DISCUSSION PAPER SERIES

No. 8237

THE JOINT BEHAVIOR OF HIRING AND INVESTMENT

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INTERNATIONAL MACROECONOMICS and LABOUR ECONOMICS



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Discussion Paper No. 8237 February 2011

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February 2011

ABSTRACT

The Joint Behavior of Hiring and Investment*

This paper explores the dynamic behavior of investment and hiring within a unified framework, stressing their mutual dependence and placing the emphasis on their joint, forward-looking behavior. Using structural estimation in aggregate, private sector U.S. data, it shows that the model, which features adjustment costs, is able to fit the data. Unlike many previous results, the fit is achieved without implying high adjustment costs. The interaction of hiring and investment costs is significant and is negatively signed, implying complementarity between investment and hiring. There is a substantial role for labor market conditions in hiring costs, whereby the latter are lower in "good times." The fit of the investment part of the model is poor if hiring is left out completely or is introduced without the interaction between the two. The results capture the not so-well known fact whereby there is negative comovement of gross investment and gross hiring, the former being pro-cyclical while the latter is countercylical. This is so as they follow the cyclical behavior of their present values. An asset-pricing type empirical analysis indicates that the hiring rate depends mostly on future labor profitability while the investment rate depends mostly on future returns.

JEL Classification: E22, E24, E32 and J23

Keywords: business cycles, complementarity, forward-looking behavior, gross hiriting, gross investment, labor market conditions and present values of hiring and investment

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Submitted 01 February 2011

The Joint Behavior of Hiring and Investment¹

1 Introduction

This paper studies the joint behavior of hiring and investment using private sector U.S. data. The importance of these decisions by firms for aggregate activity cannot be overstated. The evolution of employment and of the capital stock are essential for the understanding of macroeconomic fluctuations. Search and matching models have shown that gross hiring is a key factor for understanding employment and unemployment dynamics.² Investment is key for the understanding of the evolution of the capital stock and consequently of firm market value.³

Hiring or investment are modelled in the literature as the outcomes of a dynamic, intertemporal optimization problem of the firm. The intertemporal dimension rests on the existence of adjustment costs on capital or labor. But while the firm evidently decides on both hiring and investment, the treatment in much of the literature has either focused on the behavior of one and not the other, or has posited adjustment costs pertaining to one but not the other. Thus, for example, search and matching models focus on labor hiring and posit either no capital or costless adjustment of capital. Investment costs models, such as those in the Tobin's Q literature, follow the same route with respect to capital, usually disregarding labor. Even DSGE models,⁴ typically model adjustment costs only on one factor – capital or labor. Moreover, all too often, the empirical work that has estimated adjustment costs, especially investment costs, has reported weak results. This weakness was manifested in a lack of fit or the need to postulate implausibly large adjustment costs to explain the data.

This paper explores the dynamic behavior of investment and hiring within a unified framework, stressing their mutual dependence and placing the emphasis on their joint, forward-looking behavior. Using structural estimation

¹I thank Russell Cooper, Jordi Gali, Giuseppe Moscarini, Richard Rogerson, Gianluca Violante, and seminar participants at the NBER Summer Institute (Rogerson, Shimer and Wright EF group meeting, July 2010), the CEPR ESSIM meetings (May 2010), the LMDG meeting (Sandbjerg, Denmark, October 2009), Tel Aviv University, CREI (Pompeu Fabra), EUI (Florence), the Fundacion Rafael del Pino (Madrid) and Birbeck College (London) for helpful comments, and Ofer Cornfeld, Darina Waisman and especially Tanya Baron for research assistance. All errors are my own.

²See, for example, Hall (2007) and Rogerson and Shimer (2010).

³See Bond and van Reenen (2007).

⁴Such as those by Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2007), or Gali (2008, 2010).

of the firms' optimality equations in aggregate, private sector U.S. data, it shows that it is able to explain the negative co-movement of investment and hiring and their different cyclical properties. In particular, it explains the (not so-well known) fact that while gross investment is pro-cyclical, gross hiring is counter-cyclical. In doing so, it shows that costs matter for both capital and labor adjustment, that the interaction between them is important, and that the model is able to fit the data without implying high adjustment costs. It also indicates that labor market conditions matter for the behavior of costs on both investment and hiring. The paper shows what is lacking (empirically) when one does not cater for these features.

A major implication of the findings is that hiring and investment can be treated as forward-looking variables, reflecting the expectations of future profits from employing labor and capital. This naturally links up with stock prices that are also forward-looking and relate to the same expected future profits. Indeed, in previous work, Monika Merz and I (Merz and Yashiv (2007)) have shown that this set-up allows one to define asset values for hiring and for investment and that these can be used to explain the time variation of equity values of firms in the U.S. economy.⁵The current paper retains the focus on forward-looking behavior but does not make use of stock market data or try to explain them. Rather, it aims at the empirical characterization of hiring and investment themselves as forward-looking decisions. Using the estimation results, it employs a number of techniques used in the asset pricing literature (forecasting regressions, restricted VAR analyses and variance decompositions) to study this forward-looking aspect. The analysis suggests that investment and hiring are differentially related to their expected, future determinants. Investment is linked more to movements in future returns than to changes in the marginal product of capital. Hiring is linked more to changes in labor profitability (the marginal product less the wage) and less to the movements in future returns. In particular, in recessions, higher expected future profitability from labor leads firms to increase the rate at which they hire workers, though employment and worker job-finding rates decline.

The paper proceeds as follows: Section 2 presents the business cycle facts of investment and hiring in the U.S. economy, highlighting their differential behavior. Section 3 briefly discusses the relevant literature. Section 4 presents the firm's optimization problem and the resulting optimality conditions. Section 5 discusses estimation issues and presents the results. Section

⁵Building on Merz and Yashiv (2007), Bazdresch, Belo and Lin (2009) have further shown that hiring and investment predict stock returns in a cross-section of U.S. publicly traded firms.

6 uses the results to look at the implied magnitude of adjustment costs and to gauge the plausibility of the estimates. Section 7 uses the estimates to approximate the "asset pricing" relationships embodied in the model and analyze the links of hiring and investment with the variables affecting them in the future. Section 8 explores the implications of the results for the comovement and cyclical behavior of hiring and investment. Section 9 concludes. Technical matters and data issues are treated in the appendices.

2 Business Cycle Facts

The analysis below focuses on the gross hiring rate $\frac{h}{n}$ and the gross investment rate $\frac{i}{k}$ of the aggregate U.S. economy. Figure 1 plots these series.⁶ The figure has four panels. Figure 1a shows the raw series. Figures 1b and 1c show, in two panels each, the logged series in levels and in Hodrick-Prescott (HP) and Band Pass (BP) filter terms, together with NBER-dated recessions. Figure 1d shows in two panels the logged, HP-filtered and BP-filtered series of investment and hiring with the NBER-dated recessions.

Figure 1

Inspection of the figures reveals that the investment and hiring rates series do not move together and have markedly different cyclical behavior – investment is pro-cyclical while hiring is counter-cyclical.

Table 1 provides a quantitative summary of these features. It looks at the stochastic behavior of investment and hiring rates in logged, HP-filter terms and BP-filter terms. It presents co-movement statistics, the dynamic correlations of investment and hiring and their co-movement with three cyclical measures (real business sector GDP f, labor productivity $\frac{f}{h}$ and capital productivity $\frac{f}{h}$).

Table 1

Gross hiring and gross investment rates exhibit negative correlation, both contemporaneously and at some leads and lags. Both contemporaneously and dynamically, hiring is counter-cyclical with respect to the three cyclical variables. These correlations are stronger when using the BP filter, relative to the HP filter. With respect to the same cyclical measures, investment is pro-cyclical, sometimes strongly so. This is so both contemporaneously and at some leads and lags. In this case the filtering method does not matter much.

 $^{^6\}mathrm{The}$ data are further discussed in Section 4.2 below.

Note that hiring is positively correlated with the cyclical indicator two years ahead (of the indicator) and subsequently is negatively correlated with it; so two years before a recession hiring falls, about a year ahead it starts to rise, and it rises through the recession. For investment it is the opposite pattern. Hence, in recessions hiring rises while investment falls. Two years ahead of the recession investment rises and hiring falls; closer to the recession they switch signs. Judging by the strength of the correlation measures, investment rates are stronger leading indicators of the cycle.

The counter-cyclicality of hiring may appear counter-intuitive. To put this behavior in further perspective and show how it relates to other labor market facts, I look at labor market variables which are often discussed in the literature. Note that in steady state, hiring to employment h equals separations from employment s:

$$h = s \tag{1}$$

Non-employment in the steady state, i.e., unemployment u plus the pool out of the labor force o, is given by:

$$\frac{u+o}{pop} = \frac{\psi}{\frac{h}{u+o} + \psi} \tag{2}$$

where *pop* is the working age population and ψ is the separation rate from employment n ($s = \psi n$).

In steady state the hiring rate is the product of the job finding rate, steady state non-employment and the inverse of the employment rate:

$$\frac{h}{n} = \frac{h}{u+o} \times \frac{u+o}{pop} \times \frac{pop}{n} \tag{3}$$

Using the above formulation of steady-state non-employment:

$$\underbrace{\frac{h}{n}}_{\text{hiring rate}} = \underbrace{\frac{h}{u+o}}_{\text{job finding}_{ss \text{ non-emp}}} \times \underbrace{\frac{1}{\frac{n}{pop}}}_{\text{inv emp ratio}} \tag{4}$$

Table 2 repeats some of the moments of Table 1 for these variables.

Table 2

The table shows that the employment stock n and the job finding rate $\frac{h_t}{u_t+o_t}$ are pro-cyclical, as is well known. At the same time the gross hiring

rate $\frac{h_t}{n_t}$ is counter-cyclical, as are steady state non-employment $\frac{\psi}{\frac{h}{u+o}+\psi}$ and the inverse employment ratio $\frac{1}{\frac{n}{pop}}$. The hiring rate is counter-cyclical as the counter-cyclicality of the last two variables has a stronger effect than the pro-cylicality of the job-finding rate. In what follows, the gross hiring rate $\frac{h_t}{n_t}$ will be a key variable in the analysis. It is useful to keep in mind that, in line with these features, it behaves differently from the employment stock nand is not to be confused with the job finding rate $\frac{h_t}{u_t+o_t}$.

Some of these stylized facts are not obvious. In particular, one needs to account for the fact that hiring and investment move in opposite ways. Intuitively we may think that if investment rises, hiring should rise too, at least with a lag, but this is not what we observe. Moreover, their relationship with the cycle is different and switches sign as discussed above.

Why did the literature give little, if any, attention to these facts? This is so probably because business cycle models usually do not look at the gross hiring flows, but rather at the employment stock. Search and matching models look at gross hiring flows but typically do not consider investment. Hence the two – investment and hiring – are usually not examined together. This approach is manifest in the literature review to which I turn now.

3 Literature

The current paper relates to two major strands in the literature and provides a missing link between them.

The first is the literature on search and matching models, which feature dynamic, optimal hiring decisions by firms in the face of costs (see Pissarides (2000), Rogerson, Shimer, and Wright (2005), Yashiv (2007) and Rogerson and Shimer (2010) for overviews and surveys). Hiring costs are a major source of frictions in these models which place emphasis on the existence of frictions in the labor market. The first order condition for optimal hiring is a key ingredient in these models and this is one of the two estimating equations examined here. However, most of this literature does not include capital as a factor of production, and when it does, it is typically assumed not to be the subject of adjustment costs. A large part of this literature posits very simple hiring costs, usually a linear function of the number of job vacancies. Thus, it usually states that marginal hiring costs are constant. As indicated above, gross hiring is considered to be key in accounting for employment and unemployment dynamics. The model here features a generalization of the hiring problem considered by these models.

It should be noted too that models with labor adjustment costs have

been studied for about half a century (Hamermesh (1993) provides a useful discussion). Most of these studies typically relate to net employment changes as distinct from gross changes of the type examined here, and have ignored any interaction with capital. The distinction between net and gross flows is important, as hiring costs are incurred with respect to the gross flow of incoming workers and the stochastic properties of these various flows are substantially different (see Hamermesh and Pfann (1996), in particular pp. 1266-67).

The second strand includes investment models with adjustment costs, mostly in the Tobin's Q tradition (though Q can be related to investment without any adjustment costs, see Abel and Eberly (2010)). These models have been studied extensively for four decades, since the seminal contribution of Brainard and Tobin (1968) and Tobin (1969); Chirinko (1993) is an earlier survey and Erickson and Whited (2000) and Bond and van Reenen (2007) are more recent discussions. The idea in these models is that adjustment costs are key to the understanding of investment behavior. As in the hiring case, they endow the investment problem with its dynamic optimization aspect and are geared to capture the real world feature of gradual adjustment of the capital stock. These models have encountered a lot of empirical difficulties and have engendered much debate (see Chirinko (1993) and Bond and van Reenen (2007)). Like search and matching models, much of this literature does not feature the other factor of production, namely labor. In the current paper I present results both from the "traditional" formulation of the investment costs model and from a formulation which allows for the interaction of investment costs and hiring costs. Hence, when presenting the results I provide a comparison with the results of nine key studies in this literature. The approach here is akin to the Euler equation approach in the investment literature proposed by Abel (1980), with the important distinction that it incorporates hiring and the interaction of costs between hiring and investment.

It should also be noted that models of the business cycle (evidently) feature optimal hiring and investment decisions. Many of them do not feature adjustment costs, though a large part of the RBC literature assumes lags in the installation of capital. More recent RBC models and the latest vintage of business cycle models, such as Christiano, Eichenbaum and Evans (2005) or Smets and Wouters (2007) do posit adjustment costs for investment but no frictions in hiring. Note, too, that in business cycle models there is no explicit interaction between hiring costs and investment costs.

A key issue in the current paper is the mutual dependence of hiring and investment and the interaction of their costs. Mortensen (1973) examined the interrelation of costs in a theoretical model and over the years some empirical work was attempted; prominent examples include Nadiri and Rosen (1969), Shapiro (1986), Hall (2004) and Bloom (2009). These studies pointed to the potential importance of including adjustment costs on both capital and labor. However key differences with the current study are that these papers do not model at least one of two elements, which the empirical work below finds to be of crucial importance: an interaction term between the two costs and gross, as opposed to net, hiring flows. Hence their findings are quite different from what is reported here.

4 The Model

I delineate a partial equilibrium model which serves as the basis for estimation.⁷ There are identical workers and identical firms, who live forever and have rational expectations. It takes time and resources for firms to adjust their capital stock and hire new workers. All variables are expressed in terms of the output price level. Firms make investment (i) and hiring (h) decisions.⁸ Once a new worker is hired, the firm pays her a per-period wage w. Firms use physical capital (k) and labor (n) as inputs in order to produce output goods y according to a constant-returns-to-scale production function f with productivity shock z:

$$y_t = f(z_t, n_t, k_t), \tag{5}$$

Gross hiring and gross investment are costly activities. Hiring costs include advertising, screening, and training. In addition to the purchase costs, investment involves capital installation costs, learning the use of new equipment, etc. Adjusting labor or capital involves disruptions to production, and potentially also the implementation of new organizational structure within the firm and new production practices. All of these costs reduce the firm's profits. I represent these costs by an adjustment costs function $g[i_t, k_t, h_t, n_t]$ which is convex in the firm's decision variables and exhibits constant returnsto-scale. I allow hiring costs and capital adjustment costs to interact. I specify the functional form of g and discuss its properties in the empirical work below.

In every period t, the capital stock depreciates at the rate δ_t and is augmented by new investment i_t . The capital stock's law of motion equals:

$$k_{t+1} = (1 - \delta_t)k_t + i_t, \quad 0 \le \delta_t \le 1.$$
 (6)

⁷This formulation is consistent with the afore-cited analysis in Merz and Yashiv (2007). The parts concerned with the labor market are consistent with the prototypical search and matching model within a stochastic framework. See Pissarides (2000) and Yashiv (2007).

⁸In the standard search and matching model, gross hires are labeled new job-matches.

Similarly, workers separate at the rate ψ_t . It is augmented by new hires h_t :

$$n_{t+1} = (1 - \psi_t)n_t + h_t, \quad 0 \le \psi_t \le 1.$$
(7)

Note that hiring and separations are both gross flows and that the separation rate is time-varying.

Firms' profits before tax, π , equal the difference between revenues net of adjustment costs and total labor compensation, wn:

$$\pi_t = [f(z_t, n_t, k_t) - g(i_t, k_t, h_t, n_t)] - w_t n_t.$$
(8)

Every period, firms make after-tax cash flow payments cf to the stock owners and bond holders of the firm. These cash flow payments equal profits after tax minus purchases of investment goods plus investment tax credits and depreciation allowances for new investment goods:

$$cf_t = (1 - \tau_t)\pi_t - (1 - \chi_t - \tau_t D_t)\,\widetilde{p}_t^I\,i_t \tag{9}$$

where τ_t is the corporate income tax rate, χ_t the investment tax credit, D_t the present discounted value of capital depreciation allowances, \tilde{p}_t^I the real pre-tax price of investment goods.

The discount factor between periods t + j - 1 and t + j for $j \in \{1, 2, ...\}$ is given by:

$$\beta_{t+j} = \frac{1}{1 + r_{t+j-1,t+j}}$$

where $r_{t+j-1,t+j}$ denotes the time-varying discount rate between periods t + j - 1 and t + j. Appendix B contains a description of how alternative values of the discount rate r are computed in the empirical work.

The representative firm chooses sequences of i_t and h_t in order to maximize its *cum dividend* market value $cf_t + s_t$:

$$\max_{\{i_{t+j},h_{t+j}\}} E_t \left\{ \sum_{j=0}^{\infty} \left(\prod_{i=0}^j \beta_{t+i} \right) c f_{t+j} \right\}$$
(10)

subject to the definition of cf_{t+j} in equation (9) and the constraints (6) and (7). The firm takes the paths of the variables $w, p^I, \delta, \psi, \tau$ and β as given. The Lagrange multipliers associated with these two constraints are Q_{t+j}^K and Q_{t+j}^N , respectively. These Lagrange multipliers can be interpreted as marginal Q for physical capital, and marginal Q for employment, respectively.

The first-order conditions for dynamic optimality are the same for any two consecutive periods t + j and t + j + 1, $j \in \{0, 1, 2, ...\}$. For the sake of

notational simplicity, I drop the subscript j from the respective equations to follow:

$$Q_t^K = E_t \left\{ \beta_{t+1} \left[(1 - \tau_{t+1}) \left(f_{k_{t+1}} - g_{k_{t+1}} \right) + (1 - \delta_{t+1}) Q_{t+1}^K \right] \right\}$$
(11)

$$Q_t^K = (1 - \tau_t) \left(g_{i_t} + p_t^I \right)$$
(12)

$$P_{t}^{K} = (1 - \tau_{t}) \left(g_{i_{t}} + p_{t}^{I} \right)$$
(12)

$$Q_{t}^{N} = E_{t} \left\{ \beta_{t+1} \left[(1 - \tau_{t+1}) \left(f_{n_{t+1}} - g_{n_{t+1}} - w_{t+1} \right) + (1 - \psi_{t+1}) Q_{t+1}^{N} \right] \left\} 3 \right\}$$

$$Q_{t}^{N} = (1 - \tau_{t}) g_{h_{t}}$$
(14)

where I use the real after-tax price of investment goods, given by:

$$p_{t+j}^{I} = \frac{1 - \chi_{t+j} - \tau_{t+j} D_{t+j}}{1 - \tau_{t+j}} \, \widetilde{p}_{t+j}^{I}.$$
(15)

Dynamic optimality requires the following two transversality conditions to be fulfilled

$$\lim_{T \to \infty} E_T \left(\beta_T Q_T^K k_{T+1} \right) = 0$$

$$\lim_{T \to \infty} E_T \left(\beta_T Q_T^N n_{T+1} \right) = 0.$$
(16)

I can summarize the firm's first-order necessary conditions from equations (11)-(14) by the following two expressions:

$$(1 - \tau_t) \left(g_{i_t} + p_t^I \right) = E_t \left\{ \beta_{t+1} \left(1 - \tau_{t+1} \right) \left[\begin{array}{c} f_{k_{t+1}} - g_{k_{t+1}} \\ + (1 - \delta_{t+1}) (g_{i_{t+1}} + p_{t+1}^I) \end{array} \right] \right\}$$

$$(1 - \tau_t) g_{h_t} = E_t \left\{ \beta_{t+1} \left(1 - \tau_{t+1} \right) \left[\begin{array}{c} f_{n_{t+1}} - g_{n_{t+1}} - w_{t+1} \\ + (1 - \psi_{t+1}) g_{h_{t+1}} \end{array} \right] \right\} . (18)$$

Solving equation (11) forward and using the law of iterated expectations expresses Q_t^K as the expected present value of future marginal products of physical capital net of marginal capital adjustment costs:

$$Q_{t}^{K} = E_{t} \left\{ \sum_{j=0}^{\infty} \left(\prod_{i=0}^{j} \beta_{t+1+i} \right) \left(\prod_{i=0}^{j} \left(1 - \delta_{t+1+i} \right) \right) \left(1 - \tau_{t+1+j} \right) \left(f_{k_{t+1+j}} - g_{k_{t+1+j}} \right) \right\}$$
(19)

It is straightforward to show that in the special case of time-invariant discount factors, no adjustment costs, no taxes, and a perfectly competitive market for capital, Q_t^K equals one. Similarly, solving equation (13) forward and using the law of iterated expectations expresses Q_t^N as the expected present value of the future stream of surpluses arising to the firm from an additional hire of a new worker:

$$Q_t^N = E_t \left\{ \sum_{j=0}^{\infty} \left(\prod_{i=0}^j \beta_{t+1+i} \right) \left(\prod_{i=0}^j \left(1 - \psi_{t+1+i} \right) \right) \left(1 - \tau_{t+1+j} \right) \left(f_{n_{t+1+j}} - g_{n_{t+1+j}} - w_{t+1+j} \right) \right\}$$
(20)

In the special case of a perfectly competitive labor market and no hiring costs, Q_t^N equals zero.

5 Estimation

I estimate alternative versions of the model. The alternatives pertain to the degree of convexity of the adjustment costs function, the possibility that hiring costs may depend on labor market conditions, the formulation of the discount rate, the examination of standard specifications, and the set of instruments used. I estimate equations (17) and (18), using structural estimation. In what follows I present the parameterization of this function (as well as of the production function), the econometric methodology, the data and the results.

5.1 Methodology

5.1.1 Parameterization

To estimate the model I need to parameterize the relevant functions. For the production function I use a standard Cobb-Douglas:

$$f(z_t, n_t, k_t) = e^{z_t} n_t^{\alpha} k_t^{1-\alpha}, \ 0 < \alpha < 1.$$
(21)

For the adjustment costs function g, I use a convex function to be delineated below. Recent work by Cooper and Haltiwanger (2006), Kahn and Thomas (2008) and Bloom (2009) gives empirical support to the use of a convex adjustment costs function in aggregate data.⁹These papers show that while non-convexities matter at the micro level, a convex formulation is appropriate at the aggregate, macroeconomic level.

The specifications to be used capture the idea that adjustment costs increase with the extent of the factor adjustment relative to the size of the firm, where a firm's size is measured by its physical capital stock or its level of employment. The functions used postulate that costs are proportional to output, and that they increase in the investment and hiring rates. More specifically, the terms in the function relating to hiring may be justified as follows (drawing on Garibaldi and Moen (2008)): suppose each worker *i* makes a recruiting and training effort h_i ; as this is a convex function it is optimal to spread out the efforts equally across workers so $h_i = \frac{h}{n}$; formulating the

 $^{^{9}}$ See the discussion on pages 628 and 629 of Cooper and Haltiwanger (2006), pages 417-421 in Kahn and Thomas (2008), and page 665 in Bloom (2009).

costs as a function of these efforts and putting them in terms of output per worker I get $c\left(\frac{h}{n}\right)\frac{f}{n}$; as *n* workers do it then the aggregate adjustment cost function is $c\left(\frac{h}{n}\right)f$.

Drawing on this logic, the parametric form I use is the following, generalized convex function.

$$g(\cdot) = \begin{bmatrix} \frac{e_1}{\eta_1} (\frac{i_t}{k_t})^{\eta_1} \\ + \left[\frac{e_{20} + e_{21} \frac{v_t}{u_t + o_t}}{\eta_2} \right] (\frac{h_t}{n_t})^{\eta_2} \\ + \left[\frac{e_{30} + e_{31} \frac{v_t}{u_t + o_t}}{\eta_3} \right] (\frac{i_t}{k_t} \frac{h_t}{n_t})^{\eta_3} \end{bmatrix} f(z_t, n_t, k_t).$$
(22)

This function has a number of important properties: first, it is linearly homogenous in its arguments i, k, h, n, v, o, and u. The parameters $e_l, l =$ 1, 20, 21, 30, 31 express scale, and η_l express the elasticity of adjustment costs with respect to the different arguments. Second, the terms $e_{21} \frac{v_t}{u_t + o_t}$ and $e_{31} \frac{v_t}{u_t + o_t}$ allow for the scale of costs to depend on labor market tightness $\frac{v_t}{u_t + o_t}$. This caters for the possibility that hiring costs – by themselves and by their interaction with investment costs – depend on the state of the labor market captured by market tightness for any given hiring rate $\frac{h}{n}$. The sign of e_{21} and of e_{31} may be positive or negative, as there may be different effects of these aggregate conditions on the firm hiring process. In the empirical work below, these are unconstrained parameters to be estimated. Third, the term $\left(\frac{i_t}{k_t} \frac{h_t}{n_t}\right)^{\eta_3}$ expresses the interaction of capital and labor adjustment costs. This term, usually absent in many studies, has important implications for the complementarity of investment and hiring. It, too, is estimated without constraints.

The function encompasses the widely used quadratic case for which $\eta_1 = \eta_2 = 2$. Note that a standard Tobin's Q model of investment with adjustment costs postulates $e_{20} = e_{21} = e_{30} = e_{31} = 0$ and $\eta_1 = 2$.

In estimation, I explore a number of alternative specifications:

1) The degree of convexity of the g function. I examine restricted and free estimation of the power parameters η_1, η_2 and η_3 .

2) Scale as a function of market conditions. I examine the above as well as the case where labor market conditions do not matter, namely $e_{21} = e_{31} = 0$.

3) Standard specifications. I set $e_{20} = e_{21} = e_{30} = e_{31} = 0$ and look at investment costs only and then I set $e_1 = e_{30} = e_{31} = 0$ and look at hiring costs only. I also examine the case of both investment and hiring costs but no interaction $e_{30} = e_{31} = 0$.

4) Instrument sets. I use alternative instrument sets in terms of variables and number of lags.

Estimation of the parameters in these functions allows for the quantification of the derivatives g_{i_t} and g_{h_t} that appear in the firms' optimality equations (17) and (18).

5.1.2 Structural Estimation

I structurally estimate the firms' first-order conditions (17) and (18), using Hansen's (1982) generalized method of moments (GMM). The moment conditions estimated are those obtained under rational expectations. That is, the firms' expectational errors are orthogonal to any variable in their information set at the time of the investment and hiring decisions. The moment conditions are derived by replacing expected values with actual values plus expectational errors j and specifying that the errors are orthogonal to the instruments Z, i.e., $E(j_t \otimes Z_t) = 0$. I formulate the equations in stationary terms by dividing (17) by $\frac{f_t}{k_t}$ and (18) by $\frac{f_t}{n_t}$.

The estimating equations errors j_t are thus given by:

$$j_{t}^{1} = \frac{(1-\tau_{t})\left(g_{i_{t}}+p_{t}^{I}\right)}{\frac{f_{t}}{k_{t}}} - \left\{\frac{\frac{f_{t+1}}{k_{t+1}}}{\frac{f_{t}}{k_{t}}}\beta_{t+1}\left(1-\tau_{t+1}\right)\frac{\left[f_{k_{t+1}}-g_{k_{t+1}}+(1-\delta_{t+1})(g_{i_{t+1}}+p_{t+1}^{I})\right]}{\frac{f_{t+1}}{k_{t+1}}}\right\}$$
$$j_{t}^{2} = \frac{(1-\tau_{t})g_{h_{t}}}{\frac{f_{t}}{n_{t}}} - \left\{\frac{\frac{f_{t+1}}{n_{t+1}}}{\frac{f_{t}}{n_{t}}}\beta_{t+1}\left(1-\tau_{t+1}\right)\frac{\left[f_{n_{t+1}}-g_{n_{t+1}}-w_{t+1}+(1-\psi_{t+1})g_{h_{t+1}}\right]}{\frac{f_{t+1}}{n_{t+1}}}\right\}$$
(24)

Appendix A spells out the first derivatives included in these equations.

I compute the J-statistic test of the overidentifying restrictions proposed by Hansen (1982). I also check whether the estimated g function fulfills the convexity requirement.

5.2 The Data

The data are quarterly, pertain to the private sector of the U.S. economy, and cover the period 1976-2007. They include NIPA data on GDP and its deflator, capital, investment, the price of investment goods and depreciation, BLS CPS data on employment and on worker flows, and Fed data on the constituents of the discount factor and on tax and depreciation allowances (Fed computations). Appendix B elaborates on the sources and on data construction. These data have the following features:

(i) The data pertain to the U.S. private sector, thus not confounding the analysis with government hiring and investment.

(ii) Both hiring h and investment i refer to gross flows. Likewise, separation of workers ψ and depreciation for capital δ are gross flows.

(iii) The estimating equations take into account taxes and depreciation allowances.

Points (ii) and (iii) require a substantial amount of computation, which is elaborated in Appendix B.

Table 3 presents key sample statistics.

Table 3

5.3 Results

Table 4a presents the preferred estimates of the parameters. The table uses $\eta_1 = \eta_2 = 2, \eta_3 = 1$ and $\alpha = 0.68$ throughout, i.e. quadratic costs, linear interaction and a standard value for the labor share in production. The table reports the estimates and their standard errors, Hansen's (1982) J-statistic and its p-value. Appendix C reports the afore-cited alternative specifications and discusses them.

The first row of Table 4a presents estimates of a standard quadratic investment adjustment costs function with no role for hiring. The second row does the same for hiring with no investment. The third row allows for both but without any interaction between them. The fourth row is the same as the third but allows hiring costs to depend on labor market tightness. The fifth row allows for interaction of costs and the sixth row allows for both interaction of costs and dependence on labor market tightness.

Table 4a

All specifications yield precise estimates and the J-statistics do not reject the model. However, rows 1, 3 and 4 have low p-values. Row 1 with the standard quadratic specification has precise estimates but these imply very high adjustment costs; the e_1 point estimate is big, almost four times as high as the estimate of row 6. This has been the usual case in the literature. Row 2 provides for a reasonable estimate (of e_{20}) but does not allow for investment by construction. Row 3, which allows for both, still implies very high investment adjustment costs (high e_1). Row 4, which does not allow for interaction either, adds the dependence of hiring on labor market conditions and produces lower investment adjustment costs, though still higher than subsequent specifications. Rows 5 and 6 allow for investment and hiring costs to interact. This interaction is negatively signed (see the estimate of e_{30}) and it is the ingredient which allows the model to best fit the data. Row 6 also allows for hiring costs (and their interaction with investment) to depend on labor market conditions. This dependence $(e_{21} \text{ and } e_{31})$ turns out significant and leads to further reduction in the estimate of e_1 .

The implications thus far are as follows, taking into account also the alternative specifications discussed in Appendix C: quadratic costs and linear interaction of costs generate a fit of the data; the interaction is significant and is negatively signed, implying complementarity between investment and hiring (to be discussed below); there is a significant role for labor market conditions in hiring costs, whereby the latter are lower in "good times"; the investment part is problematic if hiring is left out or if hiring is introduced without interaction or without dependence on labor market conditions. By 'problematic' I mean that one needs high investment costs (high e_1) to fit the data. In what follows I shall refer to the results of rows 5 and 6 as the preferred specifications.

In order to further explore the implications of these estimates and characterize the joint behavior of investment and hiring, I use them in several ways. I start by looking at the magnitude of adjustment costs, comparing them to the findings in the literature (Section 6). I then look at the right hand side of the optimality equations and use the estimates to approximate and decompose the present values of hiring and investment which drive these decisions (Section 7). The following section (8) explores the implications of the results for the co-movement and business cycle behavior of investment and hiring and their determinants.

6 Gauging the Estimates: the Value of Adjustment Costs

The results of Table 4a merit inspection for plausibility and the derivation of the adjustment costs they imply. This is done by constructing the time series for total and marginal adjustment costs implied by the point estimates of the parameters of the g function and relating them to what is known on these issues. The estimated costs are interesting and important by themselves, as many models rely on their existence. The key moments are presented in Table 4b.

Table 4b

How do these compare to the literature?

Total costs as a fraction of GDP (i.e. $\frac{g}{f}$) are around 2.5% of output according to the preferred specifications (rows 5 and 6 of Table 4a), a reasonable estimate, as will be discussed below.

Marginal costs of hiring (i.e. g_h) in terms of average output per worker $\left(\frac{f}{n}\right)$ have a sample mean of 0.26 in row 5 and of 0.28 in row 6, the preferred

specifications. This is roughly equivalent to 39% (row 5) or 42% (row 6) of quarterly wages.¹⁰ In other words, firms pay the equivalent of about 5 to 5.5 weeks of wages to hire the **marginal** worker.

How does one evaluate this estimate? There is little empirical evidence on these adjustment costs in the literature. In what follows I cite some estimates on average hiring costs. Mortensen and Nagypal (2006, page 30) note that "Although there is a consensus that hiring costs are important, there is no authoritative estimate of their magnitude. Still, it is reasonable to assume that in order to recoup hiring costs, the firm needs to employ a worker for at least two to three quarters. When wages are equal to their median level in the standard model (w = 0.983), hiring costs of this magnitude correspond to less than a week of wages." The widely-cited Shimer (2005) paper calibrates these costs at 0.213 in terms similar to g_h here, using a linear cost function, which is equivalent to 1.4 weeks of wages. Hagedorn and Manovskii (2008) decompose this cost into two components: (i) the capital flow cost of posting a vacancy; they compute it to be - in steady state -47.4 percent of the average weekly labor productivity; (ii) the labor cost of hiring one worker, which, relying on micro-evidence, they compute to be 3 percent to 4.5 percent of quarterly wages of a new hire. The first component would correspond to a figure of 0.037 here; the second component would correspond to a range of 0.02 to 0.03 in the terms used here; together this implies 0.057 to 0.067in current terms or around 1.1 to 1.3 weeks of wages. Note that the results here refer to the marginal hire with convex costs; hence they are consistent with the cited estimates of average costs.

Older, micro evidence suggests a very wide range of estimates (see Hamermesh (1993, pp. 207-209)). These latter studies typically pertain to costs of net employment changes $(n_t - n_{t-1})$, as distinct from gross hiring (h_t) . Hence, there is no solid benchmark in this type of studies against which to compare the current estimates.

The marginal costs of investment (i.e. g_i) in terms of average output per unit of capital $(\frac{f}{k})$ have a sample mean of 0.61 in row 5 and of 0.72 in row 6 of Table 4b.¹¹ To give another, more intuitive, perspective on these numbers, consider how much one needs to add to the price of one unit of the investment good p^I in **marginal** adjustment costs: 5.7% on average in row 5, 6.7% on average in row 6. By contrast, the estimate of row 1 with only quadratic investment costs has a sample mean of 3.54 in terms of average output per unit of capital $(\frac{f}{k})$ or 33% to be added to the price of the investment good.

 $^{^{10}}$ Wages are 66% of output per worker on average, see Table 3.

¹¹The units of measurement – in terms of output per unit of capital $\frac{f}{k}$ – were chosen so as to facilitate comparison with existing studies, as discussed below.

How reasonable are these estimates? The most natural place to look for comparisons is the Q-literature which incorporates adjustment costs. Table 5 presents nine estimates of the investment equation from this literature. The equation links the investment-to-capital ratio to a measure of Tobin's Q. Note that these studies differ from each other and from the current study on many dimensions: the data sample used, the functional form assumed for marginal adjustment costs, additional variables included in the cost function, treatment of tax issues, and reduced form vs. structural estimation. Estimates of the curvature of the marginal cost function may be conditional on additional variables included in the analysis and reduced form estimates may be consistent with several alternative underlying structural models. The studies often came in response to previous estimates, each trying to introduce changes so as to improve on the preceding ones; some of these changes were substantial. Hence, Table 5 cannot give more than a rough idea as to the "neighborhood" of adjustment costs estimates.

Table 5

The table shows huge variation across studies: it ranges from marginal costs as low as 0.04 to as high as 60 (in terms of $\frac{f}{k}$). It should be noted that the differences in marginal cost estimates are usually due to differences in the parameter estimates, and not just due to the diversity in the rate of investment used. One can divide the results into three sets:

(i) High adjustment costs, as in studies 1 and 2. Marginal costs range between 3 to 60 in terms of average output per unit of capital. The implied total costs range between 15% to 100% of output. This set characterizes the earlier studies.

(ii) Moderate adjustment costs, as in studies 3, 5 and 6b. Marginal costs are around 1 in terms of average output per unit of capital. Total costs range between 0.5% to 6% of output.

(iii) Low adjustment costs, as in the rest of the studies, namely 4, 6a, 7, 8, and 9. Marginal costs are 0.04 to 0.50 of average output per unit of capital. Total costs range between 0.1% to 0.2% of output. The studies finding these magnitudes are micro studies, using cross-sectional or panel data.

Coming back to the initial question of comparing these estimates to the current findings, two conclusions emerge:

(i) The specification that I run that is closest to the one used in most studies of Table 5 is the one reported in row 1 of Table 4b. This is the specification positing a quadratic function and ignoring labor. The implied total costs are 4.4% of output (as in studies of the moderate adjustment costs set) and the implied marginal costs are 3.5 of average output per unit of capital (as in the high adjustment costs set). As indicated above, this is 33% of the price of a unit of investment good p^{I} . These implausible results are a major reason to reject these particular estimates here.

(ii) The preferred specifications – the GMM results of the full model, rows 5 and 6 of Table 4b – cannot be directly compared to the results of Table 5, as they take into account hiring costs through the interaction between hiring and investment costs and have a convex specification. In formal terms the marginal investment costs are specified by $\frac{g_i}{\frac{1}{k}} = \left[e_1\left(\frac{i}{k}\right)^{\eta_1-1} + \left(e_{30} + e_{31}\frac{v}{u+o}\right)\left(\frac{h}{n}\right)^{\eta_3}\left(\frac{i}{k}\right)^{\eta_3-1}\right]$ while most specifications of Table 5 posit $g_i = e_1\frac{i}{k}$. In particular, the expression in the current paper depends on $\frac{h}{n}$ in a substantial way. Nevertheless, looking at marginal costs as a fraction of output per unit of capital $\left(\frac{g_i}{\frac{1}{k}}\right)$, estimated at a mean of 0.61 or 0.72, the findings of Table 4b correspond to the third set, i.e., to low adjustment costs. Note that the estimation here uses time series, while the cited papers of the third set use cross-sectional or panel data.

Overall, then, the adjustment costs implied by the estimates are not high and are very reasonable in comparison to what is known from the literature.

Figure 2 presents plots of the estimated marginal costs functions over the sample range. The plot describes functions derived from the estimates of Table 4a.

Figure 2

Panel a shows that allowing for hiring costs has a big effect on the marginal investment costs function, moving it down substantially (compare the black line with the red and blue lines). Allowing for labor market conditions to affect the interaction of hiring costs with investment costs has a small effect, moving this function back up somewhat (compare the red and blue lines). Panel b shows similar changes for the marginal hiring cost function, albeit with different magnitudes. Hence, the figure clearly shows the importance of the interaction of investment and hiring costs and the (smaller) impact of labor market conditions.

7 The Value of Investment and Hiring

I have derived – through structural estimation – the adjustment costs function (g) which defines the present value of hiring (Q^N) and of investment (Q^K) . How are these values related to their expected future determinants, given that both hiring and investment are forward-looking variables? In this section, I follow the empirical methodology of the asset pricing literature in Finance and examine the present value relationships governing hiring and investment. This involves the study of the determinants of hiring and investment, using forecasting regressions, VARs and approximated relations. The analysis is based on the framework proposed by Campbell and Shiller (1988) and its more recent elaboration by Cochrane (2005, 2008), whose notation I follow. This model is often referred to as the dynamic, dividend-growth model.¹² Note that I do not consider stock prices here; I simply apply the empirical framework developed in the cited Finance literature to the current context. As mentioned above, the connections between the current framework and stock prices were explored in Merz and Yashiv (2007).

7.1 Asset Pricing Model

The model begins from the following two-period representation for the stock price (P) and dividends (D):

$$P_{t} = E_{t} \left(R_{t+1}^{-1} [D_{t+1} + P_{t+1}] \right)$$

$$\frac{P_{t}}{D_{t}} = E_{t} \left(R_{t+1}^{-1} [\frac{D_{t+1}}{D_{t}} + \frac{D_{t+1}}{D_{t}} \frac{P_{t+1}}{D_{t+1}}] \right)$$
(25)

where R is the gross return. Iterated forward this yields:

$$\frac{P_t}{D_t} = E_t \left(\sum_{j=0}^{\infty} \left(\prod_{k=1}^{j+1} R_{t+k}^{-1} \frac{D_{t+k}}{D_{t+k-1}} \right) \right)$$
(26)

These relationships hold true also ex-post if one defines returns as:

$$R_t = \frac{D_{t+1} + P_{t+1}}{P_t} \tag{27}$$

Using logs, this asset pricing relationship can be approximated as:

$$p_t - d_t = k + E_t \left(d_{t+1} - d_t - r_{t+1} + \rho(p_{t+1} - d_{t+1}) \right)$$
(28)

where:

¹²Lettau and Ludvigson (2009) and Koijen and Van Nieuwerburgh (2010) provide surveys of these empirical studies and a discussion of their implications for asset pricing.

$$p_t \equiv \ln P_t$$

$$d_t = \ln D_t$$

$$r_t = \ln R_t$$

$$k = \ln(1 + \frac{P}{D}) - \rho(p - d)$$

$$\rho = \frac{\frac{P}{D}}{1 + \frac{P}{D}}$$

and where P, D are steady state or long-term average values.

Equation (28) is an ex-ante formulation using conditional expectations. The following ex-post equation holds true as well, when using (27):

$$p_t - d_t = k + (d_{t+1} - d_t - r_{t+1} + \rho(p_{t+1} - d_{t+1}))$$
(29)

Based on (29), the following ex-post relations in levels and in variance hold true in approximation:

$$p_t - d_t \simeq \sum_{j=1}^{\infty} \rho^{j-1} k + \left(\sum_{j=0}^{\infty} \rho^j \left(d_{t+j+1} - d_{t+j} - r_{t+j+1} \right) \right)$$
(30)

$$var(p_t - d_t) \simeq cov \left[p_t - d_t, \sum_{j=0}^{\infty} \rho^{j-1} (d_{t+j+1} - d_{t+j}) \right]$$
 (31)
 $-cov \left[p_t - d_t, \sum_{j=0}^{\infty} \rho^{j-1} r_{t+j+1} \right]$

The current price dividend ratio $(p_t - d_t)$ is related to future dividend growth $(d_{t+j+1} - d_{t+j})$ and to future returns (r_{t+j+1}) , with the relevant discounting (using ρ^{j}). The price-dividend ratio will be higher when future dividend growth is higher and/or when future returns are lower.

7.2**Empirical Methodology**

These relationships have been examined in the Finance literature in a number of ways. One is to estimate forecasting regressions of the type:

$$(p_{t+1} - d_{t+1}) = a + \phi(p_t - d_t) + e_{p,t}$$

$$d_{t+1} - d_t = c + b_d(p_t - d_t) + e_{d,t}$$

$$r_{t+1} = d + b_r(p_t - d_t) + e_{r,t}$$
(32)

The log price dividend ratio $(p_t - d_t)$ is expected to forecast future dividend growth $(d_{t+1} - d_t)$ and/or future returns (r_{t+1}) . These equations are examined as separate regressions or within a system. The last two regressions have been estimated also using a longer horizons, so on the LHS may appear longer horizon dividend growth $(d_{t+H} - d_t)$ or compounded returns $(r_{s,t+H} = r_{s,t+1} + r_{t+1,t+2} + \dots + r_{t+H-1,t+H})$, where H is the forecast horizon. Using equation (29), the coefficients in this system should obey the restriction:

$$b_d - b_r = 1 - \rho\phi \tag{33}$$

A second, more general formulation, encompassing (32) as a special case, is to estimate a restricted VAR on the de-meaned variables:

$$\begin{pmatrix} p_{t+1} - d_{t+1} \\ d_{t+1} - d_t \\ r_{t+1} \end{pmatrix} = B \begin{pmatrix} p_t - d_t \\ d_t - d_{t-1} \\ r_t \end{pmatrix} + \begin{pmatrix} \varepsilon_{1_t} \\ \varepsilon_{2_t} \\ \varepsilon_{3_t} \end{pmatrix}$$
(34)

Defining:

$$z_{t} = \begin{pmatrix} p_{t} - d_{t} \\ d_{t} - d_{t-1} \\ r_{t} \end{pmatrix}, \text{ with the variables de-meaned}$$
$$e_{1} = (1, 0, 0)$$
$$e_{2} = (0, 1, 0)$$
$$e_{3} = (0, 0, 1)$$
$$\underline{\varepsilon}_{t} = \begin{pmatrix} \varepsilon_{1_{t}} \\ \varepsilon_{2_{t}} \\ \varepsilon_{3_{t}} \end{pmatrix}$$
This VAB can be written as:

This VAR can be written as:

$$z_{t+1} = B z_t + \underline{\varepsilon} \tag{35}$$

Equations (29) and (30) can be written in the same terms as:

$$e_{1}z_{t} = k + e_{2}z_{t+1} - e_{3}z_{t+1} + \rho e_{1}z_{t+1}$$

$$= k + e_{2}z_{t+1} - e_{3}z_{t+1} + \rho(k + e_{2}z_{t+2} - e_{3}z_{t+2} + \rho e_{1}z_{t+2})$$

$$= \sum_{j=1}^{\infty} \rho^{j-1}k + \sum_{j=0}^{\infty} \rho^{j} (e_{2} - e_{3}) B^{j+1}z_{t}$$
(36)

Hence the restrictions for this VAR are (allowing for de-meaning, hence dropping the first term on the RHS):

$$e_1 z_t = \sum_{j=0}^{\infty} \rho^j \left(e_2 - e_3 \right) B^{j+1} z_t \tag{37}$$

which gives:

$$e_1(I - \rho B) - (e_2 - e_3) B = 0 \tag{38}$$

Note that restriction (33) above is a special case of the last set of restrictions (38).

A third way used by this empirical literature is to truncate the RHS of (31) at date T and compute the components of this variance decomposition.

To connect the first and third ways, note the following (see Cochrane (2008), pp. 1544-1545). First, divide (33) by $1 - \rho \phi$ to get:

$$\frac{b_d}{1-\rho\phi} - \frac{b_r}{1-\rho\phi} = 1$$

Define:

•

$$b_d^{lr} = \frac{b_d}{1 - \rho\phi}$$
$$b_r^{lr} = \frac{b_r}{1 - \rho\phi}$$

to be the long run regression coefficients of log dividend growth on the log price -dividend ratio and of log returns on the log price -dividend ratio (i.e., coefficients of the regressions of $\sum_{j=0}^{\infty} \rho^{j-1} (d_{t+j+1} - d_{t+j})$ on $p_t - d_t$ and of $\sum_{j=0}^{\infty} \rho^{j-1} r_{t+j+1}$ on $p_t - d_t$). This means:

$$b_d^{lr} - b_r^{lr} = 1$$

From the third type of computation, divide (31) throughout by $var(p_t-d_t)$ to get:

$$1 \simeq cov \left[p_t - d_t, \sum_{j=0}^{\infty} \rho^{j-1} \left(d_{t+j+1} - d_{t+j} \right) \right]$$
$$-cov \left[p_t - d_t, \sum_{j=0}^{\infty} \rho^{j-1} r_{t+j+1} \right]$$

This too yields:

$$b_d^{lr} - b_r^{lr} \simeq 1$$

Employing the first way, one gets estimates of b_d^{lr} and b_r^{lr} by running regressions using the whole sample. Employing the third way, one gets estimates of b_d^{lr} and b_r^{lr} by computing the truncated (*T* periods) co-variance terms.

7.3 Implementing the Model for Hiring and Investment

I cast the estimated model of hiring and investment into this asset pricing framework by defining P and D for the optimal investment equation and for the optimal hiring equation. The "price" P is the value of investment or the value of hiring; this is essentially marginal Q for capital investment (Q^K) and marginal Q for labor hiring (Q^N) , each divided by the relevant productivity; the "dividend" D is the flow of net income from capital or from labor.

Consider the investment equation (see equation (17)):

$$\frac{(1-\tau_t)\left(g_{i_t}+p_t^I\right)}{\frac{f_t}{k_t}} = \left\{\frac{\frac{f_{t+1}}{k_{t+1}}}{\frac{f_t}{k_t}}\frac{\beta_{t+1}\left(1-\tau_{t+1}\right)}{\frac{f_{t+1}}{k_{t+1}}}\left[f_{k_{t+1}}-g_{k_{t+1}}+(1-\delta_{t+1})(g_{i_{t+1}}+p_{t+1}^I)\right]\right\}$$
(39)

I define the following asset pricing terms:

$$P_{t+k}^{1} = \frac{(1 - \tau_{t+k}) \left(g_{i_{t+k}} + p_{t+k}^{I}\right)}{\frac{f_{t+k}}{k_{t+k}}} \equiv \frac{Q_{t+k}^{K}}{\frac{f_{t+k}}{k_{t+k}}}$$
(40)
$$D_{t+k}^{1} \equiv \frac{(1 - \tau_{t+k}) \frac{(f_{k_{t+k}} - g_{k_{t+k}})}{\frac{f_{t+k}}{k_{t+k}}}}{(1 - \delta_{t})}$$

Likewise for the hiring equation (see equation (18)):

$$\frac{(1-\tau_t)g_{h_t}}{\frac{f_t}{n_t}} = \left\{ \frac{\frac{f_{t+1}}{n_{t+1}}}{\frac{f_t}{n_t}} \frac{\beta_{t+1}\left(1-\tau_{t+1}\right)}{\frac{f_{t+1}}{n_{t+1}}} \left[f_{n_{t+1}} - g_{n_{t+1}} - w_{t+1} + (1-\psi_{t+1})g_{h_{t+1}} \right] \right\}$$
(41)

I define:

$$P_{t+k}^{2} \equiv \frac{(1 - \tau_{t+k}) g_{h_{t+k}}}{\frac{f_{t+k}}{n_{t+k}}} \equiv \frac{Q_{t+k}^{N}}{\frac{f_{t+k}}{n_{t+k}}}$$
(42)
$$D_{t+k}^{2} = \frac{(1 - \tau_{t+k}) \left(\frac{f_{n_{t+k}} - g_{n_{t+k}} - w_{t+k}}{\frac{f_{t+k}}{n_{t+k}}}\right)}{1 - \psi_{t}}$$

These prices and "dividends" are not observed on the market, as in the Finance literature. Rather, they represent what the firm actually gets from its use of capital and labor in production. Thus, the "dividend" in the investment case is the net marginal productivity of capital; in the hiring case it is net labor profitability, i.e., the net marginal product of labor less the wage. These "dividends" do not depend on institutional or financial considerations of firms as dividends do in the Finance context.

Table 6 presents the different tests discussed above, separately for the two equations – investment and hiring.¹³

Table 6

There are a number of results which stand out, keeping in mind that "prices", "dividends" and "returns" are the variables defined above for the current context and are not to be confused with stock market variables:

(i) In the single variable forecasting regressions, most coefficients are significant. Using the terms of the asset pricing literature, this implies that dividend growth and returns are forecastable or predictable by the pricedividend ratio.

(ii) In the case of hiring, the single variable forecasting regressions adjusted R-squared (\overline{R}^2) increases with the forecast horizon for dividend growth, reaching high levels of almost 0.80 at 16 and 20 quarters. For the return forecast these adjusted R-squared decrease with the forecast horizon to around 0. At four quarters, the \overline{R}^2 is around 0.30. This means that dividend growth is highly forecastable and returns are less so. Dividend growth in the current context means the changes in log labor profitability, which is after-tax labor productivity less wages.

¹³When presenting the approximated variance decomposition, I report the error of the approximated variance equation (31) divided by the variance of the log price-dividend ratio, $\frac{e}{var(p_t-d_t)}$, namely the difference between the LHS and the RHS divided by the LHS. Note from equation (31) that this can be positive or negative. This error comes from estimation and approximation errors and from the sample truncation.

(iii) In the case of investment, the single variable forecasting regressions \overline{R}^2 increase with the forecast horizon for dividend growth and for the return forecast. At 20 quarters they reach about 0.20 for dividend growth and almost 0.40 for the return forecast.

Points (i)-(iii) indicate results which are markedly different from those typically obtained in the Finance literature (albeit, there, relating to stock market variables). Here there is far better forecasting power for the hiring equation, especially with respect to dividend growth.¹⁴ In the Finance literature, the values of the \overline{R}^2 coefficients noted above are seldom higher than 0.10 for one-period forecasts and 0.30 for long horizon forecasts in terms of future returns. They are around 0 for future dividend growth. Likewise, dividend growth coefficients are typically not statistically significant in that literature. The results for the investment equation are somewhat more similar to those obtained in this Finance literature and so is also the pattern of a rise in explanatory power with the forecast horizon.

(iv) The analysis for hiring indicates that the price-dividend ratio is persistent (ϕ and b_{pp} are estimated to be above 0.9), that a simple restricted system produces estimates similar to the single-variable regressions, and that the complete, restricted VAR analysis indicates a stronger predictive effect for dividend growth ($b_{dp} > b_d$) and a weaker one for returns ($|b_{rp}| < |b_r|$), relative to the single-variable regressions. All estimated coefficients are significant.

(v) The analysis for investment indicates that the price-divided ratio is extremely persistent (ϕ and b_{pp} are estimated to be around 0.99), a finding that is similar to many Finance studies for stock-price to dividend ratios. The simple restricted system produces estimates similar to the single-variable regressions. The complete, restricted VAR analysis indicates a stronger predictive effect for dividend growth ($b_{dp} > b_d$) and a weaker one for returns ($|b_{rp}| < |b_r|$), relative to the single-variable regressions. But the \overline{R}^2 in some of the investment VAR equations is low or even negative and the estimate of b_d is insignificant.

Points (iv) and (v) basically show that the single regressions, systems, and VARs yield similar results, but that the complete, restricted VARs assign a different strength to the predictor variable.

(vi) In the hiring case, the variance decomposition yields approximated values that have a relatively small error (see last row of panel c). There is also a close correspondence between the variance decomposition results and the estimates of long run coefficients. It indicates that hiring values co-move

¹⁴Compare, for example, the results here to those discussed by Cochrane (2008), Lettau and Ludvigson (2009) and Koijen and Van Nieuwerburgh (2010).

more with future dividend growth (around 60% to 80% of the variance of price-dividend ratios) than with future returns (the complementary 40% to 20%, in absolute value). Even a small number of periods (T = 10) suffices to get this result in the approximated relationship.

(vii) In the investment case, the variance decomposition yields approximated values that have a relatively small error only at long horizons (i.e., at a high value of T). It indicates that investment values co-move more with future returns (around 50% to 80% of the variance of price-dividend ratios in absolute value) than with future dividend growth (the complementary 50% to 20%).

Points (vi) and (vii) imply that hiring and investment relate differentially to their future determinants.

Taken together, these results imply that the connection between "pricedividend" ratios with future variables is significant and seems stronger or tighter for hiring than for investment. Both are stronger or much stronger than the typical findings in the Finance literature for stock price-dividend ratios. Hiring values are linked more to future dividend growth, i.e., to changes in labor profitability, while investment values are linked more to future returns.

8 The Co-Movement and Cyclical Behavior of Hiring and Investment

This section examines the implications of the estimates for the co-movement of hiring and investment and their cyclical behavior. It begins with a discussion of the significance of the finding of negative interaction (sub-section 8.1). This is followed by a discussion of the sensitivity of investment and hiring to their present values (8.2). Finally, the second moments related to co-movement and cyclical analysis are presented and discussed (8.3).

8.1 Negative Interaction Engenders Simultaneity

Across all specifications of Table 4a, the estimate of the coefficient of the interaction term, e_{30} , is negative. This negative point estimate implies a negative value for g_{hi} and, therefore, a positive sign for $\partial h_t/\partial Q^k$ and for $\partial i_t/\partial Q^n$ (for the full derivations of these derivatives, as well as the relevant elasticities, see Appendix A.) Note that $\partial i_t/\partial Q^k$ and $\partial h_t/\partial Q^n$ are positive due to convexity. Hence, when the marginal value of investment Q^K rises, both investment and hiring rise. A similar argument shows that they both rise when the marginal value of hiring Q^N rises.

The signs of these elasticities and derivatives imply that for given levels of investment, total and marginal costs of investment decline as hiring increases. Similarly, for given levels of hiring, total and marginal costs of hiring decline as investment increases. This finding of complementarity between investment and hiring is to be expected as it implies that they should be simultaneous. One interpretation of this result is that simultaneous hiring and investment is less costly than sequential hiring and investment of the same magnitude. This may be due to the fact that simultaneous action by the firm is less disruptive to production than sequential action. This feature is quantified by the following 'scope' statistic:

$$\frac{g(0,\frac{h}{n}) + g(\frac{i}{k},0) - g(\frac{i}{k},\frac{h}{n})}{g(\frac{i}{k},\frac{h}{n})}$$

The statistic measures how much – in percentage terms – is simultaneous investment and hiring cheaper than non-simultaneous action. Its sample mean and standard deviation are presented in the first column of Table 7.

Table 7

The scope is 0 by construction in rows 1-5 as there is no interaction there. For the preferred specifications, it is on average 61% (row 5) or 23% (row 6) out of total adjustment costs. This means that there are substantial savings of costs when adjusting both capital and labor together. Hence the preferred estimates of rows 5 and 6 in Table 4a imply that there is meaningful interrelation between hiring and investment costs. One decision by the firm is strongly dependent on the other.

8.2 The Elasticities of Hiring and Investment w.r.t Present Values

Table 7 further quantifies the relations between hiring and investment by presenting the mean and standard deviation of the elasticities of investment i and of hiring h with respect to the present values Q^k and Q^n . The table shows that the investment is very highly elastic with respect to the present value of investing Q^K ; the high elasticities of rows 4 to 6 contrast with the relatively low elasticity of row 1. This is consistent with the finding of high adjustment costs and therefore low elasticity in row 1 relative to the other formulations. Hiring has around unitary elasticity with respect to its present value Q^N . The cross elasticities are low for investment w.r.t Q^N (an elasticity of around 0.3-0.4) and high for hiring w.r.t Q^K (1.7 and 3.3).

The following distinction, however, is important. The afore-going argument favors simultaneous hiring and investment, i.e., positive levels of both $(\frac{i}{k}, \frac{h}{n} > 0)$. Thus the representative firm is hiring and investing at the same time. But it does **not** necessarily imply highly positive co-movement or correlation between hiring and investment. In other words, investment and hiring take place at the same time, but it is possible to have one rise while the other rises, stays the same or even declines. Suppose, for example, Q^K rises while Q^N declines. The rise in Q^K will lead to higher investment and higher hiring, while the fall in Q^N will lead to lower investment and lower hiring. The elasticity estimates of Table 7 imply that the Q^K movements and the Q^N movements engender different responses. Therefore it is possible that investment will rise with the rise in Q^K while hiring falls with the fall in Q^N .

8.3 Co-Movement and Cyclical Analysis

To see these relations in the data, Figure 3 shows the sample behavior of $\frac{i}{k}$ and $\frac{h}{n}$, of the estimated Q^{K} (net of p^{I}) and Q^{N} using the point estimates of row 6 of Table 4a, and of non-financial business sector GDP f. The series are all logged and HP or BP filtered. Table 8 shows the co-movement of the same series, contemporaneously and in cross-correlations.

Figure 3 and Table 8

The figure and the table show the following:

(i) The investment rate moves together with the estimated Q^K (net of p^I), contemporaneously, with a correlation of around 0.85, and at lags and leads up to a year. The hiring rate moves together with the estimated Q^N , contemporaneously, with a correlation of around 0.40, and at leads up to two years, though the relation weakens in the cross-correlations. These results are consistent with the elasticities reported in Table 7.

(ii) There is negative co-movement of Q^K (net) and Q^N , contemporaneously with a correlation of about -0.90, and at lags and leads of up to a year. This is consistent with a negative co-movement of the investment and hiring rates, contemporaneously with a correlation of -0.10 or -0.20 (depending on the filtering) and at two year lags. Investment and hiring rates thus follow similar patterns of cross correlations as do their marginal Qs, albeit with lower correlations in absolute value.

(iii) Investment rates $\frac{i}{k}$, the estimated marginal investment costs $\frac{g_i}{f/k}$, and the estimated Q^K (net of p^I) are all pro-cyclical, contemporaneously and up to 1 year lags and leads under all specifications.

(iv) Hiring rates $\frac{h}{n}$, the estimated marginal hiring costs $\frac{g_h}{f/n}$ and the estimated Q^N are all counter-cyclical, contemporaneously and usually up to 1 year lags and leads under all specifications.

(v) Comparing panels IIc and IId, which differ in modelling the effect of labor market conditions, one finds:

a. The counter-cyclicality of hiring costs and of Q^N strengthen, once labor market conditions are included. As $\frac{v}{o+n}$ is pro-cyclical and it enters with a negative sign $(e_{21} < 0)$, then in good (bad) times, hiring costs decline (increase), hence the strengthened counter-cyclicality.

b. The pro-cyclicality of investment costs and Q^K strengthen when including labor market conditions. As $\frac{v}{o+n}$ is pro-cyclical and it enters with a positive sign $(e_{31} > 0)$, then in good (bad) times investment costs increase (decline), hence strengthening their pro-cyclicality.

The key notion here is the forward-looking aspect of investment and hiring. These results imply the following cyclical "story": in a recession investment rates and their present values decline while hiring rates and their present values increase. This is so because the rates move together with their present values. These statements also apply with lags or leads of up to a year. The emerging idea is that the present values of hiring and investment are both tightly linked to the cycle. But in the U.S. data examined here, the present value of investment was pro-cyclical while that of hiring was counter-cyclical.

9 Conclusions

The paper has shown that a model of aggregate investment and hiring with adjustment costs is a consistent and reasonable model, which fits U.S. data. It was shown that it is important to examine investment and hiring together, to allow for the interaction between their costs and to allow for dependence on labor market conditions. It is difficult to capture hiring behavior and (even more) investment behavior without considering the other factor. The model fits the data even though adjustment costs are estimated to be moderate or small relative to what has been previously found in the literature. While hiring and investment decisions have a similar structure, the actual series behave differently. This has to do with the differential behavior of the driving forces, the present values of hiring and of investment and with the differential relations of investment and hiring with the relevant components of these present values. Investment is driven mostly by expected returns while hiring depends mostly on changes in labor profitability.

Importantly, in the sample period, the present value of investment (Q^K)

behaved pro-cyclically while the present value of hiring (Q^N) behaved countercyclically. These patterns engendered the behavior of investment and hiring described above, including their negative co-movement. In recessions, while employment and the worker job-finding rate declined, the present value of hiring rose, i.e., expected future marginal profitability of labor rose. In these recessionary times, firms, looking into the future, predicted higher profitability from employing labor. Hence, they increased the rate at which they hired workers. Relying on the empirical asset-pricing analysis of Section 7, we know that these expected future gains were related much more to labor profitability than to future discount rates.

This paper, purposefully, did not specify a full DSGE model. This was done in order to focus on firms' investment and hiring decisions and not let the analysis be affected by possible mis-specifications or problematics in other parts of the macroeconomy To account for firm investment and hiring behavior, one does not need to get into issues such as optimal intertemporal consumption and labor choices of the individual, with all the associated empirical difficulties. Future research may, nonetheless, take up such a model in an attempt to map the linkages between the shocks to the economy and the differential evolution of the relevant present values.

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Appendix A The Adjustment Cost Function and Its Key Derivatives and Elasticities

The Adjustment Cost Function

$$g(\cdot) = \left[\frac{e_1}{\eta_1} (\frac{i_t}{k_t})^{\eta_1} + \frac{(e_{20} + e_{21} \frac{v_t}{u_t + o_t})}{\eta_2} (\frac{h_t}{n_t})^{\eta_2} + \frac{(e_{30} + e_{31} \frac{v_t}{u_t + o_t})}{\eta_3} \left(\frac{i_t}{k_t} \frac{h_t}{n_t}\right)^{\eta_3}\right] f(z_t, n_t, k_t).$$
(43)

First Derivatives

$$g_{i_t} = \left[e_1 \left(\frac{i_t}{k_t}\right)^{\eta_1 - 1} + \left(e_{30} + e_{31} \frac{v_t}{u_t + o_t}\right) \left(\frac{h_t}{n_t}\right)^{\eta_3} \frac{i_t}{k_t}^{\eta_3 - 1} \right] \frac{f_t}{k_t}$$
(44)
$$= \left[\left(e_{20} + e_{21} \frac{v_t}{u_t}\right) \left(\frac{h_t}{t}\right)^{\eta_{2-1}} + \left(e_{30} + e_{31} \frac{v_t}{u_t}\right) \left(\frac{i_t}{t}\right)^{\eta_3} \frac{h_t}{h_t}^{\eta_3 - 1} \right] \frac{f_t}{f_t}$$

$$g_{h_t} = \left[(e_{20} + e_{21} \frac{v_t}{u_t + o_t}) (\frac{n_t}{n_t})^{\eta_{2-1}} + (e_{30} + e_{31} \frac{v_t}{u_t + o_t}) \left(\frac{u_t}{k_t}\right)^{-1} \frac{n_t}{n_t} \right] \frac{f_t}{n_t}$$
(45)

$$g_{k_{t}} = -\left[e_{1}\left(\frac{i_{t}}{k_{t}}\right)^{\eta_{1}} + \left(e_{30} + e_{31}\frac{v_{t}}{u_{t} + o_{t}}\right)\left(\frac{h_{t}}{n_{t}}\frac{i_{t}}{k_{t}}\right)^{\eta_{3}}\right]\frac{f_{t}}{k_{t}}$$

$$+(1-\alpha)\left[\frac{e_{1}}{\eta_{1}}\left(\frac{i_{t}}{k_{t}}\right)^{\eta_{1}} + \frac{\left(e_{20} + e_{21}\frac{v_{t}}{u_{t} + o_{t}}\right)}{\eta_{2}}\left(\frac{h_{t}}{n_{t}}\right)^{\eta_{2}} + \frac{\left(e_{30} + e_{31}\frac{v_{t}}{u_{t} + o_{t}}\right)}{\eta_{3}}\left(\frac{i_{t}}{k_{t}}\frac{h_{t}}{n_{t}}\right)^{\eta_{3}}\right]\frac{f_{t}}{k_{t}}$$

$$(46)$$

$$g_{n_{t}} = -\left[\left(e_{20} + e_{21} \frac{v_{t}}{u_{t} + o_{t}} \right) \left(\frac{h_{t}}{n_{t}} \right)^{\eta_{2}} + \left(e_{30} + e_{31} \frac{v_{t}}{u_{t} + o_{t}} \right) \left(\frac{h_{t}}{n_{t}} \frac{i_{t}}{k_{t}} \right)^{\eta_{3}} \right] \frac{f_{t}}{n_{t}}$$

$$+ \alpha \left[\frac{e_{1}}{\eta_{1}} \left(\frac{i_{t}}{k_{t}} \right)^{\eta_{1}} + \frac{\left(e_{20} + e_{21} \frac{v_{t}}{u_{t} + o_{t}} \right)}{\eta_{2}} \left(\frac{h_{t}}{n_{t}} \right)^{\eta_{2}} + \frac{\left(e_{30} + e_{31} \frac{v_{t}}{u_{t} + o_{t}} \right)}{\eta_{3}} \left(\frac{i_{t}}{k_{t}} \frac{h_{t}}{n_{t}} \right)^{\eta_{3}} \right] \frac{f_{t}}{n_{t}}$$

$$(47)$$

Second Derivatives

$$g_{ii_{t}} = \underbrace{\left[\begin{array}{c} e_{1}(\eta_{1}-1)\left(\frac{i_{t}}{k_{t}}\right)^{\eta_{1}-2} \\ +(e_{30}+e_{31}\frac{v_{t}}{u_{t}+o_{t}})(\eta_{3}-1)\left(\frac{i_{t}}{k_{t}}\frac{h_{t}}{n_{t}}\right)^{\eta_{3}-2}\left(\frac{h_{t}}{n_{t}}\right)^{2} \end{array}\right]}_{\widetilde{g}_{ii}} \underbrace{\frac{f(z_{z},n_{t},k_{t})}{k_{t}^{2}}}_{\widetilde{g}_{ii}}$$
(48)

$$g_{hh_{t}} = \underbrace{\begin{bmatrix} (e_{20} + e_{21}\frac{v_{t}}{u_{t}+o_{t}})(\eta_{2}-1) & \left(\frac{h_{t}}{n_{t}}\right)^{\eta_{2}-2} \\ + (e_{30} + e_{31}\frac{v_{t}}{u_{t}+o_{t}})(\eta_{3}-1) & \left(\frac{i_{t}}{k_{t}}\frac{h_{t}}{n_{t}}\right)^{\eta_{3}-2} & \left(\frac{i_{t}}{k_{t}}\right)^{2} \end{bmatrix}}_{\widetilde{g}_{hh}} \frac{f(z_{z}, n_{t}, k_{t})}{n_{t}^{2}} \quad (49)$$

$$g_{ih_{t}} = g_{hi_{t}} = \underbrace{\left[(e_{30} + e_{31}\frac{v_{t}}{u_{t}+o_{t}})\eta_{3} & \left(\frac{i_{t}}{k_{t}}\frac{h_{t}}{n_{t}}\right)^{\eta_{3}-1}\right]}_{\widetilde{g}_{ih}} \frac{f(z_{z}, n_{t}, k_{t})}{k_{t}n_{t}} \quad (50)$$

Elasticities

Starting from the F.O.C and differentiating the following is obtained:¹⁵

$$\begin{split} \frac{\partial i_t}{\partial Q^K} \frac{Q^K}{i_t} &= \frac{\widetilde{g}_{hh}}{(1-\tau_t) \left[\widetilde{g}_{ii}\widetilde{g}_{hh} - \widetilde{g}_{ih}\widetilde{g}_{hi}\right]} \frac{\frac{Q^K}{\frac{f_t}{k_t}}}{\frac{i_t}{k_t}} \\ \frac{\partial h_t}{\partial Q^k} \frac{Q^K}{h_t} &= -\frac{\widetilde{g}_{hi}}{(1-\tau_t) \left[\widetilde{g}_{ii}\widetilde{g}_{hh} - \widetilde{g}_{ih}\widetilde{g}_{hi}\right]} \frac{\frac{Q^K}{\frac{f_t}{k_t}}}{\frac{h_t}{n_t}} \\ \frac{\partial h_t}{\partial Q^N} \frac{Q^N}{h_t} &= -\frac{\widetilde{g}_{ii}}{(1-\tau_t) \left[\widetilde{g}_{ii}\widetilde{g}_{hh} - \widetilde{g}_{ih}\widetilde{g}_{hi}\right]} \frac{\frac{Q^N}{\frac{f_t}{n_t}}}{\frac{h_t}{n_t}} \\ \frac{\partial i_t}{\partial Q^N} \frac{Q^N}{i_t} &= -\frac{\widetilde{g}_{ih}}{(1-\tau_t) \left[\widetilde{g}_{ii}\widetilde{g}_{hh} - \widetilde{g}_{ih}\widetilde{g}_{hi}\right]} \frac{\frac{Q^N}{\frac{f_t}{n_t}}}{\frac{h_t}{n_t}} \end{split}$$

¹⁵The complete derivation is available upon request.

Appendix B: The Data

variable	symbol	definition
GDP	f	gross value added of NFCB
GDP deflator	p^{f}	price per unit of gross value added of NFCB
wage share	$\frac{wn}{f}$	numerator: compensation of employees in NFCB
discount rate 1	r	the rate of consumption growth minus 1
discount rate 2	r	the weighted average cost of capital – see note 1
employment	$\mid n$	employment in nonfinancial corporate business sector
hiring	h	gross hires
separation rate	ψ	gross separations divided by employment
vacancies	v	adjusted Help Wanted Index
investment	i	gross investment in NFCB sector
capital stock	k	stock of private nonresidential fixed assets in NFCB sector
depreciation	δ	depreciation of the capital stock
price of capital goods	p^{I}	real price of new capital goods

variable	symbol	source
GDP	f	NIPA accounts, table 1.14, line 40
GDP deflator	p^{f}	NIPA table 1.15, line 1
wage share	$\frac{wn}{f}$	NIPA table 1.14 , lines 17 and 20
discount rate 1	r	COMPLETE
discount rate 2	r	Fed; see note 1
employment	$\mid n$	CPS; see note 2
hiring	h	CPS; see note 2
separation rate	$ \psi $	CPS; see note 2
vacancies	v	Conference Board; see note 3
investment	i	BEA and Fed Flow of Funds; see note 4
capital stock	k	BEA and Fed Flow of Funds; see note 4
depreciation	δ	BEA and Fed Flow of Funds; see note 4
price of capital goods	p^{I}	NIPA and U.S. tax foundation; see note 5

Notes:

1. The discount rate and the discount factor

I use two alternatives for the firms' discount rate r_t and the corresponding discount factor $\beta_t = \frac{1}{1+r_t}$: a. The discount rate based on a DSGE model with logarithmic utility

 $U(c_t) = \ln c_t.$

Then in general equilibrium:

$$U'(c_t) = U'(c_{t+1}) \cdot (1 + r_t)$$

Hence:

$$\beta_t = \frac{c_t}{c_{t+1}}$$

b. Following the weighted average cost of capital approach in corporate finance, the discount rate is a weighted average of the returns to debt, r_t^b , and equity, r_t^e :

$$r_t = \omega_t r_t^b + (1 - \omega_t) r_t^e,$$

with

$$\begin{array}{lll} r^b_t &=& \left(1-\tau_t\right)r^{CP}_t-\theta_t \\ r^e_t &=& \displaystyle \frac{\widetilde{cf}_t}{\widetilde{s}_t}+\widetilde{\widehat{s}}_t-\theta_t \end{array}$$

where:

(i) ω_t is the share of debt finance. I calculate it on the basis of Level Tables of Flow of Funds accounts (files ltabs.zip). The calculations are as follows:

1. D = Credit market instruments (FL104104005 in the Coded Tables ltabs.zip, table L.102) + Trade payables (FL103170005 in the Coded Tables ltabs.zip, table L.102)

2. E = Market value of equities (FL103164003 in the Coded Tables ltabs.zip, table L.102)

3. Debt share = D/(D+E).

(ii) The definition of r_t^b reflects the fact that nominal interest payments on debt are tax deductible. r_t^{CP} is Moody's seasoned Aaa commercial paper rate (Federal Reserve Board table H15). The tax rate is τ as discussed below.

(iii) θ denotes inflation and is measured by the GDP-deflator of p^f .

(iv) For equity return I use the CRSP Value Weighted NYSE, Nasdaq and Amex nominal ex-dividend returns $(\frac{\widetilde{cf}_t}{\widetilde{s}_t} + \widetilde{s}_t$ in terms of the model, using tildes to indicate nominal variables) deflated by the inflation rate θ).

2. Employment, hiring and separations

As a measure of employment in nonfinancial corporate business sector (n) I take wage and salary workers in non-agricultural industries (series ID LNS12032187) less government workers (series ID LNS12032188), less self-employed workers (series ID LNS12032192), less unpaid family workers (series ID LNS12032193). All series originate from CPS databases. I do not subtract

workers in private households (the unadjusted series ID LNU02032190) from the above due to lack of sufficient data on this variable.

To calculate hiring and separation rates for the whole economy I use the series kindly provided by Ofer Cornfeld. This computation first builds the flows between E (employment), U (unemployment) and N (not-in-the-labor-force) that correspond to the E, U, N stocks published by CPS. The methodology of adjusting flows to stocks is taken from BLS, and is given in Frazis et al (2005). This methodology, applied by BLS for the period 1990 onward, produces a dataset that appears in <u>http://www.bls.gov/cps/cps_flows.htm</u>. Here the series have been extended back to 1976.

The quarterly separation rate (ψ) and the quarterly hiring rate (h/n) for the whole economy are defined as follows:

$$\psi = \frac{EN + EU}{E}$$
$$h/n = \frac{NE + UE}{E}$$

where the employment (E) is the quarterly average of the original seasonally adjusted total employment series from BLS (LNS12000000).

3. Vacancies and Market Tightness

In order to compute $\frac{v}{n+o}$ I use:

(i) The vacancies series based on the Conference Board Composite Help-Wanted Index that takes into account both printed and web job advertisements, as computed by Barnichon (2010), available at

http://sites.google.com/site/regisbarnichon/research.

This index was multiplied by a constant to adjust its mean to the mean of the JOLTS vacancies series over the overlapping sample period (2001Q1–2007Q4).

(ii)The unemployment and the out of labor force series are the BLS CPS data.

4. Investment, capital and depreciation

The goal here is to construct the quarterly series for real investment flow i_t , real capital stock k_t , and depreciation rates δ_t . I proceed as follows:

• Construct end-of-year fixed-cost net stock of private nonresidential fixed assets in NFCB sector, K_t . In order to do this I use the quantity index for net stock of fixed assets in NFCB (FAA table 4.2, line 28, BEA).

- Construct annual fixed-cost depreciation of private nonresidential fixed assets in NFCB sector, D_t . The chain-type quantity index for depreciation originates from FAA table 4.5, line 28. The current-cost depreciation estimates are given in FAA table 4.4, line 28.
- Calculate the annual fixed-cost investment flow, I_t :

$$I_t = K_t - K_{t-1} + D_t$$

• Calculate implied annual depreciation rate, δ_a :

$$\delta_a = \frac{I_t - (K_t - K_{t-1})}{K_{t-1} + I_t/2}$$

• Calculate implied quarterly depreciation rate for each year, δ_{qt} :

$$\delta_q + (1 - \delta_q)\delta_q + (1 - \delta_q)^2\delta_q + (1 - \delta_q)^3\delta_q = \delta_a$$

- Take historic-cost quarterly investment in private non-residential fixed assets by NFCB sector from the Flow of Funds accounts, atabs files, series FA105013005).
- Deflate it using the investment price index (the latter is calculated as consumption of fixed capital in domestic NFCB in current dollars (NIPA table 1.14, line 18) divided by consumption of fixed capital in domestic NFCB in chained 2000 dollars (NIPA table 1.14, line 41). This procedure yields the implicit price deflator for depreciation in NFCB. The resulting quarterly series, i_t_unadj , is thus in real terms.
- Perform Denton's procedure to adjust the quarterly series i_t_unadj from Federal Flow of Funds accounts to the implied annual series from BEA I_t , using the depreciation rate δ_{qt} from above. I use the simplest version of the adjustment procedure, when the discrepancies between the two series are equally spread over the quarters of each year. As a result of adjustment I get the fixed-cost quarterly series i_t .
- Simulate the quarterly real capital stock series k_t starting from k_0 (k_0 is actually the fixed-cost net stock of fixed assets in the end of 1975, this value is taken from the series K_t), using the quarterly depreciation series δ_{qt} and investment series i_t from above:

$$k_{t+1} = k_t \cdot (1 - \delta_{qt}) + i_t$$

5. Real price of new capital goods

In order to compute the real price of new capital goods, p^{I} , I use the price indices for output and for investment goods. Investment in NFCB Inv consists of equipment Eq and structures St. I define the time-t price-indices for good j = Inv, Eq, St as p_{t}^{j} and their change between t-1 and t by Δp_{t}^{j} , j = Inv, Eq, St. These price indices are chain-weighted. Thus:

$$\frac{\Delta p_t^{Inv}}{p_{t-1}^{Inv}} = \omega_t \frac{\Delta p_t^{Eq}}{p_{t-1}^{Eq}} + (1 - \omega_t) \frac{\Delta p_t^{St}}{p_{t-1}^{St}}$$

where

$$\omega_t = \frac{(\text{nominal expenditure share of } Eq \text{ in } Inv)_{t-1}}{2}.$$

The weights ω_t are calculated from the NIPA table 1.1.5, lines 8,10. The price indices p_t^j for j = Eq, St are from NIPA table 1.1.4, lines 9, 10. I divide the series by the price index for output, p_t^f , to obtain the real price of new capital goods, p^I .

Note that the price indices p^{Eq} and p^{St} and therefore p^{I} are actually adjusted for taxes. The parameter τ denotes the statutory corporate income tax rate as reported by the U.S. Tax Foundation.

Let *ITC* denote the investment tax credit on equipment and public utility structures, *ZPDE* the present discounted value of capital depreciation allowances, and χ the percentage of the cost of equipment that cannot be depreciated if the firm takes the investment tax credit. Flint Brayton has kindly provided me with the data. Then

$$p^{Eq} = \widetilde{p}^{Eq} (1 - \tau_{Eq})$$
$$p^{St} = \widetilde{p}^{St} (1 - \tau_{St}),$$

$$1 - \tau^{S_t} = \frac{\left(1 - \tau \ ZPDE^{St}\right)}{1 - \tau}$$

$$1 - \tau^{Eq} = \frac{1 - ITC - \tau ZPDE^{Eq} \left(1 - \chi ITC\right)}{1 - \tau}$$

Appendix C Alternative Specifications

As mentioned in the text, I test for alternative specifications. These are reported in Table C-1.

Table C-1

The top row gives the preferred benchmark specification of Table 4a, row 6.

The first three rows report other values of fixed powers; they look at higher convexity or allow for quadratic interaction. The problems with these specifications are that they do not fulfill all the conditions for convexity, usually failing the condition on second derivatives, and they imply high marginal investment costs.

The next row reports estimation using constant δ and ψ , set at their sample averages. This yields an insignificant e_1 estimate and negative marginal costs of investment. Fixing one of these variables only either results in the same outcome or violates the condition on second derivatives for convexity.

Using the weighted average cost of capital (wacc) for β_t – reported in rows 5, 6, 7 and 8 – results in either high marginal costs of investment or an estimate of the interaction g_{ih} which switches signs over the sample period. Also, in some specifications total and marginal costs switch signs over the sample period. In addition, the condition on second derivatives for convexity is violated.

Using a small instrument set results in a low p-value and some insignificant estimates. Using a large instrument set is fine though some sets violate the conditions on second derivatives for convexity.

Table 1

Stochastic Behavior of Hiring and Investment logged, HP-filtered and BP-filtered

a. Investment and Hiring Co-Movement $\rho(\frac{h_t}{n_t}, \frac{i_{t+i}}{k_{t+i}})$							
HP filtered $(\lambda = 1600)$							
lag/lead	-8	-4	-1	0	1	4	8
	-0.06	-0.06	-0.19	-0.16	-0.16	0.01	0.11
BP filtered (Baxter-King, 6-32)							
lag/lead	-8	-4	-1	0	1	4	8
	-0.09	-0.04	-0.27	-0.29	-0.25	0.03	0.18

b. Hiring Cyclicality $\rho(\frac{h_t}{n_t}, y_{t+i})$ HP filtered ($\lambda = 1600$)

		(/		
-8	-4	-1	0	1	4	8
-0.08	-0.12	-0.28	-0.21	-0.17	-0.01	0.10
-0.07	0.00	-0.07	-0.03	-0.08	0.01	0.03
-0.10	-0.13	-0.26	-0.18	-0.14	0.03	0.11
B	P filtere	ed (Baxt	ter-King	, 6-32)		
-8	-4	-1	0	1	4	8
-0.07	-0.17	-0.40	-0.36	-0.24	0.07	0.09
0.07	-0.05	-0.21	-0.15	-0.05	0.10	0.02
-0.12	-0.20	-0.40	-0.35	-0.22	0.11	0.10
	$-0.08 \\ -0.07 \\ -0.10 \\ B] \\ -8 \\ -0.07 \\ 0.07 \\ 0.07$	$\begin{array}{c ccc} -0.08 & -0.12 \\ -0.07 & 0.00 \\ -0.10 & -0.13 \\ \hline & \mathbf{BP \ filtere} \\ -8 & -4 \\ \hline & -0.07 & -0.17 \\ 0.07 & -0.05 \\ \end{array}$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$

c. Investment Cyclicality $\rho(\frac{i_t}{k_t}, y_{t+i})$ HP filtered ($\lambda = 1600$)

lag/lead	-8	-4	-1	0	1	4	8	
f	-0.22	0.45	0.82	0.79	0.65	0.03	-0.31	
$\frac{f}{n}$	0.05	0.60	0.62	0.51	0.32	-0.29	-0.41	
$\frac{n}{\frac{f}{k}}$	-0.10	0.57	0.83	0.75	0.56	-0.13	-0.42	
BP filtered (Baxter-King, 6-32)								
lag/lead	-8	-4	-1	0	1	4	8	
ſ								
J	-0.28	0.40	0.84	0.79	0.61	-0.03	-0.30	
$\frac{f}{\frac{f}{\frac{f}{k}}}$	-0.28 0.01	$\begin{array}{c} 0.40 \\ 0.58 \end{array}$	$\begin{array}{c} 0.84 \\ 0.67 \end{array}$	$\begin{array}{c} 0.79 \\ 0.51 \end{array}$	$\begin{array}{c} 0.61 \\ 0.28 \end{array}$	$-0.03 \\ -0.33$	$-0.30 \\ -0.39$	

Notes:

1. The variable y denotes the cyclical indicator which is f (NFCB GDP), or $\frac{f}{n}$ (labor productivity), or $\frac{f}{k}$ (capital productivity).

Table 2 Stochastic Behavior of Gross Hiring and Other Labor Market Variables

Co-Movement (contemporaneous) with Cyclical Indicators

logged, HP filtered

	n_t	$\frac{h_t}{n_t}$	$\frac{h_t}{u_t+o_t}$	$\frac{\psi}{\frac{h_t}{u_t+o_t}+\psi}$	$\frac{1}{\frac{n_t}{POP_t}}$
with GDP f		-0.20			
with labor productivity $\frac{f}{n}$	0.32	-0.03	0.35	-0.55	-0.48

logged, BP filtered						
	n_t	$rac{h_t}{n_t}$	$\frac{h_t}{u_t + o_t}$	$\frac{\psi}{\frac{h_t}{u_t+o_t}+\psi}$	$\frac{1}{\frac{n_t}{POP_t}}$	
with GDP f		-0.36		-0.80	-0.88	
with labor productivity $\frac{f}{n}$	0.36	-0.15	0.44	-0.72	-0.50	

Table 3

Descriptive Sample Statistics Quarterly, U.S. data 1976-2007

Variable	Mean	Standard Deviation
$\frac{i}{k}$	0.024	0.004
$\frac{f}{k}$	0.166	0.014
τ	0.387	0.057
δ	0.017	0.003
$\frac{wn}{f}$	0.658	0.013
$\frac{h}{n}$	0.133	0.013
$\frac{v}{u+o}$	0.057	0.012
ψ	0.132	0.012
β	0.994	0.005

Note: The sample size contains 127 quarterly observations from 1976:2 to 2007:4. For data definitions see Appendix B.

	specification	e_1	e_{20}	e_{21}	e_{30}	e_{31}	J-Statistic
1	investment costs only	144.16	—	—	—	—	76.05
		(2.29)	_	_	_	_	(0.09)
2	hiring costs only	_	2.75	_	—	_	69.21
	_		(0.13)	_	_	_	(0.22)
3	both, no interaction	121.36	0.39	_	_	_	77.32
	no market tightness	(6.12)	(0.21)	_	_	_	(0.07)
4	both, no interaction	64.62	2.45	-21.13	_	_	79.37
	with market tightness	(5.24)	(0.26)	(2.69)	_	_	(0.04)
5	both, with interaction	50.70	2.92	_	-4.87	_	67.80
	no market tightness	(9.29)	(0.35)	_	(1.43)	_	(0.20)
6	both, with interaction	39.82	4.50	-35.62	-6.10	72.75	66.10
	with market tightness	(8.94)	(0.48)	(5.95)	(1.65)	(22.57)	(0.19)

Table 4a GMM Estimates of the FOC (17) and (18)

Notes:

- 1. The table reports point estimates with standard errors in parantheses.
- 2. The J-statistic is reported with p value in parantheses.
- 3. η_1, η_2, η_3 are fixed at 2,2,1; α is fixed at 0.68. 4. The instrument set is: $\frac{h}{n}, \frac{w}{\frac{f}{n}}, \frac{v}{u}$ with 10 lags.

Table 4b
Adjustment Costs Implied by the GMM Estimation Results

	specification	$\frac{g}{f}$		$\frac{g_i}{\frac{f}{k}}$		$\frac{g_h}{\frac{f}{n}}$	
		mean	std.	\tilde{mean}	std.	mean^n	std.
1	investment costs only	0.044	0.014	3.54	0.55	_	_
2	hiring costs only	0.024	0.004	_	_	0.36	0.03
3	both, no interaction no market tightness	0.041	0.011	2.98	0.46	0.05	0.004
4	both, no interaction with market tightness	0.031	0.005	1.59	0.25	0.16	0.04
5	both, with interaction no market tightness	0.025	0.003	0.61	0.23	0.26	0.05
6	both, with interaction with market tightness	0.027	0.003	0.72	0.22	0.28	0.05

Notes:

1. Mean and std. refer to sample statistics.

2. The functions were computed using the point estimates in Table 4a.

Table 5

Estimates of the Marginal Adjustment Costs for Capital Summary of Key Studies for the U.S. Economy

Stu	ıdy	Sample	Mean $\frac{i}{k}$	Mean $\frac{g_i}{\frac{f}{k}}$
1	Summers (1981)	BEA, 1932-1978	0.13	2.5 - 60.5
2	Hyashi (1982)	Corporate, 1953-1976	0.14	3.2
3	Shapiro (1986)	Manufacturing, 1955-1980	0.08	1.33
4	Hubbard et al (1995)	Compustat, 1976-1987	0.20 - 0.23	0.15 - 0.45
5	Gilchrist and Himmelberg (1995)	Compustat, 1985-1989	0.17 - 0.18	0.50 - 0.98
6a	Gilchrist and Himmelberg (1998)	Compustat, 1980-1993	0.23	0.15 - 0.21
6b		Split Sample		0.13 - 1.1
7	Hall (2004)	Industry panel, 1958-1999	0.10	0.10
8	Cooper and Haltiwanger (2006)	LRD panel, $1972-1988$	0.12	0.04, 0.26
9	Cooper et al (2010)	LRD panel, 1972-1988	0.12	

Notes:

1. Investment rates $\frac{i}{k}$ are expressed in annual terms.

2. All studies pertain to annual data except Shapiro (1986) who uses quarterly data.

3. The entries in the last column are expressed in terms of f/k, so, they are comparable to the estimated marginal costs reported in Table 4b.

Table 6Asset Pricing Tests

Hiring

a. Single Forecasting Regressions Results

	coefficient	standard error	\overline{R}^2
b	0.11	0.03	0.08
b	0.48	0.07	0.28
b	1.01	0.09	0.50
b	1.42	0.09	0.66
b	1.62	0.08	0.78
b	1.58	0.09	0.76
b	-0.06	0.01	0.17
b	-0.22	0.03	0.28
b	-0.30	0.06	0.17
b	-0.24	0.08	0.06
b	-0.10	0.10	0.0003
b	0.09	0.10	-0.002
	b b b b b b b b b b b b b b b b	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$

b. VARs

D. VAIIS		coefficient	standard error	\overline{R}^2
restricted, forecasting system ²	ϕ	0.95	0.02	0.87
	b_d	0.12	0.02	0.08
	b_r	-0.07	0.02	0.16
restricted, complete VAR^3	b_{pp}	0.90	0.02	0.87
	b_{dd}	0.08	0.04	0.27
	b_{rr}	0.45	0.11	0.11
	b_{dp}	0.19	0.02	0.27
	b_{rp}	-0.04	0.02	0.11

c. Variance Decomopsition and Long Run Coefficients 4

Т	10	20	30	40
$rac{b_d}{1- ho\phi}=b_d^{lr}\ rac{b_r}{rac{b_r}{1- ho\phi}}=b_r^{lr}$		0.62(-0.37)	(0.09) (0.09)	
$rac{b_{dp}}{1- ho b_{pp}}=b_d^{lr}\ rac{b_{rp}}{1- ho b_{pp}}=b_r^{lr}$		0.82(-0.17)	(0.07)	
$\frac{cov\left[p_t - d_t, \sum_{j=0}^{T} \rho^{j-1}(d_{t+j+1} - d_{t+j})\right]}{var(p_t - d_t)}$ $cov\left[p_t - d_t, \sum_{j=0}^{T} \rho^{j-1}r_{t+j+1}\right]$	0.71	0.75	0.77	0.78
$\frac{\frac{cov\left[p_t - d_t, \sum_{j=0}^{p_j - 1} r_{t+j+1}\right]}{var(p_t - d_t)}}{\frac{e_t}{var(p_t - d_t)}}$	-0.21 0.08		-0.19 0.04	

Investment

		coefficient	standard error	\overline{R}^2
dividend growth forecasting ¹	b	0.01	0.01	-0.002
4 quarter $ahead^1$	b	0.03	0.02	0.02
8 quarter $ahead^1$	b	0.07	0.03	0.04
$12 \text{ quarter ahead}^1$	b	0.10	0.03	0.07
16 quarter ahead 1	b	0.15	0.04	0.11
20 quarter ahead ¹	b	0.23	0.04	0.21
return forecasting ¹	b	-0.02	0.01	0.05
4 quarter $ahead^1$	b	-0.10	0.02	0.16
8 quarter $ahead^1$	b	-0.18	0.03	0.24
$12 \text{ quarter ahead}^1$	b	-0.26	0.04	0.30
$16 \text{ quarter ahead}^1$	b	-0.31	0.04	0.32
$20 \text{ quarter ahead}^1$	b	-0.35	0.05	0.36

a. Single Forecasting Regressions Results

b. VARs

		coefficient	standard error	\overline{R}^2
restricted, forecasting $system^2$	ϕ	0.998	0.006	0.995
	b_d	0.007	0.006	-0.002
	b_r	-0.024	0.006	0.045
restricted, complete VAR^3	b_{pp}	0.997	0.005	0.994
	b_{dd}	0.060	0.047	0.438
	b_{rr}	0.343	0.043	-0.071
	b_{dp}	0.015	0.004	0.438
	b_{rp}	-0.016	0.005	-0.071

c. Variance Decompsition and Long Run Coefficients $\!\!\!^4$

Т	60	65	70	75
$rac{b_d}{1- ho\phi}=b_d^{lr}\ rac{b_r}{1- ho\phi}=b_r^{lr}$		`	(0.17) (0.17)	
$rac{b_{dp}}{1- ho b_{pp}}=b_d^{lr}\ rac{b_{rp}}{1- ho b_{pp}}=b_r^{lr}$		``	(0.13) (0.13)	
$\frac{cov \left[p_t - d_t, \sum_{j=0}^{T} \rho^{j-1} (d_{t+j+1} - d_{t+j}) \right]}{var(p_t - d_t)}$ $cov \left[p_t - d_t, \sum_{j=0}^{T} \rho^{j-1} r_{t+j+1} \right]$	0.45	0.44	0.38	0.22
$\frac{\frac{cov\left[p_t-d_t,\sum_{j=0}^{\rho^{j-1}r_{t+j+1}}\right]}{var(p_t-d_t)}}{\frac{e_t}{var(p_t-d_t)}}$	$-0.35 \\ 0.20$	$-0.40 \\ 0.16$	$-0.50 \\ 0.12$	$-0.66 \\ 0.12$

Notes:

1. Forecasting regressions:

Dividend growth forecasting

$$d_{t+H} - d_t = a + b(p_t - d_t) + e_t$$

Return forecasting

$$r_{t+H} = a + b(p_t - d_t) + e_t$$

where:

$$r_{t+H} = r_{t+1} + r_{t+2} + \dots + r_{t+H}$$

The table reports results for H = 1, 4, 8, 12, 16, 20. 2. **Restricted, forecasting system** given by:

$$(p_{t+1} - d_{t+1}) = a + \phi(p_t - d_t) + e_{p,t}$$

$$d_{t+1} - d_t = c + b_d(p_t - d_t) + e_{d,t}$$

$$r_{t+1} = d + b_r(p_t - d_t) + e_{r,t}$$

The restriction is:

$$b_d - b_r = 1 - \rho\phi$$

where

$$\rho = \frac{\frac{P}{D}}{1 + \frac{P}{D}}$$

and P, D are sample average values.

3. Restricted, complete VAR:

Using
$$z_t = \begin{pmatrix} p_t - d_t \\ d_t - d_{t-1} \\ r_t \end{pmatrix}$$
, with the variables de-meaned
 $e_1 = (1, 0, 0)$
 $e_2 = (0, 1, 0)$
 $e_3 = (0, 0, 1)$
 $\underline{e}_t = \begin{pmatrix} \varepsilon_{1_t} \\ \varepsilon_{2_t} \\ \varepsilon_{3_t} \end{pmatrix}$

The VAR is:

$$z_{t+1} = Bz_t + \underline{\varepsilon}$$

where:

$$B = \left(\begin{array}{ccc} b_{pp} & b_{pd} & b_{pr} \\ b_{dp} & b_{dd} & b_{dr} \\ b_{rp} & b_{rd} & b_{rr} \end{array}\right)$$

The restrictions are:

$$e_1(I - \rho B) - (e_2 - e_3) B = 0$$

4. Variance Decomposition and Long Run Coefficients

a. $b_d, b_r, \, \phi$, b_{dp}, b_{rp}, b_{pp} taken from panel b. Standard errors are computed using the delta method.

b. T varies according to the values indicated in the top row.

Scope and Elasticities Implied by The GMM Estimation Results								
specification	scope	$rac{\partial i_t}{\partial Q^K} rac{Q^K}{i_t}$	$rac{\partial i_t}{\partial Q^N} rac{Q^N}{i_t}$	$rac{\partial h_t}{\partial Q^k} rac{Q^K}{h_t}$	$rac{\partial h_t}{\partial Q^N} rac{Q^N}{h_t}$			
1 investment costs only	0	4.10	_	_	_			

Table 7
Scope and Elasticities Implied by The GMM Estimation Results

			(0.98)			
2	hiring costs only	0	_	—	—	1.00
						(0.00)
3	both, no interaction	0	4.68	—	—	1.00
	no market tightness		(1.16)			(0.00)
4	both, no interaction	0	7.91	_	_	1.00
	with market tightness		(2.18)			(0.00)
5	both, with interaction	0.61	11.05	0.44	3.30	0.81
	no market tightness	(0.05)	(3.18)	(0.13)	(0.35)	(0.08)
6	both, with interaction	0.23	12.54	0.25	1.69	0.89
	with market tightness	(0.09)	(3.67)	(0.14)	(0.54)	(0.04)

Notes:

- 1. All computations are based on the point estimates of Table 4a.
- 2. The scope statistic is defined as

$$\frac{g(0,\frac{h}{n}) + g(\frac{i}{k},0) - g(\frac{i}{k},\frac{h}{n})}{g(\frac{i}{k},\frac{h}{n})}$$

3. The elasticities are derived in Appendix A.

Table 8Cyclical Behavior

I Co-Movement of Investment and Hiring Rates and Their Present Values

logged, HP filtered								
lag/lead	-8	-4	-1	0	1	4	8	
$\rho(\frac{h_t}{n_t}, \frac{i_{t+j}}{k_{t+j}})$	-0.06	0.01	-0.11	-0.10	-0.11	0.03	0.07	
$\rho(Q_t^N, Q_{t+j}^K)$	0.13	-0.35	-0.81	-0.86	-0.78	-0.24	0.25	
$\rho(\frac{i_t}{k_t}, Q_{t+j}^K)$	-0.35	0.33	0.84	0.83	0.69	0.02	-0.31	
$\rho(\frac{h_t}{n_t}, Q_{t+j}^N)$	0.11	0.05	0.20	0.41	0.05	-0.06	-0.13	

logged, BP filtered									
$\log/lead$	-8	-4	-1	0	1	4	8		
$\rho(\frac{h_t}{n_t}, \frac{i_{t+j}}{k_{t+j}})$	-0.10	-0.03	-0.20	-0.22	-0.20	0.03	0.16		
$\rho(Q_t^N, Q_{t+j}^K)$		-0.31	-0.84	-0.90	-0.87	-0.35	0.25		
$ \rho(\frac{i_t}{k_t}, Q_{t+j}^K) $	-0.37	0.36	0.87	0.85	0.71	0.02	-0.33		
$\rho(\frac{h_t}{n_t}, Q_{t+j}^N)$	0.24	0.21	0.40	0.38	0.28	-0.12	-0.12		

II Co-Movement of Series with Business Sector GDP f $\rho(x_t,f_{t+j})$

	a.	Invest	tment	\mathbf{costs}	only						
	logged, HP filtered										
lag/lead	-8	-4	-1	0	1	4	8				
$\frac{i}{k}$	-0.25	0.41	0.82	0.76	0.60	0.02	-0.25				
$rac{rac{i}{k}}{rac{g_i}{f/k}}$	-0.25	0.41	0.82	0.76	0.60	0.02	-0.25				
Q^{K}	-0.19	0.36	0.82	0.79	0.67	0.10	-0.27				
		logge	ed, BP	filtere	d						
lag/lead	-8	-4	-1	0	1	4	8				
$rac{i}{k}{g_i}$	-0.32	0.45	0.85	0.79	0.63	0.02	-0.29				
$\frac{\ddot{g_i}}{f/k}$	-0.32	0.45	0.85	0.79	0.63	0.02	-0.29				
Q^{K}	-0.27	0.37	0.84	0.83	0.72	0.14	-0.31				

b. Hiring costs only

logged, HP filtered										
$\log/lead$	-8	-4	-1	0	1	4	8			
$\frac{h}{n}$	-0.06	-0.04	-0.21	-0.15	-0.14	-0.03	0.04			
$rac{g_h}{f/n}$	-0.06	-0.04	-0.21	-0.15	-0.14	-0.03	0.04			
Q^N	0.04	-0.09	-0.13	-0.04	0.02	0.11	-0.02			
		log	gged, BF	filtered						
lag/lead	-8	-4	-1	0	1	4	8			
$\frac{h}{n}$	-0.06	-0.20	-0.36	-0.32	-0.23	0.003	0.05			
$rac{\overline{g_h}}{f/n}$	-0.06	-0.20	-0.36	-0.32	-0.23	0.003	0.05			
$\hat{Q}^{\hat{N}}$	0.08	-0.28	-0.22	-0.09	0.06	0.25	-0.01			

logged, HP filtered									
$\log/lead$	-8	-4	-1	0	1	4	8		
$\frac{i}{k}$	-0.25	0.41	0.82	0.76	0.60	0.02	-0.25		
$rac{rac{i}{k}}{rac{g_i}{f/k}}$	-0.25	0.41	0.80	0.72	0.56	-0.05	-0.27		
Q^K	-0.24	0.34	0.82	0.75	0.59	-0.02	-0.29		
$\begin{array}{c} \frac{h}{n} \\ \frac{g_h}{f/n} \\ Q^N \end{array}$	-0.06	-0.04	-0.21	-0.15	-0.14	-0.03	0.04		
$\frac{g_h}{f/n}$	0.04	-0.26	-0.58	-0.51	-0.44	-0.07	0.14		
Q^N	0.09	-0.27	-0.51	-0.42	-0.32	0.01	0.10		

c. Both hiring costs and investment costs with interaction, without market tighness

logged, BP filtered									
lag/lead	-8	-4	-1	0	1	4	8		
$\frac{i}{k}$	-0.32	0.45	0.85	0.79	0.63	0.02	-0.29		
$\begin{array}{c} \frac{i}{k} \\ \frac{g_i}{f/k} \\ Q^K \end{array}$	-0.32	0.43	0.85	0.77	0.59	-0.05	-0.30		
Q^{K}	-0.31	0.41	0.87	0.81	0.63	-0.01	-0.32		
$rac{h}{g_h}{rac{g_h}{f/n}}$	-0.06	-0.20	-0.36	-0.32	-0.23	0.003	0.05		
$\frac{g_h}{f/n}$	0.15	-0.41	-0.76	-0.71	-0.57	-0.06	0.20		
Q^N	0.18	-0.44	-0.64	-0.54	-0.37	0.09	0.14		

	logged, HP filtered									
$\log/lead$	-8	-4	-1	0	1	4	8			
$\begin{array}{c} \frac{i}{k} \\ \frac{g_i}{f/k} \\ Q^K \end{array}$	-0.25	0.41	0.82	0.76	0.60	0.02	-0.25			
$\frac{g_i}{f/k}$	-0.24	0.26	0.84	0.87	0.77	0.17	-0.18			
Q^K	-0.22	0.25	0.84	0.88	0.79	0.19	-0.19			
,										
$\frac{h}{n}$	-0.06	-0.04	-0.21	-0.15	-0.14	-0.03	0.04			
$\frac{g_h}{f/n}$	0.20	-0.24	-0.81	-0.87	-0.82	-0.35	0.03			
$\begin{array}{c} \frac{h}{n} \\ \frac{g_h}{f/n} \\ Q^N \end{array}$	0.23	-0.26	-0.80	-0.84	-0.77	-0.30	0.01			
		log	gged, BF	p filtered						
lag/lead	-8	log -4	gged, BF -1	filtered 0	1	4	8			
	-8 -0.32	-		_		4	8			
		-4	-1	0	1					
$\frac{\frac{\log/\text{lead}}{\frac{i}{k}}}{Q^{K}}$	-0.32	-4 0.45	-1 0.85	0	1 0.63	0.02	-0.29			
$\frac{\frac{i}{k}}{\substack{g_i\\f/k\\Q^K}}$	$-0.32 \\ -0.30$	-4 0.45 0.31	-1 0.85 0.88	0 0.79 0.90	1 0.63 0.79	0.02 0.18	$-0.29 \\ -0.22$			
$\frac{\frac{i}{k}}{\substack{g_i\\f/k\\Q^K}}$	$-0.32 \\ -0.30$	-4 0.45 0.31	-1 0.85 0.88	0 0.79 0.90	1 0.63 0.79	0.02 0.18	$-0.29 \\ -0.22$			
	$-0.32 \\ -0.30 \\ -0.29$	$ \begin{array}{r} -4 \\ 0.45 \\ 0.31 \\ 0.29 \\ \end{array} $	-1 0.85 0.88 0.88	0 0.79 0.90 0.91	$ \begin{array}{r} 1 \\ 0.63 \\ 0.79 \\ 0.82 \end{array} $	0.02 0.18 0.21	-0.29 -0.22 -0.23			

d. Both hiring costs and investment costs with interaction, with market tighness

Notes:

1. All series are based on the point estimates of Table 4a. Panel IIa corresponds to row 1 in Table 4a; Panel IIb to row 2; Panel IIc to row 5 and panel IId to row 6. 2. $\frac{Q^K}{f/k}$ is net of p^I .

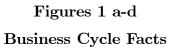
	specification	e_1	e_{20}	e_{21}	e_{30}	e_{31}	J-Stat
	Benchmark	39.8	4.50	-35.62	-6.10	72.75	66
	Table 4a Row 6	(8.9)	(0.48)	(5.95)	(1.65)	(22.57)	(0.2)
1	$\eta_1 {=} \eta_2 {=} 3$	4,717	1.53	-10.93	11.94	-128.72	70
	$\eta_3 = 1$	(401)	(2.10)	(40.72)	(1.41)	(31.11)	(0.1)
2	$\eta_1{=}\eta_2{=}3$	2,299	21.10	-45.11	-708.76	-5,711	66
	$\eta_3 = 2$	(298)	(3.74)	(53.95)	(661.13)	(8, 984)	(0.2)
3	$\eta_1{=}\eta_2{=}4$	181, 247	-2.18	173.12	16.02	-172.34	67
	$\eta_3 = 1$	(14, 269)	(13.89)	(283.93)	(1.33)	(29.76)	(0.2)
4	δ, ψ fixed	6.12	6.03	-62.45	-7.12	156.16	62
		(12.07)	(0.54)	(5.96)	(1.75)	(22.28)	(0.3)
5	β using wacc	-253.7	12.2	-55.4	-17.3	236.9	58
	$\eta_1 \!\!= 2, \eta_2 \!\!= 2, \eta_3 \!\!= 1$	(22.4)	(0.9)	(14.4)	(4.4)	(57.6)	(0.4)
6	β using wacc	-218.39	86.77	-410.97	-6,462	77,985	57
	$\eta_1 \!\!= 2, \eta_2 \!\!= 3, \eta_3 \!\!= 2$	(20.09)	(9.00)	(130.29)	(1, 467)	(19, 178)	(0.5)
7	β using wacc	8,187	-15.4	2,214	-1,031	-69,539	55
	$\eta_1 \!= 3, \eta_2 \!= 4, \eta_3 \!= 2$	(490)	(22.2)	(480)	(596)	(9, 955)	(0.5)
8	β using wacc	96,217	6.56	117.84	12.69	-199.69	63
	$\eta_1\!\!=4, \eta_2\!\!=3, \eta_3\!\!=1$	(21, 656)	(4.04)	(70.33)	(2.75)	(53.79)	(0.3)
9	small set	16.8	3.0	-26.6	4.5	-35.6	45
	4 lags of $\frac{i}{k}, \frac{h}{n}$	(16.9)	(0.9)	(17.7)	(4.4)	(63.5)	(0.00002)
10	large set, 8 lags of	108.6	-1.05	11.8	11.4	-136.5	90
	$rac{i}{k},rac{h}{n},rac{wn}{f},rac{v}{u},p^i,eta$	(4.4)	(0.19)	(2.6)	(0.69)	(11.4)	(0.6)
11	large set, 6 lags of	33.65	2.79	-16.67	-0.91	-17.47	80
	$rac{i}{k}, rac{h}{n}, rac{wn}{f}, rac{v}{u}, p^i, eta$	(7.95)	(0.27)	(4.18)	(1.18)	(21.08)	(0.2)

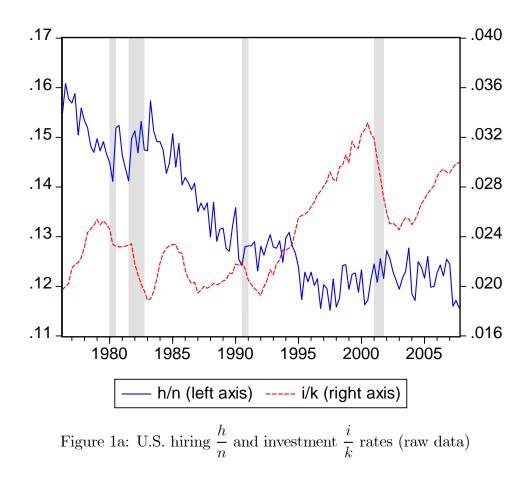
Table C-1Alternative GMM Estimates of the FOC (17) and (18)

Notes:

- 1. The table reports point estimates with standard errors in parantheses.
- 2. The J-statistic is reported with p value in parantheses.
- 3. α is fixed at 0.68

4. The instrument set is: $\frac{i}{k}$, $\frac{h}{n}$, $\frac{w}{\frac{f}{n}}$, $\frac{v}{u}$, p^{I} , β with 6 lags, except for rows 9-11 where it is indicated.





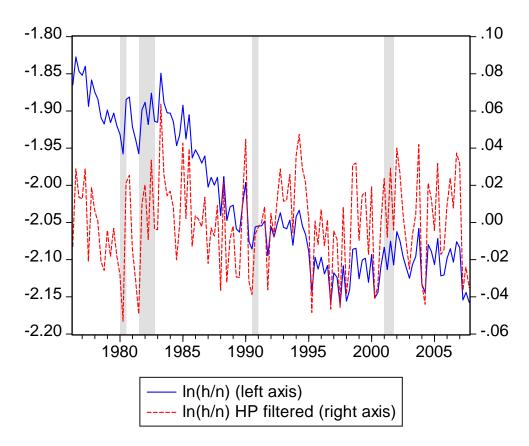


Figure 1b, Panel A: Log Hiring Rates (levels and HP filtered).

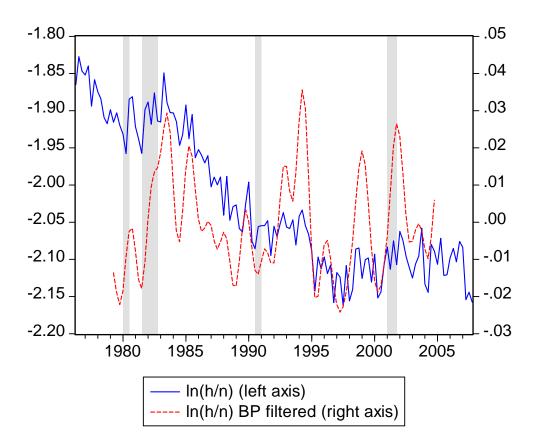


Figure 1b, Panel B: Log Hiring Rates (levels and BP filtered).

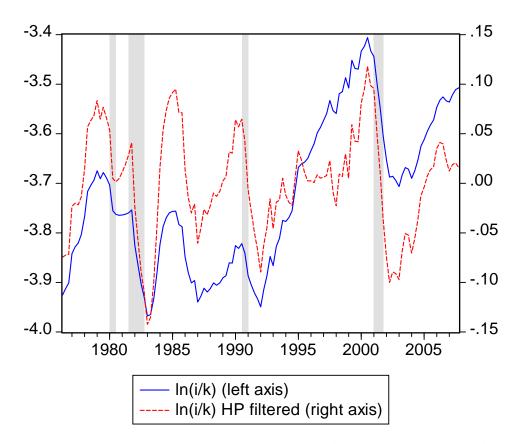


Figure 1c, Panel A: Log Investment Rates (levels and HP filtered).

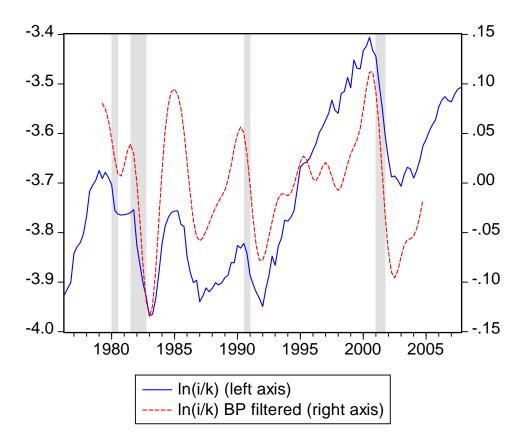
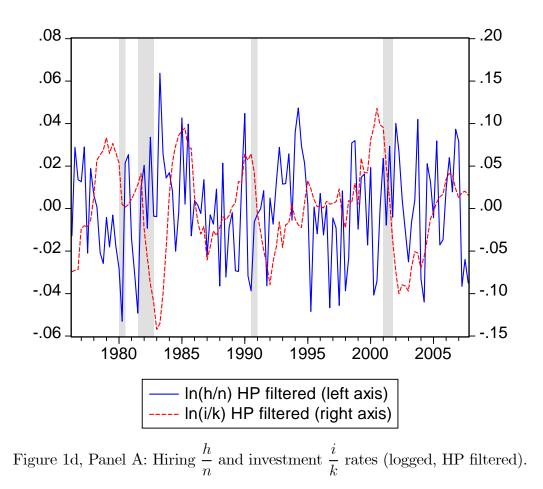


Figure 1c, Panel B: Log Investment Rates (levels and BP filtered).



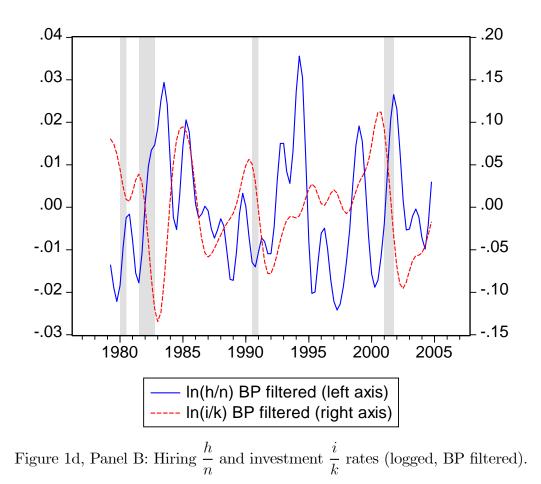
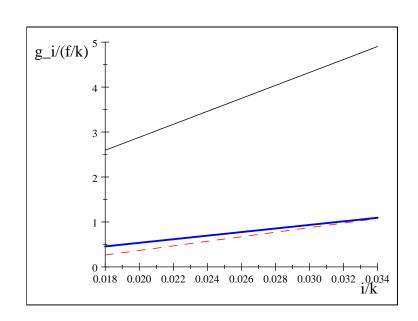


Figure 2 The Estimated Marginal Costs Functions

a. Marginal Investment Costs



$$\frac{g_{i_t}}{\frac{f_t}{k_t}} = e_1(\frac{i_t}{k_t}) + (e_{30} + e_{31}\overline{\frac{v_t}{u_t + o_t}})\left(\frac{h_t}{n_t}\right)$$

Notes:

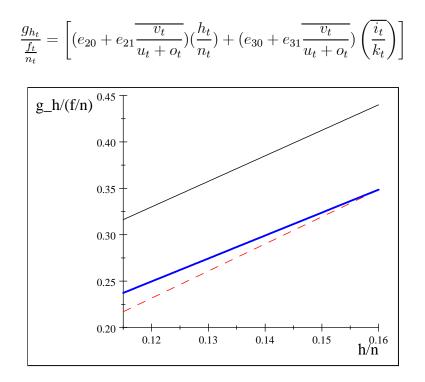
1. The graph uses the point estimates of Table 4a to plot $\frac{g_{i_t}}{\frac{f_t}{k_t}}$ as a function of $\frac{i_t}{k_t}$ according to the above equation. 2. The black line uses row 1 estimates – no hiring $e_{30} = e_{31} = 0$.

3. The red line (dashed) uses row 5 estimates with interaction but no effect for labor market conditions, $e_{30} \neq 0$; $e_{31} = 0$.

4. The blue line (thick, solid) uses row 6 estimates, the full specification $e_{30} \neq 0; e_{31} \neq 0.$

5. Throughout average sample values are used for $\frac{\overline{v_t}}{u_t+o_t}$ and for $\frac{\overline{h_t}}{n_t}$.

b. Marginal Hiring Costs



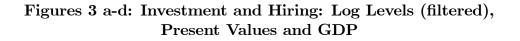
Notes:

1. The graph uses the point estimates of Table 4a to plot $\frac{g_{h_t}}{\frac{f_t}{n_t}}$ as a function of $\frac{h_t}{n_t}$ according to the above equation. 2. The black line uses row 2 estimates – no investment $e_{30} = e_{31} = 0$.

3. The red line (dashed) uses row 5 estimates with interaction but no effect for labor market conditions, $e_{30} \neq 0$; $e_{31} = 0$.

4. The blue line (solid, thick) uses row 6 estimates, the full specification $e_{30} \neq 0; e_{31} \neq 0.$

5. Throughout average sample values are used for $\frac{v_t}{u_t+o_t}$ and for $\frac{i_t}{k_t}$.



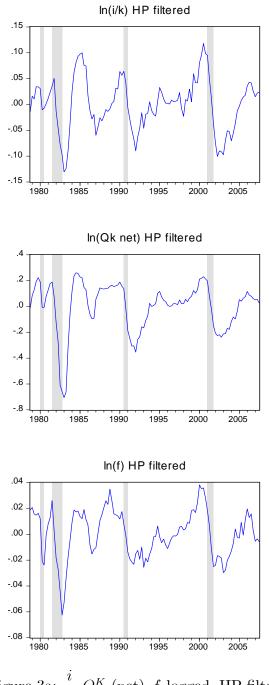


Figure 3a: $\frac{i}{k}$, Q^{K} (net), f logged, HP filtered.

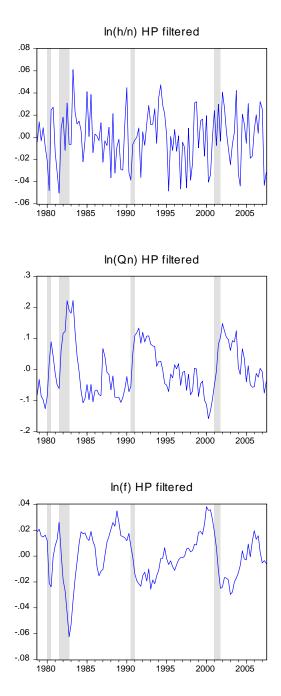


Figure 3b: $\frac{h}{n}$, Q^N , f logged, HP filtered.

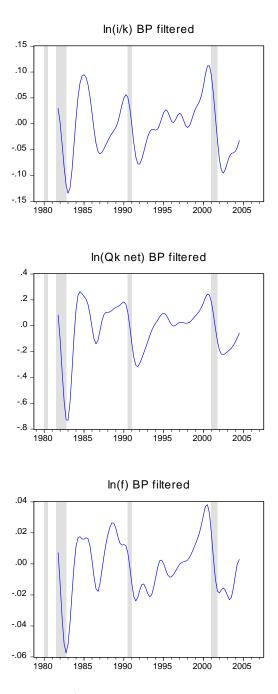


Figure 3c: $\frac{i}{k}$, Q^{K} (net), f logged, BP filtered.

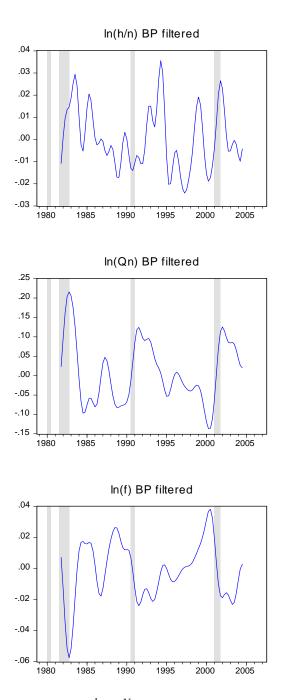


Figure 3d: $\frac{h}{n}$, Q^N , f logged, BP filtered.