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ABSTRACT

Government Debt-Threshold Contracts*

Politicians tend to push the amount of public debt beyond socially desirable levels in order to increase their reelection chances. We develop a model that provides a new explanation for this behavior: office holders undertake debt-financed public projects, but postpone the timing of part of the output to the next term. This makes it difficult to replace them. As a consequence, the office-holders' reelection chances rise -- as does public debt. As a potential remedy for this inefficiency, we allow candidates for public office to offer government debt-threshold contracts. Such a contract contains an upper limit for government debt and the sanction that an office-holder violating this limit cannot stand for reelection. We show that such competitively-offered contracts contain low debt levels that limit debt financing and improve the citizens' welfare. When negative macroeconomic events occur, government debt contracts may be violated, and such shocks are stabilized.

JEL Classification: D7, D82 and H4
Keywords: elections, government debt, macroeconomic shocks and political contracts

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1 Introduction

Controlling public debt levels is one of the most difficult policy tasks in developing and developed countries alike. Many industrialized countries have experienced rising debt levels over the last few decades, and these rises have been accelerated by the recent crisis. The tendencies for large industrialized countries are illustrated in Figure 1.

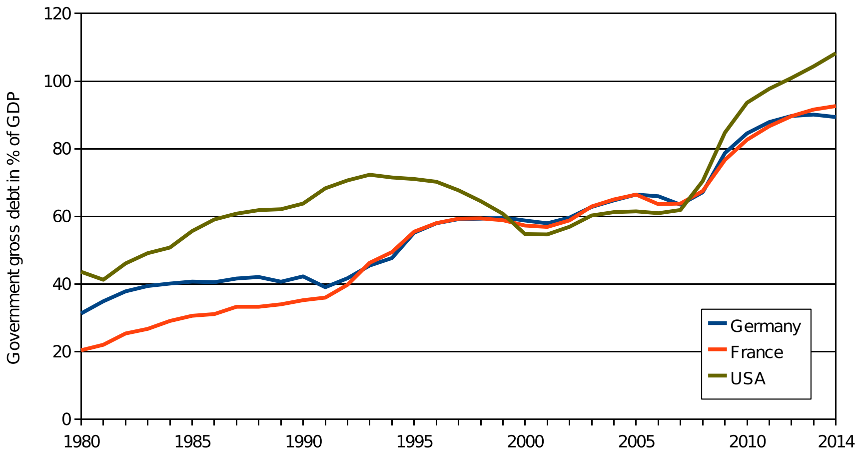

Figure 1: Government debt for selected countries (years 2009-2014 estimated)
Source: IMF World Economic Outlook Database (2009)

In this paper we provide a novel explanation for the inclination of politicians to push public debt beyond socially desirable levels. We then introduce government debt-threshold contracts as a potential remedy for this inefficiency. Such a contract contains an upper limit of government debt and the sanction that an office holder violating this limit cannot stand for reelection. We allow candidates for office to offer government debt-threshold contracts (henceforth GDTC) during campaigns.

We consider a two-period model in which office holders choose taxes, levels of public goods, and debt levels. There is asymmetric information regarding the ability of office holders. Office holders in the first period can increase their reelection chances by investing more in public good provision but postponing the timing of some of the output to the second period, where it can only be realized if the initiator remains in office. Higher debt levels and the associated output shifts are socially expensive, but they increase the incumbent's indispensability. As a consequence, office holders with below-average ability are reelected. Attempts of office holders to make their replacement costly by debt issuance is a new explanation for rising debt levels in democracies.

To limit the tendency of incumbents to push public debt beyond socially desirable levels, we allow candidates for public office to voluntarily offer GDTCs. Such contracts contain an upper limit for government debt that must be honored by the candidate if he is elected to public office. If an office holder violates the self-stipulated debt threshold in his contract, he loses the right to stand for reelection.

We show that candidates offer GDTCs with balanced budgets. In other words, they promise that the debt level will not increase. However, once elected, a candidate may violate his GDTC when negative macroeconomic shocks occur, to allow for government debt issuance to stabilize such shocks. Such behavior is desired by the electorate. GDTCs with balanced budgets maximize citizens' welfare and the election chances of candidates in a two-candidate political contest.

While government debt can be justified on normative grounds - for instance, by the famous tax-smoothing argument of Barro (1979) - other political economic forces tend to push public debt beyond socially desirable levels. Fragmented governments are prone to excessive spending when few groups benefit from public goods but the costs are distributed over society as a whole (see Weingast et al., 1981, Baron and Ferejohn, 1989, Baron, 1991, Neck and Sturm, 2008, Battaglini and Coate, 2008, and Yared, 2009). The investigations referred to suggest that there is underprovision of public goods, excessive rent-seeking, and accumulation of debt over time. Moreover, the accumulation of debts allows governments to shift fiscal burdens to future generations, which are underrepresented or not represented at all in contemporary elections (Song et al., 2009).

Several procedures have been implemented to limit excessive public-debt accumulation, including fixed budget-limits, various debt-brakes, rainy-day funds, and balanced-budget rules. These rules, however, run into credibility and flexibility problems. On the one hand, tight budget rules and debt-brakes have not proved very credible. Some were repealed temporarily, others did not sanction violations. On the other hand, when such rules allowed for room to maneuver, policy-makers exploited their flexibility even in quiet times, and debt accumulation was not slowed down. Both Canova and Pappa (2006) and Auerbach (2008) have assessed the success of fiscal rules in most states of the U.S., and their findings show that these rules have had little success in limiting debt accumulation.

Government debt-threshold contracts are a new variant of the political contracts surveyed by Gersbach (2008). The particular feature of GDTCs is that they may be violated by office holders and that this is desirable from the electorate's perspective.

Our paper is organized as follows: In Section 2 we outline the basic model. In Section 3 we derive the equilibrium if only elections are present. In Section 4 we introduce GDTCs and derive the resulting equilibrium. Macroeconomic shocks and GDTCs are presented in Section 5. Section 6 is a discussion of further extensions and ramifications, and Section 7 concludes the paper.

2 The Model

2.1 Agents

We consider a society that delegates policy making to a politician. The electorate consists of a continuum of voters. Each individual voter is indexed by $i \in [0, 1]$. At the beginning of each of the two periods $t = 1$ and $t = 2$, there is an election, and the same two candidates are competing for government office. Candidates are denoted by k or k'. ($k, k' = 1, 2$ and $k \neq k'$)

Each individual is endowed with one unit of labor, which is supplied inelastically. There is a non-storable private consumption good produced by a linear technology that generates income y for each individual in each period. We use c_t to denote the consumption of the private consumption good in period $t = 1, 2$.

There is a public good whose level is denoted by g_t ($t = 1, 2$) and which can be produced and financed by taxing the citizens and issuing government debt. The details are explained below.

2.2 Policies

There are two types of policy decisions taken by the office holder.

2.2.1 Public goods

In each period the office holder provides a public good. The result is determined by the ability of the office holder and by public investments. The amount of this public good in period t is given as

$$g_t = \gamma(a_k + e)K_t, \tag{1}$$

where a_k represents his ability and is a random variable distributed uniformly on $[-A, A]$ with $A > 0$. Furthermore, e is a constant that fulfills $e > A$, so the amount of the public good is still positive even if the candidate with the lowest ability comes into power. K_t is the amount of public investments in the public good. Finally, the productivity of public investments is affected by variable γ with $\gamma > 0$.

Public investments can be financed from two sources: taxation or issuing government debt in capital markets. In particular,

$$K_1 = \tau_1 y + d_1 \tag{2}$$

$$K_2 = \tau_2 y - (1 + r)d_1, \tag{3}$$

where τ_t is the tax rate in period t. Variable d_1 denotes the debt the government chooses in period 1. Variable r denotes the interest rate that has to be paid on government debt. As government debt and interest rate payment have to be paid back in the next period, it does not matter whether the government borrows from its citizens or from foreign investors. Either way, citizens have to pay the debt.

We assume that taxation is distortionary and that there exists a tax revenue maximizing tax rate $\overline{\tau}$, with $0 < \overline{\tau} < 1$. That is, $\overline{\tau}$ is the tax rate where the Laffer curve attains its maximum. For ease of presentation we adopt a polar scenario and assume that taxation does not reduce labor supply as long as $\tau \leq \overline{\tau}$. If $\tau > \overline{\tau}$, citizens stop working in the market and switch to home production. Tax revenues would then fall to zero.

The informational assumptions are as follows: The office holder learns how able he is after he has made the investment decision. This however will remain private information. Voters will observe g_t. The citizens derive utility from the public project in accordance with the instantaneous utility function $U^P(g_t)$.

The utility function is given by

$$U^P(g_t) = \begin{cases} -\infty & if \quad g_t < g^{min} \\ g_t & if \quad g_t \geq g^{min} \end{cases}$$

The formulation is a Stone-Geary-type utility expressing the fact that a minimal provision of public goods, such as security, property rights, and transportation infrastructure, are essential for the functioning of the state and the economy. This is captured by g^{min} ($g^{min} > 0$). If the government provides less than g^{min}, the utility for citizens is very low.

The positivity of g^{min} ensures that an office holder in the first period will plan for a positive amount of public goods in the second period to be provided, as office holders are also citizens. Accordingly, the ability of office holders will also matter for citizens in the second period.

2.2.2 Output-shift policy

After the policy maker has observed his own ability a_k in period 1, he can decide whether or not to shift the realization of a specific part of the output of size sg_t ($0 < s < 1$, s exogenously given) from period $t = 1$ to period $t = 2$. In particular, he can decide to realize amount $(1 - s)g_t$ in period $t = 1$. If he is still in office in $t = 2$, he can realize fsg_t in $t = 2$ ($0 < f < 1$) on the basis of his activities and investment in $t = 1$. We assume that in $t = 2$ a new office holder cannot reap the benefits of the effort invested by a preceding policy maker.

With this we capture policies that require policy-specific efforts by the policy maker. Examples are international negotiations and treaties, foreign policy, large-scale public infrastructure projects with time-consuming planning efforts, or new regulatory frameworks for specific industries such as the health care system. Such policies require policy-specific human capital that is lost at least partly when a new politician comes into office. Moreover, the timing for the realization of the benefits from such policies lies with the policy maker.

Output shift can also occur in the legislative sector. For instance, a member of parliament may lobby to have government funds or infrastructure projects channeled to his district. The member of the legislature may decide to postpone some of the benefits from his activities in order to make it costly to replace him. The option to shift output across time is a simple device generating an incumbency advantage. Note that our assumption $f < 1$ implies that output-shifts are socially detrimental.[1]

2.3 Utilities

In this section we describe the utilities for voters and candidates. We use ϵ_k to denote the output-shift decision of candidate k in the first period. If policy maker k shifts output in period 1 ϵ_k is equal to 1, otherwise ϵ_k is zero. The common discount factor of voters and politicians is denoted by β with $0 < \beta \leq 1$.

2.3.1 Voters

The expected utility of voter i evaluated at the beginning of the game is given by the discounted sum of the benefits from the public project plus the benefits from consumption. Assume for a moment that $g_1(1-s)$ and g_2 are at least as large as g^{min}. We distinguish two cases.

(i) If the same politician k is in office in both periods, ex ante lifetime utility is given by

$$V_i = \mathbf{E}[lnc_1 + g_1 - \epsilon_k s g_1 + \beta(lnc_2 + g_2 + \epsilon_k f s g_1)], \tag{4}$$

where $\mathbf{E}$ is the expectation operator. Variables c_1 and c_2 denote the consumption in period 1 and period 2, respectively.

[1]Our model can also be applied to case $f > 1$ when output shifting is potentially socially valuable. In such circumstances, however, higher debt levels are socially desirable.

(ii) If politician k is in office in period $t = 1$ and politician k' ($k' \neq k$) holds office in period $t = 2$, ex-ante lifetime utility is given by

$$V_i = \mathbf{E}[lnc_1 + g_1 - \epsilon_k s g_1 + \beta(lnc_2 + g_2)] \tag{5}$$

After the office holder has chosen the first tax rate in period 1, voters take consumption decisions subject to the intertemporal budget constraint

$$(y(1-\tau_1) - c_1)(1+r) = c_2 - y(1-\tau_2). \tag{6}$$

Accordingly, we assume that citizens have frictionless access to the capital market and can borrow or save at interest rate r. We note that the amount of public goods across periods will generally differ when the office holder chooses different debt levels in the first period, despite the Ricardian features that both citizens and the government can borrow at rate r and citizens have to pay back the debt. The reason is that different debt levels are associated with different amounts of the public good in the first period shifted to the next period. As output shifts entail output losses, different debt levels induce different output losses.

2.3.2 Politicians

The office holder derives utility from two sources.

- Office holding
  An office holder derives private benefits b ($b \geq 0$) from holding office, including monetary and non-monetary benefits such as power and enhanced career prospects.

- Benefits from policies
  The office holder derives the same benefits from public projects as voters.

To describe the overall utility for politicians, we have to distinguish four cases. Again assuming that $g_1(1-s), g_2 \geq g^{min}$, the lifetime utility of politician k, denoted by V_k, can be computed as follows:

(i) If k is in office over both periods,

$$V_k = \mathbf{E}[lnc_1 + b + g_1 - \epsilon_k s g_1 + \beta(lnc_2 + b + g_2 + \epsilon_k f s g_1)].$$

(ii) If k is in office in $t = 1$ only,

$$V_k = \mathbf{E}[lnc_1 + b + g_1 - \epsilon_k s g_1 + \beta(lnc_2 + g_2)].$$

(iii) If k is in office in $t = 2$ only,

$$V_k = \mathbf{E}[lnc_1 + g_1 - \epsilon_{k'} s g_1 + \beta(lnc_2 + b + g_2)].$$

(iv) If k never is in office,

$$V_k = \mathbf{E}[lnc_1 + g_1 - \epsilon_{k'} s g_1 + \beta(lnc_2 + g_2 + \epsilon_{k'} f s g_1)].$$

2.4 The overall game

We summarize the overall game in the following figure:

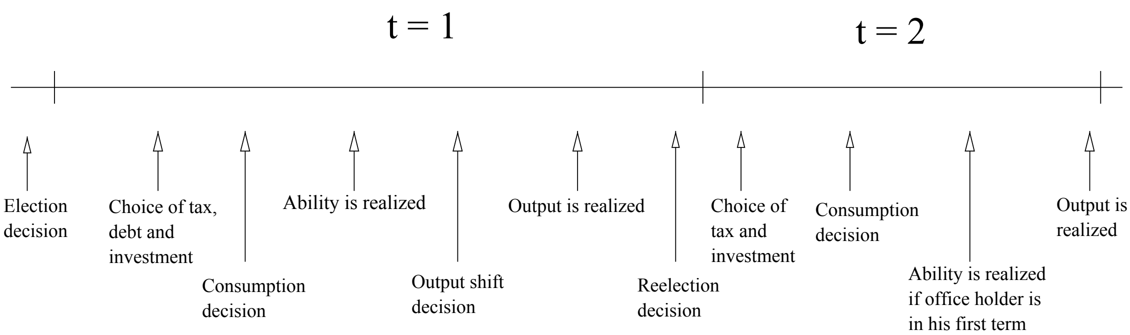

Figure 2: Time-line with standard elections

2.5 Assumptions and equilibrium concept

We assume that voters observe only output g_1 and not ability.[2] Output g_1 is not contractible, so it cannot be used to generate rewards for office holders beyond elections.

[2]This assumption follows Alesina and Tabellini (2007).

Voters observe the values of τ_1, τ_2, and d_1 and are thus able to calculate K_1 and K_2. Moreover, citizens are assumed to vote sincerely, i.e. they vote for the candidate from whom they expect higher utility.[3] We are looking for perfect Bayesian Nash equilibria for the game under these assumptions.

We assume that b is sufficiently large[4] so that policy makers will prefer to be in office under any of the circumstances we consider in sections 3 and 4. In addition, we assume in sections 3 and 4 that $(1-s)\gamma(e-A)\overline{\tau}y > g^{min}$, which implies that any office holder can shift output in period $t = 1$ when $d_1 = 0$ without risking a collapse of the state. Moreover we assume that the public good is sufficiently valuable, meaning citizens will prefer public goods to be produced with tax rates $\overline{\tau}$, which is equivalent to condition $\gamma(e-A)y - \frac{1}{(1-\bar{\tau})y}$.[5] Finally, we assume $\beta = \frac{1}{1+r}$, and to simplify the exposition, we set $\beta = 1$ and $r = 0$.

3 Elections Alone

We first examine the standard case where elections are held before the first and second terms start. The candidate with the higher share of votes will be elected. If both candidates obtain the same share of votes, the probability for each candidate of winning in the first period is 0.5. In the second period, we consider the tie-breaking rule determining that, in this case, the incumbent will be elected.

We start with the following observation.

Lemma 1

(1) An office holder in the first period will choose a debt level d_1 with

$$d_1 \leq \overline{d}_1 = \bar{\tau}y - \frac{g^{min}}{\gamma(e-A)}$$

[3]Obviously, with a continuum of voters the individual voter has no influence on the outcome of an election. The optimality of sincere voting can be justified for a model variant with a large but finite number of voters or when the act of voting generates benefits.

[4]Precise conditions for b are given in Appendix B.

[5]We require that utility from private and public good consumption be non-decreasing in τ at $\bar{\tau}$, i.e. $\frac{d}{d\tau}\ \{ln((1-\tau)y) + \gamma(e-A)(\tau y - d_1)\}|_{\tau=\bar{\tau}} = \gamma(e-A)y - \frac{1}{(1-\bar{\tau})y} \geq 0$.

(2) An office holder will choose K_1 such that $g_1 \geq g^{min}$ for any $a_k \in [-A, +A]$

The proof of Lemma 1 is given in the Appendix. We note that for $d_1 = \overline{d}_1$ we obtain $K_2 = \frac{g^{min}}{\gamma(e-A)}$ and thus $g_2 = g^{min}$ if the office holder in the second period has ability $-A$. Hence Lemma 1 puts an upper bound on debt levels chosen by office holders, as office holders are also citizens and do not want to risk a dysfunctional state in the future, i.e. they ensure that $g_2 \geq g^{min}$ even if the ability of the future office holder is very low.

From now on we neglect the case where public good provision in either period could fall below g^{min}.

3.1 The second period

We first focus on the second period and assume that a politician k with ability a_k holds office. For the moment, we assume further that a_k has become known if the office holder has already been in office in period 1. This will be proven in the next subsection. If a new politician enters office in the second period, the ability expected by all agents is zero.

Fact 1

Suppose that candidate k is elected at date $t = 2$. Then

(i) the expected utility of an office holder at the beginning of period 2 is given by

α) first-term office holder: $V^*_{k2} = b + ln((1-\bar{\tau})y) + \gamma e K_2$

β) second-term office holder: $V^*_{k2} = b + ln((1-\bar{\tau})y) + \gamma(e + a_k)K_2 + \epsilon_k f s g_1$.

(ii) The expected utility of the politician $k' \neq k$ who has lost the second election is given by

α) $V^D_{k'2} = ln((1-\bar{\tau})y) + \gamma(e + a_k)K_2 + \epsilon_k f s g_1$ if k has been in office in the first period

β) $V^D_{k'2} = ln((1-\bar{\tau})y) + \gamma e K_2$ if k' has been in office in the first period.

3.2 The first period: general consideration

We now look at the equilibria in the first period. Once in office, the politician has to choose d_1. We start with two simple observations that will hold in every equilibrium with pure strategies.

Fact 2

1. *Voters will perfectly infer the ability of the office holder at the end of period* 1.

2. *An office holder will shift output if this action is critical in guaranteeing reelection.*

The first statement follows from the informational structure of the game. As voters know parameters γ and e and as they learn value K_1, they are able to infer ability a_k.[6] The second statement follows from the informational assumptions, as the office holder observes his ability before he decides on output shift.

Now we derive the optimal choice of output shift by the office holder in the first period for a given level of public debt d_1. This analysis will help determine the equilibrium in both cases, i.e. elections alone and elections where public-debt contracts are offered. We need to distinguish three cases. First, office holder k's ability may be so high that he will be reelected even if he does not choose output-shift policies. In this case, he will not choose output-shift policies ($\epsilon_k = 0$) and will be reelected. We use p^0 to denote the probability the office holder assigns to this eventuality when he chooses d_1. Second, the office holder may have an intermediate level of ability, which implies that he will only be reelected if he chooses the output-shift policy ($\epsilon_k = 1$). Because we have assumed that b is sufficiently high, the office holder will choose the socially detrimental option $\epsilon_k = 1$, which implies reelection. We now introduce p^1, which represents the office holder's estimate of the probability of this eventuality. Third, office holder k's ability may turn out to be very low. As a result, he will never be reelected, irrespective of his decision about output-shift policies. In this case, it is optimal to choose $\epsilon_k = 0$. The probability of this happening is $1 - p^0 - p^1$.

[6]Formally, $a_k = \frac{g_t - \gamma e K_1}{\gamma K_1}$.

Finally, we introduce $\widetilde{a}_k$ as office holder k's expected level of ability conditional on the fact that he is reelected. Note that office holder k is reelected with probability $p^0 + p^1$ and dismissed with probability $1 - p^0 - p^1$. With probability p^1, net losses $sg_1(1-f)$ occur as a result of output-shift policies. It is useful for the following analysis to characterize these probabilities and the choices of the office holder for some given debt level d_1. In the Appendix we show

Proposition 1

(i) Suppose that the office holder selects d_1. Then he will choose $\tau_1 = \overline{\tau}$ and $K_1 = \overline{\tau}y + d_1$. The office holder in the second period will choose $\tau_2 = \overline{\tau}$ and $K_2 = \overline{\tau}y - d_1$.

(ii) The probability of k choosing $\epsilon_k = 0$ and being reelected is given by

$$p^0 = \frac{1}{2}. \tag{7}$$

(iii) The probability of k choosing $\epsilon_k = 1$ and being reelected is given by

$$p^1 = \frac{fsK_1e}{2A(K_2 + fsK_1)}. \tag{8}$$

(iv) The average ability level of a reelected office holder corresponds to

$$\widetilde{a}_k = \frac{A - \frac{fsK_1e}{K_2+fsK_1}}{2}. \tag{9}$$

Proposition 1 reveals two types of inefficiencies in politics. First, incumbents with a below-average ability level can ensure reelection by choosing output-shift policies. This happens with probability p^1. The socially optimal reelection rule would stipulate that an office holder k will be reelected if and only if his ability is equal or above average, i.e. if $a_k \geq 0$. This would imply that the average ability level of a reelected politician would amount to $A/2$. With standard elections this average level is lower. Second, office holders shift output, which creates output and welfare losses. This happens with probability p^1.

3.3 Equilibrium

We now derive the overall equilibrium for the elections-only case.

Proposition 2

(i) The office holder chooses $d_1^* = \overline{d}_1$, $\tau_1^* = \overline{\tau}$, $\tau_2^* = \overline{\tau}$, $K_1^* = \overline{\tau}y + \overline{d}_1$ *and* $K_2^* = \frac{g^{min}}{\gamma(e-A)}$.

(ii) The probability of k *choosing* $\epsilon_k = 0$ *and being reelected is given by*

$$p^0 = \frac{1}{2}. \tag{10}$$

(iii) The probability of k *choosing* $\epsilon_k = 1$ *and being reelected is given by*

$$p^1 = \frac{fsK_1^*e}{2A(K_2^* + fsK_1^*)}. \tag{11}$$

(iv) The average ability level of a reelected office holder corresponds to

$$\widetilde{a}_k = \frac{A - \frac{fsK_1^*e}{K_2^*+fsK_1^*}}{2}. \tag{12}$$

The proof is given in the Appendix. Proposition 2 shows that politicians exhaust the debt limit in the first period. The reason is that a higher debt level increases the potential for output shifting, and thus raises the reelection probability.

4 Government Debt Threshold Contracts

4.1 Government debt thresholds as political contracts

In this section we allow both candidates to offer contracts by stipulating a government debt threshold d_{k1}^c ($d_{k1}^c \geq 0$)[7]. The interpretation is as follows: If candidate k takes office in $t = 1$, he is not allowed to increase first-period government debt above d_{k1}^c. If he increases the debt above d_{k1}^c then he is not allowed to run for reelection, and the challenger will take office.

[7]We restrict d_{k1}^c to non-negative levels. An extension of the model would be to allow government surplus contracts as well.

The government debt threshold contract is a particular type of political contract. Generally, political contracts are verifiable election promises, associated with rewards or sanctions depending on whether promises are kept or not. As outlined in Gersbach (2008), political contracts have to be approved by an independent governmental body and are enforced by a court.[8]

4.2 The second and first period

For the first step of the analysis, we assume that candidate k has been elected after offering a government debt threshold $d^c_{k1} \leq \overline{d}_1$ in his contract. In the second period, the choice regarding public good provision by k (if he remains in office) or by k' (if he enters office) will remain the same as in Proposition 1.

In the first period, the incumbent can no longer choose $d_1 = \overline{d}_1$ if $d^c_{k1} < \overline{d}_1$ due to his government debt contract, unless he is willing to give up his right to stand for reelection. We have assumed that the value of office is so high that violating the GDTC is not in the interests of the office holder.

As an immediate consequence of Proposition 1 we obtain

Proposition 3
Suppose the office holder has offered the GDTC with d^c_{k1}. Then he chooses $d_1 = d^c_{k1}$ and $\tau_1 = \overline{\tau}$. Together with $\tau_2 = \overline{\tau}$, this yields $K_1 = \overline{\tau}y + d^c_{k1}$ and $K_2 = \overline{\tau}y - d^c_{k1}$.

The reelection probabilities p^0 and p^1 and the average ability of a reelected office holder are given by Proposition 1.

The proof of Proposition 3 is analogous to the proof of Proposition 2, except that we have to use $d_1 = d^c_{k1}$ instead of $d_1 = \overline{d}_1$.

We are now ready to describe the overall equilibrium. For that purpose, we use d^s_1 to denote the socially optimal debt level, i.e. the debt level that maximizes the welfare of voters subject to the incentive constraint that the office holder may shift output and

[8] For a detailed discussion of the certification and verification procedures of political contracts, see Gersbach (2008)

may be reelected with below-average ability. The expected level of ability of an office holder who will shift output and get reelected is denoted by $\widetilde{\widetilde{a}}_k$, which can be derived from the proof of Proposition 1.

Proposition 4

(i) There exists a unique equilibrium in which both candidates for office offer GDTCs with $\hat{d}^c_{k1} = 0$, $k = 1, 2$. The probability of being elected is $\frac{1}{2}$ for each candidate.

(ii) An elected candidate chooses $d_1 = 0$ in office.

(iii) $d_1 = d^s_1 = 0$ is the welfare-optimal debt level.

The proof of Proposition 4 is given in the Appendix. Proposition 4 shows that candidates for office offer the welfare-optimal debt level d^s_1 in their GDTCs and will choose it if they are elected. The intuition is as follows: Higher debt levels induce office holders to invest more in period 1. As a consequence, low-ability office holders will shift more output to period 2 so as to be reelected. As output shifts are costly, low-ability policy makers should not be reelected, and as fewer investments in period 1 can be compensated by investment in period 2, higher debt levels are socially undesirable.

5 Macroeconomic Shocks and Debt Contracts

A useful extension of the model is to consider macroeconomic shocks. For instance, shocks to the income of citizens or to the productivity of the public good in a particular period may make debt financing welfare-improving because it stabilizes such shocks or takes advantage of highly productive public investment opportunities. We focus on negative macroeconomic shocks.

More specifically, suppose there is a macroeconomic shock in the first period after a politician has entered office. Income y is either $y^h > y$ with probability p $(0 < p < 1)$ or $y^l < y$ with probability $1 - p$ such that $y = py^h + (1 - p)y^l$. Moreover we assume that

$$\gamma\,(e - A)\,\overline{\tau}\,y^l < g^{min}$$

which implies that if a bad shock occurs a lowest-ability office holder could not guarantee a minimal state without debt issuance.

Now the behavior of an office holder regarding debt thresholds has to be modified, as the value of office is large but finite, while there may be a risk that utility derived from public goods will become minus infinity. Suppose that an office holder has stipulated debt threshold d_{k1}^c in his contract. Then

- he will not violate the contract if the probability that $g_1 < g^{min}$ is zero,
- he will violate the contract if the probability that $g_1 < g^{min}$ is positive and the risk of a break-down of the state can be eliminated by debt issuance.

We assume that GDTCs cannot be conditioned on macroeconomic shocks, as it is difficult or impossible to identify such shocks in such a way that they are verifiable in court.[9]

We obtain

Proposition 5

(1) Both candidates offer GDTCs with $\hat{d}_{k1}^c = 0, k = 1, 2$

(2) The office holder in the first period chooses $d_1 = 0$ *in the good state and*

$$d_1^{crit} = \frac{g^{min}}{\gamma(e-A)} - \overline{\tau} y^l > 0$$

if the macroeconomic shock is negative. In this case, he cannot stand for reelection.

The proof is given in the Appendix. Proposition 5 indicates that candidates for office still offer GDTCs with balanced budgets. Candidates and voters understand that the contract will be broken when a negative macroeconomic shock occurs.

[9] This is reminiscent of the familiar commitment/flexibility tradeoff in monetary policy. Of course, one could achieve even better social results if public debt in contracts could be conditioned on macroeconomic shocks. While this is extremely difficult in practice, one could give the parliament the right to declare a GDTC null and void when severe negative macroeconomic shocks occur if a super-majority is in favor of doing so.

6 Extensions and Ramifications

We have illustrated the workings of GDTCs in a simple model. Numerous extensions can and should be pursued to address the robustness and validity of the argument for using GDTCs in a broader context.

6.1 Communication advantage

Shifting output is not the only source of incumbency advantage. Suppose a candidate can achieve a communication advantage when he is in office. For instance, the uncertainty (variance) about ideological policies is usually lower for incumbents than it is for challengers. When voters are risk-averse, they value lower uncertainty and will reelect an incumbent even if the median voter prefers the challenger in expected terms (Gersbach, 1998). Due to this communication advantage, an incumbent can move towards his own preferred ideological position in the next election and can use public debt to pursue his political objectives, e.g. by targeting government spending at interest groups that support his bid for reelection. GDTCs may help to limit such activities, which often tend to be socially wasteful.

6.2 Variable effort choice

Another fruitful extension is to introduce variable effort choice. For instance, the production function of public projects could be a function of effort, ability, and public investment, i.e. $g_t = \gamma(e_{kt} + a_k)K_t$ where e_{kt} is the effort exerted by the office holder k in period t. In such situations, public debt tends to have further benefits as it may increase the effort level office holders exert in the first period. The reason is that higher debt levels allow the office holder to generate a higher amount of public goods by putting in more effort[10] and thus increase the amount that can be shifted to the next period, which, in turn, increases his reelection chances. As a consequence, GDTCs may contain a higher level of public debt.[11]

[10] Note that the marginal gain from effort is higher when K_1 is higher.
[11] Details are available on request.

6.3 Less severe shocks

When negative shocks are less severe and the minimal state is not at risk, office holders may not violate their GDTC, as the value of office outweighs the disadvantage of inefficient public good provision. In such circumstances, when the likelihood of negative events is sufficiently high, politicians may offer GDTCs with positive debt levels. As voters anticipate that an office holder will not violate the GDTC, they will prefer an office holder who partially stabilizes negative shocks. Thus candidates will offer GDTC with moderately positive debt levels to maximize their election chances.

7 Conclusion

We have made a simple proposal for constraining government debt accumulation. Numerous issues and more reflection on other and maybe unintended consequences deserve further scrutiny. Nevertheless, government debt threshold contracts are a new instrument that liberal democracies would do well to explore.

Appendix A: Proofs

Proof of Lemma 1

As office holders are also citizens, they will choose a debt level d_1 such that the probability that $g_2 < g^{min}$ is zero. Otherwise the expected utility of the office holder is $-\infty$. To ensure that $g_2 \geq g^{min}$ in all circumstances, the investment level in the second period has to satisfy

$$\gamma(-A+e)K_2 \geq g^{min}$$

The maximal feasible level of K_2 given some d_1 is

$$K_2 = \overline{\tau}y - d_1$$

which implies

$$\overline{d}_1 = \overline{\tau}y - \frac{g^{min}}{\gamma(e-A)}$$

The second point follows the same logic.

■

Proof of Proposition 1

The first point follows from the assumptions of the game, where the office holder chooses $\tau_1 = \overline{\tau}$, $\tau_2 = \overline{\tau}$ for some debt level d_1 as this is preferred by the citizens for whom the public good is sufficiently valuable.

In the following, we consider the reelection decision of the voters. It is optimal for them to reelect k if this implies that their expected utility in the second period will be higher. Formally, this can be stated as

$$\begin{aligned} K_2\gamma(e+a_k) + \epsilon_k f s g_1 &\geq K_2\gamma e \\ \Leftrightarrow K_2\gamma a_k + \epsilon_k f s K_1 \gamma(e+a_k) &\geq 0, \\ \Leftrightarrow a_k \geq -\frac{\epsilon_k f s K_1 e}{K_2 + \epsilon_k f s K_1}. & \end{aligned} \tag{13}$$

The above condition states that k will be reelected if his ability level is equal to or above the critical level $-\frac{\epsilon_k f s K_1 e}{K_2+\epsilon_k f s K_1}$.[12]

[12] For simplicity, we use the tie-breaking rule that the incumbent is reelected if he receives exactly half of the votes.

Now we turn to k's decision about ϵ_k. If $a_k \geq 0$, then k is reelected even for $\epsilon_k = 0$. Then it is optimal to choose $\epsilon_k = 0$, which eliminates the losses from output-shift policies. Applying the fact that a_k is uniformly distributed on $[-A, +A]$, we conclude that the probability of a_k being higher than 0 amounts to $p^0 = \frac{1}{2}$. If $-\frac{fsK_1e}{K_2+fsK_1} \leq a_k < 0$, then it is optimal to choose $\epsilon_k = 1$, which prevents the office holder from being dismissed. The probability of a_k lying in this interval is given by $\frac{fsK_1e}{2A(K_2+fsK_1)}$. Finally, for $a_k < -\frac{fsK_1e}{K_2+fsK_1}$, k's ability is too low to enable him to become reelected. This will induce him to refrain from ultimately fruitless efforts to increase his reelection chances by pursuing output-shift policies ($\epsilon_k = 0$).

It remains to derive the expression for $\widetilde{a}_k$ stated in the text. Recall that this variable denotes the ability level of k, conditional on the fact that he is reelected. We have already shown that k will be reelected if and only if $a_k \geq -\frac{fsK_1e}{K_2+fsK_1}$. The arithmetical average of $-\frac{fsK_1e}{K_2+fsK_1}$ and A yields the desired expression, i.e. $\widetilde{a}_k = \frac{A-\frac{fsK_1e}{K_2+fsK_1}}{2}$.

■

Proof of Proposition 2

We need to show that the office holder will choose $d_1^* = \overline{d}_1$. Given $d_1^* = \overline{d}_1$, all other subsections of the proposition follow from Proposition 1.

The probability of being reelected is $p^0 + p^1 = \frac{1}{2} + p^1$. For an arbitrarily chosen debt level $d_1 < \overline{d}_1$ we calculate how reelection probability is affected by marginal changes in debt.

$$\begin{aligned}
\frac{dp^1}{dd_1} &= \frac{(fse)2A(\overline{\tau}y - d_1 + fs(\overline{\tau}y + d_1)) - 2A(-1+fs)fse(\overline{\tau}y + d_1)}{(2A(K_2 + fsK_1))^2} \\
&= \frac{fse2A(2\overline{\tau}y)}{(2A(K_2 + fsK_1))^2} \\
&> 0
\end{aligned}$$

Hence an office holder can increase his reelection probability by loading on more government debt. Accordingly, the office holder will choose $\overline{d}_1$ over any debt level $d_1 < \overline{d}_1$. The office holder will not choose a debt level above $\overline{d}_1$, as otherwise there is a positive probability that $g_2 < g^{min}$. The reason is that his ability may turn out to be low, which in turn will lead to his deselection, and the new office holder in period 2 will also have low ability, so the expected utility of the current office holder would be $-\infty$ if $d_1 > \overline{d}_1$. Hence $d_1^* = \overline{d}_1$ is the optimal choice for the office holder, which completes the proof.

■

Proof of Proposition 4

In the first three steps we establish that $d_1 = 0$ maximizes the welfare of voters subject to the incentive constraint that office holders may shift output, i.e. $d_1 = 0 = d_1^s$.

Step 1

From Proposition 1 we know that the office holder will choose $\tau_1 = \overline{\tau}$, $\tau_2 = \overline{\tau}$ for any debt level.

Hence the voter i will maximize his utility

$$\max_{c_1,\, c_2}\{V_i = lnc_1 + lnc_2 + \mathbf{E}[g_1 - \epsilon_k s g_1 + g_2 + \epsilon_k f s g_1]\}$$

subject to

$$y(1-\overline{\tau}) - c_1 = c_2 - y(1-\overline{\tau})$$

The solution is

$$c_1 = y(1-\overline{\tau}),\;\; c_2 = y(1-\overline{\tau})$$

and is independent of the debt level.

Step 2

The utility of voters from public good provision is

$$\begin{aligned}
& \mathbf{E}[g_1 - \epsilon_k s g_1 + g_2 + \epsilon_k f s g_1]\} \\
= \;& \mathbf{E}[\gamma(a_k + e)(\overline{\tau}y + d_1)] \\
& +(p^0 + p^1)\mathbf{E}[\gamma(a_k + e)(\overline{\tau}y - d_1)] \\
& +(1 - p^0 - p^1)\mathbf{E}[\gamma(a_{k'} + e)(\overline{\tau}y - d_1)] \\
& +p^1\mathbf{E}[-s\gamma(a_k + e)(\overline{\tau}y + d_1) + fs\gamma(a_k + e)(\overline{\tau}y + d_1)]
\end{aligned}$$

The first term is the utility from public good provision in the first period. The second and third terms reflect public good provision in the second period, when the office holder is either reelected or deselected. The fourth term represents the output losses when office holders shift output.

Evaluating each term yields

$$\begin{aligned}
&= \gamma e(\overline{\tau}y + d_1) + p^0\gamma(\tfrac{A}{2} + e)(\overline{\tau}y - d_1) + p^1\gamma(\widetilde{\widetilde{a}}_k + e)(\overline{\tau}y - d_1) \\
&\quad +(1 - p^0 - p^1)\gamma e(\overline{\tau}y - d_1) + p^1(f-1)s\gamma(\widetilde{\widetilde{a}}_k + e)(\overline{\tau}y + d_1) \\
&= \gamma e(2\overline{\tau}y) + p^0\gamma\tfrac{A}{2}(\overline{\tau}y - d_1) + p^1\gamma\widetilde{\widetilde{a}}_k(\overline{\tau}y - d_1) + p^1\gamma s(f-1)(\widetilde{\widetilde{a}}_k + e)(\overline{\tau}y + d_1) \\
&= \gamma e(2\overline{\tau}y) + p^0\gamma\tfrac{A}{2}(\overline{\tau}y - d_1) + p^1\gamma\widetilde{\widetilde{a}}_k[(1 + sf - s)\overline{\tau}y - (1 + s - sf)d_1] \\
&\quad +p^1\gamma se(f-1)(\overline{\tau}y + d_1)
\end{aligned}$$

with $\widetilde{\widetilde{a}}_k = \frac{-fsK_1e}{2(K_2+fsK_1)}$.

Step 3

We observe

$$\widetilde{\widetilde{a}}_k = \frac{-fsK_1e}{2(K_2 + fsK_1)} < 0, \;\; \frac{d\widetilde{\widetilde{a}}_k}{dd_1} < 0 \text{ and } \frac{dp^1}{dd_1} > 0$$

Hence

$$\frac{dV_i}{dd_1} < 0 \quad \text{for all } d_1 \geq 0$$

Hence $d_1 = 0$ is the welfare-optimal debt level d_1^s, which proves the third point.

Step 4

Suppose a politician has offered $\hat{d}^c_{k1} = 0$ and has been elected. In this step we show that he will choose $d_1 = 0$. His expected utility when he chooses debt level $d_1 > \hat{d}^c_{k1} = 0$ is

$$\begin{aligned}
V_k = &\quad 2ln((1 - \overline{\tau})y) + b + \gamma e(\overline{\tau}y + d_1) + \gamma e(\overline{\tau}y - d_1) \\
V_k = &\quad 2ln((1 - \overline{\tau})y) + b + 2\gamma e\overline{\tau}y
\end{aligned}$$

as he will be deselected with certainty and thus will not shift output either.

His expected utility from choosing $d_1 = \hat{d}^c_{k1} = 0$ is

$$\begin{aligned}
V_k = &\quad 2ln((1 - \overline{\tau})y) + b + \gamma e(\overline{\tau}y) + p^0\left[b + \gamma(e + \frac{A}{2})(\overline{\tau}y)\right] + p^1\left[b + \gamma(e + \widetilde{\widetilde{a}}_k)(\overline{\tau}y)\right] \\
&\quad +(1 - p^0 - p^1)\gamma e(\overline{\tau}y) - p^1\gamma(e + \widetilde{\widetilde{a}}_k)s(1-f)(\overline{\tau}y)
\end{aligned}$$

If b is sufficiently high, the office holder will always honor his contract.

Step 5

Uniqueness follows from the standard undercutting argument. We first note that GDTCs with $d_{k1}^c > \overline{d}_1$ do not impose constraints on an office holder, as he will never choose debt levels above $\overline{d}_1$. So we can assume, without loss of generality, that both candidates k and k' ($k \neq k'$) will offer GDTCs. The constellation $\{d_{k1}^c, d_{k'1}^c\}$ with $d_{k1}^c > 0$ and/or $d_{k'1}^c > 0$ with $d_{k1}^c \neq d_{k'1}^c$ cannot be an equilibrium. Since $\frac{dV_i}{dd_1} < 0$ according to Step 3, a candidate who offers a higher debt threshold than his competitor has zero probability of being elected. Accordingly, offering a higher debt threshold than the competitor cannot be optimal. Suppose both candidates offer $d_{k1}^c = d_{k'1}^c > 0$ and are elected with probability $\frac{1}{2}$. Then a candidate can marginally lower his debt threshold, thereby ensuring that he will be elected with probability 1. Hence this deviation is profitable, and the initial constellation cannot be an equilibrium. This proves uniqueness.

■

Proof of Proposition 5

Step 1

We first observe that a debt level d_1^{crit} ensures that $g_1 \geq g_{min}$ in the worst case, i.e. if a negative income shock occurs and the ability of the office holder turns out to be $-A$, as

$$\gamma(e - A)(\overline{\tau} y^l + d_1) = g^{min} \quad \text{for } d_1 = d_1^{crit}$$

In turn, any debt level below d_1^{crit} would be associated with a positive probability that $g_1 < g^{min}$ and thus would create extremely negative expected utility.
Hence the welfare-optimal debt levels are $d_1 = 0$ in the good state and d_1^{crit} in the bad state.

Step 2

Suppose the elected politician has signed a GDTC with some d_{k1}^c.

- If $d_{k1}^c \geq d_1^{crit}$, the office holder will choose $d_1 = d_{k1}^c$. The reasons are as follows:

The office holder can secure minimal public good provision in the bad state. By choosing $d_1 = d_{k1}^c$ the office holder maximizes his reelection chances, as he just fulfills the contract, and the logic derived in Proposition 4 applies in the same way.

- If $d_{k1}^c < d_1^{crit}$, the office holder will choose $d_1 = d_{k1}^c$ in the good state and $d_1 = d_1^{crit}$ in the bad state. The latter choice is conditioned by two considerations. The office holder violates the GDTC because otherwise he cannot guarantee minimal public good provision, which would hurt him extremely as he is also a citizen. Due to violation of the GDTC he will be deselected with certainty. In the second period he will be an ordinary citizen with certainty, so increasing debt levels beyond d_1^{crit} to shift output is not in the interests of the office holder.

Step 3

Given the behavior of an office holder as derived in Step 2, the welfare-optimal debt threshold is $d_{k1}^c = 0$, which leads to $d_1 = 0$ in the good and $d_1 = d_1^{crit}$ in the bad state. Using reasoning analogous to that in Proposition 4 we find that $d_{k1}^c = d_{k'1}^c = 0$ constitutes the unique equilibrium.

■

Appendix B: Conditions on b

Our results rely on the assumption that the value of office is sufficiently high. In this appendix, we make this statement more precise. In particular, we establish a lower bound $\bar{b}$ such that our results hold for $b \geq \bar{b}$. At three places in sections 3 and 4, the statement "b is sufficiently high" is used. These statements give rise to three conditions on b, denoted by $\bar{b}_1, \bar{b}_2$, and $\bar{b}_3$ respectively.

- If $b \geq \bar{b}_1$, a politician is better off running for office than remaining an ordinary citizen.
- If $b \geq \bar{b}_2$, a politician will shift output to the next period if he can thus increase his reelection chances.
- If $b \geq \bar{b}_3$, a politician will honor the GDTC in Propositions 3 and 4.

We will determine the values of $\bar{b}_1, \bar{b}_2$, and $\bar{b}_3$. Then we can conclude that our results hold if[13]

$$b \geq \bar{b} \quad \text{with} \quad \bar{b} = \max\{\bar{b}_1, \bar{b}_2, \bar{b}_3\}$$

Step 1: Incentives to run for office

We first focus on the second period and assume that a politician k with ability a_k holds office in $t = 1$. As shown in Proposition 1, the incumbent k will be reelected for $t = 2$ if $a_k \geq -\frac{\epsilon_k f s K_1 e}{K_2 + \epsilon_k f s K_1}$. As stated in Fact 2, voters perfectly infer the ability of the office holder at the end of period 1. We distinguish the following two cases:

- $a_k \geq -\frac{\epsilon_k f s K_1 e}{K_2 + \epsilon_k f s K_1}$

  If the incumbent does not run for reelection the voters' and the incumbent's expected utility for the second period is

$$V_{i2} = ln((1 - \bar{\tau})y) + \gamma e K_2$$

[13] As a tie-breaking rule, we assume that an indifferent politician will decide to run for office, shift output to the second period if it is critical for reelection, and honor the GDTC he has offered. Hence we include case $b = \bar{b}$ in the statement.

and the incumbent k's expected utility if he runs for reelection is

$$V_{k2}^{*} = ln((1-\bar{\tau})y) + \gamma(e+a_k)K_2 + \epsilon_k fs\gamma(e+a_k)K_1 + b.$$

Therefore, for any $b \geq 0$, the incumbent will run for office and will be reelected for $t = 2$.

- $a_k < -\frac{\epsilon_k fsK_1 e}{K_2+\epsilon_k fsK_1}$

  The incumbent will not be reelected with certainty, and the challenging politician k' will be elected. In this case, if challenger k' runs for office, his expected utility for the second period is

$$V_{k'2}^{*} = ln((1-\bar{\tau})y) + \gamma eK_2 + b.$$

  If k' does not run for office, the incumbent will remain in office, and k' will expect a utility of

$$V_{k'2}^{D} = ln((1-\bar{\tau})y) + \gamma(e+a_k)K_2$$

  Therefore $b \geq 0$ is sufficient for k' to run for office and be elected for $t = 2$ since $a_k < 0$.

Therefore $b \geq 0$ is a sufficient condition for politicians to run for office in $t = 2$.

We next focus on the first period. The ex ante expected life-time utility of voters is

$$\begin{aligned} V_i = \quad & 2ln((1-\bar{\tau})y) + \gamma eK_1 + p^0\gamma(e+\frac{A}{2})K_2 + p^1\gamma(e+\widetilde{\widetilde{a}}_k)K_2 \\ & +(1-p^0-p^1)\gamma eK_2 - p^1\gamma(e+\widetilde{\widetilde{a}}_k)s(1-f)K_1 \end{aligned}$$

The ex ante expected life-time utility of a candidate running for election at $t = 1$ is

$$\begin{aligned} V_k = \quad & \frac{1}{2}[2ln((1-\bar{\tau})y) + b + \gamma eK_1 + p^0\{b+\gamma(e+\frac{A}{2})K_2\} + p^1\{b+\gamma(e+\widetilde{\widetilde{a}}_k)K_2\} \\ & +(1-p^0-p^1)\gamma eK_2 - p^1\gamma(e+\widetilde{\widetilde{a}}_k)s(1-f)K_1] \\ & +\frac{1}{2}[2ln((1-\bar{\tau})y) + \gamma eK_1 + p^0\gamma(e+\frac{A}{2})K_2 + p^1\gamma(e+\widetilde{\widetilde{a}}_k)K_2 \\ & +(1-p^0-p^1)\{b+\gamma eK_2\} - p^1\gamma(e+\widetilde{\widetilde{a}}_k)s(1-f)K_1] \\ V_k = \quad & 2ln((1-\bar{\tau})y) + \gamma eK_1 + p^0\gamma(e+\frac{A}{2})K_2 + p^1\gamma(e+\widetilde{\widetilde{a}}_k)K_2 \\ & +(1-p^0-p^1)\gamma eK_2 - p^1\gamma(e+\widetilde{\widetilde{a}}_k)s(1-f)K_1 + b \\ V_k = \quad & V_i + b \end{aligned}$$

Therefore for any $b \geq 0$, both politicians would prefer to run for office in $t = 1$. Hence $b \geq \bar{b}_1 = 0$ is a sufficient condition for politicians to run for office in both periods under elections alone or with GDTCs.

Step 2: Incentives to shift output

Next we consider the office holder's decision to shift output to the second period with a view to determining $\bar{b}_2$. Assume that politician k has been elected for office in the first period and that he has learned his ability. Office holders with $a_k \in [2\widetilde{\widetilde{a}}_k, 0)$ shift output as they will only be reelected with certainty if they shift output. Assume that office holder k's ability is such that shifting output is critical for his reelection. If he shifts output his expected utility in the second period is

$$V_{k2}^{*} = ln((1-\bar{\tau})y) + b + \gamma(e + a_k)K_2 - s(1-f)\gamma(e + a_k)K_1$$

His expected utility if he does not shift output is

$$V_{k2}^{D} = ln((1-\bar{\tau})y) + \gamma e K_2$$

As $a_k \in [2\widetilde{\widetilde{a}}_k, 0)$, the office holder will shift output if

$$b \geq \quad s(1-f)\gamma(e + 2\widetilde{\widetilde{a}}_k)K_1 - \gamma 2\widetilde{\widetilde{a}}_k K_2$$

Substituting $\widetilde{\widetilde{a}}_k = -\frac{fseK_1}{2(K_2+fsK_1)}$

$$\begin{aligned} b \geq &\quad s(1-f)\gamma e K_1 + \frac{\gamma f s e K_1}{(K_2 + fsK_1)}\left[K_2 - s(1-f)K_1\right] \\ b \geq &\quad \frac{\gamma e s K_1 K_2}{K_2 + fsK_1} \end{aligned}$$

As debt levels vary between 0 and $\bar{d}_1$, let

$$\bar{b}_2 = \max_{d_1 \in [0,\ \bar{d}_1]} \frac{\gamma e s K_1 K_2}{K_2 + fsK_1}$$

We note that $\bar{b}_2$ is finite, as $\frac{\gamma e s K_1 K_2}{K_2 + fsK_1}$ is bounded on $[0,\ \bar{d}_1]$.

Step 3: Incentives to honor GDTC

Suppose that politician k has been elected to office in $t = 1$ by offering a GDTC with d_{k1}^{c}.

Since k does not know his ability when he chooses d_1, his expected utility from violating the GDTC by selecting $d_1 > d^c_{k1}$ is

$$\begin{aligned}
V_k = &\quad 2ln((1-\bar{\tau})y) + b + \gamma e(\overline{\tau}y + d_1) + \gamma e(\overline{\tau}y - d_1) \\
V_k = &\quad 2ln((1-\bar{\tau})y) + b + 2\gamma e\overline{\tau}y
\end{aligned}$$

We observe that the level of d_1 is irrelevant for the utility of a politician who violates a GDTC. His expected utility from choosing $d_1 = d^c_{k1}$ is

$$\begin{aligned}
V_k = &\quad 2ln((1-\bar{\tau})y) + b + \gamma e(\overline{\tau}y + d^c_{k1}) + p^0[b + \gamma(e + \frac{A}{2})(\overline{\tau}y - d^c_{k1})] \\
&+ p^1[b + \gamma(e + \widetilde{\widetilde{a}}_k)(\overline{\tau}y - d^c_{k1})] + (1 - p^0 - p^1)\gamma e(\overline{\tau}y - d^c_{k1}) \\
&- p^1\gamma(e + \widetilde{\widetilde{a}}_k)s(1-f)(\overline{\tau}y + d^c_{k1})
\end{aligned}$$

The office holder will always honor the GDTC if

$$b \geq \quad \frac{\gamma}{(p^0 + p^1)}\left[p^1(e + \widetilde{\widetilde{a}}_k)s(1-f)(\overline{\tau}y + d^c_{k1}) - p^0\frac{A}{2}(\overline{\tau}y - d^c_{k1}) - p^1\widetilde{\widetilde{a}}_k(\overline{\tau}y - d^c_{k1})\right]$$

As debt levels vary between 0 and $\bar{d}_1$, let

$$\bar{b}_3 = \quad \max_{d^c_{k1} \in [0,\ \bar{d}_1]} \frac{\gamma}{(p^0 + p^1)}\left[p^1(e + \widetilde{\widetilde{a}}_k)s(1-f)(\overline{\tau}y + d^c_{k1}) - (p^0\frac{A}{2} + p^1\widetilde{\widetilde{a}}_k)(\overline{\tau}y - d^c_{k1})\right]$$

We note that $\bar{b}_3$ is finite, as the above expression is bounded on $[0,\ \bar{d}_1]$.

Step 4: Determination of $\bar{b}$

We set $\bar{b} = \max\{\bar{b}_1, \bar{b}_2, \bar{b}_3\}$ and obtain

$$\bar{b} = \max\{0, \bar{b}_2, \bar{b}_3\}$$

Step 5: Condition for b in Section 5

We can perform the same exercise for the lower bound of b in Section 5. Assume that politician k has been elected to office in $t = 1$ by offering a GDTC with d^c_{k1}. Analogous to the proof of Proposition 5, we consider the following two cases in the good state and the bad state of the economy.

- **Case 1:** If $d^c_{k1} < d^{crit}_1$

(i) In the good state of the economy income is y^h. To be eligible to stand for reelection, the office holder must restrict the debt level to d^c_{k1}. His expected utility from selecting d^c_{k1} is

$$\begin{aligned} V_k = \quad & b + ln((1-\bar{\tau})y^h) + \gamma e(\bar{\tau}y^h + d^c_{k1}) + p^0 \left\{ b + \gamma(e + \frac{A}{2})(\bar{\tau}y - d^c_{k1}) \right\} \\ & + p^1 \left\{ b + \gamma(e + \tilde{\tilde{a}}_k)(\bar{\tau}y - d^c_{k1}) \right\} - p^1 \gamma(e + \tilde{\tilde{a}}_k)(\bar{\tau}y^h + d^c_{k1})s(1-f) \\ & + (1 - p^0 - p^1)\gamma e(\bar{\tau}y - d^c_{k1}) + ln((1-\bar{\tau})y) \end{aligned}$$

His expected utility from selecting d_1^{crit} is

$$V_k = \quad b + ln((1-\bar{\tau})y^h) + \gamma e(\bar{\tau}y^h + d_1^{crit}) + ln((1-\bar{\tau})y) + \gamma e(\bar{\tau}y - d_1^{crit})$$

He will select d^c_{k1} in the good state of the economy if

$$b \geq \frac{\gamma}{(p^0 + p^1)} \left[p^1(e + \tilde{\tilde{a}}_k)s(1-f)(\bar{\tau}y^h + d^c_{k1}) - (p^0 \frac{A}{2} + p^1 \tilde{\tilde{a}}_k)(\bar{\tau}y - d^c_{k1}) \right]$$

We denote by $\bar{b}_4$ the maximum of the right-hand side for every d^c_{k1} with $d^c_{k1} < d_1^{crit}$. We note that $\bar{b}_4$ is finite since the right-hand side viewed as a function of d^c_{k1} is bounded on the relevant interval $[0, \bar{d}_1]$. Hence for $b \geq \bar{b}_4$ the office holder will select d^c_{k1} in the good state of the economy if $d^c_{k1} < d_1^{crit}$.

(ii) In the bad state of the economy, income is y^l. Since $\gamma(e - A)\bar{\tau}y^l < g^{min}$ there is a positive probability that $g_1 < g^{min}$ if the office holder selects a debt level lower than d_1^{crit}. This probability for $d_1 = d^c_{k1}$ is denoted by $q > 0$. Therefore the office holder's expected utility from selecting d^c_{k1} is

$$\begin{aligned} V_k = \quad & b + ln((1-\bar{\tau})y^l) + q(-\infty) + (1-q)\gamma e(\bar{\tau}y^l + d^c_{k1}) + ln((1-\bar{\tau})y) \\ & + p^0 \left\{ b + \gamma(e + \frac{A}{2})(\bar{\tau}y - d^c_{k1}) \right\} + p^1 \left\{ b + \gamma(e + \tilde{\tilde{a}}_k)(\bar{\tau}y - d^c_{k1}) \right\} \\ & - p^1 \gamma(e + \tilde{\tilde{a}}_k)(\bar{\tau}y^l + d^c_{k1})s(1-f) + (1 - p^0 - p^1)\gamma e(\bar{\tau}y - d^c_{k1}) \\ V_k \rightarrow \quad & -\infty \end{aligned}$$

The office holder must select at least a debt level of d_1^{crit} to ensure that $q = 0$. However this means that he is not allowed to run for reelection. Therefore his expected utility from selecting d_1^{crit} is

$$V_k = \quad b + ln((1-\bar{\tau})y^l) + \gamma e(\bar{\tau}y^l + d_1^{crit}) + ln((1-\bar{\tau})y) + \gamma e(\bar{\tau}y - d_1^{crit})$$

Accordingly, for any $b < \infty$, the office holder will select d_1^{crit} in the bad state of the economy.

- **Case 2:** If $d_{k1}^c \geq d_1^{crit}$

  The office holder is able to guarantee minimal public good provision as long as he selects a debt level higher than d_1^{crit}. Also he will be allowed to stand for reelection as long as he limits the debt level to d_{k1}^c. Since $d_{k1}^c \geq d_1^{crit}$ he selects the highest debt level that allows him to stand for reelection in order to increase his relection chances. We find the lower bound on b for this condition. k's expected utility from selecting d_{k1}^c is

$$
\begin{aligned}
V_k = \quad & b + ln((1-\bar{\tau})y^x) + \gamma e(\bar{\tau} y^x + d_{k1}^c) + p^0 \left\{ b + \gamma(e + \frac{A}{2})(\bar{\tau} y - d_{k1}^c) \right\} \\
& + p^1 \left\{ b + \gamma(e + \widetilde{\widetilde{a}}_k)(\bar{\tau} y - d_{k1}^c) \right\} - p^1 \gamma (e + \widetilde{\widetilde{a}}_k)(\bar{\tau} y^x + d_{k1}^c) s(1-f) \\
& + (1 - p^0 - p^1)\gamma e(\bar{\tau} y - d_{k1}^c) + ln((1-\bar{\tau})y)
\end{aligned}
$$

  where $x = h$ with probability p and $x = l$ with probability $1-p$.

  As shown in the proof of Proposition 4, p^1 and $\widetilde{\widetilde{a}}_k$ change with the debt level such that $\frac{dp^1}{dd_1} > 0$ and $\frac{d\widetilde{\widetilde{a}}_k}{dd_1} < 0$. Let $p^1 = p^1_{crit}$ and $\widetilde{\widetilde{a}}_k = \widetilde{\widetilde{a}}_k^{\,crit}$ when $d_1 = d_1^{crit}$. The office holder's expected utility from selecting $d_1 = d_1^{crit}$ is

$$
\begin{aligned}
V_k = \quad & b + ln((1-\bar{\tau})y^x) + \gamma e(\bar{\tau} y^x + d_1^{crit}) + ln((1-\bar{\tau})y) \\
& + p^0 \left\{ b + \gamma(e + \frac{A}{2})(\bar{\tau} y - d_1^{crit}) \right\} + p^1_{crit} \left\{ b + \gamma(e + \widetilde{\widetilde{a}}_k^{\,crit})(\bar{\tau} y - d_1^{crit}) \right\} \\
& - p^1_{crit} \gamma (e + \widetilde{\widetilde{a}}_k^{\,crit})(\bar{\tau} y^x + d_1^{crit}) s(1-f) + (1 - p^0 - p^1_{crit})\gamma e(\bar{\tau} y - d_1^{crit})
\end{aligned}
$$

  The office holder will select d_{k1}^c if

$$
\begin{aligned}
b \; \geq \quad & \frac{\gamma}{(p^1 - p^1_{crit})} \Bigg\{ (d_{k1}^c - d_1^{crit}) p^0 \frac{A}{2} \\
& - p^1 \left[\widetilde{\widetilde{a}}_k (\bar{\tau} y - d_{k1}^c) - (e + \widetilde{\widetilde{a}}_k)(\bar{\tau} y^x + d_{k1}^c) s(1-f) \right] \\
& + p^1_{crit} \left[\widetilde{\widetilde{a}}_k^{\,crit} (\bar{\tau} y - d_1^{crit}) - (e + \widetilde{\widetilde{a}}_k^{\,crit})(\bar{\tau} y^x + d_1^{crit}) s(1-f) \right] \Bigg\}
\end{aligned}
$$

  We use $\bar{b}_5$ to denote the maximum of the right-hand side for every d_{k1}^c with $d_{k1}^c \geq d_1^{crit}$. We note that $\bar{b}_5$ is finite since the right-hand side viewed as a function of d_{k1}^c is bounded on the relevant interval $[0, \bar{d}_1]$. Hence for $b \geq \bar{b}_5$ the office holder will select d_{k1}^c in both the good and bad states of the economy if $d_{k1}^c \geq d_1^{crit}$.

- **Overall condition for** b

  For Section 5, a sufficient condition for b is

  $$\max\{\bar{b}, \bar{b}_4, \bar{b}_5\}$$

  where $\bar{b}$ has been determined in Step 4.

Appendix C: List of Notations

Symbol	Meaning
i	$i \in [0,1]$ identifies each individual voter
k, k'	k, k' identifies each individual candidate competing for office
y	income of each individual in a period
c_t	consumption of the private good in period $t = 1, 2$
g_t	level of the public good in period $t = 1, 2$
a_k	ability of office holder k
A	a_k is a random variable uniformly distributed on $[-A, A]$
e	a constant that fulfills $e > A$
K_t	the amount of public investments in the public good in period $t = 1, 2$
γ	the productivity of public investments
τ_t	the tax rate in period $t = 1, 2$
d_1	the debt level chosen by the government in period 1
r	the interest rate to be paid on government debt
$\overline{\tau}$	the revenue maximizing tax rate where the Laffer curve attains its maximum
$U^P(g_t)$	instantaneous utility of citizens from the public project
g^{min}	the minimal provision of public goods that is essential for the functioning of the state and the economy
s	exogenously given variable that denotes the fraction of output shifted from period 1 to period 2
f	indicates the fraction of the output shifted from period 1 to period 2 that can be realized in period 2
ϵ_k	denotes the output-shift decision of office holder k in period 1. ϵ_k equals 1 if output is shifted and 0 otherwise.
β	the discount factor of citizens and politicians
V_x	the expected utility of voters or politicians $x = i, k$
b	private benefits derived by politicians from holding office
$\overline{d}_1$	the highest debt level that can be chosen by the office holder in period 1 to ensure that $g_2 \geq g^{min}$
V^*_{k2}	the expected utility of office holder k at the beginning of period 2
$V^D_{k'2}$	the expected utility of politician k' who has lost the election at the beginning of period 2

Symbol	Meaning
V_{i2}	the expected utility of voters at the beginning of period 2
p^0	the office holder's estimate of the probability of being reelected without choosing output shift policies
p^1	the office holder's estimate of the probability of being reelected only by choosing output shift policies
$\widetilde{a}_k$	office holder k's expected level of ability conditional on the fact that he is reelected
d_1^*	the equilibrium debt level if only elections are present
τ_t^*	the equilibrium tax level in period $t = 1, 2$ if only elections are present
K_t^*	the equilibrium amount of public investments in the public good in period $t = 1, 2$ if only elections are present
d_{k1}^c	government debt threshold stipulated by candidate k for period 1 in his contract
d_1^s	the socially optimal debt level
$\hat{d}_{k1}^c$	the equilibrium government debt threshold in GDTCs
$\widetilde{\widetilde{a}}_k$	voters' expected level of ability of office holder k if he is reelected by shifting output
y^x	income in the first period resulting from a macroeconomic shock $x = h, l$ with $y^h > y$ and $y^l < y$
p	$y^h > y$ occurs with probability p, and $y^l < y$ occurs with probability $1 - p$
d_1^{crit}	the welfare-optimal debt level under a negative macroeconomic shock

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