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ABSTRACT

Calling Circles: Network Competition with Non-Uniform Calling Patterns*

We introduce a flexible model of telecommunications network competition with non-uniform calling patterns, which account for the fact that customers tend to make most calls to a small subset of people. Equilibrium call prices are distorted away from marginal cost, and competitive intensity is affected by the concentration of calling patterns. Contrary to previous predictions, jointly profit-maximizing access charges are set above termination cost in order to dampen competition, and the resulting on-net prices are below off-net prices, if calling patterns are sufficiently concentrated.

JEL Classification: L13 and L51
Keywords: network competition, non-uniform calling patterns and termination charges

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1 Introduction

Modern communication networks allow users to easily establish a large number of links, both on the same network and across networks. Still, most users' contacts are often limited to only a small fraction of all other users. Exchanges tend to be with friends and family or between people who have, for different reasons, close social links, such as members of user groups linked to business customers. While each user thus calls only a small fraction of all other users at all, the subset of links with whom he has also frequent communications may be even smaller. For instance, Shi et al. (2009) find, for a Chinese cellular network, that people make most of their calls ("80%") to a very small proportion ("20%") of their contacts.[1]

These observations contrast with a standard assumption that is frequently made in the literature on competition between telecommunications networks: That of "uniform calling patterns", whereby each subscriber is assumed to be equally likely to call any other subscriber in the market (e.g., Armstrong, 1998; Laffont et al., 1998a, 1998b). As calls between networks (called "off-net calls") involve the payments of wholesale charges (also called access charges, or termination rates, in the literature), the assumption of uniform calling patterns also bears consequences on how access charges impact on consumers' retail prices. When competing networks can set access charges strategically, Gans and King (2001) find that access charges are set below cost. Under multi-part tariffs, this leads to *higher* prices for on-net than for off-net calls. Together with the assumption of uniform calling patterns, this implies that the fraction of call minutes that a subscriber makes on-net should be *below* the market share of the network to which he subscribes. Typically, the opposite seems to be the case. For instance, for mobile networks in Italy the subscriber market shares and the share of on-net calls compare as follows (on-net shares in brackets): TIM 36% (70%), Vodafone 34% (78%), Wind 20% (70%), and H3G 10% (33%).[2] A similar picture emerges in Portugal: Over the years 2003-2009, the share of on-net calls in all national mobile-to-mobile calls has been fairly constant at 75%, while under uniform calling patterns this share should have been at most 40%.

[1] Also Birke and Swann (2006) show empirically that calling patterns are heavily on-net biased, and that consumers tend to choose networks following their calling clubs.

[2] The share of on-net calls is computed by taking into account only the total number of on-net and off-net calls to mobile phones, while calls to fixed lines are excluded (source: Relazione Annuale 2009, AGCOM, http://www.agcom.it/default.aspx?message=viewdocument&DocID=3239).

This paper introduces non-uniform calling patterns in a tractable model of network competition. Customers differ in their preferences for a particular network, and instead of stipulating that each subscriber calls any other subscriber with the same probability, we suppose that he is more likely to call other subscribers with similar preferences than those further away in preference space. For instance, the brand positioning of a network may be more appealing to a particular age group.[3] Likewise, differences in local network coverage could generate similar patterns of call preferences. We analyze how non-uniform calling patterns, with the resulting high fraction of on-net calls, affect equilibrium outcomes.

When calling patterns are uniform, economic theory predicts that under multi-part tariffs variable prices for calls should be set equal to marginal costs. Instead, with non-uniform calling patterns, we find that networks practice price discrimination by using on- and off-net prices as metering devices. We derive a simple general pricing formula. This relates the deviation from marginal-cost pricing to the difference between the calling pattern of a network's "marginal subscriber", who is just indifferent between joining this and a competitor's network, and the average calling pattern of subscribers on this network.

An important implication of our results concerns networks' choice of profit-maximizing access charges. Under a uniform calling pattern, networks would choose access charges below cost to dampen competition (Gans and King, 2001). This induces off-net prices below on-net prices, leading to negative "tariff-mediated network effects" (Laffont et al., 1998b): The average call price on a smaller network will be lower due to the larger share of off-net calls. Consumers then prefer to join a smaller rather than a larger network, which dampens competition. However, when calling patterns are no longer uniform, the proportion of on- and off-net calls of the marginal subscriber is less closely tied to the respective market shares of the two networks. This dampens the role of the aforementioned tariff-mediated network effects. We derive conditions for when, with non-uniform calling patterns, networks would choose access charges *above* cost, while off-net prices will exceed on-net prices, as observed in practice. Then, as we show, higher access charges tend to make the marginal subscriber less attractive to firms, which dampens competition.

Our finding that access charges above cost can dampen competition is related to,

[3] This seems to be the case, for instance, in the UK for customers of Virgin Mobile, who are typically young people attracted by the brand of the Virgin group. See http://www.ofcom.org.uk/research/cm/cmr09/. Also, when mobile operators sponsor different sport activities or specific clubs, subscribers may sort according to their respective preferences over sport or clubs.

but conceptually different from, the standard logic of "raising rivals' cost". When, in a standard oligopoly model without network competition, firms cross-licence an essential input to each other, a higher marginal royalty unambiguously increases *both* a firm's own true cost *and* its opportunity cost of serving more customers. As acquiring the marginal customer thus becomes less attractive, this reduces the intensity of competition. To see the difference to the case of network competition, note that when off-net and on-net prices are the same, a change in access charges would *not* affect competition, as it would not change the profits that firms can make with the marginal subscriber: When calls between networks are balanced, for both networks the higher charges for off-net calls are exactly off-set by the higher revenues from received calls.[4] We show how the endogenous price difference between on-net and off-net prices, together with the degree of concentration of calling pattern, determine when lower or higher access charges dampen competition.

By considering changes in the concentration of calling patterns, we also contribute, more broadly, to the literature that analyzes how network effects affect the intensity of competition (e.g. Katz and Shaprio, 1985). In the literature on competition between telecom networks, previous models incorporating non-uniform calling patterns have only considered the extreme case where a fraction of calls is made to exactly the same location in preference space, see, e.g., Gabrielsen and Vagstad (2008) and Calzada and Valletti (2008). In these models, only subscribers with identical preferences are part of the same calling club, and these clubs are completely disjunct from each other. Instead, in our model we allow for arbitrary overlap between calling patterns of different subscribers.

In our model, local network externalities are induced by the endogenously set tariffs, which is different in Fjeldstad et al. (2010). Still, our modeling of social interaction patterns, through "calling circles", may be useful also for wider applications in the area of network economics. In fact, Fjeldstad et al. (2010) quote Farrell and Klemperer (2007), who note: "A more general formulation (of network externalities) would allow each user i to gain more from the presence of one other user j than of another k." In our framework this is the case due to the interaction of (local) calling patterns and differences between

[4] This does no longer hold, however, when tariffs are restricted to be linear, in which case networks jointly would choose access charges above cost. Starting from a symmetric equilibrium, a deviating firm could only acquire market share through undercutting the marginal price, which would lead to an imbalance of calls (call minutes): Any of its customers would make more off-net calls than he receives (or he would call for fewer minutes). Higher access charges then dampen competition through this (out–of-equilibrium) imbalance (cf. Armstrong, 1998 and Laffont et al., 1998a).

on-net and off-net calls.

Cabral (2009) provides a recent (dynamic) analysis of the impact of various access charges on competition. This, as well as ours, contributes to a large literature, starting with Armstrong (1998) and Laffont et al. (1998a,b), on how networks gain from choosing (unregulated) reciprocal access charges. An important puzzle is the prediction of Gans and King (2001) that with two-part tariffs and discrimination between on- and off-net calls networks would jointly choose access charges below cost, which seems to be at odds with how networks set access charges in reality. Still within the constraints of uniform calling patterns, Lopez and Rey (2009) show that high access charges can serve foreclosure, Jullien et al. (2009) show how they can arise from price discrimination between heavy and light useres[5], and in Armstrong and Wright (2009) they may arise under the simultaneous presence of mobile and fixed networks. As noted previously, in our model we also obtain for non-uniform calling patterns that off-net prices may, in equilibrium, be indeed higher than on-net prices.[6]

The rest of this paper is organized as follows. Section 2 introduces the model and the necessary notions related to calling patterns. In Section 3 we determine the on- and off-net pricing structure. Section 4 determines the market equilibrium. In section 5 networks choose reciprocal access charges. In Section 6 we derive some additional results for a specific family of non-uniform calling patterns. Section 7 offers some concluding remarks.

2 A Model of Competition with Non-Uniform Calling Patterns

We consider competition between two interconnected telephone networks, $i = 1, 2$. Both networks incur a fixed cost f to serve each subscriber. The marginal cost of providing a minute of a telephone call consists of the terminating and originating cost, which are equal to c_0, and the conveying cost, c_1. As a result, the total marginal cost of an on-net call initiated and terminated on the same network i is $c_{ii} \equiv 2c_0 + c_1$. Networks pay each

[5] See, however, Hurkens and Jeon (2009), who find below-cost access charges in a model with elastic subscription demand. Dessein (2003) as well as Armstrong (2004) also allow for heterogeneity between high- and low-volume users, while the probability that any given subscriber is called by any other subscriber is independent of their respective locations.

[6] In the extant literature, when subscribers receive utility also from received calls, such a pricing structure is obtained by Jeon et al. (2004), but disappears once access charges are endogenized (Cambini and Valletti, 2008).

other a reciprocal access charge (or termination rate) a when a call initiated on network i is terminated on a different network j. Thus, for an off-net call, the economic marginal cost is still c_{ii}, but the "perceived" marginal cost for the network that initiates the call is $c_{ij} \equiv c_1 + c_0 + a$.

Networks offer multi-part tariffs and discriminate between on-net and off-net calls. As a result, network $i = 1, 2$ offers a tariff with the following structure:[7]

$$T_i(q_{ii}, q_{ij}) = F_i + p_{ii}q_{ii} + p_{ij}q_{ij},$$

where F_i is the fixed monthly subscription fee that consumers pay to network i, p_{ii} and q_{ii} are the price and quantity of on-net call minutes, and p_{ij} and q_{ij} are the respective price and quantity for off-net call minutes from network i to network $j \neq i$.

Consumer preferences over networks and call demand. The market consists of a continuum of size 1 of consumers. A consumer is indexed by his relative preference for the two networks, where we normalize the space of preferences to $x \in X \equiv [0, 1]$. To simplify expressions, we stipulate that consumers are uniformly distributed over X, though it is straightforward to extend results in this direction. The two networks' own "attributes" are represented by their locations at the two extremes $x_1 = 0$ and $x_2 = 1$, respectively. Preferences and networks' locations may relate to the brand image that networks have created through marketing and services directed at specific customer groups. If a consumer at location x subscribes to network i, he bears a disutility or "transport cost" $\tau |x - x_i|$ with $\tau > 0$. Consumers receive a fixed utility v_0 from being connected. We assume that v_0 is large enough so that all consumers connect to some network. This convenient assumption may be understood to reflect the fact that most markets for fixed or mobile telephony are highly saturated.

Once a call is placed to somebody, its length depends on the call price. Given a price per minute p, consumers demand calls of length $q_{ii} = q(p_{ii})$ and $q_{ij} = q(p_{ij})$, with demand elasticity $\eta(p) = -pq'(p)/q(p)$. The level of consumer surplus associated with this demand function is denoted by $v_{ii} = v(p_{ii})$ for on-net calls, and similarly $v_{ij} = v(p_{ij})$ for off-net

[7]With this simple tariff structure we can, as we show, go a long way in characterizing the equilibrium even with general demand and calling patterns. In future work, the analysis could be extended to allow for a menu of tariffs (or for a single non-linear tariff), which would offer firms more scope to price discriminate. (Cf. Armstrong and Vickers, 2001 and Rochet and Stole, 2002 for models of competition with nonlinear pricing.)

calls. This indirect utility function $v(\cdot)$ has standard properties. In particular, it holds for the respective price p and quantity q that $dv/dp = -q$.

Calling patterns. The novel ingredient in our model is that consumers differ in their individual calling patterns. The latter are represented by a function $G(y|x)$ on X^2, the likelihood with which a consumer of preference ("location") x will call consumers at locations $y' \leq y$, with $G(0|x) = 0$ and $G(1|x) = 1$ for all $x \in X$. Depending on whether the chosen recipient belongs to the same network or not, the respective call minutes will then equal q_{ii} or q_{ij}. $G(y|x)$ is assumed to be differentiable in (x, y), and non-decreasing in y with density $g(y|x)$. This density may be zero for certain $(x, y) \in X^2$. A uniform calling pattern is obtained when $G(y|x) \equiv G^U(y|x) = y$.

We focus on symmetric network competition and thus stipulate symmetric calling patterns: $G(y|x) = 1 - G(1-y|1-x)$ for all $(x, y) \in X^2$. In particular, this implies $G(1/2|1/2) = 1/2$ and $g(y|x) = g(1-y|1-x)$.

Below we provide conditions for when, in equilibrium, there will be a cutoff customer type $\hat{x}$ such that all $x \leq \hat{x}$ subscribe to network 1 and all $x > \hat{x}$ subscribe to network 2. For given $\hat{x}$, we define the total expected number of on-net calls on network 1 by

$$L_{11}(\hat{x}) = \int_0^{\hat{x}} G(\hat{x}|x)dx,$$

and the total expected number of off-net calls by

$$L_{12}(\hat{x}) = \int_0^{\hat{x}} [1 - G(\hat{x}|x)]\, dx.$$

Since network 1 has the mass $\hat{x}$ of subscribers who each make a unit mass of calls, it holds that $L_{11}(\hat{x}) + L_{12}(\hat{x}) = \hat{x}$. In a similar manner, for network 2 we define on-net calls $L_{22}(\hat{x}) = \int_{\hat{x}}^1 [1 - G(\hat{x}|x)]\, dx$ and off-net calls $L_{21}(\hat{x}) = \int_{\hat{x}}^1 G(\hat{x}|x)dx$, with $L_{22}(\hat{x}) + L_{21}(\hat{x}) = 1 - \hat{x}$.

Calls between networks are *balanced* when, for given $\hat{x}$, it holds that $L_{12}(\hat{x}) = L_{21}(\hat{x})$. Though we will only make use of balancedness for almost symmetric market shares, $\hat{x} \approx 1/2$, for simplicity we stipulate, more generally,

$$L_{12}(\hat{x}) = L_{21}(\hat{x}) \text{ for all } \hat{x} \in [0, 1].$$

In terms of primitives, network balancedness always holds when individual calls are balanced pairwise across all call contacts, i.e., $g(y|x) = g(x|y)$ for all x, y. Put differently, ties between two customers are then “equally strong”, e.g., they reciprocate calls, so that the probability of calling is symmetric in both directions (though the duration of the call will differ if consumers pay different call prices). Evidently, this holds under a uniform calling pattern G^U.

An immediate implication of balancedness that we will use frequently in the analysis is the following. Note first that since calling patterns are symmetric, we have that $L_{12}(\hat{x}) = L_{21}(\hat{x}) = L_{12}(1-\hat{x})$. Differentiating the latter at $\hat{x} = 1/2$ then yields

$$L'_{12}(1/2) = 0. \tag{1}$$

In words, a marginal change of $\hat{x}$ at $\hat{x} = 1/2$ does not affect the number of off-net calls.[8]

Our analysis rests on the following definition of “concentrated” calling patterns.

Definition. *Consider two calling patterns G_1 and G_2. G_2 is more concentrated than G_1 if the following conditions hold:*
i) Customers with location $0 < x < 1/2$, who have a preference for network 1, are less likely under G_2 to call customers with a stronger preference for network 2: For all $y \in [x, 1)$ it holds that $1 - G_2(y|x) < 1 - G_1(y|x)$.
ii) Customers with location $1/2 < x < 1$, who have a preference for network 2, are less likely under G_2 to call customers with a stronger preference for network 1: For all $y \in (0, x)$ it holds that $G_2(y|x) < G_1(y|x)$.

Note that by symmetry of calling patterns it would be sufficient to invoke the requirement only for customers $x < 1/2$ (condition i) or for customers $x > 1/2$ (condition ii). Note also that the definition takes into account that always $G(0|x) = 0$, $G(1|x) = 1$, and $G(1/2|1/2) = 1/2$.[9] In the following, we will call a calling pattern “concentrated” when, according to the above definition, it is more concentrated than the uniform calling pattern G^U.

In Section 6 we will obtain explicit expressions for our equilibrium characterization, as well as additional implications, by using a family of calling patterns denoted as G^λ, $\lambda \in [0,1]$. Each customer calls with probability $1-\lambda$ randomly someone else, while with

[8] From $L_{11}(\hat{x}) + L_{12}(\hat{x}) = \hat{x}$, we then have for the number of on-net calls that $L'_{11}(1/2) = 1$ and $L'_{22}(1/2) = -1$.

[9] Hence, for the sake of brevity we have not included into the definition the calling patterns at the “corners” $x = 0, 1$. There, for $y = x$, the requirements can only hold weakly, as $G(0|0) = 0$ and $G(1|1) = 1$.

probability λ he makes a call to someone in his personal "calling circle". For customers who are not too close to the "corners" $x = 0, 1$, calling circles are defined symmetrically around the customer's own location, namely by a symmetric distribution function $H(z)$ with support $z \in [-\varepsilon, \varepsilon]$ for some $\varepsilon \in (0, 1/2)$ (cf. Section 6 for more details). Then let

$$G^{\lambda}(y|x) = (1 - \lambda)\, y + \lambda H(y - x).$$

It is easily verified that G^{λ} satisfies symmetry and balancedness. It is also easy to see that a higher value of λ leads to a more concentrated calling pattern, according to our definition, while for $\lambda = 0$ the calling pattern reduces to a uniform calling pattern G^{U}.

Utility. When network 1 serves all consumers $x \leq \hat{x}$ and network 2 all consumers $x \geq \hat{x}$, the marginal consumer $\hat{x}$ is just indifferent between the two offers. Given $\hat{x}$, for any consumer x the net utility from subscribing to network 1 is given by

$$U_1(x, \hat{x}) = u_1(x, \hat{x}) + v_0 - F_1 - \tau x,$$

where

$$u_1(x, \hat{x}) = G(\hat{x}|x) v(p_{11}) + [1 - G(\hat{x}|x)]\, v(p_{12}).$$

If the consumer subscribes, instead, to network 2, then his utility is

$$U_2(x, \hat{x}) = u_2(x, \hat{x}) + v_0 - F_2 - \tau(1 - x),$$

with

$$u_2(x, \hat{x}) = [1 - G(\hat{x}|x)]\, v(p_{22}) + G(\hat{x}|x) v(p_{21}).$$

Market game. At $t = 1$, for any given reciprocal access charge, networks compete for consumers by simultaneously making contract offers T_i. At $t = 2$, consumers subscribe and place calls. At this stage, all payoffs are realized. Below in Section 5, we will also consider an initial stage $t = 0$ where networks jointly choose a profit-maximizing reciprocal access charge.

Section 3 solves for networks' optimal call prices for given market shares. Here, the focus is on networks' optimal price discrimination strategy for on-net and off-net calls. Section 4 determines the symmetric Nash equilibrium in multi-part tariffs. Section 5 considers the choice of profit-maximizing access charges, and Section 6 provides an illustration for a specific family of calling patterns.

3 On- and Off-Net Prices as Metering Devices

Given the contract T_1, each subscriber at location $x \leq \hat{x}$ of network 1 yields expected profits equal to the sum of the fixed part F_1, the expected call profits

$$\pi_1(x,\hat{x}) = G(\hat{x}|x)(p_{11} - c_{11})q(p_{11}) + [1 - G(\hat{x}|x)]\,(p_{12} - c_{12})q(p_{12}),$$

and the expected termination profits

$$R_{12}(x,\hat{x}) = (a - c_0)q(p_{21}) \int_{\hat{x}}^{1} g(x|x')dx'.$$

We can thus write the total expected profits that network 1 obtains from a given subscriber at location x as

$$\Pi_1(x,\hat{x}) = \pi_1(x,\hat{x}) + F_1 + R_{12}(x,\hat{x}) - f.$$

Total expected profits of network 1 can now be written as

$$\begin{aligned}\overline{\Pi}_1\,(\hat{x}) &= \int_0^{\hat{x}} \Pi_1(x,\hat{x})dx \\ &= \hat{x}\,(F_1 - f) + L_{11}(\hat{x})(p_{11} - c_{11})q(p_{11}) \qquad (2)\\ &\quad + L_{12}(\hat{x})(p_{12} - c_{12})q(p_{12}) + L_{21}(\hat{x})(a - c_0)q(p_{21}).\end{aligned}$$

Similar expressions can be obtained for network 2.

Optimal prices. Take the marginal subscriber $\hat{x}$, and thus the networks' market shares $\hat{x}$ and $1 - \hat{x}$, as given. We consider how networks optimally choose on- and off-net prices so as to maximize profits, holding these market shares constant.

More specifically, we consider the following program. We take as given the gross utility level that the marginal consumer must obtain: $U_1(\hat{x},\hat{x}) \geq \overline{U}$. (In equilibrium, the latter will be determined by the offer of the competing network, i.e., $\overline{U} = U_2(\hat{x},\hat{x})$.) For given $\hat{x}$ and $\overline{U}$, we then solve for the choices p_{11} and p_{12} that maximize $\bar{\Pi}_1$. We first relax this program by only considering the participation constraint of the marginal consumer $x = \hat{x}$ but not those of consumers $x < \hat{x}$, and then state a sufficient condition for when (both on- and off-equilibrium) the solution to the relaxed program is indeed a solution to the original one.

Let now $\hat{L}_{11}(\hat{x}) = \hat{x}G(\hat{x}|\hat{x})$ be the total number of on-net calls on network 1 that *would* arise if all subscribers of network 1 had the same calling pattern as the marginal subscriber

$\hat{x}$. With symmetric market shares we have $\hat{L}_{11}(1/2) = 1/4$. Define likewise the number of off-net calls that *would* obtain if all subscribers had the same calling pattern as the marginal subscriber $\hat{L}_{12}(\hat{x}) = \hat{x}(1 - G(\hat{x}|\hat{x}))$. For network 2 we define $\hat{L}_{22}(\hat{x}) = (1 - \hat{x})(1 - G(\hat{x}|\hat{x}))$ and $\hat{L}_{21}(\hat{x}) = (1 - \hat{x})G(\hat{x}|\hat{x})$.

Proposition 1 *Take the relaxed program of the two networks, where for each firm only the participation constraint of a given marginal customer $\hat{x}$ binds. Then, network i's charges for on-net calls satisfy*

$$\frac{p_{ii} - c_{ii}}{p_{ii}} = \frac{1}{\eta(p_{ii})}\left(1 - \frac{\hat{L}_{ii}(\hat{x})}{L_{ii}(\hat{x})}\right), \tag{3}$$

while those for off-net calls satisfy

$$\frac{p_{ij} - c_{ij}}{p_{ij}} = \frac{1}{\eta(p_{ij})}\left(1 - \frac{\hat{L}_{ij}(\hat{x})}{L_{ij}(\hat{x})}\right). \tag{4}$$

For any market share $\hat{x}$, if τ is sufficiently large such that for all $x \leq \hat{x}$

$$\frac{\partial G(\hat{x}|x)}{\partial x}\left(v_{11} - v_{12} + v_{22} - v_{21}\right) \leq 2\tau \tag{5}$$

holds, the above expressions characterize also the optimal prices for the two networks.

Proof. See Appendix.

While condition (5) is not stated in terms of the primitives alone, as the terms v_{ij} depend on the respective prices, note that τ does not enter call prices. Hence, holding all else constant, we can always choose the degree of horizontal differentiation τ and the fixed utility from participation v_0 large enough such that all consumers participate and (5) holds everywhere (cf. also Laffont et al., 1998b) if $\partial G/\partial x$ is bounded. In what follows, we assume that this is the case.

When the calling pattern of the average infra-marginal subscriber is the same as that of the marginal subscriber $\hat{x}$, as is the case with the uniform calling pattern G^U, then $\hat{L}_{ii}(\hat{x}) = L_{ii}(\hat{x})$ and $\hat{L}_{ij}(\hat{x}) = L_{ij}(\hat{x})$. Proposition 1 then yields the standard (perceived) marginal-cost pricing result $p_{ii} = c_{ii}$ and $p_{ij} = c_{ij}$. Yet, when the marginal subscriber makes more off-net calls and less on-net calls than the average subscriber, then it holds that $p_{ii} > c_{ii}$ and $p_{ij} < c_{ij}$. Intuitively, raising the on-net price above marginal cost and lowering the off-net price below marginal cost then allows the network to extract more of the "information rent" of all infra-marginal subscribers, who will not switch networks when they have to cede slightly more of their surplus. The pricing formula in Proposition 1 trades off the increase in profits that is made from infra-marginal subscribers with the compensation that must be given to the marginal subscriber in terms of an adjusted fixed fee.

Symmetric market shares. In light of our subsequent characterization of a market equilibrium, we now derive the metering result for the case with symmetric market shares $\hat{x} = 1/2$. Thereby, we also link the characterization of prices in Proposition 1 to our definition of concentrated calling patterns.

From the definition of more concentrated calling patterns, we have immediately that the respective numbers of on-net calls, $L_{11}(1/2)$ and $L_{22}(1/2)$, are strictly higher when calling patterns are more concentrated. Likewise, the respective numbers of off-net calls, $L_{12}(1/2)$ and $L_{21}(1/2)$, are strictly lower. As, given symmetry, the marginal customer at $\hat{x} = 1/2$ always makes half of his calls on-net and half of it off-net, we have, regardless of how concentrated calling patterns are, $\hat{L}_{11}(1/2) = \hat{L}_{12}(1/2) = 1/4$. From these observations we have immediately that, as calling patterns become more concentrated relative to G^U, prices in Proposition 1 become more distorted: The multiplier $1 - \frac{\hat{L}_{ii}(1/2)}{L_{ii}(1/2)} > 0$ in expression (3) increases, which pushes up $p_{ii} > c_{ii}$, and the multiplier $1 - \frac{\hat{L}_{ij}(1/2)}{L_{ij}(1/2)} < 0$ in expression (4) decreases, which pushes down $p_{ij} > c_{ij}$.

For our following analysis it is convenient to restate this result by introducing some additional notation. We define μ as a measure of how the average number of on-net calls per subscriber changes as the marginal subscriber is shifted away from the symmetric market share $\hat{x} = 1/2$:

$$\mu = \frac{d}{d\hat{x}} \left(\frac{L_{11}(\hat{x})}{\hat{x}} \right) \Bigg|_{\hat{x}=1/2}. \tag{6}$$

As we can show that $\mu = 4L_{12}(1/2)$,[10] we have that $\mu = 1$ for a uniform calling pattern and that μ is strictly *lower* when calling patterns are more concentrated. As we have already observed that $\hat{L}_{11}(1/2) = 1/4$, we obtain the following result for symmetric market shares.

[10]From (1) it follows that $L'_{12}(1/2) = 0$, and thus

$$\mu = -\frac{d}{d\hat{x}} \left(\frac{L_{12}(\hat{x})}{\hat{x}} \right) \Bigg|_{\hat{x}=1/2} = \frac{L_{12}(\hat{x})}{\hat{x}^2} - \frac{L'_{12}(\hat{x})}{\hat{x}} \Bigg|_{\hat{x}=1/2} = 4L_{12}(1/2).$$

Corollary 1 *With symmetric market shares, on-net and off-net prices are*

$$\begin{aligned}\frac{p_{ii}-c_{ii}}{p_{ii}} &= \frac{1}{\eta(p_{ii})}\left(\frac{1-\mu}{2-\mu}\right), \\ \frac{p_{ij}-c_{ij}}{p_{ij}} &= -\frac{1}{\eta(p_{ij})}\left(\frac{1-\mu}{\mu}\right),\end{aligned} \tag{7}$$

where $\mu < 1$ holds for concentrated calling patterns and is strictly lower as calling patterns become more concentrated.

4 Market Equilibrium

To characterize the market equilibrium, it is convenient to introduce one more piece of notation. We denote by $\widehat{G}(\hat{x}) = G(\hat{x}|\hat{x})$ the number of on-net calls of the marginal customer $\hat{x}$. Clearly, $\widehat{G}(0) = 0$, $\widehat{G}(1/2) = 1/2$, and $\widehat{G}(1) = 1$. We assume that $\widehat{G}$ is differentiable at $\hat{x} = 1/2$, and further define

$$\hat{\mu} = \widehat{G}'(1/2). \tag{8}$$

Since $\widehat{G}(\hat{x})$ is the marginal subscriber's number of on-net calls on network 1, its derivative, $\hat{\mu}$, measures how this number varies as the marginal subscriber changes. With a uniform calling pattern, we have $\hat{\mu} = 1$, as the number of calls that the marginal customer, just as any other customer, makes to a given network is just equal to the respective market share. Intuitively, when the calling pattern is concentrated, then $\hat{\mu} < 1$, while $\hat{\mu}$ *decreases* when the calling pattern becomes more concentrated.[11] After all, when the calling pattern is more concentrated, as the marginal customer shifts, the fraction of calls that he makes to either network is less closely tied to networks' market shares but more closely to the customer's own location (cf. also the discussion in Section 6).

The marginal cost of expanding market share. Throughout the subsequent analysis we assume existence of a unique symmetric equilibrium in pure strategies.[12] As the marginal consumer must be indifferent between the offers of the two networks, i.e., $U_1(\hat{x}, \hat{x}) = U_2(\hat{x}, \hat{x})$, we have

$$F_1 = F_2 + u_1(\hat{x}, \hat{x}) - u_2(\hat{x}, \hat{x}) + \tau(1 - 2\hat{x}). \tag{9}$$

[11] Formally, this follows immediately from the definition of a more concentrated calling pattern around $x = 1/2$.

[12] A proof of existence of equilibrium follows the same steps as in Laffont et al. (1998a,b) and is therefore omitted here.

From this we obtain

$$\frac{dF_1}{d\hat{x}} = \widehat{G}'(\hat{x})\left(v_{11} + v_{22} - v_{12} - v_{21}\right) - 2\tau,$$

which in a symmetric equilibrium and after substituting from (8) becomes

$$\left.\frac{dF_1}{d\hat{x}}\right|_{\hat{x}=1/2} = -2\left[\tau - \hat{\mu}\left(v_{ii} - v_{ij}\right)\right]. \tag{10}$$

Expression (10) captures how expensive it is for a network to shift the marginal subscriber and, thereby, capture market share. In the original Hotelling model the respective marginal cost would be just 2τ. This remains true when on- and off-net call prices are identical, as then $v_{ii} = v_{ij}$. On the other hand, when off-net calls are more expensive than on-net calls, as is usually the case in practice, $v_{ii} > v_{ij}$ and tariff-mediated network externalities are created. If $\hat{\mu} > 0$ it then becomes *less* expensive for a network to expand its market share. For $p_{ii} > p_{ij}$ and thus $v_{ii} < v_{ij}$ the opposite holds. Importantly, the marginal cost of expanding a network's market share (10) is affected by the value of $\hat{\mu}$ if there are tariff-mediated network externalities. The effect of the latter is therefore reduced under a more concentrated calling pattern.

Equilibrium profits. We will now derive from (2) networks' fixed fees and profits in symmetric equilibrium. Given the tariff of network 2 and the optimal structure of call prices discussed above, network 1 maximizes its profits by adjusting its fixed fee, or equivalently, its market share.

In a symmetric equilibrium, we have $p_{ii} = p_{jj}$ and $p_{ij} = p_{ji}$. It is then convenient to denote the per-call profits from on-net calls by

$$r_{ii} = (p_{ii} - c_{ii})q(p_{ii}).$$

Also denote

$$r_{ij} = (p_{ij} - c_{ij})q(p_{ij}) + (a - c_0)q(p_{ji}),$$

which represents network i's profits from an exchange of one pair of off-net calls with network j. This can be simplified, after substitution of $c_{ij} \equiv c_1 + c_0 + a$, to obtain for a symmetric equilibrium where $p_{ij} = p_{ji}$ that $r_{ij} = (p_{ij} - c_{ii})q(p_{ij})$, where $c_{ii} \equiv 2c_0 + c_1$ is the true marginal cost.

In the proof of Proposition 2 below we show how, after substituting for $dF_1/d\hat{x}$ from (10) and using symmetry, we can solve the first-order condition for profit-maximization to obtain

$$F^* = f + \tau - r_{ii} - \hat{\mu}\left(v_{ii} - v_{ij}\right). \tag{11}$$

The equilibrium fixed fee increases in the per-customer fixed cost and in the transport cost, as usual. Substituting F^* back into expression (2) for profits leads to the following outcome.

Proposition 2 *In a symmetric equilibrium, profits for each network are equal to*

$$\bar{\Pi}^* = \frac{1}{2}\left[\tau - \hat{\mu}(v_{ii} - v_{ij}) + \frac{\mu}{2}\left(r_{ij} - r_{ii}\right)\right]. \tag{12}$$

Proof. See Appendix.

Note that so far we still consider the access charge a to be exogenous. Before we make use of the result in Proposition 2 to endogenize the latter, we offer some interpretation for expression (12). In the original Hotelling model profits would be equal to $\tau/2$. Thus, the first term in expression (12) captures, in the traditional manner, how profits depend on the substitutability of networks.

The second term in expression (12) captures the effect of the tariff-mediated network externalities on the marginal subscriber. If the term $\hat{\mu}(v_{ii} - v_{ij})$ is positive, as a result of off-net prices above on-net prices, then these externalities are positive and it is easier to capture market share. When on-net prices are above off-net prices, on the other hand, then these externalities are negative, and it is more costly to capture market share. Importantly, when it is easier to capture market share, competition is more intense, and equilibrium profits are lower. The relevance of this term depends on the relevance of the calling circle of the marginal consumer, as described by $\hat{\mu}$.

We come now to the third term in expression (12), which describes how profits due to infra-marginal subscribers change with the difference between on- and off-net prices. Starting from symmetric market shares, when a network deviates and captures more market share, it increases the number of on-net calls at the expense of decreasing the number of both outgoing and incoming off-net calls. Observe also that $r_{ij} - r_{ii}$ captures the true difference in profits between off-net and on-net calls, i.e., evaluated at the true marginal cost. The third term in (12) thus captures how, through turning off-net calls into on-net calls, a marginal increase in market share, starting from $\hat{x} = 1/2$, impacts on profits. As this effect works through all subscribers on a given network, its importance depends on the average behavior of subscribers, as described by the term μ. A more concentrated calling pattern, i.e., a lower μ, reduces the profits that can be derived from tariff-mediated network externalities. Taken together, the second and third terms in expression (12) capture the tariff-induced *costs* and *benefits* from acquiring customers.[13] Both costs and benefits decrease for more concentrated calling patterns.

5 Dampening Competition through Access Charges

In this section we will determine the jointly profit-maximizing reciprocal access charge a. This is the access charge that networks would want to negotiate if they were free to choose between themselves.

When firms set the reciprocal access charge, they maximize joint profits, which in a symmetric equilibrium are equal to $2\bar{\Pi}^*$. There are now two opposing effects that need to be considered, corresponding to the second and third terms in expression (12) for profits. We referred to these terms as the costs and benefits from capturing additional market share. Take first the costs, i.e., the second term in expression (12). Decreasing the access charge pushes down off-net prices (cf. Proposition 1), leading to a decrease in the utility difference $v_{ii} - v_{ij}$. This makes joining a smaller network more attractive for customers and, thereby, dampens competition through increasing the costs of capturing market share. From this perspective alone firms should thus lower a. However, lower off-net prices decrease the profits $r_{ij} = (p_{ij} - c_{ii})q_{ij}$ from making and receiving off-net calls, at least as long as the off-net price induced by a is still below the "monopoly price"

$$p_M = \arg\max_p [(p - c_{ii})q(p)].$$

This increases the benefits from capturing market share, so that there is a countervailing effect from the third term in expression (12).

Put differently, when a increases the effect that works through a reduction of the cost of acquiring the marginal customer leads to more intense competition and thus lower profits.

[13] Note that if firms were constrained to offer *uniform* two-part tariffs, i.e., if they were to charge identical prices for on- and off-net calls, then the second and the third term in expression (12) would be zero. Profits would be constant at $\tau/2$ and thus independent of the level of the access charge. Under uniform pricing, therefore, the access charge is profit-neutral even for non-uniform calling patterns.

This effect is stronger when $\hat{\mu}$ is larger due to a less concentrated calling pattern, since then a shift of the marginal customer has a larger effect on the marginal customer's share of on-net calls. The other effect, which works through a decrease in the benefits of acquiring the marginal customer when a increases, leads to less intense competition and thus higher profits. This effect also increases with a less concentrated calling pattern, i.e., μ is larger, since more off-net calls will be transformed into on-net calls.

As shown in the proof of the Proposition 3 below, from the aforementioned two opposing effects, we obtain that the optimal off-net price that the networks wish to jointly implement through their choice of the access charge is indeed strictly decreasing in $\hat{\mu}$ and strictly increasing in μ:

$$\frac{p_{ij} - c_{ii}}{p_{ij}} = \frac{1}{\eta\left(p_{ij}\right)}\left(1 - \frac{2\hat{\mu}}{\mu}\right). \tag{13}$$

When calling patterns are uniform, such that $\hat{\mu} = 1$ and $\mu = 1$, from (13) we obtain $p_{ij} < c_{ii}$, which together with Proposition 1 results in $a < c_0$, i.e., the result of Gans and King (2001). More generally, once we substitute the equilibrium prices from Proposition 1 into (13), we obtain the following result.

Proposition 3 *The jointly profit-maximizing access charge is*

$$a^* = c_0 + c_{ii}\frac{1 - 2\hat{\mu}}{\mu\left[\eta\left(p_{ij}\right) - 1\right] + 2\hat{\mu}}. \tag{14}$$

In particular, a^* is strictly above the cost of termination ($a^* > c_0$) if calling patterns are sufficiently concentrated so that

$$\hat{\mu} < \frac{1}{2}, \tag{15}$$

while $a^* < c_0$ applies when $\hat{\mu} > \frac{1}{2}$.

Proof. See Appendix.

Whether the profit-maximizing access charge is above or below the cost of termination is thus closely tied to the parameter $\hat{\mu}$. When calling patterns are sufficiently concentrated, the result that Gans and King (2001) obtained with a uniform calling pattern is overturned, as then the access charge is chosen *above* cost in order to dampen competition. In other words, the balance between the costs and benefits of expanding market share changes for more concentrated calling patterns, with the benefits of higher off-net prices outweighing their cost.

On-net vs. off-net prices. Having determined the jointly profit-maximizing access charge, we can finally return to the question of whether on-net prices will be higher or lower than off-net prices if networks adopt this access charge. While metering makes off-net prices lower than the respective cost, the access charge effect pushes in the opposite direction, to the extent that $a^* > c_0$, as characterized by the previous Proposition. The following result shows that the outcome is determined by the relative strength of these two countervailing effects.

Proposition 4 *The on-net price is lower than the off-net price, at the profit-maximizing access charge, iff*

$$\hat{\mu} < \frac{1}{2}\frac{\mu}{2-\mu}. \tag{16}$$

Proof. See Appendix.

The effects of more concentrated calling patterns involves contradictory forces in this case. Essentially, we need to distinguish between the effects on marginal and average subscribers. If the calling pattern is sufficiently concentrated for the marginal subscriber (low $\hat{\mu}$), and at the same time sufficiently less concentrated for the average subscriber (high μ), then the on-net price will be below the off-net price at the profit-maximizing access charge. This happens because lower $\hat{\mu}$ implies a higher access charge and higher μ a higher off-net price, both of which favor setting off-net prices above on-net prices.

With a uniform calling pattern, where $\hat{\mu} = \mu = 1$, condition (16) clearly is not satisfied, thus confirming the result of Gans and King (2001). Below we further analyze condition (16) for the family of calling patterns G^{λ}.

Welfare and consumer surplus. We ask next how the networks' profit-maximizing access charge compares with the access charge that a social planner would optimally set. The consumer surplus, in a symmetric equilibrium, is

$$CS = v_0 + 2L_{ii}(1/2)v_{ii} + 2L_{ij}(1/2)v_{ij} - F^* - 2\int_0^{1/2} \tau x dx.$$

Substituting for F^* from (11), we have

$$CS = v_0 - f - \frac{5}{4}\tau + r_{ii} + \left(1 - \frac{\mu}{2} + \hat{\mu}\right) v_{ii} + \left(\frac{\mu}{2} - \hat{\mu}\right) v_{ij}. \tag{17}$$

Suppose now that the social planner would want to maximize social welfare: $W = 2\bar{\Pi}^* + CS$. As shown in the Appendix, this yields $p_{ij} = c_{ii}$ and

$$a^W = c_0 + c_{ii}\frac{1-\mu}{\mu}\frac{1}{\eta(c_{ii})}, \tag{18}$$

where $1 - \mu = 0$ in the case of a uniform calling pattern and strictly positive for any concentrated calling pattern. Intuitively, for non-uniform calling patterns, it is efficient to set the access charge above cost so as to "compensate" for the firms' using distorted prices as a "metering device".

Proposition 5 *A social planner would set $a^* = a^W$, as given by (18), when he wants to maximize social welfare. This results in an access price strictly above cost for all concentrated calling patterns. The profit-maximizing access charge is socially optimal if and only if*

$$\hat{\mu} = \mu/2.$$

Proof. See Appendix.

For completeness, suppose now instead the social planner would want to maximize consumer surplus. As only off-net prices but not on-net prices are affected by the access charge a^*, this yields a "bang-bang" solution that is immediately obtained by looking at the last term in expression (17): When $\hat{\mu} < \mu/2$, to maximize consumer surplus the social planer would want to push down a^* as far as possible, thereby decreasing p_{ij} and increasing v_{ij}. This happens when $\hat{\mu}$ is low, and the access charge has a limited effect on the fixed fee F^*, as given by (11). On the other hand, when $\hat{\mu} > \mu/2$, the regulator would want to push up a^* as far as possible. This is because, despite shutting down off-net communications and the associated surplus, in this range the externality effect intensifies competition for the market via lower fixed fees, to the benefit of consumers.

6 Implications

In this section we use our characterization of the market equilibrium to obtain some additional positive implications on readily obtainable market data from the industry. As we will make use also of our particular choice of "calling circles", which we introduced in Section 2, we first complete the specification of the respective family of calling patterns

G^{λ}. Recall that with this specification, each customer calls any other customer randomly with probability $1-\lambda$, while he calls those in his symmetric calling circle with probability λ. For the calling circle, we specified the symmetric distribution $H(z)$ with support $z \in [-\varepsilon, \varepsilon]$, where $0 < \varepsilon < 1/2$. For those customers who are not too close to the "corners", as $\varepsilon \leq x \leq 1-\varepsilon$, we then have $G^{\lambda}(y|x) = (1-\lambda) y + \lambda H(y-x)$.[14] We already noted above that for $\lambda = 0$ the calling pattern is uniform, while it becomes more concentrated the higher is λ.

It is further convenient to denote the dispersion of calls within the calling circle by[15]

$$\delta = \int_{-\varepsilon}^{\varepsilon} |x| \, h(x) dx = 2 \int_{0}^{\varepsilon} x h(x) dx \leq \varepsilon.$$

With this notation at hand, we have that, for $z \in [\varepsilon, 1-\varepsilon]$,

$$L_{11}(z) = (1-\lambda) z^2 + \lambda \int_{0}^{z} H(z-x) \, dx = (1-\lambda) z^2 + \lambda \left(z - \frac{\delta}{2} \right),$$

and from the definition (6) we obtain that $\mu = 1 - \lambda (1 - 2\delta)$ for the family G^{λ}, which is thus indeed decreasing in λ. Recall that μ measures how different is the calling pattern of the marginal customer, $\hat{x} = 1/2$, from the average calling pattern of all infra-marginal customers, e.g., customers $x < 1/2$ for network 1. Further, from $G^{\lambda}(x|x) = (1-\lambda) x$, we now have $\hat{\mu} = 1 - \lambda$. This expression is particularly revealing. With a uniform calling pattern, we already observed that as the marginal customer $\hat{x}$ increases, his fraction of calls made to network 1 increases one-by-one with the market share of this network, so that $\hat{\mu} = 1$. This clearly corresponds to the case where $\lambda = 0$. At the other extreme, when a customer *only* makes calls to his calling circle and no random calls, then the fraction of calls that the marginal customer makes to either network is completely independent of his location, so that $\hat{\mu} = 0$ when $\lambda = 1$.

[14] Though this is inconsequential for our analysis, for all other customers, $x < \varepsilon$ and $x > 1-\varepsilon$, we can complete the specification by choosing, for instance, that

$$G^{\lambda}(y|x) = (1-\lambda) y + \lambda \left[H(y-x) + H(y+x) - H(2-y-x) \right].$$

Essentially, the bits of the calling clubs that "stick out" are thereby folded back into the preference space. Note that with this specification calling patterns are always symmetric and balanced. For callers x and receivers y such that $\varepsilon \leq y + x \leq 2 - \varepsilon$, this simplifies to the expression in the main text. We also note that the results we derive below for this example could also be obtained using the Salop circle instead of the often-employed Hotelling line, with two networks placed at opposite points of a unit circumference. The only difference is that, with the Salop circle, every consumer always has people to call both on his left and on his right. This difference is immaterial for our findings.

[15] For example, if H is a uniform distribution with density $h(z) = 1/2\varepsilon$, then $\delta = \varepsilon/2$, or if H is a tent distribution with density $h(z) = (\varepsilon - |z|)/\varepsilon^2$ we have $\delta = \varepsilon/3$.

6.1 Call Imbalance Ratios

One of our motivations from the Introduction was that observed calling patterns are difficult to reconcile with the specification of uniform calling patterns that is mostly used in the literature. A useful empirical measure of inter-network calling patterns is the *call imbalance ratio*, which is defined as the ratio of on- to off-net calls per recipient. When we calculate the imbalance ratio that obtains in equilibrium, where market shares are symmetric, we obtain for either network the ratio[16]

$$r = \frac{L_{ii}(1/2)}{L_{ij}(1/2)} = \frac{2-\mu}{\mu}.$$

This is equal to one when the calling pattern is uniform, and strictly above one when the calling pattern is concentrated. It is also decreasing in μ, so that the imbalance ratio is strictly higher when the calling pattern is more concentrated. For our specification of calling circles G^λ, we have in terms of primitives

$$r = \frac{1+\lambda\,(1-2\delta)}{1-\lambda\,(1-2\delta)},$$

which is always greater than 1 for any $\lambda > 0$ and indeed increases as calling patterns become more concentrated.

This measure of call imbalances is in terms of the number of *calls* that are made. Empirically, one may also have access to the overall number of *minutes*, which in our model are sensitive to off- and on-net prices. In a symmetric equilibrium, we have for the respective minutes imbalance ratio for either network

$$r^m = \frac{2-\mu}{\mu}\frac{q(p_{ii})}{q(p_{ij})}.$$

Our specification of calling patterns G^λ allows us now to proceed further in obtaining equilibrium prices and, thereby, r^m in terms of the primitives of the model. For this we specify, in addition, that demand is isoelastic and thus captured by $\eta > 1$. From Proposition 1 we then first obtain the respective on- and off-net prices

$$\frac{p_{ii}-c_{ii}}{p_{ii}} = \frac{1}{\eta}\frac{\lambda\,(1-2\delta)}{1+\lambda\,(1-2\delta)}, \quad \text{and} \quad \frac{p_{ij}-c_{ij}}{p_{ij}} = -\frac{1}{\eta}\frac{\lambda\,(1-2\delta)}{1-\lambda\,(1-2\delta)}. \tag{19}$$

[16] For asymmetric market shares, the imbalance would be, for network 1 for instance, $r_1(\hat{x}) = \frac{L_{11}(\hat{x})/\hat{x}}{L_{12}(\hat{x})/(1-\hat{x})}$. The Italian data given in the Introduction imply imbalance ratios of, respectively, 4.15 (TIM), 6.88 (Vodafone), 9.33 (Wind) and 4.43 (H3G).

Note that as calling circles become more concentrated (higher λ), on-net prices increase and off-net prices decrease. The same holds as calls made to the calling circle become less dispersed (lower δ). In both cases, prices become more distorted as the calling pattern of the marginal customer diverges more from that of infra-marginal customers. To evaluate prices in (19) at the profit-maximizing access charge, we obtain next

$$a^* = c_0 - \frac{1 - 2\lambda}{(\eta + 1)(1 - \lambda) + 2(\eta - 1)\lambda\delta} c_{ii}, \tag{20}$$

so that we have from (19) and (20) that now the on-net price is below the off-net price when

$$\lambda \geq \lambda^* = \frac{1 - 6\delta + \sqrt{9 - 28\delta + 36\delta^2}}{4(1 - 2\delta)}. \tag{21}$$

That is, calling circles must be sufficiently relevant for off-net calls to be more expensive than on-net calls. The condition is stricter when δ is low, i.e., when calls made to the calling circles are less dispersed. This effect arises because, as we observed, less dispersion of calls made to calling circles implies lower off-net prices, which counters the effect of the rising access charge.

Taken together, we can substitute from (19) and (20) to obtain, for the chosen specification of calling circles and with isoelastic demand,

$$r^m = \frac{1 + \lambda(1 - 2\delta)}{1 - \lambda(1 - 2\delta)} \left(\frac{\eta + \frac{1 - \lambda - 2\delta\lambda}{1 - \lambda + 2\delta\lambda}}{\eta - \frac{(1 - 2\delta)\lambda}{1 + \lambda - 2\delta\lambda}} \right)^{-\eta}.$$

Note that from (21), at $\lambda = \lambda^*$, both imbalance ratios will take the same value $r^m = r > 1$, both in terms of calls and in terms of minutes, as on-net prices are identical to off-net prices and thus call minutes are also the same. More generally, for call minutes, an increase in the importance of calling circles now increases the imbalance through two channels when $\lambda > \lambda^*$ and the access price is set at its profit-maximizing level: Both as more calls will be made to the circle and as on-net calls will become relatively cheaper compared to off-net calls. Instead, when the calling pattern is uniform, off-net calls are always cheaper than on-net calls and the imbalance ratio will be below one when calculated in terms of minutes.[17]

[17] At $\lambda = 0$, it is $r^m = \left(\frac{\eta+1}{\eta}\right)^{-\eta} < 1$ for all $\eta > 1$.

6.2 Waterbed Effect

In many jurisdictions around the world, wholesale access charges are subject to some form of regulation. While changes in a should obviously directly affect the price for off-net calls and networks' termination revenues, policy-makers also have a practical interest in understanding how their intervention may influence the structure of *other* prices and, ultimately, the customer's bill, which is frequently referred to as a "waterbed" or "seesaw" effect. This possible rebalancing in the price structure is an information readily obtainable from price data.[18] In particular, with two-part tariffs a change in the access price translates into a change of off-net prices and the fixed fees.

We thus return to Section 4, and notice that inspection of (11) shows that the access charge a has an indirect effect on the equilibrium fixed fee through the off-net indirect utility, v_{ij}, since the latter depends on the off-net price p_{ij} and thus on a. With constant elasticity of demand, the off-net price $p_{ij} = (c_{ii} + a - c_0)\,\frac{\mu\eta}{\mu(\eta-1)+1}$ increases in a, and for the fixed fee

$$\frac{dF^*}{da} = -\hat{\mu} q_{ij} \frac{dp_{ij}}{da} = -\hat{\mu} \frac{p_{ij} q_{ij}}{c_{ii} + a - c_0}.$$

Recall now that a more concentrated calling pattern leads to a lower $\hat{\mu}$, which dampens the waterbed effect, i.e., it mitigates the decrease in the fixed fee. This happens because with concentrated calling patterns additional marginal subscribers will make relatively more off-net calls. These customers become even less attractive with a higher access charge, so that the compensation in the fixed fee offered is lower. Apart from the direct effect through $\hat{\mu}$ there is, however, also an indirect effect of a change in concentration of calling patterns, namely through μ: A lower value of μ, as implied by a more concentrated calling pattern, now amplifies the waterbed effect. This second effect arises because

$$\frac{\partial^2 p_{ij}}{\partial a \partial \mu} = \frac{\mu}{[\mu(\eta - 1) + 1]^2} > 0.$$

That is, a more concentrated calling pattern reduces the pass-through of the access charge to off-net prices. What ultimately matters, though, is how μ affects off-net call revenues $p_{ij} q_{ij}$. As we are in the elastic portion of the demand for calls, $\eta > 1$, total revenues decrease with p_{ij}, and hence with μ. This effect will depend on the elasticity: it is highest for high values of η, while it disappears as we approach unit-elastic demand.

[18] Cf. Genakos and Valletti (2010).

Again, for the family of calling patterns G^{λ}, we can proceed somewhat further. In particular, in the limit, when consumers make *only* calls to their circle ($\lambda = 1$), the fixed fee is *independent* of the access charge, such that there is *no* waterbed effect on the fixed fee.[19] Generally, we have from

$$\frac{dF^*}{da} = -(1-\lambda)(c_{ii} + a - c_0)^{-\eta}\left(1 - \frac{1}{\eta} + \frac{1}{\eta}\frac{1}{1-\lambda(1-2\delta)}\right)^{\eta-1}$$

that at least when calling patterns are very concentrated ($\lambda \to 1$), there should be only a marginal waterbed effect with respect to the fixed fee.

7 Conclusions

We introduce a flexible model of network competition with non-uniform calling patterns. The model allows us to analyze the implications of non-uniform calling patterns ("calling circles") on equilibrium outcomes as well as profit-maximizing reciprocal access charges.

We show how equilibrium tariffs (on- and off-net call prices, and fixed fees) vary depending on the calling pattern. If calling patterns are concentrated then networks attempt to extract more rents from subscribers by setting higher on-net and lower off-net prices than under the uniform calling pattern. Concentrated calling patterns can also help to explain the (on-net to off-net) imbalance ratios that are observed in practice.

We analyze when networks would choose reciprocal access charges above cost or below cost so as to thereby dampen competition. With uniform calling patterns, it is known that this is achieved through setting access charges below cost. We show that this result is reversed if calling patterns are sufficiently concentrated: Profit-maximizing access charges are set above cost, which sustains high off-net prices. Our results on above-cost access charges also imply that contrary to other results in the literature, at the profit-maximizing reciprocal access charge on-net prices can be below off-net prices. We analyze how these different results are obtained from the interaction of two effects: Competition is dampened when it becomes relatively more expensive to "capture" the marginal customer or when the marginal customer is made less profitable for both networks. We show how the strength of either effect changes when calling patterns become more concentrated.

[19] The access charge has, however, an effect on off-net prices also in this case. In fact, we can show generally that the *total bill* of each individual customer strictly increases in a.

As in much of the literature on network competition, we restricted consideration to a model with only two networks. This has the benefit of making our results comparable to extant results. Also, we are able to offer a simple definition of our concept of more concentrated calling patterns. Extending this concept, as well as results, to cases where more networks compete could be a fruitful avenue to bring the analysis closer to models of real markets. Likewise, consumers' preferences for networks, which may arise from marketing efforts, could be endogenized in future work.

8 Appendix: Omitted Proofs

Proof of Proposition 1. Given constant market shares, network 1's marginal customer is determined by the condition $U_1(\hat{x},\hat{x}) = \bar{U}$, which can be restated as $F_1 = u_1(\hat{x},\hat{x}) + v_0 - \tau\hat{x} - \bar{U}$. Substituting $\hat{L}_{11}(\hat{x}) = \hat{x}G(\hat{x}|\hat{x})$ and $\hat{L}_{12}(\hat{x}) = \hat{x}(1 - G(\hat{x}|\hat{x}))$ into network 1's profits leads to

$$\begin{aligned}\overline{\Pi}_1(\hat{x}) &= \hat{L}_{11}(\hat{x})v(p_{11}) + L_{11}(\hat{x})(p_{11} - c_{11})q(p_{11}) \\ &\quad + \hat{L}_{12}(\hat{x})v(p_{12}) + L_{12}(\hat{x})(p_{12} - c_{12})q(p_{12}) + const.,\end{aligned}$$

where the last term on the right-hand side does not depend on p_{11} and p_{12}. We obtain from the maximization of the relevant terms with respect to p_{11} the first-order condition

$$(p_{11} - c_{11})q'(p_{11}) + \left(1 - \frac{\hat{L}_{11}(\hat{x})}{L_{11}(\hat{x})}\right) q(p_{11}) = 0,$$

which solves for the expression presented in the proposition. The result for the off-net price is derived similarly.

Finally, we state a sufficient condition that allows us to ignore the participation constraint of all subscribers with location $x < \hat{x}$, i.e., when indeed, as presumed in the relaxed program, $U_1(x,\hat{x}) \geq U_2(x,\hat{x})$ for all $x \leq \hat{x}$. We have

$$U_1(x,\hat{x}) - U_2(x,\hat{x}) = [u_1(x,\hat{x}) - u_2(x,\hat{x})] + \tau(1 - 2x) - [F_1 - F_2],$$

where

$$\begin{aligned}u_1(x,\hat{x}) - u_2(x,\hat{x}) &= \{G(\hat{x}|x)v(p_{11}) + [1 - G(\hat{x}|x)]\, v(p_{12})\} \\ &\quad - \{[1 - G(\hat{x}|x)]\, v(p_{22}) + G(\hat{x}|x)v(p_{21})\}.\end{aligned}$$

A sufficient condition for $U_1(x,\hat{x}) \geq U_2(x,\hat{x})$ holding for all $x < \hat{x}$ is that

$$\frac{\partial}{\partial x}\left[U_1(x,\hat{x}) - U_2(x,\hat{x})\right] \leq 0,$$

which is equivalent to

$$\frac{\partial G(\hat{x}|x)}{\partial x}\left[v(p_{11}) - v(p_{12}) + v(p_{22}) - v(p_{21})\right] \leq 2\tau,$$

as stated in the text. **Q.E.D.**

Proof of Proposition 2. Since calls are balanced with $L_{12}(\hat{x}) = L_{21}(\hat{x})$, and using for the accompanying change in on-net and off-net prices the envelope theorem, we obtain for symmetric call prices

$$\frac{d\overline{\Pi}_1(\hat{x})}{d\hat{x}} = F_1 - f + \hat{x}\frac{dF_1}{d\hat{x}} + L_{11}'(\hat{x})r_{11} + L_{12}'(\hat{x})r_{12}.$$

Note next that $L_{11}'(\hat{x}) + L_{12}'(\hat{x}) = 1$, which yields the first-order condition

$$F^* - f + r_{ii} + \frac{1}{2}\left.\frac{dF_1}{d\hat{x}}\right|_{\hat{x}=1/2} + L_{ij}'(1/2)(r_{ij} - r_{ii}) = 0.$$

Using $L_{ij}'(1/2) = 0$ and substituting finally for $dF_1/d\hat{x}$ from (10) yields expression (11).

To obtain equilibrium profits, in a symmetric equilibrium, and after substituting for F^* from (11), profits (2) become

$$\begin{aligned}\bar{\Pi}^* &= \frac{1}{2}(F^* - f + r_{ii}) + L_{ij}(1/2)(r_{ij} - r_{ii}) \\ &= \frac{1}{2}\left[\tau - \hat{\mu}(v_{ii} - v_{ij})\right] + \frac{\mu}{4}(r_{ij} - r_{ii}).\end{aligned}$$

Q.E.D.

Proof of Proposition 3. When differentiating profits in (12) w.r.t. a, note first that at a symmetric equilibrium $\hat{x} = 1/2$ does not change. Moreover, from Proposition 1 only off-net but not on-net prices change with a. Note further that $dp_{ij}/da > 0$ and that profits do not directly depend on a. Therefore we can equivalently maximize $\bar{\Pi}^*$ over p_{ij}. We obtain[20]

$$\begin{aligned}2\frac{d\bar{\Pi}^*}{dp_{ij}} &= -\hat{\mu}q_{ij} + \frac{\mu}{2}\left[q_{ij} + (p_{ij} - c_{ii})\, q_{ij}'\right] \\ &= \left[\frac{\mu}{2}\left(1 - \frac{p_{ij} - c_{ii}}{p_{ij}}\eta(p_{ij})\right) - \hat{\mu}\right] q_{ij} = 0,\end{aligned}$$

[20] It is easy to show that the sufficient second-order condition for a strict local maximum holds for a constant demand elasticity, which implies also that profits are quasi-concave in p_{ij}.

or

$$\frac{p_{ij} - c_{ii}}{p_{ij}} = \frac{1}{\eta\left(p_{ij}\right)}\left(1 - \frac{2\hat{\mu}}{\mu}\right). \tag{22}$$

This condition can be solved for p_{ij} as

$$p_{ij} = c_{ii}\frac{\mu\eta\left(p_{ij}\right)}{\mu\left(\eta\left(p_{ij}\right) - 1\right) + 2\hat{\mu}}.$$

On the other hand, from (7) we obtain

$$p_{ij} = \left(c_{ii} + a - c_0\right)\frac{\mu\eta(p_{ij})}{\mu\left(\eta(p_{ij}) - 1\right) + 1}. \tag{23}$$

Equating and solving for a leads to the result stated in the Proposition. **Q.E.D.**

Proof of Proposition 4. Equating p_{ij} in (22) and p_{ii} from (7) leads to $1 - 2\hat{\mu}/\mu = (1-\mu)/(2-\mu)$, or $\hat{\mu} = \mu/(4-2\mu)$. For larger $\hat{\mu}$ we will have $p_{ij} < p_{ii}$, due to (22). **Q.E.D.**

Proof of Proposition 5. Substituting for CS and concentrating only on the off-net elements, we obtain

$$W = 2\bar{\Pi}^* + CS = \frac{\mu}{2}\left(r_{ij} + v_{ij}\right) + const. \tag{24}$$

Thus the socially optimal off-net price continues to be $p_{ij} = c_{ii}$ even for general calling patterns, as should be expected. Equating to (23) and solving for a leads to the result in the text. The profit-maximizing access charge is equal to the socially optimal one if and only if the relative weights in the objective functions (12) and (24) on r_{ij} and v_{ij} are equal, or if $\hat{\mu} = \mu/2$. **Q.E.D.**

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