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ABSTRACT

Estimating the Aggregate Consumption Euler Equation with State-Dependent Parameters*

The consumption Euler equation is a building block of modern macro theory. Yet, the existing evidence on aggregate data offers very conflicting results for the estimates of the degree of forward-lookingness and interest rate semi-elasticity. The disappointing performance can be rationalized by estimating an Euler equation in which the parameters are allowed to vary with the state of the economy. The nonlinear method reveals that during periods in which consumption is above its conditional average the estimates of the degree of forward-lookingness and interest rate semi-elasticity are significantly larger (in absolute value) than the estimates associated with periods of below-average consumption. Our evidence is consistent with models of state-dependence or heterogeneity in the discount factor and the elasticity of intertemporal substitution.

JEL Classification: E21, E32 and E52
Keywords: aggregate consumption, Euler equation, heterogeneity and state-dependence

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1 Introduction

It is hard to think of a model in modern macro theory in which the (consumption) Euler equation had no prominent role. This intertemporal optimality condition represents a cornerstone of Classical theory, Real Business Cycle and New-Keynesian models, being an essential channel for the transmission of macroeconomic disturbances as well as policy shocks. Despite the extended use in the theory, researchers using aggregate data have struggled to find a significant and stable relationship between cyclical measures of real activity, their leads and lags, and the real rate of interest.

Estimates of the degree of forward-looking behaviour in aggregate consumption or output Euler equations range from values not statistically different from one (as in Ireland, 2005) to values not significantly different from zero (as in Fuhrer and Rudebusch, 2004). The semi-elasticity of interest rate is often statistically insignificant (see Lindé, 2005), and when it is not, the point estimates are so small as to imply only modest effects through the transmission of structural shocks or economic policies (see Dennis, 2009). As for the micro evidence, Crossley and Low (2011) show that the hypothesis of a constant elasticity of intertemporal substitution is strongly rejected by demand data.

On the theoretical side, a long-standing tradition of formal models has shown how to generate forms of nonlinearity in the Investment-Saving (IS) decision which share similarities with the specification employed in this paper. Hansen and Sargent (2010, p. 129) define "*a pessimist*" as someone who "*thinks that good news is temporary and that bad news endures.*" They show that this assumption coupled with model uncertainty produces time-varying risk prices as well as a process for consumption such that periods of below-average (above-average) expenditure display above-average (below-average) persistence. On the other hand, Köszegi and Rabin (2009) show that loss-aversion in the tradition of

Kahermann and Tversky (1979) generates reference-dependent consumption plans according to which households' expenditure responds more promptly to periods of below-average consumption because the associated utility loss is greater than the gain from periods of above-average consumption.

Another important strand of the literature emphasizes that state-dependence in pricing behaviour, along the lines of Caplin and Leahy (1991) and Dotsey, King and Wolman (1999), or in investment decisions, a la Caballero and Engle (1999) and Acemoglu and Scott (1997), imply that the effects of policy shocks depends on the level of economic activity. To the extent that the share of impatient and/or liquidity constrained households vary over the business cycle, periods of below-average (above-average) real activity may be associated with below-average (above average) interest rate elasticity in Euler equation specifications in the spirit of Iacoviello (2005) and Curdia and Woodford (2009). Along similar lines, Guvenen (2006) shows that heterogeneity in the intertemporal elasticity of substitution improves the fit of an otherwise standard calibrated macro model. The financial accelerator literature pioneered by Bernanke and Gertler (1989) emphasizes that depressed collateral values at times of below-average economic activity have the potential to generate asymmetric amplification of the effects of structural shocks.[1]

This paper offers a rationale for the conflicting estimates from earlier contributions by proposing a nonlinear specification for the consumption Euler equation. This is estimated on aggregate data using the Instrumental Variable Quantile Regression (IVQR) method proposed by Chernozhukov and Hansen (2005), which is well-suited to handle simultaneously nonlinearity and endogeneity issues. In the empirical model, nonlinearity stems from

[1] Our paper is also related to the literature on the asymmetric effects of monetary policy on real activity surveyed by Ravn and Sola (2004). The focus of that strand, however, is mainly on the effects of small versus large as well as negative versus positive monetary shocks whereas our interest is on differences in the estimates of the parameters governing the transmission mechanism conditional to the state of the business cycle. Rosenblatt-Wisch (2008) and Gaffeo et. al (2010) provide evidence in favor of asymmetric effects using piecewise linear Euler equation and piecewise linear VAR respectively.

the possibility, highlighted by the theories above, that the Euler coefficients may vary with the state of the economy, the latter being determined within the estimation method. Another contribution of the paper is to map state-dependent parameters into time-varying estimates for the Euler equation coefficients, thereby complementing the evidence from the drifting parameter models popularized by Cogley and Sargent (2005) and Primiceri (2005) and from the regime switching set up used by Sims and Zha (2006).

Our results can be summarized as follows. First, there is a significant extent of nonlinearity in the estimates of the degree of forward-lookingness: periods in the bottom (top) 10% of the conditional consumption distribution are characterized by fully backward-looking (forward-looking) behaviour. Second, there is also a significant extent of nonlinearity in the estimates of the interest rate semi-elasticity with values around -0.005 below the $70th$ percentile and values between -0.02 and -0.08 above that. This suggests that monetary policy is more effective during periods of conditionally high consumption. Third, the evidence of nonlinearity using only services expenditure is more pronounced that the evidence of nonlinearity using only nondurable goods expenditure. Fourth, a linear Euler equation significantly over-estimate (under-estimate) the degree of forward-lookingness and the interest rate semi-elasticity during periods of low (high) consumption conditional on covariates. Fifth, mapping the state dependent estimates into time-varying Euler equation coefficients reveals that periods of *conditionally* low (high) consumption coincide with periods of *unconditionally* below-trend (above-trend) consumption. The implication is that all our findings can be equivalently cast in terms of phases of the business cycle. Sixth, our results are robust to employing alternative filters to isolate cyclical movements, measures of real activity, instrument sets and specifications of the lag structure in the transmission mechanism.

The paper is organized as follows. In section 2, we present the estimation method to explore nonlinearity and account for endogeneity in our empirical investigation. Section 3 introduces the data and the instrument sets. In section 4, we present the main results of the paper as well as a method to translate state-dependent parameters into time-varying parameters. A sensitivity analysis is offered in Section 5 before conclusions. The Appendices report a Montecarlo analysis to assess the small sample bias associated with the nonlinear instrumental variable method presented in section 2 as well as convergence results for our Markov chain Montecarlo algorithm.

2 A nonlinear aggregate consumption Euler equation

The empirical and theoretical considerations above suggest that the parameters of the aggregate consumption Euler equation may vary over the business cycle. In particular, models featuring state-dependence or heterogeneity in the discount factor and the elasticity of intertemporal substitution generate the predictions that the degree of forward-lookingness as well as the interest rate semi-elasticity may depend on the current level of consumption, conditional on covariates.

In this section, we lay out a nonlinear model proposed by Chernozhukov and Hansen (2005) which is well-suited to investigate empirically the asymmetries in the aggregate consumption Euler equation emphasized by the theory. More specifically, suppose that a cyclical measure of aggregate consumption c_t evolves according to the following rule $F(\cdot)$:

$$c_t = F\left(c_{t-1}, E_t c_{t+1}, i_t..i_{t-h}, E_t\pi_{t+1}..E_t\pi_{t-h+1}, u_t\right) = F(r_t, d_t) = F(D_t) \qquad (1)$$

where i denotes the nominal interest rate, π is inflation, r stands for the ex-ante real rate, d refers to leads and lags of consumption, and E represents the expectation operator. The unobserved state of the world is denoted by u_t, which represents the source of heterogeneity.

Our aim is to estimate the shape of (1) using quantile regressions (QR). Above all, this will not assume that the relationship between consumption, its leads and lags and the interest rate is linear. Furthermore, we will consider the possibility that both the ex-ante real rate and future consumption are endogenous variables. The QR approach treats aggregate consumption as a potential latent outcome. It is latent because, given the macro covariates d_t, the observed outcome in each unit of time t is only one of the possible realizations in the admissible space of outcomes. The quantiles, Q_τ, of the potential outcome distributions conditional on covariates are denoted by:

$$Q_\tau\left(c_t | r_t, d_t\right) \quad \text{with } \tau \in (0,1). \tag{2}$$

The effect of a change in the real rate (the "treatment"), for instance, on different points of the marginal distribution of the potential outcome is given by:

$$QTE_\tau = \frac{\partial Q_\tau\left(c_t | r_t, d_t\right)}{\partial r} \tag{3}$$

The quantile treatment model can then be written as:

$$c_t = q\left(r_t, d_t, u_t\right) \quad \text{with } u_t | rr_t \sim U\left(0,1\right). \tag{4}$$

where $q\left(r_t, d_t, u_t\right) = Q_\tau\left(c_t | r_t, d_t\right)$. Note that we can always work with a suitable monotonic transformation of the underlying measure of unobserved heterogeneity such that u_t is a rank variable, i.e. it measures the relative ranking of states of the world in terms of potential outcomes. According to this interpretation, QTE_τ measures the causal effect of the real rate on aggregate consumption, holding the state of the world fixed at $u_t = \tau$.

If the ex-ante real interest rate and future consumption were exogenous, then the methods outlined in Koenker and Bassett (1968) could be used to estimate quantile effects on the basis of the conditional moment restrictions:

$$Prob[c \leq q\left(r, d, \tau\right) | r, d] = Prob[u \leq \tau | r, d] = \tau \quad \text{for each } \tau \in (0,1).$$

and the empirical specification of the conditional τ-th quantile distribution would take the following form:

$$Q_\tau (c_t|\cdot) = \alpha(\tau) + [1-\mu(\tau)]\, c_{t-1} + \mu(\tau)\, c_{t+1} + \beta(\tau) \left[\frac{1}{\kappa} \sum_{j=0}^{\kappa-1} (i_{t+j+m} - \pi_{t+j+m+1}) \right] \quad (5)$$

But assuming that r_t and c_{t+1} are exogenous is not satisfactory. Following a long-standing tradition in macro and micro econometrics we model the endogenous variables as a function of lagged values of consumption, inflation and the nominal interest rate. We will refer to a generic instrument set as Z_t. Given this, we can exploit the IVQR model of Chernozhukov and Hansen (2005) and write the process for consumption as:

$$c_t = q(r_t, d_t, u_t) \quad \text{with } u_t|Z_t, d_t \sim U(0,1). \quad (6)$$

where

$$Prob[c \leq q(r,d,\tau)\,|Z] = Prob[U \leq \tau|Z,d] = \tau \quad \text{for each } \tau \in (0,1).$$

In particular, we require that, given the exogenous elements of D_t, then u_t is distributed independently of Z_t. The parameters of the model are estimated by solving the following optimisation problem

$$\min_{\Xi} \left\| \frac{1}{T} \sum_{t=1}^{T} [1(c_t - D_t \Xi) - \tau]\, Z_t \right\| \quad (7)$$

where $1(\cdot)$ is an indicator function that takes value one if $(c_t - D_t\Xi) \leq 0$ and zero otherwise.

The objective function in equation (7) is not straightforward to minimise given the discontinuity introduced by the indicator function. Chernozhukov and Hansen (2005) suggest a novel method to minimise equation (7). This involves a grid search to find values of the vector Ξ such that the QR projections of $(c_t - D_t\Xi)$ on Z_t are minimised. This method works well with one endogenous regressor. However when the number of endogenous regressors exceeds one, we found that the estimates of the parameters were highly dependent on the limits and the length of the grid. Therefore, we estimate the model using the Markov chain Montecarlo (MCMC) approach introduced by Chernozhukov and Hong (2003), who define the quasi-posterior as

$$p_n(\theta) = \frac{\exp(L_n(\Xi))\,\pi(\Xi)}{\int \exp(L_n(\Xi))\,\pi(\Xi)} \tag{8}$$

where $\pi(\Xi)$ is a prior density and $L_n(\Xi)$ is defined as

$$L_n(\Xi) = -\frac{1}{2}\left(\left(\frac{1}{\sqrt{N}}\sum_{t=1}^{N} m_t(\Xi)\right)' W_n(\Xi)\left(\frac{1}{\sqrt{N}}\sum_{t=1}^{N} m_t(\Xi)\right)\right) \tag{9}$$

with $m_t(\Xi) = (\tau - 1(c_t - D_t\Xi))\,z_t$.

Chernozhukov and Hong (2003) set out the conditions under which a random walk Metropolis-Hastings (MH) algorithm provides valid point estimates and confidence intervals for Ξ. Note that unlike the grid search approach in Chernozhukov and Hansen (2005), the MCMC approach can easily be used for applications involving a moderate to large number of endogenous regressors.

At each value of τ, we initialise the MH algorithm using standard quantile regression estimates of the model parameters. The variance of the shock to the random walk proposal density is set such that the acceptance rate remains between 20% and 40%. Note that in our benchmark specification we require some of the parameters to lie within the bounds implied by economic theory. These are reported in table 1 and they are imposed by assigning an acceptance probability of zero to any draw that violates the bounds.

An important and non-standard requirement for IVQR relative to standard instrumental variable estimators is the rank similarity condition, which says that conditional on covariates the distribution of the rank variable u_t does not vary systematically with the endogenous variables. Note that this assumption is very likely to hold in our application: u_t can be interpreted as either the latent state of the world or an exogenous structural shock and these are unlikely to vary systematically with the level of the real interest rate or future consumption.

3 Data and instruments

The data were collected in October 2010 from the website of the Federal Reserve Bank of St. Louis. The nominal rate is the three month treasury bill rate. Inflation is measured as the first difference in the logarithm of the personal consumption expenditure (PCE) deflator. In the baseline case, real consumption is measured as the logarithm of the personal expenditure on nondurable goods and services, deflated by the PCE deflator and divided by the civilian noninstitutional population. In one of the sensitivity analyses, we will also consider durable consumption (constructed as above) and the monthly estimates of GDP computed by Stock and Watson (2010). All series but population are seasonally adjusted at the source.

The Montecarlo analysis in Appendix A reveals that the small sample bias associated with the IVQR method can be large using 200 observations. In contrast, it appears far more contained using as many as 700 observations. This result encourages us to work with monthly rather than quarterly data as for the latter there would be at most 250 data points available over the post-WWII period. On the other hand, monthly consumption data are available since 1959:1 and they can be extended back to 1948:5 by linearly interpolating quarterly observations. This implies that our sample, which ends in 2010:7, includes slightly more than 700 data points.

The cyclical component of consumption/output c_t is constructed using five methods: (1) HP filter, (2) Band-Pass filter, (3) linear de-trending, (4) quadratic de-trending and (5) the approximated de-trended method proposed by Cogley and Sargent (2005) where the trend c_t^* is computed using the recursion $c_t^* = c_{t-1}^* + 0.075(c_t - c_{t-1}^*)$. The data are displayed in figure 1.

Our benchmark instrument set includes lags of the cyclical measure of consumption, inflation and the nominal interest rate: $Z_t = (c_{t-2}, c_{t-3}, r_{t-2}, \pi_{t-2}, 1)$. We also consider four alternative sets which include different lags of the endogenous variables, namely $Z1_t = (c_{t-2}..c_{t-6}, i_{t-2}..i_{t-6}, \pi_{t-2}..\pi_{t-6}, 1)$; $Z2_t = (c_{t-2}..c_{t-12}, i_{t-2}..i_{t-12}, \pi_{t-2}..\pi_{t-12}, 1)$; $Z3_t = (c_{t-1}, i_{t-1}, \pi_{t-1}, 1)$; $Z4_t = (c_{t-2}, i_{t-2}, \pi_{t-2}, 1)$. While the results are very similar across the four alternatives, the benchmark instrument set is preferred when considering instrument strength. The first row of Table 2 shows that the multivariate F-statistics for the baseline instrument set are consistently larger than the critical value of 9.5 determined by Stock and Yogo (2003).[2] The entries for Z_t compare favourably to the entries in the third row of Table 2 for the alternative instrument sets used in one of the sensitivity analysis.

4 Results

In this section, we present the main results of our investigation. They are organized around the different measures of cyclical consumption and the benchmark instrument sets. In the following section, we assess the robustness of our results to changes in the specification of the Euler equation as well as to changes in the instrument set and the measure of real activity.

4.1 State-dependent Euler equation parameters

Figure 2 presents the estimates of the nonlinear consumption Euler equation using the benchmark instrument set and assuming $\kappa = 1$ and $m = 0$:

$$c_t = \alpha(\tau) + (1 - \mu(\tau))\, c_{t-1} + \mu(\tau)\, c_{t+1} + \beta(\tau)\left[(i_t - \pi_{t+1})\right] + u_t, u_t\tilde{}\, U(0,1)$$

The model is estimated using $1,000,000$ MCMC iterations, with inference based on the last $20,000$ draws. The recursive means of the retained draws (shown in Appendix B)

[2] This critical value depends on the number of endogenous variables, $d = 3$, the number of included and excluded regressors, $k = 5$, and the bias relative to OLS, $b = 0.05$, (see Stock and Yogo, 2003).

show little fluctuation providing evidence in favour of convergence. The rows of figure 2 present results for the five filters used in section 3 to extract the cyclical component of consumption.

Our discussion begins with the left column which displays the estimated coefficient on c_{t+1}. The Two Stage Least Square (TSLS) estimates, reported as dotted lines, are centered around the value of 0.5, consistent with the findings in Fuhrer and Rudebusch (2004). On the one hand, the IVQR estimates are significantly smaller than the TSLS estimates for quantiles below 25%. On the other hand, the TSLS significantly under-estimates the presence of forward-looking behaviour at the top 25^{th} percentile of the conditional distribution of consumption. This evidence suggest that periods in which consumption is conditionally low (high) are also periods characterized by higher (lower) persistence. In other words, agents appear more forward looking in good times (as measured by high values of τ). This finding is more striking using the HP filter in the first row but it holds across the different cyclical measures.

As for the interest rate semi-elasticity β, the TSLS estimates in the second column of figure 2 are centered around zero and they are never statistically significant as it is often the case in Fuhrer and Rudebusch (2004). This finding is similar to the IVQR estimates obtained for values of $\tau < 60\%$. In the top 20^{th} percentiles of the conditional distribution of consumption, however, the sensitivity of the IS schedule to the real rate averages from values around -0.03 using the HP filter to values around -0.07 using Cogley and Sargent's approximate detrending. This evidence on nonlinearity suggests that monetary policy (and more generally the effects of real rate on consumption) exerts its maximum impact during periods in which consumption is conditionally high.

In summary, we find strong evidence in favour of nonlinearity: the heterogeneous estimates of the Euler equation parameters across the conditional distribution of consumption differ significantly from the average effect estimated using a linear specification.

4.2 Mapping state-dependence into time-variation

In this section, we link the distribution of consumption conditional on covariates (which is the basis for the IVQR estimation) to the unconditional distribution of consumption. Our goal is to establish whether periods of conditionally low (high) consumption correspond to periods of below-trend (above-trend) consumption. We find that they do, which allows us to interpret the state-dependent estimates of the previous section in terms of phases of the business cycle.

To map the evidence of state-dependence into time-varying estimates, we follow a procedure similar to the one described in Koenker (2005 pp. 295-316). In particular, we re-estimate the following version of the benchmark specification

$$c_t = \alpha^* (\tau) + (1 - \mu (\tau)) \tilde{c}_{t-1} + \mu (\tau) \tilde{c}_{t+1} + \beta (\tau) [(\tilde{\imath}_t - \tilde{\pi}_{t+1})] + u_t$$

where the superscript ˜ indicates deviations from the mean. Note that $\alpha^* (\tau)$ now captures the level of c_t in each τ, given the mean of the real rate and lagged consumption. We then use the estimated value of $\alpha^* (\tau)$ in each quantile to link the level of the dependent variable across time to the values of $\mu (\tau)$ and $\beta (\tau)$. More specifically, we define an indicator variable $I [\alpha^* (\tau_{i-1}) < c_t \leq \alpha^* (\tau_i)]$ where $i = 1..M$ indexes the M values of τ that we consider. We specify μ_t and β_t as two $T \times 1$ vectors with all elements initially equal to zero. We loop through the elements of μ_t and β_t setting $\mu_t = \mu (\tau_i)$ and $\beta_t = \beta (\tau_i)$ if $I [\alpha^* (\tau_{i-1}) < c_t \leq \alpha^* (\tau_i)] = 1$. Repeating this for $i = 1..M$ fills all elements of μ_t and β_t and produces a time-series for these coefficients.

Our findings are reported in figure 3. The top (bottom) panel shows the estimates of μ (β) over the post-WWII sample together with our baseline measure of cyclical consumption. Whenever consumption is above trend, consumption decisions appear (i) more forward-looking and (ii) more sensitive to movements in the real rate. On the other hand, during the troughs of the cycle, and especially during the recession episodes of the 1970s, 1981, 1992 and 2008, agents tend to be significantly more backward-looking and their expenditure becomes far less sensitive, if any, to the real rate. While this finding is not based on any explicit assumption on the evolution of the Euler equation parameters over time, the evidence in figure 3 is more reminiscent of the type of time-variation estimated using a model with Markov switching regime changes rather than random walk drifting coefficients.

5 Sensitivity Analysis

In this section, we perform four robustness check. First, we consider alternative measures of real activity. Second, we allow for four different instrument sets. Third, we experiment with different lags of the real rate structure. Fourth, we estimate the IVQR model without imposing the restrictions that the coefficient on the real rate should be non-positive and that the coefficients on consumption should sum up to one. Our findings are robust to all these modifications of the baseline case.

5.1 Measures of activity

In the baseline case, we have used per capita real expenditure on non-durable goods and services. In this section, we first consider non-durable goods and services consumption separately, and then perform the IVQR estimation using the real expenditure on durable goods and the monthly estimates of GDP produced by Stock and Watson (2010). The results are reported in figure 4 and they show a similar pattern to figure 2 with some important qualifications. First, for both Euler equation coefficients, the evidence of state-dependence is significantly stronger using services consumption than using non-durable goods consumption. Second, there is little evidence of heterogeneity across phases of the business cycle using durable goods consumption. Third, using Stock and Watson's monthly GDP measure the estimates of degree of forward-lookingness at the upper tail of the conditional distribution are larger than the estimates at the lower tail while the evidence for the interest rate semi-elasticity is more muted. The corresponding F-statistics for the four measures are reported in the second row of Table 2. Only services consumption and Stock and Watson's monthly GDP pass comfortably Stock and Yogo's test of instrument strength. Interestingly, these are also the only two measures for which we find strong evidence of state-dependent Euler equation parameters.

5.2 Instrument set

The results of the F-tests using the different filters together with the Montecarlo analysis in Appendix A suggest that our baseline estimates are unlikely to suffer from a weak instrument problem. Nevertheless, it is useful to assess the sensitivity of our findings to using instruments set with a different number of lags for each endogenous variables. The results are reported in figure 5 for the instrument sets $Z1_t = (c_{t-2}..c_{t-6}, i_{t-2}..i_{t-6}, \pi_{t-2}..\pi_{t-6}, 1)$; $Z2_t = (c_{t-2}..c_{t-12}, i_{t-2}..i_{t-12}, \pi_{t-2}..\pi_{t-12}, 1)$; $Z3_t = (c_{t-1}, i_{t-1}, \pi_{t-1}, 1)$; $Z4_t = (c_{t-2}, i_{t-2}, \pi_{t-2}, 1)$. The conclusions one can drawn upon the alternative instrument sets are very similar to the ones for the baseline case. It should be noted, however, that according to the F-statistics in the third row of Table 2 the alternative sets are less likely to qualify as strong instruments. Indeed, this is the reason behind our choice of focusing on the baseline set Z_t in section 4.

5.3 Timing of the transmission mechanism

Different specifications of the lag structure for the real interest rate also appear unable to overturn our evidence of state-dependence. Specifying [k=0, m=1], [k=12, m=0] and [k=12, m=1] for $\sum_{j=0}^{\kappa-1} (i_{t+j+m} - \pi_{t+j+m+1})$ in the first, second and third row respectively of figure 6 makes our conclusions even stronger. (Fully) backward-looking behaviour is predominant at the bottom (end) 30% of the conditional distribution while (fully) forward-looking behaviour is predominant at the top (end) 30%. As for the interest rate semi-elasticity, we confirm that only the top 25% observations above the consumption average are associated with significantly negative estimates ranging from values around -0.03 in the first row to values around -0.06 in the last row. The F-statistics for these specifications, reported in the last row of Table 2, are always above Stock and Yogo's critical value.

5.4 Unrestricted estimates

In line with the theory, the estimates above restrict the coefficient on the real interest to be non-positive. Furthermore, the coefficients on backward-looking and forward-looking terms are restricted to sum up to one. In this section, we assess the robustness of our finding on nonlinearity to relaxing both restrictions.

The estimated coefficients on future and lagged consumption in figure 7 are very similar to the benchmark results and their sum is never statistically larger than one. The pattern of the coefficient on the real interest rate across quantiles is very similar to the pattern for the benchmark case: the estimates of β become more negative and statistically significant at the right tail of the consumption distribution. Note, however, that for quantiles $\tau < 25\%$, the positive coefficient is hard to justify from an economic perspective. The Stock and Yogo's F-statistics for this case is 18.6.

6 Conclusions

This paper has provided empirical evidence in favour of state dependent parameters in the aggregate consumption Euler equation using instrumental variable quantile regressions. In periods of below-average consumption, households' decisions tend to be more backward-looking and less sensitive to movements in the real rate of interest. Periods at the other end of the conditional distribution of consumption are associated with fully forward-looking behaviour and with the maximum impact of the real rate onto aggregate expenditure. The average effect estimated on the basis of a linear specification, in contrast, points toward an insignificant interest rate semi-elasticity and towards roughly equal weights received by backward-looking and forward-looking components.

Our results offer empirical support for the notion that the dynamics of consumption during expansions are qualitatively and quantitatively different from the dynamics during contractions. This finding is consistent with the predictions of state dependent models of investment/saving plans as well as heterogeneous agent models of the discount factor and the elasticity of intertemporal substitution. Furthermore, it suggests that monetary policy (and any other shock channelled through the real rate of interest) has asymmetric effects over the business cycle. This implies that central bank interventions in periods of below-average consumption need to be significantly larger than the policy interventions required in periods of above-average consumption to generate effects of the same size.

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Table 1: Sign Restrictions in the benchmark model

	Lower	*Upper*
$\mu(\tau)$	0	1
$\beta(\tau)$	-1	0

Table 2: Stock and Yogo multi-variate F-statistics

specifications using different:

filters	*Hodrick-Prescott*	*band pass*	*linear DT*	*quadratic DT*	*approx. DT*
	18.6	310.7	23.3	23.4	17.6
measures of real activity	*non-durable cons.*	*services cons.*	*durable cons.*	*SW's GDP*	
	1.7	25.0	5.2	12.3	
instrument sets	*Z1*	*Z2*	*Z3*	*Z4*	
	9.9	7.0	0.1	0.8	
real rate lag structures	$k=0, m=1$	$k=12, m=0$	$k=12, m=1$		
	18.6	18.2	18.2		

Note: DT stands for de-trending and SW for Stock and Watson (2010). The instruments are $Z1_t=(c_{t-2}..c_{t-6},i_{t-2}..i_{t-6},\pi_{t-2}..\pi_{t-6},1)$; $Z2_t=(c_{t-2}..c_{t-12},i_{t-2}..i_{t-12},\pi_{t-2}..\pi_{t-12},1)$;$Z3_t=(c_{t-1},i_{t-1},\pi_{t-1},1)$;$Z4_t=(c_{t-2},i_{t-2},\pi_{t-2},1)$. k and m refers to the lag structure in the specification (5). The critical value for which the bias relative to OLS is no more than 5% is 9.5. Above this value, Stock and Yogo (2003) deem the instruments strong and the inference based on the IV estimator reliable.

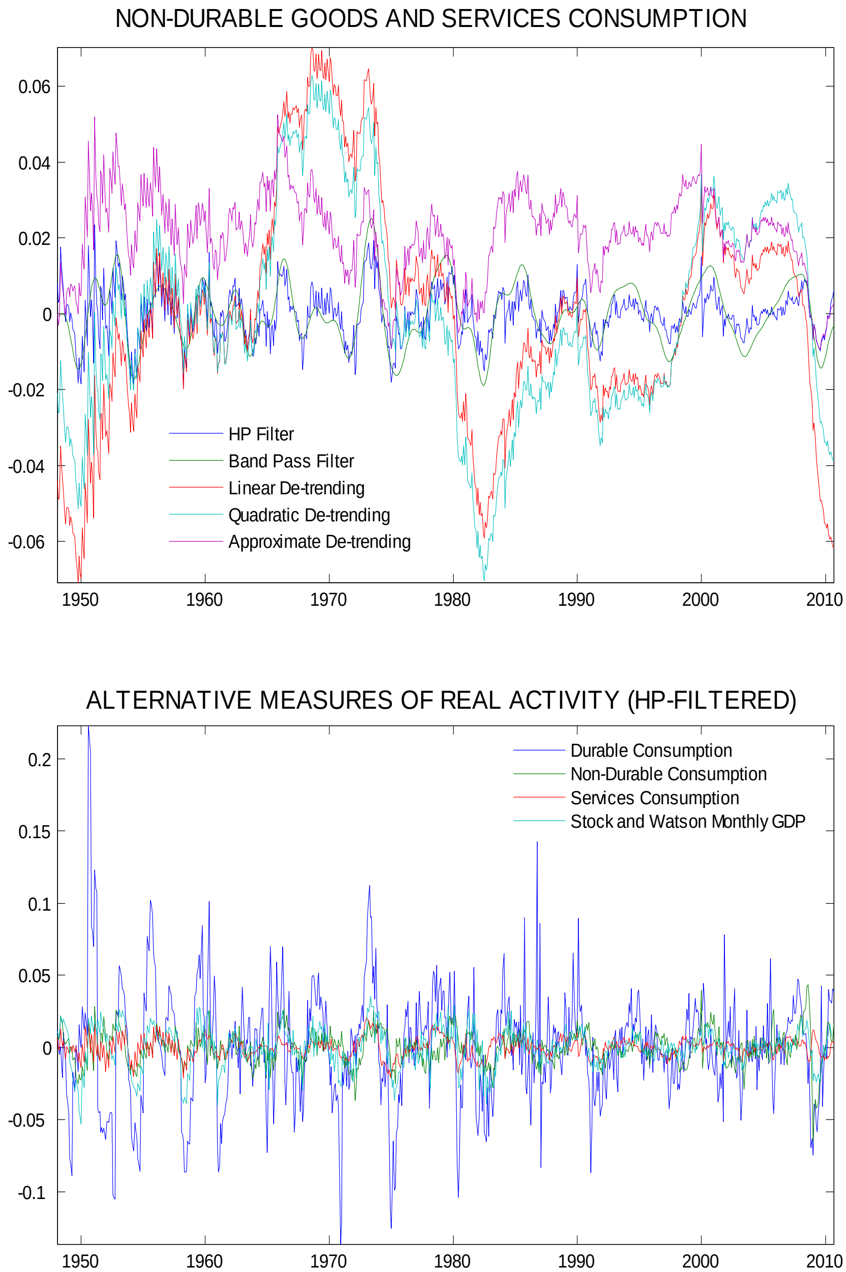

Figure 1: the data

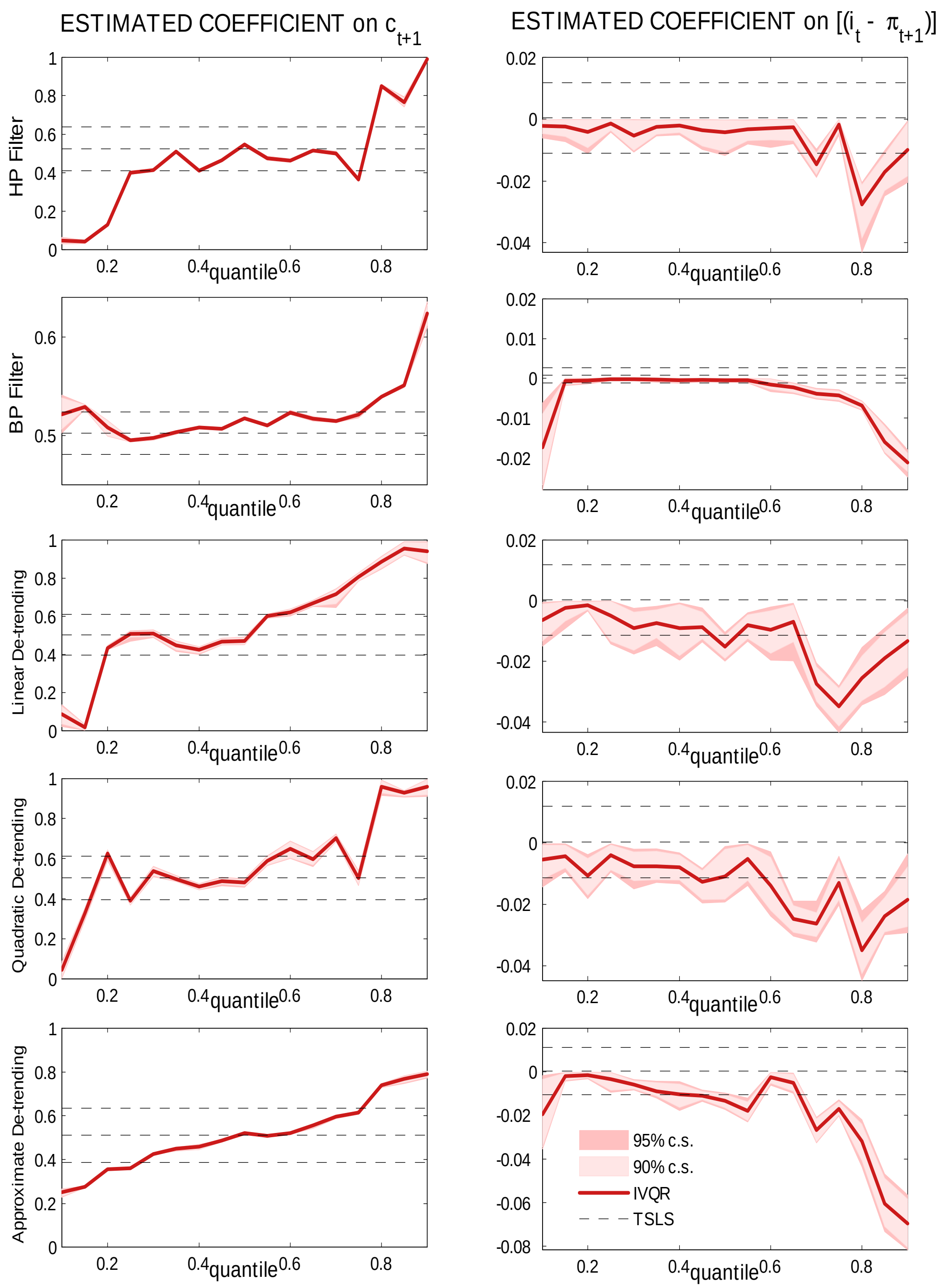

Figure 2: IVQR estimates of the parameters of the Euler equation with $\kappa = 1, m = 0$ using the benchmark instrument set $Z_t = (c_{t-2}, c_{t-3}, i_{t-2}, \pi_{t-2}, 1)$ and alternative filters on the real personal consumption expenditure of non-durable goods and services. Dotted lines represent to TSLS estimates and 95% credible sets. Sample: 1948:5-2010:7.

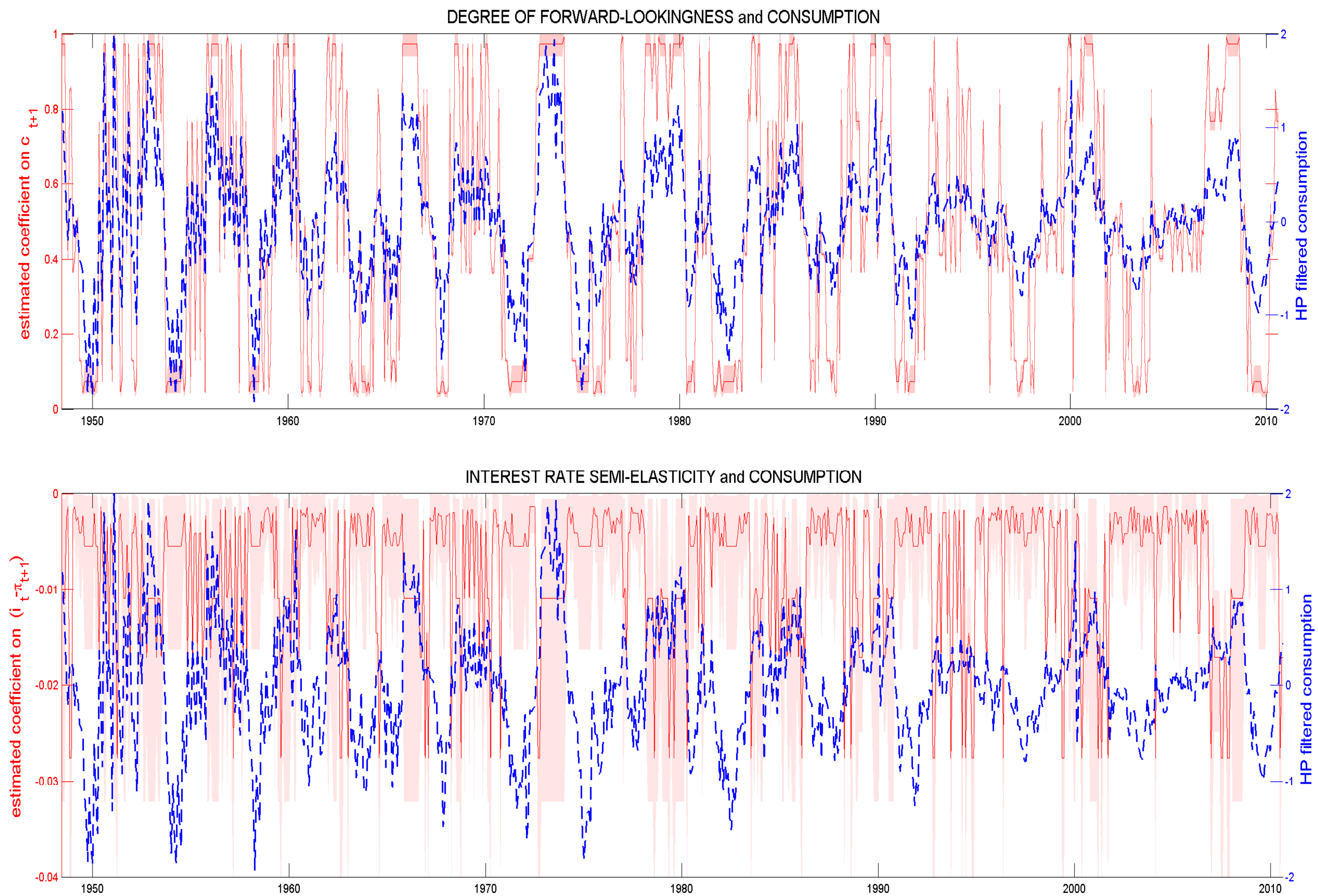

Figure 3: time series mapping of the IVQR estimates of the parameters of the Euler equation (solid lines in red with 95% credible set shaded area) with $\kappa = 1, m = 0$ using the HP filter on the real personal consumption expenditure of non-durable goods and services (dotted line in blue) and the benchmark instrument set $Z_t = (c_{t-2}, c_{t-3}, i_{t-2}, \pi_{t-2}, 1)$. Sample: 1948:5-2010:7.

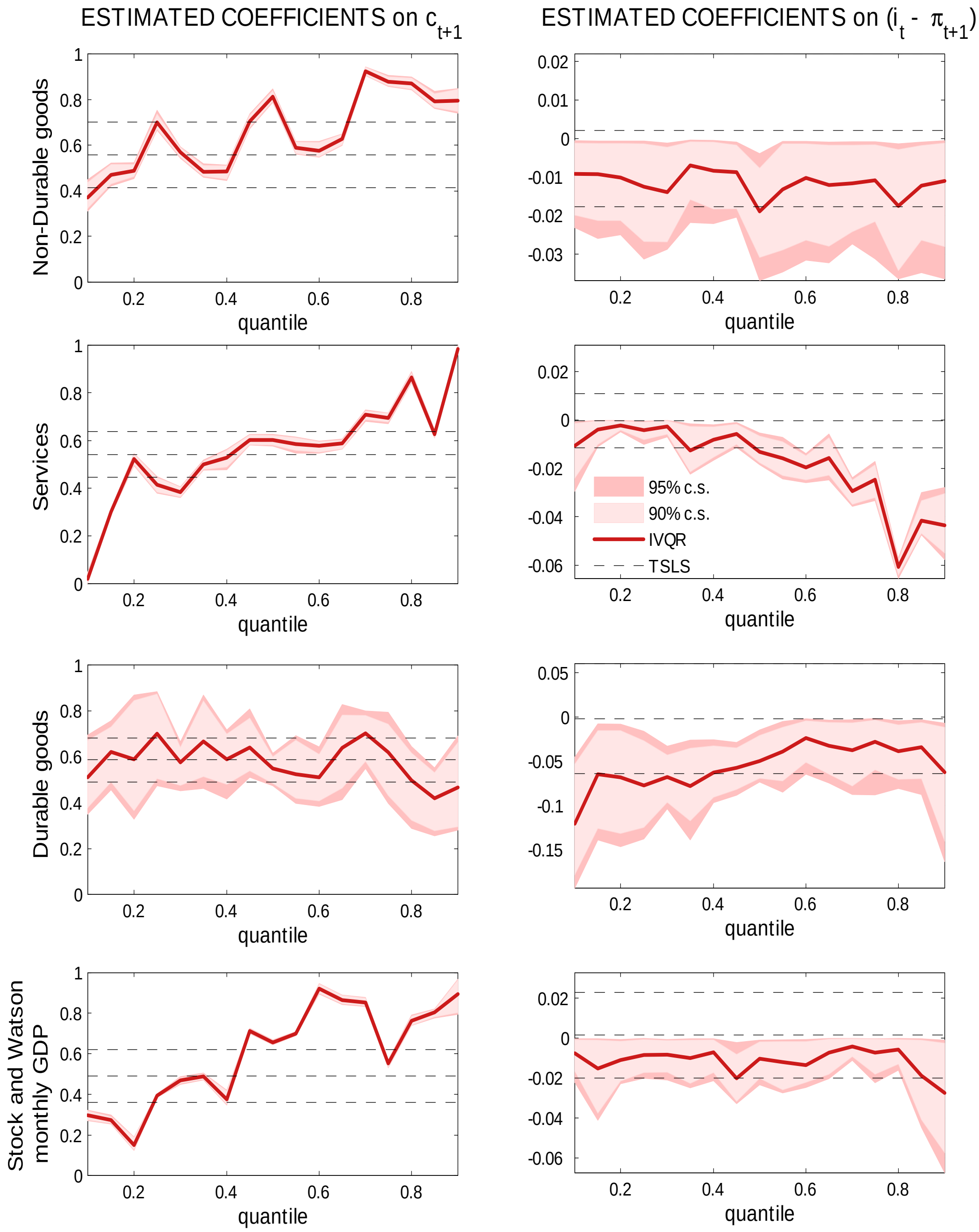

Figure 4: IVQR estimates of the parameters of the Euler equation with $\kappa = 1, m = 0$ using the benchmark instrument set $Z_t = (c_{t-2}, c_{t-3}, i_{t-2}, \pi_{t-2}, 1)$, the HP filter and alternative measures of real activity. Dotted lines represent to TSLS estimates and 95% credible sets. Sample: 1948:5-2010:7.

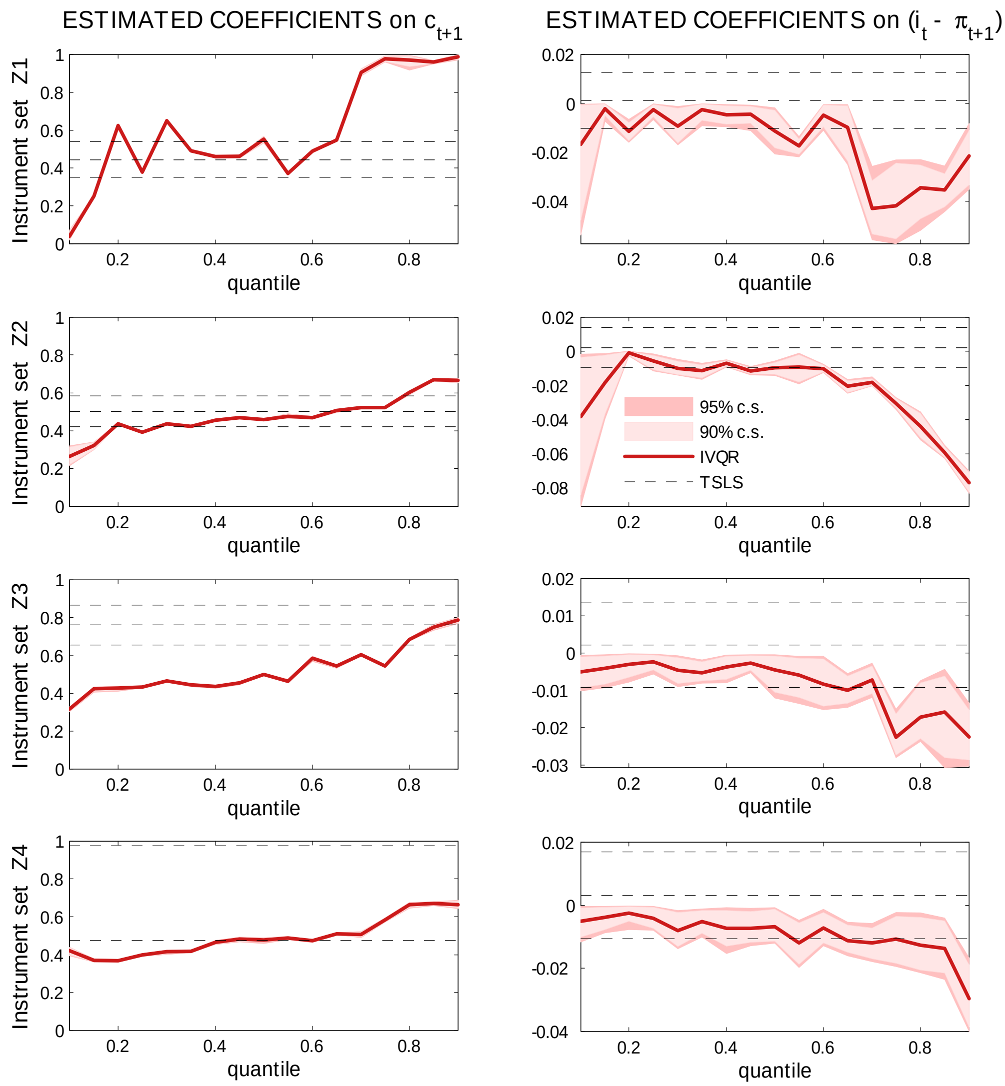

Figure 5: IVQR estimates of the parameters of the Euler equation with $\kappa = 1, m = 0$ using the HP filter on the real personal consumption expenditure of non-durable goods and services, and the alternative instrument sets $Z1_t = (c_{t-2}..c_{t-6}, i_{t-2}..i_{t-6}, \pi_{t-2}..\pi_{t-6}, 1)$; $Z2_t = (c_{t-2}..c_{t-12}, i_{t-2}..i_{t-12}, \pi_{t-2}..\pi_{t-12}, 1)$; $Z3_t = (c_{t-1}, i_{t-1}, \pi_{t-1}, 1)$; $Z4_t = (c_{t-2}, i_{t-2}, \pi_{t-2}, 1)$. Dotted lines represent to TSLS estimates and 95% credible sets. Sample: 1948:5-2010:7.

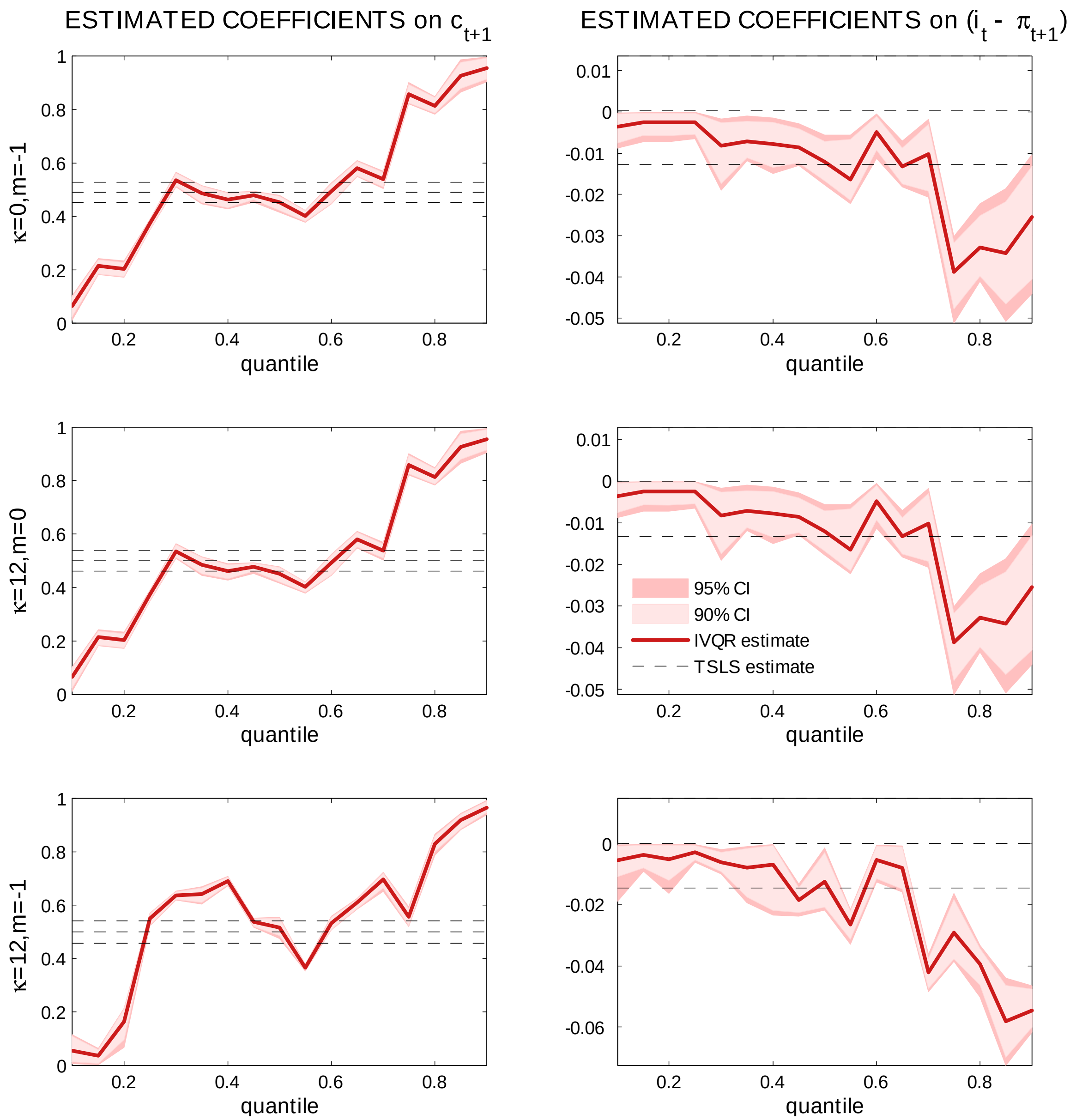

Figure 6: IVQR estimates of the parameters of the Euler equation using the HP filter on the real personal consumption expenditure of non-durable goods and services, the benchmark instrument set $Z_t = (c_{t-2}, c_{t-3}, i_{t-2}, \pi_{t-2}, 1)$ and alternative values of κ and m for the timing of the effect of the expected real interest rate. Dotted lines represent to TSLS estimates and 95% credible sets. Sample: 1948:5-2010:7.

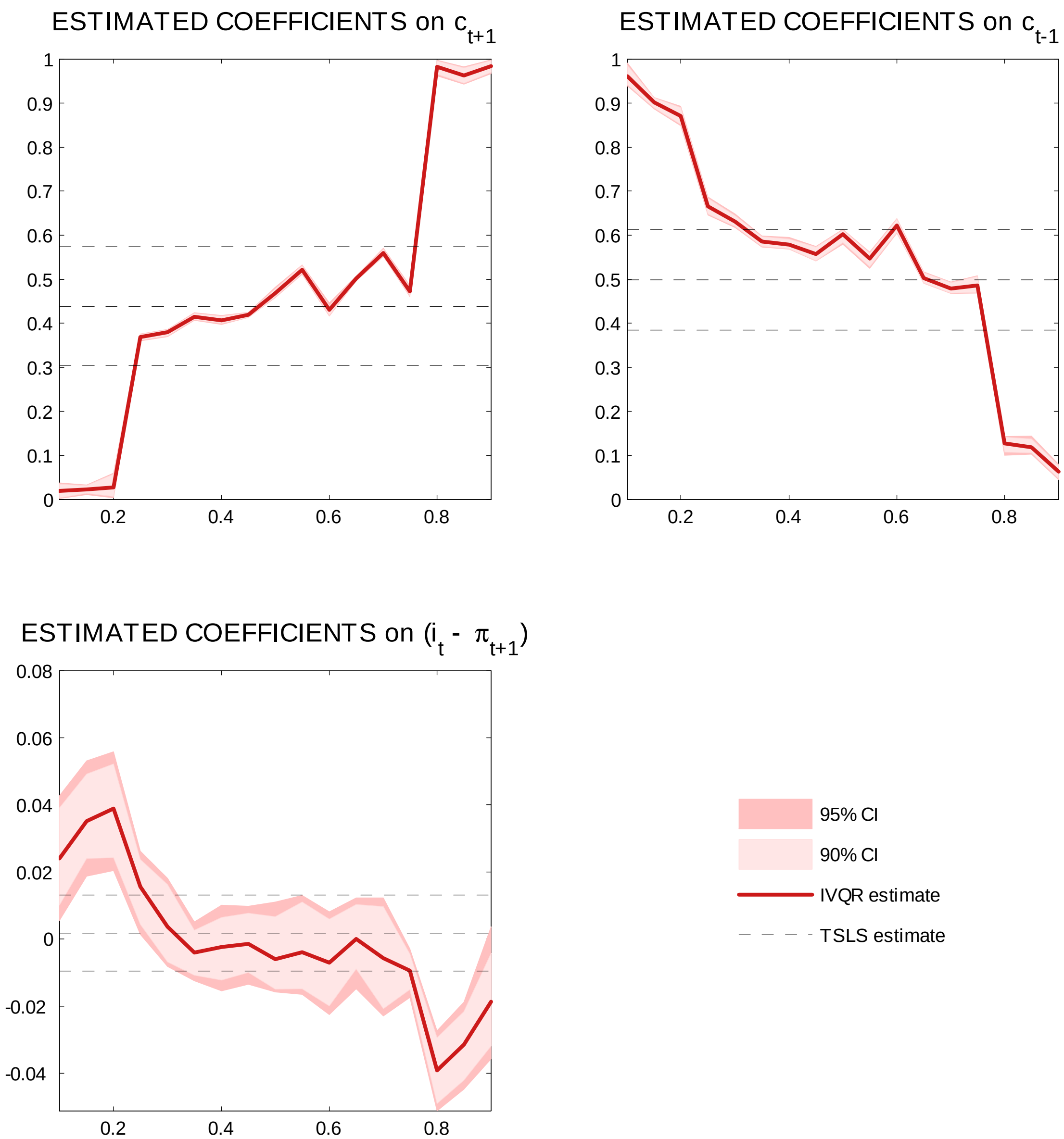

Figure 7: IVQR estimates of the parameters of the Euler equation with $\kappa = 1, m = 0$ using the HP filter on the real personal consumption expenditure of non-durable goods and services, and the benchmark instrument set $Z_t = (c_{t-2}, c_{t-3}, i_{t-2}, \pi_{t-2}, 1)$, without imposing (i) the coefficient on expected real interest rate to be non-positive and (ii) the coefficients on backward- and forward-looking components to sum up to one. Dotted lines represent to TSLS estimates and 95% credible sets. Sample: 1948:5-2010:7.

Appendix A: montecarlo analysis

Chernozhukov and Hong (2003) present montecarlo evidence to show that the MCMC estimator performs well when considering a simple quantile regression model with exogenous regressors. Here we extend their simulation to instrumental variables by considering a state-dependent parameter version of the Euler equation studied by Fuhrer and Rudebusch (2004). In particular, we consider two versions of the Euler equation: one in which both parameters vary across conditional consumption quantiles, and the other in which only the interest rate semi-elasticity is assumed to vary. The first simulation experiment is meant to assess the ability of the IVQR estimator to recover the Euler equation parameters when these vary across conditional quantiles. The second experiment assesses the ability of the IVQR estimator to recover the Euler equation parameters when one of these is, in fact, constant across the conditional distribution. We consider the following data generating process

$$c_t = \alpha_0(u) + (1 - \alpha(u))\, c_{t+1} + \alpha(u)\, c_{t-1} - \beta(u)\,[i_t - \pi_{t+1}] \tag{10}$$

where $\alpha(u) = \min(0.2 + 0.5u, 0.8)\,, \beta(u) = 0.1 + 0.5u$ and $\alpha_0(u) = \Phi^{-1}(u)$ with Φ^{-1} being the inverse normal cumulative distribution function (see Koenker and Xiao, 2006). We augment equation (10) with the following bivariate VAR(1) model which describes the dynamics of r_t and π_t.

$$\begin{pmatrix} i_t \\ \pi_t \end{pmatrix} = \begin{pmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{pmatrix} \begin{pmatrix} i_{t-1} \\ \pi_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}, \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \tilde{\ } N(0, I_2) \tag{11}$$

We generate three samples of $T + 50$ observations and discard the first 50 to reduce the influence of initial conditions. The length of the artificial samples are 100, 200 and 700 with the latter reflecting the number of observations used in the empirical investigation in the text. An important goal of this montecarlo analysis is to assess the minimum number of observations that are needed to draw reliable inference from the IVQR estimator. 200 (100) is roughly the number of observations that an econometrician would have available using quarterly data over the post-WWII (great moderation) period, while 700 is the number of observations available using monthly data.

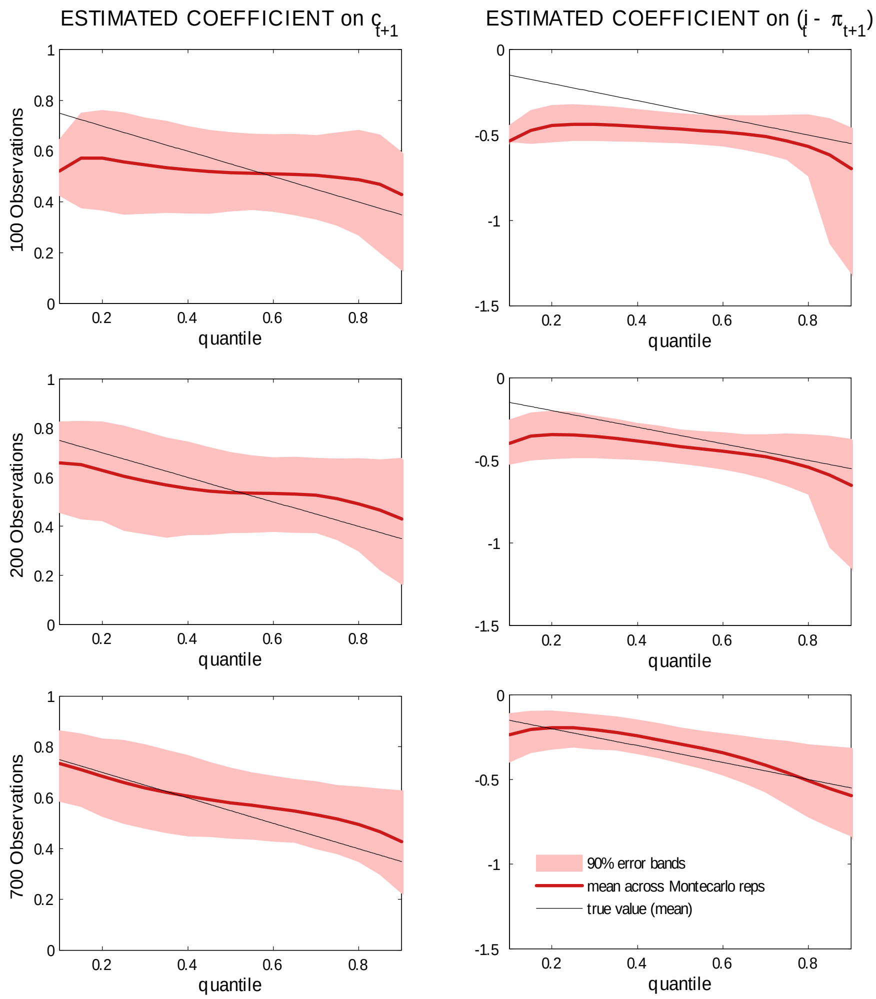

Figure 8: Montecarlo experiment with both coefficients varying across states τ.

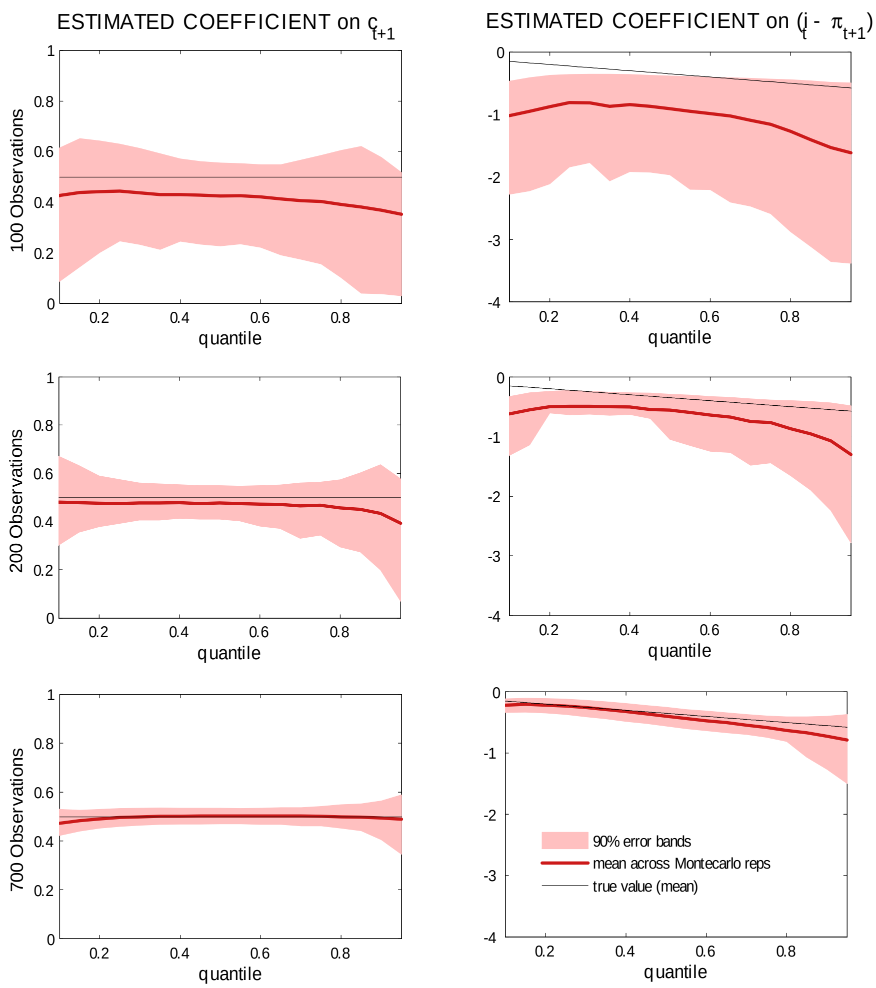

Figure 9: Montecarlo experiment with only the interest rate semi-elasticity varying across states τ.

For each observation, we draw the latent state u from a standard uniform distribution. We then solve the system given by equations (10) and (11) using the gensys.m solution algorithm proposed by Sims (2002). The reduced form representation of the structural model is used to generate data on c_t, r_t and π_t.

The model is estimated on this artificial data using $100,000$ replications of the MCMC algorithm for the 5% to 95% quantiles (with incremental steps of 5%). In line with the benchmark instrument set used in the analysis on actual data of section 4, we use the first and the second lag of c_t, and the second lag of r_t and π_t as instruments. The experiment is repeated 1000 times and the results are presented below. The panels of figure 8 refer to the case where both parameters vary across quantiles and they show the mean estimates across montecarlo replications (red line), the 90% confidence interval across the replications (shaded area) and the (sorted) mean of the true parameter values across the 1000 replications (black line).

Using 700 observations, the estimates track closely the underlying distributions with the true value always within the confidence interval. When the number of observations is lower, however, the point estimates tend to diverge from the true values. For example, the small sample bias associated with 100 observations could be as large as 50% (400%) for the estimates of the degree of forward-lookingness (interest rate semi-elasticity). Using a sample of 700 observations, in contrast, the bias is at most 12% and only at specific quantiles. The size of the bias becomes even smaller, around 5%, if the data generating process is such that only the parameter on the real rate is varying across quantiles (see figure 9). We conclude that our nonlinear specification of the aggregate consumption Euler equation is better suited for monthly (as opposed to quarterly) data where the number of available observations is about 700 (as opposed to 200).

Appendix B: convergence

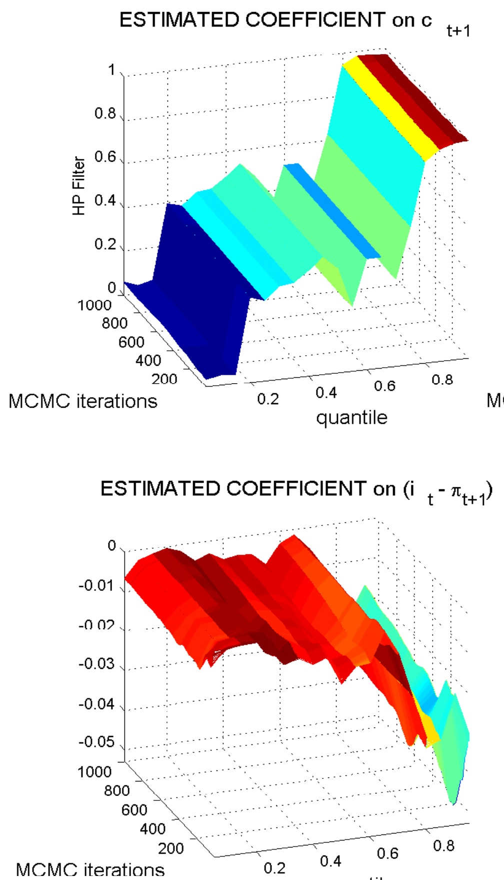

Figure 10: Recursive means of the retained MCMC draws, calculated at every 20^{th} draw.