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ABSTRACT

Liquidity shocks, roll-over risk and debt maturity*

We develop an infinite horizon model of an economy in which banks finance long term assets by placing non-tradable debt among savers. Banks choose the overall principal, interest rate, and maturity of their debt taking into account two opposite forces: (i) investors' preference for short maturities (which stems from their exposure to preference shocks) and (ii) banks' exposure to systemic liquidity crises (during which debt refinancing becomes specially expensive). Importantly, the terms of access to refinancing during crises depend endogenously on banks' aggregate refinancing needs. Due to pecuniary externalities, the unregulated equilibrium exhibits inefficiently short debt maturities. We analyze the possibility of restoring efficiency or improving welfare by means of limits to debt maturity, Pigovian taxes, and liquidity insurance schemes.

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1   Introduction

The recent financial crisis has extended the view among regulators and policy-makers that prior to the crisis maturity mismatch in the financial system was excessive and not properly addressed by the existing regulatory framework (see, for example, Tarullo, 2009). When the first losses on the subprime positions arrived in early 2007, investment banks, hedge funds and many commercial banks were heavily exposed to refinancing risk in wholesale debt markets. This exposure was a key lever in generating, amplifying, and spreading the consequences of the collapse of money markets during the crisis (Brunnermeier, 2009; Gorton, 2009).

This paper aims to improve our understanding of the linkage between maturity mismatch and roll-over risk in banking. Surprisingly, as described below, the theoretical literature on banks' funding maturity decisions is small and mostly focused on the three-date setup first explored by Diamond and Dybvig (1983).[1] This paper develops a simple infinite horizon model in which long-lived banks finance long-term assets by placing non-tradable debt among unsophisticated savers subject to preference shocks. Banks decide the overall principal, interest rate and maturity of their debt contracts taking into account savers' preferences for short maturities and the risk of facing systemic crises during which the available refinancing sources become very costly. Our main result is that, because of a pecuniary externality, banks' equilibrium debt maturities are inefficiently short.

The logic of this result is as follows. In most periods, banks can replace their maturing debt with new debt issued at the same good terms faced in most prior periods. But during *systemic liquidity crises* (modeled as rare exogenous shocks), banks' normal refinancing strategies fail and banks have to rely on expensive funds supplied by some *bridge financiers*. Guaranteeing the access to bridge financing during systemic crises (which is assumed to be superior to partly or fully liquidating the bank) imposes a constraint on banks' debt structure decisions during normal times. The negative pecuniary externality that makes unregulated debt maturities too short arises from the combination of such constraint with the competitive pricing of bridge financing during crises.[2]

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[1] See Allen and Gale (2007) for an overview. To be sure, several papers, including Bencivenga and Smith (1991), Allen and Gale (1997), and Fulghieri and Rovelli (1998), have explored the implications of embedding such three-date structure in an overlapping generations setup.

[2] Pecuniary externalities are a common source of inefficiency in models with financial constraints (e.g.

Banks offer savers debt contracts that promise the payment of an interest rate per period and the repayment of a fixed principal at maturity. As advanced above, maturity is decided by banks taking into account two opposite forces. The first force is savers' preference for short maturities, which is due to the fact that they are subject to shocks that turn them more impatient, in which case postponing consumption until the contract matures is a source of disutility, like in Diamond and Dybvig (1983). The second force derives from the fact that there are infrequent events (systemic crises) in which all ordinary savers turn impatient and banks have to (temporarily) rely on the (more expensive) funding provided by some sophisticated investors that have their own alternative outside investment opportunities and normally would not invest in a bank.[3]

Thus the key trade-off for the decisions concerning funding maturity in this model is between reducing the interest rate that has to be promised to ordinary investors to attract them in the first place (which can be achieved by choosing shorter maturities) and reducing the need for expensive bridge financing during systemic crises (which can be achieved by choosing longer maturities). The anticipated cost of funds during crises determines the relative importance of the second force and, hence, banks' maturity decisions, giving rise to a downward slopping demand for bridge financing during crises. The equilibrium cost of funds during crises emerges from the intersection of such demand schedule with the upward slopping supply associated with the heterogeneity in outside investment opportunities of the bridge financiers.[4]

By coordinating a properly chosen increase in the maturity of debt of all banks, a regulator can reduce the demand of funds during crises and, thus, the equilibrium cost of bridge financing. This implies a transfer of wealth from the bridge financiers to the banks and, quite crucially, a relaxation of banks' financing constraints which allows banks to expand

in Suarez and Sussman, 1997, and Lorenzoni, 2008). The usual emphasis in the existing papers (including the recent contributions of Bianchi and Mendoza, 2011, and Korinek, 2011) is on their potential to cause excessive fluctuations in credit and overborrowing.

[3]Resembling the conditions that justify the well-known sequential service constraint in papers about deposit runs (Wallace, 1988), we assume that banks and ordinary savers only learn about the occurrence of the systemic crisis when it is too late for banks to induce the savers (say, by improving the terms of the new debt) to rectify their plans not to roll-over maturing debt.

[4]This part of the model plays a role similar to fire-sale pricing (or cash-in-the-market pricing) in models where levered institutions accommodate their refinancing needs by selling part of their long-term assets (e.g. Allen and Gale, 1998, or Acharya and Viswanathan, 2011).

their leverage. We find that an aggregate welfare gain can be achieved by inducing banks to make a lower use of the intensive margin (short maturities) and a larger use of the extensive margin (leverage) of maturity transformation. In this sense the unregulated equilibrium is not constrained efficient.[5]

In the context of our simple model, the direct regulation of maturity is sufficient to achieve a constrained-efficient allocation. A Pigovian tax on refinancing needs can also restore efficiency but, to that effect, tax revenue should be rebated to banks in a lump-sum manner. Otherwise, the tax would induce first-best maturity decisions but its own impact on banks' financing constraints would partly off-set the beneficial effects of the intervention.[6] Finally, we find that introducing a fairly-priced liquidity insurance arrangement, if at all feasible, can definitely be welfare-increasing but is complementary to funding maturity regulation since the basic pecuniary externality that justifes the latter remains present.

Our paper is related to the corporate finance and banking literatures on funding maturity decisions and roll-over risk. The use of shocks to investors preferences as a motivation for short debt maturities is inspired in Diamond and Dybvig (1983). For analytical tractability, we consider debt contracts that mature with a constant probability per period, like in He and Xiong (2009), who take the maturity of funding as given and focus on a dynamic coordination problem among the short-term investors. He and Xiong (2010) analyze a debt-overhang problem in the context of a structural credit risk model à la Leland (1994) where debt has a short maturity. Diamond and He (2010) study the relationship between debt maturities and the debt overhang problem in a model with four dates.

In most corporate finance papers on debt maturity, short-term debt is advantageous because of its disciplinary effect on managers (e.g. Flannery, 1994, Leland, 1998) or because it allows firms with private information to profit from future rating upgrades (e.g. Flannery, 1986, and Diamond, 1991). The incentive effects of short-term maturities in a banking context are analyzed in Calomiris and Kahn (1991), Diamond and Rajan (2001) and Huberman and Repullo (2010). To keep focus and tractability, our analysis abstracts from these effects.

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[5]We restrict attention to interventions involving no net positive use of public funds and no greater informational requirements than the unregulated equilibrium.

[6]Perotti and Suarez (2011) show that Pigovian taxes may perform better than the direct regulation of maturity in the presence of unobservable heterogeneity across banks. Our insight on the need to rebate tax revenue to the banks is an important amendment to the conclusions reached in their reduced-form setup.

Like He and Xiong (2009), several other papers, including Morris and Shin (2004, 2009) and Rochet and Vives (2004), have looked at roll-over risk as the result of a coordination problem between short-term creditors, typically in simpler timing frameworks. The crisis has given raise to papers that adopt some complementary perspectives. Acharya and Viswanathan (2011) explore the effects of a deterioration of economic conditions on the relationship between risk-shifting incentives, difficulties to roller-over short-term, and fire asset sales. Acharya, Gale, and Yorulmazer (2010) show that high roll-over frequency can reduce the collateral value of risky securities; however, the authors take debt maturity as exogenous and do not emphasize the normative implications of their analysis. Brunnermeier and Oehmke (2009) show that unresolved conflicts of interests between long-term and short-term creditors during debt crises can push firms to choose debt maturities which are inefficiently short from an individual firm's value maximization perspective.

The paper is organized as follows. Section 2 presents the model and defines equilibrium. Section 3 characterizes the interest rates that savers demand for each possible debt maturity that banks may choose. Section 4 analyzes banks' individual optimal funding decisions for given terms of refinancing during systemic liquidity crises. Section 5 characterizes the equilibrium determination of those terms. Section 6 examines the social efficiency properties of equilibrium and possible regulatory interventions. Section 7 discusses robustness and several potential extensions of the analysis. Section 8 concludes. All the proofs are in the appendices.

2   The model

We consider an infinite horizon economy in which time is discrete and indexed by $t = 0, 1, 2, ...$ The economy is populated by two wide classes of long-lived risk-neutral agents: possibly-patient *savers* and impatient *experts*. Both classes of agents enter and exit the economy in an overlapping generation fashion further described below. Normally, a sufficiently large measure of savers are born patient, in which case their per-period discount rate is ρ_P, although they may randomly and irreversibly become impatient, in which case their discount rate becomes $\rho_I > \rho_P$. Experts, on the other hand, are always impatient, discounting the future at rate ρ_I, but they are the only agents with the skills needed to extract value from

some of the existing investment opportunities and to manage the *banks*.

The banks posses potentially-perpetual illiquid assets, are owned by the experts who manage them (the *bankers*), and obtain external financing by placing non-tradable debt among initially patient savers.[7] External financing allows bankers to profit from patient savers' lower opportunity cost of the funds.

A final important feature of our economy is its exposure to *systemic liquidity crises*, modeled as temporary random events in which all patient agents become impatient. In a systemic crisis, banks will face difficulties to roll-over their maturing short-term debt and will end up solving their refinancing needs by appealing to experts who will provide some costly *bridge financing* until the crisis ends. Short-term debt maturities have the advantage of allowing savers to promptly recover their funds if they become impatient (which makes them willing to accept lower interest rates in the first place), however short-term debt maturities also imply larger excess refinancing costs during systemic crises, producing a clear trade-off for banks' debt maturity decisions.[8]

In the next subsections we first describe each of the ingredients of the model in detail and then provide a formal definition of the equilibrium on which we focus.

2.1 Aggregate shocks

For the descriptions that follow, it is necessary to differentiate between periods in which the economy is in a normal state, $s_t = N$, and periods in which it is in a systemic liquidity crisis state, $s_t = C$. For analytical convenience, we assume $\Pr[s_{t+1} = C \mid s_t = N] = \varepsilon$ and $\Pr[s_{t+1} = C \mid s_t = C] = 0$, so that crises have a constant probability of following any normal period but never last for more than one period. Thus a period should be empirically interpreted as "the standard duration of a crisis."[9]

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[7]In Section 7 we justify and discuss the importance of the assumption that bank debt cannot be traded. As it stands, the only possibility for a saver to recover the principal of his debt is to wait until it matures.

[8]Our results rely on the maintained assumption that, due to unmodeled information and incentive reasons, banks cannot offer contracts contingent on the realization of the idiosyncratic and aggregate preference shocks, or that give them the option to postpone debt repayments at will. These features will have obvious value to help the banks accommodate savers' preferences for liquidity while limiting their own exposure to systemic crises.

[9]Arguably uncertainty regarding the duration of crises is an important dimension of a crises risk from which we are abstracting in this paper.

2.2 Agents

In each period t a sufficiently large continuum of new risk-neutral savers and experts enter the economy, each endowed with a unit of funds. The measures of each of these classes of new agents are large relative to the size of the refinancing and management needs of the banking sector.

2.2.1 Savers

Except during periods of systemic liquidity crisis ($s_t = C$), a sufficiently large measure of savers are born patient, with a discount rate ρ_P.[10] In normal states ($s_t = N$), patient savers have a purely idiosyncratic (independent) probability $\gamma \in [0, 1]$ of turning irreversibly impatient, with a discount rate $\rho_I > \rho_P$ from thereon. During crises, both entering and existing savers are or become impatient with probability one.

Entering savers decide on whether to invest their endowment in the assets offered by banks (described below) or to consume it. Savers who opt for the first alternative, may face similar (re)investment decisions during their lifetime. Savers who decide to consume their savings become irrelevant for the rest of the economy from thereon.

We assume that savers learn about their own preferences before learning about the aggregate state of the economy and, more importantly, that they make their consumption plans in between both stages. We assume that changing consumption plans after knowing the aggregate state (or postponing the consumption decision to that stage) will entail a cost $\varkappa$ per unit of planned consumption.[11]

2.2.2 Experts

Experts are always impatient, with a constant discount rate ρ_I. When they enter the economy they have the opportunity of undertaking some irreversible private investment project with

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[10]One can interpret ρ_P as the risk-free return of some alternative short-term asset (e.g. government bonds) in which savers can invest and disinvest without the mediation of an expert. In this case, what we call "consumption" of the patient savers might correspond to investing such asset until they become impatient, point at which they would consume.

[11]In practice, consumption planning may include the search and ordering of the goods to buy as well as arranging the access to the funds needed to pay for them (e.g. cancelling an automatically renewable term deposit).

a cost of one and a net present value (discounted at the rate ρ_I) of z. The parameter $z \in [0, \overline{\phi}]$ is heterogeneously distributed over the population of entering experts according to a differentiable and strictly increasing function $F(\phi)$, with $F(0) = 0$ and $F(\overline{\phi}) = \overline{F}$, which for each ϕ gives the measure of the population of entering experts with $z \leq \phi$.

On occasions, especially in crisis periods, *entering experts* will have the alternative of becoming part of the population of active bankers, in the terms specified below. However, we assume that experts impatience is always large enough for them not to accumulate any wealth in any form different from their private investments or their bank shares and that each expert can only devote her expertise to a single venture (private project or bank) at a time.[12]

2.3   The banking sector

The banking sector is initially made up of a measure-one continuum of banks.[13] Each bank has the same fixed amount of assets with residual value L in case of liquidation. Productive bank assets yield a constant cash flow $\mu > 0$ per period. Bank assets only remain productive if continuously managed by an expert or coalition of experts (bankers). The best use for unproductive banks assets is liquidation.

Each bank is initially managed by one or several bankers who hold 100% of its equity.[14] The bankers decide each bank's initial funding structure at some initial normal period (say, $t = 0$). For clarity of exposition we assume that the initial funding structure is held fixed in between crises and restored immediately after each crisis. As discussed in Section 7, this assumption entails no loss of generality from a dynamic optimization perspective.

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[12]These assumptions simplify the analysis by helping exclude the possibility that experts who undertook private projects in a previous period accumulate their dividends or abandom the projects so as to become bankers during systemic crises.

[13]To keep things simple we do not consider the possibility that new banks are formed. In the equilibrium on which we focus banks are never liquidated and hence their measure is always one.

[14]For simplicity, we assume away permanent ownership of bank equity by ordinary savers. This might be justified in reference to some unmodeled governance problem that would make outside equity holders to be recurrently expropriated by managers.

2.3.1 Initial funding and normal times refinancing

We assume that each bank's initial funding structure consists of a continuum of ex ante equal infinitesimal-size non-tradable debt contracts which can be collectively described as a triple (D, r, δ), where D is the overall principal (and par value of the contracts at the issuing period), r is the constant per-period interest rate paid on the non-matured contracts, and δ is the constant probability with which each infinitesimal contract matures in each period. Thus, each contract's maturity is random and has the property that the expected time to maturity, if the contract has not yet matured, is constant and equal to $1/\delta$.[15] We assume contract maturities to be independent both within banks and across banks.[16] At the level of the bank, this produces essentially the same effect as having the overall debt D made up of uniform perfectly-staggered fixed-maturity contracts which are rolled-over (or replaced by identical contracts) as they mature.[17]

Differences in discount rates make bank debt obviously more attractive to patient savers than to any impatient saver or expert. Hence all the initial holders of (D, r, δ), if issued in a normal period, will be patient savers (whose number has been assumed to be sufficiently large). Overall this debt will oblige the bank to pay interest equal to rD in each period and to refinance the amount δD resulting from the fraction of contracts that mature. In normal periods, δD will be refinanced by replacing the maturing contracts with identical contracts placed among savers who are or remain patient in that period. Thus, after each normal period, the bank will have a free cash flow of $\mu - rD$ that can be paid as a dividend to the bankers, who will consume it.[18]

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[15] This assumption produces trade-offs both for savers and banks very similar to those of (more realistic) fixed-maturity contracts but makes the analytics of the problem much more tractable.

[16] The case of perfectly correlated maturities within a bank (and independent across banks) is as tractable as our benchmark case but implies that banks are more vulnerable to systemic liquidity crises. All results are qualitatively identical to the ones reported below, but banks produce less value to their shareholders.

[17] Indeed, under this interpretation, the corresponding fixed maturity would be exactly $1/\delta$, if this were an integer. Leland and Toft (1996) and He and Xiong (2010) develop continuous-time models with fixed-maturity contracts.

[18] For sufficiently impatient bankers and a sufficiently small likelihood of suffering a systemic crisis, paying out and consuming these dividends is optimal for bankers in normal periods. This is the case even if bankers had the possibility of holding precautionary savings so as refinance their banks when a crisis comes (e.g. by investing in a risk-free asset at the rate ρ_P until a crisis occurs).

2.3.2 Refinancing during crises

In a systemic crisis, the bank cannot replace its maturing debt with identical debt contracts because there are no old or newly-born patient savers. The following assumptions help define the course of events in crisis periods:

1. *Dividends.* Bankers learn about the state of the economy after having received and consumed dividends of $\mu - rD$.

2. *Savers' consumption plans vs. expert refinancing.* The frequency of systemic crises is low enough for savers to plan to consume their entire savings as soon as they learn to be impatient. Moreover, savers' cost of rectifying their consumption plans, $\varkappa$, is larger than the opportunity cost of funds $z = \phi$ of the relevant marginal entering expert in a crisis period.

3. *Experts' bridge financing.* Experts can be offered to refinance δD in exchange for an equity stake in the bank. If this arrangement is feasible, the bank operates with lower debt, $(1 - \delta)D$, during the crisis period. In the period after the crisis, it restores the original debt structure (D, r, δ) by issuing an additional amount δD of such debt.

4. *Bankruptcy.* If the bank were unable to refinance its maturing debt δD, creditors would file for bankruptcy. Bankruptcy entails the liquidation of the bank and the division of its liquidation value L among creditors.

The logic and motivation for these assumptions is quite self-explanatory. The first simplifies the algebra and could be removed without material qualitative or quantitative effect on the results.[19] The second captures a realistic feature of systemic crises—the failure of banks' standard financing channels—and pushes banks into the bridge financing provided by experts. Such experts (that in reality might correspond to hedge funds, distant sovereign funds, and other sophisticate investors that in normal times are not important for banks' funding) are assumed to have alternative profitable investment opportunities (with their

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[19]In the numerical examples below, the dividends $\mu - rD$ end up being very small relative to the refinancing needs δD, so their omission would only reduce very marginally the (excess) refinancing costs suffered in a crisis.

NPV measured by z) that they have to give up to finance the bank. The heterogeneity in z will make excess refinancing costs during crises increasing in banks' aggregate refinancing needs.

The third assumption establishes the way the bank satisfies its refinancing needs during a crisis and how it restores its pre-crisis debt structure (D, r, δ) once it is over. The exact form of the securities supporting the bridge financing arrangement is not relevant: their overall returns must just be enough to attract the marginal *bridge financier*.

Finally, the fourth assumption sets a (sufficiently bad) outside option for the bankers who attempt to refinance their bank during the crisis. In fact, a sufficiently low L (relative to the cost of bridge financing) will not only push the bankers into trying to obtain bridge financing for δD, but it will also lead them to choose an initial debt structure (D, r, δ) that makes bridge financing feasible, ruling out bankruptcy in equilibrium. To simplify the presentation, we will directly assume that avoiding bankruptcy is optimal, relegating to Section 7 the discussion of the conditions under which this optimality holds.

2.4 Equilibrium with bridge financing

When the initial bankers choose debt structures (D, r, δ) compatible with obtaining bridge financing during crises, the bridge financiers receive some fraction α of each bank's equity in each crisis. Competition in the market for bridge financing implies that, in equilibrium, α will have to be enough to compensate some marginal entering expert for the opportunity cost of her funds, which we denote by ϕ.

The heterogeneity in the value of the private investment opportunities of the entering experts and the size δD of banks' aggregate refinancing needs implies that clearing the market for bridge financing requires $F(\phi) = \delta D$ (which in turn requires $\delta D \leq \overline{F}$). Since $F(\cdot)$ is strictly increasing, we can equivalently write this condition as $\phi = F^{-1}(\delta D) \equiv \Phi(\delta D)$, where $\Phi(\cdot)$ is strictly increasing and differentiable, with $\Phi(0) = 0$ and $\Phi(\overline{F}) = \overline{\phi}$. We will refer to ϕ as the *excess cost of liquidity* during a crisis and to $\Phi(\cdot)$ as the *inverse supply of liquidity* during a crisis.

We are now ready to define an equilibrium with bridge financing:

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Definition 1 *Given the exogenous parameters of the model ε, ρ_P, ρ_I, γ, μ, and the function $\Phi(\cdot)$, an equilibrium with bridge financing is a tuple $(\phi^e, (D^e, r^e, \delta^e))$ describing an excess cost of liquidity during a crisis ϕ^e and a debt structure for banks (D^e, r^e, δ^e) such that:*

1. *Patient savers accept the debt contracts involved in (D^e, r^e, δ^e).*

2. *Among the class of debt structures that allow banks to be refinanced during crises, (D^e, r^e, δ^e) maximizes the value of each bank to its initial owners.*

3. *The market for liquidity during crises clears in a way compatible with the refinancing of all banks, i.e. $\phi^e = \Phi(\delta^e D^e)$.*

In the next sections we undertake the steps necessary to prove the existence and uniqueness of this equilibrium, and establish its properties. We will start looking at the conditions upon which the contracts involved in some debt structure (D, r, δ) are acceptable to patient savers in normal periods. This will determine a participation constraint relevant for banks' debt structure optimization. Then, for any given excess cost of crisis liquidity, ϕ, we will write down the equation that describes the value of each bank to its shareholders in a normal period, the condition for the feasibility of bridge financing during a crisis (the *bridge financing constraint*), and finally the optimization problem that, conditional on ϕ, determines initial bankers' value maximizing choice of (D, r, δ) subject to the bridge financing constraint. Finally, we will establish the positive and normative properties of equilibrium.

3 Savers' required maturity premium

In this section we analyze the conditions upon which the debt contracts associated with a debt structure (D, r, δ) are acceptable to savers during normal times. In that case, the debt structure will be feasible when first put in place and the refinancing of its per-period maturing fraction δD will be feasible, again in normal periods, under exactly the same conditions as in the replaced contracts.

We have assumed that banks issue their debt at par, so for the purposes of this section we can abstract from D and focus on the valuation of a debt contract with a principal of one. From a saver's perspective, given that the bank will fully pay back its maturing debt

even in crisis periods, the valuation of such contract does not depend on the aggregate state of the economy per se but on whether he is patient ($i = P$) or impatient ($i = I$).

Let U_i be the value of the contract to a saver at each of these states $i = P, I$, just after the interest rate r of the current period is paid. These values must satisfy the following recursive system of equations:

$$U_P = \frac{1}{1 + \rho_P}\{r + \delta + (1 - \delta)[(1 - \varepsilon)(1 - \gamma)U_P + ((1 - \varepsilon)\gamma + \varepsilon)U_I]\}, \tag{1}$$

$$U_I = \frac{1}{1 + \rho_I}[r + \delta + (1 - \delta)U_I].$$

To explain them, notice that the different discount factors multiply the payoffs and continuation values relevant under each individual state $i = P, I$. The contract pays r with probability one in each next period. Additionally it matures with probability δ, in which case it pays also back its principal of one. With probability $1 - \delta$, it does not mature and then its continuation value depends on the investor's individual state in the next period. To understand the terms multiplying U_P and U_I in the right hand side (RHS) of the equations in (1), notice that impatience is an absorbing state that any patient saver can reach in each following period either idiosyncratically, with probability γ, if such period is normal (which happens with probability $1 - \varepsilon$) or, with probability one, if a systemic crisis arrives (which happens with probability ε).

Banks issue their debt in normal periods, when patient savers are abundant, so the relevant condition for the acceptability of some terms (r, δ) is having $U_P(r, \delta) \geq 1$, where

$$U_P(r, \delta) = \frac{r + \delta}{\rho_I + \delta} \frac{\rho_I + \delta + (1 - \delta)\pi}{\rho_P + \delta + (1 - \delta)\pi} \tag{2}$$

is the solution for U_P arising from (1) and $\pi \equiv (1 - \varepsilon)\gamma + \varepsilon$ denotes the unconditional probability that a patient saver becomes impatient in the next period. It is obvious that, for any given δ, a bank maximizing its initial owners' value will offer contracts with the minimal interest rate r that satisfies $U_P(r, \delta) = 1$. Denoting such interest rate by $r(\delta)$, we obtain:

$$r(\delta) = \frac{\rho_I \rho_P + \delta \rho_P + (1 - \delta)\pi \rho_I}{\rho_I + \delta + (1 - \delta)\pi}. \tag{3}$$

From here, we can state the following result:

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Proposition 1 *The minimal interest rates acceptable to patient savers under the debt contracts described above are given by a function $r(\delta)$ which is strictly decreasing and convex, with $r(0) = \rho_I \frac{\rho_P + \pi}{\rho_I + \pi} \in (\rho_P, \rho_I)$ and $r(1) = \rho_P$.*

This result highlights the value of offering short debt maturities to the savers in our model. The intuition is quite straightforward. A contract maturing after just one period ($\delta = 1$) would allow all patient savers to ensure that they can consume their savings as soon as they turn impatient, which implies $r(1) = \rho_P$. For any lower δ, the expected maturity of the contract, $1/\delta$, gets lengthened, which means that the saver bears the risk of turning impatient and having to postpone his consumption until his contract matures. Compensating the cost of waiting via a larger interest rate generates a maturity premium, $r(\delta) - \rho_P > 0$, increasing in the expected time to maturity $1/\delta$. Figure 1 illustrates the behavior of $r(\delta)$ under some specific values of the parameters.[20]

4 Banks' optimal debt structures

In this section, treating the excess cost of crisis liquidity, ϕ, as a given constant, we write down the equation that describes the value of each bank to its shareholders in a normal period, the condition for the feasibility of bridge financing during a crisis (the *bridge financing constraint*), and finally the optimization problem that, given ϕ, determines initial bankers' optimal choice of (D, r, δ) subject to the corresponding bridge financing constraint.

For the purposes of this section, we will take savers' participation constraint into account by assuming that the debt structures (D, r, δ) considered by the banks always set $r = r(\delta)$. This reduces the dimensionality of banks' problem and allows us to refer their debt structures as (D, δ). In the equations we will keep writing r rather than $r(\delta)$ except when presentationally convenient.

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[20] This and all other figures rely on a baseline parameterization in which one period is one month, $\Phi(x) = x^2$, and the remaining parameters take the following values: agents' annualized discount rates are $\rho_P = 2\%, \rho_I = 6\%$; the annualized yield on bank assets is $\mu = 4\%$; the expected time until the arrival of an idiosyncratic preference shock is 1 year (implying $\gamma = 1/12$); and the expected time between systemic crises is 10 years (implying $\varepsilon = 1/120$).
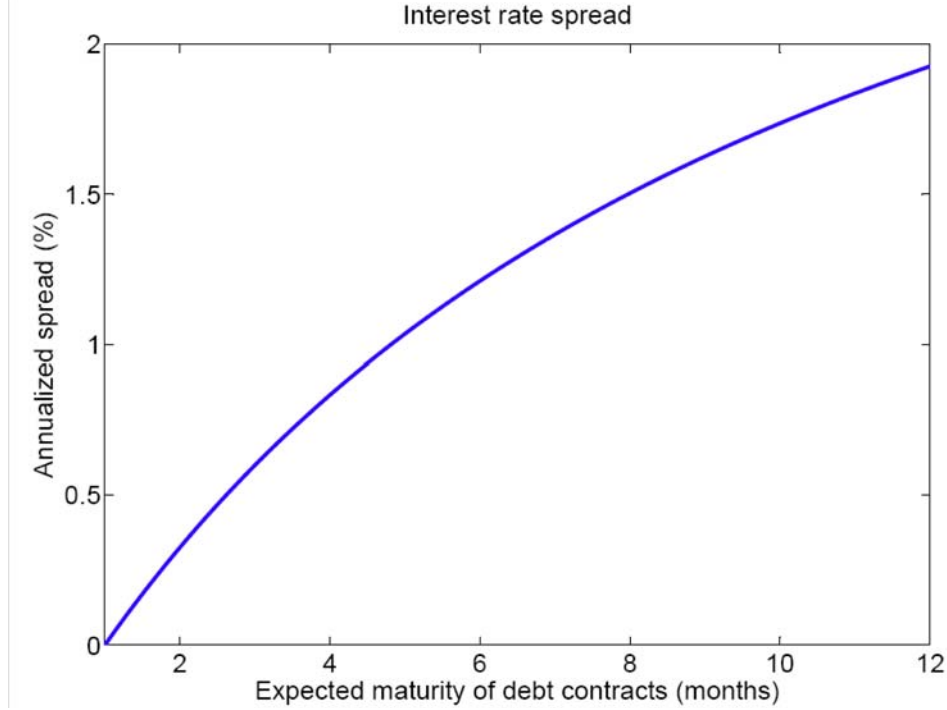
Figure 1: Interest rate spread vs. $1/\delta$

4.1 Value of a bank in normal times

Let $E(D, \delta; \phi)$ be the value of a bank at a normal period to those shareholders that hold 100% of its shares, immediately after having paid the dividends due to cash flows generated in the prior period (if applicable). And let $V(D, \delta; \phi) = D + E(D, \delta; \phi)$ be the total market value of the bank at the same stage of a normal period. Notice that when the bank at $t = 0$ adopts the structure (D, δ), the initial bankers appropriate D out of what savers pay for the corresponding debt. Hence optimal debt structures will maximize $V(D, \delta; \phi)$.

A bank's equity value in normal times $E(D, \delta; \phi)$ satisfies the following recursive equation:

$$E(D, \delta; \phi) = \frac{1}{1 + \rho_I}\{(\mu - rD) + (1 - \varepsilon)E(D, \delta; \phi) + \tag{4}$$
$$+\varepsilon(1 - \alpha)\frac{1}{1 + \rho_I}[\mu - (1 - \delta)rD + \delta D + E(D, \delta; \phi)]\}.$$

To explain the equation, recall that ρ_I is bankers' discount rate and that after each normal period bankers have been assumed to obtain (and immediately consume) the dividend $\mu - Dr$. If the next period is a normal period (i.e. with probability $1 - \varepsilon$), bankers additionally obtain

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the normal-period continuation value $E(D, \delta; \phi)$.[21] If, instead, a systemic crisis arrives (with probability ε), refinancing the bank involves accessing bridge financing, which for current shareholders implies relinquishing a fraction α of the bank's equity to the bridge financiers.

To explain the factor $\frac{1}{1+\rho_I}[\mu - (1 - \delta)rD + \delta D + E(D, \delta; \phi)]$ in the expression above, notice that this accounts for the total value of the equity of banks after being refinanced in the crisis period. Such value is expressed in terms of the payoffs and continuation values received one period ahead, once the crisis period is over. The term $\mu - (1 - \delta)rD$, within the square brackets, corresponds to the dividends paid in that period, which are inflated by the fact that, during the crisis, the bank's outstanding debt was temporarily reduced to $(1 - \delta)D$. The term δD accounts for the fact that, once back to normal times, the bank reissues the debt that was bridge financed by the new shareholders (and uses the proceeds to pay a special dividend). After completing that transaction the bank's normal-times original debt structure is fully restored and thus the bank's equity value becomes $E(D, \delta; \phi)$ again.

Before continuing, let us discuss how α is determined. Bridge financiers are called to supply their funds for the (temporary) funding of the maturing debt δD. Compensating the marginal bridge financier implies paying $(1 + \phi)\delta D$ in present value terms for the obtained funds. Using the expression for the continuation value of the bank's equity in crisis periods explained above, the condition for the participation of the bridge financiers becomes:

$$\alpha \frac{1}{1 + \rho_I}[\mu - (1 - \delta)rD + \delta D + E(D, \delta; \phi)] \geq (1 + \phi)\delta D. \tag{5}$$

Competition between bridge financiers implies that bankers will obtain bridge financing in exchange for the minimal α which satisfies (5), which will then hold with equality. Since we must have $\alpha \leq 1$, it follows from (5) that the feasibility of bridge financing eventually requires

$$\mu + E(D, \delta; \phi) \geq [(1 + \rho_I)(1 + \phi)\delta + (1 - \delta)r - \delta]D, \tag{6}$$

which we will call the *bridge financing constraint* (BF).

Now, since (5) holds with equality, we can rewrite the recursive equation (4) in the

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[21] Notice that the terms associated with a normal period do not reflect the negative cash flows due to the maturing debt δD since they are exactly cancelled out with the issuance of an identical amount of replacing debt.

following terms:

$$E(D, \delta; \phi) = \frac{1}{1 + \rho_I}\{(\mu - rD) + (1 - \varepsilon)E(D, \delta; \phi) + \tag{7}$$

$$+ \varepsilon\{\frac{1}{1 + \rho_I}[\mu - r(1 - \delta)D + \delta D + E(D, \delta; \phi)] - (1 + \phi)\delta D\}\},$$

where the term multiplied by the probability of a crisis, ε, shows how the excess cost of bridge financing is internalized by the initial bankers.

We can solve for $E(D, \delta; \phi)$ in (7), finding the following extended Gordon-type formula for equity value:

$$E(D, \delta; \phi) = \frac{1}{\rho_I}\left[\mu - r(\delta)D - \frac{\varepsilon}{1 + \rho_I + \varepsilon}\{[(1 + \rho_I)\phi + \rho_I] - r(\delta)\}\delta D\right]. \tag{8}$$

The interpretation is very intuitive:

1. $\frac{1}{\rho_I}$ is the present value of a perpetual unit cash flow discounted at bankers' discount rate.

2. μ is the unlevered cash flow of the bank; the remaining terms are proportional to the amount of debt D.

3. $r(\delta)$ is the interest rate paid on debt in normal periods.

4. $\frac{\varepsilon}{1 + \rho_I + \varepsilon}\{[(1 + \rho_I)\phi + \rho_I] - r(\delta)\}$ reflects the differential cost of refinancing the amount of maturing debt δD every time a crisis arrives. It can be decomposed in two factors:

   (a) $\frac{\varepsilon(1 + \rho_I)}{1 + \rho_I + \varepsilon}$, which is the net present value multiplier for crisis-period cash flows;

   (b) $\frac{1}{1 + \rho_I}\{[(1 + \rho_I)\phi + \rho_I] - r(\delta)\}$ reflects that the debt that matures in a crisis is bridge financed. This means that at the end of the crisis period (so the discounting) such debt costs $[(1 + \rho_I)\phi + \rho_I]$ rather than $r(\delta)$.

Using (8), the total market value of the bank can then be written as:

$$V(D, \delta; \phi) = D + E(D, \delta; \phi) = \frac{\mu}{\rho_I} + \frac{\rho_I - r(\delta)}{\rho_I}D - \frac{1}{\rho_I}\frac{\varepsilon\{[(1 + \rho_I)\phi + \rho_I] - r(\delta)\}}{1 + \rho_I + \varepsilon}\delta D, \tag{9}$$

where the first term is the value of the unlevered bank, the second term (which is positive since $r(\delta) < \rho_I$, by Proposition 1) reflects the value of financing the bank with debt claims

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held by savers potentially more patient than the bankers, and the third term reflects costs due to facing refinancing problems during systemic crises. Note that the last term can be made zero by choosing $\delta = 0$, i.e. financing the bank with perpetual debt. In fact, Proposition 1 implies $r(0) < \rho_I$, so that debt financing (or financing the bank with initially patient savers) is a source of value in this economy even with $\delta = 0$. However, unless ϕ is excessively large, banks can generate even more value by undertaking maturity transformation, i.e. choosing funding structures with $\delta > 0$. Showing this formally requires looking at banks' optimization problem in full detail, which is what we do next.

4.2 Optimal debt structure problem

The maximization problem of the bank can be written as:

$$\max_{D \geq 0, \, \delta \in [0,1]} V(D, \delta; \phi) = D + E(D, \delta; \phi) \tag{10}$$
$$\text{s.t.} \quad E(D, \delta; \phi) \geq 0 \qquad \text{(LL)}$$
$$\mu + E(D, \delta; \phi) - [(1 + \rho_I)(1 + \phi)\delta + (1 - \delta)r - \delta]D \geq 0 \quad \text{(BF)}$$

The first constraint imposes the non-negativity of the bank's equity value in normal periods, and we will refer to it as bankers' limited liability constraint (LL). It is easy to realize from equation (8) that satisfying (LL) implies in particular the non-negativity of bankers dividends, $\mu - r(\delta)D \geq 0$.

The second constraint is the already discussed bridge financing constraint (6), which comes from requiring $\alpha \leq 1$ and, thus, can be interpreted as bankers' limited liability as applicable in crisis times. It can be shown that (BF) is generally tighter than (LL) (and both impose the same constraint on D for $\delta = 0$ since in that case the bank is immune to systemic crises).[22] So in the presence of (BF), (LL) can be safely ignored.

The following technical assumptions help us prove the existence and uniqueness of the solution to the bank's optimization problem:[23]

Assumption 1 The function Φ is upper bounded by $2\frac{1+\rho_P}{1+\rho_I} - 1$.

Assumption 2 $\pi < \frac{1-\rho_I}{2}$.

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[22]For a formal argument that uses (4), see the proof of Proposition 2 in Appendix A.

[23]We have checked numerically that the results in Proposition 2 below are also true when these assumptions do not hold. In any case, these sufficient conditions do not impose tight restrictions on parameters.

Proposition 2 *For any given excess cost of liquidity during a crisis $\phi \leq 2\frac{1+\rho_P}{1+\rho_I} - 1$, the bank's maximization problem has a unique solution (D^*, δ^*). In the solution:*

1. *The bridge financing constraint is binding, i.e. in each crisis bridge financiers take 100% of the bank's equity.*

2. *Optimal debt maturity $1/\delta^*$ is increasing in ϕ and the optimal amount of maturing debt per period $\delta^* D^*$ is decreasing in ϕ. In fact, if $\delta^* \in (0,1)$, both δ^* and $\delta^* D^*$ are strictly decreasing in ϕ.*

The intuition for these results is as follows. First, even if the bank does not get involved in maturity transformation ($\delta = 0$), its value is increasing in D, making it interested in choosing the maximum feasible leverage. If maturity transformation generates value, this tendency remains, so (BF) is necessarily binding at the optimum.[24] Second, as the excess cost of liquidity in a crisis ϕ increases, the value of maturity transformation diminishes which implies the choice of a lower δ^* (i.e. a longer expected maturity). For given δ, (BF) becomes tighter, forcing banks to reduce the amount of funding $\delta^* D^*$ demanded to bridge financiers during crises. Although, we have no formal proof regarding total debt D^*, in all our numerical examples D^* is also decreasing in ϕ.[25]

Interestingly, although our primary focus is on debt maturity decisions, our theory has also implications for the choice between debt and equity. Each bank keeps the minimal equity value in normal times compatible with obtaining sufficient bridge financing during a systemic crisis. Figure 2 exhibits a bank's optimal equity to total market value ratio (i.e. its *capital ratio*) in the N state, $E(D^*, \delta^*; \phi)/V(D^*, \delta^*; \phi)$, as a function of the (expected) excess cost of liquidity in a crisis ϕ. The optimal capital ratio is tiny for $\phi = 0$ and strictly increasing in ϕ.[26] For a wide range of values of ϕ, our parameterization yields capital ratios in a realistic 4% to 8% range.[27]

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[24]The full dilution of the original equity stakes of the bank in each crisis is an implication of the fact that all crises have the same severity. If we introduce heterogeneity in this dimension, for example, by introducing random shifts in the inverse supply of liquidity curve $\Phi(x)$, the bridge financing constraint might only be binding (or even not satisfied, inducing bankruptcy) in the most severe crises.

[25]Although not stated in the proposition above, we can also prove that δ^* is independent from the asset return μ (which acts very much like a scale parameter), while D^* is increasing in μ.

[26]Even for $\phi = 0$ banks need to operate with a strictly positive equity buffer because if a systemic crisis arrives bridge financiers demand a return $\rho_I > r$ for the fraction of maturing debt that they finance.

[27]Capital ratios in actual banks may be driven by regulatory constraints. In fact the capital ratios depicted
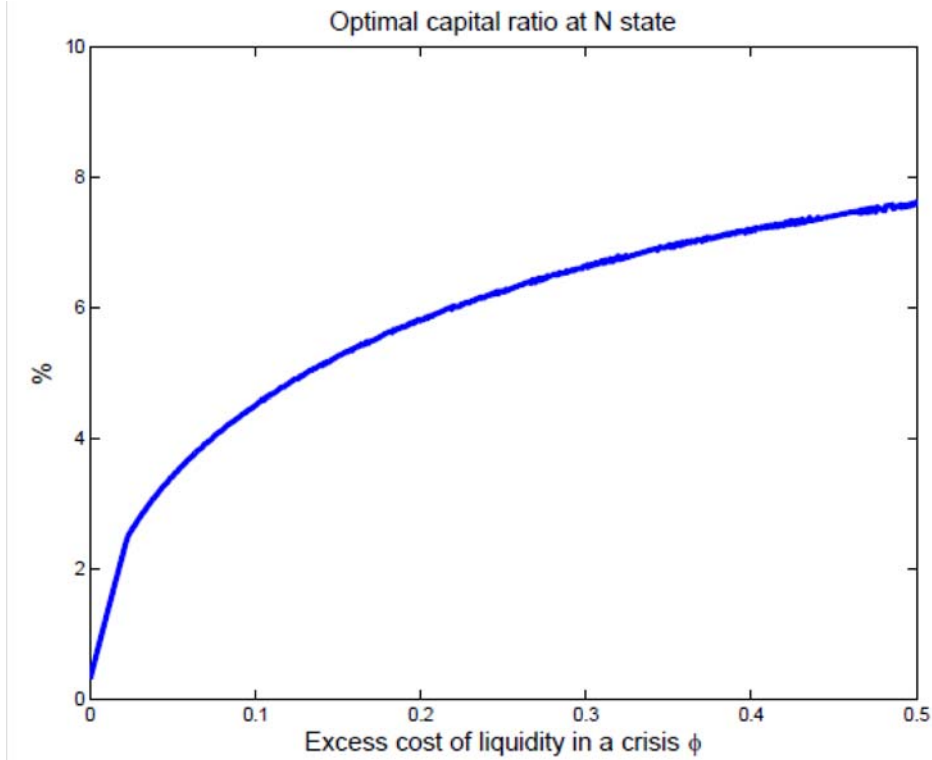
Figure 2: Banks' optimal capital ratio as a function of ϕ

5   The competitive equilibrium

We have just discussed the solution to banks' optimization problem for any given excess cost of liquidity in a crisis ϕ. Such problem embedded savers' participation constraint. The only remaining condition for equilibrium is finding the value of ϕ for which banks' funding structures are compatible with the clearing of the market for bridge financing in crisis periods. The following result relies on the continuity and monotonicity of the excess demand function in the market for liquidity during a crisis:

Proposition 3 *The equilibrium of the economy* $(\phi^e, (D^e, r^e, \delta^e))$ *exists and is unique.*

The next result shows the effects of shifts in the supply of crisis liquidity:

in Figure 2 are the minimal ones compatible with banks being able to avoid default during a systemic crises. These might be the relevant regulatory capital ratios imposed on banks in an extended version of the model in which, perhaps without fully internalizing some social costs of bank failures, bankers wanted to expose their banks to default during crises (see Appendix B for a rationalization of when they might wish to do so).

Proposition 4 *If the inverse supply of liquidity during crises $\Phi(x)$ shifts upwards, the equilibrium changes as follows: expected debt maturity $1/\delta^e$ increases, total refinancing needs $\delta^e D^e$ fall, bank debt yields r^e increase, and the cost of liquidity during crises ϕ^e increases. If initially $\delta^e \in (0,1)$, all these variations are strict.*

The results in Proposition 4 are illustrated in the first column of graphs in Figure 3 where we plot the competitive equilibrium of the economy as a function of the multiplicative factor a of the inverse supply of liquidity curve $\Phi_a(x) = ax^2$. As funds during crises become more expensive banks set longer debt maturities so as to reduce their refinancing needs (Panel A.1). At the same time, each bank generates less value per unit of debt, which through the bridge financing constraint happens to force the bank to reduce its debt (Panel A.2). Finally, banks' reaction to the change in a partially offsets the direct effect of the increase in this parameter on the cost of liquidity during the crisis (producing the concave curve depicted in Panel A.3).

The second and third columns of graphs in Figure 3 show the effects of increasing the time to the arrival of systemic and idiosyncratic shocks, respectively (i.e. the effects of reducing the frequency of each of these shocks). As systemic liquidity shocks become less frequent banks become less worried about crises and thus shorten the maturity of their debt (Panel B.1). Maturity transformation produces more value and the bridge financing constraint is relaxed, so leverage increases (Panel B.2). As a consequence, the equilibrium excess cost of liquidity in a crisis also increases (Panel B.3).[28]

When idiosyncratic liquidity shocks become less frequent, savers disutility due to delaying consumption is reduced. In this situation, the bank might react with a reduction in the interest rate. But it can do better by combining a smaller reduction in the interest rate with an increase in debt maturity, thus reducing its refinancing needs during crises (Panel C.1). The fall in funding costs allows the bank to expand its leverage and to generate more value (Panel C.2). Finally, the equilibrium excess cost of crisis liquidity falls because the effect of lengthening debt maturity dominates the effect of increasing leverage (Panel C.3).

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[28]Note that the first-round response of a bank to a reduction in the frequency of systemic crises is partially offset as all other banks also shorten their debt maturities and increase their leverage, which induces an increase in ϕ^e and gives banks second-round incentives to take stabilizing decisions in the opposite direction.
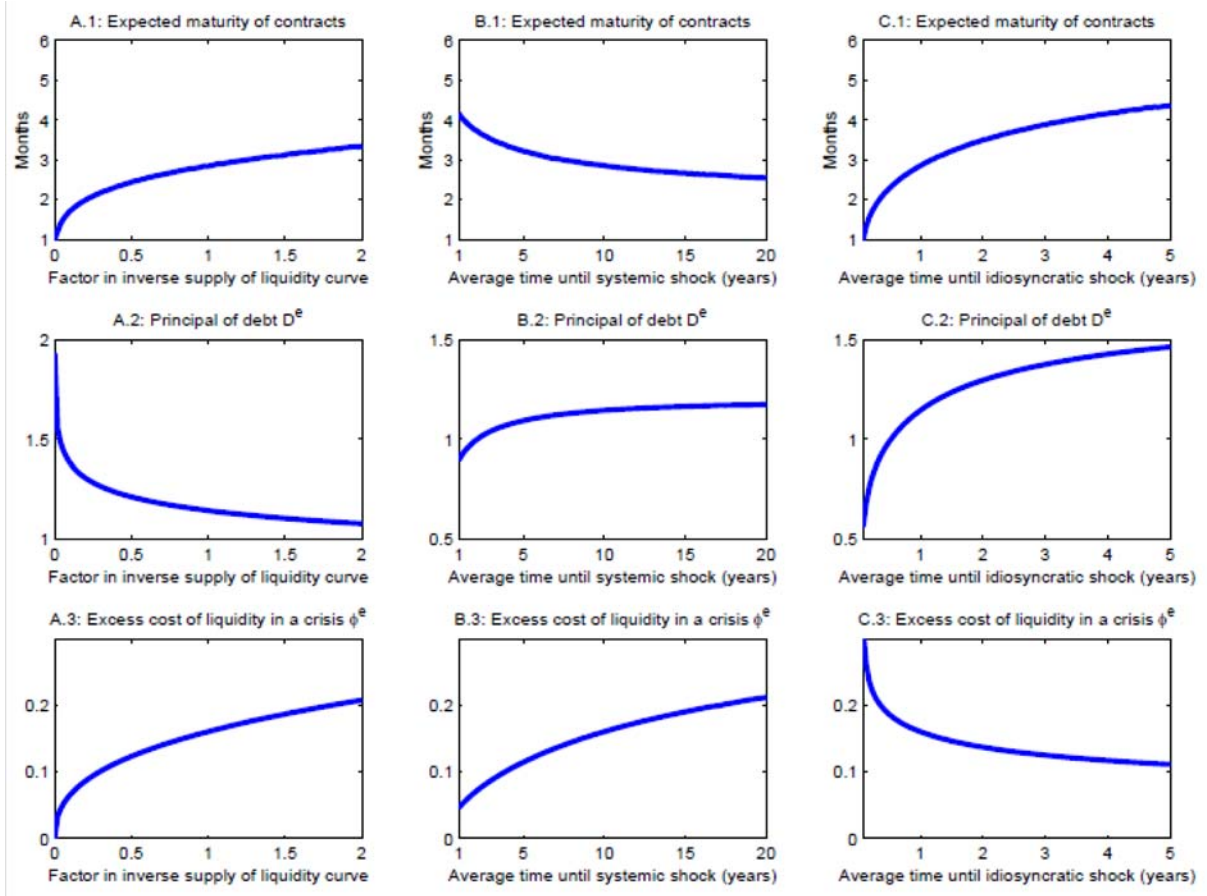
Figure 3: Effect of changes in the parameters on the competitive equilibrium

6   Efficiency properties of the competitive equilibrium

In this section we want to study the social efficiency properties of the competitive equilibrium. In the first part we solve the welfare maximization problem of a (constrained) social planner who has the ability to directly control or regulate banks' funding structure decisions subject to the same type of constraints that banks face when solving their private value maximization problems. We compare the solution of this problem with the unregulated competitive equilibrium characterized in previous sections and we find that the latter features inefficiently short debt maturities because of a pecuniary externality. In the second part, we analyze the possibility of restoring efficiency by means of a (Pigovian) tax on banks' refinancing needs. In the third part, we consider the welfare implications of introducing some

fairly-priced insurance against systemic crises—an arragement which requires relaxing in the first place our latent assumption on the lack of contractibility of the crisis event.

6.1 Inefficiency of the unregulated equilibrium

Let us suppose that a social planner can regulate both the amount D and the maturity parameter δ of the debt that banks offer. In our economy only bank shareholders (both the initial bankers and those who become bankers when providing bridge financing during systemic crises) obtain a surplus. The natural objective function for the social planner is thus the present value of the net payoffs that banks generate for current and future bankers.

Because of the heterogeneity of their alternative investment opportunities, entering experts who become bridge financiers in crisis periods obtain the difference between the competitive excess cost of liquidity during a crisis ϕ and the net present value of their alternative project z. From the condition for the clearing of the market for crisis liquidity, each choice of (D, δ) will imply some $\phi = \Phi(\delta D)$. Hence, bridge financiers' surplus in a crisis period can be computed as:

$$u(D, \delta) = \int_0^{D\delta} \left(\Phi(\delta D) - \Phi(x) \right) dx = \delta D \Phi(\delta D) - \int_0^{D\delta} \Phi(x) dx.$$

And the present value (evaluated at a normal period) of the surpluses they obtain along all future crises can be written as:

$$U(D, \delta) = \frac{1}{\rho_I} \frac{(1 + \rho_I)\varepsilon}{1 + \rho_I + \varepsilon} u(D, \delta).$$

Using this expression and our prior expression (9) for the market value of the bank to its initial owners, $V(D, \delta; \phi)$, the objective function of the social planner can be expressed as:

$$
\begin{aligned}
W(D, \delta) &= V(D, \delta; \Phi(\delta D)) + U(D, \delta) \\
&= \frac{\mu}{\rho_I} + \frac{\rho_I - r(\delta)}{\rho_I} D - \frac{1}{\rho_I} \frac{\varepsilon(\rho_I - r(\delta))}{1 + \rho_I + \varepsilon} \delta D - \frac{1}{\rho_I} \frac{(1 + \rho_I)\varepsilon}{1 + \rho_I + \varepsilon} \int_0^{D\delta} \Phi(x) dx.
\end{aligned}
$$

So social welfare is made up of four intuitive terms: the value of an unlevered bank, the value added by maturity transformation in the absence of systemic crises, the value lost due to the need of temporarily financing the bank with impatient agents during liquidity crises,

and the value lost due to the fact that the experts providing bridge financing to the banks during crises give up the NPV of their private investment projects.

Thus, the social planner's problem can be written as:[29]

$$\max_{D \geq 0, \, \delta \in [0,1]} \quad W(D, \delta) \tag{11}$$
$$\text{s.t.} \quad \mu + E(D, \delta; \Phi(\delta D)) - [(1+\rho_I)(1+\Phi(\delta D))\delta + (1-\delta)r - \delta]D \geq 0 \quad \text{(BF')}$$

This problem differs from banks' optimization problem (10) in two dimensions. First, the social planner takes into account the surplus that bridge financiers obtain. Second, she also internalizes the effect of banks' funding structure decisions on the (equilibrium) excess cost of liquidity during crises: this is why the social planner's (BF') constraint contains $\Phi(D\delta)$ in the place occupied by ϕ in the (BF) constraint of individual banks in (6).

A first interesting result that may help us understand the sources of inefficiency in this economy is the following:

Proposition 5 *If either the total amount of debt D issued by banks or the expected maturity $1/\delta$ of their debt contracts is exogenously fixed, the competitive equilibrium of the model is socially efficient.*

This "efficiency" result refers to a hypothetical situation in which the social planner were able to regulate δ (or D) without changing some given (perhaps independently regulated) D (or δ). The result shows that moving δ (D) away from the equilibrium value $\overline{\delta}^e$ ($\overline{D}^e$) that would arise in the fixed-D (fixed-δ) situation would not produce any net welfare gain. The reason for this is that changing that sole variable would amount to a pure redistribution of value between bridge financiers and the initial bankers (e.g. a lower δ would reduce the induced excess cost of crisis liquidity ϕ but the increase in bankers value V would be exactly offset, in the margin, by the decline in bridge financiers' value U).[30]

Now, let $D^s(\delta)$ be the unique principal of debt that for every δ satisfies the bridge financing constraint (11) with equality:

$$\mu + E(D, \delta; \Phi(\delta D)) - [(1 + \rho_I)(1 + \Phi(\delta D))\delta + (1 - \delta)r - \delta]D = 0.$$

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[29]Recall that the constraint called (LL) in (10) can be ignored because it is implied by the bridge financing constraint.

[30]If for whatever reasons the social planner gives more weight in the social welfare function to the initial bankers than to the potential bridge financiers, then, even for fixed D, there might be social gains from imposing some $\overline{\delta} < \overline{\delta}^e$.

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Intuitively, this is the frontier of the society's set of maturity transformation possibilities. It is possible to prove that the solution of the social planner's problem lies on this curve.

The following proposition states the main efficiency result of the paper:

Proposition 6 *If the competitive equilibrium features $\delta^e \in (0,1)$ then a social planner can increase social welfare by choosing a longer expected debt maturity than in the competitive equilibrium, i.e. some $1/\delta^s > 1/\delta^e$.*

The root of the discrepancy between the competitive and the socially optimal allocation is at the way individual banks and the social planner perceive the frontier of maturity transformation possibilities along which they optimize. Figure 4 depicts banks' (BF) constraint at the competitive equilibrium (where ϕ^e is taken as given) and the social planner's (BF') constraint (where $\phi = \Phi(\delta D)$). It can be shown that at the competitive equilibrium allocation (D^e, δ^e) both the social planner's and the initial bankers' indifference curves are tangent to (BF).[31] Also, (BF), when evaluated at ϕ^e, and (BF') intersect at (D^e, δ^e) since by definition the competitive equilibrium satisfies the market clearing condition $\phi^e = \Phi(\delta^e D^e)$. However, and most importantly, at (D^e, δ^e), the social planner's indifference curve is not tangent to (BF').

In the neighborhood of the equilibrium allocation, (BF') allows for a larger increase in D (expansion of leverage), by reducing δ (lengthening debt maturity) than what seems implied by (BF) for constant ϕ. As a result, socially efficient maturity transformation would require a larger use of its extensive margin (leverage) and a lower use of its intensive margin (short maturities), like at point (D^s, δ^s) in Figure 4.[32] Figure 5 illustrates the comparison between the equilibrium and the socially-efficient bank funding structures in some specific parameterizations of the model.

Although our discussion is focused on aggregate social welfare considerations, distributional implications are also worth mentioning. It turns out that, since we have $\delta^s D^s < \delta^e D^e$, moving from (D^e, δ^e) to (D^s, δ^s) would reduce the net present value of the surplus appro-

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[31] For individual banks, this is a trivial implication of their optimization problem.

[32] This finding offers a new perspective for the joint assessment of some of the regulatory proposals emerged in the aftermath of the recent crisis, which defend reducing *both* the leverage of the financial system *and* its reliance on short-term funding.
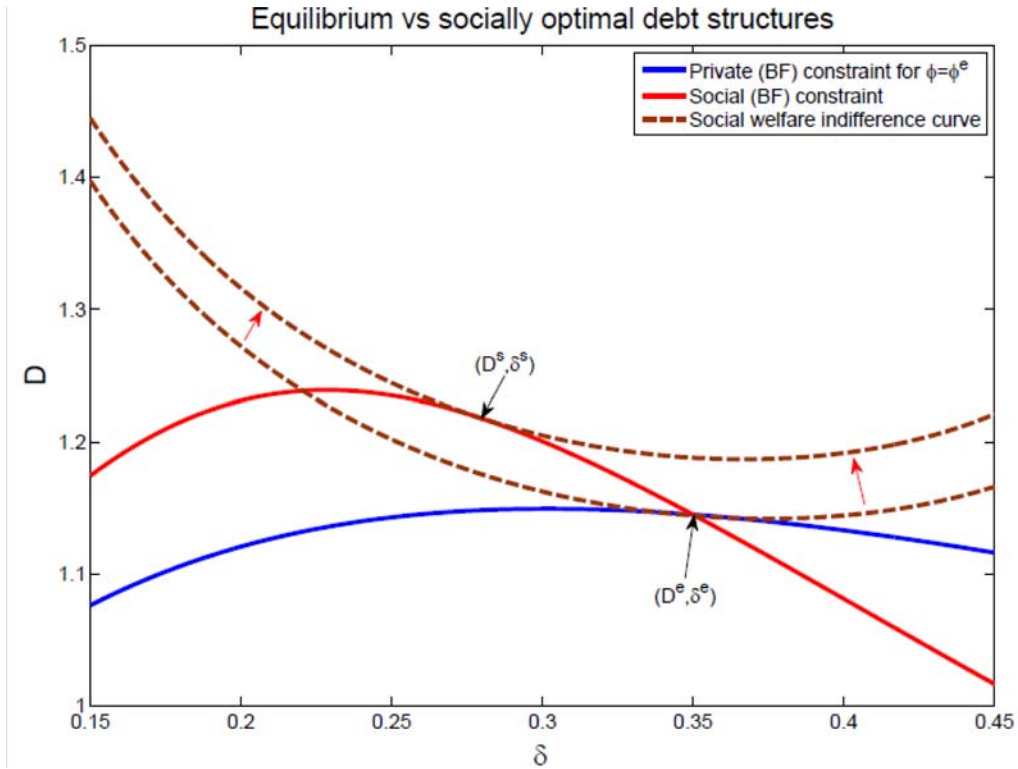
Figure 4: Maturity transformation possibilities from private and social perspectives

priated by future bridge financiers and would make the already existing bankers the great beneficiaries of debt maturity regulation.[33]

6.2   Restoring effiency with a Pigovian tax

In order to achieve the socially efficient debt structure (D^s, δ^s) the social planner can introduce a Pigovian tax on banks' per period financing needs. We consider a class of schemes characterized by two non-negative constants (τ, M) such that:

1. Each bank pays a *proportional tax* of rate τ per period on its refinancing needs δD.

2. The social planner pays a *self-financed lump-sum transfer* M to each bank.

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[33]Making sense of bankers' opposition to maturity regulation (something commonly observed in practice) within the logic of our model would require us to consider additional ingredients such as (i) regulations that constrain maturity transformation beyond what is socially optimal (e.g. by forcing a reduction in D at the same time as a reduction in δ) or (ii) regulations that reduce some implicit subsidies related to the underpricing of safety net guarantees.
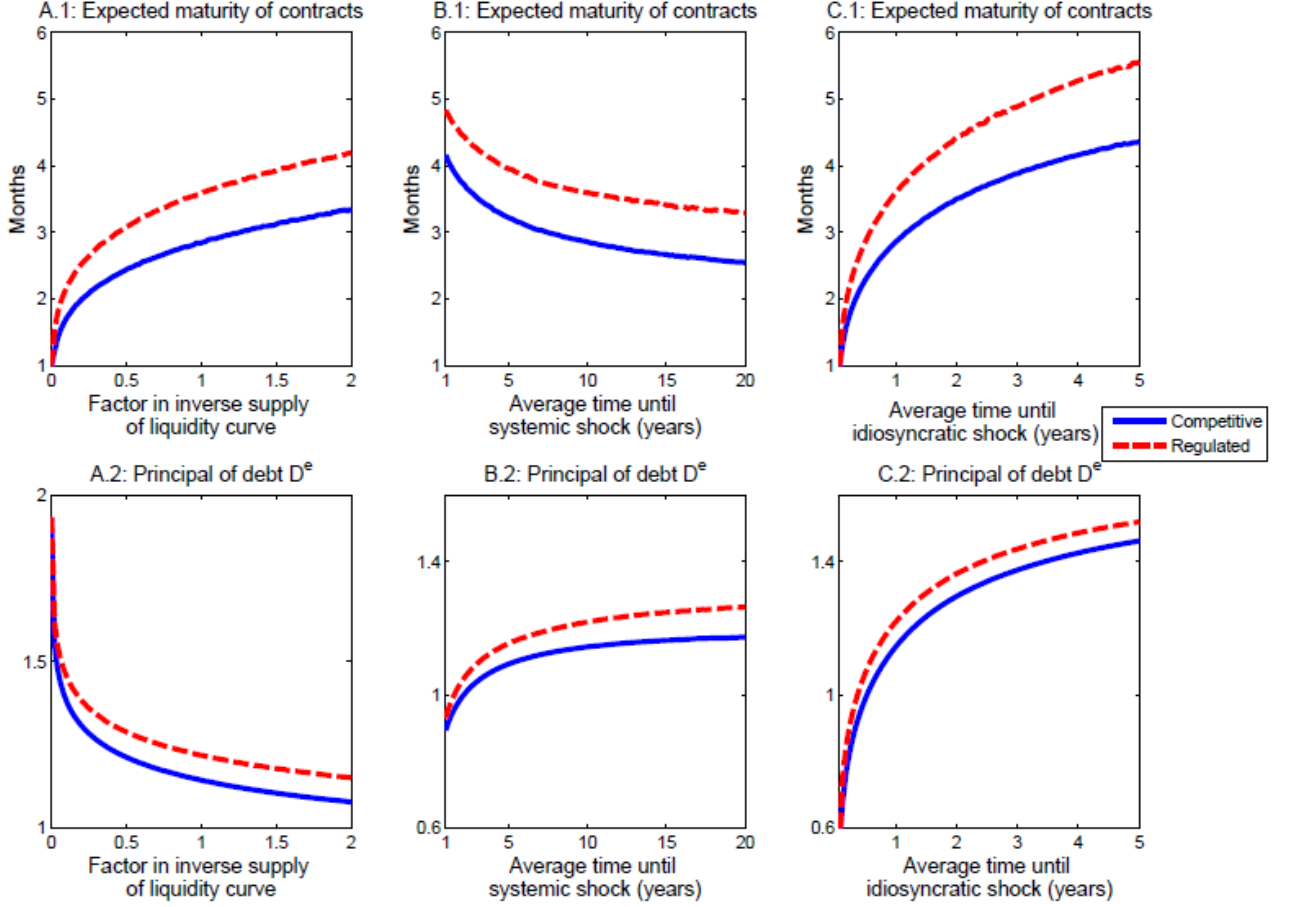
Figure 5: Competitive equilibrium vs. socially-efficient funding structures

If we denote by $\left(\phi^P, (D^P, \delta^P)\right)$ the competitive equilibrium that emerges under (τ, M), then the revenue raised by the tax is $\tau \delta^P D^P$ and the scheme is self-financed if and only if $M \leq \tau \delta^P D^P$. We can prove analytically the following result:

Proposition 7 *If the unregulated competitive equilibrium features $\delta^e \in (0,1)$, there exists a Pigovian tax scheme (τ^P, M^P) that induces the socially optimal allocation $((D^s, \delta^s), \phi^s)$. This scheme satisfies $\tau^P > 0$ and $M^P = \tau^P \delta^s D^s$, and is unique if $\delta^s > 0$.*

Intuitively, the scheme uses τ^P to push banks towards funding decisions involving lower refinancing needs than in the unregulated competitive equilibrium. It is interesting to notice that in order to reach the socially efficient allocation all the revenue raised by the tax τ^P has

to be rebated to the banks through the lump-sum transfer M^P. The reason is that any values of τ and M which induce the socially efficient maturity decision δ^s but involve $M < \tau \delta^P D^P$ would produce a lower normal-times value of bank equity than in the situation in which δ^P is directly enforced by regulation.[34] The lower equity value would tighten banks' bridge financing constraints and imply that their overall leverage D^P would be strictly lower than the socially optimal level D^S. This result constitutes a call for caution against regulations which (unintendedly) reduce banks' equity values: they may be socially counterproductive for essentially the same mechanism that justifies regulating maturity decisions in the first place—the tightening of the financial contraints at the root of the pecuniary externality.[35]

6.3 Private provision of insurance

The fact that in both the competitive and the regulated allocations banks' bridge financing constraints are binding suggests that some form of insurance against systemic liquidity crises might increase welfare. To consider the possibility of insurance, we need to relax prior assumptions about the infeasibility of writing contracts contingent on the realization of systemic crises.[36] However, to keep things tractable, we constrain that possibility to simple one-period refinancing insurance arrangements subscribed by individual banks and newly born experts at the beginning of each period, prior to the realization of uncertainty regarding the occurrence of a crisis.

Specifically, the arrangements we consider establish that:

1. Except in the period immediately after each crisis, the bank pays to a measure $\theta \delta D$ of entering experts a per-period premium $\omega \theta \delta D > 0$, where $\theta \in [0, 1]$.[37]

2. If there is a systemic crisis, the insuring experts supply the bank with funds $\theta \delta D$ in the period and receive a gross repayment of $[1 + r(\delta) + \omega]\theta \delta D$ in the following period.

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[34] The difference is the present value of the net tax revenue appropriated by the tax authorities.

[35] This insight is absent in the reduced-form analysis of Perotti and Suarez (2011), where systemic risk externalities are formalized as non-pecuniary.

[36] We may interpret the new possibility as associated with the introduction of a macroprudential authority that officially declares the existence of a systemic crisis. Introducing that type of declarations may not be desirable in practice if (for reasons not explicitly captured in our model) the announcement of a systemic crisis has the potential to contribute to the extension of panic.

[37] We exclude insurance in the periods immediately after a crisis period because, according to our assumptions, no new crisis may occur at that point.

Three comments are in place. First, θ denotes the fraction of maturing debt per period that the bank decides to insure. Second, the proposed repayment to the insurers includes an extra ω per unit of borrowing in order to offset the impact on the banks' net income of the fact that no insurance is paid in the periods immediately after a crisis (since, under our assumptions, a crisis period is never followed by another crisis period). Third, for insurance to be attractive to an entering expert with the opportunity to extract NPV from her funds equal to z in normal periods and equal to $\max\{z, \phi\}$ in crisis periods, the insurance premium cost ω must satisfy

$$\omega + (1-\varepsilon)(1+z) + \varepsilon\frac{1+r+\omega}{1+\rho_I} \geq (1-\varepsilon)(1+z) + \varepsilon\max\{1+z, 1+\phi\}, \qquad (12)$$

where ϕ is the anticipated aggregate cost of liquidity in a crisis. In good logic, competition among the entering experts with low values of z will lead to a situation in which the above condition holds with equality for the marginal provider of either insurance or bridge financing, who will have $z = \phi$.[38] Solving for ω in the corresponding equality yields

$$\omega = \frac{(1+\rho_I)\varepsilon}{1+\rho_I+\varepsilon}\left(1+\phi-\frac{1+r}{1+\rho_I}\right), \qquad (13)$$

which depends on the debt maturity decision of the contracting bank through r.

Now, let $E(D, \delta, \theta; \phi)$ be the value of equity at the N state when a bank choosing a debt structure (D, δ) and facing an excess cost of crisis liquidity ϕ decides to insure a fraction θ of its refinancing needs. One can check that

$$E(D,\delta,\theta;\phi)=\frac{1}{\rho_I}\left\{\mu-\left[\left(1-\frac{\varepsilon}{1+\rho_I+\varepsilon}(1-\theta)\delta\right)r(\delta)+\theta\omega\delta+\frac{(1+\rho_I)\varepsilon}{1+\rho_I+\varepsilon}(1-\theta)\delta\left(\phi+\frac{\rho_I}{1+\rho_I}\right)\right]D\right\},$$

which, after using (13), implies:

$$E(D,\delta,\theta;\phi) = E(D,\delta,0;\phi) = E(D,\delta;\phi). \qquad (14)$$

This result can be interpreted as Modigliani-Miller type result: for a given debt structure (D, δ) and if liquidity insurance is fairly priced, moving the fraction θ of insured funding simply redistributes some present discounted value of future "uninsured" cash flows among the insurance takers and the insurers.

[38] Notice that market clearing in the market for liquidity in a crisis will require $\phi = \Phi(\delta D)$, irrespectively of the fraction of aggregate refinancing needs δD which banks cover with insurance.

Importantly, however, such redistribution is not irrelevant from the perspective of the bank's overall optimization since it alters the bridge financing constraint. Relative to what we had in equation (6), the bridge financing constraint with insurance (BFI) is:

$$\mu + E(D, \delta; \phi) \geq \{(1 + \rho_I)(1 + \phi)(1 - \theta)\delta + r[1 - (1 - \theta)\delta] + \theta\omega\delta - (1 - \theta)\delta\}D, \quad \text{(BFI)}$$

reflecting the fact that equity value during a crisis must be enough to guarantee the "bridge financing" of just the uninsured fraction $(1 - \theta)$ of the refinancing needs δD.

It is easy to check that the RHS of (BFI) is decreasing in θ, so that $\theta = 1$ implies maximal relaxation of the constraint.[39] On the other hand, the bank's limited liability constraint (LL) does not depend on θ, by (14), and thus is identical to that in (6). Therefore, banks will solve the counterpart of the value maximization problem in (6) by getting fully insured against systemic crises ($\theta = 1$). By doing so, its per period net cash flow becomes constant and equal to $\mu - rD - \omega\delta D$ and the contraints (BFI) and (LL) collapse into a single constraint, equivalent to requiring the non-negativity of this cash flow, that will be binding at the optimum.

The following proposition describes the positive welfare implications of adding insurance in an economy where bank's funding decisions are regulated. Perhaps more surprisingly, it also shows that, even with liquidity insurance, banks in the unregulated economy will opt for inefficiently short debt maturities.

Proposition 8 *In a regulated economy, adding liquidity insurance strictly increases welfare. Even under systemic liquidity insurance, expected debt maturity in the unregulated economy is too short.*

The intuitions for this result are as follows. First, when insurance is introduced banks only need to satisfy (LL), which expands the society's set of maturity transformation possibilities (i.e. the funding structures that respect (LL) for the excess cost of crisis liquidity they induce). Thus a social planner who optimally decides on such set is definitely able to increase social welfare.

Second, the unregulated equilibrium remains inefficient in the presence of insurance because the pecuniary externality regarding banks' decisions on δ operates qualitatively in the

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[39]It suffices to realize that equation (13) implies $(1 + \rho_I)(1 + \phi) - 1 - r > \omega$.

same way as before. Individual decisions affect the excess cost of crisis liquidity ϕ, which in turn affects the cost of insurance ω, which in turn tightens the relevant financial constraint (now $\mu - rD - \omega\delta D \geq 0$). This confirms that the inefficiency of the unregulated equilibrium is not due to market incompleteness but to the presence of binding financial constraints which are affected by equilibrium prices.

Finally, although we are not able to prove that the introduction of insurance increases welfare in the unregulated economy, this is actually the case in all the numerical simulations that we have explored.[40] The main policy message from this subsection is that, if arranging for systemic liquidity insurance is at all feasible, it should be promoted but not as a substitute but as a complement to funding maturity regulation.

7   Discussion and extensions

In this section we discuss in detail some of the key assumptions in the model and comment on potential extensions to our analysis.

7.1   Optimality of not defaulting during crises

We have assumed throughout the paper that the liquidation value L of banks in case of default is small enough so that banks' find it optimal to rely on funding structures that satisfy the so-called (BF) constraint. How small L has to be (and what happens if it is not) is discussed next.

If a bank were not able to refinance its maturing debt, it would default, and we assume that this would precipitate its liquidation. For simplicity, we assume that, if the bank defaults, the liquidation value L is orderly distributed among all debtholders, which excludes the possibility of preemptive runs à la He and Xiong (2009a), where patient savers run afraid of the possibility that others run in the future. Of course, default in case of a crisis

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[40]To explain the theoretical ambiguity, notice that for given ϕ, (LL) is strictly less tight than (BF) and thus, with full insurance, banks can choose funding structures that increase their value relative to the situation without insurance. However, insured banks will tend to engage in maturity transformation more than without insurance, producing an increase in the equilibrium cost of crisis liquidity ϕ^e. In principle, this negative general equilibrium effect might be strong enough to dominate the (partial equilibrium) gains due to insurance.

is anticipated by the savers who then require the debt interest rate r to include proper compensation for credit risk.

Based on the derivations provided in Appendix B, Figure 6 depicts for each possible equilibrium excess cost of liquidity in a crisis, ϕ^e, the maximum liquidation value $L^{\max}(\phi^e)$ for which, when all other banks opt for bridge financing, an individual bank also prefers to rely on bridge financing.[41] The variation of ϕ^e in this figure must be thought of as a reduced-form representation of general shifts in the inverse supply of liquidity in a crisis $\Phi(\delta D)$. Importantly, these shifts affect individual banks' decisions and values only when opting for bridge financing and only through ϕ^e, which is independent of L. Hence both dimensions of the figure account for shifts in exogenous parameters. If for the value of ϕ^e that corresponds to a certain configuration of parameters we have $L \leq L^{\max}(\phi^e)$, then the candidate equilibrium with bridge financing gets confirmed as an equilibrium.

$L^{\max}(\phi^e)$ is decreasing, so the higher the cost of funds during crises, the stronger the incentives for banks to ex ante opt for funding structures that expose them to default in a crisis. To reinforce intuitions, Figure 6 also shows the total market value in a crisis of a bank that relies on bridge financing, $V^C(\phi^e)$.[42] The fact that $L^{\max}(\phi^e) < V^C(\phi^e)$ reflects that, for the values of L contained between the two curves exposing the bank to liquidation in case of a crisis is ex-ante optimal but ex-post inefficient. The intuition here is that, at the cost of being exposed to liquidation in a crisis, the bank can get rid of the (BF) constraint, be left with the counterpart of (LL) in (10), and expand its leverage.

In situations with $L > L^{\max}(\phi^e)$ at least some banks will opt for being exposed to liquidation during each systemic crisis. Hence one may wonder whether, given the absence of new bank formation or entry in our model, such a configuration of parameter might imply a dynamics that leads to the full collapse of the banking sector after sufficiently many crisis. For $L < L^{\max}(0)$, the answer is not, since there is a self-equilibrating mechanism implied by the upward sloping supply of crisis liquidity. Actually such mechanism guarantees convergence to a steady state in which the mass of surviving banks is some unique $m < 1$ and in which all of them rely on bridge financing during crises.

Specifically, for $L \in (L^{\max}(\phi^e), L^{\max}(0))$, we can always find a unique $\phi^m \in (0, \phi^e)$ such

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[41] These decisions are implicitly made at $t = 0$, when banks choose their funding structures.

[42] Since (BF) is binding, the value of a bank's pre-existing equity at a crisis is 0 and thus $V^C(\phi^e) = D^e(\phi^e)$.
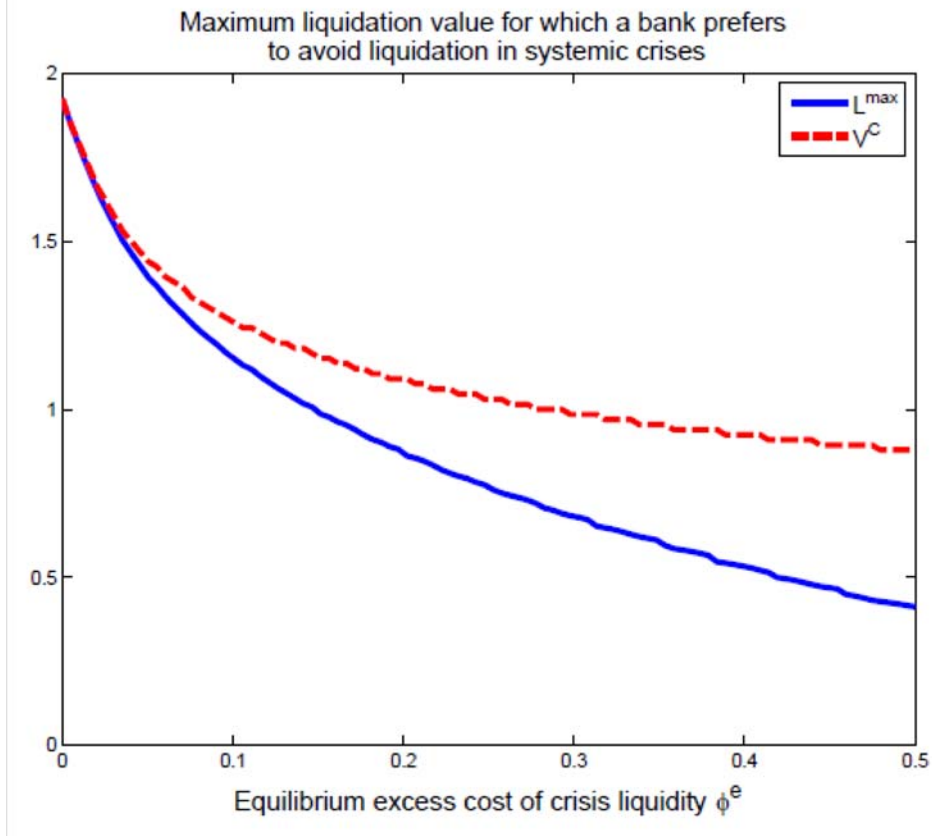
Figure 6: Conditions for the optimality of not defaulting during crises

that $L^{\max}(\phi^m) = L$. Now, let (D^m, δ^m) be the optimal funding structure for banks subject to the (BF) constraint when $\phi = \phi^m$, and let m be defined by $\Phi(m\delta^m D^m) = \phi^m$, where the properties of $\Phi(\cdot)$ guarantee that $m \in (0, 1)$ is unique. If the mass of banks is at any point larger than or equal m, then a mass m of banks will use (D^m, δ^m), which satisfies (BF) and will survive each crisis. The remaining mass of banks, if positive, will use a debt structure that expose them to default in each crisis and, by construction, yields the same ex ante value to the corresponding bankers as the debt structure used by the other banks. As crises materialize, the mass of banks exposed to each next crisis shrinks and the economy eventually converges to steady state in which the whole mass m of surviving banks use (D^m, δ^m). Both in the transition to and in this stationary equilibrium all our key positive and normative results hold.

7.2 Deterministic vs random maturity

For tractability we have assumed that debt contracts have random maturity. It would be more realistic to assume that the bank chooses the deterministic maturity T of its debt contracts, where T is the (integer) number of periods until the contract matures with certainty. In this setting it is possible to determine savers' required maturity premium $r^{\mathrm{det}}(T)$ as we did in Section 3. It can also be shown that for $T = 1/\delta$, we have $r^{\mathrm{det}}(T) < r(\delta)$ because discounting is a convex function of time and thus the random variation in maturity realizations produces disutility to impatient savers.

With deterministic maturities, the model would lose some of the Markovian properties that make it analytically tractable. In particular, in the period after a crisis the initial funding structure would not be immediately restablished since, in addition to the debt with principal $\frac{1}{T}D$ that matures and has to be refinanced, the bank would also have to issue the debt with face value $\frac{1}{T}D$ that was bridge financed during the crisis. But, then, in order for the bank to keep a constant fraction $1/T$ of its debt maturing in each period, half of the debt issued by the bank in the after-crisis period should have maturity $T - 1$, and this would introduce heterogeneity in interest rate payments across the various debts. The description would become further complicated if a new crisis arrives prior to the maturity of the debt with maturity $T - 1$.

Summing up, assuming random maturities implies some loss of efficiency but is essential for the simplicity of the recursive valuation formulas obtained above (e.g. equation (4) for the valuation of bank equity). Fortunately, there is no reason to think that having deterministic rather than random maturities should qualitatively change any of the trade-offs behind the main positive and normative results of the paper.

7.3 Resetting debt structures over time

For the sake of clarity, in our baseline model we assumed that the debt structure (D, δ) decided by initial bankers at $t = 0$ is kept constant along time (albeit for the fraction δD that is "bridge financed" for one period during each crisis). What would happen if bankers could reconsider the banks' funding decisions at some later period?

To narrow down the question, suppose, in particular, that in some given normal period

banks had the option to buy all their outstanding debt at market value and then decide on a new debt structure that would be held constant from that moment onwards. It is obvious from the simple Markovian structure of the model that, after such a single re-optimization opportunity, the bank would not deviate from the funding structure characterized in the baseline sections of the paper.[43] More generally, it is possible to prove that in the case in which current bank shareholders are allowed to buy back all outstanding debt and decide a new debt structure at every normal period, the optimal decision (taking all future optimal decisions as given) would also be the same as in our baseline model.

The more general case in which at every date the bank could decide to roll-over part of its maturing debt at perhaps some new terms, while keeping constant the structure of its non-maturing debt would not be easy to analyze. We would need a more complicated space of state variables to describe the debt structures that a bank might end up having. Modeling that space and establishing formulas for the valuation of equity and the various classes of debt in such setting is out of the scope of this paper. However, there are no reasons to believe that those apparently more general funding structures might be a net source of value to the bank relative to the simple stationary structures that we consider. Intuition from simpler models suggests that altering the terms of new debt as maturing debt is rolled over might only create value to shareholders at the expense of non-maturing debt holders, but this (i) would have a negative repercussion on the value of such a debt when issued (and hence on initial shareholder value) and (ii) could be prevented by including proper covenants in the preexisting debt contracts.

7.4 Tradability of debt

The non-tradability of banks' debt plays a key role in the model. Savers who turn impatient suffer disutility from delaying consumption until their debt matures because there is no secondary market where to sell the debt (or where to sell it at a sufficiently good price).

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[43]The formal argument goes as follows: denote the bank's current debt structure by $(\overline{D}, \overline{\delta})$. In a N state that does not follow a C state, the market value of total outstanding debt is $\overline{D}$. Current shareholders would maximize $V(D, \delta; \phi) - \overline{D}$ subject to the same financing constraints as at $t = 0$ and it is obvious that the optimal solution would be the same as at $t = 0$, since the only difference between the optimization problems is the (constant) $\overline{D}$ now subtracted from the objective function. In a N state that follows a C state, the market value of outstanding debt would be $(1-\overline{\delta})\overline{D}$ and current shareholders would maximize $V(D, \delta; \phi) - (1-\overline{\delta})\overline{D}$ but again the solution would not change.

If bank debt could be traded without frictions, impatient savers would try to sell their debts to newly born patient savers, achieving it immediately in normal periods and with one period of delay in crises. Banks could issue perpetual debt (i.e. with $\delta = 0$) at some initial period and get fully rid of refinancing concerns. In practice a lot of bank debt, starting with retail deposits, but including also certificates of deposit placed among the public, interbank deposits, debt acquired in the course of sales with repurchase agreements (repos), commercial paper issued over the counter (OTC), et cetera, is non-tradable.

Our model does not contain an explicit justification for the lack of tradability. Arguably non-tradability might stem from the administrative, legal compliance, and operational costs of organizing trade (specially centralized trade) for heterogenous debt instruments issued in small amounts, with a short life or, perhaps more importantly, among a dispersed mass of unsophisticated investors. In fact, if other banks (or some other sophisticated traders) could possess better information about banks than ordinary savers (e.g. around episodes in which some banks suffer solvency problems), then costs associated with asymmetric information (e.g. exposure to a winners' curse problem in the acquisition of bank debt) might make the secondary market for bank debt unattractive to ordinary savers (Gorton and Pennacchi, 1990). This view is consistent with the common description of interbank markets as markets where peer monitoring is important (Rochet and Tirole, 1996).

Additionally, the literature in the Diamond and Dybvig (1983) tradition has demonstrated that having markets for the secondary trading of bank claims might damage the insurance role of bank deposits.[44] Yet, the case for the complementarity between banks and markets can be made in cases where, at least for some agents, the access to markets is not guaranteed (Diamond, 1997).

We believe that our model could be extended to describe situations in which debt is tradable but in a non-centralized secondary market characterized by search frictions (like in the models of OTC markets recently explored by Duffie et al., 2005, Vayanos and Weill, 2008, and Lagos and Rocheteau, 2009). In such setting, shortening the maturity of debt would have the effect of increasing the outside option of an impatient saver who is trying to find a buyer for his non-matured debt. This could allow sellers to obtain better prices in

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[44] See von Thadden (1999) for an insightful review of the results obtained in this tradition.

the secondary market, making them willing to pay more for the debt in the first place and encouraging banks to issue short-term debt.[45] In any case developing this extension would constitute another paper.

8 Conclusion

We have developed an infinite horizon equilibrium model in which banks that invest in long-lived assets decide the overall principal, interest rate payments, and maturity of their debt. The model contains a microfoundation for savers' preference for short maturities in line with the traditional Diamond and Dybvig (1983) formulation, which is simplified and adapted to the needs of a recursive dynamic formulation. Banks' incentive not to set debt maturities as short as savers might ceteris paribus prefer, comes from the fact that there are events (called systemic liquidity crises) in which their normal financing channels fail and they have to turn to more expensive sources of funds.

We identify a pecuniary externality that renders the unregulated competitive equilibrium of the model socially inefficient. It turns out that if a social planner induces banks to choose some longer debt maturity than the one they would uncoordinatedly decide, social welfare increases. This is because longer maturities reduce banks' aggregate refinancing needs during crises and, consequently, relax banks' financing constraints.

The pecuniary externality arises from the combination of the ex-post competitive pricing of funds during crises and the ex-ante financial constraints faced by the banks. Alternatives for restoring efficiency include forcing banks to issue debt of longer maturities or inducing them to do so with a (Pigovian) tax on their refinancing needs. Interestingly, we find that the pecuniary externality and, hence, the case for regulating maturity decisions does not disappear when a fairly-priced liquidity insurance arrangement is introduced, suggesting that liquidity insurance and liquidity risk regulation are complements rather than substitutes when addressing banks funding decisions under the threat of systemic crises.

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[45] The empirical evidence in Mahanti et al (2008) and Bao, Pan, and Wang (2010), among others, shows that short-term bonds are indeed more "liquid" (as measured by the narrowness of the bid-ask spread) than long-term bonds.

Appendix

A Proofs

This appendix contains the proofs of the propositions included in the body of the paper.

Proof of Proposition 1 Using (3) it is a matter of simple algebra to obtain that:

$$r'(\delta) = \frac{-\pi(1 + \rho_I)(\rho_I - \rho_P)}{(\rho_I + \delta + (1 - \delta)\pi)^2} < 0,$$

$$r''(\delta) = \frac{2\pi(1 - \pi)(1 + \rho_I)(\rho_I - \rho_P)}{(\rho_I + \delta + (1 - \delta)\pi)^3} > 0.$$

The other properties stated in the proposition are immediate.∎

Proof of Proposition 2 The proof is organized in a sequence of steps.

1. If (BF) is satisfied then (LL) is strictly satisfied Using equation (8) we have that (LL) can be written as:

$$0 \leq E(D, \delta; \phi) = \frac{1}{\rho_I}(\mu - rD) - \frac{1}{\rho_I}\frac{(1 + \rho_I)\varepsilon}{1 + \rho_I + \varepsilon}\left(1 + \phi - \frac{1 + r}{1 + \rho_I}\right)\delta D,$$

while (BF) can be written, using (6), as

$$0 \leq \frac{1}{1 + \rho_I}(\mu - r(1 - \delta)D + \delta D + E(D, \delta; \phi)) - (1 + \phi)\delta D =$$

$$= \frac{1}{\rho_I}(\mu - rD) - \left(1 + \frac{1}{\rho_I}\frac{\varepsilon}{1 + \rho_I + \varepsilon}\right)\left(1 + \phi - \frac{1 + r}{1 + \rho_I}\right)\delta D.$$

Now, since $1 + \frac{1}{\rho_I}\frac{\varepsilon}{1 + \rho_I + \varepsilon} > \frac{(1 + \rho_I)\varepsilon}{\rho_I(1 + \rho_I + \varepsilon)}$ we conclude that whenever (BF) is satisfied, (LL) is strictly satisfied.

2. Notation and useful bounds Using equation (8) we can write:

$$V(D, \delta; \phi) = D + E(D, \delta; \phi) = \frac{1}{\rho_I}\mu + D\Pi(\delta; \phi),$$

where

$$\Pi(\delta, \phi) = 1 - \frac{1}{\rho_I}\left[\left(1 - \frac{\varepsilon}{1 + \rho_I + \varepsilon}\delta\right)r + \frac{(1 + \rho_I)\varepsilon}{1 + \rho_I + \varepsilon}\delta\left(\phi + \frac{\rho_I}{1 + \rho_I}\right)\right]$$

can be interpreted as the value the bank generates to its shareholders per unit of debt. Using Proposition 1 we can see that the function $\Pi(\delta, \phi)$ is concave in δ.

(BF) in equation (6) can be rewritten as:

$$\mu + V(D, \delta; \phi) \geq [(1 + \rho_I)(1 + \phi)\delta + (1 + r)(1 - \delta)]D,$$

and if we define $C(\delta, \phi) = (1 + \rho_I)(1 + \phi)\delta + (1 + r)(1 - \delta)$, (BF) can be written in the more compact form that will be used from now onwards:

$$\frac{1 + \rho_I}{\rho_I}\mu + (\Pi(\delta, \phi) - C(\delta, \phi))D \geq 0. \tag{15}$$

Using Proposition 1 we can see that the function $C(\delta, \phi)$ is convex in δ.
We have the following relationship:

$$\Pi(\delta, \phi) = 1 - \frac{1}{\rho_I}\frac{(1 + \rho_I)}{1 + \rho_I + \varepsilon}\left[r(\delta) + \frac{\varepsilon}{1 + \rho_I}(C(\delta, \phi) - 1)\right] \tag{16}$$

The assumption $\phi \leq 2\frac{1 + \rho_P}{1 + \rho_I} - 1$ implies $(1 + \rho_I)(1 + \phi) \leq 2(1 + \rho_P) \leq 2(1 + r(\delta))$ for all δ, and we can check that the following bounds (that are independent from ϕ) hold:

$$\begin{aligned} C(\delta, \phi) &\geq 1 + r(\delta). \\ \frac{\partial C(\delta, \phi)}{\partial \delta} &\leq 2(1 + r(\delta)) - (1 + r(\delta)) = 1 + r(\delta). \end{aligned} \tag{17}$$

Using the assumption $\pi < \frac{1 - \rho_I}{2}$ it is a matter of algebra to check that for all δ:

$$\frac{d^2 r}{d\delta^2} + \frac{dr}{d\delta} \geq 0,$$

and finally from this inequality, $\frac{dr}{d\delta} < 0$ and $r < \rho_I$ we obtain after some algebra:

$$\frac{\partial^2 \Pi(\delta, \phi)}{\partial \delta^2} + \frac{\partial \Pi(\delta, \phi)}{\partial \delta} < -\frac{1}{\rho_I}\left(1 - \frac{\varepsilon}{1 + \rho_I + \varepsilon}\delta\right)\left(\frac{dr}{d\delta} + \frac{d^2 r}{d\delta^2}\right) \leq 0. \tag{18}$$

To save on notation, we will drop from now on the arguments of these functions when it does not lead to ambiguity.

3. $D^* = 0$ is not optimal It suffices to realize that $\frac{\partial V(D, 0; \phi)}{\partial D} = \Pi(0, \phi) = 1 - \frac{r(0)}{\rho_I} > 0$.

4. The solution (D^*, δ^*) of the maximization problem in equation (10) exists, is unique, and satisfies (BF) with equality, i.e. $\frac{1 + \rho_I}{\rho_I}\mu + (\Pi(\delta^*, \phi) - C(\delta^*, \phi))D^* = 0$

We are going to prove existence and uniqueness in the particular case that there exist $\delta_\Pi, \delta_C \in [0, 1]$ such that $\frac{\partial \Pi(\delta_\Pi, \phi)}{\partial \delta} = \frac{\partial C(\delta_C, \phi)}{\partial \delta} = 0$. This will ensure that the solution of the

maximization problem is interior in δ. The other cases are treated in an analogous way but might give rise to corner solutions in δ.[46]

First, since $\Pi(\delta, \phi)$ is concave in δ we have that $\frac{\partial \Pi(\delta, \phi)}{\partial \delta} \geq 0$ iff $\delta \leq \delta_\Pi$. Since $C(\delta, \phi)$ is convex in δ we have that $\frac{\partial C(\delta, \phi)}{\partial \delta} \geq 0$ iff $\delta \geq \delta_C$. It is easy to prove from equation (16) that $\delta_C < \delta_\Pi$.

Now, let (D^*, δ^*) be a solution to the maximization problem. The first order conditions (FOC) that characterize an interior solution (D^*, δ^*) are:

$$
\begin{aligned}
(1 + \theta)\Pi - \theta C &= 0, & (19) \\
(1 + \theta)\frac{\partial \Pi}{\partial \delta} - \theta \frac{\partial C}{\partial \delta} &= 0, \\
\theta \left[\frac{1 + \rho_I}{\rho_I} \mu + (\Pi - C) D^* \right] &\geq 0, \\
\theta &\geq 0,
\end{aligned}
$$

where θ is the Lagrange multiplier associated with (BF) and we have used that $D^* > 0$ in order to eliminate it from the second equation.

If $\theta = 0$ then the second equation implies $\delta^* = \delta_\Pi$ and thus $\Pi(\delta^*, \phi) \geq \Pi(0, \phi) > 0$ and the first equation is not satisfied. Therefore we must have $\theta > 0$ so that (BF) is binding at the optimum. Now we can eliminate θ from the previous system of equations, which gets reduced to:

$$
\begin{aligned}
\frac{\partial \Pi(\delta^*, \phi)}{\partial \delta} C(\delta^*, \phi) &= \frac{\partial C(\delta^*, \phi)}{\partial \delta} \Pi(\delta^*, \phi), & (20) \\
\frac{1 + \rho_I}{\rho_I} \mu &= [C(\delta^*, \phi) - \Pi(\delta^*, \phi)] D^*. & (21)
\end{aligned}
$$

We are going to show that equation (20) has a unique solution in δ. For $\delta \leq \delta_C < \delta_\Pi$, we have $\frac{\partial C}{\partial \delta} \leq 0 < \frac{\partial \Pi}{\partial \delta}$ and thus the left hand side (LHS) of (20) is strictly bigger than the RHS. For $\delta \geq \delta_\Pi > \delta_C$, we have $\frac{\partial \Pi}{\partial \delta} \leq 0 < \frac{\partial C}{\partial \delta}$ and thus RHS of (20) is strictly bigger.

Now, the function $\frac{\partial C(\delta, \phi)}{\partial \delta} \Pi(\delta, \phi)$ is strictly increasing in the interval (δ_C, δ_Π) since both terms are positive and increasing. Thus, it suffices to prove that for $\delta \in (\delta_C, \delta_\Pi)$ the function $\frac{\partial \Pi(\delta, \phi)}{\partial \delta} C(\delta, \phi)$ is decreasing.[47] Using the the bounds in (17), inequality (18) and $\frac{\partial^2 \Pi}{\partial \delta^2} < 0, \frac{\partial \Pi}{\partial \delta} > 0$ for $\delta \in (\delta_C, \delta_\Pi)$, we have:

$$
\frac{\partial}{\partial \delta} \left(\frac{\partial \Pi}{\partial \delta} C \right) = \frac{\partial^2 \Pi}{\partial \delta^2} C + \frac{\partial \Pi}{\partial \delta} \frac{\partial C}{\partial \delta} \leq (1 + r) \left(\frac{\partial^2 \Pi}{\partial \delta^2} + \frac{\partial \Pi}{\partial \delta} \right) \leq 0.
$$

---

[46]More precisely, if for all $\delta \in [0, 1]$ $\frac{\partial C(\delta, \phi)}{\partial \delta} > 0$ we might have $\delta^* = 0$ and if for all $\delta \in [0, 1]$, $\frac{\partial \Pi(\delta, \phi)}{\partial \delta} > 0$ we might have $\delta^* = 1$.

[47]This is not trivial since $C(\delta, \phi)$ is increasing.

This concludes the proof on the existence and uniqueness of a δ^* that satisfies the necessary FOC in (20).

Now, for given δ^*, the other necessary FOC (21) determines D^* uniquely.[48]

5. δ^* is independent from μ and D^* is strictly increasing in μ Equation (20) determines δ^* and is independent from μ. Then equation (21) shows that D^* is increasing in μ.

6. δ^* is decreasing in ϕ and, if $\delta^* \in (0,1)$, it is strictly decreasing Let $\delta(\phi)$ be the solution of the maximization problem of the bank for given ϕ. Let us assume that $\delta(\phi)$ satisfies the FOC (20). The case of corner solutions is analyzed in an analogous way.

We have proved in Step 3 above that the function $\frac{\partial \Pi}{\partial \delta}C - \frac{\partial C}{\partial \delta}\Pi$ is decreasing in δ around $\delta(\phi)$. In order to show that $\delta(\phi)$ is decreasing, it suffices to show that the derivative of this function w.r.t. ϕ is negative. Using the definitions of $C(\delta, \phi), \Pi(\delta, \phi)$ after some (tedious) algebra we obtain:

$$\frac{\partial}{\partial \phi}\left[\frac{\partial \Pi}{\partial \delta}C - \frac{\partial C}{\partial \delta}\Pi\right] = -(1+\rho_I) - \frac{1}{\rho_I}\frac{1+\rho_I}{1+\rho_I+\varepsilon}\left[(1+\rho_I)\left(\frac{dr}{d\delta}\delta - r\right) + \varepsilon\right].$$

Now we have $\frac{d}{d\delta}\left(\frac{dr}{d\delta}\delta - r\right) = \frac{d^2r}{d\delta^2}\delta \geq 0$ and thus $\frac{dr}{d\delta}\delta - r \geq \frac{dr}{d\delta}\delta - r\big|_{\delta=0} = -r(0)$, and finally:

$$\frac{\partial}{\partial \phi}\left[\frac{\partial \Pi}{\partial \delta}C - \frac{\partial C}{\partial \delta}\Pi\right] \leq -(1+\rho_I) - \frac{1}{\rho_I}\frac{1+\rho_I}{1+\rho_I+\varepsilon}\left[-(1+\rho_I)r(0) + \varepsilon\right]$$

$$< -(1+\rho_I) + \frac{1}{\rho_I}(1+\rho_I)r(0) = -(1+\rho_I)\left(1 - \frac{r(0)}{\rho_I}\right) < 0.$$

This concludes the proof that $\frac{d\delta}{d\phi} < 0$.[49]

7. δ^*D^* is decreasing with ϕ. If $\delta^* > 0$ it is strictly decreasing

Let $\delta(\phi), D(\phi)$ be the solution of the maximization problem of the bank for given ϕ. We have:

$$\frac{1+\rho_I}{\rho_I}\mu = \left[C(\delta(\phi), \phi) - \Pi(\delta(\phi), \phi)\right]D(\phi).$$

Let $\phi_1 < \phi_2$. In Step 5 we showed that $\delta(\phi_1) \geq \delta(\phi_2)$. If $\delta(\phi_2) = 0$ then trivially $\delta(\phi_1)D(\phi_1) \geq \delta(\phi_2)D(\phi_2) = 0$. Let us suppose that $\delta(\phi_2) > 0$. Since trivially $\Pi(\delta(\phi_1), \phi_1)D(\phi_1) \geq \Pi(\delta(\phi_2); \phi_2)D(\phi_2)$, we must have $C(\delta(\phi_1), \phi_1)D(\phi_1) \geq C(\delta(\phi_2), \phi_2)D(\phi_2)$. Now, suppose that $\delta(\phi_1)D(\phi_1) \leq \delta(\phi_2)D(\phi_2)$, then we have the following two inequalities:

$$(1+\rho_I)(1+\phi_1)\delta(\phi_1)D(\phi_1) < (1+\rho_I)(1+\phi_2)\delta(\phi_2)D(\phi_2),$$

$$(1+r(\delta(\phi_1)))(1-\delta(\phi_1)) \leq (1+r(\delta(\phi_2)))(1-\delta(\phi_2)),$$

---

[48] Let us observe that for all δ, $C(\delta, \phi) \geq 1 > \Pi(\delta, \phi)$.

[49] In the case of corner solution $\delta^*(\phi) = 1$, we might have $\frac{d\delta^*}{d\phi} = 0$ and obviously for $\delta^*(\phi) = 0$, $\frac{d\delta^*}{d\phi} = 0$.

that imply $C(\delta(\phi_1), \phi_1)D(\phi_1) < C(\delta(\phi_2), \phi_2)D(\phi_2)$, but this contradicts our assumption. Thus, $\delta(\phi_1)D(\phi_1) > \delta(\phi_2)D(\phi_2)$.∎

Proof of Proposition 3   Let us denote $(D(\phi), \delta(\phi))$ the solution of the bank's optimization problem for every excess cost of crisis liquidity $\phi \geq 0$. Proposition 2 states that $\delta(\phi)D(\phi)$ is decreasing in ϕ. For $\phi \in [0, \overline{\phi}]$ let us define $\Sigma(\phi) = \Phi(\delta(\phi)D(\phi)) - \phi$. This function represents the difference between the excess cost of liquidity during a crisis by banks' decisions and banks' expectation on such variable. Since Φ is an increasing function on the aggregate demand of funds during a crisis the function $\Sigma(\phi)$ is strictly decreasing. Because of the uniqueness of the solution to the problem that defines $(D(\phi), \delta(\phi))$, the function is also continuous. Moreover, we trivially have $\Sigma(0) \geq 0$ and $\lim_{\phi \to \infty} \Sigma(\phi) = -\infty$. Therefore there exists a unique $\phi^e \in \mathbb{R}^+$ such that $\Sigma(\phi^e) = 0$. By construction $D(\phi^e), \delta(\phi^e), \phi^e$ is the unique equilibrium of the economy.∎

Proof of Proposition 4   We are going to follow the notation used in the proof of Proposition 3. Let Φ_1, Φ_2 be two curves describing the inverse supply of liquidity during a crisis and assume they satisfy $\Phi_1(x) > \Phi_2(x)$ for all $x > 0$. Let us denote $\Sigma_i(\phi) = \Phi_i(\delta(\phi)D(\phi)) - \phi$ for $i = 1, 2$. By construction we have $\Sigma_1(\phi_1^e) = 0$. Let us suppose that $\phi_1^e < \phi_2^e$. Then we would have:

$$\Sigma_2(\phi_2^e) = \Phi_2(\delta(\phi_2^e)D(\phi_2^e)) - \phi_2^e \leq \Phi_1(\delta(\phi_2^e)D(\phi_2^e)) - \phi_2^e < \Phi_1(\delta(\phi_1^e)D(\phi_1^e)) - \phi_1^e = \Sigma_1(\phi_1^e) = 0, \tag{22}$$

where in the first inequality we use the assumption $\Phi_2(x) \leq \Phi_1(x)$ for $x \geq 0$, and in the second inequality we use that if $\phi_1^e < \phi_2^e$ then $\delta(\phi_2^e)D(\phi_2^e) \leq \delta(\phi_1^e)D(\phi_1^e)$ (Proposition 2), and that $\Phi_1(\cdot)$ is increasing.

Notice that the sequence of inequalities in (22) implies $\Sigma_2(\phi_2^e) < 0$, which contradicts the definition of ϕ_2^e. We must therefore have $\phi_1^e \geq \phi_2^e$. Now Proposition 2 implies that $\delta_1^e \leq \delta_2^e, \delta_1^e D_1^e \leq \delta_2^e D_2^e, r_1^e \geq r_2^e$. Let us suppose that $\delta_2^e \in (0, 1)$ then the first inequality in (22) is strict, since $\delta_2^e D_2^e > 0$, and we can straightforwardly check that the previous argument implies $\phi_1^e > \phi_2^e$. Now, since $\delta_2^e \in (0, 1)$, Proposition 2 implies that $\delta_1^e < \delta_2^e, \delta_1^e D_1^e < \delta_2^e D_2^e$, and $r_1^e > r_2^e$.∎

Proof of Proposition 5   We consider two cases.
Case 1: Debt issuance exogenously fixed We are going to follow the notation in the

proof of Proposition 2. Using the definition of $W(D, \delta)$ we have

$$
\begin{aligned}
\frac{\partial W(D, \delta)}{\partial \delta} &= \frac{\partial V(D, \delta; \Phi(\delta D))}{\partial \delta} + \frac{\partial V(D, \delta; \Phi(\delta D))}{\partial \phi} D \Phi'(\delta D) + \frac{\partial U(D, \delta)}{\partial \delta} \\
&= \frac{\partial V(D, \delta; \Phi(\delta D))}{\partial \delta} = D \frac{\partial \Pi(\delta, \Phi(\delta D))}{\partial \delta}
\end{aligned}
\tag{23}
$$

and

$$
\begin{aligned}
\frac{\partial^2 W(D, \delta)}{\partial \delta^2} &= D \frac{\partial^2 \Pi(D, \delta; \Phi(\delta D))}{\partial \delta^2} + \frac{\partial^2 \Pi(D, \delta; \Phi(\delta D))}{\partial \delta \partial \phi} D^2 \Phi'(\delta D) \\
&= \frac{\partial^2 \Pi(D, \delta; \Phi(\delta D))}{\partial \delta^2} - \frac{1}{\rho_I} \frac{(1 + \rho_I)\varepsilon}{1 + \rho_I + \varepsilon} D^2 \Phi'(\delta D) < 0,
\end{aligned}
$$

where in the last inequality we have used that $\Pi(D, \delta; \phi)$ is concave in δ and that $\Phi'(\cdot) > 0$. Notice that $W(D, \delta)$ is concave in δ.

Denote the exogenous amount of debt referred in the proposition as $\overline{D} > 0$. Let (ϕ^e, δ^e) be the equilibrium of the economy in which banks do not decide $\overline{D}$. Let us suppose that $\delta^e \in (0, 1)$; the argument if $\delta^e = 0, 1$ is analogous and will be omitted for brevity. By analogy with the system of equations in (19), the competitive equilibrium is characterized by:

$$
(1 + \theta) \frac{\partial \Pi(\delta^e, \phi^e)}{\partial \delta} - \theta \frac{\partial C(\delta^e, \phi^e)}{\partial \delta} = 0,
\tag{24}
$$

$$
\theta \left[\frac{1 + \rho_I}{\rho_I} \mu + (\Pi(\delta^e, \phi^e) - C(\delta^e, \phi^e)) \overline{D} \right] \geq 0,
$$

$$
\theta \geq 0,
$$

$$
\phi^e = \Phi(\delta^e \overline{D}).
$$

Now, let δ^s be the solution to the social planner problem. We can distinguish two cases: i) $\theta = 0$. In this case the system of equations (24) implies $\frac{\partial \Pi(\delta^e, \phi^e)}{\partial \delta} = 0$. Now, if we use equation (23) we have

$$
\frac{\partial W(\overline{D}, \delta^e)}{\partial \delta} = \overline{D} \frac{\partial \Pi(\delta^e, \Phi(\overline{D} \delta^e))}{\partial \delta} = \overline{D} \frac{\partial \Pi(\delta^e, \phi^e)}{\partial \delta} = 0
$$

and, therefore, δ^e maximizes the (concave) function $W(\overline{D}, \delta)$. Thus, in this case $\delta^s = \delta^e$.
ii) $\theta > 0$. In this case, from equation (16) we conclude that $\frac{\partial \Pi(\delta, \phi)}{\partial \delta} < 0$ implies $\frac{\partial C(\delta, \phi)}{\partial \delta} > 0$. Now, the first equation in the system (24) implies $\frac{\partial \Pi(\delta^e, \phi^e)}{\partial \delta} > 0, \frac{\partial C(\delta^e, \phi^e)}{\partial \delta} - \frac{\partial \Pi(\delta^e, \phi^e)}{\partial \delta} > 0$. So we have

$$
\frac{\partial W(\overline{D}, \delta^e)}{\partial \delta} = \overline{D} \frac{\partial \Pi(\delta^e, \phi^e)}{\partial \delta} > 0,
$$

and, since $W(\overline{D}, \delta)$ is concave, we have $W(\overline{D}, \delta) < W(\overline{D}, \delta^e)$ for all $\delta < \delta^e$. Now, given that δ^e satisfies (BF) with equality, in order to prove that $\delta^s = \delta^e$ it suffices to show that for

$\delta > \delta^e$ (BF) is not satisfied. This, in turn, is an immediate consequence of the following inequality that is easily checked:

$$\frac{\partial}{\partial \delta} \left[C(\delta, \Phi(\overline{D}\delta)) - \Pi(\delta, \Phi(\overline{D}\delta)) \right] > 0, \text{ for } \delta \geq \delta^e.$$

Case 2: Average maturity exogenously fixed Once we realize that the function $W(D, \delta)$ is concave in D the proof is completely analogous to the previous one and is omitted here. ∎

Proof of Proposition 6 We are going to follow the notation used in the proof of Proposition 2. The proof is organized in five steps:

1. Preliminaries We have seen in the proof of Proposition 5 that:

$$\frac{\partial W(D, \delta)}{\partial \delta} = \frac{\partial V(D, \delta; \Phi(\delta D))}{\partial \delta} = D \frac{\partial \Pi(\delta, \Phi(\delta D))}{\partial \delta}. \tag{25}$$

Similarly we have

$$\frac{\partial W(D, \delta)}{\partial D} = \frac{\partial V(D, \delta; \Phi(\delta D))}{\partial D} = \Pi(\delta, \Phi(\delta D)). \tag{26}$$

2. (BF) is binding at the socially optimal debt structure This is a statement that has been done in the main text just before Proposition 6. The proof is analogous to the one for the maximization problem of the bank that we did in Step 4 of the proof of Proposition 2. The only difference is that ϕ is not taken as given but as the function $\Phi(\delta D)$ in D and δ.

3. Definition of function $D^c(\delta)$ and its properties Let $(\phi^e, (D^e, \delta^e))$ be the competitive equilibrium. Let us assume that $\delta^e < 1$. By definition of equilibrium we have $\phi^e = \Phi(\delta^e D^e)$. For every δ let $D^c(\delta)$ be the unique principal of debt such that (BF) is binding, i.e.:

$$\frac{1 + \rho_I}{\rho_I} \mu = [C(\delta, \phi^e) - \Pi(\delta, \phi^e)] D^c(\delta). \tag{27}$$

Differentiating w.r.t. δ:

$$\left[\frac{\partial C(\delta, \phi^e)}{\partial \delta} - \frac{\partial \Pi(\delta, \phi^e)}{\partial \delta} \right] D^c(\delta) + [C(\delta, \phi^e) - \Pi(\delta, \phi^e)] \frac{dD^c(\delta)}{d\delta} = 0. \tag{28}$$

Using the characterization of δ^e in equation (20), the inequalities $C(\delta, \phi^e) \geq 1 > \Pi(\delta, \phi^e)$ imply $\frac{\partial C(\delta^e, \phi^e)}{\partial \delta} - \frac{\partial \Pi(\delta^e, \phi^e)}{\partial \delta} > 0$ and, then, we can deduce from the equation above that $\frac{dD^c(\delta^e)}{d\delta} < 0$. Since (BF) is binding at the optimal debt structure we can think of the bank

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problem as maximizing the univariate function $V(D^c(\delta), \delta; \phi^e)$. Hence δ^e must satisfy the necessary FOC for an interior solution to the maximization of $V(D^c(\delta), \delta; \phi^e)$:

$$\frac{dV(D^c(\delta), \delta^e; \phi^e)}{d\delta} = 0 \Leftrightarrow D^c(\delta^e)\frac{\partial\Pi(\delta^e, \phi^e)}{\partial\delta} + \Pi(\delta^e, \phi^e)\frac{dD^c(\delta^e)}{d\delta} = 0, \qquad (29)$$

which multiplying by δ^e can be written as

$$D^c(\delta^e)\frac{\partial\Pi(\delta^e, \phi^e)}{\partial\delta^e}\delta^e = \Pi(\delta^e, \phi^e)\left(-\frac{dD^c(\delta^e)}{d\delta}\delta^e\right).$$

Since $\frac{\partial\left(\Pi - \frac{\partial\Pi}{\partial\delta}\delta\right)}{\partial\delta} = -\frac{\partial^2\Pi}{\partial\delta^2}\delta \geq 0$ and $\Pi(0, \phi) - \frac{\partial\Pi(0, \phi)}{\partial\delta}0 > 0$, we have $\Pi(\delta, \phi) > \frac{\partial\Pi(\delta, \phi)}{\partial\delta}\delta$ for all $\delta \in [0, 1]$ and the previous equation implies

$$D^c(\delta^e) > -\frac{dD^c(\delta^e)}{d\delta}\delta^e \Leftrightarrow \left.\frac{d\left(\delta D^c(\delta)\right)}{d\delta}\right|_{\delta=\delta^e} > 0.$$

4. Evaluation of $\left.\frac{d(D^s(\delta))}{d\delta}\right|_{\delta=\delta^e}$ **and** $\left.\frac{d(\delta D^s(\delta))}{d\delta}\right|_{\delta=\delta^e}$ For every δ, let $D^s(\delta)$ be the unique principal of debt such that (BF) is binding, i.e.

$$\frac{1+\rho_I}{\rho_I}\mu = [C(\delta, \Phi(\delta D^s(\delta))) - \Pi(\delta, \Phi(\delta D^s(\delta)))]D^s(\delta).$$

Differentiating w.r.t. δ, we obtain

$$\left[\frac{\partial C(\delta, \Phi)}{\partial\delta} - \frac{\partial\Pi(\delta, \Phi)}{\partial\delta}\right]D^s(\delta) + [C(\delta, \Phi(\delta D^s(\delta))) - \Pi(\delta, \Phi(\delta D^s(\delta)))]\frac{dD^s(\delta)}{d\delta} + \qquad (30)$$

$$+ \left[\frac{\partial C(\delta, \Phi)}{\partial\phi} - \frac{\partial\Pi(\delta, \Phi)}{\partial\phi}\right]\Phi'(\delta D^s(\delta))\frac{d\left(\delta D^s(\delta)\right)}{d\delta} = 0.$$

By construction, $D^s(\delta^e) = D^c(\delta^e) = D^e$. Now, subtracting equation (28) from equation (30) at the point $\delta = \delta^e$ we obtain

$$[C(\delta^e, \phi^e) - \Pi(\delta^e, \phi^e)]\left(\frac{dD^s(\delta^e)}{d\delta} - \frac{dD^c(\delta^e)}{d\delta}\right) + \left[\frac{\partial C(\delta^e, \phi^e)}{\partial\phi}\right. \qquad (31)$$

$$\left.- \frac{\partial\Pi(\delta^e, \phi^e)}{\partial\phi}\right]\Phi'(\delta^e D^e)\left.\frac{d\left(\delta D^s(\delta)\right)}{d\delta}\right|_{\delta=\delta^e} = 0.$$

Suppose that $\left.\frac{d(\delta D^s(\delta))}{d\delta}\right|_{\delta=\delta^e} \leq 0$, then we would have $\frac{dD^s(\delta^e)}{d\delta} \geq \frac{dD^c(\delta^e)}{d\delta}$, since trivially $\frac{\partial C(\delta^e, \phi^e)}{\partial\phi} - \frac{\partial\Pi(\delta, \phi^e)}{\partial\phi} > 0$. But then

$$\left.\frac{d\left(\delta D^s(\delta)\right)}{d\delta}\right|_{\delta=\delta^e} = D^s(\delta^e) + \frac{dD^s(\delta^e)}{d\delta}\delta^e > D^c(\delta^e) + \frac{dD^c(\delta^e)}{d\delta}\delta^e = \left.\frac{d\left(\delta D^c(\delta)\right)}{d\delta}\right|_{\delta=\delta^e} > 0,$$

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which contradicts the hypothesis. We must thus have $\left.\frac{d(\delta D^s(\delta))}{d\delta}\right|_{\delta=\delta^e} > 0$, in which case equation (31) implies $\frac{dD^s(\delta^e)}{d\delta} < \frac{dD^c(\delta^e)}{d\delta} < 0$.

5. Evaluation of $\left.\frac{dW(D^s(\delta),\delta)}{d\delta}\right|_{\delta=\delta^e}$ Using equations (25) and (26), we have:

$$
\begin{aligned}
\frac{dW(D^s(\delta),\delta)}{d\delta} &= \frac{\partial W(D^s(\delta),\delta)}{\partial\delta} + \frac{\partial W(D^s(\delta),\delta)}{\partial D}\frac{dD^s(\delta)}{d\delta} \\
&= D^s(\delta)\frac{\partial\Pi(\delta,\Phi(\delta D^s(\delta)))}{\partial\delta} + \Pi(\delta,\Phi(\delta D^s(\delta)))\frac{dD^s(\delta)}{d\delta}.
\end{aligned}
$$

And, using $\frac{dD^s(\delta^e)}{d\delta} < \frac{dD^c(\delta^e)}{d\delta}$ and (29), we obtain:

$$
\left.\frac{dW(D^s(\delta),\delta)}{d\delta}\right|_{\delta=\delta^e} < D^s(\delta^e)\frac{\partial\Pi(\delta^e,\phi^e)}{\partial\delta} + \Pi(\delta^e,\phi^e)\frac{dD^c(\delta^e)}{d\delta} = 0.
$$

Summing up, having

$$
\left.\frac{dW(D^s(\delta),\delta)}{d\delta}\right|_{\delta=\delta^e} < 0, \quad \left.\frac{dD^s(\delta)}{d\delta}\right|_{\delta=\delta^e} < 0, \text{ and } \left.\frac{d(\delta D^s(\delta))}{d\delta}\right|_{\delta=\delta^e} > 0,
$$

implies that a social planner can increase welfare by fixing some $\delta^s < \delta^e$, and suggests that doing so will produce higher leverage and lower refinancing needs than in the unregulated competitive equilibrium.∎

Proof of Proposition 7 Let $(\phi^s,(D^s,\delta^s))$ be the socially efficient equilibrium. The sketch of the proof is as follows:

1. For any fixed ϕ banks' optimal choice of δ depends only on τ (and not on the lump-sum tax rebate M) and as τ increases δ decreases. From here we can show that if banks' expectation on the excess cost of liquidity in a crisis is $\phi^s < \phi^e$ there exists a Pigovian tax $\tau^P > 0$ that induces the socially efficient choice of maturity.

2. For $\phi = \phi^s$ and Pigovian tax τ^P defined above, once banks have taken their maturity decision δ^s they issue as much debt D as (BF) allows and at this point the amount of the lump-sum transfer M matters. The effect of the net per period transfer $\tau^P\delta^s D - M \geq 0$ from banks to the SP is to reduce banks' equity value at the N state and thus to strictly tighten (BF) with respect to the situation in which the SP directly regulates maturity to its social optimum $1/\delta^s$ unless there is full rebate of the Pigovian tax, i.e. $M = \tau^P\delta^s D$. More precisely, it can be shown that D is strictly increasing with M and that $D = D^s$ if and only if $M = \tau^P\delta^s D^s$.

3. Our candidate for optimal Pigovian tax scheme is (τ^P, M^P) with $M^P = \tau^P\delta^s D^s$. By construction, under this tax scheme if banks' expectation on the excess cost of liquidity

in a crisis is ϕ^s, then banks' optimal funding structure coincides with the socially efficient structure (D^s, δ^s), which in turn satisfies $\phi^s = \Phi(\delta^s D^s)$, confirming ϕ^s as an expectation compatible with the equilibrium.

The most cumbersome details of the proof are analogous to those in the proof of Proposition 2 and are ommitted for brevity. They are available from the authors upon demand.∎

Proof of Proposition 8   Let us recall that the introduction of insurance does not change the value of equity at the N state, i.e. $E(D, \delta, \theta; \phi) = E(D, \delta; \phi)$ for all θ. In addition banks choose full insurance, $\theta = 1$, and the only financial constraint is (LL) that can be written $E(D, \delta, 1; \phi) = E(D, \delta; \phi) > 0$. For the next steps, we follow the notation introduced in the proof of Proposition 2.

1. Insurance increases social welfare in the regulated economy Let (D^s, δ^s) be the socially optimal debt structure in the absence of insurance. In the proof of Proposition 6 we showed that (BF) is binding at (D^s, δ^s). In fact, we have $\frac{\partial W(D^s, \delta^s)}{\partial D} > 0$. Step 1 in the proof of Proposition 2 states that (LL) is satisfied with slack, i.e.

$$E(D^s, \delta^s; \Phi(\delta^s D^s)) > 0,$$

and thus by continuity there are values $D' > D^s$ such that $E(D', \delta^s; \Phi(\delta^s D')) > 0$ and $W(D', \delta^s) > W(D^s, \delta^s)$. Introducing insurance makes debt structures such as (D', δ^s) feasible and, hence, increases welfare relative to the regulated economy without insurance.

2. Under insurance the competitive expected maturity is shorter than the socially optimal one When insurance is introduced the relevant financial constraint faced both by banks in the unregulated equilibrium (for given ϕ) and by the social planner (for $\phi = \Phi(\delta D)$) is (LL) and is binding. From here, the proof is analogous to that of Proposition 6 and we omit it for brevity.∎

B   Debt structures inducing default during crises

In this section we examine the possibility that a bank decides to expose itself to the risk of defaulting on its debt obligations and being (physically) liquidated during systemic crises. First, we describe the sequence of events following a bank's default. Second, we show how the debt of the bank is valued by savers that correctly anticipate this course of events (analogous to Section 4). After that, we make explicit the bank's optimal funding problem in the context of default during crises (analogous to Section 5).

Default and liquidation If at any period the bank is not able to satisfy its refinancing needs, it defaults and its liquidation yields a residual value $L \geq 0$. Partial liquidation is not allowed. We assume that under default L is distributed equally among all debtholders independently of their contract having just matured or not. This eliminates the type of preemptive runs studied by He and Xiong (2009a). It is easy to realize that if the bank does not rely on bridge financing and defaults in case of facing refinancing needs during a crisis, then it is optimal for the bank to make its debt mature in a perfectly correlated manner since this minimizes the probability of default. Hence we assume that the debt issued by the bank when getting rid of the (BF) constraint has perfectly correlated maturities.

Savers' required maturity premium when default is anticipated From a saver's perspective, there are three states relevant for the valuation of a given debt contract: personal patience ($i = P$), personal impatience in a normal period ($i = IN$), and personal impatience in a crisis period ($i = IC$).

Let $l = L/D < 1$ be the fraction of the principal of debt which is recovered in case of liquidation and let Q_i be the present value of expected losses due to default as evaluated from each of the states i just after the uncertainty regarding the corresponding period has realized and conditional on the debt not having matured in such period. Losses are measured relative to the benchmark case without default in which at maturity savers recover 100% of the principal. These values satisfy the following system of recursive relationships:

$$
\begin{aligned}
Q_P &= \frac{1}{1 + \rho_P} \left[\delta \varepsilon (1 - l) + (1 - \delta) \{ (1 - \varepsilon)[(1 - \gamma) Q_P + \gamma Q_{IN}] + \varepsilon Q_{IC} \} \right], \\
Q_{IN} &= \frac{1}{1 + \rho_I} \left[\varepsilon \delta (1 - l) + (1 - \delta) \left[(1 - \varepsilon) Q_{IN} + \varepsilon Q_{IC} \right] \right], \\
Q_{IC} &= \frac{1}{1 + \rho_I} (1 - \delta) Q_{IN}.
\end{aligned}
$$

These expressions essentially account for the principal $1 - l > 0$ which is lost whenever the saver's debt contract matures in a state of crisis. First equation reflects that default as well as any of three states i may follow state P. The second equation reflects that impatience is an absorbing state. The last equation reflects that a crisis period can only be followed by a normal period.

The value of a debt contract $(1, r, \delta)$ a patient saver in a normal period, when default is expected if the bank runs into refinancing needs during a crisis, can then be written as

$$
U_P^d(r, \delta) = U_P(r, \delta) - Q_P(\delta),
$$

where $U_P(r, \delta)$ is the value of the same contract in the scenario in which the principal is always recovered at maturity, whose expression is given in (2).

Now, let $r^d(\delta)$ be the interest rate yield that the bank offers in the default setting, which satisfies $U_P^d(r^d(\delta), \delta) = 1$. Since the non-default yield $r(\delta)$ satisfies $U_P(r(\delta), \delta) = 1$, the equation $U_P^d(r^d(\delta), \delta) = U_P(r(\delta), \delta)$ allows us to express $r^d(\delta)$ as the sum of $r(\delta)$ and a default-risk premium:

$$r^d(\delta) = r(\delta) + \frac{1}{D} \frac{(1 + \rho_I) \, \delta \varepsilon (D - L)}{1 + \rho_I + (1 - \delta) \varepsilon}.$$

It is easy to observe that the default-risk premium $r^d(\delta) - r(\delta)$ is increasing and convex in δ, increasing in ε, decreasing in L, and increasing in D. Using Proposition 1 we deduce that $r^d(\delta)$ is convex in δ but that, given that increasing δ increases the probability of default, $r^d(\delta)$ is not necessarily decreasing in δ.

Banks' optimal funding structure inducing default   If the bank does not satisfy the bridge financing constraint and thus defaults whenever it faces refinancing needs during a crisis, its equity value in normal times $E^d(D, \delta)$ will satisfy the following recursive equation:

$$E^d(D, \delta) = \frac{1}{1+\rho_I} \left[\mu - r^d D + (1-\varepsilon) E^d(D, \delta) + \{\varepsilon\delta \cdot 0 + (1-\delta) \frac{1}{1+\rho_I} [\mu - r^d D + E^d(D, \delta)]\} \right],$$

whose solution yields:

$$E^d(D, \delta) = \frac{1 + \rho_I + \varepsilon(1 - \delta)}{(1 + \rho_I)^2 - (1 + \rho_I)(1 - \varepsilon) - \varepsilon(1 - \delta)} (\mu - r^d D).$$

In this context, the problem determining the bank's optimal debt structure decision in the absence of the bridge financing constraint can be written as:

$$\max_{D \geq 0, \, \delta \in [0,1]} \quad V^d(D, \delta) = D + E^d(D, \delta), \tag{32}$$
$$\text{s.t.} \quad E^d(D, \delta) \geq 0, \quad \text{(LL)}$$

where (LL) is trivially equivalent to $\mu - r^d D \geq 0$.

Figure 6 in the main text has been generated by numerically solving this problem for each value of ϕ^e and L, and finding $L^{\max}(\phi^e)$ as the (maximum) value of L for which the total market value of the bank under the best debt structure compatible with (BF) equals the total market value that the bank can attain solving (32).

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