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Joseph Francois, Johannes Kepler Universität Linz and CEPR
Hugo Rojas-Romagosa, CPB Netherlands Bureau for Economic Policy Analysis

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Centre for Economic Policy Research
53–56 Gt Sutton St, London EC1V 0DG, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

Household Inequality, Social Welfare, and Trade*

Working with Sen social welfare functions (meaning explicit separability between mean income and income dispersion), we develop a generalized dual approach to tracking household inequality aspects of social welfare in general equilibrium. We highlight how household equity can be examined analytically alongside production efficiency in duality-based models, using our dual framework to explore potential trade-offs between efficiency and equity effects of trade policy. Our results complement the set of standard inequality results in trade theory focused on functional rather than household inequality. We also find that the relative distributional impact of tariffs on welfare is conditional on the initial level of inequality.

JEL Classification: D31, D72, F13 and O15
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Joseph Francois
Department of Economics
Johannes Kepler University Linz
Altenbergerstraße 69
A-4040 Linz
AUSTRIA
Email: joseph.francois@jku.at

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Hugo Rojas-Romagosa
Bureau for Economic Policy Analysis, CPB
Postbus 80510
2508 GM The Hague
THE NETHERLANDS
Email: H.Rojas-Romagosa@cpb.nl

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1 Introduction

A classic set of results in the theory of international trade involves the linkages between goods prices and factor prices. Indeed, basic theorems on these linkages in the Heckscher-Ohlin-Samuelson (HOS) model – Jones (1965; 1975), Mussa (1974) and Lloyd (2000) – have served as the theoretical underpinnings for the now massive literature on globalization and relative wages (cf. Feenstra and Hanson, 2003)[1]. Inequality concerns have long been theoretically regarded as a determinant of trade policy behaviour (Baldwin, 1989). More recently, policy interest has driven applied research on linkages between economic integration and household inequality.[2] This literature includes both econometric and numerical modeling approaches, building on the growing availability of comparable household survey data.[3] However, the bulk of the combined literature is focused on interactions between integration and the functional distribution of income.

In this paper, we develop a dual approach to analyzing general equilibrium (GE) relationships between trade policy and the household (as distinct from the functional) distribution of income. This includes the introduction of a social welfare function into the dual GE system grounded in the literature on social welfare and inequality. In particular, the model is built from individual household preferences and is explicitly separable between mean income and income dispersion.[4] What we highlight here is how general equilibrium distributional aspects of social welfare related to import protection may be examined alongside corresponding efficiency aspects in a dual framework. An advantage of the dual approach is that it ultimately leads to a mapping of policy-induced price changes into household inequality for a broad class of models that may have potential for empirical application.

The main contribution of this paper is that we construct a concise theoretical framework where the efficiency and equity effects of trade policy can be jointly analyzed. In addition, we find that the relative distributional impact of tariffs is conditional on the initial level of inequality. This result is clearly stated for the HOS case where we have two factors and two goods, but also applies for the Ricardo-Viner (RV) specific-factors framework. Thus, the distributional impact of trade policy is not only conditional on how the relative factor incomes are changing, but also, on how these factors are initially distributed

[1] Comprehensive surveys are also provided by Richardson (1995) and Cline (1997).

[2] Recent literature surveys on this topic are provided by Anderson (2005) and Goldberg and Pavcnik (2007).

[3] See Edwards (1997), Spilimbergo et al. (1999), Barro (2000), Higgins and Williamson (2002), Winters (2000), Winters et al. (2004), Hertel et al. (2004), Topalova (2007), Valenzuela et al. (2009), and Do and Levchenko (2009). The recent computational literature, including Hertel et al. (2004), has pioneered integration of household survey data and related measures in computational models, though welfare measurement in the computational literature, even with linkages to household data, is based on mean incomes.

[4] Also relevant is Anderson (2002). While Anderson's paper is focused on a different set of issues (his goal is to explore the public finance concept of the marginal cost of funds in general equilibrium), he does use ethical weights to stress the decomposition of general equilibrium welfare effects of raising public funds into a composition (i.e. efficiency) effect and a distributional effect.

between households. We find that, in general, for economies with initial low inequality levels the distributional effects of trade policy are smaller than in economies with initial higher inequality. This theoretical result is relevant for both empirical estimations and for economic policy analysis.[5] The dual approach also offers a possible estimating framework for decomposing policy-induced price changes into household equality effects for a broad class of general equilibrium models.

Although we focus our attention on import tariffs, the main message that follows from this approach can be applied in a more general context of trade policy instruments. The precise distributional and efficiency components may change, but in essence the trade-off and interrelation between both economic outcomes is still present. The dual approach allows us to be relatively general in terms of model structure, while also allowing a more parsimonious representation of basic relationships in the n-sector case than generalizations based on a primal approach. We follow Bourguignon and Morrisson (1989; 1990) and use an ownership matrix that allows us to move from functional to household income. We then obtain a dual representation of the household income distribution in terms of endowments, tariffs and the ownership structure. Using this analytical framework, we analyze the impact of trade and tariffs. Treating equity issues as relevant, we work with Sen (1974) type social welfare functions. This approach lets us work from micro-foundations to embed inequality indexes in the social welfare function. In particular, we work with the widely used Gini coefficient and with the Atkinson (1970) family of inequality indexes, although other indexes may be employed. Using this framework we are able to decompose the general equilibrium import protection effects into real income level and dispersion changes.

The paper is organized as follows. Section 2 develops a formal representation of social welfare inclusive of income inequality. In Section 3, we embed this social welfare function into a dual general equilibrium trade model. We also develop the equilibrium representation of inequality, based on the dual representation of general equilibrium system fundamentals. Section 4 then explores linkages between trade policy, inequality, and welfare. Using two specific-cases (HOS and the RV specific-factors trade models) it also examines theoretical linkages between country size, development, policy, and inequality. We conclude in Section 5.

[5]For instance, using a political economy setting, Francois and Rojas-Romagosa (2005) find that the factors driving protection are manifested not only in special interest politics, but also through the direct impact of inequality on a government's objective function. Therefore, equity considerations may serve to counter lobbying interests in both capital-rich and capital-poor countries, though with an opposite marginal impact on the final policy outcome. This also results in a protectionist bias on the part of welfare maximizing governments in capital rich countries based on inequality aversion, rather than the risk aversion-based protectionist bias identified by Fernandez and Rodrik (1991).

2 Defining social welfare as including inequality

Our goal in this section is to develop a functional linkage between inequality and aggregate (social) welfare. This is then be integrated in the next section into a dual general equilibrium trade model. A critical condition for inequality to have a meaningful link to aggregate (social) welfare is that the utility function be strictly concave with respect to income. Additionally, for tractability we prefer to work with a social welfare function that is symmetric and additively separable in individual utilities.

The existence of social welfare functions depends crucially on the possibility to compare interpersonal utility levels. One such possibility is offered by the 'veil of ignorance' approach first proposed by Harsanyi (1953; 1955) and further developed by Rawls (1971), where we rank different individual situations not knowing which would be the actual situation. As stated by Sen (1997) this interpersonal comparison can be defined as those situations where we make judgements of the type:

> "I would prefer to be person A rather than person B in this situation" and "while we do not really have the opportunity (or perhaps the misfortune, as the case may be) of in fact becoming A or B, we can think quite systematically about such a choice, and indeed we seem to make such comparisons frequently".

Because GDP per capita is the most common indicator of social welfare, the 'veil of ignorance' approach supports the use of an inequality measure to complement GDP per capita comparisons. If we do not know which individual household we are in a specific country, then the expected utility becomes a function of mean income and the personal distribution of income. How we evaluate the probability of receiving any given income is then determined by the functional representation of the utility function and more specifically by the degree of concavity of this function. In this context, a natural extension of cross-country welfare comparisons is to complement GDP per capita levels with some measure of inequality.[6]

Under the social welfare approach to income distribution measurement, inequality is associated with the dispersion of income around the mean. This raises two measurement problems. The first is that we cannot generally rely on first moment-based indicators. The second is that even though the concepts of Lorenz-dominance and general Lorenz-dominance (Shorrocks, 1983)are accepted as ways to impartially rank two different distributions[7], in many cases the Lorenz-curves intersect at least once, so that we obtain incomplete ranking of distributions. To solve both these problems, inequality indexes are usually

[6] This approach was formally treated by Sen (1976).
[7] See Lambert (1993) for details.

used to rank distributions in indeterminate cases and to provide a summary variable that can be used in empirical models. While the most commonly used is the Gini coefficient, most inequality measures are implicitly based on a social welfare function (Dalton, 1920; Kolm, 1969; Atkinson, 1970). As such, there is no perfect index, and any index has built in social preferences.

In this paper, we employ two representations of household utility and social welfare. Both reflect Sen's (1974) preferred definition of social welfare as:

$$SW = \overline{y}\left(1 - I\right) \tag{1}$$

where SW is the social welfare, $\overline{y}$ is mean income, and I is an index of inequality.

Starting with constant relative risk aversion (CRRA) preferences yields the well-known Atkinson inequality index directly as a natural metric for a mapping from income distribution to social welfare (see Atkinson, 1970). In this sense, Atkinson's index fits naturally into Sen's proposed social welfare function.

Sen actually offered equation (1) as defined with respect to the Gini coefficient. In this case, the social welfare function is axiomatic, in that we do not have an obvious mapping –through aggregation– from individual preferences to an aggregate social welfare function. This follows because the social welfare function is then rank sensitive. We work with both the Atkinson index and Gini coefficient here.

2.1 The Atkinson index-based social welfare function

Formally, we define a composite consumer good over the range of all consumption goods, which follows from a linear homothetic aggregation function. As such, cost minimization yields a composite consumer price index. This is defined over all consumer prices p_c.

$$p_c = f\left(p\right) \tag{2}$$

Household utility u^h is defined as a function of household consumption of the composite consumer good c^h:

$$u^h = \psi\left(c^h\right) \tag{3}$$

We next map aggregate individual utility to aggregate welfare ϕ, which is defined as the sum of household utility,

$$\phi = \sum_h u^h \tag{4}$$

while aggregate consumption c is the sum of household consumption.

$$c = \sum_h c^h \tag{5}$$

We assume that the function ψ is CRRA:[8]

$$\psi\left(c^h\right) = \begin{cases} \frac{\left(c^h\right)^{1-\theta}}{1-\theta} & \text{if } \theta \neq 1 \\ \ln c^h & \text{if } \theta = 1 \end{cases} \tag{6}$$

In general, we assume that $\theta > 0$, and in this paper we focus on the case where $\theta \neq 1$.[9] We employ a simple linear transformation, and are then able to define a social welfare index in per-capita terms.

$$SW_A = (1-\theta)\, n^{-1} \sum \psi = \frac{1}{n} \sum_h \left(c^h\right)^{1-\theta} \tag{7}$$

Simple manipulation then yields social welfare as a function of per-capita income $\overline{y}$, consumer prices, and income equality.

$$SW_A = \left(\frac{\overline{y}}{p_c}\right)^{1-\theta} E_A \tag{8}$$

With some further manipulation, our equality measure E_A can be mapped directly to the Atkinson index of income inequality, yielding a Sen-type social welfare function. In particular, taking the definition of the Atkinson index, we have the following relationships between the Atkinson index I_A, E_A, and social welfare.

$$I_A = 1 - \left[\frac{1}{n} \sum_h \left(\frac{y^h}{\overline{y}}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} = 1 - E_A^{\frac{1}{1-\theta}} \tag{9}$$

$$SW_A = \left[\frac{\overline{y}}{p_c}\left(1 - I_A\right)\right]^{1-\theta} \tag{10}$$

Note that as $\theta \to 0$ only average income matters, rather than income inequality. Alternatively, when $\theta \to \infty$, then $SW_A = \min\left(y^h\right)$ and we have the extreme Rawlsian maximin social welfare function, where the income level of the poorest individual is the only relevant variable and average income is

[8] In the present context, constant relative inequality aversion (CRIA) is a better label and acronym.

[9] One gets the same basic results with log preferences. Estimates in the macro literature are that θ is less than 1.

unimportant. Moreover, for a given distribution (measured as shares of total income) we have declining marginal utility of income.

2.2 The Gini index-based social welfare function

The Gini coefficient is defined as twice the area between the Lorenz curve and the 45-degree line. As such, $(1 - G)$ is then twice the area below the Lorenz curve. Formally, this index is defined as follows:

$$I_G = 1 + \frac{1}{n} - \frac{2}{n^2\overline{y}}\left(y^1 + 2y^2 + ... + ny^n\right) = 1 + \frac{1}{n} - \left(\frac{2}{\overline{y}n^2}\sum_h hy^h\right) \tag{11}$$

$$SW_G = \frac{\overline{y}}{p_c}\left(1 - I_G\right) \tag{12}$$

where we have arranged households so that $y^1 \geq y^2 \geq ... \geq y^n$. Unlike the Atkinson-based social welfare function, the Gini-based social welfare function embodies asymmetry not on specific individuals, but rather on relative income rankings. This ranking provides the concavity of the utility function with respect to income. The higher the income in the ranking, the less social weight it has. At the same time, equation (12) is linear in average income. As such, SW_G is relatively more sensitive to mean income than SW_A and less sensitive to inequality.

3 Inequality and trade in general equilibrium

To explore the interaction between production, trade and trade policy, and inequality, we work with a modified dual representation of trade in general equilibrium (Dixit and Norman, 1980). To do so, we first adopt the following additional set of assumptions:

- Rational behavior by households and firms.
- Complete and perfectly competitive markets.
- Convex technology, with neoclassical production functions.
- Goods are tradable and factors are not.
- Every household has the same neoclassical technology for producing the composite consumption good.

Given these assumptions, we are able to define the core general equilibrium system for demand and production in terms of expenditure and revenue functions, with expenditure defined in terms of the composite consumption good. Social welfare then follows as a set of side equations from the core general equilibrium system.

3.1 The core general equilibrium system

Because we assume that all households have the same consumption technology defined with respect to the composite consumption good, we can drop the household index from consumption and represent aggregate expenditure as a function of aggregate consumption and prices:

$$e(p, c) = c \cdot f(p) \tag{13}$$

On the production side, we assume standard neoclassical production functions with constant returns to scale: $x_i = g_i(v_{ji})$, where $g_i(\cdot)$ is the production function for good i and v_{ji} is the use of factor j in the production of good i. Defining unit input coefficients as a_{ji} we also obtain: $1 \leq g_i(a_{ji})$. Endowment constraints are then $\sum a_{ji} x_i \leq \overline{v}_j$. From these conditions, we can define the economy-wide revenue function with respect to goods prices and endowments. This is represented in equation (14).

$$r(p, v) = \max_{x_i, a_{ji}} \left\{ \sum_i p_i x_i \mid \sum_i a_{ji} x_i \leq \overline{v}_j \text{ and } 1 \leq g_i(a_{ji}) \ \forall i, j \right\} \tag{14}$$

From the envelope theorem and the properties of the revenue function r, factor incomes and goods production can be expressed in terms of the value of the partial derivatives of the revenue function, evaluated at the equilibrium set of prices:

$$\frac{\partial r(p, v)}{\partial v_j} = w_j = w_j(p, v) \qquad \forall j \tag{15}$$

$$\frac{\partial r(p, v)}{\partial p_i} = x_i = x_i(p, v) \qquad \forall i \tag{16}$$

Taking equations (15) and (16) in conjunction with equations (13) and (14), we can write the general equilibrium system for production, consumption, and trade as follows:[10]

$$\begin{aligned} c^h f(p) &= \left(\sum_j w_j(p,v) \cdot v_j^h\right) + \omega_\tau^h \cdot \tau \cdot m \qquad \forall h & (17) \\ m &= \sum_h c^h \cdot f(p) - x(p,v) & (18) \\ e(p,c) &= \sum_h \left[\left(\sum_j w_j(p,v) \cdot v_j^h\right) + \omega_\tau^h \cdot \tau \cdot m\right] & (19) \\ p &= P^* + \tau = 1 + \tau & (20) \end{aligned}$$

In equations (17) – (20), we have assumed the home country imposes a tariff of τ on imports from the rest of the world, while world prices are normalized to one. In addition, ω_τ^h is the household share of the tariff revenue and v_j^h is the household ownership share of factor j. In the first equation, household consumption is equal to the household budget. Equation (18) defines imports on which tariff revenue is generated and equation (19) sets economy wide expenditure equal to national income. Together, the system of four equations has an equally dimensioned set of unknowns: c^h, m, e and p.

3.2 Household inequality

As explained earlier, the recent literature on trade and the distribution of income has focused on the functional distribution of income. The functional distribution of income is also an important building block here for the representation of the household distribution of income. In equation (21) we define factor incomes s, which follow directly from the endowment stock and the properties of the revenue function.

$$s_j = r_{vj}(p,v)\, v_j = w_j v_j \tag{21}$$

Thus, the functional distribution of income is a function of equilibrium prices, preferences, the production technology and the endowment set. In reduced form, the functional distribution of income $F(s)$ is then an artifact of the equilibrium matching of preference and the technology set, given our endowment vector.

$$F(s) = F(p,v) \tag{22}$$

[10]A two-country general equilibrium system can readily be formalized using the same framework.

Using factor incomes w_j and the household ownership share of production factors, ω_j^h we can readily obtain household income. In addition, we include the assignment of import tariff revenue, again represented by a household share parameter. Equation (23) presents the basic definition of household income in terms of its primary components.

$$y^h = \left(\sum_j w_j v_j \omega_j^h\right) + \omega_\tau^h \cdot \tau \cdot m \tag{23}$$

$$c^h = \frac{y^h}{p_c} \tag{24}$$

where $1 \geq \omega^h \geq 0$ and $\sum \omega_j^h = \sum \omega_\tau^h = 1$. In reduced form, the personal distribution of income $F(y)$ is a consequence of the elements affecting the functional distribution and the $h \times j$ ownership matrix of coefficients ω_j^h, represented by Ω:

$$F(y) = F(p, v, \Omega) \tag{25}$$

Note that social welfare is ultimately a function of the ownership matrix in the economy, while the impact of trade policy will then depend on the interaction of the underlying economic structure and the ownership matrix.

3.3 Inequality indexes with system fundamentals

We can write our social metrics of the distribution of income –the Atkinson and Gini indexes– in terms of system fundamentals. Making a substitution from (23) into (9) and (11), we obtain the following equations:

$$I_A = 1 - \left\{ \frac{1}{n} \sum_h \left[\frac{n\left(\sum_j w_j v_j \omega_j^h\right) + n\omega_\tau^h \cdot \tau \cdot m}{y} \right]^{1-\theta} \right\}^{\frac{1}{1-\theta}}$$

$$I_A = 1 - \left\{ n^{-\theta} \sum_h \left[n^{-1} + \sum_j \beta_j \left(\omega_j^h - n^{-1}\right) \right]^{1-\theta} \right\}^{\frac{1}{1-\theta}} \tag{26}$$

$$I_G = 1+\frac{1}{n}-\frac{2}{n^2}\sum_h h\left[\frac{n\left(\sum_j w_j v_j \omega_j^h\right)+n\omega_\tau^h\cdot\tau\cdot m}{y}\right]$$

$$I_G = 1+\frac{1}{n}-\frac{2}{n}\sum_h h\left[n^{-1}+\sum_j \beta_j\left(\omega_j^h-n^{-1}\right)\right] \tag{27}$$

where $\beta_j = \frac{w_j v_j}{y}$ represents the national income share of factor j and $\sum_j \beta_j + \frac{\tau\cdot m}{y} = 1$. In what follows, we apply the additional normalization that each household receives an equal share of the tariff revenues, so that $\omega_\tau^h = n^{-1}$.[11]

The ratio of the household's income to per capita income, which accounts for income dispersion, is given by the sum of the differences between the actual ownership share of factors and equal shares for each household. From equations (26) and (27), we can make a substitution back into equations (10) and (12), yielding social welfare itself as a function of system fundamentals.

$$SW_A = \left[\frac{\overline{y}}{p_c}\left(1-I_A\right)\right]^{1-\theta}$$

$$SW_A = \left(\frac{\overline{y}}{p_c}\right)^{1-\theta} n^{-\theta}\sum_h\left[n^{-1}+\sum_j \beta_j\left(\omega_j^h-n^{-1}\right)\right]^{1-\theta} \tag{28}$$

$$SW_G = \frac{\overline{y}}{p_c}\left(1-I_G\right)$$

$$SW_G = \left(\frac{\overline{y}}{p_c}\right)\left\{\frac{2}{n}\sum_h h\left[n^{-1}+\sum_j \beta_j\left(\omega_j^h-n^{-1}\right)\right]-n^{-1}\right\} \tag{29}$$

4 Trade policy, equity, and welfare

From equations (28) and (29) above, social welfare is a function of the first two moments of the household distribution of income. In this setting the inclusion of the Atkinson index and the Gini coefficient provides different ways to weight deviations from mean income and thus, create an income dispersion component in the social welfare function.[12] Because the contributions of the income mean and dispersion components to social welfare are separable in equations (28) and (29), we can decompose the impact of trade policy as well into its impact on per-capita income (an efficiency effect), and its impact on

[11] The distributional impact of tariff revenues can be substantial. This is the emphasis of the paper by Galor (1994), which includes tariffs in his general equilibrium Overlapping-Generations model.

[12] Note that both inequality indexes do not provide a strict variance of income term, but instead, a term that provides a measure of income dispersion around mean income within the social welfare function.

the dispersion of income (a distributional effect). Together, they determine the overall social welfare impact. Formally, differentiating equations (28) and (29) with respect to tariffs, we obtain the following equations:

$$\begin{aligned}\frac{\partial SW_A}{\partial \tau_i} &= (1-\theta)\left[\frac{\overline{y}}{p_c}(1-I_A)\right]^{-\theta}(1-I_A)^{1-\theta}\left(\frac{\partial \overline{y}}{\partial \tau_i}-\frac{\partial p_c}{\partial \tau_i}\frac{\overline{y}}{p_c^2}\right) \\ &\quad -(1-\theta)\left(\frac{\overline{y}}{p_c}\right)^{1-\theta} I_A^{-\theta}\frac{\partial I_A}{\partial \tau_i} \qquad (30)\\ \frac{\partial SW_G}{\partial \tau_i} &= (1-I_G)\left(\frac{\partial \overline{y}}{\partial \tau_i}-\frac{\partial p_c}{\partial \tau_i}\frac{\overline{y}}{p_c^2}\right)-\left(\frac{\overline{y}}{p_c}\right)\frac{\partial I_G}{\partial \tau_i} \qquad (31)\end{aligned}$$

How do we interpret equations (30) and (31)? The efficiency component is well known (see for example, Dixit and Norman, 1980), and is shown here in equation (32). Basically, the impact of tariffs on per-capita income depends on the combination of terms-of-trade and allocation effects (the first set of terms in square brackets in equation (32)), and tariff revenue (the second set of terms).

$$\frac{\partial \overline{y}}{\partial \tau_i}=\frac{1}{n}\sum_h \frac{\partial y^h}{\partial \tau_i}=\frac{1}{n}\left[m\left(1-\frac{\partial p}{\partial \tau_i}\right)+\tau_i\cdot\frac{\partial m}{\partial \tau_i}\right] \qquad (32)$$

For a small country, negative allocation effects outweigh the terms-of-trade effects, so that the impact of the tariff on mean income is strictly negative. Also, for the small country, the impact on the cost of living will be to raise prices. As such, the real mean-income effect is strictly negative for a small country. With a large country, the combined income and cost-of-living effect, or in other words the real income effect of the tariff change as represented by the term $\left(\frac{\partial \overline{y}}{\partial \tau_i}-\frac{\partial p_c}{\partial \tau_i}\frac{\overline{y}}{p_c^2}\right)$ in equations (30) and (31) may be positive or negative depending on the magnitude of terms-of-trade effects.

The impact on household income distribution, the other part of equations (30) and (31), follows from differentiation of equations (26) and (27). This is shown below:

$$\begin{aligned}\frac{\partial I_A}{\partial \tau_i} &= -n^{\frac{-\theta}{1-\theta}}\left\{\sum_h\left[\sum_j \beta_j\left(\omega_j^h-n^{-1}\right)+n^{-1}\right]^{1-\theta}\right\}^{\frac{\theta}{1-\theta}} \\ &\quad \sum_h\left\{\left[\sum_j \beta_j\left(\omega_j^h-n^{-1}\right)+n^{-1}\right]^{-\theta}\left[\sum_j \frac{\partial \beta_j}{\partial \tau_i}\left(\omega_j^h-n^{-1}\right)\right]\right\} \qquad (33)\\ \frac{\partial I_G}{\partial \tau_i} &= -\frac{2}{n}\sum_h h\left[\sum_j \frac{\partial \beta_j}{\partial \tau_i}\left(\omega_j^h-n^{-1}\right)\right] \qquad (34)\end{aligned}$$

Note that we also have an inverse income weighting, by a factor of θ , in equation (33) applied to induced changes in income. The weighting of induced changes in income for the Gini index depends on the ranking of individual households on the relative income scale. Equations (33) and (34) provide an analytical mapping that we believe may prove useful, empirically, for analysis of linkages between policy-induced price changes and standard indexes of inequality (in this case the Atkinson and Gini index). One could apply such a decomposition econometrically, or apply it to adjust summary welfare measures in CGE models to include equity effects and to decompose them.

Close inspection of equations (33) and (34) reveals a more general relationship between inequality and tariffs. In particular, if we define ethical weights ψ, then for a broad class of inequality measures, we have:

$$\frac{\partial I}{\partial \tau_i} = \sum_h \psi^h \left[\sum_j \frac{\partial \beta_j}{\partial \tau_i} \left(\omega_j^h - n^{-1} \right) \right] \tag{35}$$

In this context, assuming we adopt a Sen-type social welfare function so that our ethical weights ψ map to an index of equity E, we then also have:

$$SW = \left(\frac{\bar{y}}{p_c} \right) [1 - I] \tag{36}$$

$$\frac{\partial SW}{\partial \tau_i} = [1 - I] \left(\frac{\partial \bar{y}}{\partial \tau_i} p_c^{-1} - \frac{\partial p_c}{\partial \tau_i} \frac{\bar{y}}{p_c^2} \right) - \left(\frac{\bar{y}}{p_c} \right) \sum_h \psi^h \left[\sum_j \frac{\partial \beta_j}{\partial \tau_i} \left(\omega_j^h - n^{-1} \right) \right] \tag{37}$$

In general, changes in household income depend on the set of factor price changes, filtered by the ownership matrix and our ethical weights, where factor price changes in turn depend on Stolper-Samuelson derivatives and the induced price changes that follow from tariff changes. This is expressed in equation (38), where the term $\frac{\partial \beta_j}{\partial \tau_i}$ depends on system fundamentals and Stolper-Samuelson relationships:

$$\frac{\partial \beta_j}{\partial \tau_i} = \frac{\partial w_j}{\partial p} \frac{\partial p}{\partial \tau_i} \frac{v_j}{y} - \frac{\partial y}{\partial \tau_i} \frac{w_j v_j}{y^2} \tag{38}$$

We can also represent the relationship in elasticity terms: $\varepsilon_{\beta_j,\tau_i} = \varepsilon_{w_j,p} \varepsilon_{p,\tau_i} - \varepsilon_{y,\tau_i}$.

In general terms, the use of Sen-type social welfare functions as in equation (36) conveys a broader spectrum of welfare outcomes related to trade policy. The efficiency and equity effects may not move in the same direction, and thus, the inclusion of equity considerations into welfare analysis can magnify or mitigate the known efficiency effects of trade policies that are a standard result from trade theory. For example, if inequality worsens with import protection, this magnifies the negative efficiency effects of small-country tariff incidence on welfare. Where the efficiency and dispersion effects run in opposite directions, we can have welfare effects that run counter to the message from standard trade theory. This depends on which effect dominates. We formalize this in our first theorem:

Theorem 1 *With a Sen-type social welfare function, import protection (trade liberalization) may improve (reduce) welfare even when average income falls (rises).*[13]

If the induced change in inequality due to trade policy is large enough and of the correct sign, it can offset the impact of the change in average income levels. This all depends on the underlying functional forms in the model and the parameterization of the social welfare function. Even though this becomes a theoretical possibility, the prediction of Theorem 1 is conditional on many parameters of the general equilibrium system, the ownership matrix and the ethical weights. Therefore, to convey more specific results, in the following sections we work with the simplifications embedded in two routinely used trade theory models: Heckscher-Ohlin-Samuelson (HOS) and Ricardo-Viner (RV).

4.1 Equity in the 2x2 HOS Model

For simplicity, we assume that inequality is the result of an uneven distribution of only one of the two factors in a Heckscher-Ohlin-Samuelson trade model. In particular, we assume that the first factor l (unskilled labor) is evenly distributed, i.e. $\left(\omega_l^h - n^{-1}\right) = 0 \; \forall h$. While inequality is purely a function of the allocation of assets, which is the second factor and indexed by k. Note that our discussion in terms of assets includes both the 2x2 capital-labor and 2x2 skilled-unskilled versions of the Heckscher-Ohlin model found in the literature on the functional distribution of income. The interpretation of β_k with physical capital is obvious. If we instead are working with skilled and unskilled labor in the 2x2 model, then the return to skill as an asset is $\beta_k = \left[\beta_s - n_s n^{-1} \beta_u\right]$ where β_s is the income share of skilled labor, interpreted as including both the basic labor and skill component of skilled labor income, and where s and u index skilled and unskilled workers.[14] Substituting skill for capital, we arrive at equation (39).

In this HOS framework, equation (35) can be simplified and the impact of tariff changes on household inequality is given by:

[13] All theorems and corollary proofs are presented in the Appendix.

[14] Formally, assume first that unskilled labor earns w_u and skilled labor earns w_s, where $w_u < w_s$. We can then decompose the skilled labor price into two components, such that $w_s = w_u + (w_s - w_u)$. If we define skill as an asset with return r_k, then we can now define $r_k = (w_s - w_u)$. Viewed this way, all households have been endowed with a claim on income equal to the price of a unit of basic labor earning w_u, while some have also been endowed with a claim on the income of a unit of skill. The distribution of this claim on skill income is then the source of inequality. In share terms, we will have $\beta_s = n_s n_u^{-1} \beta_u + \beta_k$, or, $\beta_k = \beta_s - n_s n_u^{-1} \beta_u$.

$$\frac{\partial I}{\partial \tau_i} = \Psi \frac{\partial \beta_k}{\partial \tau_i} \tag{39}$$

where Ψ is determined by two components: the ethical weights implicit in the inequality index/measurement used (Gini or Atkinson)[15], and the distribution of factors $\left(\omega_k^h - n^{-1}\right)$, which is defined by the ownership matrix Ω. Using equation (33) for the Gini coefficient we have:

$$\Psi = \sum_h \frac{-2h}{n} \left(\omega_k^h - n^{-1}\right) \tag{40}$$

and by equation (34), in the case of the Atkinson index we get:

$$\Psi = -n^{\frac{-\theta}{1-\theta}} \left\{ \sum_h \left[\beta_k \left(\omega_k^h - n^{-1}\right) + 2n^{-1}\right]^{1-\theta} \right\}^{\frac{\theta}{1-\theta}} \sum_h \left\{ \left[\beta_k \left(\omega_k^h - n^{-1}\right) + 2n^{-1}\right]^{-\theta} \left(\omega_k^h - n^{-1}\right) \right\} \tag{41}$$

Since any inequality index gives more weight to the lower parts of the income distribution and lower incomes have assets shares lower than the median, we have that $\Psi > 0$.[16] Thus, equation (39) provides a straightforward approach to analyze the impact of tariffs on inequality.

The results of equation (39) are determined by which sector receives tariff protection. The Heckscher-Ohlin theorem states that imports are of the scarce-factor intensive good.[17] Thus, if we assume that tariffs are levied to protect the importing sector, then we have two cases depending on the relative factor endowment of the country. First, in a rich asset-abundant country, if good 1 uses intensively unskilled labor (l) and good 2 uses assets (k) intensively, then import protection involves tariffs being levied on good 1: $\tau_1 > 0$, and not on good 2: $\tau_2 = 0$. Conversely, in a poor unskilled-labor-abundant country import protection means that $\tau_1 = 0$ and $\tau_2 > 0$. For the remainder of our discussion, we assume that import protection follows the above relation between tariff signs and relative asset-abundance.

Formally, using equations (38) and (39), we can summarize our discussion above with the following theorems about import protection and inequality in the 2x2 Heckscher-Ohlin model.

[15] This is given by the parameter ψ^h in equation (35).

[16] In the case of the Gini coefficient this is easily observed. Recalling that incomes are ranked starting with the highest: $y^1 \geq y^2 \geq ... \geq y^n$, then we have from equation (40) that households with high incomes: $\omega_k^h - n^{-1} > 0$, have low ethical weight $\left(\frac{-2h}{n}\right)$, while low income households have $\omega_k^h - n^{-1} < 0$ and high ethical weights. The same principle applies to the case of the Atkinson index. Formal proofs are provided in the Appendix.

[17] While we do not develop the point fully here, a related implication is that in the two country version of the Heckscher-Ohlin model, import protection by a capital rich country may worsen inequality in its capital poor trading partner, magnifying the negative welfare impact of trade protection on its capital poor trading partner.

Theorem 2 *In a small 2x2 Heckscher-Ohlin economy with inequality determined by an uneven distribution of assets (capital or skills), if assets in the economy are relatively abundant then a new tariff on the unskilled-labor-intensive good causes inequality to fall.*

Theorem 3 *In a small 2x2 Heckscher-Ohlin economy with inequality determined by an uneven distribution of assets (capital or skills), if assets in the economy are relatively scarce then a new tariff on the asset-intensive good causes inequality to rise.*

If we assume that assets (capital or skill) in the 2x2 model are unevenly distributed, then from equation (39), changes in inequality indexes depend strictly on a weighted sum of the change in the share of income going to those assets, $\frac{\partial \beta_k}{\partial \tau_i}$. From the Stolper-Samuelson theorem, the income share of those assets falls with a tariff on the unskilled-labor-intensive good if the economy is asset rich. The contrary happens in asset poor countries when the tariff is levied on the asset-intensive good. Weights are assigned to households that are inversely monotonic in household capital deviations from the average, $\left(\omega_k^h - n^{-1}\right)$ in both the Atkinson and the Gini case. This means that the change in incomes for households holding more capital than average or households holding skilled labor, and hence more income than average, determine the sign of the income effect. As a result, in asset-rich countries import protection leads to a drop in asset income $\left(\frac{\partial \beta_k}{\partial \tau_1} < 0\right)$ and this improves income distribution, while import protection in asset-poor countries increases asset income $\left(\frac{\partial \beta_k}{\partial \tau_2} > 0\right)$ and there is a rise in inequality.

While inequality depends on relative factor incomes, the social welfare effect depends on the trade-off between real income effects following from import protection, and the impact on inequality. In other words, it depends on the trade-off between equity and efficiency. From equations (36) and (37), this is ultimately a function of the degree of inequality aversion, combined with the structural features of the economy and its market power on world markets. For a small country, real income effects are strictly negative, while inequality effects may be positive or negative, depending on the relative endowment structure of the economy and which sectors are being protected. For a large country, it is possible for both effects to work in the same direction. However, in this case, note that positive terms-of-trade gains will slow any rise (or slow any fall) in capital income shares, from equation (38). This in turn means that terms of trade effects will tend to mitigate the inequality effects of import protection.

On the basis of Theorems 2 and 3 we can immediately make the following statements about asset rich and poor Heckscher-Ohlin economies.

Corollary 1 *In a small asset-poor 2x2 Heckscher-Ohlin economy, where the mean real-income effects of import protection are negative, we have a* ***magnification effect****. The effect of import protection on welfare through mean income is magnified by the impact through inequality. Because of this magnification effect, net effects remain unambiguous and negative.*

Corollary 2 *In a small, asset-rich 2x2 Heckscher-Ohlin economy, where the mean real-income effects of import protection are negative, we have a* ***mitigation effect****. The effect of import protection on welfare through mean income is at least partially offset by the impact through inequality.*

Corollary 3 *The effect of import protection on inequality as measured by the Atkinson and Gini indexes will be weaker, in a Heckscher-Ohlin economy, for large countries. This is because of terms of trade effects from equation* (38)*, which dampen the goods-price to factor-price transmission mechanisms at play.*

Corollary 1 flags a magnification effect, linking efficiency and inequality effects, in labour abundant economies. In contrast, we have an offsetting effect in capital-abundant economies instead , as noted in Corollary 2. This result is derived from Theorem 2 and equations (30) and (31) . It means that in the 2x2 model, the impact of import protection on welfare can be ambiguous for small economies when inequality matters. This stands in contrast to a standard result of the classic 2x2 model, where import protection are unambiguously welfare-reducing for small countries. Corollary 3 follows because our import protection analytics are driven by the transmission of tariff changes into price changes, and these are weaker in larger economies. These smaller internal price effects mean smaller inequality effects.

Moreover, we have that Ψ is a monotonic function of the initial inequality, such that: $\frac{\partial \Psi}{\partial I} > 0$.[18] Thus, low levels of inequality are associated with low levels of Ψ. This results leads us to the following theorem:

Theorem 4 *In a small 2x2 Heckscher-Ohlin economy with inequality determined by uneven distribution of assets (capital or skills), the impact of import protection on inequality is directly related to initial inequality levels. The distributional impact of import protection is greater for economies with high inequality and smaller in economies with low inequality.*

Using equation (39) we can conclude that the impact of import protection on inequality is directly related to the initial levels of inequality. This can also be clearly observed in the common

[18]The formal proof is in the Appendix.

term $\frac{\partial \beta_j}{\partial \tau_i}\left(\omega_j^h - n^{-1}\right)$ present in equations (33 and 34). The impact of a tariff on the good which uses intensively the relatively scarce factor in the economy changes the income share of factor j directly proportionally on how that factor is distributed $\left(\omega_j^h - n^{-1}\right)$.

This result has profound policy implications, since it provides an extra magnification effect for asset-poor unequal economies, while it also creates an extra mitigation effect for asset-rich equal economies. In particular, poor unequal economies that liberalize trade can expect not only the well-known efficiency gains from trade, but also a higher reduction in inequality and an accordingly, an extra boost to social welfare. For the case of poor countries with low inequality, social welfare also is increasing, but less than in unequal economies, since both efficiency and equity effects are positive in asset-poor economies. The argument, of course, is turned around if the policy instrument is to increase import protection. Then poor unequal economies suffer larger welfare decreases than poor and less unequal ones.

On the other hand, asset-rich egalitarian economies have a smaller negative inequality impact of trade, making it more probable that the efficiency gains from liberalized trade outweigh the negative equity effects and social welfare is raised. For the case of asset-rich unequal countries, the negative inequality effect is stronger and thus, the probability that the inequality effects counteracts the positive efficiency gains is higher.

4.2 Equity in the Specific Factors Model

Consider next the specific factors model. We can make a similar manipulation of equation (35) for the standard 2-good, 3-factor model. This yields the following equation:

$$\frac{\partial I}{\partial \tau_i} = \sum_h \psi^h \left[\frac{\partial \beta_{k1}}{\partial \tau_i}\left(\omega_{k1}^h - n^{-1}\right) + \frac{\partial \beta_{k2}}{\partial \tau_i}\left(\omega_{k2}^h - n^{-1}\right)\right] \tag{42}$$

Again, we assume that unskilled labor is evenly distributed in the population and that inequality follows from the ownership pattern of both (specific) assets (k_i), which are unevenly distributed. In the special case when import protection creates a shift in income shares from more to less concentrated factors (in terms of the concentration of factor ownership) this yields a reduction in inequality. In this case, we obtain the same outcomes raised before in the theorems and corollaries of the previous section, with regard to inequality effects, social welfare changes and country size in the Heckscher-Ohlin model. Otherwise, the impact of import protection on inequality depends on the pattern of relative factor prices and ownership effects. We can summarize our results with respect to the Ricardo-Viner model as follows:

Theorem 5 *The impact of import protection on inequality, like the effect of import protection on income for the mobile factor itself, is ambiguous in the Ricardo-Viner model when specific factor ownership patterns are the source of inequality. This follows from the divergent impact on different classes of (sector specific) assets.*

The standard result from the RV model is that $sgn\left(\frac{\partial\beta_{k1}}{\partial\tau_i}\right) \neq sgn\left(\frac{\partial\beta_{k2}}{\partial\tau_i}\right)$. Import protection is directed to one of the two sectors of the economy, and thus, there is an opposite effect of the tariff impact on the sector-specific factors. From equation (42), changes in the inequality indexes depend strictly on a weighted sum of the change in the share of income going to both forms of sector-specific assets, $\frac{\partial\beta_{ki}}{\partial\tau_i}$. Weights are assigned to households that are inversely monotonic in household deviations from the average portfolio, $\left(\omega_{ki}^h - n^{-1}\right)$. This means that the change in incomes for households holding more assets than average, and hence more income than average, determine the sign of the income inequality effect. As a result we have a fall in inequality as long as all asset income shares decline.

Theorem 5 follows from the need to sign the final terms in square brackets in equation (42). Depending on the distribution of ownership, functional forms, and the share of unskilled labor in total income in the benchmark, inequality may then rise or fall. For example, in a developing country where the poor have unskilled labor and land, and the rich unskilled labor and capital, protection makes the concentration of income worse, assuming the sector using capital is an import-competing sector. On the other hand, if ownership of land is very highly concentrated relative to capital, import protection may improve the distribution of income.

Finally, the initial inequality levels, implicit in the ownership matrix Ω are also determinant to the overall inequality impact of import protection. The smaller the terms $\left(\omega_j^h - n^{-1}\right)$ are, the smaller the factor distribution impact of tariffs $\left(\frac{\partial\beta_{k1}}{\partial\tau_i}\right)$ is. However, the interrelations of the two specific-factors with the overall distribution of income that is present in the Ricardo-Viner framework, does not allow us a strong statement as that given in Theorems 2 and 3.

5 Conclusions

We have developed a dual theoretical framework for exploring linkages between import protection and the household distribution of income. This complements the existing literature that links trade policy to factor incomes and the functional distribution of income. Stolper-Samuelson effects constitute a first step in our analysis. In a general equilibrium context, tariff changes ultimately affect the household distribution through variations in ownership patters in conjunction with Stolper-Samuelson effects.

To illustrate the mapping of general dual results to standard workhorse models, we have used the Heckscher-Ohlin and Ricardo-Viner trade models. Within both frameworks, we explore theoretical linkages between trade protection, country size, level of development, and personal income inequality.

Another contribution of this paper is that we examine the formal link between social welfare and the equilibrium determinants of the distribution of income. Using Sen-type social welfare functions, we decompose the general equilibrium welfare effects of import protection into real income level and distribution components. Depending on the levels of inequality aversion, the dispersion component can be represented exactly through use of the Gini or Atkinson inequality indexes. With these explicit inequality derivatives we map import protection to inequality-adjusted welfare. In addition, when standard trade models are employed this framework also yields predictions relating social welfare with import protection, country size and levels of development. In conjunction with the relevant inequality index, the general form of the decomposition of welfare and inequality we develop here may also be useful for producing summary measures of distributional impacts in applied general equilibrium applications focused on inequality.

Once the distributional effects of trade liberalization are determined, we can also analyze the political economy impact of employing a Sen-type social welfare functions. In such a framework, endogenous tariff formation models can be used to assess how the optimum tariff is affected by equity concerns. These political economy implications are extensively analyzed in Francois and Rojas-Romagosa (2005). For instance, in representative democratic systems, positive optimum tariffs can be sustained in capital-abundant countries even when the policy-maker assigns a low or zero weight to the contributions of special interests groups. In this case, the positive distributional effect of import protection can offset or compensate the efficiency losses of reduced trade. In poor countries, characterized by the relative abundance of labor, positive tariffs are explained by the influence of special interest groups (i.e. capitalists) that heavily lobby for higher tariffs. Thus, import protection in developing countries not only diminishes social welfare through efficiency and equity considerations, but also signals the economic and political weight of the capital-owners.

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A Appendix

Proof. Theorem 1. We rewrite equation (37) as: $\frac{\partial SW}{\partial \tau_i} = f\left(\alpha, \Omega, \frac{\partial I}{\partial \tau_i}, \frac{\partial \bar{y}}{\partial \tau_i}, \frac{\partial p_c}{\partial \tau_i}\right)$, where α are the ethical weights, Ω is the ownership matrix, and $\frac{\partial p_c}{\partial \tau_i}$ are the terms-of-trade effects. The first two parameters are fixed (α, Ω). Assuming first a small international price-taking country $\left(\frac{\partial p_c}{\partial \tau_i} = 0\right)$, then the $sgn\left(\frac{\partial SW}{\partial \tau_i}\right) = f\left(\frac{\partial I}{\partial \tau_i}, \frac{\partial \bar{y}}{\partial \tau_i}\right)$. If $sgn\left(\frac{\partial I}{\partial \tau_i}\right) \neq sgn\left(\frac{\partial \bar{y}}{\partial \tau_i}\right)$, i.e. we have opposing efficiency and equity effects, then for a certain combination of parameters, we can obtain $sgn\left(\frac{\partial SW}{\partial \tau_i}\right) \neq sgn\left(\frac{\partial \bar{y}}{\partial \tau_i}\right)$. This is also true when $\frac{\partial p_c}{\partial \tau_i} \neq 0$ ■

Proof. $\Psi > 0$ for the Gini coefficient. Recall that households h are ranked starting with the highest income: $y^1 \geq y^2 \geq ... \geq y^n$ and $\sum_h \omega_k^h = 1$. We start from a simple case with only two households $(n = 2)$. From equation (40) we get: $\Psi = \sum_h \frac{-2h}{n}\left(\omega_k^h - n^{-1}\right) = -\left(\omega_k^1 - \frac{1}{2}\right) - 2\left(\omega_k^2 - \frac{1}{2}\right)$. If assets k are totally concentrated in one household, $\omega_k^1 = 1$ and $\omega_k^2 = 0$. Then $\Psi = \frac{1}{2}$. The other extreme case is when assets are almost perfectly distributed, such that: $\omega_k^1 = \frac{1}{2} + \varepsilon$ and $\omega_k^1 = \frac{1}{2} - \varepsilon$, where ε is an infinitesimal small but positive number. Then we have $\Psi = \varepsilon$. Since the case when assets are perfectly distributed is trivial, then for $n = 2$ we have that $\Psi \in [\varepsilon, 0.5]$. These results are easily generalized to the case of $n > 2$. For instance, with $n = 3$ equation (40) is: $\Psi = -\frac{2}{3}\left(\omega_k^1 - \frac{1}{3}\right) - \frac{4}{3}\left(\omega_k^2 - \frac{1}{3}\right) - 2\left(\omega_k^3 - \frac{1}{3}\right)$. If k is totally concentrated then $\Psi = \frac{2}{3}$ and if k is almost perfectly distributed using $\omega_k^1 = \frac{1}{3} + \varepsilon$, $\omega_k^2 = \frac{1}{3}$, and $\omega_k^3 = \frac{1}{3} - \varepsilon$, then we have $\Psi = \frac{4}{3}\varepsilon$. Thus, the value of Ψ ranges between $\frac{4}{3}\varepsilon$ to $\frac{2}{3}$. A similar procedure can be applied to the general case of n households. With total concentration $\omega_k^1 = 1$ and $\omega_k^h = 0\ \forall h \neq 1$. Then $\Psi = \frac{-2}{n}\left(1 - \frac{1}{n}\right) + \frac{2}{n}\sum_{h=2}^{n}\frac{h}{n} = 1 - \frac{1}{n} > 0$. For the other extreme case of almost perfect distribution of k, we have that $\omega_k^1 = \frac{1}{n} + \varepsilon$, $\omega_k^h = \frac{1}{n}\forall h \neq 1, n$ and $\omega_k^n = \frac{1}{n} - \varepsilon$. This yields $\Psi = -\frac{2}{n}(\varepsilon) - 2(-\varepsilon) = \frac{2\varepsilon}{n}(n-1) > 0$. To summarize, we have that $\Psi \in \left[\frac{2\varepsilon}{n}(n-1), 1 - \frac{1}{n}\right]$ ■

Proof. $\Psi > 0$ for the Atkinson index. Again, we begin with the simple case with only two households $(n = 2)$. From equation (41) we have:

$$\begin{aligned}\Psi \quad = \quad & -2^{\frac{-\theta}{1-\theta}}\left\{\left[\beta_k\left(\omega_k^1 - \frac{1}{2}\right) + \beta_k\left(\omega_k^2 - \frac{1}{2}\right) + 2\right]^{1-\theta}\right\}^{\frac{\theta}{1-\theta}} \\ & \left\{\left(\beta_k\left(\omega_k^1 - \frac{1}{2}\right) + 1\right)^{-\theta}\left(\omega_k^1 - \frac{1}{2}\right) + \left(\beta_k\left(\omega_k^2 - \frac{1}{2}\right) + 1\right)^{-\theta}\left(\omega_k^2 - \frac{1}{2}\right)\right\}\end{aligned}$$

If assets k are totally concentrated, $\omega_k^1 = 1$ and $\omega_k^2 = 0$. Then we get:

$$\Psi = -2^{\frac{-\theta}{1-\theta}} \left\{2^{1-\theta}\right\}^{\frac{\theta}{1-\theta}} \left\{ \left(\frac{\beta_k}{2} + 1\right)^{-\theta} \left(-\frac{1}{2}\right) + \left(\beta_k \left(-\frac{1}{2}\right) + 1\right)^{-\theta} \left(-\frac{1}{2}\right) \right\}$$

Using values of $\theta \in \left[\frac{1}{2}, 2\right]$[19] and values of $\beta_k \in (0, 1]$, numerical evaluations of Ψ are always positive. Only when $\beta_k = 0$ we have $\Psi = 0$, but this is a trivial case that generates a perfect income distribution. In the case that assets are almost perfectly distributed, we use again: $\omega_k^1 = \frac{1}{2} + \varepsilon$ and $\omega_k^1 = \frac{1}{2} - \varepsilon$. Here we obtain that:

$$\Psi = -2^{\frac{-\theta}{1-\theta}} \left\{ \left[\beta_k (\varepsilon) + 1\right]^{1-\theta} + \left[\beta_k (-\varepsilon) + 1\right]^{1-\theta} \right\}^{\frac{\theta}{1-\theta}} \left\{ \left[\beta_k (\varepsilon) + 1\right]^{-\theta} (\varepsilon) + \left[\beta_k (-\varepsilon) + 1\right]^{-\theta} (-\varepsilon) \right\}$$

Again, numerical evaluations of Ψ are always positive if we use values of $\theta \in [0.5, 2]$ and $\beta_k \in (0, 1]$. Extending the number of households to $n > 2$ greatly complicates the algebraic representation of Ψ, but using the same technique as before with extreme value evaluation, we always obtain $\Psi > 0$ ■

Proof. Theorem 2. For small international price-taking countries in equation (38): $\frac{\partial \beta_k}{\partial \tau_1} < 0$ if k is the relatively abundant factor in the economy and τ_1 is a positive tariff on the unskilled-labor-intensive good. This is the standard Stolper-Samuelson effect applied to a rich asset-abundant country. The Theorem then follows from combining equations (38) and (39), since $\Psi > 0$, then $sgn\left(\frac{\partial \beta_k}{\partial \tau_l}\right) = sgn\left(\frac{\partial I}{\partial \tau_l}\right)$ ■

Proof. Theorem 3. For a small international price-taking country in equation (38): $\frac{\partial \beta_k}{\partial \tau_2} > 0$ if k is the relatively scarce factor in the economy and τ_2 is a positive tariff on the asset-intensive good. This is the standard Stolper-Samuelson effect applied to a poor labor-abundant country. Again, since $\Psi > 0$, then $sgn\left(\frac{\partial \beta_k}{\partial \tau_k}\right) = sgn\left(\frac{\partial I}{\partial \tau_k}\right)$ ■

Proof. Corollary 1. From equation (32) we have that for a small international price-taking country: $\frac{\partial \overline{y}}{\partial \tau_i} < 0$, i.e. mean real-income effects of import protection are negative. From Theorem **??** we also know that $\frac{\partial I}{\partial \tau_i} > 0$ for an asset-poor economy. Substituting these results into equations (30 and 31) we obtain than $\frac{\partial SW}{\partial \tau_i} < 0$. The negative efficiency effect (the first term in equations (30 and 31) is magnified by the negative inequality effect (the second term in equations (30 and 31) ■

Proof. Corollary 2. From equation (32) we have that for a small international price-taking country: $\frac{\partial \overline{y}}{\partial \tau_i} < 0$, i.e. mean real-income effects of import protection are negative. From Theorem **??** we also

[19] This are considered to be the extreme values of θ in the literature.

know that $\frac{\partial I}{\partial \tau_i} < 0$ for an asset-rich economy. The negative efficiency effect (the first term in equation (31)) is mitigated by the positive inequality effect (the second term in equation (31). Thus, the sign of $\frac{\partial SW}{\partial \tau_i}$ is not clear and the welfare effects of a tariff increase are ambiguous ■

Proof. Corollary 3. From equation (32) a country that is large enough to affect international prices we have that $\frac{\partial p}{\partial \tau_i} > 0$ and then from equation (38) this price effects weakens the overall efficiency effect reducing $\frac{\partial \beta_j}{\partial \tau_i}$ and, consequently the effect of tariffs on inequality from equation (39) ■

Proof. $\frac{d\Psi}{dI} > 0$ for the Gini coefficient. An infinitesimal increase in income inequality using the Gini coefficient (dI_G) is assured if there is an infinitesimal small transfer (δ) from a household with smaller income ($j + \alpha$) to a richer household (j). In this setting, j can refer to a single household or to different household groupings. What we need is that the ranking of j reflects the income ranking of households: $y^1 \geq y^2 \geq ... \geq y^n$. Thus, the parameter α signals the income rank difference between households or groups of households. We use equation (11) as the definition of I_G, such that I^0 is the initial income inequality value and I^1 is the value after the income redistribution of the δ value, when the richer household has the new income $y^\eta + \delta$ and the poorer household $y^{\eta+\alpha} - \delta$, where $\eta \in [1, h]$. Then we have that $dI_G = I_G^1 - I_G^0$, which yields the value $dI_G = \frac{2\alpha\delta}{n^2\overline{y}}$. To evaluate the changes in Ψ we rewrite equation (40) as: $\Psi = 1 + \frac{1}{n} - \sum_h \frac{h\omega_k^h}{n}$. We define Ψ^0 as the initial value of Ψ without any income transfers and Ψ^1 as the new value after the transfer, i.e. when household $\eta + \alpha$ decreases its share of assets k. Since $\omega_k^h = \frac{y^h}{n\overline{y}_k}$, then we have that a income transfer of δ is reflected in the new household $\eta + \alpha$ share: $\omega_k^{\eta+\alpha} - \frac{\delta}{n\overline{y}_k}$, while household η has the new higher share:$\omega_k^\eta + \frac{\delta}{n\overline{y}_k}$. Defining $d\Psi = \Psi^1 - \Psi^0$ and including these new shares into Ψ^1 we obtain the following result: $\Psi^1 - \Psi^0 = -\left[\frac{(\eta+\alpha)\left(-\frac{\delta}{n\overline{y}_k}\right)}{n} + \frac{\eta\frac{\delta}{n\overline{y}_k}}{n}\right] = \frac{\delta\alpha}{n^2\overline{y}_k} > 0$. Combining the results for dI_G and $d\Psi$ we get: $\frac{d\Psi}{dI_G} = \frac{1}{2}\frac{\overline{y}}{\overline{y}_k} > 0$ ■

Proof. $\frac{d\Psi}{dI} > 0$ for the Atkinson Index. By the *principle of transfers* of inequality measures, a transfer from a poor to a richer household must increase inequality (cf. Cowell, 2000). Thus, we have that $dI_A > 0$ when the transfer δ is applied.[20] For instance, using the same definition of the transfer δ as above and equation (9), we obtain:

$$\frac{\partial I_A}{\partial \delta} = \frac{-1}{1-\theta}\left[\frac{1}{n}\sum_h\left(\frac{y^h}{\overline{y}}\right)^{1-\theta}\right]^{\frac{\theta}{1-\theta}}\left[\frac{1-\theta}{n\overline{y}^{1-\theta}}\left[\left(y^h+\delta\right)^{-\theta} - \left(y^{h+\alpha}-\delta\right)^{-\theta}\right]\right]$$

[20] Note that this was also the case for the Gini coefficient.

Since $\left(y^h+\delta\right)^{-\theta}-\left(y^{h+\alpha}-\delta\right)^{-\theta}<0$, then for any value of $\theta \neq 0$, we have that $\frac{\partial I_A}{\partial \delta}>0$. The same results are obtained when we use the definition of the Atkinson index for $\theta=1$: $I_A=1-\frac{1}{\bar{y}}\left(\prod_h y^h\right)^{\frac{1}{n}}$. In this case, with $y^h+2\delta>y^{h+\alpha}$, we obtain that $\frac{\partial I_A}{\partial \delta}>0$. This proves the principle of transfers for the case of the Atkinson Index. Likewise, as above we define the new asset k shares as: $\left(\omega_k^{\eta}+\frac{\delta}{n\bar{y}_k}\right)$ and $\left(\omega_k^{\eta+\alpha}-\frac{\delta}{n\bar{y}_k}\right)$, and in combination with equation (41) we obtain $\frac{\partial \Psi}{\partial \delta}>0$. Finally, using $\frac{\partial I_A}{\partial \delta}=dI_A$ and $\frac{\partial \Psi}{\partial \delta}=d\Psi$, in conjunction with the signs of these derivatives we get: $\frac{d\Psi}{dI_A}>0$ ■

Proof. Theorem 4. With $\frac{d\Psi}{dI}>0$, this is a direct result from equation (39) ■

Proof. Theorem 5. $sgn\left(\frac{\partial \beta_{k1}}{\partial \tau_i}\right) \neq sgn\left(\frac{\partial \beta_{k2}}{\partial \tau_i}\right)$, and the ambiguity of $\frac{\partial \beta_L}{\partial \tau_i}$ are standard trade results. See for example, Dixit and Norman (1980). This implies that $sgn\left(\frac{\partial I}{\partial \tau_i}\right)=f\left(\Omega, \frac{\partial \beta_j}{\partial \tau_i}\right)$, i.e. conditional on the relative magnitudes of the opposite specific-factors income shares $\left(\frac{\partial \beta_j}{\partial \tau_i}\right)$, and how these factors are distributed (Ω) ■