DISCUSSION PAPER SERIES

No. 8279

INTERTEMPORAL MOVIE DISTRIBUTION: VERSIONING WHEN CUSTOMERS CAN BUY BOTH VERSIONS

Joan Calzada and Tommaso Valletti

INDUSTRIAL ORGANIZATION

Centre for Economic Policy Research

www.cepr.org

Available online at: **www.cepr.org/pubs/dps/DP8279.asp**

ISSN 0265-8003

INTERTEMPORAL MOVIE DISTRIBUTION: VERSIONING WHEN CUSTOMERS CAN BUY BOTH VERSIONS

Joan Calzada, Universitat de Barcelona and IREA-GiM
Tommaso Valletti, Imperial College London, University of Rome and CEPR

Discussion Paper No. 8279
March 2011

Centre for Economic Policy Research
77 Bastwick Street, London EC1V 3PZ, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

CEPR Discussion Paper No. 8279

March 2011

ABSTRACT

Intertemporal movie distribution: Versioning when customers can buy both versions*

We study a model of film production, distribution and consumption. The studio can release two goods, a theatrical and a video version, and has to decide on its versioning and sequencing strategy. In contrast with the previous literature, we allow for the possibility that consumers watch both versions. This simple extension leads to novel results. It now becomes optimal to introduce versioning if the goods are not too substitutes for one another, even when production costs are zero (pure information goods). We also demonstrate that a 'day-and-date' strategy can be optimal when the studio is integrated with the exhibition and distribution channels. In contrast, a 'video window' is typically the outcome of the negotiation between the studio and independent distributors and exhibitors.

JEL Classification: D21, L12, L82 and M31
Keywords: distribution channels, information goods, movie industry, product segmentation, sequential release and versioning

Joan Calzada
Departamento de Política Econòmica
Universitat de Barcelona
Av. Diagonal 690
08034 Barcelona
SPAIN

Email: Calzada@ub.edu

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=161156

Tommaso Valletti
Imperial College London
Tanaka Business School
South Kensington Campus
London
SW7 2AZ

Email: t.valletti@imperial.ac.uk

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=145352

* We thank the Editor-in-Chief, the Area Editor, two anonymous referees, Guillermo Caruana, Peter Claeys, Ricard Gil, Steve Wildman, David Waterman and seminar participants in Barcelona (IREA), Berlin (DIW), Ljubljana (EARIE), Madrid (NEREC), Milan (Bocconi) and Valencia (SAE) for useful comments.

Submitted 18 February 2011

1 Introduction

This paper examines the sale of movies through sequential distribution channels. Many products such as movies, books, and video games are introduced sequentially through different channels. First, they are offered in specialized stores with selected audiences and afterwards they are made widely available through chain stores. Firms use this strategy to segment consumers reluctant to buy from the first channel and to stimulate consumers to buy the product repeatedly (Lehmann and Weinberg, 2000).

In the movie industry, producers' revenues are crucially determined by the number of versions being offered and by the timing of their release. Movies are typically first shown in a theatre, followed later by their video version. This responds to the principle of the "second-best alternative", according to which the producer should initially offer the movie in the channel that generates the highest revenue in the least amount of time. Then, the movie should cascade down to markets with lower returns per unit of time. Historically, this has resulted in theatrical screening, followed by pay-TV programming, home video, network television, and finally local television syndication (Owen and Wildman, 1992; Eliashberg et al., 2006; Vogel, 2007). All these various channels have distinct features that might affect the success of the movie (Henning-Thurau et al., 2006). Theatre box office and home video represent the two major sources of revenues, with home video consistently at the top since the early 90s (Waterman, 2005).

An essential aspect in the sequencing strategy of a movie is the lapse of time between its initial release in theatres and its debut on video, which is called "video window". In the last decade, the inter-release time has decreased gradually from the 6 months that were the norm for many years to approximately 3 or 4 months (Vogel, 2007; Frank, 1999; Nelson et al. 2007). While part of this change can be attributed to advances in the home video market, there is also evidence of frictions between distribution channels, in particular between producers and exhibitors. For example, Disney decided in 2010 to accelerate the release of DVD and Blu-ray Disc versions of "Alice in Wonderland", a 3D motion picture directed by Tim Burton, shortly after its theatrical screening - both in the U.S. and in Europe.[1] In the U.K., the big exhibitors first reacted by refusing to book any movie that would not have a guaranteed 4-month run, but later had to concede and allowed Disney to cut the gap between the theatrical opening and the DVD release to 12 weeks.[2] The understanding of these changes is the main motivation of our research. We analyze how producers like Disney commercialize their movies through separate distribution channels. In particular, we examine how producers negotiate the release sequence of the theatrical and video versions with independent exhibitors and distributors who might

[1] Hollywood Reporter (Los Angeles): "Bob Iger wasn't bluffing. The Disney CEO has been telling Wall Street for months of his plans for studio executives to shorten traditional movie release schedules. The time has arrived for the first grand experiment." http://news.yahoo.com/s/nm/20100210/film_nm/us_alice

[2] "Odeon reverses Alice in Wonderland boycott", February 25, 2010. http://news.bbc.co.uk/1/hi/entertainment/8536195.stm.

have contrasting interests. The model we propose allows characterizing the versioning and sequencing decisions in a new and unified way.

The literature on versioning has shown that when a firm introduces a new version this expands the market, but also cannibalizes the existing versions, as some consumers switch to the new alternative. The seminal works of Mussa and Rosen (1978) and Moorthy (1984) show that versioning is optimal, while Stokey (1979) provides conditions under which it is not the best strategy. Salant (1989) reconciles these studies by showing that their differences stem from the marginal cost function. In fact, in the model specifications of Mussa and Rosen (1978) versioning is optimal if the marginal cost function of improving quality is sufficiently convex. By contrast, in the case of information goods, where the variable reproduction costs are constant or even zero, versioning is not profitable and only the high-quality good should be supplied. A number of recent studies in the marketing and management literature have generalized this analysis.[3] Bhargava and Choudhary (2008) show that versioning is profitable when the optimal market share of the lower quality version, offered alone, is greater than the optimal market share of the high quality version, offered alone. Anderson and Dana (2009) find that an "increased percentage differences" condition is needed for versioning to be optimal, that is, the percentage change in total joint surplus (between the consumers and the firm) associated with a product upgrade is increasing in consumers' willingness to pay.

The literature has also tackled the fundamental question of whether to introduce the different versions simultaneously or sequentially. Moorthy and Png (1992) use the framework of Mussa and Rosen (1978) to analyze the optimal introduction of a product for a monopoly seller. They demonstrate that the sequential introduction of the variants, whereby the monopoly first serves the consumers with high preferences and afterwards the consumers with lower preferences, might be profitable when consumers are relatively more impatient than the seller (i.e., they have a higher time discount rate). They employ a model with increasing marginal costs of production, which arguably does not fit information goods very well.[4] Padmanabhan et al. (1997) analyze the monopolist's distribution strategy when consumers are uncertain about the presence of demand externalities. They show that the firm can offer a credible signal of high externalities by first introducing a product with less than full quality and afterwards an upgrade. For example, a part of a software product can be given away for free to signal the attractiveness of the commercially sold version.

Why, then, another paper on versioning? All the works mentioned above assume that consumers buy at most one version of a good, e.g., one cup of coffee, or one model of car.[5] However, this assumption does not really suit the movie industry, where some consumers

[3] For a comprehensive review of the literature on product development, see Krishnan and Ulrich (2001).

[4] Riggins (2004) extends this model to consider that a seller markets its products in an online (Internet) channel and an offline (bricks-and-mortar) store. He shows that low-type consumers are served only in the offline market, when there are not enough consumers to offer a low-quality good online. Wildman (2008) and Zhang (2009) also consider distribution through the Internet and traditional channels.

[5] An exception is Gabszewicz and Wauthy (2003) who however study a duopoly model of competition and are less interested in the core questions of versioning and sequencing we focus on here.

watch both versions, especially those with a high willingness to pay. While empirical studies have demonstrated that individual consumers might watch the same movie more than once in different channels,[6] scant attention has been paid to develop theoretical models that explain how this affects the versioning and sequencing decisions of firms.

Our first contribution to the marketing literature is to extend the model of Mussa and Rosen (1978) and Moorthy and Png (1992) to analyze a simple problem of versioning, where an integrated monopolist sells two variants of a product to a continuum of consumers who can potentially consume both versions. We address a novel problem by considering versioning (the manufacturer's role) and the ability for a consumer to buy multiple items (the consumer's role). This simple innovation has important managerial implications. We show that, if the single unit purchase assumption is imposed, the firm never offers the two versions. In contrast, when consumers are allowed to buy both versions, versioning becomes optimal, with some consumers buying the high-quality good, some buying the low-quality good, and some consumers buying both. The degree of substitutability between versions is essential for this conclusion to hold, because if the two versions are too substitutes for one another, one version cannibalizes the other and our model boils down to the standard result for pure information goods where versioning is not profitable.

We then analyze when the manufacturer should use sequencing. We show that, when the seller and the consumer have the same discount factor, the monopolist will *never* introduce the two versions sequentially. Both versions are always sold at the earliest possible date when versioning is optimal, because delaying the release of one version would only discount profits into the future. However, when the consumers' time discount factor is lower than the seller's, sequencing emerges for pure information goods as long as the versions are imperfect substitutes. This result is in stark contrast to Moorthy and Png (1992), who analyze the case with a convex cost function. We also show that the video window is longer when the firm is unable to commit to the future price of the video version. When versions are imperfect substitutes, the firm has an incentive to delay the introduction of the low quality version as a mechanism to endogeneously differentiate the products, and this incentive is stronger when it lacks the ability to commit ex ante to a sufficiently high price for this version.

Since versions have to be distributed to consumers, the second contribution our paper is to probe the relevance of having separate distribution channels (the channel's role). We show that the versioning and sequencing strategies vary substantially when there is vertical separation between the producer of a movie and the distribution channels. In the movie industry, producers and video distributors might benefit from a quicker video release because potential consumers are still influenced by the publicity from the theatrical release and because

[6] Luan and Sudhir (2007) estimate a flexible structural model of versioning that allows products to be substitutes (and, possibly, even complements). They find that, on average, a consumer's utility from a DVD would be reduced after having viewed the movie in a theatre. The degree of substitutability changes with quality and genre, with highly-rated movies and animation movies showing less substitutability between the theatre and DVD versions.

this moves their video revenues ahead. However, theatre exhibitors might be worried that, if the video window is too short, consumers will wait for the video version. While some papers like Gil (2007 and 2009), Corts (2001) and Mortimer (2007) take into account the vertical separation between the producer and the channels, to our knowledge this is the first work that considers the interaction among the roles of producers, consumers, and channels.

Our model considers that the producer, as copyright holder, designs the whole commercialization strategy of movies. However, it must conduct two separate wholesale bargains, first with the exhibitor to determine a rental price for the movie and then with the video distributor to set a revenue sharing agreement for the video. The channels are strategic players, as the retail prices of the two versions are set independently by each channel. We show that both versioning and sequencing can appear, even when the two versions are perfect substitutes. Incentives are misaligned as vertical separation prevents the producer from fully internalizing the profits by selling just one version. Interestingly, our findings are supported by recent empirical analysis that shows that majors in the U.S. have longer video windows than independent integrated producers, who prefer a quicker introduction of videos (Waterman and Lee, 2010). A further finding of our model, confirmed by the empirical literature, is that the present shrinking of the video window is related both to the distribution of bargaining power of the producer vis-à-vis the various channels, and to the perceived convergence in quality offered by theatrical and video versions.

The rest of the paper is as follows. Section 2 presents relevant stylized facts about the movie industry. Section 3 analyzes the optimal versioning and sequencing strategies of an integrated monopolist. Section 4 re-assesses the main results when the two versions are sold in a vertically-separated chain. Section 5 concludes and offers directions for future research.

2 The movie industry

It is customary to divide the movie industry into three vertically-related sectors: production, distribution, and exhibition. In the U.S., production and distribution are often performed by the same studios, which we simply call "producers" in our model. Hollywood's major studios (Universal Pictures, Paramount Pictures, MGM, Fox Film Corporation, Columbia Pictures, Disney, and Warner Brothers) account for 80 to 90% of the total income from the sale of movies to theatres and other media in the United States (Waterman, 2005). "Exhibitors", by contrast, run theatres and screen movies to attract audiences.

The vertical structure of the movie industry has undergone several changes over the last century. In the 1920s and 1930s major studios owned chains of theatres and formed cartels with other theatre owners to assign the stage run and run length intervals and to set minimum prices and geographic and temporal clearances to theatres in urban areas (Orbach, 2004). In 1948, the U.S. Supreme Court in *United States v. Paramount* considered that these agreements were in violation of the Sherman Act and required the majors to divest themselves of their theatre chains. The Court prevented majors from setting admission prices and exclusivity contracts with exhibitors. These theatre-owning restrictions were partially relaxed in the 1980s and some majors acquired large theatre chains.

Producers and exhibitors typically agree to share a percentage of the theatre box-office receipts (De Vany, 2004).[7] In spite of this, their interests are not perfectly aligned, as contracts are incomplete and give rise to tensions. Vogel (2007), McKenzie (2008), and Gil and Hartmann (2009) argue that, while producers get a percentage of the revenues generated by theatrical versions, exhibitors have incentives to keep the admission prices low in order to raise their popcorn and other concession prices. This situation creates a constant struggle between producers and exhibitors over admission prices, with producers wanting higher admission prices than theatres. In a similar spirit, Gil (2009) argues that the vertical separation of producers and exhibitors causes misaligned incentives when choosing the optimal movie run length, and confirms this hypothesis using Spanish data. What is relevant for the purpose of this paper is that producers have less than perfect control over exhibitors. Exhibitors enjoy some market power because they may be the only theatre in town, because the movies they show are licensed exclusively to them in their geographic area, or because they influence how many weeks the movie will be on screen. As a result, two mark-ups appear in the vertical chain, the first imposed by producers on the rental price paid by exhibitors, and the second set by exhibitors in the admission price because of their monopolistic condition. Producers cannot avoid this with resale price maintenance or with vertical integration, given the strict anti-trust provisions. It is not coincidental that the seminal paper of Spengler (1950) on the double marginalization distortion was inspired by the 1948 *Paramount* case.

Another important channel for producers is the home video, which is sold by "distributors" in our model. In the last decade, this market has experienced extraordinary growth with the introduction of DVDs, and today video rentals and DVD sales are the largest source of domestic revenue for studios. Mortimer (2007) and Ho et al. (2008) analyze different pricing mechanisms used by producers in their contracts with video stores. They show that Blockbuster Video adopted sophisticated revenue sharing agreements with several studios already in 1998, and other retailers were quick to adopt the same instrument. In our model, we analyze the case where producers and video distributors negotiate revenue sharing agreements, and we compare this with the alternative case where producers can sell directly the video version to the consumers. Both cases are empirically relevant as there are examples where the distributor is totally independent from the producer (e.g., Blockbuster or Netflix), as well

[7] Theatres pay the distributor a fee per week and keep a "house nut" (approximately the exhibitor's weekly cost of operating the theatre). Contracts include a sliding scale for sharing box office receipts that exceed the house nut: typically a box-office percentage of 70-90% in the first two weeks, and thereafter distributors' shares decreases to 60%. Swami et al. (1999) argue that this arrangement creates management problems for exhibitors. Producers have a strong incentive to promote the movies intensely only during the first weeks. They propose a programming model to help exhibitors both select and schedule movies in their theatres. Raut et al. (2009) also consider the complexity of these contracts. Despite including a broad range of factors in the optimization problem, possible cannibalization from video releases is not considered.

as cases where the producer sells directly the version via Internet streaming or specialized stores (e.g., Disney or Warner stores).

A final but key feature of the movie industry is the video window, the time lag between the theatrical and the video release of a movie. As explained in the introduction, the recent shrinking trend in the video window might reflect the tension existing between producers and channels, and "Alice" is just one example of this conflict.[8] For many years the industry has discussed about the possibility of simultaneously releasing the theatrical and video versions, and a series of experiments have been carried out in this direction.[9] Some studios even contemplate "day-and-date" strategies, meaning that a title is released across two or more channels on the same day. For instance, the movie "Bubble" directed by Steven Soderbergh was released simultaneously across all channels already back in 2006 by 2929 Entertainment, a company that is vertically-integrated across production, distribution, and exhibition.

Several authors have empirically analyzed the factors affecting the video window. Frank (1999) and Lehmann and Weinberg (2000) show that large windows reduce the cannibalization of the first version, and theatrical marketing and word-of-mouth effects from cinema are used to increase video sales. Luan and Sudhir (2007) consider a model where consumers form expectations about the extent of the video window, and hence they adjust their behavior when producers shorten inter-release times. Hennig-Thurau et al. (2007) analyze the optimal determination of windows in a market with three or more channels to exhibit movies. They also consider how order changes will affect studio revenues and account for regional differences. August and Shin (2010) study sequencing release of movies when going early to the theatre can generate negative congestion externalities.

Most of these studies typically consider models with only two periods, and analyze whether to introduce another version of the movie in the first or in the second period. These two periods are pre-determined (an exception is Prasad et al., 2004). In contrast, we endogenize optimal sequencing in order to examine *when* to introduce a new variant. Another common feature of these studies is that they do not consider the separation of the industry between production, exhibition and distribution. Our paper shows that accounting for both the manufacturers' and the distributors' roles crucially modifies the release strategy of movies.

3 The basic model

We consider a single firm that offers two versions of a product, a high-quality version denoted as H and a low-quality version denoted as L. Our departure from the extant literature is that we allow consumers to buy both versions. Thus consumers can buy one unit of H alone, or

[8] See the comments of John Fithian, President & CEO of the National Association of Theatre Owners, in 2009. "I believe that the two biggest threats to the movie business are shrinking theatrical release windows and movie theft (or "piracy")." See http://www.natoonline.org

[9] *Before You Buy a Ticket, Why Not Buy the DVD?* The New York Times, http://www.nytimes.com/2005/12/19/business/19Theaters.html

one unit of L alone, or one unit of both versions (we refer to this case as B, a mnemonic for 'both'), or they can also decide to buy nothing (we denote this case by 0). In our application to the movie industry, the firm is an integrated producer, H represents watching a movie in a theatre, and L represents renting a DVD for watching it at home.[10]

Let u_i denote the quality of product $i = \{H, L, B, 0\}$, where $u_0 = 0$. When both H and L are bought, the resulting quality of both versions consumed jointly is

$$u_B = u_H + u_L(1 - s),$$

where s represents the level of substitutability of the products. The versions are partial substitutes for $0 \leq s < 1$. Products H and B are perfect substitutes for $s = 1$ and versions H and L are independent for $s = 0$. Notice that $s = 1$ corresponds to the standard case in the literature, where consumer are limited to a single-unit purchase of either version. In fact, in this case, if a consumer has already bought H, buying L confers no additional utility on top, and therefore she never buys the two versions. The more interesting case is when $0 < s < 1$, as it represents the situation where consumers are willing to watch a movie at home that they have already watched in a theater, though the additional benefit they enjoy is not as high as if they were watching the movie at home for the very first time.

There is a continuum of consumers who are heterogeneous in their preferences over quality. Each consumer is represented by her type θ, which is uniformly distributed over the segment $[0, 1]$. Following Mussa and Rosen (1978), the net surplus of a consumer that buys a product of quality u_i at price p_i is given by $\theta u_i - p_i$.

The quality of the two versions is taken as given and we denote by $k = u_H/u_L > 1$ the quality ratio. In order to concentrate on the more interesting case of pure information goods that can be offered in different versions, we posit that, once quality is determined and its associated developing costs have been sunk, the firm has constant marginal costs of supplying each variety of the product, which we normalize to zero. This assumption simplifies expressions and is useful for our objective of analyzing price discrimination and versioning since it implies that differences in prices are due only to differences in willingness to pay.[11]

[10] In the movie industry, the utility derived from the theatrical version is allegedly higher than the one obtained from the video version. As an example, in March 2010 Twentieth Century Fox announced the release on regular DVD and Blu-ray, but not in 3-D home video, of "Avatar", the movie directed by James Cameron. See "Avatar out on DVD April 22, but not in 3-D ", http://www.msnbc.msn.com/id/35895913/ns/entertainment-movies. It is worth noting that a feature of videos, which is not considered here explicitly, is that they can be viewed several times and by several people (Varian, 2000). In this case, there is a difference between the one-time experience of a movie, and the stream experience of a DVD. To account for this, the model can be reformulated in terms of having comparable units, so that the DVD streams are aggregated to generate a single overall stock, and the video price would realistically reflect the average price per viewer.

[11] It is possible to add an initial stage in our basic model to consider the possibility where the firm sets the product qualities u_H, u_L as well as the substitutability level s. If the cost of production of the versions increases with the level of differentiation, the firm will typically sell two versions and will release them with a strictly positive substitutability level. This occurs because although a further differentiation of the channels increases revenues in the last stage of the game, this does not compensate for the increase in the production costs of the versions. Results are available from the authors.

3.1 Simultaneous release of the two versions

We start with a single-period analysis where we study the monopolist's incentives to release both versions simultaneously. The second part of this section considers the possibility of releasing the two versions sequentially.

When the two versions are released simultaneously, the net pay-off V of a type-θ consumer, can be summarized as follows, where $i = \{H, L, B, 0\}$ denotes the set of choices available to each consumer:

$$V(\theta, i) = \begin{cases} \theta u_H - p_H \text{ if } H; \\ \theta u_L - p_L \text{ if } L; \\ \theta[u_H + u_L(1-s)] - p_H - p_L \text{ if } B; \\ 0 \text{ if } 0. \end{cases} \tag{1}$$

The two products are sold separately. This is the more interesting and realistic case for the movie industry, and is especially convenient for our analysis in Section 4 where the versions are sold sequentially and separately by exhibitors and video distributors.[12]

We can now illustrate how the market is split, taking into account that consumers make incentive-compatible decisions. Define θ_{ij} as the consumer that is indifferent between buying good i and j, where $j \neq i$, at a price $p_i = \{p_H, p_L, p_H + p_L, 0\}$ respectively. We thus obtain that $\theta_{LH} = (p_H - p_L)/(u_H - u_L)$ is the consumer that is indifferent between buying L and H separately, that is, $V(\theta_{LH}, L) = V(\theta_{LH}, H)$. Similarly, $\theta_{HB} = p_L/(u_B - u_H) = p_L/[u_L(1-s))]$ is the consumer that is indifferent between buying the high quality version and both versions, and $\theta_{i0} = p_i/u_i$ is the consumer that is indifferent between buying product $i = H, L, B$ alone, and not buying anything. Different market shares for the two products can arise according to the relative magnitude of the various indifferent consumers θ_{ij}.

The timing of the game is the following: the firm decides how many versions to release and sets the prices p_L and p_H to maximize its profits, anticipating that consumers will purchase one particular version, both, or none. The firm then releases only H, or both H and L simultaneously. The following proposition describes the optimal strategy.

Proposition 1. *The firm's optimal segmentation strategy is as follows:*

- *When* $0 < s \leq 2/3$, *it offers the product line* $L/H/B$ *at prices* $p_L = \frac{(1-s)u_L}{(2-s)}$ *and* $p_H = \frac{u_H}{2} - \frac{su_L}{2(2-s)}$ *(versioning);*
- *When* $2/3 < s \leq 1$, *it offers the product line* H *at a price* $p_H = u_H/2$ *(no versioning).*

Proof. *See the Appendix.*

As already anticipated, for $s = 1$, our model corresponds to the standard case of information goods analyzed by the previous literature, e.g., by Bhargava and Choudhary (2001).

[12] In the proof of Proposition 1 below we briefly consider the case where the firm bundles the two versions, although this possibility will be difficult to enforce in the movie industry.

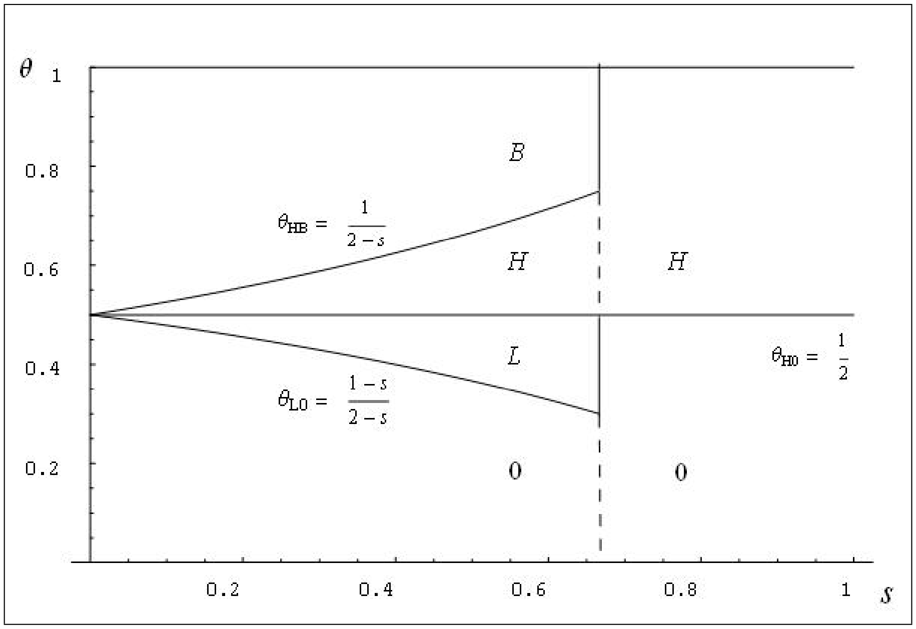

Figure 1: Market segmentation. The figure shows how the firm optimally segments the market as a function of the degree of substitutability s, taking into account the incentive-compatible choices of consumers. Consumer types in region B purchase multiple products, those in region H buy the high-quality product, those in region L buy the low-quality product, those in region 0 do not buy.

Indeed, when the degree of substitutability is sufficiently high ($s > 2/3$), the cannibalization effect of introducing the lower quality variant always prevails over the market expansion effect, and the monopolist is better off by supplying only H. When the level of substitutability is low, however, the firm finds it profitable to offer both variants, and consumers self select the variant(s) that maximize their individual utility. The product line that emerges is $L/H/B$, where this notation means that consumers with a very low θ buy nothing, those with a low θ buy only L, those with an intermediate θ buy H, and those with a high θ buy both versions, B (Figure 1). This segmentation disappears if products are completely independent ($s = 0$), in which case it is optimal to sell both versions to every buyer.

The result that market segmentation is sustainable as long as consumers are able to buy both versions and the degree of substitutability is not too high is novel in the literature. Finally, note that the commercial strategy of the firm will be more intricate but conceptually similar when the firm has a positive marginal cost of supplying each version. In the Appendix we show in greater detail the firm's optimal segmentation decision when the products offered are not pure information goods.

3.2 Sequential release

We now examine the case where the movie producer is not constrained to selling both products simultaneously, but can introduce the versions sequentially. Imagine that H is offered in the first period and L can be released simultaneously or at a later time. With this extension we are stipulating two dimensions of product substitutability: one is exogenous and is represented by s, and the other is endogenous and is created by delaying the introduction of L. In particular, we study if differences in the discount factors of consumers and the producer might affect the commercialization strategy and the sequential release of the two versions.

Imagine that the firm releases H at time t_0 and L at a possibly later time t_1. We denote by d the compound discount factor for t_1, so that choosing d implicitly determines the time t_1 when L is released. In particular, the period of time that elapses between t_0 and t_1 can be obtained from $d = \delta_p^t$, where δ_p is the producer's discount factor and $t = t_1 - t_0$ is the video window that separates the release of the two versions. If $t_0 = t_1$, then $d = 1$ and H and L are supplied simultaneously. If $t_0 < t_1$ then $d < 1$ and L is introduced some time after H. The lower the value of d, the later the release of L. The case of the release of H alone corresponds to the case of infinite delay of L, that is, $d = 0$.

We also account for possible differences in the discount factor of the producer and of the viewers. We define the consumer's discount factor as $\delta_c = x\delta_p$, where $0 \leq x \leq 1$ measures the consumer's relative impatience with respect to the firm. Hence, we can write the consumer's compound discount factor as $d_c = x^t d$.

Consumer's utility function (1) is immediately generalized to:

$$V(\theta, i) = \begin{cases} \theta u_H - p_H \ \text{ if } H; \\ \theta u_L - d_c p_L \ \text{ if } L; \\ \theta[u_H + d_c u_L(1-s)] - p_H - d_c p_L \ \text{ if } B; \\ 0 \ \text{ if } 0. \end{cases} \tag{2}$$

The expressions for the indifferent types are the same as before, with the exception of $\theta_{LH} = (p_H - d_c p_L)/(u_H - d_c u_L)$ which is the consumer that is indifferent between buying separately either H at t_0 or L at t_1.

With the possibility of sequential release, the producer's strategy depends crucially on its ability to commit to the prices and the video window announced in the first period. We begin our analysis considering the case where it can credibly commit to its subsequent policy and viewers form correct expectations about it. The case of commitment seems particularly appropriate in the movie industry where the prices for both movie tickets and DVDs are stable over relatively long periods of time and can therefore be seen as long-term choice variables. This can occur, for example, when the firm has built in the past a strong reputation on how it will introduce the products to the market, or when it designs expensive marketing campaigns that are costly to modify. Subsequently we will also analyze the case without commitment.

The timing of the game is thus the following: at time t_0, the producer sets the retail prices p_H and p_L and also chooses the time $t_1 \geq t_0$ in which L will be released. At time t_0 it then releases H, and at time t_1 it releases L. The following proposition shows the firm's optimal versioning and sequencing strategy when $0 \leq s \leq 1$.

Proposition 2. *Imagine that the firm commits to its pricing and sequencing strategy.*
1) *If* $\delta_c/\delta_p = 1$ *sequencing never arises and products are offered as in Proposition 1.*
2) *If* $1 > \delta_c/\delta_p = x > x^c$ *the firm's optimal sequencing strategy is as follows*:

- *When* $s = 1$*, the firm supplies only* H *immediately (no versioning)*;
- *When* $\hat{s}_1^c < s < 1$*, the firm offers the product line* $L/H/B$ *and* $0 < d < 1$ *(sequencing);*
- *When* $0 < s \leq \hat{s}_1^c$*, the firm offers the product line* $L/H/B$ *and* $d = 1$ *(versioning);*

3) *If* $0 < \delta_c/\delta_p = x < x^c$ *the firm's optimal sequencing strategy is as follows*:

- *When* $s = 1$*, the firm supplies only* H *immediately (no versioning)*;
- *When* $s_1^c < s < 1$*, the firm offers the product line* $L/H/B$ *and* $0 < d < 1$ *(sequencing);*
- *When* $s_2^c < s \leq s_1^c$*, the firm offers the product line* L/B *and* $0 < d < 1$ *(sequencing);*
- *When* $0 < s \leq s_2^c$*, the firm offers the product line* $L/H/B$ *and* $d = 1$ *(versioning).*

Proof. *See the Appendix.*[13]

The first part of the proposition shows that when the producer's and the viewers' discount factors do not differ ($x = 1$) the firm *never* introduces the products sequentially, because the loss generated by the postponement of profits of L does not compensate the reduction in the cannibalization over H (see panel A of Figure 2). More precisely, when screening among customers, the compound discount factor d affects only the decision of θ_{LH}. However, by deferring the introduction of the low-quality version the firm's profits are negatively affected, not only from those who indeed buy the low quality version alone in the second period, but also from those who buy one version in each period. This effect always prevails and sequencing never occurs. As in Proposition 1, when s is high enough, only H is released, otherwise there is versioning.

The second and the third parts of the proposition present the case where consumers are relatively more impatient than the firm ($x < 1$). The firm supplies H alone in the first period *only* in the limiting case where $s = 1$ (H and B are perfect substitutes) That is, when the consumption of the second version does not confer any additional utility to consumers, the firm does not release L, not even some time after the introduction of H.

[13] In the proof, we provide the expressions of the prices, of the video window and of the threshold values of s. This remark also applies to Proposition 3 below.

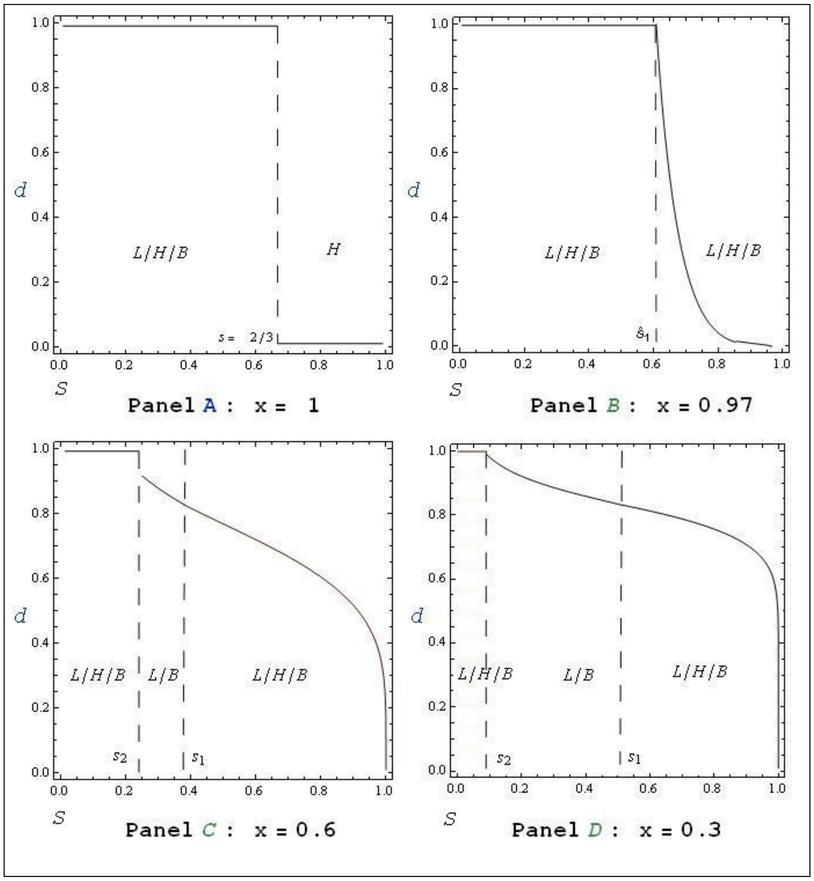

Figure 2: Video window with commitment. The figure plots the profit-maximizing value of d, as a function of the degree of substitutability s, and for different values of the relative degree of consumers' impatience x. When $d = 1$ there is immediate release, when $d = 0$ there is no versioning (other parameter values in the plots are: $u_L = 1$; $k = 3$; $\delta = 0.9$).

For $s < 1$ the strategy of the firm depends on the ratio $x = \delta_c/\delta_p$. Panel B of Figure 2 shows that, for $x > x^c$,[14] when s is low enough ($0 < s \leq \hat{s}_1^c$), still there is no sequencing, and both versions are released simultaneously. The two goods are rather independent, and the firm releases them instantaneously as there is no gain in waiting until later. For $\hat{s}_1^c < s < 1$, however, there is now both versioning and sequencing. A low segment of consumers only buys L in the second period, an intermediate segment of consumers only buys H in the first period, and a high segment of consumers buys both goods sequentially. The screening problem with a video window becomes more profitable with impatient customers: especially those with an intermediate willingness to pay prefer to buy H immediately instead of waiting for the delayed release of L. As we formally show in the proof, note that $\hat{s}_1^c$ becomes smaller (and therefore

[14] With the parameters of Figure 2, it is $x^c \simeq 0.62$.

sequencing arises more often) as x gets lower (customers are relatively more impatient) or as δ_p gets higher (the firm does not discount much the future).

Panels C and D are drawn for $x < x^c$. They present a similar story as panel B either when s is high ($s_1^c < s < 1$), in which case the firm offers the product line $L/H/B$ and the versions are introduced sequentially, and when s is low ($0 \leq s \leq s_2^c$), in which case it offers the product line $L/H/B$ and versions are introduced simultaneously. For intermediate values of s ($s_2^c < s < s_1^c$), there is an intermediate region where the firm offers the product line L/B and the versions are released sequentially. In this case, the firm finds it optimal to set prices such that *all* consumers who bought H in the first period also buy L later.

The previous case considered versioning when the producer is able to commit to its future pricing and sequencing policies. In some instances, however, the announcements made by the firm may not be credible for customers. For example, producers could adjust the price of the video version if the movie turned out to be a success in the box-office or if it won a prize in a film festival. Taking this into account, we follow Moorthy and Png (1992) and analyze next the producer's sequencing strategy when it cannot commit to the price of L at t_0, which is instead set only at its release date. We assume again that consumers anticipate the future sales policy of the firm and that the firm is aware of the consumers' expectations.

The timing of this new game is as follows: At time t_0 the producer sets the retail price p_H and releases it, and it also decides the time t_1 at which version L will be released; at time t_1 it sets p_L and releases L. The following proposition shows our results.

Proposition 3. *Imagine that the firm does not commit to the price of L, which is set at its release date.*

1) *If $\delta_c/\delta_p = 1$ the firm's optimal strategy is:*

- *When $\bar{s}^{nc} < s \leq 1$, the firm only supplies H immediately (no versioning);*
- *When $\bar{s}_1^{nc} < s \leq \bar{s}^{nc}$, the firm offers the product line $L/H/B$ and $d = 1$ (versioning);*
- *When $0 < s \leq \bar{s}_1^{nc}$, the firm offers the product line L/B and $d = 1$ (versioning).*

2) *If $1 > \delta_c/\delta_p = x > x^{nc}$ the firm's optimal sequencing strategy is*:

- *When $s = 1$, the firm supplies only H immediately (no versioning)*;
- *When $\hat{s}^{nc} < s < 1$, the firm offers the product line $L/H/B$ and $0 < d < 1$ (sequencing);*
- *When $s_1^{nc} < s \leq \hat{s}^{nc}$, the firm offers the product line $L/H/B$ and $d = 1$ (versioning);*
- *When $0 < s \leq s_1^{nc}$, the firm offers the product line L/B and $d = 1$ (versioning).*

3) *If $0 < \delta_c/\delta_p = x < x^{nc}$ the firm's optimal sequencing strategy is*:

- *When $s = 1$, the firm supplies only H immediately (no versioning)*;

- *When* $s_1^{nc} < s < 1$*, the firm offers the product line* $L/H/B$ *and* $0 < d < 1$ *(sequencing);*
- *When* $s_2^{nc} < s \leq s_1^{nc}$*, the firm offers the product line* L/B *and* $0 < d < 1$ *(sequencing);*
- *When* $0 < s \leq s_2^{nc}$*, the firm offers the product line* L/B *and* $d = 1$ *(versioning).*

Proof. *See the Appendix.*

As in the commitment case, the first part of the proposition shows that the firm never introduces the products sequentially when the consumers and the producer have the same discount factor (Panel A in Figure 3). Now, however, the firm will introduce versioning less often than in the commitment case, because $\bar{s}_1^{nc} < 2/3$. As the firm sets the price of p_L at t_1 instead of t_0, it fixes it too low, which therefore might induce some customers (that are as patient as the firm) to wait for the release of L. As the firm cannot commit to a higher price of L initially, for a wider range of values of s it prefers not to release L at all.

Notice that the impossibility of commitment for the price of L also affects the case when it results $d = 1$: while both versions are consumed at the same time, there is still a difference in the order of the moves between the commitment and the no commitment case. While in the first case the prices of both versions are set simultaneously and prior to purchasing decisions, in the no commitment case the firm first sets the price of H (and customers might buy it), and immediately after it sets the price of L (and customers might buy it). This explains why, for identical discount factors and $d = 1$ the firm follows different strategies with commitment and no commitment. In particular, with no commitment the product line L/B can emerge (the firm sells version L to all customers that have already bought H). Instead, under commitment, this strategy is never optimal as the firm would find it profitable to increase the price of L by a bit, making the more segmented product line $L/H/B$ emerge.

The rest of the proposition considers the case where consumers are more impatient than the firm ($x < 1$). Only when $s = 1$ does the firm offer H exclusively during the first period. In panel B of Figure 3 we present a case with $x > x^{nc}$.[15] For low and intermediate values of s, there is no sequencing and $d = 1$ (there are two different regions: for low values of s the product line is L/B and for intermediate values of s the product line is $L/H/B$). For higher values of s, there is sequencing, with a product line $L/H/B$. Panels C and D are plotted for $x < x^{nc}$. For sufficiently low values of s, there is again a corner solution $d = 1$ with immediate release, and a product line L/B. The difference with panel B is that now L/B can also emerge for intermediate values of s with a proper sequencing choice $0 < d < 1$. As in the commitment case, the existence and the size of the regions depends on the value of the ratio $x = \delta_c/\delta_p$.

While the case of identical discount factors shows that there is less versioning without commitment, when $x < 1$, if the seller cannot pre-commit it will use longer video windows and

[15] With the parameters of Figure 3, it is $x^{nc} \simeq 0.96$.

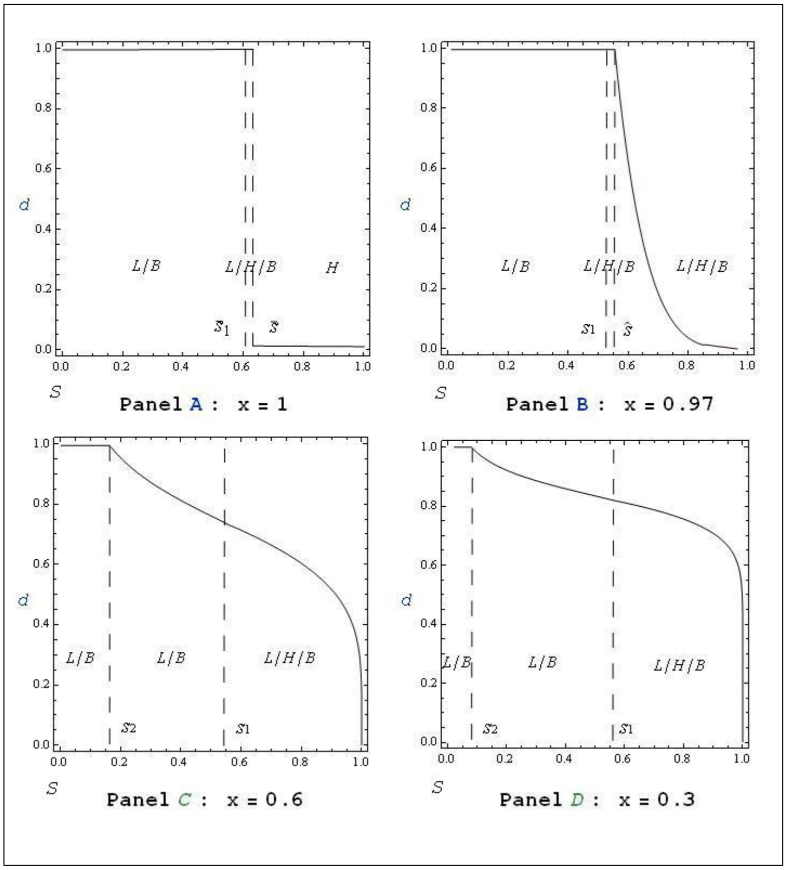

Figure 3: Video window with no commitment (same parameter values as Figure 2).

there will be more sequencing. Our results thus complete those of Moorthy and Png (1992), who show that, for $s = 1$, sequencing is less likely with no commitment. It is important to recall the differences between our models. In their model with single purchase, versioning happens because of the convex cost function they employ. In our model with pure information goods, versioning happens thanks to the multi-purchase assumption as long as the versions are imperfect substitutes, but would not arise when $s = 1$. Our models also differ in the sequencing analysis, as in Moorthy and Png (1992) the firm is constrained to release the versions only at two possible dates (in our notation, $t_1 - t_0 = t = 1$), while we considered that the video length is endogenous and could take any value. This is why we find that sequencing is more likely in the no commitment case, because the firm is able to better fine-tune the release of L with an appropriate window.

To sum up, our results show that when the consumers' discount factor is lower than the producer's, it is optimal to introduce the versions sequentially if the degree of substitution between the versions is sufficiently large. This applies to pure information goods such as movies, without any reproduction cost. However, we believe it is difficult to validate this rationale empirically in the movie industry, as it is hard to calculate the relative degree of patience of producers and viewers.[16] If instead one took the more neutral view that discount factors were the same, then sequencing would never emerge in our model. In addition, we have found that 'day-and-date' release strategies should be preferred under a wider range of circumstances when versioning is optimal. Therefore, our basic model still does not offer a complete reason for why, in practice, DVDs are often released sequentially. We propose next a new and simple reason for sequencing which relies on the vertical structure of the industry.

4 Sequential distribution of movies in a vertically-separated industry

This section focuses on the case where the versions of a product can be released through separated distribution "channels". We show that when the downstream sales channels are controlled by independent firms, the incentives for versioning change and, crucially, sequencing can appear in situations that would otherwise not be contemplated by a fully integrated monopolist. To address this more complex situation, imagine there is a producer (a studio), an exhibitor (a movie theater), and a distributor (a DVD store). The producer holds all the rights over the movie but needs to release it in theaters and/or through DVD stores. The studio bargains wholesale payments with each channel, and afterwards the channels set the price of the versions they control, i.e., the theatre exhibitor sets the price of H and the distributor sets the price of L.

Our bargaining assumption contrasts with alternative models of vertical chains where one of the parties (typically the upstream producer) makes a take-it-or-leave-it offer to the other parties. This flexible bargaining approach offers the same results that a take-it-or-leave-it model when all the bargaining power rests with one particular party. Moreover, bargaining illustrates well how the video window is usually determined in the movie industry. Indeed, although producers design the overall commercialization strategy for movies and have control over the DVD time release, exhibitors decide when to schedule the movie in the theater and the number of weeks that it will stay on screen. Therefore, both the producer and the exhibitor play a key role in determining the length of the video window.

In order to reflect the situation of the U.S. movie industry after the *Paramount* decision in 1948, we first model the case where the producer is vertically separated from every downstream channel. The producer cannot fully appropriate the revenues associated with each version

[16] De Vany and Walls (1999) analyzed the performance of 2,015 movies in the U.S. in the period 1984-1996 and concluded that the movie industry is a profoundly uncertain business. "Movies are complex products and the cascade of information among film-goers during the course of a film's run can evolve along so many paths that it is impossible to attribute the success of a movie to individual causal factors. The audience makes a movie a hit and no amount of "star power" or marketing can alter that."

and the commercialization interests of the producer, the exhibitor and the video distributor are not perfectly aligned. At the end of this section, we also present the case where the producer directly sells the DVD versions (or offers directly the movies on streaming), as this is a situation that can be of material relevance in the movie industry.

4.1 Negotiations between the producer and two independent distribution channels

Following the notation of Section 3, we call H the movie exhibited in theaters (high quality version) and L the movie viewed on DVD (low quality version). We stipulate that the prices H and L are set independently by the exhibitor and distributor, respectively. Moreover, the producer conducts two separate wholesale bargains, first with the exhibitor and then with the video distributor. The producer bargains with the exhibitor over both the rental price of the theatrical version (a) and the release time of the video version (t_1). It also separately bargains with the distributor over a revenue sharing agreement of video sales (r). These specifications and the vertical separation of the market generate a double markup distortion for the theatrical version and incentive misalignments about the video window among firms.[17]

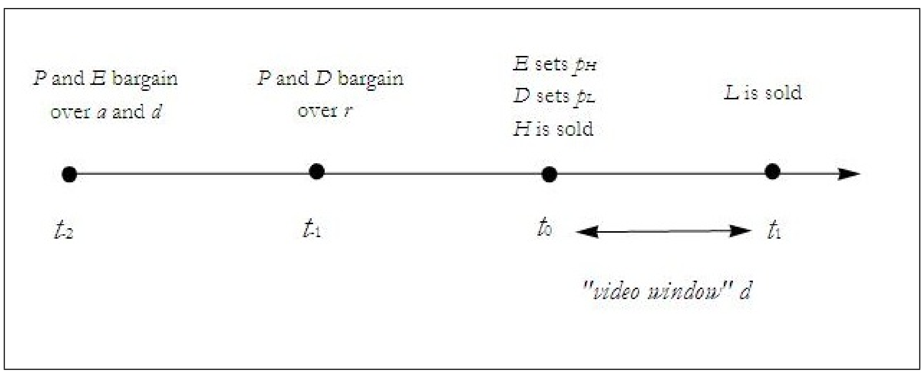

Figure 4: Time line in the channels' negotiation game.

We next describe the game that is played between the producer and the channels. We denote by t_0 the earliest possible date to show any version of the movie. Prior to this date, the timing of the negotiations is as follows (see Figure 4):

- First, at time t_{-2} the producer and the exhibitor jointly decide whether or not to show the movie in the theatre (version H). If negotiations break down, then H is not shown, the exhibitor gets nothing, and the producer still has the option of negotiating at time

[17] Notice that we have chosen a vertical contract (revenue sharing) that does not cause double markups with the distributor. The same qualitative results would arise if the producer and the DVD distributor were able to write efficient complete contracts, as commonly found in the empirical literature on DVD video stores. See for example Mortimer (2007) and Ho et al. (2008).

t_{-1} with the distributor the release of L and obtaining a share of the video sales. If instead the two parties agree to show H, they negotiate over the rental price a that must accrue to the producer and the release time $t_1 \geq t_0$ for the video version. We model bargaining using the generalized Nash axiomatic approach, whereas the two negotiating players maximize the weighted product of the payoffs received by the players in excess of their disagreement payoffs. We denote by α the degree of bargaining power of the producer in this negotiation and by $1-\alpha$ the degree of bargaining power of the exhibitor.

- Second, at time t_{-1} the producer and the video distributor jointly decide the revenue share r accruing to the producer from DVD sales and the share $1-r$ kept by the distributor. In this negotiation β is the producer's degree of bargaining power and $1-\beta$ the distributor's.[18] If at time t_{-2} an agreement with the exhibitor had been reached, then DVD sales can occur at $t_1 \geq t_0$.[19] If instead past negotiations with the exhibitor had failed, the distributor and the producer could commercialize the DVD at the earliest possible date, i.e., t_0. In either case, if the negotiation with the distributor breaks down, the distributor gets nothing while the producer might still get its rental price from each movie ticket that is sold in the theaters, in case negotiations at t_{-2} had succeeded.[20]

- Finally, after these contractual terms over a, d, and r are set, at time t_0 the exhibitor and the distributor independently set the retail prices p_H and p_L respectively.

- The exhibitor then releases H at time t_0 and the distributor offers L at t_1. As in Section 3, we denote as d the compound discount factor for t_1, so that choosing d implicitly also determines t_1. We assume that d is common to all firms and consumers. Hence, taking into account the results of the previous section, in this setting we would never find sequencing under full integration.

The next proposition presents the equilibrium strategies for the firms. In order to keep the model as simple as possible, we show the results for the extreme cases where $s = 0$ (independent products) and $s = 1$ (standard case of single-unit purchase).

Proposition 4. *Imagine that the producer negotiates independently with the exhibitor and the distributor.*
1) *When $s = 0$ the product line offered by the producer is L/B and $d = 1$ (versioning and immediate release). The negotiated wholesale contracts specify $a = \frac{\alpha u_H}{2}$ and $r = \beta$.*

[18] In general, α and β will differ as the negotiating environments with the two channel will differ too. Some of the factors that affect the negotiation between the parties include the presence of other channels in the region, the affiliation to an association of exhibitors, or the degree of patience of the players during negotiations.

[19] The "video" window we consider implies that L is sold after H. In principle, one could also analyze an alternative timing where L is released first, and then H. However, it is easy to show that in our model a "movie" window never occurs under this alternative timing, as H is always introduced together with L.

[20] For simplicity, we do not consider explicitly in this paper competition from alternative movies of other producers. This approach is justified by the fact that the producer has a “unique” title/product which comes in two versions. If the exhibitor or the distributor could also sell the movies of other producers, their outside options would not be zero.

2) *When $s = 1$ the producer's segmentation strategy is the following:*

- *If $\alpha > \alpha^1$, the product line offered is L/H and $d = 1$ (versioning and immediate release)*;
- *If max $[0, \alpha^0] < \alpha \leq \alpha^1$, the product line offered is L/H and $0 < d < 1$ (sequencing);*
- *If $\alpha \leq$ max $[0, \alpha^0]$, the producer offers H and $d = 0$ (no versioning);*

where the threshold levels satisfy $\alpha^0 < \alpha^1$.

- *The negotiated wholesale contracts specify a revenue share to the producer $r = \beta + \frac{a(1-\beta)(4k-d)[a(3k-d)-k(k-d)u_L]}{2k^2(a+u_H-du_L)^2} > \beta$, for all $\beta < 1$.*

Proof: *See the Appendix.*[21]

The proposition shows that, when there is separation between the producer and the distribution channels, the results regarding versioning and sequencing change quite dramatically. Recall from Proposition 1 that, if an integrated monopolist fully controls the commercialization of the two versions, it offers B when $s = 0$ and it introduces only H when $s = 1$ (the DVD version is delayed indefinitely). However, with channel separation, these strategies are no longer sustainable.

When $s = 0$, versions H and L are independent and there is no cannibalization when the producer commercializes L at t_0. As a result, sequencing is never profitable and a "day-and-date" strategy emerges. The producer ends up selling the product line L/B, instead of the product line B that we found in Proposition 1, because the price p_H is particularly high due to the double markup imposed by the producer and the exhibitor. Hence, customers with intermediate willingness to pay now buy only the L version. Finally, at the wholesale level, the rental price rises both with the quality of H and with the distributor's relative bargaining power, α. Similarly, the greater the producer's bargaining power β relative to the distributor's, the greater his revenue share from DVD sales.

The solution for $s = 0$ is rather intuitive, given the absence of strategic interaction between the two versions. The bargaining problem becomes considerably more involved when $s = 1$. Several equilibrium releasing strategies are now possible for the theatre and DVD versions. One or both versions might be released, and the two versions might be introduced simultaneously or sequentially. The reason for this is that now the firms have different preferences over the introduction of version L, once H is released at time t_0. The theatre exhibitor obtains more profits by delaying the introduction of L, as this reduces the cannibalization of L over H. The distributor seeks just the opposite. And the producer considers the joint

[21] In the proof, we provide the expressions of the threshold values of α, as well as the rental charge a and video window d. This remark also applies to Proposition 5 below.

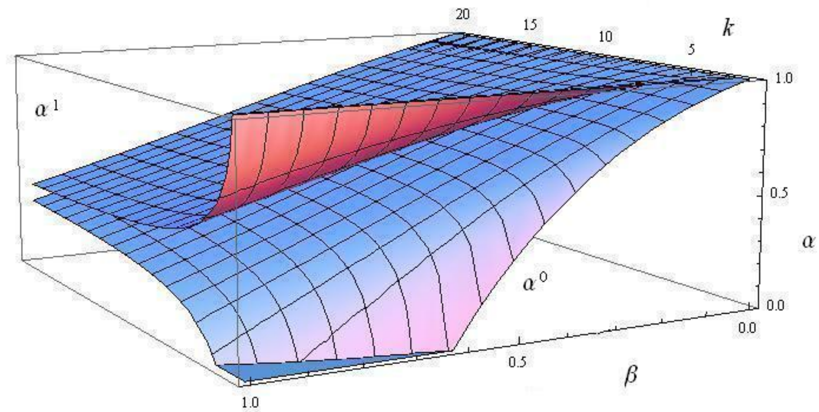

Figure 5: Versioning and sequencing. The figure plots the three regions characterized by Proposition 4, as a function of the degrees of bargaining power of the producer. When $\alpha < \alpha^0$ falls in the lowest region, only H is released. When $\alpha^0 < \alpha < \alpha^1$ falls in the intermediate region, there is both versioning and sequencing. When $\alpha > \alpha^1$ falls in the highest region, there is versioning and simultaneous release of both versions.

maximization of its profits in the two channels and conducts the negotiations taking into account the bargaining power of its counterparts.[22]

The proposition shows that, depending on the relative bargaining power of the firms, three different segmentation strategies can appear. In order to clarify the exposition of the results, Figure 5 plots the range of validity of these regions when $s = 1$. Figure 6 instead plots the equilibrium value of the video window.

When α is low enough ($\alpha < \alpha^0$) there is a region where only H is released and therefore versioning does not arise. This corresponds to the case where the exhibitor is a tough negotiator and manages to delay the introduction of the video version indefinitely. This pattern disappears as soon as the producer's bargaining power vis-à-vis the exhibitor (α) and/or vis-à-vis the distributor (β) increases. In this situation, if α is not too high ($\alpha^0 < \alpha < \alpha^1$) the versions are released sequentially, with a video window. However, for large values of α ($\alpha > \alpha^1$) we can get the striking result that the producer, in complete contrast with the exhibitor, will impose a "day-and-date" strategy and release the two version simultaneously.

[22] While for simplicity we concentrate our analysis only on the cases $s = 0$ and $s = 1$, more generally it is possible to identify a threshold value of s such that, for values of s below this level, both exhibitor and producer prefer the simultaneous introduction of the two versions. Instead, for values above this threshold the exhibitor is interested in delaying the introduction of L and thus sequencing can occur.

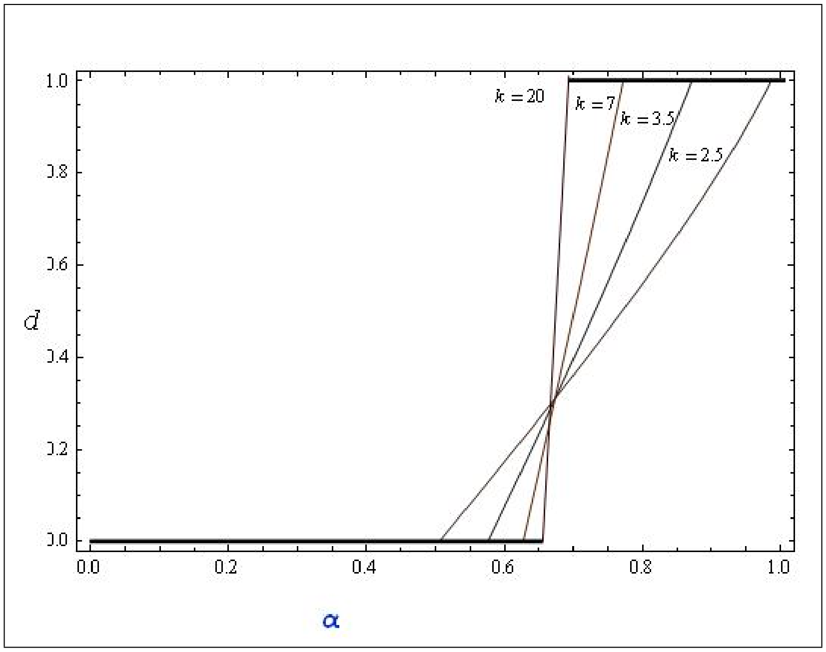

Figure 6: Negotiated video window. The figure plots the optimal length of the video window d chosen by a producer bargaining with an exhibitor. As characterized by Proposition 4, the window can fall into three areas according to the value taken by the quality ratio k: $d = 1$ corresponds to the simultaneous release of both versions ("day-and-date"), $0 < d < 1$ corresponds to a proper "video window", and $d = 0$ implies the release of the movie only.

Note that the size of these three regions is affected by the quality ratio $k = u_H/u_L$. Since $\partial\alpha^0/\partial k > 0$, it is more likely that only H is introduced when the quality of H is far superior to L (as long as $\alpha < \alpha^0$). Also, as $\partial\alpha^1/\partial k < 0$, the region with sequencing where $\alpha^0 < \alpha < \alpha^1$ becomes smaller and smaller the higher the quality of H relative to L. Finally, it is more likely that both versions are introduced simultaneously for high values of k (as long as $\alpha > \alpha^1$). Notice in particular how, for very high values of k, we can have "bang-bang" solutions almost abruptly: a small (but discrete) increase in α can bring the solution from $d = 0$ to $d = 1$ (consider for example the case where $k = 20$ in Figure 6).

We remark that, in order to obtain sequencing, we need quite crucially both the vertical structure and incomplete contracts. Intuitively, because of the double marginalization over H, the producer can not fully internalize the profits by selling just one version and it opts for versioning even when $s = 1$. Moreover, as it obtains part of the revenues generated from L, the producer may be interested in not delaying too much the release of this version (this interest increases the higher is β). Instead, the exhibitor only sells H and tries to delay as much as possible the introduction of L to reduce the cannibalization of this product. The exhibitor also has to make sure that the producer will not walk away from negotiations and release only L. This outside option is particularly relevant when k is not large, which tilts the choice in favor of versioning (see again Figure 5). The product line that ultimately emerges is determined by the relative degree of bargaining power of the various parties, as well as by the quality ratio of the two versions.

Another interesting result of the proposition is that versioning (either with immediate or staggered release of the two versions) becomes *more* likely the higher is β. With a high value of β the producer, being 'in between' the two channels, is better placed to internalize the cannibalization that L exerts over H, which should lead to no versioning when $s = 1$. However, the producer is still attracted to versioning for two reasons. First, a high value of β means that the producer directly gets more from DVD sales, making a versioning strategy more appealing. Second, the producer has also an indirect *strategic* reason for versioning. In fact, with versioning, the outside option from DVD sales becomes a valuable threat when negotiating with the exhibitor, allowing the producer to secure a better rental charge from H. Conversely, if the producer obtains a higher rental charge from H, this also increases its outside option when negotiating with the distributor, as there is a complementarity between the outside options. This strategic effect also explains why the revenue share obtained by the producer from the DVD version is strictly higher than its bargaining ability, $r > \beta$, in contrast with the case where $s = 0$ where strategic effects are absent and hence $r = \beta$.

Finally, note that, even if β is very high and the producer appropriates most of the revenues generated by L, the pricing decision of L is still taken by the independent distributor, not by the producer. In other words, even for $\beta = 1$, the price of the DVD is still set too "aggressively" by the independent distributor that has no incentive to internalize its impact on the sales of H. This raises the question of whether there would be more or less versioning if the producer were able to sell the L version *directly*. In the movie industry, the separation between the studios and theater chains was dictated by an antitrust provision, but the distribution arrangements of DVD sales have been traditionally more varied. The situation analyzed so far corresponds to the situation where the distributor is independent from the producer. However, quite frequently the producers sell directly the DVD version via their websites or specialized stores. We next analyze the effects of this alternative market configuration.

4.2 Negotiations between an integrated producer and the exhibitor

We now consider the situation where the producer can sell directly the L version, but still needs to strike an agreement with the exhibitor to show the movie in a theatre. There is thus only one negotiation between the exhibitor and the producer at time t_{-1}, while at time t_0 the exhibitor and the producer take the retail price decisions over H and L, respectively. The next proposition presents the results for this market structure.

Proposition 5. *Imagine that the producer sells L directly and negotiates with the exhibitor the commercialization of H.*

1) *When* $s = 0$*, the product line offered by the producer is* L/B *and* $d = 1$ *(versioning and immediate release). The negotiated retail charge is* $a = \frac{\alpha u_H}{2}$.
2) *When* $s = 1$*, the producer's optimal segmentation strategy is:*

- *If* $\alpha > \widehat{\alpha}^1$*, the pattern offered is* L/H *and* $d = 1$ *(versioning and immediate release)*;
- *If max* $[0, \widehat{\alpha}^0] < \alpha \leq \widehat{\alpha}^1$*, the product line offered is* L/H *and* $0<d<1$ *(sequencing);*
- *If* $\alpha \leq$ *max* $[0, \widehat{\alpha}^0]$*, the producer offers* H *and* $d = 0$ *(no versioning);*

where the threshold levels satisfy $\alpha^0 < \alpha^1 < 1$ *and, compared to the case of channel separation (Proposition 4), it holds that* $\alpha^0 > \widehat{\alpha}^0$ and $\alpha^1 > \widehat{\alpha}^1$.

Proof: *See the Appendix.*

The result for $s = 0$ is the same as Proposition 4. Not surprisingly, when versions are independent there is no cannibalization or strategic effects arising from the two versions. The case where $s = 1$ is instead more interesting. Similar to Proposition 4, if the bargaining power of the producer is high enough, the producer segments the market and offers the product line L/H. In spite of having integrated the L channel, the producer still cannot fully internalize the profits by selling just H due to the double marginalization over H. Hence, it uses the rental price charged to the theatre exhibitor to extract part of its revenues and, in addition, it directly offers L. The greater the bargaining power enjoyed by the producer, the higher the rental price that is eventually set.

What is remarkable from this case, compared to the full channel separation of Proposition 4, is that the producer will be *more* likely (i.e., for a wider range of values of α) introduce the DVD version and reduce the length of the video window. The main reason is that an independent distributor would set the price for the DVD too low, even when $\beta = 1$ and the producer gets all the profits of the video distribution. By contrast, when the producer is integrated with the video distributor, it sets the price of L and can better internalize the effects from video sales on the theater side. This implies that the region where only H is released is smaller with integration ($\alpha^0 > \hat{\alpha}^0$) while the DVD version is introduced more often. Similarly, versioning with relatively short windows is obtained by the producer more often when it negotiates with the exhibitor, making it more likely that immediate release arises ($\alpha^1 > \hat{\alpha}^1$).

5 Conclusions

We have presented a model of movie distribution and consumption across two channels, which provides insights on how studios should time the window between theatrical and video releases. The main result is that a vertically integrated producer should release several versions of a movie in different channels when consumers are able to buy both versions of the product. The key parameter for this finding is the degree of substitutability between the versions. Taking this into account, when planning their distribution strategy studio managers should determine the extent to which theatrical and non-theatrical consumer segments overlap. If DVDs deter people from going to the theater, then versioning should be less likely. However, if consumers enjoy consuming the same information goods or cultural products several times and through different channels, then day-and-date release should occur more often than previously thought by the literature. Indeed, some consumers would consider theatrical movies and DVDs partial substitutes, or even complements, because the utility they derive from consumption is not lost with each repetition, or because consumption of different versions enable them to appreciate different aspects of the movie.

The vertical structure of the supply chain is at the core of the second contribution of our paper. We characterized equilibrium outcomes when the organizational structure of the industry is accounted for. In a vertically separated movie industry, the producer should agree to supply the two versions if the quality differential between them is not too high, even when the two versions are perfect substitutes. In addition, if the producer's bargaining power is sufficiently low, versions will be introduced sequentially. In this case, intertemporal movie distribution can occur to a large extent because of inefficient vertical contracting.

The possibility of delaying the introduction of a version still raises many questions both at a theoretical and at an empirical level. Waterman et al. (2010) found that the video window was quite stable (established at around 6 months) between 1988 and 1997, however it has since fallen steadily to a current window of about 4 months. Our model predicts this trend, either when markets are not subject to cannibalization or when distributors have a stronger bargaining power than that of the exhibitors when deciding on the length of the video window. Luan and Sudhir (2007) calculate that the optimal window should be even shorter, at around 2.5 months. Their approach, though, considers the profits of an integrated producer/exhibitor, while we suggest that vertical separation might be one reason for longer video windows. Therefore, features of the vertical chain should be accounted for in future empirical studies or in numerical computations aimed at calibrating the optimal time release of different versions.

Despite our contributions to the literature, several relevant aspects for the movie industry have not been considered in full by this paper but could warrant interesting extensions aimed at increasing our understanding about the future of this industry. Piracy is an important problem that could be analyzed using our simple framework. Pirate copies are themselves a different variant (Sundararajan, 2004b). They should be a very close substitute of DVDs, but a very poor substitute for the theatrical experience. In this paper, we also focused our attention on the optimal release for a single movie title supplied by a single studio, although versions may be distributed by different firms. We foresee the introduction of competition among content producers as an essential issue for our future research.

6 Appendix

Proof of Proposition 1. When the firm only offers H, it sets the price p_H that maximizes $\pi_H = p_H(1-\theta_{H0})$, i.e., $p_H = u_H/2$ and the firm obtains $\pi_H = u_H/4$.

Next, consider the case where the firm offers L to the low segment of consumers, H to the intermediate segment, and L and H to the high segment. It then sets p_H and p_L to maximize:

$$\pi_{LHB} = (p_H + p_L)(1-\theta_{HB}) + p_H(\theta_{HB} - \theta_{LH}) + p_L(\theta_{LH} - \theta_{L0}). \tag{3}$$

Solving this problem, we obtain the following prices and corresponding firm's profits:

$$p_L = \frac{(1-s)u_L}{(2-s)} \quad ; p_H = \frac{2u_H - s(u_L+u_H)}{2(2-s)}; \quad \pi_{LHB} = \frac{u_H}{4} + \frac{(2-3s)u_L}{4(2-s)}. \tag{4}$$

It is simple to verify that $\pi_H < \pi_{LHB}$ for $s < 2/3$. In this range, it is also straightforward to confirm that at the equilibrium prices, $\theta_{HB} = \frac{1}{2-s} > \theta_{LH} = \frac{1}{2} > \theta_{L0} = \frac{1-s}{2-s}$.

Imagine now that the firm is able to bundle together the two products. In this case, the firm's problem is to set the price p_B that maximizes $\pi_B = p_B(1-\theta_{B0})$. Solving this problem we obtain that the optimal price is $p_B = (u_H + u_L(1-s))/2$ and the firm's profits are $\pi_B = (u_H + u_L(1-s))/4$. Finally, observe that $\pi_B > \pi_{LHB}$ and $\pi_B > \pi_H$ for any value of s. Thus, if feasible, the firm would bundle the two versions together. **Q.E.D.**

The following proposition generalizes the results to the case where $c \geq 0$ and $-1 \leq s \leq 1$. When $s < 0$, the two variants of the product are complements. Although this possibility can be quite exceptional in the movie industry, it is more common for other information goods such as music, where consumers obtain more utility from a concert if they have previously listened to the same music in a CD.

Generalization of Proposition 1. *Imagine $c \geq 0$. The optimal segmentation strategy and the profit maximizing prices depend on the degree of substitution s between versions*:

- *When $s^1 < s \leq 1$, the firm supplies only the H version at a price $p_H = (c+u_H)/2$;*
- *When $s^2 < s \leq s^1$, the firm supplies the product line $L/H/B$ at prices $p_H = \frac{c+u_H}{2} - \frac{su_L}{2(2-s)}$ and $p_L = \frac{c}{2} + \frac{u_L(1-s)}{(2-s)}$;*
- *When $\hat{s} < s \leq s^2$, the firm supplies the product line H/B at prices $p_H = \frac{u_H}{2} - \frac{u_H[c(s-2)+su_L]}{2[u_L+u_H(1-s)]}$ and $p_L = p_H u_L/u_H$;*
- *When $s^3 < s \leq \hat{s}$, the firm supplies the product line H/B at prices $p_H = \frac{c+u_H}{2}$ and $p_L = \frac{c+u_L(1-s)}{2}$;*
- *When $s \leq s^3$, the firm supplies the product line B at prices $p_H + p_L = (c+u_B)/2$,*

where the cut-off points are

$$s^1 = \frac{u_L\{c^2(3-4u_H)+6cu_H-5u_Lu_H+[(u_Lu_H-2cu_H+c^2)(u_Lu_H+6cu_H+c^2)]^{1/2}\}}{2c^2(u_L-u_H)+u_Lu_H(4c-6u_L)};$$

$$s^2 = \frac{2c}{c+u_L}; \quad s^3 = 1-\frac{u_H}{u_L} < 0; \quad \hat{s} = c(\frac{1}{u_L}-\frac{1}{u_H}).$$

Proof of the Generalization of Proposition 1. Following the same steps as in the proof of Proposition 1, the firm's expression for the profit when it only offers H and $c > 0$ is $\pi_H = (p_H - c)(1-\theta_{H0})$. Maximizing this with respect to the price we obtain $p_H = (c+u_H)/2$ and the corresponding profit $\pi_H = (u_H-c)^2/(4u_H)$. Consumers between $\theta_{H0} = 1/2+c/(2u_H)$ and 1 buy only H, and the others buy nothing.

When the firm offers the pattern $L/H/B$, the prices and the associated profits are:

$$p_H = \frac{c+u_H}{2} - \frac{su_L}{2(2-s)}; \qquad p_L = \frac{c}{2} + \frac{u_L(1-s)}{(2-s)}, \tag{5}$$

$$\pi_{LHB} = \frac{u_H}{4} + \frac{(2-3s)u_L}{4(2-s)} + \frac{c[c(2-s)-4(1-s)u_L]}{4(1-s)u_L}.$$

It can be computed that the value of s that equals π_H and π_{LHB} is

$$s^1 = \frac{u_L\{c^2(3-4u_H)+6cu_H-5u_Lu_H+[(u_Lu_H-2cu_H+c^2)(u_Lu_H+6cu_H+c^2)]^{1/2}\}}{2c^2(u_L-u_H)+u_Lu_H(4c-6u_L)}. \tag{6}$$

Therefore, for $s^1 < s \leq 1$ there is a region A where only H is provided. For $s < s^1$ the prices in (5) are a candidate solution, as long as $\theta_{HB} = \frac{1}{2-s} + \frac{c}{2u_L(1-s)} > \theta_{LH} = \frac{1}{2} > \theta_{L0} = \frac{1-s}{2-s} + \frac{c}{2u_L}$. Notice, however, that when $s = s^2 = \frac{2c}{c+u_L}$, at the prices given by (5) it is $\theta_{L0} = \theta_{H0} = \theta_{LH}$. Therefore, for $s^2 < s \leq s^1$ there is a region B where the product line $L/H/B$ is offered. For $s < s^2$, the first marginal buyers will choose H instead of L, since now $\theta_{H0} < \theta_{L0}$, and no one buys L alone. When this occurs, the firm will choose p_H and p_L to maximize:

$$\pi_{HB} = (p_H + p_L - 2c)(1-\theta_{BH}) + (p_H - c)(\theta_{HB} - \theta_{H0}).$$

The prices and the associated profit would be:

$$p_H = \frac{c+u_H}{2}; \qquad p_L = \frac{c+u_L(1-s)}{2}, \tag{7}$$

$$\pi_{HB} = \frac{u_H+u_L(1-s)}{4} - c + \frac{c^2[(1-s)u_L+u_H]}{4(1-s)u_Hu_L}.$$

In this region, which we shall call C, the product line is H/B and the indifferent types are $\theta_{HB} = \frac{1}{2} + \frac{c}{2u_L(1-s)}$, $\theta_{H0} = \frac{1}{2} + \frac{c}{2u_H}$. This solution holds as long as $\theta_{HB} \geq \theta_{H0}$, which results in $s \geq s^3 = 1 - \frac{u_H}{u_L} < 0$. Notice, however, that we also have to check that consumers may not want to buy L: $\theta_{L0} u_L - p_L \leq 0$ at the prices given by (7). This is satisfied when $s \leq \hat{s} = c(1/u_L - 1/u_H) < s^2$, and (7) is the solution for $s^3 \leq s \leq \hat{s}$.

When $\hat{s} \leq s \leq s^2$, we are still in region C as the product line is H/B, but the prices take a different expression. In particular, the firm sets $p_L = p_H(u_L/u_H)$ in order to make sure that $\theta_{L0} = \theta_{LH}$. The prices that satisfy this condition and maximize the profit in (7) are:

$$p_H = \frac{u_H}{2} - \frac{u_H[c(s-2) + su_L]}{2[u_L + u_H(1-s)]}; \quad p_L = p_H(u_L/u_H). \tag{8}$$

In this case the indifferent types are $\theta_{HB} = \frac{1}{2} + \frac{c(2-s)+(1-s)u_H}{2(1-s)[u_L+u_H(1-s)]}$, $\theta_{H0} = \frac{1}{2} + \frac{c(2-s)-su_L}{u_L+u_H(1-s)}$.

Finally, when s is very negative (strong complementarity), all consumers who buy prefer to buy B. For $s < s^3$ the firm maximizes the following profit $\pi_B = (p_H + p_L - 2c)(1 - \theta_{B0})$. The optimal price is $p_H + p_L = c + u_B/2$ and the firm's associate profit is $\pi_B = (u_B - 2c)^2/(4u_B)$, where $u_B = u_H + u_L(1-s)$. The indifferent type is $\theta_{B0} = \frac{1}{2} + \frac{c}{u_B}$. This is region D.

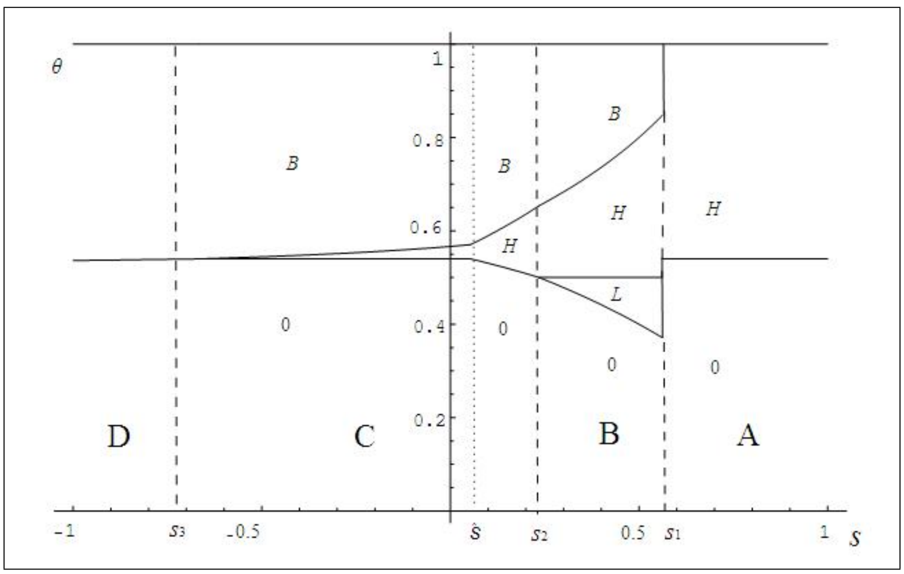

Figure 7: Market segmentation when $c > 0$ and $-1 < s < 1$.

The various regions A, B, C and D are plotted in Figure 7. Their existence and size depend on the value of c and on the utility generated by each version. (When $c = 0$, $s^2 = 0$ and region C disappears. As a result, for $0 \leq s < s^1 = 2/3$ there is the product line $L/H/B$, as already found in Proposition 1.) We have already shown that $s^2 > \hat{s} > s^3$. We need therefore only discuss when $1 > s^1 > s^2$. The expression for s^1 given by (6) is a bit cumbersome, but it can be shown to decrease in c in the relevant range. Thus it takes a maximum when $c = 0$, in which case it simplifies to $s^1 = 2/3 < 1$. Secondly, still at $c = 0$, $s^2 = \frac{2c}{c+u_L}$ simplifies to

$s^2 = 0 < s^1$. By continuity, the four regions always exist for sufficiently low levels of c. As c increases, region B shrinks, until it disappears when $s^1 = s^2$.[23] Also notice that, in order for the problem to make economic sense, $c < u_L$, thus s^2 is always bounded below 1, and regions A, C, D are always non-empty. **Q.E.D.**

Proof of Proposition 2. We first consider the case where $x = 1$, which implies that $d_c = d$. Imagine that the firm releases the two versions sequentially: it offers H at t_0 to the intermediate and the high segment of consumers, and L at t_1 to the low and high segment of consumers. It then maximizes the following profit:

$$\pi_{LHB}^{SEQ} = (p_H + dp_L)(1 - \theta_{HB}) + p_H(\theta_{HB} - \theta_{LH}) + dp_L(\theta_{LH} - \theta_{L0}). \tag{9}$$

The prices that solve this problem and the corresponding profits are:

$$p_H = \frac{ku_L}{2} - \frac{dsu_L}{2(2-s)}; \qquad p_L = \frac{(1-s)u_L}{2-s};$$

$$\pi_{LHB}^{SEQ} = \frac{[k(2-s) + d(2-3s)]u_L}{4(2-s)}. \tag{10}$$

Next, in order to determine the firm's sequencing policy, note that $\frac{\partial \pi_{LHB}^{SEQ}}{\partial d} = \frac{(2-3s)u_L}{4(2-s)}$. This implies that, for $s \leq 2/3$, the firm optimally sets $d = 1$ and there is simultaneous release. For $s > 2/3$, the firm sets $d = 0$ and offers H alone at t_0, with a resulting profit of $\pi_H = u_H/4$, as in Proposition 1.

The second part of the proposition analyzes the case where $x < 1$ and $d_c = x^t d$. When the firm releases the products sequentially, it maximizes the profit as in (9). The optimal prices and the associated profits are now:

$$p_L = \frac{(1-s)(3+x^t)(k-dx^t)u_L}{4k(2-s) - d[1 + 6x^t + x^{2t} - s(1+x^t)^2]}; p_H = \frac{(k-dx^t)[2k(2-s) + d(1-s-x^t(1+s))]u_L}{4k(2-s) - d[1 + 6x^t + x^{2t} - s(1+x^t)^2]};$$

$$\pi_{LHB}^{SEQ} = \frac{(k-dx^t)[k(2-s) + d(2-s(2+x^t))]u_L}{4k(2-s) - d[1 + 6x^t + x^{2t} - s(1+x^t)^2]}. \tag{11}$$

The next step is to analyze how the firm establishes the video window. The analytical solution is rather complex, but we can easily establish limiting cases. First note that, from $d = \delta_p^t$, it follows that $t = Log(d)/Log(\delta_p)$. Next, by evaluating the FOC with respect to d at $d = 1$ we obtain:

$$\left.\frac{\partial \pi_{LHB}^{SEQ}}{\partial d}\right|_{d=1} = \frac{[(3s-2)Log(\delta_p) + sLog(x)]u_L}{4(s-2)Log(\delta_p)}.$$

[23] By way of a numerical example, when $u_H = 1$ and $u_L = 0.6$, region B exists as long as $c < 0.18$. This numerical set of parameters is just for illustration and results hold over a wide parameter range.

The sign of this expression is positive for $s < \hat{s}_1^c = \frac{2Log(\delta_p)}{3Log(\delta_p)+Log(x)} < 1$, hence the corner solution $d = 1$ exists in the range $0 \leq s \leq \hat{s}_1^c$. For $\hat{s}_1^c < s < 1$ there will be an interior solution with $0 < d < 1$ (sequential release). An example is plotted in Panel B of Figure 2. Also note that the FOC calculated for $d = 0$ is

$$\left.\frac{\partial \pi_{LHB}^{SEQ}}{\partial d}\right|_{d=0} = \frac{9(1-s)u_L}{16(2-s)}$$

which is never negative, unless $s = 1$. Thus, the firm sets $d = 0$ and offers H alone *only* in the limiting case where $s = 1$, while in all other cases with $s < 1$ it always sets $d > 0$. Finally, it can be seen immediately that $\hat{s}_1$ gets smaller as x is low (customers are relatively more patient) or δ_p is high (the firm does not discount much the future).

Therefore, for $\hat{s}_1^c < s < 1$, the prices in (11) are a candidate solution, as long as the ranking of indifferent types follows $\theta_{HB} > \theta_{LH} > \theta_{L0}$. In this region the indifferent types are:

$$\begin{aligned}
\theta_{HB} &= \frac{(3+x^t)(dx^t-k)}{4k(s-2)+d[1+6x^t+x^{2t}-s(1+x^t)^2]};\\
\theta_{LH} &= \frac{2k(s-2)+d[(s-1)-2(s-2)x^t-(s-1)x^{2t}]}{4k(s-2)+d[1+6x^t+x^{2t}-s(1+x^t)^2]};\\
\theta_{L0} &= \frac{(1-s)(3+x^t)(dx^t-k)}{4k(s-2)+d[1+6x^t+x^{2t}-s(1+x^t)^2]}.
\end{aligned}$$

It is always $\theta_{LH} > \theta_{L0}$. Also, if $d = 1$, it is always $\theta_{HB} > \theta_{LH}$. More in general, in the $\{s, d\}$ space, the condition $\theta_{HB} > \theta_{LH}$ is always satisfied by the candidate solution obtained from $\partial \pi_{LHB}^{SEQ}/\partial d = 0$ as long as x is higher than a limiting value, denoted as x^c. For lower values of x, however, the ranking cannot be preserved. Consider therefore the condition such that $\theta_{HB} = \theta_{LH}$ which gives

$$d[1 - x^t + s(2x^t + (x^t)^2 - 1)] = k(x^t + 2s - 1). \tag{12}$$

The limiting value x^c is obtained by looking at the first point of tangency between the curve that describes $\partial \pi_{LHB}^{SEQ}/\partial d = 0$ and the curve that describes (12). After computations, it is the highest value of x such that the equation

$$\left[k\left(3+x^{2t}\right)+d\left(1-4x^t-x^{2t}\right)\right]\log(\delta_p)-2(d+k)x^t\left(1-x^t\right)=0$$

can have a root in the plausible domain $0 < d < 1$, $0 < x < 1$.

When $x < x^c$, it can be shown that the curve that describes $\partial \pi_{LHB}^{SEQ}/\partial d = 0$ intersects the curve (12) twice in the the $\{s, d\}$ space: call these roots (s_1^c, d_1^c) and $(s_2^c, d_{2,}^c)$, where $s_1^c > s_2^c$. In particular, when $s < s_2^c$ or $s > s_1^c$, the ranking $\theta_{HB} > \theta_{LH} > \theta_{L0}$ is preserved, and therefore the firm will offer the product line $L/H/B$ with the video window previously described. For $s_2^c \leq s \leq s_1^c$, the second marginal buyer will buy B instead of H, and no consumer buys H alone. Thus, the firm offers the product line L/B and sets $p_L = \frac{(1-s)p_H}{k-dsx^t}$ to make sure that $\theta_{HB} = \theta_{LH}$. In particular, the firm now maximizes:

$$\pi_{LB}^{SEQ} = (p_H + dp_L)(1 - \theta_{LB}) + dp_L(\theta_{LB} - \theta_{L0}).$$

The prices and the associated profits are:

$$p_L = \frac{(1-s)[k + d(1 - s(1 + x^t))]u_L}{2[k + d(1 + s^2 - s(2 + x^t))]}; \quad p_H = \frac{(k - dsx^t)[k + d(1 - s(1 + x^t))]u_L}{2[k + d(1 + s^2 - s(2 + x^t))]};$$

$$\pi_{LB}^{SEQ} = \frac{[k + d(1 - s(1 + x^t))]^2 u_L}{4[k + d(1 + s^2 - s(2 + x^t))]}. \tag{13}$$

With these prices, the indifferent types are $\theta_{LB} = \frac{k+d[1-s(1+x^t)]}{2[k+d(1+s^2-s(2+x^t))]}$ and $\theta_{L0} = \frac{(1-s)[k+d(1-s(1+x^t))]}{2[k+d(1+s^2-s(2+x^t))]}$. Finally, in this intermediate range of s the video window chosen by the firm at t_0 is obtained from $\partial\pi_{LB}^{SEQ}/\partial d = 0$. Examples of strictly interior solutions with $0 < d < 1$ and a product line L/B are plotted in panels C and D of Figure 2. **Q.E.D.**

Proof of Proposition 3. Imagine that the firm does not commit to the price of L. If it releases the two versions sequentially, at t_1 it can sell L to consumers with a low θ that have bought nothing in the first period, and to consumers with a high θ that have already bought H. It thus sets p_L to maximize $\pi_L = [(1 - \theta_{HB}) + (\theta_{LH} - \theta_{L0})]p_L$, where $\theta_{L0} = \frac{p_L}{u_L}$ and $\theta_{HB} = \frac{p_L}{u_L(1-s)}$. The analysis at the second stage is independent of the particular value taken by x since in the expressions that define the market shares there is no discounting. Also notice that, at time t_1, θ_{LH} should be taken as given, as its value is determined at t_0.

When there is an interior solution that preserves the ranking $\theta_{HB} \geq \theta_{LH} \geq \theta_{L0}$ the firm offers the product line $L/H/B$ and sets the price $p_L = \frac{(1-s)[1+\theta_{LH}]u_L}{2(2-s)}$. At t_0, it offers H and sets p_H and d to maximize the following profit:

$$\pi_{LHB}^{SEQ} = (p_H + dp_L)(1 - \theta_{HB}) + p_H(\theta_{HB} - \theta_{LH}) + dp_L(\theta_{LH} - \theta_{L0}), \tag{14}$$

where p_L is the price defined at t_1 and $\theta_{LH} = \frac{p_H - d_c p_L}{u_H - d_c u_L}$. After solving the problem we obtain that the optimal prices determined by the firm and the associated profits are:

$$p_H = \frac{[k(2k(s-2) + d(s-1))(s-2) - d(k(s-5) + d(s-1))(s-2)x^t - d^2(s-3)x^{2t}]u_L}{(s-2)[4k(s-2) - d(s - 1 + 2(s-3)x^t)]};$$

$$p_L = \frac{[3k(s-2) + d(5-2s)x^t](s-1)u_L}{(s-2)[4k(s-2) - d(s - 1 + 2(s-3)x^t)]}; \tag{15}$$

$$\pi_{LHB}^{SEQ} = \frac{[k^2(s-2)^2 + 2dk(s-2)(s-1+x^t) + d^2x^t(x^t - 4 - 2(s-3)s)]u_L}{(s-2)[4k(s-2) - d(s-1+2(s-3)x^t)]},$$

where $t = Log(d)/Log(\delta_p)$.

The first part of the proposition considers the simpler case of identical discount factors, $x = 1$. This simplifies the above profit function as follows:

$$\pi_{LHB}^{SEQ} = \frac{[k^2(s-2)^2 + 2dk(s-2)s - d^2(3 + 2s(s-3))]u_L}{(2-s)[4k(2-s) + d(3s-7)]}.$$

This expression is strictly convex in d. Hence the solution is either at $d = 0$ (i.e., only H is sold) yielding $\pi_H = \frac{ku_L}{4}$, or at $d = 1$ (simultaneous release) yielding $\pi_{LHB}^{SEQ} = \frac{[k^{\ 2}(s-2)^2 - 3 + 6s + 2k(s-2)s - 2s^2]u_L}{(2-s)[3s-7-4k(s-2)]}$. The latter is preferred as long as

$$s < \bar{s}^{nc} = \frac{29k - 24[192 - 416k + 225k^2]^{1/2}}{22k - 16}.$$

Notice that $\bar{s}^{nc}$ is equal to $2/3$ only in the limiting case when k tends to 1, otherwise it is strictly lower than $2/3$ as it decreases in k, reaching a limit of $7/11$ for $k \to \infty$.

Prices in (15) are a candidate solution when $d = 1$ as long as $\theta_{HB} \geq \theta_{LH} \geq \theta_{L0}$, where:

$$\begin{aligned}
\theta_{HB} &= \frac{(2s-5) - 3k(s-2)}{(s-2)[(3s-7) - 4k(s-2)]}; \\
\theta_{LH} &= \frac{(s-3) - 2k(s-2)}{(3s-7) - 4k(s-2)}; \\
\theta_{L0} &= \frac{(s-1)[(2s-5) - 3k(s-2)]}{(s-2)[(3s-7) - 4k(s-2)]}.
\end{aligned}$$

It is always $\theta_{LH} > \theta_{L0}$. Simple computations show that if $s > \bar{s}_1^{nc}$ then $\theta_{HB} > \theta_{LH}$, where

$$\bar{s}_1^{nc} = \frac{5k - 3 - [5 - 14k + 9k^2]^{1/2}}{4k - 2}.$$

It can now be verified that $\bar{s}^{nc} > \bar{s}_1^{nc}$ for $k > 4/3$. This implies that for $s > \bar{s}^{nc}$ the solution is $d = 0$, and for $\bar{s}_1^{nc} < s \leq \bar{s}^{nc}$ it is $L/H/B$ and $d = 1$. Instead, for $s \leq \bar{s}_1^{nc}$ the second marginal buyer will buy B instead of H, since now $\theta_{HB} < \theta_{LH}$, and no consumer buys H alone. In this range, the firm has two options: It can set the price p_L such that either $\theta_{HB} = \theta_{LH}$ (the product line L/B emerges), or that $\theta_{L0} = \theta_{LH}$ (the product line H/B emerges). The former is the most profitable strategy, hence the firm sets $p_L = (1-s)u_L\theta_{LH}$.[24] This implies that in the second period the firm caters to all the previous customers, plus a bit more on the lower

[24] To be precise, this is true as long as $\theta_{LH} > (1-s)/(3 - 3s + s^2)$ which is satisfied at equilibrium.

end. With this pattern, at t_0 the firm offers H and sets p_H and d to maximize:

$$\pi_{LB}^{SEQ} = (p_H + dp_L)(1 - \theta_{LB}) + dp_L(\theta_{LB} - \theta_{L0}).$$

where $\theta_{L0} = (1-s)\theta_{LH}$, $\theta_{LB} = \theta_{LH}$ and $\theta_{LH} = \frac{p_H - dp_L}{u_H - du_L} = \frac{p_H}{u_L(k-ds)}$. The optimal prices and the associated profits are:

$$p_L = \frac{(1-s)[k + d(1-2s)]u_L}{2[k + d(1-3s+s^2)]}; \qquad p_H = \frac{(k-ds)[k + d(1-2s)]u_L}{2[k + d(1-3s+s^2)]},$$

$$\pi_{LB}^{SEQ} = \frac{[k + d(1-2s)]^2 u_L}{4[k + d(1-3s+s^2)]}. \tag{16}$$

This profit function is again strictly convex in d. Hence the firm's optimal sequencing strategy is either at $d = 0$ yielding $\pi_H = \frac{ku_L}{4}$, or at $d = 1$ yielding $\pi_{LB}^{SEQ} = \frac{(1+k-2s)^2 u_L}{4[k+1-3s+s^2]}$. The latter option is preferred by the firm as long as $s < \bar{s}_2^{nc} = \frac{4+k-[k(5k-4)]^{1/2}}{8-2k}$. It is also $\bar{s}_2^{nc} > \bar{s}_1^{nc}$ and therefore, when $s \leq \bar{s}_1^{nc}$, the firm sets $d = 1$ and offers the product line L/B.

The second part of the proposition analyzes the case where $x < 1$ and $d_c = x^t d$. Starting from (15) we now determine how the firm sets the video window. In analogy with Proposition 2, we first evaluate the FOC with respect to d at $d = 1$:

$$\left.\frac{\partial \pi_{LHB}^{SEQ}}{\partial d}\right|_{d=1} = \frac{[(51s - 21 - 32s^2 + 6s^3 + k^2(s-2)^2(11s-7) - 8k(15s - 6 - 10s^2 + 2s^3)Log(\delta^p) + 2(k-1)(s-2)(1+2s-s^2-k(2+s-s^2))Log(x)]u_L}{[7 + 4k(s-2) - 3s]^2(s-2)Log(\delta^p)}.$$

We call $\hat{s}^{nc}$ the value of s such that $\left.\frac{\partial \pi_{LHB}^{SEQ}}{\partial d}\right|_{d=1} = 0$ (we do not report the explicit value as this involves a long expression). The sign of this FOC is positive for $s \leq \hat{s}^{nc}$, hence the corner solution $d = 1$ (simultaneous release) can exist for $0 < s < \hat{s}^{nc}$. For $\hat{s}^{nc} < s < 1$ there can exist an interior solution with $0 < d < 1$ where the firm releases H and L sequentially. Indeed, as with commitment, it is easily verified that the FOC calculated for $d = 0$ is never negative for any value of $x < 1$, unless $s = 1$.

This implies that for $\hat{s}^{nc} < s < 1$ the firm offers $L/H/B$ and the prices in (15) are a candidate solution, as long as the indifferent types are ranked as $\theta_{HB} > \theta_{LH} > \theta_{L0}$, where:

$$\begin{aligned}
\theta_{HB} &= \frac{d[2s-5]x^t - 3k(s-2)}{(s-2)(4k(s-2) + d[1-s-2(s-3)x^t])}; \\
\theta_{LH} &= \frac{2k(s-2) - d[1-s+2(s-2)x^t]}{4k(s-2) + d[1-s-2(s-3)x^t]}; \\
\theta_{L0} &= \frac{(s-1)[3k(s-2) + d(5-2s)x^t}{(s-2)(4k(s-2) + d[1-s-2(s-3)x^t])}.
\end{aligned}$$

It is always $\theta_{LH} > \theta_{L0}$. Solving $\theta_{HB} = \theta_{LH}$ with respect to s we obtain:

$$s_1^{nc} = \frac{5k + d(3-6x^t) - [9k^2 - 2dk(10x^t - 3) + d^2(1 - 8x^t + 12x^{2t})]^{1/2}}{2(d + 2k - 2dx^t)}.$$

For $s > s_1^{nc}$ the ranking $\theta_{HB} > \theta_{LH}$ is preserved. Two further case distinctions can arise. If $s_1^{nc} < \hat{s}^{nc}$, then for $s_1^{nc} < s < \hat{s}^{nc}$ there is a corner solution $d = 1$ with the product line $L/H/B$ and for $\hat{s}^{nc} < s < 1$ there is an interior solution with the product line $L/H/B$. This case can arise only for values of x higher than a threshold value that we denote x^{nc} (Part 2) in Proposition 3). If instead $x < x^{nc}$, it is $s_1^{nc} > \hat{s}^{nc}$ and therefore there cannot be a corner solution with the product line $L/H/B$, but there is an interior solution for $0 < d < 1$ (Part 3) in Proposition 3). Conversely, for $s \leq s_1^{nc}$ the second marginal buyer will buy B instead of H, since now $\theta_{HB} \leq \theta_{LH}$, and no consumer buys H alone. The firm thus offers the product line L/B. At t_1 it sets the price $p_L = (1-s)\theta_{LH} u_L$ to make sure that $\theta_{HB} = \theta_{LH}$. At t_0 it offers H and sets p_H and d to maximize the following profit:

$$\pi_{LB}^{SEQ} = (p_H + dp_L)(1 - \theta_{LB}) + dp_L(\theta_{LB} - \theta_{L0}),$$

where $\theta_{LH} = \frac{p_H - d_c p_L}{u_H - d_c u_L} = \frac{p_H}{k - dsx^t}$. Notice that this gives the same price that would be offered in the commitment case when $x \leq x^c$ and the firm offers the L/B. Taking this into account, the firm's optimal profits are thus again as in (13). The analysis with respect to d is also the same when there is an interior solution. The difference with the commitment case is that now there can be a corner solution with $d = 1$ and a product line L/B. This is characterized by FOC with respect to d, when calculated at $d = 1$:

$$\left.\frac{\partial \pi_{LB}^{SEQ}}{\partial d}\right|_{d=1} = \frac{[(1-2s)(1-3s+s^2) + k(1-s-s^2))Log(\delta_p) - s(1+k+2s(s-2))Log(x)](1+k-2s)u_L}{4[1+k+(s-3)s]^2 Log(\delta_p)}. \tag{17}$$

We call s_2^{nc} the value of s that satisfies $\left.\frac{\partial \pi_{LB}^{SEQ}}{\partial d}\right|_{d=1} = 0$. For $0 \leq s \leq s_2^{nc}$ the sign of (17) is positive and hence the corner solution $d = 1$ exists and the firm offers the pattern L/B. Since it is $s_2^{nc} < s_1^{nc}$ when $x < x^{nc}$, this corner solution is in the admissible range. **Q.E.D.**

Proof of Proposition 4. Imagine that $s = 0$ and the producer reaches wholesale agreements with the exhibitor and the distributor. If the producer offers the product line L/B, at t_0 the exhibitor's retail problem is to set p_H to maximize $\pi_{LB}^e = (p_H - a)(1 - \theta_{LB})$ and the distributor's is to set p_L to maximize $\pi_{LB}^d = (1-r)dp_L(1-\theta_{L0})$, where the indifferent types are $\theta_{LH} = (p_H - dp_L)/(u_H - du_L)$ and $\theta_{L0} = p_L/u_L$. Computing the Nash equilibrium in prices and substituting them in the profits yields:

$$\pi_{LB}^d = (1-r)d\frac{u_L}{4}; \quad \pi_{LB}^e = \frac{(u_H - a)^2}{4u_H}.$$

As a consequence of these prices, the producer would get

$$\pi_{LB}^p = a(1 - \theta_{LB}) + drp_L(1 - \theta_{L0}) = \frac{a}{2}(1 - \frac{a}{u_H}) + rd\frac{u_L}{4}. \tag{18}$$

At time t_{-1}, the producer negotiates with the distributor over the revenue share r in a Nash bargain. Let $0 < \beta < 1$ be the producer's degree of bargaining power vis-à-vis the distributor. In case the negotiation breaks down, the distributor's outside option is zero, while the producer can still set an agreement with the exhibitor at t_{-2} to sell H, in which case it obtains $a(1 - \frac{u_H + a}{2u_H})$. Thus the pair of firms solve the following problem:

$$\max_r \Omega^{p,d}_{LB} = \left(\pi^p_{LB} - a(1 - \frac{u_H + a}{2u_H})\right)^\beta \left(\pi^d_{LB}\right)^{1-\beta}.$$

This gives the equilibrium share $r = \beta$. Instead, in case negotiations with the exhibitor had previously broken down, the producer's outside option would be zero in the current negotiation with the distributor, and the revenue share, as well as the release date of L, would be determined from selling L alone, that is

$$\max_{r,d} \Omega^{p,d}_L = (\pi^p_L)^\beta \left(\pi^d_L\right)^{1-\beta} = d(1-r)^\beta r^{1-\beta}\frac{u_L}{4},$$

from which we get again the result that $r = \beta$ since H and L are independent ($s = 0$), as well as $d = 1$. That is, *if* version H is not commercialized due to an earlier breakdown, the producer and the distributor release immediately L and share the monopoly profits according to their relative degree of bargaining power. This result determines the value of the producer's outside option $\pi^p_L = \beta u_L/4$ used in its negotiation with the exhibitor.

At time t_{-2}, the producer and the exhibitor bargain to determine the length of the video window and the rental price a, where the outside option for the exhibitor is zero, while the outside option for the distributor amounts to π^p_L, as just described. Let $0 < \alpha < 1$ now denote the producer's degree of bargaining power vis-à-vis the exhibitor. The firms solve the following Nash problem:

$$\max_{a,d} \Omega^{p,e}_{LB} = \left(\pi^p_{LB} - \frac{\beta u_L}{4}\right)^\alpha \left(\pi^e_{LB}\right)^{1-\alpha},$$

where $r = \beta$ is used into the expression (18) for π^p_{LB}. First, notice that π^p_{LB} overall depends positively on d, while π^e_{LB} is independent of it. Thus d is set at its highest possible value, $d = 1$. Second, after solving the FOC with respect to the rental price we get $a = \alpha u_H/2$.

The proof of the case where $s = 1$ follows the same reasoning, although the solution is now considerably more complex. If the producer offers the product line L/H, at time t_0 the exhibitor sets p_H to maximize $\pi^e_{LH} = (p_H - a)(1 - \theta_{LH})$ and the distributor sets p_L to maximize $\pi^d_{LH} = dp_L(\theta_{LH} - \theta_{L0})$. Solving these problems and substituting the prices in the profit functions yields:

$$\pi^d_{LH} = (1-r)\frac{du_L u_H (a + u_H - du_L)^2}{(4u_H - du_L)^2(u_H - du_L)}; \qquad \pi^e_{LH} = \frac{[2u_H(u_H - du_L) - a(u_H - du_L)]^2}{(4u_H - du_L)^2(u_H - du_L)}.$$

The producer thus would get

$$\begin{aligned}\pi^p_{LH} &= a(1-\theta_{LH}) + drp_L(\theta_{LH}-\theta_{L0}) \\ &= a(1-\frac{(a+u_H-du_L)(2u_H-du_L)}{(4u_H-du_L)(u_H-du_L)}) + r\frac{du_Lu_H(a+u_H-du_L)^2}{(4u_H-du_L)^2(u_H-du_L)}. \qquad (19)\end{aligned}$$

At time t_{-1}, the producer negotiates with the distributor over the revenue share for version L in a Nash bargain. They solve the following problem

$$\max_r \Omega^{p,d}_{LH} = \left(\pi^p_{LH} - a(1-\frac{u_H+a}{2u_H})\right)^{\beta} \left(\pi^d_{LB}\right)^{1-\beta}.$$

Notice that the producer's outside option is the same as for $s=0$, as it corresponds to the case where H is sold monopolistically. This gives the revenue share:

$$r = \beta + \frac{a(1-\beta)(4k-d)[a(3k-d)-k(k-d)u_L]}{2k^2(a+u_H-du_L)^2}, \qquad (20)$$

where $k = u_H/u_L > 1$. Notice that the revenue share is $r=1$ when $\beta=1$, but in all other cases it is $r>\beta$, that is, the producer gets a revenue share in excess of its bargaining power (the square bracket in (20) is always positive to the extent that a is high enough, which will happen at equilibrium). The revenue share to the producer also increases in d.

In case negotiations with the exhibitor had previously broken down, the producer and the distributor release immediately version L ($d=1$) and share the monopoly profits according to their relative degree of bargaining power ($r=\beta$). This determines the producer's outside option $\pi^p_L = \beta u_L/4$ used in its negotiation with the exhibitor at time t_{-2}.

At t_{-2}, the length of the window and the rental price of H are determined from the following Nash bargain between the producer and the exhibitor:

$$\max_{a,d} \Omega^{p,e}_{LH} = \left(\pi^p_{LH} - \frac{\beta u_L}{4}\right)^{\alpha} \left(\pi^e_{LH}\right)^{1-\alpha},$$

where (20) is used into the expression (19) for π^p_{LH}. The interior solution of this problem ($0<d<1$) is characterized by the following FOCs:

$$\begin{aligned}\frac{\alpha}{1-\alpha}\frac{\pi^e_{LH}}{\pi^p_{LH}-\frac{\beta u_L}{4}} &= -\frac{\partial\pi^e_{LH}/\partial a}{\partial\left(\pi^p_{LH}-\frac{\beta u_L}{4}\right)/\partial a}, \qquad (21) \\ \frac{\alpha}{1-\alpha}\frac{\pi^e_{LH}}{\pi^p_{LH}-\frac{\beta u_L}{4}} &= -\frac{\partial\pi^e_{LB}/\partial d}{\partial\left(\pi^p_{LH}-\frac{\beta u_L}{4}\right)/\partial d}. \qquad (22)\end{aligned}$$

Taking the ratio, the FOCs simplify to

$$\frac{\partial \pi^e_{LH}/\partial a}{\partial \left(\pi^p_{LH} - \frac{\beta u_L}{4}\right)/\partial a} = \frac{\partial \pi^e_{LH}/\partial d}{\partial \left(\pi^p_{LH} - \frac{\beta u_L}{4}\right)/\partial d}, \tag{23}$$

which, after substitutions, results in the following expression:

$$\begin{aligned} 0 \quad = \quad & (1-\beta)[a(2k-d) - 2ku_L(k-d)][2a(10k^2 - 4dk + d^2) + ku_L(4k^2 - 12dk - d^2)] + \\ & 4k^2[a^2(3d-2k) - 2a(d^2 + 5dk - 6k^2)u_L + (d-k)k(7d+2k)u_L^2]. \end{aligned} \tag{24}$$

Equation (24) is of second degree in a, but has only one positive root and therefore can be easily solved to find the value a^*. Substituting a^* into (22) results in one last equation in d, which can be solved as a function of the parameters to get d^*. This solves the problem completely. We do not report here the explicit value for d^*, as this involves a long expression, but this solution takes values in the appropriate interval [0, 1] when parameters k, α and β are in a relevant range (see also Figure 6 in the text for a plot of d^* at equilibrium.

We now discuss what happens when an interior solution does not exist for some parameter configuration. Take first the corner solution $d = 0$, i.e., version L is not released. In this case, the rental price that maximizes $\Omega^{p,e}_H$ is obtained only from (21), which simplifies to

$$2a^2 - a(2+\alpha)k + (\beta - \alpha\beta + \alpha k)k = 0.$$

The previous equation has only one valid root, which defines the value for the rental charge a^0. To define the range of validity of this solution, substitute a^0 and $d = 0$ into (22), to get

$$sign[\frac{\partial \Omega^{p,e}_{LH}}{\partial d}] < 0 \text{ iff } \alpha < \alpha^0, \text{ where}$$

$$\alpha^0 = \frac{1}{1 + \frac{\beta(1+15\beta-5A)(5-3\beta-A)^3 k}{\{25\beta^3 + 4k(7-3A) + \beta^2(-40+6k) + 2\beta[8+(A-3)k]\}[3+6\beta^2+A+\beta(2A-17)]}}; \quad A = \sqrt{9 - 10\beta + 9\beta^2}.$$

Hence the corner solution $d = 0$ can occur as long as α^0 takes plausible values (between 0 and 1). It can be shown that α^0 is decreasing in β, hence it takes its maximum value $\alpha^0 = 1$ for $\beta = 0$. α^0 is also increasing in k. In particular, when $\beta = 1$, it reaches the limit $\alpha^0 = 6 - 4\sqrt{2} \approx 0.343$ when $k \to \infty$. Figure 5 reports a three-dimensional plot of α^0.

Consider now the corner solution $d = 1$ (both versions are released simultaneously). In this case, the rental price that maximizes $\Omega^{p,e}_{LH}$ is obtained only from (21), which simplifies to

$$\begin{aligned} 0 \quad = \quad & 2(2k-1) \\ & - \frac{2\alpha(k-1)k(4k-1)u_L[a(3+2k(4k-7)+\beta(6k-3)) + u_L(\beta - 2k + 2(5+\beta)k^2 - 8k^3)]}{2a^2[(4k-1)^2(k-1) + \beta(1-7k+10k^2)] - 2a(k-1)k[(1-4k)^2 + \beta(-1+8k)]u_L + \beta(k-1)k(1+4k(3k-1))u_L^2}. \end{aligned}$$

The previous equation is of second degree in a, and has only one valid root, which we call

a^1. To define the range of validity of this solution, we substitute a^1 and $d = 1$ into (22), to get

$$sign[\frac{\partial \Omega_{LB}^{p,e}}{\partial d}] > 0 \text{ iff } \alpha > \alpha^1 = \frac{1}{1 + \frac{Num}{Den}}, \text{ where}$$

$$\begin{aligned} Num &= 2\beta k^2[3 + 56k^2 - 40k^3 + 3\beta(8k - 1 - 16k^2 + 8k^3) + 2k(B - 8) - B]\{87 + 1104k^6 + 320k^7 - 3\beta^2(-29 + 302k \\ &\quad -1455k^2 + 4096k^3 - 7300k^4 + 7872k^5 - 4464k^6 + 960k^7 - 10B - B^2 - 32k^5(222 + 13B) + 4k^4(2903 + 226B) \\ &\quad -k^3(8688 + 548B - 20B^2) + 2k(-413 + 16B + 5B^2) - k^2(-3549 - 74B + 23B^2)[2\beta - 87 + 15296k^5 \\ &\quad -8912k^6 + 2304k^7 + k(866 - 8B) + 5B - 28k^4(563 + 5B) - k^2(3989 + 45B) + 2k^3(5116 + 85B)]\}; \\ Den &= [\beta(3 - 8k - 20k^2 + 128k^3 - 112k^4) - (1 - 2k + 4k^2)(3 + 6k - 12k^2 - B)]\{\beta^3(77k - 8 - 375k^2 + 1346k^3 - 3284k^4 + 5016k^5 \\ &\quad -4496k^6 + 1760k^7) + (4k - 1)^2(k - 1)k[5 - 80k^3 + 80k^4 + 6B + B^2 - 12k^2(7 + 2B) + 4k(13 + 3B)] + \beta^2[16 - 5200k^5 \\ &\quad +2608k^6 + 1472k^7 - 1536k^8 + k^4(3764 - 72B) - 3k(53 + 2B) + k^2(647 + 6B) + 4k^3(12B - 421)] - \beta[8 + 2464k^7 - 1792k^8 \\ &\quad -16k^6(45 + 4B) + 8k^5(117 + 44B) - 4k^4(301 + 104B) - k(87 + 12B + B^2) + 2k^3(47 + 34B - 5B^2) + k^2(265 + 48B + 7B^2)]\}; \\ B &= \sqrt{9\,(1 + 2k - 4k^2)^2 + 9\beta^2\,(1 - 6k + 4k^2)^2 - 2\beta\,(9 - 36k + 148k^2 - 192k^3 + 80k^4)}. \end{aligned}$$

Hence the corner solution $d = 1$ can occur as long as α^1 takes plausible values (between 0 and 1). The expression for α^1 is complex, yet it can be shown that α^1 is decreasing in β, hence it takes its maximum value $\alpha^1 = 1$ for $\beta = 0$. α^1 is also decreasing in k. In particular, when $\beta = 1$, it reaches the limit value $\alpha^1 = 6 - 4\sqrt{2} \approx 0.343$ when $k \to \infty$. Since α^0 is instead increasing in k and for $\beta = 1$ it reaches a limit value for $\alpha^0 = 6 - 4\sqrt{2}$ when $k \to \infty$ this imply that $\alpha^1 > \alpha^0$ for *all* parameters range. Figure 5 reports the three-dimensional plot of α^1. **Q.E.D.**

Proof of Proposition 5. The proof for $s = 0$ is identical to Proposition 4, simply after substituting $\beta = 1$. Consider now $s = 1$ and imagine firms offer L/H. At time t_0 the exhibitor sets p_H to maximize $\pi_{LH}^e = (p_H - a)(1 - \theta_{LH})$ and the producer sets p_L to maximize $\pi_{LH}^p = dp_L(\theta_{LH} - \theta_{L0}) + a(1 - \theta_{LH})$. Solving these problems yields:

$$\pi_{LH}^p = \frac{(u_H - a)[du_L(u_H - du_L) + a(du_L + 8u_H)]}{(4u_H - du_L)^2}; \qquad \pi_{LH}^e = \frac{4(u_H - a)^2(u_H - du_L)}{(4u_H - du_L)^2}.$$

Contract terms are established at time t_{-1} by maximizing the following expression:

$$\max_{a,d} \Omega_{LH}^{p,e} = \left(\pi_{LB}^p - \frac{u_L}{4}\right)^\alpha (\pi_{LB}^e)^{1-\alpha}.$$

As in Proposition 4, assume first there is an interior solution with $0 < d < 1$. Taking the ratio of the FOCs, yields again (23) which now simplifies and, after substitutions, results in the following rental charge:

$$a = (2k^2 + 8d^*k - d^{*2})\frac{u_L}{18k}, \tag{25}$$

where d^* is the solution of the following equation, obtained after substituting (25) into (22):

$$d^{*3} + 9(2-\alpha)d^{*2}k + 6(3\alpha - 14)d^*k^2 + k^2[8k(9\alpha - 2) + 81(1-\alpha)] = 0. \qquad (26)$$

We do not report the explicit value for d^* as this involves a long expression, but Figure 8 below shows when the solution takes interior values.

We now consider what happens when there is the corner solution $d = 1$. In this case, the rental price that maximizes $\Omega_{LH}^{p,e}$ is obtained only from (21), which gives:

$$a^1 = \frac{2 + 16k^2 - (4k-1)\{\alpha + 2\alpha k - [16k(k-1)(1-\alpha) + (\alpha + 2\alpha k)^2]^{1/2}\}}{4 + 32k} u_L.$$

To define the range of validity of this solution, substitute $d = 1$ into (26), to get

$$(k-1)u_L^3[8(9\alpha - 2)k^2 + (9\alpha - 19)k - 1].$$

This expression is positive for any value of $\alpha > \widehat{\alpha}^1 = \frac{16k^2+19k+1}{72k^2+9k}$. This implies that for $\alpha > \widehat{\alpha}^1$ we are in a corner solution where $d = 1$ and $a = a^1$. As $\widehat{\alpha}^1$ decreases in k, then $\widehat{\alpha}^1$ takes values from $2/9$ (when $k \to \infty$) to $4/9$ (when $k \to 1$). It is also verified that $\widehat{\alpha}^1 < \alpha^1$, where α^1 is defined in Proposition 4 (compare also Figures 6 and 8).

Take now the corner solution $d = 0$. In this case, we get the rental price from (21):

$$a^0 = [(2+\alpha)k - k^{1/2}[k(2-\alpha)^2 - 8(1-\alpha)]^{1/2}]\frac{u_L}{4}.$$

To define the range of validity of this solution, substitute $d = 0$ into (22), to get

$$k^2[81 - 16k + 9\alpha(8k - 9)].$$

This corner solution exists for values of $\alpha < \max[0, \widehat{\alpha}^0]$, where $\widehat{\alpha}^0 = \frac{16k-81}{72k-81}$. As $\widehat{\alpha}^0$ is increasing in k, then it takes admissible values from 0 to $2/9$ (when $k \to \infty$). It is also $\widehat{\alpha}^0 < \alpha^0$, where α^0 is defined in Proposition 4 (compare also Figures 6 and 8). **Q.E.D.**

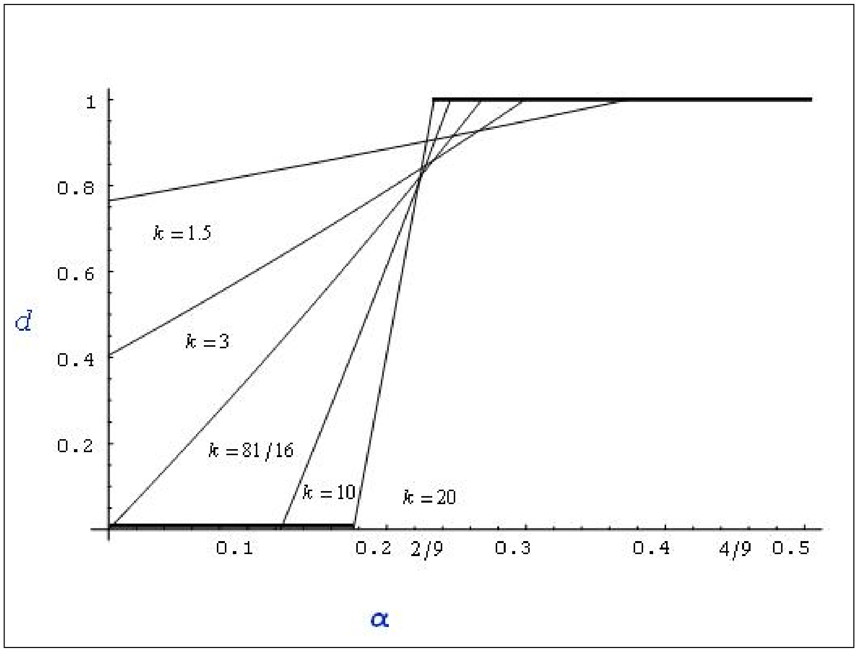

Figure 8: Negotiated video window. This figure is the analogue to Figure 6, in case the producer also distributes the L version directly.

References

[1] Anderson, E. and J. Dana (2009), When is Price Discrimination Profitable?, Management Science, 55(6), 980-989.

[2] August, T. and H. Shin (2010), Optimal Timing of Sequential Distribution: The Impact of Congestion Externalities and Day-and-Date Strategies, mimeo.

[3] Bhargava, H.K. and V. Choudhary (2001), Information Goods and Vertical Differentiation, Journal of Management Information Systems, 18(2), 85-102.

[4] Bhargava, H.K. and V. Choudhary, (2008), Research Note: When is Versioning Optimal for Information Goods? Management Science, 54(5), 1029-1035.

[5] Corts, K.S. (2001), The Strategic Effects of Vertical Market Structure: Common Agency and Divisionalization in the US Motion Picture Industry, Journal of Economics and Management Strategy, 10 (4), 509-28.

[6] Choudhary, V. (2010), Use of Pricing Schemes for Differentiating Information Goods, Information System Research, 21, 78-92.

[7] Dana, J. and K. Spier (2001), Revenue Sharing and Vertical Control in the Video Rental Industry, Journal of Industrial Economics, 49(3), 223-245.

[8] De Vany, A. and D. Walls (1999), Uncertainty in the Movie Industry: Does Star Power Reduce the Terror of the Box Office?, Journal of Cultural Economics, 23. 285-318.

[9] De Vany, A. (2004), Hollywood Economics. How Extreme Uncertainty Shapes the Film Industry, New York, NY: Routledge.

[10] Eliashberg, J., A. Elberse and M. Leenders (2006), The Motion Picture Industry: Critical Issues in Practice, Current Research, and New Research Directions, Marketing Science, 25(6), 638-661.

[11] Frank, B. (1994), Optimal Timing of Movie Releases in Ancillary Markets: The Case of Video Releases, Journal of Cultural Economics, 18, 125-133.

[12] Gabszewicz, J.J. and X. Wauthy (2003), The option of joint purchase in vertically differentiated markets, Economic Theory, 22, 817-829.

[13] Gil, R. (2007), "Make-or-buy" in movies: integration and ex-post renegotiation", International Journal of Industrial Organization, 25(4), 643-655.

[14] Gil, R. (2009), Revenue Sharing Distortions and Vertical Integration in the Movie Industry, Journal of Law, Economics, and Organization, 25(2), 579-610.

[15] Gil, R. and W. Hartmann (2009), Empirical Analysis of Metering Price Discrimination: Evidence from Concession Sales at Movie Theaters, Marketing Science, 28(6), 1046-1062.

[16] Hennig-Thurau, T., V. Henning, H. Sattler, F. Eggers, and M. Houston (2007), The Last Picture Show? Timing and Order of Movie Distribution Channels, Journal of Marketing, 71(4), 63-83.

[17] Hennig-Thurau, T., M.B. Houston, and G. Walsh (2006), The Differing Roles of Success Drivers Across Sequential Channels: An Application to the Motion-Picture Industry, Journal of the Academy of Marketing Science, 34, 559-575.

[18] Ho, J., K. Ho, and J. Mortimer (2008), Welfare Effects of Full-line Forcing Contracts in the Video Rental Industry, mimeo.

[19] Huang, K.-W. and A. Sundararajan (2011), Pricing Digital Goods: Discontinuous Costs and Shared Infrastructure, Information System Research, 22(4), forthcoming.

[20] Krishnan, U. and K. Ulrich (2001), Product Development Decisions: A review of the literature, Management Science, 47(1), 1-21.

[21] Lehmann, D. and C. Weinberg (2000), Sales Through Sequential Distribution Channels: An Application to Movies and Videos, Journal of Marketing, 64(3), 18-33.

[22] Levinthal, D. and D. Purohit (1989), Durable goods and product obsolescence, Marketing Science, 8(1), 35-56.

[23] Luan, J. and K. Sudhir (2007), Optimal Inter-Release Time Between Sequential Products: Application to Theatre Movies and DVD, mimeo.

[24] McKenzie, R. (2008), Why Popcorn Cost So Much at the Movies: And Other Pricing Puzzles, New York, NY: Springer.

[25] Moorthy, K. (1984), Marketing Segmentation, Self-Selection, and Product Line Design, Marketing Science, 3(4), 288-307.

[26] Moorthy, K. and I. Png (1992), Market Segmentation, Cannibalization, and the Timing of Product Introductions, Management Science, 38(3), 345-359.

[27] Mortimer, J., (2007), Price Discrimination, Copyright Law and Technological Innovation: Evidence form the Introduction of DVDs, Quarterly Journal of Economics, 122, 1307-50.

[28] Mussa, M. and S. Rosen (1978), Monopoly and Product Quality, Journal of Economic Theory, 18(2), 301-317.

[29] Nelson, R., C. Reid, and O. Gilmore (2007), An Analysis of the out-of-market gap for DVDs in the US, Review of Industrial Organization, 30, 303-23.

[30] Orbach, B. (2004), Antitrust and Pricing in the Motion Picture Industry, Yale Journal on Regulation, 21, 317-366.

[31] Owen, B. and S. Wildman (1992), Video Economics. Cambridge, MA: Harvard University Press.

[32] Padmanabhan, V., S. Rajiv and K. Srinivasan (1997), New Products, Upgrades, and New Releases: A Rationale for Sequential Product Introduction, Journal of Marketing Research, 34, 456-472.

[33] Prasad, A., B. Bronnenberg and V. Mahajan (2004), Product Entry Timing in Dual Distribution Channels: The Case of the Movie Industry, Review of Marketing Science, 2, article 4.

[34] Riggins, F. (2004), A Multichannel Model of Separating Equilibrium in the Face of the Digital Divide, Journal of Management Information Systems, 21(2), 161-179.

[35] Raut, S., S. Swami, E. Lee and C. Weinberg (2008) How complex Do movie Channel contracts need to be?, Marketing Science, 27(4), 627-641.

[36] Salant, S.W. (1989), When is Inducing Self-Selection Suboptimal for a Monopolist?, Quarterly Journal of Economics, 104, 391-397.

[37] Spengler, J. (1950), Vertical Integration and Antitrust Policy, Journal of Political Economy, 58, 347–52.

[38] Stokey, N. (1979), Intertemporal Price Discrimination, Quarterly Journal of Economics, 93, 355-371.

[39] Sundararajan, A. (2004a), Nonlinear Pricing of Information Goods, Management Science, 50(12), 1660-1673.

[40] Sundararajan, A. (2004b), Managing Digital Piracy: Pricing and Protection, Information Systems Research, 15(3), 287-308.

[41] Swami, S., J. Eliashberg, C. B. Weinberg (1999), SilverScreener: A modeling approach to movie screens management, Marketing Science, 18(3), 352-372.

[42] Thatcher, M.E. and D.E. Pingry (2004), An Economic Model of Product Quality and IT Value, Information Systems Research, 15(3), 268-286.

[43] Varian, H. (2000), Buying, Sharing and Renting Information Goods, The Journal of Industrial Economics, 58(4), 473-488.

[44] Vogel, H. L., (2007), Entertainment Industry Economics. Cambridge: Cambridge University Press.

[45] Waterman, D. (2005), Hollywood's Road to Riches, Cambridge, MA: Harvard University Press.

[46] Waterman, D., A. Weiss and S.C. Lee (2010), Time Consistency and Seller Commitment in Intertemporal Movie Distribution: An Empirical Study of the Movie Window, Journal of Industrial Economics, forthcoming.

[47] Wildman, S. (2008), Interactive Channels and the Challenge of Content Budgeting, The International Journal of Media Management, 10, 91-101.

[48] Zhang, X. (2009), Retailers' Multichannel and Price Advertising Strategies, Marketing Science, 28(6), 1080-1094.