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# ABSTRACT

## Endogenous debt crises*

We distinguish two types of debt crises: those that are the outcome of exogenous shocks (to productivity growth for instance) and those that are endogenously created, either by self-fulfilling panic in financial markets or by the reckless behavior of "Panglossian" borrowers. After Krugman, we characterize as "Panglossian" those borrowers who only focus on their best growth prospects, anticipating to default on their debt if hit by an adverse shock, rationally ignoring the risk of default. We apply these categories empirically to the data. We show that, taken together, endogenous crises are powerful explanations of debt crises, more important for instance than the sheer effect of growth on a country's solvency.

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# 1 Introduction

International debt crises are (very) costly. Why do we observe that so many countries fall into their trap? Should we not expect more prudent behavior from such countries? The theoretical answer in fact is: it depends. Take the simplest form of financial crisis driven by an exogenous shock. Spreads on sovereign bonds are high because the country is expected to be vulnerable to an earthquake or to a long-lasting commodity shock that is beyond its control. The country should then indeed behave with increased prudence: the greater the debt the country might have to repay, the heavier the cost of the earthquake relative to a favorable state of the nature. Yet, on the other hand, if the expected earthquake is so large that the country knows that it will actually default on its debt, then a "Panglossian attitude" (as Krugman has coined it) may become rational: the debt will lose all value after the earthquake, and it would then be absurd not to have borrowed more beforehand. The country then behaves as if the risk of unfavorable shocks can be ignored. Following Dr. Pangloss, the character of Voltaire's book *Candide*, the country acts as if only "the best of all possible worlds" will occur. In this case, debt endogenously leads to debt; we call this the *self-enforcing* case.

Let us now consider the case when crises are driven by the lack of confidence of financial markets towards a given country, making the country financially fragile through self-fulfilling behavior. Self-fulfilling debt crises have been analyzed in different forms. In the model of Cole and Kehoe (1996, 2000), self-fulfilling crises are a variant of a liquidity crisis, by which a lack of coordination among creditors leads a solvent country to default. As argued by Chamon (2007), however, such crises can readily be avoided when lenders manage to offer contingent loans of the kind organized by venture capitalists. If any individual creditor offers a line of credit, conditionally on other creditors following suit, then liquidity crises can be easily avoided.

Self-fulfilling crises have also been analyzed as the perverse outcome of a snowball effect through which the buildup of debt becomes unmanageable, out of the endogenous fear that it can indeed become unmanageable (Calvo, 1988). Relying on an intuition developed in a simpler model in Cohen and Portes (2004), we show that snowball spirals can only occur in cases where a debt crisis has the potential of damaging the fundamentals of the indebted country. If a crisis reduces the GDP of a country by say 10%, then it is clear that the lack of confidence toward a country can degenerate into a self-fulfilling crisis. If instead the fundamentals are not altered by the crisis, we show that self-fulfilling crises of the Calvo type are (theoretically) impossible.

At the end of this argument, we chose to focus on a simple characterization of a self-fulfilling debt crisis as one that is the outcome of an endogenous weakening of the country's fundamentals. In the self-fulfilling case so defined, it is the crisis that reduces GDP, originating from the various disruptions that a weakening of the confidence in a country may bring about (capital flight, exchange rate crisis...). In the "earthquake case", the sequence of causation is inversed: the fundamentals are first destroyed, then the crisis occurs.

From the theoretical model that we present, a simple typology of cases is obtained. Below a critical level of debt, a country tends to act prudently, aiming for instance to reduce its debt in response to a permanent adverse shock. Past a critical level of the debt-to-GDP ratio, which can be the outcome of a sequence of repeated unfavorable exogenous shocks, a country will begin to behave in the Panglossian mode, rationally ignoring the bad news, increasing the level of debt to its upper limit in a self-enforcing process. A crisis may then occur either because of the occurrence of another adverse exogenous shock, or because of a self-fulfilling shock, one that endogenously weakens the ability of a country to service its debt.

We approach the data with this type of typology. We use a slightly modified version of the database that has been compiled by Kraay and Nehru (2004), which we updated to cover all debt crises that have occurred until 2004. Following and adapting the work of these authors, we show that the likelihood of a debt crisis is well explained by three factors: the debt-to-GDP ratio, the level of real income per capita, and a measure of overvaluation of the domestic currency.

In order to estimate the risk of a self-fulfilling debt crisis, we then distinguish the law of motion of debt in tranquil times from the motion triggered by the onset of the crisis. We define a self-fulfilling crisis as one that would not have happened, had debt simply been driven along the pre-crisis path. We find that self-fulfilling crises, so defined, correspond to a small minority of cases. On average, less than a fifth of debt crises appear to be self-fulfilling. This proportion is clearly not negligible, however, and deserves to be taken seriously.

We also calibrate the strength of the Panglossian effect. We show that countries do appear to have behaved as if the distribution of the risk was truncated, leading the country to ignore risk. The influence of this mechanism on the debt buildup is tested through Monte-Carlo simulation. We show that it is substantial, and about twice as large as the self-fulfilling effect itself (see Arellano (2008) for similar insights applied to the the case of Argentina).

In this paper, we proceed as follows: in section 2, we set up an infinite-horizon model that we solve in section 3.3 and use to analyze the logic of each crisis. We then turn in section 4 to the presentation of the data supporting our econometric analyzes. In section 5, we build and estimate an econometric model which enables us to quantify the importance of both self-fulfilling and self-enforcing crises. Section 6 concludes.

# 2 A Panglossian theory of debt

In this section we develop a modeling framework which shares many features with the seminal work of Eaton and Gersovitz (1981) and with more recent works such as Aguiar and Gopinath (2006) and Arellano (2008). Our model mainly differs in the way we specify the stochastic process for output: instead of assuming that it is totally exogenous, we suppose that there is a feedback effect of a default upon output (in addition to the conventional penalty imposed by creditors upon the defaulting country). The joint determination of default and output is a feature shared with the model of Mendoza and Yue (2008), who take into account the negative effect of high interest rate spreads on the domestic economy, in a general equilibrium framework. In our model, the effect of a default upon output is modeled in a rather *ad hoc* manner, since we don't want to focus on a specific channel of transmission: beside the channel exhibited by Mendoza and Yue (2008), we want this feedback effect to also be a proxy for other potential channels such as exchange rate crises, capital flights, political instability...

## 2.1 The economy

We consider a one-good exchange economy. The country is inhabited by a representative consumer who can tilt consumption away from autarky by borrowing or lending on the international financial markets.

Output produced at time $t$ is a random variable $Q_t$, driven by a Markovian process. More precisely, the (gross) growth rate of output $g_t = \frac{Q_t}{Q_{t-1}}$ is assumed to be an *i.i.d.* variable, with a cumulative distribution function $F(g)$ and probability distribution function $f(g)$. In other words, $\ln Q_t$ is a random walk. For the sake of simplicity, we also assume that $g$ has a bounded support, and we denote by $g^{\max}$ its maximum value.

The world financial markets are characterized by a riskless rate of interest, which is a constant $r$. Lenders are risk-neutral and subject to a zero-profit condition by competition. We further suppose that debt is short-term and needs to be refinanced every year.

In order to ensure that the wealth of the country is finite, we make the assumption that the average growth rate $\mathbb{E}(g)$ is less than the gross interest rate $1 + r$.

At any time $t$, a country that has accumulated a debt $D_t$ may decide to default upon it. When it does so, we assume that the country suffers forever after a negative productivity shock. One can say that default creates a panic that destroys capital either through an exchange-rate or a banking crisis. We simply assume that post default output can be written:

$$Q_t^d = \mu Q_t$$

in which $\mu < 1$. As another cost, we assume that the country is subject to financial autarky, being unable to borrow again later on (a milder form of a sanction would be, more realistically, that the country is barred from the financial market for some time only; analytically, the outcome is formally equivalent).

Once the country has defaulted, creditors will attempt to recover some of their losses. In order to do so, they further reduce the resources of the country, in a way which, we assume, is socially efficient: the fraction that they grab is simply substracted, one for one, from the country's post-default output. Call $\lambda_t$ the fraction so reduced. We assume that $\lambda_t$ is itself an *i.i.d.* stochastic variable, in the domain $[0, 1]$ and independent of $g_t$, which varies with the (legal) strength of the international financial community. We denote by $H(\lambda_t)$ the cumulative distribution function of $\lambda_t$, and $h(\lambda_t)$ its probability distribution function. Creditors then capture:

$$P_t = \lambda_t Q_t^d,$$

while the country consequently consumes (given financial autarky):

$$C_t = (1 - \lambda_t) Q_t^d.$$

In the case when $\mu$ is equal to one, the outcome may be characterized as an efficient restructuring of the debt, at least from a static point of view (we return to this issue below): creditors are able to capture a fraction of output, which is less than what they are owed, but without imposing a social cost to the economy. When instead, at the other extreme, $\lambda_t$ is nil or very low and $\mu < 1$, then the implication is that default is socially costly, imposing a social loss, and no fraction of output can be captured by anyone.

## 2.2 Financial markets

The timing of events unfolds as follows. First assume that the country has incurred a debt obligation $D_t$, falling due at time $t$, and has always serviced it in full in previous years. At the beginning of period $t$, the country learns the value of its output $Q_t$ and the fraction of output $\lambda_t$ that it would lose were it to default. After observing these variables, the country decides to default or to reimburse its debt.

If the debt is reimbursed in full, the country can contract a new loan, borrowing $L_t$, which must be repaid at time $t+1$, in the amount of $D_{t+1}$. In order to avoid coordination problems, we assume, following Chamon (2007), that creditors can commit on $L_t$ and $D_{t+1}$ before the decision to service the debt is known, conditionally on the decision to service the debt being made.

Such financial agreements being concluded, the country eventually consumes, in the event it services its debt in full:

$$C_t = Q_t + L_t - D_t$$

Alternatively, in the event of a debt crisis the country's consumption is nailed down to $C_t^d = (1-\lambda_t)Q_t^d$.

We denote by $\mathscr{D}(D_{t+1}, Q_t)$ the default set, *i.e.* the set consisting of all realizations $(g_{t+1}, \lambda_{t+1})$ for which the country will decide to default in $t+1$, conditionally on the level of current debt $D_{t+1}$ and past output $Q_t$. We denote by $\mathscr{R}(D_{t+1}, Q_t)$ the repayment set, *i.e.* the complementary to the default set.

We can then define the risk of a debt crisis in $t+1$, as it is perceived from the perspective of date $t$:

$$\pi_{t+1|t} = \mathbb{P}(\mathscr{D}(D_{t+1}, Q_t)).$$

The zero-profit condition for creditors may be written as:

$$L_t(1+r) = D_{t+1}(1-\pi_{t+1|t}) + \int\limits_{\mathscr{D}(D_{t+1},Q_t)} V_{t+1}(g\,Q_t, \lambda)\mathrm{d}F(g)\mathrm{d}H(\lambda) \qquad (1)$$

in which $V_{t+1}(Q_{t+1}, \lambda_{t+1})$ is the discounted present value of all cash-flows that the banks will be able to extract from the country, when they expect to receive forever after $t+1$ an amount $P_s = \lambda_s Q_s^d$ in every period.

Finally, we also assume that the usual no-Ponzi game condition holds, at all time $t$:

$$\lim_{s\to+\infty} \mathbb{E}_t \frac{D_{t+s}}{(1+r)^{t+s}} = 0.$$

## 2.3 Preferences

The decision to default or to stay current on the financial markets involves a comparison of two paths that implies expectations over the entire future.

In order to address this problem, we assume that the country seeks to solve:

$$J^*(D_t, Q_t, \lambda_t) = \max_{\{C_s\}_{s\geq t}} \mathbb{E}_t \left\{ \sum_{s=t}^{\infty} \beta^{(s-t)} u(C_s) \right\}$$

where $C_t$ must be positive, while $D_t$ can be negative if the country builds up foreign assets.

We assume that utility is isoelastic, of the form:

$$u(x) = \frac{x^\varepsilon}{\varepsilon}$$

where $\frac{1}{1-\varepsilon}$ is the intertemporal elasticity of substitution.

We shall call:

$$J^d(Q_t, \lambda_t) = \mathbb{E}_t \left\{ \sum_{s=t}^{\infty} \beta^{(s-t)} u((1-\lambda_s) Q_s^d) \right\}$$

the post-default level of utility, which becomes by definition independent of debt, and to which the country is nailed down in case of servicing difficulties.

If it were to stay current on its debt obligation, it would obtain:

$$\begin{aligned} J(D_t, Q_t) = \max_{L_t, D_{t+1}} \Bigg\{ & u(Q_t - D_t + L_t) + \beta \int_{\mathscr{D}(D_{t+1}, Q_t)} J^d(g\,Q_t, \lambda) \mathrm{d}F(g) \mathrm{d}H(\lambda) \\ & + \beta \int_{\mathscr{R}(D_{t+1}, Q_t)} J(D_{t+1}, g\,Q_t) \mathrm{d}F(g) \mathrm{d}H(\lambda) \Bigg\} \end{aligned}$$

subject to the zero-profit condition (1).

Note that $J(D_t, Q_t)$ does not depend on the current value of $\lambda_t$.

When comparing how much it can get by staying on the markets and the post-default level of welfare, the country picks up its optimum level:

$$J^*(D_t, Q_t, \lambda_t) = \max \left\{ J(D_t, Q_t), J^d(Q_t, \lambda_t) \right\}$$

Note that $J^*(D_t, Q_t, \lambda_t)$ is clearly a function of the current value $\lambda_t$ through the influence of $J^d$.

# 3 Recursive equilibrium

## 3.1 Definition and basic properties

We now turn to the formal definition of a recursive equilibrium in this model. Such an equilibrium consists of a set of policy functions for the country and the investors. We assume that agents act sequentially, and that the government does not have commitment.

We also make the assumption that in the process of negociating debt contracts, the country first announces the amount $L$ that it wants to borrow today, and the investors reply with the amount $D'$ that they ask tomorrow for that loan.

**Definition 1 (Recursive equilibrium)** *A recursive equilibrium is defined by default and repayment sets $\mathscr{D}$ and $\mathscr{R}$ and value functions $J$, $J^d$, $J^*$ for the country, policy function $\tilde{D}'$ and a default value function $V$ for the investors, such as:*

- *The value function $J^d$ in case of default satisfies*

$$J^d(Q,\lambda) = u((1-\lambda)\mu Q) + \beta \int J^d(g'\,Q,\lambda')\mathrm{d}F(g')\mathrm{d}H(\lambda')$$

- *Given default and repayment sets $\mathscr{D}$ and $\mathscr{R}$ and investors' policy function $\tilde{D}'$, the value function $J$ in case of repayment satisfies:*

$$J(D,Q) = \max_{L\in\mathscr{L}(Q),\, L\geq D-Q} \Bigg\{ u(Q-D+L) + \beta \int_{\mathscr{D}(\tilde{D}'(L,Q),Q)} J^d(g'\,Q,\lambda')\mathrm{d}F(g')\mathrm{d}H(\lambda')$$
$$+ \beta \int_{\mathscr{R}(\tilde{D}'(L,Q),Q)} J(\tilde{D}'(L,Q), g'\,Q)\mathrm{d}F(g')\mathrm{d}H(\lambda') \Bigg\} \quad (2)$$

  *where $\mathscr{L}(Q)$ characterizes the domain of definition of $\tilde{D}'$.*

- *$J^*$ is the maximum of $J$ and $J^d$, and the default and repayment sets verify:*

$$(g',\lambda') \in \mathscr{D}(D',Q) \;\Leftrightarrow\; (g',\lambda') \notin \mathscr{R}(D',Q) \;\Leftrightarrow\; J^d(g'\,Q,\lambda') > J(D',g'\,Q)$$

- *The value that investors can extract in case of default satisfies:*

$$V(Q,\lambda) = \lambda\mu Q + \frac{1}{1+r}\int V(g'\,Q,\lambda')\mathrm{d}F(g')\mathrm{d}H(\lambda')$$

- *Given the default and repayment sets $\mathscr{D}$ and $\mathscr{R}$, the policy function of investors satisfies the zero-profit condition for all $L \in \mathscr{L}(Q)$:*

$$L(1+r) = \tilde{D}'(L,Q)\mathbb{P}[\mathscr{R}(\tilde{D}'(L,Q),Q)] + \int_{\mathscr{D}(\tilde{D}'(L,Q),Q)} V(g'Q,\lambda')\mathrm{d}F(g')\mathrm{d}H(\lambda') \tag{3}$$

At this point, it is important to note that our model is constructed in such a way that homogenous equilibria, as defined below, are possible.

**Definition 2 (Homogenous recursive equilibrium)** *A recursive equilibrium is said* homogenous *if it satisfies the following relationships for $\alpha > 0$:*

$$\begin{aligned}
J(\alpha D, \alpha Q) &= \alpha^{\varepsilon} J(D,Q) \\
J^d(\alpha Q, \lambda) &= \alpha^{\varepsilon} J^d(Q,\lambda) \\
J^*(\alpha D, \alpha Q, \lambda) &= \alpha^{\varepsilon} J^*(D,Q,\lambda) \\
\tilde{D}'(\alpha L, \alpha Q) &= \alpha \tilde{D}'(L,Q) \\
V(\alpha Q, \lambda) &= \alpha V(Q,\lambda) \\
\mathscr{L}(\alpha Q) &= \alpha \mathscr{L}(Q) \text{ (with obvious notation)}
\end{aligned}$$

The possibility of homogenous recursive equilibria stems from three specific features of our model: the isoelasticity of the utility function, the specific form of the output process (*i.i.d.* in growth rates), and the proportionality of default costs.

It is theoretically possible that our model has recursive equilibria that are not homogenous, but such equilibria are more of the nature of mathematical curiosities rather than economically relevant objects. In the following, we will therefore assume that there exists at least a homogenous equilibria, which satisfies standard regularity conditions, and we establish several results that apply to these homogenous recursive equilibria.

**Lemma 1** *The following functions have a closed-form solution:*

$$\begin{aligned}
J^d(Q,\lambda) &= \frac{u((1-\lambda)\mu Q)}{1-\beta\mathbb{E}(g^{\varepsilon})} \\
V(Q,\lambda) &= \frac{\lambda\mu Q}{1-\frac{\mathbb{E}(g)}{1+r}}
\end{aligned}$$

**Proof.** Immediate using the homogeneity of the functions in their definition. ■

**Lemma 2** *Default occurs if and only if debt-to-GDP ratio is higher than a given threshold* $d^*(\lambda)$, i.e. *one has:*

$$(g', \lambda') \in \mathscr{D}(D', Q) \Leftrightarrow \frac{D'}{g'Q} > d^*(\lambda')$$

**Proof.** Immediate consequence of the homogeneity of value functions. ■

Finally, we introduce a definition, which will be useful for characterizing the case of multiple equilibria:

**Definition 3 (Smooth default)** *We call the* smooth default *case the situation where the default threshold is equal to what the investors can extract in case of default,* i.e. *when* $d^*(\lambda) = V(1, \lambda)$.

Clearly, a smooth default is only possible when $\mu = 1$, *i.e.* when a default leads to an efficient restructuring of the debt from a static point of view (there is still an inefficiency related to the loss of access to financial markets). The reciprocal is not true: it is possible to have statically efficient defaults which are not smooth.Think of a country with a low rate of time preference (lower than the riskless interest rate), and with a linear utility function. It is easy to show that, in that case, the country will be willing to repay a debt higher than what investors would extract in case of default (simply because the country has a lower discount rate than investors).

## 3.2 The risk of multiple equilibria

In a standard setup, the interest rate charged by investors is entirely determined the probability of default, via the risk premium: the higher the risk, the higher the interest rate.

But the reverse causality can very well be also at work. One may have situations where two equilibria are possible: a "good equilibrium" where the investors ask for a low interest rate, leading to a low debt-to-GDP tomorrow – and therefore a low risk of default (consistent with the low interest rate), and a "bad equilibrium" where investors ask for a high interest rate, consistently leading to a high level of risk.

This kind of multiple equilibria in the interest rate, also called the "snowball effect", have been studied by Calvo (1988).

Note that multiple equilibria in the interest rate are possible in our model because the country announces $L$ and the investors reply with some $D'$ which satisfies the zero-profit condition; as noted by Chamon (2007, footnote 7), such multiple equilibria are impossible in the reverse setup, where the country announces $D'$ and the investors reply with the corresponding $L$.

Formally, a multiple equilibrium in the interest rate is a situation where, for a given $L_t$, there are two values $D^1_{t+1} < D^2_{t+1}$ verifying the zero-profit condition, and such that $\mathscr{D}(D^1_{t+1}, Q_t) \subset \mathscr{D}(D^2_{t+1}, Q_t)$.

**Proposition 3** *Multiple equilibria in the interest rate are impossible in the smooth default case.*

The proof is in appendix.

This result is the generalization of the result obtained by Cohen and Portes (2004) in a simpler model, who show that multiple equilibria are ruled out when default is statically efficient. Their intuition is simple: for a given set of fundamentals there can only be one equilibrium, in the simplest settings at least. What drives the multiple equilibrium case is the fact that the crisis endogenously destroys part of the fundamentals upon which the debt is repaid (since after default, in the previous case, creditors receive nothing). This may be the key reason why corporate self-fulfilling debt crises are a curiosity. To the extent that an appropriate bankruptcy procedure exists, the risk that a financial crisis can, out of its own making, endanger the value of a firm is much reduced.

## 3.3 Dynamics for non-defaulters

We now derive the Euler equation of non defaulters. Call:

$$q(D, Q) = -\frac{\partial J}{\partial D}(D, Q)$$

Using the enveloppe theorem in equation (2), we get:

$$q(D, Q) = u'(Q - D + L^*)$$

where $L^*$ is the optimal level of borrowing in case of repayment.

The first order condition of the maximisation in (2) leads to:

$$u'(Q - D + L^*) = \beta \frac{\partial \tilde{D}'}{\partial L}(L^*, Q) \int\limits_{\mathscr{R}(\tilde{D}'(L^*,Q),Q)} q(\tilde{D}'(L^*, Q), g'\, Q) \mathrm{d}F(g') \mathrm{d}H(\lambda')$$

(using the fact that $J$ and $J^d$ are equal at the default threshold).

The derivative of the investors' decision rule $\tilde{D}'$ can be obtained from equation (3), using the implicit function theorem:[1]

$$\frac{\partial \tilde{D}'(L, Q)}{\partial L} = \frac{1 + r}{\mathbb{P}[\mathscr{R}(\tilde{D}'(L, Q), Q)] - \gamma(\tilde{D}'(L, Q), Q)}$$

[1]See the proof of proposition 3 in appendix for some elements of the computation.

where:
$$\gamma(D',Q) = \frac{D'}{Q}\int (d^*(\lambda') - V(1,\lambda'))\mathrm{d}H(\lambda')$$

Note that $\gamma(D',Q) \geq 0$ because $d^*(\lambda') \geq V(1,\lambda')$ (easy to show). Also note that $\gamma(D',Q) = 0$ in the smooth default case.

In order to get the intuition behind this result, assume that $\lambda$ is a constant. With obvious notations, the zero-profit condition for creditors may be written as:

$$L_t(1+r) = D_{t+1}(1-\pi_{t+1}) + \int_{\underline{Q}_{t+1}}^{Q^*_{t+1}} V_{t+1}(\tilde{Q})dF_{t+1}(\tilde{Q})$$

in which $V_{t+1}(\tilde{Q})$ is the discounted present value of all cash-flows that the banks will be able to extract from the country, when they expect to receive forever after $t+1$ an amount $P_s = \lambda Q^d_s$ in every period.

One can then write :

$$(1+r)\frac{\partial L_t}{\partial D_{t+1}} = (1-\pi_{t+1}) - \frac{\partial \pi_{t+1}}{\partial D_{t+1}}D_{t+1} + \frac{\partial Q^*_{t+1}}{\partial D_{t+1}}V_{t+1}(Q^*_{t+1})$$

*i.e.*
$$\frac{\partial L_t}{\partial D_{t+1}} = \frac{1}{1+r}(1-\pi_{t+1}-\gamma_{t+1})$$

which corresponds to the more general solution obtained above. In the smooth repayment case ($\gamma = 0$), we simply find the marginal price of debt to equal to the probability of default.

Returning to the general case, we can rewrite the Euler equation as:

$$q(D,Q) = \frac{\beta(1+r)}{\mathbb{P}[\mathscr{R}(\tilde{D}'(L,Q),Q)] - \gamma(\tilde{D}'(L,Q),Q)}\int_{\mathscr{R}(\tilde{D}'(L^*,Q),Q)} q(\tilde{D}'(L^*,Q),g'\,Q)\mathrm{d}F(g')\mathrm{d}H(\lambda')$$

Along an equilibrium path, this means that we have (switching back to the notation using time subscripts):

$$q_t = \beta(1+r)\left(\frac{1-\pi_{t+1|t}}{1-\pi_{t+1|t}-\gamma_{t+1|t}}\right)\mathbb{E}_t\left[q_{t+1}\left|\mathscr{R}(D_{t+1},Q_t)\right.\right]$$

where $q_t = u'(C_t)$, $\gamma_{t+1|t} = \gamma(D_{t+1},Q_t)$, $\pi_{t+1|t}$ is the probability of default in $t+1$ from the perspective of date $t$, and the term $\mathbb{E}_t\left[q_{t+1}\left|\mathscr{R}(D_{t+1},Q_t)\right.\right]$ stands for the expectation of $q_{t+1}$, from the perspective of date $t$, *conditionally on the decision to repay at date* $t+1$.

This equation reveals the core of the Panglossian theory. First consider the smooth default case where $\gamma_{t+1|t} = 0$. In that case, the equation boils down to:

$$q_t = \beta(1+r)\mathbb{E}_t\left[q_{t+1}\left|\mathscr{R}(D_{t+1}, Q_t)\right.\right]$$

When its decides its level of indebtment, the country only takes into account the consequences of its decision for the subset of events where growth is high and makes default non-optimal. It then rationally ignores risk: this is the Panglossian effect.

In the general case where $\gamma_{t+1|t}$ is positive, the Panglossian motive is reduced. Taking a linear approximation of the term $\frac{1-\pi_{t+1|t}}{1-\pi_{t+1|t}-\gamma_{t+1|t}}$, one can write:

$$q_t = \beta(1+r)(1+\gamma_{t+1|t})\mathbb{E}_t\left[q_{t+1}\left|\mathscr{R}(D_{t+1}, Q_t)\right.\right]$$

It is indeed evident that the term $\gamma_{t+1|t}$ tends to raise the marginal utility of consumption at time $t$, and consequently reduces the propensity to borrow. The intuition is straightforward: to the extent that default entails a social loss, the benefit of borrowing against future risk is reduced, decreasing the desirability of debt in consequence.

## 3.4 A linear approximation

We now write the first-order linear approximation of the model presented so far.

Let us note: $q_t = -\frac{\partial J}{\partial D}(D_t, Q_t) = a\,Q_t - b\,D_t$

We may then write the Euler equation as:

$$a\,Q_t - b\,D_t = \delta\left[a\,Q^+_{t+1|t} - b\,D^*_{t+1}\right]$$

in which $Q^+_{t+1|t} = \mathbb{E}_t\left[Q_{t+1}\left|\mathscr{R}(D^*_{t+1}, Q_t)\right.\right]$, $\delta = \beta(1+r)(1+\gamma)$ (neglecting here the variability of the factor $\gamma$), and $D^*_{t+1}$ is the corresponding first best decision regarding debt.

Furthermore, let us denote $M_{t+1|t} = \mathbb{E}_t Q_{t+1}$, the expected value of output at time $t+1$, so that we may also write:

$$M_{t+1|t} = (1-\pi_{t+1|t})Q^+_{t+1|t} + \pi_{t+1|t}Q^-_{t+1|t}$$

in which $Q^-_{t+1|t} = \mathbb{E}_t\left[Q_{t+1}\left|\mathscr{D}(D^*_{t+1}, Q_t)\right.\right]$.

Denoting

$$\Gamma_{t+1|t} = Q^+_{t+1|t} - M_{t+1|t} = \pi_{t+1|t}(Q^+_{t+1|t} - Q^-_{t+1|t})$$

the Euler equation can then be written as:

$$a\,Q_t - b\,D_t = \delta\left[a\,M_{t+1|t} - \varrho\,D^*_{t+1}\right] + \delta\,a\,\Gamma_{t+1|t}$$

or again as:

$$D^*_{t+1} = \theta D_t + \frac{a}{b}\Gamma_{t+1|t} + \frac{a}{b}\left[M_{t+1|t} - \theta\,Q_t\right]$$

with $\theta = \frac{1}{\delta}$.

The term $M_{t+1|t} - \theta\,Q_t$ may be interpreted as a business-cycle component of the debt buildup. When output is low compared to the expected mean of next year's output, it borrows in order to smooth out consumption. The term is neglected in the Markovian model presented above, and show not much empirical relevance below.

The term $\frac{a}{b}\Gamma_{t+1|t}$ is the Panglossian term, which measures the way creditors truncate their forecasting set.

For practical matters, we shall also assume that the level of debt is not carefully derived from this first order equation. As shown by Campos et al. (2006), there is a lot of extrinsic noise in the level of debt, due to either unforeseen contingencies debt, or unpredicted valuation effects. In other words, we simply assume that $D^*_{t+1}$ differs from actual debt by a noisy term, and write:

$$D_{t+1} = D^*_{t+1} + u_{t+1}Q_{t+1}$$

where $u_{t+1}$ is an *i.i.d.* shock.

Let us then write:

$$\frac{Q_{t+1}}{Q_t} = 1 + g_{t+1}$$

the growth rate of the economy

With an obvious change of notation, we are able to redefine the Panglossian effect as:

$$\frac{\Gamma_{t+1|t}}{Q_t} = \pi_{t+1|t}(g^+_{t+1|t} - g^-_{t+1|t}). \quad (4)$$

We can then write:

$$d_{t+1} = \theta d_t - g_{t+1}d_t + c\,\pi_{t+1|t}(g^+_{t+1|t} - g^-_{t+1|t}) + u_{t+1} \quad (5)$$

Note importantly that the growth rate itself will be negatively affected by the occurence of the crisis, which is the essence of the risk of self-fulfilling equilibria. It is such an equation that we now apply to the data.

# 4 Dataset

Our empirical strategy relies on a dataset of "debt distress" and "normal times" episodes, following the methodology of Kraay and Nehru (2004).

More precisely, for a given year, a country is considered to be in debt crisis if at least one of the following three conditions holds:

1. The country receives debt relief from the Paris Club in the form of a rescheduling and/or a debt reduction.

2. The sum of its principal and interest in arrears is large relative to the outstanding debt stock.

3. The country receives substantial balance of payments support from the IMF through a non-concessional Standby Arrangement (SBA) or an Extended Fund Facility (EFF).

For the last two conditions, we choose the same thresholds as do Kraay and Nehru (2004); that is, a country is considered to be in crisis if its arrears are above 5% of the total stock of its outstanding debt, or if the total amount agreed to under SBA/EFF arrangements is above 50% of the country's IMF quota. Moreover, a country receiving Paris Club relief for a given year is also considered to be in crisis for the following two years since the relief decision is typically based on three-year balance of payments projections by the IMF.

Having defined when a country is considered to be in crisis or not, we then define "debt distress" episodes as periods of at least three consecutive years of crisis. Moreover, we impose the restriction that a distress episode should be preceded by at least three years without crisis, so that we are able to consider macroeconomic variables before a crisis episode as being exogenous to the crisis.

We also define "normal times" episodes as five consecutive years without any crisis (imposing no other restriction).

For identifying "debt distress" and "normal times" episodes, we use the following data sources:

- the World Bank's *Global Development Finance 2006* for data on debt levels and payment arrears,

- the Paris Club website[2] for information on debt reliefs,

- the IMF's *International Financial Statistics 2006* for data on SBA/EFF commitments.

[2] http://www.clubdeparis.org

In our subsequent econometric estimations, we also use two other sources:

- the World Bank's *World Development Indicators 2006* for general macroeconomic variables,
- the *Penn Word Tables* (version 6.2) for data on Purchasing Power Parity (PPP) variables.

The set of countries over which are made the computations consists of the 135 developing countries defined by the World Bank, from which we removed the 38 countries that have absolutely no access to private financial markets.[3]

We choose to remove them since their situation of indebtedness is somewhat different from that of the rest of the developing world (in particular, they have a much higher proportion of concessional lending). From the standpoint of the model, they probably fall into the category of countries that have no access to risky markets, and their debt dynamics must consequently be different.

We are therefore left with a sample of 97 countries. From the time angle, our data cover the period 1970-2004.

Prior to the elimination of certain observations in our econometric estimations (due to missing data), our largest sample of episodes consists of 70 distress episodes, and 223 normal times episodes.

To summarize, the differences between our dataset and that of Kraay and Nehru are twofold: first, we update their data to 2004, which is relatively minor but allows us to include the Ecuadorian debt crisis of 2000 for instance. Second, we restrict our analysis to the emerging countries that have access to private credit markets.

# 5 The econometric model

## 5.1 The estimated equations

Our empirical framework is given by the following system of three simultaneous equations. Since these three equations exhibit a circular dependency, there is an identification issue, which is dealt with in the following section.

$$d_{it} = X_{1,i,t-1}\beta_{X_1} + g_{it}\, X_{2,i,t-1}\beta_{X_2} + u_{it} \tag{6}$$

[3]We define market access as in Gelos et al. (2004). The countries we removed are those that never accessed international credit markets between 1980 and 2000, in accordance with the authors' definition. The complete country list can be found on page 29 of their paper.

$$g_{it} = Y_{1,i,t-1}\beta_{Y_1} + c_{it}\, Y_{2,i,t-1}\beta_{Y_2} + v_{it} \tag{7}$$

$$c_{it} = \mathbf{1}_{\{Z_{1,i,t-1}\beta_{Z_1} + d_{it}\, Z_{2,i,t-1}\beta_{Z_2} + \varepsilon_{it} > 0\}} \tag{8}$$

where $i$ indexes countries, $t$ indexes time, $d_{it}$ is the debt-to-GDP ratio, $g_{it}$ is the percentage year-on-year growth rate of nominal US$ GDP, $c_{it}$ is a dummy indicating a debt crisis, $\alpha$ is a scalar parameter, $X_{j,i,t-1}$, $Y_{j,i,t-1}$ and $Z_{j,i,t-1}$ for $j = 1, 2$ are row-vectors of exogenous variables, $\beta_{X_j}$, $\beta_{Y_j}$ and $\beta_{Z_j}$ are column-vectors of parameters, and $u_{it}$, $v_{it}$, $\varepsilon_{it}$ are stochastic exogenous shocks.

Estimated equation (6) reflects the theoretical debt dynamics equation (5), and we therefore interpret the shock $u_{it}$ as a deviation from the Euler equation, for the reasons explained in section 3.4. In the growth equation (7), the shock $v_{it}$ is the driver of the country's growth exogenous uncertainty. Depending on the occurence of a debt crisis, growth can be endogenously reduced, as captured by the incidence of the $c_{it}$ variable on growth. Finally, in the debt crisis equation (8), the shock $\varepsilon_{it}$ corresponds to the variability of the threshold level of debt default, such as (negatively) driven by the strength of the international community.

We suppose the following normal distribution for these shocks (which are assumed to be independent and identically distributed over periods and countries):

$$\begin{pmatrix} u_{it} \\ v_{it} \\ \varepsilon_{it} \end{pmatrix} \rightsquigarrow \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_u^2 & 0 & 0 \\ 0 & \sigma_v^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right)$$

The crisis dummy is therefore defined by a probit-like equation. Identifiability is guaranteed by setting the variance of $\varepsilon_{it}$ to unity.

## 5.2 Identification and multiple equilibria

Since there is a circular dependency between the three endogenous variables, our model can not be identified at this stage. Indeed, for a given set of exogenous $X_{j,i,t-1}$, $Y_{j,i,t-1}$, $Z_{j,i,t-1}$, and for a given draw of the random variables $u_{it}$, $v_{it}$ and $\varepsilon_{it}$, the model does not rule out the possibility of having two vectors $(d_{it}, g_{it}, c_{it})$ satisfying equations (6), (7) and (8): of these two vectors, one would be a no-crisis scenario ($c_{it} = 0$), and the other a crisis scenario ($c_{it} = 1$).

This feature is precisely the possibility of multiple equilibria that we are trying to modelize.

In order to address this identification issue, we make two extensions to the model: first, we add restrictions over the parameters, stemming from economic theory, which eliminate multiple equilibria for some values of the exogenous; and for the remaining cases where multiple equilibria are possible, we introduce a stochastic variable (with only two possible values) which determines which equilibrium to choose: it is a *sunspot* variable, as it is sometimes called in the litterature, *i.e.* a variable with no relation to economic fundamentals but which makes agents coordinate on one equilibrium when several are possible.

Let $g_{it}^0$ and $d_{it}^0$ be the growth and the debt-to-GDP ratio conditional to no crisis occurring ($c_{it} = 0$). Conversely, let $g_{it}^1$ and $d_{it}^1$ be the growth and the debt-to-GDP ratio conditional to a crisis occurring ($c_{it} = 1$).

One can easily see that:

$$\begin{aligned} g_{it}^0 &= Y_{1,i,t-1}\beta_{Y_1} + v_{it} \\ g_{it}^1 &= Y_{1,i,t-1}\beta_{Y_1} + Y_{2,i,t-1}\beta_{Y_2} + v_{it} = g_{it}^0 + Y_{2,i,t-1}\beta_{Y_2} \\ d_{it}^0 &= X_{1,i,t-1}\beta_{X_1} + g_{it}^0 X_{2,i,t-1}\beta_{Z_2} + u_{it} \\ d_{it}^1 &= X_{1,i,t-1}\beta_{X_1} + g_{it}^1 X_{2,i,t-1}\beta_{Z_2} + u_{it} = d_{it}^0 + Y_{2,i,t-1}\beta_{Y_2} X_{2,i,t-1}\beta_{Z_2} \end{aligned} \tag{9}$$

With these notations, a solution to equations (6), (7) and (8) is necessarily $(d_{it}^0, g_{it}^0, 0)$ or $(d_{it}^1, g_{it}^1, 1)$.

In relation to economic theory, we make the following assumptions over the parameters of the model:

$$\forall i,t :: X_{2,i,t-1}\beta_{X_2} < 0 \tag{10}$$

$$\forall i,t :: Y_{2,i,t-1}\beta_{Y_2} < 0 \tag{11}$$

$$\forall i,t :: Z_{2,i,t-1}\beta_{Z_2} > 0 \tag{12}$$

Constraint (11) implies that $g_{it}^1 < g_{it}^0$: growth is always lower in a crisis scenario than in a no-crisis scenario, *ceteris paribus.*

Constraint (10) means that the debt-to-GDP ratio is a decreasing function of growth. Combined with (11), it implies that $d_{it}^0 < d_{it}^1$: the debt-to-GDP ratio is always worse in a crisis scenario than in a no-crisis scenario, *ceteris paribus.*

Constraint (12) simply states that the probability of a debt crisis – as given by equation (8) – is an increasing function of the debt-to-GDP ratio.

Finally, we introduce a fourth random variable $\delta_{it}$ following a Bernouilli distribution of parameter $p$ (that is: $\mathbb{P}(\delta_{it} = 1) = p$ and $\mathbb{P}(\delta_{it} = 0) = 1 - p$). The variable $\delta_{it}$ is a sunspot: its role is to discriminate between the two equilibria when both are possible.

Given these extensions, we are now able to describe how the model behaves. For a given set of exogenous $X_{j,i,t-1}$, $Y_{j,i,t-1}$, $Z_{j,i,t-1}$, and for a given draw of random variables $u_{it}$, $v_{it}$, $\varepsilon_{it}$ and $\delta_{it}$, three cases are possible:

- The *crisis equilibrium*, inexorably driven by economic fundamentals, when $Z_{1,i,t-1}\beta_{Z_1} + d_{it}^0 Z_{2,i,t-1}\beta_{Z_2} + \varepsilon_{it} > 0$. In that case, a no-crisis equilibrium is impossible, and because of equations (10), (11) and (12), we have $Z_{1,i,t-1}\beta_{Z_1} + d_{it}^1 Z_{2,i,t-1}\beta_{Z_2} + \varepsilon_{it} > 0$, *i.e.* a crisis is triggered.

- The *no-crisis equilibrium*, when $Z_{1,i,t-1}\beta_{Z_1} + d_{it}^1 Z_{2,i,t-1}\beta_{Z_2} + \varepsilon_{it} < 0$. A crisis equilibrium is impossible, and because of equations (10), (11) and (12), we have ${}_{Z_1} + d_{it}^0 Z_{2,i,t-1}\beta_{Z_2} + \varepsilon_{it} < 0$, *i.e.* no crisis occurs.

- The *multiple equilibria case*, when $Z_{1,i,t-1}\beta_{Z_1} + d_{it}^1 Z_{2,i,t-1}\beta_{Z_2} + \varepsilon_{it} > 0 > Z_{1,i,t-1}\beta_{Z_1} + d_{it}^0 Z_{2,i,t-1}\beta_{Z_2} + \varepsilon_{it}$. Both equilibria are possible. The outcome is given by the sunspot: $c_{it} = \delta_{it}$ (and $g_{it}$ and $d_{it}$ are set accordingly). A *self-fulfilling crisis* can occur if $\delta_{it} = 1$: it could have been avoided (if the sunspot had been different), since the fundamentals are compatible with a no-crisis equilibrium.

The derivation of the likelihood function of the model can be found in appendix C.

## 5.3 Estimating the self-fulfilling effect

The econometric model presented in sections 5.1 and 5.2 is estimated with full information maximum likelihood (FIML) on the dataset of debt crisis episodes presented in section 4. Details about the estimation procedure can be found in appendix D.

Table 1 reports the results for various specifications. In this section we only deal with columns (1) and (2); the remaining ones will be discussed in the following section.

All exogenous variables are taken in $t-2$ (*i.e.* two years before the beginning of the episode). The parameter $p$ is calibrated: its estimation has not been possible with a reasonable accuracy. We try two different values for its calibration: $p = 1$, which reflects the assumption that, when two equilibria are possible, the market always chooses the worst of the two; and $p = 0.5$, which means that, when there is a possibility of a self-fulfilling crisis, a coin is flipped and the crisis takes place half of the time.

The upper part of the table reports the debt-to-GDP ratio dynamics (equation (6)), the middle part reports the growth dynamics (equation (7)), and the lower part reports the crisis probability (equation (8)). In particular, recall that the coefficient lines beginning with $\beta_{X_2}$ (resp. $\beta_{Y_2}$, $\beta_{Z_2}$) present regressors that are interacted with growth (resp. the crisis dummy and the debt-to-GDP ratio).

Table 1: Estimation results

| | (1) | (2) | (3) | (4) | (5) |
|---|---|---|---|---|---|
| **Debt/GDP ratio dynamics** | | | | | |
| $\beta_{X1}$: Debt/GDP (t-2) | 1.204*** | 1.205*** | 1.104*** | 1.197*** | 1.104*** |
| | (0.023) | (0.023) | (0.075) | (0.066) | (0.073) |
| $\beta_{X1}$: Crisis prob * $\Delta$ (t/t-2) | | | 0.821** | 1.313* | 0.825** |
| | | | (0.262) | (0.559) | (0.266) |
| $\beta_{X1}$: Crisis prob * Debt/GDP (t/t-2) | | | | -0.321 | |
| | | | | (0.244) | |
| $\beta_{X1}$: Growth (t-2) - Mean Growth (t-2/t-4) | | | | | -0.017 |
| | | | | | (0.212) |
| $\beta_{X2}$: Debt/GDP (t-2) | -1.722*** | -1.719*** | -1.651*** | -1.897*** | -1.669*** |
| | (0.214) | (0.210) | (0.320) | (0.318) | (0.317) |
| $\sigma_u$ | 0.124*** | 0.125*** | 0.120*** | 0.116*** | 0.121*** |
| | (0.006) | (0.006) | (0.008) | (0.011) | (0.008) |
| **Growth dynamics** | | | | | |
| $\beta_{Y1}$: Log per capita PPP real GDP (t-2) | -0.023** | -0.025** | -0.023** | -0.022** | -0.023** |
| | (0.008) | (0.008) | (0.007) | (0.007) | (0.007) |
| $\beta_{Y1}$: Growth (t-2) | 0.281** | 0.277** | 0.281** | 0.284*** | 0.278** |
| | (0.101) | (0.101) | (0.086) | (0.077) | (0.087) |
| $\beta_{Y1}$: Constant | 0.268*** | 0.290*** | 0.271*** | 0.263*** | 0.270*** |
| | (0.064) | (0.064) | (0.059) | (0.059) | (0.060) |
| $\beta_{Y2}$: Constant | -0.059*** | -0.077*** | -0.062*** | -0.059*** | -0.061*** |
| | (0.015) | (0.014) | (0.015) | (0.014) | (0.015) |
| $\sigma_v$ | 0.094*** | 0.093*** | 0.094*** | 0.094*** | 0.094*** |
| | (0.004) | (0.004) | (0.004) | (0.005) | (0.004) |
| **Debt crisis determinants** | | | | | |
| $\beta_{Z1}$: Log per capita PPP real GDP (t-2) | -0.365** | -0.426** | -0.356** | -0.363* | -0.363** |
| | (0.132) | (0.133) | (0.135) | (0.141) | (0.135) |
| $\beta_{Z1}$: US$ GDP / PPP GDP (t-2) | 1.477** | 1.582** | 1.454** | 1.387* | 1.475** |
| | (0.535) | (0.530) | (0.525) | (0.542) | (0.525) |
| $\beta_{Z1}$: Constant | 0.237 | 0.705 | 0.202 | 0.313 | 0.261 |
| | (1.071) | (1.070) | (1.085) | (1.108) | (1.084) |
| $\beta_{Z2}$: Constant | 2.883*** | 2.971*** | 2.815*** | 2.748*** | 2.801*** |
| | (0.456) | (0.465) | (0.429) | (0.454) | (0.430) |
| p: Sunspot Bernouilli parameter | 1.000 | 0.500 | 1.000 | 1.000 | 1.000 |
| **Self-fulfilling probability** | 0.111 | 0.077 | 0.111 | 0.119 | 0.110 |
| **Self-enforcing probability** | | | 0.124 | 0.192 | 0.124 |
| Number of observations | 253 | 253 | 251 | 251 | 248 |
| Log-likelihood | 301.683 | 300.872 | 306.744 | 313.844 | 300.338 |
| AIC | -579.366 | -577.744 | -587.489 | -599.688 | -572.675 |

The current debt-to-GDP ratio is explained by past debt-to-GDP ratio, and in addition by the interaction of *current growth* with past debt-to-GDP ratio (under the category $\beta_{X_2}$). This second term is meant to capture the accounting effect of growth in the denominator of the debt-to-GDP ratio.

Current growth is explained by three factors: past growth, the occurrence of a crisis – where a crisis lowers the level of growth by a constant level (the constant term under the category $\beta_{Y_2}$), and the level of real GDP per capita – in order to capture the international convergence effect.

The occurrence – or not – of a debt crisis is explained by the *current debt-to-GDP ratio* (the constant term under the category $\beta_{Z_2}$), the level of real GDP per capita, and the overvaluation of the exchange rate (using as a proxy the ratio of GDP expressed in current US$ over GDP expressed in international PPP US$). The level of real GDP per capita is included because in the data richer countries seem to exhibit less crisis; the overvaluation of the exchange rate is meant to capture the fact that currency misalignment increases the risk of currency crisis, which in turn increases the risk of debt crisis since debt is generally denominated in foreign currency.

In table 1, one can see in columns (1) and (2) that most of the parameters of interest are estimated with the expected sign, and with a good accuracy.

As expected, the debt dynamics exhibits high inertia (the coefficient on past debt-to-GDP is close to unity), and the interaction of *current growth* with past debt-to-GDP ratio has a strong effect (with a coefficient close to -2, which is logical given the fact that lagged variabled are taken two periods in the past).

The growth dynamics has some serial autocorrelation, though not very high. The convergence effect of poor countries appears clearly. And, as expected, a debt crisis lowers the level of growth by more than 5% on average.

Our estimators for the determinants of debt crises are also consistent: debt crises are made more likely by a high *current* debt-to-GDP ratio, low real income level and overvaluation of local currency.

In addition to the results of parameter estimations, the tables also report information about the percentage of crises that were of a self-fulfilling nature. Indeed, with our model, it is possible for a given crisis, to compute the *a posteriori* probability that it was of a self-fulfilling nature, by opposition to a crisis solely driven by fundamentals and exogenous shocks (see below section 5.5). The line entitled "Self-fulfilling probability" in the tables reports the mean of that probability over all the crises in the dataset. In column (1) where $p$ is calibrated to 1 – that is, when the markets are considered as "panic prone" – about 11% of debt crises are reported as being self-fulfilling. In column (2), where $p$ is calibrated to 0.5, the proportion of self-fulfilling crises is almost halved, being around 7%.

## 5.4 Estimating the Panglossian effect

Since our theoretical model predicts that some countries will adopt a prudent behavior, while others will accumulate debt, ignoring the risk of a crisis in the Panglossian mode, we test this hypothesis in the data. More precisely, we construct a proxy variable for the Panglossian effect, $\pi_{it}(g_{it}^{+} - g_{it}^{-})$ – see equation (4), which appears in the debt dynamics equation (5).

The first step consists of estimating $\pi_{it}$, the probability of a debt crisis for country $i$ at date $t$, given variables in $t-2$. For that purpose, we estimate a simple probit on our dataset of episodes, where the probability of a debt crisis is a function of several exogenous variables.[4] This enables us to compute for every date the probability of a debt crisis two periods ahead, as predicted by the probit model, independently of the actual realization or not of a crisis.[5]

The second step consists of estimating expected growth $\Delta_{it} = g_{it}^{+} - g_{it}^{-}$, conditionally on the absence of a crisis occurring, minus expected growth, conditionally on the occurrence of a crisis. For a given $\pi_{it}$, we compute the corresponding difference $g_{it}^{+} - g_{it}^{-}$ by taking the mean growth rate (accross the whole data sample) above and below the quantile $\pi_{it}$. This method is rigorously true when a common factor drives (up to uncorrelated disturbances) the determinant of growth and that of the probability of default.

We use the Panglossian variable thus constructed in the estimations of columns (3), (4) and (5) of table 1. Note that since our Panglossian effect is a generated regressor, the standard errors of our parameter estimates – as generated by the FIML estimator – need to be corrected to take into account the sampling error of the first step probit. For that purpose, we implemented the generic method proposed by Murphy and Topel (1985) for

[4]Those variables are: the debt-to-GDP ratio, the log of per capita real PPP GDP, the total debt service to exports ratio and the overvaluation of exchange rate (measured by US$ GDP to PPP GDP ratio). All exogenous are taken two years before the beginning of the episode. The methodology is exactly that of Kraay and Nehru (2004), using a slightly different set of exogenous variables.

[5]One possible criticism against our methodology is that the crisis probability as defined by a probit is not consistent with the crisis probability as defined by our larger estimated simultaneous equations model. The main reason for adopting this methodology is that estimating a model-consistent probability is a very difficult problem from a computational point of view: it involves the computation of a fixed point in the maximum likelihood estimation, and there is no well-known methodology for computing the standard errors of estimated coefficients in that case. From an economic point of view, our methodology is equivalent to the hypothesis that agents in the economy only know the probit model, but not the simultaneous equations model, and use the probit model to form their expectations about the future. This hypothesis is not fully satisfactory, but can nevertheless be justified by the fact that crisis forecasting is generally done with very simple models, as the probit one, both in policy institutions and in credit rating agencies.

two stage maximum likelihood estimation.

The estimation reported in column (3) shows that the Panglossian effect enters in the debt dynamics equation, consistently with our theoretical model. Its coefficient has the expected sign, and is significant at the 0.2% level.

The table also reports information about the percentage of crises that are of a self-enforcing nature, *i.e.* that are the direct consequence of the Panglossian effect. More precisely, after having cancelled the self-fulfilling effect, one can compute the probability that a crisis would not have occurred if the Panglossian effect had not been operative between $t-2$ and $t$. Note that the self-fulfilling and the self-enforcing probabilities thus computed are additive by construction. This leads, on average, to a self-enforcing probability of about 12%.

Note that the self-enforcing probabilities reported here only take into account the Panglossian effect on the law of motion of debt between dates $t-2$ and $t$. In section 5.6, we will compute a quantitative measure of the importance of the Panglossian effect, which takes into account its cumulative effect over several periods.

We also perform robustness checks in order to show that what we are measuring with our Panglossian variable is indeed the effect exhibited in our theoretical model, and not a proxy for another economic mechanism.

First, one may argue that what we are capturing in the Panglossian effect is simply the mechanical effect of the risk premium ask by investors associated to a higher level of risk. In column (4), we test our Panglossian variable against the variable $\pi_{it}\frac{D_{i,t-2}}{Q_{i,t-2}}$, which is a proxy for the risk premium effect (since the risk premium is supposed to be highly correlated with the crisis probability). The results show that our Panglossian variable remains significant – though at a lower level, while the risk premium variable is not significant, and has the wrong sign.

Secondly, one may argue that our Panglossian variable is simply a proxy for "bad news", and that the increase in debt that we are measuring when such a bad news occurs would also be predicted by a standard intertemporal consumption smoothing when a temporary bad shock hits. To test that hypothesis, we construct a measure of the business cycle, equal to growth in $t-2$ minus mean growth over $t-4$ to $t-2$. If the intertemporal consumption smoothing was true, this variable should enter in the debt dynamics, since it captures temporary shocks. On the contrary, the results in column (5) show that this variable is not significant, and does not diminish the explanatory power of the Panglossian variable.

### 5.5 *A posteriori* self-fulfilling probabilities

In tables 2 and 3, we report, for each crisis in the sample, the *a posteriori* probability that it was self-fulfilling. These probabilities are computed as the measure of the set of events where the (unobservable) trigger of default $\varepsilon_{it}$ is such that $Z_{1,i,t-1}\beta_{Z_1}+d^1_{it}\, Z_{2,i,t-1}\beta_{Z_2}+\varepsilon_{it} > 0 > Z_{1,i,t-1}\beta_{Z_1}+d^0_{it}\, Z_{2,i,t-1}\beta_{Z_2}+\varepsilon_{it}$. Since we consider only crisis episodes, the value of $d^1_{it}$ is directly observable, and that of $d^0_{it}$ can be easily found using equation (9). It is then straightforward to compute the probability since $\varepsilon_{it}$ is assumed to be normally distributed.[6]

The crises are ordered by their likelihood of being self-fulfilling episodes. The figures given are computed from the means of those computed for model (3) of table 1. Note that in this specification, the self-fulfilling parameter $p$ is calibrated to 1; a lower value would give lower self-fulfilling probabilities. The values reported can therefore be considered as upper bounds of the real probabilities.

In words, the Jordan crisis of 1989 or the Rwandan crisis of 1994 were almost surely not created by a self-fulfilling process. They could not have been avoided by simply restoring confidence.

In contrast, the crises of Argentina in 1983, El Salvador in 1990 or Indonesia in 1997 may have been self-fulfilling. There is about one chance in five that they could have been avoided if confidence had been maintained and panic avoided.

### 5.6 Simulating the model

We now turn to simulation results of our estimated model. Our strategy is that we simulate the dynamic model described by equations (6), (7) and (8) over several periods, for a given trajectory of random draws $u_{it}$, $v_{it}$ and $\varepsilon_{it}$, of the exogenous values in the $X$, $Y$ and $Z$ matrices, and of the various parameters.

More precisely, we simulate the specification reported in column (3) of table 1, for given values of both the set of exogenous and of parameters (as obtained by maximum-likelihood estimation). For the log of per capital PPP real GDP, and of the US\$ GDP to PPP GDP ratio, we set them constant across time and equal to the sample mean. We start from an initial debt-to-GDP ratio of 60%. The probability $\pi_{it}$ used for the Panglossian effect is recomputed at each period, using the simple probit described in section 5.4. We simulate 2500 series of 5 periods (*i.e.* of 10 years, since lagged variables are taken 2 years earlier).

[6] See equation (13) in appendix C.

Table 2: Individual crises self-fulfilling probabilities

| Country | Year | Crisis length | Self-fulfill prob. (in %) |
|---|---|---|---|
| Jordan | 1989 | 16 | 0.2 |
| Somalia | 1981 | 24 | 1.4 |
| Rwanda | 1994 | 11 | 1.4 |
| Congo, Rep. | 1985 | 20 | 1.6 |
| Nigeria | 1986 | 19 | 1.9 |
| Cote d'Ivoire | 1981 | 16 | 3.1 |
| Guinea-Bissau | 1981 | 23 | 3.7 |
| Madagascar | 1980 | 25 | 4.5 |
| Congo, Dem. Rep. | 1976 | 29 | 4.6 |
| Turkey | 1978 | 7 | 4.6 |
| Uruguay | 1983 | 4 | 5 |
| Ethiopia | 1991 | 14 | 5.1 |
| Benin | 1983 | 16 | 5.4 |
| Benin | 1970 | 9 | 5.9 |
| Chile | 1983 | 7 | 6.5 |
| India | 1981 | 3 | 6.6 |
| Egypt, Arab Rep. | 1977 | 4 | 7.8 |
| Uruguay | 2002 | 3 | 7.9 |
| Mexico | 1983 | 10 | 8 |
| Sudan | 1977 | 28 | 9 |
| Gabon | 1986 | 19 | 9.1 |
| Peru | 1977 | 4 | 9.9 |
| Ghana | 1970 | 7 | 10 |
| Solomon Islands | 2002 | 3 | 10.3 |
| Brazil | 1998 | 7 | 10.3 |
| Kenya | 1975 | 3 | 10.6 |
| Pakistan | 1972 | 5 | 10.8 |
| Senegal | 1980 | 23 | 10.8 |
| Philippines | 1976 | 3 | 10.9 |
| Paraguay | 1986 | 9 | 11.5 |
| Brazil | 1983 | 3 | 11.6 |

Table 3: Individual crises self-fulfilling probabilities (continued)

| Country | Year | Crisis length | Self-fulfill prob. (in %) |
|---|---|---|---|
| Niger | 1983 | 22 | 11.9 |
| Ecuador | 2000 | 5 | 12.4 |
| Kenya | 1992 | 5 | 12.6 |
| Bangladesh | 1979 | 3 | 12.9 |
| Honduras | 1979 | 23 | 13 |
| Egypt, Arab Rep. | 1984 | 12 | 13 |
| Colombia | 1999 | 3 | 13.2 |
| Dominican Republic | 1983 | 17 | 14.2 |
| Turkey | 1999 | 6 | 14.5 |
| Kenya | 2000 | 3 | 14.7 |
| Indonesia | 1970 | 3 | 14.7 |
| Ecuador | 1983 | 14 | 14.8 |
| Jamaica | 1977 | 24 | 14.9 |
| Comoros | 1987 | 18 | 15.2 |
| Tunisia | 1986 | 6 | 15.3 |
| Ghana | 1996 | 3 | 15.5 |
| Algeria | 1994 | 4 | 15.7 |
| Chile | 1972 | 5 | 15.8 |
| Morocco | 1980 | 15 | 15.9 |
| Trinidad and Tobago | 1988 | 5 | 16 |
| Thailand | 1997 | 3 | 16.3 |
| Costa Rica | 1980 | 16 | 16.3 |
| Cameroon | 1987 | 18 | 16.6 |
| Pakistan | 1980 | 4 | 17 |
| Kyrgyz Republic | 2002 | 3 | 18.1 |
| Pakistan | 1994 | 10 | 18.7 |
| Venezuela, RB | 1989 | 4 | 19.3 |
| Indonesia | 1997 | 8 | 19.6 |
| El Salvador | 1990 | 3 | 19.9 |
| Argentina | 1983 | 13 | 20.3 |

Table 4: Simulated contributions of shocks and Panglossian effect to crises

| Effect | Contribution |
|---|---:|
| Crisis shock | 55.8% |
| Debt shock | 15.2% |
| Panglossian effect | 12.0% |
| Growth shock | 11.0% |
| Self-fulfilling effect | 6.1% |
| Total | 100.0% |

The dynamics of the model are affected by four shocks that may be switched off for comparison purposes: shocks to the law of motion of debt ($u_{it}$), to growth ($v_{it}$), to the crisis equation ($\varepsilon_{it}$), plus the self-fulfilling shock ($\delta_{it}$). We also consider simulations where the Panglossian effect is switched off (just by removing the corresponding term in the debt equation). Thus, there is a total of $2^5 = 32$ possible combinations according to whether or not we active these five effects.

When the five effects are activated, 89.4% of the simulations exhibit a crisis episode in at least one of the 5 simulation periods. This high occurrence rate of crisis is the consequence of the relatively high level of the debt-to-GDP ratio that we have chosen as the starting point for simulations.

In order to compute the contribution of each of these five effects to these crises, we shut off each of them one by one, and observe by how much the number of crises diminishes, which gives the contribution of each one. An issue is that the results depend on the order in which the effects are shut down: we solve this problem by making these computations for the 120 possible orders, and by computing the average contributions.

We present the results in table 4 that reports the contribution of each effect: it shows the percentage of crisis episodes that can be considered a direct consequence of each effect.

One can see that the largest contributor is by far the crisis shock $\varepsilon_{it}$ which explains more than 55% of crises: this means that most crises are triggered by events not related to the level of the debt-to-GDP ratio. For the remaining crises, the Panglossian effect comes third, explaining about 12% of the crises, while the self-fulfilling effect accounts for about 6%.

# 6 Conclusion

We have tried to distinguish two attitudes towards debt: the attitude of prudent borrowers, who attempt to stabilize their debt at low levels, even in the event of an adverse shock, and Panglossian borrowers, who only take into account the best scenarios possible, rationally anticipating to default on their debt if hit by an unfavorable shock (or by a sequence of them). We have shown empirically that this distinction is consistent with the data.

We also have distinguished two types of debt crises: those that are the effect of an exogenous shock, and those that are created in a self-fulfilling manner by the financial markets themselves. We have shown that the large majority of crises are of the first kind, although the probability of self-fulfilling cases is not negligible.

These results have a few policy implications that we leave to future work. For one thing, if the earthquake model is correct, then there is room for improving the stability of financial markets by the use of more conditional sovereign lending, contingent on other lenders following suit. It indeed remains a question to understand why sovereign debt arrangements contain so few contingency clauses.

Regarding the self-fulfilling case, if our results can be trusted, while the now old debate on sovereign debt restructuring remains important, it may be relatively so than finding more innovative source of finance.

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# A Debt crises

The following tabulations present the complete list of the crisis episodes identified according to the methodology of section 4.

For each crisis episode, the first three columns give the country, the year of the crisis outbreak, and the number of years it lasted. The columns labeled “type of crisis” give some detail about the type of debt crisis, whether it was characterized by a Paris Club relief, accumulated arrears or IMF intervention (or several of these options).

The remaining columns give several macroeconomic indicators about the country: the debt-to-GDP ratio at three points in time (3 years before the outbreak, the year of the outbreak and three years later), the debt-to-PPP-GDP ratio (at the same dates), the debt-service-to-exports ratio, the mean annual growth before the crisis and the mean effective interest rate charged on the debt before the crisis.

| Country | Year | Length | Type of crisis | | | D/GDP | | |
|---|---|---|---|---|---|---|---|---|
| | | | Paris Club | Arrears | SBA/EFF | t-3 | t | t+3 |
| Indonesia | 1970 | 3 | Y | N | N | 46.9 | 46.9 | 42.3 |
| Benin | 1970 | 9 | N | Y | N | 12.5 | 12.5 | 11.7 |
| Ghana | 1970 | 7 | N | Y | N | 25.8 | 25.8 | 30.6 |
| Guinea | 1970 | 35 | Y | Y | Y | NA | NA | NA |
| Chile | 1972 | 5 | Y | Y | Y | 33.1 | 30.7 | 76.4 |
| Pakistan | 1972 | 5 | Y | N | N | 34.0 | 43.7 | 50.7 |
| Tanzania | 1972 | 33 | Y | Y | Y | NA | NA | NA |
| Kenya | 1975 | 3 | N | N | Y | 27.6 | 39.6 | 41.0 |
| Congo, Dem. Rep. | 1976 | 29 | Y | Y | Y | 13.2 | 30.2 | 30.0 |
| Philippines | 1976 | 3 | N | N | Y | 27.4 | 35.3 | 48.3 |
| Jamaica | 1977 | 24 | Y | Y | Y | 60.4 | 51.7 | 71.4 |
| Egypt, Arab Rep. | 1977 | 4 | N | N | Y | 24.5 | 80.2 | 83.5 |
| Peru | 1977 | 4 | Y | N | Y | 38.8 | 64.4 | 45.4 |
| Sudan | 1977 | 28 | Y | Y | Y | 28.9 | 35.1 | 68.0 |
| Panama | 1978 | 3 | N | N | Y | 50.4 | 93.8 | 77.7 |
| Turkey | 1978 | 7 | Y | N | Y | 10.9 | 22.3 | 28.9 |
| Honduras | 1979 | 23 | Y | Y | Y | 27.1 | 52.6 | 63.5 |
| Bangladesh | 1979 | 3 | N | N | Y | 19.8 | 19.5 | 28.0 |
| Mauritius | 1979 | 3 | N | N | Y | NA | NA | 53.3 |
| Costa Rica | 1980 | 16 | Y | Y | Y | 42.9 | 56.8 | 133.1 |
| Madagascar | 1980 | 25 | Y | Y | Y | 33.1 | 30.6 | 57.9 |
| Senegal | 1980 | 23 | Y | Y | Y | 31.7 | 49.3 | 83.8 |
| Morocco | 1980 | 15 | Y | Y | Y | 50.8 | 51.7 | 93.5 |
| Pakistan | 1980 | 4 | Y | N | Y | 50.0 | 41.9 | 41.9 |

| Country | Year | Length | Type of crisis | | | D/GDP | | |
|---|---|---|---|---|---|---|---|---|
| | | | Paris Club | Arrears | SBA/EFF | t-3 | t | t+3 |
| India | 1981 | 3 | N | N | Y | 12.4 | 12.1 | 16.5 |
| Romania | 1981 | 5 | Y | N | Y | NA | NA | NA |
| Cote d'Ivoire | 1981 | 16 | Y | Y | Y | 48.6 | 96.5 | 124.9 |
| Guinea-Bissau | 1981 | 23 | Y | Y | N | 43.9 | 97.8 | 183.1 |
| Somalia | 1981 | 24 | Y | Y | Y | 91.8 | 151.0 | 190.0 |
| Argentina | 1983 | 13 | Y | Y | Y | 35.3 | 44.2 | 47.3 |
| Niger | 1983 | 22 | Y | Y | Y | 34.4 | 52.7 | 74.3 |
| Benin | 1983 | 16 | Y | Y | N | 30.2 | 68.1 | 74.0 |
| Brazil | 1983 | 3 | Y | N | Y | 30.4 | 48.5 | 40.7 |
| Chile | 1983 | 7 | Y | N | Y | 43.8 | 90.7 | 119.3 |
| Dominican Republic | 1983 | 17 | Y | Y | Y | 30.2 | 34.0 | 60.2 |
| Ecuador | 1983 | 14 | Y | Y | Y | 50.4 | 67.9 | 90.5 |
| Mexico | 1983 | 10 | Y | N | Y | 29.5 | 62.5 | 77.9 |
| Uruguay | 1983 | 4 | N | N | Y | 16.4 | 64.8 | 66.7 |
| Egypt, Arab Rep. | 1984 | 12 | Y | Y | Y | 94.3 | 105.1 | 109.0 |
| Congo, Rep. | 1985 | 20 | Y | Y | Y | 91.8 | 141.2 | 184.9 |
| Lebanon | 1986 | 6 | N | Y | N | NA | NA | 37.7 |
| Sao Tome and Principe | 1986 | 19 | Y | Y | N | 83.9 | 122.9 | 291.9 |
| Gabon | 1986 | 19 | Y | Y | Y | 27.0 | 57.1 | 80.0 |
| Nigeria | 1986 | 19 | Y | Y | Y | 50.2 | 109.9 | 126.3 |
| Paraguay | 1986 | 9 | N | Y | N | 25.2 | 58.9 | 54.6 |
| Tunisia | 1986 | 6 | N | N | Y | 48.6 | 65.9 | 69.0 |
| Cameroon | 1987 | 18 | Y | Y | Y | 37.2 | 37.9 | 59.7 |
| Comoros | 1987 | 18 | N | Y | N | 97.5 | 103.5 | 71.8 |

| Country | Year | Length | Type of crisis | | | D/GDP | | |
|---|---|---|---|---|---|---|---|---|
| | | | Paris Club | Arrears | SBA/EFF | t-3 | t | t+3 |
| Trinidad and Tobago | 1988 | 5 | Y | Y | Y | 19.6 | 46.7 | 46.7 |
| Vietnam | 1988 | 17 | Y | Y | Y | 0.4 | 2.4 | 243.4 |
| Jordan | 1989 | 16 | Y | Y | Y | 78.2 | 177.2 | 150.0 |
| Venezuela, RB | 1989 | 4 | N | N | Y | 58.3 | 76.8 | 64.7 |
| El Salvador | 1990 | 3 | Y | N | N | 50.2 | 44.8 | 29.3 |
| Seychelles | 1990 | 15 | N | Y | N | 69.4 | 49.7 | 38.7 |
| Ethiopia | 1991 | 14 | Y | Y | N | 99.9 | 95.8 | 181.7 |
| Kenya | 1992 | 5 | Y | Y | N | 71.2 | 83.9 | 80.8 |
| Algeria | 1994 | 4 | Y | N | Y | 62.3 | 71.1 | 64.5 |
| Rwanda | 1994 | 11 | Y | Y | N | 42.4 | 126.6 | 60.1 |
| Pakistan | 1994 | 10 | Y | N | Y | 51.4 | 52.8 | 48.2 |
| Ghana | 1996 | 3 | Y | N | N | 76.7 | 83.6 | 83.3 |
| Indonesia | 1997 | 8 | Y | Y | Y | 61.0 | 63.1 | 87.5 |
| Thailand | 1997 | 3 | N | N | Y | 45.3 | 72.7 | 64.9 |
| Brazil | 1998 | 7 | N | N | Y | 22.8 | 30.7 | 45.5 |
| Colombia | 1999 | 3 | N | N | Y | 29.7 | 39.9 | 40.7 |
| Turkey | 1999 | 6 | N | N | Y | 44.1 | 55.6 | 71.3 |
| Kenya | 2000 | 3 | Y | N | N | 49.3 | 48.4 | 45.6 |
| Ecuador | 2000 | 5 | Y | N | Y | 65.2 | 86.0 | 62.0 |
| Kyrgyz Republic | 2002 | 3 | Y | N | N | 139.0 | 115.3 | NA |
| Solomon Islands | 2002 | 3 | N | Y | N | 49.7 | 79.5 | NA |
| Uruguay | 2002 | 3 | N | N | Y | 35.3 | 86.4 | NA |

| Country | Year | D/PPP-GDP | | | TDS/X | Growth | Interest rate |
|---|---|---|---|---|---|---|---|
| | | t-3 | t | t+3 | | avg(t-3...t-1) | avg(t-3...t-1) |
| Indonesia | 1970 | 16.0 | 16.0 | 16.0 | 13.0 | 6.9 | 1.0 |
| Benin | 1970 | 5.7 | 5.7 | 6.0 | 3.4 | 2.6 | 1.0 |
| Ghana | 1970 | 12.6 | 12.6 | 13.4 | 10.8 | 3.2 | 2.1 |
| Guinea | 1970 | 14.6 | 14.6 | 22.8 | NA | NA | 1.3 |
| Chile | 1972 | 17.2 | 17.5 | 27.6 | 27.3 | 4.9 | 3.5 |
| Pakistan | 1972 | 14.5 | 14.8 | 14.3 | 33.2 | 5.8 | 2.0 |
| Tanzania | 1972 | 8.6 | 49.4 | 63.7 | NA | NA | 0.9 |
| Kenya | 1975 | 12.0 | 21.3 | 23.6 | 8.6 | 9.0 | 3.5 |
| Congo, Dem. Rep. | 1976 | 9.8 | 28.9 | 26.0 | 9.2 | 2.1 | 3.4 |
| Philippines | 1976 | 7.3 | 11.2 | 16.9 | 22.5 | 6.0 | 3.3 |
| Jamaica | 1977 | 43.8 | 41.8 | 43.2 | 37.2 | -3.7 | 6.9 |
| Egypt, Arab Rep. | 1977 | 7.3 | 26.0 | 25.9 | 16.3 | 8.7 | 1.7 |
| Peru | 1977 | 19.7 | 27.0 | 18.6 | 42.5 | 4.9 | 5.0 |
| Sudan | 1977 | 18.7 | 29.2 | 46.1 | 12.7 | 14.6 | 1.8 |
| Panama | 1978 | 30.2 | 53.5 | 47.8 | NA | 1.5 | 4.4 |
| Turkey | 1978 | 9.2 | 19.3 | 18.2 | 19.1 | 7.0 | 5.6 |
| Honduras | 1979 | 13.2 | 28.1 | 33.4 | 29.1 | 10.3 | 5.1 |
| Bangladesh | 1979 | 5.0 | 5.4 | 6.7 | 28.8 | 5.1 | 1.9 |
| Mauritius | 1979 | 2.7 | 12.0 | 14.9 | NA | NA | 3.3 |
| Costa Rica | 1980 | 20.6 | 31.5 | 43.8 | 22.0 | 6.7 | 5.0 |
| Madagascar | 1980 | 18.4 | 24.3 | 32.2 | 37.7 | 3.2 | 2.5 |
| Senegal | 1980 | 15.8 | 29.6 | 31.2 | 7.1 | 0.1 | 3.8 |
| Morocco | 1980 | 24.6 | 29.6 | 30.6 | 19.1 | 4.4 | 4.8 |
| Pakistan | 1980 | 14.8 | 12.7 | 10.6 | 31.9 | 5.3 | 2.5 |

| Country | Year | D/PPP-GDP | | | TDS/X | Growth | Interest rate |
|---|---|---|---|---|---|---|---|
| | | t-3 | t | t+3 | | avg(t-3...t-1) | avg(t-3...t-1) |
| India | 1981 | 4.2 | 4.0 | 4.5 | 16.1 | 2.4 | 2.7 |
| Romania | 1981 | 1.9 | 13.5 | 7.8 | NA | NA | 4.3 |
| Cote d'Ivoire | 1981 | 37.6 | 62.3 | 47.5 | 16.4 | 0.8 | 6.5 |
| Guinea-Bissau | 1981 | 22.9 | 44.9 | 67.5 | 10.5 | -0.3 | 1.0 |
| Somalia | 1981 | 16.1 | 27.3 | 36.2 | 3.0 | -1.0 | 0.3 |
| Argentina | 1983 | 15.9 | 23.7 | 24.6 | 107.4 | -2.2 | 8.8 |
| Niger | 1983 | 23.4 | 19.0 | 26.9 | 22.9 | -0.0 | 9.2 |
| Benin | 1983 | 21.5 | 30.8 | 29.3 | 9.1 | 6.3 | 3.0 |
| Brazil | 1983 | 15.2 | 18.9 | 15.1 | 69.4 | 1.8 | 12.1 |
| Chile | 1983 | 29.1 | 42.1 | 41.4 | 43.0 | 0.9 | 11.7 |
| Dominican Republic | 1983 | 15.6 | 15.9 | 17.6 | 29.8 | 3.9 | 9.1 |
| Ecuador | 1983 | 22.0 | 24.5 | 28.0 | 34.0 | 2.4 | 9.4 |
| Mexico | 1983 | 20.2 | 25.8 | 26.8 | 52.7 | 5.8 | 12.0 |
| Uruguay | 1983 | 12.3 | 24.4 | 23.7 | 19.6 | -0.8 | 9.2 |
| Egypt, Arab Rep. | 1984 | 26.1 | 29.5 | 37.6 | 19.9 | 7.0 | 4.3 |
| Congo, Rep. | 1985 | 56.3 | 75.8 | 123.5 | 20.2 | 12.1 | 5.7 |
| Lebanon | 1986 | NA | NA | NA | NA | NA | 8.0 |
| Sao Tome and Principe | 1986 | 46.5 | 76.5 | 117.5 | 24.2 | NA | 1.7 |
| Gabon | 1986 | 19.1 | 35.7 | 51.0 | 11.7 | 3.6 | 7.9 |
| Nigeria | 1986 | 37.5 | 42.2 | 36.9 | 53.8 | -0.1 | 9.4 |
| Paraguay | 1986 | 12.8 | 16.3 | 14.1 | 13.7 | 1.3 | 4.0 |
| Tunisia | 1986 | 17.1 | 22.0 | 21.5 | 22.2 | 5.4 | 5.9 |
| Cameroon | 1987 | 15.4 | 21.0 | 28.1 | 15.9 | 7.4 | 6.2 |
| Comoros | 1987 | 20.4 | 34.0 | 26.9 | 28.9 | 2.8 | 1.2 |

| Country | Year | D/PPP-GDP | | | TDS/X | Growth | Interest rate |
|---|---|---|---|---|---|---|---|
| | | t-3 | t | t+3 | | avg(t-3...t-1) | avg(t-3...t-1) |
| Trinidad and Tobago | 1988 | 13.8 | 23.9 | 24.0 | 11.0 | -4.0 | 7.6 |
| Vietnam | 1988 | NA | NA | 28.2 | NA | 3.4 | 0.5 |
| Jordan | 1989 | 44.9 | 70.3 | 63.2 | 32.8 | 2.7 | 6.0 |
| Venezuela, RB | 1989 | 38.9 | 29.7 | 27.7 | 43.7 | 5.3 | 8.6 |
| El Salvador | 1990 | 14.7 | 14.2 | 10.5 | 34.6 | 1.8 | 3.9 |
| Seychelles | 1990 | 43.8 | 31.8 | 24.0 | 9.1 | 7.0 | 5.2 |
| Ethiopia | 1991 | 46.7 | 47.2 | 37.2 | 48.6 | 0.9 | 1.0 |
| Kenya | 1992 | 25.8 | 25.6 | 22.3 | 37.2 | 3.4 | 4.6 |
| Algeria | 1994 | 24.7 | 26.8 | 22.4 | 68.9 | -0.5 | 7.1 |
| Rwanda | 1994 | 10.3 | 25.0 | 16.9 | 16.5 | -1.6 | 1.3 |
| Pakistan | 1994 | 10.3 | 10.1 | 9.5 | 25.4 | 4.8 | 3.6 |
| Ghana | 1996 | 23.6 | 25.7 | 23.9 | 24.4 | 4.1 | 2.1 |
| Indonesia | 1997 | 16.9 | 16.5 | 17.1 | 30.4 | 7.9 | 5.0 |
| Thailand | 1997 | 18.4 | 26.3 | 19.7 | 14.0 | 8.0 | 4.3 |
| Brazil | 1998 | 15.1 | 20.7 | 18.0 | 39.7 | 3.4 | 6.2 |
| Colombia | 1999 | 13.5 | 15.2 | 13.0 | 36.6 | 2.0 | 6.4 |
| Turkey | 1999 | 25.0 | 28.1 | 33.6 | 28.0 | 5.9 | 5.7 |
| Kenya | 2000 | 17.7 | 16.0 | 16.7 | 22.1 | 2.0 | 3.0 |
| Ecuador | 2000 | 29.3 | 24.6 | 26.7 | 31.2 | -0.0 | 6.0 |
| Kyrgyz Republic | 2002 | 12.3 | 10.7 | NA | 20.9 | 4.8 | 3.3 |
| Solomon Islands | 2002 | 15.5 | 20.5 | NA | 5.0 | -7.9 | 1.9 |
| Uruguay | 2002 | 20.7 | 34.3 | NA | 27.6 | -2.6 | 6.8 |

# B Proof of proposition 3

**Proof.** Formally, for a given default set $\mathscr{D}$ (such that $(g', \lambda') \in \mathscr{D}(D', Q) \Leftrightarrow \frac{D'}{g'Q} > d^*(\lambda')$), there exists a unique continuous function $\tilde{D}'(L, Q)$ satisfying the zero-profit condition (3) in the smooth default case.The function $\tilde{D}'(L, Q)$ is determined by the implicit equation (3). This equation can be rewritten as:

$$f(L, Q, \tilde{D}'(L, Q)) = 0$$

where:

$$f(L, Q, D') = D'\mathbb{P}[\mathscr{R}(D', Q)] + \int\limits_{\mathscr{D}(D',Q)} V(g'Q, \lambda')\mathrm{d}F(g')\mathrm{d}H(\lambda') - L(1 + r)$$

The implicit function theorem states that there is a unique solution to this implicit equation if the derivative of $f$ with respect to $D'$ is non negative.

Using the specific structure of $\mathscr{D}$ and $\mathscr{R}$, this can be rewritten as:

$$f(L, Q, D') = \int \left( \int\limits_{\frac{D'}{d^*(\lambda')Q}}^{g^{\max}} D'\mathrm{d}F(g') + \int\limits_{0}^{\frac{D'}{d^*(\lambda')Q}} V(g'Q, \lambda')\mathrm{d}F(g') \right) \mathrm{d}H(\lambda') - L(1+r)$$

Taking the derivative with respect to $D'$, one gets:

$$\frac{\partial f}{\partial D'}(L, Q, D') = \mathbb{P}[\mathscr{R}(D', Q)] - \frac{D'}{Q} \int (d^*(\lambda') - V(1, \lambda'))\mathrm{d}H(\lambda')$$

In the general case the sign of this derivative is not constant, since both terms in the expression are positive (the second term is positive because of $d^*(\lambda') \geq V(1, \lambda')$). But since we assumed that $d^*(\lambda') = V(1, \lambda')$ (smooth default), we have:

$$\frac{\partial f}{\partial D'}(L, Q, D') = \mathbb{P}[\mathscr{R}(D', Q)] \geq 0$$

So the derivative is non null, except for the points where $\mathbb{P}[\mathscr{R}(D', Q)] = 0$. But in this latter case, the zero profit condition implies that:

$$L = \frac{1}{1 + r} \int V(g'Q, \lambda')\mathrm{d}F(g')\mathrm{d}H(\lambda').$$

Hence the derivative is non null everywhere except on a set of points of empty interior. Using the implicit function theorem, this implies that there is a unique continuous function verifying the zero-profit condition. ■

# C Likelihood derivation

In this section we derive the likelihood function:

$$\mathcal{L}_\theta(d_{it}, g_{it}, c_{it}|X_{1,i,t-1}, X_{2,i,t-1}, Y_{1,i,t-1}, Y_{2,i,t-1}, Z_{1,i,t-1}, Z_{2,i,t-1})$$

of a single observation $(d_{it}, g_{it}, c_{it})$ given the exogenous values and the vector of parameters $\theta = (\beta_{X_1}, \beta_{X_2}, \beta_{Y_1}, \beta_{Y_2}, \beta_{Z_1}, \beta_{Z_2}, \sigma_u, \sigma_v, p)$.

For the remaining of this subsection, we drop the $i$ and $t$ subscripts for the sake of simplicity. We note $\mathcal{I}$ the information set containing $X_1, X_2, Y_1, Y_2, Z_1, Z_2$.

We note $\varphi$ the density function of the normal distribution with zero mean and unit variance, and $\Phi$ its cumulative distribution.

Given $(d, g, c)$, $u$ and $v$ can be immediately inferred. The likelihood function is therefore, by independence of the four shocks $(u, v, \varepsilon, \delta)$:

$$\mathcal{L}_\theta(d, g, c|\mathcal{I}) = \mathbb{P}_\theta(u = d - X_1\beta_1 - gX_2\beta_2)\,\mathbb{P}_\theta(v = g - Y_1\beta_1 - cY_2\beta_2)\,\mathbb{P}_\theta(c|d, \mathcal{I})$$

The first two factors are:

$$\mathbb{P}_\theta(u = d - X_1\beta_1 - gX_2\beta_2) = \frac{1}{\sigma_u}\varphi\left(\frac{d - X_1\beta_1 - gX_2\beta_2}{\sigma_u}\right)$$

$$\mathbb{P}_\theta(v = g - Y_1\beta_1 - cY_2\beta_2) = \frac{1}{\sigma_v}\varphi\left(\frac{g - Y_1\beta_1 - cY_2\beta_2}{\sigma_v}\right)$$

We discuss the third factor below.

## C.1 Crisis case

If $c = 1$, we know that $d = d^1$ and $g = g^1$. Then:

$$\begin{aligned}\mathbb{P}(c = 1|d^1, \mathcal{I}) &= \mathbb{P}(Z_1\beta_{Z_1} + d^0 Z_2\beta_{Z_2} + \varepsilon > 0) + \\ &\quad p\,\mathbb{P}(Z_1\beta_{Z_1} + d^1 Z_2\beta_{Z_2} + \varepsilon > 0 > Z_1\beta_{Z_1} + d^0 Z_2\beta_{Z_2} + \varepsilon)\end{aligned}$$

In this equation, the first term corresponds to a crisis driven solely by fundamentals and exogenous shocks, and the second term to the self-fulfilling case.

Using (9), it can be rewritten as:

$$\begin{aligned}\mathbb{P}(c = 1|d^1, \mathcal{I}) &= \Phi[Z_1\beta_{Z_1} + (d^1 - Y_2\beta_{Y_2}X_2\beta_{Z_2})Z_2\beta_{Z_2}] + \\ &\quad p\left\{\Phi(Z_1\beta_{Z_1} + d^1 Z_2\beta_{Z_2}) - \Phi[Z_1\beta_{Z_1} + (d^1 - Y_2\beta_{Y_2}X_2\beta_{Z_2})Z_2\beta_{Z_2}]\right\}\end{aligned}$$

For a given crisis observation, it is therefore possible to compute the *a posteriori* probability that the crisis is of a self-fulfilling nature (by opposition to a crisis solely driven by fundamentals and exogenous shocks). This probability is given by:

$$\tau_\theta(d^1, \mathcal{I}) = \frac{p\left\{\Phi(Z_1\beta_{Z_1} + d^1 Z_2\beta_{Z_2}) - \Phi[Z_1\beta_{Z_1} + (d^1 - Y_2\beta_{Y_2}X_2\beta_{Z_2})Z_2\beta_{Z_2}]\right\}}{\mathbb{P}_\theta(c = 1|d^1, \mathcal{I})} \tag{13}$$

### C.2 No-crisis case

If $c = 0$, we know that $d = d^0$. Then:

$$\begin{aligned} \mathbb{P}(c = 0|d^0, \mathcal{I}) \;\; = \;\; & \mathbb{P}(Z_1\beta_{Z_1} + d^1 Z_2\beta_{Z_2} + \varepsilon < 0) + \\ & (1-p)\,\mathbb{P}(Z_1\beta_{Z_1} + d^1 Z_2\beta_{Z_2} + \varepsilon > 0 > Z_1\beta_{Z_1} + d^0 Z_2\beta_{Z_2} + \varepsilon) \end{aligned}$$

In the this equation, the first term corresponds to the no-crisis equilibrium driven by strong fundamentals, and the second term to the self-fulfilling case in which the country escapes the crisis.

Using (9), it can be rewritten as:

$$\begin{aligned} \mathbb{P}(c = 0|d^0, \mathcal{I}) \;\; = \;\; & 1 - \Phi[Z_1\beta_{Z_1} + (d^0 + Y_2\beta_{Y_2}X_2\beta_{Z_2})Z_2\beta_{Z_2}] + \\ & (1-p)\left\{\Phi[Z_1\beta_{Z_1} + (d^0 Z_2 + Y_2\beta_{Y_2}X_2\beta_{Z_2})\beta_{Z_2}] - \Phi(Z_1\beta_{Z_1} + d^0 Z_2\beta_{Z_2})\right\} \end{aligned}$$

## D Estimation

The model is estimated with full information maximum (log-)likelihood, *i.e.* by computing the following:

$$\underset{\theta\in\mathcal{B}}{\operatorname{argmax}} \sum_{(i,t)} \log \mathcal{L}_\theta(d_{it}, g_{it}, c_{it}|X_{1,i,t-1}, X_{2,i,t-1}, Y_{1,i,t-1}, Y_{2,i,t-1}, Z_{1,i,t-1}, Z_{2,i,t-1})$$

where $\mathcal{B}$ is a set of constraints over parameters to ensure that constraints (10), (11) and (12) are satisfied and that $\sigma_u > 0$, $\sigma_v > 0$, and $p \in [0, 1]$.

### D.1 Constraints

Because of software requirements, the only constrained-optimization algorithm at our disposal is the L-BFGS-B method (see Byrd et al., 1995), which allows box constraints, that is each variable can be given a lower and/or upper bound.

The constraints over $\sigma_u$, $\sigma_v$ and $p$ already fit into this category.

We deal with constraints (10), (11), (12) (respectively over $\beta_{X_2}$, $\beta_{Y_2}$, $\beta_{Z_2}$) by replacing them by tighter constraints, in the following way:

- First, we only choose constant sign regressors in $X_2$, $Y_2$, $Z_2$ (that is, all elements of a given column in these matrices have a constant sign).
- Second, we constrain every component of $\beta_{X_2}$, $\beta_{Y_2}$, $\beta_{Z_2}$ to have the sign that will enforce the constraint.

Therefore, constraints (10), (11), (12) are clearly satisfied, and the constraints over $\beta_{X_2}$, $\beta_{Y_2}$, $\beta_{Z_2}$ can be dealt with by the L-BFGS-B algorithm.

## D.2 Non-concavity

The second issue is the fact that the log-likelihood function is not globally concave, which implies that different initial values in the optimization algorithm can lead to different local maxima.

We deal with this problem with a simple randomization algorithm.

The following procedure is repeated 50,000 times:

- Generate a random initial value for the maximisation algorithm. We alternate between two algorithms for generating this point (each algorithm is used half of the time):
  - Draw a totally random point. For unconstrained parameters $\beta_{X_1}$, $\beta_{Y_1}$, $\beta_{Z_1}$, a standard normal distribution is used. For sign-constrained parameters $(\beta_{X_2}, \beta_{Y_2}, \beta_{Z_2}, \sigma_u, \sigma_v)$, a $\chi^2_1$ distribution is used (multiplied by $-1$ for the relevant components of $\beta_{X_2}$, $\beta_{Y_2}$, $\beta_{Z_2}$). The parameter $p$ is either calibrated or drawn from a uniform distribution over $[0, 1]$.
  - Draw a point in the neighborhood of the point which has the highest likelihood so far. For all parameters, we use a normal distribution centered around that point, and with the same standard error than the maximum likelihood estimator.
- Run the L-BFGS-B algorithm using the initial value thus generated.
- If the result has a greater log-likelihood than the previous best point, keep it, otherwise discard it.

The results obtained in this way exhibit good numerical stability.