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Torben M Andersen

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Torben M Andersen, Aarhus University, CESifo, IZA, and CEPR

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Centre for Economic Policy Research
53–56 Gt Sutton St, London EC1V 0DG, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

Incentive and insurance effects of tax financed unemployment insurance*

The potential distortions of job-search incentives caused by unemployment benefits and their financing are well known. However, a benefit-tax scheme also provides insurance having direct utility effects as well as indirect effects on risk taking. The latter mitigates or may even dominate standard incentive effects to produce a non-monotone relation between efficiency (incentives) and equity (insurance). This implies that an increase in both benefits and the tax rate up to some point may increase average income and reduce inequality, i.e., there is not necessarily a trade-off between considerations for efficiency and equity. However, optimal utilitarian policies always position the economy at a point where marginal policy changes involve a trade-off, otherwise policies would not be optimal.

JEL Classification: D80, J20 and J65
Keywords: incentives, risk sharing, search, unemployment benefits

Torben M Andersen
Department of Economics
School of Economics and
Management
University of Aarhus
Nordre Ringgade 1
DK-8000 Aarhus C
Denmark

Email: tandersen@econ.au.dk

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1 Introduction

The role of unemployment insurance benefits for labour market performance is an important policy issue due to the incentive effects arising from the direct role of benefits but also due to the indirect effect arising when benefits are tax financed[1]. There is a vast literature analysing these effects of unemployment insurance both theoretically and empirically[2]. Many policy discussions of unemployment benefits take outset in these incentive effects, and benefit generosity is often mentioned as a reason for high unemployment rates.

However, the recent debate on flexicurity[3] has suggested that unemployment benefits via the insurance they provide may actually have some effects conducive for labour market performance. The argument being that flexible hiring rules in combination with generous unemployment benefits provide both flexibility to firms and security to workers. To the extent that agents are risk averse, the insurance effect may have both a direct utility effect via risk pooling[4] and an indirect effect by making job search activities less risky, which in turn may increase job search and contribute to higher employment. Two aspects of the flexicurity argument are particularly important, namely that there is a generous (mainly) tax financed unemployment insurance scheme for all (universal) and that flexible firing rules imply that there is a large turnover in the labour market. Since firms can easily adjust their work force, a high incidence of short term unemployment is to be expected, and a (short-term) unemployment risk is present for most workers, i.e., a large share of workers receive benefits for some part of the year[5]. The essence of the flexicurity debate is the balance between flexibility and security which in more general terms refers to the balance between incentives and insurance.

The aim of this paper is to discuss the incentive and insurance effects of a tax financed unemployment scheme in a labour market with a high level of job turnover and thus risk of (short or long-term) unemployment. This paper does not address all aspects associated with the flexicurity debate[6] but focuses on the security aspect and the effects of unemployment insurance on incentives and insurance as a necessary condition for the argument that flexicurity can be conducive for labour market performance. The model structure of the paper is purposely kept simple to capture the main mechanisms of unemployment insurance and taxation for labour market performance. Agents choose search effort

[1] Moreover, unemployment benefits may affect the wage setting by imposing a lower bound on wages.

[2] For references and a critical discussion of the literature see e.g. Howell et al. (2007).

[3] Much of the flexicurity debate is focussed on the experience in Denmark with rather flexible hiring and firing rules combined with a generous unemployment insurance scheme. See Andersen and Svarer (2007) for an account and discussion of the Danish flexicurity model.

[4] Which may also be important for ensuring political support for flexible hiring/firing rules.

[5] In Denmark, often highlighted as a "flexicurity" country, 28.9 % of the unemployed in 2007 where unemployed for less than one month within the year, in contrast to only 8% for OECD-Europe. For a period of up to three months within a year the share is 50.9 % in Denmark and 25% in OECD-Europe according to www.sourceoecd.org.

[6] See also Lommerud and Straume (2008), Brown and Snower (2009) and Keuschnigg and Davoine (2010) for analyses of aspects of flexicurity.

which has a risky return due to unemployment risk driven by uncertain length of employment spells (job tenure). Unemployment benefits are financed (in part or fully) by an income tax. In this way, the model includes the standard channels analysed in the literature on unemployment benefits and income taxation. The paper considers both the positive aspect of how the design of unemployment insurance scheme affects labour market performance and the normative issue of the optimal design of unemployment insurance schemes.

The debate on flexicurity raises issues on unemployment benefits in particular and social insurance in general. The incentive effects of unemployment insurance have been extensively studied both theoretically and empirically. Most work focuses on the disincentive effects of unemployment insurance (and taxation) for job search (less search and/or higher reservation demands) and therefore the higher unemployment following from generous benefit levels[7]. However, since unemployment benefits subsidize search, there may also be beneficial effects if there are frictions in the labour market implying that more search may lead to better matches (see e.g. Diamond (1981) and Marimon and Zilibotti (1999)). Acemoglu and Shimer (1999, 2000) show that risk sharing via unemployment insurance may lead to creation of jobs which are more productive, but also more difficult/risky to match. These papers show how unemployment insurance may effect the qualitative composition of employment, while the present paper shows that it also has an effect on employment in the quantitative dimension.

A seminal contribution on the normative aspects of unemployment insurance design is Bailey (1978) considering a tax financed unemployment insurance scheme[8]. The unemployment insurance scheme provides insurance which has a positive welfare effect by smoothing consumption, but it also distorts search effort (causing higher unemployment), and hence a standard trade-off between efficiency and equity arises[9]. The optimal benefit level is thus determined by trading off the insurance and incentive effects. This relates to the literature on social insurance[10]. An important starting point of this literature is the observation that schemes which in an ex post sense perform a redistributive role, ex ante will have an insurance effect to the extent that the ex post situation depends on risky factors. Hence, taxation not only distorts incentives but also provides insurance (the Domar-Musgrave effect), see e.g. Varian (1980), Eaton and Rosen (1980)). This tends to lower the efficiency costs which in turn have implications for the trade-off between efficiency and equity. Sinn (1995) argues in the case of a tax-transfer scheme that this may imply that the relation

[7] This may be countered by entilement effects if benefits have fixed duration.

[8] The present paper may be seen as a generalization of Bailey (1978), see also Chetty (2006). Note however, that the Bailey (1978) model relies on approximations and particular assumptions (including that the third derivate of the utility function is zero). The approach taken in this paper allows a much more precise characterization of optimal policies and comparative static results. Moreover risk sharing has no implications for incentives in Bailey (1978), while they have here.

[9] See also Gruber (1997) for an empirical analysis of this mechanism.

[10] See e.g. Barr (2001) for a general interpretation of welfare state arrangements as means of risk diversification or social insurance.

between efficiency and equity does not involve a trade-off but rather is hump-shaped such that more redistribution/insurance up to some point may lead to both more efficiency and equity (less risk).

This paper can be seen as a merger of labour market and social insurance aspects in a basic model clarifying the interaction between incentives and insurance for a tax financed unemployment benefit scheme. The present analysis has several distinctive features. First, it is shown that the insurance effect of unemployment benefits not only relates to consumption but also to the return of search, i.e., if the return to job search is risky, it follows that a benefit-tax system may make job search less risky, and therefore be conducive to job search. Second, since the model has ex ante risk and ex post differences in the position of individuals, it can be used to make inferences on the relation between efficiency and equity facing policy makers. It is shown that the interaction between the incentive and insurance effects differ significantly between the individual behavioural response and the equilibrium response to policy changes. Actually the non-monotone relationship more easily arises at the equilibrium level than at the individual level, and the precise conditions under which a non-monotone relationship between efficiency and equity arises in policy choices are identified. Finally, the paper uses a somewhat neglected method utilizing a location-scale condition allowing expected utility to be written as an indirect utility function specified over two moments (see Meyer (1989) and Sinn (1991)). This allows more clear-cut results on the role of risk aversion and risk compared to standard approaches based on the direct utility function.

The specific model in which to explore the insurance and incentive effects of both welfare policies and their interaction is laid out in section 2. Section 3 considers the relation between mean income (efficiency) and risk/inequality (equity) for given policies, while section 4 analyses how benefits and taxes affect efficiency and equity measures and looks at optimal policies for a utilitarian policy maker. Section 5 provides some concluding remarks.

2 A labour market model with risk and market power

Consider a labour market where workers search for jobs which have a stochastic duration, i.e., there is some unemployment risk. Workers are entitled to unemployment benefits b during periods of unemployment, and this scheme is tax financed (universal unemployment insurance scheme) via a proportional income tax τ in possible combination with lump sum taxes (T). The sequence is such that workers search for jobs, the state of nature determines the job duration and thus unemployment, and then consumption possibilities are determined. We proceed in the usual sequential way by first considering determination of search activity by individuals, and then turning to the role of the design of the unemployment insurance scheme.

2.1 Labour market

Consider a labour market where job creation and destruction is very easy (costless). Hence, firms easily create jobs, but also easily dismiss workers. Jobs offer a given wage w equal to productivity (constant returns to scale)[11] but may be terminated at any time, i.e., ex ante all jobs are alike but ex post they differ in actual duration, i.e., there is an unemployment risk. The duration or tenure of a given job[12] is determined by

$$d = m(e)\theta$$

Job duration has two parts, a deterministic part depending on search/matching effort[13] e and a stochastic part θ. Better matching is assumed to prolong job duration[14], and hence it is assumed[15] that $m_e > 0$, and $m_{ee} < 0$. The stochastic variable θ reflects exogenous shocks to firms and thus job duration and it has support on the interval $\left[\underline{\theta}, \overline{\theta}\right]$ with mean $E(\theta)$ and standard deviation $S(\theta)$. The cumultative distribution function F for θ is assumed to fulfil the location and scale condition (see e.g. Feller (1966)), $F(\theta) = F(\alpha + \beta\theta)$ for all $\beta > 0$). The time period is normalized to one, and hence $d \in [0, 1]$ [16],

Measuring employment in a given firm by the flow of labour within the period we have

$$L(\theta) = m(e)\theta$$

Note that all sectors/firms are alike except for the realization of the shock θ. Hence, the shock variable can be used to identify a specific firm (sector). There is an infinity of firms, hence the distribution function (ex ante) for θ at the firm/individual level is the frequency function (ex post) at the aggregate level. It follows that average employment across firms is

$$L = m(e)E(\theta)$$

2.2 Public sector

As noted, the public sector offers an unemployment benefit b to all unemployed, and it is financed by a proportional income tax τ and a lump sum tax T. The

[11] In an earlier version of this paper, wage formation was endogenized via imperfect competition in the labour market. While this has implication for the level of employment it did not have any qualitative effects on the trade-off between insurance and incentives, and therefore wages are here assumed exogenous to simplify.

[12] For an analysis of job replacement risk in a business cycle context see e.g. Krebs (2007).

[13] The assumption that the quality of the match depends on search intensity is similar to the one made in e.g. Baily (1978), Diamond (1981), and Acemoglu and Shimer (1999,2000).

[14] Can also be interpreted in an efficiency wage context. In this case the effort choice is related to shirking which in turn affects job duration.

[15] In addition it is assumed that $m(0) = 0$ and $m(e) \to \overline{m}$ for $e \to \infty$.

[16] Since the time period is normalized to have unit length it follows that a job does not imply lay-off if $m(e)\theta \geq 1$. In principle it is possible to allow for a fraction of jobs with no unemployment risk. However, the fraction of jobs for which this applies, i.e., $\theta \in \left[\widetilde{\theta}, \overline{\theta}\right]$ where $m(e)\widetilde{\theta} > 1$, will be endogenous and thus change with policy changes. This effect is a second order effect which is disregarded by assuming that $\overline{m}\underline{\theta} \leq 1$. Note that even if some jobs did not terminate, there would still be an ex ante risk.

budget constraint for this scheme reads[17]

$$T + \tau w m(e)E(\theta) = [1 - m(e)E(\theta)]\,[1 - \tau]\,b \tag{1}$$

Allowing for lump sum taxation (subsidies) makes it possible to treat benefits and the income tax rate as independent instruments to clarify their specific roles, before turning to the more realistic case of fully financing via the income tax $(T = 0)$.

2.3 Preferences

Utility if defined as a standard concave utility function[18] $U(y)$ defined over the full income metric

$$y = m(e)\theta w\,[1 - \tau] + [1 - m(e)\theta]\,b\,[1 - \tau] - T - e \tag{2}$$

where the first term is the flow income from the fraction of time being in employment, the second term the flow income from the fraction of time being unemployed, the third term lump sum taxes paid (or subsidies received), and the last term is the disutility from search effort[19]. The utility function is to simplify assumed separable in consumption and leisure[20].

Since full income (2) is multiplicative in the stochastic variable θ, the location and scale condition is fulfilled and expected utility $EU(y)$ can be written as an indirect utility function defined over the mean and standard deviation of the full income metric[21], i.e.,

$$V(\mu, \sigma) \equiv EU(y) \quad ; \quad V_\mu > 0, V_\sigma < 0 \tag{3}$$

The slope of the indifference curve giving the marginal rate of substitution between risk and expected income (implicit price of risk) is labelled

$$h(\mu, \sigma) \equiv \frac{\partial \mu}{\partial \sigma} \mid_V = -\frac{V_\sigma(\mu, \sigma)}{V_\mu(\mu, \sigma)} > 0$$

[17] Other revenue requirements and administrative costs of the scheme are disregarded to simplify.

[18] That is, $U'(y) > 0$, and $U''(y) < 0$. Moreover, $U'(y) \to \infty$ for $y \to 0$.

[19] At the cost of more complexity the disutility of effort could be written as a convex function of effort, e.g. $k(e)$, $k_e > 0$, $k_{ee} > 0$.

[20] Allowing for non-separability implying that expected utility is given as $EU(c, e)$ would imply an indirect utility function $V(\mu, \sigma, e)$ with the same qualititative properties as arising in the separable case, see Andersen (2010).

[21] Let $U(y)$ be a standard concave utility function $(U' > 0, U'' < 0)$ defined over a stochastic variable y distributed according to some cumultative distribution function $F(y)$. It can be shown (see e.g. Meyer (1987)) that expected utility $EU(y)$ can be represented by a concave indirect utility function $V(\mu, \sigma)$ where $\mu = E(y)$ and $\sigma = STD(y)$ provided that the cumultative distribution function F belongs to the class satifying the the location and scale condition implying for two cumultative distribution functions G_1 and G_2 that $G_1(y) = G_2(\alpha + \beta y)$, with $\beta > 0$. The utility function V has the properties, $V_\mu > 0$, $V_\sigma < 0$. Moreover, denote by $h(\mu, \sigma) \equiv \frac{\partial \mu}{\partial \sigma} \mid_V$ the slope of the indifference curve in (μ, σ)-space, it follows that $\frac{\partial h(\mu, \sigma)}{\partial \mu} \lesseqgtr 0$ depending on whether the utility function U displays decreasing, constant or increasing absolute risk aversion. If $\frac{\partial h(\lambda\mu, \lambda\sigma)}{\partial \lambda} \lesseqgtr 0$ (for any $\lambda > 0$) the utility function displays decreasing, constant or increasing relative risk aversion. Moreover $h(0, 0) = 0$.

A higher $h(\mu, \sigma)$ is associated with higher risk aversion in the sense of Arrow-Pratt[22] (see Lajeri and Nielsen (2000)). In the following $h(\mu, \sigma)$ is referred to as the "price" of risk.

The ex-ante expected income is

$$\mu \equiv E(y) = [1 - \tau] b + m(e)E(\theta) [1 - \tau] [w - b] - T - e \tag{4}$$

and its standard deviation is

$$\sigma \equiv S(y) = [1 - \tau] [w - b] m(e)S(\theta) \tag{5}$$

It is seen that we have the standard *incentive* effects that higher unemployment benefits increase expected income ($\frac{\partial \mu}{\partial b} > 0$) and thereby lower the incentive to search for jobs, and that an increase in the tax rate reduces the return to job search ($\frac{\partial \mu}{\partial \tau} < 0$) and thus search incentives[23]. At the same time, both instruments affect risk[24] since both a higher benefit level ($\frac{\partial \sigma}{\partial b} < 0$) and tax rate ($\frac{\partial \sigma}{\partial \tau} < 0$) reduces income risk, and this will be referred to as the *insurance effect*.

Note, all agents are ex-ante identical; that is, before the realisation of the shock (θ) is known, there are no differences between agents. Effort (=search) is chosen before the realization of the state of nature (=job tenure) is known; that is, agents exert effort with a deterministic cost (=disutility of effort) and a risky return (=market income). Ex-post employment histories differ with some individuals being unemployed most of the time (low market income), while others are employed most of the time (high market income). The realization of shocks thus cause ex-post differences in income and utility across individuals, i.e., there is inequality generated by unemployment. It follows that the ex-ante income risk for inviduals is the ex-post distribution of income in the economy.

2.4 Individual search effort

The individual is choosing search effort e so as to maximize

$$V(\mu, \sigma)$$

subject to (4) and (5), and taking policies (b, τ, T) and wages (w) as given. The first order condition for the effort choice e equates the marginal benefits to the marginal costs of exerting effort, i.e.,

[22] One decision maker is said to be more risk averse than another decision maker iff any risky prospect he prefers is also to be preferred by the other.

[23] Or alternatively that $\frac{\partial}{\partial b}\left(\frac{\partial \mu}{\partial e}\right) < 0; \frac{\partial}{\partial \tau}\left(\frac{\partial \mu}{\partial e}\right) < 0$.

[24] Note that since income over the whole period is relevant for utility, it is assumed that some smoothing of income is possible via the capital market. Hence, the argument does not depend on absence of capital markets. Note that it is a general property that with an unemployment risk the variability of income is falling in the benefit level. To see this denote the employment probability by p then the expected income $E(y) = pw + (1 - p)b$ and the variance is $Var(y) = 2p(1 - p)(w - b)^2$ and hence $\frac{\partial E(y)}{\partial b} < 0$ and $\frac{\partial Var(y)}{\partial b} < 0$.

$$V_\mu \left[m_e(e) E(\theta) \left[1-\tau \right] \left[w-b \right] - 1 \right] + V_\sigma \left[\left[1-\tau \right] \left[w-b \right] m_e(e) S(\theta) \right] = 0 \qquad (6)$$

and the second order condition is assumed to hold (see Appendix B). From (6) it follows that

$$m_e(e) = \frac{1}{\left[1-\tau \right] \left[w-b \right] \left[E(\theta) - h(\mu,\sigma) S(\theta) \right]},$$

which implies that e is increasing in $\left[1-\tau \right] \left[w-b \right] \left[E(\theta) - h(\mu,\sigma) S(\theta) \right]$. In the deterministic case ($S(\theta) = 0$) it is thus trivial that e is decreasing in τ and b due to the standard incentive effects. However, policies also have implications for risk sharing and therefore the price of risk $h(\mu,\sigma)$. Other things being equal an increase in risk ($S(\theta)$) or the price of risk ($h(\mu,\sigma)$) leads to lower job search. It follows that if policies tend to make job search less risky and thus reduce the price of risk this works to increase job search, i.e. the insurance effect has an effect on effort choice. Denoting $\eta \equiv - \left[\frac{m_{ee}(e) e}{m_e(e)} \right]^{-1} > 0$, we have

$$\frac{\partial e}{\partial b} \frac{b}{e} = -\eta \left[\frac{\frac{b}{w}}{1-\frac{b}{w}} + \frac{h(\mu,\sigma) S(\theta)}{E(\theta) - h(\mu,\sigma) S(\theta)} \frac{\partial h}{\partial b} \frac{b}{h} \right] \qquad (7)$$

$$\frac{\partial e}{\partial \tau} \frac{\tau}{e} = -\eta \left[\frac{\tau}{1-\tau} + \frac{h(\mu,\sigma) S(\theta)}{E(\theta) - h(\mu,\sigma) S(\theta)} \frac{\partial h}{\partial \tau} \frac{\tau}{h} \right] \qquad (8)$$

giving the elasticity of search effort wrt the benefit level and the tax rate, respectively. This elasticity is made up of the usual incentive effect (the first term) which is unambiguously negative and an ambiguously signed effect on the willingness to accept risk (the second term). The latter depends on how the policy change affects the price of risk ($\frac{\partial h}{\partial z} \frac{z}{h}$ for $z = b, \tau$) weigthed by the importance of risk ($\frac{h(\mu,\sigma) S(\theta)}{E(\theta) - h(\mu,\sigma) S(\theta)}$). Hence, if $\frac{\partial h}{\partial z} \frac{z}{h} < 0$ ($z = b, \tau$) higher benefits and taxes lower the price of risk and this goes in the direction of increasing search effort. Hence, the net effect of benefits and taxes on job search depends on the balance between the incentive and the insurance effect.

For later reference note that the optimum condition can also be written

$$\frac{E(\theta)}{S(\theta)} - \frac{1}{m_e(e) \left[1-\tau \right] \left[w-b \right]} = h(\mu,\sigma) \qquad (9)$$

i.e., the optimal effort level is where the possibilities of substituting between risk and expected income equals the marginal rate of substitution between the two (h). The optimal effort level can be summarized by the implicit function

$$e = e(w, b, \tau, T, E(\theta), S(\theta)) \qquad (10)$$

Considering the effort responses to the various exogenous variables we have

Proposition 1 *Job search effort is stricly positive, $e > 0$, for $m_e(0)\left[1-\tau\right]\left[w-b\right]E(\theta) > 1$, and it depends on (i) Lump sum tax: sign $\frac{\partial e}{\partial T} = sign\ h_\mu$, (ii)Expected return: $h_\mu \leq 0$ is a sufficient condition that $\frac{\partial e}{\partial E(\theta)} > 0$, (ii) Risk: $h_\sigma \geq 0$ is a sufficient condition that $\frac{\partial e}{\partial S(\theta)} < 0$,(iv) Wage rate: $\frac{\partial e}{\partial w} \geq 0$ under the sufficient condition that $h_\mu \frac{\partial \mu}{\partial w} + h_\sigma \frac{\partial \sigma}{\partial w} \leq 0$, and $\frac{\partial e}{\partial w} < 0$ has as necessary conditions that $h_\mu \frac{\partial \mu}{\partial w} + h_\sigma \frac{\partial \sigma}{\partial w} > 0$ and $S(\theta) > \overline{S}^w$,(v) tax rate: $\frac{\partial e}{\partial \tau} \leq 0$ under the sufficient condtion that $\left[h_\mu \frac{\partial \mu}{\partial \tau} + h_\sigma \frac{\partial \sigma}{\partial \tau}\right] \geq 0$, $\frac{\partial e}{\partial \tau} > 0$ requires as necessary conditions $h_\mu \frac{\partial \mu}{\partial \tau} + h_\sigma \frac{\partial \sigma}{\partial \tau} < 0$ and $S(\theta) > \overline{S}^\tau$, (vi) benefit level: sign $\frac{\partial e}{\partial b} = sign\ S(\theta) - \overline{S}^b$ since $h_\mu \frac{\partial \mu}{\partial b} + h_\sigma \frac{\partial \sigma}{\partial b} < 0$.*

Proof. Proof: See Appendix B ■

Since the deterministic part is standard, it is most interesting to interpret the effects specifically related to risk and risk aversion. They depend both on the level of risk ($S(\theta)$) and the willingness to accept risk measured by the price of risk ($h(\mu, \sigma)$). Consider first the lump sum tax which in a deterministic setting has no behavioural consequences. It has so under risk since an increase in the lump sum tax reduces expected income (μ) which in turn affects the willingness to carry risk. If $h_\mu < 0$ a higher (lower) expected income leads to more (less) willingness to accept risk (decreasing absolute risk aversion). Since a higher T lowers expected income, it follows that it leads to lower search effort for $h_\mu < 0$. An increase in the expected value of the shock ($E(\theta)$) has a direct effect on expected income and the same effect on the willingness to accept risk as the lump sum tax. Hence $h_\mu \leq 0$ is a sufficient condition that $\frac{\partial e}{\partial E(\theta)} > 0$. More risk (higher $S(\theta)$) has a direct effect on risk, but does also lower the willingness to accept risk if $h_\sigma \geq 0$, and hence the latter is a sufficient condition that $\frac{\partial e}{\partial S(\theta)} < 0$.

In the following we make the assumption

Assumption 1: $h_\mu \leq 0$ **and** $h_\sigma \geq 0$.

The first part implies non increasing absolute risk aversion, i.e., either constant or declining absolute risk aversion. The second part implies that the price of risk or compensation needed for agents to accept more risk, is increasing in risk. Assumption 1 ensures that $\frac{\partial e}{\partial T} \leq 0$, $\frac{\partial e}{\partial E(\theta)} > 0$ and $\frac{\partial e}{\partial S(\theta)} < 0$.

The effects of the wage rate, the benefit level and the tax rate are more complicated since they affect both mean income and its risk, i.e., we have that the effect on the price of risk is $h_\mu \frac{\partial \mu}{\partial z} + h_\sigma \frac{\partial \sigma}{\partial z}$, $z = w, \tau, b$. To interpret this, take the tax rate as an example. A higher tax rate tends - other things being equal - to decrease expected income ($\frac{\partial \mu}{\partial \tau} < 0$) which in turn increases the price of risk ($h_\mu \frac{\partial \mu}{\partial \tau} \geq 0$). But market income also becomes less risky ($\frac{\partial \sigma}{\partial \tau} < 0$) and this tends to decrease the price of risk ($h_\sigma \frac{\partial \sigma}{\partial \tau} < 0$). Hence, a change in the tax rate releases two opposite signed effects on the price of risk and the net effect is therefore ambiguous. If the net effect is a non decreasing price of risk ($h_\mu \frac{\partial \mu}{\partial \tau} + h_\sigma \frac{\partial \sigma}{\partial \tau} \geq 0$) it follows that the standard incentive effect of higher taxes is strengthened by the insurance effect and we have unambiguously that the standard sign applies

($\frac{\partial e}{\partial \tau} < 0$). However, if the change lowers the price of risk ($h_\mu \frac{\partial \mu}{\partial \tau} + h_\sigma \frac{\partial \sigma}{\partial \tau} < 0$), this induces more risk taking and thus more labour market search running counter to the incentive effect. This has two implications, first that the incentive effects are muted by the insurance effect, and second that this effect may be so strong as to imply more job search ($\frac{\partial e}{\partial \tau} > 0$) if the level of risk is sufficiently high ($S(\theta) > \overline{S}^b$)[25]. The latter condition arises because the price of risk is weighted by the underlying level of risk, and hence this has to be sufficiently large for the insurance effect to dominate the incentive effect. Similar reasoning applies, mutatis mutandis, to changes in the wage rate. However, for the benefit level there is a difference since we unambiguously have that an increase in benefits lowers the price of risk ($h_\mu \frac{\partial \mu}{\partial b} + h_\sigma \frac{\partial \sigma}{\partial b} < 0$), i.e., the insurance effect of benefits always affects job search in the opposite direction of the incentive effect. It is thus a general finding that higher benefits via the insurance effect work in the direction of increasing job search. The insurance effect dominates the incentive effect if risk is sufficiently important ($S(\theta) > \overline{S}^b$) implying that $\frac{\partial e}{\partial b} > 0$, and vice versa.

3 Insurance and incentives

The risk sharing or diversification established via the tax financed unemployment benefit scheme is seen clearly by noting that in equilibrium we have for given policies (b and τ are given, and T adjusts endogenously to clear the public budget constraint) that the full income for a given household is (using (1) in (2))

$$y = m(e)\theta w + [[1-\tau]\, b + \tau w]\, m(e)\, [E(\theta) - \theta] - e$$

The net-transfer from the benefit-tax system is

$$[[1-\tau]\, b + \tau w]\, m(e)\, [E(\theta) - \theta]$$

Hence, agents for whom $\theta < E(\theta)$ receive a net-transfer, and agents for whom $\theta > E(\theta)$ pay a net-tax. This shows how the benefit-tax scheme ex-post redistributes from those with employment above (unemployment below) the average ($\theta - E\theta > 0$) to those with employment below (unemployment above) the average ($\theta - E\theta < 0$). Ex ante this is perceived by agents and this works as an insurance mechansism towards the risk associated with job duration and unemployment. The extent of risk sharing depends on the factor $[1-\tau]\, b + \tau w$ given as the sum of the after tax benefit and the wage tax. Full income can also be written

$$y = wm(e)E(\theta) + [1-\tau]\, [1-r]\, wm(e)(\theta - E\theta) - e$$

where $r \equiv \frac{b}{w}$ is the replacement rate for the benefit level. This shows that the scheme implies that income is given as mean income plus a term correcting for

[25] Observe that the critical level of risk $\overline{S}^z(z = w, \tau, b)$ depends on the willingness to accept risk (the implicit price of risk) in the intuitive way, that larger the willingness to accept risk (higher implicit price of risk), the lower the critical risk level.

the individual position. Individuals with $\theta > E\theta$ have income above the mean, and vice versa. Note that the effective rate of taxation is given by the total tax wedge $[1-\tau][1-r] \simeq 1-\tau-r$.

A moral hazard problem arises due to the well-known common pool problem created by a tax financed benefit scheme. This is seen when noting that in equilibrium, and thus taking into account the budget constraint of the unemployment benefit scheme, that mean income and its standard deviation are given as

$$\widehat{\mu} = m(\widehat{e})E(\theta)w - \widehat{e} \tag{11}$$
$$\widehat{\sigma} = [1-\tau][w-b]\, m(\widehat{e})S(\theta) \tag{12}$$

where equilibrium effort $\widehat{e}$ is determined by (see Appendix C)

$$\widehat{e} = F(w, \tau, b, E(\theta), S(\theta))$$

The expression for expected income (11) shows that the scheme is a pure redistribution scheme, that is, the revenue collected is also handed out, and therefore mean income is not directly affected by the parameters of the policy package (b, τ, T). There is however an indirect effect to the extent that the scheme affects the search effort $(\widehat{e})$. In addition the scheme contributes to a reduction in risk for given effort ($[1-\tau]\left[1-\frac{b}{w}\right] \leq 1$ for all $\tau \in [0,1]$ and $\frac{b}{w} \in [0,1]$) .

3.1 Equilibrium effort - insurance and incentives

The unemployment insurance scheme and its financing clearly affect incentives, but since the effort choice is made under uncertainty wrt the return to work (employment) there is a risk/insurance effect also. Hence, the equilibrium response of search effort to the various exogenous variables is far from obvious. We have

Proposition 2 *Equilibrium search effort is (i) unambiguously increasing in the expected value of the shock* $\frac{\partial \widehat{e}}{\partial E(\theta)} > 0$, *(ii) decreasing in its standard deviation* $\frac{\partial \widehat{e}}{\partial S(\theta)} < 0$, *(iii) ambiguously depending on the wage rate* $\frac{\partial \widehat{e}}{\partial w} \lesseqgtr 0$ *, and (iv)* $\frac{\partial \widehat{e}}{\partial z} \lesseqgtr 0$ *for* $S(\theta) \lesseqgtr \widehat{S}^z$ *for* $z = b, \tau$.

Proof. Proof: See Appendix C ■

Unsurprisingly we find that the direction in which equilibrium effort responds to the expected value and standard deviation of the shock variable ($E(\theta)$ and $S(\theta)$, respectively) corresponds to the effects found for the individual effort function, cf. Proposition 1. The ambiguity wrt the wage rate reflects the finding in Proposition 1, since the wage is exogenous. More interesting is the response to the tax rate and the benefit level since the equilibrium response differs from the direct individual response due to the effects running via the budget for the unemployment insurance scheme. As noted above the source of the ambiguity

is the insurance effect. At the equilibrium level the effect of policy changes (b and τ) on the price of risk runs only via variability of income (σ) since there is no direct effect on mean income. Hence, for both the benefit level and the tax rate we have that they reduce the price of risk, and this tends to increase job search ($h_\sigma \frac{\partial \sigma}{\partial z} < 0$ for $z = b, \tau$). Since the incentive effect goes in the opposite direction, it follows that the incentive effect dominates if risk is low ($S(\theta) < \widehat{S}^z$ for $z = b, \tau$), and vice versa. While the sign reversal is interesting in its own sake, it is probably more important to note that even when the conventional signs apply, it is an implication that the insurance effect works to mitigate the incentive effect of changes in benefits or taxes on search effort (see (7) and (8)), and this may contribute to explain why observed behavioural responses are small.

3.2 Efficiency-equity locus

The opportunity locus of feasible levels of mean income and risk can now be found. Using (11) and (12), it follows that the mean income can be expressed in terms of its standard deviation and the effort level, i.e.,

$$\widehat{\mu} = \frac{\widehat{\sigma}}{1-\tau}\frac{w}{w-b}\frac{E(\theta)}{S(\theta)} - e$$

Using (12), we have that market risk and effort are related as

$$\begin{aligned} \widehat{e} &= m^{-1}\left[\frac{\widehat{\sigma}}{1-\tau}\frac{1}{w-b}\frac{1}{S(\theta)}\right] \\ &\equiv g(\frac{\widehat{\sigma}}{1-\tau}\frac{1}{w-b}\frac{1}{S(\theta)}) \qquad g' > 0, g'' > 0 \end{aligned}$$

It is an implication that a higher effort level ($\widehat{e}$) is associated with more market risk ($\widehat{\sigma}$), or phrased differently, a higher level of market risk is associated with a higher effort level. We now have that the expected income can be written as a function of its standard deviation and various parameters of the model, i.e.,

$$\widehat{\mu} = \frac{\widehat{\sigma}}{1-\tau}\frac{w}{w-b}\frac{E(\theta)}{S(\theta)} - g(\frac{\widehat{\sigma}}{1-\tau}\frac{1}{w-b}\frac{1}{S(\theta)}) \tag{13}$$

This gives the risk-return locus for given policies, and its slope is

$$\frac{\partial \widehat{\mu}}{\partial \widehat{\sigma}} = \frac{1}{1-\tau}\frac{w}{w-b}\frac{E(\theta)}{S(\theta)} - g'(\frac{\widehat{\sigma}}{1-\tau}\frac{1}{w-b}\frac{1}{S(\theta)})\frac{1}{1-\tau}\frac{1}{w-b}\frac{1}{S(\theta)} \lessgtr 0$$

Defining[26] (μ^t, σ^t) as the expected income and risk prevailing for $\frac{\partial \widehat{\mu}}{\partial \widehat{\sigma}} = 0$, it follows that

[26] That is, σ^t is the solution to

$$wE(\theta) - g'(\frac{\sigma^t}{\chi}) = 0$$

$$\frac{\partial \widehat{\mu}}{\partial \widehat{\sigma}} = \begin{cases} > 0 & \text{for } \widehat{\sigma} < \sigma^t \\ = 0 & \text{for } \widehat{\sigma} = \sigma^t \\ < 0 & \text{for } \widehat{\sigma} > \sigma^t \end{cases}$$

The relation between expected income and market risk is shown in figure 1[27]. To any point on the locus is associated an effort level, and the further to the right on the curve, the higher the effort level. We thus have that the relationship between mean income and risk is non-monotone. This suggests the possibility of having a non-monotone relationship between a measure of efficiency (effort or mean total income) and equity (risk, which is equal to the dispersion of income across individuals). Depending on the incentives underlying effort choices, society may thus be in a position where more efficiency goes hand in hand with less risk/inequality, or it may face a trade-off where more efficiency is attained at the cost of more risk/inequality. Note, however, that the non-monotone relationship given here depends on the policy instruments, and it does not straightforwardly give the relation available for policy choices, see below.

Figure 1a: The risk-return locus

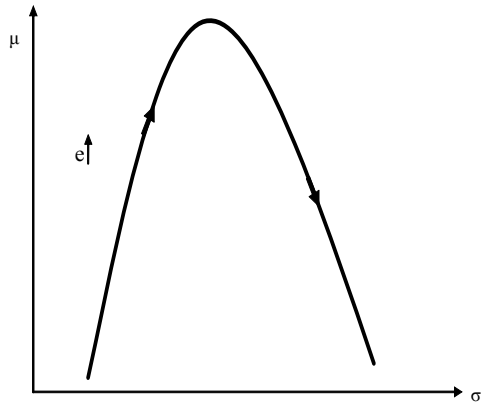
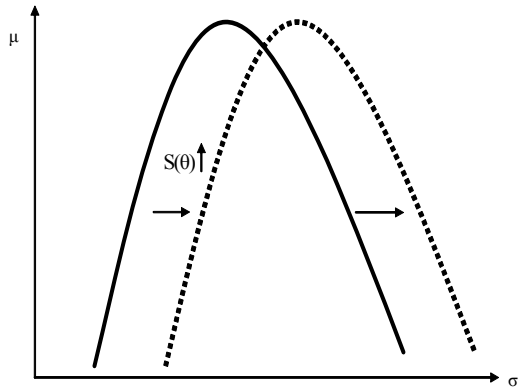


Figure 1b: The effect of higher risk ($S(\theta)$)



The position of the risk-return locus depends on the various underlying fac-

where $\varkappa \equiv [1-\tau]\,[w-b]\,S(\theta)$ and hence

$$\mu^t = \frac{\sigma^t}{\chi} wE(\theta) - g(\frac{\sigma^t}{\chi})$$

Since $m'(e) \to \infty$ for $e \to 0$, $g' \to 0$ for $\sigma \to 0$ this ensures that the critical level of risk σ^t is positive. It follows that $\frac{\sigma^t}{\chi}$ and hence μ^t is determined by $wE(\theta)$. As a consequence an increase in χ for given $wE(\theta)$ will lead to a proportional increase in σ^τ and leave μ^t unaffected. Therefore an increase in e.g. $S(\theta)$ shifts the risk-return locus to the right in the (μ, σ)-space. Note that the slope change is found by using that $\frac{\partial \mu}{\partial \sigma} = \frac{wE(\theta)}{\varkappa} - g'(\frac{\sigma}{z})\frac{1}{\varkappa}$ and hence $\frac{\partial}{\partial \varkappa}\frac{\partial \mu}{\partial \sigma} = -\frac{1}{\varkappa}\frac{\partial \mu}{\partial \sigma} + g''(\frac{\sigma}{\varkappa})\frac{\sigma}{\varkappa^3}$. It follows that $\frac{\partial}{\partial \varkappa}\frac{\partial \mu}{\partial \sigma} > 0$ for $\frac{\partial \mu}{\partial \sigma} < 0$ and $\frac{\partial}{\partial \varkappa}\frac{\partial \mu}{\partial \sigma} < 0$ for $\frac{\partial \mu}{\partial \sigma} > 0$ if $\sigma \leq \overline{\sigma}$ and $\frac{\partial}{\partial \varkappa}\frac{\partial \mu}{\partial \sigma} > 0$ for $\frac{\partial \mu}{\partial \sigma} > 0$ if $\sigma > \overline{\sigma}$.

[27] Note that market income ($= m(e)wE(\theta)$) is monotonously increasing in effort and thus risk.

tors, and figure 1b shows that the curve shifts rightwards for a higher level of risk ($S(\theta)$). In section 4, we turn to the optimal policy choices and the question of the trade-off between return and risk (efficiency and equity) available for policy makers.

3.3 Social optimal effort level for given policies

To clarify the inefficiencies arising under the benefit-tax scheme, it is useful to consider both the unconstrained and constrained social optimal effort choice. The latter is the social optimal search effort given a policy regime (b, τ).

The unconstrained social optimum is easily characterised since complete risk sharing ($\tau = 1$) is feasible leaving all with a certain consumption level ($m(e)wE(\theta)$), and the optimal effort (e^{**}) level mazimixes full income under certainty, and is determined by the condition

$$m_e(e^{**})E(\theta)w = 1$$

This allocation cannot be decentralized because complete redistribution/insurance affects incentives and thus the effort choice (no return to work as perceived by the individual).

For given policy (b, τ) the constrained social optimal effort level (e^*) is characterized by the solution to

$$Max_{\sigma}\ V(\mu, \sigma) \text{ subject to (13)}$$

yielding the first order condition

$$\frac{1}{1-\tau}\frac{w}{w-b}\frac{E(\theta)}{S(\theta)} - g'(\frac{\sigma^*}{1-\tau}\frac{1}{w-b}\frac{1}{S(\theta)})\frac{1}{1-\tau}\frac{1}{w-b}\frac{1}{S(\theta)} = h, \qquad (14)$$

i.e., in this case the planner internalizes the public budget, but the policy instruments (b, τ) are taken as given.

We now have

Proposition 3 *The effort levels can be ranked:* $\widehat{e} < e^* < e^{**}$ *and* $\frac{\partial \mu}{\partial e} > 0$ *for all* $e < e^{**}$.

Proof. The constrained optimal effort level e^* is determined by the conditon (14) which can be written

$$m_e(e^*)wE(\theta) - 1 = h\left[1-\tau\right]\left[w-b\right]S(\theta)$$

It follows immediately that $e^* < e^{**}$. To compare this effort choice to the individual effort choice $\widehat{e}$, we note that the first order condition (6) can be written

$$m_e(\widehat{e})E(\theta)\left[1-\tau\right]\left[w-b\right] - 1 = h\left[\left[1-\tau\right]\left[w-b\right]m_e(\widehat{e})S(\theta)\right]$$

or

$$\frac{E(\theta)}{S(\theta)} - \frac{1}{m_e(\widehat{e})}\frac{1}{1-\tau}\frac{1}{w-b}\frac{1}{S(\theta)} = h$$

It follows that the decentralized chosen effort level $\widehat{e}$ is lower than the social optimum e^* (for given policies), i.e.,

$$\widehat{e} < e^*$$

Finally, note that

$$\frac{\partial \mu}{\partial e} = w m_e(e) E(\theta) - 1$$

where $\frac{\partial^2 \mu}{\partial e^2} = w m_{ee}(e) E(\theta) < 0$ and hence $\frac{\partial \mu}{\partial e} > 0$ for all $e < e^{**}$. ■

This result is interesting since it makes it possible to decompose the difference between the social optimum and the decentralized effort level (equilibrium) $(e^{**} - \widehat{e})$ into the part due to incomplete risk sharing $(e^{**} - e^*)$ and the part caused by the distortion or moral hazard (common pool) problem created by the insurance mechanism $(e^* - \widehat{e})$. Moreover, it is an implication that the decentralized equilibrium is always on the upward slope of the risk-return locus depichted in figure 1[28].

Another way of expressing the result that the decentralized search effort is too low $(\widehat{e} < e^*)$ is to note that when comparing the constrained social optimum to the decentralized outcome we have from (9) and (14)

$$\frac{\partial \mu}{\partial \sigma} \mid_{\text{const. soc. opt.}} = \frac{1}{1-\tau}\frac{1}{1-\frac{b}{w}}\frac{E(\theta)}{S(\theta)} - \frac{1}{m_e(e)\left[1-\tau\right]\left[w-b\right]}$$

$$\frac{\partial \mu}{\partial \sigma} \mid_{\text{decentralized eq.}} = \frac{E(\theta)}{S(\theta)} - \frac{1}{m_e(e)\left[1-\tau\right]\left[w-b\right]}$$

Hence,

$$\frac{\partial \mu}{\partial \sigma} \mid_{\text{const. soc. opt.}} > \frac{\partial \mu}{\partial \sigma} \mid_{\text{decentralized eq.}},$$

i.e., for the social planner more risk (effort) will at the margin increase mean income more than perceived by individual decision makers. The reason is the distortion (relative to constrained optimum) created by the benefit-tax wedge, and captured by the term

$$\frac{\partial \mu}{\partial \sigma} \mid_{\text{const. soc. opt.}} - \frac{\partial \mu}{\partial \sigma} \mid_{\text{decentralized eq.}} = \left[\frac{1}{1-\tau}\frac{1}{1-r} - 1\right]\frac{E(\theta)}{S(\theta)} = \psi \frac{E(\theta)}{S(\theta)} > 0$$

where

$$\psi(r,\tau) \equiv \frac{1}{1-\tau}\frac{1}{1-r} - 1 = \frac{\tau + \left[1-\tau\right] r}{\left[1-\tau\right]\left[1-r\right]} \simeq \frac{\tau + r}{1-\tau-r} \tag{15}$$

[28] This locus is in Sinn (1995) denoted the redistribution line, and the possibility that the economy is on the downward sloping part is discussed. Note that the locus is drawn for given policies, and that all decentralized equilibria are on the upward sloping part.

i.e., the distortion arises from the tax rate ($\frac{1}{1-\tau} > 1$ for $\tau < 1$) and the replacement rate ($\frac{1}{1-r} > 1$ for $r < 1$) and is magnified by $\frac{E(\theta)}{S(\theta)}$. The distortion arises via mean income only since the social planner takes into account that the scheme is a pure redistribution system, whereas the individual takes benefits and taxes as given. It is seen that the total distortion is created by the product of the tax and benefit distortion, but also that this effect is scaled by the expected value of the shock relative to its standard deviation ($\frac{E(\theta)}{S(\theta)}$). Hence a higher level of risk ($S(\theta)$) reduces the importance of the wedge (ψ). It is an implication that assessments of distortions in a deterministic setting may have an upward bias by neglecting risk effects (See also Andersen (2010)).

4 Policies: incentives vs. insurance

The preceding analysis has clarified individual effort choices and the implied relation between return and risk in equilibrium. The next step is to analyse the relationship between mean income (efficiency) and risk (equity) faced by the policy maker in the choice of the properties of the welfare arrangement (benefit levels and tax rates). The policy choice is addressed in two steps. First, the return-risk locus arising in each policy scenario is considered to clarify how the policy instrument(s) affects mean income and risk/inequality in the economy. Second, the optimal policy choice of a utilitarian policy maker is considered. The latter has the advantage of selecting a particular policy outcome, but at the cost of a specific assumption of policy preferences and a neglect of the political process[29].

In the following we consider first optimal benefits for given tax rates, and next the optimal tax rates for given benefits, and finally the joint determination of benefit levels and tax rates where lump-sum taxes are feasible or not feasible.

4.1 Optimal benefits b for given tax rate τ

Consider first the case where the benefit level is chosen for a given tax rate τ to focus on the direct effect of unemployment benefits for a given financing method ($\tau = 0$ corresponds to fully lump sum financing). We start by working out the trade-off between mean income and risk (distribution) the policy maker faces in deciding on the benefit level. We have that mean income and its risk can be written

$$\mu \equiv \widehat{\mu}(\widehat{e}, w, \widehat{e}(\theta)) = wm(\widehat{e})E(\theta) - \widehat{e} \tag{16}$$

$$\sigma \equiv \widehat{\sigma}(\widehat{e}, w, S(\theta), \tau, b) = [1-\tau]\,[w-b]\,m(\widehat{e})S(\theta) \tag{17}$$

$$\widehat{e} = F(w, \tau, b, E(\theta), S(\theta)) \tag{18}$$

[29] Note that since agents are ex-ante alike, this may be less problematic in the present setting.

It follows straightforwardly that

$$\begin{aligned}\frac{\partial \mu}{\partial b} &= \frac{\partial \widehat{\mu}}{\partial e}\frac{\partial \widehat{e}}{\partial b} = [wm_e(\widehat{e})E(\theta) - 1]\,\frac{\partial \widehat{e}}{\partial b} \\ \frac{\partial \sigma}{\partial b} &= \frac{\partial \widehat{\sigma}}{\partial e}\frac{\partial \widehat{e}}{\partial b} + \frac{\partial \widehat{\sigma}}{\partial b} = [1-\tau]\,S(\theta)\left[-m(\widehat{e}) + [w-b]\,m_e(\widehat{e})\frac{\partial \widehat{e}}{\partial b}\right]\end{aligned}$$

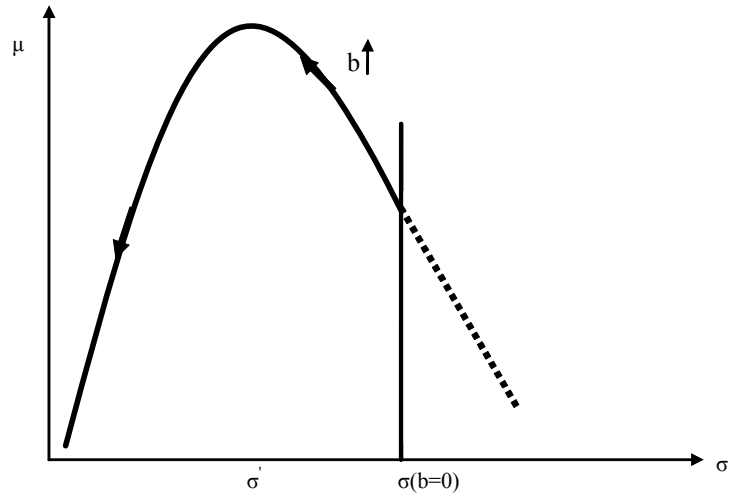
Lemma 4 $\frac{\partial \widehat{\mu}}{\partial b} \lesseqgtr 0$ *for* $S \lesseqgtr \overline{S}^b$, *and* $\frac{\partial \widehat{\sigma}}{\partial b} < 0$

Proof. From Appendix C it follows that $\frac{\partial \widehat{e}}{\partial b} = sign\ \Phi_b$, where

$$\Phi_b \equiv -m'(\widehat{e})\,[1-\tau]\left[[E(\theta) - h(\mu,\sigma)S(\theta)] + [w-b]\,S(\theta)\left[h_\sigma \frac{\partial \widehat{\sigma}}{\partial b}\right]\right] \qquad (19)$$

First, $\frac{\partial \sigma}{\partial b} < 0$ is proved by contradiction. Assume $\frac{\partial \sigma}{\partial b} > 0$ which requires $\frac{\partial \widehat{e}}{\partial b} > 0$. $\frac{\partial \sigma}{\partial b} > 0$ implies $\Phi_b < 0$ which in turn implies $\frac{\partial \widehat{e}}{\partial b} < 0$, hence a contradiction. Note that (see Appendix B) $wm_e(\widehat{e})E(\theta) - 1 > 0$. For $S(\theta) < \overline{S}^b$, we have $\frac{\partial \widehat{e}}{\partial b} < 0$, hence $\frac{\partial \mu}{\partial b} < 0$ and $\frac{\partial \widehat{e}}{\partial b} > 0$ for $S(\theta) > \overline{S}^b$. Let $\overline{\sigma}^b$ denote the standard deviation for income corrsponding to $S(\theta) = \overline{S}^b$. ■

Figure 2: Return-Risk locus available when setting the benefit level



The opportunity line or relation between expected income and risk available when setting the benefit level is as shown in figure 2. Note that the relevant part of the locus depends on the underlying level of risk, and the negatively sloped part (and thus the hump-shape) is only relevant if $\sigma(b=0) > \sigma'$. This

condition can also be written as[30]

$$E(\theta) - h(\mu,\sigma)S(\theta)\left[1 + \frac{h_\sigma \sigma}{h}\right] < 0$$

which is more likely to hold for a given expected return ($E(\theta)$), the higher the implicit price of risk ($h(\mu,\sigma)$), the underlying risk ($S(\theta)$) and the sensitivity of the risk premium to risk ($\frac{h_\sigma \sigma}{h}$). For $\sigma(b=0) < \sigma'$ only the upward sloping part of the locus is relevant, and we have a standard trade-off between expected income and risk.

The non-monotone relation is particularly interesting since it implies that it is possible over some range to increase both expected income and reduce risk by increasing the benefit level. It is important to note that despite their similar look the non-monotone relation in figure 1 (variations in effort level) and 2b (variation in benefit level) are not showing the same thing. One important difference is that the top point in figure 1a corresponds to the unconstrained social optimal effort level (e^{**}) and therefore all effort levels arising in a decentralized equilibrium are on the upward sloping segment of the curve in figure 1a. This is not the case in figure 2 where the non-monotonicity arises via the effort response to a change in policy. Another important difference of empirical importance is that the non-monotone relation in figure 1a only applies to full income (income less disutility of effort), while the hump shaped relation in figure 2a applies both to full income (as depichted) and to market income ($wm(e)E(\theta)$). The insurance effect implies that the relation between expected income and risk/inequlity may be flat or even hump-shaped. In the latter case it suggests that "low" and "high" benefit levels may be associated with the same mean income, but obviously different levels of risk/inequality. This may be suggestive in interpreting why countries with very dissimilar social safety nets have rather similar economic performance assessed on the basis of average indicators.

Optimal policy - utilitarian policy maker

The optimal utilitarian choice of the benefit level is given as the solution to the problem

$$Max_b \quad V(\mu,\sigma)$$

given (16) and (17). The first order condition reads

$$\frac{\frac{\partial \mu}{\partial b}}{\frac{\partial \sigma}{\partial b}} = h(\mu,\sigma)$$

[30] The requirement is that $\Phi_b < 0$ for $b = 0$. We have (see proof of lemma 4) that evaluated for $b = 0$ that

$$\begin{aligned} sign\ \Phi_b &\equiv -sign\left[[E(\theta) - h(\mu,\sigma)S(\theta)] + [w-b]\,S(\theta)\left[h_\sigma \frac{\partial \widehat{\sigma}}{\partial b}\right]\right] \\ &= -sign\left[[E(\theta) - h(\mu,\sigma)S(\theta)] - wS(\theta)h_\sigma\,[1-\tau]\,m(e)S(\theta)\right] \\ &= -sign\left[E(\theta) - h(\mu,\sigma)S(\theta)\left[1 + \frac{h_\sigma \sigma}{h}\right]\right] \end{aligned}$$

or

$$-\frac{\partial \mu}{\partial b} = -\frac{\partial \sigma}{\partial b} h\left(\mu, \sigma\right) \tag{20}$$

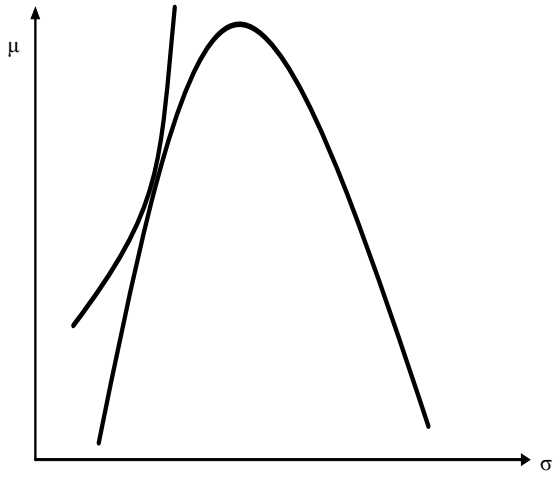
The lhs of (20) can be interpreted as the marginal costs of providing benefits measured by the change in expected income (the incentive effect). The rhs gives the marginal benefits as the change in risk (the insurance effect) multiplied by the price of risk.

Proposition 5 *The optimal benefit level is always determined at a point where there is a trade-off between insurance and incentives on the margin, i.e.* $\frac{\partial \sigma}{\partial b} < 0$ *and* $\frac{\partial \mu}{\partial b} < 0$.

Proof. We have that $h > 0$ and since $\frac{\partial \sigma}{\partial b} < 0$, cf proposition 2, it follows that $\frac{\partial \mu}{\partial b} < 0$ which also implies $\frac{\partial \widehat{e}}{\partial b} < 0$. ■

This result is trivial if the return-risk locus is monotone[31]. In the non-monotone case we have a segment where higher benefits increase mean income and lowers risk. However, the optimal policy always implies that marginal changes in the benefit level involves a trade-off between mean income (efficiency) and risk (equity). Figure 3 illustrates in the case of a non-monotone return-risk locus.

Figure 3: The risk return locus and optimal benefits



The condition determining the optimal benefit level (10) can be written in a more easily interpretable way as (see Appendix D)

$$\psi(r, \tau) = \frac{h(\mu, \sigma)}{\varepsilon\left(\widehat{e}\right)} \frac{S(\theta)}{E(\theta)} \tag{21}$$

[31] Note that for $\sigma(b = 0) < \sigma'$ a corner solution is possible where $b = 0$, this is the case if for $b = 0$ we have $-\frac{\partial \mu}{\partial b} > -\frac{\partial \sigma}{\partial b} h\left(\mu, \sigma\right)$.

where $\psi(r,\tau)$ is given from (15) and it is increasing in both r and τ, and

$$\varepsilon\left(\widehat{e}\right) \equiv -\frac{\partial m(\widehat{e})}{\partial \widehat{e}} \frac{\widehat{e}}{m(\widehat{e})} \frac{\partial \widehat{e}}{\partial\left[1-r\right]} \frac{1-r}{\widehat{e}} > 0$$

Note that $\epsilon\left(\widehat{e}\right)$ measures the elasticity of employment wrt the net gain from work (one minus the replacement ratio), where $\psi(r,\tau)$ is capturing the distortion of job search from the benefit level. In equation (21) note that the LHS is increasing in r (τ is exogenous), and hence an increase in the price of risk ($h(\mu,\sigma)$) or the level of risk ($S(\theta)$), tends to imply a higher optimal replacement rate, and the higher the elasticity of employment ($\varepsilon\left(\widehat{e}\right)$) and the expected return ($E(\theta)$), the lower the optimal replacement rate. In this way the optimal replacement rate balances the insurance effect ($h(\mu,\sigma)S(\theta)$) and the incentive effect ($\varepsilon\left(\widehat{e}\right)E(\theta)$). Note that (21) is not a closed form solution since both $h(\mu,\sigma)$ and $\varepsilon\left(\widehat{e}\right)$ depend on the replacement rate, and the interpretation given here is therefore only suggestive.

An interesting question is how the optimal benefit level is affected by the underlying risk. Intuitively as explained above, it is to be expected that more risk leads to more need for insurance and thus a higher benefit level. However, the answer is more complicated, since

$$\frac{\partial \sigma}{\partial S(\theta)} = \left[1-\tau\right]\left[w-b\right] m(\widehat{e}) \left[1 + \frac{m_e(\widehat{e})\widehat{e}}{m(\widehat{e})} \frac{\partial \widehat{e}}{\partial S(\theta)} \frac{S(\theta)}{\widehat{e}}\right] \lesseqgtr 0$$

More risk has an ambiguous effect on income risk (σ). The direct effect of an increase in the underlying risk ($S(\theta)$) is to increase income risk, but this is counteracted by the fact that effort is lowered as a reponse to the increase in risk ($\frac{\partial \widehat{e}}{\partial S(\theta)} < 0$). Hence if the effort response is sufficiently strong, it is possible that total risk declines, but under the following assumption

Assumption 2: $\frac{\partial \widehat{e}}{\partial S(\theta)} \frac{S(\theta)}{\widehat{e}} > -\left[\frac{m_e(\widehat{e})\widehat{e}}{m(\widehat{e})}\right]^{-1}$

we have that $\frac{\partial \sigma}{\partial S(\theta)} > 0$.

Proposition 6 *With an optimal benefit level for a given tax rate, (i) An increase in expected return ($E(\theta)$) unambigously leads to an increase in expected/average income (μ) and in risk/inequality (σ). The effect on the optimal benefit level is ambiguous,(ii) An increase in risk ($S(\theta)$) unambiguously leads to a fall in expected/average income (μ) but has an ambiguous effect on risk/inequality (σ). Under assumption 2, a sufficient condition that the optimal benefit level increases with more risk ($\frac{\partial b}{\partial S(\theta)} > 0$) is that $\frac{\partial \varepsilon(\widehat{e})}{\partial S(\theta)} \leq 0$*

Proof. See Appendix D. ■

The response of the optimal benefit rate to changes in either expected return ($E(\theta)$) or risk ($S(\theta)$) obviously depends both on how these changes affect the marginal rate of substitution between risk and expected income ($\frac{\partial \mu/\partial b}{\partial \sigma/\partial b}$) and the price of risk ($h(\mu,\sigma)$). Again an important difference arises in how the price of

risk is affected[32]. For a change in the expected return there is an ambgiuous effect, since

$$\frac{\partial h(\mu,\sigma)}{\partial E(\theta)} = h_\mu(\mu,\sigma)\frac{\partial \mu}{\partial E(\theta)} + h_\sigma(\mu,\sigma)\frac{\partial \sigma}{\partial E(\theta)} \lesseqgtr 0$$

where the first term is negative, capturing that higher expected income reduces the price of risk, but the implied higher exposure to market risk (the second term) goes in the opposite direction. For an increase in risk ($S(\theta)$) we have that the price of risk unambiguously increases (under Assumption 2), i.e.,

$$\frac{\partial h(\mu,\sigma)}{\partial S(\theta)} = h_\mu(\mu,\sigma)\frac{\partial \mu}{\partial S(\theta)} + h_\sigma(\mu,\sigma)\frac{\partial \sigma}{\partial S(\theta)} > 0$$

Hence, a sufficient condition that more risk leads to a higher benefit level (replacement rate) is that the employment does not become more sensitive to the gain from work ($\frac{\partial \varepsilon(\widehat{e})}{\partial S(\theta)} \leq 0$).

4.2 Choosing the income tax rate τ for given benefits b

Consider next the choice of the tax rate for given benefits, and the budget is balanced via changes in the lump sum tax (observe that in the special case where $b = 0$, we have a pure income transfer scheme, cf. Sinn (1995)). This case is interesting in its own right because it addresses the question of whether it is optimal to use an income tax to finance unemployment benefits given that lump sum taxation is available.

The return-risk choice set

We start by working out the choice set in terms of mean income and risk (distribution) that the policy maker faces when deciding on the income tax rate for a given benefit level. It follows from (11) and (12) that

$$\begin{aligned}
\frac{\partial \widehat{\mu}}{\partial \tau} &= [m_e(\widehat{e})E(\theta)w - 1]\frac{\partial \widehat{e}}{\partial \tau} \\
\frac{\partial \widehat{\sigma}}{\partial \tau} &= [w-b]\,S(\theta)\left[-m(\widehat{e}) + [1-\tau]\,m_e(\widehat{e})\frac{\partial \widehat{e}}{\partial \tau}\right]
\end{aligned}$$

where (see Appendix B) $m_e(\widehat{e})E(\theta)w - 1 > 0$.

Lemma 7 *$\frac{\partial \widehat{\mu}}{\partial \tau} \lesseqgtr 0$ for $S \lesseqgtr \overline{S}_\tau$, and $\frac{\partial \widehat{\sigma}}{\partial \tau} < 0$*

Proof. *The proof is by contradiction. Assume $\frac{\partial \widehat{\sigma}}{\partial \tau} > 0$ which requires $\frac{\partial \widehat{e}}{\partial \tau} > 0$ which in turn requires*

$$-\left[[E(\theta) - h(\widehat{\mu},\widehat{\sigma})S(\theta)] + [1-\tau]\,S(\theta)h_\sigma\frac{\partial \widehat{\sigma}}{\partial \tau}\right] > 0$$

[32] As shown in Appendix E, there is also a qualitative difference in respect to the effects on the marginal rate of substitution between expected income and risk ($\frac{\partial \mu/\partial b}{\partial \sigma/\partial b}$). While $\frac{\partial \eta}{\partial S(\theta)} \leq 0$ is sufficient to imply that $\frac{\partial \mu/\partial b}{\partial \sigma/\partial b}$ is decreasing in $S(\theta)$, $\frac{\partial \eta}{\partial E(\theta)} \geq 0$ is not sufficient to imply that it is increasing in $E(\theta)$.

However, this inequality is violated for $\frac{\partial \widehat{\sigma}}{\partial \tau} > 0$*, hence a contradiction. Note* $[E(\theta) - h(\widehat{\mu}, \widehat{\sigma})S(\theta)] > 0$ ■

Accordingly, in choosing the tax rate, the policy maker qualitatively faces the same situation as in choosing the benefit rate, cf figure 2, since the return-risk relation may be either monotoneoulsy increasing or hump-shaped, and where a position further to the left along the curve is associated with a lower tax rate.

Note that a tax increase has an ambiguous effect on the dispersion in pre-tax income,

$$\frac{\partial \left[[w-b]\, S(\theta) m(e) \right]}{\partial \tau} = [w-b]\, S(\theta) m'(e) \frac{\partial e}{\partial \tau} \lesseqgtr 0.$$

However, dispersion in post-tax income always decreases ($\frac{\partial \sigma}{\partial \tau} < 0$). This rules out the redistribution paradox by Sinn (1995) that tax increases lead to more post-tax inequality via more risk taking.

Optimal policy - utilitarian policy maker

To address this question, note that the optimal tax rate maximising $V(\mu, \sigma)$ is characterized by the first order condition

$$V_\mu \left[\frac{\partial \widehat{\mu}}{\partial \tau} - h(\widehat{\mu}, \widehat{\sigma}) \frac{\partial \widehat{\sigma}}{\partial \tau} \right] = 0$$

implying that

$$\frac{\frac{\partial \widehat{\mu}}{\partial \tau}}{\frac{\partial \widehat{\sigma}}{\partial \tau}} = h(\widehat{\mu}, \widehat{\sigma}).$$

We have

Proposition 8 *The tax rate is positive* ($\tau > 0$) *and always determined at a point where there is a trade-off between insurance and incentives on the margin, i.e.,* $\frac{\partial \widehat{\sigma}}{\partial b} < 0$ *and* $\frac{\partial \widehat{\mu}}{\partial b} < 0$ *provided* $b < \underline{b}$.

Proof. The first part follows directly from the fact that $h > 0$ and $\frac{\partial \sigma}{\partial b} < 0$. The second part follows by noting that

$$\begin{aligned} \frac{\partial \widehat{\mu}}{\partial \tau} &= [w m_e(\widehat{e}) E(\theta) - 1] \frac{\partial \widehat{e}}{\partial \tau} \\ \frac{\partial \widehat{\sigma}}{\partial \tau} &= [w-b]\, S(\theta) \left[-m(\widehat{e}) + [1-\tau]\, m_e(\widehat{e}) \frac{\partial \widehat{e}}{\partial \tau} \right] \end{aligned}$$

Implying that

$$\begin{aligned} \frac{\partial \widehat{\mu}}{\partial \tau} - h \frac{\partial \widehat{\sigma}}{\partial \tau} &= [w m_e(\widehat{e}) E(\theta) - h\, [w-b]\, S(\theta) m_e(\widehat{e}) - 1] \frac{\partial \widehat{e}}{\partial \tau} + h\, [w-b]\, S(\theta) m(\widehat{e}) \text{ for } \tau = 0 \\ &= [b m_e(\widehat{e}) E(\theta)] \frac{\partial \widehat{e}}{\partial \tau} + h\, [w-b]\, S(\theta) m(\widehat{e}) \text{ for } \tau = 0 \end{aligned}$$

which is positive for $b = 0$ and hence there exists a $\underline{b}$ such that $\tau > 0$ for $b < \underline{b}$.
■

Although lump sum taxation is assumed to be feasible, it is interesting to note that the optimal tax is positive, and the reason is the insurance effect. The result here resembles the one found for the optimal benefit level (for given tax rate) but has the interesting additive feature that it shows that it is optimal to include a (linear) income tax in the financing of unemployment benefits, provided that the benefits are not too large.

4.3 Optimal benefit rate b and tax rate τ with lump sum taxation

In the present setting there is a close complementarity between the tax rate and the benefit level when lump sum taxation is feasible. To see this, note that both individual effort choice (see (10)) and the income risk (see (5)) depend on the composite variable

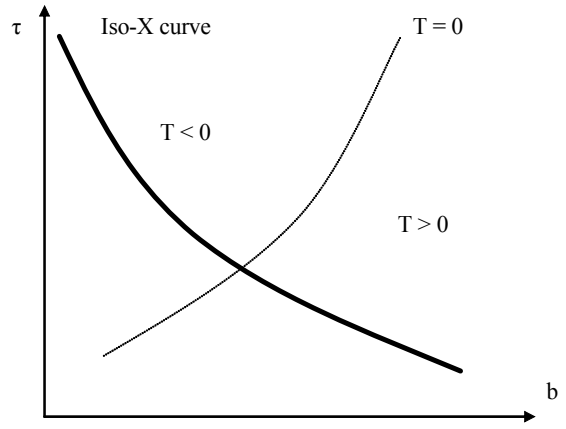
$$\chi \equiv [1-\tau]\,[1-r]$$

hence combinations of τ and r leaving χ unchanged imply the same effort level as well as the same expected income (μ) and risk of income (σ). Therefore they are equivalent in utility terms. However, the particular combinations of τ and r have different implications for the public budget, and may imply that the lump-sum tax is either positive or negative (a lump sum subsidy). Combinations of τ and b leaving $T = 0$ are from (1) given by

$$\tau w m(\widehat{e})E(\theta) = [1 - m(\widehat{e})E(\theta)]\,[1-\tau]\,b \tag{22}$$

Figure 4 depichts an iso χ curve in (b, τ) space as well as the $T = 0$ locus.

Figure 4: Benefits and tax rates with lump-sum taxes/transfers



Hence depending on the policy constellation the outcome may imply either a lump sum tax or a lump sum subsidy but leave effort, expected income and income risk unaffected. This points to a basic equivalence between a tax-transfer

scheme and a benefit-tax scheme when lump sum taxation is feasible. An unemployment insurance scheme provides support to the unemployed (low income) effectively by taxing the employed (high income). A pure tax-transfer scheme provides a lump sum transfer to everybody financed by taxes proportional to market income, and therefore effectively makes transfers from high income to low income groups. Hence, when the budget can be balanced by a lump-sum tax, it follows that the same level of insurance can be attained by either instrument.

4.4 Optimal benefit rate b and tax rate τ without lump sum taxation

In the preceding analyses, lump-sum financing plays an important role, and it is therefore of interest to consider the case where the unemployment benefits are entirely financed by an income tax (imposing the requirement $T = 0$). If we take the benefit level to be the policy instrument, it follows that the tax rate is determined from the public sector budget constraint (1), which implies that the tax rate is given as

$$\tau = \tau\left(e, b\right) \equiv \frac{\left[1 - m(e)E(\theta)\right] b}{wm(e)E(\theta) + b\left[1 - m(e)E(\theta)\right]} \quad ; \frac{\partial \tau}{\partial e} < 0, \frac{\partial \tau}{\partial r} > 0$$

Hence individual effort is determined by

$$\Phi(e, w, b, \tau, E(\theta), S(\theta)) \equiv m_e(e)\left[1 - \tau\left(e, b\right)\right]\left[w - b\right]\left[E(\theta) - h(\mu, \sigma)S(\theta)\right] - 1 = 0.$$

Equilibrium effort $\widetilde{e}$ in this policy regime is given as

$$\widetilde{e} = G\left(b, w, E(\theta), S(\theta)\right)$$

Hence, mean income and its standard deviation are determined as

$$\begin{aligned} \widetilde{\mu} &= m(\widetilde{e})E(\theta)w \quad - \widetilde{e} \\ \widetilde{\sigma} &= \left[1 - \tau(e, b)\right]\left[w - b\right] m(\widetilde{e})S(\theta) \end{aligned}$$

It follows that both the effect on expected income and risk are ambiguous, i.e.,

$$\begin{aligned} \frac{\partial \widetilde{\mu}}{\partial b} &= \left[m_e(\widetilde{e})E(\theta)w \quad - 1\right] \frac{\partial \widetilde{e}}{\partial b} \gtreqless 0 \\ \frac{\partial \widetilde{\sigma}}{\partial b} &= \left[-\tau_b\left[w - b\right] m(\widetilde{e}) - \left[1 - \tau(e, b)\right]\left[m(\widetilde{e}) - \left[w - b\right] m_e(\widetilde{e})\frac{\partial \widetilde{e}}{\partial b}\right]\right] S(\theta) \gtreqless 0 \end{aligned}$$

Proposition 9 *$\frac{\partial \widetilde{\mu}}{\partial b} \lesseqgtr 0$ for $S \lesseqgtr \widetilde{S}_b$, and $\frac{\partial \widetilde{\sigma}}{\partial b} < 0$*

Proof. Note first that $sign \frac{\partial \widetilde{e}}{\partial b} = sign\ \Phi_b$ where

$$\Phi_b = -\left[m_e(e)\left[1 - \tau\right] + \tau_b(w - b)\right]\left[E(\theta) - h(\widehat{\mu}, \widehat{\sigma})S(\theta)\right] + m_e(e)\left[1 - \tau\right]\left[w - b\right] h_\sigma S(\theta)\frac{\partial \widetilde{\sigma}}{\partial b}$$

The proof that $\frac{\partial \widetilde{\sigma}}{\partial b} < 0$ is by contradiction. Assume that $\frac{\partial \widetilde{\sigma}}{\partial b} < 0$ this requires $\frac{\partial \widetilde{e}}{\partial b} > 0$, however, from the expression for Φ_b it follows that $\frac{\partial \widetilde{e}}{\partial b} < 0$, and hence a contradiction. $\widetilde{S}_b$ denotes the level of $S(\theta)$ for which $\frac{\partial \widetilde{\mu}}{\partial b} = 0$. ■

It follows immediately that the same qualitative results carry over to this case of a unemployment benefit financed by an income tax. In particular that the return-risk locus may be hump-shaped.

Optimal policy - utilitarian policy maker

The optimality condition for the benefit level is

$$\frac{\partial \widetilde{\mu}}{\partial \tau} - h(\widehat{\mu}, \widehat{\sigma}) \frac{\partial \widetilde{\sigma}}{\partial \tau} = 0 \tag{23}$$

For completeness we thus have

Proposition 10 *The optimal benefit level fully financed by an income tax is always determined at a point where there is a trade-off between insurance and incentives on the margin, i.e.,* $\frac{\partial \widetilde{\sigma}}{\partial \tau} < 0$ *and* $\frac{\partial \widetilde{\mu}}{\partial \tau} < 0$.

Finally note (See Appendix) that the optimality condition reads

$$\frac{[\tau + [1-\tau]\, r]}{[1-\tau]\,[1-r]} = \frac{h}{\varepsilon(\widetilde{e})} \frac{S(\theta)}{E(\theta)} \left[\frac{\tau_b b}{\tau} \frac{\tau}{[1-\tau]} [r-1] + 1 \right] \tag{24}$$

The only difference between (24) and the similar condition (21) for the case of a constant tax rate is that the marginal insurance value of an increase in the benefit level (the RHS of (24)) now also takes into account that this leads to a tax increase which in turn also has an insurance effect ($\tau_b > 0$).

5 Concluding remarks

In an unemployment insurance scheme with collective risk sharing, it has been shown that the insurance effect mitigates the standard incentive effects. This supports the reasoning underlying the flexicurity debate that unemployment insurance (in a labour market with flexible hiring and firing rules) up to some point may be beneficial for job search and thus labour market performance. The basic reason is that unemployment benefits in combinination with its financing via taxes reduces the risk associated with job search. This implies that invididual responses to benefit levels and tax rates are determined by counteracting incentive and insurance effects. At the ecoonomy wide level this implies that the relation between efficiency and equity being faced in deciding on the policy instruments (benefit level, tax rate) may be relatively flat or even non-monotone, that is, making the system more generous may up to some point improve both efficiency and equity. The intuition for this result is that the insurance effect of these policy instruments over this interval dominates the standard incentive effects. While this dominance result is possible, it is more interesting to note that the insurance effect always mitigates the incentive effect. In standard models of

labour supply it is well-known that ambiguities arises due to the interplay between substitution and income effects. In the present setting there is no income effect by definition since the benefit-tax scheme is a pure redistribution scheme. In this sense the substitution or standard incentive effect has it highest influence, and yet there is an insurance effect running in the opposite direction. This suggest that low estimated elasticities based on deterministic models may arise due to a failure to account separately for the incentive and insurance effects.

While a non-monotone relationship between efficiency and equity in setting the benefit-tax system suggests that there over some interval is no policy conflict between efficiency and equity, it is important to note that optimal (utilitarian) policies always imply that there is a trade-off between efficiency and equity on the margin. The intuition for this result is simply that otherwise there would be an unexploited scope to improve welfare.

The model of this paper was purposely chosen to be very simple to allow a focus on the basic effects. Interesting items on the list for future extensions include wage formation and multi-period settings.

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Appendix A

The following gives a two period example to show that the basic risk reducing effect of unemployment benefits also arises in this setting.

Let income be $y = w$ if employed and $y = b$ if unemployed. The probability of finding a job in the first period is $m(e)$ where e is search effort ($m_e > 0, m_{ee} < 0$). If employed in the first period, there is a probability p that the job is terminated after one period. If either unemployed in the first period or laid off after one period, the worker may find a job in period two for a job with probability n (exogenous). Hence, at the end of period 1, the expected period 2 income is w if not laid-off, and $nw + (1-n)b$ if laid-off. The period one problem is thus

$$Max_{e_1} \quad V = n(e)U(I^e) + (1 - n(e))U(I^u) - e$$

where

$$\begin{aligned} I^e &= w + R(1-p)w + Rp\left[nw + (1-n)b\right] \\ I^u &= b + R\left[nw + (1-n)b\right] \end{aligned}$$

and R is the discount factor. The first order condition for the optimal effort choice is

$$= n_e(e)\left[u(I^e) - u(I^u)\right] = 1$$

and the second order condition is $\Gamma_e < 0$. Hence

$$sign\frac{\partial e_1}{\partial b} = sign\Gamma_b$$

where

$$\Gamma_b = n_e(e)\left[u_I(I^e)Rp(1-n)b - u_I(I^u)\left(1 + R(1-n)\right)\right] \lesseqgtr 0.$$

It is seen that there is an ambiguity due to the interplay between the insurance offered by lowering the risk exposure if a job is found, and the incentive effect of the benefit provided.

Appendix B: Individual effort choice

The first order condition to the optimal effort choice reads

$$\Theta(e, w, b, \tau, T, E(\theta), S(\theta)) \equiv V_\mu\left[m_e(e)\left[1-\tau\right]\left[w-b\right]\left[E(\theta) - h(\mu,\sigma)S(\theta)\right] - 1\right] = 0,$$

where

$$\begin{aligned} \mu &= E(y) = \left[1-\tau\right]b + m(e)E(\theta)\left[1-\tau\right]\left[w-b\right] - T - e \\ \sigma &= \left[1-\tau\right]\left[w-b\right]m(e)S(\theta). \end{aligned}$$

The second order condition reads

$$\Theta_e = \left[1-\tau\right]\left[w-b\right]V_\mu\left[m_{ee}(e)\left[E(\theta) - h(\mu,\sigma)S(\theta)\right] - m_e(e)S(\theta)\left[h_\mu\frac{\partial\mu}{\partial e} + h_\sigma\frac{\partial\sigma}{\partial e}\right]\right] < 0$$

and is assumed to hold.

First, note that $e \to 0$ implies that $\mu \to 0$, and $\sigma \to 0$ implying that $h \to 0$. Hence $e > 0$ if $m_e(0)\left[1-\tau\right]\left[w-b\right]E(\theta) > 1$.

Using the second order condition, it follows that

$$sign\frac{\partial e}{\partial z} = sign\ \Theta_z \text{ for } z = w, b, \tau, E(\theta), S(\theta)$$

where

$$\begin{aligned}
\Theta_T &= V_\mu m_e(e)\left[1-\tau\right]\left[w-b\right]S(\theta)h_\mu \\
\Theta_S &= V_\mu m_e(e)\left[1-\tau\right]\left[w-b\right]\left[-h - h_\sigma\frac{\partial\sigma}{\partial S}\right] \\
\Theta_E &= V_\mu m_e(e)\left[1-\tau\right]\left[w-b\right]\left[1 - h_\mu(\mu,\sigma)\frac{\partial\mu}{\partial E(\theta)}S(\theta)\right] \\
\Theta_w &= V_\mu m_e(e)\left[1-\tau\right]\left[\left[E(\theta) - h(\mu,\sigma)S(\theta)\right] - \left[w-b\right]S(\theta)\left[h_\mu\frac{\partial\mu}{\partial w} + h_\sigma\frac{\partial\sigma}{\partial w}\right]\right] \\
\Theta_\tau &= -V_\mu m_e(e)\left[w-b\right]\left[\left[E(\theta) - h(\mu,\sigma)S(\theta)\right] + \left[1-\tau\right]S(\theta)\left[h_\mu\frac{\partial\mu}{\partial\tau} + h_\sigma\frac{\partial\sigma}{\partial\tau}\right]\right] \\
\Theta_b &= -V_\mu m_e(e)\left[1-\tau\right]\left[\left[E(\theta) - h(\mu,\sigma)S(\theta)\right] + \left[w-b\right]S(\theta)\left[h_\mu\frac{\partial\mu}{\partial b} + h_\sigma\frac{\partial\sigma}{\partial b}\right]\right]
\end{aligned}$$

It follows immediately that (i) lump sum tax: $sign\ \frac{\partial e}{\partial T} = sign\ h_\mu$, (ii) Expected value: $h_\mu \leq 0$ is a sufficient condition that $\frac{\partial e}{\partial E(\theta)} > 0$, (ii) Risk: $h_\sigma \geq 0$ is a sufficient condition that $\frac{\partial e}{\partial S(\theta)} < 0$. (iv) Wage rate: Note that $\frac{\partial\mu}{\partial w} > 0$ and $\frac{\partial\sigma}{\partial w} > 0$ and hence $h_\mu\frac{\partial\mu}{\partial w} + h_\sigma\frac{\partial\sigma}{\partial w} \lesseqgtr 0$. Considering Θ_w we have that this expression is positive under the sufficient condition that $\left[h_\mu\frac{\partial\mu}{\partial w} + h_\sigma\frac{\partial\sigma}{\partial w}\right] \leq 0$. For Θ_w to become negative requires as necessary conditions $h_\mu\frac{\partial\mu}{\partial w} + h_\sigma\frac{\partial\sigma}{\partial w} > 0$ and that this terms is weighted sufficiently strongly i.e. $S(\theta) > \overline{S}^w$. (v) Tax rate: Note that $\frac{\partial\mu}{\partial\tau} < 0$ and $\frac{\partial\sigma}{\partial\tau} < 0$ and hence $h_\mu\frac{\partial\mu}{\partial\tau} + h_\sigma\frac{\partial\sigma}{\partial\tau} \lesseqgtr 0$. We have that $\Theta_\tau < 0$ under the sufficient condition $h_\mu\frac{\partial\mu}{\partial\tau} + h_\sigma\frac{\partial\sigma}{\partial\tau} \geq 0$. For Θ_τ to become positive requires as necessary conditions $h_\mu\frac{\partial\mu}{\partial\tau} + h_\sigma\frac{\partial\sigma}{\partial\tau} < 0$ and that this terms is weighted sufficiently strongly i.e., $S(\theta) > \overline{S}^\tau$. (vi) benefit level: Note that $\frac{\partial\mu}{\partial b} > 0$ and $\frac{\partial\sigma}{\partial b} > 0$ and hence $h_\mu\frac{\partial\mu}{\partial b} + h_\sigma\frac{\partial\sigma}{\partial b} \lesseqgtr 0$. For $\Theta_b < 0$ requires as a sufficient condition that $h_\mu\frac{\partial\mu}{\partial b} + h_\sigma\frac{\partial\sigma}{\partial b} \geq 0$. For Θ_b to become positive requires as necessary conditions $h_\mu\frac{\partial\mu}{\partial b} + h_\sigma\frac{\partial\sigma}{\partial b} < 0$ and that this term is weighted sufficiently strongly i.e., $S(\theta) > \overline{S}^b$.

Appendix C: Equilibrium effort responses

To find the equilibrium effort we use the foc for the optimal effort level in equilibrium $\widehat{e}$ satisfies

$$\Phi(\widehat{e}, w, b, \tau, E(\theta), S(\theta)) \equiv V_\mu m_e(\widehat{e})\left[1-\tau\right]\left[w-b\right]\left[E(\theta) - h(\widehat{\mu},\widehat{\sigma})S(\theta)\right] - 1 = 0,$$

assessed for the mean income and its standard deviation found by consolidating budget constraints implying:

$$\begin{aligned}\widehat{\mu}(\widehat{e},w,E(\theta)) &= wm(\widehat{e})E(\theta)-\widehat{e} &(25)\\ \widehat{\sigma}(\widehat{e},w,S(\theta),\tau,b) &= (1-\tau)(w-b)m(\widehat{e})S(\theta) &(26)\end{aligned}$$

Note that

$$\begin{aligned}\frac{\partial\widehat{\mu}}{\partial\widehat{e}} &= wm_e(\widehat{e})E(\theta)-1>0\\ \frac{\partial\widehat{\sigma}}{\partial\widehat{e}} &= (1-\tau)(w-b)m_e(\widehat{e})S(\theta)>0\end{aligned}$$

The first inequality follows from the fact that individual effort falls short of the social optimum (both constraint and unconstrained). Hence, at the level of effort chosen by individuals. it is always the case that higher effort will lead to both high mean income and risk.

For later reference it is useful to write equilibrium search by the implicit function

$$\widehat{e}=F(w,\tau,bE(\theta),S(\theta))$$

It is readily found that

$$\begin{aligned}\Phi_e &= V_\mu m_{ee}(\widehat{e})\left[1-\tau\right]\left[w-b\right]\left[E(\theta)-h(\widehat{\mu},\widehat{\sigma})S(\theta)\right]-m_e(\widehat{e})\left[1-\tau\right]\left[w-b\right]S(\theta)\left[h_\mu\frac{\partial\widehat{\mu}}{\partial e}+h_\sigma\frac{\partial\widehat{\sigma}}{\partial e}\right]\\ \Phi_E &= V_\mu m_e(\widehat{e})\left[1-\tau\right]\left[w-b\right]\left[1-h_\mu S(\theta)wm(\widehat{e})\right]>0\\ \Phi_S &= -V_\mu m_e(\widehat{e})\left[1-\tau\right]\left[w-b\right]\left[h+h_\sigma\frac{\partial\widehat{\sigma}}{\partial S(\theta)}\right]<0\\ \Phi_w &= V_\mu m_e(\widehat{e})\left[1-\tau\right]\left[\left[E(\theta)-h(\widehat{\mu},\widehat{\sigma})S(\theta)\right]-\left[w-b\right]S(\theta)\left[h_\mu\frac{\partial\widehat{\mu}}{\partial w}+h_\sigma\frac{\partial\widehat{\sigma}}{\partial w}\right]\right]\\ \Phi_\tau &= -V_\mu m_e(\widehat{e})\left[w-b\right]\left[\left[E(\theta)-h(\widehat{\mu},\widehat{\sigma})S(\theta)\right]+\left[1-\tau\right]S(\theta)h_\sigma\frac{\partial\widehat{\sigma}}{\partial\tau}\right]\\ \Phi_b &= -V_\mu m_e(\widehat{e})\left[1-\tau\right]\left[\left[E(\theta)-h(\widehat{\mu},\widehat{\sigma})S(\theta)\right]+\left[w-b\right]S(\theta)h_\sigma\frac{\partial\widehat{\sigma}}{\partial b}\right]\end{aligned}$$

Note that the soc on the individual maximisation problem ($\Theta_e<0$) does not automatically ensure that $\Phi_e<0$. The reason is that the perceived effects of effort on expected income and its risk are different from the ones arising in equilibrium. Note that a sufficient condition that $\Phi_e<0$ is $h_\mu\frac{\partial\widehat{\mu}}{\partial e}+h_\sigma\frac{\partial\widehat{\sigma}}{\partial e}\geq 0$.

First we show that under Assumption 1 we have that $\frac{\partial\widehat{e}}{\partial E(\theta)}>0$ and $\frac{\partial\widehat{e}}{\partial S(\theta)}<0$. The first part follows readily. To see the second part note that

$$\frac{\partial\widehat{\sigma}}{\partial S(\theta)}=\left[1-\tau\right]\left[w-b\right]\left(m(\widehat{e})+m_e(\widehat{e})\frac{\partial\widehat{e}}{\partial S(\theta)}S(\theta)\right) \qquad (27)$$

For $\frac{\partial\widehat{e}}{\partial S(\theta)}<0$ it is required that $h+h_\sigma\frac{\partial\widehat{\sigma}}{\partial S(\theta)}>0$. The proof of this is by contradiction. Assume that $h+h_\sigma\frac{\partial\widehat{\sigma}}{\partial S(\theta)}<0$ which requires $\frac{\partial\widehat{\sigma}}{\partial S(\theta)}<0$ and

implies $\frac{\partial \widehat{e}}{\partial S(\theta)} > 0$, but this implies from (27) that $\frac{\partial \widehat{\sigma}}{\partial S(\theta)} > 0$ and hence a contradiction.

Considering the effects of wages, benefit and taxes on equilibrium effort we have

$$\begin{aligned} sign\frac{\partial \widehat{e}}{\partial w} &= sign\ \Phi_w \\ sign\frac{\partial \widehat{e}}{\partial \tau} &= sign\ \Phi_\tau \\ sign\frac{\partial \widehat{e}}{\partial b} &= sign\ \Phi_b \end{aligned}$$

We have that

$$\Phi_w = m_e(\widehat{e})\left[1-\tau\right]\left[\left[E(\theta)-h(\widehat{\mu},\widehat{\sigma})S(\theta)\right]-\left[w-b\right]S(\theta)\left[h_\mu\frac{\partial \widehat{\mu}}{\partial w}+h_\sigma\frac{\partial \widehat{\sigma}}{\partial w}\right]\right] \lesseqgtr 0$$

Note that $h_\mu\frac{\partial \widehat{\mu}}{\partial w}+h_\sigma\frac{\partial \widehat{\sigma}}{\partial w} \lesseqgtr 0$. For $S(\theta)=0$ we have $\Phi_w > 0$ implying that there exists a $\widehat{S}^w$ such that $\Phi_w > 0$ for $S(\theta) < \widehat{S}^w$.

We have that

$$\Phi_\tau \lesseqgtr 0 \ \text{ for } \left[E(\theta)-h(\widehat{\mu},\widehat{\sigma})S(\theta)\right]+\left[1-\tau\right]S(\theta)h_\sigma\frac{\partial \widehat{\sigma}}{\partial \tau} \gtreqless 0$$

where $\left[E(\theta)-h(\widehat{\mu},\widehat{\sigma})S(\theta)\right] > 0$ and $\frac{\partial \widehat{\sigma}}{\partial \tau} = -\left[w-b\right]m(\widehat{e})S(\theta) < 0$. Clearly $\Phi_\tau < 0$ for $S(\theta)=0$ and hence it follows that there exists a $\widehat{S}^\tau$ such that $\Phi_\tau \lesseqgtr 0$ for $S(\theta) \lesseqgtr \widehat{S}^\tau$. Similarly we have that

$$\Phi_b \lesseqgtr 0 \ \text{ for } \left[E(\theta)-h(\widehat{\mu},\widehat{\sigma})S(\theta)\right]-\left[w-b\right]S(\theta)h_\sigma\frac{\partial \widehat{\sigma}}{\partial b} \gtreqless 0$$

where $\left[E(\theta)-h(\widehat{\mu},\widehat{\sigma})S(\theta)\right] > 0$ and $\frac{\partial \widehat{\sigma}}{\partial b} = -\left[1-\tau\right]m(\widehat{e})S(\theta) < 0$. Clearly $\Phi_b < 0$ for $S(\theta)=0$ and hence it follows that there exists an $\widehat{S}^b$ such that $\Phi_b \lesseqgtr 0$ for $S(\theta) \lesseqgtr \widehat{S}^b$.

Appendix D: Optimal policies and comparative statics

Policy instrument is b and the aim is to maximize

$$V(\mu,\sigma)$$

where

$$\begin{aligned} \mu &\equiv \widehat{\mu}(\widehat{e},w,E(\theta)) = wm(\widehat{e})E(\theta)-\widehat{e} \\ \sigma &\equiv \widehat{\sigma}(\widehat{e},w,S(\theta),\tau,b) = \left[1-\tau\right]\left[w-b\right]m(\widehat{e})S(\theta) \end{aligned}$$

The first order condition to this problem reads

$$\Psi \equiv V_\mu\left[\frac{\partial \mu}{\partial b}-h(\mu,\sigma)\frac{\partial \sigma}{\partial b}\right]=0 \tag{28}$$

and the second order condition is $\Psi_b < 0$ and is assumed to hold.

We have that

$$\begin{aligned}\frac{\partial \mu}{\partial b} &= [w m_e(\widehat{e}) E(\theta) - 1] \frac{\partial \widehat{e}}{\partial b} \\ \frac{\partial \sigma}{\partial b} &= [1-\tau] S(\theta) \left[-m(\widehat{e}) + [w-b] m_e(\widehat{e}) \frac{\partial \widehat{e}}{\partial b} \right] \\ &= [1-\tau] S(\theta) m(\widehat{e}) \left[-1 + [w-b] \frac{m_e(\widehat{e})}{m(\widehat{e})} \frac{\partial \widehat{e}}{\partial b} \right]\end{aligned}$$

and using the foc for individual effort choice

$$m_e(\widehat{e}) [1-\tau] [w-b] [E(\theta) - h(\mu,\sigma) S(\theta)] - 1 = 0$$

it is found that

$$\begin{aligned}\frac{\partial \mu}{\partial b} &= [w m_e(\widehat{e}) E(\theta) - m_e(\widehat{e}) \ [1-\tau] [w-b] [E(\theta) - h(\mu,\sigma) S(\theta)]] \frac{\partial \widehat{e}}{\partial b} \\ &= m_e(\widehat{e}) [[\tau w + [1-\tau] b] E(\theta) + \ [1-\tau] [w-b] h(\mu,\sigma) S(\theta)] \frac{\partial \widehat{e}}{\partial b}\end{aligned}$$

Hence,

$$\begin{aligned}&\frac{\partial \mu}{\partial b} - h(\mu,\sigma) \frac{\partial \sigma}{\partial b} \\ = \ & m(\widehat{e}) [1-\tau] S(\theta) \left[\left[\frac{\tau w + [1-\tau] b}{[1-\tau] b} \frac{E(\theta)}{S(\theta)} \right] \frac{\partial \widehat{e}}{\partial b} \frac{b}{\widehat{e}} \frac{m_e(\widehat{e}) e}{m(\widehat{e})} + h(\mu,\sigma) \right]\end{aligned}$$

which implies that (21) can be written as

$$\begin{aligned}\Psi &\equiv V_\mu \left[\frac{\partial \mu}{\partial b} - h(\mu,\sigma) \frac{\partial \sigma}{\partial b} \right] \\ &= V_\mu m(\widehat{e}) [1-\tau] S(\theta) \left[h(\mu,\sigma) - \left[\frac{\tau w + [1-\tau] b}{[1-\tau] b} \frac{E(\theta)}{S(\theta)} \right] \eta \right]\end{aligned}$$

where

$$\eta \equiv -\frac{\partial \widehat{e}}{\partial b} \frac{b}{\widehat{e}} \frac{\partial m(\widehat{e})}{\partial \widehat{e}} \frac{\widehat{e}}{m(\widehat{e})} > 0$$

Which shows that the optimal policy is always where $\frac{\partial \widetilde{\mu}}{\partial \tau} < 0$, i.e. on the optimal sloping part of the return-risk locus

$$[1-\tau] [w-b] [E(\theta) - h(\mu,\sigma) S(\theta)]$$

Optimal b - fully financed by the income tax

The optimality condition (23) can be written

$$[m_e(\widetilde{e}) E(\theta) w \quad - 1] \frac{\partial \widetilde{e}}{\partial b} = h \left[-\tau_b [w-b] m(\widetilde{e}) - [1-\tau(e,b)] \left[m(\widetilde{e}) - [w-b] m_e(\widetilde{e}) \frac{\partial \widetilde{e}}{\partial b} \right] \right] S(\theta)$$

and using the foc for individual effort choice

$$m_e(\widehat{e})\,[1-\tau]\,[w-b]\,[E(\theta)-h(\mu,\sigma)S(\theta)]-1=0$$

we have that this can be reduced to

$$-E(\theta)\frac{[\tau w+(1-\tau)b]}{(1-\tau)}\frac{\partial\widetilde{e}}{\partial b}\frac{m_e(\widetilde{e})}{m(\widetilde{e})}=h\left[\frac{\tau_b}{(1-\tau)}\,[w-b]+\right]S(\theta)$$

or

$$\frac{[\tau+(1-\tau)r]}{(1-\tau)(1-r)}=\frac{h}{\varepsilon(\widetilde{e})}\frac{S(\theta)}{E(\theta)}\left[\frac{\tau_b b}{\tau}\frac{\tau}{(1-\tau)}\,[r-1]+1\right]$$

Appendix E: Response of optimal benefits to expected return $E(\theta)$ and risk $S(\theta)$.

We have that the rate of transformation between expected income and risk is given by

$$\frac{\partial\mu}{\partial\sigma}\mid_b=\frac{\frac{\partial\mu}{\partial b}}{\frac{\partial\sigma}{\partial b}}=\frac{[wm_e(\widehat{e})E(\theta)-1]\,\frac{\partial\widehat{e}}{\partial b}}{[1-\tau]\,S(\theta)m(\widehat{e})\left[-1+[w-b]\,\frac{m_e(\widehat{e})}{m(\widehat{e})}\frac{\partial\widehat{e}}{\partial b}\right]}\tag{29}$$

Consider an intial situation for which $E(\theta)=E_0$, $S(\theta)=S_0$ and that the optimal effort and benefit level in this case are $e=e_0$ and $b=b_0$, respectively. The expected income and risk are μ_0 and σ_0, and we have the optimum characterized by

$$h(\mu_0,\sigma_0)=\frac{\partial\mu}{\partial\sigma}(\mu_0,\sigma_0)$$

Change in expected return $E(\theta)$:

For given (S_0,E_0,b_0) we have that an increase in $E(\theta)$ from E_0 to E_1 implies that expected income increases to μ_1. We have immediately that $h(\mu_1,\sigma_0)<h(\mu_0,\sigma_0)$ and from (29) it follows that

$$\frac{\partial\mu}{\partial\sigma}(\mu_0,\sigma_0)<\frac{\partial\mu}{\partial\sigma}(\mu_1,\sigma_0)$$

and it follows that $h(\mu_1,\sigma_0)<\frac{\partial\mu}{\partial\sigma}(\mu_1,\sigma_0)$. Hence, benefits and effort are changed in the direction of higher expected income and risk. Since higher expected return induces more effort which in turn both increases expected income and risk it is in general ambigious how the optimal benefit level changes. This is seen by noting that

$$sign\ \frac{\partial b}{\partial E\,(\theta)}=sign\ \Psi_E$$

where

$$sign\ \Psi_E=sign\left[\begin{array}{c}h_\mu(\mu,\sigma)\frac{\partial\mu}{\partial E(\theta)}+h_\sigma(\mu,\sigma)\frac{\partial\sigma}{\partial E(\theta)}\\ -\left[\frac{\tau w+[1-\tau]b}{[1-\tau]b}\right]\eta\epsilon\frac{1}{S(\theta)}-\left[\frac{\tau w+[1-\tau]b}{[1-\tau]b}\frac{E(\theta)}{S(\theta)}\right]\frac{\partial(\epsilon\eta)}{\partial E(\theta)}\end{array}\right]$$

Note that

$$\begin{aligned}\frac{\partial \mu}{\partial E(\theta)} &= [w m_e(\widehat{e}) E(\theta) - 1] \frac{\partial \widehat{e}}{\partial E(\theta)} > 0 \\ \frac{\partial \sigma}{\partial S(\theta)} &= [1-\tau][w-b] S(\theta) m_e(\widehat{e}) \frac{\partial \widehat{e}}{\partial E(\theta)} > 0\end{aligned}$$

Change in risk $S(\theta)$:

For given (S_0, E_0, b_0) we have that an increase in $S(\theta)$ from S_0 to S_1 implies that income risk increases to σ_1. We have immediately that $h(\mu_0, \sigma_0) < h(\mu_0, \sigma_1)$ and from (29) it follows that

$$\frac{\partial \mu}{\partial \sigma}(\mu_0, \sigma_0) > \frac{\partial \mu}{\partial \sigma}(\mu_1, \sigma_0)$$

and it follows that $h(\mu_1, \sigma_0) > \frac{\partial \mu}{\partial \sigma}(\mu_1, \sigma_0)$. Hence, benefits and effort are changed in the direction of lower expected income and risk. Since the underlying change goes in the direction of increasing risk, it follows that mean income falls and the effect on risk is ambiguous.

To work out how risk affects the optimal benefit level note that

$$sign\ \frac{\partial b}{\partial S(\theta)} = sign\ \Psi_S$$

where

$$sign \Psi_S = sign \left[\begin{array}{c} h_\mu(\mu,\sigma)\frac{\partial \mu}{\partial S(\theta)} + h_\sigma(\mu,\sigma)\frac{\partial \sigma}{\partial S(\theta)} \\ + \left[\frac{\tau w + [1-\tau]b}{[1-\tau]b}\right] \eta \frac{E(\theta)}{(S(\theta))^2} - \left[\frac{\tau w + [1-\tau]b}{[1-\tau]b} \frac{E(\theta)}{S(\theta)}\right] \frac{\partial \eta}{\partial S(\theta)} \end{array} \right]$$

We have that

$$\begin{aligned}\frac{\partial \mu}{\partial S(\theta)} &= [w m_e(\widehat{e}) E(\theta) - 1] \frac{\partial \widehat{e}}{\partial S(\theta)} < 0 \\ \frac{\partial \sigma}{\partial S(\theta)} &= [1-\tau][w-b] m(\widehat{e}) \left[1 + S(\theta) \frac{m_e(\widehat{e})}{m(\widehat{e})} \frac{\partial \widehat{e}}{\partial S(\theta)}\right]\end{aligned}$$

assuming that $S(\theta)\frac{m_e(\widehat{e})}{m(\widehat{e})}\frac{\partial \widehat{e}}{\partial S(\theta)} > -1$ we have $\frac{\partial \sigma}{\partial S(\theta)} > 0$. Hence a sufficient condition that $\frac{\partial b}{\partial S(\theta)} > 0$ is that $\frac{\partial \eta}{\partial S(\theta)} \leq 0$.