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ABSTRACT

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Using synthetic data generated by a prototypical stochastic growth model, we explore the quantitative extent of measurement error of the Solow residual (Solow 1957) as a measure of total factor productivity (TFP) growth when the capital stock is measured with error and when capacity utilization and depreciation are endogenous. We propose two alternative measurements of TFP growth which do not require capital stocks: one measure eliminates the capital stock by direct substitution, while the other is based on generalized differences of detrended data and the Malmquist index. The root mean squared errors of these alternatives are as low as one third of those for the Solow-Törnqvist residual. Our comparative evaluations on artificial data indicate that measurement problems are severe, in particular for economies still far from their steady state. This drawback of the Solow residual is thus most acute in applications in which its accuracy is most highly valued. As an application, we compute and compare TFP growth estimates using data from the new and old German federal states.

JEL Classification: D24, E01, E22, O33 and O47
Keywords: Malmquist index, measurement error, Solow residual and total factor productivity

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1 Introduction

For more than fifty years, the Solow decomposition has served as the standard measurement of total factor productivity (TFP) growth in economics and management.[1] Among its central attractions are its freedom - as a first approximation - from assumptions regarding the production function, statistical model or econometric specification.[2] In his seminal paper, Robert Solow (1957) first employed the decomposition to demonstrate the bounds of accounting for economic growth using changes in observable inputs. In macroeconomics, the Solow residual has motivated research not only on the sources of long-run growth, but also on the sources of business cycle fluctuations.[3] According to the Social Sciences Citation Index, the Solow paper has been referenced more than 1,300 times since its publication.[4]

Despite this considerable prominence, the goodness or robustness of the Solow residual as a measurement tool has yet to be systematically evaluated. This is because the "true" evolution of total factor productivity is fundamentally unknown. Yet there are several reasons to suspect the quality of both microeconomic and macroeconomic TFP measurements. First, the capital stock is fundamentally unobservable; in practice, it is estimated more or less as a perpetual-inventory function of past investment expenditures plus an estimate of a unknown initial condition. Uncertainty surrounding that initial condition as well as the depreciation, obsolescence and decommissioning of subsequent investment can imply significant measurement error. Second, as many scholars of productivity analysis have stressed, the Solow residual assumes full efficiency, and thus really represents a mix of changes in total factor productivity and efficiency of factor utilization.[5] Intertemporal variation of the utilization of capital will bias an unadjusted calculation of the Solow residual as a measure of total factor productivity (Burnside, Eichenbaum, and Rebelo (1993, 1995)). Because the perpetual inventory method (PIM) is the backbone of capital measurement for the OECD and national accounting agencies in practice (Pritchett (2000), Schreyer (2009), O'Mahony and Timmer (2009), McGrattan and Prescott (2009)), capital mismeasurement continues to pose a potential problem for growth accounting, especially for developing and transition countries and when "new" types of capital are studied (e.g. research and development (R&D), information and communication technology (ICT), intangibles and public capital).

In this paper, we evaluate the extent of measurement error of the Solow decomposition using quantitative macroeconomic theory. We employ a prototypical stochastic growth model as a laboratory for studying the robustness of the Solow residual computed using capital stocks constructed, as in practice, from relatively short series of observed investment expenditures and an initial guess of an unobservable capital stock. To generate the synthetic data, we consider a version of the model with endogenous depreciation or obsolescence for all capital in place. Our comparative evaluations on artificial data indicate that measurement problems are severe, in particular for economies still far from

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[1] See, for example, Jorgenson and Griliches (1967), Kuznets (1971), Denison (1972), Maddison (1992), Hulten (1992), O'Mahony and van Ark (2003).

[2] See Griliches (1996).

[3] See the references in Hulten, Dean, and Harper (2001).

[4] Source: Social Sciences Citation Index, July 2010.

[5] For a representation of the Solow residual as difference between TFP growth and efficiency, see Mohnen and ten Raa (2002). Macroeconomists have raised this issue as well; see e.g. Summers (1986), Burnside, et al. (1993), and King and Rebelo (1999).

their steady state. This drawback of the Solow residual is thus most acute in applications in which its accuracy is most highly valued.

To deal with capital stock measurement error, we propose two alternative measurements of TFP growth. Both involve the elimination of capital stocks from the Solow calculation, while introducing their own, different sources of error. The first measurement, based on direct substitution, requires an estimate of the user cost of capital. The second, based on generalized first differences of national accounts data, requires an estimate of an initial condition for TFP growth (as opposed to an initial condition for the capital stock). To implement the latter approach, we improve on the choice of starting value of TFP growth by exploiting the properties of the Malmquist index. We then evaluate the extent of these competing errors in a horse race using the synthetic data described above. In almost all cases, our measures outperform the traditional Solow residual and reduce the root mean squared error, in some cases by as much as two-thirds.

The rest of paper is organized as follows. In Section 2, we review the Solow residual and the relationship between the Solow decomposition and the capital measurement problem. Section 3 proposes a prototypical stochastic dynamic general equilibrium model - the stochastic growth model with variable capacity utilization - as a laboratory for evaluating the quality of the Solow residual as a measure of TFP growth. In Section 4, we propose two alternative TFP growth measurements and present the results of a comparative quantitative evaluation of these measurements under varying assumptions concerning data available to the analyst. Section 5 applies the new methods to the federal states of post-reunification Germany as an unusual case of TFP growth measurement for regional economies which, while sharing a common economic environment, are presumably both close to and far from their respective steady-state paths, and for which the potential for capital mismeasurement is particularly large. Section 6 concludes.

2 The Solow Residual and the Capital Measurement Problem

2.1 The Solow Residual after a Half-Century: A Brief Review

Solow (1957)[6] considered a standard neoclassical production function $Y_t = F(A_t, K_t, N_t)$ expressing output (Y_t) in period t as a constant returns function of a homogeneous physical capital stock (K_t) and employment (N_t), while A_t represents the level of total factor productivity. Solow approximated TFP growth as $\frac{\dot{Y}_t}{Y_t} - \alpha_t \frac{\dot{K}_t}{K_t} - (1-\alpha_t)\frac{\dot{N}_t}{N_t}$, the difference of the observable growth rate of output and a weighted average of the growth of the two inputs, where α_t and $1-\alpha_t$ are local output elasticities of capital and labor; a dot denotes the time derivative (e.g. $\dot{A} = dA/dt$). In practice, the Solow decomposition is generally implemented in discrete time as (see Barro (1999) and Barro and Sala-I-Martin (2003)):

$$TFPG_t = \frac{\Delta Y_t}{Y_{t-1}} - \alpha \frac{\Delta K_t}{K_{t-1}} - (1-\alpha)\frac{\Delta N_t}{N_{t-1}} \tag{1}$$

[6] Because it is so widely used in economics and management, we offer only a cursory survey of the growth accounting methods anticipated by Tinbergen (1942) and pioneered by Solow (1957) and Denison (1962), focusing on the capital stock as a source of measurement error in this framework. For more detailed reviews of the Solow decomposition, see Diewert and Nakamura (2003, 2007) and ten Raa and Shestalova (2006).

where K_t denotes capital available at the beginning of period t. In competitive factor markets, output elasticities of capital and labor equal aggregate factor income shares, which are constant in the case of the Cobb-Douglas production function; for other technologies which allow for factor substitution, equation (1) gives a reasonable first order approximation. A central reason for the Solow residual's enduring popularity as a measure of TFP growth is its robustness; it measures the contribution of observable factor inputs to output growth solely on the basis of theoretical assumptions (constant returns to scale, perfect competition in factor markets) and external information (factor income shares), and without recourse to statistical techniques.

Yet the Solow residual itself is hardly free of measurement error.[7] Jorgenson and Griliches (1967, 1972) argued that the Solow residual is only a "measure of our ignorance". Denison (1972) and others extended the TFP measurement paradigm to a larger set of production factors, and confirmed that the unexplained residual is the most important factor driving output growth. Ever since Christensen, Jorgenson, and Lau (1973) raised concerns about the choice of weights α and $1-\alpha$, it has become commonplace to employ the so-called Törnqvist index specification of the Solow residual, presented here as a logarithmic approximation:

$$TFPG_t\,(ST) = \Delta \ln Y_t - \bar{\alpha}_{t-1}\Delta \ln K_t - (1-\bar{\alpha}_{t-1})\,\Delta \ln N_t \tag{2}$$

where $\bar{\alpha}_{t-1} = \frac{\alpha_{t-1}+\alpha_t}{2}$ (see Törnqvist (1936)). This formulation reduces measurement error and is exact if the production function is translog (Diewert (1976)). Denison (1962) and Hall and Jones (1999) have employed the Solow approximation across space as opposed to time to assess the state of technical progress relative to a benchmark economy.

Measurement error can arise for other reasons besides the initial condition of the capital stock. While output and employment are directly observable and readily quantifiable, capital must be estimated relying on a number of controversial assumptions. In this context it is worth recalling the famous capital controversy between Cambridge University, led by Joan Robinson, and the Massachusetts Institute of Technology and in particular, Paul Samuelson. Essentially our paper lends more credence to the position taken by Robinson, for reasons somewhat different from those she adduced (see Robinson (1953)).

2.2 The Capital Measurement Problem

The capital stock poses a particular problem in growth accounting because it is not measured or observed directly, but is constructed by statistical agencies using time series of investment expenditures and ancillary information. At some level, capital stocks are constructed by integrating the "Goldsmith equation" (Goldsmith (1955))

$$K_{t+1} = (1-\delta_t)\,K_t + I_t, \qquad t = 0, 1, \ldots \tag{3}$$

[7] Solow himself wrote:
"[L]et me be explicit that I would not try to justify what follow by calling on fancy theorems on aggregation and index numbers. Either this kind of aggregate economics appeals or it doesn't.[...] If it does, one can draw some useful conclusions from the results." Solow (1957: 312).

forward from some initial condition K_0, given sequences of investment expenditures $\{I_t\}$ and depreciation rates $\{\delta_t\}$. Formally, (3) can be solved from period 0 to period $t+1$ to yield

$$K_{t+1} = \left[\prod_{i=0}^{t}(1-\delta_{t-i})\right] K_0 + \sum_{j=0}^{t}\left[\prod_{i=0}^{j}(1-\delta_{t-i})\right] I_{t-j} \tag{4}$$

The current capital stock is the weighted sum of an initial capital value, K_0, and subsequent investment expenditures, with weights corresponding to their underpreciated components. If the depreciation rate is constant and equal to δ, (4) collapses to

$$K_{t+1} = (1-\delta)^{t+1} K_0 + \sum_{j=0}^{t}(1-\delta)^j I_{t-j}. \tag{5}$$

which is identical to an expression found in Hulten (1990).

From the perspective of measurement theory, four general problems arise with using capital stock data estimated by statistical agencies.[8] First, the construction of capital stocks presumes an accurate measurement of the initial condition K_0. The shorter the series under consideration, the more likely such measurement error will affect the construction of the Solow residual. Second, it is difficult to distinguish truly utilized capital at any point in time from that which is idle. Solow (1957) also anticipated this issue, arguing that the appropriate measurement should be of *"capital in use, not capital in place"*. Third, depreciation is also fundamentally unobservable. For some sectors and some types of capital, it is difficult if not impossible to apply an appropriate depreciation rate; this is especially true of the retail sector. Fourth, many intangible input stocks such as cumulated research and development effort and advertising goodwill are not included in measured capital.

The Goldsmith equation (3) implies that mismeasurement of the initial capital stock casts a long shadow on the construction of the Solow residual. The problem can only be solved by pushing the initial condition sufficiently far back into the past; yet with the exception of a few countries,[9] sufficiently long time series for investment are unavailable. The perpetual inventory approach to constructing capital series was thus criticized by Ward (1976) and Mayes and Young (1994), who proposed alternative approaches grounded in estimation methods.[10]

Even today, capital stock estimation relies heavily on PIM and the implied error remains a widely-recognized problem in growth accounting as well as productivity measurement.[11] Employing long time series for the US, Gollop and Jorgenson (1980) equate capital at time $t = 0$ to investment in that period. Jacob, Sharma, and Grabowski

[8] See Diewert and Nakamura (2007) for more a detailed discussion of these issues.

[9] For example, Denmark and the US Statistical Office have data on investment from 1832 and 1947 respectively; most industrialized economies only report data since the 1960s or afterwards.

[10] Schreyer (2001) suggests comparing initial capital estimates with five different benchmarks: 1) *population census* data, which take into account different types of dwellings; 2) *fire insurance records*; 3) *company accounts*; 4) *administrative property records*, which provides residential and commercial buildings valued at current market prices; and 5) *company share valuation*. Because these data are generally unavailable on a regular basis, they are normally used to check the plausibility of capital stocks estimates constructed from investment time series.

[11] See, for example, the recent OECD manual on measuring capital (Schreyer (2009)), the US Bureau of Economic Analysis (`http://www.bea.gov/national/pdf/NIPAhandbookch1-4.pdf`) and the methodological appendix contained therein, which observe that initial conditions can affect capital measurement if time series are short.

(1997) estimated the initial capital stock using artificial investment series for the previous century and assume that the investment grows at the average same rate of output. The US Bureau of Economic Analysis (Reinsdorf and Cover (2005) and Sliker (2007)) set capital equal to investment in the initial period I_0, reflecting a steady state in which expenditures grow at rate g and are depreciated at rate δ, so a natural estimate of K_0 is $\left(\frac{1+g}{\delta+g}\right) I_0$. Griliches (1980) used an initial condition $K_0 = \rho \frac{I_0}{Y_0}$ for measuring R&D capital stocks, where ρ is a parameter to be estimated. Caselli (2005) assessed the quantitative importance of capital measurement and found that measurement error induced by the initial guess is most severe for the poorest countries. Rather than employing the standard steady-state condition $K_0 = \frac{I_0}{(g+\delta)}$ (see e.g., Kohli (1982)), he estimates capital stocks of the poorest countries using:[12]

$$K_0 = K_0^* \left(\frac{Y_0}{Y_0^*}\right)^{\frac{1}{\alpha}} \left(\frac{N_0^*}{N_0}\right)^{\frac{1-\alpha}{\alpha}} \tag{6}$$

where the star refers to data from the benchmark economy presumed to be in the steady state (here, the United States). The precision of Caselli's innovative approach will depend, among other things, on the distance of the benchmark economy from its steady state. In addition, the key assumption in (6) that total factor productivity levels are identical to those in the US in the base year are inconsistent with the findings of Hall and Jones (1999). Most important, the US capital stock estimate for 1950 is likely subject to large measurement error.

2.3 Measurement Error, Depreciation and Capital Utilization

The initial condition problem noted by Caselli (2005) applies *a fortiori* to a more general setting in which the initial value of capital is measured with error, if depreciation is stochastic, or is unobservable. Suppose that the elements of the sequence of depreciation rates δ_t move around some constant value δ. It is possible to rewrite (4) as:

$$\begin{aligned} K_{t+1} = {} & (1-\delta)^{t+1} K_0 + \sum_{j=0}^{t} (1-\delta)^{j+1} I_{t-j} \\ & + \left[\prod_{i=0}^{t} \frac{(1-\delta_{t-i})}{(1-\delta)} - 1\right] (1-\delta)^{j+1} K_0 \\ & + \sum_{j=0}^{t} \left[\prod_{i=0}^{j} \frac{(1-\delta_{t-i})}{(1-\delta)} - 1\right] (1-\delta)^{t+j} I_{t-j} \end{aligned} \tag{7}$$

Equation (7) expresses the true capital stock as the sum of three components: 1) an initial capital stock, net of assumed depreciation at some constant rate δ, plus the contribution of investment $\{I_s\}_{s=0}^{t}$, also expressed net of depreciation at rate δ; 2) mismeasurement of the initial condition's contribution due to fluctuation of depreciation about the assumed constant value; and 3) mismeasurement of the contribution of all investment expenditures from period 0 to t. Each of these three components represents a potential source of measurement error. The first component contains errors involving the initial valuation of the capital stock. For the most part, the second and third components are unobservable.

[12] In his original formulation Caselli (2005) considers an extended production function with human capital.

Ignored in most estimates of capital, they represent a potentially significant source of mismeasurement which would contaminate a Solow residual calculation.

The interaction between the depreciation of capital and capacity utilization is also important for macroeconomic modeling. From a growth accounting perspective, Hulten (1986) criticized the assumption that all factors are fully utilized. Time-varying depreciation rates imply changing relative weights of old and new investment in the construction of the capital stock. In dynamic stochastic general equilibrium models, the depreciation rate is generally assumed constant, despite empirical evidence to the contrary.[13] In addition, as argued by Corrado and Mattey (1997) and Burnside, Eichenbaum, and Rebelo (1995), capacity utilization is highly procyclical. A positive link between depreciation and capacity utilization is a central feature of the artificial economy used to generate the artificial data used in the next two sections.

3 Capital Measurement and the Solow Residual: A Quantitative Assessment

3.1 The Stochastic Growth Model as a Laboratory

The central innovation of this paper is its assessment of alternative TFP growth measurement methods using synthetic data generated by a known, prototypical model of economic growth and fluctuations. We extend the standard, neoclassical framework (King and Rebelo (1999)), in which the first and second welfare theorems hold and markets are complete, to allow for variable capacity utilization, following Greenwood, Hercowitz, and Huffman (1988), Burnside, Eichenbaum, and Rebelo (1995), and Wen (1998). By using this well-understood model as a laboratory, we are able to assess quantitatively the limitations of the Solow residual measurement. In this section we first briefly describe this standard model and the data which it generates. Details can be found in Appendix A.

3.1.1 Technology

Productive opportunities in this one-good economy evolve as a trend-stationary stochastic process. Total factor productivity $\{A_t\}$ is embedded in a standard constant returns production function in capital services and labor inputs and evolves for $t = 1, 2, \ldots$ according to

$$A_t = \psi^{t(1-\rho)} A_{t-1}^{\rho} e^{\epsilon_t} \tag{8}$$

where $\psi > 1$, $|\rho| < 1$, A_0 is given and ϵ_t is white noise. Output is given by the Cobb-Douglas production technology

$$Y_t = A_t \left(U_t K_t\right)^{\alpha} N_t^{1-\alpha} \tag{9}$$

where $U_t \in (0, 1)$ denotes the utilization rate of capital ("capacity utilization").

[13] See the OECD manual (Schreyer (2009)) on capital stock estimation.

In this version of the model, output can either be consumed or invested in productive capacity ("capital"). Starting from a given initial K_0, capital evolves according to (3), where the rate of depreciation is an increasing, convex function of capacity utilization

$$\delta_t = \frac{B}{\chi} U_t^{\chi} \tag{10}$$

where $B > 0$ and $\chi > 1$. We depart from Wen (1998) and Harrison and Weder (2002) by adding a scale parameter B, which allows us to match both the mean and variance of the model's simulated capacity utilization to the data.

3.1.2 Households

Households own capital and labor and sell factor services to firms in competitive factor markets. Facing sequences of wages $\{\omega_t\}_{t=0}^{\infty}$ and user cost of capital $\{\kappa_t\}_{t=0}^{\infty}$, the representative household chooses paths of consumption $\{C_t\}_{t=0}^{\infty}$, labor supply $\{N_t\}_{t=0}^{\infty}$, capital in the next period $\{K_{t+1}\}_{t=0}^{\infty}$, and capital utilization $\{U_t\}_{t=0}^{\infty}$ to maximize the expected present value of lifetime utility (see e.g. Prescott (1986), Greenwood, Hercowitz, and Huffman (1988), Cooley and Prescott (1995), King and Rebelo (1999)):

$$\max_{\{C_t\},\{N_t\},\{K_{t+1}\},\{U_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln C_t + \frac{\theta}{1-\eta} \left[(1-N_t)^{1-\eta} - 1 \right] \right\} \tag{11}$$

subject to an initial condition for the capital stock held by household K_0, the periodic budget restriction for $t = 0, 1, ...$

$$C_t + K_{t+1} - (1-\delta_t)K_t = \omega_t N_t + \kappa_t U_t K_t, \tag{12}$$

and the dependence of capital depreciation on utilization given by (10). The period-by-period budget constraint restricts consumption and investment to be no greater than gross household income from labor $\omega_t N_t$ and capital $\kappa_t U_t K_t$.

3.1.3 Firms

Firms in this perfectly competitive economy are owned by the representative household. The representative firm employs labor N_t and hires capital services $U_t K_t$ to maximize profits subject to the constant returns production function given by (9). Note that for the firm, capital service input is the product of the capital stock and its utilization rate; the firm is indifferent to whether these originate from extensive or intensive use of the capital stock.

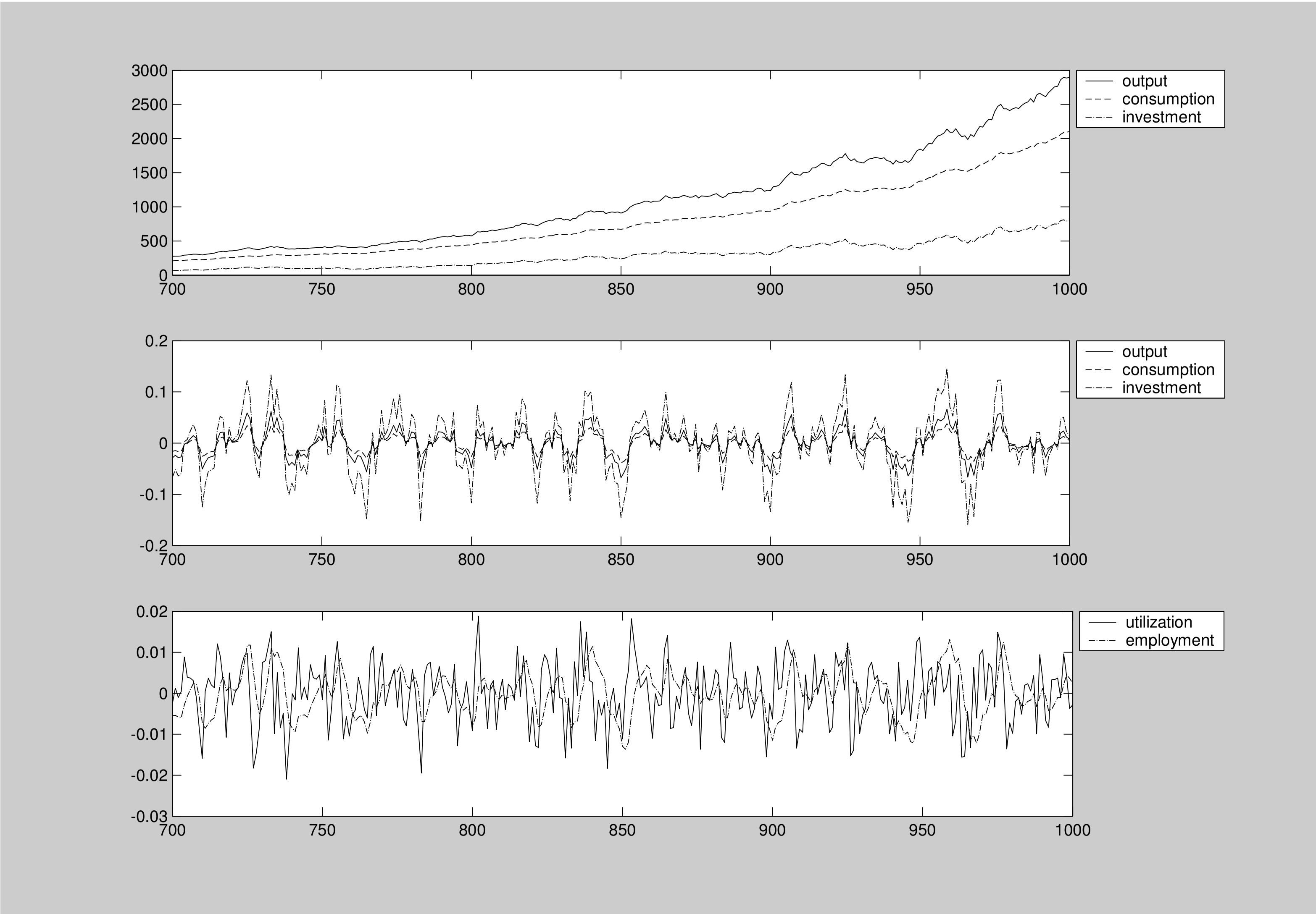
3.1.4 First Order Conditions, Decentralized Equilibrium and Steady State

In Appendix A, we summarize the first order conditions for optimal behavior of households and firms and characterize the decentralized market equilibrium, in which this regular economy is unique. Dynamic behavior can be approximated by log-linearized versions of these equilibrium conditions around the model's unique steady state growth path. Along that path, output, consumption, investment and capital stock all grow at a constant rate $g = \psi^{\frac{1}{1-\alpha}} - 1$, while total factor productivity grows at rate $\psi - 1$. Employment, capital utilization and interest rates are trendless.

3.2 Construction of the Data Sets

The model was simulated as a quarterly calibration to the US economy with standard parameter values described in Appendix A. Each realization of the artificial economy is a set of time series $\{Y_t\}, \{A_t\}, \{K_t\}, \{N_t\}, \{C_t\}, \{I_t\}, \{U_t\}, \{\kappa_t\}$, $\{\omega_t\}$ of 1,200 observations. The initial condition for TFP (A_0) was drawn from a normal distribution with mean zero and standard deviation one and the capital stock in period zero (K_0) is set to its steady-state value; the model is allowed to run 100 periods before samples were drawn. For each realization, samples were drawn for both "mature" and for "transition" economies. A mature economy corresponds to data after period 700, while a transition economy consists of the same realization as a mature economy until period 699, when the capital stock is reset to half its original value. The economy's equilibrium is then re-computed with this lower initial capital stock from period 700 to 1200. In Figure 1 we display a representative time series realization of the mature economy in original and H-P detrended form with detrending parameter set at 1600.

Fig. 1 A typical time series realization in levels and in H-P detrended form, periods 700-1000



The model's properties are summarized in Table 1 and compared with moments of the Hansen (1985) stochastic growth model as well as of the data as reported by Stock and Watson (1999) and Dejong and Dave (2007). Our benchmark model thus generates data which roughly replicates key features of the US economy required for the evaluation of the Solow residual.

Table 1 Comparative statistical properties of the model economy

Series	Benchmark model economy (200 Quarters)	Hansen (1985) Divisible labor model	Hansen (1985) Indivisible labor model	US DATA 1953Q1-1996Q4 (Stock & Watson (1999))	US DATA 1948Q1-2004Q4 (Dejong & Dave (2007))
Cross-correlations with output					
Consumption	0.99	0.89	0.87	0.90	-
Investment	1.00	0.99	0.99	0.89	-
Employment	0.48	0.98	0.98	0.89	-
Productivity	1.00	0.98	0.87	0.77	-
Std. dev. normalized by std. dev. of output					
Consumption	0.52	0.46	0.29	0.76	0.46
Investment	2.08	2.38	3.24	2.99	4.23
Employment	0.62	0.34	0.77	1.56	1.05

3.3 Evaluating Measurement Error of the Solow Residual

We now use the data from the artificial economy to evaluate the precision of the Solow residual. The basis of comparison is the average root mean squared error (RMSE) computed for samples of either 50 or 200 observations, from 100 independent realizations starting at period 700, for both the mature and the transition economy. The Solow residual measure is calculated as a Törnqvist index described in equation (2).[14] As in reality, our central assumption is that the true capital stock data are always unobservable to the analyst, who estimates them by applying PIM to investment data and an initial capital stock, which is estimated using methods described above. In the baseline scenario A, the analyst is unable to observe either the rate of capacity utilization or the depreciation rate. Alternatively, we assume that the analyst can observe the utilization rate only (B) or both the utilization and the depreciation rate (C). In (B) and (C) a modified Solow residual calculation is used.[15] Caselli's measure is computed using a BEA estimate of capital K_0^* in the simulated benchmark economy assumed to be in the steady state.

The results of this evaluation are presented in Table 2 as the average RMSE (in percent) for each estimate. Standard errors are computed across 100 realizations and are presented in parentheses. The results show that the initial condition of the capital stock is an important source of error. Of the different methods, the BEA and Caselli approaches perform best, yet are still characterized by significant measurement error. As expected, average RMSE declines with sample size. Yet even at a sample length of 50 years (200 quarters), the annualized root mean squared error remains high at about 2%.

[14] Note that for the Cobb-Douglas production and competitive factor markets, factor shares and output elasticities are constant, so the Törnqvist Index and lagged factor share versions are equivalent.

[15] That is, $TFPG_t\,(ST) = \frac{\Delta Y_t}{Y_{t-1}} - \alpha\left(\frac{\Delta K_t}{K_{t-1}} + \frac{\Delta U_t}{U_{t-1}}\right) - (1-\alpha)\,\frac{\Delta N_t}{N_{t-1}}$

Table 2 Avg. RMSE (%) for Solow residuals using different capital stock estimates (standard errors in parentheses).

Mature Economy (100 realizations)						
	A		B		C	
Initial capital stock estimate	**T=50**	**T=200**	**T=50**	**T=200**	**T=50**	**T=200**
-BEA	3.56	1.96	3.50	1.85	3.50	1.85
$K_0 = I_0 \frac{g+1}{g+\delta}$, g growth of investment rate	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.02)
-Caselli (2005)/BEA	3.55	1.95	3.49	1.84	3.49	1.85
$K_0 = K_0^* \left(\frac{Y_0}{Y_0^*}\right)^{\frac{1}{\alpha}} \left(\frac{N_0^*}{N_0}\right)^{\frac{1-\alpha}{\alpha}}$	(0.26)	(0.13)	(0.27)	(0.13)	(0.28)	(0.13)
-Gollop and Jorgenson (1980)	4.98	2.62	4.94	2.54	4.94	2.54
$K_0 = I_0$	(0.04)	(0.02)	(0.03)	(0.08)	(0.03)	(0.08)
Transition Economy (100 realizations)						
	A		B		C	
Initial capital stock estimate	**T=50**	**T=200**	**T=50**	**T=200**	**T=50**	**T=200**
-BEA	4.64	2.87	1.69	1.42	3.97	2.08
$K_0 = I_0 \frac{g+1}{g+\delta}$, g growth of investment rate	(0.24)	(0.15)	(1.14)	(0.40)	(0.16)	(0.08)
-Caselli (2005)/BEA	5.26	2.76	5.27	2.78	5.28	2.79
$K_0 = K_0^* \left(\frac{Y_0}{Y_0^*}\right)^{\frac{1}{\alpha}} \left(\frac{N_0^*}{N_0}\right)^{\frac{1-\alpha}{\alpha}}$	(0.27)	(0.13)	(0.27)	(0.13)	(0.27)	(0.13)
-Gollop and Jorgenson (1980)	5.49	2.87	2.84	1.80	2.84	1.85
$K_0 = I_0$	(0.21)	(0.10)	(0.57)	(0.19)	(0.19)	(0.10)

A: Analyst observes $\{Y_t, N_t, I_t, \bar{\kappa}, \omega_t\}$ only
B: Analyst observes $\{Y_t, N_t, I_t, \bar{\kappa}, \omega_t, U_t\}$
C: Analyst observes $\{Y_t, N_t, I_t, \bar{\kappa}, \omega_t, U_t, \delta_t\}$

4 TFP Growth Measurement without Capital Stocks: Two Alternatives

We have shown that the Solow residual can be subject to considerable measurement error. In scenario A, about 40% of this error in the short dataset is due to the estimated initial condition of the capital stock, while the rest is due to unobservable depreciation and capacity utilization. Measurement error in K_0 will bias TFPG computations when 1) depreciation is low and 2) the time series under consideration is short. For conventional rates of depreciation, errors in estimating the initial condition can have long-lasting effects on estimated capital stocks. In simulated data, it takes more than 100 periods to reach convergence within 10% of the steady state. In the following two sections, we propose two capital stock-free alternatives to the Solow residual. The first, the DS measure, is appropriate when the economy is far from its steady-state. The second, the GD measure, relies on the economy‘s proximity to a steady-state path.

4.1 Direct Substitution (DS)

The first strategy for estimating TFP relies on direct substitution to eliminate the capital stock from the equation generally used to construct the Solow residual. Differentiation of the production function $Y_t = F\left(A_t, U_t K_t, N_t\right)$ with respect to time yields

$$\dot{Y}_t = F_A \dot{A}_t + F_K \left(\dot{K}_t + \dot{U}_t\right) + F_N \dot{N}_t. \tag{13}$$

Substitution of the capital transition equation $\dot{K}_t = I_t - \delta_t K_t$ and rearrangement yields:

$$TFPG_t\left(DS\right) = \frac{\dot{Y}_t}{Y_t} - F_K \frac{I_t}{Y_t} + \alpha_t \left(\delta_t - \frac{\dot{U}_t}{U_t}\right) - \left(1-\alpha_t\right) \frac{\dot{N}_t}{N}_t. \tag{14}$$

In an economy with competitive conditions in factor markets, the marginal product of capital F_K is equated to κ_t, the user cost of capital in t. This equation is adapted to a discrete time context as

$$TFPG_t\left(DS\right) = \frac{\Delta Y_t}{Y_{t-1}} - \kappa_{t-1} \frac{I_{t-1}}{Y_{t-1}} + \alpha_{t-1} \left(\delta_{t-1} - \frac{\Delta U_t}{U_{t-1}}\right) - \left(1-\alpha_{t-1}\right) \frac{\Delta N_t}{N_{t-1}}. \tag{15}$$

The substitution eliminates the capital stock from the TFP calculation. The DS approach will be a better measurement of TFP growth to the extent that 1) the capital stock is unobservable or poorly measured; 2) capital depreciation varies from period to period and is better measured from other sources; 3) the last gross increment to the capital stock is more likely to be completely utilized than older capital. Once TFP growth is estimated, the total contribution of capital to growth can be calculated as $\frac{\Delta Y_t}{Y_{t-1}} - TFPG_t\left(DS\right) - \left(1-\alpha_{t-1}\right) \frac{\Delta N_t}{N_{t-1}} - \alpha_{t-1} \frac{\Delta U_t}{U_{t-1}}$.

4.2 Generalized Differences of Deviations from the Steady State (GD)

If an economy or sector is close to its steady state, it may be more appropriate to measure growth in total factor productivity as deviations from a long-term deterministic trend path estimated using the entire available data set, e.g. trend regression estimates, moving averages or Hodrick-Prescott filtered series. If $\tilde{X}_t$ denotes the deviation of X_t around a steady state value $\overline{X}_t$, then the production function (1) and the Goldsmith equation (3) can be approximated as

$$\tilde{Y}_t = \tilde{A}_t + s_K \left(\tilde{K}_t + \tilde{U}_t\right) + (1 - s_K)\,\tilde{N}_t \tag{16}$$

and

$$\tilde{K}_t = \frac{(1-\delta)}{(1+g)}\tilde{K}_{t-1} + \iota \tilde{I}_{t-1}, \tag{17}$$

respectively where $\iota = \frac{\overline{(I/K)}}{(1+g)}$, g is the deterministic steady state growth rate, and the capital elasticity $s_K \equiv \frac{F_K(A_t,K_t,N_t)K}{Y_t}$ is assumed constant, following the steady state restrictions on grand ratios emphasized by King, Plosser, and Rebelo (1988). Multiplying both sides of (16) by $\left(1 - \frac{(1-\delta)}{(1+g)}L\right)$ and substituting (17) yields the following estimate of a generalized difference of TFP growth:

$$\left(1 - \frac{(1-\delta)}{(1+g)}L\right)\widehat{A}_t = \left(1 - \frac{(1-\delta)}{(1+g)}L\right)\widetilde{Y}_t - \iota s_K \widetilde{I}_{t-1} \tag{18}$$

$$- \left(1 - \tfrac{(1-\delta)}{(1+g)}L\right) s_K \widetilde{U}_t - \left(1 - \tfrac{(1-\delta)}{(1+g)}L\right)(1 - s_K)\,\widetilde{N}_t$$

In (18) the capital stock has been eliminated completely from the computation. Given an initial condition, TFP growth estimates may be recovered recursively for the logarithmic approximation of TFP growth $t = 2, ..., T$:

$$TFPG_t\,(GD) = \varDelta\theta_t + \left(\frac{1-\delta}{1+g}\right) TFPG_{t-1}\,(GD) \tag{19}$$

where $\varDelta\theta_t = \left(1 - \frac{(1-\delta)}{(1+g)}L\right)\widetilde{Y}_t - s_K\left[\iota \widetilde{I}_{t-1} + \left(1 - \frac{(1-\delta)}{(1+g)}L\right)\widetilde{U}_t\right] - \left(1 - \frac{(1-\delta)}{(1+g)}L\right)(1 - s_K)\,\widetilde{N}_t$.

The computation of productivity growth estimates using the GD procedure will thus require an estimate of the initial condition, $TFPG_0\,(GD) = \ln\left(\frac{A_1}{A_0}\right)$. Our estimate, which is based on the Malmquist index, is described in detail in Appendix B and equals the geometric mean of labor productivity growth and output growth in the first period.

4.3 The Need for Numerical Evaluation

The central difference between the two alternatives to the Solow residual is the point around which the approximation is taken. In the DS approach, the point of approximation is the levels of factor inputs in the previous period. In the GD approach, the point of approximation is a balanced growth path for which the capital elasticity, s_K, the growth rate g, and the grand ratio I/K are constant. The advantages and disadvantages of each measurement will depend on the application at hand. If the economy is far from the steady state, the GD is likely to yield a poor approximation. On the other hand, it is likely to be more appropriate for business cycle applications involving OECD countries.

While both measurements eliminate capital from the TFP measurement, they introduce other forms of measurement error. The DS method replaces the capital stock with a relatively accurately measured gross investment flow plus a depreciation rate which is likely to be time-varying. The capital rental price κ_t can be obtained from independent sources or economic theory, but is likely to be measured with error.[16] Similarly, the GD procedure measures the marginal contribution of new capital but substitutes another form of measurement error (the growth of TFP in the first period). Given that the GD method necessarily assumes a constant rate of depreciation, it will tend to do worse when the depreciation rate is in fact endogenous and procyclical. It should perform poorly for economies or sector which are far from their steady states. In the end, it is impossible to see which type of measurement error is lower without resorting to simulation methods.

4.4 Assessing Alternative Measures of TFP Growth: A Horse Race

We now employ the same artificial data produced by the stochastic growth model in Section 3.1 to compare our alternative measurements with the most precise versions of the Solow-Törnqvist calculation, which estimate initial capital stocks along the lines of the BEA (Reinsdorf and Cover (2005), Sliker (2007)) and Caselli (2005). It is important to state carefully the assumptions behind the construction of the TFP growth measures. As before, the analyst is assumed never to observe the true capital stock, but does observe gross investment, employment, GDP, and real wages in each period. Under alternative scenarios, the analyst can or cannot observe the rate of capacity utilization or the depreciation rate in each period. When not observable, a constant quarterly value of the depreciation rate was assumed, equal to 0.015. For the DS method, we assume that the analyst cannot observe the user cost of capital (κ_t) in each period, but rather uses a constant $\bar{\kappa}$, its average value over the entire sample realization. For the GD estimates computed using equation (19), values of the constants δ and ι are set equal to 0.015 and 0.0112. respectively. We employed the Malmquist index described in Section 4.2 to estimate the initial condition of TFP growth as described in Appendix B.

As in the previous section, the basis of comparison is the root mean squared error (RMSE) for sample time series of 50 or 200 observations taken from 100 independent realizations of the stochastic growth model described in Section 3. The RMSE of this horse race along with standard errors are presented in Table 3 for both the "mature" economy (first panel) as well as the "transition economy" (second panel).

The horse race suggests that elimination of the capital stock is associated with substantial improvement of TFP growth measurement over the conventional Solow-Törnqvist residual. This improvement is significant for samples of both 50 and 200 observations, and for both mature and traditional economies. The DS outperforms both alternatives under all assumptions, and by as much as 63% (BEA versus DS, T=50). For the GD approach, the estimate of initial TFP growth based on of the Malmquist index makes a substantial contribution to RSME compared with assuming $\ln\left(\frac{A_1}{A_0}\right) = 0$.[17]

[16] The user cost of capital p_{kt} can be expressed as a function of the depreciation rate δ, of the nominal rate of return i and the unit capital at investment price p_i:

$$p_{kt} = p_{it-1} i_t + \delta p_{it-1} - [p_{it-1} - p_{it}]$$

As Balk (2009) has noted, user cost is more difficult to compute once the usual neoclassical assumptions are relaxed.

[17] We also considered the Malmquist index (41) itself as an alternative measure of TFP in each period. We obtained very similar, but inferior, results compared with the GD measure.

Table 3 A horse race: Stock-less versus traditional Solow-Törnquist estimates of TFP growth.

Mature economy (100 realizations, standard errors in parentheses)						
	A		B		C	
TFP growth estimates	**T=50**	**T=200**	**T=50**	**T=200**	**T=50**	**T=200**
TFPG(DS)	0.90	0.89	0.64	0.64	0.65	0.64
	(0.10)	(0.06)	(0.09)	(0.04)	(0.09)	(0.04)
TFPG(GD)	2.62	2.57	2.21	2.14	2.19	2.12
	(0.19)	(0.11)	(0.14)	(0.08)	(0.20)	(0.07)
TFPG(ST)/BEA estimate of K_0	3.56	1.96	3.50	1.85	3.50	1.85
	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.02)
TFPG(ST)/Caselli (2005)/BEA	3.55	1.95	3.49	1.84	3.49	1.85
estimate of K_0	(0.26)	(0.13)	(0.27)	(0.13)	(0.28)	(0.13)
Transition economy (100 realizations, standard errors in parentheses)						
	A		B		C	
TFP growth estimates	**T=50**	**T=200**	**T=50**	**T=200**	**T=50**	**T=200**
TFPG(DS)	3.22	1.78	1.50	1.33	2.34	1.31
	(0.22)	(0.10)	(0.67)	(0.18)	(0.11)	(0.08)
TFPG(GD)	4.85	3.40	1.75	1.72	3.08	2.55
	(0.35)	(0.15)	(1.31)	(1.21)	(0.21)	(0.09)
TFPG(ST)/BEA estimate of K_0	4.64	2.87	3.16	1.89	3.97	2.08
	(0.24)	(0.15)	(1.14)	(0.40)	(0.03)	(0.02)
TFPG(ST)/Caselli (2005)/BEA	5.26	2.76	5.27	2.78	5.28	2.79
estimate of K_0	(0.27)	(0.13)	(0.27)	(0.13)	(0.27)	(0.13)

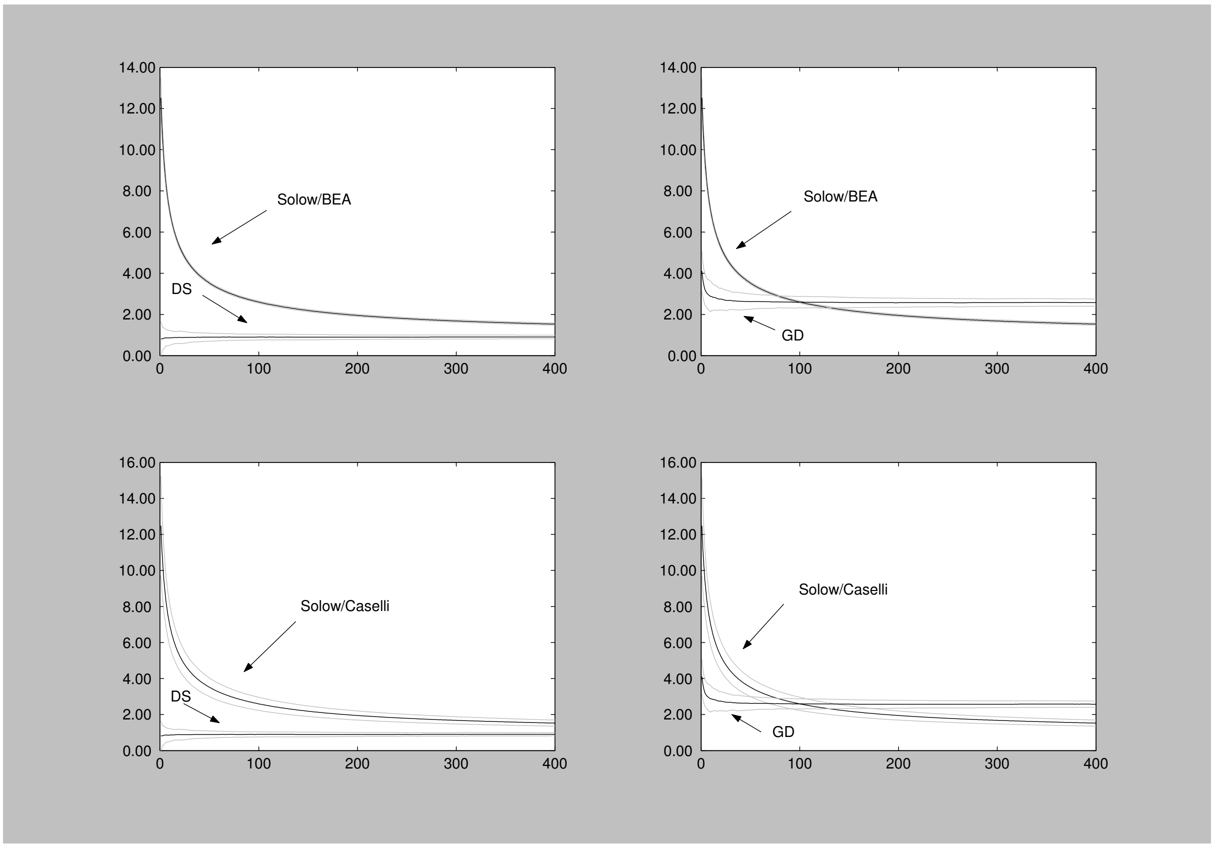
A: Analyst observes $\{Y_t, N_t, I_t, \bar{\kappa}, \omega_t\}$ only
B: Analyst observes $\{Y_t, N_t, I_t, \bar{\kappa}, \omega_t, U_t\}$
C: Analyst observes $\{Y_t, N_t, I_t, \bar{\kappa}, \omega_t, U_t, \delta_t\}$

As would be expected, the RMSE improvement of the stock-less measures over the conventional Solow-Törnqvist residual estimates is inversely related to the relative importance of the initial condition and thus to the length of the sample time series. This relationship in our synthetic data set of 100 realizations of the artificial economy is displayed in Figure 2, which presents four graphical comparisons of average RMSE for the mature economy case, along with two standard-deviation confidence bands, as a function of sample size. The graphs demonstrate how the stock-less measurements perform significantly better in a root mean squared error sense for a small sample, and that this advantage tends to die out at a slow rate. Only with samples sizes of 400 or more observations - a century of data - are the ST, DS and GD measures equally accurate.

5 Application: TFP Growth in the German Federal States

We now apply the two new measures to study the source of economic growth in the federal states of Germany ("Bundesländer") after reunification. Income and product account data are available beginning with 1991 for 16 states: 11 "old" Western states (Bavaria, Baden-Württemberg, Bremen, Hamburg, Hesse, Lower Saxony, North Rhine-Westphalia, Rhineland-Palatinate, Saarland, Schleswig-Holstein), 6 "new" Eastern states (Berlin, Brandenburg, Mecklenburg-

Fig. 2 Dependence of RMSE (%) on sample size (with two standard error bands)



West Pommerania, Saxony-Anhalt, Saxony, and Thuringia).[18] We employ the income and product accounts and capital stock estimates at the level of the federal states published by the Working Group for State Income and Product Accounts (*Arbeitsgemeinschaft Volkswirtschaftliche Gesamtrechnung der Länder*).[19] This dataset allows us to revisit the findings of Burda and Hunt (2001), who studied labor productivity and total factor productivity growth using the conventional Solow residual measure and their own estimates of the states' capital stocks. Given the poor measurement of the capital stock in the new states, especially for structures, the alternative DS and GD methods offer an opportunity to investigate TFP growth measurements with a "treatment" group (East Germany) as well as a "control" group (West Germany), where the treatment is an unusually bad measurement of initial capital stocks. Reunification - due to both market competition and the revaluation of the east German mark - rendered about 80% of East German production noncompetitive (Akerlof, Rose, Yellen, and Hessenius (1991)), implying a large loss of value of existing equipment and structures. At the same time, many structures measured initially at minimal book value have since been

[18] Berlin is counted as a new state consisting of the union of East and West Berlin, because the western half of Berlin, while under the protection and economic aegis of Western Germany until 1989, never enjoyed full status as a *Bundesland*.

[19] The data can be downloaded at the website `http://www.vgrdl.de/Arbeitskreis_VGR/ergebnisse.asp`. Capital stocks for the new states in the period 1991-1993 were computed by backcasting the perpetual inventory method from the 1994 estimates.

re-employed by businesses, implying higher capital stock value than conventionally measured. Depreciation rates and capacity utilization data do not exist at the state level, further compounding already severe measurement problems.

In Table 4, we present Solow-Törnqvist residuals and our stock-free TFP measurements for both new and old German states averaged over two sub-periods 1994-1999 and 2000-2006. We also present the same calculations based on macroeconomic aggregates constituted by the Eastern states, the Western and all of Germany. The Solow residual estimates utilize an estimate of capital stocks provided by the state (*Bundesland*) statistical agencies and the working group involved in collecting and standardizing the state income and product accounts. A constant capital share (0.33) was assumed. For the DS method, the annual rental price of capital (κ) was set to a constant value over the entire period (0.11). For the GD approach, a simple two-sided moving average of three years was used to estimate the trend. For both approaches, a constant rate of capital depreciation δ equal to 7.52% per annum was employed. Capacity utilization and depreciation at the *Bundesland* level is not available, so the equivalent of scenario A was adopted throughout. Lacking data on hours worked, we used total employment as a measure of labor input.

Table 4 TFP Measurement in German Federal States: A Comparison

	$TFPG\,(ST)$		$TFPG\,(DS)$		$TFPG\,(GD)$	
	1994-1999	2000-2006	1994-1999	2000-2006	1994-1999	2000-2006
Berlin	0.1	-0.2	0.4	0.7	1.9	1.0
Brandenburg	0.7	1.0	1.8	1.7	4.4	2.1
Mecklenburg-Western Pomerania	-0.4	0.7	0.1	1.2	4.9	2.1
Saxony	-0.3	1.7	1.3	2.4	4.6	2.0
Saxony-Anhalt	-1.0	1.4	0.1	2.1	3.8	1.8
Thuringia	0.2	1.5	1.0	2.2	4.0	1.8
All East German states	**0.4**	**1.0**	**0.8**	**1.7**	**3.4**	**1.6**
Baden-Württenberg	1.6	0.8	2.7	1.6	1.8	1.0
Bavaria	1.4	1.4	2.5	2.2	1.0	0.7
Bremen	1.5	1.6	2.8	2.7	1.8	1.1
Hamburg	0.9	0.0	2.5	0.3	1.2	0.9
Hesse	1.4	0.8	2.6	1.6	1.3	0.8
Lower Saxony	0.5	0.5	1.5	1.2	0.5	0.2
North Rhine-Westphalia	0.7	0.4	1.9	1.3	1.6	0.8
Rhineland-Palatinate	0.5	0.4	1.5	1.1	1.0	0.5
Saarland	0.9	1.6	1.8	2.3	2.0	1.0
Schleswig-Holstein	0.7	0.6	1.7	1.5	1.2	0.7
All West German states	**1.0**	**0.8**	**2.2**	**1.6**	**1.3**	**0.7**
Germany	**1.0**	**0.8**	**2.0**	**1.6**	**1.6**	**0.8**

Table 5 Growth accounting using the three methods, 1994-1999 (% per annum).

	$\frac{\Delta Y}{Y}$	$(1-\alpha)\frac{\Delta N}{N}$	$TFPG\,(ST)$	$\frac{\Delta K}{K}$	$TFPG\,(DS)$	$\alpha\frac{\Delta K^{DS}}{K^{DS}}$	$TFPG\,(GD)$	$\alpha\frac{\Delta K^{GD}}{K^{GD}}$
Berlin	-0.3	-0.9	0.1	0.5	0.4	0.2	1.9	-1.3
Brandenburg	4.8	0.2	0.7	3.9	1.8	2.8	4.4	0.1
Mecklenburg-Western Pomerania	4.5	0.2	-0.4	4.8	0.1	4.3	4.9	-0.5
Saxony	4.3	0.3	-0.3	4.2	1.3	2.6	4.6	-0.7
Saxony-Anhalt	3.7	-0.4	-1.0	5.0	0.1	3.9	3.8	0.3
Thuringia	4.4	0.4	0.2	3.7	1.0	3.0	4.0	0.0
All East German states	**3.0**	**0.0**	**0.4**	**2.6**	**0.8**	**2.3**	**3.4**	**-0.4**
Baden-Württenberg	2.2	0.3	1.6	0.3	2.7	-0.7	1.8	0.1
Bavaria	2.3	0.3	1.4	0.6	2.5	-0.5	1.0	1.0
Bremen	1.0	-0.6	1.5	0.0	2.8	-1.3	1.8	-0.3
Hamburg	1.1	-0.2	0.9	0.4	2.5	-1.1	1.2	0.2
Hesse	1.8	0.0	1.4	0.3	2.6	-0.8	1.3	0.5
Lower Saxony	1.3	0.4	0.5	0.4	1.5	-0.7	0.5	0.3
North Rhine-Westphalia	1.2	0.3	0.7	0.2	1.9	-1.1	1.6	-0.8
Rhineland-Palatinate	1.4	0.5	0.5	0.4	1.5	-0.5	1.0	-0.1
Saarland	1.7	0.5	0.9	0.3	1.8	-0.6	2.0	-0.8
Schleswig-Holstein	1.4	0.2	0.7	0.5	1.7	-0.5	1.2	0.0
All West German states	**1.7**	**0.3**	**1.0**	**0.4**	**2.2**	**-0.8**	**1.3**	**0.1**
Germany	**1.9**	**0.2**	**1.0**	**0.7**	**2.0**	**-0.3**	**1.6**	**0.1**

Note: Components may not add exactly due to rounding error.

Table 6 Growth accounting using the three methods, 2000-2006 (% per annum).

	$\frac{\Delta Y}{Y}$	$(1-\alpha)\frac{\Delta N}{N}$	$TFPG\,(ST)$	$\frac{\Delta K}{K}$	$TFPG\,(DS)$	$\alpha\frac{\Delta K^{DS}}{K^{DS}}$	$TFPG\,(GD)$	$\alpha\frac{\Delta K^{GD}}{K^{GD}}$
Berlin	-0.4	-0.3	-0.2	0.1	0.7	-0.7	1.0	-1.1
Brandenburg	1.2	-0.8	1.0	1.1	1.7	0.3	2.1	-0.1
Mecklenburg-Western Pomerania	0.7	-0.8	0.7	0.8	1.2	0.3	2.1	-0.7
Saxony	1.7	-0.6	1.7	0.7	2.4	0.0	2.0	0.4
Saxony-Anhalt	1.0	-1.0	1.4	0.6	2.1	-0.1	1.8	0.3
Thuringia	1.6	-0.9	1.5	1.0	2.2	0.3	1.8	0.7
All East German states	**0.9**	**-0.7**	**1.0**	**0.6**	**1.7**	**-0.1**	**1.6**	**0.0**
Baden-Württenberg	1.6	0.3	0.8	0.4	1.6	-0.4	1.0	0.3
Bavaria	2.2	0.3	1.4	0.5	2.2	-0.3	0.7	1.2
Bremen	1.8	0.0	1.6	0.2	2.7	-0.9	1.1	0.7
Hamburg	1.3	0.0	0.8	0.4	1.6	-0.4	0.8	0.4
Hesse	1.2	0.3	0.0	1.0	0.3	0.6	0.9	0.0
Lower Saxony	1.2	0.1	0.8	0.3	1.6	-0.5	0.8	0.3
North Rhine-Westphalia	1.1	0.2	0.5	0.4	1.2	-0.3	0.2	0.6
Rhineland-Palatinate	0.8	0.1	0.4	0.3	1.3	-0.6	0.8	-0.1
Saarland	1.1	0.3	0.4	0.4	1.1	-0.4	0.5	0.2
Schleswig-Holstein	1.9	0.2	1.6	0.1	2.3	-0.6	1.0	0.7
All West German states	**0.9**	**0.0**	**0.6**	**0.3**	**1.5**	**-0.6**	**0.7**	**0.2**
Germany	**1.4**	**0.2**	**0.8**	**0.4**	**1.6**	**-0.4**	**0.7**	**0.4**

Note: Components may not add exactly due to rounding error.

We first turn to TFP growth estimates for the aggregated regions East, West and all Germany. The qualitative predictions of the DS measure is broadly consistent with those of the Solow residual, which indicate a pick-up of TFP growth over the period in the East and a decline in the West. Evidently, the GD measure is inappropriate for the new states, given their initial condition at the start of reunification. Both DS and GD estimates are considerably less volatile than the Solow residual; the coefficient of variation for the period 1994-2006 were 0.42 and 0.48 for the former, respectively, compared with 0.78 for the latter. To the extent that all three measures are estimating the same phenomenon, the alternative we propose appear to provide a tighter estimate of the temporal evolution of TFP in the two regions.

The same conclusion can be drawn from the cross-sectional dimension of our TFP measurements. The prior expectation is that measurement error should be most severe in the new states, given the limited statistical basis for computing capital stocks. Yet there is little reason to expect wide variation across space within the East or West during during these seven-year intervals. Indeed, the coefficients of variation for Solow residuals in the East are almost an order of magnitude larger than the DS and GD estimates in the early period 1994-1999 (5.0 versus 0.9 and 0.3). In the latter half of the sample, the coefficients of variation of the three measures are similar across the East-West divide (0.7, 0.4, and 0.2 for Solow, DS, and GD in the East, versus 0.7, 0.4, 0.3 in the West, respectively). The consistently lower coefficients of variation of our alternative measures in cross section is further evidence that they are subject to less measurement error than the Solow-Törnqvist counterpart.

The DS and GD estimates can be used to back out an implied contribution of capital to real growth, or, given a capital share, to growth in the "true" (i.e. actually utilized) capital stock. These estimates are presented in Tables 5 and 6. They indicate indeed a larger degree of fluctuation than implied by official estimates of capital stock growth. They support the findings of Burnside, Eichenbaum, and Rebelo (1995) and others, that the fluctuation of capital in use is an important source of measurement error and should be taken seriously when computing the Solow residual. The GD and DS measures reduce this source of mismeasurement to the extent that the utilization of recent capital formation more closely tracks the "true" utilization rate. It is striking that both alternative measurements imply little or no contribution of growth in capital input to the evolution of East German GDP in the latter period, despite impressive investment rates in the 1990s (Burda and Hunt (2001)).

6 Conclusion

Over the past half-century, the Solow residual has achieved widespread use in economics and management as a measurement of total factor productivity. Its popularity can be attributed to its simplicity and independence of statistical methods. Despite this acceptance, there has been no effort to evaluate systematically the quality of this measurement tool. This complacency is remarkable in light of potentially severe measurement problems associated with capital stock data. We have documented the significance of this error, as measured by the root mean squared error is in a synthetic data set. Application of our TFP growth measures to the federal German states after reunification yields estimates which are more stable across time and in space.

While the measurement error of the Solow residual decreases with sample size, it is a serious problem for short data sets or economies in transition. Thus, the Solow residual is least accurate in applications for which TFP measurements are most valuable. Such applications include the transition to a market economy, the introduction of ICT capital in the production process, and assessing the role of weightless assets such as advertising goodwill and research and development knowledge (Corrado, Hulten, and Sichel (2006)).

Both of our proposed alternatives to the Solow-Törnqvist measures can be thought of as a "marginalization" of the error carried forward in the capital stock across time. Most recent investment is most likely to be properly valued at acquisition cost and to be fully utilized. Our results suggest that these methods could be applied to a number of investment context and types, thus widening the scope and appeal of applied TFP measurement.

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References

Akerlof, G. A., A. K. Rose, J. L. Yellen, and H. Hessenius (1991): "East Germany in from the Cold: The Economic Aftermath of Currency Union," *Brookings Papers on Economic Activity*, 22(1991-1), 1–106.

Balk, B. M. (2009): "Measuring Productivity Change without Neoclassical Assumptions: A Conceptual Analysis," Discussion paper 09023, Statistics Netherlands.

Barro, R. J. (1999): "Notes on Growth Accounting," *Journal of Economic Growth*, 4, 119–137.

Barro, R. J., and X. Sala-I-Martin (2003): *Economic Growth*. MIT Press.

Burda, M. C., and J. Hunt (2001): "From Reunification to Economic Integration: Productivity and Labor Market in Eastern Germany," *Brookings Papers on Economic Activity*, 2, 1–71.

Burnside, C., M. Eichenbaum, and S. Rebelo (1993): "Labor Hoarding and the Business Cycle," *Journal of Political Economy*, 101(2), 245–73.

——— (1995): "Capital Utilization and Returns to Scale," *NBER Macroeconomics Annual*.

Caselli, F. (2005): "Accounting for Cross-Country Income Differences," *Handbook of Economic Growth*.

Caves, D. W., L. R. Christensen, and W. E. Diewert (1982): "The Economic Theory of Index Numbers and the Measurement of Input, Output, and Productivity," *Econometrica*, 50(6), 1393–1414.

Christensen, L., D. Jorgenson, and L. Lau (1973): "Trascendental Logarithmic Production Frontiers," *Review of Economics and Statistics, 55, 26-45.*

Cooley, T., and E. Prescott (1995): "Economic Growth and Business Cycles, in Frontiers of Business Cycle Research (ed. Thomas F. Cooley)," *Princeton University Press*, Princeton University Press.

Corrado, C., and J. Mattey (1997): "Capacity Utilization," *Journal of Economic Perspectives*, 11(1), 151–67.

Corrado, C. A., C. R. Hulten, and D. E. Sichel (2006): "Intangible Capital and Economic Growth," NBER Working Papers 11948, National Bureau of Economic Research, Inc.

De Borger, B., and K. Kerstens (2000): "The Malmquist Productivity Index and Plant Capacity Utilization," *Scandinavian Journal of Economics*, 102(2), 303–10.

Dejong, D., and C. Dave (2007): "Structural Macroeconometrics," *Princeton University Press.*

Denison, E. (1962): "Source of Economic Growth in the United States and the Alternative before Us," *Committee for Economic Development.*

——— (1972): "Some Major Issues in Productitivy Analysis: An Examination of the Estimates by Jorgenson and Griliches," *Survey of Current Business*, 5(2), 1–27.

Diewert, W. E. (1976): "Exact and superlative index numbers," *Journal of Econometrics*, 4(2), 115–145.

Diewert, W. E., and K. J. Fox (2010): "Malmquist and Törnqvist Productivity Indexes: Returns to Scale and Technical Progress with Imperfect Competition," Ubc departmental archives, UBC Department of Economics.

Diewert, W. E., and A. O. Nakamura (2003): "Index Number concepts, Measures and Decomposition of Productivity Growth," *Journal of Productivity Analysis*, 19, 127–159.

——— (2007): "The Measurement of Productivity for Nations," in *Handbook of Econometrics*, ed. by J. Heckman, and E. Leamer, vol. 6 of *Handbook of Econometrics*, chap. 66. Elsevier.

Färe, R., S. Grosskopf, B. Lindgren, and P. Ross (1989): "Productivity Developments in Swedish Hospitals: A Malmquist Output Index Approach," *Southern Illinois University at Carbondale. Department of Economics. Working*

Paper.

——— (1992): "Productivity Changes in Swedish Pharmacies 1980-1989: A Nonparametric Malmquist Approach," *Journal of Productivity Analysis*, 3(1:2), 85–101.

Goldsmith, R. (1955): "The National Wealth of the United States in the Post-war Period," *Princeton, N.J.: Princeton University Press.*

Gollop, F., and D. W. Jorgenson (1980): "United States Factor Productivity by Industry, 1947-1973," *in John W. Kendrick and Beatrice Vaccara (eds.), New Developments in Productivity Measurement and Analysis, Conference on Research in Income and Wealth, Chicago: University of Chicago Press for the National Bureau of Economic Research*, 44(3).

Greenwood, J., Z. Hercowitz, and G. W. Huffman (1988): "Investment, Capacity Utilization, and the Real Business Cycle," *American Economic Review*, 78(3), 402–17.

Griliches, Z. (1980): "R&D and the Productivity Slowdown," NBER Working Papers 0434, National Bureau of Economic Research, Inc.

——— (1996): "The Discovery of the Residual: A Historical Note," *Journal of Economic Literature*, 34(3), 1324–1330.

Hall, R. E., and C. I. Jones (1999): "Why Do Some Countries Produce So Much More Output Per Worker Than Others?," *The Quarterly Journal of Economics*, 114(1), 83–116.

Hansen, G. D. (1985): "Indivisible labor and the business cycle," *Journal of Monetary Economics*, 16(3), 309–327.

Harrison, S., and M. Weder (2002): "Did Sunspot Cause the Great Depression?," *Journal of Economic Theory.*

Hulten, C., E. Dean, and M. Harper (2001): "New Developments in Productivity Analysis (eds.)," (63).

Hulten, C. R. (1986): "Productivity change, capacity utilization, and the sources of efficiency growth," *Journal of Econometrics*, 33(1-2), 31–50.

——— (1990): "The Measurement of Capital," *Fifty Years of Economic Measurement, E.R. Berndt and J.E. Triplett (eds.), Studies in Income and Wealth, The National Bureau of Economic Research, Chicago: The University of Chicago Press*, 54(June), 119–152.

Hulten, C. R. (1992): "Growth Accounting When Technical Change Is Embodied in Capital," *American Economic Review*, 82(4), 964–80.

Jacob, V., S. C. Sharma, and R. Grabowski (1997): "Capital Stock Estimates for Major Sectors and Disaggregated Manufacturing in Selected OECD Countries," *Applied Economics*, 29(5), 563–79.

Jorgenson, D., and Z. Griliches (1967): "The Explanation of Productivity Change," *Review of Economic Studies*, 34, 249–283.

——— (1972): "Issue in Growth Accounting: A Reply to Edward F. Denison," *Survey of Current Business*, 52, 65–94.

King, R. G., C. I. Plosser, and S. T. Rebelo (1988): "Production, growth and business cycles : I. The basic neoclassical model," *Journal of Monetary Economics*, 21(2-3), 195–232.

King, R. S., and S. Rebelo (1999): "Resuscitating Real Business Cycles," *Handbook of Macroeconomics, edited by J. B. Taylor and M. Woodford, Volume 1B, 9271007. Amsterdam: Elsevier.*

Kuznets, S. (1971): "Economic Growth of Nations," *Harvard University Press.*

Maddison, A. (1992): "A Long-Run Perspective on Savings," *Scandinavian Journal of Economics*, 94(2), 181–196.

Malmquist, S. (1953): "Index numbers and indifference surfaces," *Trabajos de Estadistica*, 4, 209–242.

MAYES, D., AND G. YOUNG (1994): "Improving Estimates of the U.K. Capital Stock," *National Institute Economic Review*, pp. 84–96.

MCGRATTAN, E. R., AND E. C. PRESCOTT (2009): "Unmeasured Investment and the Puzzling U.S. Boom in the 1990s," Discussion paper.

MOHNEN, P., AND T. TEN RAA (2002): "Neoclassical Growth Accounting and Frontier Analysis: A Synthesis," *Journal of Productivity Analysis*, 18, 111–28.

O'MAHONY, M., AND M. TIMMER (2009): "Output, Input and Productivity Measures at the Industry Level: The EU KLEMS Database," *Economic Journal*, 119(538), F374–F403.

O'MAHONY, M., AND B. VAN ARK (2003): "EU Productivity and Competitiveness: An Industry Perspective Can Europe Resume the Catching-up Process?," *Office for Official Publications of the European Communities, Luxembourg.*

PRESCOTT, E. C. (1986): "Theory ahead of business cycle measurement," *Quarterly Review, Federal Reserve Bank of Minneapolis*, pp. 9–22.

PRITCHETT, L. (2000): "The Tyranny of Concepts: CUDIE (Cumulated, Depreciated, Investment Effort) Is Not Capital," *Journal of Economic Growth*, 5(4), 361–84.

REINSDORF, M., AND M. COVER (2005): "Measurement of Capital Stocks, Consumption of Fixed Capital, and Capital Services," *BEA, mimeo.*

ROBINSON, J. (1953): "The Production Function and the Theory of Capital," *The Review of Economic Studies.*

SCHREYER, P. (2001): "OECD Manual: Measuring Capital," *OECD.*

——— (2009): "OECD Manual: Measuring Capital," *OECD.*

SHEPARD, R. (1970): *Theory of Costs and Production Functions.* Princeton University Press, Princeton.

SLIKER, B. (2007): "2007 R&D Satellite Account Methodologies: R&D Capital Stocks and Net Rates of Return," R&d satellite account background paper, Bureau of Economic Analysis/National Science Foundation.

SOLOW, R. M. (1957): "Technical Change and the Aggregate Production Function," *Review of Economics and Statistics,39:312-320.*

STOCK, J. H., AND M. W. WATSON (1999): "Business cycle fluctuations in us macroeconomic time series," *Handbook of Macroeconomics*, 1, 3–64.

TEN RAA, T., AND V. SHESTALOVA (2006): "Alternative Measures of Total Factor Productivity Growth," CentER Discussion Papers 54, CentER.

THANASSOULIS, E., M. PORTELA, AND O. DESPIC (2008): "Data Envelopment Analysis: The Mathematical Programming Approach to Efficiency Analysis," in *The Measurement of Productive Efficiency and Productivity Growth*, ed. by H. Fried, K. Lovell, and S. Schmidt, chap. 3, pp. 251–420. Oxford University Press.

TINBERGEN, J. (1942): "Zur Theorie der Langfristigen Wirtschaftstsentwicklung," *Weltwirtschaftliches Archiv*, 55(1), 511–49.

TÖRNQVIST, L. (1936): "The Bank of Finland's Consumption Price Index," *Bank of Finland Monthly Bulletin, 10, 1-8.*

WARD, M. (1976): "The Measurement of Capital. The Methodology of Capital Stock Estimates in OECD Countries.," *OECD Paris.*

WEN, Y. (1998): "Capacity Utilization under Increasing Returns to Scale," *Journal of Economic Theory*, 81(1), 7–36.

Appendix A: The Stochastic Growth Model

A1. First Order Conditions and Decentralized Market Equilibrium

Let λ_t denote the Lagrange multiplier corresponding to the periodic resource constraint (12). The first-order conditions for the household are, for $t \geq 0$:

$$C_t : \ \lambda_t = \frac{1}{C_t} \tag{20}$$

$$K_{t+1} : \ \lambda_t = \beta E_t \left[\lambda_{t+1} \left((1-\delta_{t+1}) + \kappa_{t+1} U_{t+1} \right) \right] \tag{21}$$

$$N_t : \ \theta (1-N_t)^{-\eta} = \lambda_t \omega_t \tag{22}$$

$$U_t : \ B U_t^{\chi - 1} = \kappa_{t+1} \tag{23}$$

First-order conditions for the firms

$$N_t : \ (1-\alpha) A_t \left(U_t K_t \right)^{\alpha} N_t^{-\alpha} = \omega_t \tag{24}$$

$$K_t : \ \alpha A_t U_t^{\alpha} K_t^{\alpha - 1} N_t^{1-\alpha} = \kappa_t U_t \tag{25}$$

the production function

$$Y_t = A_t U_t^{\alpha} K_t^{\alpha} N_t^{1-\alpha} \tag{26}$$

and the aggregate resource constraint (since $\omega_t N_t + \kappa_t U_t K_t = Y_t$).

$$K_{t+1} = (1-\delta_t) K_t + Y_t - C_t \tag{27}$$

The equilibrium of this decentralized economy is defined as the sequences of wages $\{\omega_t\}$, rental prices for capital $\{\kappa_t\}$, output $\{Y_t\}$, consumption $\{C_t\}$, employment $\{N_t\}$, capital stocks $\{K_{t+1}\}$, and the capacity utilization rate $\{U_t\}$ such that equations (21)-(27) hold for $t \geq 0$ plus a suitable transversality condition to guarantee that the capital stock path is indeed consistent with utility maximization. The equilibrium of the problem will be, due to the first and second welfare theorems, unique and equivalent to the one chosen by a social planner with the objective of maximizing the utility of the representative household.

A2. Detrended Version of Equilibrium

Steady state values of the model's variables are denoted by an upper bar. In the steady state $\bar{X}_{t+1} = (1+g)\, \bar{X}_t$ for $X \in \{C, I, Y, K\}$ and $\bar{A}_{t+1} = \psi \bar{A}_t$. We define detrended values of the variables of interest such that $\widetilde{X}_t \equiv X_t / \overline{X}_t$. The following equations characterize the equilibrium of this transformed economy:

$$\frac{\theta \tilde{C}_t}{(1-N_t)^{\eta}} = (1-\alpha)\,\gamma_t U_t^{\alpha} \tilde{K}_t^{\alpha} N_t^{-\alpha} \tag{28}$$

$$1 = E_t\left[\beta \frac{\tilde{C}_t}{\psi \tilde{C}_{t+1}} R_{t+1}\right] \tag{29}$$

$$\alpha\gamma_t\left(\frac{\tilde{K}_t}{N_t}\right)^{\alpha-1} = BU_t^{\chi-\alpha} \tag{30}$$

$$\psi \tilde{K}_{t+1} = (1-\delta_t)\,\tilde{K}_t + \tilde{Y}_t - \tilde{C}_t$$

The first equation characterizes intratemporal optimality of time across alternative uses in production and leisure; the second is the familiar Euler equation which arbitrages expected intertemporal rates of substitution and transformation in expectation, where the latter is defined by $R_{t+1} = \alpha\gamma_t \left(U_t\widetilde{K_t}\right)^{\alpha-1} N_t^{1-\alpha}$ and represents the gross rate of return on holding a unit of capital from period t to period $t+1$. The last equation is the periodic resource constraint of the economy, given the production function and competitive factor remuneration. Given that this economy fulfills the conditions of the first welfare theorem, it would also characterize the optimal choice of a central planner maximizing (11) subject to the resource constraints (12) and the initial condition K_0.

A3. The Steady State

To solve for the non-stochastic steady state, let $\gamma_t = 1$ and $\tilde{K}_{t+1} = \tilde{K}_t = \bar{K}$. We obtain the following equations:

$$\frac{\theta \bar{C}}{(1-\bar{N})^{\eta}} = (1-\alpha)\,\bar{U}^{\alpha} \bar{K}_t^{\alpha} \bar{N}^{-\alpha} \tag{31}$$

$$1 = \frac{\beta}{\psi}\bar{R} \tag{32}$$

$$\alpha\left(\frac{\bar{K}}{\bar{N}}\right)^{\alpha-1} = B\bar{U}^{\chi-\alpha} \tag{33}$$

A4. Log Linearization

Using the convention that $\widehat{x} = (x - \overline{x})/\overline{x}$ denote deviations from steady state values, the log-linearized first order condition for labor supply can be written as

$$\hat{c}_t - \left(\alpha + \frac{N}{1-N}\eta\right)\hat{n}_t = \gamma_t + \alpha\left(\hat{u}_t + \hat{k}_t\right) \tag{34}$$

The resource constraint is:

$$\frac{\bar{C}}{\bar{K}}\hat{c}_t + \psi\hat{k}_{t+1} = (1-\delta)\,\hat{k}_t - \chi\hat{u}_t + \alpha\frac{\bar{Y}}{\bar{K}}\hat{k}_t + (1-\alpha)\,\frac{\bar{Y}}{\bar{K}}\bar{N}\hat{n}_t + \frac{\bar{Y}}{\bar{K}}\hat{\gamma}_t \tag{35}$$

and the Euler equation becomes

$$0 = E_t\left[\hat{c}_t - \hat{c}_{t+1} + \beta\bar{r}\left[\hat{\gamma}_{t+1} - (1-\alpha)\left(\hat{k}_{t+1} - \hat{n}_{t+1}\right) - \chi\hat{u}_t\right]\right] \tag{36}$$

A5. Model Calibration and Generation of the Synthetic Dataset

We calibrate the model to a quarterly setting using values typically used for simulating the US time series in the literature and discussed in Prescott (1986) or Rebelo and King (1999). The values chosen for the parameters are presented in Table 7.

Table 7 Stochastic growth model: parameters and calibration values

Parameter	Definition	Value	Source
β	utility discount factor (quarterly)	0.985	Data
$\bar{R}$	average real interest factor (quarterly)	1.015	Data
$\bar{\gamma}$	technology	1	Theory
$\bar{\delta}$	depreciation rate of physical capital	0.015	Data
α	capital elasticity in production	0.36	Data
η	elasticity of periodic utility to leisure	0.85	Theory
θ	utility weight for leisure/consumption	2.1	Theory
$\psi = (1+g)^{1-\alpha}$	constant growth factor of technology	1.0075	Data
B	level parameter for capital depreciation rate	0.0255	Data
χ	elasticity of depreciation to capacity utilization	1.9	Data
ρ	autocorrelation of TFP term A_t	0.95	Theory

Appendix B: The Malmquist Index

B1. The Basics

The Malmquist index represents one of the most commonly used indices in data envelopment analysis and an alternative way for computing productivity and efficiency changes in the production functions.[20] Proposed by Caves, Christensen, and Diewert (1982) reinterpreting an index introduced by Malmquist (1953), it is defined in the original version by the ratio of two distance output functions $D_O^t(x, y)$ (Shepard (1970)) at time t and $t+1$:

$$M_{CCD}^{t} = \frac{D_O^t\left(x^{t+1}, y^{t+1}\right)}{D_O^t\left(x^t, y^t\right)} \tag{37}$$

where the numerator is represented by the maximal proportional change in outputs required to obtain the combination (x^{t+1}, y^{t+1}) feasible in relation to the technology at time t. Färe, Grosskopf, Lindgren, and Ross (1989) consider an alternative measure of (37), as

$$M_{FGLR}^{t+1} = \frac{D_O^{t+1}\left(x^{t+1}, y^{t+1}\right)}{D_O^{t+1}\left(x^t, y^t\right)} \tag{38}$$

and propose a new version of the Malmquist index, defined as the geometric mean of (37) and (38):

$$M_0\left(x^{t+1}, y^{t+1}, x^t, y^t\right) = \left[\left(\frac{D_O^t\left(x^{t+1}, y^{t+1}\right)}{D_O^t\left(x^t, y^t\right)}\right)\left(\frac{D_O^{t+1}\left(x^{t+1}, y^{t+1}\right)}{D_O^{t+1}\left(x^t, y^t\right)}\right)\right]^{\frac{1}{2}}. \tag{39}$$

In addition, Färe, Grosskopf, Lindgren, and Ross (1992) rewrite (39) yielding an efficiency and a technological term:

$$M_0\left(x^{t+1}, y^{t+1}, x^t, y^t\right) = \left(\frac{D_O^{t+1}\left(x^{t+1}, y^{t+1}\right)}{D_O^t\left(x^t, y^t\right)}\right)\left[\left(\frac{D_O^t\left(x^t, y^t\right)}{D_O^{t+1}\left(x^t, y^t\right)}\right)\left(\frac{D_O^t\left(x^{t+1}, y^{t+1}\right)}{D_O^{t+1}\left(x^{t+1}, y^{t+1}\right)}\right)\right]^{\frac{1}{2}} \tag{40}$$

where the term $\left[\left(\frac{D_O^t(x^t, y^t)}{D_O^{t+1}(x^t, y^t)}\right)\left(\frac{D_O^t(x^{t+1}, y^{t+1})}{D_O^{t+1}(x^{t+1}, y^{t+1})}\right)\right]^{\frac{1}{2}}$ measures the contribution of technological change and it is equivalent to the Törnqvist index.[21]

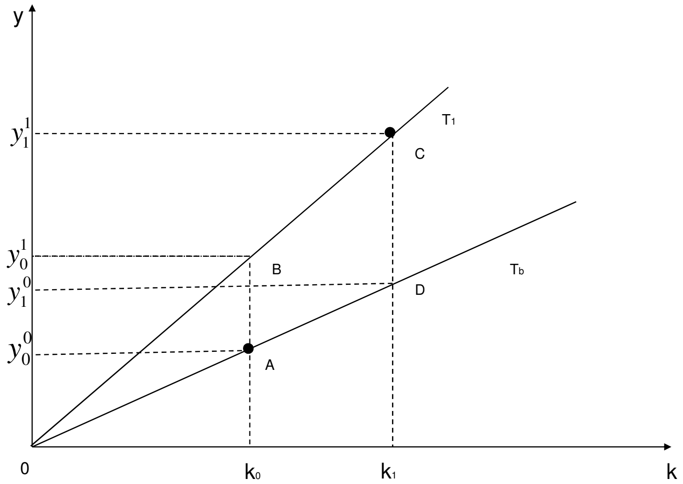
Assuming a case with one output and two inputs, it is possible to normalize by labor so as only one input in the production function, so that $y_t = \frac{Y_t}{N_t}$ and $k_t = \frac{K_t}{N_t}$.

Figure 3 depicts a graphical representation of the Malmquist index for an economy in the presence of constant return to scale and full efficiency: four data points provide a measure of technology change (from T_0 to T_1), which contributes to move from point A, i.e., the amount of output produced at time 0 $y_0^0 \equiv f_0(k_0)$, to point C, i.e., the production in the second period $y_1^1 \equiv f_1(k_1)$. To do so, TFP growth is decomposed into the input accumulation and the information on the counterfactuals, point D, which represents the production using the technology at time 0 with the amount of input used at time 1 ($y_1^0 \equiv f_0(k_1)$), and point B, i.e., the amount produced with input at time 0 and technology used

[20] For a review of the indices used in productivity analysis, see Thanassoulis, Portela, and Despic (2008).

[21] Recently, Diewert and Fox (2010) shows that a relationship between the Malmquist and the Törnqvist under imperfect returns to scale.

Fig. 3 Construction of the Malmquist index in the full efficiency case



at time 1 ($y_0^1 \equiv f_1(k_0)$), where, for each y_i^j is the amount produced with input at time j and technology at time k. Assuming constant returns and full efficiency, the log of the Malmquist index equals the log of the geometric mean of the average products in the first two periods, or

$$\ln M_0^1 = \frac{1}{2}\ln\left(\frac{y_1^1}{y_0^0}\frac{y_1^0}{y_0^1}\right) = \underbrace{\frac{1}{2}\ln\left(\frac{y_1^1}{y_0^0}\right)}_{KNOWN} + \underbrace{\frac{1}{2}\ln\left(\frac{y_1^0}{y_0^1}\right)}_{UNKNOWN} \tag{41}$$

The Malmquist index puts a bound on possible evolution of TFP from period 0 to period 1, even when the capital stock is poorly measured or unobservable. Consider first the extreme case in which there is no capital accumulation in period 0, i.e. $k_0 = k_1$ and $\ln M_0^1 = \frac{1}{2}\ln\left(\frac{y_1^1}{y_0^0}\right)$; in the other extreme, capital accumulation is identical to the growth of labor productivity, i.e. $\ln M_0^1 = \ln\left(\frac{y_1^1}{y_0^0}\right)$. We will employ the midpoint between these two values. We also consider the construction of the index when of negative technological progress: in this case, the lower bound is represented by the extreme case when capital accumulation is equal to the growth of labor productivity, i.e. $\ln M_0^1 = \ln\left(\frac{y_1^1}{y_0^0}\right)$, while the upper bound is represented by the case where there is no capital accumulation between the two periods.

B3. The Malmquist Index when Capacity Utilization is Observed

If data on capacity utilization are available, we can rewrite (41) for the first two periods in the case of full efficiency extending De Borger and Kerstens (2000):

$$M_0^1 = \frac{CU_0\left(k_0, U_0 k_0, y_0\right)}{CU_1\left(k_1, U_1 k_1, y_1\right)} \sqrt{\frac{D_0\left(k_1, y_1\right)}{D_0\left(k_0, y_0\right)}} \tag{42}$$

where $CU_t\left(k_t, U_t k_t, y_t\right) = \frac{D_t(U_t k_t, y_t)}{D_t(k_t, y_t)} \leq 1$ is the output efficiency measure removing any existing technical inefficiency. If production function is given by (9), we can rewrite $CU_t\left(k_t, U_t k_t, y_t\right) = \frac{A_t\left[(U_t K_t)^\alpha N_t^{1-\alpha}\right]/N_t}{A_t\left[K_t^\alpha N_t^{1-\alpha}\right]/N_t} = U_t^\alpha$ and recompute (41) as

$$\ln M_0^1 = \frac{1}{2}\ln\left(\frac{y_1^1}{y_0^0}\frac{y_1^0}{y_0^1}\right) + \alpha \ln\frac{U_0}{U_1} = \underbrace{\frac{1}{2}\ln\left(\frac{y_1^1}{y_0^0}\right) + \alpha\ln\frac{U_0}{U_1}}_{KNOWN} + \underbrace{\frac{1}{2}\ln\left(\frac{y_1^0}{y_0^1}\right)}_{UNKNOWN} \tag{43}$$