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INFORMATIONAL RENTS, MACROECONOMIC RENTS, AND EFFICIENT BAILOUTS

Thomas Philippon and Philipp Schnabl

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Thomas Philippon, Stern School of Business, New York University, NBER and CEPR
Philipp Schnabl, Stern School of Business, New York University

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Centre for Economic Policy Research
77 Bastwick Street, London EC1V 3PZ, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

Informational Rents, Macroeconomic Rents, and Efficient Bailouts*

We analyze government interventions to alleviate debt overhang among banks. Interventions generate two types of rents. Informational rents arise from opportunistic participation based on private information while macroeconomic rents arise from free riding. Minimizing informational rents is a security design problem and we show that warrants and preferred stocks are the optimal instruments. Minimizing macroeconomic rents requires the government to condition implementation on sufficient participation. Informational rents always impose a cost, but if macroeconomic rents are large, efficient recapitalizations can be profitable.

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Thomas Philippon
Stern Business School
New York University
44 West 4th Street, Suite 9-190
New York, NY 10012
USA

Email: tphilipp@stern.nyu.edu

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Philipp Schnabl
Stern School of Business
New York University
44 W. 4th St.
New York, NY 10012
USA

Email: pschnabl@stern.nyu.edu

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It has been well understood since the seminal work of Myers (1977) that debt overhang can lead to under-investment. Firms in financial distress find it difficult to raise capital for new investments because the proceeds from these new investments end up increasing the value of the existing debt instead of the value of equity.

This paper asks whether and how the government should intervene if there is debt overhang in the financial sector. We first show that debt overhang in the financial sector can generate negative externalities at the aggregate level. In particular, one bank's decision to forgo profitable lending or investment opportunities (due to debt overhang) reduces payments to households, which can increase household defaults and thus worsen other banks' debt overhang. If the household sector is sufficiently weak, this mechanism can generate equilibria in which banks do not invest because they expect other banks not to invest.

If an economy suffers from such negative externalities, the social costs of debt overhang exceed the private costs, and there may be room for a government intervention. By directly providing capital to banks, the government can alleviate debt overhang and possibly improve economic efficiency. The goal of our paper is to characterize the optimal form of an intervention under such circumstances.

In our model banks differ along two dimensions: the quality of their existing assets and the quality of their investment opportunities. Asset quality determines the severity of debt overhang and missed investment opportunities generate welfare losses. The objective of the government is to increase socially valuable investments while minimizing the deadweight losses from raising new taxes.

We first show that the cost of government interventions comes in the form of 'macroeconomic' and 'informational' rents. Macroeconomic rents occur because of general equilibrium effects. These rents accrue to banks that do not participate in an intervention but benefit from the rise in asset values because of other banks' participation. As a result, there is a free-rider problem among banks. Informational rents, on the other hand, occur because of private information. These rents accrue to banks that participate opportunistically. In general, macroeconomic rents imply that there is insufficient uptake of the program, while informational rents mean that there is excessive participation.

We analyze the design of intervention to deal both with free-riding and with opportunistic participation. To address free-riding, the government must condition the implementa-

tion of an intervention on sufficient participation by banks. The intuition for this result is that banks have an incentive to coordinate participation because each bank's participation increases asset values in the economy. By conditioning on sufficient participation, the government makes each bank pivotal in whether the intervention is implemented and therefore reduces banks' outside options. In the limit, the government can completely solve the free-rider problem and extract the entire value of macroeconomic rents from banks.

To address opportunistic participation, the government must request preferred stock and warrants in exchange for new capital. The intuition for this result is that banks' equity holders receive informational rents if the financial sector is better informed about asset values and investment opportunities than the government. Indeed, we first show that the form of the intervention is irrelevant if the government and the financial sector have the same information about uncertain asset values and investment opportunities. In this case, the government can extract all informational rents by keeping banks to their participation constraints using different forms of intervention such as equity injections, debt guarantees, or asset purchases.

However, if the government has less information about asset values and investment opportunities than the financial sector, then the government can lower informational rents by requesting equity in return for capital. The reason is that equity is costly for banks with good investment opportunities because they have to share some of the upside of their investments with the government. Thus, banks with good investment opportunities prefer to invest without government support which reduces the cost of an intervention.

We show that the government can further reduce informational rents by asking for warrants at a strike price of bank asset values without investment. Such a program is *only* profitable for banks with good investment opportunities that do not invest in the absence of an intervention, which is exactly the set of banks the government wants to attract. In the limit, the government uses preferred stock with warrants to completely eliminate opportunistic participation and extracts the entire value of informational rents from banks.

Finally, the government's cost of the efficient intervention depends on the severity of the debt overhang relative to the macroeconomic rents. Severe debt overhang increases the cost because the efficient intervention provides an implicit subsidy to bank debt holders. Larger macroeconomic rents reduce the cost because they allow the government to extract

the value of investment externalities from banks. If the macroeconomic rents are small, then the intervention is costly and the government trades off the benefit of new investments with the deadweight loss of additional taxation. If the macroeconomic rents are large, then the government can recapitalize banks at a profit.

We discuss three extensions of the model. First, the government should start the implementation of the efficient intervention with a small number of large banks. The reason is that large banks are more likely to internalize the positive impact of their participation decision on asset values and that a small number facilitates coordination among banks. Second, we show that deposit insurance decreases the cost of the intervention because the government is partly reducing its own expected insurance payments. However, deposit insurance does not change the optimal form of the intervention. Third, heterogeneity among assets *within* banks generates additional informational rents and, as a result, equity injections become even more attractive relative to asset purchases.

We emphasize three contributions of our analysis. First, the conditional participation requirement can be interpreted as a mandatory intervention. Our paper thus provides a novel explanation of why governments should require participation in their recapitalization efforts and why there seems to be insufficient take-up in the absence of such a requirement.[1] Second, the preferred stock-warrants combination also limit risk-shifting and therefore emerges as the optimal solution in other studies of optimal security design (Green (1984)). In our model, banks cannot risk shift with their new investments since they are riskless as in Myers (1977), but risk shifting occurs through the reluctance to sell risky assets.[2] Our paper thus provides a novel mechanism for the optimality of preferred stock with warrants under asymmetric information. Third, other work on bank recapitalization mostly focuses on bank run externalities on the liabilities side of bank balance sheets. In contrast, our model focuses on investment externalities on the asset side of bank balance sheets. Our model therefore provides a novel motivation for government intervention even in the absence of bank runs.

Our results can shed light on the form of bank bailouts during the financial crisis of 2007-2009. In October 2008, the US government decided to inject cash into banks under

[1] Mitchell (2001) reviews the empirical evidence and suggests that there is often too little take-up of government interventions

[2] Selling risky assets for cash is formally equivalent to reverse-risk-shfiting. Our result that purchasing risky assets from banks is expensive is based on this insight.

the Troubled Asset Relief Program. Initial attempts to set up an asset purchase program failed and, after various iterations, the government met with the nine largest US banks and strongly urged *all* of them to participate in equity injections. Even though some banks were reluctant, all nine banks agreed to participate and the intervention was eventually implemented using a combination of preferred stock and warrants. This intervention was then offered to all other banks.

Our model extends the existing literature on debt overhang. Debt overhang arises because renegotiations are hampered by free-rider problems among dispersed creditors and by contract incompleteness (Bulow and Shoven (1978), Gertner and Scharfstein (1991), and Bhattacharya and Faure-Grimaud (2001)). A large body of empirical research has shown the economic importance of renegotiation costs for firms in financial distress (Gilson, John, and Lang (1990), Asquith, Gertner, and Scharfstein (1994), Hennessy (2004)). Moreover, from a theoretical perspective, one *should* expect renegotiation to be costly for at least two reasons. First, the covenants that protect debt holders from risk shifting (Jensen and Meckling (1976)) are precisely the ones that can create debt overhang. Second, debt contracts are able to discipline managers only because they are difficult to renegotiate (Hart and Moore (1995)). Our model takes large renegotiation costs as given and analyzes how to resolve debt overhang in this situation.

Our paper relates to the theoretical literature on bank bailouts. Gorton and Huang (2004) argue that the government can bail out banks in distress because it can provide liquidity more effectively than private investors. Diamond and Rajan (2005) show that bank bailouts can backfire by increasing the demand for liquidity and causing further insolvency. Diamond (2001) emphasizes that governments should only bail out the banks that have specialized knowledge about their borrowers. Aghion, Bolton, and Fries (1999) show that bailouts can be designed so as not to distort ex-ante lending incentives. Bebchuk and Goldstein (2009) study bank bailouts in a model where banks may not lend because of self-fulfilling credit market freezes. Farhi and Tirole (2009) examine bailouts in a setting in which private leverage choices exhibit strategic complementarities due to the monetary policy reaction. Corbett and Mitchell (2000) discuss the importance of reputation in a setting where a bank's decision to participate in a government intervention is a signal about asset values, and Philippon and Skreta (2009) formally analyze optimal interventions

when outside options are endogenous and information-sensitive. Mitchell (2001) analyzes interventions when there is both hidden actions and hidden information. Landier and Ueda (2009) provide an overview of policy options for bank restructuring. Bhattacharya and Nyborg (2010) examine bank bailouts in a model where the government wants to eliminate bank credit risk. In contrast, our paper focuses on the form of efficient recapitalization under debt overhang.

Two other theoretical papers share our focus on debt overhang in the financial sector. Kocherlakota (2009) analyzes a model where it is the insurance provided by the government that generates debt overhang. He analyzes the optimal form of government intervention and finds an equivalence result similar to our symmetric information equivalence theorem. Our papers differ because we focus on debt overhang generated within the private sector and we consider the problem of endogenous selection into the government's programs. In Diamond and Rajan (2009) as in our model, debt overhang makes banks unwilling to sell their toxic assets. In effect, refusing to sell risky assets for safe cash is a form of risk shifting. But while we use this initial insight to characterize the general form of government interventions, Diamond and Rajan (2009) study its interactions with trading and liquidity. In their model, the reluctance to sell leads to a collapse in trading which increases the risks of a liquidity crisis.

The paper also relates to the empirical literature on bank bailouts. Allen, Chakraborty, and Watanabe (2009) provide empirical evidence consistent with the main predictions of our model: they find that interventions work best when they target equity injections into the banks that have material risks of insolvency. Giannetti and Simonov (2009) find that bank recapitalizations result in positive abnormal returns for the clients of recapitalized banks as predicted by our debt overhang model. Glasserman and Wang (2009) develop a contingent claims framework to estimate market values of securities issued during bank recapitalizations such as preferred stock and warrants.

The paper proceeds as follows. Section 1 sets up the formal model. Section 2 solves for the decentralized equilibrium with and without debt overhang. Section 3 analyzes macroeconomic rents. Section 4 analyzes informational rents. Section 5 describes two extensions to our baseline model. Section 6 discusses the relation of our results to the financial crisis of 2007-2009. Section 7 concludes.

1 Model

We present a general equilibrium model with a financial sector and a household sector. We refer to all financial firms as banks and we assume that banks own industrial projects. The model has a continuum of households, a continuum of banks, and three dates, $t = 0, 1, 2$.

1.1 Banks

All banks are identical at $t = 0$, with existing assets financed by equity and long term debt with face value D due at time 2. At time 1, banks become heterogenous along two dimensions: they learn about the quality of their existing assets and they receive investment opportunities. Figure 1 summarizes the timing, technology, and information structure of the model.

The assets deliver a random payoff $a = A$ or $a = 0$ at time 2. The probability of a high payoff depends on the idiosyncratic quality of the bank's portfolio and on the aggregate performance of the economy. We capture macroeconomic outcomes by the aggregate payoff $\bar{a}$, and idiosyncratic differences across banks by the random variable ε. At time 1, all private investors learn the realization of ε for each bank. We define the probability of a good outcome conditional on the information at time 1 as:

$$p(\bar{a}, \varepsilon) \equiv \Pr(a = A | \varepsilon, \bar{a}).$$

The variables are defined so that the probability $p(\varepsilon, \bar{a})$ is increasing in ε and in $\bar{a}$. Note that p is also the expected payoff per unit of face value for existing assets of quality ε in the aggregate state $\bar{a}$. The average payoff in the economy is simply

$$\bar{p}(\bar{a}) \equiv \int_{\varepsilon} p(\bar{a}, \varepsilon)\, dF_{\varepsilon}(\varepsilon),$$

where F_{ε} is the cumulative distribution of asset quality across banks. The variable $\bar{a}$ is a measure of common performance for all banks' existing assets and satisfies the accounting constraint:

$$\bar{p}(\bar{a}) A = \bar{a}. \tag{1}$$

Banks receive new investment opportunities at time 1. All new investments cost the same fixed amount x at time 1 and deliver income $v \in [0, V]$ at time 2. The payoff v is

heterogenous across banks and is known to the financial sector at time 1.[3] A bank's type is therefore defined by ε, the bank-specific deviation of asset quality from average bank asset quality, and v, the quality of its investment opportunities.

Let i be an indicator for the bank's investment decision: $i = 1$ if the bank invests at time 1, and zero otherwise. The decision to invest depends on the banks's type and on the aggregate state, so we have $i(\varepsilon, v, \bar{a})$. Banks must borrow an amount l in order to invest. We normalize the banks' cash balances to zero, so that the funding constraint is $l = x \cdot i$. We will later allow the government to inject cash in the banks to alleviate this funding constraint. At time 2 total bank income y is:

$$y = a + v \cdot i,$$

There are no direct deadweight losses from bankruptcy. Let r be the gross interest rate between $t = 1$ and $t = 2$. Under the usual seniority rules at time 2, we have the following payoffs for long term debt holders, new lenders, and equity holders:

$$y^D = \min(y, D); \quad y^l = \min(y - y^D, rl); \quad y^e = y - y^D - y^l.$$

We assume that banks suffer from debt overhang, or equivalently, that long term debt is risky.

Assumption A1 (Risky Debt): $V < D < A$.

Under assumption A1, in the high payoff state ($a = A$) all liabilities are fully repaid ($y^D = D$ and $y^l = rl$) and equity holders receive the residual ($y^e = y - D - rl$). In the low payoff state ($a = 0$) long term debt holders receive all income ($y^D = y$) and other investors receive nothing: ($y^l = y^e = 0$). Figure 2 summarizes the payoffs to investors by payoff state.

1.2 Households

At time 0 all consumers are identical. Each consumer owns the same portfolio of long term debt and equity of banks. They also have various types of loans due to the banks at time 2 with face value A. These loans could be mortgages, auto loans, student loans, credit card debt, or other consumer loans.

[3] As in the original Myers (1977) model.

At time 1, each consumer receives an identical endowment $\bar{w}_1$ and they have access to a storage technology which pays off one unit of time-2 consumption for an investment of one unit of time-1 endowment. Consumers can also lend to banks. Consumers are still identical at time 1 and we consider a symmetric equilibrium where they make the same investment decisions. They lend $\bar{l}$ to banks and they store $\bar{w}_1 - \bar{l}$. At time 2 they receive income w_2 which is heterogenous and random across households. Let $\bar{y}^e$, $\bar{y}^D$, and $\bar{y}^l$ be the aggregate payments to holders of equity, long term debt, and short term debt. The total income of the household is therefore:

$$n_2 = \underbrace{\bar{w}_1 - \bar{l}}_{\text{safe storage}} + \underbrace{w_2}_{\text{risky labor income}} + \underbrace{\bar{y}^e + \bar{y}^D + \bar{y}^l}_{\text{financial income}} \tag{2}$$

The household defaults if and only if $n_2 < A$. There are no direct deadweight losses of default so the bank recovers n_2 in case of default. The aggregate payments (or average payment) from households to banks are therefore:

$$\bar{a} = \int \min(n_2, A)\, dF_w(w_2)\,. \tag{3}$$

Note that the mapping from household debt to bank assets endogenizes the aggregate payoff $\bar{a}$ but leaves room for heterogeneity of banks' assets quality captured by the parameter ε. This heterogeneity is needed to analyze the consequences of varying quality of assets across banks. Finally, we need to impose the market clearing conditions. Let $\mathcal{I}$ be the set of banks that invest at time 1: $\mathcal{I} \equiv \{(\varepsilon, v) \mid i = 1\}$. Aggregate investment at time 1 must satisfy $\bar{l} = \bar{x}(\mathcal{I}) \equiv x \iint_{\mathcal{I}} dF(\varepsilon, v)$ and consumption (or GDP) at time 2 is:

$$\bar{c} = \bar{w}_1 + \bar{w}_2 + \iint_{\mathcal{I}} (v - x)\, dF(\varepsilon, v)\,. \tag{4}$$

2 Equilibrium

2.1 First best equilibrium

We assume that households have sufficient endowment to finance all positive NPV projects.

Assumption A2 (Excess Savings): $\bar{w}_1 > \bar{x}(1_{v>x})$

Under assumption A2, the time-1 interest rate is pinned down by the storage technology, which is normalized to 1.

In the first best equilibrium, banks choose investments at time 1 to maximize firm value $V_1 = E_1[a] + v \cdot i - E_1[y^l]$ subject to the time 1 budget constraint $l = x \cdot i$, and the break even constraint for new lenders $E_1[y^l] = l$. This implies that firm value $V_1 = E_1[a] + (v - x) \cdot i$. Therefore, investment takes place when a banks has a positive NPV project, or equivalently, when $v > x$.

The unique first best solution is for investment to take place if and only if $v > x$, irrespective of the value of ε and $E_1[\bar{a}]$. The first best equilibrium is unique and first-best consumption is $\bar{c}^{FB} = \bar{w}_1 + \bar{w}_2 + \int_{v>x} (v - x)\, dF_v(v)$. We can think of the first best as a world in which banks can pledge the PV of new projects to households (no debt overhang). Hence, positive NPV projects can always be financed. Figure 3 illustrates investment under the first best. [4]

2.2 Debt overhang equilibrium without intervention

Under debt overhang, we assume that banks maximize equity value $E_1[y^e|\varepsilon] = E_1[y - y^D - y^l|\varepsilon]$ taking as given the priority of senior debt $y^D = \min(y, D)$. Recall that the idiosyncratic shock ε is known at time 1. With probability $p(\bar{a}, \varepsilon)$ the bank is solvent and repays its creditors, and shareholders receive $A - D + (v - rl) \cdot i$. With probability $1 - p(\bar{a}, \varepsilon)$ the bank is insolvent, and shareholders get nothing. Using the break even constraint for new lenders, $r = 1/p(\bar{a}, \varepsilon)$, equity holders solve:

$$\max_i \ p(\bar{a}, \varepsilon) \left(A - D + \left(v - \frac{x}{p(\bar{a}, \varepsilon)} \right) \cdot i \right).$$

The condition for investment is $p(\bar{a}, \varepsilon)\, v > x$, which is more restrictive than under the first best because of debt overhang. The investment domain without government intervention is therefore:

$$\mathcal{I} = I(\bar{a}, 0) \equiv \left\{ (\varepsilon, v) \mid p(\bar{a}, \varepsilon) > \frac{x}{v} \right\}. \tag{5}$$

[4] Notice the equivalence between maximizing firm value and maximizing equity value with efficient bargaining. We can always write $V_1 = E_1[y - y^l] = E_1[y^e + y^D]$. The maximization program for firm value is equivalent to the maximization of equity value $E_1[y^e]$ as long as we allow renegotiation and transfer payments between equity holders and debt holders at time 1.

The index 0 in the investment set indicates that there is no intervention by the government. At time 2, we aggregate across all banks and we have the accounting identity:

$$\underbrace{\bar{a} + \iint_{\mathcal{I}} v dF(\varepsilon, v)}_{\text{aggregate bank income}} = \underbrace{\bar{y}^e + \bar{y}^D + \bar{y}^l}_{\text{payments to households}} \tag{6}$$

Using (2) and (6), we can write household income n_2 as:

$$n_2 = w_2 + \bar{w}_1 + \bar{a} + \iint_{\mathcal{I}} (v - x)\, dF(\varepsilon, v) \tag{7}$$

With the exception of risky time 2 income w_2, all terms in household income are identical across households. The three unknowns in our model are the repayments from households to banks $\bar{a}$, the investment set I, and the income of households n_2. The three equilibrium conditions are therefore (3), (5), (7). We solve the model backwards. First, we examine the equilibrium at time 2, when the investment set is given. We then solve for the equilibrium at time 1, when investment is endogenous.

Equilibrium at time 2

Let us define the sum of time 1 endowment and investment as

$$K(\mathcal{I}) = \bar{w}_1 + \iint_{\mathcal{I}} (v - x) \cdot dF(\varepsilon, v). \tag{8}$$

Note that K is fixed at date 2 because investment decisions are taken at time 1. Using equation (8), we can write equation (7) as $n_2 = w_2 + \bar{a} + K$. Using (3) we obtain the equilibrium condition for $\bar{a}$:

$$\bar{a} = \int \min(w_2 + \bar{a} + K, A)\, dF^w(w_2). \tag{9}$$

We now make a technical assumption:

Assumption 3: $\int \min(w_2 + \bar{w}_1, A)\, dF^w(w_2) > 0$.

Assumption A3 rules out a multiple equilibria at time 1 Allowing for multiple equilibria complicates the analysis but does not affect our main results.

The following Lemma gives the properties of the aggregate performance of existing assets at time 2:

Lemma 1 *There exists a unique equilibrium* $\bar{a}(K)$ *at time 2. Moreover,* $\bar{a}$ *is increasing and concave in* K*.*

Proof. The slope on the left hand side of equation (9) is 1. The slope on the right hand side is $F^w(\hat{w}_2) \in [0,1]$ where $\hat{w}_2 = A - \bar{a} - K$ is the income of the marginal household (the differential of the boundary term is zero since the integrated function is continuous). There is therefore at most one solution. Moreover, under assumption A3 the RHS is strictly positive when $\bar{a} = 0$. When $\bar{a} \to \infty$ the RHS goes to A, which is finite. Therefore the equilibrium exists and is unique. At the equilibrium, the slope of the RHS must be strictly less than one, so the solution must satisfy $F(\hat{w}_2) < 1$. The comparative statics with respect to K is:

$$\frac{\partial \bar{a}}{\partial K} = \frac{F(\hat{w}_2)}{1 - F(\hat{w}_2)} > 0$$

So the function $\bar{a}$ is increasing in K. Moreover we have

$$\frac{\partial \hat{w}_2}{\partial K} = -1 - \frac{\partial \bar{a}}{\partial K} < 0$$

Since $\hat{w}_2$ is decreasing in K, the slope of $\bar{a}$ is decreasing and the function is concave. ■

The shape of the function $\bar{a}$ is intuitive because the impact of additional income only increases payment of households in default. Hence, if the share of households in default decreases with income K, the impact of additional income K decreases.

Equilibrium at time 1

We can now turn to the equilibrium at time 1. We have just seen in equation (9) that $\bar{a}$ increases with K at time 2. At time 1, K depends on the anticipation of $\bar{a}$ because investment depends on the expected value of existing assets through the debt overhang effect. To see this, let us rewrite equation (8) as:

$$K(\bar{a}) = \bar{w}_1 + \iint_{\varepsilon > \hat{\varepsilon}(\bar{a}, v)} (v - x) dF(\varepsilon, v), \tag{10}$$

The cutoff $\hat{\varepsilon}$ is defined implicitly by $p(\bar{a}, \hat{\varepsilon})\, v = x$, which implies $\frac{\partial \hat{\varepsilon}}{\partial \bar{a}} = -\frac{\partial p / \partial \bar{a}}{\partial p / \partial \varepsilon}$ and therefore:[5]

$$\frac{\partial K}{\partial \bar{a}} = \int_{v > x} (v - x) \frac{\partial p}{\partial \bar{a}} \frac{f(v, \hat{\varepsilon}(v))}{\partial p / \partial \varepsilon} dv.$$

[5] We can restrict our analysis to the space where $v > x$ since from (5) we know that there is no investment outside this range.

This last equation shows that K is increasing in $\bar{a}$ since all the terms on the right-hand-side are positive. The economic intuition is straightforward. When banks anticipate good performance on their assets, they are less concerned with debt overhang and are more likely to invest. The sensitivity of K to $\bar{a}$ depends on the extent of the NPV gap $v - x$, the elasticity of p to $\bar{a}$, and the density evaluated at the boundary of marginal banks (the term $\partial p / \partial \varepsilon$ is simply a normalization given the definition of ε). Figure 4 illustrates investment under the debt overhang equilibrium.

The important question here is whether the equilibrium is efficient. The simplest way to answer this question is to see if a pure transfer program can lead to a Pareto improvement. This is what we do in the next section.

2.3 Debt overhang equilibrium with cash transfers

We study here a simple cash transfer program. The government announces at time 0 that it gives $m \geq 0$ to each bank. The government raises the cash by imposing a tax m on households' endowments $\bar{w}_1$. The deadweight loss from taxation at time 1 is χm. Non distorting transfers correspond to the special case where $\chi = 0$.

Consider the investment decision for banks. Banks receive cash injection m. It is straightforward to show that if a bank is going to invest, it will first use its cash m, and borrow only $x - m$. The break even constraint for new lenders remains $r = 1/p(\bar{a}, \varepsilon)$. If the bank does not invest it can simply keep m on its balance sheet. Equity holders therefore maximize:

$$\max_i \; p(\bar{a}, \varepsilon) \left[A - D + i \cdot \left(v - \frac{x - m}{p(\bar{a}, \varepsilon)} \right) + (1 - i) \cdot m \right].$$

This yields the investment condition $p(v - m) > x - m$ which defines the investment domain:

$$\mathcal{I} = I(\bar{a}, m) \equiv \left\{ (\varepsilon, v) \mid p(\bar{a}, \varepsilon) > \frac{x - m}{v - m} \right\}. \tag{11}$$

Households do not care about transfers because they are residual claimants: what they pay as taxpayers, they receive as bond and equity holders. We therefore only need to modify the definition of K to include the deadweight losses at time by replacing $\bar{w}_1$ by $\bar{w}_1 - \chi m$ in equation (8). Conditional on K, the equilibrium at time 2 is unchanged and equation (9)

gives the same solution $\bar{a}(K)$. At time 1 we now have:

$$K(\bar{a}, m) = \bar{w}_1 + \iint_{I(\bar{a};m)} (v - x) f(v, \varepsilon)\, d\varepsilon dv - \chi m. \tag{12}$$

The cutoff $\hat{\varepsilon}$ is defined implicitly by $p(\bar{a}, \hat{\varepsilon})(v - m) = (x - m)$. The system is therefore described by the increasing and concave function $\bar{a}(K)$ in (9) which implies $d\bar{a} = \bar{a}_K dK$ and the function $K(\bar{a}, m)$ in (12) which implies $dK = K_{\bar{a}} d\bar{a} + K_m dm$.[6]

At this point, we need to discuss briefly the issue of multiple equilibria. Without debt overhang, K would not depend on $\bar{a}$ and there would be only one equilibrium. With debt overhang, however, there is a positive feedback between investment, the net worth of households, and the performance of outstanding assets. We can rule out multiple equilibria when $\bar{a}_K K_{\bar{a}} < 1$. A simple way to ensure unicity is to have enough heterogeneity in the economy (either in labor income, or in asset quality). When the density f is small, the slope of K is also small, and the condition $\bar{a}_K K_{\bar{a}} < 1$ is satisfied.[7] Since multiple equilibria are not crucial for the insights of this paper, we proceed under the assumption that the debt overhang equilibrium is unique.

The impact of cash injection m on average repayment $\bar{a}$ is

$$\frac{d\bar{a}}{dm} = \frac{\bar{a}_K K_m}{1 - \bar{a}_K K_{\bar{a}}},$$

and from (4), we see that consumption at time 2 satisfies

$$d\bar{c} = dK(\bar{a}, m) = \frac{K_m}{1 - \bar{a}_K K_{\bar{a}}} dm.$$

From the definition of the cutoff we get $\frac{\partial \hat{\varepsilon}}{\partial m} \frac{\partial p}{\partial \varepsilon} = -\frac{(v-x)}{(v-m)^2}$. Differentiating (12) we therefore have:

$$\frac{\partial K}{\partial m} = \int_{v>x} \frac{(v - x)^2}{(v - m)^2} \frac{f(v, \hat{\varepsilon})}{\partial p / \partial \varepsilon} dv - \chi. \tag{13}$$

The sensitivity of K to m increases in the NPV gap $v - x$ and the density evaluated at the boundary of marginal banks and decreases in deadweight loss of taxation χ. Importantly, the equilibrium always improves when $\chi = 0$, which shows that the decentralized equilibrium is not efficient.

[6] We are using the standard notations $\bar{a}_K = \frac{\partial \bar{a}}{\partial K}$ and $K_{\bar{a}} = \frac{\partial K}{\partial \bar{a}}$.

[7] In any case, multiple equilibria simply correspond to the limiting case when $\bar{a}_K K_{\bar{a}}$ goes to one, and, as will be seen shortly, they only reinforce the efficiency of government interventions.

Proposition 1 *The decentralized equilibrium under debt overhang is inefficient. Non distorting transfers from households to banks at time 1 lead to a Pareto superior outcome.*

Figure 5 illustrates investment in the debt overhang equilibrium with cash transfers. If tax revenues can be raised without costs – i.e., if taxes do not create distortions and if tax collection does not require any labor or capital – then these revenues should be used to provide cash to the banks until debt overhang is eliminated. In such a world the issue of efficient recapitalization does not arise, since the government has in effect access to infinite resources.

If government interventions are costly, however, we see from (13) that the benefits of cash transfers are reduced. The overall impact of the cash transfers can even be negative if deadweight losses are large. In such a world, it become critical for the government to minimize the costs of its interventions. This is the issue we address now.

3 Macroeconomic rents

We consider first interventions at time 0 when the government and firms have the same information about uncertain asset values and investment opportunities. This allow us to focus on macroeconomic rents and abstract from informational rents. For interventions at time 0, we show that the critical feature is to allow the government to design programs conditional on aggregate participation. However, the form of the intervention does not matter.

3.1 Government and shareholders

The objective of the government is to maximize the expected utility of the representative agent. All consumers are risk neutral and identical as of $t = 0$ and $t = 1$. Hence, the government simply maximizes

$$\max_{\Gamma} E\left[\bar{c}\left(\Gamma\right)\right] \tag{14}$$

where Γ describes the specific intervention. Let $\Psi\left(\Gamma\right)$ be the expected net transfer from the government to financial firms. We assume that raising taxes is inefficient and leads to a deadweight loss at time 1 equal to $\chi\Psi\left(\Gamma\right)$. The government takes into account this deadweight loss in its maximization program.

We assume the government can make a take-it-or-leave-it offer to bank equity holders. Equity holders then decide whether they want to participate in the intervention. The government faces the same debt overhang problem as the private sector, that means the government cannot renegotiate the claims of long term debt holders. Moreover, we assume the government can restrict dividend payments to shareholders at time 1. This is necessary because under debt overhang the optimal action for equity holders is to return cash injections to equity holders.

At time 0, banks do not yet know their idiosyncratic asset value ε and investment opportunities v. Hence, all banks are identical and when participation is decided at time 0, we can without loss of generality consider programs where all banks participate. To be concrete, we first consider three empirically relevant interventions: equity injections, asset purchases, and debt guarantees.

In an asset purchase program, the government purchases an amount Z of risky assets at a per unit price of q. If a bank decides to participate, its cash balance increases by $m = qZ$ and the face value of its assets becomes $A - Z$. In an equity injection program, the government offers cash m against a fraction α of equity returns. In a debt guarantee program the government insures an amount S of debt newly issued at time 0 for a per unit fee of ϕ. The rate on the insured debt is one and the cash balance of the banks becomes $m = S - \phi S$.

To study efficient interventions it is critical to understand the participation decisions of equity holders. The following value function will prove useful throughout our analysis. Conditional on a cash injection m, the time 0 value of equity value is:

$$E_0\left[y^e|\bar{a}, m\right] = \bar{p}\left(\bar{a}\right)\left(A - D + m\right) + \iint_{I(\bar{a},m)} \left(p\left(\bar{a}, \varepsilon\right)v - x + \left(1 - p\left(\bar{a}, \varepsilon\right)\right)m\right)dF(\varepsilon, v), \qquad (15)$$

In this equation, one must of course also recognize that in equilibrium $\bar{a}$ depends on m, as explained earlier. The first term is the expected equity value of long term assets plus the cash injection using the unconditional probability of solvency $\bar{p}\left(\bar{a}\right)$. The second term is the time 0 expected value of new investment opportunities. This value is positive when the bank's type belongs to the investment set I defined in Equation (11). Note that cash adds an extra term to the expected value of investment opportunities because the cash spent on investment is not given to debt holders at time 2. For bank equity holders, the opportunity

cost of using cash for investment is therefore less than the opportunity cost of raising funds from lenders at time 1.

3.2 Free participation

In this section we study interventions in which the implementation of an intervention is independent of a bank's decision. We refer to this setup as interventions with free participation:

Definition 1 *An intervention satisfies free participation if the program offered to a bank only depends on that bank's participation decision.*

We first study an asset purchase program. Banks sell assets with face value Z and receive cash $m = qZ$. It is easy to see that the government does not want to buy assets to the point that default occurs in both states. We can therefore restrict our attention to the case where $A - Z > D$. After the intervention, the equilibrium takes place as in the decentralized debt overhang equilibrium. We know that the investment domain in the equilibrium where all the banks participate is $I(\bar{a}(m), m)$ defined in (11). From the perspective of the government, we can define the *equilibrium* investment set as:

$$\hat{I}(m) \equiv I(\bar{a}(m), m),$$

which recognizes that the cash injection determines the macro state $\bar{a}$. Let $\mathcal{T} = [\varepsilon_{\min}, \varepsilon_{\max}] \times [0, V]$ be the state space. We then have the following Lemma:

Lemma 2 *Consider an asset purchase program (Z, q) with free participation at time 0. Let $m = qZ$. This program implements the investment set $\hat{I}(m)$ at the strictly positive cost:*

$$\Psi_0^{free}(m) \equiv m \iint_{\mathcal{T} \backslash \hat{I}(m)} (1 - p(\bar{a}(m), \varepsilon)) dF(\varepsilon, v) - \iint_{\hat{I}(m) \backslash I(\bar{a}(m), 0)} (p(\bar{a}(m), \varepsilon) v - x) \, dF(\varepsilon, v). \quad (16)$$

Proof. The cost to the government is $m - \bar{p}(\bar{a})Z$. The participation constraint of banks is $E_0[y^e|\bar{a}, m] - \bar{p}(\bar{a})Z \geq E_0[y^e|\bar{a}, 0]$. Using (15), we can write a binding constraint as

$$\bar{p}(\bar{a})(Z - m) = m \iint_{\hat{I}(m)} (1 - p(\bar{a}, \varepsilon)) dF(\varepsilon, v) + \iint_{\hat{I}(m) \backslash I(\bar{a}, 0)} (p(\bar{a}, \varepsilon) v - x) dF(\varepsilon, v)$$

From the definition of $\bar{p}(\bar{a})$ we then get the cost function $\Psi_0^{free}(m)$. Finally, both terms on the RHS of (16) are positive. The first is obvious. The second is also positive because $p(\bar{a},\varepsilon)v - x$ is negative over the domain $\hat{I}(m) \setminus I(\bar{a},0)$. ■

The government's cost under symmetric information has a natural interpretation in terms of the two terms on the right-hand side of equation (16). The first term reflects the transfer of wealth from the government to the debt holders of banks that do not invest: debt value simply increases by $(1-\bar{p})\,m$ over the domain $\mathcal{T} \setminus \hat{I}(m)$. The second term measures the subsidy needed to induce investment over the expanded domain $\hat{I}(m)$ compared to the investment domain $I(\bar{a},0)$.

We can now compare asset purchases with equity injections and debt guarantees.

Proposition 2 *Under symmetric information, the type of financial security used in the intervention is irrelevant.*

Proof. See Appendix. ■

Proposition 2 says that an asset purchase program (Z,q) is equivalent to a debt guarantee program with $S = Z$ and $q = 1 - \phi$. It is also equivalent to an equity injection program (m,α), where $m = qZ$ and q and α are chosen such that at time 0 all banks are indifferent between participating and not participating in the program. All programs implement the same investment set $\hat{I}(m)$ and have the same expected cost $\Psi_0^{free}(m)$

The key to this irrelevance theorem is that banks decide whether to participate before they receive information about investment opportunities and asset values. The government thus optimally chooses the program parameters such that bank equity holders are indifferent between participating and not participating. The cost to the government is thus independent of whether banks are charged through assets sales, debt guarantee fees, or equity injections.

3.3 Conditional participation

We now focus on the participation decision. So far we assumed that banks can decide whether to participate independently of other banks' participation decisions. We now allow the government to condition the program offered to one bank on the participation of other

banks. We call this a program with conditional participation. In effect, the offer by the government holds only if all banks participate in the program. The key is that if a bank that was supposed to participate decides to drop out, then the program is cancelled for all banks. It is straightforward to see that the equivalence result of Proposition 2 holds for conditional programs, and we have the following proposition:

Proposition 3 *A program with conditional participation implements the investment set* $\hat{I}(m)$ *at cost*

$$\Psi_0^{cond}(m) = \Psi_0^{free}(m) - \mathcal{M}(m)$$

where $\mathcal{M}(m) \equiv E_0[y^e|\bar{a}(m), 0] - E_0[y^e|\bar{a}(0), 0] \geq 0$ *measures macroeconomic rents.*

Proof. The government offers a program that is implemented only if all the banks opt in. If they do, the equilibrium is $\bar{a}(m)$. If anyone drops out, the equilibrium is $\bar{a}(0)$. Let $E[y^g]$ be the expected payments to the government. The participation constraint is $E_0[y^e|\bar{a}(m), m] - E[y^g] \geq E_0[y^e|\bar{a}(0), 0]$. By definition, we have $E_0[y^e|\bar{a}(0), 0] = E_0[y^e|\bar{a}(m), 0] - \mathcal{M}(m)$. The cost to the government is $mE[y^g]$. Using a binding participation constraint, we therefore obtain $\Psi_0^{cond}(m) = \Psi_0^{free}(m) - \mathcal{M}(m)$. ■

The key point is that free riding occurs because banks do not internalize the impact of their participation on the health of other banks. The program with conditional participation is less costly because the government appropriates the macroeconomic rents created by its intervention. We can use equation (15) to study these rents. Let $\Delta_p(\varepsilon) = p(\bar{a}(m), \varepsilon) - p(\bar{a}(0), \varepsilon)$. We then have

$$\mathcal{M}(m) = \bar{\Delta}_p(A - D) + \iint_{\hat{I}(0)} \Delta_p(\varepsilon)\, dF(\varepsilon, v) + \iint_{I(\bar{a}(m),0)\setminus\hat{I}(0)} (p(\bar{a}, \varepsilon)v - x)\, dF(\varepsilon, v).$$

This expression decomposes the macroeconomic rents to shareholders into three components. The first term is higher repayment rate on assets in place, the second term is the higher expected value of investments that would have been made even without intervention, and the third term is the expected benefit of expanding the equilibrium investment set. Finally, the costs of the conditional participation program can be negative when the

macroeconomic rents are large. In this case, the government can recapitalize banks and end up with a profit. We can therefore summarize our results in the following theorem.

Theorem 1 *The government must use a conditional participation program in order to capture the macroeconomic value of its intervention. Under symmetric information, the type of security used in the intervention is irrelevant.*

We note that the conditional participation requirement makes each bank pivotal for the implementation for the program. This mechanism may be difficult to implement when there is a large number of banks and if some bank equity holders decide against participation for reasons outside of our model. Also, there exists an equilibrium in which no bank participates because each bank expects other banks not to participate. To alleviate these implementation concerns, the government should first implement the efficient intervention with a small number of banks. A small number of banks reduces the likelihood that banks may deviate from the optimal participation decision and facilitates the coordination among banks. The government should target the largest banks for participation because then the program has the greatest impact on the macro state $\bar{a}$ for a given number of participating banks. Moreover, the largest banks are more likely to internalize the positive impact of their participation decision on the macro state $\bar{a}$ (because they are not complete price takers) and thus are more willing to participate in government interventions than small banks.

4 Informational rents

In this section we consider interventions at time 1, when banks know their types but the government does not. The macroeconomic rents that we have studied in the previous section still exist but we do not need to repeat our analysis. For brevity, we study only programs with free participation and we focus on the consequences of information asymmetry.

4.1 Complete information benchmark

We first discuss participation and investment under perfect information and derive the minimum cost of an intervention. We note that this setting is different from the time 0 setting where banks and the government have the same information but they still face uncertainty

about asset values and investment opportunities. Instead we assume that the government is perfectly informed about each bank's asset values and investment opportunities. For example, this would be the case if banks can credibly reveal their information to the government.

Under perfect information, the government simply decides which banks should participate and provides enough capital such that bank equity's participation constraint is binding. We can thus provide a general characterization of the minimum cost of any intervention with free participation:

Lemma 3 *Consider a program with free participation that implements the investment set* $\mathcal{I}$*. Let* $\Omega^{\min} = \mathcal{I} \backslash I(\bar{a}, 0)$*. The cost of the program cannot be lower than:*

$$\Psi_1^{\min} = -\iint_{\Omega^{\min}} \left(p(\bar{a}, \varepsilon) v - x \right) dF(\varepsilon, v).$$

Proof. Note that $I(\bar{a}, 0)$ is the set of banks that can invest alone, and $\Omega^{\min}$ is the set of types that invest only thanks to the program. The best the government can do with $I(\bar{a}, 0)$ is to make sure they do not participate. Voluntary participation means that equity holders in $\Omega^{\min}$ must get at least $p(A - D)$. The government and old equity holders must share the residual surplus whose value is

$$p(A - D) + p(\bar{a}, \varepsilon) v - x$$

Hence the expected net payments to the government must be at least $\iint_{\Omega^{\min}} \left(p(\bar{a}, \varepsilon) v - x \right) dF(\varepsilon, v)$. These payments are negative by definition of $\Omega^{\min}$, and therefore the lower bound $\Psi_1^{\min}$ is strictly positive. ■

A simple way to understand this result is to imagine what would happen if the government could write contracts contingent on investment. For the shareholders of type (ε, v), the value of investment is $p(\bar{a}, \varepsilon) v - x$, which is negative outside the private investment region $I(\bar{a}, 0)$. If the government has perfect information, it can offer a contract with a type-specific payment contingent on investment. The minimum the government would have to offer type (ε, v) would be $-(p(\bar{a}, \varepsilon) v - x)$. We define an intervention's informational rents as the subsidy provided to bank equity holders in excess of this amount.

We note that the government cannot simply use observed assets prices to implement the intervention because the expectation of an intervention may in turn affect prices (see Bond, Goldstein, and Prescott (2010) and Bond and Goldstein (2010)). Credit default swap prices of US banks during the financial crisis of 2007 to 2009 provide clear evidence of this issue. Most market participants expected some form of intervention if a crisis became sufficiently severe and indeed the government intervened several times after credit default swaps reached critical levels. Hence, it is unlikely that credit default swaps reflected the probability of default in the absence of government interventions.

4.2 Participation and investment under asymmetric information

We now examine participation and investment under asymmetric information at time 1. We first compare asset purchases, debt guarantees, and equity injections. The objective function of the government is the same as in the previous section. The participation decisions are based on equity value which is now conditional on each bank's type (ε, v). The structure of the programs is the same as at time 0, but the government must now take into account the endogenous participation decisions of banks. Under free participation, banks opt in if and only if $E_1[y^e|\bar{a}, \varepsilon, v, \Gamma]$ is greater than $E_1[y^e|\bar{a}, \varepsilon, v, 0]$.

There are several cases to consider: opportunistic participation, inefficient participation, and efficient participation. Consider opportunistic participation first. It happens when a bank takes advantage of a program even though it would have invested without it. We define the net value of opportunistic participation as:

$$U\left(\bar{a}, \varepsilon, v; \Gamma\right) \equiv E_1[y^e|\bar{a}, \varepsilon, v, \Gamma, i = 1] - E_1[y^e|\bar{a}, \varepsilon, v, 0, i = 1]. \tag{17}$$

Consider now inefficient participation. It happens when a bank participates but fails to invest. We define the net value of inefficient participation as:

$$NIP\left(\bar{a}, \varepsilon, v; \Gamma\right) \equiv E_1[y^e|\bar{a}, \varepsilon, v, \Gamma, i = 0] - E_1[y^e|\bar{a}, \varepsilon, v, 0, i = 0] \tag{18}$$

It is straightforward to show that the government should always prevent inefficient participation, and that it can do so by charging a small fee. We always make sure that out program satisfy $NIP < 0$ for all types. Finally, efficient participation occurs when a bank that would not invest alone opts in the program. We define the net value of opportunistic

participation as:

$$L(\bar{a}, \varepsilon, v; \Gamma) \equiv E_1[y^e|\bar{a}, \varepsilon, v, \Gamma, i = 1] - E_1[y^e|\bar{a}, \varepsilon, v, 0, i = 0]. \tag{19}$$

We will see that $U = 0$ defines an *upper* participation schedule and that $L = 0$ defines a *lower* participation schedule (hence our choice of notations).

The participation set of any program Γ is therefore

$$\Omega(\bar{a}, \Gamma) = \{(\varepsilon, v) \mid L(\bar{a}, \varepsilon, v; \Gamma) > 0 \wedge U(\bar{a}, \varepsilon, v; \Gamma) > 0\}. \tag{20}$$

Note that $L > 0$ and $NIP < 0$ implies that there is always investment conditional on participation. The investment domain under the program is the combination of the investment set $I(\bar{a}, 0)$ (banks that would invest without government intervention) and the participation set $\Omega(\bar{a}, \Gamma)$. With a slight abuse of notation, we define:

$$I(\bar{a}, \Gamma) = I(\bar{a}, 0) \cup \Omega(\bar{a}, \Gamma). \tag{21}$$

Note that the overlap between the two sets, $I(\bar{a}, 0) \cap \Omega_1(\bar{a}, \Gamma)$, represents opportunistic participation. Opportunistic participation is participation by banks that would invest even without the program.

4.3 Comparison of standard interventions

We now compare the relative efficiency of the three standard interventions (described earlier) under asymmetric information. We study first the asset purchase program. The upper participation curve (17) is defined by $U^a(\bar{a}, \varepsilon, v; Z, q) = (q - p(\bar{a}, \varepsilon)) Z$. Banks participate only if the price q offered by the government exceeds the true asset value $p(\bar{a}, \varepsilon)$. This is the adverse selection problem between the government and the financial sector. The NIP-constraint (18) only requires $q < 1$, which is always satisfied by efficient interventions. The lower bound schedule (19) is given by $L^a(\bar{a}, \varepsilon, v; Z, q) = p(\bar{a}, \varepsilon) v - x + (q - p(\bar{a}, \varepsilon)) Z$. The lower- and the upper-schedules define the participation set $\Omega_1^a(\bar{a}, Z, q)$ from (20). The expected cost of the asset purchase program is:

$$\Psi_1^a(\bar{a}, q, Z) = Z \iint_{\Omega^a(\bar{a}, Z, q)} (q - p(\bar{a}, \varepsilon))\, dF(\varepsilon, v). \tag{22}$$

Figure 6 shows the investment and participation sets for asset purchases under asymmetric information. The figure distinguishes three regions of interest: efficient participation, opportunistic participation, and independent investment. The efficient participation region comprises the banks that participate in the intervention and that invest because of the intervention. The opportunistic region comprises the banks that participate in the intervention but would have invested even in the absence of the intervention. The independent investment region comprises the banks that invest without government intervention. As is clear from the figure, the government's trade-off is between expanding the efficient participation region and reducing the opportunistic participation region.

From cost equation (22) we see that an asset purchase qZ is less costly than an equivalent a cash transfer qZ for three reasons. First, the independent investment region reduces opportunistic participation without reducing investment. Second, the pricing $q < 1$ excludes banks that would not invest. Third, the government receives Z in the high-payoff state which lowers the government's cost without affecting investment. Let us now compare asset purchases to debt guarantees:

Proposition 4 ***Equivalence of asset purchases and debt guarantees****. An asset purchase program (Z, q) with participation at time 1 is equivalent to a debt guarantee program with $S = Z$ and $q = 1 - \phi$.*

Proof. See Appendix. ■

The equivalence of asset purchases and debt guarantees comes from the fact that both programs make participation contingent on asset quality $p(\bar{a}, \varepsilon)$ but not investment opportunity v. To see this result, consider the upper-bound schedule. If $q = 1 - \phi$, banks with asset quality $p \in [1 - \phi, 1]$ choose not to participate. Hence, asset purchase program and debt guarantees have the same upper-bound schedule. Next, note that the net benefit of asset purchases is $(q - p)$, whereas the net benefit of debt guarantees is $(1 - \phi - p)$. Hence, asset purchases and debt guarantees have the same lower bound schedule. The NIP constraint for asset purchases is $p < 1$, which is equivalent to $\phi > 0$. The last step is to show that both asset purchases and debt guarantees have the same cost to the government, which

is true since they yield the same net benefit to participants. We can finally compare debt guarantees and asset purchases to equity injections:

Proposition 5 ***Dominance of equity injection***. *For any asset purchase program (Z, q) with participation at time 1, there is an equity program that achieves the same allocation at a lower cost for the government.*

Proof. See Appendix. ■

The dominance of equity injection over debt guarantees and asset purchases comes from the fact the equity injections are dependent both on asset quality ε and investment opportunity v. To understand this result, it is helpful to define the function $X(\bar{a}, \varepsilon; m, \alpha)$ as the part of the net benefit from participation that is tied to existing assets:

$$X(\bar{a}, \varepsilon; m, \alpha) \equiv (1-\alpha)\, m - \alpha p(\bar{a}, \varepsilon)(A - D). \tag{23}$$

In words, a participating bank receives net cash injection $(1-\alpha)\, m$ and gives up share α of the bank's expected equity value $p(\bar{a}, \varepsilon)(A - D)$. To compare equity injections with other programs, start by choosing an arbitrary asset purchase program. Then choose $X(\bar{a}, \varepsilon; m, \alpha)$ such that the lower-bound schedule of the asset purchase program coincides with the lower-bound schedule of the equity injection program. Under both programs, equity holders at the lower-bound schedule receive no surplus and are indifferent between participating and not participating. For given level of asset quality ε, the cost of participation for banks with a good investment opportunity v is higher under the equity injection program than under the asset purchase program because the government receives a share in both existing assets and new investments. As a result, there is less opportunistic participation with equity injections than with asset purchases.

Figure 7 shows the participation and investment regions under the equity injection program. The increase in cost of participation relative to the asset purchase program has two effects. First, conditional on participation, the cost to the government is smaller because the government receives a share in the investment opportunity v. Second, there is less opportunistic participation because participation is more costly. As a result, equity injections and asset purchases implement the same level of investment but equity injections

are less costly to the government relative to asset purchases.[8] The macroeconomic feedback from equation (12) only reinforces the dominance of equity injection. Finally, we note that participating banks receive informational rents in an equity injection program. It is therefore straightforward to show that equity injections do not achieve the minimum cost under perfect information.

4.4 Efficient interventions

We now analyze the efficient intervention in our setting. In particular, we examine whether an intervention with warrants and preferred stock can eliminate informational rents and achieve the minimum cost under perfect information Under the efficient intervention, the government injects cash m at time 1 in exchange for state contingent payoffs at time 2. New lenders at time 1 must break even and we can without loss of generality restrict our attention to the case where the government payoffs depend on the residual payoffs $y - y^D - y^l$. As in previous sections, we analyze cost minimization for a given investment set.[9]

It is far from obvious whether the government can reach the minimum cost under perfect information. The surprising result is that it can do so with warrants and preferred stock.

Theorem 2 *Consider the family of programs $\Gamma = \{m, h, \eta\}$ where the government provides cash m at time 1 in exchange for preferred stock with face value $(1+h)\,m$ and a portfolio of $(1-\eta)/\eta$ warrants at the strike price $A - D$. These programs implement the same set of investment domains as equity injections, but at a lower cost. In the limit $\eta \to 0$, opportunistic participation disappears and the program achieves the minimum cost:*

$$\lim_{\eta \to 0} \Psi_1(\bar{a}, \Gamma) = \Psi_1^{\min}.$$

Proof. See Appendix. ■

[8] The final step in the proof is to show that the NIP constraint is the same under both programs. This is true because the equity injection provides lower rents to participating banks than the asset purchase programs. Hence, if the no-efficient participation holds under the asset purchase program, it also holds under the equity injection program. We can also show that equity programs at time 1 cannot be improved by mixing them with a debt guarantee or asset purchase program. Pure equity programs always dominate. The proof is available upon request.

[9] In general, the government can offer a menu of contracts to the banks in order to obtain various investment sets. The actual choice depends on the distribution of types $F(p, v)$ and the welfare function W but we do not need to characterize it. We simply show how to minimize the cost of implementing any particular set.

Figure 8 shows the investment and participation region under the optimal intervention. The efficient intervention completely eliminates informational rents. The intuition for this result is that the initial shareholders receive the following payment in the high-payoff state:

$$f(y^e) = \min(y^e, A - D) + \eta \max(y^e - A - D, 0) \tag{24}$$

Shareholders are full residual claimants up to the face value of old assets $A - D$ and η residual claimants beyond. When η goes to zero, the entire increase in equity value due to investment is extracted by the government via warrants. As a result, the opportunistic participation region disappears and only the banks that really need the capital injection to invest participate in the program.[10]

Four properties of this optimal program are worth mentioning. First, we use preferred stock because it is junior to new lenders at time 1 and senior to common equity, but the program could also be implemented with a subordinated loan. Second, it is important that the government also takes a position that is *junior* to equity holders. The warrants give the upside to the government, which limits opportunistic participation. Third, the use of warrants limits risk shifting incentives since the government owns the upside, not the old equity holders (see, for instance, Green (1984)). Fourth, the use of warrants may allow the government to credibly commit to protecting new equity holders. This may be important for reasons outside the model if investors worry about outright nationalization of the banks.

We also note that our results describe the optimal intervention for a *given* investment set. The optimal investment set is the solution to the government objection function (14) and depends on the distribution of asset values and investment opportunities $F(\varepsilon, v)$ and the deadweight losses of taxation χ. We note that implementing the optimal investment set may require a menu of programs.

Our results on the efficient intervention can be extended to alternative payoff structures. It is straightforward to allow for uncertainty in asset values in the low payoff state. We think this type of uncertainty is the most relevant for banks because banks usually hold loans and debt securities with a fixed face value A and thus primarily face downside risk. It is clear

[10] In practice, there might be a lower bound on ε because the government might not want to own the banks. An approximate optimal program could then be implemented at this lower bound ε. Similarly, the rate h is chosen to rule out inefficient participation (the NIP constraint). In theory, any $h > 0$ would work, but in practice, parameter uncertainty could prevent h from being too close to zero.

from the discussion above that such downside risk in the low payoff state does not affect banks' participation and investment choices in the high-payoff state and therefore does not affect our results. We also examine more general asset distributions in the extensions of our model.

5 Extensions

In this section we present three extensions to our baseline model. We consider the consequences of heterogenous assets within banks and of deposit insurance.[11]

5.1 Heterogeneous assets within banks

We consider an extension of our model to allow for asset heterogeneity within banks. Suppose that the face value of assets at time 0 is $A + A'$. All these assets are ex-ante identical. At time 1, the bank learns which assets are A' and which assets are A. The A assets are just like before, with probability $p(\bar{a}, \varepsilon)$ of A and $1 - p(\bar{a}, \varepsilon)$ of 0. The A' assets are worth zero with certainty. The ex-ante problems are unchanged, so all programs are still equivalent at time 0.

The equity and debt guarantee programs are unchanged at time 1. So equity still dominates debt guarantee. But the asset purchase program at time 1 is changed. For any price $q > 0$ the banks will always want to sell their A' assets. This will be true in particular of the banks without profitable lending opportunities.

Proposition 6 *With heterogeneous assets inside banks, there is a strict ranking of programs: equity injection is best, debt guarantee is intermediate, asset purchase program is worse.*

The main insight from this extension is that adverse selection across banks is different from adverse selection across assets within banks. Adverse selection within banks increases the cost of the asset purchase program but does not affect the other programs.

[11] Other extensions to continuous asset distributions and the sale of safe assets are available upon request. They do not generate new insights that justify their inclusion in this section.

5.2 Continuous asset distribution

Our benchmark model assumes a binary payoff structure for assets in place. We can generalize our model to a distribution $F(a|\varepsilon)$ over $[0,\infty)$. As before, we assume that ε parameterizes the quality of assets in place. We now discuss how our main results change in this more general setup.

It is clear that Theorem 1 continues to hold. It is relatively easy to check that Proposition 5 also carries over to a general distribution of asset values. The issue of optimal interventions under asymmetric information is more delicate. Consider a generalization of the program described in Theorem 2. The government offers cash m in exchange for preferred stock with face value $(1+h)\,m$ and a portfolio of $(1-\eta)/\eta$ warrants with strike price $\max(a-D,0)$. The key difference is that the strike price of the warrants is conditional on the realized asset payoff. The schedule of payoffs to the old shareholders is now

$$f(y^e) = \min\left(y^e, \max(a-D,0)\right) + \eta \max\left(y^e - \max(a-D,0), 0\right)$$

and total shareholder payoff is

$$y^e = \max\left(a - D + i\cdot(v - rx + (r-1)\,m) - hm, 0\right)$$

where r is the break even rate conditional on ε, v and m (in the binary case, it was simply $r = 1/p_\varepsilon$). The NIP constraint is $h > 0$ and for simplicity we take the limit $h \to 0$. The first key point is that $E[y^e]$ is increasing in i if and only if $v + (r-1)\,m > rx$. This pins down the investment constraint conditional on participation. For any $\eta > 0$ it also pins down the lower participation schedule $L(\varepsilon, v; \Gamma) = 0$. By not participating, these types get $\max(a-D,0)$. It is easy to check that they prefer to participate (and invest) if and only if $v + (r-1)\,m > rx$.

Consider now the upper participation schedule $U(\varepsilon, v; \Gamma) = 0$. These types invest alone and a fortiori with the help of the government. For these types, $y^e > \max(a-D,0)$ so $f(y^e) = \max(a-D,0) + \eta\max(y^e - \max(a-D,0), 0)$. As we decrease η towards 0, the schedule $f(y^e)$ becomes $\max(a-D,0)$ which is the payoff without investment. It is strictly lower than the outside option of any type that would invest alone. In other words, in the (ε, v) space, the schedule $U(\varepsilon, v; \Gamma) = 0$ converges to the schedule $L(\varepsilon, v; \varnothing) = 0$ and opportunistic participation disappears. By the same argument, the rents of all participating types

also disappear since their payoff $f(y^e)$ converges to their outside option $\max(a-D,0)$. We therefore obtain our last result:

Proposition 7 *Theorem 1 and Proposition 5 hold for any asset distribution. Theorem 2 holds when the strike price of the warrants is set ex-post to* $\max(a-D,0)$.

The key point of the implementation is that banks whose assets perform well ex-post are rewarded by a lower dilution of their equity. Technically, by adjusting the strike price based on the realized asset value a, the government can provide the same incentives as in the model with binary asset payoffs. While the security design is not standard, it is perfectly consistent with the assumptions we have made regarding information and contracts. If assets trade, the government only need to use ex-post market prices. Even if assets do not trade, the government can implement the optimal intervention by buying a small random sample of assets, observe the ex-post performance, and set the strike price accordingly.

5.3 Deposit insurance

Suppose long term debt consists of two types of debt: deposits Δ and unsecured long term debt B such that

$$D = \Delta + B.$$

Suppose that the government provides insurance for deposit holders and that deposit holders have priority over unsecured debt holders. Then the payoffs are are:

$$y^{\Delta} = \min(y,\Delta)\,; y^{B} = \min\left(y - y^{\Delta}, B\right)$$

Proposition 8 *The costs of time 0 and time 1 programs decrease. The equivalence results and ranking of both time 0 and time 1 programs remain unchanged.*

Proof. See Appendix. ■

The intuition is that the government has to pay out deposit insurance in the low-payoff state. Hence, every cash injection lowers the expected cost of deposit insurance in the low-payoff state one-for-one. As a result, the government recoups the cash injection both in the high- and low-payoff state. Put differently, a cash injection represents a wealth transfer to depositors and, because of deposit insurance, a wealth transfer to the government. Hence, the equivalence results and the ranking of interventions remain unchanged..

6 Discussion of financial crisis of 2007/09

The financial crisis of 2007-2009 has underlined the importance of debt overhang. There is agreement among many observers that debt overhang is an important reason for the decline in lending and investment during the crisis (see Allen, Bhattacharya, Rajan, and Schoar (2008) and Fama (2009), among others).

For example, Ivashina and Scharfstein (2008) show that new lending was 68% lower in the three-month period around the Lehman bankruptcy relative to the three-month period before the Lehman bankruptcy. Using cross-sectional variation in bank access to deposit financing, the authors show that the reduction in lending reflects a reduction in credit supply by banks rather than a reduction in credit demand by borrowers.

The crisis has also shown the difficulty of finding effective solutions to the debt overhang problem. Several experts have expressed concerns that existing bankruptcy procedures for financial institutions are insufficient for reorganizing the capital structure. As an alternative, Zingales (2008) argues for a law change that allows for forced debt-for-equity swaps. Coates and Scharfstein (2009) suggest to restructure bank holding companies instead of bank subsidiaries. Ayotte and Skeel (2009) argue that Chapter 11 proceedings are adequate if managed properly by the government. Assuming that restructuring can be carried effectively, these approaches reduce debt overhang at low cost to the government. However, Swagel (2009) argues that the government lacks the legal authority to force restructuring and that changing bankruptcy procedures is politically infeasible once banks are in financial distress.

Moreover, concerns for systemic risk and contagion make it difficult to restructure financial balance sheets in the midst of a financial crisis. Aside from the costs of its own failure,

the bankruptcy of a large financial institution may trigger further bankruptcies because of counterparty risks and runs by creditors. For example, Heider, Hoerova, and Holthausen (2008) emphasize the role of counterparty risk in the interbank market.

The government may therefore decide to avoid restructuring because there is a positive probability of a breakdown of the entire financial system. Even if the government decides to let some institutions restructure, the government also has to address debt overhang among the financial institutions that do not restructure. In fact, even proponents of restructuring suggest to rank banks based on their financial health and only restructure banks below a cut-off. Hence, independent of whether the government restructures some banks, the optimal form of government intervention outside restructuring remains an important question.

Surprisingly however, while there is at least some agreement regarding the diagnostic (debt overhang), there is considerable disagreement about the optimal form of government intervention outside restructuring. The original bailout plan proposed by former Treasury Secretary Paulson favors asset purchases over other forms of interventions. Stiglitz (2008) argues that equity injections are preferable to asset purchases because the government can participate in the upside if financial institutions recover. Soros (2009a) also favors equity injections over asset purchases because otherwise banks sell their least valuable assets to the government. Diamond, Kaplan, Kashyap, Rajan, and Thaler (2008) argue that the optimal government policy should be a combination of both asset purchases and equity injections because asset purchases establish prices in illiquid markets and equity injections encourage new lending. Bernanke (2009) suggests that in addition to equity injections and debt guarantees the government should purchase hard-to-value assets to alleviate uncertainty about bank solvency. Geithner (2009) argues that asset purchases are necessary because they support price discovery of risky assets.

Other observers have pointed out common elements among the different interventions without necessarily endorsing a specific one. Ausubel and Cramton (2009) argue that both asset purchases and equity injections require to put a price on hard-to-value assets. Bebchuk (2008) argues that both asset purchases and equity injections have to be conducted at market values to avoid overpaying for bad assets. Soros (2009b) argues that bank recapitalization has to be compulsory rather than voluntary. Kashyap and Hoshi (2008) compare the financial crisis of 2007-2009 with the Japanese banking crisis and argue that in Japan

both asset purchases and capital injections failed because the programs were too small. Scharfstein and Stein (2008) argue that government interventions should restrict banks from paying dividends because, if there is debt overhang, equity holders favor immediate payouts over new investment. Acharya and Backus (2009) suggest that public lender of last resort interventions would be less costly if they borrowed some of the standard tools used in private contracts for lines of credit.

We believe our results in this paper make three contributions to this debate. First, we believe an analytical approach to this question is helpful because it allows the government to implement a principled approach in which financial institutions are treated equally and government actions are predictable. This approach is preferable to a trial-and-error approach in which the government adjust interventions depending on current market conditions and tailors interventions to requests of individual financial institutions. In fact, such a trial-and-error approach may create more uncertainty for private investors, which makes them even less willing to invest. Uncertainty also generates an option to wait for future interventions, which further undermines private recapitalizations. Moreover, tailor-made interventions are more likely to be influenced and distorted by powerful incumbents (see Hart and Zingales (2008), Johnson (2009)).

Second, we distinguish the economic forces that matter from the ones that do not by providing a benchmark in which the form of government interventions is irrelevant. Under symmetric information, all interventions implement the same level of lending at the same expected costs. In contrast, under asymmetric information buying equity dominates other forms of intervention because buying equity reduces the extent of adverse selection across banks. Our analysis also shows how the government can use warrants to minimize the expected cost to taxpayers, an important element which has not been emphasized in the public debate on the financial crisis. Interestingly, Swagel (2009) notes that the terms of the Capital Purchase Program, the first round of government intervention, consisted of providing a loan in exchange for preferred stock and warrants. This structure is qualitatively consistent with the optimal intervention.

Third, our analysis clarifies why government interventions are costly. Under symmetric information, equity holders are held to their participation constraint but debt holders receive an implicit transfer. Hence, the same economic force that generates debt overhang in

the first place, also generate the cost to the government. Under asymmetric information, participating banks receive informational rents because otherwise they would choose not to participate. Hence, under asymmetric information government interventions are costly because the government has to recapitalize at above market rates.

Our analysis focuses on one specific market failure, debt overhang, and its negative consequences on lending and investment. We have emphasized the importance of asymmetric information between the government and the private sector, but we have maintained the assumption of symmetric information within the private sector. Philippon and Skreta (2009) solve for the optimal form of intervention when the market failure is adverse selection among private agents. They find that debt guarantees are optimal and that the government should always aim for pooling interventions where all banks participate.

Finally, we note that our analysis does not address why the banking system entered financial distress and whether government bailouts affect future bank actions. In our model, we take debt overhang as given and rely on other research that links the financial crisis to securitization (Mian and Sufi (2009), Keys, Mukherjee, Seru, and Vig (2010)) and the tendency of banks to become highly levered (Adrian and Shin (2008), Acharya, Schnabl, and Suarez (2010), Kacperczyk and Schnabl (2009)). Regarding the impact of government interventions on future bank actions, we recognize that bailouts can create expectations of future bailouts which may cause moral hazard. However, if the government decides to intervene, then it is optimal for the government to choose the intervention with the lowest costs. Also, the optimal intervention minimizes rents to equity and debt holders, so the optimal intervention also minimizes moral hazard conditional on the decision to intervene.

7 Conclusion

In this paper, we study the efficiency and welfare implications of different government interventions in a standard model with debt overhang. We find that government interventions generate informational and macroeconomic rents for banks. Informational rents accrue to banks that participate in an intervention but do not change their level of investment as a result. macroeconomic rents accrue to banks that do not participate but benefit from the rise in asset values because of other banks' participation. We show that the efficient inter-

vention minimizes informational rents by using preferred stock with warrants and minimizes macroeconomic rents by conditioning implementation on sufficient bank participation. The first feature allows the government to extract the upside of new investments and the second feature reduces banks' outside options. If macroeconomic rents are large, then the efficient intervention recapitalizes banks at a profit.

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Proof of Proposition 2

Cash injection
The government offers cash m against fraction α of equity capital. The government recognizes that the equilibrium is $\bar{a}(m)$ which yields the investment domain $I^i(m)$. At time 0, equity holders participate in the voluntary intervention if

$$(1-\alpha) E_0 [y^e|\bar{a}, m] \geq E_0 [y^e|\bar{a}, 0]. \tag{25}$$

The cost of the program to the government is

$$\Psi_0^e (m, \alpha) = m - \alpha E_0 [y^e|\bar{a}, m].$$

Because the investment domain does not depend on α, the government chooses equity share α such that the participation constraint (25) binds. Using the participation constraint (25) to eliminate α from the cost function yields

$$\Psi_{0,free}^e (m, \alpha) = m - (E_0 [y^e|\bar{a}, m] - E_0 [y^e|\bar{a}, 0]).$$

Using expected shareholder value at time 0

$$E_0 [y^e|\bar{a}, m] - E_0 [y^e|\bar{a}, 0] = \bar{p}(\bar{a}) m + m \iint_{I^i(m)} (1 - p(\bar{a}, \varepsilon)) dF(\varepsilon, v) + \int\int_{I^i(m)\backslash I(\bar{a},0)} (p(\bar{a}, \varepsilon) v - x) dF(\varepsilon, v).$$

Therefore the cost of the government is

$$\begin{aligned}
\Psi_{0,free}^e (m, \alpha) &= m - \alpha E_0 [y^e|\bar{a}, m] \\
&= (1 - \bar{p}(\bar{a})) m - m \iint_{I^i(m)} (1 - p(\bar{a}, \varepsilon)) dF(\varepsilon, v) - \int\int_{I^i(m)\backslash I(\bar{a},0)} (p(\bar{a}, \varepsilon) v - x) dF(\varepsilon, v) \\
&= \Psi_0^{free}(m)
\end{aligned}$$

Debt guarantee
The government recognizes that the equilibrium conditional on intervention is given by the function $\bar{a}((1-\phi) S)$. Using equation (15), we see that conditional on participation, the equity value at time 0 is $E_0[y^e|\bar{a}, (1-\phi) S] - \bar{p}(\bar{a})S$. If a bank opts out, equity value becomes $E_0[y^e|\bar{a}, 0]$. Since participation only depends on $m = (1-\phi) S$, the government chooses the program such that the participation constraint binds:

$$\bar{p}(\bar{a}) S = \bar{p}(\bar{a}) m + (1-\phi) S \iint_{I^i(m)} (1 - p(\bar{a}, \varepsilon)) dF(\varepsilon, v) + \iint_{I^i(m)\backslash I(\bar{a},0)} (p(\bar{a}, \varepsilon) v - x) dF(\varepsilon, v)$$

The cost to the government is

$$\Psi_{0,free}^e = (1 - \bar{p}(\bar{a})) S - \phi S$$

Plugging the participation constraint into the cost function yields the expected cost $\Psi_0^{free}(m)$ defined in equation (16) The programs is equivalent to an asset purchase program when $Zq = (1-\phi) S$.

Proof of Proposition 4

We omit $\bar{a}$ to shorten the notations but all the calculations are conditional on the equilibrium value of $\bar{a}$. We must show equivalence along four dimensions: (i) the NIP constraint, (ii) the upper schedule, (iii) the lower schedule, and (iv) the cost function. Upon participation and investment, equity value is

$$E_1[y^e|i = 1; S, \phi] = p(\varepsilon)(A - D) + p(\varepsilon)v - x + (1 - \phi - p(\varepsilon))S$$

Participation without investment yields

$$E_1[y^e|i = 0; S, \phi] = p(\varepsilon)(A - D - \phi S).$$

Now consider the three constraints:

- NIP: $E_1[y^e|i = 0; S, \phi] < E_1[y^e|i = 0; 0, 0]$ or
$$\phi > 0.$$
- Upper schedule: $E_1[y^e|i = 1; S, \phi] > E_1[y^e|i = 1; 0, 0]$ or
$$U(\varepsilon, v; S, \phi) = (1 - \phi - p(\varepsilon))S.$$
- Lower schedule: $E_1[y^e|i = 1; S, \phi] > E_1[y^e|i = 0; 0, 0]$ or
$$L(\varepsilon, v; S, \phi) = p(\varepsilon)v - x + (1 - \phi - p(\varepsilon))S$$

  Using the notations of the asset purchase program, the participation set is $\Omega_1^a(S, 1-\phi)$, the investment domain is $I_1^a(S, 1-\phi)$ and the expected cost of the program is

$$\Psi_1(S, 1-\phi) = \phi S - S \int\int_{\Omega^a(S,1-\phi)} (1 - p(\varepsilon))\, dF(\varepsilon, v)$$

  Now if we set $S = Z$ and $q = 1 - \phi$, we see that the NIP constraint, the upper and lower schedules, and the cost functions are the same as for the asset purchase program. The two programs are therefore equivalent.

Proof of Proposition 5

Equity value at time 1 with cash injection m is:

$$E_1[y^e|\varepsilon, v, \bar{a}, m] = p(\bar{a}, \varepsilon)(A - D + m) + 1_{(\varepsilon,v)\in I(\bar{a},m)}(p(\bar{a}, \varepsilon)v - x + (1 - p(\bar{a}, \varepsilon))m)$$

We omit $\bar{a}$ to shorten the notations but all the calculations are conditional on the equilibrium value of $\bar{a}$. We first analyze the equity injection program at time 1. Upon participation and investment, equity value (including the share going to the government) is

$$E_1[y^e|i = 1; m] = p(\varepsilon)(A - D) + p(\varepsilon)v - x + m$$

Participation without investment yields

$$E_1[y^e|i = 0; m] = p(\varepsilon)(A - D + m)$$

Now consider the three constraints

- NIP: $(1-\alpha) E_1[y^e|i=0;m] < E_1[y^e|i=0,0]$ or:

$$(1-\alpha) m < \alpha (A-D).$$

- Upper schedule: $(1-\alpha) E_1[y^e|i=1;m] - E_1[y^e|i=1;0]$ or:

$$U^e = (1-\alpha) m - \alpha (p(\varepsilon)(A-D) + p(\varepsilon) v - x).$$

- Lower schedule: $(1-\alpha) E_1[y^e|i=1;m] - E_1[y^e|i=0;0]$ or:

$$L^e = (1-\alpha)(p(\varepsilon) v - x + m) - \alpha p(\varepsilon)(A-D).$$

If we define the function $X(\varepsilon;m,\alpha) \equiv (1-\alpha) m - \alpha p(\varepsilon)(A-D)$ as in equation (23), we can rewrite the program as:

$$\begin{aligned} L^e &= (1-\alpha)(p(\varepsilon) v - x) + X(\varepsilon;m,\alpha) \\ U^e &= \alpha (p(\varepsilon) v - x) - X(\varepsilon;m,\alpha) \end{aligned}$$

The participation set is

$$\Omega^e(m,\alpha) = \{(\varepsilon, v) \mid L^e > 0 \wedge U^e > 0\}.$$

The cost function is therefore

$$\Psi_1^e(m,\alpha) = \int\int_{\Omega^e(m,\alpha)} (m - \alpha E_1[y^e|i=1;m]) \, dF(\varepsilon, v).$$

We can rewrite the cost function such that

$$\Psi_1^e(m,\alpha) = \int\int_{\Omega^e(m,\alpha)} X(\bar{a},\varepsilon;m,\alpha) \, dF(\varepsilon,v) - \alpha \int\int_{\Omega^e(m,\alpha)} (p(\varepsilon) v - x) \, dF(\varepsilon,v).$$

The following table provides a comparison of the government interventions:

	Asset purchase	Equity injection
Participation	$\Omega^a(\bar{a}, Z, q)$	$\Omega^e(\bar{a}, m, \alpha)$
Investment	$I(\bar{a}) \cup \Omega^a(\bar{a}, Z, q)$	$I(\bar{a}) \cup \Omega^e(\bar{a}, m, \alpha)$
NIP-constraint	$q < 1$	$(1-\alpha) m < \alpha (A-D)$
Cost function	$\Psi_1^a(\bar{a}, Z, q)$	$\Psi_1^e(\bar{a}, m, \alpha)$

Now let us prove that the equity injection program dominates the other two programs. Take an asset purchase program (Z, q). We are going to construct an equity program that

has same investment at lower cost. To get equity with same lower bound graph we need to ensure that:

$$L^e(\varepsilon, v; m, \alpha) = L^g(\varepsilon, v; q, Z) \text{ for all } \varepsilon, v.$$

It is easy to see that this is indeed possible if we identify term by term: $\frac{\alpha}{1-\alpha} = \frac{Z}{A-D}$ and $m = qZ$. In this case we also have $I^a(\bar{a}, S, \phi) = I^e(\bar{a}, m, \alpha)$. The NIP constraint are also equivalent since $(1-\alpha)m < \alpha(A-D) \Leftrightarrow q < 1$.

Now consider the upper bound. Consider the lowest point on the upper schedule of the asset purchase program, i.e., the intersection of $U^e = 0$ with $p(\varepsilon)v - x = 0$. At that point $(\widetilde{\varepsilon}, \widetilde{v})$, we have $p(\widetilde{\varepsilon}) = q$ and $\widetilde{v} = x/q$. Using the fact that lower bounds are equal to zero, we can write $X(\varepsilon; m, \alpha) = (1-\alpha)(1-p(\varepsilon))qZ$ for all ε, v. This implies that $X(\tilde{\varepsilon}; m, \alpha) = 0$, and therefore $U^e(\widetilde{\varepsilon}, \widetilde{v}; m, \alpha) = \alpha(p(\tilde{\varepsilon})\tilde{v} - x) - X(\bar{a}, \tilde{\varepsilon}; m, \alpha) = 0$. Therefore the upper schedule $U^e(\varepsilon, v; m, \alpha) = 0$ also passes by this point. But the schedule $U^e(\varepsilon, v; m, \alpha) = 0$ is downward slopping in (ε, v), so the domain of inefficient participation is smaller (see Figure 7) than in the asset purchase case. Formally, we have just shown that:

$$\Omega^e(m, \alpha) \subset \Omega^g(S, \phi).$$

As an aside, it is also easy to see that the schedule $U^e(\varepsilon, v; m, \alpha) = 0$ is above the schedule $pv - x = 0$ so it does not get rid completely of opportunistic participation, but it helps. The final step is to compare the cost functions $\Psi_1^a(q, Z)$ and $\Psi_1^e(m, \alpha)$. By definition of the participation domain, we know that lower bound $L^e(\bar{a}, \varepsilon, v; m, \alpha) > 0$. Therefore:

$$-\int\int_{\Omega^e(m,\alpha)} (p(\varepsilon)v - x)\, dF(p, v) < \frac{X(\varepsilon; m, \alpha)}{1-\alpha} \text{ for all } (\varepsilon, v) \in \Omega^e(m, \alpha)$$

Therefore

$$\Psi_1^e(m, \alpha) < \frac{1}{1-\alpha}\int\int_{\Omega^e(m,\alpha)} X(\varepsilon; m, \alpha)\, dF(\varepsilon, v) = Z\int\int_{\Omega^e(m,\alpha)} (q - p(\varepsilon))\, dF(\varepsilon, v)$$

Since $q - p(\varepsilon) > 0$ for all $(\varepsilon, v) \in \Omega^e(m, \alpha)$, and since $\Omega^e(m, \alpha) \subset \Omega^a(q, Z)$, we have

$$\Psi^e(m, \alpha) < \Psi^a(q, Z).$$

Finally, note that the the comparison is conditional on equilibrium $\bar{a}$. However, the equity injection requires lower taxes and therefore lead to higher equilibrium level $\bar{a}$, only reinforcing our proposition.

Proof of Theorem 2

Consider the equity payoffs for a bank in the program:

$$y^e = \max\left(a - D + i \cdot \left(v - \frac{x - m}{p_\varepsilon} - m\right) - hm, 0\right).$$

In the good state, as soon as $y^e > A - D$, the warrants are in the money and the number of shares jumps to $1 + \frac{1-\eta}{\eta} = \frac{1}{\eta}$. So the old shareholders get only a fraction η of the value beyond $A - D$. The payoff function for old shareholders is therefore:

$$f(y^e) = \min(y^e, A - D) + \eta \max(y^e - A - D, 0).$$

Old shareholders are full residual claimants up to the face value of old assets $A - D$ and η residual claimants beyond. Now let us think about their decisions at time 1.

The NIP-constraint is simply $h > 0$. The value for old shareholders conditional on participation and investment is

$$E_1 \left[f\left(y^e\right) | \varepsilon, v, \Gamma, i = 1 \right] = p_\varepsilon \left(A - D + \eta \left(v - \frac{x - m}{p_\varepsilon} - (1 + h)\, m \right) \right)$$

The lower schedule (efficient participation) is therefore

$$L\left(\varepsilon, v; \Gamma\right) = \eta \left(p_\varepsilon v - x + m \left(1 - (1 + h)\, p_\varepsilon\right)\right).$$

For any $\eta > 0$, we can see that the lower schedule is equivalent to that of an equity injection with $\frac{\alpha}{1-\alpha} = \frac{m(1+h)}{A-D}$, and that of an asset purchase with $m = qZ$ and $q = \frac{1}{1-h}$. If we take $h \to 0$ we get the lower bound of a simple cash injection program, with an investment set simply equal to $I\left(\bar{a}, m\right)$. In general, we have an investment set $\mathcal{I}$.

The upper schedule (opportunistic participation) is:

$$U\left(\bar{a}, \varepsilon, v; \Gamma\right) = \eta m \left(1 - (1 + h)\, p_\varepsilon\right) - (1 - \eta)\left(p_\varepsilon v - x\right)$$

When $\eta \to 0$, the upper bound schedule $\{U = 0\}$ converges to the schedule $\{p_\varepsilon v - x = 0\}$. In this limit, there is no opportunistic participation and

$$\lim_{\eta \to 0} \Omega\left(\Gamma\right) = \mathcal{I} \backslash I\left(\bar{a}, 0\right) = \Omega^{\min}$$

Finally, the expected payments to the old shareholders converge to $p_\varepsilon \left(A - D\right)$. So the government receives expected value $p_\varepsilon v - x + m$ by paying m at time 1. The total cost therefore converges to:

$$\lim_{\eta \to 0} \Psi_1\left(\bar{a}, \Gamma\right) = -\iint_{\Omega^{\min}} \left(p_\varepsilon v - x\right) dF\left(\varepsilon, v\right) = \Psi_1^{\min}.$$

QED.

Proof of Proposition 8

Time 0 Programs

Full transfer: $v < \Delta$

For simplicity, we suppress the macro state $\bar{a}$ in all expressions. The expected values of deposits at time 1 and time 0 are

$$\begin{aligned} E_1\left[y^\Delta(m)\right] &= p(\varepsilon)\,\Delta + (1 - p(\varepsilon))\, m \text{ if } (\varepsilon, v) \in T \backslash I(m) \\ &= p(\varepsilon)\,\Delta + (1 - p(\varepsilon))\, v \text{ if } (\varepsilon, v) \in I(m) \\ E_0\left[y^\Delta(m)\right] &= \overline{p}\Delta + (1 - \overline{p})\, m + \iint_{I(m)} (1 - p)(v - m)\, dF(\varepsilon, v). \end{aligned}$$

The expected cost of deposit insurance at time 0 is

$$\begin{aligned}\Psi_0^F(m) &= \Delta - E_0\left[y^{\Delta}(m)\right] \\ &= (1-\overline{p})(\Delta - m) - \iint\limits_{I(m)} (1-p(\varepsilon))(v-m)\,dF(\varepsilon, v)\end{aligned}$$

The change in the expected cost of deposit insurance as a result of the cash injection m is

$$\begin{aligned}\Lambda_0^F(m) &= \Psi_0^F(m) - \Psi_0^F(0) \\ &= -(1-\overline{p})\,m + m\int\int\limits_{I(m)\backslash I(0)} (1-p)\,dF(\varepsilon, v) - \int\int\limits_{I(m)\backslash I(0)} (1-p(\varepsilon))\,v dF(\varepsilon, v)\,.\end{aligned}$$

The net cost of government intervention is

$$\bar{\Psi}_0(m) + \Lambda_0^F(m) = -\int\int\limits_{I(m)\backslash I(0)} v dF(\varepsilon, v).$$

Note that this term is negative because the benefits of incremental investments accrue to the government.

Partial Transfer: $\Delta < v$
The expected values of deposits at time 1 and time 0 are

$$\begin{aligned}E_1\left[y^{\Delta}(m)\,|\varepsilon, v\right] &= p(\varepsilon)\,\Delta + (1-p(\varepsilon))\max(\Delta, m) \text{ if } (p, v) \in T\backslash I(m) \\ &= \Delta \text{ if } (\varepsilon, v) \in I(m) \\ E_0\left[y^{\Delta}(m)\right] &= \Delta - \int\int\limits_{T\backslash I(m)} (1-p(\varepsilon))(\Delta - \max(\Delta, m))\,dF(\varepsilon, v)\end{aligned}$$

The expected cost of deposit insurance is

$$\Psi_0^F(m) = \int\int\limits_{T\backslash I(m)} (1-p(\varepsilon))(\Delta - \max(\Delta, m))\,dF(\varepsilon, v).$$

The change in the expected cost of deposit insurance

$$\Lambda_0^F(m) = \int\int\limits_{T\backslash I(m)} (1-p(\varepsilon))(\Delta - \max(\Delta, m))\,p(\varepsilon) - \int\int\limits_{T\backslash I(0)} (1-p(\varepsilon))(\Delta)\,dF(\varepsilon, v).$$

Note that this expression is negative. The government cost is $\Lambda_0^F(m) + \bar{\Psi}_0(m)$. The results apply to all programs because all programs have the same cost function at time 0.

Time 1 programs

Full Transfer: $v < \Delta$
The expected values of deposits at time 1 and time 0 given an asset purchase program (q, Z) are

$$\begin{aligned}
E_1\left[y^{\Delta}(q,Z)\right] &= p(\varepsilon)\Delta \text{ if } (\varepsilon,v) \in T\backslash\left(I(0)\cup\Omega_1(q,Z)\right) \\
&= p(\varepsilon)\Delta + (1-p(\varepsilon))v \text{ if } (\varepsilon,v) \in I(0)\cup\Omega_1(q,Z) \\
E_0\left[y^{\Delta}(q,Z)\right] &= \overline{p}\Delta + \int\int_{I(0)\cup\Omega_1(q,Z)} (1-p(\varepsilon))\,vdF(\varepsilon,v)
\end{aligned}$$

The expected cost of deposit insurance is

$$\Psi_0^F(q,Z) = (1-\overline{p})\Delta - \int\int_{I(0)\cup\Omega_1(q,Z)} (1-p(\varepsilon))\,vdF(\varepsilon,v)$$

The change in the cost of deposit insurance due to the injection is

$$\Lambda_0^F(q,Z) = -\int\int_{\Omega_1^g(q,Z)/I(0)} (1-p(\varepsilon))\,vdF(\varepsilon,v)$$

The expected cost for the government is $\Psi_1^a(q,Z) + \Lambda_0^F(q,Z)$.

Partial Transfer: $\Delta < v$
The expected values of deposits at time 1 and time 0 are

$$\begin{aligned}
E_1\left[y^{\Delta}(Z,q)\right] &= p\Delta \text{ if } (\varepsilon,v) \in T\backslash\left(I(0)\cup\Omega_1(q,Z)\right) \\
&= \Delta \text{ if } (\varepsilon,v) \in I(0)\cup\Omega_1(q,Z) \\
E_0\left[y^{\Delta}(Z,q)\right] &= \Delta - \int\int_{T\backslash(I(0)\cup\Omega_1(q,Z))} (1-p(\varepsilon))\,\Delta dF(\varepsilon,v)
\end{aligned}$$

The expected cost of government insurance is

$$\Psi_0^F(Z,q) = \int\int_{T\backslash(I(0)\cup\Omega_1(q,Z))} (1-p(\varepsilon))\,\Delta dF(\varepsilon,v).$$

The change in expected cost of deposit insurance is

$$\Lambda_0^F(Z,q) = -\int\int_{\Omega_1(q,Z)/I(0)} (1-p(\varepsilon))\,\Delta dF(\varepsilon,v).$$

The cost to the government is$\Psi_1^a(q,Z)+\Lambda_0^F(q,Z)$. The results also apply to debt guarantees at time 1 because asset purchases and debt guarantees have the same cost function at time1.

Cash against equity at time 1

Note that we can compute the expected cost of time 1 cash against equity similarly to the time 1 asset purchase program. The only difference is the participation region for cash against equity $\Omega^e(m, \alpha)$ and the participation region for asset purchase $\Omega_1^g(q, Z)$. It turns out that the change in the expected cost of deposit insurance $\Lambda_0^F(m)$ is equivalent under both programs because both in the full and partial transfer case the difference in the participation region cancels out when computing the difference in expected cost of deposit insurance. It follows that the relative ranking of programs is unchanged.

Fig 1: Information & Technology

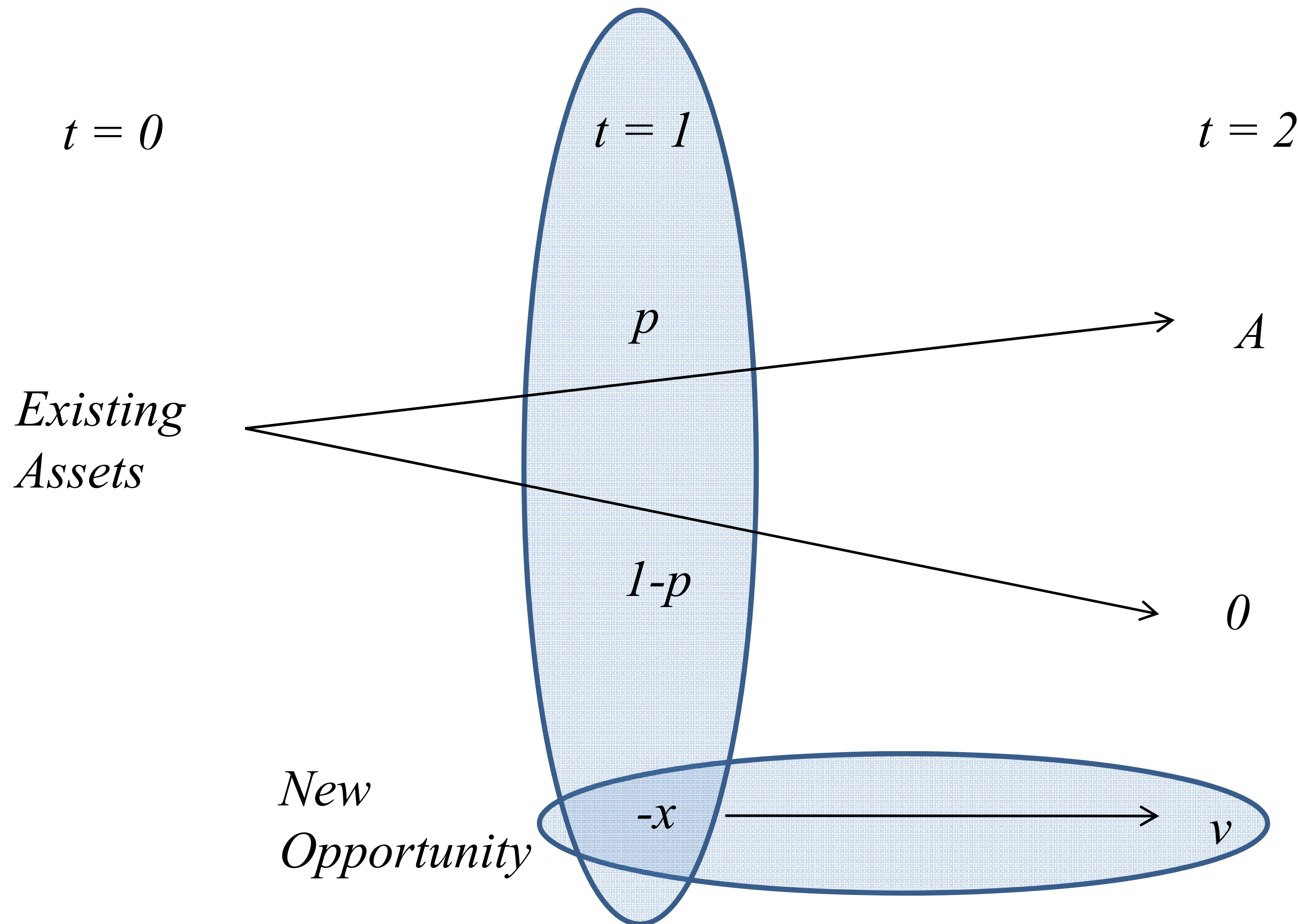

Fig 2: Payoffs

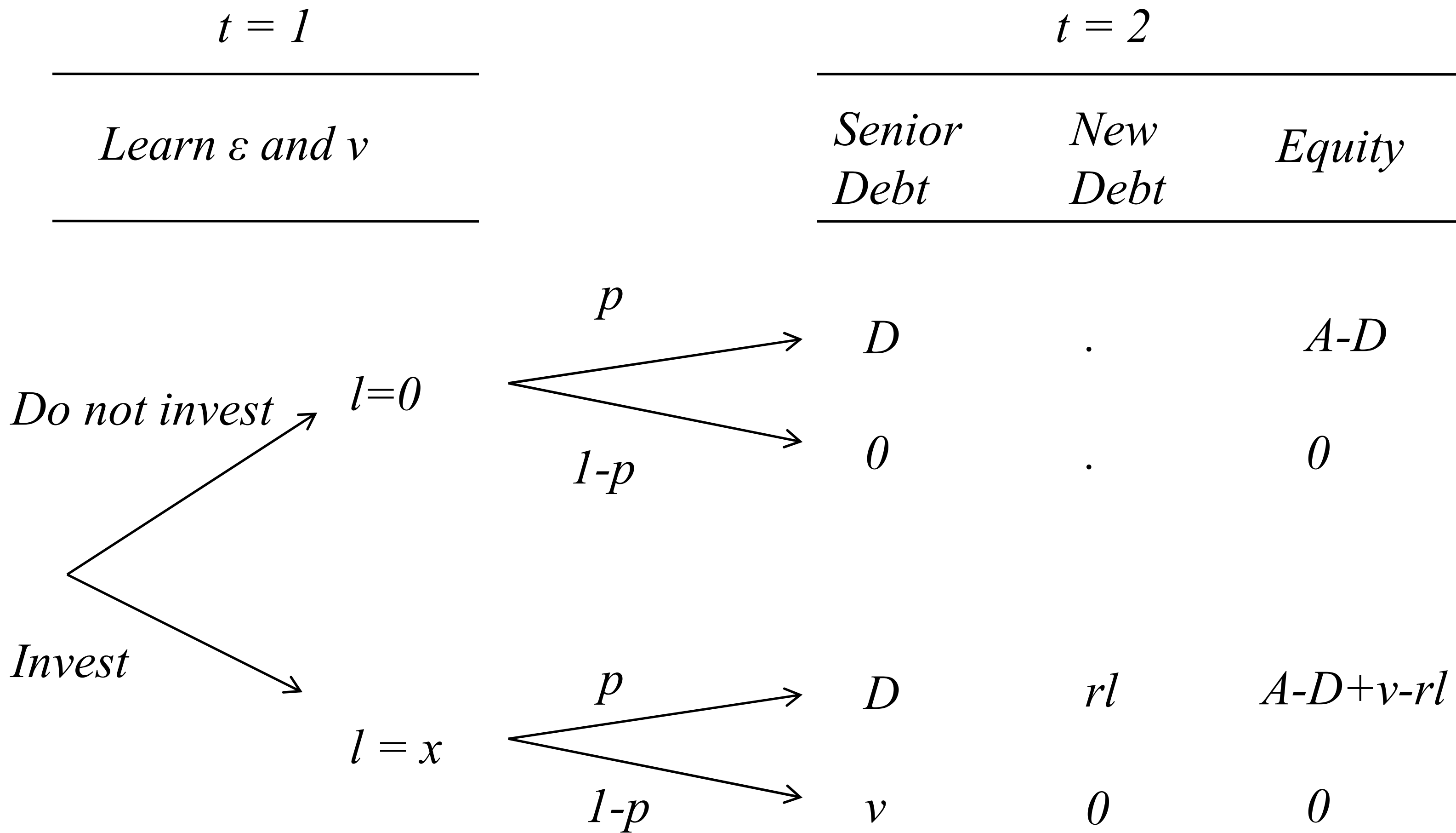

Fig 3: First Best

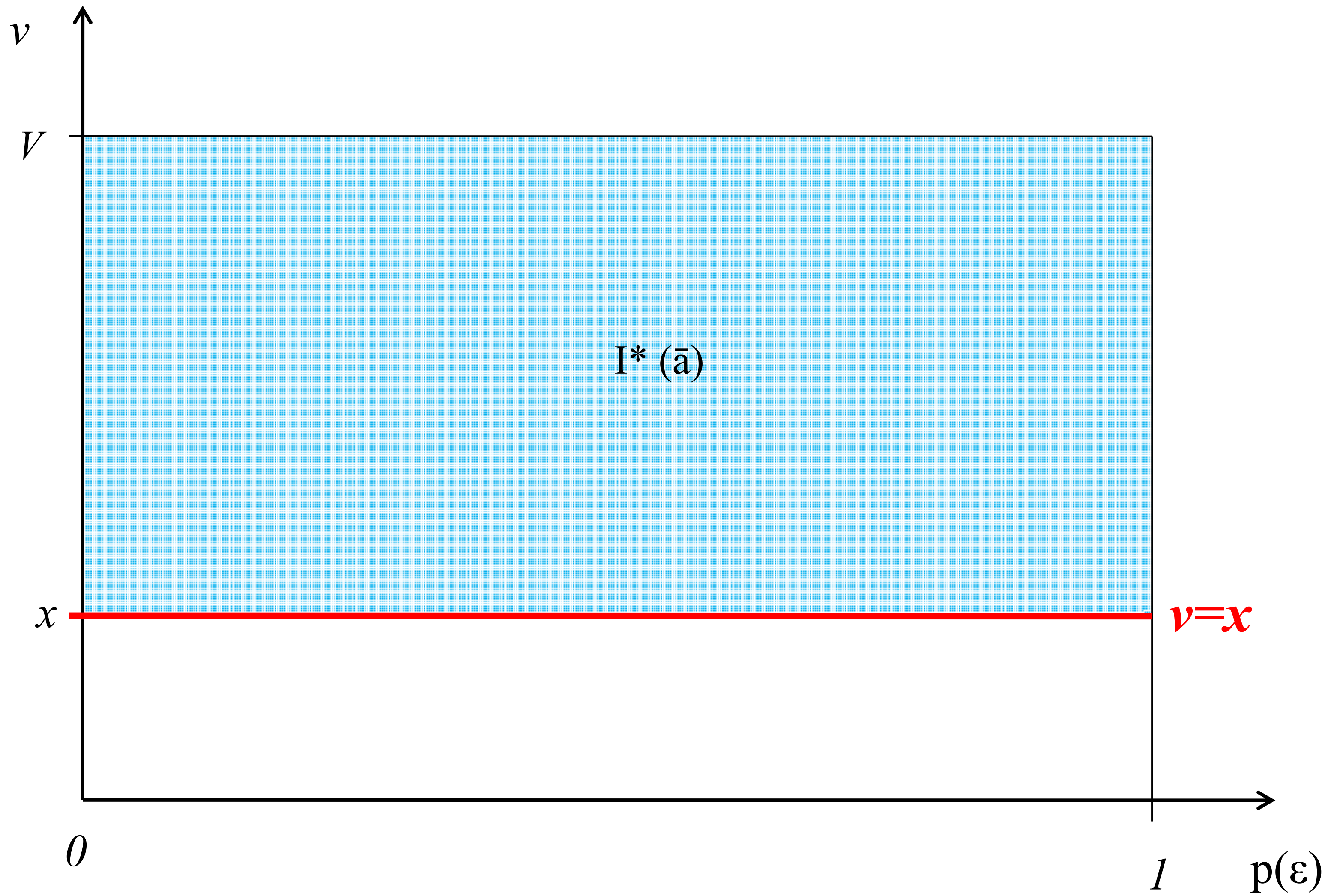

Fig 4: Debt Overhang

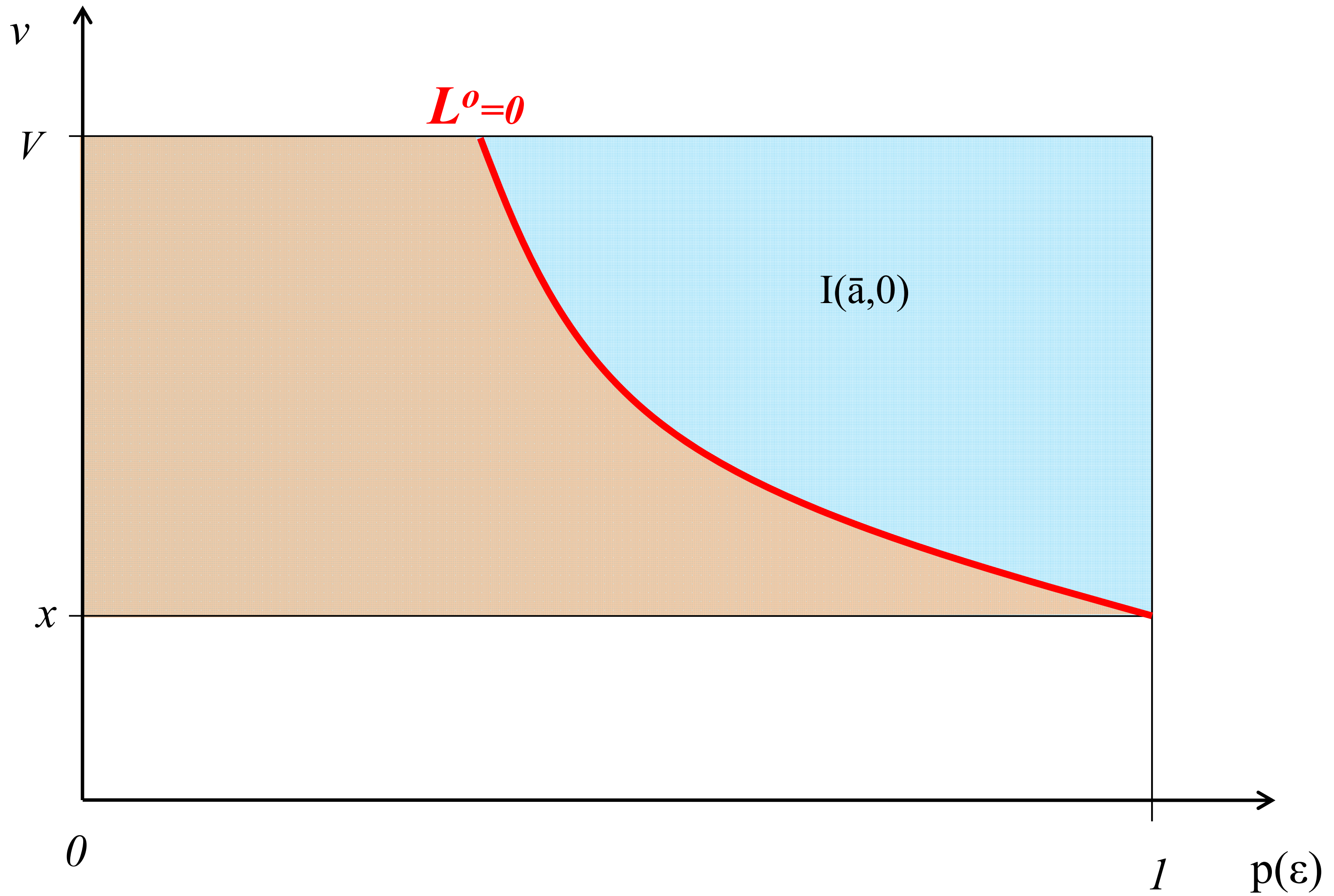

Fig 5: Cash at time 0

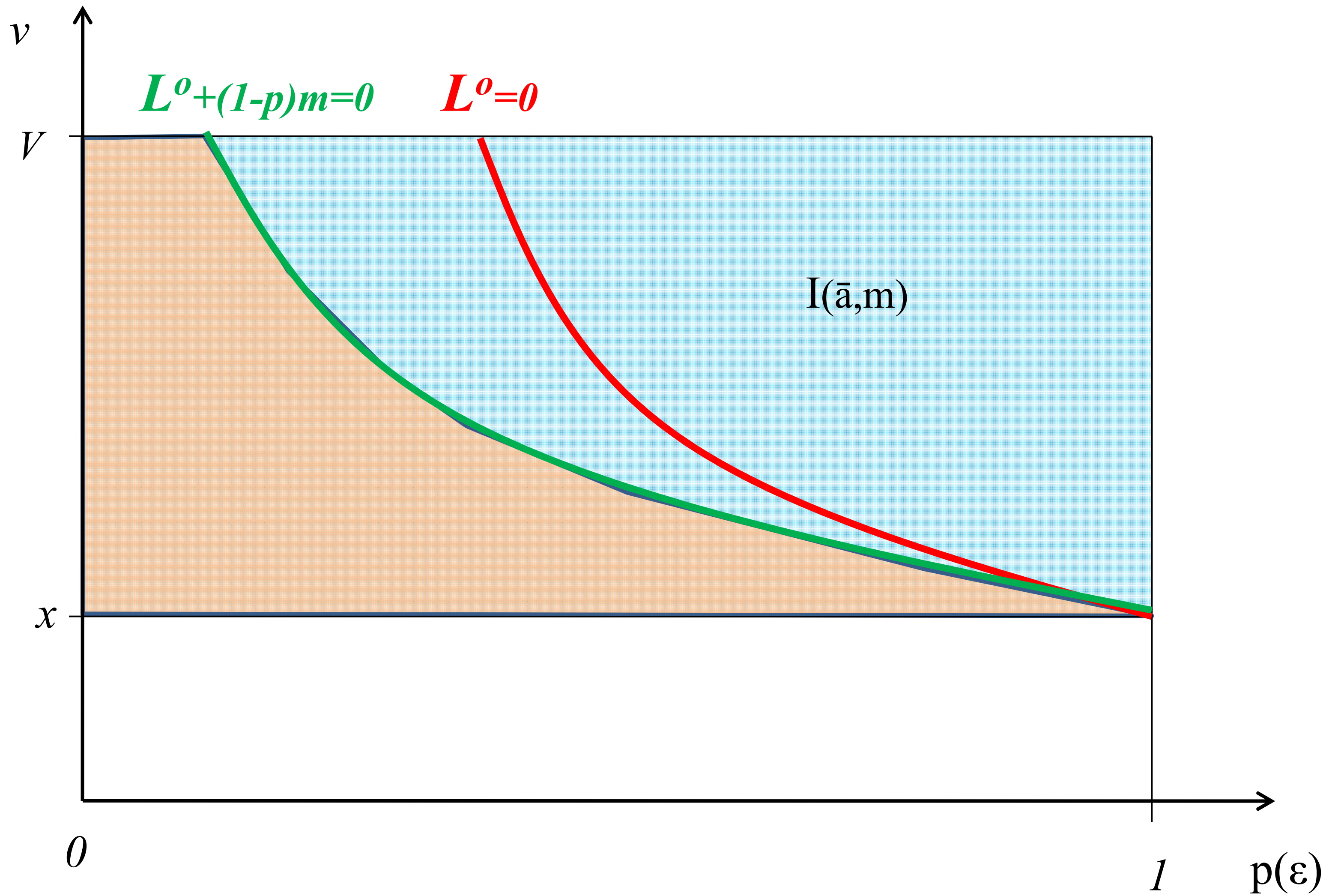

Fig 6: Asset Purchase at time 1

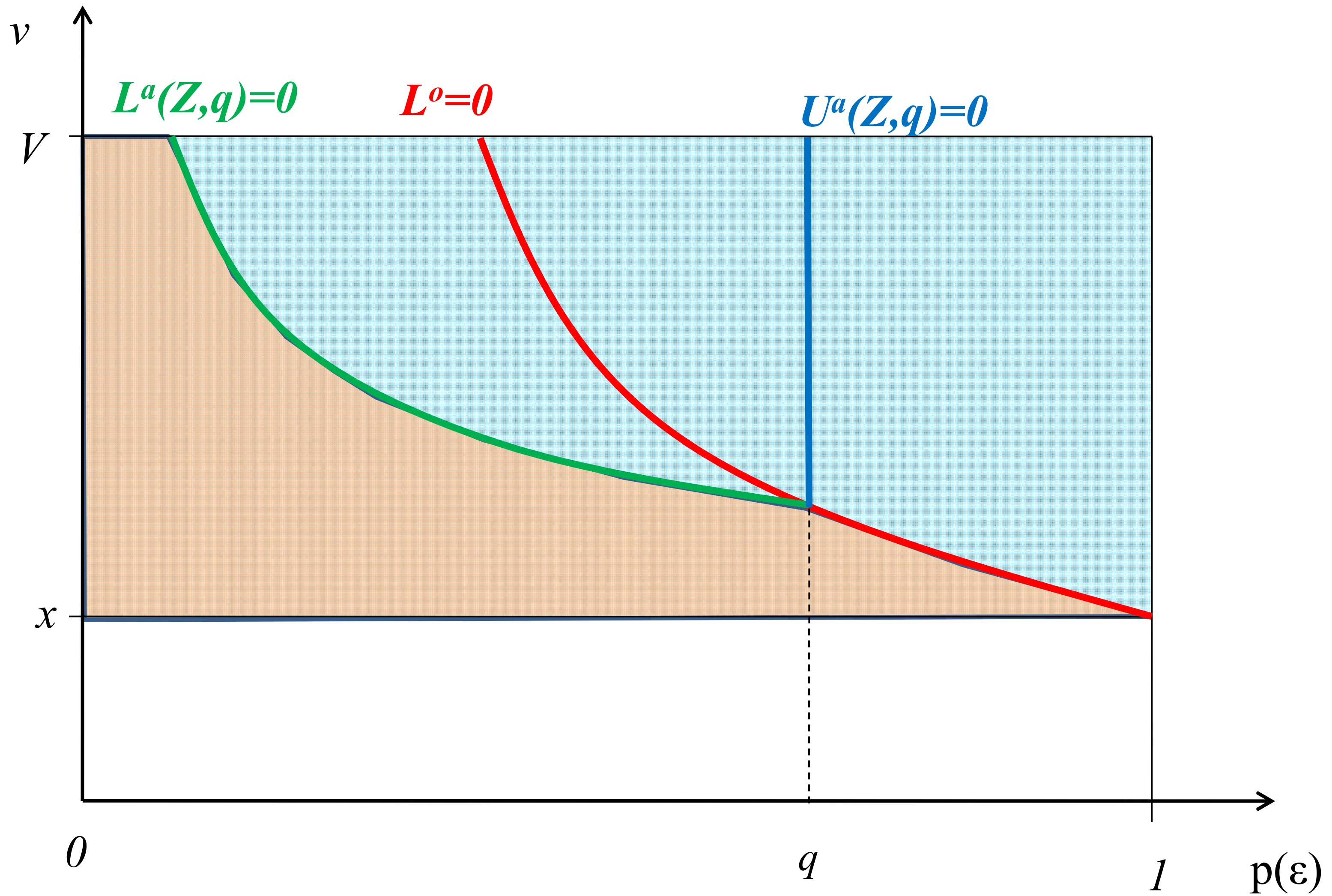

Figure 7: Equity injection at time 1

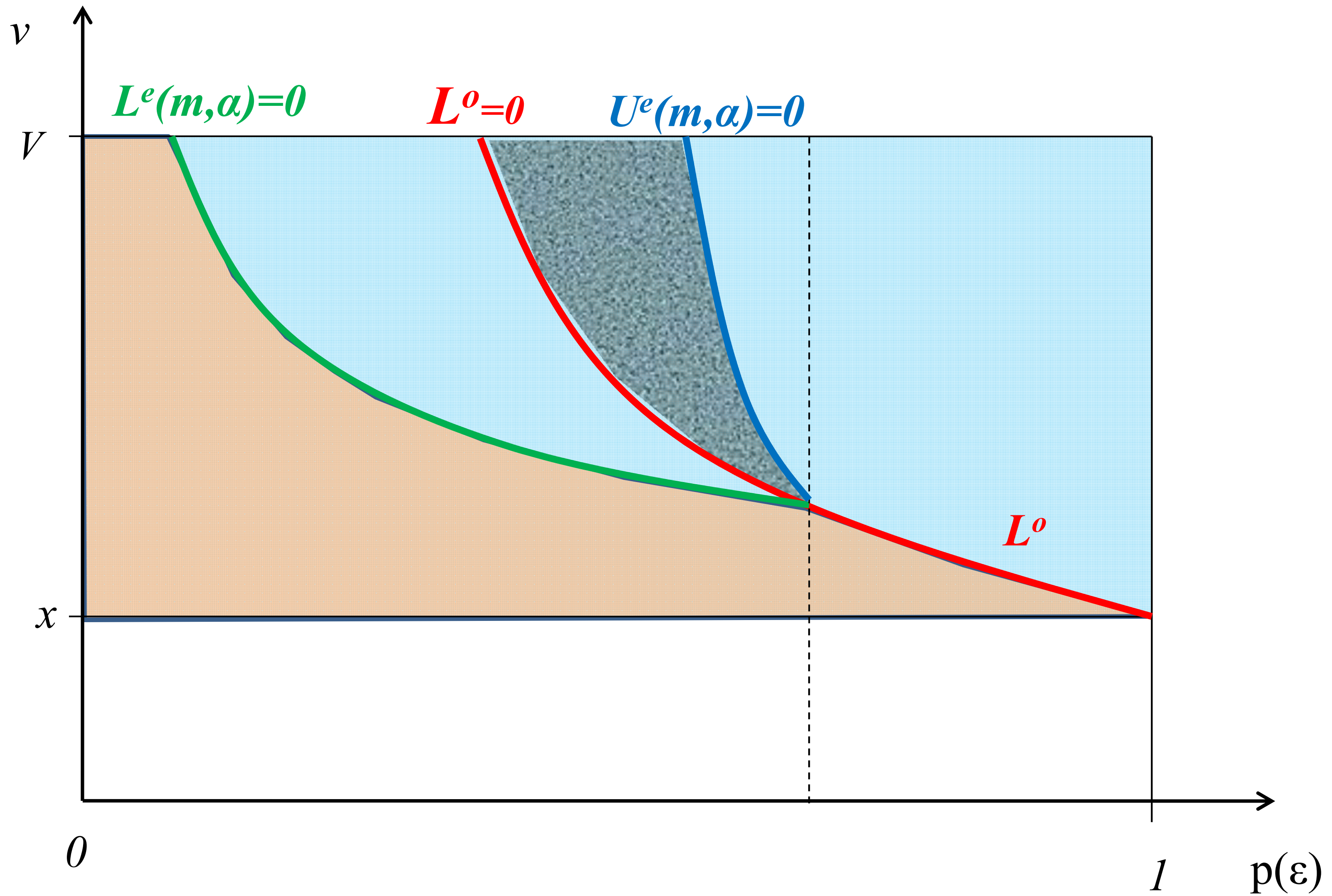

Figure 8: Efficient Mechanism