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ABSTRACT

Variance risk, financial intermediation, and the cross-section of expected option returns*

We explore the pricing of variance risk by decomposing stocks' total variance into systematic and idiosyncratic return variances. While systematic variance risk exhibits a negative price of risk, common shocks to the variances of idiosyncratic returns carry a large positive risk premium. This implies investors pay for insurance against increases (declines) in systematic (idiosyncratic) variance, even though both variances comove countercyclically. Common idiosyncratic variance risk is an important determinant for the cross-section of expected option returns. These findings reconcile several phenomena, including the pricing differences between index and stock options, the cross-sectional variation in stock option expensiveness, the volatility mispricing puzzle, and the significant returns earned on various option portfolio strategies. Our results are consistent with theories of financial intermediation under capital constraints.

JEL Classification: G12, G13 and G24
Keywords: asset pricing, cross-section of option returns, financial intermediation and variance risk

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Variance is increasingly being viewed by market participants not only as a risk characteristic, but as an investable asset class.[1] The motives for trading variance range from hedging of macroeconomic uncertainty to speculation on the arrival of news in financial markets. In response, the financial industry has developed various investment strategies offering exposure to variance risk, with equity options being the commonly used instruments due to their liquidity.[2] Despite these trends, little is known about the risk-return characteristics of variance as an investment vehicle, the pricing of the different components of variance risk—systematic and idiosyncratic return variances, and the link between correlation and variance risk premia.

In this paper, we study the risk premia on systematic and idiosyncratic return variance using an extensive panel data set on stock and index options. We first decompose stocks' total variance risk into variance risk from systematic and idiosyncratic returns. We then construct model-free variance swaps that allow observing the risk premium on each source of variance risk. Our analysis on all S&P 100 and Nasdaq 100 stocks shows that systematic and idiosyncratic variance risk command sizeable premia and significantly affect expected option returns. The prices of risk have opposite signs, even though both variances comove countercyclically and spike during crisis periods. Since correlations rise when systematic (idiosyncratic) variances rise (drop), correlation trading strategies earn a combination of the risk premia on systematic and idiosyncratic return variance. These findings suggest a risk-based explanation for several option pricing anomalies.

Equity option prices exhibit a number of empirical regularities. First, as documented by

[1]See, e.g., "Volatility: Investment Characteristic or an Investable Asset Class?" by Eric Brandhorst, State Street Global Advisors.

[2]A prominent hedge fund strategy is to short variance, generating a high propensity of small gains and infrequent large losses (known as "picking up nickels in front of a steamroller"). Malliaris and Yan (2010) find that approximately 40% of hedge fund assets are invested in nickel-picking type strategies, while only 0.6% are in the reverse strategies. Malliaris and Yan (2010) and Makarov and Plantin (2010) provide models explaining this behavior. Mutual funds, by engaging in covered call writing, are also heavily exposed to time variation in stock variance.

Carr and Wu (2009) and Driessen, Maenhout, and Vilkov (2009), there exist sizeable differences between the prices of index and single-stock options. Index options are "expensive," that is, the future variance implied by index option prices exceeds the variance subsequently realized, meaning that index options carry a negative variance risk premium. By contrast, stock options tend to be "cheap," that is, the risk premium on stock variance is on average positive or close to zero, depending on the sample. Second, variance risk premia extracted from stock option prices exhibit sizeable cross-sectional variation that is not captured by known risk factors (Di Pietro and Vainberg (2006), Carr and Wu (2009)). Specifically, option returns are larger for stocks with higher past realized variance, implied variance, past option returns, and stock index betas. Third, as documented by Goyal and Saretto (2009), portfolios sorted on the ratio of past realized volatility to implied volatility earn abnormal returns, suggesting "volatility mispricing" in stock options.

To explain these patterns, we develop a model for pricing variance swaps on the components of stock return variance that allows measuring the risk premia on each source of variance risk. Beyond the assumptions of factor structures in stock returns and variances (see Andersen et al. (2001)), we do not impose parametric restrictions on the stochastic processes for (co)variances and correlations. The setting is general enough to capture the stylized facts that variances and correlations vary stochastically over time and are related to macroeconomic conditions and that systematic variances, idiosyncratic variances, and return correlations comove countercyclically and tend to spike during crisis periods and when the stock market performs poorly.[3] Expected returns on variance swaps on the components of total stock variance provide us with model-independent estimates for systematic

[3]In a seminal study, Black (1976) shows that volatility rises in falling financial markets. Andersen, Bollerslev, Diebold, and Ebens (2001) document that individual stock variances tend to move together and correlations are high when variance is high. Longin and Solnik (2001) also document that correlations rise in periods of high volatility. Erb, Harvey, and Viskanta (1994) show that stock market correlations tend to be higher when several countries are simultaneously in recession. Campbell, Lettau, Malkiel, and Xu (2001) document that idiosyncratic variances are countercyclical and positively related to market variance. Goyal and Santa-Clara (2003) find that stocks' average idiosyncratic variance positively predicts market returns.

and idiosyncratic variance risk premia.

We estimate the risk premia associated with the various sources of variance risk by combining data on S&P 100 and Nasdaq 100 index options with stock option data for the index constituents. We first establish that the risk premium on total stock variance is zero on average for S&P 100 stocks (as in, e.g., Driessen et al. (2009)) but positive for Nasdaq 100 stocks. The total variance risk premium rises with stock index betas, in contrast to what one would expect if only systematic variance risk is priced. Dispersion trades with zero exposure to index variance shocks are highly profitable, generating Sharpe ratios between 1.7 and 2.2. These features demonstrate that stock options are exposed to priced (idiosyncratic variance) risk factors that are absent from index options.

Decomposing the total variance risk premium into systematic and idiosyncratic variance risk premia, we observe that both variance risks command sizeable risk premia but the prices of risk have opposite signs. Consistent with Carr and Wu's (2009) negative estimates for the risk premium on systematic variance risk (SVR), the prices of variance risk on the return factors are significantly negative. By contrast, common idiosyncratic variance risk ($CIVR$)—the risk of common movements in the variances of idiosyncratic returns—carries a large positive risk premium.

These differences in the pricing of systematic and idiosyncratic variance risk allow reconciling several stylized facts about equity option returns. While index options load on SVR only and therefore earn the negative SVR premium, stock option returns are exposed to both SVR and $CIVR$. The bulk of the returns earned on the Goyal and Saretto (2009) strategy and on option portfolios constructed on the basis of other criteria can be attributed to the portfolios' exposure to $CIVR$. Thus, while idiosyncratic returns can be diversified in large portfolios, the variances of idiosyncratic returns are a crucial determinant of expected returns on option investments.

In the cross-section, the risk premium on idiosyncratic variance increases with firms' market-to-book ratio, the amount of employee stock options, and mutual fund ownership. It decreases with firm profitability and financial leverage and is largely unrelated to the bid-ask spread in the options, the bid-ask spread in the underlying, and several other liquidity proxies. These findings contradict the hypothesis that the premium on *CIVR* reflects compensation for the illiquidity of stock options and the costs associated with hedging them (see Cao and Han (2010)).

To explain the observed patterns, we explore the role of capital-constrained financial intermediaries in the pricing of variance risk. Financial intermediaries play a pivotal role as counterparties in the options market, providing liquidity to hedgers and speculators and absorbing much of the trading. Idiosyncratic movements in the variances of idiosyncratic returns are diversified in a dealer's large portfolio of options. What remains is the risk that the variances of idiosyncratic returns move in a systematic way. As a result, financial intermediaries are sensitive to common idiosyncratic variance risk and require compensation for bearing it. A simple equilibrium model yields that *CIVR* commands a positive risk premium when investors have aggregate short exposure to the common component in idiosyncratic return variances (see Garleanu, Pedersen, and Poteshman (2009) for evidence that investors are net suppliers of individual stock options). Equilibrium pricing implies further that *CIVR* is a priced factor in the cross-section of options. The price of risk for *CIVR* is larger at times when the total net supply of stock options is larger and when the riskiness of *CIVR* is larger, and an individual asset's variance risk premium is higher the larger the net supply of variance for that asset and the more variable the asset's idiosyncratic variance. We find empirical support for these predictions.

We also investigate alternative macroeconomic explanations for the sign and size of systematic and idiosyncratic variance risk premia. Taking positions in variance is attractive for investors when states of the economy in which aggregate consumption is low and state prices are high coincide with high stock variances and correlations.[4] To test this hypothesis, we consider Merton's (1973) ICAPM and investigate whether systematic and idiosyncratic variances predict the future state of the economy (see Campbell (1993)). We confirm that systematic variance negatively predicts GDP and investment growth. However, we find no support that idiosyncratic variances have strong predictive power for either macroeconomic variables or stock market conditions, once we control for systematic variance. Thus, the ICAPM can account for the negative risk premium on systematic variance but not for the positive risk premium on idiosyncratic variance.[5]

The paper is related to several strands of literature. Bakshi and Kapadia (2003a) provide early evidence of a negative market volatility risk premium in S&P 500 index options.[6] Carr and Wu (2009) document the existence of a systematic variance risk factor that carries a negative price of risk, suggesting that "investors are willing to pay a premium to hedge away upward movements in the return variance of the stock market." By contrast, Driessen et al. (2009) find that variance risk is not priced, and instead emphasize the importance of priced correlation risk as a separate source of risk. These opposing findings can be reconciled once one accounts for the pricing of *CIVR* and the no-arbitrage relationship between variance and correlation risk premia. Correlations rise when systematic variances increase or idiosyncratic variances decline. Correlation risk premia are therefore a composite of the premia on *SVR* and *CIVR*.

The differential pricing of systematic and idiosyncratic variance risk is also consistent

[4] Augmenting investors' portfolios with traded instruments that allow hedging (co)variance/correlation risk may yield substantial welfare gains (see Driessen et al. (2009), Egloff, Leippold, and Wu (2009), DaFonseca, Grasselli, and Ielpo (2009), and Buraschi, Porchia, and Trojani (2010)).

[5] See Campbell et al. (2001) and Goyal and Santa-Clara (2003) for related evidence. Several general equilibrium models have generated negative variance risk premia by incorporating time variation in economic uncertainty in the long-run risks model of Bansal and Yaron (2004). See Bollerslev, Tauchen, and Zhou (2009), Eraker (2009), Drechsler and Yaron (2011).

[6] Using options on 25 individual stocks, Bakshi and Kapadia (2003b) find that individual equity option prices also embed the negative market volatility risk premium, and that idiosyncratic volatility does not appear to be priced. More recently, Serban, Lehoczky, and Seppi (2010) estimate a one-factor model with stochastic volatilities for the S&P 500 index and 13 constituent stocks.

with the finding in Duan and Wei (2009) that, controlling for total risk, implied volatility is positively related to the systematic portion in stock returns. Finally, the paper relates to the literature on the pricing of non-systematic risks (Green and Rydqvist (1997)), the use of option strategies by investors (Lakonishok et al. (2007)), and the impact of investors' demand for options on their prices (Garleanu et al. (2009)). Most notably, while Garleanu et al. (2009) investigate how investors' supply of individual options affects their prices, we document that investors' aggregate exposure to *CIVR* is a crucial determinant for the cross-section of expected option returns.

The paper is organized as follows. Section 1 describes the model and its implications for variance swap pricing. Section 2 describes the data and our empirical methodology. Section 3 provides descriptive statistics and a specification analysis. In Section 4 we compute systematic and idiosyncratic variance risk premia and document the pricing of variance risk in the cross-section of equity options. This section also establishes the relationship between variance and correlation risk premia. Section 5 tests our explanation for the observed patterns, and Section 6 describes our robustness checks. Section 7 concludes. Technical developments are gathered in the Appendix.

1. Pricing Model with Systematic and Idiosyncratic Variance Risk

This section develops a financial market model with factor structures in both stock returns and (co)variances. Section 1.1 describes our assumptions on asset returns and variances. Section 1.2 discusses the arbitrage-free pricing of (co)variance swaps and the identification of variance risk premia.

1.1. The model

Consider an economy with N risky assets indexed by $n = 1, \ldots, N$. Asset prices $S_{n,t}$ are driven by J systematic return factors $F_t = (F_{1,t}, \ldots, F_{J,t})'$ and an idiosyncratic return component. The instantaneous excess return on risky asset n under the risk-neutral measure Q is given by

$$\frac{dS_{n,t}}{S_{n,t}} - r_{f,t}dt = \underbrace{\beta'_{n,t}dF_t}_{\text{Systematic return}} + \underbrace{(V^{\epsilon}_{n,t})^{\frac{1}{2}}dZ^{\epsilon}_{n,t}}_{\text{Idiosyncratic return}} , \tag{1}$$

where $r_{f,t}$ denotes the riskless interest rate, $\beta_{n,t}$ the J-dimensional vector of factor exposures, and the last term captures asset n's idiosyncratic return. Assume the instantaneous factor returns dF_t exhibit the following risk-neutral dynamics:

$$dF_t = (\Sigma_t)^{\frac{1}{2}}dZ_t \ . \tag{2}$$

For ease of exposition, $(Z^{\epsilon}_{n,t})_{n=1,\ldots,N}$ and Z_t are standard Brownian motion vectors.[7] As is standard in factor models, assume that all comovements in returns are caused by exposure to the common factors, so that $dZ^{\epsilon}_{m,t}dZ^{\epsilon}_{n,t} = 0$ for all $m \neq n$, and that the idiosyncratic returns are independent of the factor returns, $dF_t dZ^{\epsilon}_{n,t} = 0$ for all $n = 1, \ldots, N$. Under these assumptions, the excess returns under Q on different stock indices $I_{p,t}$, $p = 1, \ldots, P$,

[7] The results generalize to the case where stock returns contain jumps. To handle this case, it suffices to replace factor and idiosyncratic variances $\Sigma_t dt$ and $V^{\epsilon}_{n,t}dt$ by the increment in their quadratic variation. Letting $[R_n, R_n]_t$, $[F, F]_t$ and $[R^{\epsilon}_n, R^{\epsilon}_n]_t$ denote the quadratic (co)variation of asset returns, factor returns, and idiosyncratic returns, the quadratic variation of asset and index returns can be decomposed (just like variances in (5) and (6) below) as $d[R_n, R_n]_t = \beta'_{n,t}d[F,F]_t\beta_{n,t} + d[R^{\epsilon}_n, R^{\epsilon}_n]_t$ and $d[R_{I,p}, R_{I,p}]_t = \beta'_{I,p,t}d[F,F]_t\beta_{I,p,t} + \sum_n w^2_{n,p,t}d[R^{\epsilon}_n, R^{\epsilon}_n]_t$. Therefore, the decomposition of asset and index variance swap rates $VS_{n,t} = \frac{1}{\tau}E^Q_t[\int_t^{t+\tau} d[R_n, R_n]_u]$ and $VS_{I,p,t} = \frac{1}{\tau}E^Q_t[\int_t^{t+\tau} d[R_{I,p}, R_{I,p}]_u]$ and that of asset and index variance risk premia $VRP_{n,t} = E^P_t[d[R_n, R_n]_t] - E^Q_t[d[R_n, R_n]_t]$ and $VRP_{I,p,t} = E^P_t[d[R_{I,p}, R_{I,p}]_t] - E^Q_t[d[R_{I,p}, R_{I,p}]_t]$ hold. The factor variance swap extraction also goes through.

comprising the N constituent assets with index weights $w_{n,p,t}$, are equal to

$$\frac{dI_{p,t}}{I_{p,t}} - r_{f,t}dt = \beta'_{I,p,t}dF_t + \sum_{n=1}^{N} w_{n,p,t}(V^{\epsilon}_{n,t})^{\frac{1}{2}}dZ^{\epsilon}_{n,t} \ , \tag{3}$$

where $\beta_{I,p,t} = \sum_{n=1}^{N} w_{n,p,t}\beta_{n,t}$ is index p's exposure to the return factors.

The instantaneous factor variance-covariance matrix, Σ_t, and the instantaneous variances of the assets' idiosyncratic returns, $V^{\epsilon}_{n,t}$, follow stochastic processes that may be correlated with each other, with Z_t and $Z^{\epsilon}_{n,t}$, and be associated with risk premia. Specifically, given the evidence in Campbell et al. (2001) that assets' idiosyncratic return variances are time varying and related to market variance, assume the following factor structure on the idiosyncratic return variances $V^{\epsilon}_{n,t}$:

$$dV^{\epsilon}_{n,t} = \underbrace{\gamma'_{n,t}d\Gamma_t}_{\text{Common idiosyncratic variance}} + \underbrace{d\tilde{V}^{\epsilon}_{n,t}}_{\text{Truly idiosyncratic variance}} \ , \tag{4}$$

where Γ_t is a G-dimensional stochastic process of common idiosyncratic variance factors that may be correlated with Σ_t and Z_t; $\gamma_{n,t}$ is a G-dimensional vector of factor exposures; and $\tilde{V}^{\epsilon}_{n,t}$ is a stochastic process that denotes the part of asset n's idiosyncratic return variance specific to asset n (we will refer to $\tilde{V}^{\epsilon}_{n,t}$ as asset n's "truly idiosyncratic" variance). Beyond these factor structure assumptions, we do not specify parametric dynamics for Γ_t and $\tilde{V}^{\epsilon}_{n,t}$ in eq. (4).

In our empirical analysis, we use a nonparametric approach to estimate the risk premia on Σ_t, Γ_t, and $\tilde{V}^{\epsilon}_{n,t}$. To gain intuition on the identification, note that under the above assumptions the variances of asset returns $dS_{n,t}/S_{n,t}$ and of index returns $dI_{p,t}/I_{p,t}$ are given by

$$\sigma^2_{n,t} = \beta'_{n,t}\Sigma_t\beta_{n,t} + V^{\epsilon}_{n,t}\ , \tag{5}$$

$$\sigma^2_{I,p,t} = \beta'_{I,p,t}\Sigma_t\beta_{I,p,t} + \sum_{n=1}^{N} w^2_{n,p,t}V^{\epsilon}_{n,t}\ . \tag{6}$$

The first (second) term on the right-hand sides captures the systematic (idiosyncratic) variance component of asset n and, respectively, index p. Eqs. (5) and (6) yield a linear system that can be solved for Σ_t and $V^{\epsilon}_{n,t}$, so long as a sufficient number of stock indices are available. For uncorrelated return factors, identification requires $P \geq J$, that is, at least as many indices as return factors. For correlated factors, the corresponding condition is $P \geq J(J+1)/2$. As we will see in the next section, the same conditions hold for the prices of contracts on variance (so called variance swaps).

Following Driessen et al. (2009), the instantaneous variance risk premia on asset n and index p are given by $VRP_{n,t} \equiv E^P_t[d\sigma^2_{n,t}] - E^Q_t[d\sigma^2_{n,t}]$ and, respectively, $VRP_{I,p,t} \equiv E^P_t[d\sigma^2_{I,p,t}] - E^Q_t[d\sigma^2_{I,p,t}]$, where P denotes the physical probability measure and Q the risk-neutral measure (the ordering is irrelevant). Similarly, the variance risk premia on the return factors are $VRP_t \equiv E^P_t[d\Sigma_t] - E^Q_t[d\Sigma_t]$ and the variance risk premium on asset n's idiosyncratic return component is $VRP^{\epsilon}_{n,t} \equiv E^P_t[dV^{\epsilon}_{n,t}] - E^Q_t[dV^{\epsilon}_{n,t}]$. Using eqs. (5) and (6) and assuming deterministic factor exposures or, alternatively, imposing the assumptions in Appendix A, variance risk premia inherit the linear factor structure:

$$VRP_{n,t} = \beta'_{n,t}VRP_t\beta_{n,t} + VRP^{\epsilon}_{n,t}\ , \tag{7}$$

$$VRP_{I,p,t} = \beta'_{I,p,t}VRP_t\beta_{I,p,t} + \sum_{n=1}^{N} w^2_{n,p,t}VRP^{\epsilon}_{n,t}\ . \tag{8}$$

Expressions (7) and (8) yield that the total variance risk premium on a stock is the composite of the systematic and idiosyncratic variance risk premia, while the variance risk premium on a stock index is given predominantly by the systematic variance risk premium arising from the index loading on the return factors. Short-dated variance swaps on total stock variance and index variance provide the means to empirically identify the risk premia on systematic and, respectively, idiosyncratic variance in eqs. (7) and (8). The next section describes the pricing of variance contracts in this setting and shows how one can identify factor and idiosyncratic variance swap rates.

1.2. Identifying variance swap rates on systematic and idiosyncratic variance

Variance swaps are contracts that at maturity pay the realized variance, RV, over a fixed horizon net of a premium called the *variance swap rate*, VS. The latter is set by the contracting parties such that the variance swap has zero net market value at entry. Denote by $VS_{n,t}$ and $VS^{\epsilon}_{n,t}$ the arbitrage-free variance swap rates on the total and, respectively, idiosyncratic return of asset $n = 1, \ldots, N$ at time t with maturity $t + \tau$ (we drop the dependence on τ from the notation to save space). That is, $VS^{\epsilon}_{n,t}$ is the variance swap rate of a synthetic asset exposed only to asset n's idiosyncratic risk. Similarly, denote by VS_t the matrix of arbitrage-free (co)variance swap rates on the systematic return factors at time t with maturity $t + \tau$.[8] By absence of arbitrage, one has

$$\begin{aligned} &\text{Total variance swaps:} && VS_{n,t} = \tfrac{1}{\tau} E^{Q}_{t}[\textstyle\int_{t}^{t+\tau} \sigma^{2}_{n,u} du] \\ &\text{Factor (co)variance swaps:} && VS_{t} = \tfrac{1}{\tau} E^{Q}_{t}[\textstyle\int_{t}^{t+\tau} \Sigma_{u} du] \\ &\text{Idiosyncratic variance swaps:} && VS^{\epsilon}_{n,t} = \tfrac{1}{\tau} E^{Q}_{t}[\textstyle\int_{t}^{t+\tau} V^{\epsilon}_{n,u} du] \end{aligned} \tag{9}$$

Individual stock variance swap rates $VS_{n,t}$, $n = 1, \ldots, N$, and index variance swap rates $VS_{I,p,t} = \frac{1}{\tau} E^{Q}_{t}[\int_{t}^{t+\tau} \sigma^{2}_{I,p,u} du]$, $p = 1, \ldots, P$ exhibit the same linear factor structure as vari-

[8] That is, the (j, j) component of VS_t is the variance swap rate of a synthetic asset with unit exposure to factor j, zero exposure to all other factors, and no idiosyncratic risk, and the (i, j) component of VS_t is the covariance swap rate between factors i and j.

ances. Assuming constant index weights and factor exposures over the life of the variance contract (i.e., $w_{n,p,u} = w_{n,p,t}$, $\beta_{n,u} = \beta_{n,t}$, and $\beta_{I,p,u} = \beta_{I,p,t}$ over $u \in [t, t+\tau)$), one has

$$VS_{n,t} = \beta'_{n,t} VS_t \beta_{n,t} + VS^{\epsilon}_{n,t} \ , \tag{10}$$

$$VS_{I,p,t} = \beta'_{I,p,t} VS_t \beta_{I,p,t} + \sum_{n=1}^{N} w^2_{n,p,t} VS^{\epsilon}_{n,t} \ . \tag{11}$$

Conditions (10) and (11) yield that variance swap rates on individual stocks and on stock indices are linked by absence of arbitrage to factor variance swap rates and idiosyncratic variance swap rates, but with different weights, and these weights are known functions of the index constituents' exposures to the return factors and their weights in the index. VS_n and VS_I can therefore be used to back out the swap rates on systematic and idiosyncratic variance.

Combining the expressions for all stocks and indices yields a linear system of $N + P$ equations that can be solved for the N idiosyncratic variance swap rates $VS^{\epsilon}_{n,t}$ and the J factor variance swap rates VS_t, so long as the number of indices exceeds the number of return factors ($P \geq J$).[9] The key feature for identification is that the relation between the observables, $VS_{n,t}$ and $VS_{I,p,t}$, and the unobservables, $VS^{\epsilon}_{n,t}$ and VS_t, in eqs. (10) and (11) is exact. In our empirical implementation we can accommodate approximation error in eq. (11) and measurement error in the data by applying the Kalman filter.

Appendix A contains details on how the decomposition (10)-(11) (and consecutive results) have to be modified in the presence of time-varying factor exposures over the life of the variance swap and parameter uncertainty.[10] Internet Appendix IA.A contains simula-

[9] In the general case of correlated return factors, VS_t is a J-by-J matrix of factor (co)variance swap rates. The corresponding identification condition becomes $P \geq J(J+1)/2$.

[10] As we show in Appendix A, sufficient conditions for the analysis to go through are that individual assets' factor exposures are martingales and changes in factor exposures are uncorrelated with factor variances and covariances Σ_t. We also show empirically that accounting for time variation in factor exposures does not substantively affect our results.

tion evidence that the constant-weight approximation in eq. (11) is very accurate. Table 1 summarizes the notation and formulas used throughout.

[**Table 1 about here**]

2. Data and Empirical Methodology

The data for our empirical analysis consists of option price data from OptionMetrics and daily index and constituent stock returns from CRSP. We also employ data from Compustat, Thomson Financial, and other sources. The sample period ranges from January 2, 1996 to October 31, 2009. For most of the analysis, we use data on two stock market indices, the S&P 100 index (OEX) and the Nasdaq 100 index (NDX), for which liquid options are available throughout the sample, and on their constituent stocks.[11] For all indices, we obtain historical index weights of the constituent stocks on each trading day in the sample period as described in Appendix B.

Variance swaps are traded over-the-counter and swap quotes are difficult to obtain at low cost and associated with large bid-ask spreads. To avoid the associated liquidity issues, we employ synthetic variance swap rates. We derive the swap rates from equity option prices using the methodology outlined in Demeterfi, Derman, Kamal, and Zou (1999) and Carr and Wu (2009). Appendix C provides a more detailed description of the methodology and our implementation. In addition, Section 6 examines how discreteness in option strike prices, limitations in data availability over a wide range of moneyness, and jumps in underlying prices affect the accuracy of the synthetic variance swap rates.

With data on returns and variance swap rates on two broad stock indices and their

[11] We have also considered the Dow Jones Industrial Average index (DJX), for which options are available since September 24, 1997. The DJX's exposure to the common return factors is very similar to that of the S&P 100 index and, as a result, the DJX index adds little explanatory power to identify variance swap rates on the common return factors. The same limitation arises with the S&P 500 index (SPX), for which options are available for the entire sample.

constituents, we can estimate the model by consecutively applying the following steps. First, we specify the factor model for expected stock returns. There are several ways to go about this. One can either specify observable factors that are known to explain the cross-section of stock returns (such as the Fama-French-Carhart $FF4$ factors) or extract empirical factors from the data. The advantage of the latter approach is that one can construct orthogonal return factors. We therefore perform a latent factor analysis using quasi-maximum likelihood. We extract $J = 2$ latent factors from the panel of stock returns and perform the optimal factor rotation to determine the factor realizations F_t.[12]

While one cannot rule out additional return factors in the residuals, we find that two latent return factors have explanatory power comparable to the $FF4$ factors. Table 2, Panel A reports the explained variation in stock and, respectively, index returns. The two extracted factors explain 26.37% of the cross-sectional variation in stock returns, compared to 25.61% for the $FF4$ factors. More than two factors do not produce any material improvement in explanatory power for index returns beyond 92% for the S&P and 89% for Nasdaq. Table 2, Panel B reports the correlations between the two optimally rotated return factors and the $FF4$ factors. The first rotated return factor is strongly correlated with the market return and close to uncorrelated with the HML factor. The second return factor is weakly correlated with the market and strongly negatively correlated with HML. Both factors are weakly correlated with SMB and UMD.

Internet Appendix IA.B conducts a number of additional specification tests to ensure that two factors are sufficient for our analysis. The main conclusions from these tests are that, first, factor variances capture index variances and their dynamics almost perfectly and that, second, the two return factors accurately capture correlations in the returns of the index constituents and their time variation. Figure 1, Panel A reports the decomposition

[12]A known issue in standard factor analysis is that estimated factor loadings and scores are unique only up to scale and rotation. The appropriate rotation is uniquely identified when the factors have stochastic variances. The optimal rotation minimizes the time-series standard deviation of the factor covariances.

of index variance into three components, the component due to (1) the factor variances, (2) the covariance in assets' residual returns, and (3) the variances of assets' residual returns. The decomposition illustrates that factor variances account for an overwhelming part of the level [98% (96%) for S&P (Nasdaq)] and the variability [98% (79%)] of index variances. The contribution of both the average covariance between the return residuals and the idiosyncratic return variances to index variances are negligible. Figure 1, Panel B reports the average correlation of stock returns and return residuals. The residual correlations are close to zero throughout the sample period for both indices.

[**Table 2 and Figure 1 about here**]

In the second step, we compute variance swap rates on the return factors, VS_t, and on the N stocks' idiosyncratic return component, $VS^{\epsilon}_{n,t}$, by applying the Kalman filter. Technical details of the implementation are provided in Internet Appendix IA.C. Figure 2 plots the time-series of the extracted variance swap rates on the return factors and illustrates the outcomes in each step of the estimation. The left panels present the case of constant asset factor exposures, the right panels that of time-varying factor exposures. The findings in both settings are similar throughout.

[**Figure 2 about here**]

Finally, we compute the stocks' realized variance, $RV_{n,t} \equiv \frac{1}{\tau}\int_t^{t+\tau} \sigma^2_{n,u} du$, index variances $RV_{I,p,t}$, idiosyncratic variances $RV^{\epsilon}_{n,t}$, and factor variances RV_t as the variances of daily returns over the holding period $t+1$ day to $t+\tau$ days. Throughout the empirical analysis, we compute realized variances using rolling one-month ($\tau = 21$ trading days) windows and use variance swap rates and variance swap returns over one-month horizons.

3. Descriptive Statistics and a Basic Analysis

We start by establishing a number of stylized facts about the dynamics of systematic and idiosyncratic variances in our sample. We also provide evidence that the main empirical pricing patterns documented in the paper can be found in model-free dispersion trades.[13] Hence, our findings do not rely in an obvious way on the modeling assumptions in Sections 1 and 2.

3.1. Variance and variance swap dynamics

Figure 3 illustrates individual and index variances over the sample period. The figure plots realized index variance, the average variance of the index constituents, the average correlation between index constituents, and their product. The top panel depicts the series for the S&P 100, the bottom panel for the Nasdaq 100. Figure 3 highlights that individual variances, index variances, and return correlations all comove; they increase during crisis periods and when the stock market performs poorly ("leverage effect", Black (1976)). Individual variances also exhibit pronounced commonality, that is, individual stock variances move together (see Andersen et al. (2001)), exhibit spikes and mean-revert—consistent with the findings in the GARCH literature. Correlations are high when variances are high (Longin and Solnik (2001)). The time variation in the difference between average constituent variances and index variances shows that idiosyncratic variances are highly time-varying, countercyclical, and positively related to market variance (Campbell et al. (2001)). The figure also reveals that index variance is driven by both constituent stock variances and their correlations—as predicted by the model in Section 1.

[Figure 3 about here]

[13] Dispersion trading strategies are portfolios with long positions in options (or variance swaps) on the index constituents and short positions in options (or variance swaps) on the underlying index.

Figure 4 plots variance swap rates on the index against the average rate of the index constituents for the S&P 100 and Nasdaq 100 indices. As for realized variances depicted in Figure 3, asset and index variance swap rates strongly co-move. Regressing the S&P 100 and Nasdaq 100 index variance swap rates on the average variance swap rate of their constituents yields R^2 values of 79.47% and 93.88%, respectively.[14] Thus, idiosyncratic variance swap rates are time-varying and imperfectly correlated with index variance swap rates.

[**Figure 4 about here**]

Another feature of the model in Section 1 is that individual asset variances, variance swap rates, and variance risk premia exhibit a factor structure. To validate this assumption, we perform factor analyses of the panels of realized variances, variance swap rates and, respectively, variance swap returns.[15] The results in Table 3 reveal, first, the presence of strong co-movements in variances, variance swap rates, and variance swap returns and, second, common variance movements in the cross-section that are absent from the index. A single factor explains 44% of the variation in individual assets' realized variances—as can be seen in the first row of Panel A. Two (four) factors explain up to 55% (62%). The first row of Panel B reveals that commonality is even stronger for variance swap rates, suggesting that shocks to truly idiosyncratic variances $\tilde{V}^{\epsilon}_{n,t}$ are short-lived and only marginally alter (risk neutral) expectations of future variance. A single factor explains around 56% of the variation in individual assets' variance swap rates, and two (four) factors account for about

[14] Performing the same analysis on variance swap returns produces similar results: regressing the variance swap return of the S&P 100 and Nasdaq 100 index on the average variance swap return of their constituents yields R^2 values of 74.53% and 78.38%, respectively.

[15] To our knowledge, the factor structure of individual assets' variance swap rates has not been documented previously. Carr and Wu (2009) compute variance betas for each of the stocks they consider by regressing these stocks' realized variances on the realized variance of the S&P 500 index, which they take as a proxy for the market portfolio variance. They document that stocks with higher variance betas have variance risk premia that are more strongly negative. However, they do not investigate co-movements in asset variance swap rates. Vilkov (2008) investigates the factor structure of variance swap returns, but not that of variance swap rates.

70% (78%) of the variation. The first row of Panel C reveals that common factors are also present in individual assets' variance swap returns. A single factor explains about 26%, and two factors explain about 29% of the variation in variance swap returns. These values are comparable to those for stock returns reported in Section 2.

[**Table 3 about here**]

3.2. Variance risk premia on index vs. constituents

To gauge whether both systematic and idiosyncratic return variances are priced, we now compare model-free variance risk premia on the index with those on the constituents. Following Carr and Wu (2009), we measure variance risk premia on the N stocks (and, analogously, index variance risk premia) using the expected returns on short-dated variance swaps, computed as the average holding period return from a long variance swap position over the holding period t to $t + \tau$ (τ is chosen to be 21 trading days):

$$r_{n,t} = \frac{RV_{n,t} - VS_{n,t}}{VS_{n,t}} \ . \tag{12}$$

Table 4 reveals that, consistent with the prior literature (Carr and Wu, 2009, Driessen et al., 2009), the variance risk premia on the S&P 100 and Nasdaq 100 indices are strongly negative, with average values of -15.11% and -5.36% per month, respectively (the Newey-West t-statistics with 20 lags are -3.13 and -1.27). Also consistent with Driessen et al. (2009), the average variance swap returns on the S&P 100 constituents are marginally positive but statistically insignificant, with a value of 3.70% per month (NW t-stat 1.06). By contrast, the average variance swap returns on the Nasdaq 100 index constituents are economically and statistically positive, with a value of 9.64% per month (NW t-stat 3.21). Hence, the stylized fact of a zero variance risk premium on individual stock variances in Driessen et al. (2009)

does not generalize to Nasdaq 100 stocks, emphasizing the need to separately investigate the variance risk premia on the systematic and idiosyncratic components of returns.

[**Table 4 about here**]

A simple and robust way to establish that both systematic and idiosyncratic variance risk are priced is to consider dispersion trading strategies with different exposures to these two sources of variance risk. Consider a dispersion trading strategy exposed to idiosyncratic variance risk but unexposed to systematic variance risk. Such a strategy can be constructed by purchasing variance swaps on individual stocks in proportion to the stocks' weights in the index and setting the size of the short position in the index variance swap such that the portfolio has zero exposure to the returns on the index variance swap. The strategy should earn negligible returns if idiosyncratic variance risk is unpriced.

Table 5, Panel A, reports summary statistics for the returns on dispersion trading strategies constructed in this way, separately for the S&P 100 and Nasdaq 100 indices. The strategy is highly profitable in spite of zero exposure to index variance swap returns, generating Sharpe ratios of 1.7 for S&P and 2.2 for Nasdaq. These results suggest that single stock variances are exposed to priced (idiosyncratic variance) risk factors that are absent from index variances and that these idiosyncratic variance risk factors carry a positive risk premium. Notably, as can be seen in Panel B of the table, strategies constructed to be uncorrelated with the average constituent variance swap return are profitable as well. The corresponding Sharpe ratios are 1.8 for S&P and 2 for Nasdaq. These stylized facts constitute a challenge not only for the view that only systematic variance risk is priced, but also for the view—common among practitioners—that correlation risk is priced while variance risk is not, and that dispersion trades earn the correlation risk premium (see Driessen et al. (2009)). We show in Internet Appendix IA.D that dispersion trades earn a combination of the risk premia on systematic and idiosyncratic variance.

[**Table 5 about here**]

4. The Pricing of Systematic and Idiosyncratic Variance Risk

In this section, we decompose total variance risk premia into the components due to systematic and idiosyncratic variance risk. We establish the relationship between variance and correlation risk premia and demonstrate that common idiosyncratic variance risk (*CIVR*) is an important factor in explaining the cross-section of equity option returns.

4.1. Decomposition of variance risk premia

Total variance risk premia equal the sum of systematic and idiosyncratic variance risk premia (see condition (7)). One can identify the systematic and idiosyncratic variance components by decomposing variance swap returns $r_{n,t}$ (given in eq. (12)) as follows:

$$r_{n,t} = \beta'_{n,t} \underbrace{\frac{RV_t - VS_t}{VS_{n,t}}}_{\text{Factor variance swap return}} \beta_{n,t} + \underbrace{\frac{RV^{\epsilon}_{n,t} - VS^{\epsilon}_{n,t}}{VS_{n,t}}}_{\text{Idiosyncratic variance swap return}}, \qquad (13)$$

where RV_t denotes realized factor (co)variances and $RV^{\epsilon}_{n,t}$ individual assets' realized idiosyncratic variances, computed over the life of the contract.

Table 6 reports monthly variance risk premia (*VRP*), weighted according to the corresponding index composition, for the constituents of the S&P 100 and, respectively, Nasdaq 100 indices. The table also reports the decomposition of total variance risk premia into systematic and idiosyncratic variance components for the entire sample period as well as split by calendar year.[16] *Specification 1* (on the left) assumes constant factor exposures β_n and *Specification 2* (on the right) allows for time-varying factor exposures $\beta_{n,t}$. For the

[16]The values reported for the entire sample differ slightly from the average of the yearly values because the data for 2009 ends on October 31.

constituents of either stock index, both the systematic and idiosyncratic components of the total variance risk premium are economically sizeable and statistically significant. For S&P stocks, the average monthly systematic variance risk premium (VRP) is -19.73% (NW t-stat = -8.19) and the average idiosyncratic VRP is 23.43% (NW t-stat = 14.28). For Nasdaq stocks, the average systematic VRP is -11.99% (NW t-stat = -6.00) and the average idiosyncratic VRP 21.64% (NW t-stat = 12.99). Thus, the total variance risk premium is about zero for S&P 100 stocks because idiosyncratic and systematic variance risk premia roughly offset each other. By contrast, the idiosyncratic variance risk premium for Nasdaq stocks is about twice as large in absolute value as the average systematic component, resulting in a positive risk premium on total variance risk. Hence, by splitting variances and variance swap rates into systematic and idiosyncratic components, we uncover a negative risk premium on systematic variance and a *positive risk premium on idiosyncratic variance.* These results are robust when we split the data by year and/or allow for time-varying factor exposures (*Specification 2* reported in the second set of columns in Table 6).[17]

[**Table 6 about here**]

These results suggest that index options are expensive because they hedge increases in factor variances and single-stock options are cheap once their exposure to the common return factors is accounted. That is, idiosyncratic variance swaps sell for less than their expected discounted payoffs. The expected monthly return is around 20%, which suggests that an insurer against increases in idiosyncratic volatility loses substantial amounts on average.

[17]For the members of either index, there is sizeable variation in the variance risk premia across years, both for total variance and for its components. Nonetheless, a clear pattern emerges from Table 6. The systematic variance component is negative with two exceptions (years 2000 and 2008) and the idiosyncratic variance component is positive in every year, while the total variance risk premium is positive in about half of the years for both indices (seven out of fourteen years).

4.2. The relationship between variance and correlation risk premia

It is instructive to contrast these results with those reported in the existing literature. Carr and Wu (2009) find that only systematic variance carries a risk premium, while Driessen et al. (2009) find that total variance does not carry a risk premium but correlation does. In terms of option prices, the results in Carr and Wu (2009) imply that index options are expensive (i.e., implied variance exceeds realized variance) because they allow hedging increases in market variance, and single-stock options are expensive only to the extent that they are exposed to shifts in market variance. The results in Driessen et al. (2009) imply that single-stock options are not expensive (i.e., implied variance matches realized variance on average) but index options are expensive because the index is subject to correlation shocks.

By contrast, we show that both systematic and idiosyncratic variance risk are priced, but with opposite signs. As a result, total variance appears unpriced in situations when the premia on the two sources of variance risk offset each other (as in the case of S&P 100 stocks). Since correlations rise when systematic variances rise or idiosyncratic variances drop, correlation risk premia are a composite of systematic and idiosyncratic variance risk premia.[18] This can be seen as follows. Let $\rho_{m,n,t} = \frac{\beta'_{m,t}\Sigma_t\beta_{n,t}}{\sigma_{m,t}\sigma_{n,t}}$ denote the instantaneous return correlation between assets m and n. Using Itô's Lemma, one can show that correlation risk premia $E_t^P[d\rho_{m,n,t}] - E_t^Q[d\rho_{m,n,t}]$ are given by the following combination of the (co)variance risk premia on the common return factors, VRP_t, and the stocks' total variance risk premia, $VRP_{n,t}$:

$$E_t^P[d\rho_{m,n,t}] - E_t^Q[d\rho_{m,n,t}] = \frac{1}{\sigma_{m,t}\sigma_{n,t}}\beta'_{m,t}VRP_t\beta_{n,t} - \frac{\rho_{m,n,t}}{2}\left(\frac{VRP_{m,t}}{\sigma^2_{m,t}} + \frac{VRP_{n,t}}{\sigma^2_{n,t}}\right). \quad (14)$$

The variance risk premia of the individual assets on the right-hand side of eq. (14) de-

[18] Time-varying correlations and correlation risk premia have been studied, for instance, by Erb et al. (1994), Longin and Solnik (2001), and Buraschi et al. (2010).

pend, in turn, on the (co)variance risk premia on the return factors and on idiosyncratic return variance risk premia (eq. (7)). Thus, correlation risk premia are a combination of the (co)variance risk premia on the return factors and the risk premia on stocks' idiosyncratic variance.[19] Hence, by explicitly accounting for the factor structure of stock returns, expressions (7), (8) and (14) allow decomposing correlation risk premia into their economic sources. By contrast, the correlation risk premia estimated in the literature only identify a composite of different (co)variance risk premia.[20]

4.3. Can priced idiosyncratic variance risk explain the cross-section?

The stylized facts established in Section 3 and the decomposition in Section 4.1 suggest a unifying risk-based explanation for the return differences between index and constituent options and for pricing anomalies in the cross-section of equity options. To save space, in the remainder we report results for the case of time-varying factor exposures. The results obtained when assuming constant factor exposures are similar.

Table 7 reports return and turnover statistics (monthly average returns and standard deviations μ and σ, annual Sharpe ratio SR and monthly turnover) as well as monthly

[19]It is possible to reconcile expression (8) with the expression for the index variance risk premium derived by Driessen et al. (2009),

$$VRP_{I,p,t} = \sum_{n=1}^{N} x_{n,p,t} VRP_{n,t} + \sum_{n=1}^{N} \sum_{\substack{m=1 \\ m \neq n}}^{N} w_{n,p,t} w_{m,p,t} \sigma_{n,t} \sigma_{m,t} (E_t^P[d\rho_{m,n,t}] - E_t^Q[d\rho_{m,n,t}]) \,,$$

where $x_{n,p,t} \equiv w_{n,p,t}^2 + \sum_{m \neq n} w_{n,p,t} w_{m,p,t} \rho_{m,n,t} \frac{\sigma_{m,t}}{\sigma_{n,t}}$. This expression can be shown to be equivalent with condition (8) once one accounts for the relationship between correlation risk premia and variance risk premia in eq. (14).

[20]The relationship between variance and correlation risk premia becomes particularly intuitive when the common return factors are uncorrelated. In this case, correlation risk premia among assets can only arise if variance risk (either factor variance risk, or idiosyncratic variance risk, or both) are priced. Specifically, the first term in (14) becomes $\frac{1}{\sigma_{m,t}\sigma_{n,t}} \sum_{j=1}^{J} \beta_{m,t}(j) \beta_{n,t}(j) (E_t^P[d\Sigma_t^{jj}] - E_t^Q[d\Sigma_t^{jj}])$, where $\beta_{m,t}(j)$ denotes the jth component of $\beta_{m,t}$. Thus, correlation risk premia $E_t^P[d\rho_{m,n,t}] - E_t^Q[d\rho_{m,n,t}]$ are driven *only* by the variance risk premia on the common return factors, $E_t^P[d\Sigma_t^{jj}] - E_t^Q[d\Sigma_t^{jj}]$, and the variance risk premia of the individual assets, $VRP_{n,t}$.

abnormal returns (Alpha) and factor loadings of long-short portfolios of single stock variance swaps constructed by sorting on different characteristics. Each equal-weighted portfolio is long quintile 5 and short quintile 1. The portfolios are benchmarked against three expected return models, (i) the Fama-French four-factor model ($FF4$), (ii) the Fama-French model augmented by two systematic variance risk factors ($FF4$+SVR), and (iii) the Fama-French model augmented by two systematic variance risk factors and two idiosyncratic variance risk factors ($FF4$+VR). The systematic variance risk factors are measured by the S&P and, respectively, Nasdaq index variance swap return, and the proxies for common idiosyncratic variance risk are constructed as the cross-sectional average idiosyncratic variance swap return on the index constituents for each of the indices. The characteristics used to construct the long-short portfolios in Table 7 are the ratio of realized variance in the previous month to the variance swap rate at the end of the previous month (Panel A), variance swap returns in the previous month (Panel B), stocks' realized variance in the previous month (Panel C), stocks' variance swap rate at the end of the previous month (Panel D), and stocks' exposure to the S&P 100 and Nasdaq 100 index returns (Panels E-F).[21]

[**Table 7 about here**]

The strategy in Panel A corresponds to the "volatility mispricing" strategy in Goyal and Saretto (2009, *GS* henceforth) and generates a sizeable annual Sharpe ratio of 1.61.[22] Panel B reveals that variance swap returns on individual stocks exhibit high short-term persistence. Panels C and D report the returns of portfolios constructed using the components of the *GS* ratio. Panel C reveals that variance swap returns are significantly higher for stocks that

[21] In order to avoid survival bias issues, only those stocks that are members of the S&P 100 or Nasdaq 100 index as of the portfolio formation date are considered in the analysis. The results including all the stocks in our sample are similar.

[22] *GS* find that a trading strategy that is long (short) options with a large (small) ratio of realized volatility during the previous twelve months to at-the-money implied volatility as of the portfolio formation date earns large abnormal returns. The strategy in Panel A differs from *GS* in that we compute realized variance over the previous month rather than over the previous 12 months and use the variance swap rate instead of at-the-money implied volatility.

had high realized variance in the previous month. At 43.56% per month, average portfolio turnover is much lower than in panels A and B. Panel D reveals that similar returns are achieved when the portfolios are constructed on the basis of the variance swap rate. Average turnover is about half that of the realized variance strategy in Panel C, reflecting that variance swap rates are more persistent than realized variances. Panels E and F focus on a different return pattern. If only systematic variance risk is priced and commands a negative risk premium, higher beta stocks should exhibit lower variance risk premia. As can be seen, however, variance swap returns are higher for high-beta stocks, irrespective of whether beta is computed with respect to the S&P 100 or the Nasdaq 100 index. Importantly, average monthly turnover is less than 2%, suggesting that different risk exposures rather than temporary mispricing give rise to the return patterns.[23]

In Table 7, all sort portfolios exhibit large and significant abnormal returns when measured against the $FF4$ model, ranging from 8.70% to 18.30% per month. Abnormal returns are also large and significant when measured against the $FF4$+SVR model. When both systematic and common idiosyncratic variance risk factors are included ($FF4$+VR model), abnormal returns decline to between -2.56% and 1.94% per month and become insignificant. The same pattern arises for the returns on the long-short portfolios constructed on the basis of firm characteristics (return on assets, firm size, and market-to-book ratio). Thus, the returns on the sort portfolios can almost entirely be attributed to the portfolios' exposures to variance factors. While both systematic and common idiosyncratic variance factors are priced, the *CIVR* factors appear to be the key driver of option returns. Consistent with this interpretation, all sort portfolios in Table 7 load strongly on the variance risk factors.

[23] Long-short portfolios constructed by sorting on several firm characteristics—return on assets, firm size, and market-to-book ratio—also earn sizable returns, with Sharpe ratios of 0.9436, 0.9908 and, respectively, 0.6534. Di Pietro and Vainberg (2006) provide a detailed analysis for firm size and the market-to-book ratio using a larger sample of over 1400 firms for the period 1996-2004.

5. Equilibrium Pricing of Idiosyncratic Variance Risk

In this section, we propose an explanation for the positive premium on idiosyncratic variance risk and for the fact that it is heavily priced in the cross-section of expected option returns.

5.1. Investor clienteles and limited risk bearing capacities

Financial intermediaries play a pivotal role as counterparties in the options market. They provide liquidity to hedgers and speculators and absorb much of the demand and supply from investors. Idiosyncratic movements in the variances of idiosyncratic returns are diversified away in a dealer's large portfolio of options. What remains is the risk that the variances of idiosyncratic returns move in a systematic way. Since intermediaries cannot hedge options perfectly, they are sensitive to risk. Thus, to the extent that investors' net supply or demand for options causes intermediaries to be exposed to common movements in idiosyncratic variances, *CIVR* will be priced in equilibrium.

Many investors in principal-agent relations have preferences for negative skewness and, therefore, are natural suppliers of stock options.[24] Hedge funds are a typical example. For instance, a prominent hedge fund strategy is to short stock variance, generating a high propensity of small gains and infrequent large losses when volatility spikes (known as "picking up nickels in front of a steamroller").[25] Mutual funds and retail investors provide an other source of stock option supply. Covered call writing—allowed under SEC regulations—is a

[24]Using a unique dataset, Garleanu, Pedersen, and Poteshman (2009) provide evidence that end-users of options on individual names are net short while option market makers are net long. In cross-sectional tests, they show that end-users' net demand for a stock option impacts its expensiveness.

[25]In a recent paper, Malliaris and Yan (2010) show that reputation concerns induce fund managers to adopt strategies with negatively skewed payoffs, even if such strategies generate inferior returns. They show that four out of the ten style indices in the Credit Suisse/Tremont Hedge Fund Index, representing more than 40% of total hedge fund assets, have negatively skewed returns. Scott and Horvath (1980) demonstrate that preferences for moments of higher order than the variance yield "lotto" behavior, i.e., investors prefer positive skewness in return distributions. Mitton and Vorkink (2007) show that heterogeneity in preferences is required so that skewness induces investors to underdiversify in equilibrium.

popular investment strategy among mutual funds and individual investors.[26] Option-based compensation in public corporations gives rise to an additional source of exchange-traded options when corporate executives and employees hedge their exposure.[27] Between 1996 and 2009, the outstanding amount of company-issued options (Compustat variable *optosey*) constituted on average about 77% (139%) of the call (put) open interest in front-month exchange-traded options closest to at-the-money, and about 25% (58%) of the total call (put) open interest of all front-month exchange-traded options.

5.2. Equilibrium pricing implications

A simple model of equilibrium variance risk pricing formalizes the above intuition and allows quantifying variance risk premia. Suppose investors supply index and individual variance risk with notional amounts $(x_t, \mathbf{y}_t)$. Financial intermediaries (FI) take positions in the riskless asset and in index and individual variance swaps to maximize their expected utility of terminal wealth W_T. To determine the properties of equilibrium variance risk premia, we proceed by solving the FI's portfolio problem for the optimal variance swap positions and requiring that in equilibrium the FI's demand equals investors' supply.

Assume for simplicity there exist a single return factor F_t and a single common id-

[26]Using accounts from a sample of retail investors at a discount brokerage house, Lakonishok et al. (2007) document that a large fraction of call writing is part of covered-call strategies. As described in CBOE (2001), SEC regulations and no-action letters provide that a mutual fund seeking to take a short option position must either (1) hold the underlying security or an offsetting option position, i.e., "cover" the option position, or (2) set aside in a segregated, custodial account consisting of cash, U.S. government securities, or high-grade debt securities in an amount at least equal in value to the optioned securities, i.e., "segregation of assets".

[27]Both CBOE and ISE have amended rules to allow employee stock options to be used as collateral for writing exchange-traded options (SEC, 2009a and 2009b). A number of intermediaries provide covered call writing and escrow services to corporate managers. While employment contracts often prohibit such trading activity, these provisions are subject to enforcement issues. The Securities Exchange Act of 1934 and SEC regulations do not prohibit managers from hedging their employee stock and stock options, so long as the delta of their overall position—including employee stock, employee stock options, exchange-traded stock, and exchange-traded options—remains positive. For details, see the Securities and Exchange Commission in its opinion letter "Response of the Office of Chief Counsel, Division of Corporation Finance, Re: Credit Suisse First Boston ("CSFB") Incoming letter dated March 16, 2004."

iosyncratic variance factor Γ_t. Stock return variances have the factor structure $\sigma^2_{n,t} = \beta^2_{n,t}\Sigma_t + \gamma_{n,t}\Gamma_t + \tilde{V}^{\epsilon}_{n,t}$, $n = 1, ..., N$. Collect terms in $\mathbf{b}_t = (\beta^2_{1,t}, \ldots, \beta^2_{N,t})'$, $\gamma_t = (\gamma_{1,t}, \ldots, \gamma_{N,t})'$. Letting $A_t = \frac{-J_{WW,t}W_t}{J_{W,t}}$ denote the financial intermediary's wealth risk aversion, the first-order conditions derived from the financial intermediary's HJB equation yield the following equilibrium condition for variance risk premia $\mathbf{VRP}_t = (VRP_{1,t}, \ldots, VRP_{N,t})'$ (see Appendix D):

$$\mathbf{VRP}_t \quad = \quad (\mathbf{b}_t + \gamma_t \frac{\psi_{\Gamma\Sigma,t}}{\psi_{\Sigma,t}})\frac{VRP_{I,t}}{\beta^2_{I,t}} + A_t \left[\gamma_t \left(\mathbf{y}'_t\gamma_t\right) \psi^2_{\Gamma,t} + \text{diag}(\psi^{\epsilon}_t)^2 \mathbf{y}_t\right] \;, \tag{15}$$

where $VRP_{I,t}$ denotes the index variance risk premium given by expression (D.11), $\beta_{I,t}$ the index factor exposure, $(\psi_{\Sigma,t}, \psi_{\Gamma,t}, \psi^{\epsilon}_t)$ the standard deviations of factor variance Σ_t, common idiosyncratic variance Γ_t and, respectively, truly idiosyncratic variance $(\tilde{V}^{\epsilon}_{n,t})_{n=1,\ldots,N}$, and $\frac{\psi_{\Gamma\Sigma,t}}{\psi_{\Sigma,t}}$ the sensitivity of changes in common idiosyncratic variance Γ_t to changes in factor variance Σ_t.

Condition (15) yields that total variance risk premia are driven by a premium for exposure to factor variance (first term) plus a term reflecting the premium on idiosyncratic variance (second term). The second term equals the FI's risk aversion A_t scaled by two components: (1) exposure γ_t to *CIVR*, times the FI's total exposure to *CIVR*, $(\mathbf{y}'_t\gamma_t)$, and the riskiness of *CIVR*, $\psi^2_{\Gamma,t}$, and (2) the assets' contribution to the FI's bearing of diversifiable variance risk. The first component in brackets tends to dominate the second, since the total exposure to *CIVR* across all assets is generally much larger than that of any single asset n.

The equilibrium pricing condition (15) yields a number of testable predictions. An asset's idiosyncratic variance risk premium will be higher (1) the larger the net supply of variance for that asset, y_n, and (2) the higher the riskiness of the asset's truly idiosyncratic variance, $(\psi^{\epsilon}_{n,t})^2$. Further, (3) *CIVR* is priced in the cross-section, and the price of risk for common idiosyncratic variance is larger at times (4) when the total net supply of stock options/variance swaps $\mathbf{y}'\gamma_t$ is larger, and (5) when the riskiness of common idiosyncratic variance $\psi^2_{\Gamma,t}$ is larger. We now empirically assess predictions (1)-(3) in the cross section and predictions (4)-(5) in the time series.

5.3. Empirical evidence for the model predictions

In order to test the cross-sectional predictions (1)-(3), we estimate the equilibrium pricing condition (15) by means of Fama-MacBeth cross-sectional regressions. The dependent variables are the stocks' total or, alternatively, idiosyncratic variance swap returns constructed from eqs. (12) and (13). The explanatory variables are: exposure to the market, value, size, and momentum factors; exposure to the common idiosyncratic variance risk factor (the factor is constructed as the cross-sectional average of individual stocks' idiosyncratic variance swap returns); proxies for supply by investment funds and holders of firm-issued options (plus indicator variables for when the variables cannot be constructed); and the riskiness of each stock's "truly idiosyncratic" variance (computed as the time-series variance of the residuals from the first-pass regression of each stock's idiosyncratic variance swap return on the four Fama-French factors plus the common idiosyncratic variance factor). The proxy for option writing by investment funds to enhance yields and attract fund flows is the ratio of the number of shares held by mutual funds at the end of each quarter (obtained from the ThomsonReuters mutual fund holdings database) divided by the number of shares outstanding. As proxy for supply by holders of firm-issued options—such as employee stock options, we use the ratio of the number of firm-issued options outstanding at the end of each year (Compustat variable *optosey*) divided by the number of shares outstanding.

Table 8 reports the estimation results. The left (right) two columns investigate total (idiosyncratic) variance risk premia. Consistent with the model's predictions, idiosyncratic variance risk premia are larger for stocks that have larger exposure to movements in common idiosyncratic variance, larger option compensation, larger mutual fund holdings, and more variable "truly idiosyncratic" variance. All coefficients are statistically significant. These effects are in part also apparent in total variance risk premia, which are significantly positively related to stocks' exposure to shifts in common idiosyncratic variance and to option compensation.

[**Table 8 about here**]

The time-series predictions of the model are that the risk premium on common idiosyncratic variance should be larger in periods when the riskiness of common idiosyncratic variance is larger and when the aggregate net supply of stock options is larger.[28] To test whether there exists a positive relationship between the riskiness of common idiosyncratic variance and the risk premium on common idiosyncratic variance, we estimate a GARCH-in-mean model on the cross-sectional average idiosyncratic variance swap return. The estimation results are presented in Table 9. As predicted by the model, the in-mean effect in the variance risk premium is positive and highly significant. This holds true for both the total variance risk premium (left) and the idiosyncratic variance risk premium (right)—the coefficients on conditional variance are 1.53 (t-stat=9.01) and 10.09 (t-stat=5.87), respectively. This suggests that variance risk premia are higher at times when there is more variance risk in the financial system.

[**Table 9 about here**]

In summary, the estimates reveal that the positive sign of the idiosyncratic variance risk premium is consistent with supply pressure in stock options from investors. Financial intermediaries need to be compensated for absorbing this supply. Consistent with this hypothesis, we find that idiosyncratic variance risk premia are higher, the greater the number

[28]Data on the number of firm-issued options outstanding and on mutual fund holdings are only available on a yearly and quarterly basis, respectively, and the aggregate series are highly persistent. This persistence, together with entry and exit into option market making, renders the relationship between aggregate supply and variance risk premia predicted by the model more complex in real data.

of firm-issued options outstanding and the larger mutual fund ownership. The idiosyncratic variance risk premium is larger at times when there is more variance risk, the larger the exposure of the underlying stock to shifts in common idiosyncratic variance, and the greater the riskiness of a stock's "truly idiosyncratic" variance.

6. Alternative Explanations and Robustness Checks

6.1. Systematic and idiosyncratic variance as ICAPM state variables

An alternative explanation for the results in Section 4 is that systematic and idiosyncratic variances are state variables whose innovations contain information about the future state of the economy and/or changes in investment opportunities. Campbell (1993) shows that conditioning variables that forecast the return or variance on the market portfolio will be priced. Conditioning variables for which positive shocks are associated with good (bad) news about future investment opportunities have a positive (negative) risk price so long as the coefficient of relative risk aversion exceeds unity. Hence, the signs of the risk premia suggest that increases in systematic variance are bad news and increases in idiosyncratic variance are good news.[29] The question is whether we can find direct support for the predictive power of systematic and idiosyncratic variances?

Table 10 investigates the statistical relationship between market returns, market variance, idiosyncratic variance, and a number of macroeconomic variables. The dependent variables used in the various specifications (across columns) are the quarterly market excess return $r_{M,t} - r_{f,t}$, market variance $\sigma^2_{M,t}$ (used as proxy for systematic variance), total variance (computed as the cross-sectional average of stocks' total variance), common idiosyncratic

[29] Campbell et al. (2001) provide evidence that idiosyncratic variances are countercyclical, positively related to market variance, and negatively predict GDP growth, suggesting that idiosyncratic variance should command a negative price of risk in equilibrium.

variance $\overline{\sigma}^2_{\epsilon,t}$ (computed as the cross-sectional average of idiosyncratic variance), quarterly GDP growth, investment growth, consumption growth, the 3-month T-bill rate, the term spread (computed as the difference between the yield on 10-year Treasury bonds and the 3-month T-bill rate), and the default spread (computed as the difference between the yield on BAA and AAA corporate bonds). Gross domestic product, investment, and consumption data are taken from the Federal Reserve Bank of St. Louis' FRED system, and interest rate data are from the Federal Reserve Statistical Release. We conduct the regressions using both levels (Panels A and C) and innovations (Panels B and D) of the dependent and independent variables. For each series, innovations are computed as the residuals from an AR specification with the number of lags selected optimally using Schwarz' Bayesian Information Criterion (BIC). The number of lags used when computing the innovations in each series is reported in the row labeled "AR Lags (BIC)" in Panels B and D. Panels A and B report estimates from contemporaneous regressions, and Panels C and D the results of predictive regressions for quarterly horizons.

[**Table 10 about here**]

The contemporaneous regressions reported in Panel A (on levels) and B (on innovations) confirm the strong leverage effect (negative contemporaneous relation between market return and market variance in columns 2-3) and the positive comovement in market and idiosyncratic variances documented in prior studies (columns 3 and 5). There is no statistical link between market returns and idiosyncratic variance after controlling for the correlation between market return and variance (columns 2 and 5). Now consider the macroeconomic variables. Market variance is negatively related to GDP growth (column 6) and to investment growth (column 7). There is also a negative relationship between market variance and consumption growth and the T-bill rate (columns 8 and 9), and a positive relationship with the term spread and the default spread (columns 10 and 11). The reverse contemporaneous relation holds between idiosyncratic variance and the macroeconomic indicators. In particular, idiosyncratic variance is positively correlated with GDP growth (t-stat = 1.35), investment growth (t-stat = 0.58), consumption growth (t-stat = 4.62), T-bill (t-stat = 3.46), and negatively correlated with term spread (t-stat = -1.13) and default spread (t-stat = -3.87). Table 10, Panel B reveals a similar picture for innovations with a few exceptions.[30]

The predictive regressions over a one-quarter horizon reported in Panels C and D reveal a different pattern.[31] Market variance does not significantly predict market returns, but predicts itself and negatively forecasts idiosyncratic variance, GDP growth, and investment growth. We find little forecasting power for future consumption or interest rate variables. By contrast, idiosyncratic variance is also positively autocorrelated, but does not significantly predict any of the macroeconomic indicators or interest rate variables. When considering innovations (Panel D), market variance negatively predicts GDP and investment growth. Idiosyncratic variance, again, does not have any predictive power. Summarizing, the results in Table 10 show that increases in market variance are indeed bad news, but we find no evidence that idiosyncratic variance predicts the future state of the economy.[32]

In summary, we find evidence of contemporaneous correlation between systematic and idiosyncratic variances and several macroeconomic indicators, and the sign of the correlations differ between systematic and idiosyncratic variances. However, in contrast to Campbell et al. (2001), we find no significant predictive power of idiosyncratic variances for indicators of

[30] The estimates in Panel B differ from Panel A in that the relationships between market variance and GDP growth, investment growth and, respectively, the term spread become statistically insignificant. In addition, there is a negative relationship between idiosyncratic variance innovations and the term spread.

[31] In Panel C, in order to account for the autocorrelation of the dependent variable, we include as regressor the lagged dependent variable up to a number of lags selected optimally using Schwarz' Bayesian Information Criterion (BIC). The number of lags used is reported in the row labeled "AR Lags (BIC)" in Panel C.

[32] For robustness, we have also estimated a vector autoregression as suggested in Campbell (1993). We include in the system the monthly market excess return, $r_{M,t} - r_{f,t}$, market variance $\sigma^2_{M,t}$ (as proxy for systematic variance), and common idiosyncratic variance $\overline{\sigma}^2_{\epsilon,t}$ (computed as the cross-sectional average of idiosyncratic variance). Consistent with Table 10, the coefficients (available from the authors) indicate that neither systematic variance nor idiosyncratic variance are related to future market returns. Market variance is positively related to future market variance, but idiosyncratic variance is not. Thus, increases in systematic variance are bad news about future investment opportunities, while increases in idiosyncratic variance seem irrelevant for the investment opportunity set.

macroeconomic conditions. Thus, the results in Table 10 confirm that intertemporal hedging demands by investors can account for the negative risk premium on systematic variance, but it seems unlikely that they generate the observed positive price of idiosyncratic variance risk.

6.2. The effect of option illiquidity and hedging costs on variance risk premia

An other alternative is that the positive idiosyncratic variance risk premium reflects compensation for the illiquidity of individual stock options or the difficulty in delta-hedging them. Market makers have been shown to be net long individual stock options (Garleanu, Pedersen, and Poteshman (2009)) and may therefore be willing to pay less for options that are more difficult to (delta-)hedge. In this case, we would expect a positive relationship between the variance risk premium and measures of market imperfections.

In Table 11, we conduct Fama-MacBeth regressions of variance risk premia on various characteristics and proxies for market frictions to investigate if frictional costs explain our findings. We include the quoted bid-ask spread on the underlying (in percent), the quoted bid-ask spread on options (average of the bid-ask spreads on at-the-money or, alternatively, out-of-the-money put and call options in percent), trading volume in the underlying and the options, and option open interest as explanatory variables in our regression specification. Additional control variables are also included in the three specifications.

[**Table 11 about here**]

The results reveal that variance risk premia are largely unrelated to bid-ask spreads on the underlying stock and on options. This result holds both for the total variance risk premium (left columns) and the idiosyncratic variance risk premium (right columns). Stock volume is positively related to the variance risk premium, while option volume has a hump-shaped impact. In summary, there is little evidence that the positive sign of the idiosyncratic variance risk premium represents compensation for the illiquidity of stock options or the costs involved in hedging them.

6.3. *Transaction costs, discrete strikes, stock price jumps*

The remainder of this section discusses the robustness of our findings to limitations of the data.

Transaction costs: Transaction costs may limit market participants' ability to arbitrage any variance swap/option mispricing and to take exposures to different components of variance risk. The bid and ask quotes available in OptionMetrics are only indicative end-of-day quotes, hence of limited informativeness about actual transaction costs. While we have no information on spreads on OTC single-stock variance swap rates, bid-ask spreads on OTC index variance swap rate quotes from major broker dealers are on average around 50-100 basis points (Egloff, Leippold, and Wu (2009)) and, hence, an order of magnitude lower than the returns documented in this paper.

Discrete option strike prices and jumps in underlying prices: The replication of variance swaps using a portfolio of out-of-the-money options and delta-hedging in the underlying is exact only if options with an unlimited number of strikes are available and underlying asset prices do not jump. Nonetheless, Jiang and Tian (2005) show that variance swap rates can be computed accurately from option prices even if the underlying price process jumps and a limited number of strikes are available. Carr and Wu (2009) show that the approximation error introduced by jumps is of third order and find that the combined error introduced by jumps and the availability of a limited number of strikes is small. Broadie and Jain (2007) show that under realistic parameterizations the jump-induced error in variance swap rates computed using the replicating portfolio is less than 2%. Dividends or stochastic interest rates are another source of approximation error. Though, Torné (2010) shows that the absolute error from these issues is less than 1%.

7. Conclusion

We decompose the variance risk of all S&P 100 and Nasdaq 100 stocks into the components due to systematic and idiosyncratic stock returns. We find that both systematic and idiosyncratic variance risk command sizeable risk premia, but the prices of risk have opposite signs. While systematic variance risk (SVR) exhibits a negative price of risk, common idiosyncratic variance risk ($CIVR$)—the risk of common movements in the variances of idiosyncratic returns—has a large positive price of risk. Both types of variance risk are shown to be heavily priced risk factors in the cross-section of equity option returns and not accounted for by standard risk factor models.

These findings help explain several phenomena, including the relative expensiveness of index options and inexpensiveness of individual options, the sizeable cross-sectional variation in risk premia on individual stock variances, the volatility mispricing puzzle documented by Goyal and Saretto (2009), and the substantial returns earned on various option portfolio strategies. Dispersion trading strategies—common among hedge funds—earn a combination of the negative risk premium on systematic variance and the positive risk premium on idiosyncratic variance, since correlation risk is a composite of SVR and $CIVR$.

A simple model of financial intermediation under capital constraints can account for the observed risk premia. The model can simultaneously account for the negative market price of SVR and the positive market price of $CIVR$. The model yields predictions that find support in the data. By contrast, we find limited support for macroeconomic and liquidity-based explanations of the observed patterns. Overall, these results help improve our understanding of the sources of risk in the options market which is essential for managing investment risks, educating investors, and making informed policy choices.

Appendix

A. Stochastic Factor Exposures and Parameter Uncertainty

This appendix shows that our variance swap extraction methodology holds with minor modifications when factor exposures are time-varying and there is parameter uncertainty. Assume that under Q, (1) changes in factor exposures are uncorrelated with factor variances and covariances; (2) changes in factor exposures are uncorrelated across assets and factors; and (3) factor exposures follow a random walk without drift. With time-varying factor exposures, the variance swap rate on asset n and, respectively, on index p can be written:

$$VS_{n,t} = \frac{1}{\tau}E_t^Q[\int_t^{t+\tau}(\beta'_{n,u}\Sigma_u\beta_{n,u} + V^{\epsilon}_{n,u})du], \quad \text{(A.1)}$$

$$VS_{I,p,t} = \frac{1}{\tau}E_t^Q[\int_t^{t+\tau}((\sum_{n=1}^{N} w_{n,p,t}\beta'_{n,u})\Sigma_u(\sum_{n=1}^{N} w_{n,p,t}\beta_{n,u}) + \sum_{n=1}^{N} w^2_{n,p,t}V^{\epsilon}_{n,u})du] \ . \quad \text{(A.2)}$$

Index variance swap rate that are neutral to idiosyncratic variances can be constructed as:

$$\begin{aligned} y_{p,t} &\equiv VS_{I,p,t} - \sum_{n=1}^{N} w^2_{n,p,t}VS_{n,t} \\ &= \frac{1}{\tau}E_t^Q[\int_t^{t+\tau}((\sum_{n=1}^{N} w_{n,p,t}\beta_{n,u})'\Sigma_u(\sum_{n=1}^{N} w_{n,p,t}\beta_{n,u}) - \sum_{n=1}^{N} w^2_{n,p,t}\beta'_{n,u}\Sigma_u\beta_{n,u})du] \ . \quad \text{(A.3)} \end{aligned}$$

Using i to index the factors and using assumptions 1 to 3, one obtains:

$$\begin{aligned} y_{p,t} &= \frac{1}{\tau}\int_t^{t+\tau}(\sum_j E_t^Q[(\sum_{n=1}^{N} w_{n,p,t}\beta_{n,u}(i))^2 - \sum_{n=1}^{N} w^2_{n,p,t}\beta_{n,u}(j)^2]E_t^Q[\Sigma_u^{jj}])du \\ &+\frac{2}{\tau}\int_t^{t+\tau}(\sum_i\sum_{j\neq i} E_t^Q[(\sum_{n=1}^{N} w_{n,p,t}\beta_{n,u}(i))(\sum_{n=1}^{N} w_{n,p,t}\beta_{n,u}(j)) - \sum_{n=1}^{N} w^2_{n,p,t}\beta_{n,u}(i)\beta_{n,u}(j)]E_t^Q[\Sigma_u^{ij}])du \\ &= \frac{1}{\tau}E_t^Q[\int_t^{t+\tau}(\sum_j((\sum_{n=1}^{N} w_{n,p,t}\tilde{\beta}_{n,t}(j))^2 - \sum_{n=1}^{N} w^2_{n,p,t}\tilde{\beta}_{n,t}(j)^2)\Sigma_u^{jj})du] \\ &+\frac{2}{\tau}E_t^Q[\int_t^{t+\tau}(\sum_i\sum_{j\neq i}((\sum_{n=1}^{N} w_{n,p,t}\tilde{\beta}_{n,t}(i))(\sum_{n=1}^{N} w_{n,p,t}\tilde{\beta}_{n,t}(j)) - \sum_{n=1}^{N} w^2_{n,p,t}\tilde{\beta}_{n,t}(i)\tilde{\beta}_{n,t}(j))\Sigma_u^{ij})du] \\ &= \tilde{\beta}'_{I,p,t}VS_t\tilde{\beta}_{I,p,t} - \sum_{n=1}^{N}\tilde{\beta}'_{n,t}VS_t\tilde{\beta}_{n,t}, \quad \text{(A.4)} \end{aligned}$$

where $\tilde{\beta}_{n,t}$, $n = 1, \ldots, N$ denote the conditional expectation of the factor exposures. Thus, when the factor exposures are time-varying and there is (potentially) parameter uncertainty, the relationship between adjusted index variance swap rates and factor variance swap rates (IA.C.2) will still hold;

it suffices to compute $A_{p,j,t}$ and $B_{p,i,j,t}$ in (IA.C.3) using estimated factor exposures $\tilde{\beta}_{n,t}$.

Idiosyncratic variance swap rates then equal

$$VS^{\epsilon}_{n,t} = VS_{n,t} - \frac{1}{\tau}E^Q_t[\int_t^{t+\tau} \beta'_{n,u}\Sigma_u\beta_{n,u}du] \quad \text{(A.5)}$$

$$= VS_{n,t} - \frac{1}{\tau}E^Q_t[\int_t^{t+\tau} (\sum_j \beta_{n,u}(j)^2\Sigma^{jj}_u + 2\sum_i\sum_{j\neq i}\beta_{n,u}(i)\beta_{n,u}(j)\Sigma^{ij}_u)du]$$

$$= VS_{n,t} - \frac{1}{\tau}\int_t^{t+\tau} (\sum_j(\tilde{\beta}_{n,t}(j)^2 + \mathrm{var}_t(\beta_{n,u}(j)))E^Q_t[\Sigma^{jj}_u] + 2\sum_i\sum_{j\neq i}\tilde{\beta}_{n,t}(i)\tilde{\beta}_{n,t}(j)E^Q_t[\Sigma^{ij}_u])du$$

$$= VS_{n,t} - \tilde{\beta}'_{n,t}VS_t\tilde{\beta}_{n,t} - \sum_j\frac{1}{\tau}\int_t^{t+\tau} \mathrm{var}_t(\beta_{n,u}(j))E^Q_t[\Sigma^{jj}_u]du. \quad \text{(A.6)}$$

By the mean value theorem, there exists a value of u in $[t, t+\tau]$, denoted $\overline{u}$, such that

$$\int_t^{t+\tau} \mathrm{var}_t(\beta_{n,u}(j))E^Q_t[\Sigma^{jj}_u]du = \mathrm{var}_t(\beta_{n,\overline{u}}(j))\int_t^{t+\tau} E^Q_t[\Sigma^{jj}_u]du. \quad \text{(A.7)}$$

In our implementation, since $\mathrm{var}_t(\beta_{n,u}(j))$ is linear in u, we can approximate this expression using $\overline{u} = t + \tau/2$. One can therefore extract the idiosyncratic variance swap rate using

$$VS^{\epsilon}_{n,t} = VS_{n,t} - \tilde{\beta}'_{n,t}VS_t\tilde{\beta}_{n,t} - \sum_j \mathrm{var}_t(\beta_{n,t+\tau/2}(j))VS^{jj}_t\ . \quad \text{(A.8)}$$

B. Sample Selection, Index Constituents, and Index Weights

For both the S&P 100 index and the Nasdaq 100 index, we obtain historical index weights on each trading day in the sample period as follows. First, we compute the weight of each stock in each index on January 2, 1996, on each date in which constituent changes occurred, as well as on the regular quarterly index rebalance dates, which occur on the third Friday of March, June, September and December.[33] Starting from each of these rebalance or constituent change dates, we then compute the weights on the next trading day by multiplying them with one plus each stock's realized return and normalizing them such that they sum to unity. We do this until we reach the next rebalance or constituent change date.

For the S&P 100 index, we obtain the list of the constituents on January 2, 1996, the list of index constituent changes that took place during our sample period (there were 84 such changes), and the dates at which they occurred directly from Standard and Poor's. For the period from January 2, 1996 to December 31, 2000, historical weights for the S&P 100 index are not available at reasonable cost, so we compute the weights on each rebalance or constituent change date by normalizing the S&P 500 index weights, which are available from Bloomberg. This approach is

[33]For the S&P 100 index, most index constituent changes do not occur on the regular quarterly rebalance dates, which deal primarily with adjustments to the number of shares included in the index to account for share repurchases, seasoned equity offerings, and similar corporate events. For the Nasdaq 100 index, a larger fraction of constituent changes takes place on the quarterly rebalance dates than for the S&P 100 index, but a sizeable fraction does not.

accurate because Standard and Poor's accounts for free float, dual classes of stock, etc. in the same way for both indices. For the period from January 2, 2001 to September 30, 2008, we obtain the exact index weights on each rebalance or constituent change date directly from Bloomberg. For the period from October 1, 2008 to October 31, 2009, we compute the weights on each rebalance or constituent change date based on stocks' market capitalization.

For the Nasdaq 100 index, we obtain the list of constituent changes during our sample period (there were 227 such changes) and their dates from the Nasdaq website.[34] For the period from January 2, 1996 to December 31, 2000, exact index weights are not available at reasonable cost, so we estimate them based on the market capitalization of the constituent stocks. For the period from January 2, 2001 to October 31, 2009, we obtain the exact index weights directly from Bloomberg.

C. Variance Swap Computation

We compute synthetic variance swap rates from equity option prices using the methodology outlined in Demeterfi, Derman, Kamal, and Zou (1999) and Carr and Wu (2009). For both indices and the 452 stocks that were members of one of the two indices at some point during our sample period, we extract daily put and call option implied volatilities for a constant maturity of one month (30 calendar days) from the OptionMetrics database. The OptionMetrics volatility surface file provides option implied volatilities for deltas between 0.2 and 0.8 in absolute value in steps of 0.05. The data are adjusted for early exercise. On the basis of these implied volatilities, we compute variance swap rates using the methodology described by Carr and Wu (2009). We linearly interpolate the volatility surface between the points provided in the OptionMetrics database using log moneyness $k \equiv \ln(K/F)$, where K is the strike price and F the futures price, to obtain the Black-Scholes implied volatility for moneyness level k, $\sigma(k)$. We then use these implied volatilities to evaluate the cost of the replicating portfolio of a variance swap with maturity $t+\tau$ as (here we drop from the notation the dependence on n or p):[35]

$$VS_t = \frac{2}{\tau}[\int_{-\infty}^{0} \left(-e^{-k}N(-d_1(k)) + N(-d_2(k))\right) dk + \int_{0}^{\infty} \left(e^{-k}N(d_1(k)) - N(d_2(k))\right) dk] \ , \quad \text{(C.1)}$$

where $N(\cdot)$ denotes the standard normal cumulative distribution function and

$$d_1(k) = \frac{-k + \sigma^2(k)\tau/2}{\sigma(k)\sqrt{\tau}} \ , \quad d_2(k) = d_1(k) - \sigma(k)\sqrt{\tau}. \quad \text{(C.2)}$$

D. An Equilibrium Model of Variance Risk Pricing

Consider a financial intermediary (FI) with capital W_t. Assume the FI deals in variance swaps—this assumption greatly simplifies the derivations. Alternatively, the FI could be buying or selling stock options and delta hedging. Consider for tractability the case with a single return factor and a single common idiosyncratic variance factor. Using the notation from Section 1, the instantaneous variance of returns on each stock $n = 1, \ldots, N$ is given by $\sigma_{n,t}^2 = \beta_{n,t}^2 \Sigma_t + \gamma_{n,t}\Gamma_t + \tilde{V}_{n,t}^{\epsilon}$, where Σ_t denotes the variance of the common return factor, Γ_t the common idiosyncratic variance factor,

[34] The data are available at http://www.nasdaq.com/indexshares/historical_data.stm.

[35] Eq. (C.1) is derived, for instance, in Carr and Wu (2004, p. 26).

and $\tilde{V}^{\epsilon}_{n,t}$ stock n's truly idiosyncratic return variance. Let $w_{n,t}$ denote stock n's weight in the index. The instantaneous variance of index returns is given by

$$\sigma^2_{I,t} = \beta^2_{I,t}\Sigma_t + \gamma_{I,t}\Gamma_t + \sum_{n=1}^{N} w^2_{n,t}\tilde{V}^{\epsilon}_{n,t} \ , \tag{D.1}$$

where $\beta_{I,t} = \sum_n w_{n,t}\beta_{n,t}$ denotes the index's exposure to the return factor, $\gamma_{I,t} = \sum_n w^2_{n,t}\gamma_{n,t}$ its exposure to common idiosyncratic variance shocks, and $\sum_{n=1}^{N} w^2_{n,t}\tilde{V}^{\epsilon}_{n,t}$ index variance resulting from stocks' truly idiosyncratic variances. The last two terms in (D.1) are small if the index is well-balanced.

Assume that the variance of the common return factor Σ_t, the common idiosyncratic variance factor Γ_t and the N truly idiosyncratic return variances $\tilde{V}^{\epsilon}_{n,t}$, $n = 1, \ldots, N$ follow diffusion processes

$$\begin{aligned} d\Sigma_t &= \mu_{\Sigma,t}dt + \sigma_{\Sigma,t}dB_{\Sigma,t} \ , \\ d\Gamma_t &= \mu_{\Gamma,t}dt + \sigma_{\Gamma,t}dB_{\Gamma,t} + \sigma_{\Gamma\Sigma,t}dB_{\Sigma,t} \ , \\ d\tilde{V}^{\epsilon}_{n,t} &= \mu_{n,t}dt + \sigma_{n,t}dB_{n,t} \ . \end{aligned} \tag{D.2}$$

Consistent with the empirical evidence in Campbell et al. (2001) and in Section 3.1, we allow factor variance Σ_t and common idiosyncratic variance Γ_t to be correlated. Consistent with the fact that $\tilde{V}^{\epsilon}_{n,t}$ are truly idiosyncratic variances, we assume that for all assets $n = 1, \ldots, N$, $dB_{n,t}$ is independent of all other sources of uncertainty.

The FI takes positions in the riskless asset, the variance swap on the stock market index, and the N individual stock variance swaps in order to maximize his expected utility of terminal wealth. The rate of return on the riskless asset is $r_{f,t}$. The dollar return on a notional investment of \$1 in the index variance swap and, respectively, individual asset variance swaps are

$$r_{I,t} = \beta^2_{I,t}r_{\Sigma,t} + \gamma_{I,t}r_{\Gamma,t} + \sum_{n=1}^{N} w^2_{n,t}r^{\epsilon}_{n,t} \ , \tag{D.3}$$

$$r_{n,t} = \beta^2_{n,t}r_{\Sigma,t} + \gamma_{n,t}r_{\Gamma,t} + r^{\epsilon}_{n,t} \ , \tag{D.4}$$

where $r_{\Sigma,t} = \phi_{\Sigma,t}dt + \psi_{\Sigma,t}dB_{\Sigma,t}$ denotes the return on the factor variance swap, $r_{\Gamma,t} = \phi_{\Gamma,t}dt + \psi_{\Gamma,t}dB_{\Gamma,t} + \psi_{\Gamma\Sigma,t}dB_{\Sigma,t}$ that on common idiosyncratic variance, and $r^{\epsilon}_{n,t} = \phi^{\epsilon}_{n,t}dt + \psi^{\epsilon}_{n,t}dB_{n,t}$ that on truly idiosyncratic variance.

To simplify matters, assume that at any time t, the FI settles any open variance swap position from the previous instant (causing him to realize a gain or loss) and initiates new variance swap positions which cost zero to enter (see Egloff, Leippold and Wu (2009)). Thus, the FI has a 100% cash position. Letting x denote the notional investment in the index variance swap and $\mathbf{y}$ the notional investments in the individual asset variance swaps normalized by total wealth, wealth dynamics are given by

$$\begin{aligned} \frac{dW_t}{W_t} &= r_{f,t}dt + x[\beta^2_{I,t}(\phi_{\Sigma,t}dt + \psi_{\Sigma,t}dB_{\Sigma,t}) + \gamma_{I,t}(\phi_{\Gamma,t}dt + \psi_{\Gamma,t}dB_{\Gamma,t} + \psi_{\Gamma\Sigma,t}dB_{\Sigma,t}) + \mathbf{z}'_t(\phi^{\epsilon}_t dt + \mathrm{diag}(\psi^{\epsilon}_t)d\mathbf{B}_t)] \\ &+ \mathbf{y}'[\mathbf{b}_t(\phi_{\Sigma,t}dt + \psi_{\Sigma,t}dB_{\Sigma,t}) + \gamma_t(\phi_{\Gamma,t}dt + \psi_{\Gamma,t}dB_{\Gamma,t} + \psi_{\Gamma\Sigma,t}dB_{\Sigma,t}) + (\phi^{\epsilon}_t dt + \mathrm{diag}(\psi^{\epsilon}_t)d\mathbf{B}_t)], \end{aligned} \tag{D.5}$$

where $\mathbf{b}_t \equiv (\beta_{1,t}^2, \ldots, \beta_{N,t}^2)'$, $\gamma_t \equiv (\gamma_{1,t}, \ldots, \gamma_{N,t})'$, $\phi_t^\epsilon \equiv (\phi_{1,t}^\epsilon, \ldots, \phi_{N,t}^\epsilon)'$, $\psi_t^\epsilon \equiv (\psi_{1,t}^\epsilon, \ldots, \psi_{N,t}^\epsilon)'$ and $\mathbf{z}_t \equiv (w_{1,t}^2, \ldots, w_{N,t}^2)'$. Letting $J(W,t)$ denote the indirect utility of wealth, $\Psi_{I,t} = \beta_{I,t}^2 \psi_{\Sigma,t} + \gamma_{I,t}\psi_{\Gamma\Sigma,t}$, and $\mathbf{\Psi_t} = \mathbf{b}_t\psi_{\Sigma,t} + \gamma_t\psi_{\Gamma\Sigma,t}$, the Bellman equation is

$$
\begin{aligned}
0 = \max_{x,\mathbf{y}} J_t + J_W W_t [r_{f,t} + x\left(\beta_{I,t}^2\phi_{\Sigma,t} + \gamma_{I,t}\phi_{\Gamma,t} + \mathbf{z}_t'\phi_{\epsilon,t}\right) + \mathbf{y}'\left(\mathbf{b}_t\phi_{\Sigma,t} + \gamma_t\phi_{\Gamma,t} + \phi_{\epsilon,t}\right)] \\
+ \frac{1}{2} J_{WW} W_t^2 [(x\Psi_{I,t} + \mathbf{y}'\mathbf{\Psi_t})^2 + (x\gamma_{I,t} + \mathbf{y}'\gamma_t)^2\psi_{\Gamma,t}^2 + \left(\mathbf{z}_t' + \mathbf{y}'\right)\operatorname{diag}(\psi_t^\epsilon)^2\left(\mathbf{z}_t + \mathbf{y}\right)]. \quad \text{(D.6)}
\end{aligned}
$$

The first-order optimality conditions for x and $\mathbf{y}$ are

$$
0 = J_W W_t\left(\beta_{I,t}^2\phi_{\Sigma,t} + \gamma_{I,t}\phi_{\Gamma,t} + \mathbf{z}_t'\phi_{\epsilon,t}\right) + J_{WW}W_t^2[\Psi_{I,t}\left(x\Psi_{I,t} + \mathbf{y}'\mathbf{\Psi_t}\right) + \gamma_{I,t}\left(x\gamma_{I,t} + \mathbf{y}'\gamma_t\right)\psi_{\Gamma,t}^2] \quad \text{(D.7)}
$$

and

$$
\begin{aligned}
0 = J_W W_t\left(\mathbf{b}_t\phi_{\Sigma,t} + \gamma_t\phi_{\Gamma,t} + \phi_t^\epsilon\right) \\
+ J_{WW}W_t^2[\mathbf{\Psi_t}\left(x\Psi_{I,t} + \mathbf{y}'\mathbf{\Psi_t}\right) + \gamma_t\left(x\gamma_{I,t} + \mathbf{y}'\gamma_t\right)\psi_{\Gamma,t}^2 + \operatorname{diag}(\psi_t^\epsilon)^2\left(\mathbf{z}_t + \mathbf{y}\right)]. \quad \text{(D.8)}
\end{aligned}
$$

In equilibrium, FI's net demand equals investors' net supply. That is, given net demand for index variance swaps $-x$ and net demand for individual stock variance swaps $-\mathbf{y}$ by investors, equilibrium variance risk premia on the index and the individual assets are given by

$$
\begin{aligned}
VRP_{I,t} &= \beta_{I,t}^2\phi_{\Sigma,t} + \gamma_{I,t}\phi_{\Gamma,t} + \mathbf{z}_t'\phi_t^\epsilon \\
&= A_t[\Psi_{I,t}\left(x\Psi_{I,t} + \mathbf{y}'\mathbf{\Psi_t}\right) + \gamma_{I,t}\left(x\gamma_{I,t} + \mathbf{y}'\gamma_t\right)\psi_{\Gamma,t}^2] \quad \text{(D.9)}
\end{aligned}
$$

and

$$
\begin{aligned}
\mathbf{VRP}_t &= \mathbf{b}_t\phi_{\Sigma,t} + \gamma_t\phi_{\Gamma,t} + \phi_t^\epsilon \\
&= A_t[\mathbf{\Psi_t}\left(x\Psi_{I,t} + \mathbf{y}'\mathbf{\Psi_t}\right) + \gamma_t\left(x\gamma_{I,t} + \mathbf{y}'\gamma_t\right)\psi_{\Gamma,t}^2 + \operatorname{diag}(\psi_t^\epsilon)^2\left(\mathbf{z}_t + \mathbf{y}\right)], \quad \text{(D.10)}
\end{aligned}
$$

where $A_t = \frac{-J_{WW,t}W_t}{J_{W,t}}$ denotes the FI's wealth risk aversion. For ease of interpretation, consider the case $\gamma_{I,t} = 0$ and $\mathbf{z}_t = \mathbf{0}$ (these two components will be very small in practice). Then, the variance risk premia are given by

$$
\begin{aligned}
VRP_{I,t} &= A_t[x\beta_{I,t}^2 + \mathbf{y}'(\mathbf{b}_t + \gamma_t\frac{\psi_{\Gamma\Sigma,t}}{\psi_{\Sigma,t}})]\beta_{I,t}^2\psi_{\Sigma,t}^2, \quad \text{(D.11)} \\
\mathbf{VRP}_t &= (\mathbf{b}_t + \gamma_t\frac{\psi_{\Gamma\Sigma,t}}{\psi_{\Sigma,t}})\frac{VRP_{I,t}}{\beta_{I,t}^2} + A_t[\gamma_t\left(\mathbf{y}'\gamma_t\right)\psi_{\Gamma,t}^2 + \operatorname{diag}(\psi_t^\epsilon)^2\mathbf{y}]. \quad \text{(D.12)}
\end{aligned}
$$

The first expression yields that the index variance risk premium equals the product of the FI's risk aversion, his net exposure to factor variance (accounting for both his index and his single-stock variance position) $x\beta_{I,t}^2 + \mathbf{y}'(\mathbf{b}_t + \gamma_t\frac{\psi_{\Gamma\Sigma,t}}{\psi_{\Sigma,t}})$, the index's exposure to factor variance $\beta_{I,t}^2$, and the riskiness of factor variance $\psi_{\Sigma,t}^2$. The second expression yields that variance risk premia on individual stocks are driven by their exposure to factor variance, plus the FI's risk aversion A_t multiplied with the sum of two components: (1) the assets' exposure to common idiosyncratic variance γ_t, times the FI's total exposure to common idiosyncratic variance $\mathbf{y}'\gamma_t$, times the riskiness of common idiosyncratic variance, $\psi^2_{\Gamma,t}$, and (2) the assets' contribution to the FI's bearing of diversifiable variance risk, which is simply the product of the riskiness of truly idiosyncratic variance for each asset, $(\psi^{\epsilon}_{n,t})^2$, and the net supply of variance for that asset, y_n.

Internet Appendix

IA.A Approximation Accuracy in Expression (11)

In order to assess the accuracy of the approximation in expression (11), we compare the variance swap rates obtained from (11) with those obtained by simulating the system and thereby accounting for the random variation in weights. We do this in a parametric setting with a single common factor. The variance of the factor is assumed to follow a CIR process:

$$dV_t = \kappa^Q(\overline{v}^Q - V_t)dt + \sigma\sqrt{V_t}dZ^Q_{V,t} \,. \tag{IA.A.1}$$

The correlation between the factor and variance innovations is denoted by ρ.

We take the parameter values of the factor's variance process to be those estimated by Aït-Sahalia and Kimmel (2007) for the S&P 500 index using daily data for the period from January 2, 1990 until September 30, 2003, namely $\kappa = 5.07$, $\overline{v} = 0.0457$, $\sigma = 0.48$ and $\rho = -0.767$ (see Table 6, column (2) of their paper). We assume that the variances of the idiosyncratic noise terms follow independent square root processes with the same parameters as the factor variance. We assume that the index comprises 100 securities with identical initial weights and spread the exposure of the individual assets to the common factor uniformly around 1 using values from 0.505 to 1.495. This allows for heterogeneity in factor exposures, while guaranteeing an average exposure of 1. We consider a variance swap with a maturity of three months (the approximation is more accurate than reported below for shorter maturities), and compute the simulation-based variance swap rate using 20,000 simulation runs and a 1-day discretization interval.

The upper panel of Figure IA.1 shows the variance swap rates obtained from the simulation and using the approximation (11) for initial variances between 10% and 200% of the long-term mean estimated by Aït-Sahalia and Kimmel (2007) (we use the same initial variances for the factor and all assets' idiosyncratic noises). The lower panel reports the relative approximation errors. Observe that the approximation is almost indistinguishable from the variance swap rate obtained from the simulation. The approximation has a slight downward bias for very low initial variances, and a slight upward bias for very large initial variances. However, even in the worst cases, the approximation error is of the order of 1%. We conclude that the approximation (11) is very accurate.

[**Figure IA.1 about here**]

IA.B Specification Analysis

In this appendix, we ascertain whether a factor model with a small number of return factors is sufficiently accurate for our analysis. For our return model to be well specified, three requirements need to be met: First, the latent return factors need to reproduce the time series of index returns; second, the variance of these factors needs to capture realized index variances and their movement through time; finally, the factors need to account for correlations in individual asset returns and their movement through time. We find that a factor model with two common return factors ($J = 2$) meets all three requirements.

Time series of index vs. factor returns: In order to test whether the factors reproduce the time series of index returns, we first extract the realizations of the common return factors from

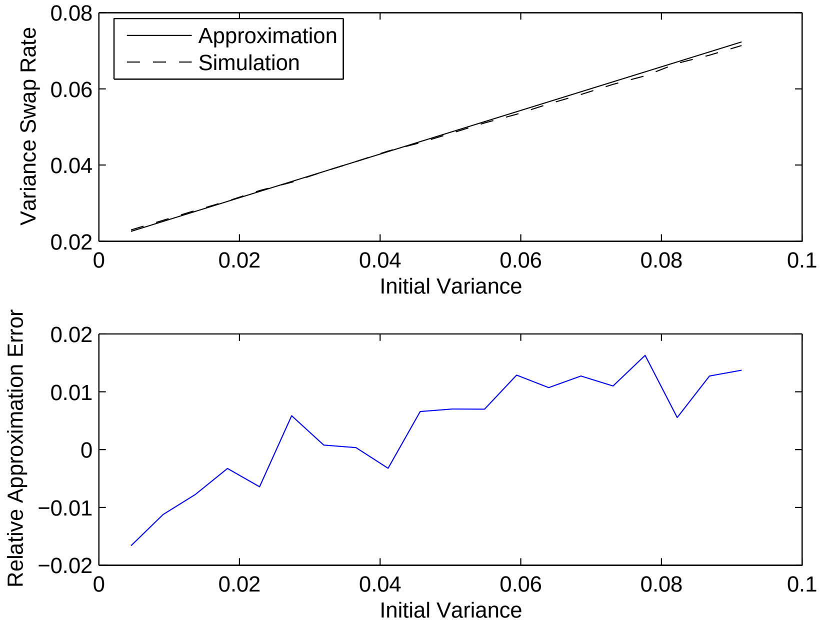

Figure IA.1
Accuracy of the index variance swap rate approximation (11)

The figure compares the index variance swap rates obtained from the approximation (11) with those obtained by simulating the system and thereby accounting for the random variation in index weights. The variances of the return factor and each stock's idiosyncratic return are assumed to follow CIR processes with the parameters estimated by Aït-Sahalia and Kimmel (2007).

our panel of asset returns using a standard factor analysis allowing for one, two, and three common factors, and then regress the returns of the two indices on these common factors.[36] Table 2, Panel A reports the coefficients of determination from these regressions. We conclude that a model with two common return factors is appropriate for our analysis.

Time series of index vs. factor variances: We next verify that the variances of the common return factors can account for index variances and their movement through time. When the number of common return factors is too small, residual returns will be correlated across assets. Denote by $\rho^{\epsilon}_{m,n,t}$ the correlation in residual returns between assets m and n. The variance of index returns

[36]Sentana and Fiorentini (2001) and Sentana (2004) show that standard factor analysis can be used even in the presence of stochastic volatility. Specifically, Sentana (2004) shows that if the factor loadings are constant over time and the unconditional variances of common and idiosyncratic factors are constant, then the unconditional covariance matrix of return innovations will inherit the factor structure.

$dI_{p,t}/I_{p,t}$ is given by

$$\sigma^2_{I,p,t} = \beta'_{I,p,t}\Sigma_t\beta_{I,p,t} + \sum_{n=1}^{N}\sum_{m\neq n}^{N} w_{m,p,t}w_{n,p,t}\rho^{\epsilon}_{m,n,t}\sqrt{V^{\epsilon}_{m,t}V^{\epsilon}_{n,t}} + \sum_{n=1}^{N} w^2_{n,p,t}V^{\epsilon}_{n,t} \ . \qquad \text{(IA.B.1)}$$

Thus, index variance consists of three components: (i) a component reflecting the variances and covariances of the common return factors Σ_t and assets' average factor exposures $\beta_{I,p,t}$, (ii) the weighted-average covariance between the return residuals, and (iii) the sum of the assets' idiosyncratic variances multiplied with the square of their weight in the index.[37]

A simple specification test is whether any significant covariances in residual returns manifest themselves in index variance. In order to assess the relative magnitude of the three variance components, we compute each using historical asset returns and the factor realizations obtained from the factor analysis of stock returns. For each date we compute the variances and covariances of the common factors Σ_t, the residual covariances $\sigma^{\epsilon}_{m,n,t} = \rho^{\epsilon}_{m,n,t}\sqrt{V^{\epsilon}_{m,t}V^{\epsilon}_{n,t}}$, and the residual variances $V^{\epsilon}_{n,t}$ using trailing one-month (21 trading day) windows. For each stock index, we apply the index weights on the last day of the estimation window in order to obtain the three components in (IA.B.1) for that index.

[**Table IA.I about here**]

Table IA.I provides summary statistics, and Panel A of Figure 1 plots the three components of index variance as well as the total index variance for both indices over our sample period. The conclusion emerging from these results is striking: the variances of the two common return factors account for the overwhelming part of the level and the variability of index variances. As can be seen in Figure 1, Panel A, the contribution of both the average covariance between the return residuals and the idiosyncratic return variances to index variances are extremely small. Even at their peak during the burst of the Internet bubble in the year 2000, the two components make up only a small share of overall index variance. In Table IA.I, the level of the factor component amounts to over 98% (96%) of index variance and its standard deviation to over 98% (79%) of the standard deviation of index variance for the S&P 100 and Nasdaq 100 indices, respectively. The average residual covariance amounts to less than 3% (5%) of index variance for the S&P 100 and Nasdaq 100 indices, respectively. Finally, the correlation between the factor component and index variance exceeds 95% for both indices. Thus, two common return factors are able to capture the realized variances of both indices and their movements through time.

Return Correlations: The final test is whether the common return factors can account for the correlation in asset returns and its time-variation. Figure 1, Panel B shows the average return correlation and the average residual correlation of the index constituents over the sample period when using two return factors. For both indices, the average return correlation among assets fluctuates significantly through time, but the average residual correlation is almost zero throughout the sample period. Thus, the two return factors accurately capture correlations in the returns of the index constituents and their time-variation. We conclude that a model with two return factors appears sufficiently accurate for our analysis.

[37] As will be shown in Internet Appendix IA.C, our methodology allows adjusting for idiosyncratic variances when extracting the variance swap rates on the common return factors, so the magnitude of (iii) is not a concern. In addition, it turns out that (iii) is small.

Table IA.I
Specification analysis: Components of realized variance in the S&P 100 and Nasdaq 100

The table reports summary statistics for realized index variance $\sigma^2_{I,p,t}$ and its components in the decomposition

$$\sigma^2_{I,p,t} = \beta'_{I,p,t}\Sigma_t\beta_{I,p,t} + \sum_{n=1}^{N}\sum_{m\neq n}^{N} w_{m,p,t}w_{n,p,t}\rho^{\epsilon}_{m,n,t}\sqrt{V^{\epsilon}_{m,t}V^{\epsilon}_{n,t}} + \sum_{n=1}^{N} w^2_{n,p,t}V^{\epsilon}_{n,t}\ ,$$

namely (i) $\beta'_{I,p,t}\Sigma_t\beta_{I,p,t}$, the component reflecting the variances and covariances of the common return factors Σ_t and assets' average factor exposures $\beta_{I,p,t}$, (ii) $\sum_{n=1}^{N}\sum_{m\neq n}^{N} w_{m,p,t}w_{n,p,t}\rho^{\epsilon}_{m,n,t}\sqrt{V^{\epsilon}_{m,t}V^{\epsilon}_{n,t}} = \sum_{n=1}^{N}\sum_{m\neq n}^{N} w_m w_n \sigma^{\epsilon}_{m,n,t}$, the component reflecting the weighted-average covariance between the return residuals, and (iii) $\sum_{n=1}^{N} w^2_{n,p,t}V^{\epsilon}_{n,t}$, the component reflecting the sum of the assets' idiosyncratic variances multiplied with the square of their weight in the index. The sample period is January 1996 to October 2009.

			Correlation with		
	Mean	Std.	Residual Covariances	Residual Variances	Index Variance
A. S&P 100 Index					
Factors $\beta'_{I,p,t}\Sigma_t\beta_{I,p,t}$	0.0442	0.0723	36.95%	59.49%	**97.78%**
Residual covariances $\sum_{n=1}^{N}\sum_{m\neq n}^{N} w_m w_n \sigma^{\epsilon}_{m,n,t}$	0.0012	0.0019		55.94%	41.22%
Residual variances $\sum_{n=1}^{N} w^2_n V^{\epsilon}_{n,t}$	0.0018	0.0017			60.58%
Index variance $\sigma^2_{I,p,t}$	0.0458	0.0736			
B. Nasdaq 100 Index					
Factors $\beta'_{I,p,t}\Sigma_t\beta_{I,p,t}$	0.0979	0.1134	60.59%	56.40%	**95.96%**
Residual covariances $\sum_{n=1}^{N}\sum_{m\neq n}^{N} w_m w_n \sigma^{\epsilon}_{m,n,t}$	0.0054	0.0081		60.81%	69.81%
Residual variances $\sum_{n=1}^{N} w^2_n V^{\epsilon}_{n,t}$	0.0047	0.0041			62.86%
Index variance $\sigma^2_{I,p,t}$	0.1155	0.1435			

IA.C Identifying Factor and Idiosyncratic Variance Swap Rates

Methodology: We compute variance swap rates on the factor and idiosyncratic return components by combining information on individual assets' and indices' variance swap rates as follows.[38] For each stock index $p = 1, \ldots, P$, we first construct *adjusted* index variance swap rates $y_{p,t}$ that are robust to imperfect diversification of the index (i.e., completely unaffected by constituents' idiosyncratic variances) by combining index variance swap rates $VS_{I,p,t}$ and individual variance swap

[38]We present results for the case of constant and known factor exposures. As shown in Appendix A, our methodology can be applied with minor modifications to situations with time-varying factor exposures and parameter uncertainty.

rates $VS_{n,t}$ as follows:

$$y_{p,t} \equiv VS_{I,p,t} - \sum_{n=1}^{N} w_{n,p,t}^2 VS_{n,t} = \beta_{I,p,t}' VS_t \beta_{I,p,t} - \sum_{n=1}^{N} w_{n,p,t}^2 \beta_{n,t}' VS_t \beta_{n,t} \ . \tag{IA.C.1}$$

Combining the expressions for all indices yields a linear system in $y_t = (y_{1,t}, \ldots, y_{P,t})'$:

$$y_t = X_t \Phi_t + \epsilon_t, \tag{IA.C.2}$$

where we have allowed for an error term $\epsilon_t = (\epsilon_{1,t}, \ldots, \epsilon_{P,t})'$ that reflects the approximation error in (11) and measurement error in the data, and

$$X_t = \begin{pmatrix} A_{1,1,t} & \ldots & A_{1,J,t} & B_{1,1,2,t} & \ldots & B_{1,J-1,J,t} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ A_{P,1,t} & \ldots & A_{P,J,t} & B_{P,1,2,t} & \ldots & B_{P,J-1,J,t} \end{pmatrix}, \tag{IA.C.3}$$

$$\Phi_t = \begin{pmatrix} VS_t^{11} & \ldots & VS_t^{JJ} & VS_t^{12} & \ldots & VS_t^{J-1,J} \end{pmatrix}', \tag{IA.C.4}$$

and (with i and j in parenthesis denoting the vector component),

$$A_{p,j,t} \equiv [\beta_{I,p,t}(j)]^2 - \sum_{n=1}^{N} w_{n,p,t}^2 \beta_{n,t}(j)^2, \quad j = 1, \ldots, J, \tag{IA.C.5}$$

$$B_{p,i,j,t} \equiv 2[\beta_{I,p,t}(i)\beta_{I,p,t}(j) - \sum_{n=1}^{N} w_{n,p,t}^2 \beta_{n,t}(i)\beta_{n,t}(j)], \quad i = 1, \ldots, J-1, \ j = i+1, \ldots, J. \tag{IA.C.6}$$

Using expression (IA.C.2) as the measurement equation, the linear Kalman filter can be used to extract the factor variance swap rates VS_t from the time-series of adjusted index variance swap rates y_t. Once the factor variance swap rates are known, the idiosyncratic variance swap rates are simply given by the no-arbitrage relation $VS_{n,t}^{\epsilon} = VS_{n,t} - \beta_{n,t}' VS_t \beta_{n,t}$.

Implementation: The methodology described above allows for oblique factors. Since the latent factor analysis we use in our empirical implementation produces orthogonal factors and the optimal rotation minimizes the time-series variation of the factor covariance, we can drop the factor covariance swap rate from our estimation problem, which corresponds to dropping the VS^{ij} terms in (IA.C.4) and the $B_{p,i,j,t}$ terms in (IA.C.3). After factor rotation we compute the time series of the factor exposures for the S&P 100 and Nasdaq 100 index (the $A_{p,j,t}$ terms in (IA.C.3)). For robustness we perform this task two ways—with constant or time-varying factor exposures: In the basic specification, we assume that the assets' factor exposures are constant through time and estimate β_n using OLS by regressing stocks' log returns $r_{n,t}$ on the rotated factor scores F_t. Alternatively, we allow the assets' factor exposures to be time-varying. We assume they follow a random walk, $\beta_{n,t} = \beta_{n,t-1} + \eta_{n,t}$, and estimate $\beta_{n,t}$ using the Kalman filter based on the measurement equation $r_{n,t} = \beta_{n,t}' F_t + \epsilon_{n,t}$. We then combine the assets' estimated factor exposures obtained using each approach with the index weights to compute $A_{p,j,t}$ using (IA.C.5).

With the time series of adjusted factor exposure matrices X_t in hand, we compute the factor variance swap rates from the adjusted index variance swap rates $y_{p,t}$ with the Kalman filter, using (IA.C.2) as the measurement equation and specifying that the factor variance swap rates $VS_t^{ii} = \Phi_t(i)$ follow mean-reverting processes

$$\Phi_t(i) = \kappa_i(\overline{\Phi}(i) - \Phi_t(i)) + \sigma_i \sqrt{\Phi_{F,t}(i)}\zeta_{i,t} \ , i \in \{1,2\} \tag{IA.C.7}$$

where $\zeta_{i,t}$ is noise.

Empirical Estimates from Factor Variance Swap Rate Extraction: Figure 2 illustrates the results from each step in the factor variance swap rate extraction. The left panels present the case of constant asset factor exposures, the right panels the case of time-varying factor exposures. The top panels show the adjusted factor exposures of the S&P 100 and Nasdaq 100 indices (the $A_{p,j,t}$ terms in (IA.C.5)). Both indices have similar exposures to the first factor, but very different exposures to the second. This is expected as the first factor loads heavily on the market factor and the second on the price/book factor (see Table 2). Even though individual assets' factor exposures in the left panel are constant by assumption, both indices' factor exposures vary significantly through time because of changes in index constituents and weights. The middle panels in Figure 2 report the adjusted index variance swap rates (the $y_{p,t}$ terms in (IA.C.2)); these are the same in the left and right panels. The bottom panels show the time series of the two factor variance swap rates estimated using the Kalman filter. Importantly, the assumption of constant or time-varying factor exposures has only a minor impact on the factor variance swap rate estimates; the correlation of the factor variance swap rates obtained using the two approaches is 97.89% for the first factor and 94.74% for the second.

The results in the bottom panels of Figure 2 reveal the importance of allowing for two factor variance swap rates. Indeed, the correlation between the two factor variance swap rates is only 10%, and their peaks do not occur concurrently. For instance, the peaks in index variance swap rates that occurred during the 1997 Asia financial crisis, the 1998 financial crisis, September 2001 and the 2008-2009 financial crisis are all primarily driven by the first factor, while the extremely large variance swap rates on the Nasdaq 100 index during the burst of the Internet bubble in the years 2000 and 2001 are mostly driven by the second factor.

IA.D Profitability of Dispersion Trading

In this appendix, we show that dispersion trading strategies earn a combination of systematic and idiosyncratic variance risk premia. Assume for simplicity that there is a single common return factor. Let $\overline{w}_{n,t} \geq 0$ be the weight of the variance swap on the nth stock in the dispersion trading portfolio and $\overline{w}_{I,p,t} \leq 0$ be the weight of the variance swap on index p when the dispersion trade is entered at time t. The excess return on the portfolio at time $t+\tau$ is

$$R_t = \sum_{n=1}^{N} \overline{w}_{n,t} \frac{\frac{1}{\tau}\int_t^{t+\tau} \sigma_{n,u}^2 du - VS_{n,t}}{VS_{n,t}} + \overline{w}_{I,p,t} \frac{\frac{1}{\tau}\int_t^{t+\tau} \sigma_{I,p,u}^2 du - VS_{I,p,t}}{VS_{I,p,t}} \ . \tag{IA.D.1}$$

Substituting the asset and index variances (5) and (6) and variance swap rates (10) and (11) yields

$$
\begin{aligned}
R_t &= \left(\sum_{n=1}^{N} \frac{\overline{w}_{n,t}\beta_n^2}{VS_{n,t}} + \frac{\overline{w}_{I,p,t}\beta_{I,p,t}^2}{VS_{I,p,t}}\right)\left(\frac{1}{\tau}\int_t^{t+\tau} \Sigma_u du - VS_t\right) \\
&\quad + \sum_{n=1}^{N}\left(\left(\frac{\overline{w}_{n,t}}{VS_{n,t}} + \frac{\overline{w}_{I,p,t} w_{n,p,t}^2}{VS_{I,p,t}}\right)\left(\frac{1}{\tau}\int_t^{t+\tau} V_{n,u}^{\epsilon} du - VS_{n,t}^{\epsilon}\right)\right) \qquad \text{(IA.D.2)}
\end{aligned}
$$

Thus, the excess return of the dispersion trading strategy is a combination of the variance risk premium on the common return factor, $\frac{1}{\tau}\int_t^{t+\tau} \Sigma_u du - VS_t$, and of the idiosyncratic variance risk premia on the individual assets, $\frac{1}{\tau}\int_t^{t+\tau} V_{n,u}^{\epsilon} du - VS_{n,t}^{\epsilon}$.

The relative importance of these two components in the strategy's profitability depends on the weights of the individual asset and index variance swaps in the portfolio. By selecting the weights, one can construct portfolios that are exposed only to factor variance risk, only to idiosyncratic variance risk, or to both. For example, setting $\overline{w}_{I,p,t} = -1$ and letting $\overline{w}_{n,t} = w_{n,p,t}^2 VS_{n,t}/VS_{I,p,t}$ yields a portfolio that only earns the factor variance risk premium. Similarly, letting $\overline{w}_{n,t} = w_{n,p,t}$ and setting $\overline{w}_{I,p,t} = -\sum_{n=1}^{N} \frac{\overline{w}_{n,t}\beta_n^2}{VS_{n,t}} \frac{VS_{I,p,t}}{\beta_{I,p,t}^2}$ yields a portfolio that only earns the idiosyncratic variance risk premium.

In general, dispersion trading strategies will earn a combination of both. This is the case even for the strategy where $\overline{w}_{I,p,t} = -1$ and the weights of the variance swaps on the individual assets are set to match the index weights $w_{n,p,t}$. In this case, (IA.D.2) becomes

$$
\begin{aligned}
R_t &= \left(\sum_{n=1}^{N} \frac{w_{n,p,t}\beta_n^2}{VS_{n,t}} - \frac{\beta_{I,p,t}^2}{VS_{I,p,t}}\right)\left(\frac{1}{\tau}\int_t^{t+\tau} \Sigma_u du - VS_t\right) \\
&\quad + \sum_{n=1}^{N}\left(\left(\frac{w_{n,p,t}}{VS_{n,t}} - \frac{w_{n,p,t}^2}{VS_{I,p,t}}\right)\left(\frac{1}{\tau}\int_t^{t+\tau} V_{n,u}^{\epsilon} du - VS_{n,t}^{\epsilon}\right)\right). \qquad \text{(IA.D.3)}
\end{aligned}
$$

Observe that using (10) and (11), the term in the first bracket can be rewritten as

$$
\frac{1}{VS_{I,p,t}}[(\sum_{n=1}^{N} w_{n,p,t}^2 VS_{n,t}^{\epsilon}) \sum_{n=1}^{N} \frac{w_{n,p,t}\beta_n^2}{VS_{n,t}} - \beta_I^2 \sum_{n=1}^{N} \frac{w_{n,p,t} VS_{n,t}^{\epsilon}}{VS_{n,t}}].
$$

This expression typically takes negative values, while the term in the first bracket in the second summand will typically be positive (unless $w_{n,p,t} > VS_{I,p,t}/VS_{n,t}$ which is unlikely for reasonably broad indices).

A portfolio constructed by buying a number of variance swaps on each asset that is proportional to this asset's weight in the index also earns a combination of factor and idiosyncratic variance risk premia (here, both the weight of the factor variance swap and those of the idiosyncratic variance swaps are unambiguously positive). In this case, $\overline{w}_{I,p,t} = -1/VS_{I,p,t}$, $\overline{w}_{n,t} = w_{n,p,t}/VS_{n,t}$, and

(IA.D.2) becomes

$$R_t = \left(\sum_{n=1}^{N} w_{n,p,t} \beta_n^2 - \beta_{I,p,t}^2 \right) \left(\frac{1}{\tau} \int_t^{t+\tau} \Sigma_u du - VS_t \right) + \sum_{n=1}^{N} \left(w_{n,p,t} (1 - w_{n,p,t}) \left(\frac{1}{\tau} \int_t^{t+\tau} V_{n,u}^{\epsilon} du - VS_{n,t}^{\epsilon} \right) \right). \tag{IA.D.4}$$

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Table 1
Notation

Notation	Definition	Decomposition	Description
Σ_t			Instantaneous return factor (co)variance matrix
$V^{\epsilon}_{n,t}$			Instantaneous variance of asset n's idiosyncratic return
$\sigma^2_{n,t}$		$\beta'_{n,t}\Sigma_t\beta_{n,t} + V^{\epsilon}_{n,t}$	Instantaneous variance of asset n's total return
$\sigma^2_{I,p,t}$		$\beta'_{I,p,t}\Sigma_t\beta_{I,p,t} + \sum_{n=1}^{N} w^2_{n,p,t}V^{\epsilon}_{n,t}$	Instantaneous variance of stock index p
VS_t	$\frac{1}{\tau}E^Q_t[\int_t^{t+\tau}\Sigma_u du]$		(Co)variance swap rates on return factors
$VS^{\epsilon}_{n,t}$	$\frac{1}{\tau}E^Q_t[\int_t^{t+\tau}V^{\epsilon}_{n,u}du]$		Variance swap rate on asset n's idiosyncratic return
$VS_{n,t}$	$\frac{1}{\tau}E^Q_t[\int_t^{t+\tau}\sigma^2_{n,u}du]$	$\beta'_{n,t}VS_t\beta_{n,t} + VS^{\epsilon}_{n,t}$	Variance swap rate on asset n's total return
$VS_{I,p,t}$	$\frac{1}{\tau}E^Q_t[\int_t^{t+\tau}\sigma^2_{I,p,u}du]$	$\beta'_{I,p,t}VS_t\beta_{I,p,t} + \sum_{n=1}^{N} w^2_{n,p,t}VS^{\epsilon}_{n,t}$	Variance swap rate on stock index p
RV_t	$\frac{1}{\tau}\int_t^{t+\tau}\Sigma_u du$		Realized (co)variance of factor returns
$RV^{\epsilon}_{n,t}$	$\frac{1}{\tau}\int_t^{t+\tau}V^{\epsilon}_{n,u}du$		Realized variance on asset n's idiosyncratic return
$RV_{n,t}$	$\frac{1}{\tau}\int_t^{t+\tau}\sigma^2_{n,u}du$	$\beta'_{n,t}RV_t\beta_{n,t} + RV^{\epsilon}_{n,t}$	Realized variance on asset n's total return
$RV_{I,p,t}$	$\frac{1}{\tau}\int_t^{t+\tau}\sigma^2_{I,p,u}du$	$\beta'_{I,p,t}RV_t\beta_{I,p,t} + \sum_{n=1}^{N} w^2_{n,p,t}RV^{\epsilon}_{n,t}$	Realized variance on stock index p
VRP_t	$E^P_t[d\Sigma_t] - E^Q_t[d\Sigma_t]$		Instantaneous factor (co)variance risk premium
$VRP^{\epsilon}_{n,t}$	$E^P_t[dV^{\epsilon}_{n,t}] - E^Q_t[dV^{\epsilon}_{n,t}]$		Instantaneous idiosyncratic variance risk premium
$VRP_{n,t}$	$E^P_t[d\sigma^2_{n,t}] - E^Q_t[d\sigma^2_{n,t}]$	$\beta'_{n,t}VRP_t\beta_{n,t} + VRP^{\epsilon}_{n,t}$	Instantaneous total variance risk premium
$VRP_{I,p,t}$	$E^P_t[d\sigma^2_{I,p,t}] - E^Q_t[d\sigma^2_{I,p,t}]$	$\beta'_{I,p,t}VRP_t\beta_{I,p,t} + \sum_{n=1}^{N} w^2_{n,p,t}VRP^{\epsilon}_{n,t}$	Instantaneous variance risk premium on stock index p
r_t	$\frac{RV_t - VS_t}{VS_t}$ (component-wise)		Realized factor (co)variance swap return
$r^{\epsilon}_{n,t}$	$\frac{RV^{\epsilon}_{n,t} - VS^{\epsilon}_{n,t}}{VS_{n,t}}$		Realized idiosyncratic variance swap return on asset n
$r_{n,t}$	$\frac{RV_{n,t} - VS_{n,t}}{VS_{n,t}}$	$\beta'_{n,t}(\frac{RV_t - VS_t}{VS_{n,t}})\beta_{n,t} + r^{\epsilon}_{n,t}$	Realized variance swap return on asset n
$r_{I,p,t}$	$\frac{RV_{I,p,t} - VS_{I,p,t}}{VS_{I,p,t}}$	$\beta'_{I,p,t}(\frac{RV_t - VS_t}{VS_{I,p,t}})\beta_{I,p,t} + \sum_{n=1}^{N} w^2_{n,p,t}r^{\epsilon}_{n,t}$	Realized variance swap return on stock index p

Table 2
Specification analysis

Panel A reports the fraction of the variation in individual asset returns (average R^2) explained by the common return factors extracted from the panel of stock returns when using one, two and three common return factors. For comparison, we also report the fraction of the variation explained by the Fama-French factors and the Carhart momentum factor ($FF4$ factors). The second and third rows report the coefficients of determination (R^2) from regressions of the returns on the S&P 100 and Nasdaq 100 indices on the common return factors and on the $FF4$ factors. Panel B reports the correlations between the common return factors and the three Fama-French factors and the Carhart momentum factor when using two latent factors. The common return factors are extracted from the panel of asset returns and optimally rotated. The optimal rotation minimizes the time-series standard deviation of the factor covariances.The sample period is January 1996 to October 2009.

A. Relationship between asset/index returns and factor returns				
	Number of Return Factors			$FF4$ Factors
Asset/Index	1	2	3	
Individual assets	22.38%	26.37%	26.84%	25.61%
S&P 100 index	91.78%	92.23%	92.05%	95.33%
Nasdaq 100 index	71.33%	89.22%	89.23%	83.66%

B. Correlation between common return factors and $FF4$ factors				
	MKT	HML	SMB	UMD
Factor 1	84.89%	11.81%	−24.71%	−37.92%
Factor 2	43.77%	−60.13%	25.45%	−10.66%

Table 3
Factor structure in individual asset realized variances, variance swap rates, and variance swap returns, and the relationship between common factors and index quantities

The table reports the fraction of the variation explained by common factors extracted from the panel of individual assets. The second and third rows report the coefficient of determination from regressions of the index quantity for the S&P 100 and Nasdaq 100 indices on the common factors. Panel A reports the quantities for realized variances. Panels B and C report the quantities for variance swap rates and variance swap returns, respectively. The sample period is January 1996 to October 2009.

	Number of Variance/VS/VRP Factors					
Asset/Index	1	2	3	4	5	6
			A. Realized Variances			
Individual assets	44.00%	55.16%	58.51%	61.91%	64.29%	65.43%
S&P 100 index	78.64%	88.20%	92.02%	94.16%	94.31%	94.35%
Nasdaq 100 index	57.88%	75.66%	75.66%	81.69%	84.07%	85.37%
			B. Variance Swap Rates			
Individual assets	56.06%	70.44%	75.77%	77.92%	79.52%	81.31%
S&P 100 index	74.94%	86.30%	88.36%	89.77%	90.82%	94.51%
Nasdaq 100 index	76.33%	88.58%	91.13%	92.46%	95.32%	95.58%
			C. Variance Swap Returns			
Individual assets	25.59%	28.57%	30.84%	31.83%	34.85%	35.72%
S&P 100 index	68.11%	68.99%	76.48%	76.42%	76.93%	76.83%
Nasdaq 100 index	66.10%	66.62%	67.45%	67.32%	74.53%	73.67%

Table 4
Returns on index and single-stock variance swaps

The table reports average monthly percentage returns of index variance swaps and index-weighted single-stock variance swap portfolios. The sample period is January 1996 to October 2009.

Monthly % return (NW t-stat)	S&P 100	Nasdaq 100
Index	-15.11	-5.36
	(-3.13)	(-1.27)
Constituents (index-weighted)	3.70	9.64
	(1.06)	(3.21)

Table 5
Profitability of dispersion trading strategies

This table reports summary statistics of the monthly returns earned by a dispersion trading strategy that takes short positions in index variance swaps and long positions in individual stocks' variance swaps. The strategies in panel A have returns that are uncorrelated with index variance swap returns, while those in panel B have returns that are uncorrelated with the average variance swap return of the index constituents. Sharpe ratios are reported in annual terms. The sample period is January 1996 to October 2009.

Index	Mean	S.D.	Sharpe	Min	Max
A. Portfolio uncorrelated with index variance swap returns					
S&P 100	1.27	2.52	1.74	-12.79	19.21
Nasdaq 100	1.27	2.03	2.17	-11.77	13.48
B. Portfolio uncorrelated with average constituent variance swap returns					
S&P 100	1.87	3.62	1.79	-27.72	14.51
Nasdaq 100	1.69	2.93	1.99	-31.83	14.91

Table 6
Decomposition of variance risk premia

This table reports monthly index-weighted average variance risk premia (VRP) for the constituents of the S&P 100 and, respectively, Nasdaq 100 indices, as well as the decomposition of total variance risk premia into systematic and idiosyncratic variance components for the entire sample period and split by calendar year. Following Carr and Wu (2009), we measure realized variance risk premia by the returns on short-dated variance swaps and compute variance swap returns over the period t to $t+\tau$ (τ is chosen to be one month) using the following decomposition on the n stocks:

$$\underbrace{\frac{RV_{n,t} - VS_{n,t}}{VS_{n,t}}}_{\text{Total variance risk premium}} = \underbrace{\frac{\beta'_{n,t}(RV_t - VS_t)\beta_{n,t}}{VS_{n,t}}}_{\text{Systematic variance risk premium}} + \underbrace{\frac{RV^{\epsilon}_{n,t} - VS^{\epsilon}_{n,t}}{VS_{n,t}}}_{\text{Idiosyncratic variance risk premium}},$$

where $RV_{n,t}$ denotes the realized variance and $VS_{n,t}$ the variance swap rate on stock $n = 1, \ldots, N$. RV_t, VS_t, $RV^{\epsilon}_{n,t}$, and $VS^{\epsilon}_{n,t}$ are defined analogously. Newey-West t-statistics are reported in parentheses. The sample period is January 1996 to October 2009.

A. S&P 100 Members

			Specification 1: Constant factor exposures				*Specification 2*: Time-varying factor exposures			
	Total variance		Systematic variance		Idiosyncratic variance		Systematic variance		Idiosyncratic variance	
Year	VRP	t-stat	VRP	t-stat	VRP	t-stat	VRP	t-stat	VRP	t-stat
96-09	3.70	(1.06)	-19.73	(-8.19)	23.43	(14.28)	-24.03	(-8.19)	27.73	(19.78)
1996	2.14	(0.39)	-27.30	(-7.41)	29.44	(6.39)	-33.62	(-7.89)	35.75	(7.38)
1997	11.11	(1.39)	-25.73	(-3.75)	36.84	(12.11)	-29.48	(-3.85)	40.59	(13.16)
1998	24.63	(2.10)	-23.67	(-3.17)	48.30	(8.05)	-26.67	(-3.09)	51.30	(10.10)
1999	11.43	(1.85)	-20.04	(-8.79)	31.47	(6.19)	-22.70	(-6.84)	34.13	(7.87)
2000	34.48	(3.74)	3.07	(0.62)	31.41	(5.51)	10.18	(1.70)	24.31	(5.17)
2001	-3.25	(-0.29)	-18.07	(-2.58)	14.83	(2.24)	-27.39	(-3.08)	24.14	(3.96)
2002	16.47	(1.40)	-12.22	(-1.92)	28.70	(4.72)	-19.36	(-2.36)	35.84	(7.85)
2003	-25.57	(-11.48)	-41.62	(-20.34)	16.05	(7.58)	-55.36	(-22.47)	29.79	(12.21)
2004	-20.01	(-3.75)	-34.38	(-17.23)	14.38	(3.17)	-47.35	(-18.68)	27.34	(5.98)
2005	-19.41	(-4.66)	-27.06	(-13.84)	7.64	(2.66)	-33.07	(-13.28)	13.66	(5.23)
2006	-20.09	(-4.27)	-29.74	(-11.12)	9.65	(3.04)	-39.02	(-11.28)	18.93	(6.72)
2007	-4.16	(-0.54)	-12.69	(-1.94)	8.53	(3.24)	-20.11	(-2.81)	15.95	(8.35)
2008	55.43	(1.85)	24.61	(1.14)	30.82	(3.51)	32.34	(1.28)	23.10	(4.54)
2009	-17.25	(-2.24)	-35.44	(-9.40)	18.19	(4.03)	-25.33	(-4.41)	8.08	(2.96)

continued

Table 6
Decomposition of variance risk premia—continued

B. Nasdaq 100 Members

			Specification 1: Constant factor exposures				*Specification 2*: Time-varying factor exposures			
	Total variance		Systematic variance		Idiosyncratic variance		Systematic variance		Idiosyncratic variance	
Year	VRP	t-stat	VRP	t-stat	VRP	t-stat	VRP	t-stat	VRP	t-stat
96-09	9.64	(3.21)	-11.99	(-6.00)	21.64	(12.99)	-12.42	(-5.61)	22.06	(15.85)
1996	-0.26	(-0.05)	-14.13	(-4.27)	13.87	(4.75)	-16.12	(-4.22)	15.86	(5.97)
1997	16.48	(2.15)	-11.81	(-2.06)	28.30	(7.03)	-9.67	(-1.57)	26.16	(7.83)
1998	33.58	(2.88)	-8.65	(-0.91)	42.23	(12.58)	-6.75	(-0.67)	40.33	(15.43)
1999	19.63	(3.90)	-22.64	(-8.87)	42.27	(8.76)	-21.28	(-7.53)	40.91	(8.92)
2000	56.76	(4.79)	-2.86	(-0.30)	59.62	(12.26)	6.13	(0.60)	50.63	(11.53)
2001	12.18	(1.28)	-6.60	(-1.12)	18.78	(4.35)	-5.22	(-0.75)	17.40	(4.77)
2002	16.09	(2.00)	-4.87	(-0.81)	20.96	(6.39)	-3.93	(-0.61)	20.02	(6.60)
2003	-16.91	(-6.58)	-26.33	(-10.68)	9.42	(6.23)	-30.81	(-12.69)	13.90	(8.89)
2004	-11.44	(-2.41)	-18.41	(-7.95)	6.97	(2.16)	-20.56	(-7.90)	9.13	(3.13)
2005	-8.45	(-1.57)	-15.40	(-6.91)	6.95	(1.42)	-20.20	(-7.15)	11.75	(2.44)
2006	-5.99	(-1.03)	-13.59	(-6.25)	7.60	(1.71)	-16.81	(-5.85)	10.82	(2.73)
2007	-1.90	(-0.29)	-13.32	(-3.74)	11.42	(2.53)	-20.43	(-5.17)	18.53	(4.83)
2008	37.70	(1.74)	14.66	(0.88)	23.04	(3.66)	16.02	(0.88)	21.68	(4.46)
2009	-20.36	(-3.49)	-28.15	(-7.79)	7.79	(2.83)	-28.29	(-6.39)	7.93	(3.95)

Table 7
Can priced idiosyncratic variance risk explain the cross-section?

This table reports return and turnover statistics (monthly average returns and standard deviations μ and σ, annual Sharpe ratio SR and monthly turnover) as well as monthly abnormal returns (Alpha) and factor loadings of long-short portfolios of single stock variance swaps constructed by sorting on different characteristics. Each equal-weighted portfolio is long quintile 5 and short quintile 1. The portfolios are benchmarked against three expected return models, (i) the Fama-French four-factor model ($FF4$), (ii) the $FF4$ model augmented by two systematic variance risk factors ($FF4$+SVR), and (iii) the $FF4$ model augmented by two systematic and two idiosyncratic variance risk factors ($FF4$+VR). The two SVR factors $SVR\ F1$ and $SVR\ F2$ are proxies for systematic variance risk (measured by S&P and, respectively, Nasdaq index variance swap returns), and the two $CIVR$ factors $CIVR\ F1$ and $CIVR\ F2$ are proxies for common idiosyncratic variance risk (measured as the cross-sectional average variance swap return on the index constituents for each of the indices). The estimates are from time-series regressions. t-statistics are reported in parentheses. The sample period is January 1996 to October 2009.

	Alpha	R^2	Factor loadings							
			MKT	SMB	HML	MOM	SVR F1	SVR F2	CIVR F1	CIVR F2
A. Sort on Realized Variance over Variance Swap Rate ($\mu = 8.84\%$, $\sigma = 19.02\%$, $SR = 1.61$, Turnover = 75%)										
$FF4$	10.01	0.08	-1.33	-0.16	-0.94	-0.51	–	–	–	–
	(6.81)		(-3.93)	(-0.40)	(-2.12)	(-2.00)	–	–	–	–
$FF4$+SVR	9.00	0.40	-0.28	0.35	-0.92	-0.47	0.35	1.10	–	–
	(7.57)		(-0.90)	(1.06)	(-2.47)	(-2.29)	(1.23)	(6.62)	–	–
$FF4$+VR	1.94	0.51	-0.09	0.51	-0.49	-0.35	-0.13	0.99	2.11	-0.14
	(1.17)		(-0.31)	(1.68)	(-1.38)	(-1.81)	(-0.44)	(6.08)	(4.94)	(-0.62)
B. Sort on Variance Swap Return ($\mu = 8.12\%$, $\sigma = 19.40\%$, $SR = 1.45$, Turnover = 74%)										
$FF4$	9.00	0.04	-1.17	0.00	-0.59	-0.42	–	–	–	–
	(5.89)		(-3.32)	(0.00)	(-1.27)	(-1.57)	–	–	–	–
$FF4$+SVR	7.77	0.45	-0.03	0.56	-0.64	-0.36	0.19	1.33	–	–
	(6.68)		(-0.09)	(1.73)	(-1.75)	(-1.80)	(0.69)	(8.20)	–	–
$FF4$+VR	1.28	0.53	0.14	0.65	-0.38	-0.30	-0.09	1.17	1.53	0.15
	(0.77)		(0.51)	(2.13)	(-1.07)	(-1.56)	(-0.31)	(7.18)	(3.59)	(0.68)
C. Sort on Realized Variance ($\mu = 15.81\%$, $\sigma = 35.35\%$, $SR = 1.55$, Turnover = 44%)										
$FF4$	18.30	0.14	-3.07	-0.98	-1.56	-1.10	–	–	–	–
	(6.97)		(-5.09)	(-1.36)	(-1.97)	(-2.39)	–	–	–	–
$FF4$+SVR	16.31	0.60	-0.69	0.12	-1.44	-0.99	1.01	2.29	–	–
	(9.01)		(-1.46)	(0.24)	(-2.55)	(-3.17)	(2.30)	(9.06)	–	–
$FF4$+VR	0.23	0.75	-0.26	0.39	-0.76	-0.83	0.22	1.92	3.86	0.29
	(0.10)		(-0.68)	(0.95)	(-1.60)	(-3.27)	(0.56)	(8.83)	(6.80)	(0.97)
D. Sort on Variance Swap Rate ($\mu = 14.84\%$, $\sigma = 36.11\%$, $SR = 1.42$, Turnover = 21%)										
$FF4$	16.99	0.12	-2.96	-0.70	-1.10	-0.98	–	–	–	–
	(6.25)		(-4.73)	(-0.93)	(-1.34)	(-2.06)	–	–	–	–
$FF4$+SVR	14.85	0.55	-0.73	0.36	-1.13	-0.87	0.62	2.43	–	–
	(7.57)		(-1.41)	(0.67)	(-1.84)	(-2.56)	(1.30)	(8.89)	–	–
$FF4$+VR	-2.56	0.71	-0.26	0.62	-0.44	-0.71	-0.18	2.00	4.03	0.43
	(-1.07)		(-0.63)	(1.41)	(-0.85)	(-2.57)	(-0.41)	(8.48)	(6.53)	(1.30)
E. Sort on Exposure to S&P 100 Returns ($\mu = 7.78\%$, $\sigma = 19.63\%$, $SR = 1.37$, Turnover = 2%)										
$FF4$	8.70	0.07	-1.22	-0.40	-0.30	-0.50	–	–	–	–
	(5.73)		(-3.50)	(-0.96)	(-0.66)	(-1.88)	–	–	–	–
$FF4$+SVR	7.19	0.59	-0.23	0.13	-0.63	-0.42	-0.47	1.75	–	–
	(7.10)		(-0.88)	(0.47)	(-1.99)	(-2.39)	(-1.92)	(12.40)	–	–
$FF4$+VR	0.82	0.67	-0.04	0.13	-0.58	-0.43	-0.53	1.49	0.85	0.58
	(0.59)		(-0.19)	(0.50)	(-1.93)	(-2.69)	(-2.15)	(10.89)	(2.39)	(3.04)
F. Sort on Exposure to Nasdaq 100 Returns ($\mu = 8.61\%$, $\sigma = 22.03\%$, $SR = 1.35$, Turnover = 2%)										
$FF4$	8.92	0.03	-0.78	-0.38	0.47	-0.11	–	–	–	–
	(5.13)		(-1.96)	(-0.79)	(0.91)	(-0.36)	–	–	–	–
$FF4$+SVR	6.83	0.66	-0.30	0.04	-0.44	0.00	-2.02	2.45	–	–
	(6.62)		(-1.10)	(0.15)	(-1.35)	(0.03)	(-8.03)	(17.06)	–	–
$FF4$+VR	0.53	0.77	-0.09	-0.11	-0.68	-0.11	-1.73	2.05	-0.09	1.21
	(0.41)		(-0.38)	(-0.46)	(-2.41)	(-0.75)	(-7.38)	(15.81)	(-0.26)	(6.71)

Table 8
Testing the model-implied equilibrium pricing of variance risk

This table reports the results of Fama-MacBeth regressions of individual stocks' variance swap returns on the following explanatory variables: their exposure to the market, size, value and momentum factors, their exposure to a factor constructed as the cross-sectional average of individual stocks' idiosyncratic variance swap returns, two proxies for supply pressure arising from option compensation and covered call writing by mutual funds (as well as dummy variables capturing cases where these variables are missing), and the riskiness of each stock's "truly idiosyncratic" variance, which is computed as the time series variance of the residuals from the first pass regression of each stock's variance swap return on the market, size, value, momentum and common idiosyncratic variance factors. The dependent variable is the total variance swap return (left) or the idiosyncratic variance swap return (right); accordingly, the left (right) columns report estimates of the drivers of total (idiosyncratic) variance risk premia. The sample period is January 1996 to October 2009.

	Total variance risk premium		Idiosyncratic variance risk premium	
	Coefficient	*t*-stat	Coefficient	*t*-stat
Constant	-3.74	(-6.83)	-0.35	(-0.87)
Market	-2.43	(-4.18)	-0.91	(-1.53)
SMB	-1.71	(-4.27)	0.13	(0.31)
HML	1.33	(3.34)	0.37	(0.97)
MOM	-2.28	(-2.99)	-1.15	(-1.50)
Common Idiosyncratic Variance	2.47	(3.62)	3.39	(5.00)
Supply proxies:				
Option Compensation	23.81	(6.03)	16.52	(5.33)
Option Compensation Missing	0.41	(0.74)	0.76	(1.73)
Mutual Fund Holdings	2.86	(1.35)	4.16	(2.24)
Mutual Fund Holdings Missing	-0.19	(-0.33)	0.59	(1.32)
Truly Idiosyncratic Variance Risk	3.56	(0.98)	13.40	(2.86)
Observations	55,306		54,637	
F-statistic	10.27		10.21	
Average R^2	0.20		0.16	

Table 9
Dynamics of variance risk premia on individual stocks

This table reports the estimates from a GARCH-M specification for the time-series behavior of the cross-sectional average variance risk premium on individual stocks. The dependent variable is the total variance swap return (left) or the idiosyncratic variance swap return (right). The sample period is January 1996 to October 2009.

	Total variance risk premium		Idiosyncratic variance risk premium	
	Coefficient	*t*-stat	Coefficient	*t*-stat
Mean equation				
Constant	-3.02	(-5.89)	-0.41	(-1.02)
Conditional variance	1.53	(9.02)	10.00	(5.87)
Variance equation				
Constant	0.04	(2.48)	0.00	(0.59)
ARCH (1)	0.92	(6.83)	0.23	(5.97)
GARCH (1)	0.47	(7.42)	0.81	(41.03)
Observations	165		165	
χ^2-statistic	81.44		34.45	
Log likelihood	150.11		257.90	

Table 10
Predictive power of systematic and idiosyncratic variances

This table reports the OLS coefficient estimates from contemporaneous and predictive regressions of macroeconomic variables on the quarterly market excess return $r_{M,t} - r_{f,t}$, market variance $\sigma^2_{M,t}$ (used as proxy for systematic variance), and common idiosyncratic variance $\overline{\sigma}^2_{\epsilon,t}$ (computed as the cross-sectional average of idiosyncratic variance). The dependent variables used in the specifications reported across columns are the quarterly market excess return, market variance, total variance (computed as the cross-sectional average of stocks' total variance), common idiosyncratic variance, GDP growth, investment growth, consumption growth, the 3-month T-bill rate, the term spread between 10-year Treasury bonds and 3-month T-bills, and the default spread between BAA and AAA corporate bonds. We conduct the regressions using both levels (Panels A and C) and innovations (Panels B and D) of the dependent and independent variables. For each series, innovations are computed as the residuals from an AR specification with the number of lags selected optimally using Schwarz' Bayesian Information Criterion (BIC). The number of lags used when computing the innovations in each series is reported in the row labeled "AR Lags (BIC)" in Panels B and D. Panels A and B report estimates from contemporaneous regressions. Panels C and D report the results of predictive regressions for a one-quarter horizon. In Panel C, in order to account for the autocorrelation of the dependent variable, we include as regressor the lagged dependent variable up to a number of lags selected optimally using Schwarz' Bayesian Information Criterion (BIC). The number of lags used is reported in the row labeled "AR Lags (BIC)" in Panel C. Newey-West t-statistics are reported in parentheses. The sample period is January 1996 to October 2009.

	Market Return$_t$	Market Variance$_t$	Total Variance$_t$	Idiosyncratic Variance$_t$	GDP Growth$_t$	Invest. Growth$_t$	Cons. Growth$_t$	3-Month T-Bill$_t$	Term Spread$_t$	Default Spread$_t$
A. Contemporaneous Regression, Levels										
Market Return$_t$	–	-0.27	-0.10	-0.07	0.02	0.02	0.01	-0.02	0.02	0.01
	–	(-1.94)	(-2.10)	(-0.35)	(1.59)	(0.43)	(1.33)	(-0.86)	(1.19)	(1.28)
Market Variance$_t$	-0.73	–	1.13	0.94	-0.06	-0.28	-0.05	-0.18	0.06	0.08
	(-4.15)	–	(15.26)	(2.77)	(-3.12)	(-2.01)	(-4.04)	(-3.48)	(2.44)	(6.30)
Idiosyncratic Variance$_t$	-0.04	0.21	1.29	–	0.01	0.02	0.02	0.06	-0.02	-0.01
	(-0.35)	(3.06)	(25.17)	–	(1.35)	(0.58)	(4.62)	(3.46)	(-1.13)	(-3.87)
Observations	54	54	54	54	54	54	54	54	54	54
F-statistic	18.24	5.08	1188.31	5.54	6.71	1.94	10.99	5.63	2.02	13.86
R^2	0.27	0.41	0.99	0.27	0.41	0.24	0.42	0.27	0.08	0.73
B. Contemporaneous Regression, Innovations										
Market Return$_t$	–	-0.21	-0.07	0.00	0.03	0.12	0.01	0.01	0.01	0.00
	–	(-2.44)	(-1.24)	(-0.04)	(3.06)	(1.74)	(1.49)	(0.86)	(0.67)	(1.14)
Market Variance$_t$	-0.91	–	1.01	0.81	-0.01	0.02	-0.02	-0.02	0.02	0.05
	(-2.74)	–	(6.49)	(4.21)	(-0.52)	(0.11)	(-2.62)	(-2.74)	(1.92)	(10.97)
Idiosyncratic Variance$_t$	-0.01	0.42	1.45	–	0.00	0.02	0.01	0.01	-0.03	0.00
	(-0.04)	(2.34)	(9.68)	–	(0.20)	(0.24)	(1.85)	(1.05)	(-2.62)	(-0.41)
Observations	53	53	53	53	53	53	53	51	53	53
AR Lags (BIC)	0	1	1	1	1	1	2	4	1	2
F-statistic	8.15	3.31	258.99	12.06	5.90	1.94	6.71	4.97	2.34	82.96
R^2	0.29	0.53	0.96	0.43	0.24	0.11	0.16	0.11	0.08	0.77

continued

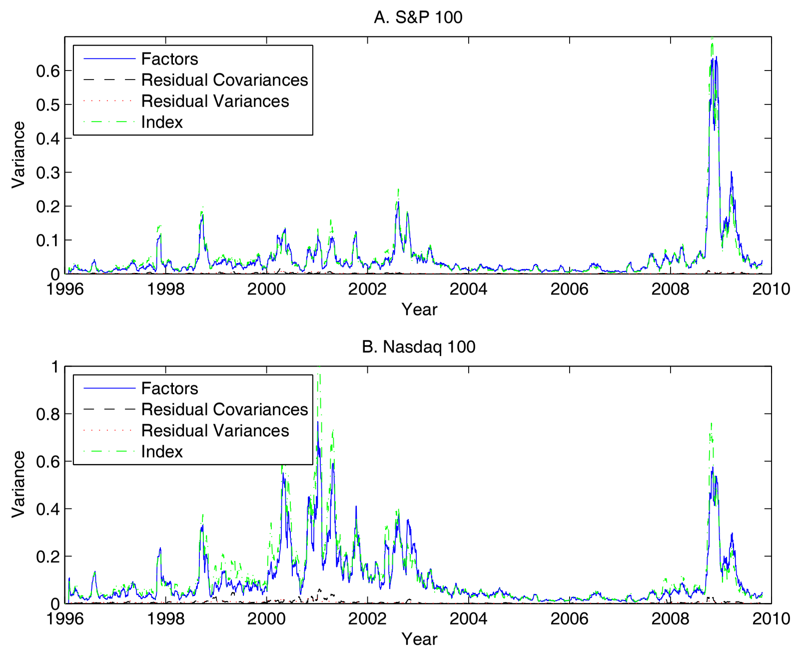
Table 10
Predictive power of systematic and idiosyncratic variances—continued

	Market Return_t	Market Variance_t	Total Variance_t	Idiosyncratic Variance_t	GDP Growth_t	Invest. Growth_t	Cons. Growth_t	3-Month T-Bill_t	Term Spread_t	Default Spread_t
				C. Predictive Regression, Levels						
Market Return_{t-1}	0.01	-0.10	-0.48	-0.22	0.01	0.07	0.01	0.01	-0.01	-0.01
	(0.07)	(-1.27)	(-2.23)	(-1.75)	(1.08)	(1.50)	(1.05)	(2.74)	(-0.94)	(-2.16)
Market Variance_{t-1}	-0.05	0.38	-0.09	-0.40	-0.06	-0.40	0.02	0.00	0.01	-0.01
	(-0.17)	(2.35)	(-0.27)	(-2.34)	(-5.17)	(-7.59)	(0.82)	(0.14)	(0.59)	(-1.26)
Idiosyncratic Variance_{t-1}	-0.08	0.02	1.17	0.88	0.01	0.05	0.00	0.00	0.01	0.00
	(-0.78)	(0.43)	(6.41)	(8.51)	(1.29)	(1.19)	(0.04)	(0.30)	(0.71)	(-0.62)
Observations	53	53	53	53	53	53	53	51	53	53
AR Lags (BIC)	–	–	0	–	0	0	2	4	2	2
F-statistic	0.44	7.49	35.67	40.79	17.68	94.08	11.08	261.85	79.24	21.40
R^2	0.02	0.25	0.57	0.71	0.39	0.55	0.43	0.95	0.83	0.70
				D. Predictive Regression, Innovations						
Market Return_{t-1}	-0.08	-0.08	-0.33	-0.16	0.00	0.05	0.01	0.01	-0.01	-0.01
	(-0.43)	(-1.18)	(-1.33)	(-1.16)	(-0.06)	(0.94)	(1.13)	(1.86)	(-1.19)	(-2.10)
Market Variance_{t-1}	-0.19	-0.20	-0.85	-0.30	-0.04	-0.31	0.02	0.01	-0.01	-0.01
	(-0.68)	(-1.04)	(-1.74)	(-1.34)	(-2.79)	(-4.18)	(1.52)	(0.97)	(-0.54)	(-0.84)
Idiosyncratic Variance_{t-1}	-0.33	0.22	0.59	0.16	0.00	0.01	-0.01	0.00	0.01	0.01
	(-1.19)	(1.84)	(1.40)	(0.70)	(0.08)	(0.06)	(-0.79)	(-0.39)	(0.44)	(1.18)
Observations	52	52	52	52	52	52	52	51	52	52
AR Lags (BIC)	0	1	1	1	1	1	2	4	1	2
F-statistic	2.40	1.41	1.33	0.75	9.66	48.09	0.83	1.18	1.35	1.51
R^2	0.09	0.06	0.07	0.05	0.15	0.37	0.06	0.07	0.06	0.08

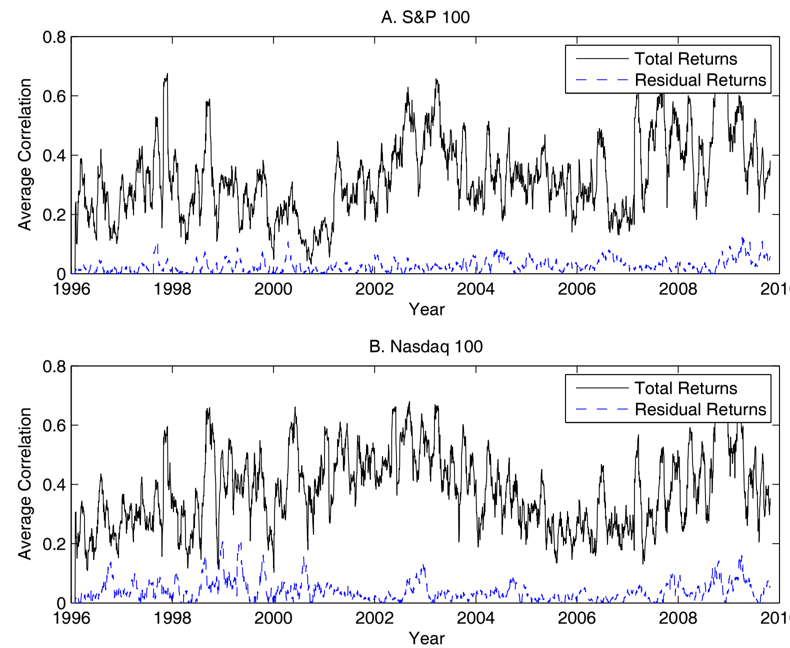
Table 11
The effect of option illiquidity and hedging costs on variance risk premia

This table reports estimation results from Fama-MacBeth regressions of individual stocks' variance swap returns on a number of explanatory variables. The dependent variable is the total variance swap return (left) or the idiosyncratic variance swap return (right); accordingly, the left (right) columns report estimates of the drivers of total (idiosyncratic) variance risk premia. Specifications (1), (2), (3) use different sets of explanatory variables. Statistically significant coefficients at the 10%, 5% and 1% level are marked with *, ** and ***, respectively. The sample period is January 1996 to October 2009.

	Total variance risk premium			Idiosyncratic variance risk premium		
	(1)	(2)	(3)	(1)	(2)	(3)
Stock bid-ask spread	-0.83	-0.57	-0.93	0.42	0.35	-0.49
Option bid-ask spread (atm)	0.00	0.01	0.01	-0.00	0.01	0.01*
Option bid-ask spread (oom)	-0.02***	-0.00	-0.00	-0.02***	0.00	0.00
Stock volume	1.05***	0.28	0.15	1.32***	0.18	0.42
Option volume > 0	1.60***	1.13***	0.53	1.62***	0.57**	0.14
Option volume	-0.35	-0.52**	-1.11***	-0.21	-0.42**	-0.81**
Open interest > 0 (atm)	-0.01	1.08	0.69	-0.25	1.18	0.30
Open interest (atm)	-0.15***	-0.07***	-0.00	-0.17***	-0.04**	-0.02
Stock skewness	–	-0.20	-0.24	–	-0.02	-0.01
Stock kurtosis	–	-0.40***	-0.33***	–	-0.61***	-0.56***
Lagged dependent variable	–	0.09***	0.11***	–	0.18***	0.20***
Realized variance	–	-0.52	-3.34	–	0.99	-2.12
β S&P	–	-1.25	-0.94	–	0.87	0.39
β Nasdaq	–	5.61***	4.76***	–	2.54***	2.81**
Market-to-book	–	–	0.20***	–	–	0.17***
Firm size	–	–	-0.08	–	–	0.04
Profitability	–	–	-3.02	–	–	-3.93**
Book leverage	–	–	-0.33**	–	–	-0.29***
Capital expenditure	–	–	1.78	–	–	0.07
Cash holding	–	–	2.15**	–	–	0.98
Dividend payer	–	–	-0.74**	–	–	-1.04***
Observations	48,166	47,330	32,265	47,982	47,060	32,093
F-statistic	6.96	9.51	4.92	14.75	45.52	18.74
R^2	0.06	0.15	0.27	0.07	0.18	0.29



(a) Components of realized index variance.



(b) Average return correlations and residual return correlations.

Figure 1
Specification analysis

Panel A plots the components of index variance in the S&P 100 and Nasdaq 100 indices over the sample period. Index variance consists of three components: (i) a component reflecting (co)variances of the common return factors Σ_t and assets' average factor exposures $\beta_{I,p,t}$, (ii) the weighted-average covariance between return residuals, and (iii) the sum of the assets' idiosyncratic variances multiplied with the square of their weight in the index, and is given by $\sigma^2_{I,p,t} = \beta'_{I,p,t}\Sigma_t\beta_{I,p,t} + \sum_{n=1}^{N}\sum_{m\neq n}^{N} w_{m,p,t}w_{n,p,t}\rho^{\epsilon}_{m,n,t}\sqrt{V^{\epsilon}_{m,t}V^{\epsilon}_{n,t}} + \sum_{n=1}^{N} w^2_{n,p,t}V^{\epsilon}_{n,t}$. Panel B shows the average return correlation and the average residual correlation of the index constituents over the sample period.

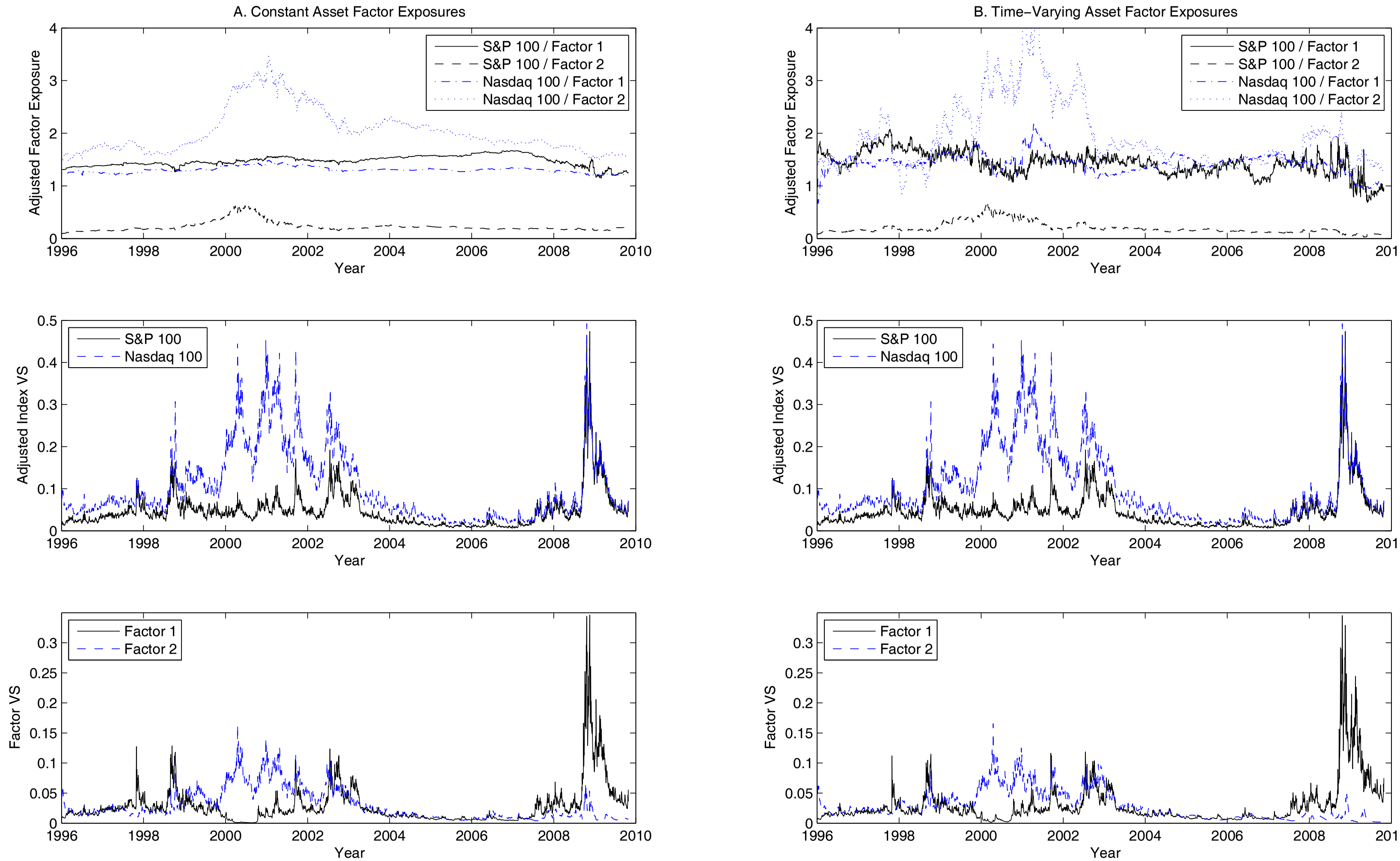


Figure 2
The time series of factor variance swap rates

This figure illustrates the results from each step in the factor variance swap rate extraction. The left panels present the case of constant asset factor exposures, the right panels the case of time-varying factor exposures. The top panels show the adjusted factor exposures of the S&P 100 and Nasdaq 100 indices (the $A_{p,j,t}$ terms in (IA.C.3)). The middle panels report the adjusted index variance swap rates (the $y_{p,t}$ terms in (IA.C.2)). The bottom panels show the time series of the two factor variance swap rates estimated using the Kalman filter.

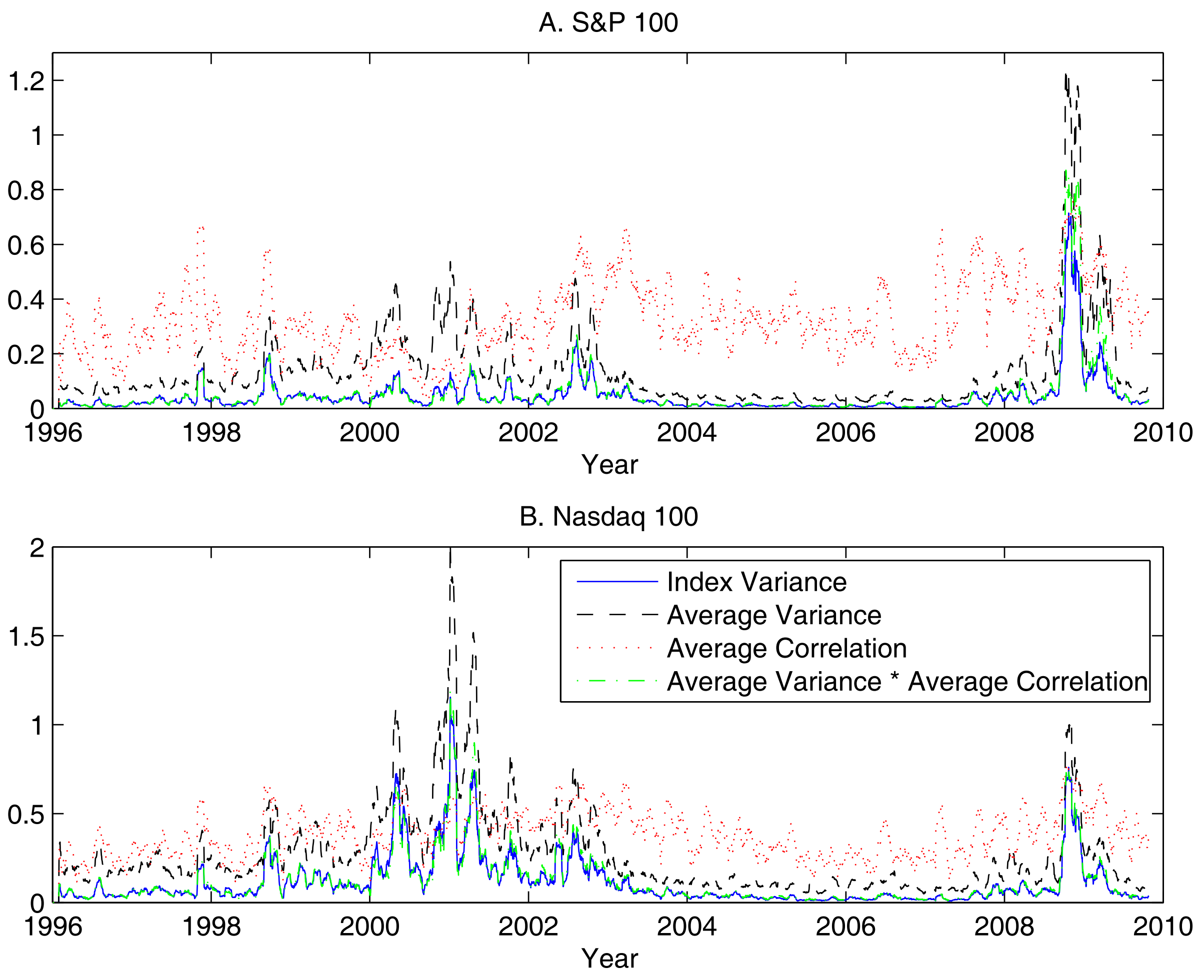


Figure 3
Relationship between index variances, constituent variances, and constituent correlations

The figure reports realized index variance, the average realized variance of the index constituents, the average correlation between index constituents, and the product of the average realized variance and the average correlation for the S&P 100 and Nasdaq 100 indices. All series are computed using trailing 1 month (21 trading day) windows, and the averages are based on the index weights. The sample period is January 1996 to October 2009.

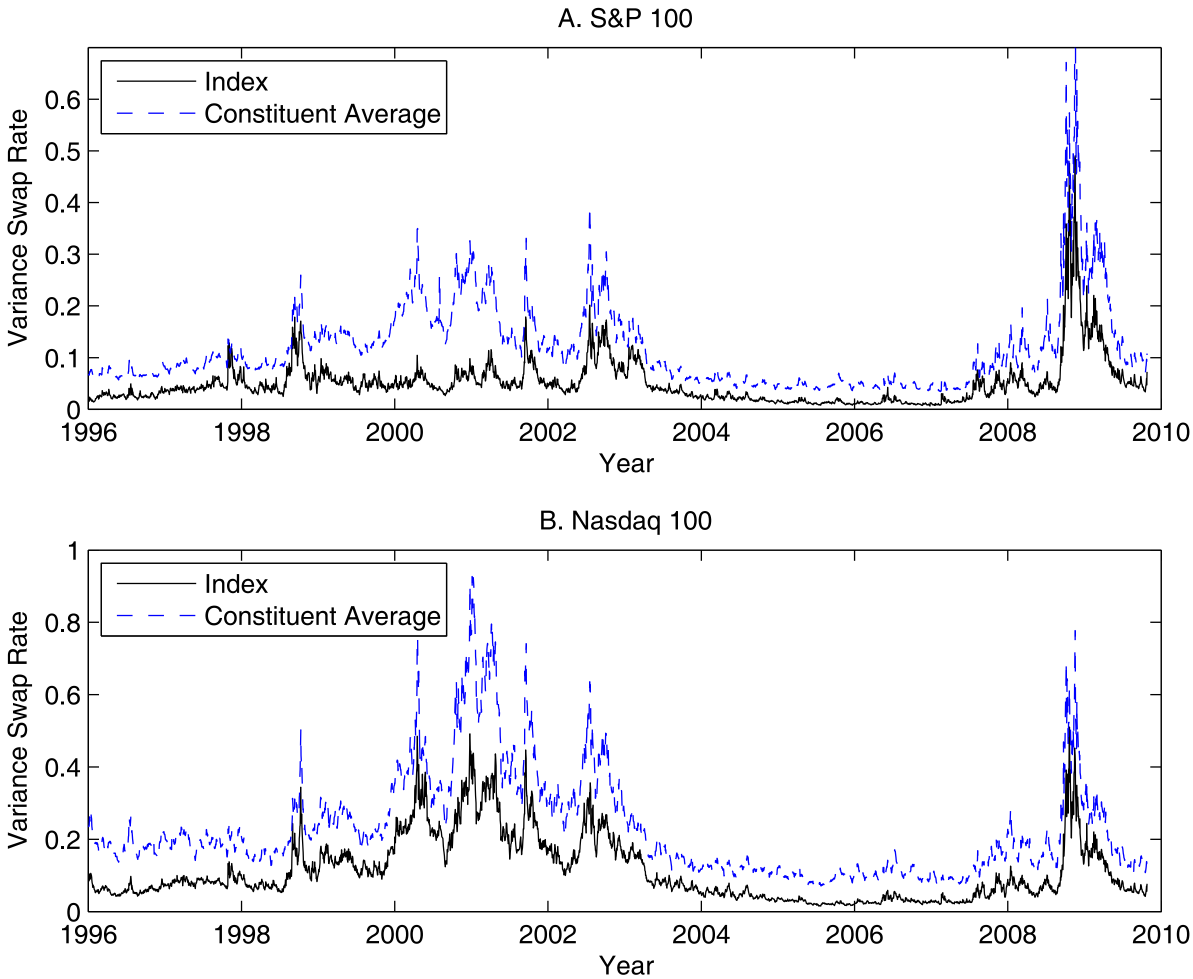


Figure 4
Relationship between index and constituent variance swap rates

The figure shows the index variance swap rates and the average variance swap rates of the index constituents for the S&P 100 and Nasdaq 100 indices. The averages are computed using the index weights. The sample period is January 1996 to October 2009.