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ABSTRACT

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JEL Classification: D83 and E40
Keywords: equilibrium selection, money, risk-dominance and search

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Equilibrium selection in a fundamental model of money*

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Abstract

Fundamental models of money always exhibit autarkic equilibria where money has no value. In this paper we propose a simple procedure to select among equilibria in such models. Our procedure unveils a natural mapping between equilibrium behavior and the primitives of the economy, thus offering insights on the conditions under which money emerges as a medium of exchange. Overall, our results favour money over autarky, especially if agents are patient.

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Keywords: Money, search, equilibrium selection, risk-dominance.

"For the importance of money essentially flows from its being a link between the present and the future."

Keynes, The General Theory of Employment, Interest and Money (1936)

1 Introduction

A common feature of any fundamental model of money (e.g., search, turnpike, overlapping generations) is the existence of multiple equilibria. In particular, besides equilibria where money is accepted and circulates as a medium of exchange, there always exist autarkic equilibria where money has no value. Although inefficient, such equilibria are viewed as an inevitable implication of the fact that money is a belief-driven phenomenon. In this paper we propose a simple way to select among equilibria in a fundamental model of money.

We cast our analysis in a standard search model of money along the lines of Kiyotaki and Wright (1993). This model is appealing for being tractable and rendering an essential role for money as

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a medium of exchange.[1] Our environment is basically the same as in Kiyotaki and Wright but for one difference. We assume that the economy experiences different states, and these states evolve according to a continuous random process. This process allows for the existence of a region where money has a negative intrinsic value and another region where money has a positive intrinsic value. Importantly, the probability of ever reaching one of these regions might be made arbitrarily small with virtually no effect on any of the results. Throughout our analysis, we are interested in describing how agents behave when they are not in a state where money has intrinsic value.

The remote states where money carries a negative intrinsic value can be interpreted in different ways: there might exist states where exchange is impossible for some reason (transporting money becomes dangerous or storing money becomes expensive); or there might be states where agents simply fail to coordinate on using money. Likewise, the remote states where money is valued may capture the possibility that at some faraway states, agents will somehow find ways to coordinate in the use of money; or perhaps the technological progress throughout the centuries might allow for the productive use of the seashells employed as medium of exchange (that are currently intrinsically useless).

The key implication of the existence of remote regions is that it imposes a condition on beliefs held by agents. In one direction, it rules out the belief that money is always going to be employed in all states of the world. In another direction, it rules out the belief that money is never going to be employed in any state of the world. In our view, equilibria that depend on such extreme beliefs are tenuous for relying on agents being sure about how they will coordinate on the use of money in all possible states at every point in time. As discussed by Morris and Shin (2000), in models with multiple equilibria, it is not at all clear why agents would be certain that everyone would always coordinate in a particular set of beliefs.

We first show that the economy exhibits a unique equilibrium. In this equilibrium, money is always accepted if there are enough gains from trade, and is never accepted if gains from trade are small. Importantly, the condition that separates between money and autarky does not depend on the intrinsic value of money on the remote states. It only depends on an agent's incentives to accept money at a hypothetical state that divides the set of states in two, a set where money is accepted and another where money is not accepted.

Our second result is that if the random process that governs the evolution of states is equally

[1] In the conclusion, we discuss the conditions under which our results extend to turnpike and overlapping generations models, as well as to search models with divisible goods and divisible money.

likely to reach either of the remote regions, as agents become infinitely patient, the condition that ensures the uniqueness of the monetary equilibrium is the same condition that ensures the existence of the monetary equilibrium in Kiyotaki and Wright (1993). Hence if the time discount factor is large enough and there is any gain from trade, autarkic equilibria are tenuous for relying on agents being sure money will never be accepted in any state of nature. Intuitively, if agents are very patient, money does not have to be immediately used for it to have value. In the presence of gains from trade, an agent is willing to accept money even if it takes time for him to use it. Since everyone knows everyone thinks like this, they end up coordinating on accepting money. Importantly, the probability of reaching the state where money is valuable can be made arbitrarily small with negligible effects on the equilibria. What matters for an agent's decision is not the probability of reaching a remote region where money has an intrinsic value but the probability of reaching a region where money is accepted as a medium of exchange.

We also consider the case where the random process that describes the evolution of states is discrete. We show that money is the unique equilibrium if the gains from trade are large, and autarky is the unique equilibrium if the gains from trade are small. There exists a region where money and autarky coexist. However, as the discrete distribution gets closer to a continuous distribution, the region of multiple equilibria shrinks to a set of measure zero. Like in the continuous case, as agents become infinitely patient, there is an equivalence between the condition for uniqueness of the monetary equilibrium and the condition for existence of the monetary equilibrium in Kiyotaki and Wright (1993).

The paper is organized as follows. In the next section we discuss the related literature. In section 3 we present the model and deliver our main result. Some examples are presented in section 4 and in section 5 we conclude.

2 Related Literature

The paper is related to the literature on equilibrium selection in coordination games. The literature on global games (Carlsson and Van Damme (1993), Morris and Shin (2000, 2003), Frankel, Morris and Pauzner (2003)) shows that the multiplicity of equilibria disappears once the information structure of the game is slightly perturbed. A related argument applies to dynamic games with complete information where a state variable is subject to shocks (Frankel and Pauzner (2000), Burdzy, Frankel, and Pauzner (2001)). Those papers assume the existence of dominant regions and of strategic complementarities. Both assumptions hold in our model: there exist regions where either accepting money or not accepting money is a dominant strategy; and the use of money intrinsically relies on coordination.

In our setup, agents have complete information about all relevant features of the environment. It is in this sense close to the dynamic games literature but quite distant from the global games literature. In the class of dynamic games, the paper that most closely compares to ours is Burdzy, Frankel, and Pauzner (BFP) (2001). BFP considers a dynamic economy where a continuum of agents meet randomly and in pairs to play a 2x2 coordination game. A key assumption in their setting is that each agent has only a small chance of changing his action in between matches, so that an agent may be locked into an action when he enters a match. This prevents agents from shifting from one action to another purely for coordination reasons and leads to a unique equilibrium in their model. It turns out that this lock-in assumption is not necessary in our environment, and the reason for that has a lot to do with money being a link between the present and the future, as stated by Keynes. Since the value of money today comes from its future use as a medium of exchange, an agent deciding about accepting money has to take into account whether other agents will accept money tomorrow, or at some point in the future, but not whether they will coordinate on accepting money today.

One key result of the global games literature (Carlsson and Van Damme (1993), Morris and Shin (2000, 2003)) is that in static coordination games the risk-dominant equilibrium is selected. The same goes through in the model considered by BFP: the risk-dominant equilibrium of the static coordination game is played all the time if agents meet frequently enough and frictions are small. In contrast, the equilibrium condition in our model is very different from what would be obtained in a static (or 2-period) setup with asymmetric information and dominant regions. The key distinction is related to the effect of the time discount factor on the risk-dominant action. Since money earned today can be saved and spent at any time in the future, the benefit of having money depends on how long one has to wait to spend it, an inherently dynamic issue. If agents are more patient, the cost of waiting until money can be spent is lower, hence accepting money is less risky. In consequence, the time discount factor not only directly affects the gains from trade as in Kiyotaki and Wright (1993), it also affects how agents coordinate on an equilibrium.

Finally, another strand of literature related to our paper studies how the addition of an intrinsic value to money may help to reduce the set of equilibria. In overlapping generations models, the focus is on the elimination of monetary equilibria that exhibit inflationary paths (Brock and Scheinkman

(1980), Scheinkman (1980)).[2] In search models of money, the objective is to characterize the set of fiat money equilibria that are limits of commodity-money equilibria when the intrinsic value of money converges to zero (Zhou (2003), Wallace and Zhu (2004), Zhu (2003, 2005)). A result that comes out of this literature is that, as long as goods are perfectly divisible and the marginal utility is large at zero consumption, autarky is not the limit of any commodity money equilibria. This result critically depends on the assumption that there is a sufficiently high probability that the economy reaches a state where fiat money acquires an intrinsic value. In contrast, our results hold even if the probability that money ever acquires an intrinsic value is arbitrarily small and the economy is eventually in states where money is not accepted.

3 Model

Our environment is a version of Kiyotaki and Wright (1993). Time is discrete and indexed by t. There are k indivisible and perishable goods, and the economy is populated by a unit continuum of agents uniformly distributed across k types. A type i agent derives utility u per unit of consumption of good i and is able to produce good $i+1$ (modulo k) at a unit cost of c, with $u > c$. Agents maximize expected discounted utility with a discount factor $\beta \in (0, 1)$. There is also a storable and indivisible object, which we denote as money. An agent can hold at most one unit of money at a time, and money is initially distributed to a measure m of agents.

Trade is decentralized and agents face frictions in the exchange process. We formalize this idea by assuming that there are k distinct sectors, each one specialized in the exchange of one good. In every period, agents choose which sector they want to join but inside each sector they are anonymously and pairwise matched under a uniform random matching technology. Each agent faces one meeting per period, and meetings are independent across agents and independent over time. For instance, if an agent wants money he goes to the sector which trades the good he produces and searches for an agent with money. If he has money he goes to the sector that trades the good he likes and searches for an agent with the good. Due to the unit upper bound on money holdings, a transaction usually happens only when an agent with money (buyer) meets an agent without money (seller).

We depart from the standard search model of money by assuming that, in any given period, the economy is in some state $z \in \mathbb{R}$. States evolve according to a random process $z_t = z_{t-1} + \Delta z_t$,

[2] See Obstfeld and Rogoff (1983) for a similar analysis in a model with money in the utility function.

where Δz_t follows a probability distribution that is independent of t, with expected value $E(\Delta z)$ and variance $V(\Delta z)$. The fundamentals of the economy describing preferences (u and β) and technology (k and c) are invariant across states. However, there are some states where money generates an intrinsic value. Precisely, there exists $\widehat{z} > 0$ such that money generates a positive intrinsic value for all states $z \geq \widehat{z}$, a negative intrinsic value for all $z \leq -\widehat{z}$ and no value at all for $z \in (-\widehat{z}, \widehat{z})$. The economy starts at $z = 0$. Throughout, we think of z as being finite but very large.

There are several ways in which one can model the intrinsic value of money. For analytical convenience we adopt the following approach. In any state $z \geq \widehat{z}$, if an agent holds one unit of money at the beginning of the period, he can choose between keeping this unit throughout the period, in which case he obtains a positive flow payoff γ; and bringing this unit into a trading post, in which case he obtains no flow payoff but he can use the unit as a medium of exchange. In turn, in any state $z \leq -\widehat{z}$, if an agent holds one unit of money at the beginning of the period, he can choose between keeping this unit throughout the period, in which case he obtains a flow payoff zero; and bringing this unit into a trading post, in which case he obtains a negative flow payoff $-\xi$ but he can use the unit as a medium of exchange. Throughout, we assume that

$$\frac{\beta}{1-\beta}\gamma > c, \tag{1}$$

$$\xi > u. \tag{2}$$

The assumption that $\frac{\beta}{1-\beta}\gamma > c$ ensures that, for large enough z, an agent always produces in exchange for money. In turn, the assumption that $\xi > u$ implies that, for small enough z, an agents never uses money as a medium of exchange. We further assume that $(1-m)u > \gamma$. This assumption simplifies the exposition by ensuring that, if an agent believes that all other agents accept money, he prefers to bring money into the trading post that trades the good he likes instead of keeping the money in order to enjoy its intrinsic benefit.

3.1 Benchmark

We initially consider the problem of an agent when $V(\Delta z) = 0$. In this case, the economy never reaches a state where money has intrinsic value. First, there always exists an equilibrium where an agent does not accept money simply because he believes no other agent will ever accept money. In this case, the economy is in permanent autarky. Now, assume that an agent believes that all other agents always accept money. Let V_0 be his value function if he does not have money, and let V_1 be the corresponding value function if he has money. We have

$$V_1 = m\beta V_1 + (1-m)(u + \beta V_0),$$

and

$$V_0 = m[\sigma(-c + \beta V_1) + (1-\sigma)\beta V_0] + (1-m)\beta V_0,$$

where $\sigma \in [0,1]$ is the probability that the agent accepts money. For example, if an agent has money he goes to the sector that trades the good he likes. In this sector, there is a probability m that he meets another agent with money and no trade happens. There is also a probability $(1-m)$ that he meets an agent without money. In this case they trade, the agent obtains utility u, and moves to the next period without money. A similar reasoning holds for an agent without money.

Assume that $\sigma = 1$. This implies that

$$V_1 - V_0 = (1-m)u + mc.$$

It is indeed optimal to always accept money as long as $-c + \beta V_1 \geq \beta V_0$, i.e.,

$$\beta[(1-m)u + mc] \geq c. \tag{3}$$

In summary, as long as (3) holds, the economy exhibits multiple equilibria.[3]

3.2 General case

We now consider the case where $V(\Delta z) > 0$ and $z \in \mathbb{R}$. The economy starts at $z = 0$. Suppose the economy in period s is in state z^* and denote by $\varphi(t)$ the probability that any state $z \geq z^*$ will be reached at time $t + s$, and not before. We are ready to present our first result.

Proposition 1 *Fix any $\widehat{z} > 0$. For all states $z \in (-\widehat{z}, \widehat{z})$, there is a unique equilibrium. Money is always accepted if*

$$\left(\sum_{t=1}^{\infty} \beta^t \varphi(t)\right)[(1-m)u + mc] > c, \tag{4}$$

and is never accepted if the inequality is reversed.

Proof. First, we prove that money is accepted if (4) holds. Fix $\widehat{z} > 0$. Note that for some $z' > \widehat{z}$, an agent will find it optimal to always produce in exchange for money. Indeed, as $z \to \infty$,

[3] Kiyotaki and Wright (1993) prove that there exists an equilibrium where agents accept money with probability between zero and one. A similar equilibrium also exists here.

the expected payoff of producing in exchange for money is at least $-c+\frac{\beta}{1-\beta}\gamma$, which is positive given the assumption in (1). Moreover, the assumption $(1-m)u>\gamma$ implies that, for $z>z'$, an agent believes that all other agents will always produce in exchange for money. As a result, when $z>z'$, the agent always uses money as a medium of exchange in the trading post that trades the good he likes.

The proof is done by induction, where at each step strictly dominated strategies are eliminated. First, fix $z^*\in(-\widehat{z},\widehat{z})$ and assume that all agents accept money if and only if $z\geq z^*$. We need to check an agent's incentives to accept money in some state $z=z^*-\epsilon$ for some $\epsilon>0$. If an agent accepts money in exchange for his good in state $z^*-\epsilon$, he obtains

$$-c+\sum_{t=1}^{\infty}\beta^t\varphi^{\epsilon}(t)\left(\int_0^{\infty}f(z^*+s|t_{\phi}=t)V_{1,z^*+s}ds\right)\equiv V^a_{z^*-\epsilon},$$

where $\varphi^{\epsilon}(t)$ is the probability that a state $z\geq z^*$ will be reached at time t, and not before, when z departs from $z^*-\epsilon$; t_{ϕ} denotes the period a state larger than or equal to z^* is reached; and $f(z|t_{\phi}=t)$ denotes the probability density function that the state z is reached conditional on t_{ϕ} equal to t. Since no agent is accepting money when $z<z^*$, the money received by the agent will not be useful (or harmful) until a state $z\geq z^*$ is reached. When such a state is reached, the agent's value function is $V_{1,z}$. The term in brackets is the average of such value functions, weighted by their densities. The expected payoff of an agent that accepts money equals the discounted value of such averages, weighted by their own probabilities, minus c. In turn, if an agent does not accept money in state $z^*-\epsilon$, he obtains

$$\sum_{t=1}^{\infty}\beta^t\varphi^{\epsilon}(t)\left(\int_0^{\infty}f(z^*+s|t_{\phi}=t)V_{0,z^*+s}ds\right)\equiv V^n_{z^*-\epsilon}.$$

This implies that the agent accepts money in state $z^*-\epsilon$ as long as

$$V^a_{z^*-\epsilon}-V^n_{z^*-\epsilon}=-c+\sum_{t=1}^{\infty}\beta^t\varphi^{\epsilon}(t)\left(\int_0^{\infty}f(z^*+s|t_{\phi}=t)\left[V_{1,z^*+s}-V_{0,z^*+s}\right]ds\right)>0. \tag{5}$$

Now, since all other agents are accepting money in any state $z\geq z^*$, the value function of an agent with money in some state $z\geq z^*$ is

$$V_{1,z}=m\beta E_zV_1+(1-m)\left(u+\beta E_zV_0\right),$$

while the value function of an agent without money in some state $z\geq z^*$ is

$$V_{0,z}=m\left(-c+\beta E_zV_1\right)+(1-m)\beta E_zV_0,$$

where $\beta E_z V_1$ and $\beta E_z V_0$ are the expected value of holding, respectively, one and zero unit of money at the end of the period, when the current state is z. Now, note that

$$V_{1,z} - V_{0,z} = (1-m)u + mc. \tag{6}$$

Substituting (6) into (5) yields

$$V^a_{z^*-\epsilon} - V^n_{z^*-\epsilon} = -c + \sum_{t=1}^{\infty} \beta^t \varphi^\epsilon(t) \left[(1-m)u + mc\right],$$

which used the fact that $\int_0^\infty f(z^* + s | t_\phi = t) ds = 1$ and $(1-m)u + mc$ is a constant. Therefore, $V^a_{z^*-\epsilon} - V^n_{z^*-\epsilon} > 0$ if

$$\sum_{t=1}^{\infty} \beta^t \varphi^\epsilon(t) \left[(1-m)u + mc\right] > c.$$

This argument holds for any $\epsilon > 0$. As $\epsilon \to 0$, $\varphi^\epsilon(t) \to \varphi(t)$, and we obtain the expression in (4).

The induction argument requires $V^a_{z^*-\epsilon} - V^n_{z^*-\epsilon} > 0$ for all $z^* \in (-\widehat{z}, z')$. The above reasoning only considered $z^* \in (-\widehat{z}, \widehat{z})$. However, if $z^* > \widehat{z}$, incentives for accepting money are at least as high as in the case $z^* \in (-\widehat{z}, \widehat{z})$, owing to the positive intrinsic value of holding money. Hence the condition in (4) is sufficient for all $z^* \in (-\widehat{z}, z')$. Moreover, the argument has assumed that agents will not accept money in states smaller than z^*, but if that were not the case, incentives for holding money would only increase, owing to the strategic complementarities in using money. Hence, if condition (4) holds, accepting money in state $z^* - \epsilon$ is a strictly dominant strategy given that all agents are accepting money in states larger than or equal to z^*. Thus, as (i) it is a strictly dominant strategy to accept money if $z \geq z'$, and (ii) for all $z^* > -\widehat{z}$, if all agents accept money whenever $z \geq z^*$, accepting money at $z = z^* - \epsilon$ is a strictly dominant strategy, accepting money is the only strategy that survives iterative elimination of strictly dominated strategies for all $z \in (-\widehat{z}, \widehat{z})$.

It remains to show that money is not accepted if the inequality in (4) is reversed. The argument is analogous to the one above. For some $z'' < -\widehat{z}$, an agent will find it optimal to never bring money into the trading post that trade the good he likes. Indeed, as $z \to -\infty$, if an agent brings money into the trading post his expected payoff is smaller than $-\xi + u$, which is negative given the assumption in (2). Since the use of money exhibits strategic complementarities, it is a dominant strategy for all agents not to bring money into trading posts for $z < z''$.

Again, we proceed by induction. Fix $z^* \in (-\widehat{z}, \widehat{z})$ and suppose that all agents accept money if and only if $z \geq z^*$. We need to compare payoffs from accepting and not accepting money at state

$z^* + \epsilon < \widehat{z}$, for some $\epsilon > 0$. An agent that accepts money in exchange for his good in state $z^* + \epsilon$ obtains

$$-c + \sum_{t=1}^{\infty} \beta^t \varphi_\epsilon(t) \left(\int_0^\infty f(z^* + s | t_\phi = t) V_{1,z^*+s} ds \right) \equiv V^a_{z^*+\epsilon},$$

where $\varphi_\epsilon(t)$ is the probability that a state $z \geq z^*$ will be reached at time t, and not before, when z departs from $z^* + \epsilon$; t_ϕ denotes the period a state larger than or equal to z^* is reached and $f(z|t_\phi = t)$ denotes the probability density function that the state z is reached conditional on t_ϕ equal to t. If an agent does not accept money in state $z^* + \epsilon$, he obtains

$$\sum_{t=1}^{\infty} \beta^t \varphi_\epsilon(t) \left(\int_0^\infty f(z^* + s | t_\phi = t) V_{0,z^*+s} ds \right) \equiv V^n_{z^*+\epsilon}.$$

This implies that the agent does not accept money in state z^* as long as

$$V^n_{z^*+\epsilon} - V^a_{z^*+\epsilon} = \sum_{t=1}^{\infty} \beta^t \varphi_\epsilon(t) \left(\int_0^\infty f(z^* + s | t_\phi = t) \left[V_{0,z^*+\epsilon} - V_{1,z^*+\epsilon} \right] ds \right) + c > 0.$$

Following the same steps as above, we obtain that not accepting money is optimal if

$$\sum_{t=1}^{\infty} \beta^t \varphi_\epsilon(t) \left[(1-m)u + mc \right] < c.$$

Taking the limit $\epsilon \to 0$, $\varphi_\epsilon(t)$ converges to $\varphi(t)$. We have considered $z^* \in (-\widehat{z}, \widehat{z})$ but for $z < z^*$, incentives to accept money could only be smaller, so the above condition is sufficient for the argument. Following the same reasoning as above, we get the claim. ■

In the benchmark model, if an agent believes that everyone will always accept money, he finds it optimal to accept money if the cost of producing the good (c) is smaller than the benefit ($(1-m)u + mc$) discounted by the discount rate β (condition (3)). The condition for the monetary equilibrium being the unique equilibrium substitutes the discount factor β by a weighted average of β^t for all t. The weights come from the following exercise: assuming that at time 0 the economy is in state $z^* \in (-\widehat{z}, \widehat{z})$, and everyone accepts money only if $z \geq z^*$, the weight of β^t is the probability of reaching a state where money is accepted at time t (and not before). If condition (4) holds, there cannot be a threshold z^* such that money is accepted only if $z \geq z^*$ because agents would find it optimal to accept money in a state $z^* - \epsilon > -\widehat{z}$ for some $\epsilon > 0$. Conversely, if condition (4) is reversed, there cannot be a threshold $z^* \in (-\widehat{z}, \widehat{z})$ such that money is accepted if $z \geq z^*$, because agents would find it optimal not to accept money in a state $z^* + \epsilon < \widehat{z}$ for some $\epsilon > 0$.

The result in Proposition 1 holds for any value of $\widehat{z}$, no matter how large it is. Indeed, the role of the regions $z \geq \widehat{z}$ and $z \leq \widehat{z}$ is simply to rule beliefs that money will never be accepted in any state, and money will always be accepted in any state. This allows us to rule out either autarky or money in every state $z \in (-\widehat{z}, \widehat{z})$ by iterative deletion of strictly dominated strategies. There is no mention of the (positive or negative) intrinsic value of money in condition (4).[4]

Remark 1 *Proposition 1 holds even if the probability that $z \geq \widehat{z}$ is ever reached is arbitrarily small.*

The probability that $-\widehat{z}$ and $\widehat{z}$ will ever be reached depends on the stochastic process of Δz. If the expected value of Δz is zero (and its variance is positive), both $-\widehat{z}$ and $\widehat{z}$ will eventually be reached with probability one. If $E(\Delta z)$ is positive, $\widehat{z}$ will eventually be reached with probability 1 regardless of how far $z = 0$ is from $\widehat{z}$, but the probability that $-\widehat{z}$ will ever be reached depends on the distance between the initial state $z = 0$ and $-\widehat{z}$. Likewise, if $E(\Delta z)$ is negative, the probability that $\widehat{z}$ will ever be reached depends on how far $z = 0$ is from $\widehat{z}$. If this distance is large enough, the probability that $\widehat{z}$ is ever reached can be arbitrarily small. Such long term probabilities are not important in the computation for the condition in (4). All that matters is the set of probabilities $\varphi(t)$ of reaching a nearby state in the following periods, while the discount rate is still not too low. Hence, two very similar stochastic processes, one with $E(\Delta z) = 0$ and another with a slightly negative $E(\Delta z)$ will yield very similar conditions for equilibria, although the difference between the probabilities of ever reaching $\widehat{z}$ can be arbitrarily close to 1.[5]

3.3 General Case: discrete state space

The analysis up to now has considered a continuous state space, but the results are easily extended to a discrete state space. Consider $V(\Delta z) > 0$ as before but now $z \in \mathbb{Z}$. Suppose the economy in period s is in state z^*. Denote by $\phi(t)$ the probability of reaching any state strictly larger than z^* at time $s+t$ and not before; and by $\phi_+(t)$ the probability of reaching any state larger or equal than z^* at time $s+t$ and not before. An argument similar to Proposition 1 yields the following result.

Proposition 2 *For all states $z \in (-\widehat{z}, \widehat{z})$,*

[4] Thus the role of the assumption on the value of money here is different from the models in Zhou (2003), Wallace and Zhu (2004) and Zhu (2003, 2005). In those models, if goods are perfectly divisible and the marginal utility at zero consumption is very large, then a small intrinsic value of money is enough to generate a monetary equilibrium. A small intrinsic value of money ensures agents will produce at least some amount of the good, and the fact that money can then be further exchanged for goods increases the amount agents are willing to produce.

[5] Under the usual assumption of common knowledge of rationality, the distance between the current state z and $\widehat{z}$ can be disregarded from the analysis. That distance could have some effect on the conditions if boundedly rational agents were not able to think too far ahead, for example.

1. If

$$\left(\sum_{t=1}^{\infty}\beta^{t}\phi\left(t\right)\right)\left[\left(1-m\right)u+mc\right]>c, \tag{7}$$

then there exists a unique equilibrium. In this equilibrium, money is always accepted.

2. If

$$\left(\sum_{t=1}^{\infty}\beta^{t}\phi_{+}\left(t\right)\right)\left[\left(1-m\right)u+mc\right]<c, \tag{8}$$

then there exists a unique equilibrium in which money is never accepted.

Proof. See Appendix. ■

Equations (7) or (8) are versions of (4) with $\phi(t)$ or $\phi_{+}(t)$ instead of $\varphi(t)$. In the continuous case we consider the probabilities of reaching z^{*} when the economy starts in a state that is arbitrarily close to z^{*}. Here, we have to start from the closest state where money is not accepted or the closest state where money is accepted, depending on which strategies we want to eliminate. As there is some distance between them, the probabilities $\phi(t)$ and $\phi_{+}(t)$ will not be the same. Hence there will be a region with multiple equilibria. But as the support of Δz increases, the discrete distribution gets closer to a continuous distribution, and $\phi(t)$ and $\phi_{+}(t)$ get closer and closer to each other.

3.4 Convenient parametrization

In order to make easy the comparison between the condition for existence of the monetary equilibrium in the benchmark model (3) and the conditions for uniqueness of the monetary equilibrium in this model, it is worth rewriting the condition in (4) as

$$\lambda\beta\left[\left(1-m\right)u+mc\right]>c$$

so that the only difference between the condition for existence of a monetary equilibrium in (3) and the condition in (4) is the factor λ, given by

$$\lambda=\sum_{t=1}^{\infty}\beta^{t-1}\varphi\left(t\right). \tag{9}$$

The factor λ is a number between 0 and 1. If $\lambda=0$, condition (7) is never satisfied and autarky is always the unique equilibrium. The larger the value of λ, the larger the region where money is the unique equilibrium. If $\lambda=1$, money is the unique equilibrium whenever it is an equilibrium in the benchmark case. Thus λ provides us with a convenient way to describe the changes on the set of equilibria. The key question is whether λ is closer to 0 or to 1.

Analogouly, we can rewrite condition (7) as

$$\lambda_M \beta \left[(1-m)\, u + mc \right] > c,$$

and condition (8) as

$$\lambda_A \beta \left[(1-m)\, u + mc \right] < c,$$

where

$$\lambda_M = \sum_{t=1}^{\infty} \beta^{t-1} \phi(t) \tag{10}$$

$$\lambda_A = \sum_{t=1}^{\infty} \beta^{t-1} \phi_+(t) \tag{11}$$

The benchmark case corresponds to $\lambda_M = 0$, which means autarky is always an equilibrium, and $\lambda_A = 1$, which means money is an equilibrium as long as it is an equilibrium in the benchmark model. As λ_M increases and λ_A decreases, the multiple equilibrium shrinks, so one question is whether they are close to each other (implying a small multiple-equilibrium region). Besides, we want to know whether they are closer to 0 (autarky wins) or to 1 (money wins).

Equations (9), (10) and (11) show that λ_M, λ_A and λ are increasing in β. As agents become more patient, the region where money is the unique equilibrium increases and the region where autarky is the unique equilibrium decreases. For more patient agents, the cost of waiting until money can be spent is smaller, so accepting money is less risky. Knowing everyone will think like that, an agent will be more willing to accept money. In sum, patience helps agents to coordinate in the money equilibrium.

3.5 There will be money

Now let's consider the limit $\beta \to 1$. One important property of the model is that as agents get more patient, money is the unique equilibrium in the whole region where money is an equilibrium in the benchmark model.

Proposition 3 *If $E(\Delta z) = 0$, as $\beta \to 1$, the conditions for a unique equilibrium in (4) and (7) converge to that in (3).*

Proof. *If* $E(\Delta z) = 0$*,* $\sum_{t=1}^{\infty} \phi(t) = 1$*,* $\sum_{t=1}^{\infty} \phi_+(t) = 1$ *and* $\sum_{t=1}^{\infty} \varphi(t) = 1$ *(since there is no drift, the threshold* z^* *will eventually be crossed). As* $\beta \to 1$*, the values of* λ_M*,* λ_A *and* λ *in equations (9), (10) and (11) converge to 1.* ■

A very patient agent close to a threshold that determines whether money is accepted will always accept money if there are gains from trade, because he does not mind waiting to spend his money. Knowing that, other agents will accept money, expecting that the patient guys ahead, facing the same decisions, will also accept.

It is possible to construct an example where the probability of ever getting to the region where holding money is a dominant strategy might be made arbitrarily small, and still as $\beta \to 1$, the above result holds. Consider $E(\Delta z) = \eta$ for some $\eta < 0$. As $\eta \to 0_-$, $\lambda \to 1$ (result of the proposition plus continuity). But for any $\eta < 0$ there exists a large enough $\widehat{z}$ so that the probability of ever reaching the region where money has positive intrinsic value is arbitrarily small. Note that in this case money is the unique equilibrium as long as $u > c$; the probability of ever reaching $\widehat{z}$ is arbitrarily small; and the probability of ever reaching $-\widehat{z}$ is one.

The above example illustrates the fact that the likelihood of reaching the region where accepting money is a dominant strategy has no effect on the results. The dominant region excludes the belief that money will never ever be accepted, but the equilibrium condition depends on the behavior of an agent close to a hypothetical threshold that determines whether money is accepted and is far enough from the dominant regions. The bottom line is that as meeting between agents becomes more frequent and the discount rate gets closer to 1, even small gains from trade imply that there will be money in equilibrium.

4 Examples

4.1 Binary case

Consider a simple stochastic process where

$$\Pr(\Delta z = 1) = p \quad \text{and} \quad \Pr(\Delta z = -1) = 1 - p.$$

The process is illustrated in figure 1. Departing from state $z^* - 1$ in period s, the probability of reaching state z^* in period $s+1$ is p. Otherwise, the economy moves to state $z^* - 2$. Then, state z^* can only be reached in period $s + 3$. The stochastic process until state z^* is reached is illustrated

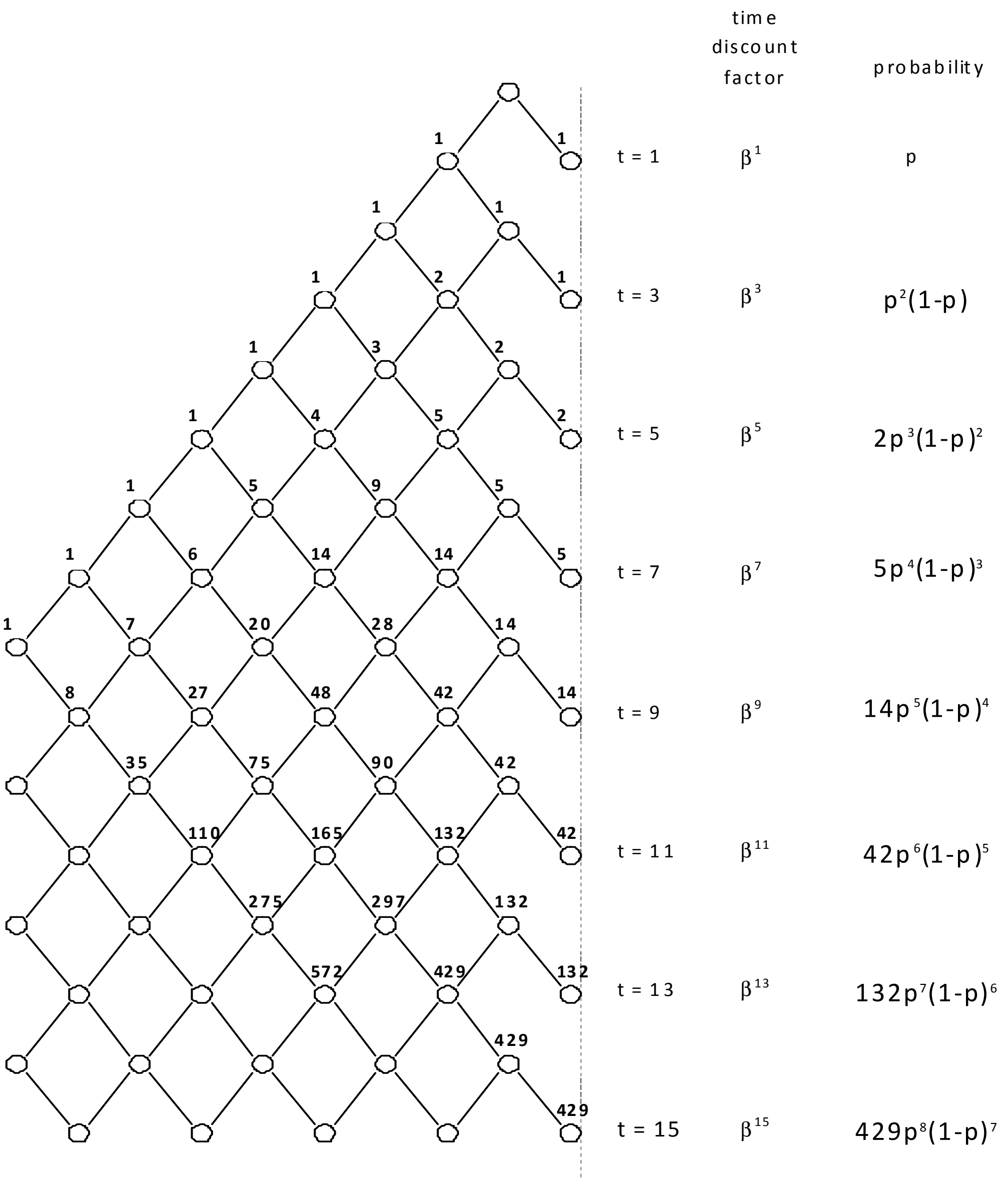

Figure 1: Binary process

in Figure 1. The probabilities that state z^* will be reached for the first time at time $s+t$ are given by (for all $i \geq 0$)

$$\begin{aligned}\phi(2i+1) &= \frac{(2i)!}{i!\,(i+1)!}p^{i+1}(1-p)^i, \\ \phi(2i) &= 0.\end{aligned}$$

Remember that $\phi(t)$ is the probability of reaching for the first time state z^* at time $s+t$, when the initial state is z^*-1. The formula for $\phi(2i+1)$ resembles a binomial distribution, but the usual combination is replaced with the Catalan numbers.[6] The value of λ_M is given by

$$\lambda_M = \sum_{i=0}^{\infty} \beta^{(2i)} \left(\frac{(2n)!}{n!\,(n+1)!}p^{i+1}(1-p)^i \right). \tag{12}$$

Departing from state z^* in period s, the probability of reaching a state larger than z^* in period $s+1$ is p. Otherwise, the economy moves to state z^*-1, which happens with probability $1-p$. At z^*-1, we are at the previous case. Thus (for all $i \geq 0$)

$$\begin{aligned}\phi_+(1) &= p, \\ \phi_+(2i+2) &= (1-p)\frac{(2i)!}{i!\,(i+1)!}p^{i+1}(1-p)^i, \\ \phi_+(2i+3) &= 0.\end{aligned}$$

Hence

$$\begin{aligned}\lambda_A &= p + (1-p)\sum_{i=0}^{\infty} \beta^{(2i+1)} \left(\frac{(2n)!}{n!\,(n+1)!}p^{i+1}(1-p)^i \right) \\ \lambda_A &= p + (1-p)\beta\lambda_M \end{aligned} \tag{13}$$

4.1.1 The case $p = 0.5$

If $p = 0.5$, λ_M becomes:

$$\lambda_M = \sum_{i=0}^{\infty} \beta^{(2i)} \left(\frac{(2i)!}{i!\,(i+1)!} \left(\frac{1}{2}\right)^{2i+1} \right) \tag{14}$$

which is a function of β only, and λ_A is then

$$\lambda_A = \frac{1+\beta\lambda^M}{2}$$

[6] See, e.g., http://mathworld.wolfram.com/CatalanNumber.html.

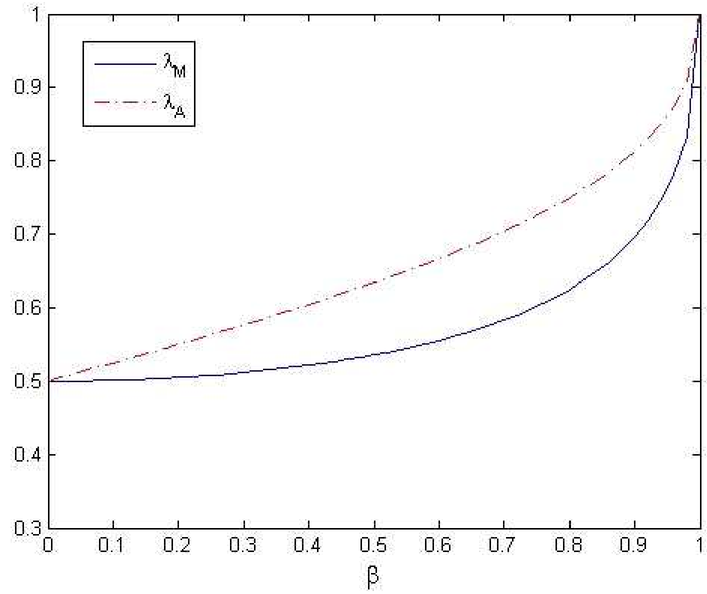

Figure 2: Binary case: λ_M and λ_A

Figure 2 shows λ_A and λ_M as a function of β. The factors λ_A and λ_M depend on the discounted sum of probabilities that state z^* will be reached when departing from z^* and $z^* - 1$, respectively. As β approaches 0, both λ_A and λ_M approach 0.5, which is the probability that z^* will be reached in the next period. The probabilities that z^* will be reached after 2 or more periods are not important if β is small. For very small values of β, the multiple equilibrium region is small, and the factor λ_M is around 0.5, so the gains from trade have to be twice as large as in the model with no dominant regions for money to become an equilibrium – and if it is an equilibrium, it is unique. For intermediate values of β, the multiple-equilibrium region is larger and the factors λ_A and λ_M are also larger.

As discussed before, both λ_M and λ_A are increasing in β and converge to 1 as β approaches 1 – the discounted sum of probabilities that state z^* will be reached approaches the sheer probability that the economy will at some point be at z^*, and if $p = 0.5$, that probability is 1. As β approaches 1, the region where the only equilibrium is the monetary one converges to the region where the monetary equilibrium exists.

In Kiyotaki and Wright (1993), a larger β implies gains from trade are also larger because the benefits of selling a good are only enjoyed in the future. Here, β also affects the set of equilibria

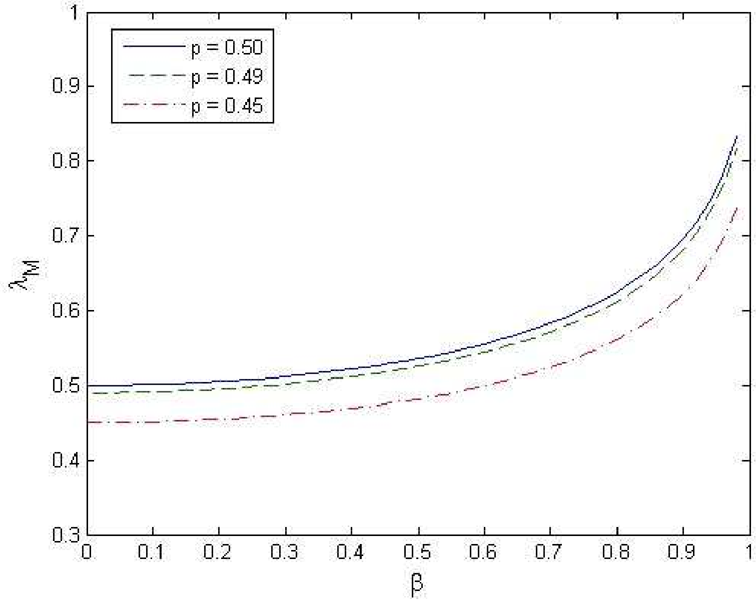

Figure 3: Case $p < 0.5$

through the *coordination channel*: in a situation where money is only accepted in some states, an agent might have to wait much longer to exchange his unit of money for something he values. If β is large, that is not much of a problem. Since all agents know they all think this way, a larger β helps agents to coordinate in the money equilibrium.

4.1.2 The case $p < 0.5$

Assume now that $p = 0.5 - \varepsilon$. For a sufficiently small ε, a value of λ_M very similar to the one implied by (14) would be obtained, but the probability that $\widehat{z}$ would ever be reached could be made arbitrarily close to 0 for some $\widehat{z}$. For lower values of p, the factor λ_M is given by equation (12). Figure 3 shows the relation between β and λ_M for different values of p. As before, as β approaches 0, λ approaches p. However, as β approaches 1, λ_M does not approach 1 since there is a positive probability, bounded away from zero, that z^* will never be reached by a process departing from $z^* - 1$. But the results are similar, the factor λ_M is increasing in β, since late arrivals at z^* are worth more for larger values of β, and is not far from 1 for high values of β.

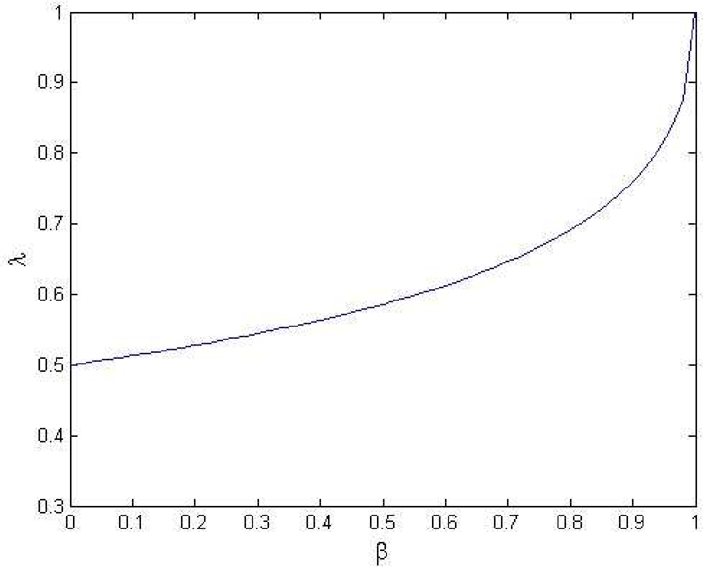

Figure 4: Normal case

4.2 Normal case

Figure 4 shows λ for a normal process assuming $E(\Delta z) = 0$. The probabilities $\varphi(t)$ for the normal case are obtained from Monte Carlo simulations (the do not depend on the variance $V(z)$). It turns out that the factor λ is between the lines for λ_A and λ_M as one would expect.

The conditions for existence of a monetary equilibrium in the benchmark model (3) and the conditions for existence and uniqueness of a monetary equilibrium in our model (4) depend on β, m, u and c. Normalizing $c = 1$ and assuming $m = 1/2$, which maximizes the amount of exchanges, the possible equilibria are drawn in figure 5. The solid curve depicts the condition for existence of a monetary equilibrium in the benchmark model (a version of Kiyotaki and Wright (1993)): autarky is the unique equilibrium in the region below the solid curve, and there are multiple equilibria above the solid curve. The dotted curve shows the equilibrium condition in our model. Autarky is the unique equilibrium below the dotted line and money is the unique equilibrium above the dotted line. The distance between both lines decreases with β and vanishes if agents meet often enough (β is close to 1).

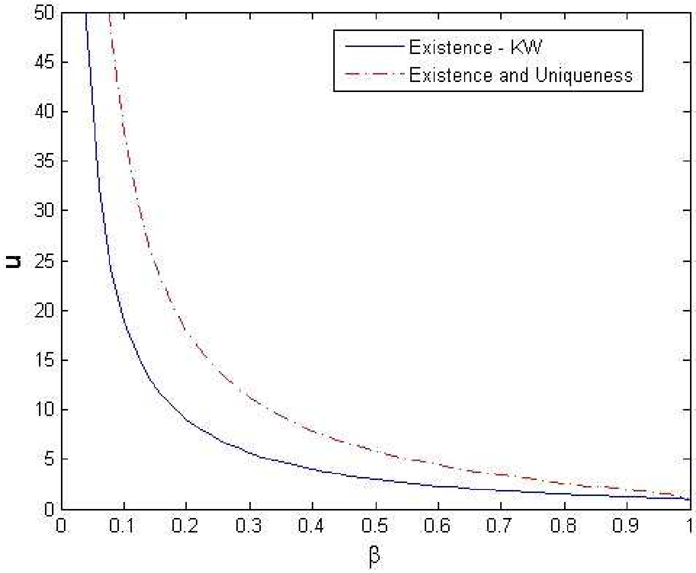

Figure 5: Equilibrium conditions in (3) and (4)

5 Conclusion

We have introduced a small modification in the model of Kiyotaki and Wright (1993) with the intent of ruling out beliefs that (i) money will always be used in any state of nature, and (ii) money will never be employed in any state. A unique equilibrium arises in the model. If agents are sufficiently patient, in the presence of any gains from trade, autarky is tenuous in the sense that it critically relies on agents being sure that money will never be accepted in any state of nature, no matter how unlikely the state is. In contrast, money is an equilibrium even if agents believe that there are states where money will not be accepted.

We have conveyed our message in a search model of money along the lines of Kiyotaki and Wright (1993), but we believe that our results might be more general than that and arise in other settings that meet two requirements. First, there must exist some states of the world where accepting money is a dominant strategy and some other states where accepting money is a dominated strategy. These states might be as unlikely as we want, and their unique role is to rule out extreme beliefs about the value of money. Second, money has to be a link between the present and the future, that is, the value of money must come from its future use as a medium of exchange. This last requirement is satisfied by other fundamental models of money such as turnpike models, and by variants of search models (such as Trejos and Wright (1995) and Lagos and Wright (2005)). It is also satisfied by overlapping generations models. However, while in turnpike and search models money earned today can be spent at *any* time in the future, in overlapping generations models an agent has *fewer* opportunities to spend his money. In particular, in a two-period overlapping generations model, a young agent is willing to produce in exchange for money only if he believes that he will be able to spend his money with a high probability when old. This difference should not matter for our uniqueness result. However, it should matter for our result on the equivalence between the condition for uniqueness of the monetary equilibrium and the condition for existence of the monetary equilibrium. Intuitively, money becomes more risky and thus autarky becomes more likely if there are fewer opportunities for money to be spent.

Finally, since the focus of our analysis was on the selection between autarky and money, we have considered an economy with only one variety of money (say, seashells). In reality, many varieties may be available at any point in time (e.g., seashells, stones, salt, gold). We believe that the selection mechanism proposed in this paper can also be extended to such environments. However, such extension is beyond the objectives of the present paper and is left for future work.

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A Appendix - Proof of Proposition 2

Proof. The proof is very similar to the proof of Proposition 1. First, we prove the first statement. Again, for some $z' > \widehat{z}$, an agent will find it optimal to accept money even if everyone else does not and since $(1-m)u > \gamma$, the agent always uses money as a medium of exchange in the trading post that trade the good he likes as opposed to keeping the money in order to enjoy its intrinsic benefit.

The proof is done by induction, but now, starting from a threshold $z^* \in (-\widehat{z}, \widehat{z})$, it is shown that an agent finds it optimal to accept money in state $z^* - 1 > -\widehat{z}$. Suppose that all agents accept money if and only if $z \geq z^*$. We need to compare the payoff of such an agent with the one received by someone who accepts money if $z \geq z^* - 1$. An agent that accepts money in exchange for his good in state $z^* - 1$ obtains

$$-c + \sum_{t=1}^{\infty} \beta^t \phi(t) \left(\sum_{i=0}^{\infty} \pi(z^* + i | t_\phi = t) V_{1,z^*+i} \right) \equiv V^a_{z^*-1},$$

and

$$\sum_{t=1}^{\infty} \beta^t \phi(t) \left(\sum_{i=0}^{\infty} \pi(z^* + i | t_\phi = t) V_{0,z^*+i} \right) \equiv V^n_{z^*-1}.$$

and following the reasoning in the proof of Proposition, 1, we get the first statement.

The proof of the second statement is also analogous: for some $z'' < -\widehat{z}$, it is a dominant strategy for all agents not to accept money for all $z < z''$. Now, suppose that all agents accept money if and only if $z \geq z^*$, where $z^* < \widehat{z}$. We need to compare the payoff of accepting and not accepting money at state z^*. An agent that accepts money in exchange for his good in state z^* obtains

$$-c + \sum_{t=1}^{\infty} \beta^t \phi_+(t) \left(\sum_{i=0}^{\infty} \pi(z^* + i | t_\phi = t) V_{1,z^*+i} \right) \equiv V^a_{z^*},$$

If an agent does not accept money in state z^*, he obtains

$$\sum_{t=1}^{\infty} \beta^t \phi_+(t) \left(\sum_{i=0}^{\infty} \pi(z^* + i | t_\phi = t) V_{0,z^*+i} \right) \equiv V^n_{z^*}.$$

and following the reasoning in the proof of Proposition, 1, we get the proof of the second statement.

■