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ABSTRACT

Markov-switching MIDAS models*

This paper introduces a new regression model - Markov-switching mixed data sampling (MS-MIDAS) - that incorporates regime changes in the parameters of the mixed data sampling (MIDAS) models and allows for the use of mixed-frequency data in Markov-switching models. After a discussion of estimation and inference for MS-MIDAS, and a small sample simulation based evaluation, the MS-MIDAS model is applied to the prediction of the US and UK economic activity, in terms both of quantitative forecasts of the aggregate economic activity and of the prediction of the business cycle regimes. Both simulation and empirical results indicate that MSMIDAS is a very useful specification.

JEL Classification: C22, C53 and E37
Keywords: business cycle, forecasting, mixed-frequency data, non-linear models and nowcasting

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1 Introduction

The econometrician often faces a dilemma when observations are sampled at different frequencies. One solution consists in estimating the model at the lowest frequency, temporally aggregating the high-frequency data. However, this solution is not fully satisfactory since important information can be discarded in the aggregation process. A second solution is to temporally disaggregate (interpolate) the low frequency variables. However, there is no agreement on the proper interpolation method, and the resulting high frequency variables would be affected by measurement error.

The third option is represented by regression models that combine variables sampled at different frequencies. They are particularly attractive since they can use the information of high-frequency variables to explain variables sampled at a lower frequency without any prior aggregation or interpolation. In this context, the MIDAS (Mixed Data Sampling) model of Ghysels et al. (2004, 2007) has recently gained considerable attention. A crucial feature of this class of models is the parsimonious way of including explanatory variables through a weighting function, which can take various shapes depending on the value of its parameters.

MIDAS models have been applied for predicting both macroeconomic and financial variables. Ghysels, Santa-Clara and Valkanov (2006) use the MIDAS framework to predict the volatility of equity returns, while Clements and Galvão (2008, 2009) successfully apply MIDAS models to the prediction of quarterly US GDP growth using monthly indicators as high frequency variables. Andreou, Ghysels and Kourtellos (2010) exploit the informational content of daily financial variables to predict quarterly GDP and inflation in the US. In particular, they extend the standard MIDAS model to include factors in a dynamic framework, along the lines of Marcellino and Schumacher (2010).

MIDAS models are generally used as single-equation models where the dynamics of the indicator is not modelled. By contrast, system-based models such as the mixed-frequency VAR (MF-VAR) explicitly model the dynamics of the indicator. Kuzin, Marcellino and Schumacher (2009) compare the forecasting performance of MIDAS and MF-VAR models for the prediction of the quarterly GDP growth in the Euro area. They find that MIDAS models outperform MF-VAR for short horizons (up to five months), while MF-VAR tend to perform better for longer horizons. A similar comparison is provided in Bai, Ghysels and Wright (2010).

An issue that has attracted so far limited attention in the MIDAS literature is the stability of the relationship between the high and low frequency variables. Time-variation in MIDAS models has been only introduced by Galvão (2009) via a smooth transition function governing the change in some parameters of the model. This Smooth Transition MIDAS is applied to the prediction of quarterly US GDP using weekly and daily financial variables.

In this paper, we propose an alternative way to allow for time-variation in the MIDAS model, introducing the Markov-switching MIDAS (MS-MIDAS) model. Regime changes may result from asymmetries in the process of the mean or variance. From an economic point of view, the predicting ability of the higher frequency variables could change across regimes following, e.g., changes in market conditions or business cycle phases. For example, the slope of the yield curve is often considered as a strong predictor of US recessions, an inverted yield curve signaling a forthcoming recession. However, Galbraith and Tkacz (2000) argue that the predictive power of the slope of the yield curve is limited in normal times. Therefore, it could be important to permit time-variation in the predictive ability of the high-frequency data. Indeed, our empirical applications show that in general the predictions from MS-MIDAS models are more accurate than those from simple MIDAS models.

An additional attractive feature of Markov-switching models is the possibility of estimating and predicting the probabilities of being in a given regime. The literature (e.g., Estrella and Mishkin (1998), Birchenhall et al. (1999)) often uses binary response models to predict the state of the economy using the NBER dating of expansions and contractions as a dependent variable. However, this method can be problematic since the announcements of turning points may be published up to twenty months after the turning point has actually occurred. Our MS-MIDAS model instead allows for real time evaluation and forecasting of the probability of being in a given regime.

Finally, MS-MIDAS is also a convenient approach to allow for the use of mixed frequency information in standard Markov-switching models. Hamilton (2010) pointed out the importance of using models with mixed frequency data for predicting recessions in real time. In our applications, the forecasting performance of standard MS models is indeed improved by the use of higher frequency information.

The paper is organized as follows. Section 2 reviews the MIDAS approach, introduces the MS-MIDAS, and discusses the estimation method. Section 3 presents Monte-Carlo simulations to assess the accuracy of the proposed estimation method in finite samples and its forecasting accuracy. Section 4 discusses an empirical application to the prediction of quarterly GDP growth and business cycle turning points in the US and the UK. Both empirical applications use financial variables as indicators. Section 5 concludes.

2 Markov-switching MIDAS

2.1 MIDAS approach

2.1.1 Basic MIDAS

The MIDAS approach of Ghysels et al. (2004, 2007) involves the regression of variables sampled at different frequencies. Following the notation of Clements and Galvão (2008, 2009), and assuming that the model is specified for h-step ahead forecasting, the basic univariate MIDAS model is given by:

$$y_t = \beta_0 + \beta_1 B(L^{1/m}; \theta) x_{t-h}^{(m)} + \epsilon_t \tag{1}$$

where $B(L^{1/m}; \theta) = \sum_{j=1}^{K} b(j; \theta) L^{(j-1)/m}$ and $L^{s/m} x_{t-1}^{(m)} = x_{t-1-s/m}^{(m)}$. Note that t refers to the time unit of the dependent variable y_t and m to the time unit of the higher frequency variables $x_{t-h}^{(m)}$.

The forecasts of the MIDAS regression are computed directly so that no forecasts for the explanatory variables are required. However, unlike iterated forecasts, direct forecasts require to re-estimate the model when the forecasting horizon changes, see Chevillon and Hendry (2005) and Marcellino, Stock and Watson (2006) for a comparison of the relative merits of iterated and direct forecasts.

The crucial difference between MIDAS and Autoregressive Distributed Lag models is that the content of the higher frequency variable is exploited in a parsimonious way through the polynomial $b(j;\theta)$, which allows to have a rich variety of shapes with a limited number of parameters. Ghysels et al. (2007) detail various specifications for the polynomial of lagged coefficients $b(j;\theta)$. A popular choice for the weighting scheme is the exponential Almon lag:

$$b(j;\theta) = \frac{exp(\theta_1 j + ... + \theta_Q j^Q)}{\sum_{j=1}^{K} exp(\theta_1 j + ... + \theta_Q j^Q)} \tag{2}$$

Note that the weighting function of the exponential Almon lag implies that the weights are always positive. In the empirical applications, we employ the exponential Almon lag scheme with two parameters $\theta = \{\theta_1, \theta_2\}$.

2.1.2 Autoregressive MIDAS

Introducing an autoregressive lag in the MIDAS specification is not straightforward as pointed out by Clements and Galvão (2008), who show that a seasonal response of y to x can appear. However, this can be done without generating any seasonal patterns if autoregressive dynamics is introduced through a common factor, so that equation 1 becomes:

$$y_t = \beta_0 + \lambda y_{t-d} + \beta_1 B(L^{1/m};\theta)(1-\lambda L^d)x_{t-h}^{(m)} + \epsilon_t \tag{3}$$

2.2 Markov-switching MIDAS

2.2.1 The model

The basic idea behind Markov-switching models is that the parameters of the underlying data generating process (DGP) depend on an unobservable discrete variable S_t, which represents the probability of being in a different state of the world (see Hamilton (1989)). The basic version of the Markov-switching MIDAS (MS-MIDAS) regression model we propose is:

$$y_t = \beta_0(S_t) + \beta_1(S_t)B(L^{1/m};\theta)x_{t-h}^{(m)} + \epsilon_t(S_t) \tag{4}$$

where $\epsilon_t|S_t \sim NID(0, \sigma^2(S_t))$.

The MS-MIDAS that includes autoregressive dynamics is instead defined as:

$$y_t = \beta_0(S_t) + \lambda y_{t-d} + \beta_1(S_t)B(L^{1/m};\theta)(1-\lambda L^d)x_{t-h}^{(m)} + \epsilon_t(S_t) \tag{5}$$

The regime generating process is an ergodic Markov-chain with a finite number of states $S_t = \{1, ..., M\}$ defined by the following transition probabilities:

$$p_{ij} = Pr(S_{t+1} = j|S_t = i) \tag{6}$$

$$\sum_{j=1}^{M} p_{ij} = 1 \forall i, j \epsilon \{1, ..., M\} \tag{7}$$

Here the transition probabilities are constant. This assumption has been originally relaxed by Filardo (1994), who used time-varying transition probabilities modelled as a logistic function, while Kim et al. (2008) model them as a probit function. However, we stick to the assumption of constant transition probabilities to keep the model tractable.

The parameters that can switch are the intercept of the equation, β_0, the parameter entering before the weighting scheme, β_1, and the variance of the disturbances, σ^2. Changes in the intercept β_0 are important since they are one of the most common sources of forecast failure, see e.g. Clements and Hendry (1999). The switch in the parameter β_1 allows the predictive ability of the higher frequency variable to change across the different states of the world [1]. Besides, we also allow the variance of the disturbances σ^2 to change across

[1] Galvão (2009) proposed a regression model (STMIDAS) that captures changes in β_1 with a smooth transition function. This so-called STMIDAS model performs well for the prediction of the US GDP using financial variables as high frequency data.

regimes. This proves to be useful not only for modelling financial variables but also for applications with macroeconomic variables.

Another attractive feature of the Markov-switching models is that they allow the estimation of the probabilities of being in a given regime. This is relevant, for example, when one wants to predict business cycle regimes. Indeed, studies about the identification and prediction of the state of the economy have gained attention over the last decade (see e.g. Estrella and Mishkin (1998), Berge and Jordá (2009), Stock and Watson (2010), and the literature review in Marcellino (2006)).

2.2.2 Estimation and model selection

In the literature, MIDAS models are usually estimated by nonlinear least squares (NLS). However, for implementing the filtering procedure described in Hamilton (1989), we estimate the MS-MIDAS via (pseudo) maximum likelihood. We thus need to make a normality assumption about the distribution of the disturbances, which is not required with the NLS estimation. We aim at maximizing the log-likelihood function given by:

$$L = \sum_{t=1}^{T} ln f(y_t|\Omega_{t-1}) \tag{8}$$

where $f(y_t|\Omega_{t-1})$ is the conditional density of y_t given the information available up to time $t-1$, Ω_{t-1}. Note that $f(y_t|\Omega_{t-1})$ can be rewritten as:

$$f(y_t|\Omega_{t-1}) = \sum_{j=1}^{M} P(S_t = j|\Omega_{t-1}) f(y_t|S_t = j, \Omega_{t-1}) \tag{9}$$

The computations are carried out with the optimization package OPTMUM of GAUSS 7.0 using the BFGS algorithm. Appendix A provides more details about the estimation method we use.

Choosing the number of regimes for Markov-switching models is a tricky problem. Indeed, the econometrician has to deal with two problems: first, some parameters are not identified under the null hypothesis and, second, the scores are identically equal to zero under the null. Hansen (1992) considers the likelihood function as a function of unknown parameters and uses empirical processes to bound the asymptotic distribution of a standardized likelihood ratio test statistic. García (1998) pointed out that the test is computationally expensive if the number of parameters and regimes is high. Carrasco et al. (2009) recently proposed a new method for testing the constancy of parameters in Markov-switching models. Their procedure is attractive since it only requires to estimate the model under the null hypothesis of constant parameters. However, this testing procedure does not allow one to discriminate between Markov-switching models with different number of regimes since the parameters must be constant under the null hypothesis.

Psaradakis and Spagnolo (2006) study the performance of information criteria based on the optimization of complexity-penalized likelihood for model selection. They find that the AIC, SIC and HQ criteria perform well for selecting the correct number of regimes and lags as long as the sample size and the parameter changes are large enough. Smith, Naik and Tsai (2006) propose a new information criterion for selecting simultaneously the number of variables and lags of the Markov-switching models. However, both studies run their analysis with models where all parameters switch across regimes, which might not always be desirable. For example, in equations 4 and 5, we do not consider switches in the vector of parameters θ since we encountered serious convergence problems in the empirical applications due to the relatively small size of our sample (T=200). However, with larger sample sizes, the MS-MIDAS model could easily accommodate changes in the θ vector. In addition, Driffill et al. (2009) show that a careful study of the parameters that can switch is crucial for forecasting accurately bond prices with the CIR model for the term structure.

In the empirical part, we will follow Psaradakis and Spagnolo (2006) and use the Schwarz information criterion for selecting the number of regimes and deciding whether the variance of the disturbances should also change across regimes. We will also report results for different parameterizations of the Markov-switching models.

3 Monte Carlo experiments

3.1 In-sample estimates

The first purpose of the Monte Carlo experiments is to assess the accuracy of the maximum likelihood estimation procedure we propose for the MS-MIDAS model. The DGP used in the Monte Carlo experiments is the MSHAR(2)-MIDAS model defined by equations (5) to (7), i.e. it is a model with two regimes and switches in the intercept β_0, in the parameter entering before the weighting function β_1 and in the variance σ^2, since models with two regimes are often used in the literature. We consider two sample sizes for the simulated series $T = 200$ and $T = 500$. The matrix of explanatory variables includes a constant and the process for $x_t^{(m)}$ is an AR(1) with a large autoregressive coefficient (0.95) and a small drift (0.025). We are primarily interested in the predictive content of monthly variables for forecasting quarterly variables, so we set $K = 3$ and $K = 13$. We use the following true parameter values:

$$(\beta_{0,1}, \beta_{0,2}) = (-1, 1), (\beta_{1,1}, \beta_{1,2}) = (0.6, 0.2), (\sigma_1, \sigma_2) = (1, 0.67) \tag{10}$$

$$(\theta_1, \theta_2) = (2 * 10^{-1}, -3 * 10^{-2}) \tag{11}$$

These parameter values are similar to those used in Kim, Piger and Startz (2008) and closely match the in-sample parameter estimates of our empirical application for the UK (see Table B in the appendix). The transition probabilities are first set such that both regimes are equally persistent ($p_{11} = 0.95$, $p_{22} = 0.95$). We also consider another set of transition probabilities: $p_{11} = 0.85$ and $p_{22} = 0.95$. Indeed, with these transition probabilities, if one thinks of y_t as quarterly observations, the duration of the first regime (6.67 quarters) is lower than the duration of the second regime (20 quarters), which roughly corresponds to the average duration of recessions and expansions experienced by the US and the UK.

We first simulate a Markov chain with two regimes using one of the two sets of transition probabilities. The dependent variable y_t is then constructed depending on the outcome of the simulated Markov chain using the above parameter values and the simulated series for x_t. The first 100 data points are discarded to eliminate start-up effects[2]. We repeat the estimation 1000 times and report the means of the maximum likelihood point estimates [3]. In addition, we report the standard deviations of the point estimates from the true parameter values.

We do not show the point estimates for θ_1 and θ_2 but rather the approximation error computed as the sum of the squared error between the estimated and the true weighting function, normalized by the squared weights of the true weighting function. We proceed this way since it is the shape of the weighting function which is important rather than the point estimates for θ_1 and θ_2. The approximation error is defined by:

$$\frac{\sum_{j=1}^{M=K}[b(j,\hat{\theta})-b(j,\theta)]^2}{\sum_{j=1}^{M=K}b(j,\theta)^2} \tag{12}$$

Table 1 shows that the parameter estimates for the intercepts $\beta_{0,1}$ and $\beta_{0,2}$, the autoregressive parameter λ and the transition probabilities p_{11} and p_{22} are very close to their true values. The estimates for $\beta_{1,1}$ and $\beta_{1,2}$ - the parameters entering before the weighting function - are slightly downward biased. The standard deviations for the estimates are lower when the sample size is large, as expected. Similarly, the shape of the weighting function is better approximated for $T = 500$ and the average R^2 is higher.

Overall, the Monte Carlo experiments suggest that maximum likelihood estimation of this specification of the MSHAR(2)-MIDAS provides accurate estimates of the model parameters, including the transition probabilities.

3.2 Forecasting exercise

We carry out another Monte Carlo experiment to assess the forecasting accuracy of the MS-MIDAS model. To this end, we generate data from the D.G.P. used in the previous

[2]Discarding more than 100 initial observations leads to identical results.

[3]Note that we do not initialize the algorithm with the true parameter values. Instead, we use the same rule of thumb than in the empirical applications for the initialization of the parameters (i.e. we run OLS regressions on sub-samples after sorting the x_t variable with respect to the dependent variable y_t).

Table 1: Monte Carlo Results, In Sample Estimates

		$p_{11}=0.95$	$p_{11}=0.95$	$\beta_{0,1}=-1$	$\beta_{0,2}=1$	$\beta_{1,1}=0.6$	$\beta_{1,2}=0.2$	$\lambda=0.2$	R^2	Approx. error
K=3										
	T=200	0.936	0.946	-1.043	1.042	0.504	0.170	0.165	0.769	0.262
		(0.039)	(0.032)	(0.224)	(0.140)	(0.103)	(0.056)	(0.074)		
	T=500	0.944	0.949	-1.042	1.028	0.532	0.181	0.174	0.795	0.157
		(0.018)	(0.016)	(0.116)	(0.074)	(0.045)	(0.023)	(0.042)		
K=13										
	T=200	0.935	0.945	-1.047	1.041	0.501	0.174	0.163	0.768	0.256
		(0.040)	(0.028)	(0.242)	(0.137)	(0.104)	(0.046)	(0.074)		
	T=500	0.944	0.949	-1.036	1.037	0.532	0.181	0.172	0.793	0.174
		(0.019)	(0.016)	(0.117)	(0.077)	(0.044)	(0.022)	(0.044)		
		$p_{11}=0.85$	$p_{11}=0.95$	$\beta_{0,1}=-1$	$\beta_{0,2}=1$	$\beta_{1,1}=0.6$	$\beta_{1,2}=0.2$	$\lambda=0.2$	R^2	Approx. error
K=3										
	T=200	0.808	0.951	-1.015	1.030	0.455	0.176	0.178	0.734	0.237
		(0.114)	(0.025)	(0.455)	(0.116)	(0.211)	(0.033)	(0.069)		
	T=500	0.832	0.952	-1.025	1.026	0.506	0.180	0.181	0.769	0.133
		(0.053)	(0.014)	(0.185)	(0.065)	(0.081)	(0.017)	(0.040)		
K=13										
	T=200	0.816	0.950	-0.993	1.035	0.451	0.175	0.176	0.730	0.234
		(0.097)	(0.028)	(0.423)	(0.121)	(0.193)	(0.032)	(0.070)		
	T=500	0.833	0.951	-1.032	1.022	0.507	0.181	0.185	0.772	0.149
		(0.051)	(0.014)	(0.191)	(0.065)	(0.076)	(0.017)	(0.041)		

This table reports the average of the 1000 point estimates of the Monte Carlo experiments. The last column reports the average approximation error for the weighting scheme as defined by equation 12. Standard deviations of the 1000 point estimates are reported in brackets.

subsection with the parameter values defined in equations 10 and 11 with a sample size of T=200 and T=500 using 13 lags for the high frequency indicator x_t^m. The sample size T is split between an estimation sample and an evaluation sample. We choose three different sizes H for the evaluation sample, $H = \{20, 50, 100\}$. We then run the following out-of-sample forecasting experiment: we use the first T-H observations and compute one-step ahead forecasts [4]. We recursively expand the estimation sample until we reach the end of the sample T so that we compute H forecasts. The design of this forecasting experiment is very close to the empirical application we run later in the paper.

We use seven different models to compute the forecasts: the MSHAR(2)-MIDAS model (i.e. the true model), the MSH(2)-MIDAS model (i.e. the true model without an autoregressive lag), a standard MIDAS and AR-MIDAS models as defined in equations 1 and 3 respectively. We also consider an AR(1) model and a standard Markov-switching model with two regimes, a switch in the intercept and in the variance of the disturbances and one autoregressive lag (i.e. MSIHAR(2) model). Finally, we also show the results for an MSIHAR(2)-MIDAS model (i.e. a model with a constant β_1). We always use the true number of lags ($K = 13$) for the high frequency variable x_t^m when the models have mixed-frequency data.

We repeat the forecasting experiment N times for each evaluation sample and report in Table 2 the average of the mean square forecast error over the number of replications for the seven models under consideration. We also report the Quadratic Probability Score (QPS) and Log Probability Score (LPS) for the models with Markov-switching features in order to check how well these models can predict the true regimes. Here, the MSFE is a criterion that allows us to assess the quantitative forecasting abilities of the model under scrutiny, whereas QPS and LPS criteria evaluate their qualitative forecasting abilities, i.e. to what extent the true regimes are predicted.

[4] Note that we use a different estimation sample for each H in order to consider the common trade-off in empirical analysis between a longer estimation sample or a longer evaluation sample. Our results suggest that, as long as the estimation sample remains long enough, a longer evaluation sample is to be preferred to a longer estimation sample.

Note that the QPS is bounded between 0 and 2 and the range of LPS is 0 to ∞. LPS penalizes large forecast errors more than QPS. LPS and QPS are computed as follows:

$$QPS = \frac{2}{H}\sum_{t=1}^{H}(P(S_{t+1}=1) - S_{t+1})^2 \tag{13}$$

$$LPS = -\frac{1}{H}\sum_{t=1}^{H}(1-S_{t+1})log(1-P(S_{t+1}=1)) + S_{t+1}log(P(S_{t+1}=1)) \tag{14}$$

where S_{t+1} is a dummy variable that takes on a value of 1 if the true regime is the first regime and $P(S_{t+1}=1)$ is the predicted probability of being in the first regime in period t+1.

For $T = 200$, Table 2 shows that the true model (i.e. the MSHAR(2)-MIDAS) gets the best results for the MSFE, QPS and LPS when the size of the evaluation sample is $H = 100$. The MSH(2)-MIDAS model obtains the best results for $H = 50$ in terms of QPS and LPS, while the AR(1) model gets the lowest MSFE. For $H = 20$, the true model yields the best performance in terms of LPS and MSFE but it is slightly outperformed by the MSIHAR model according to the QPS criterion.

For $T = 500$ and $H = 100$, the true model obtains the best performance for both discrete and continuous forecasts. For $H = 50$, the true model obtains the best results in terms of MSFE, whereas it is outperformed by the MSH(2)-MIDAS model in terms of QPS and LPS criteria. Finally, for $H = 20$, the MSIHAR(2) model obtains the best regime forecasts but it is outperformed by the MSIHAR(2)-MIDAS according to the MSFE criterion.

Overall, the true MS-MIDAS model is either ranked first or exhibits a performance very close to the best model for both discrete and continuous forecasts. Besides, given the DGP we use, the simple MIDAS model has a poor forecasting performance as compared to the AR-MIDAS model. Finally, the MS and the AR(1) models yield rather inaccurate continuous forecasts.

Table 2: Monte Carlo Results: Forecasting exercise

Number of out-sample forecasts H:	20			50			100		
First estimation sample size									
	QPS	LPS	MSFE	QPS	LPS	MSFE	QPS	LPS	MSFE
MSHAR(2)-MIDAS	0.398	**0.609**	**1.432**	0.300	0.447	1.782	**0.295**	**0.464**	**1.207**
MSIHAR(2)	**0.392**	0.642	1.709	0.370	0.584	1.617	0.872	1.690	1.301
MSH(2)-MIDAS	0.412	0.635	1.468	**0.258**	**0.398**	1.860	0.304	0.469	1.226
MSIHAR(2)-MIDAS	0.444	0.683	1.646	0.356	0.561	1.671	0.821	1.629	1.237
AR-MIDAS	-	-	1.563	-	-	1.722	-	-	1.423
MIDAS	-	-	2.510	-	-	2.432	-	-	1.738
AR(1)	-	-	1.596	-	-	**1.563**	-	-	1.456
Second estimation sample size									
MSHAR(2)-MIDAS	0.456	0.653	1.385	0.238	0.404	**1.487**	**0.387**	**0.581**	**1.121**
MSIHAR(2)	**0.358**	**0.543**	1.787	0.365	0.547	2.034	0.628	0.861	1.622
MSH(2)-MIDAS	0.524	0.736	1.442	**0.216**	**0.374**	1.519	0.403	0.605	1.183
MSIHAR(2)-MIDAS	0.361	0.546	**1.326**	0.326	0.498	1.625	0.536	0.758	1.342
AR-MIDAS	-	-	1.410	-	-	1.711	-	-	1.469
MIDAS	-	-	1.893	-	-	2.202	-	-	1.549
AR(1)	-	-	1.832	-	-	1.931	-	-	1.668

This table reports the average QPS, LPS and MSFE over 200 Monte Carlo replications. In the first estimation sample, the initial estimation sample size is $T - H$ where $T = 200$. In the second estimation sample, the initial estimation sample size is $T - H$ where $T = 500$. Both estimation samples are recursively expanded until the end of the sample is reached. Entries in bold outline the model with the lowest QPS, LPS or MSFE for each combination of the evaluation sample H and sample size T. The true model is the MSHAR(2)-MIDAS model. A classification of the models is provided in Table E.

4 An application to the prediction of quarterly GDP

4.1 Prediction of the US GDP

4.1.1 In-sample results

We analyze quarterly data for the US GDP, taken from the real-time dataset of the Philadelphia Federal Reserve[5], which originates from the work of Croushore and Stark (2001). Quarterly vintages reflect the information available in the middle month of each quarter. The dependent variable is taken as 100 times the quarterly change in the log of the US real GDP from t=1959:Q1 to 2009:Q4. For the in-sample analysis, we use the 2010:Q1 vintage.

We first consider the slope of the yield curve as high frequency indicator since its predictive power for GDP growth has been widely documented (Estrella and Hardouvelis (1991), Galvão (2006), Rudebusch and Williams (2009)). We use the difference between the 10-year Treasury bond and the 3-month Treasury-bill as a proxy for the slope of the yield curve. We also consider stock returns as a monthly indicator for forecasting quarterly aggregate economic activity. Stock returns are taken as 100 times the monthly change in the log of the S&P500 index. We finally consider the Federal Funds as a monthly indicator to take into account the stance of the monetary policy, which is often considered as an important determinant of economic activity. We take the first difference for both the slope of the yield curve and the Federal Funds since we achieve better forecasting results with this transformation. The data for the 10-year Treasury bond yields, the 3-month Treasury bill and the Federal Funds are taken from the Federal Reserve website, while the data for the S&P500 index are downloaded from Yahoo Finance.

For selecting the number of regimes and whether there is also switching in the variance of the disturbances, we use the SIC with a maximum number of regimes of $M = 3$. For $M = \{2, 3\}$, we then estimate a model with or without a switch in the variance and in

[5]The real-time vintage quarterly data for the US GDP are available at http://www.philadelphiafed.org/research-and-data/real-time-center/real-time-data/data-files/ROUTPUT/

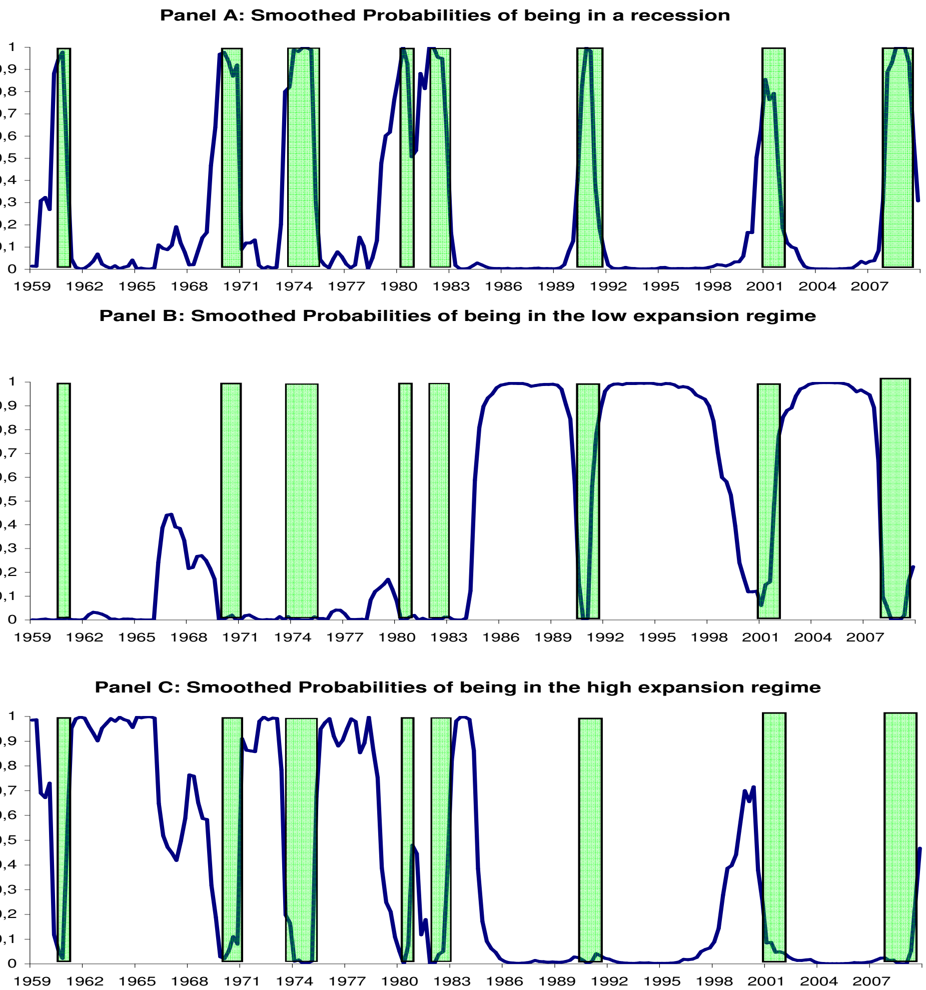
the parameter β_1. Whatever indicator we use, the model that gets the best fit has three regimes and switches in the intercept β_0, in the parameter entering before the weighting function β_1 and in the variance σ^2.

Table A in the appendix reports the in-sample results for each indicator for the models with three regimes, a switch in the variance of the disturbances, in the intercept and with or without a switch in β_1. Note that the intercept in the first regime $\beta_{0,1}$ is negative, whereas the intercept in the second regime $\beta_{0,2}$ is positive, but smaller than the intercept in the third regime $\beta_{0,3}$. Therefore, the first regime can be interpreted as the recessionary regime, the second regime is instead the low but positive growth regime, while the third regime is the strong growth regime. As expected, the variance in the second regime is always the lowest among the three regimes. Moreover, there are noticeable differences across regimes in the coefficient β_1, which measures the impact of the monthly indicators on quarterly GDP growth, while β_1 remains statistically significant in most cases. These results highlight the importance of allowing for parameter changes in MIDAS models, but also the relevance of including high frequency information in MS models.

Figure 1 depicts the estimated smoothed probabilities resulting from the MSIHAR(3)-MIDAS model with the slope of the yield curve as a monthly indicator. The shadow areas represent the recessions identified by the National Bureau of Economic Research (NBER). First, one can see that the estimated probabilities of recession match quite well the actual recessions, including the recession that started in December 2007 (panel A). Interestingly, the probability of recession falls in the third quarter of 2009, which confirms the NBER dating of the end of the last recession. The first (moderate) expansion regime - depicted in panel B - is predominant in the post-1984 era and is characterized by a much lower variance than the second (stronger) expansion regime reported in Panel C. This finding is in line with the great moderation phenomenon and supports the McConnell and Perez-Quiroz (2001) dating of the break in volatility experienced by the US. Panel C reports the estimated probabilities of being in a high growth regime, this regime is predominant in the 1960s and 1970s and is shortly resurgent in the late 1990s, reflecting the high growth experienced by the US thanks to the technology boom.

Consequently, the MS-MIDAS model with three regimes seems to be a proper specification for describing quarterly US GDP, and its forecasting performance will be assessed in the next subsection.

Figure 1: MS-MIDAS Quarterly GDP and monthly slope of the yield curve
Sample 1959:Q1 - 2009:Q4

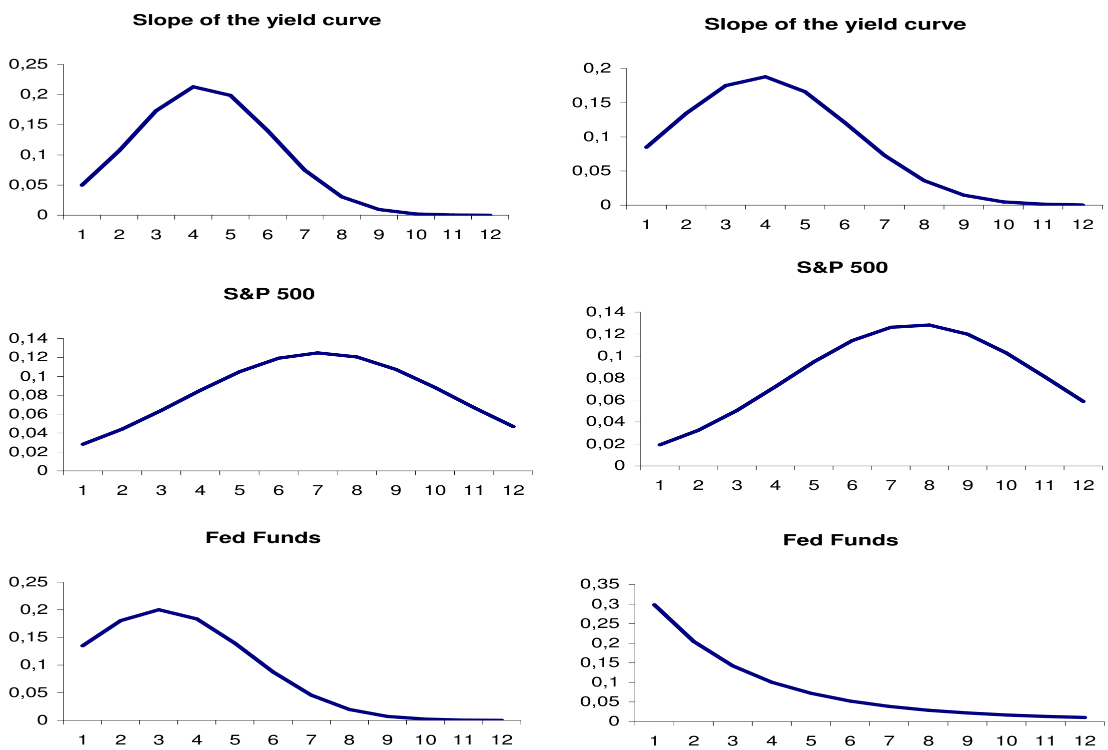


To conclude, Figure 2 plots the estimated weights corresponding to the three monthly indicators for the AR-MIDAS and MSHAR(3)-MIDAS models [6]. The figure illustrates the

[6]The MSHAR(3)-MIDAS model is a model with three regimes, an autoregressive parameter and switches in β_0 , β_1 and in the variance σ^2. Table E, which is the last one in the Appendix, reports the labels we used for each model.

variety of weights that can be attached to the indicators thanks to the MIDAS specification.

Figure 2: Weights of the AR-MIDAS (LHS) and MSHAR(3)-MIDAS (RHS) exponential lag polynomial



4.1.2 Design of the real-time forecasting exercise

The sample is split into an estimation sample and an evaluation sample. The evaluation sample consists of quarterly GDP growth in the quarters 1998:Q1 to 2009:Q4. For each of these quarters, we generate forecasts with horizons $h = \{0, 1/3, 2/3, 1, 4/3, 5/3, 2\}$. The initial estimation sample goes from 1959:Q1 to 1997:Q4 and is recursively expanded over time until 2009:Q2[7]. The design of the exercise is similar to the one described in section 3.1.1 of Clements and Galvão (2008). We denote $y_{\tau,\nu}$ as output growth in period τ released in the vintage ν data set. We aim at forecasting final estimates of the output growth $y_{t,T}$ as defined in the latest vintage available to us T= 2010:Q1. Note that for GDP, the vintage data set released in quarter t + 1 contains data up to quarter t, and quarterly vintages reflect information available in the middle month of each quarter. We use financial variables as higher frequency variables, which are available without any delays and are not subject to data revisions.

A few additional comments are required. First, forecasts for the regime probabilities k quarters ahead are computed recursively as:

$$P(S_{t+k} = j) = \sum_{i=1}^{M} p_{ij} P(S_{t+k-1} = i) \tag{15}$$

Note that the predicted probabilities only depend on the transition probabilities and on the filtered probabilities.

Second, forecasts with an horizon $h = 0$ (i.e. nowcasts) imply that we want to forecast output growth for the current quarter knowing the values of the monthly indicators for all months of the current quarter. The nowcasts are computed as follows: we first regress $y_{t|t+1}$ on $B(L^{(1/3)};\theta)x_{t|t+1}$ and $y_{t-1|t+1}$, where $y_{t|t+1} = [y_{1|t+1}, y_{2|t+1}, ..., y_{t-1|t+1}, y_{t|t+1}]$ and $x_{t|t+1} = [x_{1|t+1}, ..., x_{t-1|t+1}, x_{t|t+1}]$. We then use these estimates, the forecasts for the regime probabilities $P(S_{t+1} = j|x_t), y_{t|t+1}$ and $x_{t+1|t+1}$ to compute the forecasts $\hat{y}_{t+1|t+1}$.

Forecasts with an horizon $h = 1/3$ imply that we only know the values for the first two months of the monthly indicator. To obtain these forecasts, we first regress $y_{t|t+1}$ on $B(L^{(1/3)};\theta)x_{t-1/3|t+1}$ and $y_{t-1|t+1}$, where $x_{t-1/3|t+1} = [x_{1-1/3|t+1}, ..., x_{t-4/3|t+1}, x_{t-1/3|t+1}]$. We then use these estimates, the forecasts for the regime probabilities $P(S_{t+1} = j|x_t), y_{t|t+1}$ and

[7]The last observation used in the estimation sample is 2009:Q2 since we need the actual values of GDP for the next two quarters to compute the MSFE. Therefore, there are 47 forecasts computed for each forecast horizon h.

$x_{t+2/3|t+1}$ to obtain forecasts for y_{t+1}, which is conditioned on $x_{t+2/3|t+1}$ and $y_{t|t+1}$. Forecasts with an horizon $h = 4/3$ are generated from a regression of $y_{t|t+1}$ on $B(L^{(1/3)}; \theta)x_{t-4/3|t+1}$ and $y_{t-2|t+1}$.

Similarly, forecasts with an horizon $h = 2/3$ imply that we only know the values for the first month of the monthly indicator. To obtain these forecasts, we first regress $y_{t|t+1}$ on $B(L^{(1/3)}; \theta)x_{t-2/3|t+1}$ and $y_{t-1|t+1}$ where $x_{t-2/3|t+1} = [x_{1-2/3|t+1}, ..., x_{t-5/3|t+1}, x_{t-2/3|t+1}]$. We then use these estimates, the forecasts for the regime probabilities $P(S_{t+1} = j|x_t)$ and $x_{t+1/3|t+1}$ to obtain forecasts for y_{t+1}, which is conditioned on $x_{t+1/3|t+1}$. Forecasts with an horizon $h = 5/3$ are generated from a regression of $y_{t|t+1}$ on $B(L^{(1/3)}; \theta)x_{t-5/3|t+1}$ and $y_{t-2|t+1}$.

Finally, forecasts with an horizon $h = 1$ are computed from the regression of $y_{t|t+1}$ on $B(L^{(1/3)}; \theta)x_{t-1|t+1}$ and $y_{t-1|t+1}$, while forecasts with an horizon $h = 2$ come from the regression of $y_{t|t+1}$ on $B(L^{(1/3)}; \theta)x_{t-2|t+1}$ and $y_{t-2|t+1}$. Hence, for example, forecasts for the first quarter Q1 of a given year are generated as described in Table 3.

Table 3: Forecasting scheme for $Q1$

Forecast horizon h	0	1/3	2/3	1	4/3	5/3	2
Data up to month	$March_t$	Feb_t	Jan_t	Dec_{t-1}	Nov_{t-1}	Oct_{t-1}	$Sept_{t-1}$

4.1.3 Out-of-sample results

Table 4 reports the relative mean square forecast errors (MSFE) for seven different models for different forecast horizons using an AR(1) model as a benchmark[8]. The MIDAS

[8]We do not report tests of equal forecast accuracy since it is not straightforward to implement them in the context of nested MIDAS models with real-time data. Indeed it is uncertain whether the test proposed by Clark and McCracken (2009) for nested models with real-time data can be applied in the context of MIDAS models. Furthermore, the test for nested models by Clark and McCracken (2005)

model is the standard MIDAS as defined in equation 1. The AR-MIDAS is the model defined in equation 3. The MSIH(3)-MIDAS is a model with three regimes, a switch in the intercept and in the variance of the shocks. The MSIHAR(3)-MIDAS is an MSIH(3)-MIDAS model with an autoregressive lag introduced through a common factor as described in Section 2. The MSH(3)-MIDAS and MSHAR(3)-MIDAS are similar to the MSIH(3)-MIDAS and MSIHAR(3)-MIDAS apart from the fact that they also include a switch in the parameter β_1. We also report results for a standard Markov-switching model with three regimes, one autoregressive lag, a switch in the intercept and in the variance (MSIHAR(3) model). The number of lags included in the weighting function is selected using the SIC.

Table 4 reports the out-of-sample forecasting results for the period 1998:Q1 to 2009:Q4. Note first that the AR-MIDAS always outperforms the MIDAS with the Federal Funds and slope of the yield curve as a monthly indicator. When using stock prices, AR-MIDAS and MIDAS yield comparable forecasting performance. In the Markov-switching case, including an autoregressive lag seems to be of less importance. Second, the S&P500 index is the best indicator among the three variables considered and it also largely outperforms the AR(1) model. For forecast horizons $h = \{0, 1/3, 2/3, 1\}$, the MSIHAR-MIDAS model with stock prices turns out to be the best model for predicting quarterly GDP growth across all models under consideration. Third, the slope of the yield curve exhibits a poor forecasting performance as compared to the AR(1) model: this is a disappointing result but it is in line with the findings of Galvão (2009). Fourth, within the class of models that use the Federal Funds as a monthly indicator, the AR-MIDAS yields a better forecasting performance than models with Markov-switching features for $h = \{0, 2/3, 1\}$, while the MSHAR(3)-MIDAS model is the best forecasting model for $h = \{1/3, 4/3, 2\}$. Finally, the standard Markov-switching model is slightly better than the AR(1) for two-quarter ahead predictions but slightly worse for one-quarter ahead predictions. It is beaten by several MS-MIDAS specifications, which confirms the usefulness of introducing higher frequency

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is computationally very expensive since Monte Carlo simulations should be undertaken for each model and forecast horizon. In addition, the usual approach of adopting the Giacomini and White (2006) test combined with rolling estimation is not suited in our context, since we want to use all the available sample in each point in time to improve inference on the regimes. Hence we leave the issue of testing equal forecast accuracy for future research.

information into MS models.

In addition to predicting quarterly GDP growth, Markov-switching MIDAS models can endogenously generate probabilities of being in a given regime. Tables 5 and 6 below provide the quadratic probability score (QPS) and the log probability score (LPS) as defined in equations 13 and 14. We use the classification of the economic activity from the NBER so that S_t is a dummy variable that takes on a value of 1 if the economy is in recession in quarter t according to the NBER, while $P(S_t = 1)$ is the probability of being in the recession regime in period t. Forecasts with an horizon $h = \{0, 1/3, 2/3, 1\}$ predict the regime of the economy one quarter ahead, while forecasts with an horizon $h = \{4/3, 5/3, 2\}$ predict the state of the economy two quarters ahead.

In contrast to forecasting the level of GDP growth, the slope of the yield curve and the Federal Funds tend to better predict the state of the economy than stock prices. This is in line with the results from binary recession models that emphasize the predictive power of the slope of the yield curve (see e.g. Estrella and Mishkin (1998)). Moreover, using information from the monthly indicators produces better regime forecasts than a pure MS model for quarterly GDP.

Additional evidence on the predictive ability of the Markov-switching MIDAS specification is presented in Figure 3, where we report the nowcasted probability of being in a recession with the slope of the yield curve as a monthly indicator using the MSH(3)-MIDAS model with $h = 0$. These probabilities are generated from the recursive exercise and correspond to the filtered probabilities for the last observation T, where T is recursively expanded over time from t=1997:Q4 to 2009:Q4.

Figure 3 shows that there is a first signal of recession in the second quarter of 2001 using information up to August 2001. The probability of recession then rises above .90 in the third and fourth quarter of 2001. Interestingly, the probability of recession stays above .35 until the second quarter of 2003 and only fall below .10 in the third quarter of 2003. This illustrates the slow economic recovery that followed the 2001 recession. Figure 3 also shows that there is a first peak in the probability of recession in the fourth quarter

Table 4: Relative Mean Squared Forecast error for forecasting US GDP growth 1998:Q1-2009:Q4

		Forecast horizon (h)						
	Model	0	1/3	2/3	1	4/3	5/3	2
Slope of the yield curve	MSIH(3)-MIDAS	1.080	1.061	1.066	1.066	1.022	1.002	1.009
	MSIHAR(3)-MIDAS	**0.960**	**1.019**	1.038	**0.964**	1.032	1.049	1.041
	MSH(3)-MIDAS	1.080	1.043	1.059	1.025	1.067	1.073	1.049
	MSHAR(3)-MIDAS	1.073	1.073	1.044	0.992	0.996	1.019	1.015
	AR-MIDAS	1.037	1.022	**1.022**	1.010	**0.995**	**0.997**	**0.997**
	MIDAS	1.242	1.231	1.231	1.252	1.065	1.069	1.276
S&P 500	MSIH(3)-MIDAS	0.691	0.739	0.712	0.765	0.740	0.784	0.785
	MSIHAR(3)-MIDAS	**0.680**	**0.690**	**0.639**	**0.726**	0.775	0.776	0.792
	MSH(3)-MIDAS	0.913	0.876	0.881	1.078	0.872	0.902	0.861
	MSHAR(3)-MIDAS	0.871	1.160	0.932	1.128	0.873	0.885	0.845
	AR-MIDAS	0.769	0.726	0.715	0.746	0.715	0.754	0.779
	MIDAS	0.762	0.728	0.713	0.729	**0.684**	**0.727**	**0.767**
Fed Funds	MSIH(3)-MIDAS	0.909	0.996	0.983	1.005	1.074	1.086	1.284
	MSIHAR(3)-MIDAS	0.944	0.952	0.995	1.044	1.009	**1.046**	1.185
	MSH(3)-MIDAS	1.041	0.916	0.963	0.997	1.126	1.110	1.119
	MSHAR(3)-MIDAS	0.974	**0.886**	0.983	0.971	**0.966**	1.172	**1.009**
	AR-MIDAS	**0.874**	0.942	**0.945**	**0.945**	1.081	1.124	1.278
	MIDAS	1.041	1.078	1.171	1.174	1.237	1.237	1.442
	MSIHAR(3)	-	-	-	1.071	-	-	0.984

Real-time data set. Relative Mean Squared Forecast Error for US output growth in the quarters 1998:Q1-2009:Q4. Benchmark: AR(1) model. Recursive forecasting scheme. Entries in bold outline the model with the lowest MSFE for each indicator and forecast horizon. A classification of the models is reported in Table E.

Table 5: Quadratic Probability Score for forecasting US business cycle regimes 1998:Q1-2009:Q4

		Forecast horizon (h)						
	Model	0	1/3	2/3	1	4/3	5/3	2
Slope of the yield curve	MSIH(3)-MIDAS	0.391	0.395	0.395	0.395	0.436	**0.448**	0.469
	MSIHAR(3)-MIDAS	0.384	0.374	0.374	0.383	**0.433**	0.481	0.468
	MSH(3)-MIDAS	**0.270**	0.296	**0.297**	0.407	0.485	0.461	**0.402**
	MSHAR(3)-MIDAS	0.270	**0.287**	0.306	**0.370**	0.448	0.494	0.491
S&P 500	MSIH(3)-MIDAS	0.399	0.449	0.464	0.439	0.465	**0.437**	**0.457**
	MSIHAR(3)-MIDAS	0.434	0.436	0.443	0.408	0.520	0.519	0.509
	MSH(3)-MIDAS	0.442	0.414	**0.382**	0.412	0.473	0.495	0.517
	MSHAR(3)-MIDAS	**0.391**	**0.287**	0.410	**0.353**	**0.456**	0.466	0.635
Fed Funds	MSIH(3)-MIDAS	**0.391**	0.375	0.388	0.391	**0.385**	**0.390**	0.463
	MSIHAR(3)-MIDAS	0.394	0.401	0.400	0.385	0.459	0.462	**0.435**
	MSH(3)-MIDAS	0.415	**0.369**	**0.374**	**0.337**	0.483	0.389	0.495
	MSHAR(3)-MIDAS	0.423	0.403	0.436	0.411	0.489	0.502	0.445
	MSIHAR(3)	-	-	-	0.357	-	-	0.452

Entries in bold outline the model with the lowest QPS for each indicator and forecast horizon. QPS is computed as follows:

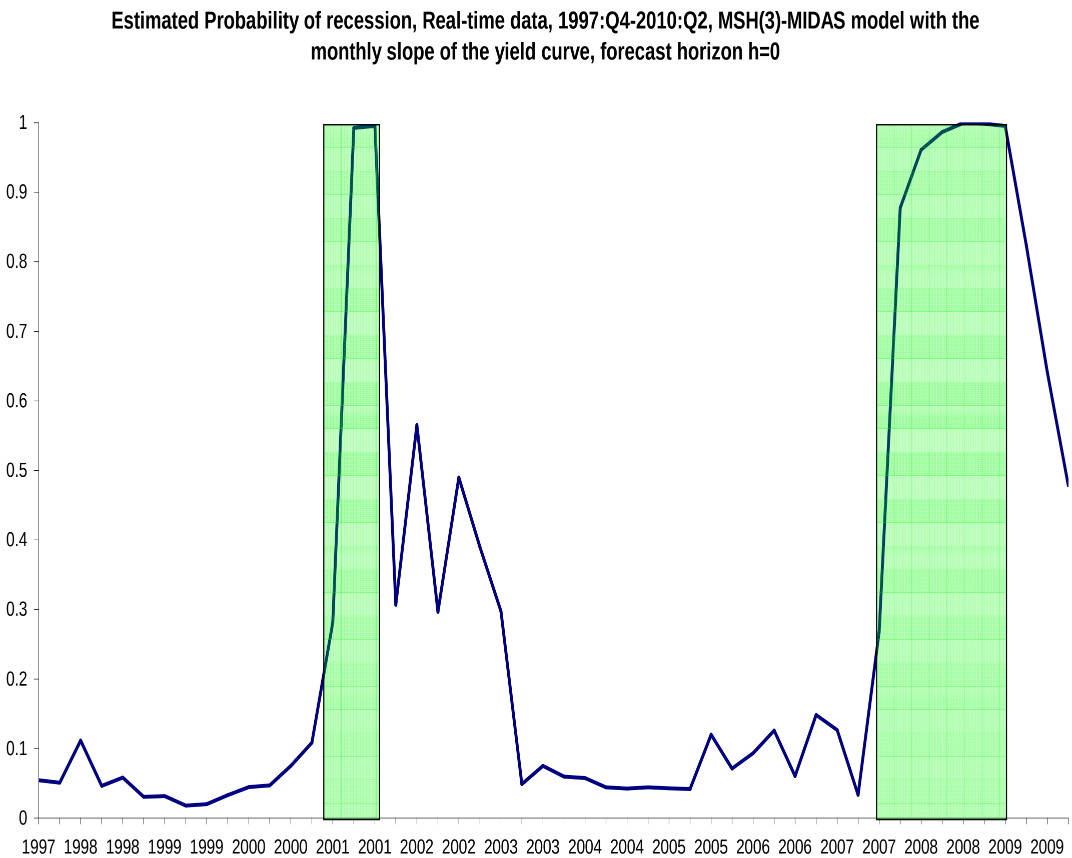
$$QPS = \frac{2}{F}\sum_{t=1}^{T}(P(S_{t+h}=1) - NBER_{t+h})^2$$

where F is the number of forecasts, $P(S_{t+h})$ are the predicted regime probabilities of being in the first regime and $NBER_{t+h}$ is a dummy variable that takes on a value of 1 if the US economy is in recession in quarter $t+h$ according to the NBER. For h={0,1/3,2/3,1}, we predict business cycle regimes one quarter ahead, whereas for h={4/3,5/3,2} we predict business cycle regimes two quarters ahead. A classification of the models is reported in Table E.

of 2007 using information up to February 2008. The probability of recession then jumps above .85 from the end of the first quarter of 2008 until the second quarter of 2009 and it starts declining in the third quarter of 2009. This confirms the NBER dating of the end of the last recession in June 2009. Note that our model gives the first signal of recession well before the announcement of the recession by the NBER that occurred in December 2008.

A crucial point of the MS-MIDAS specification is that the quarterly probabilities of recession can be updated on a monthly basis (i.e. at the frequency of the x_t^m variable). This makes this class of models very attractive for real-time estimation of business cycle conditions. Indeed, Table C in the appendix reports the nowcasted probability of recession for the MSH(3)-MIDAS model with the slope of the yield curve as an indicator for three different forecast horizons h=$\{0,1/3,2/3\}$. The table confirms that using the slope of the yield curve as a monthly indicator provides strong calls of recession in the first quarter of 2008 since the probability of recession gradually increased in the first quarter of 2008 to reach .87 in March 2008 (using information available up to May 2008). Table C also shows that the probability of recession is decreasing in the third quarter of 2009 in line with the NBER datation of the end of the last recession. However, the probability of recession remains fairly high in the third and fourth quarters of 2009, reflecting the moderate growth path experienced by the US.

In summary, Markov-switching MIDAS models not only generate good forecasting results for the level of GDP growth, but they also provide relevant information about the state of the economy. The combination of high frequency information and parameter switching performs better than using each of these two features separately, as in standard MIDAS and MS models, respectively.



4.2 Prediction of the UK GDP

4.2.1 In-sample results

The data for the UK GDP are taken from the Bank of England Real-Time Database.[9] We retain only the vintages corresponding to the first estimates of GDP. This database is

[9]The Bank of England Real-Time Database is available at: http://www.bankofengland.co.uk/statistics/gdpdatabase/

Table 6: Log Probability Score for forecasting US business cycle regimes 1998:Q1-2009:Q4

	Model	Forecast horizon (h)						
		0	1/3	2/3	1	4/3	5/3	2
Slope of the yield curve	MSIH(3)-MIDAS	0.739	0.749	0.747	0.748	**0.738**	**0.745**	0.775
	MSIHAR(3)-MIDAS	0.708	0.680	0.681	0.693	0.899	0.991	0.963
	MSH(3)-MIDAS	**0.437**	0.473	**0.465**	**0.690**	0.901	0.746	**0.604**
	MSHAR(3)-MIDAS	0.438	**0.454**	0.474	0.807	0.841	0.941	0.905
S&P 500	MSIH(3)-MIDAS	0.781	0.924	1.050	0.914	0.838	**0.697**	**0.736**
	MSIHAR(3)-MIDAS	0.853	0.856	0.891	0.815	0.981	0.985	0.920
	MSH(3)-MIDAS	**0.707**	0.756	**0.633**	0.823	**0.723**	0.889	1.052
	MSHAR(3)-MIDAS	0.741	**0.442**	0.714	**0.558**	0.919	0.923	1.127
Fed Funds	MSIH(3)-MIDAS	**0.734**	0.621	0.722	0.722	**0.581**	0.601	0.671
	MSIHAR(3)-MIDAS	0.805	0.736	0.734	0.693	0.959	0.961	**0.661**
	MSH(3)-MIDAS	0.924	**0.616**	**0.677**	**0.518**	0.768	**0.585**	0.822
	MSHAR(3)-MIDAS	0.785	0.760	0.870	0.748	0.850	0.850	0.904
	MSIHAR(3)	-	-	-	0.594	-	-	0.875

Entries in bold outline the model with the lowest LPS for each indicator and forecast horizon. LPS is computed as follows:

$$LPS = -\frac{1}{F}\sum_{t=1}^{T}(1 - NBER_{t+h})log(1 - P(S_{t+h} = 1)) + NBER_{t+h}log(P(S_{t+h} = 1))$$

where F is the number of forecasts, $P(S_{t+h})$ are the predicted regime probabilities of being in the first regime and $NBER_{t+h}$ is a dummy variable that takes on a value of 1 if the US economy is in recession in quarter $t+h$ according to the NBER. For h={0,1/3,2/3,1}, we predict business cycle regimes one quarter ahead, whereas for h={4/3,5/3,2} we predict business cycle regimes two quarters ahead. A classification of the models is reported in Table E.

updated every year following the publication of the ONS Blue Book. For the in-sample analysis, the dependent variable is taken as 100 times the quarterly change in the log of the UK real GDP from t=1975:Q1 to 2010:Q1. We consider comparable predictors as in the application for the US GDP: the slope of the yield curve, the Financial Times All Shares Index and the Bank of England base rate. The slope of the yield curve is taken as the difference between a bond with a 10-year maturity and a bond with a 1.5-year maturity. We applied the same data transformation as in the US case. The data for the FT All Shares Index and the Bank of England base rate are taken from Datastream, while the data for the UK yield curve are taken from the Bank of England database.

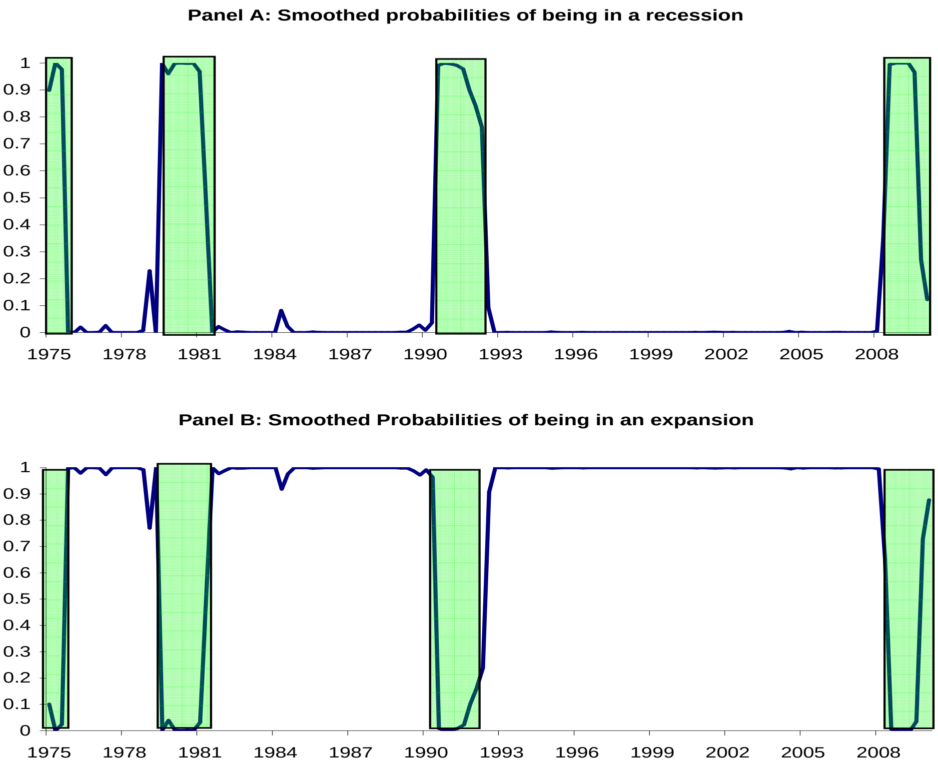
We select a model with two regimes and no switch in the variance of the error term since this model matches well the business cycle regimes experienced by the UK (see Figure 4). Information criteria (SIC and HQ) selected a model with three regimes. However, very few observations were associated with the third regime so that we decided to keep the model with two regimes for the sake of parsimony and for ease of information.

Table B in the appendix presents the in-sample results for each indicator for the MSI(2)-MIDAS and MS(2)-MIDAS models. The intercept $\beta_{0,1}$ in the first regime is always negative, while the intercept in the second regime $\beta_{0,2}$ is always positive. Both coefficients are highly significant in all cases. The coefficient β_1 is significant in most of the cases, which emphasizes the importance of including variables sampled at a monthly frequency for predicting quarterly GDP.

Figure 4 reports the estimated smoothed probabilities. The shadow areas are the recessions identified by the ECRI [10]. As mentioned, the four recessions experienced by the UK are all very well matched by the model with two regimes and no switch in the variance of the error term. We therefore use this class of models in the out-of-sample forecasting exercise. The different MS-MIDAS specifications are recursively estimated in each forecasting period and are used to predict not only the regime but also the level of GDP growth, so that the influence of the full sample regime fitting based specification is very small.

[10]The ECRI business cycle chronology is available at: http://www.businesscycle.com/resources/cycles/ The ECRI provides a business cycle chronology at the monthly frequency. We transformed it into quarterly frequency by considering that the UK economy is in recession in quarter t if the ECRI indicates so for at least one of the months of quarter t.

Figure 4: MSIAR(2)-MIDAS Quarterly GDP and monthly slope of the yield curve
Sample 1975:Q1 - 2010:Q1



4.2.2 Out-of-sample results

The design of the real-time forecasting exercise is identical to the one described in section 4.1.2. The actual values for GDP are taken from the last vintage of data available to us T=2010:Q2. Table 7 reports the Mean Square Forecast Error relative to an AR(1) model. The main results are the following.

First, note that the Markov-switching MIDAS models always outperform the MIDAS and AR-MIDAS models across all indicators for one-step ahead predictions of GDP, confirming the importance of allowing for time variation in the MIDAS regression for nowcasting and short-term forecasting. Second, each of the three indicators in the MS-MIDAS specification yield relevant information since they produce better forecasting results than the AR(1) model. Third, the AR-MIDAS and MIDAS models always obtain the best performance for two-step ahead predictions. Fourth, unlike for the US, share prices do not clearly outperform the slope of the yield curve and the short-term interest rate. Fifth, the AR-MIDAS model always outperforms the standard MIDAS with the slope of the yield curve and the short-term interest rate, while the standard MIDAS model performs better than the AR-MIDAS with share prices as an indicator for two-step ahead predictions. Finally, the standard Markov-switching model gets better forecasts than the AR(1) model for one-step ahead predictions but worse for two-step ahead predictions. At each horizon the MS model is beaten by at least one MS-MIDAS specification, confirming the importance of introducing higher frequency information in the MS model.

Tables 8 and 9 show the QPS and LPS criteria that allow us to assess the regime prediction ability of the models under scrutiny. First, the slope of the yield curve and the BoE base rate perform better than share prices for regime prediction: this confirms what we have found in the empirical application for the US. Besides, the standard Markov-switching model exhibits a good performance by itself, suggesting that the use of mixed frequency data does not improve regime prediction as much as in the case for the US. However, while the standard MS model can be only implemented at quarterly level, the MS-MIDAS specifications allow for a timely monthly update of the forecasts.

Figure 5 shows the estimated nowcasted probability of being in a recession for the model with the slope of the yield curve. These probabilities come from the forecasting exercise and correspond to the estimated filtered probabilities of being in a recession for the last observation T, where T is recursively expanded over time from t=2003:Q4 to 2010:Q1. The ECRI business cycle chronology indicates that the last recession started in May 2008. The chart shows that there is a peak in the probability of recession for the third quarter of 2008 and it indicates that the probability of recession sharply declined in the first quarter of 2010 [11].

Table D in the appendix provides further insight on the ability of the MS-MIDAS models to detect recessions in real time, and illustrates how the probability of recession can be updated on a monthly basis. This table shows that there is signal of recession in July 2008 using data for GDP from the October 2008 vintage as the probability of recession amounts to .68 before rising to 1 in the last quarter of 2008. The probabilities of recession are equal or close to 1 in 2009, but decline significantly in the first quarter of 2010. Indeed, the probability of recession in March 2010 is .39 suggesting that the UK recession that started in May 2008 according to the ECRI and two months later according to us, came to an end in the first quarter of 2010.

[11] At the time we have completed this draft, the ECRI has not announced yet the end of the last recession for the UK.

Table 7: Relative Mean Squared Forecast error for forecasting UK GDP growth 2004:Q1-2010:Q1

		Forecast horizon (h)						
	Model	0	1/3	2/3	1	4/3	5/3	2
Slope of the yield curve	MSI(2)-MIDAS	**0.621**	0.714	**0.617**	**0.693**	1.088	1.151	1.130
	MSIAR(2)-MIDAS	0.684	**0.684**	0.681	0.869	1.069	1.063	1.167
	MS(2)-MIDAS	0.801	0.896	0.819	0.844	1.070	1.168	1.164
	MSAR(2)-MIDAS	0.698	0.686	0.684	0.802	1.097	1.139	1.133
	AR-MIDAS	0.857	0.917	0.880	0.939	**0.954**	**0.953**	**0.975**
	MIDAS	0.906	0.969	0.897	1.017	1.115	1.174	1.165
Share prices	MSI(2)-MIDAS	**0.697**	**0.699**	0.861	**0.704**	1.019	1.009	1.006
	MSIAR(2)-MIDAS	0.814	0.810	**0.810**	0.806	1.070	0.998	0.994
	MS(2)-MIDAS	0.861	0.791	1.118	1.201	1.113	1.129	1.129
	MSAR(2)-MIDAS	0.750	0.801	0.916	0.766	1.190	1.041	1.040
	AR-MIDAS	0.868	0.868	0.850	0.933	0.958	0.939	0.938
	MIDAS	0.957	0.897	0.899	0.887	**0.956**	**0.925**	**0.921**
BoE base rate	MSI(2)-MIDAS	**0.633**	**0.685**	0.828	0.752	1.099	1.095	1.137
	MSIAR(2)-MIDAS	0.746	0.788	0.833	0.859	1.088	1.070	1.105
	MS(2)-MIDAS	0.637	0.767	**0.698**	**0.698**	0.925	0.925	1.149
	MSAR(2)-MIDAS	0.792	0.757	0.833	0.843	0.917	0.920	1.113
	AR-MIDAS	0.953	0.957	0.954	1.078	**0.862**	**0.903**	**1.079**
	MIDAS	1.034	1.028	1.032	1.089	1.131	1.132	1.280
	MSIAR(2)	-	-	-	0.866	-	-	1.082

Real-time data set. Relative Mean Squared Forecast Error for output growth in the quarters 2004:Q1-2010:Q1. Benchmark: AR(1) model. Recursive forecasting scheme. Entries in bold outline the model with the lowest MSFE for each indicator and forecast horizon. A classification of the models is reported in Table E.

Table 8: Quadratic Probability Score for forecasting UK business cycle regimes 2004:Q1-2010:Q1

		Forecast horizon (h)						
	Model	0	1/3	2/3	1	4/3	5/3	2
Slope of the yield curve	MSI(2)-MIDAS	0.204	0.215	0.203	0.201	0.434	0.379	0.345
	MSIAR(2)-MIDAS	**0.174**	0.174	0.174	0.184	0.403	**0.329**	0.447
	MS(2)-MIDAS	0.227	0.232	0.237	0.234	**0.354**	0.383	**0.323**
	MSAR(2)-MIDAS	0.182	**0.173**	**0.173**	**0.178**	0.356	0.341	0.391
Share prices	MSI(2)-MIDAS	0.263	0.259	0.244	0.237	**0.320**	**0.333**	**0.345**
	MSIAR(2)-MIDAS	**0.206**	0.206	**0.189**	0.192	0.398	0.411	0.410
	MS(2)-MIDAS	0.260	**0.196**	0.321	**0.180**	0.383	0.388	0.389
	MSAR(2)-MIDAS	0.367	0.469	0.326	0.432	0.488	0.474	0.474
BoE base rate	MSI(2)-MIDAS	0.214	0.215	0.213	0.187	0.332	0.339	0.328
	MSIAR(2)-MIDAS	0.176	0.177	0.176	**0.170**	0.371	0.337	**0.287**
	MS(2)-MIDAS	**0.154**	0.209	0.175	0.175	**0.239**	**0.239**	0.335
	MSAR(2)-MIDAS	0.267	**0.174**	**0.167**	0.175	0.316	0.250	0.346
	MSIAR(2)	-	-	-	0.175	-	-	0.315

Entries in bold outline the model with the lowest QPS for each indicator and forecast horizon. QPS is computed as follows:

$$QPS = \frac{2}{F}\sum_{t=1}^{T}(P(S_{t+h}=1) - ECRI_{t+h})^2$$

where F is the number of forecasts, $P(S_{t+h})$ are the predicted regime probabilities of being in the first regime and $ECRI_{t+h}$ is a dummy variable that takes on a value of 1 if the UK economy is in recession in quarter $t+h$ according to the ECRI. For $h = \{0, 1/3, 2/3, 1\}$, we predict business cycle regimes one quarter ahead, whereas for $h = \{4/3, 5/3, 2\}$ we predict business cycle regimes two quarters ahead. A classification of the models is reported in Table E.

Table 9: Log Probability Score for forecasting UK business cycle regimes 2004:Q1-2010:Q1

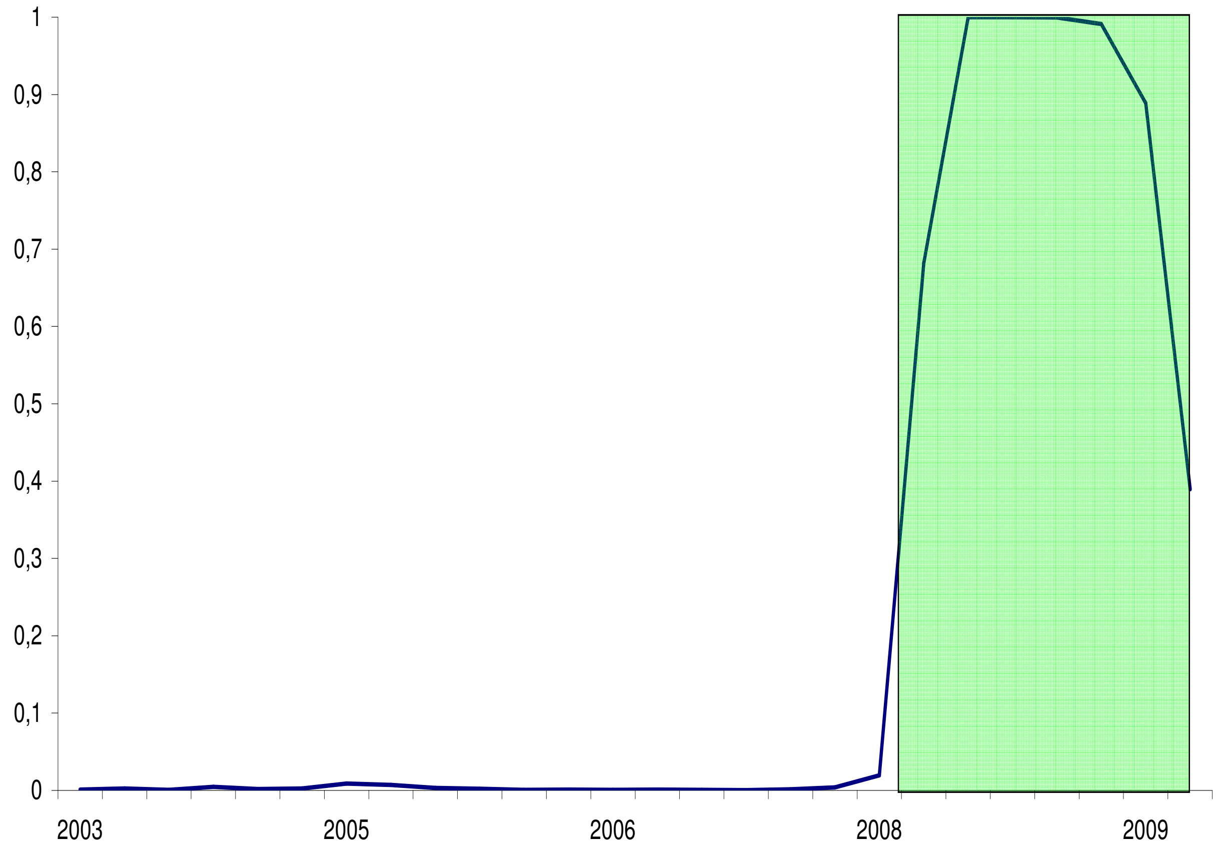
		Forecast horizon (h)						
	Model	0	1/3	2/3	1	4/3	5/3	2
Slope of the yield curve	MSI(2)-MIDAS	0.370	0.383	0.368	0.367	0.656	0.578	0.544
	MSIAR(2)-MIDAS	**0.345**	0.345	0.344	0.353	0.621	**0.532**	0.653
	MS(2)-MIDAS	0.397	0.404	0.388	0.387	0.555	0.558	**0.511**
	MSAR(2)-MIDAS	0.348	**0.342**	**0.342**	**0.346**	**0.553**	0.521	0.593
Share prices	MSI(2)-MIDAS	0.442	0.437	0.414	0.409	**0.554**	**0.568**	**0.579**
	MSIAR(2)-MIDAS	**0.369**	0.367	**0.353**	0.357	0.683	0.700	0.697
	MS(2)-MIDAS	0.586	**0.351**	0.650	**0.335**	0.677	0.663	0.664
	MSAR(2)-MIDAS	0.659	0.855	0.589	0.768	0.885	0.812	0.812
BoE base rate	MSI(2)-MIDAS	0.381	0.379	0.365	0.343	0.532	0.545	0.533
	MSIAR(2)-MIDAS	0.341	0.338	0.338	0.334	0.607	0.542	**0.485**
	MS(2)-MIDAS	**0.275**	0.372	0.346	0.346	**0.406**	**0.406**	0.536
	MSAR(2)-MIDAS	0.524	**0.333**	**0.335**	**0.330**	0.520	0.419	0.549
	MSIAR(2)	-	-	-	0.340	-	-	0.517

Entries in bold outline the model with the lowest LPS for each indicator and forecast horizon. LPS is computed as follows:

$$LPS = -\frac{1}{F}\sum_{t=1}^{T}(1 - ECRI_{t+h})log(1 - P(S_{t+h} = 1)) + ECRI_{t+h}log(P(S_{t+h} = 1))$$

where F is the number of forecasts, $P(S_{t+h})$ are the predicted regime probabilities of being in the first regime and $ECRI_{t+h}$ is a dummy variable that takes on a value of 1 if the UK economy is in recession in quarter $t + h$ according to the ECRI. For $h = \{0, 1/3, 2/3, 1\}$, we predict business cycle regimes one quarter ahead, whereas for $h = \{4/3, 5/3, 2\}$ we predict business cycle regimes two quarters ahead. A classification of the models is reported in Table E.

Figure 5: Nowcasted Probability of recession, Real-time data, 2003:Q4-2010:Q1, MSIHAR(2)-MIDAS model with the monthly slope of the yield curve, forecast horizon h=0



5 Conclusions

Mixed data sampling (MIDAS) models are attracting considerable attention in the literature for their ability to combine in a rather simple regression framework variables sampled at different frequencies. Time-varying parameter models, with changes both in the conditional mean and in the variance, are also more and more used in applied macroeconomics. In this paper we combine these two strands of literature, and introduce the Markov-switching (MS-)MIDAS model, which allows for time-variation in the parameters of MIDAS models, and for the use of high frequency information in standard MS models.

The MS-MIDAS model can be estimated by maximum likelihood, and Monte Carlo experiments indicate that the resulting estimates are rather accurate. Information criteria can then be used for the selection of the number of lags and regimes. Two empirical applications to nowcasting and forecasting quarterly GDP growth for the US and the UK using monthly financial indicators confirm the good performance of the MS-MIDAS model. It can also rather accurately predict changes in regimes.

Due to its generality and ease of implementation, we believe that the MS-MIDAS model can provide a convenient specification for a large class of empirical applications in applied macroeconomics and finance.

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Appendix A

All the models are estimated by maximum likelihood. The computations are carried out with the optimization library OPTMUM of Gauss 7.0. selecting the BFGS algorithm.

The algorithm we use is described by the following steps:

Denote ω the parameters of the models to be estimated.

- STEP 1: Give initial values to all parameters of the model ω^0.
- STEP 2: If there is regime switching, implement the Hamilton (1989) filtering procedure using in the first iteration ω^0 and in the following iterations ω^j. We thus obtain an estimate of the filtered probabilities - if there is regime switching - and the value of the log-likelihood function.
- STEP 3: Maximize the log-likelihood function to obtain an updated version of the parameters ω^j
- STEP 4: Iterate over STEP 2 and STEP 3 until the algorithm has converged.

Hamilton (1994, chapter 22) pointed out that this algorithm is a special case of the EM algorithm: the expectation (E) step is step 2 and the maximization (M) step is step 3. Note that the expectation step aims at the formulation of guesses about the latent variables given the data and the initial or updated values of the parameters, while the maximization step yields the values of the parameters that maximize the log-likelihood function over the iterations.

Table A: In-sample results, Quarterly US GDP growth rate 1959:Q1-2009:Q4

	Slope of the yield curve	S&P 500	Fed Funds	Slope of the yield curve	S&P 500	Fed Funds
$\beta_{0,1}$	-0.273	-0.024	-0.193	0.009	-0.030	-0.063
	[0.207]	[0.206]	[0.205]	[0.143]	[0.206]	[0.206]
$\beta_{0,2}$	0.778***	0.718***	0.801***	0.799***	0.781***	0.797***
	[0.053]	[0.058]	[0.052]	[0.052]	[0.052]	[0.052]
$\beta_{0,3}$	1.366***	1.271***	1.360***	1.352***	1.363***	1.410***
	[0.140]	[0.136]	[0.149]	[0.154]	[0.194]	[0.130]
$\beta_{1,1}$	-0.303**	0.098***	0.363**	-1.323***	0.121***	0.638***
	[0.145]	[0.026]	[0.159]	[0.329]	[0.039]	[0.208]
$\beta_{1,2}$	-	-	-	0.030	0.032**	0.513***
	-	-	-	[0.225]	[0.017]	[0.147]
$\beta_{1,3}$	-	-	-	0.215	0.068	-0.919***
	-	-	-	[0.464]	[0.047]	[0.243]
σ_1	0.651***	0.647***	0.618***	0.592***	0.578***	0.560***
	[0.187]	[0.215]	[0.230]	[0.154]	[0.212]	[0.155]
σ_2	0.214***	0.197***	0.213***	0.205***	0.198***	0.207***
	[0.036]	[0.038]	[0.036]	[0.036]	[0.036]	[0.033]
σ_3	0.633***	0.626***	0.653***	0.668***	0.626***	0.680***
	[0.126]	[0.131]	[0.134]	[0.148]	[0.165]	[0.128]
$P(S_t = 1)$	0.191	0.237	0.209	0.239	0.247	0.189
SIC	491.473	479.164	490.113	487.364	484.587	485.800

The first three columns report in-sample results for the MSIH(3)-MIDAS model (i.e. the model with no switch in β_1), while the last three columns report in-sample results for the the MSH(3)-MIDAS model(i.e. the model with a switch in β_1). ***, ** and * indicate significance at 1%, 5% and 10%. Standard deviations are reported in brackets. SIC is the Schwarz Information Criterion.

Table B: In-sample results, Quarterly UK GDP growth rate 1975:Q1-2010:Q1

	Slope of the yield curve	Share prices	BoE base rate	Slope of the yield curve	Share prices	BoE base rate
$\beta_{0,1}$	-0.724***	-0.825*	-0.675***	-0.847***	-0.782***	-0.971***
	[0.214]	[0.378]	[0.173]	[0.177]	[0.262]	[0.185]
$\beta_{0,2}$	0.746***	0.704***	0.747***	0.763***	0.664***	0.755***
	[0.065]	[0.071]	[0.062]	[0.061]	[0.068]	[0.059]
$\beta_{1,1}$	-0.925*	0.055**	-0.538	1.236***	0.301**	-1.462***
	[0.485]	[0.023]	[0.188]	[0.421]	[0.118]	[0.405]
$\beta_{1,2}$	-	-	-	0.388	0.097**	-0.069
	-	-	-	[0.238]	[0.030]	[0.131]
σ	0.393***	0.352***	0.394***	0.391***	0.364***	0.376***
	[0.054]	[0.081]	[0.052]	[0.050]	[0.056]	[0.048]
$P(S_t = 1)$	0.166	0.141	0.197	0.195	0.142	0.160
SIC	325.058	320.831	326.327	327.942	321.760	324.844

The first three columns report in-sample results for the MSI(2)-MIDAS model (i.e. the model with no switch in β_1), while the last three columns report in-sample results for the the MS(2)-MIDAS model(i.e. the model with a switch in β_1). ***, ** and * indicate significance at 1%, 5% and 10%. Standard deviations are reported in brackets. SIC is the Schwarz Information Criterion. QPS is the quadratic probability score using the ECRI business cycle datation and the estimated probabilities of being in the first regime.

Table C: US Estimated Quarterly Probabilities of Recession updated on a monthly basis

Quarter	Month	$P(S_t = 1)$	$NBER_T$	Quarter	Month	$P(S_t = 1)$	$NBER_T$
	January	0.146	0		January	0.995	1
Q1 2007	February	0.137	0	Q1 2009	February	1.000	1
	March	0.148	0		March	0.999	1
	April	0.128	0		April	0.994	1
Q2 2007	May	0.123	0	Q2 2009	May	0.987	1
	June	0.127	0		June	0.995	1
	July	0.053	0		July	0.829	0
Q3 2007	August	0.036	0	Q3 2009	August	0.822	0
	September	0.033	0		September	0.824	0
	October	0.283	0		October	0.663	0
Q4 2007	November	0.241	0	Q4 2009	November	0.639	0
	December	0.268	1		December	0.642	0
	January	0.636	1		January	0.486	0
Q1 2008	February	0.742	1	Q1 2010	February	0.468	0
	March	0.878	1		March	0.479	0
	April	0.931	1		April	0.339	0
Q2 2008	May	0.940	1	Q2 2010	May	0.343	0
	June	0.961	1		June	0.345	0
	July	0.985	1				
Q3 2008	August	0.983	1				
	September	0.987	1				
	October	1.000	1				
Q4 2008	November	1.000	1				
	December	1.000	1				

$P(S_t = 1)$ are the estimated probabilities of being in the first regime from the MSH(3)-MIDAS model with the monthly slope of the yield curve. $NBER_t$ is a dummy variable that takes on a value of 1 if the economy is in recession and 0 otherwise. For the months of March, June, September and December, the probabilities are obtained from the model with a forecast horizon $h = 0$. For the months of February, May, August and November, the probabilities are obtained from the model with a forecast horizon $h = 1/3$. For the months of January, April, July and October, the probabilities are obtained from the model with a forecast horizon $h = 2/3$.

Table D: UK Estimated Quarterly Probabilities of Recession updated on a monthly basis

Quarter	Month	$P(S_t = 1)$	$ECRI_T$	Quarter	Month	$P(S_t = 1)$	$ECRI_T$
	January	0.001	0		January	1.000	1
Q1 2007	February	0.001	0	Q1 2009	February	1.000	1
	March	0.001	0		March	1.000	1
	April	0.001	0		April	1.000	1
Q2 2007	May	0.001	0	Q2 2009	May	0.999	1
	June	0.001	0		June	1.000	1
	July	0.000	0		July	0.993	1
Q3 2007	August	0.000	0	Q3 2009	August	0.991	1
	September	0.000	0		September	0.991	1
	October	0.001	0		October	0.913	1
Q4 2007	November	0.001	0	Q4 2009	November	0.892	1
	December	0.001	0		December	0.889	1
	January	0.004	0		January	0.523	1
Q1 2008	February	0.004	0	Q1 2010	February	0.437	1
	March	0.004	0		March	0.390	1
	April	0.019	0				
Q2 2008	May	0.019	1				
	June	0.019	1				
	July	0.683	1				
Q3 2008	August	0.691	1				
	September	0.683	1				
	October	1.000	1				
Q4 2008	November	1.000	1				
	December	1.000	1				

$P(S_t = 1)$ are the estimated probabilities of being in the first regime from the MSIHAR(2)-MIDAS model with the monthly slope of the yield curve. $ECRI_t$ is a dummy variable that takes on a value of 1 if the economy is in recession and 0 otherwise. Note that at the time we have written this paper, the ECRI has not announced yet the end of the last recession. For the months of March, June, September and December, the probabilities are obtained from the model with a forecast horizon $h = 0$. For the months of February, May, August and November, the probabilities are obtained from the model with a forecast horizon $h = 1/3$. For the months of January, April, July and October, the probabilities are obtained from the model with a forecast horizon $h = 2/3$.

Table E: Classification of the models. The general Markov-switching MIDAS model with M regimes we consider is:

$$y_t = \beta_0(S_t) + \beta_1(S_t)B(L^{1/m};\theta)x_{t-h}^{(m)} + \epsilon_t(S_t)$$

where $\epsilon_t|S_t \sim NID(0, \sigma^2(S_t))$.

Model	regime changes in	AR component
MSI(M)-MIDAS	β_0	NO
MS(M)-MIDAS	β_0 and β_1	NO
MSIH(M)-MIDAS	β_0 and σ^2	NO
MSH(M)-MIDAS	β_0, β_1 and σ^2	NO
MSIAR(M)-MIDAS	β_0	YES
MSAR(M)-MIDAS	β_0 and β_1	YES
MSIHAR(M)-MIDAS	β_0 and σ^2	YES
MSHAR(M)-MIDAS	β_0, β_1 and σ^2	YES
MSIAR(M)	β_0	YES
MSIHAR(M)	β_0 and σ^2	YES

The suffix "H" refers to models with a switch in the variance of the shocks. The suffix "I" refers to models with a switch in the intercept β_0. The suffix "AR" means that we include an AR component in the model through a common factor to avoid a seasonal response of y to x as it is described in equation (5). The last two rows of the table show the labels we use for the standard Markov-switching models.