T^*} \left(\frac{1+r_{U,n}}{1+r_{FE,n}} \right) \times A \right\} \quad (24)$$

which reduces to⁸

$$\hat{R}_L = \hat{R}_U - \hat{Y}_D LT^* \frac{1-T_C}{1-T^*}. \quad (25)$$

Equation (25) thus provides us with the (continuously compounded) tax adjusted discount rate under continuous rebalancing.

This is exactly the same formula as derived by Cooper and Nyborg (2008), starting from (C&N 12). That partial default does not alter the formula for the tax adjusted discount rate under continuous rebalancing is surprising.

To see the intuition for this, recall that in the continuous rebalancing model, we either have $\delta = 0$ or $\delta = 1$. If $\delta = 1$, equation (22) reduces to

$$r_{L,n} = r_{U,n} - \frac{Ly_{D,n}T^*(1-T_C)}{1-T^*} \frac{1+r_{U,n}}{1+r_{FE,n}}. \quad (26)$$

⁸Since each of the last two terms converge to 1.

As seen, α has dropped out. This is intuitive, since $\delta = 1$ means that the interest tax shield is unaffected by the recovery in default.

If $\delta = 0$, the term A in (23) becomes

$$\frac{r_{F,n}(1 - T_{PD}) - \alpha(1 - T_{PE})}{y_{D,n}(1 - T_{PD}) - \alpha(1 - T_{PE})}. \quad (27)$$

This clearly converges to 1, implying that α drops out of the analysis. More intuitively, when $\delta = 0$, the recovery rate only affects the capital loss in default and this gets squeezed towards zero over an arbitrarily short time horizon since the implied probability of default must converge to zero.

To see this, note that in the basic discrete rebalancing model, it must be true that

$$(1 - p)(1 + Y_D) + p(1 + \alpha) = 1 + R_F, \quad (28)$$

where p is the risk-neutral probability of default. When we rebalance more frequently, to keep our model arbitrage-free, the risk-neutral probability of default must adjust according to

$$p_n = \frac{y_{D,n} - r_{F,n}}{y_{D,n} - \alpha}. \quad (29)$$

Thus, the probability of default in a small interval approaches zero in the limit.

The continuous rebalancing tax adjusted discount rate formula itself is intuitive, especially when rewritten in the following form [using (3) and (6)]:

$$\hat{R}_L = \hat{R}_U - \frac{\hat{Y}_D L T_S}{1 - T_{PE}}. \quad (30)$$

This shows clearly that the tax adjusted discount rate is the unlevered discount rate less the tax saving per dollar of firm value. The “raw” tax saving, $\hat{Y}_D L T_S$, is grossed up by $1 - T_{PE}$, reflecting that \hat{R}_L is a discount rate that is applied to after corporate tax, but before personal tax, unlevered cash flows, as seen in (2).

3.4 How accurate is the continuous approximation? Example

Table 4 provides a numerical example of the error arising from using the continuous approximation formula (25) rather than (19). The table shows values of R_L calculated from

(19) for different values of α . The corresponding value of R_L estimated from (25) with the same parameter values as in the table is 6.56%. We see that the continuous approximation formula (25) works well given the chosen parameter values, except for when α is close to zero and the bankruptcy code treats the principal as being paid first.⁹

Table 4: Values of R_L using (19) for different values of α .

Parameter values are: $R_U = 8\%$, $R_F = 4\%$, $T_C = 40\%$, $T_{PD} = 40\%$, $T_{PE} = 40\%$, $L = 60\%$, $Y_D = 6\%$. $R_{L,princ}$ and $R_{L,int}$ refer to tax systems where the principal and interest, respectively, are viewed as being paid in default. (25) yields $R_L = 6.56\%$ if one were to use it with the same annually compounded rates and the same values for the other parameters.

α	0	-0.1	-0.3	-0.5	-0.7	-0.9	-1
$R_{L,princ}$	7.00 %	6.69 %	6.59 %	6.56 %	6.54 %	6.54 %	6.53 %
$R_{L,int}$	6.50 %	6.50 %	6.50 %	6.50 %	6.50 %	6.50 %	6.53 %

4 Summary

We have provided a general formula for tax adjusted discount rates under a constant leverage ratio debt policy. The formula allows for personal taxes, risky debt, and partial default. It also handles different rules with respect to the order of priority of interest payments versus repayment of principal in default. In doing this, we have expanded on the analysis of Cooper and Nyborg (2008), who assumed complete default (zero recovery in default). This is important because recovery rates in practice typically are significantly larger than zero. Our general formula differs from that of Cooper and Nyborg because recovery rates affect the tax adjusted discount rate.

We have also shown that the effect of nonzero recovery rates can be quite small, and if debt rebalancing is continuous, the effect disappears altogether. Our analysis thus

⁹Note that if we were to use continuously compounded rates in (25) – i.e. $\hat{R}_U = 7.70\%$ and $\hat{Y}_D = 5.83\%$, we would get $\hat{R}_L = 6.30\%$. Annually compounded, this is equivalent to 6.50%, which is exactly the same as $R_{L,int}$ in the table in all cases except for when $\alpha = -1$.

shows that Cooper and Nyborg's tax adjusted discount rate formula under continuous rebalancing holds under more general conditions than those under which it was originally derived. We have provided an intuition for why this is so. The usefulness of the continuous approximation formula is that it is easy to use and does not require estimates of recovery rates in default. In the context of a numerical example, we have illustrated that the errors from using it are quite small, even for large recovery rates.

References

- [1] Cooper, Ian A. and Nyborg, Kjell G., 2008, Tax-Adjusted Discount Rates With Investor Taxes and Risky Debt, *Financial Management*, Summer 2008, 365 - 379
- [2] Miles, J. and J.R. Ezzell, 1980, The Weighted Average Cost of Capital, Perfect Capital Markets and Project Life: A Clarification, *Journal of Financial and Quantitative Analysis*, 15 (3), 719-730.
- [3] Miller, M.H., 1977, Debt and Taxes, *Journal of Finance* 32, 261-276.
- [4] Sick, G.A., 1990, Tax-Adjusted Discount Rates, *Management Science*, 36 (12), 1432-1450.
- [5] Taggart, R.A., 1991, Consistent Valuation and Cost of Capital Expressions with Corporate and Personal Taxes, *Financial Management*, (Autumn), 8-20.