DISCUSSION PAPER SERIES

No. 8104

DYNAMIC LIMIT PRICING

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INDUSTRIAL ORGANIZATION

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ISSN 0265-8003

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Discussion Paper No. 8104
November 2010

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CEPR Discussion Paper No. 8104

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ABSTRACT

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JEL Classification: D43, D82, L11 and L41
Keywords: dynamic limit pricing, dynamic signaling and entry deterrence

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* I thank Yair Tauman for constructive conversations and exchanges of ideas at an early stage of this project. I also thank Alex Gershkov, Paul Heidhues, George Mailath, Romans Pancs, Heski Bar-Isaac and seminar participants at the Hebrew University of Jerusalem, Universidad Torcuato Di Tella, Universidad de San Andres, the University of Bonn and the University of Cambridge for incisive and useful comments. Last, I wish to thank participants at the following conferences for feedback and comments: the Econometric Society European Meetings, Vienna (2006), the annual meetings of the American Economic Association, Chicago (2007), the Royal Economic Society, Warwick (2008), the annual meetings of the European Association for Research in Industrial Economics, Toulouse (2008) and the Latin American Meetings of the Econometric Society, Buenos Aires (2009).

Submitted 01 November 2010

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1. Introduction

Since Bain's (1949) pioneering work, limit pricing has been a staple of industrial economics. In a nutshell, limit pricing is the practice by which an incumbent firm (or cartel) deters potential entry to an industry by pricing below the profit maximizing price level. Early work on the subject took its cue from the observation that in some industries, firms price below the myopic profit maximizing price level *on a persistent basis.* This observation lead to the notion that by doing so, incumbent firms could somehow discourage potential entry which would otherwise have occurred, in effect by sacrificing profits in the short run in return for a maintenance of the monopoly position.[1]

The present work revisits received wisdom on equilibrium limit pricing in dynamic contexts, by way of a dynamic extension of a simple static model of one-sided incomplete information

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[1] Bain (1949) noted that "[...] established sellers persistently or "in the long run" forego prices high enough to maximize the industry profit for fear of thereby attracting new entry to the industry and thus reducing the demands for their outputs and their own profits".

in the spirit of Milgrom and Roberts (1982). I demonstrate that while some aspects of the standard (static) analysis are preserved qualitatively when moving to dynamic contexts, the quantitative results may radically differ. Furthermore, new predictions on equilibrium price paths that have no static counterpart are obtained.

There are two main reasons for considering the dynamics of limit pricing. First, it is of interest purely as a matter of business strategy to determine assumptions on the primitives under which it is possible to deter entry through limit pricing. Second, the answer to this question may have interesting policy implications. Unlike instances of predatory pricing, limit pricing does not leave behind a visible victim. This means that it is very difficult to determine empirically if limit pricing has been attempted and if so, to what extent it has been successful. This means that in understanding limit pricing, economists as well as competition authorities must rely heavily on the insights gained through formal analysis. This fact makes it all the more important to check the received wisdom from economic theory.

In the dynamic version of the model, the basic entry stage game is repeated as long as entry has not occurred. In this model, I identify two distinct regimes, a *monopoly price regime* and a *limit price regime*. In the monopoly price regime, limit price equilibria exist but all such equilibria are ruled out by using Cho and Kreps' (1987) notion D1 at all information sets off the equilibrium path, as compared to a natural benchmark equilibrium in which the uninformed player makes use of all available information (in a sense that will be made precise). The unique reasonable equilibrium, using this notion, is one of immediate separation on monopoly prices. In the limit price regime, both reasonable pooling and separating equilibria exist and all these involve limit pricing. I find that in the limit price regime, the basic logic of separating equilibria of the static setting carries over to the separating equilibria of the dynamic setting. In particular, I find that by sacrificing enough at some (single) stage of the game, the efficient incumbent may credibly convey his identity to the entrant and thus maintain his monopoly position. One important difference with the static setting is that in the static setting, the benefits from deterring entry are bounded while this is no longer the case in the dynamic setting, if the players are sufficiently patient. For large enough discount factor and sufficiently long horizon, the efficient incumbent needs to press the inefficient incumbent to make strictly negative profits from mimicking.

In the most interesting case, i.e. the limit price regime, all reasonable equilibria belong to a single class, consisting of (i) a (possibly non-zero and possibly infinite) number of periods during which the two types of incumbent pool, (ii) a period in which the efficient type engages in costly signaling while the inefficient type reveals himself and invites entry and (iii) continuation play in which the efficient type charges monopoly prices in all subsequent periods and deters entry while the inefficient type competes against the entrant. The welfare properties of these equilibria are not straightforward. It is true, as is the case in the static model, that in the period where separation takes place, welfare is unambiguously higher than it would be under symmetric information. This is because entry occurs under the same states of nature as under symmetric information, but the efficient type sets lower prices than would a monopolist. But if separation is preceded by periods with pooling, the conclusion is less clear cut. This is because pooling deters entry, which counterweights the benefits of lower prices set by the incumbent.[2] The only unambiguous welfare statement that one may make without making specific assumptions about the market game is that the incumbent, irrespective of his type, prefers later separation to earlier separation.

1.1. The Literature on Limit Pricing. In his analysis, Bain (1949) identified two possible channels through which current prices may deter entry: (i) a low current price may signal to potential entrants that existing and future market conditions are unfavorable to entry and that (ii) a low current price may signal to potential entrants something about the incumbent's response to entry. The first generation of contributions focused almost exclusively on explanation (ii) and featured models that were fully dynamic in nature, an approach which is suitable for the study of ongoing relationships between competing firms. This literature expounded a number of interesting characterizations of equilibrium limit price paths that could in principle be confronted with the data (see Carlton and Perloff, 2004). Nonetheless, most contributions had the unsatisfactory feature that entrants' decisions were not the outcome of rational deliberations, but rather mechanistic (assumed) responses to the incumbent's pricing behavior.[3] Furthermore, incumbents in these models were endowed with perfect ability to commit to future (i.e. post-entry) pricing behavior. This critique, first articulated by Friedman (1979), cast serious doubt on the received wisdom from Bain's insights. Milgrom and Roberts (1982) confronted this challenge by reformulating the situation as one of incomplete information and drew on both explanations (i) and (ii). In doing so, they succeeded in validating Bain's insights. The main features and results of the Milgrom-Roberts analysis will be reproduced in the benchmark model of Section 2. Unlike the early theory on limit pricing, the Milgrom-Roberts analysis is essentially static in nature (since signaling through prices occurs once and for all). This raises the question as to how robust their findings are to dynamic extensions and to what extent their predictions are reconcilable with those of the previous literature.

The second generation of models (distinguished by featuring incomplete information as initiated by Milgrom and Roberts), have greatly shaped the way economists think about limit pricing. As such, it is important to determine to what extent the lessons are robust to variations in the modeling approach. Matthews and Mirman (1983) consider the possibility that the incumbent's price only provide noisy information to the entrant about the profitability of entry. Under certain conditions, they find that limit pricing can be successfully employed by the incumbent to limit entry. Harrington (1986) considers a variation of the basic model in which the entrant is uncertain of his own costs, which are in turn correlated with those of the incumbent. This modeling approach means that a high pre-entry price may signal that the entrant's costs are likely to be high, thereby making entry less appealing. In turn, this may imply that in equilibrium, the incumbent charges a price higher than the monopoly price and also deters entry. Jun and Park (2007) consider a dynamic setup where the incumbent faces a sequence of

[2] For a nice discussion of the welfare properties of such equilibria, see Tirole (1988).

[3] A model in which entry is an increasing deterministic function of pre-entry price is that of Gaskins (1971). In Berck and Perloff (1988), the rate of entry is proportional to future profitability. Models featuring stochastic entry which is increasing in the pre-entry price are those of Kamien and Schwartz (1971), Baron (1973), De Bondt (1976) and Lippman (1980). Flaherty (1980) takes a different approach and assumes increasing returns to scale in production, while Judd and Petersen (1986) consider the possibility that entrants must fund operations by means of internal finance. See Carlton and Perloff (2004) for a thorough review and further references.

entrants that can be either weak or strong, of which only the former can be deterred. Rather than having a strong opponent enter, the incumbent may wish to appear weak by charging a price higher than the monopoly price, thereby encouraging entry by weak entrants. This conclusion should be contrasted to that gained from the Milgrom-Roberts analysis.

The third generation of work on limit pricing seeks to come full circle by integrating the dynamic nature of the first generation with a careful treatment of informational issues as emphasized in second generation models. The present paper is a contribution to this branch of the literature. Closest to my analysis is the work of Kaya (2009), who studies repeated signaling in a reduced form setup. She assumes one-sided asymmetric information and focuses on separating equilibria. Her work complements the current analysis, focusing on somewhat different issues. In particular, she does not select between equilibria and focuses on the least cost separating equilibrium which allows the informed party to smooth costly signaling intertemporally.[4] In unpublished work, Saloner (1984) extends the Matthews and Mirman (1983) setup of noisy signaling to multi-period settings. This approach is also taken by Roddie (2010), who treats signaling games with a particular monotone structure. A recent paper by Gryglewicz (2010) treats a continuous-time signaling model in which the informed party's type is constant across time. He adopts the support restriction approach and focuses on pooling equilibria. Sorenson (2004) treats a dynamic model of limit pricing similar to the present one, but implicitly assumes that the informed party is unable to credibly signal his type in a single period. This gives rise to repeated signaling over time. While these papers all contribute to the literature on limit pricing, they also belong to the separate literature on signaling in dynamic settings. Because this literature is still in its infancy, below I will give a brief review of it an emphasize the contribution of the present analysis.

1.2. The Literature on Signaling in Dynamic Settings. In the vast majority of signaling models, there is only one instance of signaling, even if the model is otherwise dynamic. When there are multiple opportunities for the informed party to engage in signaling, the details of how (and if) private information changes over time and its observability by the uninformed party become crucial. The most conventional analyses are those of models in which private information is regenerated each period or in which the uninformed party's observations are imperfect signals of the informed party's actions. In either case, no signal will ever be deemed off the equilibrium path and hence updating on the equilibrium path can always be achieved by application of Bayes' rule. Papers of this type include Mester (1992) and Vincent (1998) as well as Saloner (1984) and Roddie (2010) in the context of limit pricing.

When private information is perfectly persistent over time and the informed party's actions are perfectly observable, then the modeler must confront the issue of assigning out of equilibrium beliefs. There have been two different approaches to deal with such beliefs in the existing literature, namely (i) *support restrictions* and (ii) *belief resetting*. Both approaches rely on the fact that the solution concept perfect Bayesian equilibrium does not impose any restrictions on beliefs after probability zero events. In the former approach, once posterior beliefs are concentrated entirely on some state of nature, no possible observation will prompt a shift of probability towards alternative states of nature. In other words, once posterior beliefs are

[4] This approach to repeated signaling was first adopted by Cho (1990).

degenerate, the game is treated as one of perfect information regardless of how it subsequently unfolds. In the latter approach, posteriors are allowed to fluctuate over time. In particular, this approach allows beliefs that assign positive probability to outcomes that previously were assigned zero probability.

Support restrictions have been used in different contexts by Rubinstein (1985), Grossman and Perry (1986) and LeBlanc (1992) and in a limit pricing context by Gryglewicz (2010). While such restrictions may be perfectly appropriate for some analyses, there are instances in which they are clearly inappropriate. Madrigal et al. (1987), Noldeke and van Damme (1990a) and Vincent (1998) discuss the treatment of degenerate posteriors in depth and show that such support restricted equilibria may fail to exist.

As an alternative to support restrictions, some authors have resorted to repeated resetting of beliefs. In actual fact, beliefs are degenerate along the entire equilibrium path when employing this approach but the equilibria are constructed *as if* beliefs are reset to their prior values.[5] This is the avenue taken by Cho (1990), by Kaya (2009) in a limit pricing context and discussed by Vincent (1998).

In terms of applications to economic modeling such as limit pricing, these two approaches differ radically in their predictions in that support restrictions effectively make repeated signaling impossible (by definition) while belief resetting allows for a potentially very rich set of equilibria in which signaling occurs repeatedly. The precise assumptions adopted by the modeler therefore have profound implications for the analysis at hand and therefore merit scrutiny. What support restrictions and belief resetting have in common is that with neither approach does the observation of out of equilibrium play prompt the uninformed party to make sense of the deviation. This is at odds with the way that static signaling models are habitually analyzed. In such settings, out of equilibrium beliefs are not all treated equally, some being deemed more reasonable than others. In this way, equilibrium selection techniques are useful in that they reduce the equilibrium set significantly, sometimes even to a unique reasonable equilibrium. Hitherto, equilibrium selection techniques have not been widely applied to dynamic settings of signaling. This is unfortunate since equilibrium selection obviates the need to choose between support restrictions and belief resetting. Furthermore, it is entirely consistent with the way that static signaling models are analyzed.

Last, I should mention some papers that do feature repeated signaling but which do not rely on the arbitrariness of out of equilibrium beliefs. These papers include Noldeke and van Damme (1990b), Bar-Isaac (2003) and Sorenson (2004). In these papers, the informed party is unable to effectively separate in a single period and is hence forced to distribute costly signaling over several periods.

The remainder of the paper is structured as follows. In Section 2, I introduce the basic static model that will constitute the building block of the dynamic analysis. I characterize separating and pooling equilibria, determine sufficient conditions under which the former exist and last I perform equilibrium selection analysis. In Section 3, I analyze the dynamic setting and compare the outcomes of this analysis to the static setting. Furthermore, I perform comparative statics

[5] This distinction is immaterial since under this approach, beliefs are not fully incorporated into subsequent behavior.

analysis with respect to the length of the horizon and give conditions for equilibrium existence. Section 4 concludes.

2. The Static Model

Although the main focus will be on the multi-period problem, I start out by analyzing the static problem in some detail.[6] I do this for two reasons. First, the static model will serve as the benchmark against which the dynamic model will be compared. Second, the analysis of the static problem plays an important role in the analysis of the equilibria of the dynamic game.

Consider an incumbent monopolist I and a potential entrant E. The monopolist serves a market with demand $Q(p)$ and the entrant can enter the market at cost $K > 0$ to compete with the incumbent. The monopolist can be one of two types, high cost (H) or low cost (L), with probability μ and $(1-\mu)$ respectively. The incumbent knows his type, but his type is unknown to the entrant. Let $C_H(q)$ and $C_L(q)$ be the cost functions of H and L respectively. Denote by $\pi_i(p)$ the profit function of the incumbent of type $i = H, L$ when he sets price p. These profits are given by

$$\pi_i(p) = pQ(p) - C_i(Q(p)), \quad i = H, L \tag{1}$$

Let D_i be the duopoly profit of the incumbent of type $i = H, L$ when competing against E and let $D_E(i)$ be the duopoly profits of E when competing against the incumbent of type $i = H, L$. Denote by p_H^M and p_L^M the monopoly prices under the technologies $C_H(\cdot)$ and $C_L(\cdot)$ respectively.

Throughout, I make the following assumptions:

Assumptions

1 $C_i(q)$, $i = H, L$ and $Q(p)$ are differentiable, for $q > 0$ and $p > 0$ respectively.

2 $C'_H(q) > C'_L(q), \forall q \in \mathbb{R}_+$, with $C_H(0) \geq C_L(0)$.

3 $Q'(p) < 0, \forall p \geq 0$.

4 $D_E(L) - K < 0$.

5 $D_E(H) - K > 0$.

6 $\pi_i(p)$ is strictly increasing for $p < p_i^M$ and strictly decreasing for $p > p_i^M$, $i = H, L$.

7 $\pi_i(p_i^M) > D_i$, $i = H, L$.

Assumption 2 makes precise the sense in which type L is more efficient than type H. Assumption 3 simply states that demand is downward sloping. Assumptions (2)-(3) jointly imply that $\pi_i(p)$ has the single crossing property, i.e. that $\pi_H(p)$ and $\pi_L(p)$ cross only once. Assumptions (4)-(5) imply that E will not enter if he knows that I is of type L while he will enter if he knows that I is of type H. Assumption 6 means that the incumbent's profit function is single peaked while Assumption 7 ensures that entry deterrence is desirable for the incumbent, ceteris paribus.

[6] Although this benchmark model has two periods, it is static in the sense that signaling can take place only once.

The game between I and E is played in three stages. At the first stage, I sets a price that will serve as a signal for E of I's type. After observing the price set by I, E decides at the second stage wether or not to enter (incurring the entry cost K). Denote E's entry decision by $s_E \in \{0,1\}$, where $s_E = 0$ stands for *stay out* and $s_E = 1$ stands for *enter*. At the third stage, if E enters he will learn I's type and compete against him in complete information fashion. Both incumbent and entrant discount the future by a factor $\delta \in [0,1]$. The payoff to E is given by

$$\Pi_E(p) \equiv \begin{cases} 0 & \text{if } s_E = 0 \\ D_E(H) - K & \text{if } s_E = 1, \quad i = H \\ D_E(L) - K & \text{if } s_E = 1, \quad i = L \end{cases} \tag{2}$$

A strategy for I is a price for each of his two types, p_H or p_L, at the first stage, a price at the second stage if the entrant stays out and a quantity or price to set at the third stage if the entrant enters (depending on the mode of competition), both as functions of his type and the decisions made at the first stage. A strategy for E is a decision rule to enter or not as a function of the price set by I at the first stage and a quantity or price to set at the third stage in case he enters (again, depending on the mode of competition).

If E enters at the second stage, then at the third stage I and E play a duopoly game of complete information. Hence in any subgame perfect equilibria of the game after E's entry, I's equilibrium payoffs in the third stage are D_H or D_L. If E stays out, then I's equilibrium payoffs at the third stage are $\pi_H(p_H^M)$ or $\pi_L(p_L^M)$, depending on his type.[7] That is, the payoffs to the incumbent of type $i = H, L$ are given by

$$\Pi_i(p) \equiv \begin{cases} \pi_H(p) + \delta\pi_H(p_H^M) & \text{if } s_E = 0, \quad i = H \\ \pi_H(p) + \delta D_H & \text{if } s_E = 1, \quad i = H \\ \pi_L(p) + \delta\pi_L(p_L^M) & \text{if } s_E = 0, \quad i = L \\ \pi_L(p) + \delta D_L & \text{if } s_E = 1, \quad i = L \end{cases} \tag{3}$$

Next, I state some key definitions that will be used throughout this section. Let $\sigma \equiv (p_L, p_H, \overline{p})$ denote a triple of pure strategies of the game, i.e. a price charged by each type of I and a threshold price governing E's entry decision (details are given below). Throughout this paper, attention will be restricted to pure strategy perfect Bayesian equilibria. Denote by p_H^* and p_L^* the equilibrium prices charged by the H type and the L type respectively.

Definition 1 σ is a separating equilibrium if $p_H^* \neq p_L^*$ and a pooling equilibrium if $p_H^* = p_L^*$.

Definition 2 σ is a limit price equilibrium if $p_H^* < p_H^M$ or $p_L^* < p_L^M$ or both.

The aim of the analysis that follows is to characterize separating and pooling limit price equilibria of the game. Before continuing with the analysis, I will first state the following useful results:

Lemma 1 (i) $\pi_L(p) - \pi_H(p)$ is strictly decreasing in p and (ii) $p_H^M > p_L^M$.

[7] In the dynamic version of the model, stages one and two will together constitute a period and stage three will be a separate period.

Proof. See Appendix A ■

Lemma 1 shows that under the maintained assumptions, the profit function $\pi_i(p)$ has the single crossing property, a necessary condition for signaling to be feasible.

Perfect Bayesian equilibrium requires that beliefs be derived from Bayes' rule whenever possible. This means that one must assign beliefs after out of equilibrium (i.e. probability zero) events have been observed. For simplicity, the out of equilibrium beliefs of E will be assumed to have the following monotone structure:

$$\mu'(p) = \begin{cases} 1 \text{ if } & p \leq p' \\ 0 \text{ if } & p > p' \end{cases}$$

where μ' is the probability assigned to the incumbent being of type L and p' is the L type's equilibrium strategy (i.e. either the separating price in a separating equilibrium or the common price in a pooling equilibrium).[8] That is, for any observed price above the L type's equilibrium price, the entrant will assign probability one to the incumbent being of type H. For prices below the L type's equilibrium price, the entrant will assign probability one to the incumbent being of type L.

This structure on beliefs implies that the entrant's optimal second-stage decision rule can be written in terms of the observed price as

$$s_E(p) = \begin{cases} 1 \text{ if } & p > \overline{p} \\ 0 \text{ if } & p \leq \overline{p} \end{cases} \tag{4}$$

for some appropriately chosen threshold price $\overline{p}$ (determined by the entrant). The equivalence is straightforward[9] and is shown below for each of the two types of equilibria respectively.

2.1. Separating Limit Price Equilibria.

Characterization. In a separating equilibrium, the entrant can, by definition, infer the incumbent's type merely by observing its chosen equilibrium price. Hence assume that $p_H^* \neq p_L^*$. The best reply strategy of E in this case is to enter if $p = p_H^*$ and to stay out if $p = p_L^*$, i.e. $s_E(p_H^*) = 1$ and $s_E(p_L^*) = 0$. Therefore the H type incumbent is best off setting $p = p_H^M$, knowing that entry will occur in the second period, so $s_E(p_H^M) = 1$. Hence

$$p_H^* = p_H^M \tag{5}$$

To obtain a limit price equilibrium, it is thus required that

$$p_L^* \neq p_L^M \tag{6}$$

Next, the entrant's cutoff price can be characterized as follows:

Lemma 2 $\overline{p} = p_L^*$ and $\overline{p} < p_L^M$.

[8] This is for simplicity only. Any off equilibrium beliefs that favor entry would do.
[9] It follows from Assumptions 3 and 4.

Proof. Suppose to the contrary that $\overline{p} \geq p_L^M$. Then $s_E(p_L^M) = 0$ and L is therefore best off switching from p_L^* to p_L^M, contradicting (6). Next, observe that in a separating equilibrium, $s_E(p_L^*) = 0$ (E knows that L set p_L^*) and hence $p_L^* \leq \overline{p}$. Suppose that $p_L^* < \overline{p}$. Since $\overline{p} < p_L^M$, it follows by Assumption 6 that L is better off by increasing his price from p_L^* to $\overline{p}$, which is a contradiction. Thus, $\overline{p} = p_L^*$ ■

The characterization so far of the separating equilibrium prices may be summarized in the following way:

Corollary 1 (i) In any separating limit price equilibrium, $p_L^* < p_L^M$ and (ii) in any separating equilibrium, either $p_L^* = p_L^M \leq \overline{p}$ or $p_L^* = \overline{p} < p_L^M$.

These results completely characterize the entrant's equilibrium behavior. I proceed by further analyzing the incumbent's equilibrium strategy.

The Incentive Compatibility Constraints. Since $p_H^* = p_H^M$, the following incentive compatibility constraint for H should hold:

$$\Pi_H(p_H^M) \geq \Pi_H(p), \quad \forall p \tag{7}$$

This simply means that the H type's equilibrium strategy is globally optimal. Clearly, (7) holds for $p > \overline{p}$ since in this case, E enters and I can do no better than to set the monopoly price. Consider p such that $p \leq \overline{p}$. By Lemma 2, in a separating limit pricing equilibrium $\overline{p} = p_L^*$ and hence by Assumption 6 and Lemma 1 (ii), it follows that $p \leq \overline{p} = p_L^* < p_L^M < p_H^M$ and thus it is sufficient to consider the following inequality:

$$\Pi_H(p_H^M) \geq \Pi_H(p_L^*) \tag{8}$$

By the definition of Π_H given in (3), (8) is equivalent to

$$\pi_H(p_L^*) \leq (1-\delta)\pi_H(p_H^M) + \delta D_H \tag{9}$$

For later reference, note that the right-hand side of (9) is strictly positive. This means that for the incentive compatibility constraint (9) to be satisfied, it is not *necessarily* the case that the H type's profits from mimicking the L type are negative. As shall be shown in Section 3, this result does not carry over to the dynamic setting.

To write (9) in terms of prices, first define the set

$$A_H \equiv \left\{p : \pi_H(p) = (1-\delta)\pi_H(p_H^M) + \delta D_H\right\} \tag{10}$$

This set is simply the set of prices for which the H type's incentive compatibility constraint is binding. Since $D_H = \pi_H(p)$ for some p, then by Assumptions 6 and 7 the set A_H is non-empty and contains at most two points. Next, define

$$\widehat{p} \equiv \min A_H, \quad \widehat{q} \equiv \max A_H \tag{11}$$

where $\widehat{p} < \infty$ and $\widehat{q} \leq \infty$. Hence, according to (9), p_L^* must satisfy

$$p_L^* \leq \widehat{p} \text{ or } p_L^* \geq \widehat{q} \tag{12}$$

For later use, note that by definition,

$$\pi_H(\widehat{p}) = \delta D_H + (1-\delta)\pi_H(p_H^M) = \pi_H(\widehat{q}) \tag{13}$$

Observe that $p_L^* < p_L^M < p_H^M < \widehat{q}$. In conclusion, for the H type's incentive compatibility constraint to hold, it must be that

$$p_L^* \leq \widehat{p} \tag{14}$$

I now turn to the L type. The incentive compatibility constraint for L is given by

$$\Pi_L(p_L^*) \geq \Pi_L(p), \quad \forall p \tag{15}$$

Again, this inequality simply states that the L type's equilibrium strategy is globally optimal. But the relevant p is only $p = p_L^M$ (since deterring entry is only optimal if it yields higher payoffs than setting the monopoly price in the first period and accommodating entry). Hence (15) becomes

$$\Pi_L(p_L^*) \geq \Pi_L(p_L^M) \tag{16}$$

By the definition of Π_L given by (3), inequality (16) is equivalent to

$$\pi_L(p_L^*) + \delta\pi_L(p_L^M) \geq \pi_L(p_L^M) + \delta D_L \tag{17}$$

Consequently,

$$\pi_L(p_L^*) \geq (1-\delta)\pi_L(p_L^M) + \delta D_L \tag{18}$$

is the relevant incentive compatibility constraint for L. Define the set

$$A_L \equiv \left\{p : \pi_L(p) = (1-\delta)\pi_L(p_L^M) + \delta D_L\right\} \tag{19}$$

Again, this set is the set of prices for which the L type's incentive compatibility constraint is binding. Since $D_L = \pi_L(p)$ for some p, then by Assumptions 6 and 7 the set A_L is non-empty and contains at most two points. Let

$$p_0 \equiv \min A_L, \quad q_0 \equiv \max A_L \tag{20}$$

where $p_0 < \infty$ and $q_0 \leq \infty$.

In terms of prices, the L type's incentive compatibility (18) can then be written as

$$p_0 \leq p_L^* \leq q_0 \tag{21}$$

where, by definition, it is the case that

$$\pi_L(p_0) = \delta D_L + (1-\delta)\pi_L(p_L^M) = \pi_L(q_0) \tag{22}$$

The previous results can be summarized as follows:

Proposition 1 Any separating limit price equilibrium is a triple $(p_H^*, p_L^*, \overline{p})$ such that (i) $p_H^* = p_H^M$, (ii) $\overline{p} = p_L^*$, (iii) $p_0 \leq p_L^* \leq \widehat{p}$ and (iv) $p_L^* < p_L^M$.

Hence, to show existence of a separating limit price equilibrium, I need to show that $p_0 < \widehat{p}$. Before stating the sufficient condition for $p_0 < \widehat{p}$ to hold, I characterize the set of separating equilibria without the limit price requirement.

Separating Equilibria. If one eliminates the limit pricing requirement, then in addition to the set

$$\{(p_H^*, p_L^*, \overline{p}) : \text{ (i), (ii), (iii), (iv) satisfied}\} \tag{23}$$

there are other separating equilibrium points if $\widehat{p} \geq p_L^M$. In particular, the following result holds:

Proposition 2 Any separating equilibrium is a triple $(p_H^*, p_L^*, \overline{p})$ such that (i) $p_H^* = p_H^M$, (ii) $\overline{p} = p_L^*$ and (iii) p_L^* satisfies

$$p_0 \leq p_L^* \leq \min\{\widehat{p}, p_L^M\} \tag{24}$$

Existence of Separating Limit Price Equilibria. The existence of separating limit price equilibria is now considered. Fortuitously, existence is secured under very mild conditions on the primitives of the model, as the following result shows:

Proposition 3 Suppose that

$$\pi_L(p_L^M) - D_L > \pi_H(p_H^M) - D_H \tag{25}$$

Then $\widehat{p} > p_0$ and the set of separating limit pricing equilibria is non-empty.

Proof. From (25), (13) and (22), it follows that

$$\pi_L(p_L^M) - \pi_H(p_H^M) > \pi_L(p_0) - \pi_H(\widehat{p}) \tag{26}$$

Adding and subtracting $\pi_H(p_L^M)$ yields

$$\pi_L(p_L^M) + \pi_H(p_L^M) + \left[\pi_H(p_L^M) - \pi_H(p_H^M)\right] > \pi_L(p_0) - \pi_H(\widehat{p}) \tag{27}$$

By the definition of p_H^M, it follows that $\pi_H(p_L^M) - \pi_H(p_H^M) \leq 0$. It thus follows from (27) that

$$\pi_L(p_L^M) + \pi_H(p_L^M) > \pi_L(p_0) - \pi_H(\widehat{p}) \tag{28}$$

Since $p_0 \leq p_L^M$, it follows by Lemma 1 that

$$\pi_L(p_0) - \pi_H(p_0) \geq \pi_L(p_L^M) - \pi_H(p_L^M) \tag{29}$$

Combined with (28), this implies that $\pi_H(p_0) < \pi_H(\widehat{p})$. Finally, $p_0 \leq p_H^M$ and $\widehat{p} \leq p_H^M$ and therefore it follows by Assumption 6 that $p_0 < \widehat{p}$ ■

It can be shown that condition (25) holds for the cases of Cournot competition with linear demand and fixed marginal costs and Bertrand competition with or without product differentiation (see Tirole, 1988).

Equilibrium Selection. As seen above, the solution concept perfect Bayesian equilibrium fails to uniquely determine L's equilibrium price p_L^*. To get a sharp characterization of equilibrium behavior, I make use of the notion of *equilibrium dominance*. This entails using notions of both backward and forward induction. Specifically, it requires that the incumbent's equilibrium strategy (at the signaling stage) form part of a perfect Bayesian equilibrium of the game obtained after deletion of strategies that are not a weak best response to any of the entrant's possible equilibrium strategies (at the entry stage). In other words, a deviation from an equilibrium price will be interpreted as coming from the H type whenever he cannot possibly benefit from such a deviation (for any best response of the entrant) while the L type incumbent would stand to benefit from such a deviation. An equilibrium strategy that is undominated in this sense will be said to satisfy the *forward induction criterion*. An equilibrium will be called *reasonable* if the strategies satisfy this criterion. This refinement yields a unique equilibrium, as the following proposition shows:

Proposition 4 (i) Suppose that $p_0 < \widehat{p} \leq p_L^M$. Then only $p_L^* = \widehat{p}$ satisfies the forward induction criterion. (ii) Suppose that $p_0 < p_L^M \leq \widehat{p}$. Then only $p_L^* = p_L^M$ satisfies the forward induction criterion.

Proof. (i) Suppose that $p_0 < \widehat{p} \leq p_L^M$ and let p' satisfy $p_L^* < p' < \widehat{p}$. Whichever strategy E picks, it is a strictly dominated strategy for H to choose p'. If $s_E(p') = 1$, then since $p' < \widehat{p} \leq p_L^M \leq p_H^M$ it follows that

$$\pi_H(p') + \delta D_H < \pi_H(\widehat{p}) + \delta D_H \tag{30}$$

If in turn $s_E(p') = 0$, then

$$\pi_H(p') + \delta\pi_H(p_H^M) < \pi_H(\widehat{p}) + \delta\pi_H(p_H^M) = \pi_H(p_H^M) + \delta D_H \tag{31}$$

Hence, even if H fools E to believe that he is L, he will obtain less than $\pi_H(p_H^M) + \delta D_H$ which he would obtain under the equilibrium strategy $p_H^* = p_H^M$. In the game obtained after eliminating the strategy p' from H's strategy set, E must play $s_E(p') = 0$ since p' can have been set only by L and thus by backward induction staying out at the price p' is a best response for E. But in the new reduced game, L can profitably deviate from p_L^* to p' and obtain $\pi_L(p') - \pi_L(p_L^*) > 0$, which follows from Assumption 6 and the fact that $p' \leq p_L^M$. For completeness, note that no type of incumbent can benefit from deviations to prices such that $p' \in [p_0, p_L^*]$. The proof of (ii) follows similar steps as that of (i) ■

The price selected by the forward induction criterion is known as the least-cost separating equilibrium price, as it is the equilibrium price which involves the lowest possible cost for the L type in terms of foregone profits. In other words, it is the highest price (lower than the monopoly price) consistent with the incentive compatibility constraints. This outcome is known in the literature as the *Riley outcome*.

2.2. Pooling Limit Price Equilibria. In a pooling equilibrium, it is by definition the case that

$$p_L^* = p_H^* = p^* \tag{32}$$

This means that the entrant cannot infer the incumbent's type merely by observing his chosen equilibrium price. Observe first that if

$$\mu D_E(H) + (1-\mu) D_E(L) - K > 0 \tag{33}$$

then pooling equilibria cannot exist, since the expected profit of E when he cannot distinguish between the incumbent's types is positive and he thus enters regardless of p^*. By backward induction, each type of incumbent is better off setting his monopoly price. Since $p_H^M > p_L^M$, I thus have that $p_L^* \neq p_H^*$, contradicting the supposition that the types pool. I thus assume that

Assumption

8 $\mu D_E(H) + (1-\mu) D_E(L) - K < 0$

Under this assumption, the entrant thus expects to make negative profits against the incumbent if he cannot distinguish between the two types.

Characterization. Before characterizing the incumbent's equilibrium price, the entrant's cutoff rule can be characterized in the following way:

Lemma 3 $\overline{p} = p^*$ and $p^* \leq p_L^M$.

Proof. Clearly $\overline{p} \geq p^*$. Otherwise, E's decision rule dictates entry if p^* is charged. That is, $s_E(p^*) = 1$ if $p^* > \overline{p}$ and thus each type of incumbent would benefit from deviating to their respective monopoly prices, contradicting (32). Next observe that if $p^* > p_L^M$, then L is best off setting the price p_L^M and entry will still be deterred (i.e. $s_E(p_L^M) = 0$). Consequently, $p^* \leq p_L^M$ as claimed. Finally, suppose to the contrary that $\overline{p} > p^*$. Since $p^* \leq p_L^M < p_H^M$, it follows by Assumption 6 that the H type is better off increasing his price slightly above p^* to increase profits while still deterring E's entry. Therefore $\overline{p} = p^*$ must hold as claimed ■

The Incentive Compatibility Constraints. The incentive compatibility constraints for the H type and the L type are given by

$$\pi_H(p^*) + \delta \pi_H(p_H^M) \geq \pi_H(p_H^M) + \delta D_H \tag{34}$$

$$\pi_L(p^*) + \delta \pi_L(p_L^M) \geq \pi_L(p_L^M) + \delta D_L, \quad p^* < p_L^M \tag{35}$$

Note that for each type, the best alternative strategy to choosing the entry deterring pooling price is to set the monopoly price and inviting entry. Also note that if $p^* = p_L^M$, then there is no incentive compatibility constraint for the L type.[10] The two incentive compatibility constraints (34)-(35) can be rewritten as

$$\pi_H(p^*) \geq (1-\delta)\pi_H(p_H^M) + \delta D_H \tag{36}$$
$$\pi_L(p^*) \geq (1-\delta)\pi_L(p_L^M) + \delta D_L, \quad p^* < p_L^M \tag{37}$$

Using (11) and (20), inequality (36) holds if and only if $\widehat{p} \leq p^* \leq \widehat{q}$ while (37) holds for $p^* \geq p_0$ as long as $p^* < p_L^M$. Combining these constraints, I obtain:

Proposition 5 Any pooling equilibrium is a tuple $(p^*, \overline{p})$ such that (i) $\overline{p} = p^*$ and (ii) p^* satisfies

$$\max\{p_0, \widehat{p}\} \leq p^* \leq p_L^M < p_H^M \tag{38}$$

It should be noted that a pooling equilibrium necessarily involves limit pricing, because at least the H type (and potentially the L type) charges below his monopoly price.

Equilibrium Selection. As was the case with the set of separating limit price equilibria, there is a continuum of pooling limit price equilibria. Again, the forward induction criterion can be used to select a unique reasonable equilibrium as follows:

Proposition 6 The only pooling equilibrium price that satisfies the forward induction criterion is $p^* = p_L^M$.

Proof. The set of pooling equilibrium prices is the set

$$\left\{p^* : \max\{p_0, \widehat{p}\} \leq p^* \leq p_L^M\right\} \tag{39}$$

Suppose that $p^* < p_L^M$. First note that $s_E(p_L^M) = 1$, for otherwise the L type is better off switching from p^* to p_L^M. Thus it is a strictly inferior strategy for H to select p_L^M or p_H^M. Indeed, by H's incentive compatibility constraint (34) I have

$$\pi_H(p^*) + \delta\pi_H(p_H^M) \geq \pi_H(p_H^M) + \delta D_H > \pi_H(p_L^M) + \delta D_H \tag{40}$$

Consider the new reduced game which is obtained from the original game by eliminating p_L^M from H's strategy set. In the equilibrium of the new game, $s_E(p_L^M) = 0$, since this price can only have been set by the L type. Hence L, in the new game, is better off deviating from p^* to p_L^M ■

As was the case with the selected separating limit price equilibrium, the forward induction criterion selects the least-cost pooling limit price equilibrium.

Last, note the following result, which further reduces the set of reasonable pooling limit price equilibria:

Proposition 7 If $\widehat{p} > p_L^M$, then no pooling equilibrium satisfying the forward induction criterion exists.

[10] Throughout the paper, the qualifier $p^* < p_L^M$ will reappear in connection with constraints on pooling prices. It will henceforth be implicit that if $p_t^* = p_L^M$ in some period t, then there is no incentive compatibility constraint for the L type in that period.

Proof: First note that $\widehat{p} \geq p_L^M$ if and only if

$$(1-\delta)\pi_H(p_H^M) + \delta D_H \geq \pi_H(p_L^M) \tag{41}$$

To see this, note that from (13) it follows that

$$\delta D_H = \pi_H(\widehat{p}) - (1-\delta)\pi_H(p_H^M) \tag{42}$$

Substituting this in (41) yields

$$\pi_H(\widehat{p}) \leq \pi_H(p_L^M) \tag{43}$$

Since $\widehat{p} < p_H^M$ and $p_L^M < p_H^M$, the result then follows from Assumption 6. Next, recall that for pooling on $p^* = p_L^M$ to be incentive compatible, inequality (36) must hold, i.e.

$$\pi_H(p_L^M) \geq (1-\delta)\pi_H(p_H^M) + \delta D_H \tag{44}$$

The result then follows immediately ■

For the knife's edge case $\pi_H(p_L^M) = (1-\delta)\pi_H(p_H^M) + \delta D_H$, pooling on $p^* = p_L^M$ is incentive compatible.

2.3. Existence of Reasonable Limit Price Equilibria. Before continuing the analysis, some comments on the existence of reasonable limit price equilibria are in order. Note that the above existence result concerns itself only with the existence of limit price equilibria and *not* with the existence of *reasonable* limit price equilibria. After performing equilibrium selection, the set of equilibria can, if non-empty, be divided into two distinct regimes, namely a *limit price regime* and a *monopoly price regime*. The former obtains if $\widehat{p} < p_L^M$ and the latter if $\widehat{p} \geq p_L^M$. These regimes will reappear in an important way in the dynamic game. In the monopoly price regime, the unique reasonable equilibrium is characterized by firms separating by setting their respective monopoly prices while in the limit price regime, both reasonable pooling and reasonable separating limit price equilibria coexist. For later reference, it should be reiterated that the condition determining the regimes is given by (41). That is, the monopoly price regime obtains if and only if

$$\pi_H(p_H^M) + \frac{\delta D_H}{1-\delta} \geq \frac{\pi_H(p_L^M)}{1-\delta}$$

This inequality has an interesting interpretation. The left-hand side is the profit for the H type of revealing his type by earning monopoly profits in this period and then earning duopoly profits in perpetuity thereafter. The right-hand side is the profit stream for the H type from mimicking the L type's post-separation equilibrium price.

3. The Dynamic Model

In this section I consider a model in which the basic game of Section 2 is repeated $T-1$ times as long as entry has not occurred (so that period $T \leq \infty$ is the last period and period $T-1$ is the last period in which signaling and/or entry may occur). Note that this is not a repeated game as entry can only occur once and thus the stage game is not unvarying across periods.

Next, I formally define what is meant by a separating and a pooling equilibrium in this dynamic setting. Let $\sigma^T \equiv \{p_{t,L}, p_{t,H}, \overline{p}_t\}_{t=1}^{T-1}$ denote a triple of pure strategies of the game. Denote by $\left\{p_{t,L}^*\right\}_{t=1}^{T-1}$ and $\left\{p_{t,H}^*\right\}_{t=1}^{T-1}$ the equilibrium price sequences.

Definition 3 σ^T is a separating equilibrium if $\left\{p_{t,H}^*\right\}_{t=1}^{T-1} \neq \left\{p_{t,L}^*\right\}_{t=1}^{T-1}$ and a pooling equilibrium if $\left\{p_{t,H}^*\right\}_{t=1}^{T-1} = \left\{p_{t,L}^*\right\}_{t=1}^{T-1}$.

These definitions are the natural generalizations of the static counterparts. In essence, they extend the notion that upon observing the incumbent's equilibrium strategy, the entrant can infer the incumbent's type. Importantly though, it is quite possible that such an inference can only be made upon observing the *entire* strategy. One reason for adopting this definition is that if the incumbent's type has to be recognizable after all partial (i.e. non-terminal) histories, as is the case in Kaya (2009) and Noldeke and van Damme (1990b), then there cannot *by assumption* be any delay in separation. I shall not impose such a restriction as it may rule out interesting equilibria with delayed information revelation.

As in the static setting, out of equilibrium beliefs must be assigned. An optimal decision rule for the entrant will prescribe entry if the incumbent is believed to be of the H type and no entry otherwise, that is if either the incumbent is believed to be of the L type or the two types cannot be distinguished.[11] As in the static setting, I will assume for simplicity that the incumbent will be interpreted to be of the H type for any observed price above the L type's equilibrium price (either separating or pooling) and to be the L type otherwise. These beliefs will turn out to amount to the following (optimal) monotone decision rule as a function of the observed price p_t for $t = 1, ..., T-1$, if entry has not occurred by time t,

$$s_E(p_t) = \begin{cases} 1 \text{ if } & p_t > \overline{p}_t \\ 0 \text{ if } & p_t \leq \overline{p}_t \end{cases} \tag{45}$$

for an appropriately chosen sequence $\{\overline{p}_t\}_{t=1}^{T-1}$ (determined by E).

Before embarking on the detailed analysis of the dynamic game, consider the following modified version of the one-shot signaling game. First, nature chooses a type $i = H, L$ for the incumbent. Before he gets the opportunity to set his price, both I and E observe a perfectly informative public signal such that the incumbent's type is commonly assigned probability one. After the public signal has been observed, the game proceeds as before, with the incumbent setting a price and the entrant deciding on entry. How should we expect this game to be played? This question may seem as sophistry, but answering it is instructive in understanding the issues involved in the dynamic game at hand.

A good case can be made that since the incumbent's type is common knowledge, the game ceases to be a meaningful signaling game altogether and hence should be treated as one of perfect information. In fact, it seems quite unnatural to treat this as a signaling game. The fact that the incumbent has an opportunity to set a price before entry may take place should be regarded as a mere formality. Having observed the perfectly informative public signal, E would not a priori expect the incumbent's price to carry any useful information. The natural

[11] Assumptions on the primitives of the model that ensure the optimality of this decision rule will be imposed below.

expectation is that incumbent $i = H, L$ will set price p_i^M since this price would maximize current profits and the subsequent entry decision may be regarded as pre-determined. By implication, if an incumbent of type $i = H, L$ sets a price $p_i^M \neq p_i^M$, this would be naturally regarded as an unexpected deviation from equilibrium play that needs to be made sense of.

The equilibria studied in Kaya (2009) and Noldeke and van Damme (1990b) exploit the fact that the players may simply disregard the public information contained in past play and proceed "as if" they had not observed past play at all. The point here is not that the equilibria studied by these authors are not equilibria (which they clearly are). Rather, I argue that the reliance of such equilibria on the players ignoring past evidence can serve as a useful feature to help choose between different kinds of equilibria in this type of setting.

To be more specific, suppose the signal L is observed and that the incumbent subsequently charges price p_H^M. This sequence of events should confound the entrant since an L type could have set the preferred price p_L^M without suffering adverse consequences. There are different ways to interpret the situation. One is to insist on the veracity of the public signal and to simply ascribe p_H^M to a “mistake” by the L type incumbent. This type of obstinacy in updating is the essence of support restrictions. A second way to proceed is to suppose that the public signal was in fact “mistaken” and to infer from the observation of p_H^M that the incumbent is in fact not an L type after all. In the former approach, the public signal is given all weight while in the latter, the incumbent’s action is given all weight. But if the prior is ignored, then the L type incumbent should set his price such as to credibly convey the information that he is in fact an L type incumbent and thus deter entry. This is exactly the way in which belief resetting makes repeated signaling possible.

A third approach is to consider the two pieces of conflicting evidence together and to make sense of the conflict by using heuristics familiar to the equilibrium refinement literature. This approach consists of asking which type of incumbent, *given the public belief that he is type* L, would stand to gain from setting price p_H^M? It turns out that answering this question gives a very natural prediction in this game. The key is to observe that *given* that the entrant already assigns probability one to the incumbent being an L type, the L type *cannot possibly benefit from setting any price different from* p_L^M, as long as observing p_L^M does not prompt the entrant to revise his belief that the incumbent if of type L. On the other hand, an H type incumbent *would* benefit from this price if E disregards this piece of confounding evidence (which he is entitled to do since out of equilibrium beliefs are arbitrary in a perfect Bayesian equilibrium).

Extending this reasoning to the dynamic game, the natural benchmark equilibrium price sequence after separation has occurred is $(p_L^M, p_L^M, ..., p_L^M)$. Next, *given* this benchmark equilibrium price sequence, all deviations can be dealt with by using reasoning similar to that inherent in criterion D1. Note that this procedure is not quite a direct application of D1 since I do not compare two arbitrary equilibria. Rather, the present approach accords special significance to the equilibrium in which the uninformed party at each information set makes full use of available information (i.e. acts without ignoring available evidence). Returning to the above discussion of the perfectly informative public signal, given equilibrium strategies, separation in a given period is exactly a perfect signal of the incumbent’s type and should be treated as such in subsequent play by the uninformed player.

Reiterating, note that ex post separation, the sequence $(p_L^M, p_L^M, ..., p_L^M)$ yields the *highest possible* payoff to the L type incumbent (as long as the entrant's beliefs that he is facing the L type are not disturbed). In other words, as long as the L type sticks to this price sequence, there is no possible deviation that can yields a higher payoff to him for any beliefs that a deviation could feasibly induce. On the other hand, there are deviations that would make the H type strictly better off. For example, consider the sequence $(p_H^M, p_L^M, ..., p_L^M)$. If the entrant gives the incumbent the benefit of the doubt and ignores the out of equilibrium price p_H^M (which he is entitled to do), then the H type is strictly better off under sequence $(p_H^M, p_L^M, ..., p_L^M)$ than under $(p_L^M, p_L^M, ..., p_L^M)$. But this means that the set of best responses of the entrant that makes the H type want to deviate is strictly larger than the set of best responses that would induce the L type to deviate. This is essentially the heuristic embodied in criterion D1. In what follows, the analysis will be confined to equilibria that are selected using this approach.

3.1. Separating Limit Price Equilibria. The magnitude of the entry costs in the dynamic setting plays a more delicate role than it does in the static setting. A necessary condition for a separating limit price equilibrium with separation in any period $t = 1, ..., T-1$ to exist is that

$$D_E(L) < \left(\frac{1-\delta}{1-\delta^{T-t+1}}\right) K < D_E(H), \quad t = 2, ..., T \tag{46}$$

Since the coefficient on the entry cost K is decreasing in the number of remaining periods, the condition may fail to hold for some t.[12] In order to avoid time varying necessary conditions, I instead impose the following restrictions which ensure that separation is feasible in an arbitrary period $t = 1, ..., T-1$:

Assumptions

4′ $D_E(L) < \left(\frac{1-\delta}{1-\delta^T}\right) K$

5′ $D_E(H) > K$

It should be pointed out that these conditions are stronger than Assumptions 4-5.[13]

Characterization. The characterization of equilibria of the dynamic model follows similar steps as that of the static model, although the analysis is complicated by the dynamic nature of the problem. Based on the discussion above, a reasonable separating equilibrium price sequence $\left\{p_{t,L}^*\right\}_{t=1}^{T-1}$ for the L type is of the general form $(p_1^*, ..., p_{t-1}^*, p_{t,L}^*, p_L^M, ..., p_L^M)$, with separation occurring in period $t = 1, ..., T-1 \leq \infty$.[14]

Next, the entrant's strategy can be characterized as follows:

[12] In particular, it may be the case that the necessary condition for a separating limit price equilibrium to be feasible is that the remaining number of periods be small. Since I shall also be studying the infinite horizon limit of the game, I disregard this possibility.

[13] Note that in the static setting, Assumptions 4-5 can be replaced by the assumptions that $D_E(L) < 0$ and $D_E(H) > 0$ (and that $\mu D_E(H) + (1-\mu) D_E(L) < 0$ for a pooling equilibrium to exist). See e.g. Tirole (1988) for such a setup. In the dynamic setting however, these assumptions would mean that an entrant discovering that he has entered against the L type would immediately leave the market. The L type may in turn find it optimal to allow entry in the first period, knowing that E will subsequently leave the market.

[14] In fact, in a reasonable separating equilibrium it must also be the case that $p_s^* = p_L^M$ for $s = 1, ..., t-1$ as will be shown below.

Lemma 5 Consider the equilibrium price sequence $(p_1^*, ..., p_{t-1}^*, p_{t,L}^*, p_L^M, ..., p_L^M)$ in which separation occurs in period $t = 1, ..., T-1$. Then (i) $\overline{p}_s = p_s^*$ and $\overline{p}_s \leq p_L^M, \quad s = 1, ..., t-1$, (ii) $\overline{p}_t = p_{t,L}^*$ and $\overline{p}_t < p_L^M$ and (iii) $\overline{p}_s = p_{s,L}^* = p_L^M, \quad s = t+1, ..., T-1$.

Proof: The proofs of (i) and (ii) parallel those in the static setting while that of (iii) follows from Lemma 4 ■

It should be emphasized that I do not make any use of support restrictions in the post separation game. With a support restriction and assuming that prices have revealed that the incumbent is of type L, the two different sequences of post separation prices $(p_L^M, p_L^M, p_L^M, p_H^M, ...)$ and $(p_L^M, p_L^M, p_L^M, p_L^M, ...)$ would be treated equivalently in terms of beliefs and entry decisions whereas with equilibrium selection, the former sequence would prompt the entrant to update his beliefs and subsequently enter.

As in the analysis of the static benchmark, I shall proceed by first analyzing the incentive compatibility constraints of each type of incumbent and then move on to the issues of equilibrium existence and selection. While matters are complicated somewhat by the dynamic nature of the model (there are in each case two regimes to consider, which depend on parameter values), the basic progression of the analysis is unchanged. The analysis of the separating limit price equilibria then concludes with some comparative statics results.

The Incentive Compatibility Constraints. Next, note that as in the static setting, the best alternative for the L type to setting the separating equilibrium price is to set his monopoly price. In contrast, the best alternative for the H type to setting the separating equilibrium price, i.e. his monopoly price, is to mimic the L type's equilibrium price. With this in mind, the following partial characterization of the separating equilibrium price can be given:

Lemma 6 For the price sequence $(p_1^*, ..., p_{t-1}^*, p_{t,L}^*, p_L^M, ..., p_L^M)$ to constitute a separating limit price equilibrium, it must satisfy

$$\pi_L(p_s^*) \geq (1-\delta)\pi(p_L^M) + \delta D_L, \quad p_s^* < p_L^M, \quad s = 1, ..., t-1 \tag{47}$$

$$\pi_L(p_{t,L}^*) \geq \left(1 - \frac{\delta - \delta^{T-t+1}}{1-\delta}\right)\pi_L(p_L^M) + \left(\frac{\delta - \delta^{T-t+1}}{1-\delta}\right) D_L \tag{48}$$

Proof: See Appendix B ■

To write the incentive compatibility constraint for the separation period in terms of prices, define the following set, which is the dynamic counterpart of the set A_L:

$$A_L(T,t) \equiv \left\{ p : \pi_L(p) = \left(1 - \frac{\delta - \delta^{T-t+1}}{1-\delta}\right)\pi_L(p_L^M) + \left(\frac{\delta - \delta^{T-t+1}}{1-\delta}\right) D_L \right\} \tag{49}$$

Since $\pi_L(p) = D_L$ for some p, then by Assumptions 6 and 7 it follows that the set $A_L(T,t)$ is non-empty and contains at most two points. Let

$$p_0(T,t) \equiv \min A_L(T,t), \quad q_0(T,t) \equiv \max A_L(T,t) \tag{50}$$

where $p_0(T,t) < \infty$ and $q_0(T,t) \leq \infty$.

In terms of prices, the L type's incentive compatibility constraint for the separation period then requires that

$$p_0(T,t) \leq p^*_{t,L} \leq q_0(T,t) \tag{51}$$

For later use, note that by definition it is the case that

$$\pi_L(p_0(T,t)) = \left(1 - \frac{\delta - \delta^{T-t+1}}{1-\delta}\right)\pi_L(p^M_L) + \left(\frac{\delta - \delta^{T-t+1}}{1-\delta}\right)D_L = \pi_L(q_0(T,t)) \tag{52}$$

I next consider the incentive compatibility constraints for the H type. These are slightly more complicated, due to the fact that the H type may in general wish to mimic the behavior of the L type for an arbitrary number of periods after the L type has chosen to separate. To see this more clearly, consider an equilibrium price sequence for the L type given by $(p^*_1, ..., p^*_{t-1}, p^*_{t,L}, p^M_L, ..., p^M_L)$ for $t = 1, ..., T-1$. In equilibrium, the H type's strategy is given by a sequence $(p^*_1, ..., p^*_{t-1}, p^M_H, x, ..., x)$ where x is shorthand for H's post entry equilibrium strategy. Next, consider possible deviations for the H type. First, H may wish to deviate during periods with pooling and so these pooling prices must respect appropriate incentive compatibility constraints. Next, H may deviate in the period where separation is prescribed by mimicking the L type's strategy. Last, H may deviate by not only mimicking the L type's separating price but also by mimicking L's post separation strategy p^M_L for an arbitrary number of periods. It turns out that the optimal amount of mimicking undertaken by the H type out of equilibrium depends in a simple way on parameter values, as the following results show:

Lemma 7 (i) Suppose that (41) is satisfied. Then mimicking only once is the optimal off equilibrium strategy. Furthermore, for the price sequence $(p^*_1, ..., p^*_{t-1}, p^*_{t,L}, p^M_L, ..., p^M_L)$ for $t = 1, ..., T-1$ to constitute a separating limit price equilibrium, it must satisfy

$$\pi_H(p^*_s) \;\geq\; (1-\delta)\pi_H(p^M_H) + \delta D_H, \quad p^*_s < p^M_L, \quad s = 1, ..., t-1 \tag{53}$$

$$\pi_H(p^*_{t,L}) \;\leq\; (1-\delta)\pi_H(p^M_H) + \delta D_H \tag{54}$$

(ii) Suppose that (41) is violated. Then mimicking perpetually is the optimal off equilibrium strategy. Furthermore, for the price sequence $(p^*_1, ..., p^*_{t-1}, p^*_{t,L}, p^M_L, ..., p^M_L)$ for $t = 1, ..., T-1$ to constitute a separating limit price equilibrium, it must satisfy

$$\pi_H(p^*_s) \;\geq\; (1-\delta)\pi_H(p^M_H) + \delta D_H, \quad p^*_s < p^M_L, \quad s = 1, ..., t-1 \tag{55}$$

$$\pi_H(p^*_{t,L}) \;\leq\; (1-\delta^{T-t})\pi_H(p^M_H) + \left(\frac{\delta - \delta^{T-t+1}}{1-\delta}\right)D_H - \left(\frac{\delta - \delta^{T-t}}{1-\delta}\right)\pi_H(p^M_L) \tag{56}$$

Proof: See Appendix C ■

To express these incentive compatibility constraints in terms of prices, define the following set, which is the dynamic counterpart of A_H:

$$A_H(T,t) \equiv \left\{p : \pi_H(p) = (1-\delta^{T-t})\pi_H(p^M_H) + \left(\frac{\delta - \delta^{T-t+1}}{1-\delta}\right)D_H - \left(\frac{\delta - \delta^{T-t}}{1-\delta}\right)\pi_H(p^M_L)\right\} \tag{57}$$

Note that the coefficients on $\pi_H(p_H^M)$, D_H and $\pi_H(p_L^M)$ in the definition of $A_H(T,t)$ sum to one. It then follows from Assumptions 6 and 7 and the fact that $\pi_H(p_H^M) > \pi_H(p_L^M)$ that the set $A_H(T,t)$ contains at most two points. Let

$$\widehat{p}(T,t) \equiv \min A_H(T,t), \quad \widehat{q}(T,t) \equiv \max A_H(T,t) \tag{58}$$

where $\widehat{p}(T,t) < \infty$ and $\widehat{q}(T,t) \leq \infty$.

For periods with pooling, the price sequence must thus satisfy

$$\max\{p_0, \widehat{p}\} \leq p_s^* \leq p_L^M < p_H^M, \quad s = 1, ..., t-1 \tag{59}$$

For the period in which separation is prescribed, the H type's incentive compatibility constraint when condition (41) is satisfied is that

$$p_{t,L}^* \leq \widehat{p} \text{ or } p_{t,L}^* \geq \widehat{q} \tag{60}$$

which is as in the static setting. In this case, only the inequality $p_{t,L}^* \leq \widehat{p}$ is relevant, since $p_{t,L}^* < p_L^M < p_H^M < \widehat{q}$. For the period in which separation is prescribed, the H type's incentive compatibility constraint when condition (41) is violated is that

$$p_{t,L}^* \leq \widehat{p}(T,t) \text{ or } p_{t,L}^* \geq \widehat{q}(T,t) \tag{61}$$

In this case, only the inequality $p_{t,L}^* \leq \widehat{p}(T,t)$ is relevant, since $p_{t,L}^* < p_L^M < p_H^M < \widehat{q}(T,t)$. For later use, note that by definition it is the case that

$$\pi_H(\widehat{p}(T,t)) = (1-\delta^{T-t})\pi_H(p_H^M) + \left(\frac{\delta - \delta^{T-t+1}}{1-\delta}\right) D_H - \left(\frac{\delta - \delta^{T-t}}{1-\delta}\right) \pi_H(p_L^M) = \pi_H(\widehat{q}(T,t)) \tag{62}$$

Before summarizing the analysis of the dynamic limit price equilibria, I will briefly discuss the issue of equilibrium existence.

Existence of Separating Limit Price Equilibria. In the static setting, I showed that (25) was a sufficient condition for the set of separating limit pricing equilibria to be non-empty, since it implied that $\widehat{p} > p_0$. I now derive the dynamic counterparts of (25). Note that when (41) is satisfied, the existence of separating limit price equilibria is ensured if $\widehat{p} > p_0(T,t)$ whereas if (41) is violated, then existence is ensured if $\widehat{p}(T,t) > p_0(T,t)$.

The relevant sufficient conditions for the existence of separating limit price equilibria are given as follows:

Proposition 8 (i) Suppose that (41) is satisfied. If

$$\pi_L(p_L^M) - D_L > \left(\frac{1-\delta}{1-\delta^{T-t}}\right)\left[\pi_H(p_H^M) - D_H\right], \quad t = 1, ..., T-1 \tag{63}$$

then $\widehat{p} > p_0(T,t)$ and the set of separating limit price equilibria is non-empty.

(ii) Suppose that (41) is violated. If

$$\pi_L(p_L^M) - D_L > \left[\frac{(1-\delta)\delta^{T-t-1}}{1-\delta^{T-t}}\right]\pi_H(p_H^M) - D_H + \left[\frac{\delta-\delta^{T-2}}{1-\delta^{T-t}}\right]\pi_H(p_L^M), \quad t = 1, ..., T-1 \quad (64)$$

then $\widehat{p}(T,t) > p_0(T,t)$ and the set of separating limit price equilibria is non-empty.

Proof. (i) Solving (52) and (13) for D_L and D_H respectively, substituting into (63) and rearranging yields

$$\pi_L(p_L^M) - \pi_H(p_H^M) > \pi_L(p_0(T,t)) - \pi_H(\widehat{p}) \quad (65)$$

which is equivalent to condition (26). The remainder of the proof follows the same steps as in the static setting.

(ii) Solving (52) and (62) for D_L and D_H respectively, substituting into (64) and rearranging yields

$$\pi_L(p_L^M) - \pi_H(p_H^M) > \pi_L(p_0(T,t)) - \pi_H(\widehat{p}(T,t)) \quad (66)$$

which again is equivalent to condition (26) ■

Note that in both regimes, the relevant sufficient condition for existence becomes easier to satisfy as the horizon recedes, i.e. equilibrium may exist in the dynamic setting even if none exist in the static setting.

Equilibrium Selection. I now determine which of the equilibria in the dynamic game can be deemed reasonable in that they satisfy the forward induction criterion. I do this explicitly for the case where (41) is violated. The case where (41) is satisfied is similar, with $\widehat{p}(T,t)$ replaced by $\widehat{p}$.

Proposition 9 (i) Suppose that $p_0(T,t) < \widehat{p}(T,t) \leq p_L^M$. Then only $p_{t,L}^* = \widehat{p}(T,t)$ satisfies the forward induction criterion. (ii) Suppose that $p_0(T,t) < p_L^M \leq \widehat{p}(T,t)$. Then only $p_{t,L}^* = p_L^M$ satisfies the forward induction criterion.

Proof: (i) Suppose that $p_0(T,t) < \widehat{p}(T,t) \leq p_L^M$ and let p' satisfy $p_L^* < p' < \widehat{p}(T,t)$. Whichever strategy E picks, it is a strictly dominated strategy for H to choose p'. If $s_E(p') = 1$, then since $p' < \widehat{p}(T,t) \leq p_L^M \leq p_H^M$, Assumption 6 implies that the H type can benefit from switching to p_H^M, thereby earning $\pi_H(p_H^M) - \pi_H(p') > 0$. Next, suppose that $s_E(p') = 0$. In equilibrium, the H type should set the price p_H^M and can never earn more out of equilibrium than by playing his optimal off equilibrium strategy. But the first element of this strategy is precisely given by $\widehat{p}(T,t)$. It follows that the H type is better off by switching from p' to $\widehat{p}(T,t)$. After deleting the price p' from the H type's strategy set, E must set $s_E(p') = 0$, since p' could only have been set by the L type. But since $p' < \widehat{p}(T,t) \leq p_L^M$, it follows from Assumption 6 that the L type is better off by increasing his price to $\widehat{p}(T,t)$. The proof of (ii) follows similar steps as that of (i) ■

Before turning to the comparative statics analysis, the following result is shown:

Proposition 10 (i) If (41) is satisfied, all reasonable equilibria are characterized by separation in the first period. (ii) If (41) is violated, then all reasonable equilibria are of a form where, for $t = 1.,,,,T-1$, the L type's strategy is given by a sequence

$(p_L^M, ..., p_L^M, p_{t,L}^*, p_L^M, ..., p_L^M)$ and the H type's strategy is given by a sequence $(p_L^M, ..., p_L^M, p_H^M, x, ..., x)$ where x is the H type's post entry strategy.

Proof: (i) It is known from the static analysis that in the monopoly price regime, i.e. when (41) is satisfied, no reasonable pooling equilibria exist. The result then follows immediately from observing that the incentive compatibility constraint of the H type in periods of pre-separation pooling are identical to the incentive compatibility constraint in the static setting. (ii) The proof follows directly from the lemmata proved above ■

This means that in the monopoly price regime, the unique reasonable prediction is that each type of incumbent will set its corresponding monopoly price in the first period and deter entry in case of type L and invite entry in case of type H.

Returning to the two regimes discussed earlier, I now show the following:

Lemma 8 Condition (41) holds if and only if $\widehat{p}(T,t) \geq p_L^M$.

Proof: From (62) it follows that

$$\delta D_H = \left(\frac{1-\delta}{1-\delta^{T-t}}\right)\left[\pi_H(\widehat{p}(T,t)) - (1-\delta^{T-t})\pi_H(p_H^M) + \left(\frac{\delta-\delta^{T-t}}{1-\delta}\right)\pi_H(p_L^M)\right] \tag{67}$$

Substituting this in (41) yields

$$\pi_H(\widehat{p}(T,t)) \leq \pi_H(p_L^M) \tag{68}$$

Since $\widehat{p}(T,t) < p_H^M$ and $p_L^M < p_H^M$, the result follows from the inequality and Assumption 6 ■

Comparative Statics. It is immediately clear that the constraints in pre-separation periods are unaffected by the length of the horizon. In periods where separation is prescribed however, the constraints do explicitly depend on the remaining number of periods (if $T < \infty$). First, consider the L type's incentive compatibility constraint $p_{t,L}^* \geq p_0(T,t)$. The cutoff $p_0(T,t)$ is decreasing in T since $p_0(T,t) \leq p_L^M$ and the right-hand side of the equality defining the set $A_L(T,t)$ is decreasing in T. In the limit $T \to \infty$, $p_0(T,t)$ is implicitly given by

$$\lim_{T\to\infty} \pi_L(p_0(T,t)) = \left(1 - \frac{\delta}{1-\delta}\right)\pi_L(p_L^M) + \left(\frac{\delta}{1-\delta}\right) D_L \tag{69}$$

This means that as the horizon recedes, the L type's incentive compatibility constraint becomes easier to satisfy.

Now turn to the H type. Under condition (41), the H type's incentive compatibility constraints are unaffected by changes in T. When (41) is violated however, the appropriate constraint is $p_{t,L}^* \leq \widehat{p}(T,t)$. The cutoff $\widehat{p}(T,t)$ is decreasing in T since $\widehat{p}(T,t) \leq p_H^M$ and the right-hand side of the equality defining the set $A_H(T,t)$ is decreasing in T. In the limit $T \to \infty$, $\widehat{p}(T,t)$ is implicitly given by

$$\lim_{T\to\infty} \pi_H(\widehat{p}(T,t)) = \pi_H(p_H^M) + \left(\frac{\delta}{1-\delta}\right)\left(D_H - \pi_H(p_L^M)\right) \tag{70}$$

As the horizon recedes, the H type's incentive compatibility constraint becomes more difficult to satisfy.

In turn, this means that, in the monopoly price regime, the set of separating limit price equilibria expands with the length of the horizon, but the only reasonable equilibrium remains unchanged, namely immediate separation on monopoly prices. In the limit price regime, both critical cutoffs $\widehat{p}(T,t)$ and $p_0(T,t)$ decrease in the length of the horizon T and so both the largest and the smallest separating (and limiting) equilibrium price decrease. Although the effect of an increase in T on the set of equilibrium prices is ambiguous, the unique reasonable equilibrium limit price is unambiguously decreasing.

The Cost of Signaling. In the static setting, the H type's profits from mimicking the L type's separating equilibrium strategy was positive. Interestingly, this is no longer necessarily the case in the dynamic version of the game. In particular, I have the following result:

Proposition 11 If (41) is violated, then $\lim_{T\to\infty}\pi_H(\widehat{p}(T,t))<0$.

Proof: For $\lim_{T\to\infty}\pi_H(\widehat{p}(T,t))\leq 0$ to hold, it follows from (70) that

$$\pi_H(p_L^M)-D_H\geq\left(\frac{1-\delta}{\delta}\right)\pi_H(p_H^M) \tag{71}$$

must be true. This inequality can be rewritten as

$$\delta\geq\frac{\pi_H(p_H^M)}{\pi_H(p_H^M)-D_H+\pi_H(p_L^M)}\equiv\delta^* \tag{72}$$

Next, note that if $\pi_H(p_L^M)>D_H$, then (41) can be rewritten as

$$\delta\leq\frac{\pi_H(p_H^M)-\pi_H(p_L^M)}{\pi_H(p_L^M)-D_H}\equiv\delta^{**} \tag{73}$$

From Assumption 7 it follows that the violation of (41) is a sufficient (but not necessary) condition for $\pi_H(p_L^M)>D_H$ to hold. Last, note that $\delta^{**}\geq\delta^*$ if and only if $\pi_H(p_L^M)>D_H$, which is implied by the assumption that (41) is violated ■

In fact, the result becomes even stronger as the future becomes increasingly important, as the next result demonstrates:

Corollary 2 If (41) is violated, then $lim_{\delta\to 1}\lim_{T\to\infty}\pi_H(\widehat{p}(T,t))=-\infty$.

Proof: The result follows from taking the limit $\delta\to 1$ of (70) and again noting that $\pi_H(p_L^M)>D_H$ when (41) is violated ■

The consequences of these results are worth emphasizing. They imply that in the infinite horizon limit of the limit price regime, as the discount factor approaches one, the efficient type must force the inefficient type to make arbitrarily large losses in order to credibly signal that he is indeed efficient. This is because in this scenario, the benefits to H of perpetual incumbency approach infinity. This gives a very lopsided intertemporal profile of costs and benefits which is worth empathizing. The costs of signaling are all borne in a single period while the benefits of effectively deterring entry accrue over an infinite number of periods.

For T and δ sufficiently large, it may well be that the set

$$A_H^+(T,t) \equiv \{p \in A_H(T,t) \cap \mathbb{R}_+\}$$

is empty. In other words, depending on the details of the product market, it may be that there are no positive prices that satisfy the H type's incentive compatibility constraint. As $T \to \infty$ and $\delta \to 1$, positive prices can only be secured if demand has a vertical asymptote at $p = 0$, i.e. if $\lim_{p\to 0} Q(p) = \infty$. Even in this case, the equilibrium separating price may run afoul of the Areeda and Turner (1975) rule, requiring pricing above marginal cost.

3.2. Pooling Limit Price Equilibria. In this dynamic setting, a pooling equilibrium consists of a price sequence $\sigma^T = \{p_t^*\}_{t=1}^{T-1}$ set by both types of incumbent. This means that in every period, the entrant cannot distinguish the two types. For pooling to be feasible in period $t = 1.,,,T-1$, the following conditions need to be imposed:

$$\mu D_E(H) + (1-\mu)D_E(L) < \left(\frac{1-\delta}{1-\delta^{T-t+1}}\right) K, \quad t = 2,...,T \tag{74}$$

These constraints are easier to satisfy the farther away the final period is. In order to avoid time varying necessary conditions, I instead impose the following condition that ensures that pooling is feasible in any arbitrary period $t = 1,...,T-1$:

Assumption

8′ $\mu D_E(H) + (1-\mu)D_E(L) - (1-\delta)K < 0$

Interestingly, this condition is more difficult to satisfy than that in Assumption 8. Furthermore, it becomes increasingly more difficult the more patient the entrant becomes, i.e. the larger the discount factor δ becomes.

As in the static setting, the following characterization of the entrant's decision rule holds:

Lemma 9 $\overline{p}_t = p_t^*$ and $p_t^* \leq p_L^M, \quad t = 1,...T-1.$

Proof: The proof parallels that of the static analysis ■

The Incentive Compatibility Constraints. As was the case in the static setting, the best alternative for each type to setting the pooling price is to set the monopoly price and inviting entry. With this in mind, the following can be shown to hold:

Lemma 10 For the price sequence $\{p_t^*\}_{t=1}^{T-1}$ to constitute a pooling limit price equilibrium, it must satisfy

$$\pi_L(p_t^*) \geq (1-\delta)\pi(p_L^M) + \delta D_L, \quad p_t^* < p_L^M, \quad t = 1,...,T-1 \tag{75}$$

$$\pi_H(p_t^*) \geq (1-\delta)\pi(p_H^M) + \delta D_H, \quad t = 1,...,T-1 \tag{76}$$

Proof: See Appendix D ■

These results can be collected as follows:

Proposition 12 In any pooling limit price equilibrium, it must be the case that

$$\max\{p_0,\widehat{p}\}\leq p_t^*\leq p_L^M<p_H^M,\quad t=1,...,T-1 \tag{77}$$

Equilibrium Selection. Note that the incentive compatibility constraints in the dynamic pooling equilibrium are in fact equivalent to their static counterparts (37) and (36). It then follows from the same arguments as in the static analysis that only $p_t^*=p_L^M$ is a reasonable pooling equilibrium in that it satisfies the forward induction criterion.

Comparative Statics. As is the case in pre-separation periods in the separating equilibria, the constraints in the pooling equilibria do not depend explicitly on the remaining number of periods. It follows that the pooling equilibria are in fact not affected by the dynamic extension of the model.

3.3. Existence of Reasonable Limit Price Equilibria. As was the case in the static analysis, one may characterize two distinct regimes, namely a monopoly price regime and a limit price regime. In the monopoly price regime, the only reasonable outcome is separation on monopoly prices in the first period, whereas in the limit price regime, both reasonable pooling and separating equilibria coexist. In the reasonable pooling equilibrium, both types of incumbent set the efficient type's monopoly price and thus the equilibrium involves limit pricing. One may see this as a repetition of the static outcome. In the reasonable separating limit price equilibrium however, since the benefits from entry deterrence increase with the horizon and the patience of the players, credibly signaling to be of the efficient type may involve incurring arbitrarily large losses in the period in which separation is prescribed. Depending on the model specification and mode of competition in the market game, this may actually involve setting negative prices.[15]

The equilibrium price paths of the dynamic model should be contrasted to those of the early limit pricing literature. As Carlton and Perloff (2004) nicely show, some models predict that equilibrium prices will increase over time, others that they will decrease and others that price paths are not necessarily monotone. Because of the relatively weak restrictions on equilibrium behavior imposed by the incentive compatibility constraints, many different price profiles can be sustained in equilibrium. But not all such profiles are reasonable.

In the *monopoly price regime*, the analysis predicts immediate separation on monopoly prices, with resulting entry against the H type incumbent and no entry against the L type incumbent (who will subsequently charge monopoly prices indefinitely). In the *limit price regime*, all equilibria share the same overall structure. Namely, they are characterized by a non-negative and possibly infinite number $N=0,1,...$ of periods in which the two types of incumbent pool on the efficient type's monopoly price p_L^M, followed by a period $N+1$ in which the firms separate. In case I is of type L, prices will dip in order to signal strength, after which prices will resume to the pre-separation level p_L^M. In case I is of type H, prices will jump to p_H^M and then fall to some level $p<p_H^M$ (because of the ensuing entry and competition).

[15] This will be the case, e.g., in a model with constant marginal costs and linear demand as that considered by Tirole (1988). In fact, the efficient incumbent would have to give its customers infinitely large subsidies to credibly convey his identity.

Interestingly, in this model the timing of separation is indeterminate in the sense that in equilibrium, signaling can happen in any period, if ever. In other words, equilibrium does not pin down if and when signaling will take place. Note that this result is entirely unrelated to the equilibrium multiplicity created by choosing different off equilibrium beliefs in usual signaling games (e.g. the benchmark game analyzed in Section 2). Instead, the multiplicity is related to the coexistence of different classes of equilibria, i.e. pooling and separating equilibria. In the static setting, if both types of equilibria exist, there is no way to determine which of such different equilibria will be played. A similar situation arises in the dynamic setting, where separation may be preceded by multiple rounds of pooling. This indeterminacy effectively means that there are multiple equilibria (among which we cannot select) which differ in their predictions.

Note that both types of incumbent are better off the later separation occurs. The efficient type earns monopoly profits as long as entry does not occur and is not called upon to engage in costly signaling. The inefficient type effectively deters entry as long as pooling takes place. While pooling is indeed costly for the inefficient type, it still dominates entry. Therefore, it is not possible to use separation date as a screening device.

One possible approach to determining the timing of separation is to rule out negative prices (or prices below marginal cost). If this is done, then for sufficiently patient players, no separation can occur in equilibrium. This means that equilibrium involves either instant separation on monopoly prices (with ensuing entry and competition against the inefficient incumbent and perpetual incumbency for the efficient incumbent) or perpetual pooling on the efficient firm's monopoly price and no resulting entry.

4. Discussion

In this paper, I analyzed a dynamic model of limit pricing and compared it with the outcome of a static model. I showed that there are two regimes of interest. In one, the *monopoly price regime*, the only reasonable equilibrium involves separation in the first period on monopoly prices. In the other, the *limit price regime*, reasonable pooling limit price equilibria and reasonable separating limit price equilibria coexist. While the dynamic pooling equilibrium is essentially a repetition of the static outcome, with both types of incumbent pooling on the efficient type's monopoly price, the latter may differ quantitatively from the separating limit price equilibrium in the static setting. While the basic forces at work are similar, the fact that the game is dynamic may make the benefits from entry deterrence arbitrarily large. In turn, this means that the efficient incumbent must, in order to credibly convey his identity, make arbitrarily large (possibly infinite) losses in some period. For some standard specifications of market demand, this result may imply that firms must set negative prices. If such pricing behavior is ruled out (either because of law or because it is deemed otherwise inappropriate or unrealistic), separating equilibria may fail to exist. Consequently, the only reasonable equilibrium outcome is either immediate separation or perpetual pooling with no resulting entry.

Appendix

A. Proof of Lemma 1

(i) First, note that $\pi_L(p) - \pi_H(p) = C_H(Q(p)) - C_L(Q(p))$ and thus

$$\frac{\partial}{\partial p}\left[\pi_L(p) - \pi_H(p)\right] = Q'(p)\left[C'_H(Q(p)) - C'_L(Q(p))\right] \tag{78}$$

By Assumption 3, $Q'(p) < 0$. Thus, by Assumption 2 it follows that $\frac{\partial}{\partial p}\left[\pi_L(p) - \pi_H(p)\right] < 0$.

(ii) By the definition of monopoly prices and Assumption 6, it follows that

$$p_L^M Q_L^M - C_L(Q_L^M) > p_H^M Q_H^M - C_L(Q_H^M) \tag{79}$$

$$p_H^M Q_H^M - C_H(Q_H^M) > p_L^M Q_L^M - C_H(Q_L^M) \tag{80}$$

Adding these inequalities, I obtain

$$C_H(Q_L^M) - C_L(Q_L^M) > C_H(Q_H^M) - C_L(Q_H^M) \tag{81}$$

Hence, by Assumption 2, $Q_L^M > Q_H^M$ and by Assumption 3, $p_L^M < p_H^M$ ■

B. Proof of Lemma 6

I first derive the condition for the separating equilibrium price. The incentive compatibility constraints for the L type are given by

$$\pi_L(p_{1,L}^*) + \sum_{i=2}^{T} \delta^{i-1}\pi_L(p_L^M) \geq \pi_L(p_L^M) + \sum_{i=2}^{T} \delta^{i-1} D_L \tag{82}$$

$$\sum_{i=1}^{K+1} \delta^{i-1}\pi_L(p_i^*) + \delta^{K+1}\pi_L(p_{K+2,L}^*) + \sum_{i=K+3}^{T} \delta^{i-1}\pi_L(p_L^M) \tag{83}$$

$$\geq \sum_{i=1}^{M+1} \delta^{i-1}\pi_L(p_i^*) + \delta^{M+1}\pi_L(p_L^M) + \sum_{i=M+3}^{T} \delta^{i-1} D_L$$

for $0 \leq M \leq K = 0, 1, ..., T-3$. The first constraint (82) reduces to

$$\pi_L(p_{1,L}^*) \geq \left(1 - \frac{\delta - \delta^T}{1-\delta}\right)\pi_L(p_L^M) + \left(\frac{\delta - \delta^T}{1-\delta}\right) D_L \tag{84}$$

Next, evaluate (83) at $M = K$ and rearrange to get

$$\pi_L(p_{K+2,L}^*) \geq \left(1 - \frac{\delta - \delta^{T-K-1}}{1-\delta}\right)\pi_L(p_L^M) + \left(\frac{\delta - \delta^{T-K-1}}{1-\delta}\right) D_L \tag{85}$$

which determines the separating prices. Next, evaluate (83) at two arbitrary consecutive periods $M = K - j$ and $M = K - j - 1$ respectively, with $j = 1, ..., K - 1$. These yield

$$\delta^{K-j+1}\pi_L(p^*_{K-j+2}) - \left(\delta^{K-j+1} - \sum_{i=K+3}^{T}\delta^{i-1}\right)\pi_L(p^M_L) \geq \tag{86}$$

$$\sum_{i=K-j+3}^{T}\delta^{i-1}D_L - \delta^{K+1}\pi_L(p^*_{K+2,L}) - \sum_{i=K-j+3}^{K+1}\delta^{i-1}\pi_L(p^*_i)$$

$$\delta^{K-j}\pi_L(p^*_{K-j+1}) - \left(\delta^{K-j} - \sum_{i=K+3}^{T}\delta^{i-1}\right)\pi_L(p^M_L) \geq \tag{87}$$

$$\sum_{i=K-j+2}^{T}\delta^{i-1}D_L - \delta^{K+1}\pi_L(p^*_{K+2,L}) - \sum_{i=K-j+3}^{K+1}\delta^{i-1}\pi_L(p^*_i) - \delta^{K-j+1}\pi_L(p^*_{K-ji+2})$$

Substituting (86) in (87), rearranging and reducing yields

$$\pi_L(p^*_{K-j+1}) \geq (1-\delta)\pi_L(p^M_L) + \delta D_L \tag{88}$$

Last, if the equilibrium requires pooling in only the first period, then it must be that

$$\pi_L(p^*_1) \geq \left(1 - \sum_{i=3}^{T}\delta^{i-1}\right)\pi_L(p^M_L) + \sum_{i=2}^{T}\delta^{i-1}D_L - \delta\pi_L(p^*_{K+2,L}) \tag{89}$$

Substituting for the value of $\pi_L(p^*_{K+2,L})$ given by (85) and rearranging yields

$$\pi_L(p^*_1) \geq (1-\delta)\pi_L(p^M_L) + \delta D_L$$

■

C. Proof of Lemma 7

The constraints are as follows:

$$\sum_{i=1}^{K+1} \delta^{i-1}\pi_H(p_i^*) + \delta^{K+1}\pi_H(p_H^M) + \sum_{i=K+3}^{T} \delta^{i-1}D_H \geq$$
$$\sum_{i=1}^{M+1} \delta^{i-1}\pi_H(p_i^*) + \delta^{M+1}\pi_H(p_H^M) + \sum_{i=M+3}^{T} \delta^{i-1}D_H, \quad 0 \leq M < K = 0, ..., T-3 \tag{90}$$

$$\pi_H(p_H^M) + \sum_{i=2}^{T} \delta^{i-1}D_H \geq \pi_H(p_{1,L}^*) + \delta\pi_H(p_H^M) + \sum_{i=3}^{T} \delta^{i-1}D_H \tag{91}$$

$$\sum_{i=1}^{K+1} \delta^{i-1}\pi_H(p_i^*) + \delta^{K+1}\pi_H(p_H^M) + \sum_{i=K+3}^{T} \delta^{i-1}D_H \geq$$
$$\sum_{i=1}^{K+1} \delta^{i-1}\pi_H(p_i^*) + \delta^{K+1}\pi_H(p_{K+2,L}^*) + \delta^{K+2}\pi_H(p_H^M) + \sum_{i=K+4}^{T} \delta^{i-1}D_H, \quad K = 0, ..., T-4 \tag{92}$$

$$\pi_H(p_H^M) + \sum_{i=K+2}^{T} \delta^{i-1}D_H \geq \pi_H(p_{1,L}^*) + \sum_{i=2}^{K+2} \delta^{i-1}\pi_H(p_L^M) + \delta^{K+2}\pi_H(p_H^M) + \sum_{i=K+4}^{T} \delta^{i-1}D_H,$$
$$K = 0, ..., T-4 \tag{93}$$

$$\sum_{i=1}^{K+1} \delta^{i-1}\pi_H(p_i^*) + \delta^{K+1}\pi_H(p_H^M) + \sum_{i=K+3}^{T} \delta^{i-1}D_H \geq$$
$$\sum_{i=1}^{K+1} \delta^{i-1}\pi_H(p_i^*) + \delta^{K+1}\pi_H(p_{K+2,L}^*) + \sum_{i=K+3}^{M+2} \delta^{i-1}\pi_H(p_L^M) + \delta^{M+2}\pi_H(p_H^M) + \sum_{i=M+4}^{T} \delta^{i-1}D_H,$$
$$0 \leq K < M = 0, ..., T-4 \tag{94}$$

$$\pi_H(p_H^M) + \sum_{i=2}^{T} \delta^{i-1}D_H \geq \pi_H(p_{1,L}^*) + \sum_{i=2}^{T-1} \delta^{i-1}\pi_H(p_L^M) + \delta^{T-1}\pi_H(p_H^M) \tag{95}$$

$$\sum_{i=1}^{K+1} \delta^{j-1}\pi_H(p_i^*) + \delta^{K+1}\pi_H(p_H^M) + \sum_{i=K+3}^{T} \delta^{i-1}D_H \geq$$
$$\sum_{i=1}^{K+1} \delta^{i-1}\pi_H(p_i^*) + \delta^{K+1}\pi_H(p_{K+2,L}^*) + \sum_{i=K+3}^{T-1} \delta^{i-1}\pi_H(p_L^M) + \delta^{T-1}\pi_H(p_H^M),$$
$$K = 0, ..., T-4 \tag{96}$$

These sets of constraints will be explained in turn. Roughly, the H type's off equilibrium behavior can be described by the sequence *pool-mimic-reveal*. That is, first H pools whenever the L type pools, then the H type mimics L's behavior for some number of periods and then he reveal his type, subsequently earning duopoly profits following entry by E. The first set (90) considers the possibility of the H type revealing his type by setting the monopoly price earlier than the period in which the L type separates. These constraints will determine the pooling constraints for the H type. Next, the constraints (91) and (92) consider the H type mimicking the L type for a single period, in the cases of no prior pooling and an arbitrary number of prior periods with pooling respectively. Constraints (93) and (94) consider the H type mimicking the L type for a number of periods, in the cases of no prior pooling and an arbitrary number of prior periods with pooling respectively. Last, constraints (95) and (96) consider the possibility of the H type perpetually mimicking the L type, again in the cases of no prior pooling and an arbitrary number of prior periods with pooling respectively.

The first equations in parts (i) and (ii) of the Lemma follow from the constraints (90) and the same steps as those leading to the incentive compatibility constraints for the L type.

The next step is to order the magnitudes of the right-hand sides of constraints (91)-(96). Straightforward comparison shows that the order depends on whether or not

$$(1-\delta)\pi_H(p_H^M) + \delta D_H \geq \pi_H(p_L^M) \tag{97}$$

If (97) is satisfied, then (91)-(92) imply (93)-(96), whereas if (97) is violated, then (91)-(94) are implied by (95)-(96). Note that condition (97) is in fact just a restatement of condition (41), i.e. the condition that delineates the monopoly price regime and the limit price regime respectively.

The incentive compatibility constraints if (97) is satisfied are thus (92), which reduce to

$$\pi_H(p^*_{K+2,L}) \leq (1-\delta)\pi_H(p_H^M) + \delta D_H \tag{98}$$

for $K = 0, ..., T-4$, while the equivalent constraint for the first period follows from (91). If (97) is violated, then the relevant incentive compatibility constraints are (96), which reduce to

$$\pi_H(p^*_{K+2,L}) \leq \left(1-\delta^{T-K-2}\right)\pi_H(p_H^M) + \left(\frac{\delta - \delta^{T-K-1}}{1-\delta}\right) D_H - \left(\frac{\delta - \delta^{T-K-2}}{1-\delta}\right)\pi_H(p_L^M) \tag{99}$$

for $K = 0, ..., T-4$, while the equivalent constraint for the first period follows from (95) ■

D. Proof of Lemma 10

The incentive compatibility constraints for the L type are given by[16]

$$\sum_{i=1}^{T-1}\delta^{i-1}\pi_L(p_i^*) + \delta^{T-1}\pi_L(p_L^M) \geq \pi_L(p_L^M) + \sum_{i=2}^{T}\delta^{i-1}D_L \tag{100}$$

$$\sum_{i=1}^{T-1}\delta^{i-1}\pi_L(p_i^*) + \delta^{T-1}\pi_L(p_L^M) \geq \sum_{i=1}^{K+1}\delta^{i-1}\pi_L(p_i^*) + \delta^{K+1}\pi_L(p_L^M) + \sum_{i=K+3}^{T}\delta^{i-1}D_L \tag{101}$$

for $K = 0, 1, ..., T-3$. The set of constraints (101), (one for each K) compares the equilibrium strategy with a strategy that pools until (and including) period $K+1$ and deviates in period $K+2$. Solving (100) for $\pi_L(p_1^*)$ yields

$$\pi_L(p_1^*) \geq (1-\delta^{T-1})\pi_L(p_L^M) + \sum_{i=2}^{T}\delta^{i-1}D_L - \sum_{i=3}^{T-1}\delta^{i-1}\pi_L(p_i^*) - \delta\pi_L(p_2^*) \tag{102}$$

[16] It is without loss of generality to consider a deviation in period 1, since if there is pooling in periods $s = 1, ..., t-1$, then the period t problem is essentially the same as that faced in period 1.

Evaluating (101) at $K = 0$ and rearranging yields

$$\delta\pi_L(p_2^*) \geq \sum_{i=K+3}^{T} \delta^{i-1} D_L + \delta\pi_L(p_L^M) - \sum_{i=3}^{T-1} \delta^{i-1}\pi_L(p_i^*) \tag{103}$$

Substituting this in (102) and rearranging yields

$$\pi_L(p_1^*) \geq (1-\delta)\pi_L(p_L^M) + \delta D_L$$

For arbitrary K, (101) reduces to

$$\sum_{i=K+2}^{T-1} \delta^{i-1}\pi_L(p_i^*) + \delta^{T-1}\pi_L(p_L^M) \geq \delta^{K+1}\pi_L(p_L^M) + \sum_{i=K+3}^{T} \delta^{i-t} D_L \tag{104}$$

Straightforward manipulation yields that this inequality can be rewritten as

$$\sum_{i=0}^{T-K-3} \delta^{i}\pi_L(p_{i+K+2}^*) + \delta^{T-K-2}\pi_L(p_L^M) \geq \pi_L(p_L^M) + \sum_{i=0}^{T-K-3} \delta^{i+1} D_L \tag{105}$$

In particular, this implies that

$$\pi_L(p_{K+2}^*) \geq \left(1-\delta^{T-K-2}\right)\pi_L(p_L^M) + \sum_{i=0}^{T-K-3} \delta^{i+1} D_L - \sum_{i=2}^{T-K-3} \delta^{i}\pi_L(p_{i+K+2}^*) - \delta\pi_L(p_{K+3}^*) \tag{106}$$

But the constraint on $\pi_L(p_{K+3}^*)$ is in turn given by

$$\delta\pi_L(p_{K+3}^*) \geq \left(\delta-\delta^{T-K-2}\right)\pi_L(p_L^M) + \sum_{i=1}^{T-K-3} \delta^{i+1} D_L - \sum_{i=2}^{T-K-3} \delta^{i}\pi_L(p_{i+K+2}^*) \tag{107}$$

Substituting this back in (106) and rearranging, yields the following constraints:

$$\pi_L(p_{K+2}^*) \geq (1-\delta)\pi_L(p_L^M) + \delta D_L \tag{108}$$

for $K = 0, 1, ..., T-3$. Similar steps yield the equivalent constraints for the H type ■

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