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ABSTRACT

Rural-Urban Migration and Unemployment: Theory and Policy Implications*

We develop a regional model where, in the city, unemployment prevails because of too high (efficiency) wages, while, in the rural area, workers are paid at their marginal productivity. We characterize the steady-state equilibrium and show that it is unique. We then consider two policies: decreasing urban unemployment benefits and subsidizing urban employment. We find that decreasing the unemployment benefit in the city creates urban jobs and reduces rural-urban migration since new migrants have to spend some time unemployed before they can find a job in the city. On the other hand, raising employment subsidies increases urban employment but may also increase urban unemployment because it triggers more rural-urban migration. In this respect, the employment subsidy policy can backfire by raising rather than reducing urban unemployment.

JEL Classification: D83, J41, J64 and O15

Keywords: efficiency wages, policy, rural-urban migration and todaro paradox

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1. Introduction

The study of migration, in general, and rural-urban migration, in particular, has for long been an important area of research in development economics. The Harris-Todaro framework (Todaro, 1969; Harris and Todaro, 1970) has become a cornerstone of models of rural-urban migration. The aim of the Harris-Todaro framework is to explain the persistent rural-urban migration in developing countries despite the high unemployment rates in cities. The main idea is quite simple since it says that migration will occur as long as the urban expected income (i.e. income times the probability of finding an urban job) is higher than the rural one. The original model has been extended in different directions (see the literature surveys by Basu, 1997, Part III; Ray, 1998, Chap. 10) to incorporate different aspects of labor markets and migration in less developed countries.¹

An important issue in this literature is the study of the policy implications of these models. In particular, one of the conclusions of the Harris-Todaro model is that creating urban jobs is an insufficient solution to the urban unemployment problem because of the induced negative effect on rural migration, which may outweigh the positive effect of creating jobs (Todaro, 1976). This is referred to as the Todaro paradox. Because rural risk-neutral agents consider expected wages when deciding to migrate to the city, inter-labor market (rural-urban) equilibrium mandates urban unemployment. This unemployment ensures that the expected urban wage is equal to the rural wage. Contrary to what one would expect, the migration response to several policies aimed at reducing urban unemployment is to raise rather than to reduce urban unemployment.

There is a long line of papers, including Zarembka (1970), Blomqvist (1978), Arellano (1981), Takagi (1981), Nakagome (1989), Brueckner (1990), Stark et al. (1991), Raimondos (1993), Brueckner and Zenou (1999), Brueckner and Kim (2001), that have investigated further this policy issue. Most of these papers give conditions under which the Todaro paradox exists. In the recent literature, a new force has been added by explicitly introducing the land market in a Harris-Todaro model (Nakagome, 1989; Brueckner, 1990; Brueckner and Zenou, 1999; Brueckner and Kim, 2001). In that case, the urban-land-rent escalation provides an additional force that limits migration and the Todaro paradox does not in general exist.²

¹See, in particular, Ortega (2000) and Sato (2004) for the welfare implications of the Harris-Todaro model.

²Labor-supply constraints can also limit rural-urban migration (see, in particular, Par-

All these papers do not really analyze a government policy with a budget constraint but rather look at comparative statics results. On the other hand, most of the papers that deal with policy issues in a Harris-Todaro framework do not use a “general equilibrium” model where both workers’ and firms’ behaviors are modeled so that wages in both sectors, urban job creation, and urban unemployment are endogenous (see e.g. Fields, 2005). The contribution of the present paper is to study two distinct policies aiming at creating urban jobs in a “general equilibrium” framework.

To be more precise, we develop an efficiency wage model³ (see Stiglitz, 1974; 1976; Moene, 1988; Smith and Zenou, 1995; Brueckner and Zenou, 1999; Brueckner and Kim, 2001; for its utilization in the context of rural-urban migration),⁴ and characterize the steady-state rural-urban migration equilibrium. We show that this equilibrium is unique. We then investigate two different policies, a supply-side and a demand-side policy. In the first one, the government reduces the level of unemployment benefits, which is paid to the unemployed workers in the city. In the context of a third world country, it may be interpreted as institutional support in the urban sector. A country like China for example has important social benefit policies. The state-sponsored social insurance system, which accounts for the bulk of public social spending, is now being gradually improved and extended to cover all urban workers (see e.g. Knight and Song, 2005).⁵ The second policy consists in subsidizing urban jobs to stimulate employment. Both policies are financed by a tax on firms’ profits.

Decreasing unemployment benefits has a *direct negative effect on urban wages* and thus more urban jobs are created. This is the attraction force to the city. There is also a *direct negative effect on rural-urban migration* because the expected utility of new migrants is reduced due to the fact that they need to spend some time unemployed before finding an urban job. There is also an *indirect effect on urban wages* since, when there are more jobs in the city,

tridge and Rickman, 2008).

³We use an efficiency wage model for mainly two reasons. First, it is a convenient way of endogenizing wages and unemployment while still obtaining closed-forms solutions. Second, it has strong empirical support. For example, recent research uses a natural experiment setting in which monitoring levels are exogenously varied across similar sites and substantial resources are devoted to tracking the behavior of employees. Fehr et al. (1996), Nagin et al. (2002), Fehr and Goette (2007) show that higher wages indeed sharply reduce shirking.

⁴For a literature overview, see Zenou (2009).

⁵All our analysis would be unchanged if we interpret the “unemployment” sector as the “informal” sector (like for example in Brueckner and Zenou, 1999).

urban firms can reduce their efficiency wage because unemployment acts as a worker's discipline device. Because the two repulsion forces are strong enough, the net effect is such that creating urban jobs via a reduction in unemployment benefit increases urban employment while reducing urban unemployment.

Concerning the employment subsidy policy, there is only a *positive direct effect on job creation* (i.e. labor demand) since it becomes cheaper to employ urban workers. However, this policy also triggers rural-urban migration since the expected wages (and thus the expected utility) of migrants increase. As a result, the perverse effect of this policy is that it can increase both urban employment and unemployment. When taxes on profits are introduced in the model, these effects are mitigated and the impact on urban unemployment depends on the level of taxes compared to that of the subsidy.

The rest of the paper is organized as follows. In the next section, we present the basic model while in Section 3, we characterize the steady-state equilibrium. In Section 4, we analyze the unemployment benefit policy while, in Section 5, the employment subsidy policy is studied. Finally, Section 6 concludes. All proofs can be found in the Appendix.

2. The model

There are two regions: Rural and urban. It is assumed that the rural wage is flexible enough to guarantee that there is no rural unemployment; this wage is denoted by w_L^R . There is a continuum of ex ante identical workers whose mass is N . Among the N workers, N^C and N^R live respectively in cities and rural areas, i.e. $N = N^C + N^R$, and

$$N^C = L^C + U^C$$

$$N^R = L^R$$

where L^g and U^g are respectively the total employment and unemployment levels in region $g = C, R$ (C for cities and R for rural areas). As stated above, there is no unemployment in rural areas. Thus, by combining these two equations, we obtain:

$$U^C = N - L^C - L^R \tag{2.1}$$

The unemployment rate is then given by:

$$u^C = \frac{U^C}{U^C + L^C} = \frac{N - L^C - L^R}{N - L^R} \tag{2.2}$$

We assume a small open economy. There are two different goods, one produced in the urban area and the other in the rural area. All goods are traded at given world prices. Since these two goods are traded goods and the country is small enough, there is no loss of generality if we normalize all international prizes to be equal to unity. In area g , y^g units of output are produced and L^g workers are employed. This is a short-run model where capital is fixed and the production function in region $g = C, R$ is given thus by

$$y^g = F^g(L^g) \quad , \quad F'^g(L^g) > 0 \text{ and } F''^g(L^g) \leq 0 \quad (2.3)$$

We also assume that the Inada conditions hold, that is $\lim_{L^g \rightarrow 0} F'^g(L^g) = +\infty$ and $\lim_{L^g \rightarrow +\infty} F'^g(L^g) = 0$.

We assume that, in the city, firms use an efficiency wage policy while it is not the case in the rural area. This is because urban firms are larger than rural firms so that the problem of monitoring and shirking are more acute in cities (see, e.g. Lanjouw and Lanjouw, 2001). To be more precise, we use the standard efficiency wage model, as proposed by Shapiro and Stiglitz (1984). Each individual supplies one unit of labor. As in the standard efficiency wage model, there are only two possible levels of effort: either the worker shirks, exerting zero effort, $e = 0$, and contributing zero to production, or he/she does not shirk, providing full effort.

3. Steady-state equilibrium

The model is dynamic and we assume that, if rural workers want to get an urban job, they have first to move to the city, be unemployed and gather information about jobs, and then can eventually obtain an urban job. In the urban labor market, *firms cannot perfectly monitor workers* so that there is a probability of being detected shirking, denoted by m . If a worker is caught shirking, he/she is automatically fired. Time is continuous and workers live forever. We assume that changes in employment status are governed by a Poisson process in which a is the (endogenous) job acquisition rate and δ the (exogenous) destruction rate. Let us denote by r the common discount rate of all workers. Then, the standard *steady-state* Bellman equations for the non-shirkers, the shirkers and the unemployed are given by:

$$r I_L^{NS} = w_L^C - e - \delta (I_L^{NS} - I_U) \quad (3.1)$$

$$r I_L^S = w_L^C - (\delta + m) (I_L^S - I_U) \quad (3.2)$$

$$r I_U = w_U^C + a^C(I_L - I_U) \quad (3.3)$$

where w_L^C, w_U^C are the urban wage and the unemployment benefit, respectively, e is the effort level, r the discount rate, δ and m denote respectively the job-destruction and the monitoring rates while a^C is the endogenous job-acquisition rate. Firms set the efficiency wage such that $I_L^{NS} = I_L^S = I_L$ and we obtain that $I_L - I_U = e/m$. This is the surplus of being employed and it is strictly positive. As in Shapiro and Stiglitz (1984), this a pure incentive effect to deter shirking. This surplus depends on the monitoring technology and on the effort level provided by workers.

Equation (3.1) can be written as:

$$w_L^C = e + rI_L + \delta(I_L - I_U) = e + rI_U + (\delta + r)(I_L - I_U)$$

Furthermore, using (3.3) and the fact that $I_L - I_U = e/m$, this can be written as:

$$w_L^C = w_U^C + e + \frac{e}{m}(a^C + \delta + r)$$

Finally, at the steady state, flows out of unemployment equal flows into unemployment, i.e.

$$a^C = \frac{\delta L^C}{N - L^C - L^R} \quad (3.4)$$

so that the efficiency wage is finally given by:

$$w_L^C = w_U^C + e + \frac{e}{m} \left[\frac{\delta (N - L^R)}{N - L^C - L^R} + r \right] \quad (3.5)$$

We have the standard effects of the efficiency wage (Shapiro and Stiglitz, 1984). What is new here is the fact that rural employment affects the efficiency wage. Indeed, L^R positively affects w_L^C because more employment in rural areas implies a higher urban job acquisition rate a^C (indeed higher L^R leads to a decrease in urban unemployment since there are less competition for urban jobs) and thus urban firms have to increase their wages to meet the Non-Shirking Condition (3.5). In cities, firms' profits are given by:

$$\Pi^C = F^C(L^C) - w_L^C L^C \quad (3.6)$$

Firms decide their employment level by maximizing their profit and we obtain the following labor demand:

$$w_L^C = F'^C(L^C) \quad (3.7)$$

In rural areas, we assume that jobs are mainly menial and wages are flexible and equal to marginal product, so that there is no rural unemployment. We thus have:

$$w_L^R = F'^R(L^R) \quad (3.8)$$

We assume that the Inada conditions on both production functions hold. Concerning rural-urban migration, as stated above, a rural worker cannot search from home but must first be unemployed in the city and then search for a job. Thus, the equilibrium migration condition can be written as:

$$I_U = \int_0^{+\infty} w_L^R e^{-rt} dt = \frac{w_L^R}{r} \quad (3.9)$$

The left-hand side is the intertemporal utility of moving to the city (remember that a migrant must first be unemployed) while the right-hand side corresponds to the intertemporal utility of staying in rural areas. Using (3.1)–(3.4), $I_L^{NS} = I_L^S = I_L$ and (3.8), we can write condition (3.9) as:

$$w_U^C + \frac{e}{m} \frac{\delta L^C}{N - L^C - L^R} = F'^R(L^R) \quad (3.10)$$

where L^C is determined by (3.7).

Definition 1. A Harris-Todaro equilibrium with efficiency wages is a 5-tuple $(w_L^C, L^C, w_L^R, U^C, L^R)$ such that (3.5), (3.7), (3.8), (2.1) and (3.10) are satisfied.

In this model, given that $w_U^C, e, m, \delta, N, r$ are exogenous, an equilibrium is calculated as follows. First, from (3.5), one can calculate the urban efficiency wage as a function of L^C and L^R , that is $w_U^C(L^C, L^R)$. Second, by plugging this value $w_U^C(L^C, L^R)$ in (3.7), one obtains a relationship between L^C and L^R , that we write $L_w^C(L^R)$ and is given by

$$w_U^C + e + \frac{e}{m} \left[\frac{\delta (N - L^R)}{N - L^C - L^R} + r \right] = F'^C(L^C) \quad (3.11)$$

By totally differentiating (3.11) and using the Inada conditions, we easily obtain:

$$\frac{\partial L_w^C}{\partial L^R} < 0, \quad \lim_{L^R \rightarrow 0} L_w^C = L_0^C, \quad \lim_{L_w^C \rightarrow 0} L^R = N$$

where $0 < L_w^C(L^R) < L_0^C < N$ is the unique solution of the following equation

$$w_U^C + e + \frac{e}{m} \left[\frac{\delta N}{N - L_0^C} + r \right] = F'^C(L_0^C)$$

Third, the equilibrium-migration condition (3.10) gives another relationship between L^C and L^R , that we denote by $L_h^C(L^R)$ and has the following properties:

$$\frac{\partial L_h^C}{\partial L^R} < 0, \quad \lim_{L^R \rightarrow 0} L_h^C = N, \quad \lim_{L_h^C \rightarrow 0} L^R = L_0^R = F'^{-1}(rw_U^C)$$

where $0 < L_h^C(L^R) < L_0^R < N$. Figure 1 describes the two curves (3.11) (labor demand equation) and (3.10) (migration equilibrium condition) in the plane (L^R, L^C) and it is easy to see that there exists a unique equilibrium that gives a unique value of L^C and a unique value of L^R that we denote by (L^{R*}, L^{C*}) .

Finally, plugging L^{R*} and L^{C*} in (3.5), (3.8) and (2.1) gives respectively the equilibrium values of $w_L^{C*}, w_L^{R*}, U^{C*}$.

[Insert Figure 1 here]

We now consider two policies aiming at creating urban jobs: a supply-side policy which decreases unemployment benefit and a demand-side policy, which subsidizes urban jobs.

4. Unemployment benefit policy

4.1. Decreasing unemployment benefits

Let us now study the unemployment benefit policy, which consists in reducing the unemployment benefit w_U^C . As stated above and described by Figure 1, the equilibrium is determined by two equations (3.10) and (3.11). If we differentiate (3.10), we obtain

$$L^R = L^R \left(w_U^C, e, m, \delta, N, r, L^C \right) \quad (4.1)$$

Indeed, a higher unemployment benefit, w_U^C , or effort level, e , or job-destruction rate, δ , or discount rate, r , or a lower monitoring rate, m , or total population, N , makes the city more attractive because of higher intertemporal utility of being unemployed in the city, I_U (remember that $I_L - I_U = e/m$). Thus more workers leave the rural area, which reduces L^R . When L^C increases, the urban job acquisition rate a^C increases and again more rural workers migrate to the city, thus reducing L^R .

If we now differentiate (3.11), we get:

$$L^C = L^C \left(w_U^C, e, m, \delta, N, r, L^R \right) \quad (4.2)$$

where

$$\frac{\partial L^C}{\partial L^R} = -\frac{\frac{e}{m}\delta L^C}{\frac{e}{m}\delta(N-L^R) - (N-L^C-L^R)^2 F''^C(L^C)} < 0 \quad (4.3)$$

Indeed, a higher w_U^C , or e , or δ , or r , or a lower m , or N , shifts upward the Non-Shirking Condition (3.5), so firms have to pay a higher efficiency wage to prevent shirking. This, in turn, reduces employment since, because of higher wage costs, maximizing-profit firms have to reduce the number of employed. For L^R , the effect is through the job-acquisition rate a^C . Indeed, a higher rural employment L^R increases a^C , which obliges firms to increase their urban efficiency wages, which in turn reduces urban labor demand L^C because firms maximize their profit. We obtain the following result:

Proposition 1. *In an Harris-Todaro model with urban efficiency wages, decreasing unemployment benefit leads to*

- (i) *an increase in urban employment L^C , i.e. $\partial L^{C*}/\partial w_U^C < 0$;*
- (ii) *an increase in rural employment L^R , i.e. $\partial L^{R*}/\partial w_U^C < 0$;*
- (iii) *a decrease in urban unemployment (both in level and rate) U^C and u^C , i.e. $\partial U^{C*}/\partial w_U^C > 0$ and $\partial u^{C*}/\partial w_U^C > 0$.*

Even if the proof is tedious, the intuition of this result is quite simple. Indeed, when the government decreases the unemployment benefit, this has a direct negative effect on urban wages and thus more urban jobs are created. This is the first attraction force to the city. The second one occurs because, by decreasing unemployment benefits, the government reduces the expected utility of the urban unemployed workers and thus that of the new rural migrants (who have to spend some time unemployed before obtaining an urban job). There is also a repulsion force. Indeed, since there are more jobs in the city since unemployment act as a worker's discipline device, urban firms reduce their wages because it becomes more difficult of finding a job. Because the repulsion forces are strong enough, the net effect is that creating urban jobs via a reduction in unemployment benefit increases urban employment and reduces urban unemployment. As a result, there is no Todaro paradox here.

Let us see what happens in the case of no mobility between rural and urban areas. Indeed, imagine now that migration was totally controlled and

that workers, especially rural workers could *not* migrate to cities. In that case, it is easily verified that:

$$\frac{\partial L^C}{\partial w_U^C} < 0, \quad \frac{\partial U^C}{\partial w_U^C} > 0, \quad \frac{\partial u^C}{\partial w_U^C} > 0$$

This result is not surprising since when w_U^C decreases, firms can reduce their efficiency wages and thus hire more workers. There is no effect on rural workers. However, even when rural-urban migration is authorized, we obtain the same results because the repulsion forces are sufficiently strong to thwart the attraction force of a reduction of the unemployment benefit.

4.2. Unemployment policy with a government budget constraint

We have now some intuition of the unemployment benefit policy. We would like to go further by introducing a government budget constraint. For that, we assume that the unemployment benefit w_U^C is financed by a tax t^C on urban firms. This means that when a firm hires a worker, it is taxed and thus its instantaneous profit (3.6) can now be written as:

$$\Pi^C = F^C(L^C) - (w_L^C + t^C) L^C \quad (4.4)$$

The government's budget constraint is given by:

$$t^C L^C = w_U^C (N^C - L^C) \quad (4.5)$$

The fiscal policy is such that unemployment benefits w_U^C are kept constant and the budget adjustment is realized through a decrease or increase in taxes t^C . By (2.1), $N^C - L^C = U^C = N - L^C - L^R$, and thus, using (4.5), this means that, for a constant value of w_U^C , the tax level that balances the budget is given by:

$$t^C = \frac{w_U^C (N - L^C - L^R)}{L^C} \quad (4.6)$$

One can see that, for a given unemployment benefit level w_U^C , a higher urban employment level or a higher rural employment level is associated with a decrease in taxes. Indeed, when L^C or L^R increases, then less workers are unemployed and thus a lower tax t^C is needed to balance the budget.

Let us now see how the steady-state equilibrium is affected when a government budget constraint is introduced. It is easy to verify that only the labor demand will be directly affected and, using (4.6), it will now be given by:

$$w_L^C + \frac{w_U^C (N - L^C - L^R)}{L^C} = F'^C(L^C) \quad (4.7)$$

The steady state equilibrium is a 2-tuple (L^{C*}, L^{R*}) such that the two following equations are satisfied (using (3.5)):

$$w_U^C \left(\frac{N - L^{R*}}{L^{C*}} \right) + e + \frac{e}{m} \left[\frac{\delta (N - L^{R*})}{N - L^{C*} - L^{R*}} + r \right] = F'^C(L^{C*}) \quad (4.8)$$

$$w_U^C + \frac{e}{m} \frac{\delta L^{C*}}{N - L^{C*} - L^{R*}} = F'^R(L^{R*}) \quad (4.9)$$

We have the following result:

Proposition 2. *In a Harris-Todaro model with urban efficiency wages and a government budget constraint, decreasing unemployment benefit leads to:*

(i) *an ambiguous effect on urban employment L^C . However, if*

$$\left(\frac{L^{C*}}{N - L^{C*} - L^{R*}} \right)^2 > \frac{m w_U^C}{e \delta} \quad (4.10)$$

holds, then $\partial L^{C} / \partial w_U^C < 0$.*

(ii) *an ambiguous effect on rural employment L^R . However, if*

$$\left(\frac{L^{C*}}{N - L^{C*} - L^{R*}} \right)^2 < \frac{m w_U^C}{e \delta} \quad (4.11)$$

holds, then $\partial L^{R} / \partial w_U^C < 0$.*

(iii) *an ambiguous effect on urban employment (both in level and rate) U^C and u^C .*

Observe that conditions (4.10) and (4.11) cannot hold together since they are opposite. They are however sufficient conditions and thus do not prevent that a Todaro paradox prevails. Indeed, contrary to the previous case, a Todaro paradox can now exist because of the additional effect due to the government's budget constraint. When the government decreases the unemployment benefit w_U^C the two previously mentioned opposite mechanisms are present but there is now a new effect. When w_U^C is reduced, for given L^C and L^R , taxes on profits t^C decrease to balance the city budget (see (4.7)) and thus urban firms tend to create more jobs. With general equilibrium effects due to the fact that L^C and L^R are themselves ultimately a function of t^C , this effect is amplified. As a result, because of this last effect, a Todaro paradox can now emerge since decreasing unemployment benefits can increase both urban employment and unemployment.

5. Employment subsidy policy

We now consider a policy where *urban employment* is subsidized at a rate $S^C > 0$ per job and the employment subsidy S^C is paid to urban firms throughout the duration of the job. In that case, the profit equation (3.6) is changed and becomes

$$\Pi^C = F^C(L^C) - w_L^C L^C + S^C L^C \quad (5.1)$$

Observe that it is the firm who receives the subsidy and not the worker so that L_L is not affected and thus the efficiency wage w_L^C is still given by (3.5). Labor demand is modified and is now equal to:

$$F'^C(L^C) + S^C = w_L^C$$

which using (3.5) leads to:

$$w_U^C + e + \frac{e}{m} \left[\frac{\delta (N - L^R)}{N - L^C - L^R} + r \right] = F'^C(L^C) + S^C \quad (5.2)$$

Let us close the model as before. The equilibrium migration condition is still given by (3.9) and rural employment L^R by (3.10). So basically, the model is exactly as before with only one difference: urban employment L^C is now given by (5.2).

5.1. Increasing employment subsidies

Let us see what happens to the equilibrium if S^C , the employment subsidy per worker, increases. We have the following result:

Proposition 3. *In an Harris-Todaro model with urban efficiency wages, increasing urban employment subsidies leads to*

- (i) *an increase in urban employment L^C , i.e. $\partial L^{C*}/\partial S^C > 0$;*
- (ii) *an increase in rural employment L^R , i.e. $\partial L^{R*}/\partial S^C < 0$;*
- (iii) *an ambiguous effect on urban unemployment (both in level and rate) U^C and u^C .*

The employment subsidy policy has a direct *positive* impact on labor demand since the cost of hiring a worker becomes cheaper. As a result, urban firms hire more urban workers, and thus L^{C*} increases. This, in turn, triggers

a migration from rural areas to urban areas, which decreases L^{R*} . This policy is much more simpler than an unemployment benefit policy since the former only acts on job creation whereas the latter impacts on both labor demand and migration decision. Concerning the effect on the urban unemployment level, which is given by $U^C = N - L^C - L^R$, increasing the subsidy S^C has an ambiguous impact on U^C because it increases urban employment but decreases rural employment. As a result, it is possible in this case, that increasing employment subsidies raises both urban employment and unemployment, generating a Todaro paradox. This is due to the induced effect on migration since by creating new urban jobs more rural workers migrate to the city. This was not true in the case of the unemployment benefit policy because when the government reduced w_U^C , more urban jobs were created (which triggered more migration) but the expected utility of the unemployed workers decreased (which reduced migration since new migrants had first be unemployed before obtaining an urban job). Because there were a third effect on the urban efficiency wage, which was reduced due to an increase in urban employment, the level (and rate) of urban unemployment was always reduced following an unemployment benefit policy.

5.2. Steady-state equilibrium with a government budget constraint

Let us now introduce a government budget constraint. For that, we assume that unemployment benefits as well as employment subsidies are financed by a tax t^C on urban firms. This means that when a firm hires a worker, it is taxed and thus its profit can be written as:

$$\Pi^C = F^C(L^C) - (w_L^C + t^C - S^C) L^C \quad (5.3)$$

The government's budget constraint is given by:

$$t^C L^C = w_U^C (N^C - L^C) + S^C L^C \quad (5.4)$$

Contrary to the unemployment benefit policy, the fiscal policy is now such the taxes t^C are kept constant and the budget adjustment is realized through a decrease or increase in unemployment benefits w_U^C .⁶ This means that, for a

⁶It is easily verified that if a government has a policy such that the unemployment benefits w_U^C are kept constant and the budget adjustment is realized through a decrease or increase in taxes t^C , then there will be no effect of the subsidy S^C on the steady-state equilibrium. It is because what is given to firms (employment subsidy) is taken from them through a tax on profits.

constant value of t^C , the unemployment benefit level that balances the budget is given by:

$$w_U^C = \frac{(t^C - S^C) L^C}{(N - L^C - L^R)} \quad (5.5)$$

Let us now see how the steady-state equilibrium is affected when a government budget constraint is introduced. Again, the labor demand will be directly affected and, using (5.5), it will now be given by:

$$w_L^C + t^C - S^C = F'^C(L^C)$$

The steady state equilibrium is a 2-tuple (L^{C*}, L^{R*}) such that the two following equations are satisfied (using (3.5)):

$$\frac{(N - L^R)}{(N - L^C - L^R)} (t^C - S^C) + e + \frac{e}{m} \left[\frac{\delta (N - L^R)}{N - L^C - L^R} + r \right] = F'^C(L^{C*}) \quad (5.6)$$

$$w_U^C + \frac{e}{m} \frac{\delta L^{C*}}{N - L^{C*} - L^{R*}} = F'^R(L^{R*}) \quad (5.7)$$

The steady-state equilibrium is characterized by (5.6) and (5.7). We have the following result:

Proposition 4. *In a Harris-Todaro model with urban efficiency wages and a government budget constraint, increasing urban employment subsidies leads to:*

(i) *an ambiguous effect on urban employment L^C . However, if*

$$t^C + \frac{e}{m} \delta > S^C \quad (5.8)$$

then $\partial L^{C} / \partial S^C > 0$.*

(ii) *an ambiguous effect on rural employment L^R . However, if (5.8) holds, then $\partial L^{R*} / \partial S^C < 0$.*

(iii) *an ambiguous effect on urban employment (both in level and rate) U^C and u^C .*

These results are interesting and new. When a government budget constraint is introduced, there is a new force affecting rural-urban migration. Indeed, when the subsidy S^C increases, there is a direct positive effect on urban employment L^C . Because less people are employed, the government increases

the unemployment benefit w_U^C to balance the budget (see (5.5)). This encourages rural workers to migrate to the city. The net effect on urban employment L^C is thus ambiguous. However, if the tax on firms' profits t^C is large enough (i.e. condition (5.8) holds), then this is enough to discourage rural-urban migration and, in that case, an increase in the subsidy S^C will increase L^C . The same reasoning can be used to explain why when condition (5.8) holds, an increase in the subsidy S^C decreases L^R . Finally, because there are opposite effects on L^C and L^R , the impact of S^C on unemployment is ambiguous, even if (5.8) holds.

6. Concluding remarks

In this paper, we develop an efficiency wage model with rural-urban migration. Urban unemployment is due to too high and rigid wages and rural workers migrate to the city up to the point where their expected utility in the city is equal to their utility in the rural area. We characterize the steady-state equilibrium and show that it is unique. We then consider two policies aiming at increasing urban employment, a supply-side policy and a demand-side one. In the first one, the government decreases the unemployment benefits given to urban unemployed workers while, in the second one, it subsidizes urban jobs. We find that the unemployment benefit policy can be more effective in creating jobs without increasing urban unemployment than the employment subsidy policy. Indeed, the former policy has a direct positive impact on job creation since efficiency wages decrease following a cut in unemployment benefits. It has also another positive effect since it reduces the utility of urban unemployed workers, which, in turn, decreases the incentives for rural workers to migrate to the city because they need first to be unemployed before obtaining an urban job. As a result, when this policy is not financed, both urban employment increases and urban unemployment decreases. On the other hand, the employment subsidy policy “only” increases job creation in the city without affecting directly the rural-urban migration. However, because there are more jobs in the city, the expected utility of moving to the city increases for rural workers and thus rural-urban migration increases, which negatively affects urban unemployment. In that case, increasing employment subsidies in the city can increase both urban employment and unemployment. When these two policies are financed by a tax on firms' profits, there are general equilibrium effects due to the fact that the increase in the cost of the policy has to be financed by an increase in the tax on firms, which reduces job creation. However, under some condition, we

can show that the results previously obtained still hold in this more general framework.

The general lessons that we can learn from this paper is that urban job creation policies can backfire by increasing rather than decreasing urban unemployment. One way out is to have a policy that simultaneously increases urban employment and reduces rural-urban migration, such as the unemployment policy. Another possibility would be to combine two policies. For example, one could imagine that the government could subsidy the employment not only in the city (creating urban jobs) but also in the rural area (detering rural-urban migration).

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APPENDIX

Proof of Proposition 1

The Harris-Todaro equilibrium is defined by equations (3.11) and (3.10). From (3.11), we obtain a $L^C(L^R, w_U^C)$, whose properties are given by (4.2). Plugging this value in (3.10), we obtain the following equation:

$$w_U^C + \frac{e}{m} \frac{\delta L^C(L^{R*}, w_U^C)}{N - L^C(L^{R*}, w_U^C) - L^{R*}} = F'^R(L^{R*})$$

that gives a unique L^R , which is a function of exogenous parameters only, and in particular a function of w_U^C . This is why we denote the equilibrium value that we obtain by $L^{R*} \equiv L^R(w_U^C)$. By totally differentiating this equation, we obtain:

$$\frac{\partial L^{R*}}{\partial w_U^C} = - \frac{[N - L^C(L^{R*}, w_U^C) - L^{R*}]^2 + \frac{e\delta}{m} \frac{\partial L^C(L^{R*}, w_U^C)}{\partial w_U^C} (N - L^{R*})}{\frac{e\delta}{m} \left[\frac{\partial L^C}{\partial L^{R*}} (N - L^{R*}) + L^C(L^{R*}, w_U^C) \right] - [N - L^C(L^{R*}, w_U^C) - L^{R*}]^2 F''^R(L^{R*})} \quad (6.1)$$

where, using (4.2), we have $\frac{\partial L^C(L^{R*}, w_U^C)}{\partial w_U^C} < 0$ and $\frac{\partial L^C(L^{R*}, w_U^C)}{\partial L^{R*}} < 0$, so we cannot sign this derivative.

Now, plugging this value $L^{R*} \equiv L^R(w_U^C)$ in (3.11), we obtain a unique $L^{C*} \equiv L^C(w_U^C)$, which is only function of parameters and given implicitly by the following equation:

$$w_U^C + e + \frac{e}{m} \left[\frac{\delta (N - L^{R*})}{N - L^{C*} - L^{R*}} + r \right] = F'^C(L^{C*})$$

where $L^{C*} \equiv L^C(L^{R*}, w_U^C)$. Again, by totally differentiating this equation, we obtain:

$$\frac{\partial L^{C*}}{\partial w_U^C} = - \frac{(N - L^{C*} - L^{R*})^2 + \frac{e}{m} \delta \frac{\partial L^{R*}}{\partial w_U^C} L^{C*}}{\frac{e}{m} \delta (N - L^{R*}) - (N - L^{C*} - L^{R*})^2 F''^C(L^{C*})} \quad (6.2)$$

where $\frac{\partial L^{R*}}{\partial w_U^C}$ is given by (6.1).

Let us now calculate the exact value of $\frac{\partial L^{R*}}{\partial w_U^C}$. By plugging (4.3) and (6.2) in (6.1) and solving in $\frac{\partial L^{R*}}{\partial w_U^C}$, we obtain:

$$\frac{\partial L^{R*}}{\partial w_U^C} = \frac{(N - L^C - L^{R*})^4 F''^C(L^{C*})}{\left(\frac{e\delta}{m}\right)^2 L^C(N - L^{R*}) - (N - L^{C*} - L^{R*})^2 B} < 0 \quad (6.3)$$

where

$$B \equiv \frac{e}{m} \delta L^C F''^C(L^C) + F''^R(L^{R*}) \left[\frac{e}{m} \delta L^C (N - L^{R*}) - (N - L^{C*} - L^{R*})^2 F''^C(L^{C*}) \right] < 0$$

We can now calculate $\frac{\partial L^{C*}}{\partial w_U^C}$. By plugging (6.3) in (6.2), we obtain:

$$\begin{aligned} \frac{\partial L^{C*}}{\partial w_U^C} &= - \frac{(N - L^{C*} - L^{R*})^2 + \frac{e}{m} \delta \frac{\partial L^{R*}}{\partial w_U^C} L^{C*}}{\frac{e}{m} \delta (N - L^{R*}) - (N - L^{C*} - L^{R*})^2 F''^C(L^{C*})} \\ &= - \frac{(N - L^{C*} - L^{R*})^2 - \frac{(N - L^{C*} - L^{R*})^4 F''^C(L^{C*})}{(N - L^C - L^R)^2 B - \left(\frac{e}{m} \delta\right)^2 L^{C*} (N - L^{R*})} \frac{e}{m} \delta L^{C*}}{\frac{e}{m} \delta (N - L^{R*}) - (N - L^{C*} - L^{R*})^2 F''^C(L^{C*})} \end{aligned}$$

Let us show that $\frac{\partial L^{C*}}{\partial w_U^C} < 0$. Since the denominator is positive, we have

$$\begin{aligned} \frac{\partial L^{C*}}{\partial w_U^C} &< 0 \\ \Leftrightarrow (N - L^{C*} - L^{R*})^2 &> \frac{(N - L^{C*} - L^{R*})^4 F''^C(L^{C*})}{(N - L^C - L^R)^2 B - \left(\frac{e}{m} \delta\right)^2 L^{C*} (N - L^{R*})} \frac{e}{m} \delta L^{C*} \\ \Leftrightarrow (N - L^{C*} - L^{R*})^2 &\left[B - F''^C(L^{C*}) \frac{e}{m} \delta L^{C*} \right] - \left(\frac{e}{m} \delta\right)^2 L^{C*} (N - L^{R*}) < 0 \end{aligned}$$

Since

$$\begin{aligned} &B - F''^C(L^{C*}) \frac{e}{m} \delta L^{C*} \\ &= F''^R(L^{R*}) \left[\frac{e}{m} \delta (N - L^{R*}) - (N - L^{C*} - L^{R*})^2 F''^C(L^{C*}) \right] < 0 \end{aligned}$$

This implies that

$$(N - L^C - L^R)^2 \left[B - F''^C(L^{C*}) \frac{e}{m} \delta L^{C*} \right] - \left(\frac{e}{m} \delta\right)^2 L^{C*} (N - L^{R*}) < 0$$

which is always true. Thus

$$\frac{\partial L^{C*}}{\partial w_U^C} < 0$$

Let us now calculate $\frac{\partial U^{C*}}{\partial w_U^C}$. By differentiating (2.1), we have:

$$\frac{\partial U^{C*}}{\partial w_U^C} = - \frac{\partial L^{C*}}{\partial w_U^C} - \frac{\partial L^{R*}}{\partial w_U^C} > 0$$

Moreover, since the unemployment rate is defined as

$$u^{C*} = \frac{U^{C*}}{U^{C*} + L^{C*}}$$

then

$$\frac{\partial u^{C^*}}{\partial w_U^C} = \frac{\frac{\partial U^{C^*}}{\partial w_U^C} L^{C^*} - U^{C^*} \frac{\partial L^{C^*}}{\partial w_U^C}}{(U^{C^*} + L^{C^*})^2} > 0$$

■

Proof of Proposition 2

(i) Equation (4.9) can be written as: $L^{R^*}(L^{C^*}, w_U^C)$. By totally differentiating (4.9), we then have:

$$\frac{\partial L^{R^*}}{\partial L^{C^*}} = -\frac{N - L^{R^*}}{L^{C^*} - \frac{m}{e\delta} F''R(L^{R^*}) (N - L^{C^*} - L^{R^*})^2} < 0 \quad (6.4)$$

$$\frac{\partial L^{R^*}}{\partial w_U^C} = -\frac{1}{\frac{e}{m} \frac{\delta L^{C^*}}{(N - L^{C^*} - L^{R^*})^2} - F''R(L^{R^*})} < 0 \quad (6.5)$$

Now, using equation (4.9), we can write (4.8) as follows: $L^{C^*}(L^{R^*}(L^{C^*}, w_U^C), w_U^C)$. By totally differentiating (4.8), we obtain:

$$\frac{\partial L^{C^*}}{\partial w_U^C} = -\frac{\frac{N - L^{R^*}}{L^{C^*}} + \frac{\partial L^{R^*}}{\partial w_U^C} L^{C^*} \left[\frac{e\delta}{m(N - L^{C^*} - L^{R^*})^2} - \frac{w_U^C}{(L^{C^*})^2} \right]}{\left[\frac{e\delta}{m(N - L^{C^*} - L^{R^*})^2} - \frac{w_U^C}{(L^{C^*})^2} \right] \left[\frac{\partial L^{R^*}}{\partial L^{C^*}} L^{C^*} + N - L^{R^*} \right] - F''C(L^{C^*})}$$

Using (6.4) and (6.5), we can rewrite this last equation as:

$$\frac{\partial L^{C^*}}{\partial w_U^C} = \frac{\frac{N - L^{R^*}}{L^{C^*}} - \frac{\frac{e\delta L^{C^*}}{m(N - L^{C^*} - L^{R^*})^2} - \frac{w_U^C}{(L^{C^*})^2}}{\frac{e\delta}{m} \frac{L^{C^*}}{(N - L^{C^*} - L^{R^*})^2} - F''R(L^{R^*})}}{F''C(L^{C^*}) - \left[\frac{e\delta}{m(N - L^{C^*} - L^{R^*})^2} - \frac{w_U^C}{(L^{C^*})^2} \right] \left[N - L^{R^*} - \frac{(N - L^{R^*})L^{C^*}}{L^{C^*} - \frac{m}{e\delta} F''R(L^{R^*})(N - L^{C^*} - L^{R^*})^2} \right]}$$

Let us first study the numerator and show that it is positive. This can be written as:

$$\begin{aligned} & \left(\frac{N - L^{R^*}}{L^{C^*}} \right) \left[\frac{e\delta L^{C^*}}{m(N - L^{C^*} - L^{R^*})^2} - F''R(L^{R^*}) \right] > \left[\frac{e\delta L^{C^*}}{m(N - L^{C^*} - L^{R^*})^2} - \frac{w_U^C}{L^{C^*}} \right] \\ \Leftrightarrow & \frac{e\delta L^{C^*}}{m(N - L^{C^*} - L^{R^*})^2} \left(\frac{N - L^{R^*} - L^{C^*}}{L^{C^*}} \right) - \left(\frac{N - L^{R^*}}{L^{C^*}} \right) F''R(L^{R^*}) > -\frac{w_U^C}{L^{C^*}} \end{aligned}$$

which is always true since $F''R(L^{R^*}) \leq 0$. As a result,

$$\begin{aligned} & \text{sgn} \frac{\partial L^{C^*}}{\partial w_U^C} \\ = & \text{sgn} \left\{ - \left[\frac{e\delta}{m(N - L^{C^*} - L^{R^*})^2} - \frac{w_U^C}{(L^{C^*})^2} \right] \left[N - L^{R^*} - \frac{(N - L^{R^*})L^{C^*}}{L^{C^*} - \frac{m}{e\delta} F''R(L^{R^*})(N - L^{C^*} - L^{R^*})^2} \right] \right\} \end{aligned}$$

Let us see if this expression is strictly negative, i.e.

$$F''^C(L^{C*}) - \left[\frac{e\delta}{m(N-L^{C*}-L^{R*})^2} - \frac{w_U^C}{(L^{C*})^2} \right] \left[N - L^{R*} - \frac{(N-L^{R*})L^{C*}}{L^{C*} - \frac{mF''^R(L^{R*})(N-L^{C*}-L^{R*})^2}{e\delta}} \right] < 0$$

After some calculations, this is equivalent to:

$$F''^C(L^{C*}) \left[L^{C*} - \frac{m}{e\delta} F''^R(L^{R*}) (N - L^{C*} - L^{R*})^2 \right] + (N - L^{R*}) \left[\frac{1}{(N - L^{C*} - L^{R*})^2} - \frac{mw_U^C}{e\delta(L^{C*})^2} \right] F''^R(L^{R*}) (N - L^{C*} - L^{R*})^2 < 0$$

It should be clear that this expression cannot be signed. This sign is thus indeterminate. However, since $F''^C(L^{C*}) \leq 0$ and $F''^R(L^{R*}) \leq 0$, then if (4.10) holds, then this expression is strictly negative and thus $\partial L^{C*}/\partial w_U^C < 0$.

(ii) Equation (4.8) can be written as: $L^{C*}(L^{R*}, w_U^C)$. By totally differentiating (4.8), we then have:

$$\frac{\partial L^{C*}}{\partial L^{R*}} = - \frac{\frac{e\delta}{m} \frac{L^{C*}}{(N-L^{C*}-L^{R*})^2} - \frac{w_U^C}{L^{C*}}}{\frac{w_U^C(N-L^{R*})}{(L^{C*})^2} + \frac{e}{m} \frac{\delta(N-L^{R*})}{(N-L^{C*}-L^{R*})^2} - F''^C(L^{C*})} \quad (6.6)$$

$$\frac{\partial L^{C*}}{\partial w_U^C} = - \frac{N - L^{R*}}{\frac{w_U^C(N-L^{R*})}{(L^{C*})} + \frac{e}{m} \frac{\delta(N-L^{R*})L^{C*}}{(N-L^{C*}-L^{R*})^2} - L^{C*}F''^C(L^{C*})} < 0 \quad (6.7)$$

Now, using equation (4.8), we can write (4.9) as follows: $L^{R*}(L^{C*}(L^{R*}, w_U^C), w_U^C)$. By totally differentiating (4.9), we obtain:

$$\frac{\partial L^{R*}}{\partial w_U^C} = - \frac{1 + \frac{e\delta}{m} \frac{(N-L^{R*})}{(N-L^{C*}-L^{R*})^2} \frac{\partial L^{C*}}{\partial w_U^C}}{\frac{e\delta}{m} \frac{L^{C*} + \frac{\partial L^{C*}}{\partial L^{R*}}(N-L^{R*})}{(N-L^{C*}-L^{R*})^2} - F''^R(L^{R*})}$$

Let us show first that the numerator is strictly positive. Using (6.7), and after some calculations, the numerator is strictly positive if and only if:

$$\frac{w_U^C (N - L^{R*}) (N - L^{C*} - L^{R*})^2}{(L^{C*})} [1 - L^{C*} F''^C(L^{C*})] \\ + \frac{e}{m} \delta (N - L^{R*}) L^{C*} > \frac{e\delta}{m} (N - L^{R*})^2$$

which is always true if $L^{C*} > 1$. As a result,

$$\text{sgn} \frac{\partial L^{R*}}{\partial w_U^C} = \text{sgn} \left\{ F''^R(L^{R*}) - \frac{e\delta L^{C*} + \frac{\partial L^{C*}}{\partial L^{R*}} (N - L^{R*})}{m (N - L^{C*} - L^{R*})^2} \right\}$$

Let us see if this expression is strictly negative. Using (6.6), this is equivalent to:

$$F''^R(L^{R*}) - \frac{e\delta}{m} \frac{L^{C*} - \frac{\frac{e\delta}{m} \frac{L^{C*}}{(N - L^{C*} - L^{R*})^2} - \frac{w_U^C}{L^{C*}}}{\frac{w_U^C (N - L^{R*})}{(L^{C*})^2} + \frac{e}{m} \frac{\delta (N - L^{R*})}{(N - L^{C*} - L^{R*})^2} - F''^C(L^{C*})} (N - L^{R*})}{(N - L^{C*} - L^{R*})^2} < 0$$

After some calculations, we obtain:

$$\left[\frac{m F''^R(L^{R*}) (N - L^{C*} - L^{R*})^2}{e\delta} - L^{C*} \right] \left[\frac{w_U^C}{(L^{C*})^2} + \frac{e\delta}{m (N - L^{C*} - L^{R*})^2} - \frac{F''^C(L^{C*})}{(N - L^{R*})} \right] \\ + \frac{e\delta}{m} \frac{L^{C*}}{(N - L^{C*} - L^{R*})^2} - \frac{w_U^C}{L^{C*}} < 0$$

This can clearly not be signed. However if (4.11) holds, then this expression is strictly negative and thus $\partial L^{R*} / \partial w_U^C < 0$.

(iii) Since

$$\frac{\partial U^{C*}}{\partial w_U^C} = - \frac{\partial L^{C*}}{\partial w_U^C} - \frac{\partial L^{R*}}{\partial w_U^C}$$

then the result is straightforward. ■

Proof of Proposition 4

(i) Let us study the impact of S^C on L^C .

Observe that equation (5.7) does not depend on S^C and that it can be denoted as $L^R(L^C)$, with

$$\frac{\partial L^R}{\partial L^C} = - \frac{\frac{e\delta}{m} \frac{N - L^R}{(N - L^C - L^R)^2}}{\frac{e\delta}{m} \frac{L^C}{(N - L^C - L^R)^2} - F''^R(L^R)} < 0 \quad (6.8)$$

As a result, plugging $L^R(L^C)$ into (5.6) and differentiating this equation leads to:

$$\frac{\partial L^{C*}}{\partial S^C} = \frac{\frac{N - L^R}{N - L^C - L^R}}{\left[\frac{N - L^R + \frac{\partial L^R}{\partial L^C} L^C}{(N - L^C - L^R)^2} \right] (t^C - S^C + \frac{e}{m} \delta) - F''^C(L^C)}$$

Using (6.8), this can be written as:

$$\frac{\partial L^{C*}}{\partial S^C} = - \frac{\frac{N-L^R}{N-L^C-L^R}}{\frac{F''^R(L^R)(N-L^R)(t^C-S^C+\frac{e}{m}\delta)}{[\frac{e\delta}{m}L^C-(N-L^C-L^R)^2 F''^R(L^R)]} + F''^C(L^C)}$$

Thus if $t^C + \frac{e}{m}\delta > S^C$, then $\frac{\partial L^{C*}}{\partial S^C} > 0$. Otherwise, it is ambiguous.

(ii) Let us now study the impact of S^C on L^R .

Equation (5.6) can be denoted by $L^C(L^R, S^C)$, with

$$\frac{\partial L^C}{\partial L^R} = - \frac{L^C \frac{(t^C-S^C+\frac{e}{m}\delta)}{(N-L^C-L^R)^2}}{(N-L^R) \frac{(t^C-S^C+\frac{e}{m}\delta)}{(N-L^C-L^R)^2} - F''^C(L^C)} \quad (6.9)$$

$$\frac{\partial L^C}{\partial S^C} = \frac{\frac{(N-L^R)}{(N-L^C-L^R)}}{\frac{(N-L^R)}{(N-L^C-L^R)^2} (t^C - S^C + \frac{e\delta}{m}) - F''^C(L^C)} \quad (6.10)$$

Thus, if $t^C + \frac{e}{m}\delta > S^C$, then both $\frac{\partial L^C}{\partial L^R} < 0$ and $\frac{\partial L^C}{\partial S^C} > 0$. Otherwise, both signs are ambiguous.

Consider now equation (5.7) and replace L^C by $L^C(L^R, S^C)$. By differentiating this last equation, we obtain:

$$\frac{\partial L^{R*}}{\partial S^C} = - \frac{\frac{(N-L^R)}{(N-L^C-L^R)^2} \frac{e\delta}{m} \frac{\partial L^C}{\partial S^C}}{\frac{e}{m} \delta \frac{\frac{\partial L^C}{\partial L^R} (N-L^R) + L^C}{N-L^C-L^R} - F''^R(L^R)}$$

Using (6.9), we obtain:

$$\frac{\partial L^{R*}}{\partial S^C} = \frac{\frac{(N-L^R)}{(N-L^C-L^R)^2} \frac{e\delta}{m} \frac{\partial L^C}{\partial S^C}}{\frac{e}{m} \delta L^C \left[\frac{F''^C(L^C)(N-L^C-L^R)}{(N-L^R)(t^C-S^C+\frac{e}{m}\delta) - F''^C(L^C)(N-L^C-L^R)^2} \right] + F''^R(L^R)}$$

Thus, if $t^C + \frac{e}{m}\delta > S^C$, then $\frac{\partial L^{R*}}{\partial S^C} < 0$ since $\frac{\partial L^C}{\partial S^C} > 0$. Otherwise, it is ambiguous. ■

Figure 1: Harris-Todaro equilibrium with efficiency wages

