DISCUSSION PAPER SERIES

No. 8027

**ARE CENTRAL BANKS'
PROJECTIONS MEANINGFUL?**

Jordi Galí

INTERNATIONAL MACROECONOMICS

Centre for Economic Policy Research

www.cepr.org

ARE CENTRAL BANKS' PROJECTIONS MEANINGFUL?

Jordi Galí, CREI, Universitat Pompeu Fabra and CEPR

This Discussion Paper is issued under the auspices of the Centre's research programme in **INTERNATIONAL MACROECONOMICS**. Any opinions expressed here are those of the author(s) and not those of the Centre for Economic Policy Research. Research disseminated by CEPR may include views on policy, but the Centre itself takes no institutional policy positions.

The Centre for Economic Policy Research was established in 1983 as an educational charity, to promote independent analysis and public discussion of open economies and the relations among them. It is pluralist and non-partisan, bringing economic research to bear on the analysis of medium- and long-run policy questions.

These Discussion Papers often represent preliminary or incomplete work, circulated to encourage discussion and comment. Citation and use of such a paper should take account of its provisional character.

CEPR Discussion Paper No. 8027

October 2010

ABSTRACT

Are Central Banks' Projections Meaningful?*

Central banks' projections--i.e. forecasts conditional on a given interest rate path--are often criticized on the grounds that their underlying policy assumptions are inconsistent with the existence of a unique equilibrium in many forward-looking models. Here I describe three alternative approaches to constructing projections that are not subject to the above criticism, using two different versions of New Keynesian model as reference frameworks. Most importantly, I show how the three approaches generate different projections for inflation and output, even though they imply an identical path for the interest rate. The latter result calls into question the meaning and usefulness of such projections.

Jordi Galí
CREI
Universitat Pompeu Fabra
Ramon Trias Fargas 25-27
08005 Barcelona
SPAIN

Email: jgali@crei.cat

1   Introduction

The periodic construction and publication of macroeconomic forecasts is a common activity among central banks in advanced industrialized economies. Those forecasts get considerable attention from analysts, market participants and the financial press. Among the central banks that have adopted an explicit inflation-forecast targeting strategy those forecasts play a central role in the internal decision process for, in many cases, policy settings are chosen in order to attain a certain inflation target at a specified horizon. Those forecasts are also viewed as an important element in central banks' communication policy. More generally, and using the words of the ECB, "macroeconomic projections play an important role as a tool for aggregating and organizing existing information on current and future economic developments" (ECB (2001)).

The methods and assumptions behind those forecasts vary considerably across central banks. In particular, practices differ regarding the assumption on the path for the short-term nominal interest rate (henceforth, the interest rate) underlying those forecasts. The relative merits of alternative assumptions on which to condition the published forecasts remains the subject of considerable debate, partly sparked by the recent decision by some central banks to change their practice in that regard.[1]

At the risk of oversimplification, one can distinguish three alternative assumptions underlying central banks' forecasting practices. In some cases the interest rate is assumed to remain constant at its current level over the forecasting horizon, giving rise to so-called *constant interest rate (CIR)* forecasts. Other central banks construct their forecasts under the assumption that the interest rate will follow a path consistent with current market expectations, with the latter being approximated with the forward rates implicit in the yield curve prevailing at the time the projection is made.[2] We refer to those as *market expectations (ME)* forecasts. A third practice found among central banks consists in constructing forecasts based on the assumption that the interest rate will follow whatever path the central bank expects it to follow, i.e. a path consistent with the central bank's "own policy rule," independently of whether the latter had been made explicit or not. We refer to those forecasts

---

[1]See, e.g. the recent announcements by the Riksbank (2007) and the Federal Reserve Board (2007).

[2]Of course, it is not easy in practice to disentangle expected rates from liquidity and term premia, but this complication is orthogonal to the point made in the present paper.

as *central bank's expectations (CBE)* forecasts.[3] Since the latter forecasts are based on the central bank's best assessment of what the interest rate path will be, they can be interpreted as unconditional forecasts. By contrast, CIR or ME forecasts are conditioned on a path of interest rates that does not generally coincide with the central bank's own expectations on that path. As a result, those forecasts are not necessarily the best predictors of future outcomes, as is implicitly reflected in their common labeling as "conditional forecasts" or "projections". For convenience, in the remainder of the paper (as well as in the title) I use the term "projections" to refer to forecasts conditional on a *given* interest rate path.

Each of the forecasting procedures has its own advantages and disadvantages, which may explain the observed diversity of practices.[4] That diversity is illustrated in Table 1, which summarizes the assumptions on the interest rate path underlying the forecast practices of seventeen major central banks.

The present paper's starting motivation is a concern frequently voiced regarding central banks' projections: the inconsistency of their underlying assumptions with the existence of a unique equilibrium in a variety of forward-looking models, including the new vintage of optimizing models commonly used for monetary policy analysis.[5] That argument hinges on the notion that, in order to construct such projections, the path of the interest rate is taken *as given* by the central bank, i.e. it is assumed that the interest rate will vary according to a pre-specified path, independently of how inflation or other endogenous variables may end up evolving. In other words, the model is fed with an exogenous interest rate path as a description of monetary policy, which is known to generate an indeterminate equilibrium (and, hence, indeterminate forecasts) in many dynamic models with forward-looking components.[6]

Here I seek to contribute to that debate by clarifying the sense in which macroeconomic projections conditional on a given interest rate path are or are not feasible or well defined. First, I show that it is indeed possible to overcome the curse of indeterminacy and to generate *determinate* macroeconomic projections consistent with an arbitrary path for the interest rate. This is

---

[3]Many central banks that publish CBE-type forecasts for inflation and output also report the associated forecasts for the interest rate path (e.g. the RB of New Zealand, Norway's Norges Bank and Sweden's Riksbank). Others do not (the U.S. Federal Reserve).

[4]See, e.g., Svensson (2006) for a detailed discussion.

[5]See, e.g., Leitemo (2003), Honkapohja and Mitra (2005), and Woodford (2005, 2007).

[6]See, e.g. Woodford (2003) or Galí (2008) for a detailed discussion.

done using a canonical version of the New Keynesian (NK) model as a reference framework. As many as three approaches are proposed to construct the desired projections. The first approach, based on the "modest interventions" model of Leeper and Zha (2003), consists of adding unexpected policy shocks to the central bank's rule in order to generate the desired interest rate path.[7] The second and third approaches rely on two alternative interest rate rules designed to generate the desired path of the interest rate as an endogenous equilibrium outcome, while responding systematically to inflation and other macro variables in a way that guarantees the determinacy of the equilibrium (and, hence, uniqueness of the projections).

Thus, by showing how projections can be constructed using *any* of the above methods, the paper refutes the claim that such forecasts are not feasible due to the indeterminacy underlying the associated equilibrium. The latter finding, however, can hardly provide any consolation to advocates of conditional projections: I show by means of a simple example that the three methods will generally *yield different projections for variables other than the interest rate itself*, despite the fact that, by construction, the latter exhibits an identical path across methods. Thus, the problem is not necessarily one of multiplicity of equilibria resulting from an exogenous interest rate path, but one of *multiplicity of rules* that are consistent with the latter path, each having its own implications for other variables. That observation calls into question the usefulness of projections conditional on a given interest rate path since, in principle, there is no obvious reason to prefer one method over another in order to generate the same path.

That problem does not arise when constructing CBE forecasts, which are based on the rule effectively followed by the central bank when constructing those forecasts. More generally, the problem disappears whenever the projections are conditional on *a rule* (as long as the latter is consistent with a unique equilibrium), and *not* on an *interest rate path* (which may be supported by a multiplicity of rules). It is in that sense that the present paper provides an additional argument against the reporting of central banks' projections conditional on a given interest rate path, while endorsing instead rule-based projections. But once this is accepted, it is not clear why the central bank would want to base its projections on a rule *other than the actual rule it follow*s for, among other things, in that case the projections would

---

[7]As shown below, the "modest interventions" approach is equivalent to assuming a switch to the usual interest rate rule right after the forecast horizon

also correspond to the best unconditional forecasts.

The remainder of the paper is organized as follows. Section 2 describes the reference model used in the subsequent analysis. Section 3 revisits the problem of indeterminacy when the interest rate follows an exogenous path, and shows how this indeterminacy is inherited by the associated projections. Section 4 describes three approaches that overcome that indeterminacy problem. Section 5 illustrates, using a calibrated version of the reference model, how those three approaches lead to significantly different inflation and output projections conditional on an unchanged interest rate, at any finite horizon. Section 6 concludes.

2   A Baseline Model

I use a simple version of the New Keynesian model as a reference framework for the analysis of alternative macro projections.[8] The non-policy block of the model is made up of the following two equations:

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\}) \tag{1}$$

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa y_t + u_t \tag{2}$$

where y_t denotes the output gap, i_t is the short-term nominal interest rate, $\pi_t \equiv p_t - p_{t-1}$ is the rate of inflation between $t-1$ and t (with p_t denoting the log of the price level), and u_t.is an exogenous cost-push shock which follows an AR(1) process with autoregressive coefficient $\rho_u \in (0, 1]$. All variables are expressed in deviations from steady state values. Equation (1) can be obtained by log-linearizing the representative household's Euler equation and a market clearing condition that equates consumption to output.[9] Equation (2) is a version of the so-called New Keynesian Phillips curve, which can be derived by aggregating the price-setting decisions of monopolistically competitive firms subject to Calvo-type constraints on the frequency of price adjustment, combined with standard assumptions on technology and labor

---

[8]This is the model used in the optimal policy analysis of Clarida, Galí and Gertler (1999). The reader is referred to King and Wolman (1996), Woodford (2003) or Galí (2008a) for a detailed description of that model and a derivation of (1) and (2).

[9]We implicitly assume a constant natural rate of interest, and ignore constant terms, to lighten the notation. The analysis below carries over, with suitable modifications, to the case of a time-varying natural rate of interest.

markets. In that context, parameter σ corresponds to the coefficient of relative risk aversion, β is the household's discount factor and κ is a coefficient which is inversely related to the degree of price rigidities. All variables are expressed in terms of deviations from their values in a zero inflation steady state.

A block describing how monetary policy is conducted completes the model. We assume that the central bank follows a simple interest rate rule of the form

$$i_t = \phi \pi_t \tag{3}$$

where $\phi > 1$. As is well known, the latter condition guarantees uniqueness of the equilibrium.[10]

The remainder of the paper examines three alternative approaches to constructing forecasts for inflation and output in the above economy, under a (counterfactual) assumption regarding how interest rates evolve over the forecast horizon. But before turning to that analysis I take a brief detour to describe the basic concern associated with the construction of macroeconomic forecasts conditional on an arbitrary, exogenously given interest rate path.

3 Projections Conditional on an Exogenous Interest Rate Path: The Indeterminacy Problem

For the sake of concreteness, let us assume that at the central bank is interested in the k-period ahead forecasts of inflation and the output gap *conditional* on an arbitrary, exogenous path for the interest rate, $\{i_t^*\}$. As mentioned above, both the CIR and ME projections constructed and published by many central banks can be viewed as particular examples of such an exercise since, in both cases, the path of the interest rate is given to the forecaster, with no room allowed for possible adjustments in response to developments in the economy, at least over the horizon for which the interest rate path is defined.

---

[10]As discussed in Svensson and Woodford (2005), if the central bank were to follow instead a "targeting rule," an additional complication would arise as a result of the need to determine the interest rate rule that would be consistent with the central bak's desired allocation but with no other (i.e. the one implying a determinate equilibrium).

Combining the assumed interest rate path with equations (1) and (2) yields the system of difference equations:

$$\begin{bmatrix} \pi_t \\ y_t \end{bmatrix} = \mathbf{A}_0 \begin{bmatrix} E_t\{\pi_{t+1}\} \\ E_t\{y_{t+1}\} \end{bmatrix} + \mathbf{B}_0 \begin{bmatrix} u_t \\ i_t^* \end{bmatrix} \tag{4}$$

where

$$\mathbf{A}_0 \equiv \begin{bmatrix} \beta + \frac{\kappa}{\sigma} & \kappa \\ \frac{1}{\sigma} & 1 \end{bmatrix} \qquad ; \qquad \mathbf{B}_0 \equiv \begin{bmatrix} 1 & -\frac{\kappa}{\sigma} \\ 0 & -\frac{1}{\sigma} \end{bmatrix}$$

Letting $\mathbf{x}_t = [\pi_t, \ y_t]'$ and $\mathbf{z}_t \equiv [u_t, i_t^*]'$ we can rewrite the previous dynamical system in a more compact way as:

$$\mathbf{x}_t = \mathbf{A}_0 E_t\{\mathbf{x}_{t+1}\} + \mathbf{B}_0 \mathbf{z}_t \tag{5}$$

Iterating forward on (5) we obtain the following expression for k-horizon projections as of time t:

$$E_t\{\mathbf{x}_{t+k}\} = \mathbf{A}_0^{-k}(\mathbf{x}_t - \mathbf{f}_t) \tag{6}$$

where $\mathbf{f}_t \equiv \mathbf{B}_0 \mathbf{z}_t + \mathbf{A}_0 \mathbf{B}_0 E_t\{\mathbf{z}_{t+1}\} + ... + \mathbf{A}_0^{k-1} \mathbf{B}_0 E_t\{\mathbf{z}_{t+k-1}\}$. Thus, in order for the projections at any $k-$horizon to be well defined, the current values of $\mathbf{x}_t$ (and, thus, of inflation and the output gap) must be determined uniquely, which in turn requires that the solution to (5) exist and be unique. A necessary and sufficient condition for that to be the case is that the two eigenvalues of $\mathbf{A}_0$ lie within the unit circle, given that $\mathbf{x}_t$ consists of two non-predetermined variables. [11] But as is well known, and formally re-stated in the following lemma, that condition is not satisfied under the exogenous interest rate path regime assumed here.[12]

Lemma. Let λ_1 and λ_2 denote the eigenvalues of $\mathbf{A}_0$, where $\lambda_1 \leq \lambda_2$. Both λ_1 and λ_2 are real and satisfy $0 < \lambda_1 < 1 < \lambda_2$.

Proof: see appendix.

The multiplicity of non-explosive solutions to (5) associated with the exogenous interest rate path implies that the equilibrium value for $\mathbf{x}_t$ is not

---

[11] See, e.g. Blanchard and Kahn (1980). Throughout we restrict the analysis to equilibria that remain in a neighborhood of the zero inflation steady state.

[12] The argument involves a straighforward variation of the analysis in Bullard and Mitra (2002). See also chapter 4 in Galí (2008a).

uniquely determined. In other words there exist a continuum of values for $\mathbf{x}_t$ that are consistent with a rational expectations equilibrium. It follows from (6), that there is also a continuum of k-horizon projections for the output gap and inflation, $E_t\{y_{t+k}\}$ and $E_t\{\pi_{t+k}\}$, consistent with the assumed interest rate path.

Next I describe three alternative approaches that allow one to construct inflation and output gap projections that are not subject to "the curse of indeterminacy" described above. A feature common to the three approaches is their reliance on some rule that turns the interest rate into an endogenous variable, while guaranteeing that the desired interest rate path $\{i_t^*\}$ is realized *in equilibrium.* In all cases it is assumed that the interest rate along that path converges to its value consistent with a zero inflation steady state, at least in expectation. Given that variables are expressed in terms of deviations from steady state, this implies that $\lim_{j\to\infty} E_t\{i_{t+j}^*\} = 0$.

4   Macroeconomic Projections without the Curse of Indeterminacy

4.1   Interest Rate Rule I

The first approach proposed in order to generate unique, well defined projections assumes that the central bank adopts the interest rate rule

$$i_t = \phi\pi_t + v_t \tag{7}$$

where $\phi > 1$ and v_t is perceived to be an exogenous, zero mean *i.i.d.* monetary policy shock. Notice that the latter shock is appended to the "usual" rule (3), in order to captures the deviations ("modest interventions," using Leeper and Zha's terminology) required in order to keep the nominal interest rate on the desired path $\{i_t^*\}$.[13]

It is easy to check that there exists a unique stationary solution to the system made up of (1), (2), and (7), given the assumption of an inflation coefficient greater than one.[14] The form of that solution can be guessed to

---

[13]See also the application in Smets and Wouters (2005).
[14]See, e.g., Bullard and Mitra (2002).

be:

$$y_t = au_t + cv_t$$
$$\pi_t = bu_t + dv_t$$

Using the method of undetermined coefficients it is straightforward to determine the values of the four coefficients, which are given by:

$$a = -(\phi - \rho_u)\Lambda_u$$
$$b = \sigma(1 - \rho_u)\Lambda_u$$
$$c = -\frac{1}{\sigma + \kappa\phi}$$
$$d = -\frac{\kappa}{\sigma + \kappa\phi}$$

where $\Lambda_u \equiv \frac{1}{\sigma(1-\rho_u)(1-\beta\rho_u)+\kappa(\phi-\rho_u)}$.

Combining (1) with the above solution we obtain the following expression for $t + k$ output gap and inflation

$$
\begin{aligned}
y_{t+k} &= E_{t+k}\{y_{t+k+1}\} + \frac{1}{\sigma}E_{t+k}\{\pi_{t+k+1}\} - \frac{1}{\sigma}i^*_{t+k} \\
&= -(\phi - 1)\Lambda_u\rho_u u_{t+k} - \frac{1}{\sigma}i^*_{t+k} \qquad (8)
\end{aligned}
$$

and

$$
\begin{aligned}
\pi_{t+k} &= \beta\,E_{t+k}\{\pi_{t+k+1}\} + \kappa y_{t+k} + u_{t+k} \\
&= (\beta b\rho_u - \kappa(\phi - 1)\Lambda_u\rho_u + 1)\,u_{t+k} - \frac{\kappa}{\sigma}i^*_{t+k} \\
&= (1 - \rho_u)(\sigma + \kappa\phi)\Lambda_u u_{t+k} - \frac{\kappa}{\sigma}i^*_{t+k} \qquad (9)
\end{aligned}
$$

Thus, the central bank's k-horizon forecasts are given by

$$E_t\{y_{t+k}\} = -(\phi - 1)\Lambda_u\rho_u^{k+1}u_t - \frac{1}{\sigma}i^*_{t+k}$$

$$E_t\{\pi_{t+k}\} = (1 - \rho_u)(\sigma + \kappa\phi)\Lambda_u\rho_u^k u_t - \frac{\kappa}{\sigma}i^*_{t+k}$$

Note that, ex-post, the central bank imposes a sequence of realizations for v_t that guarantee that the desired interest rate path $\{i_t^*\}$ is attained. Formally, this requires

$$v_t = i_t^* - \phi\pi_t$$

for all t. Combining the previous condition with the expression for equilibrium inflation above we obtain an expression for v_t in closed form

$$
\begin{aligned}
v_t &= \left(1 + \frac{\kappa\phi}{\sigma}\right) i_t^* - \phi(\sigma + \kappa\phi)(1 - \rho_u)\Lambda_u u_t \\
&= \left(1 + \frac{\kappa\phi}{\sigma}\right)(i_t^* - i_t^r)
\end{aligned}
$$

where $i_t^r \equiv \phi\sigma(1 - \rho_u)\Lambda_u u_t$ is the interest rate that would prevail under the baseline or "usual" rule. Thus, it is clear that in general $\{v_t\}$ will not satisfy the $i.i.d.$ assumption ex-post, thus violating the rationality of expectations. This should be recognized by agents if the "intervention" were to last long enough.[15]

The second and third methods described below are not subject to the previous shortcoming, being fully consistent with the assumption of rational expectations. Before we turn to them I take a brief detour to describe an equivalence result.

4.1.1 Modest Interventions vs. Switching Rules: An Equivalence Result

If the interest rate path on which projections must be conditioned is assumed to revert back after the forecast horizon to a level determined by the usual rule, the problem of indeterminacy can be shown to go away, allowing the central bank to construct well defined conditional projections.

To see this assume that the central bank sets the interest rate according to the rule

$$i_{t+j} = i_{t+j}^*$$

for $j = 0, 1, 2, ..., k$ and

$$i_{t+j} = \phi\pi_{t+j}$$

---

[15] This is acknowledged by Leeper and Zha (2003) who argue that the deviations from the rule should be small enough not to induce a change in agents' expectations about the regime in place.

for $j = k + 1, k + 2, ...$, i.e. it sets the interest rate at the level determined by the given exogenous path up to the desired forecast horizon, and switches to its regular interest rate rule after that.[16] Such an approach is used by Laséen, Lindé and Svensson (2008), as a way to simulate arbitrary time-varying interest rate rules in the context of the Riksbank's estimated DSGE model (Ramses).

Note that from time $t + k + 1$ onward the equilibrium dynamics are described by (1), (2), and (3), which are associated with a unique equilibrium. We can solve for that equilibrium using the method of undetermined coefficients, after guessing that both the output gap and inflation will be proportional to the cost-push shock. This yields the following expressions

$$y_{t+k+1} = a u_{t+k+1} \quad ; \quad \pi_{t+k+1} = b u_{t+k+1}$$

where a and b are given by the same expressions as above.

Combining the previous result with (1), (2) and the fact that $i_{t+k} = i^*_{t+k}$, we can write the equilibrium conditions corresponding to period $t + k$ as

$$
\begin{aligned}
y_{t+k} &= E_t\{y_{t+k+1}\} + \frac{1}{\sigma} E_t\{\pi_{t+k+1}\} - \frac{1}{\sigma} i^*_{t+k} \\
&= -(\phi - 1)\rho_u \Lambda_u u_{t+k} - \frac{1}{\sigma} i^*_{t+k}
\end{aligned}
$$

$$
\begin{aligned}
\pi_{t+k} &= \beta \ E_t\{\pi_{t+k+1}\} + \kappa y_{t+k} + u_{t+k} \\
&= (1 - \rho_u)(\sigma + \kappa\phi)\Lambda_u u_{t+k} - \frac{\kappa}{\sigma} i^*_{t+k}
\end{aligned}
$$

which are expressions identical to (8) and (9) above. The corresponding projections as of time t are thus uniquely determined and correspond to those generated by the "modest interventions" method.[17]

---

[16]In that sense, the switching rule approach is more restrictive than the modest interventions approach (or the second and third approaches described below), since it cannot generate finite horizon projections conditional on an arbitrary infinite horizon interest rate path.

[17]One should note, however, that the previous equivalence result hinges critically on $E_t\{y_{t+k+1}\}$ and $E_t\{\pi_{t+k+1}\}$ corresponding in both cases to the expected value conditional on the baseline rule, and hence on being history-independent. The previous observation suggests that the equivalence result presented here is unlikely to carry over to a more general setting with endogenous state variables.

4.2 Interest Rate Rule II

Consider next an interest rate rule of the form

$$i_t = i_t^* - \gamma i_{t-1}^* + \gamma \left(\pi_t + \sigma \Delta y_t \right) \tag{10}$$

where γ is a constant coefficient satisfying $\gamma > 1$.[18] Combining (10) with (1) we obtain the difference equation:

$$i_t - i_t^* = \frac{1}{\gamma} E_t \{ i_{t+1} - i_{t+1}^* \} \tag{11}$$

Note that under the assumption that $\gamma > 1$ the only non-explosive solution to (11) is $i_t = i_t^*$ for all t. In other words, by following rule (10) the central bank can support any desired interest rate rate path $\{ i_t^* \}$.

The equilibrium dynamics under rule (10) are described by (2) and

$$y_t = E_t \{ y_{t+1} \} - \frac{1}{\sigma} \left(i_t^* - \gamma i_{t-1}^* + \gamma \left(\pi_t + \sigma \Delta y_t \right) - E_t \{ \pi_{t+1} \} \right) \tag{12}$$

where the latter equation can be obtained by using (10) to eliminate the interest rate in (1). Equivalently, and more compactly, we can write (2) and (12) as:

$$
\begin{bmatrix} \pi_t \\ y_t \\ y_{t-1} \end{bmatrix} = \mathbf{A}_1 \begin{bmatrix} E_t \{ \pi_{t+1} \} \\ E_t \{ y_{t+1} \} \\ y_t \end{bmatrix} + \mathbf{B}_1 \begin{bmatrix} u_t \\ i_t^* - \gamma \; i_{t-1}^* \end{bmatrix} \tag{13}
$$

where

$$
\mathbf{A}_1 \equiv \begin{bmatrix} \beta & 0 & \kappa \\ 0 & 0 & 1 \\ \frac{\beta\gamma - 1}{\sigma\gamma} & -\frac{1}{\gamma} & 1 + \frac{1}{\gamma} + \frac{\kappa}{\sigma} \end{bmatrix} \quad ; \quad \mathbf{B}_1 \equiv \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ \frac{1}{\sigma} & \frac{1}{\sigma\gamma} \end{bmatrix}
$$

Note that the system (13) involves one predetermined and two non-predetermined variables. Thus, it has a unique non-explosive solution if and only if two eigenvalues of $\mathbf{A}_1$ lie inside, and one outside, the unit circle. The following proposition establishes a necessary and sufficient condition for that property to obtain.

---

[18] The present rule generalizes the one considered in Galí (2003, 2008b) to the case of an arbitrary interest rate path.

Proposition 1. A necessary and sufficient condition for (13) to have a unique non-explosive solution is given by $\gamma > 1$

Proof. see appendix.

More generally, one can show that an interest rate rule of the form

$$i_t = \gamma \pi_t + \varphi \Delta y_t + v_t$$

generates a unique equilibrium, where $\gamma > 1$ and $\{v_t\}$ is an arbitrary exogenous process. By setting $\varphi \equiv \sigma \gamma$ and $v_t = i_t^* - \gamma i_{t-1}^*$ we guarantee that such a unique equilibrium is associated with the desired interest rate path $\{i_t^*\}$.

Letting $\mathbf{x}_t = [\pi_t, \ y_t, y_{t-1}]'$ and $\mathbf{z}_t \equiv [u_t, i_t^* - \gamma i_{t-1}^*]'$, we can compute projections $E_t\{\pi_{t+k}\}$ and $E_t\{y_{t+k}\}$ using

$$E_t\{\mathbf{x}_{t+k}\} = \mathbf{A}_1^{-k}(\mathbf{x}_t - \mathbf{g}_t)$$

where $\mathbf{g}_t \equiv \mathbf{B}_1\mathbf{z}_t + \mathbf{A}_1\mathbf{B}_1 E_t\{\mathbf{z}_{t+1}\} + ... + \mathbf{A}_1^{k-1}\mathbf{B}_1 E_t\{\mathbf{z}_{t+k-1}\}$, and where $\mathbf{x}_t$ is the unique solution to (13), which can be obtained using standard formulae (see, e.g. Blanchard and Kahn (1980)).

It should be noted here that the role of parameter γ is restricted to guaranteeing that the target path for the interest rate is attained, which is the case for *any* value of γ larger than one. The particular choice of γ has no influence, however, on the resulting equilibrium path of inflation and the output gap and, hence, on the corresponding projections. To see this, note that under rule (10) the linear combination $\pi_t + \sigma \Delta y_t$ will be equal to i_{t-1}^*, for all t. The latter condition can in turn be combined with inflation equation (2) to obtain a difference equation for the output gap, which can be solved independently of γ.

4.3 Interest Rate Rule III

Consider finally an interest rate rule of the form

$$i_t = i_t^* - \gamma i_{t-1}^* + \gamma \left(\pi_t + r_{t-1} \right) \tag{14}$$

where $r_t \equiv i_t - E_t\{\pi_{t+1}\}$ is the (ex-ante) real interest rate and $\gamma > 1$. Combining (14) with the definition of the real interest rate, yields again a difference equation of the form

$$i_t - i_t^* = \frac{1}{\gamma} E_t\{i_{t+1} - i_{t+1}^*\}$$

12

whose only stationary solution is $i_t = i_t^*$ for all t given our assumption that $\gamma > 1$.

The equilibrium dynamics under rule (14) are described by three equations: inflation equation (2), the dynamic IS equation (1) and the interest rate rule (14), with the latter two rewritten in terms of the real interest rate, that is,

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}r_t \tag{15}$$

$$r_t + E_t\{\pi_{t+1}\} = i_t^* - \gamma i_{t-1}^* + \gamma\left(\pi_t + r_{t-1}\right) \tag{16}$$

The previous equilibrium conditions can be written in compact form as:

$$\begin{bmatrix} \pi_t \\ y_t \\ r_{t-1} \end{bmatrix} = \mathbf{A}_2 \begin{bmatrix} E_t\{\pi_{t+1}\} \\ E_t\{y_{t+1}\} \\ r_t \end{bmatrix} + \mathbf{B}_2 \begin{bmatrix} u_t \\ i_t^* - \gamma i_{t-1}^* \end{bmatrix} \tag{17}$$

$$\mathbf{A}_2 \equiv \begin{bmatrix} \beta & \kappa & -\frac{\kappa}{\sigma} \\ 0 & 1 & -\frac{1}{\sigma} \\ \frac{1}{\gamma} - \beta & -\kappa & \frac{1}{\gamma} + \frac{\kappa}{\sigma} \end{bmatrix} \quad ; \quad \mathbf{B}_2 \equiv \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ -1 & -\frac{1}{\gamma} \end{bmatrix}$$

Once again, the system of difference equations (17) involves two non-predetermined and one predetermined variables. Thus, it has a unique non-explosive solution if and only if two eigenvalues of $\mathbf{A}_2$ lie inside, and one outside, the unit circle. The following proposition establishes that the condition $\gamma > 1$ is both necessary and sufficient condition for uniqueness to obtain.

Proposition 2. A necessary an sufficient condition for (17) to have a unique non-explosive solution is given by $\gamma > 1$

Proof: see appendix.

Letting now $\mathbf{x}_t = [\pi_t, \ y_t, r_{t-1}]'$ and $\mathbf{z}_t \equiv [u_t, i_t^* - \gamma i_{t-1}^*]'$, we can compute conditional forecasts $E_t\{\pi_{t+k}\}$ and $E_t\{y_{t+k}\}$ using

$$E_t\{\mathbf{x}_{t+k}\} = \mathbf{A}_2^{-k}(\mathbf{x}_t - \mathbf{g}_t)$$

where $\mathbf{g}_t \equiv \mathbf{B}_2\mathbf{z}_t + \mathbf{A}_2\mathbf{B}_2 E_t\{\mathbf{z}_{t+1}\} + ... + \mathbf{A}_2^{k-1}\mathbf{B}_2 E_t\{\mathbf{z}_{t+k-1}\}$, and where $\mathbf{x}_t$ is the unique solution to (17).

Note finally that under rule (14) the linear combination $\pi_t + r_{t-1}$ is (ex-post) equal to i_{t-1}^*, for all t. That condition can be combined with (2) and

13

(15) to obtain a system of three difference equations with three endogenous variables (π_t, y_t, and r_t), which is independent of γ. Hence, and as it was the case for interest rate rule II, the specific choice of γ has no influence on the resulting projections.

5 Multiple Determinate Projections: An Illustration using the Basic New Keynesian Model

The previous section has described three alternative approaches to the construction of projections conditional on a given path for the nominal interest rate which are not subject to the problem of indeterminacy. Such a multiplicity of approaches raises a natural question: Are the inflation and output gap projections generated by the different approaches identical, if they are conditioned on the same interest rate path? The answer to that question is negative, as the exercises shown next make clear. Furthermore, and most importantly, the differences across projections obtained using different rules are quantitatively large.

For the purposes of the present section I assume that the economy is described by the basic New Keynesian model used above. In the next section I illustrate the same point using the estimated DSGE model of Smets and Wouters (2007), a more realistic framework and, hence, one for which quantitative predictions can be taken more seriously.

Let us consider the case of a central bank which, as of time t, wants to produce conditional forecasts of inflation and the output gap for period $t+k$. As above, the non-policy block of the economy is described by equations (1) and (2). For simplicity, we assume that the economy was at its steady state position in period $t-1$, i.e. $y_{t-1} = \pi_{t-1} = i_{t-1} = 0$. A cost-push shock of unit size is assumed to hit the economy in period t, vanishing over time in proportion to ρ_u^k, for $k = 1, 2, 3, ...$

What are the model-based projections for inflation and the output gap in period $t+k$ conditional on the central bank keeping the interest rate unchanged? Next I compute those projections under each of the three approaches discussed above for a calibrated version of the baseline model. For the purposes of this exercise, I assume the following parameter values, which are similar to those often used in the literature: $\beta = 0.99$, $\sigma = 1$, $\kappa = 0.1$,

$\phi = 1.5$ and $\rho_u = 0.5$.[19] I normalize the size of the shock so that the response of (annualized) inflation on impact under the true rule is of one percentage point. It should be clear that the main finding of this section is a qualitative one, and does not hinge on the details of the calibration.

Figures 1 and 2 display the inflation and output gap projections at horizons up to 12 quarters associated with the three rules described in the previous section. The figures also display the (unconditional) forecast associated with the "true" rule (3). Note that, even though the three rules support an unchanged interest rate through the forecast horizon (and beyond, in the case of rules II and III), their associated projections for inflation and output are very different. The differences among them involve both the size of the projected changes in inflation and the output gap as a result of the assumed cost-push shock, but also in the patterns of those responses and, in one case, even the sign of those responses. We also see that none of them tracks the unconditional forecast well, but this was to be expected since the latter implies a different interest rate path. Interestingly, the constant interest rate projections differ even in terms of the sign of their deviation from the unconditional forecasts.

Why do the three rules considered above generate different projections despite being associated with an identical nominal interest rate path? Put simply, the intuition behind that result is that any given nominal interest rate path is consistent with different paths of the real rate and expected inflation. In the New Keynesian model, the presence of sticky prices makes it possible for the central bank to influence real variables, including the real interest rate. The three rules considered achieve the same nominal rate path through different combinations of real rates and expected inflation. Not surprisingly, each of those combinations is associated with different paths for the output gap and, as a result, for inflation as well. This leads, in turn, to different projections.[20]

---

[19]As argued above, the specific choice of γ has no influence on the projections, as long as it is larger than one, which we assume here.

[20]In the case of ME-based projections, one might argue that there should be a preference for the rule that does a better job at matching private sector expectations of other macro variables, in addition to the nominal interest rate. But, any model-based private sector forecasts would have to be conditioned on some interest rate rule. A well defined approach would thus consist in computing projections conditional on the policy rule assumed by the private sector. But then it is not clear that those forecasts should be attached much value if the latter rule differs from the true one.

The previous finding clearly calls into question the usefulness of projections conditional on a given interest rate path, since there is no obvious reason to prefer one method over another in order to generate the same path. Put differently, the information required in order to compute well defined macroeconomic projections goes beyond the specification of the interest rate path and the horizon of the projections. A complete description would need to include the nature and specification of the *policy rule* that will be followed in order to support the desired interest rate path.

6   Multiple Determinate Projections: An Illustration using the Smets-Wouters Model

Next I conduct an exercise similar to the one described in the previous section but using a more realistic version of the New Keynesian model, namely, the estimated DSGE model of Smets and Wouters (2007). Relative to the basic New Keynesian model above, the Smets-Wouters model incorporates a number of features, including endogenous capital accumulation (subject to adjustment costs), habit formation in consumption, variable capital utilization, staggered wage and price setting with partial indexation, and as many as seven different structural shocks. That model, as well as the related models in Smets and Wouters (2003) and Christiano, Eichenbaum and Evans (2005), can be viewed as the backbone of the estimated DSGE models developed at central banks in recent years and used for monetary policy analysis and forecasting. The reader is referred to the original Smets and Wouters (2007) paper (and its companion technical appendix) for details.

The construction of inflation and output projections under alternative rules using the Smets-Wouters model requires two main changes relative to the analysis above. First, and given the presence of endogenous state variables, the exogenous monetary policy shocks $\{v_t\}$ that must be fed into rule I ("modest interventions") in order to keep the nominal interest rate along the desired path (a constant one, in the present exercise) must be computed using an iterative procedure, as described in the appendix. Secondly, rule II has to be modified in a way consistent with the specification of the Euler equation in the Smets-Wouters model, which is given by

$$x_t = E_t\{x_{t+1}\} - \frac{1}{\widetilde{\sigma}}(i_t - E_t\{\pi_{t+1}\}) \tag{18}$$

where $x_t \equiv (1-\chi)c_t - \chi c_{t-1} - \psi n_t$, with c_t and n_t denoting, respectively, (log) consumption and (log) hours, and where χ and ψ are themselves functions of exogenous parameters. Note that (18) differs from (1) due to three factors: (i) consumption and output no longer coincide due to the presence of capital accumulation, (ii) habit formation is allowed for ($\chi \geq 0$, $\widetilde{\sigma} \geq \sigma$), and (iii) the utility function is (potentially) nonseparable in consumption and hours ($\psi \geq 0$). Accordingly, the interest rate rule corresponding to (10), and which combined with (18) will support an arbitrary exogenous interest rate path $\{i_t^*\}$ is now given by:

$$i_t = i_t^* - \gamma i_{t-1}^* + \gamma \left(\pi_t + \widetilde{\sigma}\Delta x_t\right) \tag{19}$$

Finally, I note that rule III des not require any modification and can be implemented "as is" in the Smets-Wouters model.

Figures 3 through 6 display the inflation and output projections over a twelve-quarter horizon implied by rules I through III under the assumption of a constant interest rate path ($i_{t+j}^* = 0$, $j = 0, 1, 2, ...$), and in response to four different structural shocks: a neutral technology shock ("productivity"), an investment-specific technology shock ("investment"), a price markup shock, and a wage markup shock. These four shocks account for the bulk of the forecast error variance decomposition of inflation and output at a ten-quarter horizon in Smets and Wouters (2007). In each case, the inflation and output forecasts consistent with the "true" interest rate rule are also displayed, though now the latter rule corresponds to the estimated interest rate rule in Smets and Wouters (2007), which is substantially richer than (3) above.[21] Again, the size of each shock is normalized so that the response of (annualized) inflation on impact under the true rule is of one percentage point. For the purposes of the present exercise the model is calibrated according to the baseline calibration/estimation in Smets and Wouters (2007).

The projections based on the Smets-Wouters model reported in Figures 3 through 6 confirm the main finding obtained in the context of the basic New Keynesian model, as discussed in the previous section: even though the three rules considered support the same path for the nominal interest rate (a flat path in this case), their implied projections for both inflation and output are generally very different, with the gaps among them being economically significant and often of a size of one-hundred or more basis points. Of

---

[21]In particular, the nominal interest rate is allowed to respond, in addition to inflation, to its own lag, as well as the level and first-difference of the output gap.

course, those projections also display important differences relative to those generated by the actual rule, though this should come as no surprise since the latter implies a different path for the nominal interest rate. As can be seen in Figure 5, Rules I and II, when conditioned on price markup shocks, possibly provide the only example of quantitatively similar –though far from identical– projections (which in turn happen to be similar to those generated by the actual rule). Yet, even in that case, Rule III involves very different projections, with no reason to prefer of discard the latter relative to those implied by Rules I and II, as argued above.

7   Concluding Remarks

There appears to be a growing tendency among central banks to construct and report macroeconomic projections consistent with their own views about the future evolution of nominal interest rates. Many economists have welcome that development on different grounds, including its likely benefits from the viewpoint of transparency and ease of communication of the monetary authority's decisions and overall strategy to the public.

In the present paper I have provided an additional argument for the adoption of projections based on the central bank's own interest rate forecasts. I have done so by arguing that the alternative, i.e. conditioning projections on an exogenously given interest rate path, rests on shaky theoretical grounds. The latter assessment does not follow from the often heard argument that such projections will typically be indeterminate in forward-looking models. On the contrary, I have described as many as three different approaches to construct determinate projections conditional on an arbitrary interest rate path. Instead, I have argued that the main shortcoming of those projections lies precisely in the multiplicity of methods (each associated with a different rule) that are available to generate them, together with the fact that the different methods generally yield divergent projections for variables other than the interest rate itself. That observation calls into question the usefulness of projections conditional on a given interest rate path since, in principle, there is no obvious reason to prefer one method over another in order to generate that path.

Appendix

Proof of Lemma

From the properties of matrices and their eigenvalues, we have $tr(\mathbf{A}_0) = 1 + \beta + \frac{\kappa}{\sigma} = \lambda_1 + \lambda_2$ and $\det(\mathbf{A}_0) = \beta = \lambda_1 \lambda_2$. Thus we have $\lambda_1 = \frac{1+\beta+\frac{\kappa}{\sigma}-\sqrt{(1+\beta+\frac{\kappa}{\sigma})^2-4\beta}}{2}$ and $\lambda_2 = \frac{1+\beta+\frac{\kappa}{\sigma}+\sqrt{(1+\beta+\frac{\kappa}{\sigma})^2-4\beta}}{2}$. Note that

$$
\begin{aligned}
\Delta & \equiv (1+\beta+\frac{\kappa}{\sigma})^2 - 4\beta \\
& > (1+\beta)^2 - 4\beta \\
& = (1-\beta)^2 \quad\quad\quad\quad\quad\quad\quad\quad (20)
\end{aligned}
$$

from which it follows that both eigenvalues are real. Also, (20) implies $\lambda_2 > \frac{1+\beta+\frac{\kappa}{\sigma}+\sqrt{(1-\beta)^2}}{2} > 1$.

Note also that $\sqrt{(1+\beta+\frac{\kappa}{\sigma})^2-4\beta} < 1+\beta+\frac{\kappa}{\sigma}$, from which we have $\lambda_1 > 0$. Finally, $\lambda_1 < 1$ follows from the fact that $\lim_{\frac{\kappa}{\sigma}\to 0} \lambda_1 = \beta < 1$, and $\frac{\partial \lambda_1}{\partial(\kappa/\sigma)} < 0$. *QED.*

Proof of Proposition 1

The characteristic polynomial of $\mathbf{A}_1$ is given by

$$
\begin{aligned}
p_{A_1}(z) & = z^3 - \left(1+\beta+\frac{\kappa}{\sigma}+\frac{1}{\gamma}\right)z^2 + \left(\frac{1}{\gamma}\left(1+\beta+\frac{\kappa}{\sigma}\right)+\beta\right)z - \frac{\beta}{\gamma} \\
& = (z-\lambda_1)\,(z-\lambda_2)\,(z-\gamma^{-1})
\end{aligned}
$$

where $\lambda_1 \equiv \frac{1+\beta+\frac{\kappa}{\sigma}-\sqrt{(1+\beta+\frac{\kappa}{\sigma})^2-4\beta}}{2}$ and $\lambda_2 \equiv \frac{1+\beta+\frac{\kappa}{\sigma}+\sqrt{(1+\beta+\frac{\kappa}{\sigma})^2-4\beta}}{2}$. Using the same logic as in the proof of the Lemma above we conclude that both λ_1 and λ_2 are real, and satisfy the inequality $0 < \lambda_1 < 1 < \lambda_3$. Thus, and as long as $\gamma > 1$, two eigenvalues of $\mathbf{A}$ lie inside and one outside, the unit circle. *QED.*

Proof of Proposition 2.

The characteristic polynomial of $\mathbf{A}_2$ is given by

$$
p_{A_2}(z) = z^3 - \left(1+\beta+\frac{\kappa}{\sigma}+\frac{1}{\gamma}\right)z^2 + \left(\frac{1}{\gamma}\left(1+\beta+\frac{\kappa}{\sigma}\right)+\beta\right)z - \frac{\beta}{\gamma}
$$

which is identical to that of $\mathbf{A}_1$. Hence, the same condition for uniqueness of the equilibrium applies.

References

Bank for International Settlements (2007): *Monetary Policy Frameworks and Central Bank Market Operations.*

Benhabib, Jess, Stephanie Schmitt-Grohé, and Martin Uribe (2001): "The Perils of Taylor Rules," *Journal of Economic Theory* 94, 40-69.

Blanchard, Olivier and Charles Kahn (1980), "The Solution of Linear Difference Models under Rational Expectations", *Econometrica*, 48, 1305-1311.

Bullard, James, and Kaushik Mitra (2002): "Learning About Monetary Policy Rules," *Journal of Monetary Economics*, vol. 49, no. 6, 1105-1130.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, vol. 113, no. 1, 1-45

Clarida, Richard, Jordi Galí, and Mark Gertler (1999): "The Science of Monetary Policy: A New Keynesian Perspective," *Journal of Economic Literature*, vol. 37, 1661-1707.

European Central Bank (2001): *A Guide to Eurosystem Staff Macroeconomic Projection Exercises.*

Federal Reserve Board (2007): *Minutes of the Federal Open Market Committee, October 30-31, 2007.*

Galí, Jordi (2003): "New Perspectives on Monetary Policy, Inflation, and the Business Cycle," in *Advances in Economics and Econometrics*, volume III, edited by M. Dewatripont, L. Hansen, and S. Turnovsky, Cambridge University Press.

Galí, Jordi (2008a): *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*, Princeton University Press (Princeton, NJ).

Galí, Jordi (2008b): "Constant Interest Rate Projections without the Curse of Indeterminacy: A Note," *International Journal of Economic Theory*, forthcoming.

Honkapohja, Seppo and Kaushik Mitra (2005): "Performance of Inflation Targeting based on Constant Interest Rate Projections," *Journal of Economic Dynamics and Control*, vol. 29, 1867-1892.

King, Robert G., and Alexander L. Wolman (1996): "Inflation Targeting in a St. Louis Model of the 21st Century," *Federal Reserve Bank of St. Louis Review*, vol. 78, no. 3.

Leeper, Eric M. and Tao Zha (2003): "Modest Policy Interventions,"

Journal of Monetary Economics 50, 1673-1700.

Leitemo, Kai (2003): "Targeting Inflation by Constant Interest Rate Forecasts," *Journal of Money, Credit and Banking*, vol. 35, no. 4, 609-626.

Riksbank (2007): "Riksbank to publish its own forecast for the repo rate," *Monetary Policy Report*, issue 1, 19-21.

Smets, Frank, and Raf Wouters (2003): "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area," *Journal of the European Economic Association*, vol 1, no. 5, 1123-1175.

Smets, Frank and Rafael Wouters (2005): "Bayesian New Neoclassical Synthesis (NNS) Models: Modern Tools for Central Banks," *Journal of the European Economic Association* 3(2-3), 422-433.

Smets, Frank, and Raf Wouters (2007): "Shocks and Frictions in US Business Cycles: a Bayesian DSGE Approach," *American Economic Review*, vol. 97 (3), 586-606.

Svensson, Lars E.O and Michael Woodford (2005): "Implementing Optimal Policy through Inflation-Forecast Targeting," in B. Bernanke and M. Woodford (eds.), *The Inflation Targeting Debate*, University of Chicago Press (Chicago, IL).

Svensson, Lars E.O. (2006): "The instrument-rate projection under inflation targeting: The Norwegian Example," in *Stability and Economic Growth: The Role of Central Banks*, Banco de Mexico.

Woodford, Michael (2003): *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.(Princeton, New Jersey)

Woodford, Michael (2005): "Central Bank Communication and Policy Effectiveness," in *The Greenspan Era: Lessons for the Future*, Federal Reserve Bank of St. Louis.

Table 1 Central Banks' Forecasting Practices	
Reserve Bank of Australia	CIR
Central Bank of Brazil	CIR and ME
European Central Bank	ME
Bank of Japan	ME
Reserve Bank of New Zealand	CBE
Norges Bank	CBE
Monetary Authority of Singapore	Global rates
Riksbank	CBE
Swiss National Bank	CIR
Bank of England	ME
Federal Reserve System	CBE (individual FOMC members)

Note. CIR: constant interest rate; ME: market expectations ;
CBE: central bank expectations. Source: Bank for International
Settlements (2007) and national

Figure 1
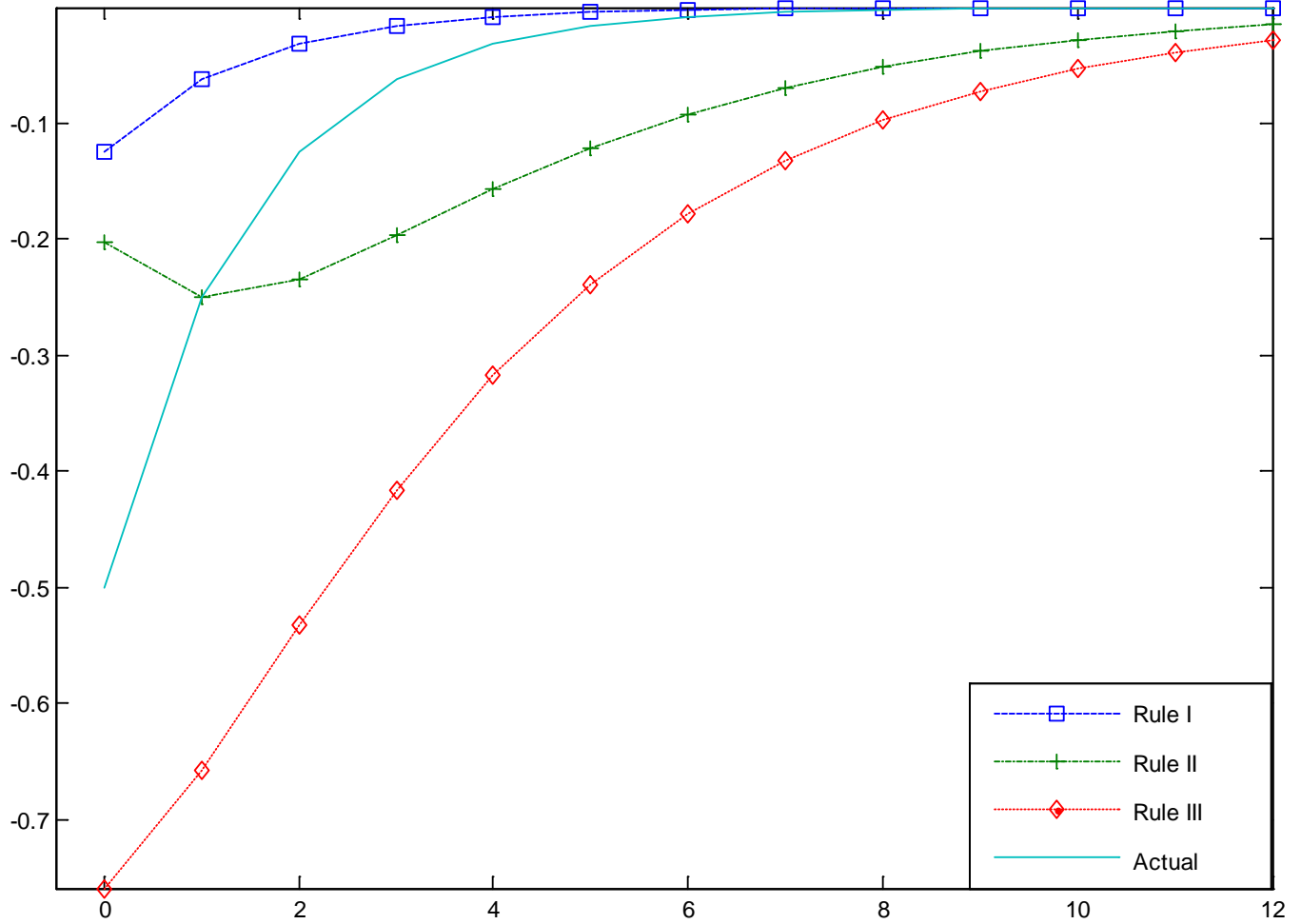CIR Projections in the Basic NK Model: Inflation
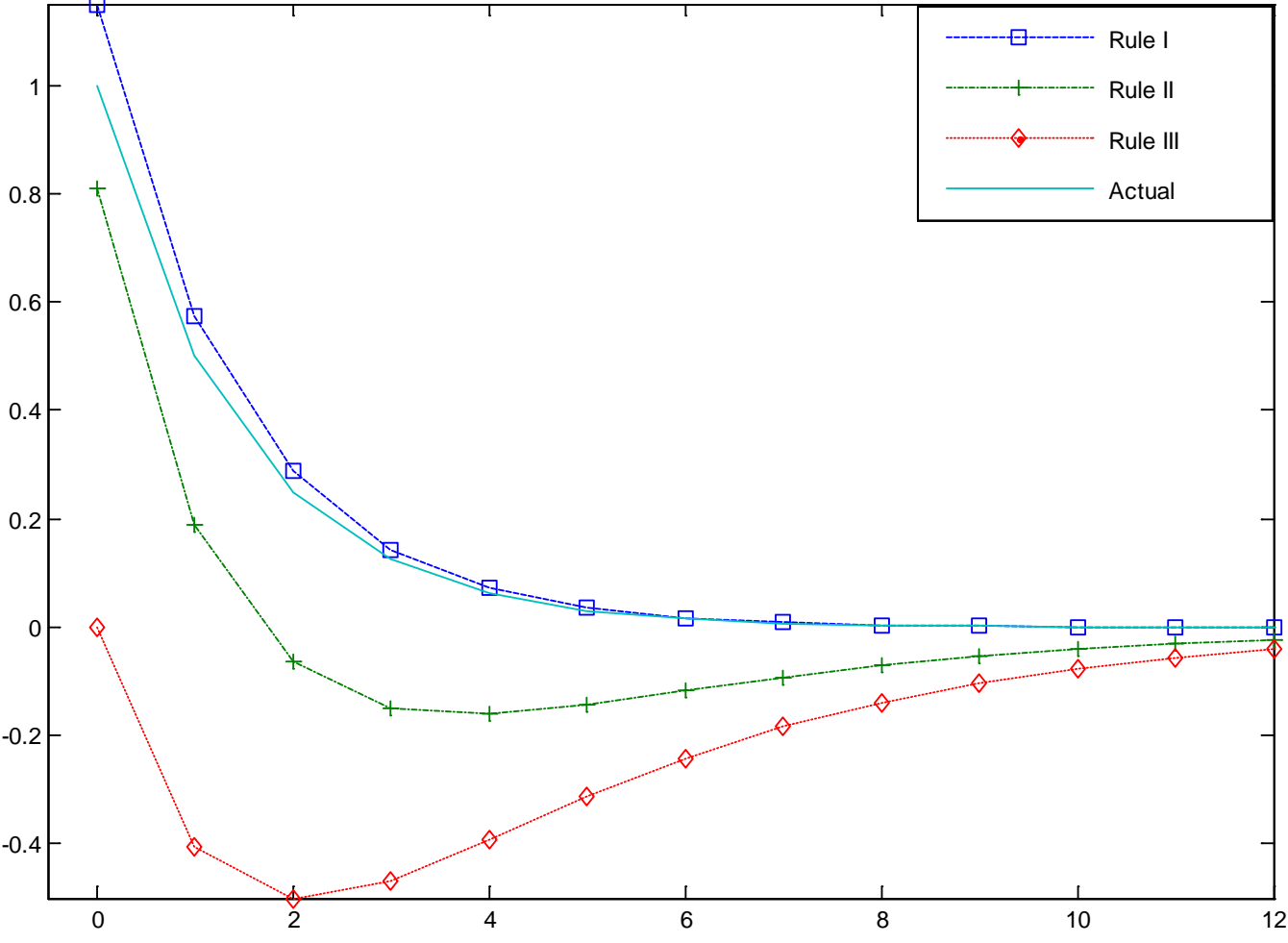
Figure 2
Constant Interest Rate Projections: Output

Figure 3
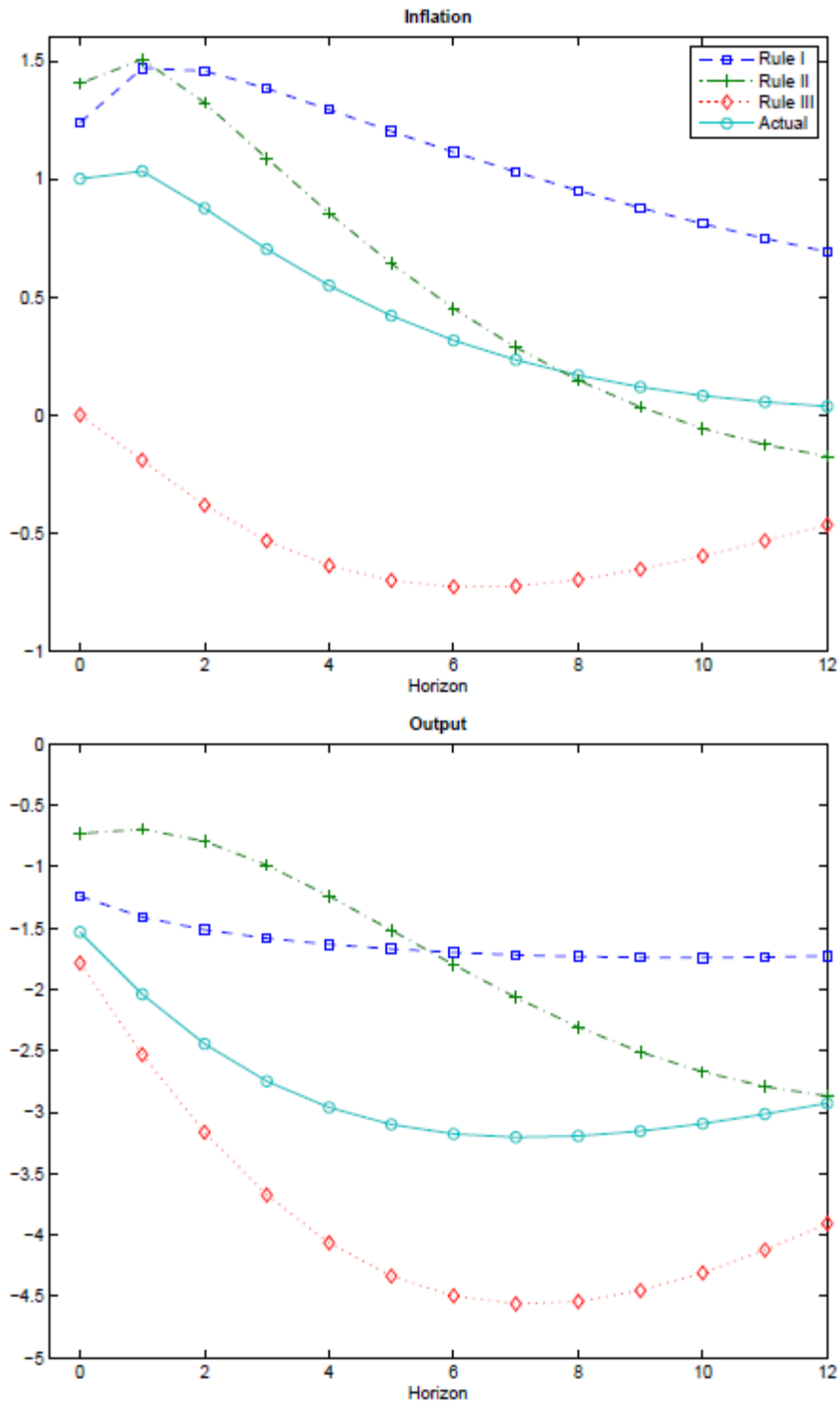## CIR Projections in the Smets-Wouters Model: Productivity Shocks

Figure 4
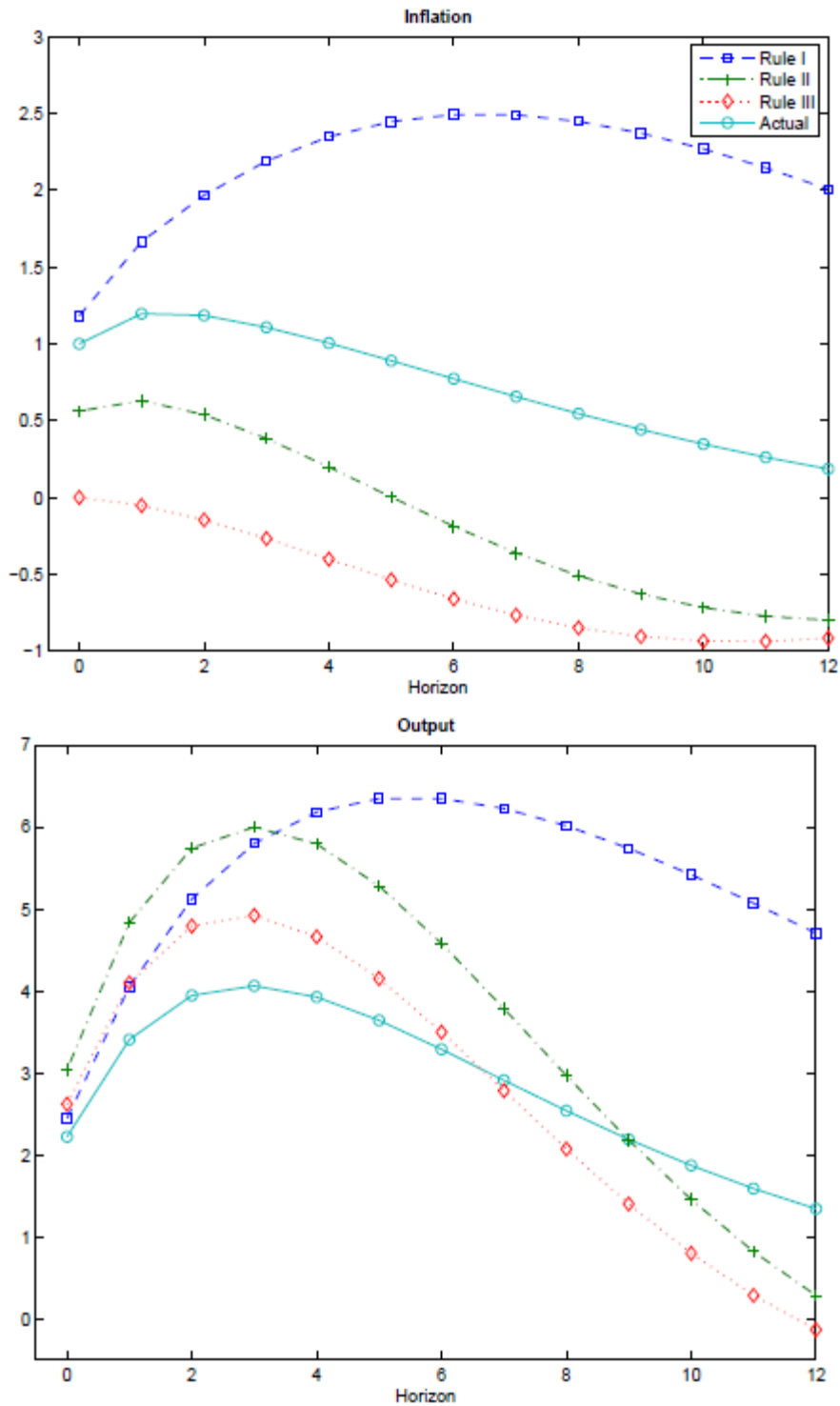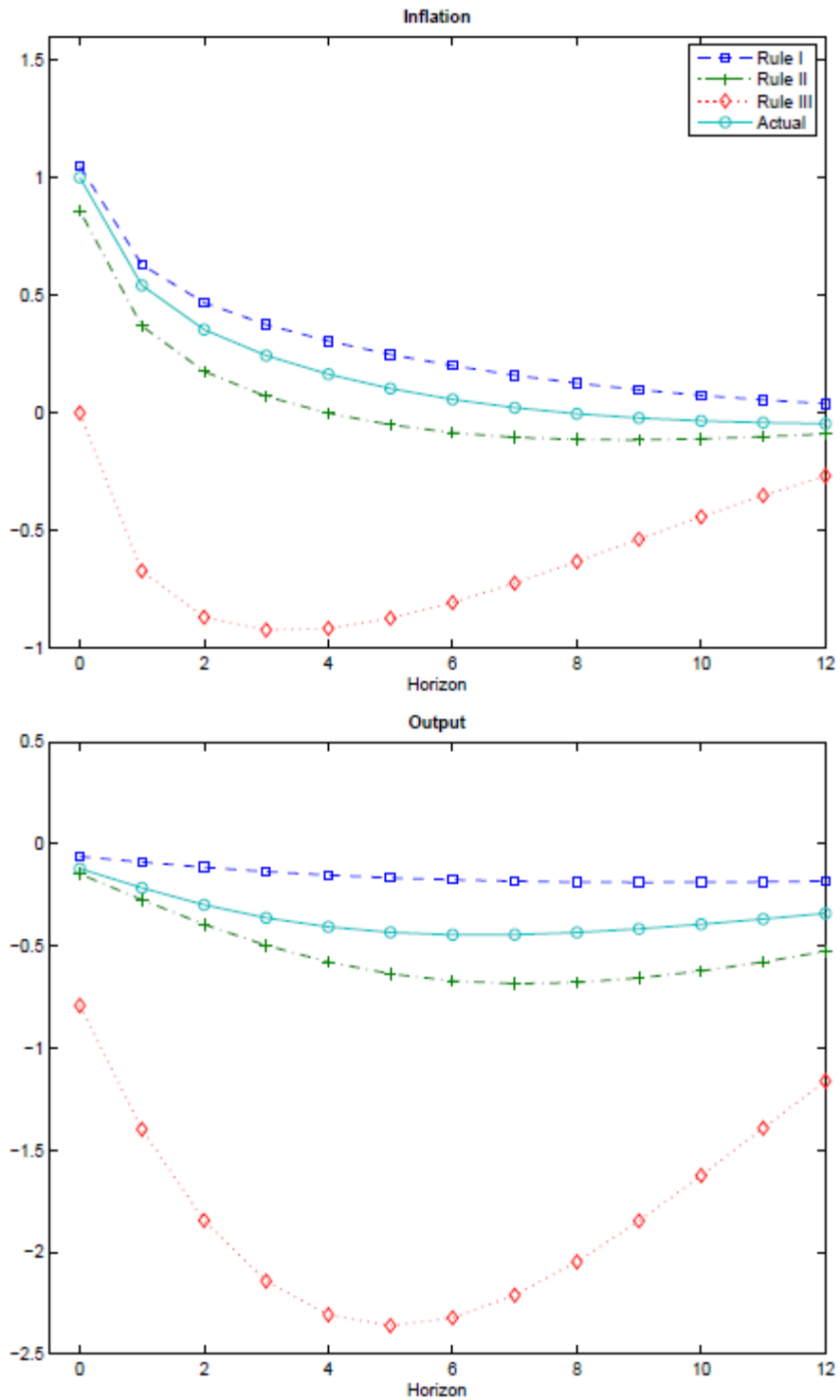## CIR Projections in the Smets-Wouters Model: Investment Shocks

Figure 5
CIR Projections in the Smets-Wouters Model: Price Markup Shocks

Figure 6
CIR Projections in the Smets-Wouters Model: Wage Markup Shocks