DISCUSSION PAPER SERIES

No. 8072

DIFFERENCE IN INTERIM PERFORMANCE AND RISK TAKING WITH SHORT-SALE CONSTRAINTS

Suleyman Basak and Dmitry Makarov

FINANCIAL ECONOMICS

CE PR

Centre for Economic Policy Research

www.cepr.org

DIFFERENCE IN INTERIM PERFORMANCE AND RISK TAKING WITH SHORT-SALE CONSTRAINTS

Suleyman Basak, London Business School and CEPR
Dmitry Makarov, New Economic School

This Discussion Paper is issued under the auspices of the Centre's research programme in **FINANCIAL ECONOMICS**. Any opinions expressed here are those of the author(s) and not those of the Centre for Economic Policy Research. Research disseminated by CEPR may include views on policy, but the Centre itself takes no institutional policy positions.

The Centre for Economic Policy Research was established in 1983 as an educational charity, to promote independent analysis and public discussion of open economies and the relations among them. It is pluralist and non-partisan, bringing economic research to bear on the analysis of medium- and long-run policy questions.

These Discussion Papers often represent preliminary or incomplete work, circulated to encourage discussion and comment. Citation and use of such a paper should take account of its provisional character.

ABSTRACT

Difference in Interim Performance and Risk Taking with Short-Sale Constraints*

Absent much theory, empirical works often rely on the following informal reasoning when looking for evidence of a mutual fund tournament: If there is a tournament, interim winners have incentives to decrease their portfolio volatility as they attempt to protect their lead, while interim losers are expected to increase their volatility so as to catch up with winners. We consider a rational model of a mutual fund tournament in the presence of short-sale constraints and find the opposite - interim winners choose more volatile portfolios in equilibrium than interim losers. Several empirical works present evidence consistent with our model, however based on the above informal argument they appear to conclude against the tournament behavior. We argue that this conclusion is unwarranted. We also demonstrate that tournament incentives lead to differences in interim performance for otherwise identical managers, and that mid-year trading volume is inversely related to mid-year stock return.

Suleyman Basak
Professor of Finance
Institute of Finance and Accounting
London Business School (LBS)
Regents Park
London NW1 4SA
UNITED KINGDOM

Dmitry Makarov
New Economic School
Nakhimovsky pr. 47
Moscow 117418
RUSSIA

Email: sbasak@london.edu

Email: dmakarov@nes.ru

1. Introduction

In an influential paper, Brown, Harlow, and Starks (1996) (BHS, hereafter) used the term "tournament" to describe the mutual fund market, meaning that fund managers attempt to outperform each other. A common justification for why managers care about relative performance is that they seek to attract higher money inflows by exploiting the positive relation between relative performance and money flows (Chevalier and Ellison (1997), Sirri and Tuffano (1998)). Given that managers are typically paid a fixed percentage of assets under management, a manager's compensation increases with the amount of inflows, and so caring about relative returns is consistent with rational self-interest.

To test for tournament behavior, BHS suggest looking at how managers change the riskiness of their portfolios over the second half of a year depending on their interim mid-year performance. According to BHS, mid-year losers are expected to increase the volatility of their portfolios to a greater extent than mid-year winners. The justification seems quite convincing: interim losers gamble in an attempt to catch up with interim winners, while interim winners play it safe in order to protect their lead. Apart from having an intuitive appeal, the idea that losers adopt riskier strategies than winners is consistent with the predictions of tournament models in other settings (Ehrenberg and Bognanno (1990), McLaughlin (1988)) and also with "gambling on resurrection" by troubled banks (Dewatripont and Tirole (1995)).

Is it indeed the case that the risk-taking incentives faced by fund managers are similar to those observed in other tournament settings? To address this question, we postulate (for the bulk of the paper) that managers have different interim performance, and investigate how the equilibrium level of risk taken by a manager depends on her interim performance. While we analyze a series of settings with different degrees of generality (discussed later), we demonstrate our key result in the context of an intentionally stylized baseline model so as to convey the main intuition as clearly as possible. The baseline model is as follows. The economy is populated by a continuum of risk-neutral fund managers with different interim performance. The investment opportunities are given by two assets, a risky stock and a riskless bond, and the managers face portfolio constraints in that they can only take long positions in the assets. What makes this a tournament setting is the assumption that at

the terminal date each manager receives money inflows which depend on her performance relative to the industry-average performance via an increasing and convex flow-performance relationship, as widely documented in the empirical literature.[1]

Solving for equilibrium in the baseline model, we obtain the main result of our paper that interim winners choose *more volatile* portfolios than interim losers. This is the opposite of the tournament hypothesis proposed by BHS and widely used in subsequent works (Busse (2001), Qiu (2003), Goriaev, Nijman, Werker (2005), Reed and Wu (2005)). The intuition is as follows. The convexity of the flow-performance relationship leads to the convexity of a manager's objective function, implying that a manager seeks to maximize the volatility of her tracking error (difference between own and industry-average returns). Hence, each manager holds only one asset in her portfolio, either stock or bond, whichever differentiates her portfolio more from the industry portfolio. This implies that in equilibrium it has to be the case that some managers invest in the stock while the remainder invest in the bond.

Why is it the interim winners who invest in the stock in equilibrium? If an interim winner invests in the risky stock, she is able to convert her high interim performance into a high volatility of the year-end return, which is due to the basic fact that the year-end return volatility is proportional to the interim performance. If on the other hand an interim winner invests in the riskless bond, then her year-end return is constant, meaning that the mechanism which converts interim performance into return volatility is effectively switched off. Hence, in equilibrium interim winners invest in the stock so as to "leverage" their high interim performance, thus driving interim losers into the bond.

To demonstrate the robustness of our results, we consider several generalizations of our baseline model. First, to account for risk aversion and also for heterogeneity in risk aversion (Koijen (2008)), we consider a setting with two types of managers, relatively risk tolerant and relatively risk averse. The analysis of this model reveals that it is the interplay between the convexity of flow-performance relation and the concavity of managers' objective function that determines how risk taking depends on interim performance. For relatively risk tolerant managers for whom the flows convexity is dominant, we find that the relation between

---

[1]We take the flow-performance relation as given, and so do not address the issue of why retail investors reward past performers despite the lack of conclusive evidence on performance persistence (see Carhart (1997), Bollen and Busse (2005), and references therein) or relatedly, on the link between past performance and stock-picking skills (Berk and Green (2004), Chen, Jegadeesh, and Wermers (2000)).

interim performance and the choice of portfolio volatility is as in the baseline model. As for relatively risk averse managers for whom the objective function concavity is dominant, their equilibrium portfolios are virtually insensitive to interim performance as these managers do not care much about winning the tournament, and hence about their interim standings. Second, we generalize our baseline model to feature multiple risky stocks and positive risk premium. With multiple stocks, we show that managers with higher interim performance invest in more volatile stocks in equilibrium. For the case of a single risky stock with positive risk premium, we demonstrate that, as with zero risk premium, interim winners invest in the stock while interim losers invest in the bond, whereby the only effect of the risk premium is that the mass of interim winners goes up due to the stock becoming more attractive. Analyzing an example where multiple stocks, risk premium, and heterogeneous risk aversion are jointly present reveals that the equilibrium outcome is similar to that in the heterogenous risk aversion generalization, the only difference being that the more risk tolerant managers now invest in a large number of stocks. In summary, our main insights and empirical predictions in the baseline model remain valid with above generalizations: interim winners (losers) increase (decrease) the riskiness of their portfolios.

To better understand the role of portfolio constraints behind our implications, and to also see whether our results may readily be applicable to hedge funds, which are largely unconstrained, we analyze how the behavior of risk averse managers is affected once the constraints are lifted. We find that when the flows convexity is dominant, our main result still obtains: interim winners choose more volatile portfolios than interim losers. However, for relatively risk averse managers whose behavior is not much affected by convexity, our analysis reveals that their portfolio volatility can be insensitive to or even decreasing with interim performance, depending on the economic setting.

Finally, we generalize our baseline model to two trading periods, corresponding to start and middle of the year. We here demonstrate that the difference in managers' interim performance at mid-year arises endogenously since the managers, though identical, choose different equilibrium portfolios at year-start. In particular, we find that the majority of the managers invest in the stock while the remainder invest in the bond, and also that the fraction of stockholders increases with the stock volatility. Since in this extended model trading happens more than once, we are also able to look at the trading volume implications

of tournament behavior, which have not been explored in the extant literature. Here, our analysis leads to a novel prediction that the mid-year trading volume is inversely related to the mid-year stock return.

1.1.   Related Literature

Although the results of our baseline model and its generalizations are in contrast to the widely-used "intuitive" tournament hypothesis, the predictions of our model are supported by several empirical studies. Busse (2001) finds "no evidence that mid-year losers increase end of year risk more than winners. If anything, the results indicate the opposite." Employing a different dataset, Qiu (2003) documents a similar pattern that "mid-year loser funds have less incentives to increase their funds risk relative to mid-year winner funds." Relying on the "intuitive" tournament hypothesis, both Busse and Qiu seem to view their results as being at odds with the tournament behavior. Our analysis reveals that their findings are in fact consistent with a rational tournament model.

The most related to our theoretical work are Goriaev, Palomino, and Prat (2003), Taylor (2003), and Chen and Pennacchi (2009) who look at the effect of interim performance on managers' risk taking. A key difference is that both Goriaev et al. and Taylor look at a strategic setting with two managers, an interim winner and a loser, and characterize their behavior by appealing to Nash equilibrium. From the viewpoint of the actual fund industry comprised by hundreds of funds, the settings in these papers correspond to a scenario where all interim winners or losers cooperatively decide on their investment strategy. In ours, managers choose their portfolios alone and are not affected by strategic motives as they recognize they are competing against a large number of managers. Being built on different premises, the predictions of these papers are considerably different. In Goriaev et al., an interim loser takes on more risk than a winner, the opposite of our result. Moreover, at year-start both their managers choose the same level of risk, while our managers, though identical, choose different levels of risk. In Taylor, both managers resort to mixed strategies in equilibrium as each tries to "confuse" the opponent, and hence only with some probability an interim winner chooses a riskier strategy than a loser. Chen and Pennacchi (2009) differ from our work in that they only consider sufficiently risk averse managers whose risk aversion dominates the fund flows convexity, and such managers who are unconstrained in their

4

portfolio choice. Their model does not generate a clear-cut empirical prediction as to how portfolio volatility is related to interim performance – they find that the relation can go either way. Finally, the trading volume implications of tournament behavior are not investigated in Goriaev et al., Taylor, and Chen and Pennacchi.

Also related is the literature investigating other aspects pertaining to a fund tournament. Palomino (2005) studies the effect of relative performance concerns on the degree of competition, measured by the number of competing funds, and also on the trading strategies. Li and Tiwari (2008) focus on the welfare implications of tournament behavior. Loranth and Sciubba (2006) investigate how the riskiness of fund strategies is affected by the (threat of) entry by new funds. Basak and Makarov (2009) study strategic interactions among a small number of top-performing funds.

Our paper also contributes to the literature establishing that convexities in managers' objectives with relative concerns have important and often unexpected implications for the volatility of optimal portfolios. Examples are Carpenter (2000), Basak, Pavlova, and Shapiro (2007), Panageas and Westerfield (2009), Cuoco and Kaniel (2010). Basak et al., Cuoco and Kaniel, as well as Chen and Pennacchi, also note the point made in our paper that fund managers' behavior can be directed towards increasing the tracking error volatility. However, these works do not investigate the link between interim performance and portfolio volatility or the trading volume implications. Models with relative concerns and no convexities have also been useful in explaining a number of empirical regularities (Abel (1990), DeMarzo, Kaniel, and Kremer (2007, 2008)).

More broadly, our paper is related to the work on tournaments in other environments (Lazear and Rosen (1981), Green and Stokey (1983), Bhattacharya and Guasch (1988), Taylor (1995), Zwiebel (1995), among many others). It is worth noting that this literature often looks at "winner-take-all" reward functions, while in the context of a mutual fund tournament the rewards (i.e., money flows) accrue to a large number of managers. Huang, Wei, and Yan (2007) formally show that such a reward function arises in equilibrium due to information acquisition and participation costs faced by retail investors.

The paper proceeds as follows. Section 2 describes the baseline economy, and Section 3 characterizes the equilibrium in this economy. Section 4 generalizes the baseline model to accommodate risk aversion, while Section 5 accommodates multiple risky stocks and

positive risk premium. Section 6 demonstrates how differential interim performance arises endogenously, and also investigates the trading volume implications of tournament behavior. Section 7 concludes. The Appendix contains all proofs.

2. Baseline Economy

In this Section, we describe our baseline economy where we intentionally abstract away from some pertinent features of a fund tournament (incorporated later in Sections 4–6). With such a stylized model, we are able to characterize equilibrium in closed form and describe our main insights in the clearest way possible.

The economy is populated by a continuum of risk neutral fund managers, indexed by $i \in [0, 1]$. Hereafter, we use the terms "fund", "manager", and "fund manager" interchangeably. Financial investment opportunities are given by a riskless bond and a risky stock. The bond return is normalized to 1, while the stock return x is normally distributed with mean 1 and variance σ^2.

There are two time periods with no discounting, $t = 1$ and $t = 2$, which we refer to as mid-year and year-end, respectively.[2] At time 1, manager i inherits a certain return $r^0(i)$ accumulated between the year-start and mid-year which we label as manager i's interim performance. Without loss of generality, we assume that all managers have a unit wealth at year start, implying that manager i's time 1 wealth equals her interim performance $r^0(i)$. In the current economy, the difference in managers' interim performance is exogenously assumed, however Section 6 formally establishes that this difference arises endogenously as a result of the managers' choosing different portfolios at year-start. We assume that $r^0(i)$ is continuous and increasing in index i; that is, we assign a higher index i to a manager with a higher interim performance.

Manager i chooses a portfolio strategy $\alpha(i)$, where $\alpha(i)$ denotes the fraction of wealth invested in the stock at time 1. The managers choose their strategies at the same time, and hence whether a manager is able to observe the other managers' strategies or not is inconsequential in our setting. We assume no-short-sale constraints on both assets, i.e.,

---

[2]Year-end is an important date since around this time many popular fund rankings are published in the media, and based on them households choose funds for investing money.

$\alpha(i) \in [0, 1]$ for all $i \in [0, 1]$, as observed in the mutual fund industry (Almazan, Brown, Carlson, and Chapman (2004)). Manager i's performance at time 2 , $R(i)$, is given by

$$R(i) = r^0(i)(\alpha(i)x + 1 - \alpha(i)). \tag{1}$$

The *industry performance* $\bar{R}$ is defined as the average of all managers' performances $R(i)$, and so is obtained by integrating the right-hand side of equation (1) over $i \in [0, 1]$. This yields after some algebra

$$\bar{R} = \bar{r}^0(\bar{\alpha}x + 1 - \bar{\alpha}), \tag{2}$$

where

$$\bar{\alpha} = \frac{\int_0^1 r^0(i)\alpha(i)di}{\bar{r}^0}, \qquad \bar{r}^0 = \int_0^1 r^0(i)di. \tag{3}$$

Given the form of expression (2), the industry performance $\bar{R}$ is equal to the performance of a hypothetical fund whose interim performance is $\bar{r}^0$ and whose portfolio strategy is $\bar{\alpha}$, where $\bar{\alpha}$ and $\bar{r}^0$ are as given in equation (3). Henceforth, we refer to $\bar{\alpha}$ as the *industry portfolio strategy*, the fraction of industry wealth invested in the stock.

We formulate tournament behavior by postulating that fund i receives money flows at year-end depending on its relative performance $R(i) - \bar{R}$ via an increasing and convex fund flows function $f(\cdot)$, which is consistent with empirical findings (Chevalier and Ellison (1997), Sirri and Tufano (1998)). We assume that managers have common knowledge of the function $f(\cdot)$. Consequently, manager i's year-end wealth $W(i)$ is given by $W(i) = R(i) + f(R(i) - \bar{R})$, which after substituting expressions (1)–(2) yields

$$W(i) = r^0(i)(\alpha(i)x + 1 - \alpha(i)) + f(r^0(i)(\alpha(i)x + 1 - \alpha(i)) - \bar{r}^0(\bar{\alpha}x + 1 - \bar{\alpha})). \tag{4}$$

When choosing her portfolio, each manager conjectures that the industry performance $\bar{R}$ has a certain distribution, being aware that her choice does not affect this distribution since she is atomistic. In equilibrium, it must be the case that aggregating over individual portfolios leads to the distribution of $\bar{R}$ as conjectured by every manager. Throughout the paper, a variable with a hat ^ denotes a best response quantity, and a variable with an asterisk * an equilibrium quantity. From (2), the distribution of $\bar{R}$ is completely characterized by the

industry portfolio strategy $\bar{\alpha}$, and so an equilibrium in our economy is defined as follows.

Definition 1. *An equilibrium is a pair $(\alpha^*(i), \bar{\alpha}^*)$ such that that the following two conditions are satisfied.*

   (i) ***Best response condition.*** *Given an industry portfolio strategy $\bar{\alpha}^*$, $\alpha^*(i)$ maximizes manager i's expected wealth*

$$\alpha^*(i) = \arg \max_{\alpha(i) \in [0,1]} \quad E\left[W(i)\right], \tag{5}$$

   where $W(i)$ is as given in (4).

  (ii) ***Aggregation condition.*** *Aggregating individual portfolio strategies $\alpha^*(i)$ yields the industry portfolio strategy $\bar{\alpha}^*$:*

$$\bar{\alpha}^* = \frac{\int\limits_0^1 r^0(i)\alpha^*(i)di}{\bar{r}^0}. \tag{6}$$

While our main focus is on equilibrium, we will also find it helpful to describe the managers' best response strategies. For a given industry strategy $\bar{\alpha} \in [0,1]$, manager i's best response $\hat{\alpha}(i)$ is a solution of the best response condition (5), i.e., $\hat{\alpha}(i)$ maximizes manager i's expected wealth (4). Note that the first term on the right-hand of (4) is zero in expectation since the expected stock and bond returns are equal. As a result, in our baseline economy we are able to easily isolate the effect of tournament incentives since the managers' behavior is driven purely by the second "tournament" term $f(\cdot)$ in (4).

3. Equilibrium

In this Section, we analytically characterize the managers' best response and equilibrium portfolio strategies in the baseline economy. We specify an exponential form for the fund flows function $f(\cdot)$, thus satisfying the above requirements that for $f(\cdot)$ to be plausible it needs to be increasing and convex. In particular, we let

$$f(R(i) - \bar{R}) = \exp(c(R(i) - \bar{R})), \tag{7}$$

8

where $c > 0$ controls the convexity of f. In fact, all our results hold for a much larger class of flow functions (see Remark 1), and it is only for notational convenience that we consider the simple specification (7). Proposition 1 presents the managers' best response strategies.

Proposition 1. *For a given industry portfolio strategy* $\bar{\alpha} \in [0, 1]$, *manager i's best response strategy is given by*

$$\hat{\alpha}(i) = \begin{cases} 1, & r^0(i) \geq 2\bar{\alpha}\bar{r}^0, \\ 0, & r^0(i) < 2\bar{\alpha}\bar{r}^0. \end{cases} \tag{8}$$

Proposition 1 reveals that there exists a threshold value of interim performance, $2\bar{\alpha}\bar{r}^0$, which divides all funds into two categories. Those whose interim performance is above the threshold, the *interim winners*, invest fully in the risky stock. The remainder, the *interim losers*, invest in the bond. This result is at odds with the tournament hypothesis, first formulated by BHS and subsequently used in many other studies.

The intuition behind our result is as follows. The convexity of fund flows with respect to relative performance leads to gambling behavior, whereby managers seek to maximize the tracking error volatility, i.e., the volatility of relative performance $R(i) - \bar{R}$. Given the convexity, the tracking error volatility is maximized by either investing fully in the stock or in the bond. Suppose that manager i invests in the stock, and thus maximizes her portfolio volatility. This has the following two effects. First, investing in the stock induces a positive correlation between her own return $R(i)$ and the industry return $\bar{R}$,[3] which has a negative effect on the tracking error volatility. Second, investing in the stock maximizes the volatility of her own return $R(i)$ which has a positive effect on the tracking error volatility. The first negative effect may well be dominant, and so maximizing the portfolio volatility is *not* synonymous to maximizing the tracking error volatility, as also noted in existing studies (Chen and Pennacchi (2009), Cuoco and Kaniel (2010)). Since the volatility of $R(i)$ is the product of manager i's interim performance $r^0(i)$ and the stock volatility σ, the magnitude of the second positive effect increases with interim performance. As a result, when a manager's interim performance is high enough, the second effect more than offsets the first negative effect, and so interim winners' best response strategy is to invest in the stock. For interim

---

[3] We here disregard the knife-edge scenario where all managers invest in the bond, in which case the industry return $\bar{R}$ is completely riskless ($\bar{\alpha} = 0$). Such a scenario can never occur in equilibrium.

losers, the first effect dominates, implying that their best response is to invest in the bond.

From the above discussion, we see that one of the mechanisms behind our result is that fund managers have incentives to increase the tracking error volatility. This mechanism is also present and noted in previous works (Basak, Pavlova, and Shapiro (2007), Chen and Pennacchi (2009), Cuoco and Kaniel (2010)). However, these studies do not investigate how a manager's portfolio volatility depends on interim performance, which is the main focus of our paper. To complete the analysis of the tournament, Proposition 2 fully characterizes the equilibrium in the baseline economy.

Proposition 2. *The equilibrium industry portfolio strategy $\bar{\alpha}^*$ and the equilibrium interim performance threshold $r^0(i^*)$ are implicitly given by*

$$\bar{\alpha}^* = \frac{\int\limits_{i^*}^{1} r^0(i)di}{\bar{r}^0}, \quad r^0(i^*) = 2\bar{\alpha}^*\bar{r}^0, \tag{9}$$

where the solution exists and is unique. The equilibrium threshold $r^0(i^*)$ lies strictly within the range of values of $r^0(i)$. The equilibrium portfolio strategies $\alpha^*(i)$ are obtained by setting $\bar{\alpha} = \bar{\alpha}^*$ in the best response expression (8).

Proposition 2 demonstrates that in equilibrium there exist managers (with positive mass) both above and below the interim performance threshold $r^0(i^*)$. This implies that managers cannot *all* invest in the stock and all be interim winners, nor can they *all* invest in the bond and all be interim losers. The reason is that if all managers invested in the same asset, each individual manager would be able to increase her tracking error volatility by switching to the other asset. Proposition 2 also reveals that the interim performance threshold $r^0(i^*)$ depends on the shape of the interim performance relation $r^0(i)$, as evident from equation (9). That is, the threshold fund is determined by the realized cross-section of interim performances, and so does not in general coincide with the median fund as assumed in BHS and other works. Hence, our model generates a novel prediction concerning the fractions of managers choosing high volatile versus low volatile strategies at mid year: the higher the threshold i^* is, the lower the share of managers who opt for high volatility. A possible way to test this implication is as follows. Given panel data containing information about fund strategies and mid-year performances over a number of years, calculate for each year the following two

10

variables: i) the threshold i^*, computed by solving condition (9) from the observed interim performances, and ii) the fraction of managers who increase their portfolio volatilities over the second year-half. Our model predicts a negative correlation between these two variables.

Remark 1. *More general flow-performance specifications.* All the results of this Section remain valid for a much larger class of flow performance specifications than the exponential specification (7). In particular, suppose that the flow function is a linear combination of exponential basis functions:

$$f(R(i) - \bar{R}) = \sum_{j=1}^{J} a_j \exp(c_j(R(i) - \bar{R})), \tag{10}$$

where $J > 0$ is an integer and $a_j > 0, c_j > 0$ for all $j = 1, ..., J$. Looking at each individual term in the sum of the right-hand side of equation (10), its maximand is given by expression (8), as proved in Proposition 1. Since $a_j > 0$, $j = 1, ..., J$, the maximand of the sum equals the maximand of the individual terms, and so is also given by (8). Hence, the managers' best responses (Proposition 1) remain the same under specification (10), and so does the equilibrium (Proposition 2). Moreover, since J, a_j, and c_j are arbitrary positive numbers (the only restriction is that J is an integer), specification (10) potentially spans a large set of increasing convex flow-performance functions. Indeed, Vasicek and Fong (1982) demonstrate that in the context of yield curve modelling, exponential spline fitting "exhibits ... sufficient flexibility to fit a wide variety of shapes of the term structure." This suggests that our specification (10) can generate a rich set of possible flow-performance functions. Note that since the exponential basis functions are smooth, specification (10) may fail to adequately approximate functions exhibiting kinks. In Section 4, we present an example that shows that our main predictions are still valid for an empirically plausible fund flow function with a kink.

4.   Risk Aversion and Convexity of Fund Flows

In this Section, we extend our baseline model of Sections 2–3 to incorporate managerial risk aversion. Since a manager's year-end wealth (4) depends on both her own performance and

11

relative performance, she in general accounts for both when choosing her portfolio.[4] However, to provide some basic intuition on the kind of incentives that drive risk averse managers, we focus on the latter consideration – the desire to maximize relative performance, and hence fund flows. As discussed in Section 3, absent risk aversion the implication of the fund flows convexity is that each manager seeks to diverge from the industry-average return so as to increase her expected inflows. If convexity were absent, on the other hand, then risk averse managers would want to mimic the industry-average portfolio so as to minimize their tracking error volatility.[5] With both features – risk aversion and convexity – now being present, it is mainly the interplay of these two mechanisms which determines the patterns of managerial risk taking.

To provide a unified analysis of possible behaviors resulting from this interplay, we consider a setting with two types of managers who differ in risk aversion, type-L (low risk aversion) and type-H (high risk aversion); the economy is otherwise as in Section 2. Intuitively, the behavior of type-L managers is going to be primarily driven by the convexity of fund flows while the behavior of type-H managers by their risk aversion. If all managers were of the same type, either type-L or type-H, then their equilibrium portfolios would be similar to the equilibrium portfolios of the corresponding type of managers in the heterogenous risk aversion setting described below. Apart from generality, considering managers with different risk aversion is consistent with Koijen (2008), who finds substantial heterogeneity in attitudes towards risk across mutual fund managers. We assume that both types of managers have a standard CRRA utility function $u(\cdot)$ defined over terminal wealth W:

$$u(W) = \frac{W^{1-\gamma}}{1-\gamma},\tag{11}$$

where $\gamma > 0$ is the relative risk aversion coefficient. The risk aversion of type-L and type-H managers is $\gamma = \gamma_L$ and $\gamma = \gamma_H$, respectively, with $\gamma_L < \gamma_H$. Since this richer model with risk

---

[4]As noted in Section 2, in the baseline economy the managers' behavior is driven purely by the desire to increase relative performance, i.e., second term $f(\cdot)$ in expression (4). With risk aversion, a manager's objective is a concave function over (4), and so the first term in (4) does not disappear from the managers' optimization, implying that managers also care about increasing their absolute performance.

[5]Indeed, given that the mean stock and bond returns are equal, the managers cannot change their own expected portfolio return, and hence cannot change their expected relative performance. They can only control the volatility of their relative performance, and so they aim to minimize this volatility since the combination of risk aversion and non-convex fund flows leads to a concave objective function over relative performance.

aversion turns out to not be tractable analytically, we consider two examples with different, empirically plausible fund flow functions $f(\cdot)$, and solve for the equilibrium numerically.[6]

Leading Example 1. *Exponential flow specification.* We set $\gamma_L = 1$ (i.e., logarithmic utility) and $\gamma_H = 5$, implying an empirically reasonable average risk aversion of three. The exponential flow function is given by equation (7) for which the convexity parameter c is set at $c = 4$, meaning that a 10% excess return leads to a 50% inflow of new investments into the fund. This is broadly consistent with Chevalier and Ellison (1997) who find that a mutual fund is expected to "grow by approximately 55 percent if its return is 10 points greater than the market return." We abandon the assumption of Section 2 that the stock return x is normally distributed, and assume that x is lognormally distributed with mean 1 and volatility 10.5%, corresponding to volatility 15% per annum. Retaining normality would lead to a degenerate model with essentially one asset, the riskless bond.[7] Each type of manager has unit mass, and we use the same index $i, i \in [0, 1]$ to refer to both types. We assume that the interim performance within each type is uniformly spread between 0.9 and 1.1, i.e., the interim performance of type-L and type-H managers with index i is given by $r^0(i) = 0.9 + 0.2 * i$.

We solve numerically for the equilibrium portfolios of type-L and type-H managers, $\alpha_L^*(i)$ and $\alpha_H^*(i)$, respectively, and present the results in Figure 1. Looking at the relatively risk tolerant type-L managers, we see that in equilibrium interim winners invest in the stock while interim losers invest in the bond, analogous to the equilibrium in our baseline setting of Section 3. The reason is that the effect of fund flows convexity dominates the effect of risk aversion, in which case risk averse managers behave similarly to risk neutral baseline managers. The relatively risk averse type-H managers choose almost the same portfolio regardless of their interim performance (the slope of the dashed line in Figure 1 is positive but close to zero). For these managers, risk aversion outweighs the convexity of fund flows,

---

[6] We implement the following approach to compute the equilibrium. For each possible value of the industry strategy $\underline{\alpha} \in [0, 1]$, we find the optimal strategies of type-L and type-H managers, $\alpha_L^*(i)$ and $\alpha_H^*(i)$, which maximize the respective expected utilities. Then, we substitute these optimal $\alpha_L^*(i)$ and $\alpha_H^*(i)$ into equation (3) to compute the industry strategy $\bar{\alpha}$ as implied by the managers portfolios. The equilibrium obtains when $\underline{\alpha} = \bar{\alpha}$, and the equilibrium industry strategy is $\bar{\alpha}^* = \underline{\alpha} = \bar{\alpha}$. As we already know the managers' optimal strategies for each $\underline{\alpha} \in [0, 1]$, the equilibrium strategies are the optimal strategies obtained for $\underline{\alpha} = \bar{\alpha}^*$.

[7] Indeed, no CRRA manager would want to invest in the risky stock with normally distributed return, as doing so may lead to negative wealth over which CRRA utility is not well-defined. Our main predictions are not affected if we consider other distributions of x such that the model is not degenerate, e.g., truncated normal or uniform.
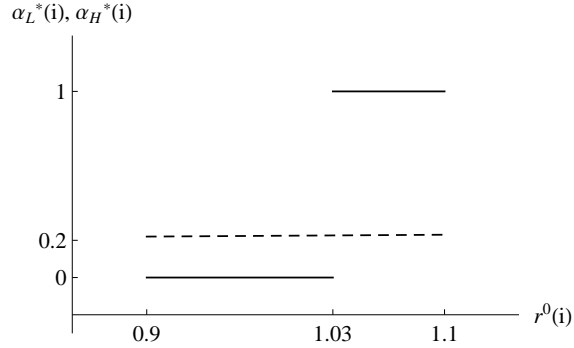
$\alpha_L^*(i),\ \alpha_H^*(i)$

0.9   1.03   1.1 $r^0(i)$

Figure 1: Equilibrium portfolios under heterogeneous risk aversion. The equilibrium portfolios of type-L (solid line) and type-H (dashed line) managers. Their relative risk aversion coefficients are $\gamma_L = 1$ and $\gamma_H = 5$, respectively. The remaining parameter values are as presented in Example 1.

and so they are not driven by the desire to win the tournament and get inflows. Consequently, a manager's interim standing in the competition for flows has little effect on her equilibrium behavior.

Solving the model for different risk aversions γ_L and γ_H reveals that we obtain a similar equilibrium as in Figure 1 as long as $\gamma_L < 1.2$ and $\gamma_H > 1.2$ (1.2 is obtained by rounding the actual threshold to one decimal). That is, given the level of convexity $c = 4$ in the leading Example 1, the fund flows convexity dominates the effect of a given risk aversion γ when $\gamma < 1.2$, and is dominated by risk aversion when $\gamma > 1.2$. Though inconsequential to our main message, we note that when $\gamma_L > 1$ and the range of interim performance is considerably wide (as quantified below for several values of γ_L), we may potentially have the relation between portfolio volatility and interim performance for type-L managers to not fully coincide with that in Example 1 (solid line Figure 1). In particular, type-L managers with extremely high or low interim performance may find that most realizations of their terminal wealth are outside the convex region of their objective functions, and so they would choose portfolios close to those of type-H managers.[8] This only happens when the difference in interim performance between the best and worst performing managers (i.e., $r^0(1) - r^0(0)$) is higher than 1.5 (15000 basis points) when $\gamma_L = 1.05$, higher than 1.1 when $\gamma_L = 1.1$, higher than 0.5 when $\gamma_L = 1.15$, and higher than 0.3 (3000 bps) when $\gamma_L = 1.2$. The higher the risk aversion γ_L the smaller the effect of the convexity on the behavior of type-L managers, and

---

[8]The reason why this behavior occurs for $\gamma_L > 1$ is a technical one: CRRA utility function is bounded when $\gamma_L > 1$, implying that the composite function $u(f(\cdot))$ can be locally but never globally convex however high the convexity of $f(\cdot)$ is.

so the critical level of the performance differential $r^0(1) - r^0(0)$ (under which the outcome is as in the baseline model) decreases in γ_L.

To better understand how the risk aversion threshold depends on the shape of the flow-performance relationship, we consider here only the following (slightly more general) flow-performance relation

$$f(R(i) - \bar{R}) = a \exp(c(R(i) - \bar{R})),\tag{12}$$

where the scaling parameter $a > 0$ captures the proportional change of money flows occurring at all levels of relative performance.[9] Figure 2 depicts how the risk aversion threshold depends on parameters c and a. Figure 2(a) reveals that the risk aversion threshold is positively related to the convexity parameter c, consistent with the above discussion of the interplay between convexity and risk aversion. Combining this result with Chevalier and Ellison (1997) who find that the flow-performance relation is more convex for young funds than for old funds, we get that the tournament behavior suggested in our analysis is likely to be more pronounced among young funds.[10] From Figure 2(b), it turns out that the threshold monotonically tends to unity as the scaling parameter a increases, and so whether the threshold increases or decreases depends on whether it is above or below the pivotal level of one (corresponding to logarithmic objective function). This result resonates with other portfolio choice applications where the case of logarithmic utility often acts as a pivotal case. For relatively high convexities $c = 4$ and $c = 6$, the risk aversion threshold is higher than one, and so it decreases with a (solid and dashed lines in Figure 2(b)). For relatively low convexity $c = 2$, the risk aversion threshold is lower than one, and so it increases with a (dotted line in Figure 2(b)).

Solving Example 1 under more general flow specifications of the form (10) leads to equilibria that are similar to the equilibrium with the baseline specification (7) depicted in Figure

---

[9]Since specification (12) is a special case of (10), all predictions of the baseline model remain valid under (12), as explained in Remark 1. As for the risk aversion case, we calibrate (12) to match Chevalier and Ellison (1997), which yields $a = 1.15$ and $c = 3$, and then solve the risk averse Example 1 under this calibration. The resulting equilibrium portfolios of type-L and type-H managers remain as depicted in Figure 1.

[10]Indeed, from Figure 2(a) a higher flows convexity results in a higher risk aversion threshold for young funds, and so the range of risk aversions for which a manager exhibits type-L behavior is wider for young funds. Provided that the cross-sectional distribution of risk aversions across young managers is the same or relatively similar to that across old managers, the higher threshold implies that the fraction of type-L managers is higher among young funds than old funds, and so young funds are expected to more strongly exhibit the tournament pattern.
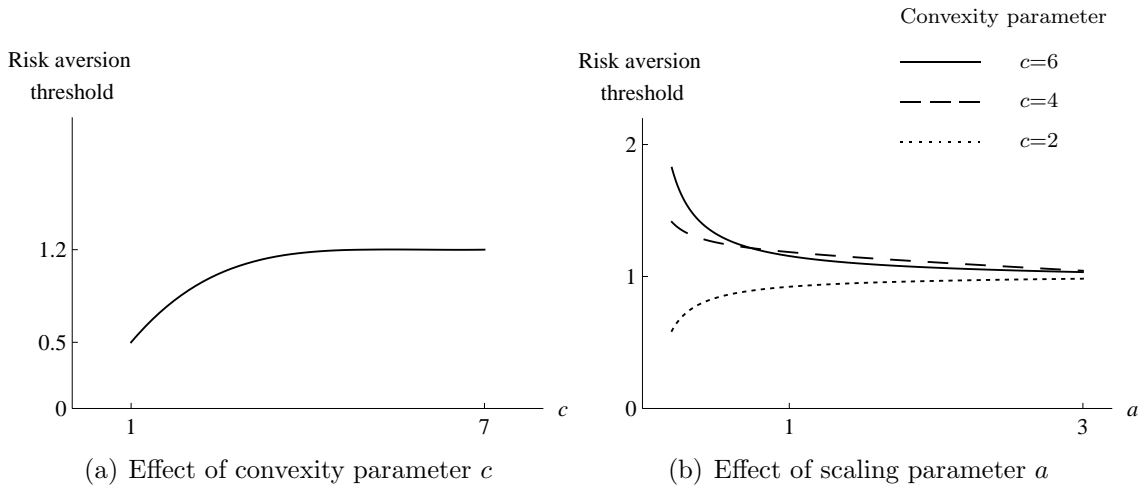
Figure 2: **Risk aversion threshold**. The risk aversion threshold for varying levels of the convexity c and scaling a parameters. In panel (a), the scaling parameter is $a = 1$. In panel (b), dotted line corresponds to $c = 2$, dashed line to $c = 4$, solid line to $c = 6$. The remaining parameter values are as presented in Example 1.

1.[11] To investigate the robustness of our results to another plausible flow function that cannot be generated by specification (10), in Example 2 we consider an option-like specification with a kink, which is consistent with the evidence of Sirri and Tufano (1998).

Example 2. *Flow specification with a kink.* Sirri and Tufano (1998) find that "the performance-flow relationship documented here...gives fund complexes a payout that resembles a call option." According to their evidence, the flow-performance relationship is almost flat when the excess return is not in the top quintile, and increases at rate 1.47 when the excess return is in the top quintile (Table 3 in Sirri and Tufano). Accordingly, we consider a flow specification $f(R(i) - \bar{R})$ that is flat until relative performance $R(i) - \bar{R}$ reaches a threshold, beyond which it is linearly increasing in $R(i) - \bar{R}$:

$$f(R(i) - \bar{R}) = \begin{cases} 1 & R(i) - \bar{R} < 0.1, \\ 1 + 1.5 * (R(i) - \bar{R} - 0.1) & R(i) - \bar{R} \geq 0.1. \end{cases} \tag{13}$$

All other parameters are as in Example 1. Solving the model numerically for various levels of risk aversions γ_L and γ_H, we find that the structure of the ensuing equilibria is similar to that in Example 1, in that there are the same two types of behavior depending on whether

---

[11]In particular, we consider all possible flow functions obtained by setting $J = 3$ in (10), letting a_1, a_2, and a_3 to take on values from the grid $[0.2, 0.5, 1, 1.5, 2, 3]$, and c_1, c_2, and c_3 to take on values from the grid $[1, 2, ..., 7]$. Solving for equilibrium under each calibration yields that there exists a certain risk aversion threshold such that portfolio volatilities of managers below the threshold are as in our baseline model, while the volatilities of managers above the threshold are virtually insensitive to interim performance.

the manager's risk aversion is above or below a risk aversion threshold. One difference from Example 1 is that the threshold now equals five while in Example 1 it equals 1.2, though both thresholds play the same role and have similar properties. That the threshold is higher now means that the range of γ_L and γ_H for which the equilibrium outcomes under risk aversion and risk neutrality are similar is considerably wider under an option-like flows function (13) than under the baseline function (7). The reason is that specification (13) features a kink, around which convexity is relatively high, while specification (7) is smooth, implying a comparatively low convexity.

From the above discussion, we have that if we were to depict the equilibrium for the heterogenous risk aversion setting of Example 1, with $\gamma_L = 1$ and $\gamma_H = 5$, then the shape of the relation between interim performance and risk taking would be similar for type-L and type-H managers as neither type would be above the threshold.[12] Given this, for illustrative purposes we assume away the heterogeneity and set $\gamma_L = \gamma_H = 3$. Figure 3 presents the resulting equilibrium portfolios. We see that the equilibrium outcome is as in the risk-neutral baseline model of Section 3.
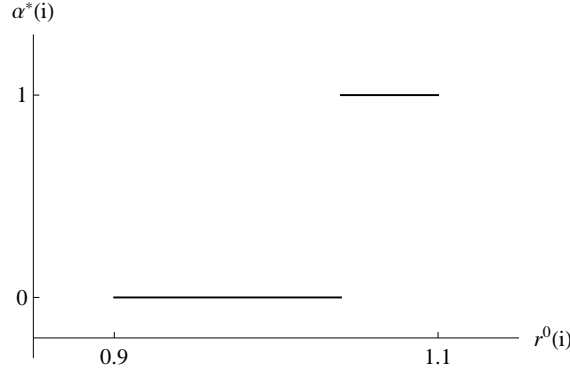

Figure 3: Equilibrium portfolios under the option-like flow specification. The equilibrium portfolios of CRRA managers with relative risk aversion $\gamma = 3$. Other parameter values are as described in Example 2. The plot is typical for other risk aversion coefficients, $\gamma \in (0, 5]$.

In summary, while the equilibria in the two settings – baseline without risk aversion and this Section with risk aversion – are somewhat different, the empirical implications are in fact very similar. Namely, dividing the managers into two groups based on interim performance,

---

[12]If we set γ_H to be higher than five, then the equilibrium portfolios of type-H managers would be virtually insensitive to interim performance, similarly to the corresponding result in Example 1 (dashed line in Figure 1).

both models predict that the portfolio volatility of interim winners is higher than that of interim losers. The presence of managers with relatively high risk aversion can only make this result less pronounced since these managers' portfolios exhibit little sensitivity to interim performance.

4.1. Role of Portfolio Constraints

Given our focus on understanding the *mutual fund* tournament, for the bulk of our paper we assume no-short-sale constraints since such constraints are prevalent in the mutual fund industry. Indeed, Almazan, Brown, Carlson, and Chapman (2004) document that 70% of mutual funds reported to the SEC that short-selling is not permitted under their investment policy, and that among the remaining 30%, only 3% in fact engaged in short-selling. From Figure 1, we observe that the constraint binds for type-L managers (solid line) and does not bind for type-H managers (dashed line), and also that the volatility-interim performance relation for "constraint-bound" type-L managers is notably different from that for "unconstrained" type-H managers. This prompts us to take a closer look at the role of portfolio constraints in our setting, which is of interest not only from from a theoretical perspective but also since it can shed some light on whether our results may be readily applicable to the hedge fund industry where the managers are largely unconstrained.

With risk aversion present, there are three forms of a manager's objective function that may arise in our model: convex (as for type-L managers in Example 1), concave (as for type-H managers in Example 1), or locally convex (as for managers in Example 2). As established above, under a no-short-sale constraint managers with convex objectives behave similarly to those with locally convex objectives (compare the solid plot in Figure 1 with the plot in Figure 3). When the constraint is now lifted, convex and locally convex objectives no longer lead to similar behaviors. In particular, while an unconstrained manager with a locally convex objective function chooses a bounded position in the risky stock as the risk aversion is dominant outside the convexity region, an unconstrained manager with a convex objective would seek an unbounded position in the stock. Hence, we would not obtain an equilibrium if some unconstrained managers in the economy have convex objectives (as in Example 1). There are, however, several considerations absent in our model that may prevent actual fund managers from taking very big gambles even when the managers are able to short, e.g.,

reputational concerns.[13] Given our focus, we do not introduce such additional features into our analysis, and so describe the unconstrained behavior of managers with concave or locally convex objectives only.

We now look at how the managers' equilibrium portfolios in Examples 1 and 2 are affected once we lift the no-short-sale constraints. From the above, in the unconstrained Example 1 we assume that all managers are of type-H and have the same risk aversion $\gamma_H = 5$, meaning that the economy is populated by managers with concave objectives. The resulting equilibrium is simple (and does not warrant a separate figure): all managers invest fully in the bond, implying that their portfolio volatility is not sensitive to interim performance. Given that the risk aversion dominates the flows convexity, the managers are essentially ignoring the tournament incentives created by fund flows and so invest in the bond as the risky stock offers no premium for risk. As demonstrated later, the managers with concave objectives do not choose the same portfolio volatility in an economy with positive risk premium (see the unconstrained Example 3 and Figure 6 in Section 5).

Turning to Example 2, we are able to describe the equilibrium portfolios with no constraints since the managers' objectives are locally convex. Figure 4 depicts manager i's equilibrium portfolio volatility, $\sigma^*(i)$, as a function of her interim performance, $r^0(i)$. First, from Figure 4 we see that when short-selling is allowed, some managers do use the opportunity to take short positions, as evident from the volatility of interim winners being around 15% while the maximum volatility under a no-short sale constraint equals the stock return volatility and so is 10.5%. Figure 4 also reveals that all managers can be divided into interim losers and interim winners, with the interim winners choosing a higher portfolio volatility than the losers, implying that the empirical predictions of the unconstrained Example 2 are similar to those when the constraint is present (Figure 3). The portfolio volatility of the unconstrained interim losers is zero, as in the constrained Example 2. For the unconstrained interim winners, however, the relation between interim performance and volatility is hump-shaped, unlike the flat relation obtained under the constraint. The intuition for the hump-shape is that the winners in the middle of the convexity region choose the highest volatility as they have the highest incentives to leave the region, while for winners closer to

---

[13]Brown, Goetzmann, and Park (2001) argue that there is "a clear tension between risk taking and the desire to develop or preserve a reputation." Supporting this argument, their empirical analysis reveals that there exist "reputational externalities that may prevent big gambles."
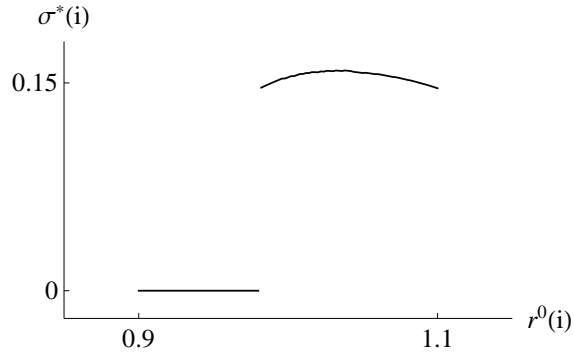
Figure 4: Equilibrium unconstrained portfolio volatilities under the option-like flow specification. The equilibrium portfolio volatilities of risk averse managers when a no-short-sale constraint is absent. The economic setting and parameter values are as described in Example 2.

the boundaries of the region the corresponding incentives are weaker, and so they choose lower portfolio volatilities.

5. Multiple Stocks and Risk Premium

In this Section, we investigate the robustness of our results to the case of multiple risky assets and positive risk premium, and also as an ultimate robustness check we look at a general case when these two features are combined with heterogenous risk aversion. We generalize the setting of Section 2 as follows. The investment opportunities are now given by N uncorrelated risky stocks, where $N \geq 1$, and a riskless bond.[14] The return on stock k, $k = 1, ..., N$, denoted by x_k, has mean $\mu \geq 1$ and volatility σ_k, where $\sigma_1 < \sigma_2 < ... < \sigma_N$ meaning that the stocks are sorted by volatility. For notational convenience, we may refer to the bond as stock 0, and so $\mu_0 = 1$ and $\sigma_0 = 0$. The assumption that all risky stocks have the same expected return μ ensures that our model is close in spirit to the tournament hypothesis discussed in the literature.[15] Up until Example 3, we assume that the risky assets are normally distributed. We let $\alpha_k(i)$, $k = 1, ..., N$, denote manager i's fraction of wealth

---

[14]The subsequent analysis remains fully applicable if stocks were correlated. Indeed, we can always combine the stocks into uncorrelated portfolios and treat these portfolios as individual stocks. The number of such uncorrelated portfolios equals the number of sources of uncertainty in the economy.

[15]In particular, from the discussion in Brown, Harlow, and Starks (1996) and Chevalier and Ellison (1997), the main driving force behind portfolio rebalancing in response to interim performance is the desire to change the portfolio volatility, and not the portfolio expected return. Given this, allowing for differential risk premia across risky stocks would distance our theoretical framework from empirical works.

invested in stock k, and $\sigma(i)$ manager i's portfolio volatility. As before, a variable with a hat $\hat{}$, an asterisk $*$, and an overbar $\bar{}$ represents a best response quantity, an equilibrium quantity, and an aggregate quantity, respectively.

Following the same steps as those leading to expression (2), we obtain that the industry performance $\bar{R}$ that enters into the fund flow function (7) is now

$$\bar{R} = \bar{r}^0 \left[\sum_{k=1}^{N} \bar{\alpha}_k x_k + (1 - \sum_{k=1}^{N} \bar{\alpha}_k) \right], \tag{14}$$

where

$$\bar{\alpha}_k = \frac{\int_0^1 r^0(i)\alpha_k(i)di}{\bar{r}^0}, \quad \bar{r}^0 = \int_0^1 r^0(i)di. \tag{15}$$

The equilibrium with multiple risky stocks is defined as in Definition 1 of Section 2 with the scalars $\alpha^*(i)$ and $\bar{\alpha}^*$ now being replaced by the vectors $(\alpha_1^*(i), ..., \alpha_N^*(i))$ and $(\bar{\alpha}_1^*, ..., \bar{\alpha}_N^*)$, respectively. The equilibrium can be characterized analytically either when there are multiple risky stocks but with zero risk premium or when the risk premium is positive but there is one risky stock. Proposition 3 reports the corresponding best responses and equilibrium outcomes.

Proposition 3. *In the case of multiple risky stocks with zero risk premium ($N > 1, \mu = 1$), for given industry strategies $\bar{\alpha}_k$, $k = 1, ..., N$, the best response of manager i is to fully invest all her wealth in stock $k(i)$, where $k(i)$ is determined from*

$$k(i) = \arg \max_{k \in \{0,1,...,N\}} (\sigma_k^2 r^0(i) - 2\sigma_k^2 \bar{\alpha}_k \bar{r}^0), \tag{16}$$

and where $\bar{\alpha}_k$ and $\bar{r}^0$ are as given in (15). In equilibrium, the managers' portfolio volatility $\sigma^*(i)$ is (weakly) increasing in the index i and, hence, in the interim performance $r^0(i)$. That is, the higher a manager's interim performance is the more volatile stock she holds in equilibrium.

In the special case of a single risky stock with positive risk premium ($N = 1, \mu > 1$), for

a given industry strategy $\bar{\alpha}$, the best response of manager i is given by

$$\hat{\alpha}(i) = \begin{cases} 1, & r^0(i) \geq 2\bar{\alpha}\bar{r}^0 - 2(1+c)r^0(i)(\mu-1)/(c\sigma)^2 \\ 0, & \text{otherwise.} \end{cases} \tag{17}$$

Manager i's equilibrium portfolio $\alpha^*(i)$ is obtained by substituting the equilibrium industry strategy $\bar{\alpha}^*$ in (17), where $\bar{\alpha}^*$ is the solution to the system of equations

$$\bar{\alpha}^* = \frac{\int_{i^*}^1 r^0(i)di}{\bar{r}^0}, \quad r^0(i^*) = 2\bar{\alpha}^*\bar{r}^0 - 2(1+c)r^0(i)(\mu-1)/(c\sigma)^2. \tag{18}$$

Proposition 3 reveals that our main prediction, that managers with higher interim performance take on more risk, is robust to the two generalizations considered. Under multiple risky stocks and zero risk premium, the managers' best responses admit an analytical representation (16), just as the best responses in our baseline model of Section 3 (see equation (8). However, unlike Section 3, it is not straightforward to see how equilibrium portfolios depend on interim performance by just eyeballing expression (16), and so we need to conduct an equilibrium analysis. Although a full characterization of equilibrium in closed form is not possible, as stated in Proposition 3, we are able to prove that in equilibrium managers with higher interim performance $r^0(i)$ choose portfolios with (weakly) higher volatility $\sigma^*(i)$, which is consistent with the predictions of the baseline model. Under the single risky stock case with positive risk premium, the best response equation (17) reveals that the interim winners invest in the risky stock while the remaining managers invest in the bond, which is as in the baseline model. Comparing with equation (9) describing the baseline equilibrium with no risk premium, we observe the presence of a negative term in the right-hand side of the second equation in (18), implying that the threshold i^* is now lower with positive risk premium. Hence, a positive risk premium increases the number of managers who invest in the stock, which is the consequence of the stock becoming relatively more attractive when it commands a positive risk premium.[16]

---

[16]Inspecting (17)–(18), we see that all managers may invest in the risky stock provided that the stock expected return μ is sufficiently high. The value of μ, however, would be determined by an equilibrium market clearing mechanism, and so it seems reasonable to focus on the case when μ is low enough to ensure a positive demand for the bond. We plan to investigate a general equilibrium version of our model in future research, which would enable us see how asset pricing implications of relative concerns differ when portfolio constraints are present (our setting) and absent (Cuoco and Kaniel (2010)).

When we consider a setting with multiple stocks and positive risk premium, the model is no longer analytically tractable, and so numerical analysis is required. In addition to these two features, we also incorporate heterogenous risk aversion since doing so strengthens the generality of our analysis without significantly complicating the computational algorithm. Example 3 presents the corresponding setting and its analysis.

Example 3. *Multiple stocks, positive risk premium, and heterogenous risk aversion.* To account for heterogeneous risk aversion, we adopt a setting described in Section 4 whereby there are two types of managers: type-L (relatively less risk averse) and type-H (relatively more risk averse), and we calibrate the model parameter values as in Example 1. We denote by $\sigma_L^*(i)$ and $\sigma_H^*(i)$ the equilibrium portfolio volatilities of type-L and type-H managers with index i, respectively. To account for multiple risky stocks with positive risk premium, we consider three risky stocks, i.e., $N = 3$, whose excess returns are lognormally distributed with mean $\mu = 6\%$ per annum and the per annum volatilities $\sigma_1 = 10\%$, $\sigma_2 = 15\%$, and $\sigma_3 = 20\%$, which are empirically plausible (see footnote 7 explaining why we abandon normality). Figure 5 plots the equilibrium volatilities, and so we again see that our main findings are robust under the generalized setting of this Example.
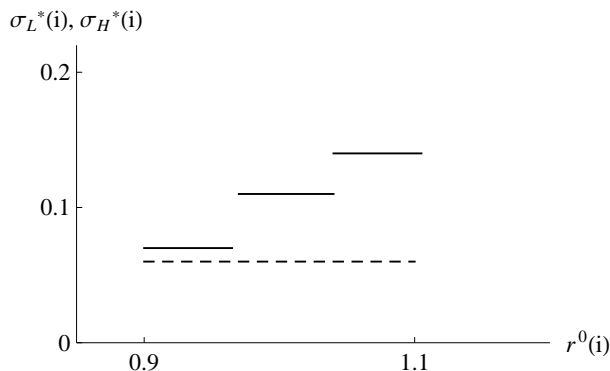

Figure 5: Equilibrium volatility under heterogeneous risk aversion, multiple stocks, and positive risk premium. The equilibrium portfolio volatility of type-L (solid line) and type-H (dashed line) managers. The calibration is as described in Example 3.

Finally, we investigate how the results of Example 3 are affected once we lift the no-short-sale constraints, which complements our analysis of the role of portfolio constraints in Section 4.1. The objectives of type-L managers are convex, and so we analyze the unconstrained Example 3 where the economy is populated by type-H managers only (as explained in Section 4.1). Figure 6 depicts the equilibrium in this economy, and we see that the relation between

interim performance and portfolio volatility is negative, which is contrary to the result in the baseline economy of Section 3. Intuitively, the key economic mechanisms at work in the baseline economy and in the unconstrained Example 3 are the reverse of each other – maximizing and minimizing the tracking error volatility, respectively, and so the results are the opposite too.
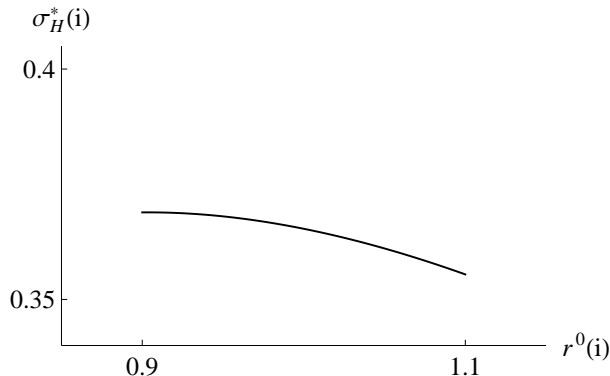

Figure 6: Equilibrium volatility in the unconstrained Example 3. The equilibrium portfolio volatility of type-H managers in the setting of Example 3 but with no type-L managers. The parameter values are as described in Example 3.

Our model is tailored towards investigating a *mutual fund* tournament, and so we leave for future work a rigorous analysis of a hedge fund tournament. However, based on our analysis of the unconstrained economies, we may provide some preliminary thoughts on whether one should expect the tournament behavior of (largely unconstrained) hedge fund managers to be different from our main predictions pertaining to mutual fund managers. While there is extant literature documenting that the flows-performance relation in the mutual fund industry is convex, there is less consensus on the shape of this relation in the hedge fund industry, with some studies documenting that the relation may be concave (Getmansky (2005)). Given the concavity, the objective functions of risk averse hedge fund managers would be concave in their relative performance. As a result, among the three unconstrained Examples, it is presumably Example 3 which is more appropriate for describing a hedge fund tournament. Supporting this conjecture, the equilibrium in the unconstrained Example 3 depicted in Figure 6 is broadly consistent with Brown, Goetzmann, and Park (2001) who look at the hedge fund industry and find "a significant reduction in variance conditional upon having performed well and limited evidence that managers who perform less well increase their risk exposure."

24

6. Endogenizing the Difference in Interim Performance

In this Section, we demonstrate that managers' differential interim performance at time 1 (middle of the year), which was exogenously specified till now, arises endogenously due to the managers' following different investment strategies at time 0 (beginning of the year). While our main focus is on time-0 equilibrium, we also outline the ensuing equilibrium at time 1, confirming that the relation between interim performance and portfolio volatility is positive, as in the baseline model. Finally, we investigate the trading volume implications of tournament behavior by analyzing how managers rebalance their time-0 portfolios at time 1. This aspect has not yet been studied in related works (Taylor (2003), Goriaev, Palomino, and Prat (2003)), and so our analysis offers novel implications.

We retain all the features and assumptions of our baseline setting of Section 2, but extend the timeline to include time 0 when managers can also trade. Since time 0 corresponds to beginning of the year and interim performance has not yet accumulated, managers are identical at time 0. Analogous to Section 2, we denote $\bar{\alpha}_0$ to be the aggregate share of wealth invested in the risky stock at time 0. Equivalently, we refer to $\bar{\alpha}_0$ as the fraction of managers who invest fully in the stock, as it turns out that no manager finds it optimal to hold both stocks and bonds in her time-0 portfolio (as proved in Proposition 4). Consequently, to show that the managers' time-0 portfolios – and hence their interim performances – are different, it suffices to demonstrate that the equilibrium fraction of stockholders $\bar{\alpha}_0^*$ is neither zero nor one, i.e., $\bar{\alpha}_0^* \in (0, 1)$. We assume that the stock return between times 0 and 1, denoted by x^0, has the same distribution as between times 1 and 2, i.e., x_0 is normally distributed with mean 1 and variance σ^2.

The definition of time-0 equilibrium is analogous to that at time 1 (Definition 1 in Section 2). Namely, taking $\bar{\alpha}_0$ as given, manager i chooses her time-0 fraction of wealth invested in the stock, denoted by $\alpha_0(i)$, so as to maximize her expected terminal wealth. The equilibrium $\bar{\alpha}_0^*$ is such that the fraction of managers investing in the stock indeed equals $\bar{\alpha}_0^*$. Proposition 4 reports the time-0 equilibrium.

Proposition 4. *In equilibrium at time 0, each manager invests either fully in the stock or in the bond, i.e., $\alpha_0^*(i) = 0$ or $\alpha_0^*(i) = 1$ for all $i \in [0, 1]$. The equilibrium fraction of managers*

investing in the stock, $\bar{\alpha}_0^*$, is always higher than 1/2 and is implicitly given by

$$\int_{-\infty}^{2(1-\bar{\alpha}_0^*)} e^{(1-\bar{\alpha}_0^*)(z-1)+((1-\bar{\alpha}_0^*)\sigma)^2/2-(z-1)^2/(2\sigma^2)}dz + \int_{2(1-\bar{\alpha}_0^*)}^{\infty} e^{(1-\bar{\alpha}_0^*)(z-1)+(z\sigma)^2/8-(z-1)^2/(2\sigma^2)}dz$$

$$= \int_{-\infty}^{2(1-\bar{\alpha}_0^*)} e^{-\bar{\alpha}_0^*(z-1)+(\bar{\alpha}_0^*\sigma)^2/2-(z-1)^2/(2\sigma^2)}dz + \int_{2(1-\bar{\alpha}_0^*)}^{1} e^{-\bar{\alpha}_0^*(z-1)+((z/2-1)\sigma)^2/2-(z-1)^2/(2\sigma^2)}dz$$

$$+ \int_{1}^{\infty} e^{-\bar{\alpha}_0^*(z-1)+(z\sigma)^2/8-(z-1)^2/(2\sigma^2)}dz. \tag{19}$$

Figure 7 plots the equilibrium fraction of time-0 stockholders $\bar{\alpha}_0^*$ as a function of the stock volatility σ, obtained by solving equation (19). From Figure 7, we see that $\bar{\alpha}_0^*$ is different from zero and one, indicating that identical managers – with the same preferences, initial wealth, and fund flow functions – choose different portfolios at time 0. To see why, suppose to the contrary that all managers invest, say, in the stock. Given the convexity of the fund flows, it is optimal for any individual manager to invest in the bond so as to differentiate herself from the rest, thus increasing the expected inflows. Another consequence of the convexity is that the risky stock becomes more attractive when its volatility σ increases since a higher volatility allows a manager to deviate more from the average manager's portfolio. Hence, the higher the volatility is the higher the fraction of managers investing in the stock in equilibrium, and so $\bar{\alpha}_0^*$ increases in σ, as depicted in Figure 7.

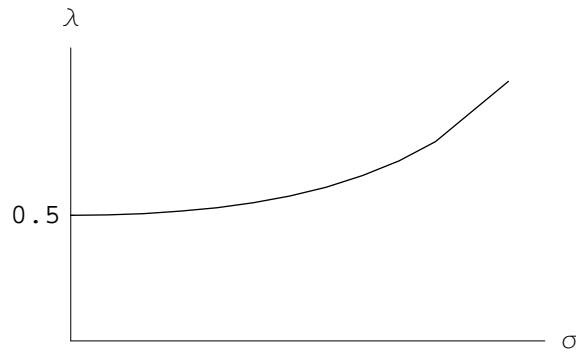**Figure 7: Time-0 equilibrium.** The equilibrium fraction of time-0 stockholders, $\bar{\alpha}_0^*$, as a function of the stock volatility, σ.

The equilibrium at time 1 is essentially as in the baseline mode and is detailed in the proof of Proposition 4 in the Appendix. There, the endogenous interim performance function $r^0(i)$

is a (weakly) increasing function (equations (A20) or (A24), depending on the stock return x^0). Consequently, the managers' time-1 equilibrium portfolios and the industry strategy essentially share the same features as in the baseline model (Propositions 1 and 2). Like in the baseline analysis, all managers in equilibrium are divided into two groups based on interim performance (the threshold is given by (A21), (A25), or (A26) depending on x^0), with the interim winners investing in the risky stock and the interim losers investing in the riskless bond. That is, the main predictions of our one period model remain valid when we endogenize the differential interim performance in this extended model.

When testing for the presence of a tournament among fund managers, existing literature tends to focus exclusively on studying the relation between interim performance and risk. The tournament incentives may, however, may well affect the managers' behavior in other dimensions. We uncover one such dimension concerning the trading volume, by investigating the equilibrium at two sequential times, beginning and middle of the year ($t = 0, 1$). All managers who rebalance their portfolios at time 1 follow a similar strategy: they sell all of the asset they have bought at time 0 and use the proceeds to buy the other asset. Hence, each manager generates a similar amount of trading, and so the total trading volume can be proxied by the fraction of managers π who rebalance. Corollary 1 characterizes the trading volume at time 1.

Trading volume	Within-group share of trading		Mid-year stock return
π	time-0 bondholders	time-0 stockholders	x^0
$\bar{\alpha}_0^* - 1/2$	0	$1 - 1/(2\bar{\alpha}_0^*)$	$x^0 \geq 1$
$1/2 + (1 - \bar{\alpha}_0^*)/x^0$	1	$(1/2 + (1 - \bar{\alpha}_0^*)(1/x^0 - 1))/\bar{\alpha}_0^*$	$2(1 - \bar{\alpha}_0^*) < x^0 < 1$
1	1	1	$x^0 \leq 2(1 - \bar{\alpha}_0^*)$

Table 1: **Trading volume implications of tournament behavior.** The fraction of managers π who rebalance their portfolios at time 1 depending on the realization of the stock return x^0 over the first half of the year. The equilibrium share of time-0 stockholders $\bar{\alpha}_0^*$ is as implicitly given in equation (19).

Corollary 1. *The trading volume π, and the fractions of time-0 bondholders and stockholders generating the trading volume are given in Table 1. Consequently, the trading volume is*

negatively related to the mid-year stock return x^0.

Corollary 1 reveals that the mid-year trading volume π decreases in the mid-year stock return x^0. Indeed, when the stock return is higher than the bond return at time 1, $x^0 \geq 1$, the majority of managers do not trade while the remainder fraction, $\pi = \bar{\alpha}_0^* - 1/2 < 1/2$, rebalances (top row, first cell in Table 1). To see this, first note that time-0 bondholders are interim losers at time 1 when $x^0 \geq 1$, and so from Proposition 1 it is optimal for them to keep their investment in bonds, implying no trade (top row, second cell in Table 1). Time-0 stockholders, on the other hand, are interim winners, and so most of them keep their risky stockholdings while only the fraction $1 - 1/(2\bar{\alpha}_0^*)$ rebalances (top row, third cell in Table 1).[17] Combining the behavior of time-0 bondholders and stockholders yields the relatively low total trading volume, $\pi = \bar{\alpha}_0^* - 1/2$. When the stock return is moderately lower than the bond return, $2(1 - \bar{\alpha}_0^*) < x^0 < 1$, the majority of managers, $\pi = 1/2 + (1 - \bar{\alpha}_0^*)/x^0 > 1/2$, rebalances at mid year (middle row, first cell in Table 1). Here, time-0 bondholders are interim winners, and so they all sell bonds and buy stocks (middle row, second cell in Table 1). Time-0 stockholders are now interim losers, and so the majority of them rebalances, as seen from $(1/2 + (1 - \bar{\alpha}_0^*)(1/x^0 - 1))/\bar{\alpha}_0^* > 1/2$, while the remainder do not trade (middle row, third cell in Table 1). Finally, when the stock return is sufficiently low, $x^0 \leq 2(1 - \bar{\alpha}_0^*)$, all managers rebalance (bottom row, Table 1).

7. Conclusion

This article investigates the validity of the so-called tournament hypothesis widely used in empirical work. According to this hypothesis, interim winners are expected to decrease, while interim losers are likely to increase, their portfolio volatility. We characterize the managers' equilibrium portfolios in a model of mutual fund tournament in the presence of short-sales constraints, and uncover the opposite result: interim winners opt for a higher portfolio volatility than interim losers. We demonstrate how differential interim performance arises endogenously in the presence of tournament incentives, and also investigate the trading volume implications of these incentives.

---

[17]From equations (9), it is straightforward to demonstrate that the equilibrium threshold i^* lies within the set of time-0 stockholders, dividing them into two groups. That is why they do not choose the same portfolio at time 1 even though they have the same interim performance.

It would be of interest to investigate a general equilibrium version of our model so as to derive the asset pricing implications of tournament behavior. Cuoco and Kaniel (2010) derive such implications in a model where portfolio constraints are absent. Comparing the results of these two models would lead to a better understanding of the role of portfolio constraints in the context of delegated portfolio management. Our brief analysis of the risk averse examples with no constraints reveals that the predictions of the unconstrained and constrained settings may be different, which gives some indication that the tournament incentives affecting largely unconstrained hedge funds and constrained mutual funds may not coincide. It would be interesting to examine this question further by analyzing a model specifically built around the pertinent features of the hedge fund industry. While challenging, it would also be valuable to analyze a fund tournament where the payoff function is of the "winner-takes-all" type.

Appendix

Proof of Proposition 1. Since $E[x] = 1$, maximizing the expected value of (4) is equivalent to

$$\max_{\alpha(i)\in[0,1]} E\left[\exp(r^0(i)(\alpha(i)x + 1 - \alpha(i)) - \bar{r}^0(\bar{\alpha}x + 1 - \bar{\alpha}))\right]. \tag{A1}$$

The argument of the exponent in (A1) is a linear transformation of a normally distributed return x, and so it has a normal distribution with mean and variance

$$E[r^0(i)(\alpha(i)x + 1 - \alpha(i)) - \bar{r}^0(\bar{\alpha}x + 1 - \bar{\alpha})] \;=\; r^0(i) - \bar{r}^0, \tag{A2}$$

$$Var[r^0(i)(\alpha(i)x + 1 - \alpha(i)) - \bar{r}^0(\bar{\alpha}x + 1 - \bar{\alpha})] \;=\; \sigma^2(\alpha(i)r^0(i) - \bar{\alpha}\bar{r}^0)^2. \tag{A3}$$

We now use the following property: If v is normally distributed, $v \sim N(\mu_v, \sigma_v^2)$, then $exp(v)$ is log-normally distributed with mean $E[\exp(v)] = \exp(\mu_v + \sigma_v^2/2)$. Using this property to express the expectation in (A1) in terms of (A2)–(A3), we obtain that manager i's best response is given by

$$\hat{\alpha}(i) = \arg\max_{\alpha(i)\in[0,1]} \; (\alpha(i)r^0(i) - \bar{\alpha}\bar{r}^0)^2. \tag{A4}$$

Because the objective function in (A4) is convex in $\alpha(i)$, the maximum is achieved at either $\alpha(i) = 0$ or $\alpha(i) = 1$. Evaluating the objective function at $\alpha(i) = 0$ and $\alpha(i) = 1$ and subtracting the former from the latter, we obtain

$$(r^0(i) - \bar{\alpha}\bar{r}^0)^2 - (\bar{\alpha}\bar{r}^0)^2 = (r^0(i))^2 - 2r^0(i)\bar{\alpha}\bar{r}^0, \tag{A5}$$

implying that manager i's best response is $\bar{\alpha}(i) = 1$ when $r^0(i) \geq 2\bar{\alpha}\bar{r}^0$, and $\bar{\alpha}(i) = 0$ otherwise. *Q.E.D.*

Proof of Proposition 2. First, we demonstrate that the solution of (9) exists and is unique. Expressing $\bar{\alpha}^*$ from the second equation in (9) and substituting it into the first equation, we obtain

$$r^0(i^*)/2 = \int_{i^*}^{1} r^0(i)di. \tag{A6}$$

If $i^* = 0$, (A6) is not an equality since its left-hand side (LHS) is less than right-hand side (RHS). Indeed, since $r^0(i)$ is increasing in i, $r^0(0)$ is the minimal value of $r^0(\cdot)$ while the integral in RHS computes the average value of $r^0(\cdot)$. If $i^* = 1$, (A6) is again not an equality since LHS is positive and RHS is zero. Since both LHS and RHS are continuous and monotonic in i, there exists a unique $i^* \in (0,1)$ such that (A6) holds.

We now demonstrate that i^* and $\bar{\alpha}^*$ that solve (9) are indeed the equilibrium values. Substituting the second equation in (9) into (A5) yields zero, meaning that manager i^* is indifferent between investing in the stock and the bond. Hence, from (8) the (candidate) equilibrium portfolio strategies are

$$\alpha^*(i) = \begin{cases} 1, & \text{if} \quad i \geq i^*, \\ 0, & \text{otherwise.} \end{cases} \tag{A7}$$

Substituting (A7) into the aggregation condition (6), we see that (6) is satisfied as it is equivalent to the first equation in (9). That the equilibrium threshold $r^0(i^*)$ lies strictly within the range of values of $r^0(i)$ follows from the earlier result that $i^* \in (0,1)$ coupled with $r^0(i^*)$ being increasing. *Q.E.D.*

Proof of Proposition 3. We first prove the results for the case of multiple stocks and zero risk premium, and then for the case of single stock and positive risk premium.

Multiple stocks and zero risk premium

Best response. Manager i's time-2 wealth under multiple risky stocks is

$$W(i) = r^0(i) \left(\sum_{k=1}^{N} \alpha_k(i) x_k + \left(1 - \sum_{k=1}^{N} \alpha_k(i)\right) \right)$$
$$+ \exp \left[c \left(\sum_{k=1}^{N} (r^0(i)\alpha_k(i) - \bar{r}^0 \bar{\alpha}_k) x_k + r^0(i) - r^0(i) \sum_{k=1}^{N} \alpha_k(i) - \bar{r}^0 + \bar{r}^0 \sum_{k=1}^{N} \bar{\alpha}_k \right) \right],$$
$$\tag{A8}$$

which replaces expression (4) obtained in the baseline model. Manager i, $i \in [0,1]$, optimally holds only one of the available assets in her portfolio due to the the convexity of her objective function. Using the property of log-normal distribution (as in proof of Proposition 1), we obtain that the expected value of manager i's wealth (A8) if she fully invests in stock l, i.e.,

if $\alpha_l(i) = 1$, is

$$E[W(i)]\big|_{\alpha_l(i)=1} = c^2\sigma_l^2(r^0(i) - \bar{\alpha}_l\bar{r}^0)^2 + c^2 \sum_{\substack{j=1,\dots,N \\ j\neq l}} \sigma_j^2(\bar{\alpha}_j\bar{r}^0)^2. \qquad \text{(A9)}$$

For given aggregate strategies $\bar{\alpha}_k$, $k = 1, \dots, N$, computing manager i's best response amounts to finding an asset $k(i)$, $k(i) = 0, 1, \dots, N$, such that investing in it maximizes (A9). Hence, the best response condition is $E[W(i)]\big|_{\alpha_{k(i)}(i)=1} \geq E[W(i)]\big|_{\alpha_l(i)=1}$ for all $l = 0, \dots N$, which from (A9) is equivalent to

$$\sigma_{k(i)}^2(r^0(i) - \bar{\alpha}_{k(i)}\bar{r}^0)^2 + \sigma_l^2(\bar{\alpha}_l\bar{r}^0)^2 \geq \sigma_l^2(r^0(i) - \bar{\alpha}_l\bar{r}^0)^2 + \sigma_{k(i)}^2(\bar{\alpha}_{k(i)}\bar{r}^0)^2, \quad l = 0, \dots, N. \quad \text{(A10)}$$

Rearranging (A10) yields

$$\sigma_{k(i)}^2 r^0(i) - 2\sigma_{k(i)}^2\bar{\alpha}_{k(i)}\bar{r}^0 \geq \sigma_l^2 r^0(i) - 2\sigma_l^2\bar{\alpha}_l\bar{r}^0, \quad l = 0, \dots, N,$$

leading to (16).

Equilibrium. We consider two arbitrary managers $i_1, i_2 \in [0, 1]$, and without loss of generality assume that $i_1 < i_2$, implying that manager i_1 has a lower interim return than manager i_2 (as $r^0(i)$ increases in i). We prove by contradiction that manager i_1 cannot choose a more volatile asset than manager i_2 in equilibrium. Suppose that managers i_1 and i_2 choose assets k_1 and k_2, respectively, where $k_1 > k_2$ and $\sigma_{k_1} > \sigma_{k_2}$. Since manager i_1 chooses asset k_1 over asset k_2, we have that $E[W(i_1)]\big|_{\alpha_{k_1}(i)=1} \geq E[W(i_1)]\big|_{\alpha_{k_2}(i)=1}$, which from (A10) is equivalent to

$$\sigma_{k_1}^2 r^0(i_1) - 2\sigma_{k_1}^2\bar{\alpha}_{k_1}\bar{r}^0 \geq \sigma_{k_2}^2 r^0(i_1) - 2\sigma_{k_2}^2\bar{\alpha}_{k_2}\bar{r}^0. \qquad \text{(A11)}$$

Similarly, since manager i_2 chooses asset k_2 over asset k_1, we have

$$\sigma_{k_2}^2 r^0(i_2) - 2\sigma_{k_2}^2\bar{\alpha}_{k_2}\bar{r}^0 \geq \sigma_{k_1}^2 r^0(i_2) - 2\sigma_{k_1}^2\bar{\alpha}_{k_1}\bar{r}^0. \qquad \text{(A12)}$$

Subtracting (A11) from (A12) yields

$$\sigma_{k_2}^2(r^0(i_2) - r^0(i_1)) \geq \sigma_{k_1}^2(r^0(i_2) - r^0(i_1)), \qquad \text{(A13)}$$

implying $\sigma_{k_2}^2 \geq \sigma_{k_1}^2$, which is a contradiction.

<div align="right">*Q.E.D.*</div>

Single stock and positive risk premium

First, we compute the expected value of manager i's wealth $W(i)$ for a given portfolio $\alpha(i) \in [0, 1]$. The expectation of the first term in (4) is

$$E[R(i)] = r^0(i)(\alpha(i)(\mu - 1) + 1). \tag{A14}$$

To find the expectation of the second term in (4), we first compute the mean and variance of $c(R(i) - \bar{R})$:

$$E[c(R(i) - \bar{R})] = c(\alpha(i)r^0(i)(\mu - 1) - \bar{\alpha}\bar{r}^0(\mu - 1) + r^0(i) - \bar{r}^0), \tag{A15}$$

$$Var[c(R(i) - \bar{R})] = c^2\sigma^2(\alpha(i)r^0(i) - \bar{\alpha}\bar{r}^0)^2, \tag{A16}$$

and then rely on the above property of lognormal distribution to obtain

$$E[\exp(c(R(i) - \bar{R}))] = c(\alpha(i)r^0(i)(\mu - 1) - \bar{\alpha}\bar{r}^0(\mu - 1) + r^0(i) - \bar{r}^0) + c^2\sigma^2(\alpha(i)r^0(i) - \bar{\alpha}\bar{r}^0)^2/2. \tag{A17}$$

Combining (A14) and (A17) and dropping the constants, we get that manager i's best response $\hat{\alpha}(i)$ yields the maximum of the objective function

$$(1 + c)r^0(i)\alpha(i)(\mu - 1) + c^2\sigma^2(\alpha(i)r^0(i) - \bar{\alpha}\bar{r}^0)^2/2. \tag{A18}$$

Since (A18) is convex, its solution is either $\alpha(i) = 0$ or $\alpha(i) = 1$, and to determine which of the two values constitutes the best response we subtract (A18) evaluated at $\alpha(i) = 0$ from (A18) evaluated at $\alpha(i) = 1$:

$$(1 + c)r^0(i)(\mu - 1) + c^2\sigma^2(r^0(i) - \bar{\alpha}\bar{r}^0)^2/2 - c^2\sigma^2(\bar{\alpha}\bar{r}^0)^2/2 =$$
$$(1 + c)r^0(i)(\mu - 1) + (c\sigma r^0(i))^2/2 - c^2\sigma^2 r^0(i)\bar{\alpha}\bar{r}^0. \tag{A19}$$

Simple rearrangement reveals that (A19) is positive or equals zero when $r^0(i) \geq 2\bar{\alpha}\bar{r}^0 - 2(1 + c)r^0(i)(\mu - 1)/(c\sigma)^2$ and is negative otherwise, leading to the best response (17). Combining the best response (17) with the aggregation condition (6) leads to (18).

<div align="right">*Q.E.D.*</div>

<div align="center">33</div>

Proof of Proposition 4. We conjecture that the structure of equilibrium at time 0 is the same as that at time 1. Namely, all managers are divided into two groups, whereby one group invests only in the stock and the other group invests only in the bond. We verify this conjecture at the end of this Proof. For notational convenience, we define the new variable λ representing the fraction of managers investing in the bond at time 0, $\lambda \equiv 1 - \bar{\alpha}_0^*$. Given that the managers are identical at time 0, they can optimally invest in two different assets only if the expected wealth from these two strategies are the same. Hence, we look for λ such that manager i is indifferent between investing in the bond, $\alpha_0(i) = 0$, and in the stock, $\alpha_0(i) = 1$. To compute manager i's expected wealth (as of time 0) under the strategies $\alpha_0(i) = 0$ and $\alpha_0(i) = 1$, we work backwards starting at time 1. The analysis of time-1 equilibrium depends on what asset, stock or bond, is chosen by the majority of managers at time 0, i.e., whether $\lambda \le 0.5$ or $\lambda > 0.5$. We present a detailed analysis for the case $\lambda \le 0.5$, and then comment on the case $\lambda > 0.5$.

Time-1 equilibrium when $\lambda \le 0.5$

There are three different equilibrium outcomes depending on whether the realization of the stock return x^0 is higher than 1 (bond return), between 2λ and 1, or lower than 2λ.

Outcome 1: $x^0 > 1$. The interim performance function $r^0(i)$ is given by

$$r^0(i) = \begin{cases} 1 & i \in [0, \lambda] \\ x^0 & i \in (\lambda, 1]. \end{cases} \tag{A20}$$

In this case, the equilibrium values of the threshold and the aggregate portfolio strategy are

$$i^* = 0.5, \ \bar{\alpha}^* = x^0/2\bar{r}^0. \tag{A21}$$

Indeed, plugging $i^* = 0.5$ into (A20) we obtain $r^0(i^*) = x^0$ and $\int_{i^*}^1 r^0(i)di = x^0/2$, and substituting along with equation (A21) into (9) yields that (9) is satisfied. Sitting at time 1, we now compute manager i's time-1 indirect utility function in equilibrium, denoted by $v(i)$. Substituting (A21) and $\alpha^*(i) = 1$ (for an interim winner) or $\alpha^*(i) = 0$ (for an interim

loser) in (5) and taking the expectation, we obtain

$$
v(i) = \begin{cases} r^0(i) + e^{r^0(i) - \bar{r}^0 + ((r^0(i) - x^0/2)\sigma)^2/2} & \text{if } \alpha^*(i) = 1 \quad\quad\text{(A22)} \\ r^0(i) + e^{r^0(i) - \bar{r}^0 + (x^0\sigma)^2/8} & \text{if } \alpha^*(i) = 0. \quad\quad\text{(A23)} \end{cases}
$$

Outcome 2: $x^0 \in [2\lambda, 1]$. The interim performance function $r^0(i)$ is given by

$$
r^0(i) = \begin{cases} x^0 & i \in [0, 1 - \lambda] \\ 1 & i \in (1 - \lambda, 1]. \end{cases} \quad\quad\text{(A24)}
$$

Note that in the above Outcome 1, lower values of the index i, $i \in [0, \lambda]$, correspond to time-0 bondholders while higher values, $i \in [\lambda, 1]$, correspond to time-0 stockholders. Now the order is reversed, and the region $i \in [0, 1 - \lambda]$ corresponds to time-0 stockholders, with $i \in [1 - \lambda, 1]$ corresponding to time-0 bondholders. Here and henceforth, the indexing order is chosen to ensure that the interim performance function $r^0(i)$ is (weakly) increasing in i for $i \in [0, 1]$, enabling us to apply the results of Proposition 2 in Section 3. From (9), we obtain the equilibrium threshold and aggregate strategy as

$$
i^* = 0.5 - \lambda + \lambda/x^0, \quad \bar{\alpha}^* = x^0/2\bar{r}^0. \quad\quad\text{(A25)}
$$

Since $\bar{\alpha}^*$ is the same as for Outcome 1, manager i's time-1 indirect utility function $v(i)$ is also the same, and is given by (A22)–(A23).

Outcome 3: $x^0 < 2\lambda$. From (9), we obtain the equilibrium quantities

$$
i^* = 1 - \lambda, \quad \bar{\alpha}^* = \lambda/\bar{r}^0. \quad\quad\text{(A26)}
$$

Substituting above into (5) and computing the expectation, we get

$$
v(i) = \begin{cases} r^0(i) + e^{r^0(i) - \bar{r}^0 + ((r^0(i) - \lambda)\sigma)^2/2} & \text{if } \alpha^*(i) = 1 \quad\quad\text{(A27)} \\ r^0(i) + e^{r^0(i) - \bar{r}^0 + (\lambda\sigma)^2/2} & \text{if } \alpha^*(i) = 0. \quad\quad\text{(A28)} \end{cases}
$$

Time-0 equilibrium when $\lambda < 0.5$

At time 0, for a given λ manager i chooses $\alpha_0(i)$ so as to maximize her expected indirect utility $v(i)$. As described above, the equilibrium λ is such that manager i is indifferent

between $\alpha_0(i) = 0$ and $\alpha_0(i) = 1$, i.e., the expected value of $v(i)$ under these two strategies is the same. If $\alpha_0(i) = 0$, manager i is an interim loser if Outcome 1 occurs, but an interim winner if Outcomes 2 or 3 occurs. Hence, $v(i)$ is given by (A23) under Outcome 1, by (A22) under Outcome 2, and by (A27) under Outcome 3, and so the expected value of $v(i)$ is

$$E_0[v(i)|\alpha_0(i) = 0] = 1 + \int_{-\infty}^{2\lambda} e^{(\lambda-1)(z-1)+((\lambda-1)\sigma)^2/2-(z-1)^2/(2\sigma^2)}dz$$

$$+ \int_{2\lambda}^{1} e^{(\lambda-1)(z-1)+((z/2-1)\sigma)^2/2-(z-1)^2/(2\sigma^2)}dz + \int_{2\lambda}^{\infty} e^{(\lambda-1)(z-1)+(z\sigma)^2/8-(z-1)^2/(2\sigma^2)}dz \quad . \quad \text{(A29)}$$

Analogously, if $\alpha_0(i) = 1$ then $v(i)$ is given by (A22) under Outcome 1, by (A23) under Outcome 2, and by (A28) under Outcome 3, from which we obtain

$$E_0[v(i)|\alpha_0(i) = 1] = 1 + \int_{-\infty}^{2\lambda} e^{\lambda(z-1)+(\lambda\sigma)^2/2-(z-1)^2/(2\sigma^2)}dz + \int_{2\lambda}^{\infty} e^{\lambda(z-1)+(z\sigma)^2/8-(z-1)^2/(2\sigma^2)}dz.$$

$$\text{(A30)}$$

Equating (A29) and (A30), and recalling that $\lambda \equiv 1 - \bar{\alpha}_0^*$, leads to (19). Solving (19) numerically for a range of plausible volatilities σ reveals that its solution indeed satisfies the condition $\bar{\alpha}_0^* > 0.5$.

Following the same steps as above, we obtain that if there exists an equilibrium λ such that $\lambda > 0.5$, then it solves

$$\int_{-\infty}^{1} e^{\lambda(z-1)+\sigma^2/8-(z-1)^2/(2\sigma^2)}dz + \int_{1}^{1/(2(1-\lambda))} e^{\lambda(z-1)+((z-1/2)\sigma)^2/2-(z-1)^2/(2\sigma^2)}dz$$

$$+ \int_{1/(2(1-\lambda))}^{\infty} e^{\lambda(z-1)+(\lambda z\sigma)^2/2-(z-1)^2/(2\sigma^2)}dz = \int_{-\infty}^{1} e^{(\lambda-1)(z-1)+\sigma^2/8-(z-1)^2/(2\sigma^2)}dz$$

$$+ \int_{1}^{1/(2(1-\lambda))} e^{(\lambda-1)(z-1)+\sigma^2/8-(z-1)^2/(2\sigma^2)}dz + \int_{1/(2(1-\lambda))}^{\infty} e^{(\lambda-1)(z-1)+((\lambda-1)z\sigma)^2/2-(z-1)^2/(2\sigma^2)}dz \quad . \quad$$

$$\text{(A31)}$$

Solving (A31) numerically reveals that its solution does not satisfy the condition $\lambda > 0.5$, or, equivalently, $\bar{\alpha}_0^* < 0.5$, meaning that we have only one equilibrium described by (19).

Finally, for a wide range of plausible levels of volatility σ, we compute numerically the expected values of $v(i)$ at the interior portfolios, $\alpha_0(i) \in (0,1)$, which reveals that the expected $v(i)$ for these interior portfolios is lower than for $\alpha_0(i) = 0$ and $\alpha_0(i) = 1$. This confirms the conjecture made at the beginning of this Proof.

Q.E.D.

Proof of Corollary 1. As in the proof of Proposition 4, for convenience we use the change of variable $\lambda \equiv 1 - \bar{\alpha}_0^*$. There, we established that when in equilibrium $\lambda < 1/2$, for Outcome 1 the equilibrium threshold is $i^* = 1/2$ and that time-0 bondholders lie on the interval $i \in [0, \lambda]$. Hence, from Proposition 2 all time-0 bondholders as well as a share $(1/2 - \lambda)/(1 - \lambda)$ of time-0 stockholders invest in the bond at time 1 while the remaining time-0 stockholders invest in the stock. This leads to the top row in Table 1. Applying analogous reasoning for Outcomes 2 and 3 in the proof of Proposition 4 yields the middle and bottom rows in Table 1.

Q.E.D.

References

Abel, A., 1990, "Asset Prices under Habit Formation and Catching Up with the Joneses," *American Economic Review*, 80, 38–42.

Almazan, A., K. Brown, M. Carlson, and D. Chapman, 2004, "Why Constrain Your Mutual Fund Manager?," *Journal of Financial Economics*, 73, 289–321.

Basak, S., and D. Makarov, 2009, "Strategic Asset Allocation in Money Management," working paper, LBS and NES.

Basak, S., A. Pavlova, and A. Shapiro, 2007, "Optimal Asset Allocation and Risk Shifting in Money Management," *Review of Financial Studies*, 20, 1583–1621.

Berk, J., and R. Green, 2004, "Mutual Fund Flows and Performance in Rational Markets," *Journal of Political Economy*, 112, 1269–1295.

Bhattacharya, S., and J. Luis Guasch, 1988, "Heterogeneity, Tournaments, and Hierarchies," *Journal of Political Economy*, 96, 867–881.

Bollen, N., and J. Busse, 2005, "Short-term Persistence in Mutual Fund Performance," *Review of Financial Studies*, 18, 569–597.

Brown, S., W. Goetzmann, and J. Park, 2001, "Careers and Survival: Competition and Risk in the Hedge Fund and CTA Industry," *Journal of Finance*, 56, 1869–1886.

Brown, K., W. Harlow, and L. Starks, 1996, "On Tournaments and Temptations: An Analysis of Managerial Incentives in the Mutual Fund Industry," *Journal of Finance*, 51, 85–110.

Busse, J., 2001, "Another Look at Mutual Fund Tournament," *Journal of Financial and Quantative Analysis*, 36, 53–73.

Carhart, M., 1997, "On Persistence in Mutual Fund Performance," *Journal of Finance*, 52, 57–82.

Carpenter, J., 2000, "Does Option Compensation Increase Managerial Risk Appetite?," *Journal of Finance*, 55, 2311–2331.

Chen, H., N. Jegadeesh, and R. Wermers, 2000, "The Value of Active Mutual Fund Management: an Examination of the Stockholdings and Trades of Fund Managers," *Journal of Financial and Quantitative Analysis*, 35, 343–368.

Chen, H., and G. Pennacchi, 2009, "Does Prior Performance Affect a Mutual Fund's Choice of Risk? Theory and Further Empirical Evidence," *Journal of Financial and Quantitative Analysis*, 44, 745–775.

Chevalier, J., and G. Ellison, 1997, "Risk Taking by Mutual Funds as a Response to Incentives," *Journal of Political Economy*, 105, 1167–1200.

Chiang, K., 1999, "Survival, Tournament, and Asset Allocation Strategy in the Mutual Fund Industry," working paper, Louisiana State University.

Cuoco, D., and R. Kaniel, 2010, "Equilibrium Prices in the Presence of Delegated Portfolio Management," *Journal of Financial Economics*, forthcoming.

DeMarzo, P., R. Kaniel, and I. Kremer, 2007, "Technological Innovation and Real Investment Booms and Busts," *Journal of Financial Economics*, 85, 735–754.

DeMarzo, P., R. Kaniel, and I. Kremer, 2008, "Relative Wealth Concerns and Financial Bubbles," *Review of Financial Studies*, 21, 19–50.

Dewatripont, M., and J. Tirole, 1995, *The Prudential Regulation of Banks*, MIT Press.

Ehrenberg, R., and M. Bognanno, 1990, "Do Tournaments Have Incentive Effects?," *Journal of Political Economy*, 98, 1307–1324.

Getmansky, M., 2005, "The Life Cycle of Hedge Funds: Fund Flows, Size and Performance," working paper, University of Massachusetts.

Goriaev, A., T. Nijman, and B. Werker, "Yet Another Look at Mutual Fund Tournaments," *Journal of Empirical Finance*, 12, 127–137.

Goriaev, A., F. Palomino, and A. Prat, 2003, "Mutual Fund Tournament: Risk Taking Incentives Induced by Ranking Objectives," working paper, LSE.

Green, J., and N. Stokey, 1983, "A Comparison of Tournaments and Contracts," *Journal of Political Economy*, 91, 349–364.

Huang, J., K. Wei, and H.Yan, 2007, "Participation Costs and the Sensitivity of Fund Flows to Past Performance," *Journal of Finance*, 62, 1273–1311.

Koijen, R., 2008, "The Cross-Section of Managerial Ability and Risk Preferences," working paper, University of Chicago.

Lazear, E., and S. Rosen, 1981, "Rank-order Tournaments as Optimum Labor Contracts," *Journal of Political Economy*, 89, 841–64.

Li, W., and A. Tiwari, 2008, "On The Consequences of Mutual Fund Tournaments," working paper, University of Iowa.

Loranth, G., and E. Sciubba, 2006, "Relative Performance, Risk and Entry in The Mutual Fund Industry," *B.E. Journal of Economic Analysis & Policy*, 6, article 19.

McLaughlin, K., 1988, "Aspects of Tournament Models: A Survey," *Research in Labor Economics*, 9, 225–256.

Palomino, F., 2005, "Relative Performance Objectives in Financial Markets," *Journal of Financial Intermediation*, 14, 351–375.

Panageas, S., and M. Westerfield, 2009, "High Water Marks: High Risk Appetites? Hedge Fund Compensation and Portfolio Choice," *Journal of Finance*, 64, 1–36.

Qiu, J., 2003, "Termination Risk, Multiple Managers and Mutual Fund Tournaments," *European Finance Review*, 7, 161–190.

Reed, A., and L. Wu, 2005, "Racing the Clock: Benchmarking or Tournaments in Mutual Fund Risk-shifting?," working paper, University of North-Carolina.

Sirri, E., and P. Tufano, 1998, "Costly Search and Mutual Funds Flows," *Journal of Finance*, 54, 359–375.

Taylor, C., 1995, "Digging for Golden Carrots: an Analysis of Research Tournaments," *American Economic Review*, 85, 872–890.

Taylor, J., 2003, "A Role for the Theory of Tournaments in Studies of Mutual Fund Behavior," *Journal of Economic Behavior and Organization*, 50, 373–383.

Vasicek, O. and H. Fong, 1982, "Term Structure Modeling Using Exponential Splines," *Journal of Finance*, 37, 339–348.

Zwiebel, J., 1995, "Corporate Conservatism and Relative Compensation," *Journal of Political Economy*, 103, 1–25.